Fast Quantification of Uncertainty and Robustness with Variational Bayes

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MIT

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Uncertainty & robustness quantification

- Bayesian inference

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- Bayesian inference
  - Complex, modular models

Uncertainty & robustness quantification

- Bayesian inference
  - Complex, modular models; posterior distribution

Uncertainty & robustness quantification

- Bayesian inference $p(\theta)$
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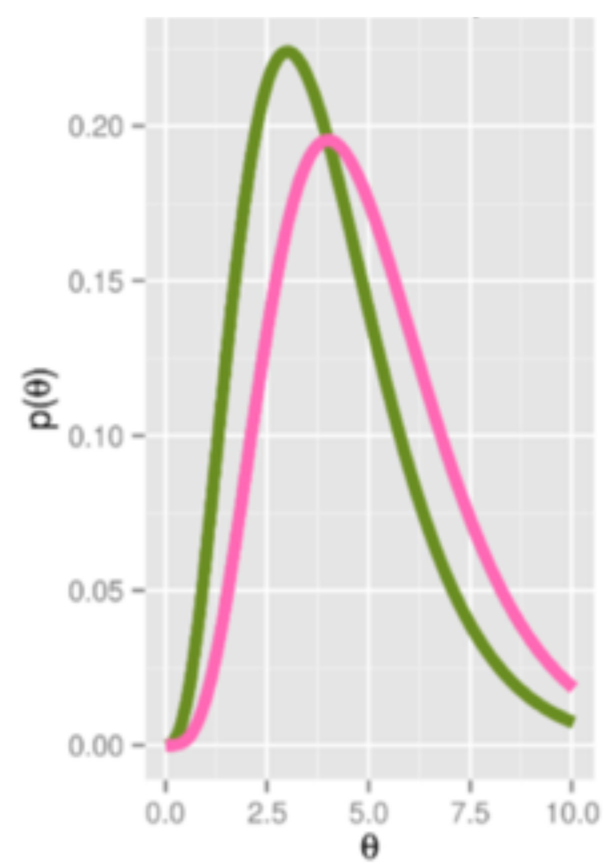
Some reasonable priors



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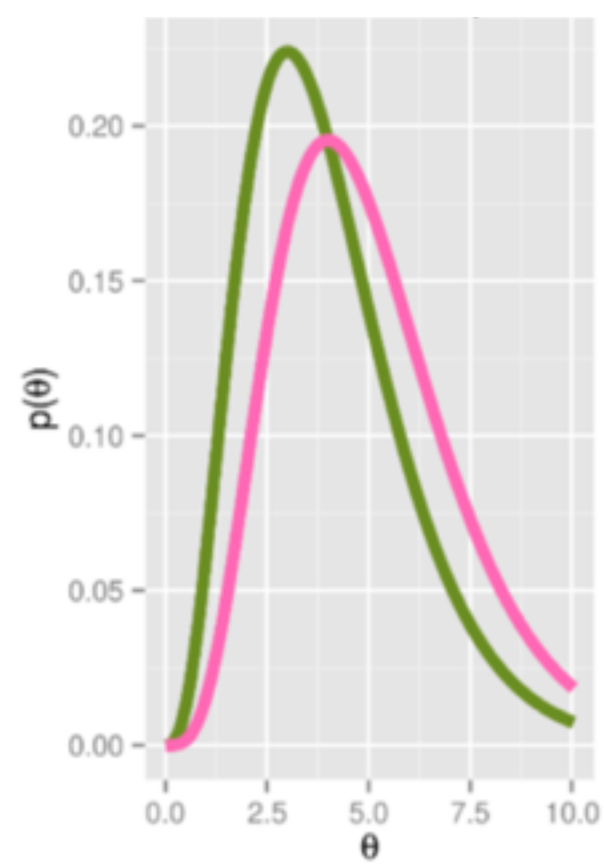

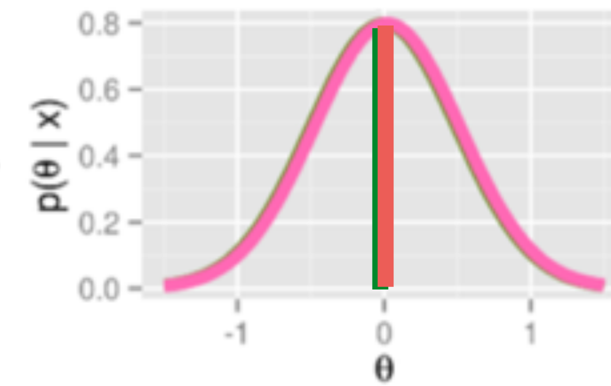

Bayes Theorem

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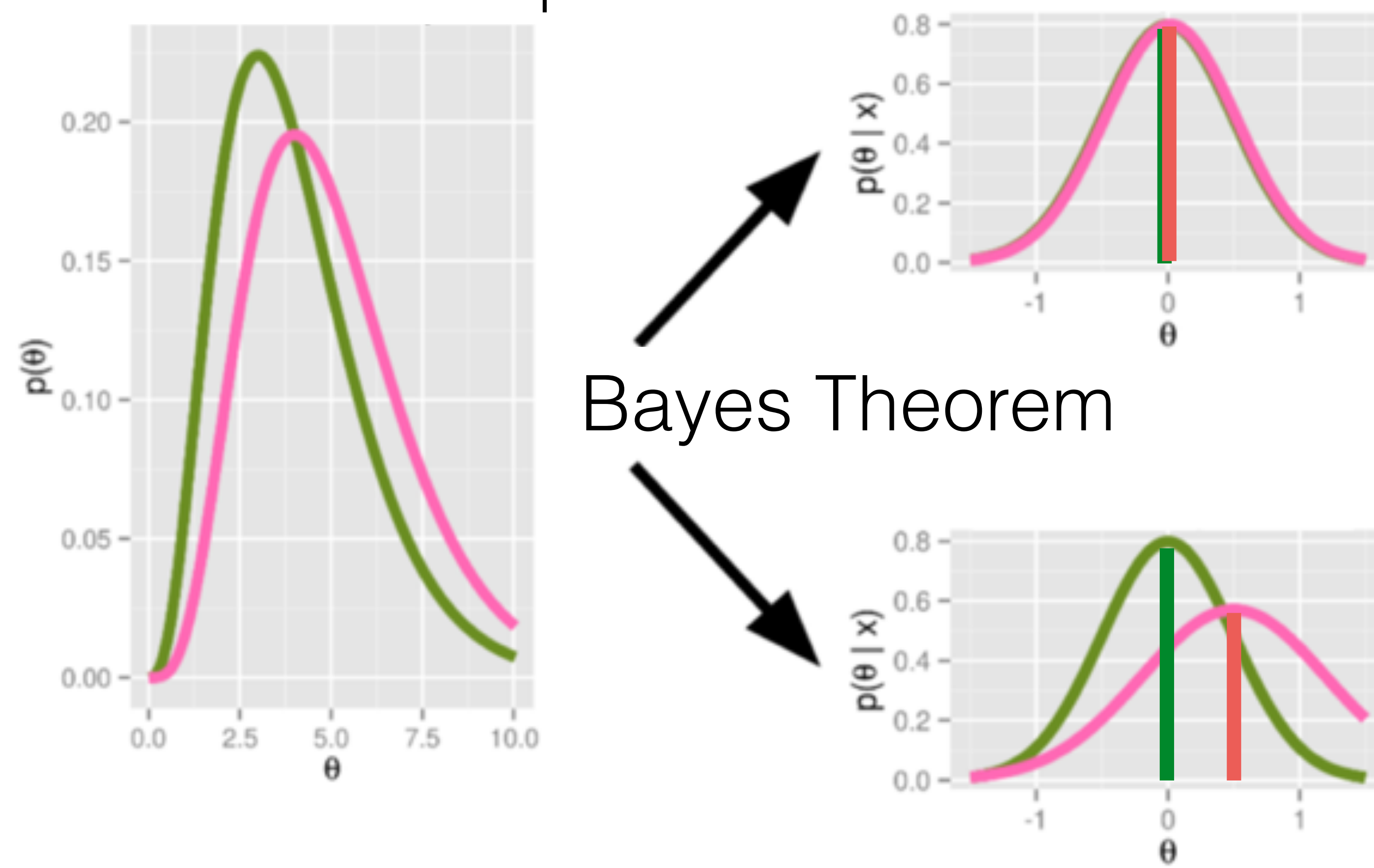

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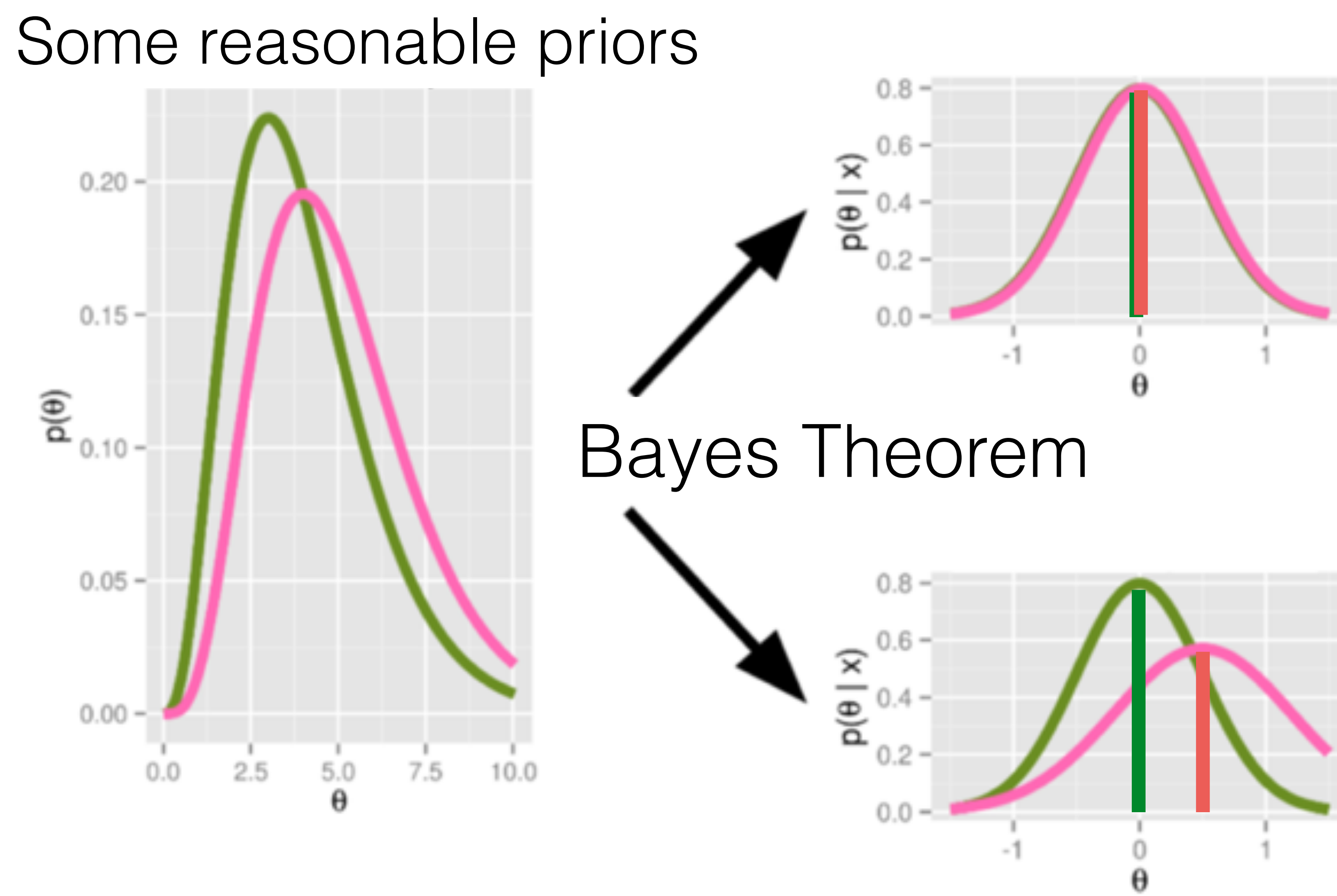

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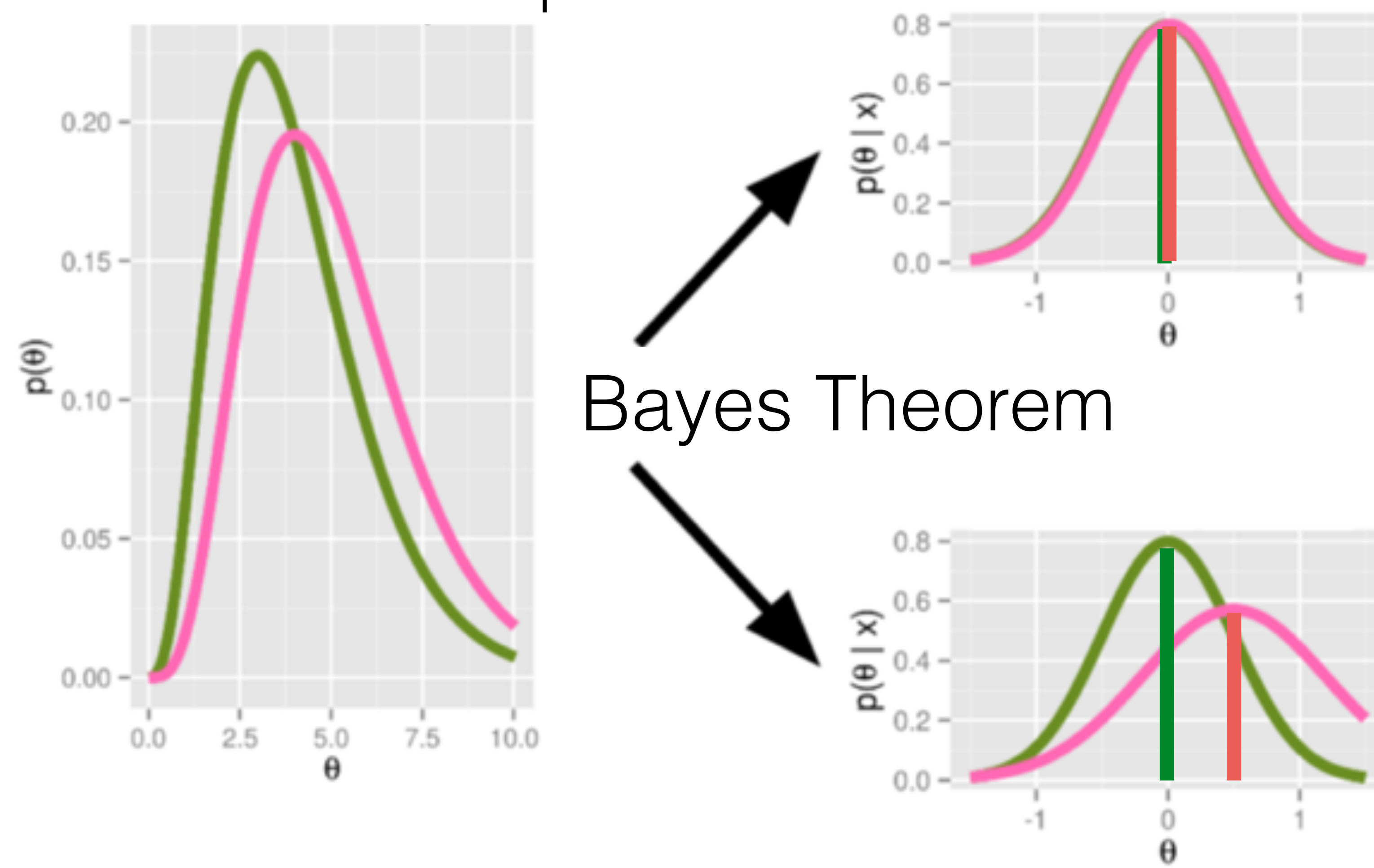

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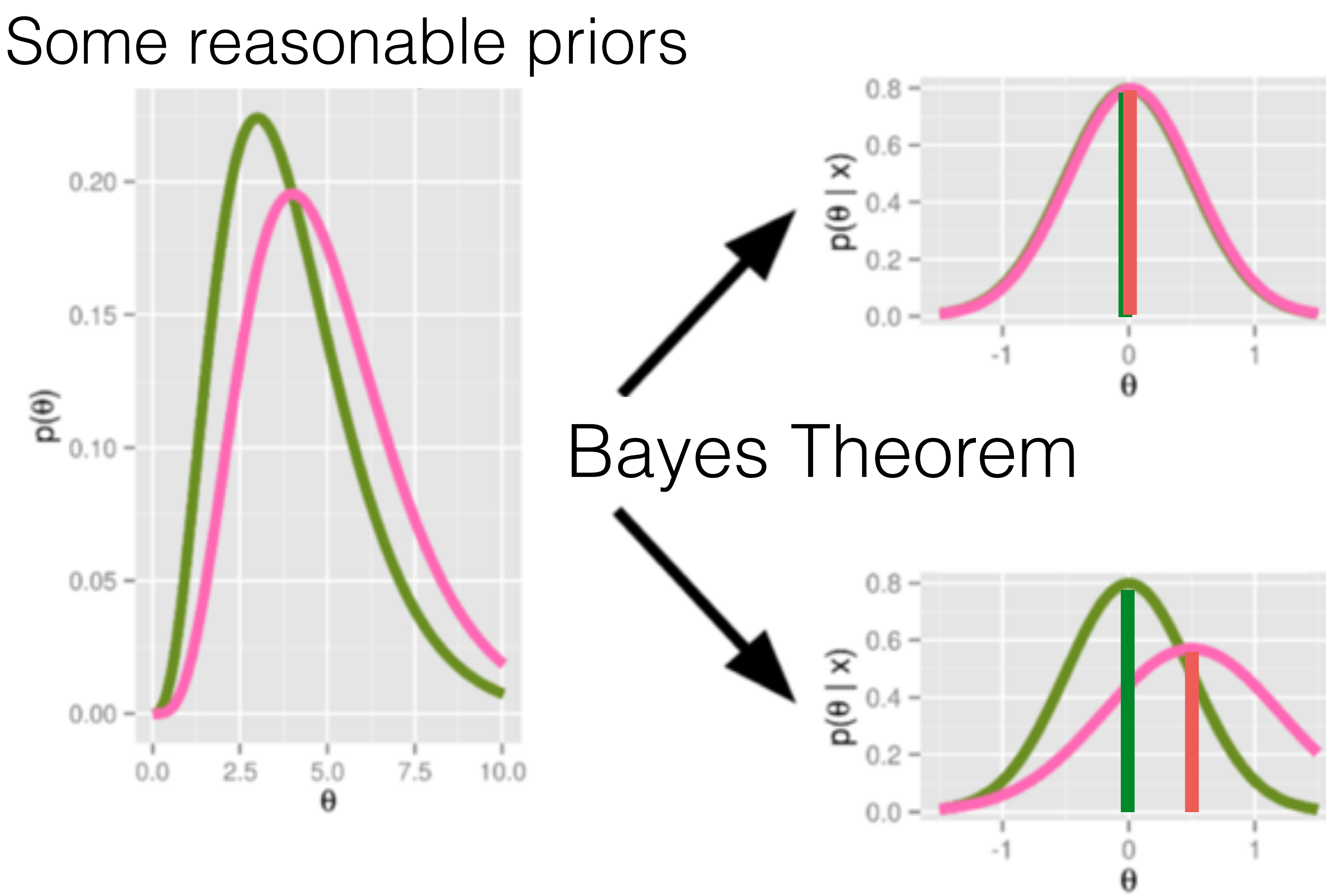

- We propose: *linear response variational Bayes*

Roadmap

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- Variational Bayes as an alternative to MCMC

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- Challenges of VB
- Accurate uncertainties from VB
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- Big idea: derivatives/perturbations are relatively easy in VB

Variational Bayes

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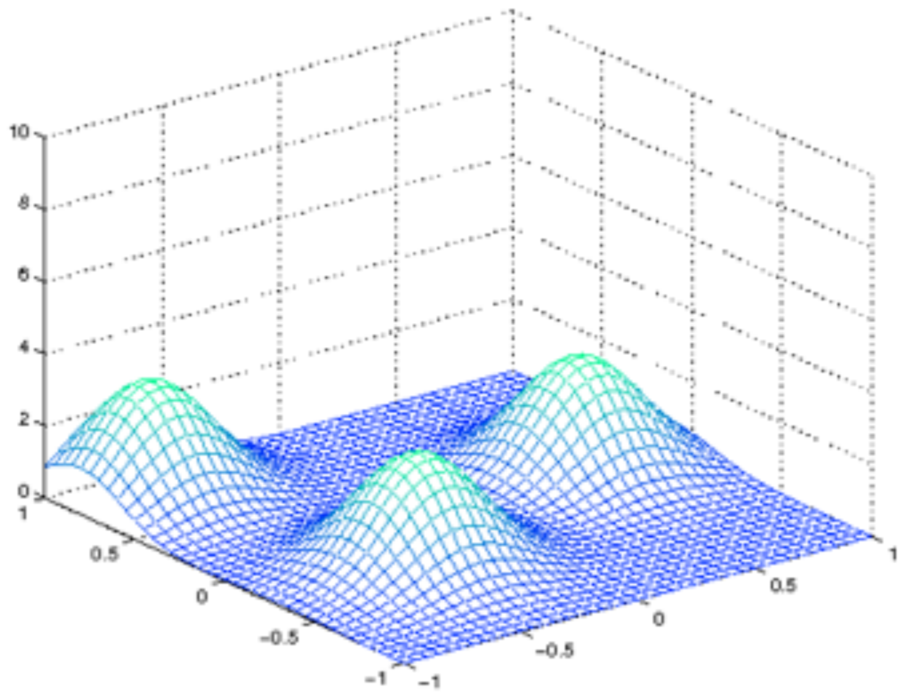
- Variational Bayes (VB)

Variational Bayes



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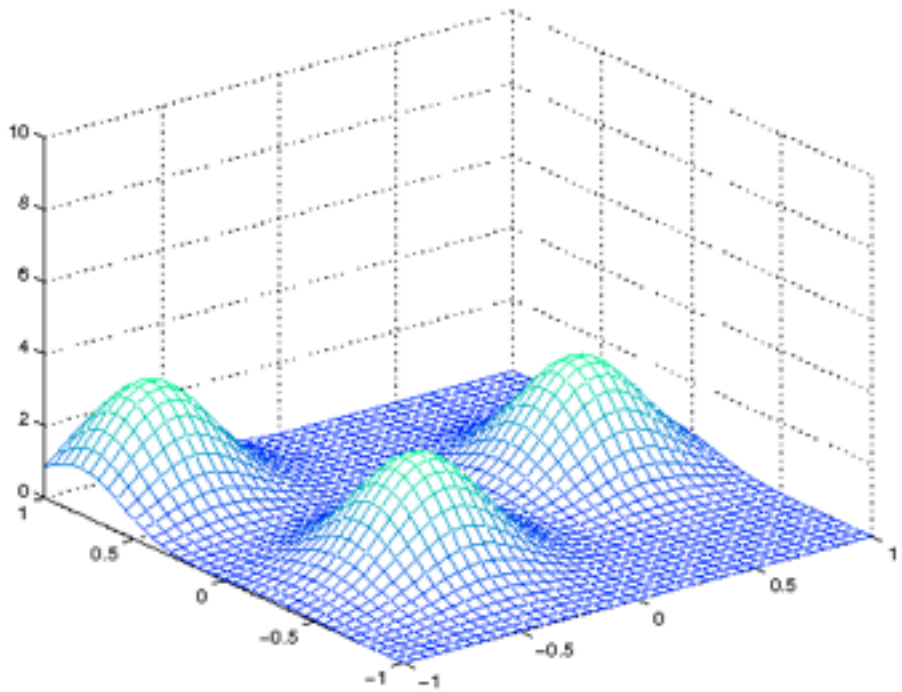

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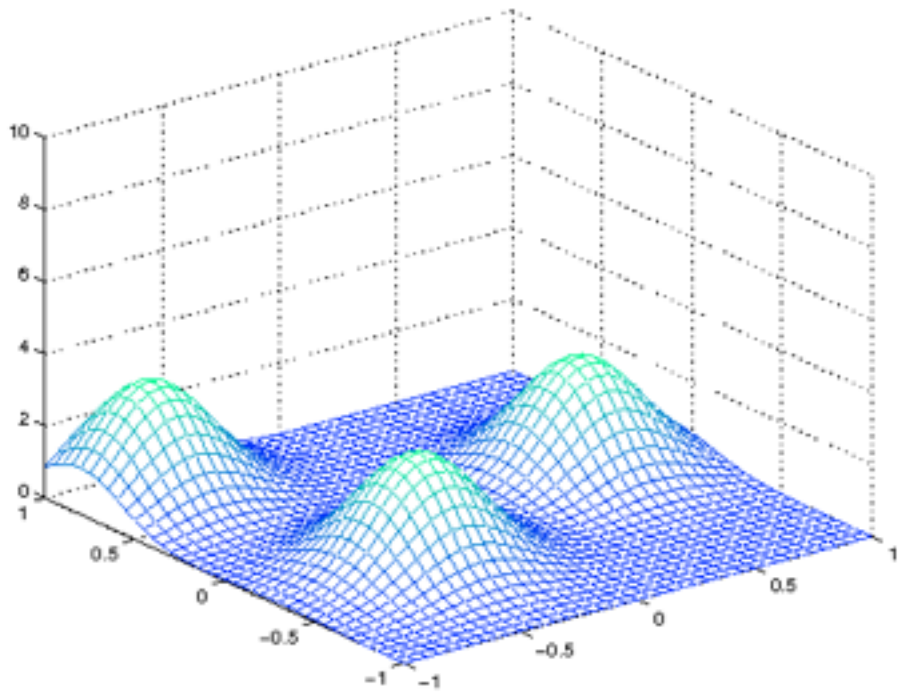

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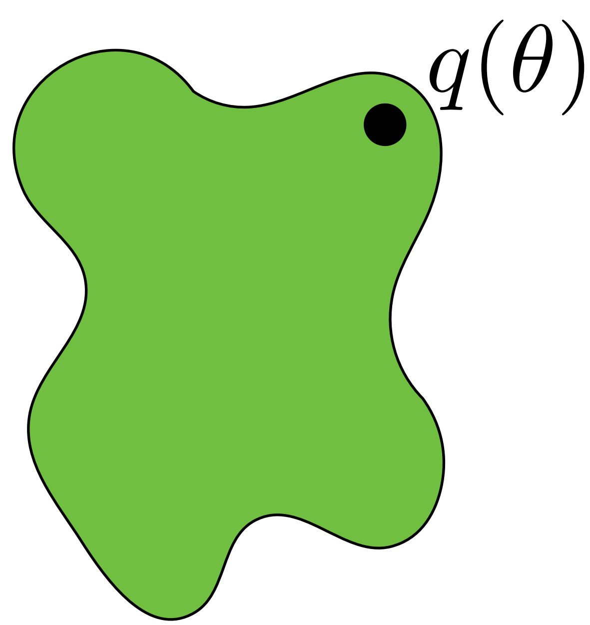

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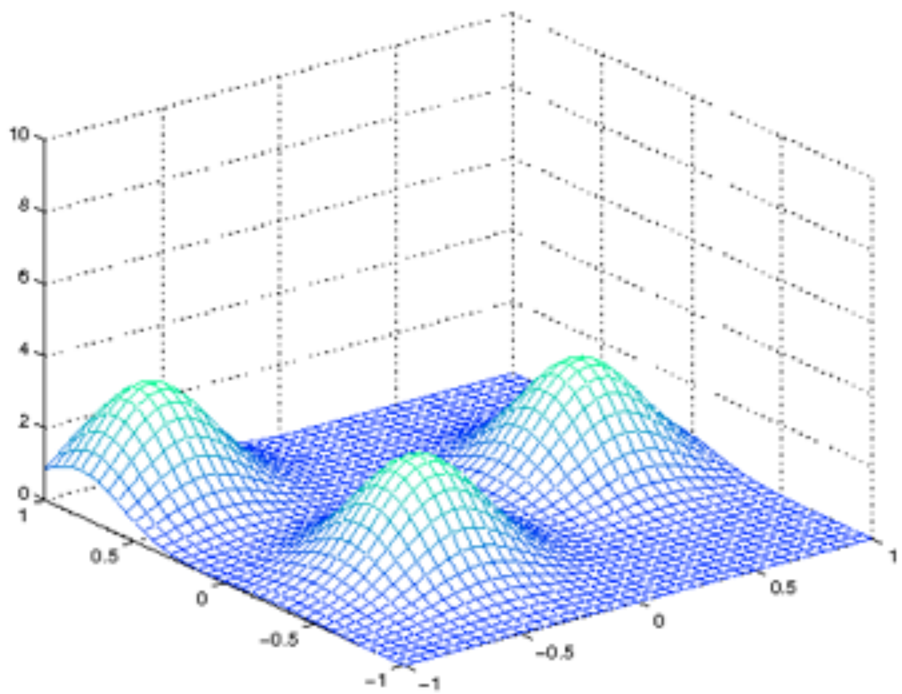

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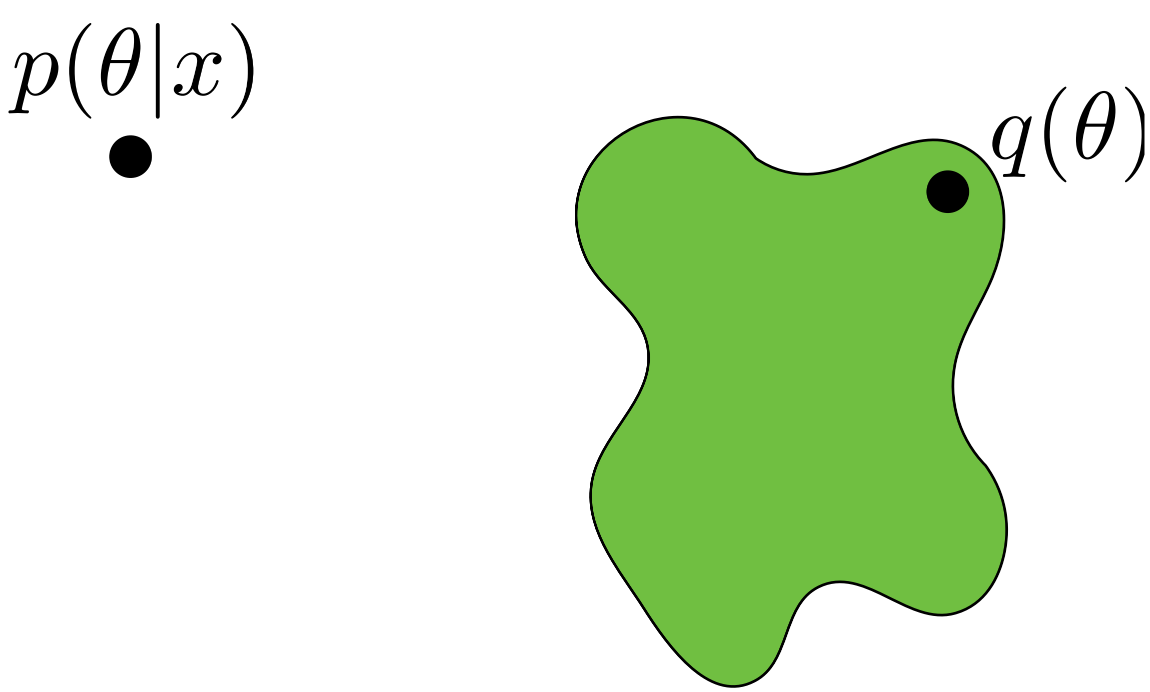

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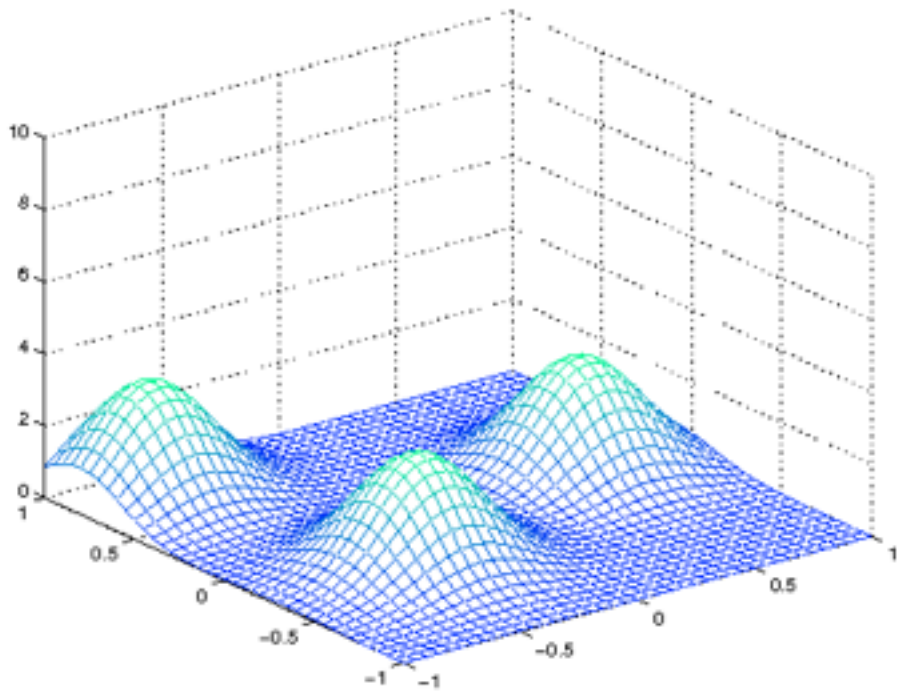

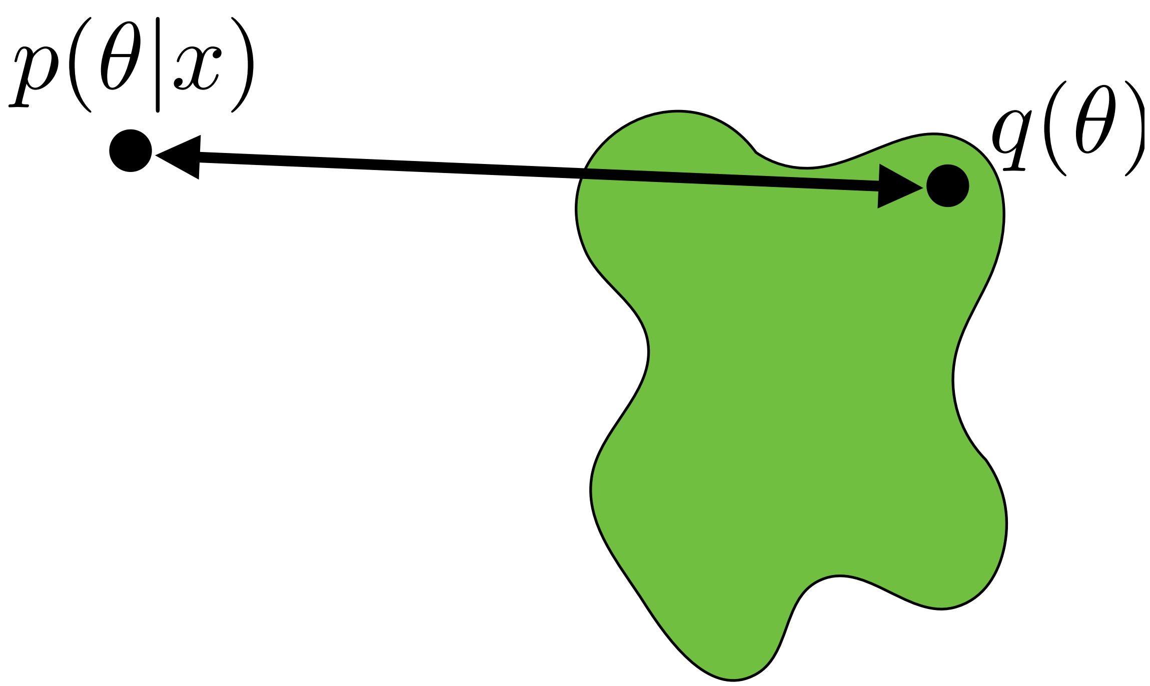

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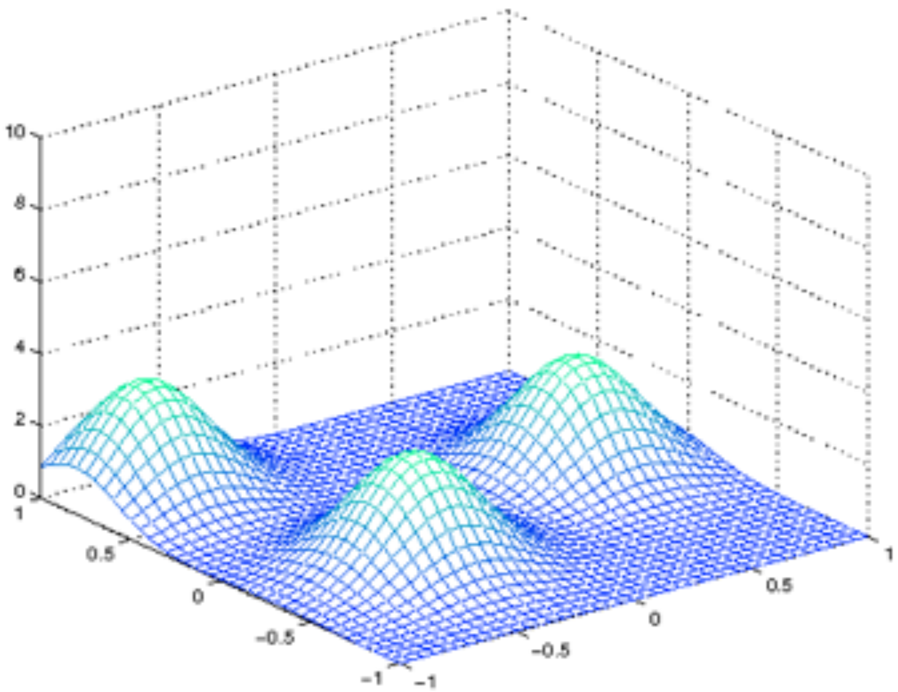

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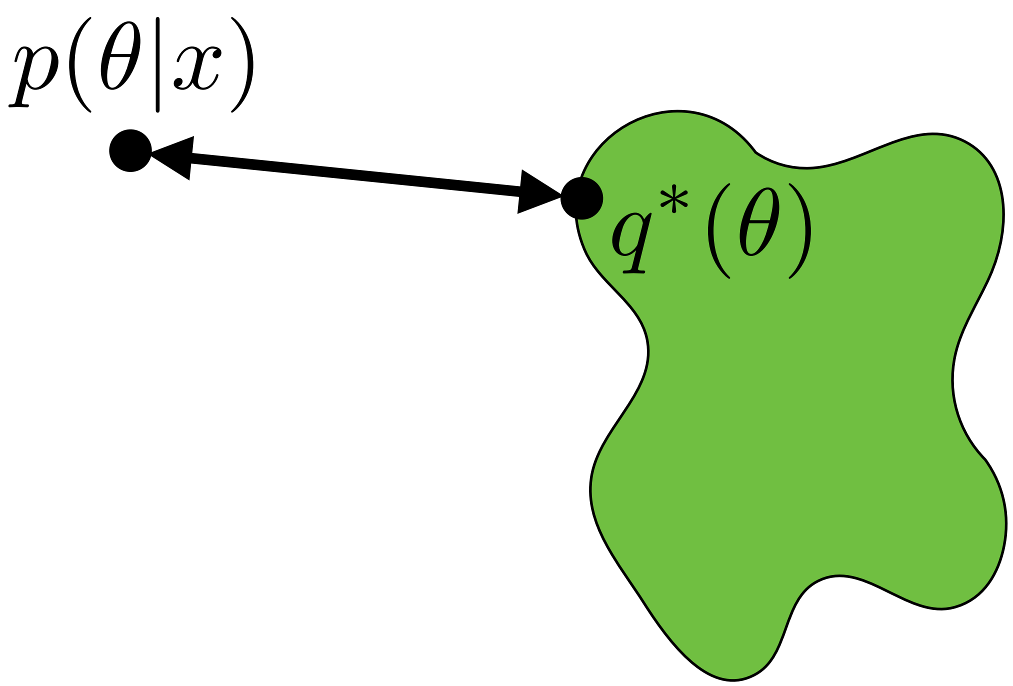

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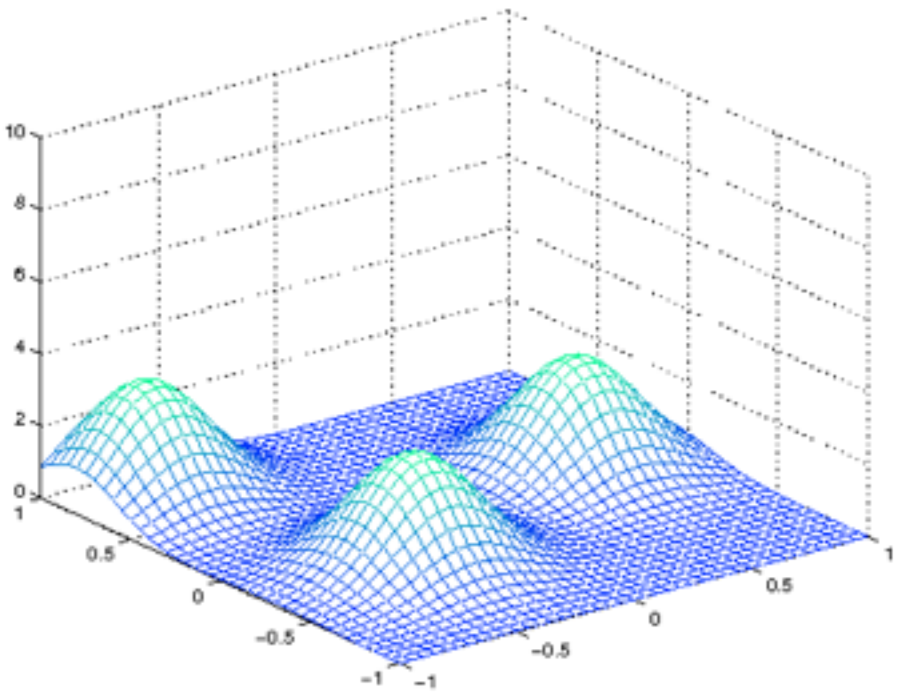

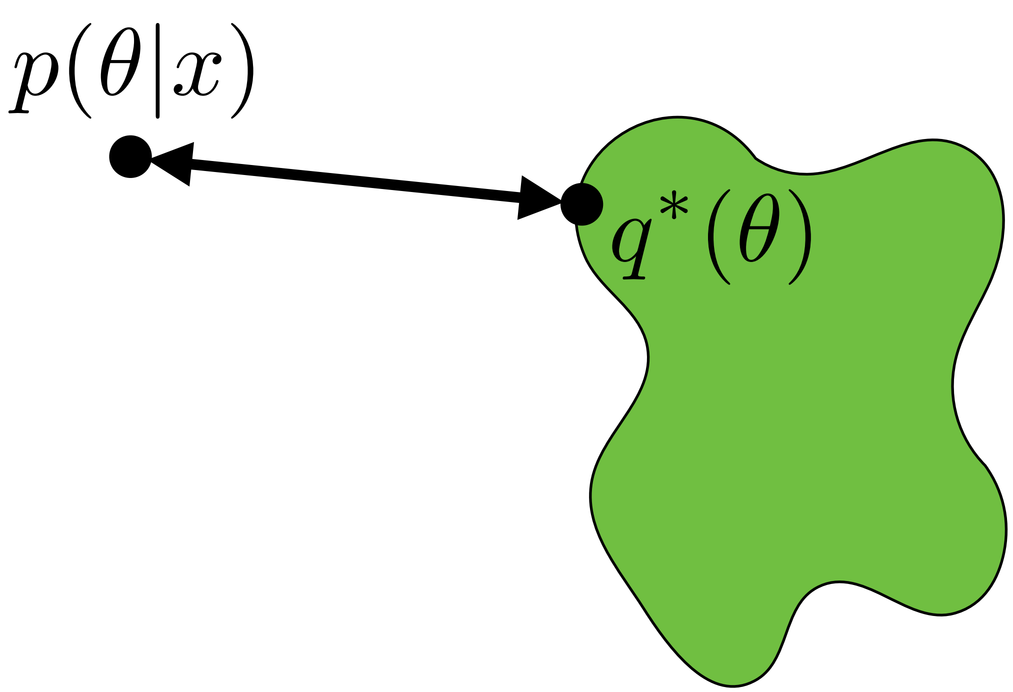

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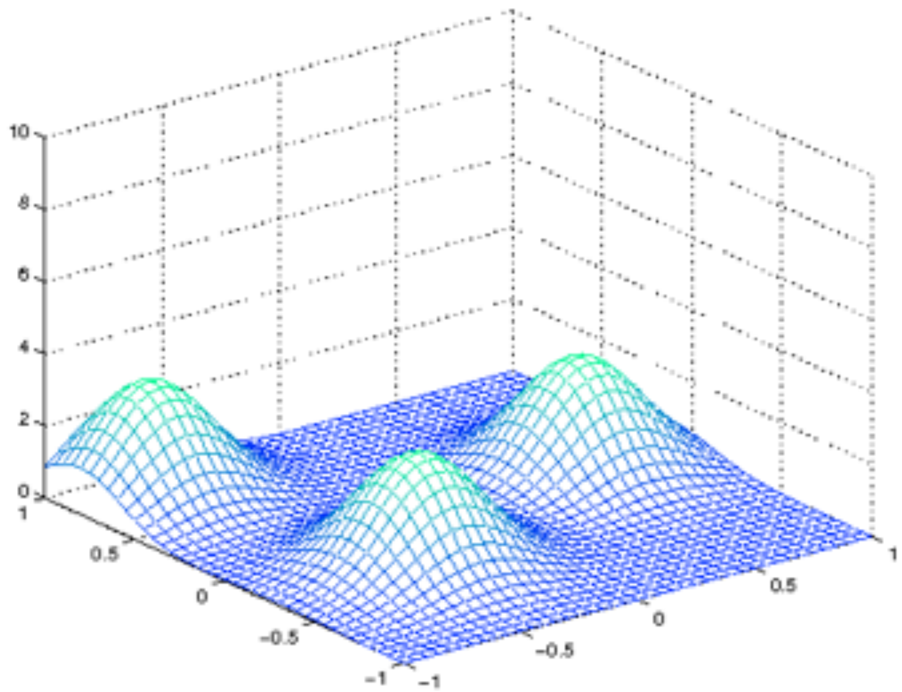

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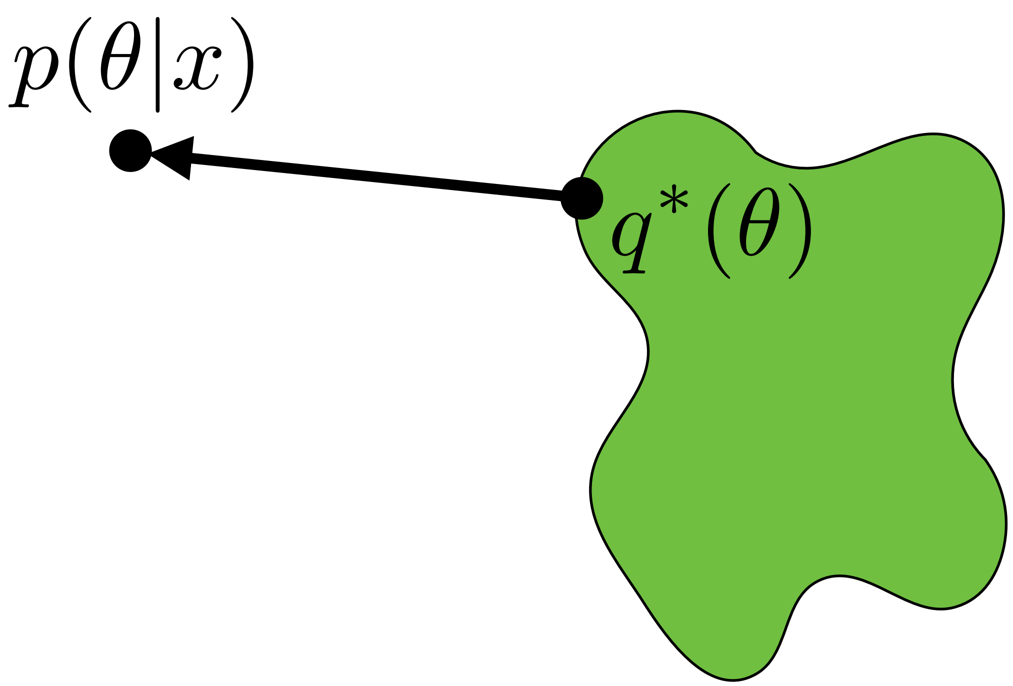

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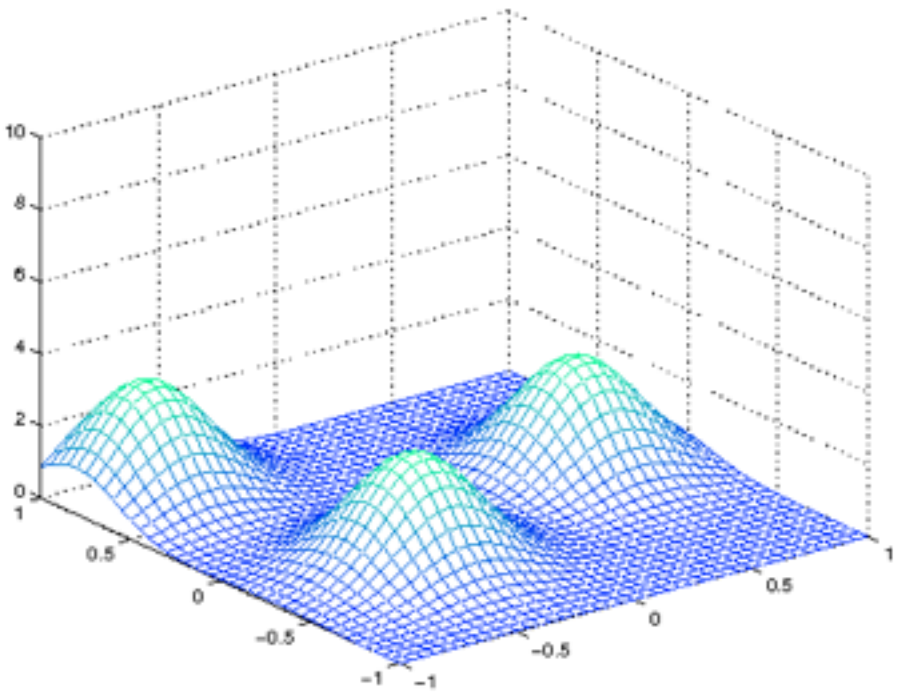

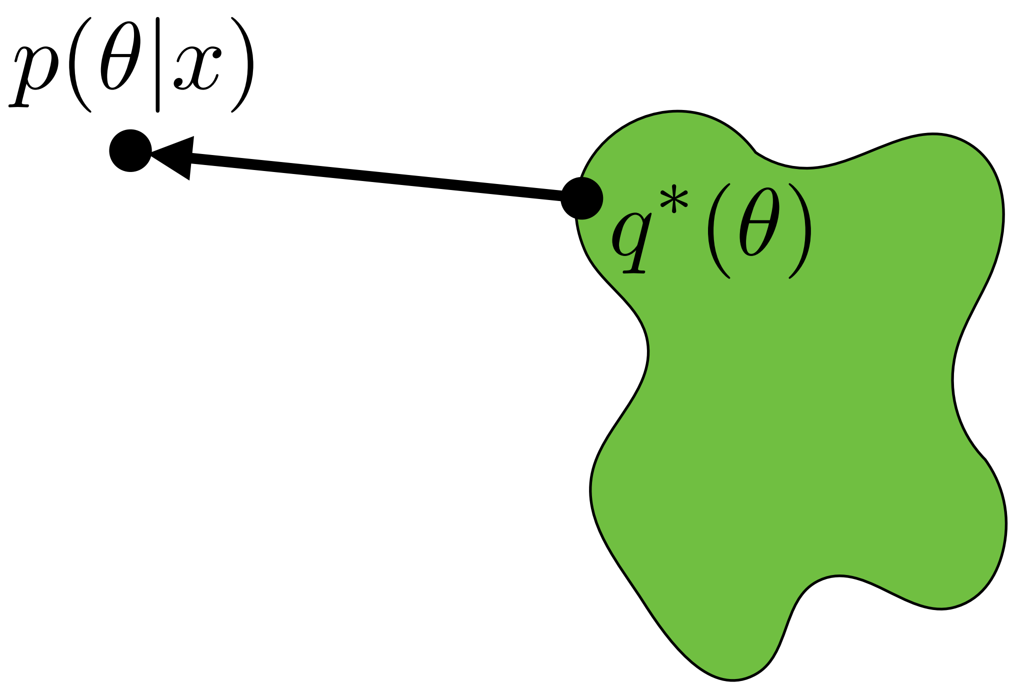

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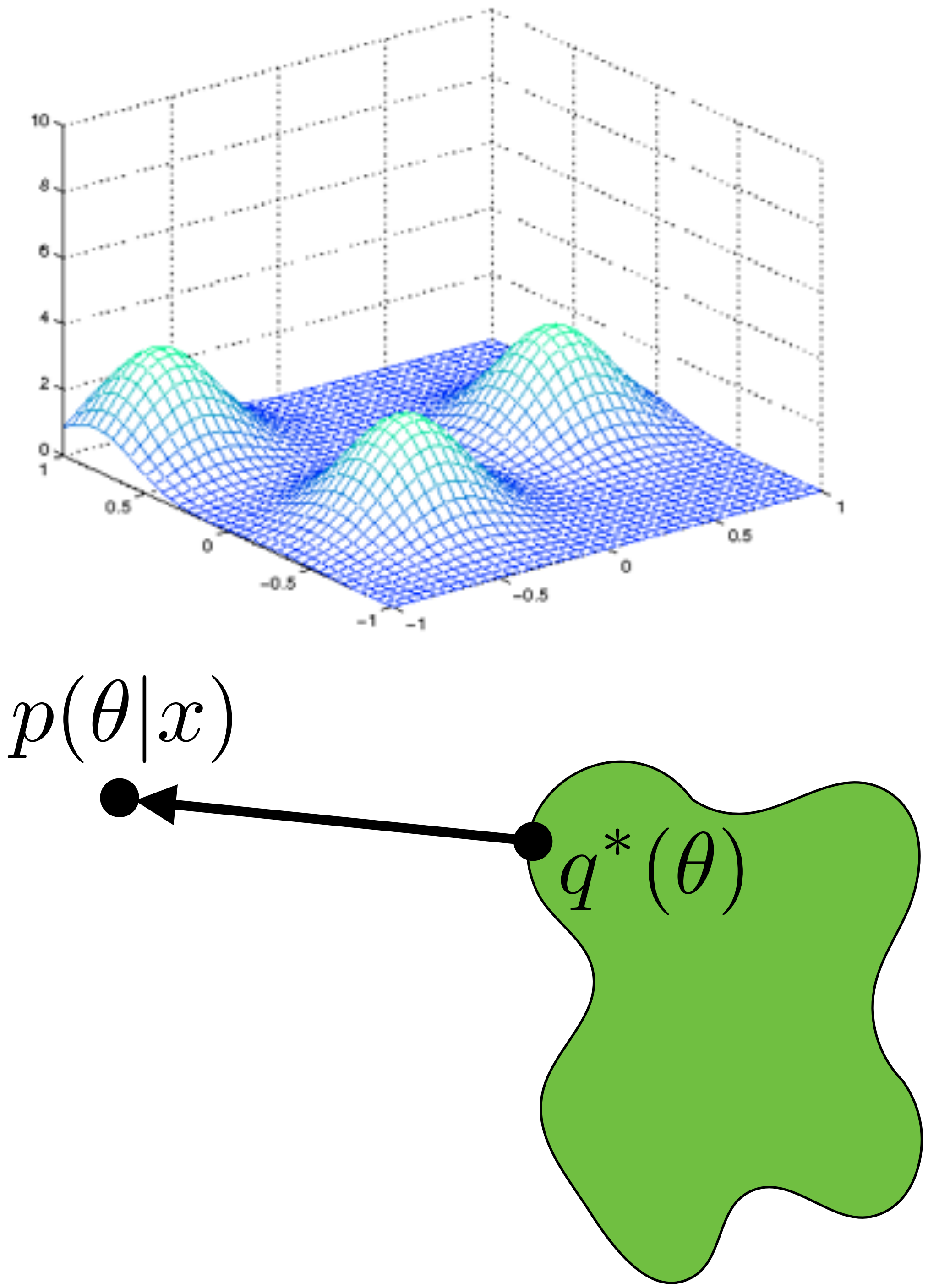

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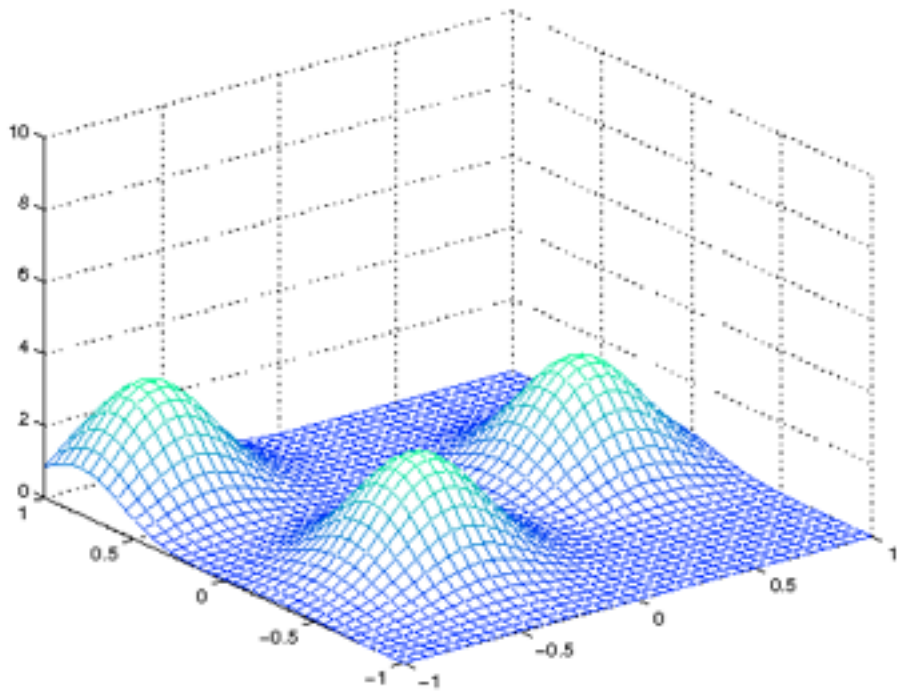

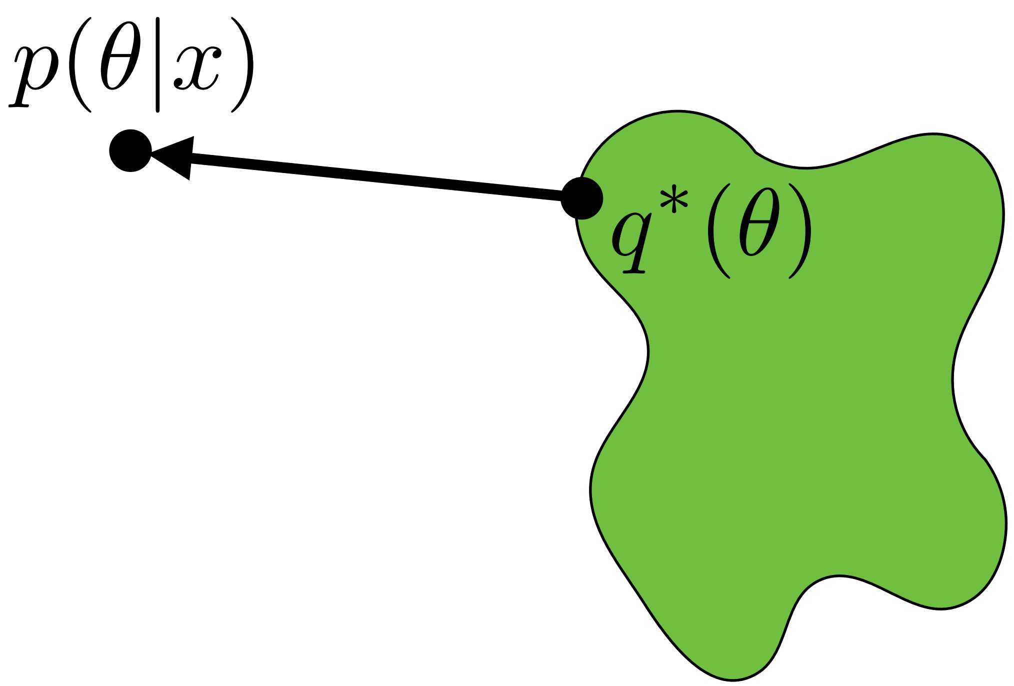

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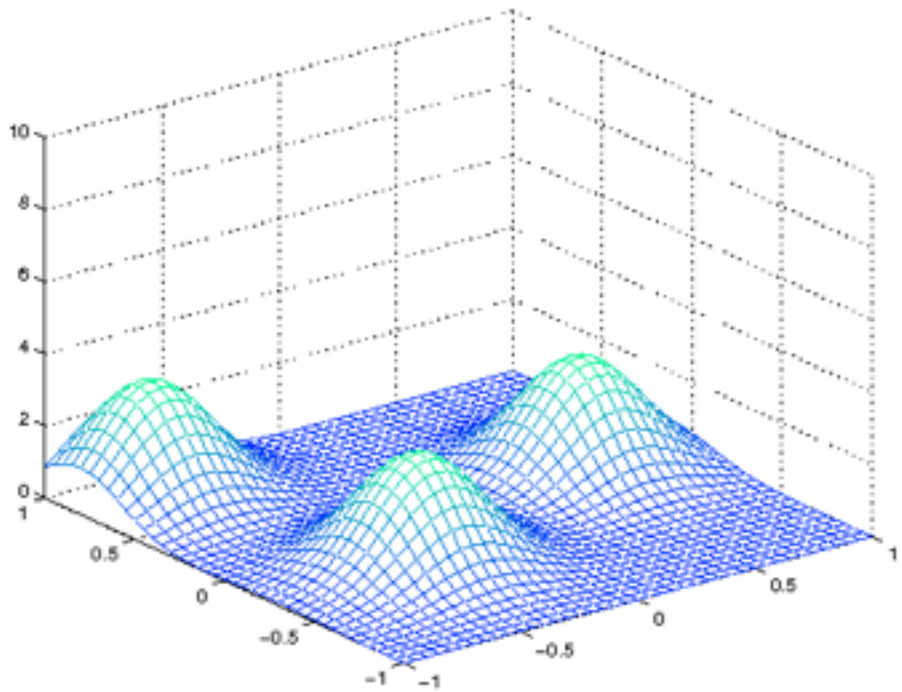
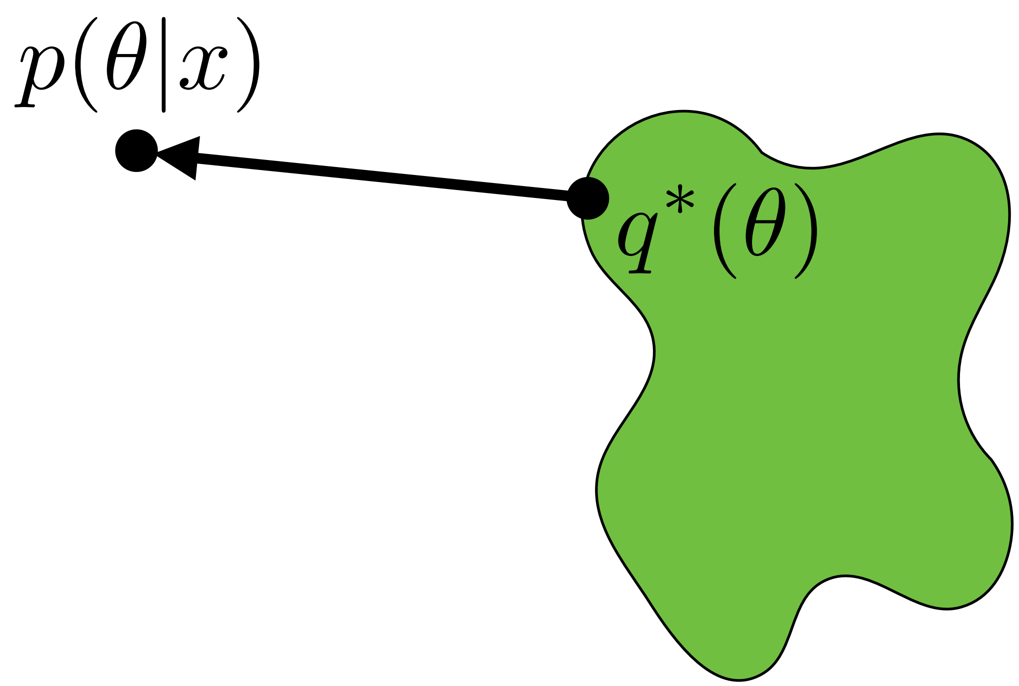
[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

Variational Bayes





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[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

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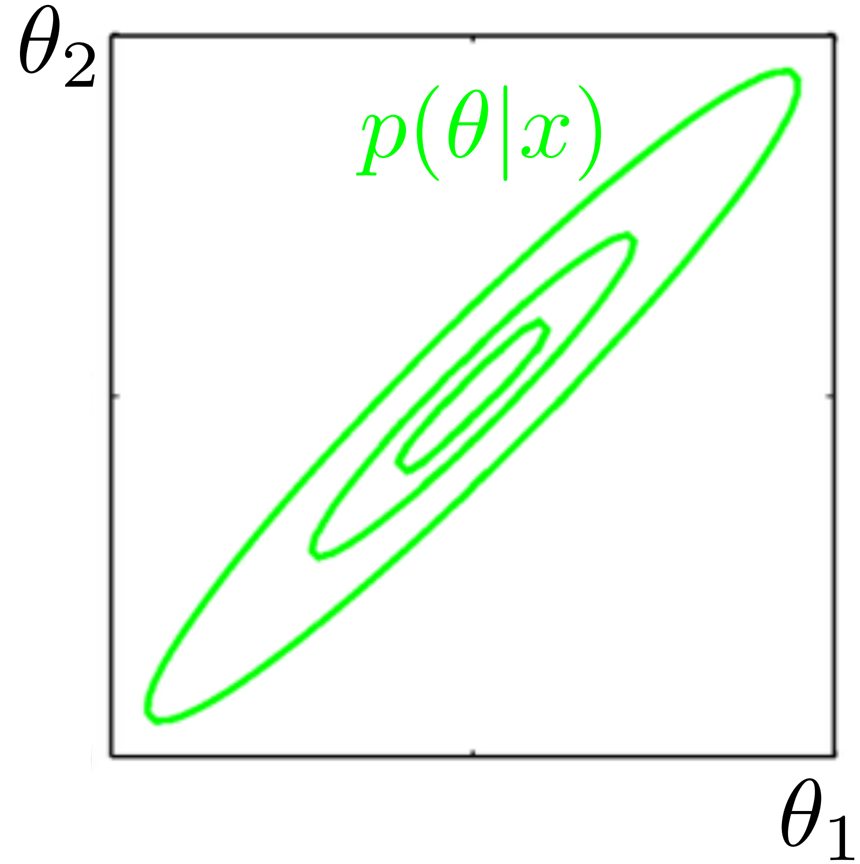

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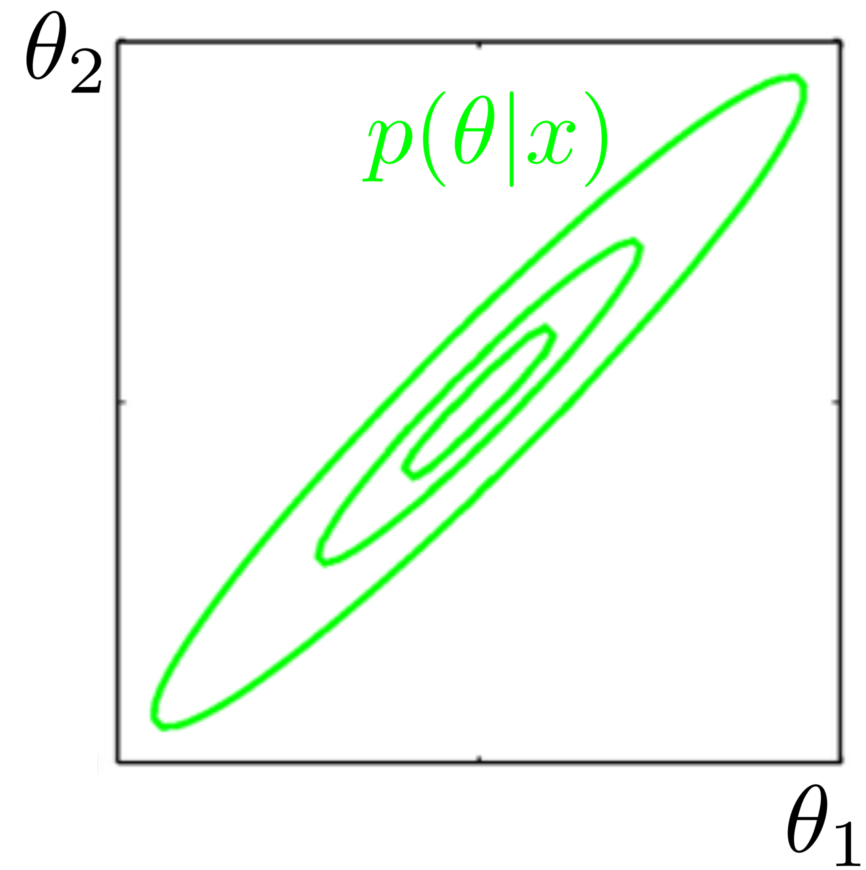

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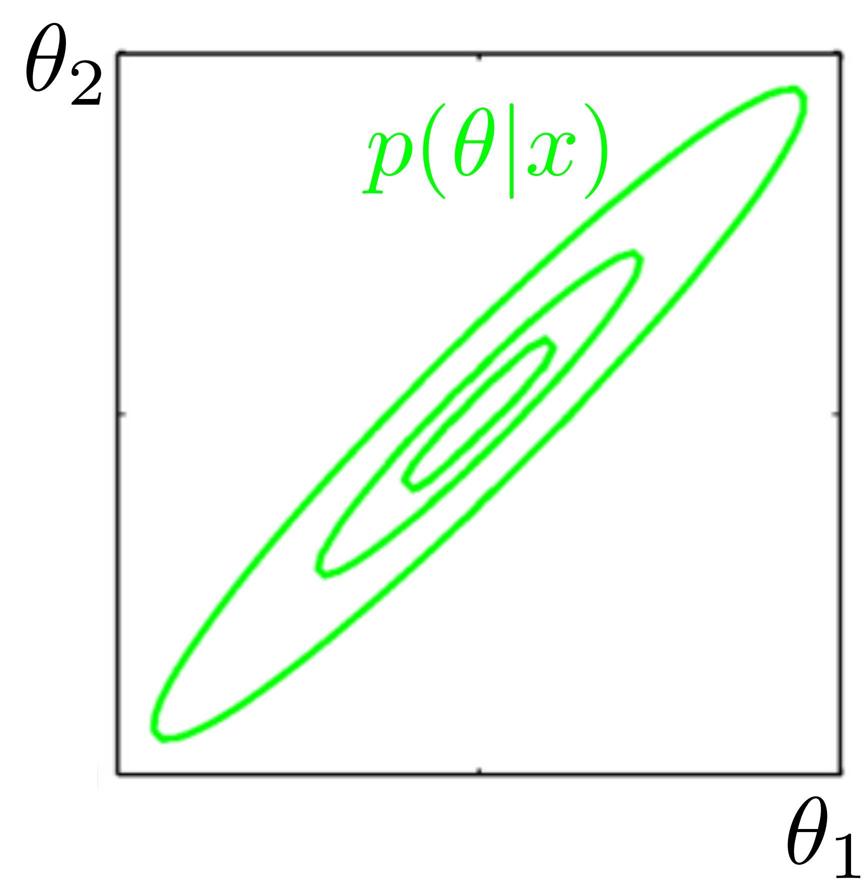

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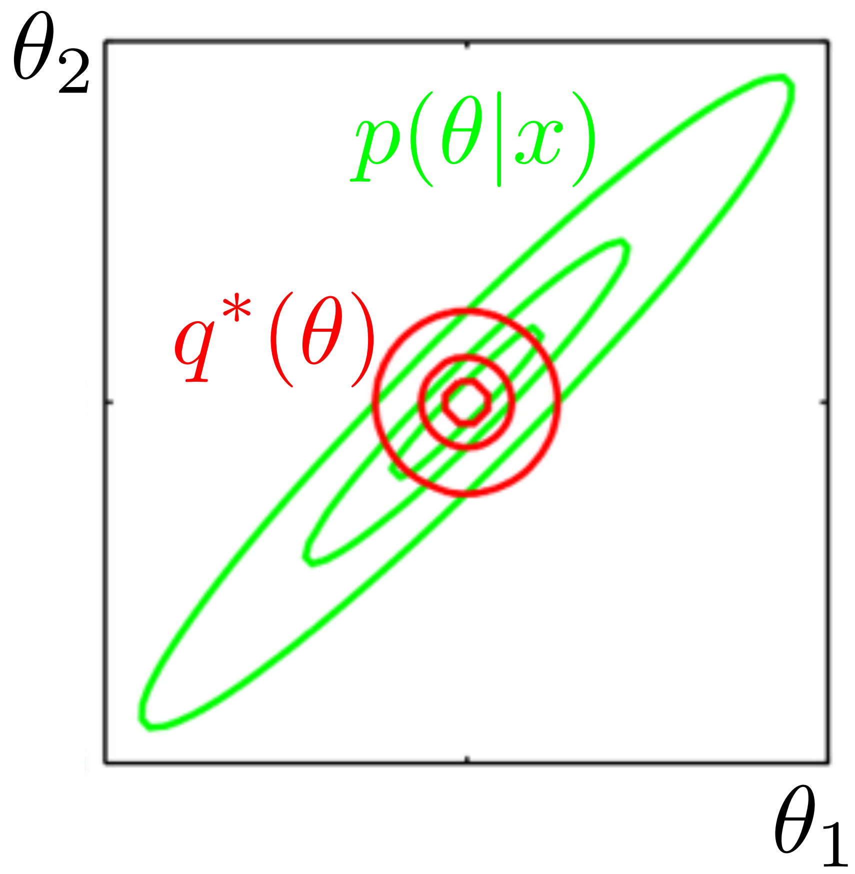

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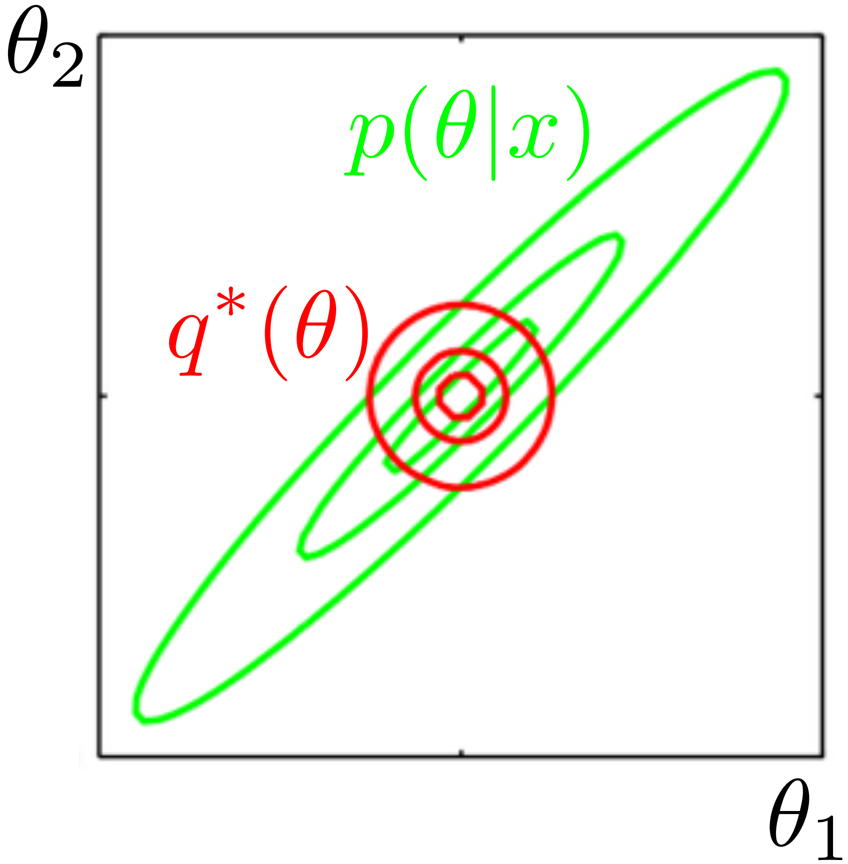

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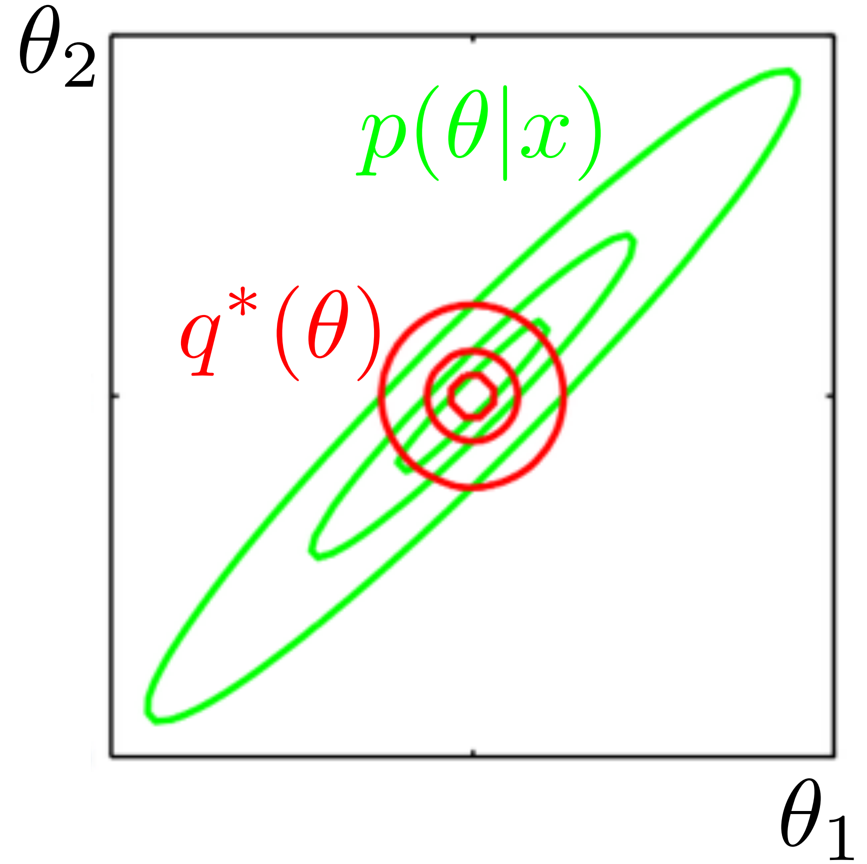

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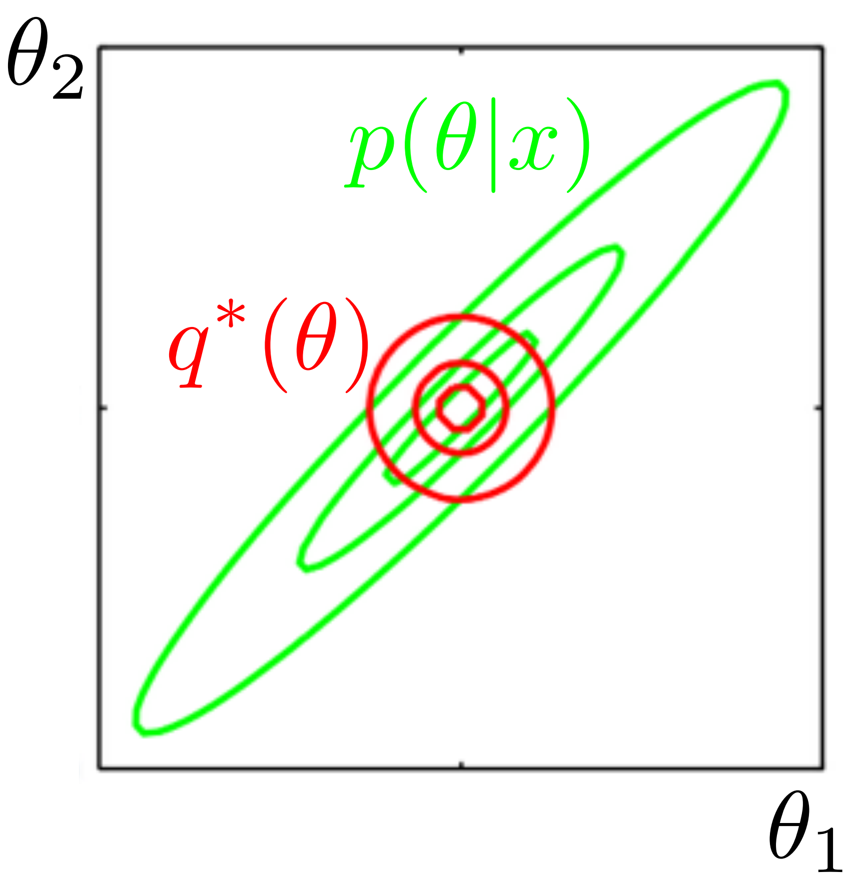

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[MacKay 2003; Bishop 2006; Wang, Titterington 2004; Turner, Sahani 2011]

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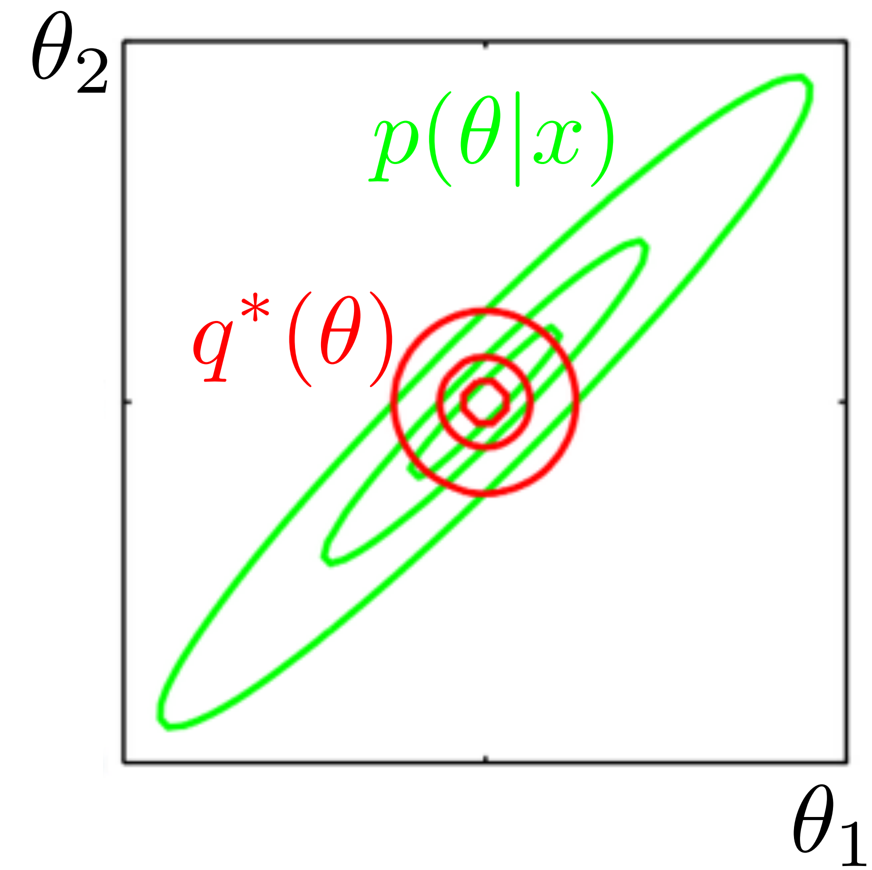

[MacKay 2003; Bishop 2006; Wang, Titterington 2004; Turner, Sahani 2011]
[Fosdick 2013; Dunson 2014; Bardenet, Doucet, Holmes 2015]

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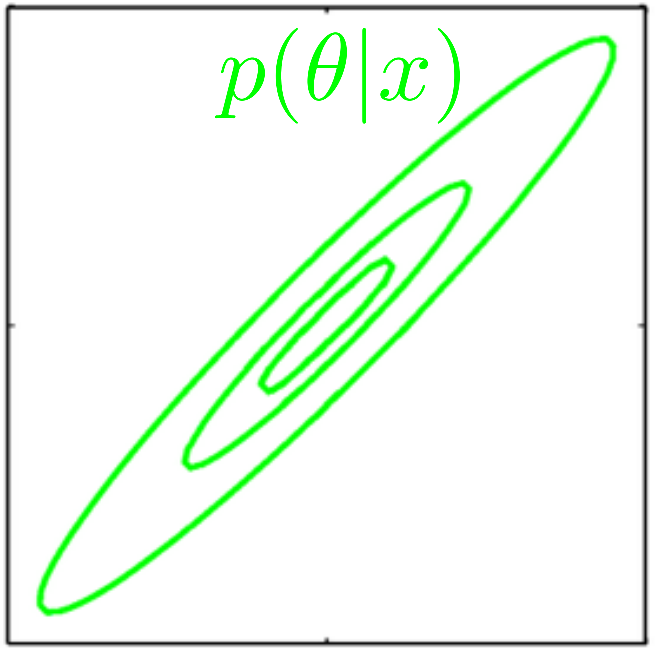

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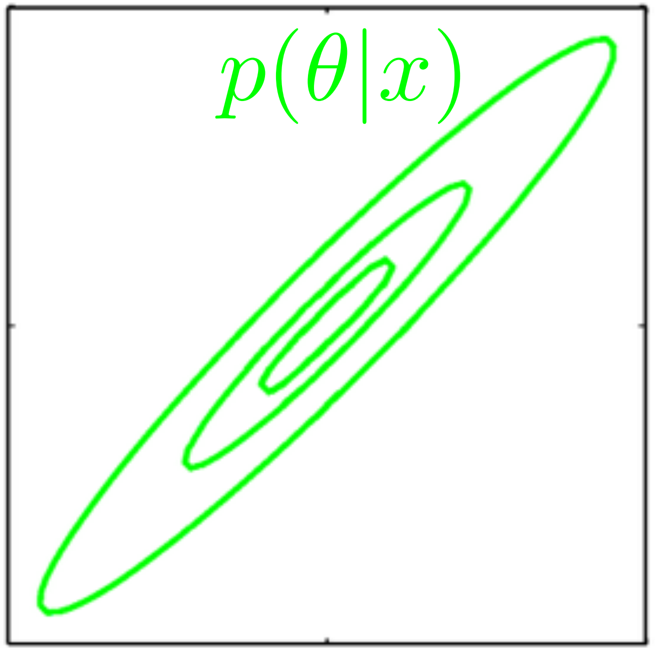

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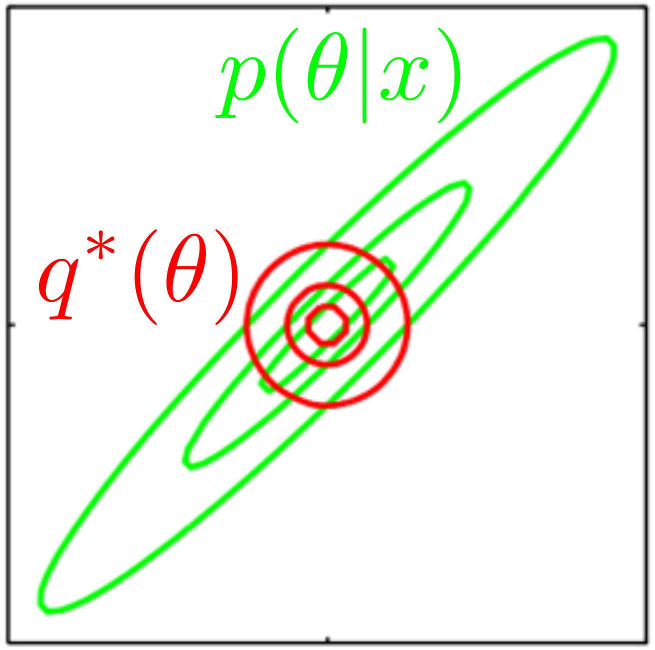

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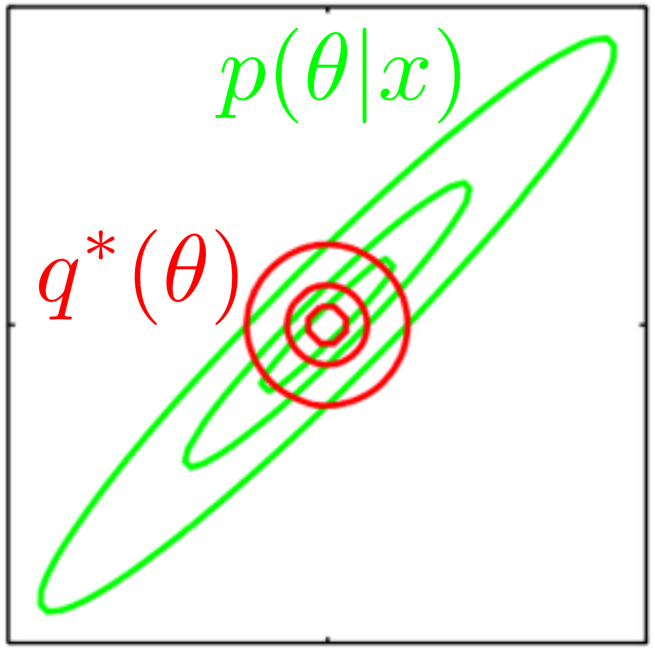

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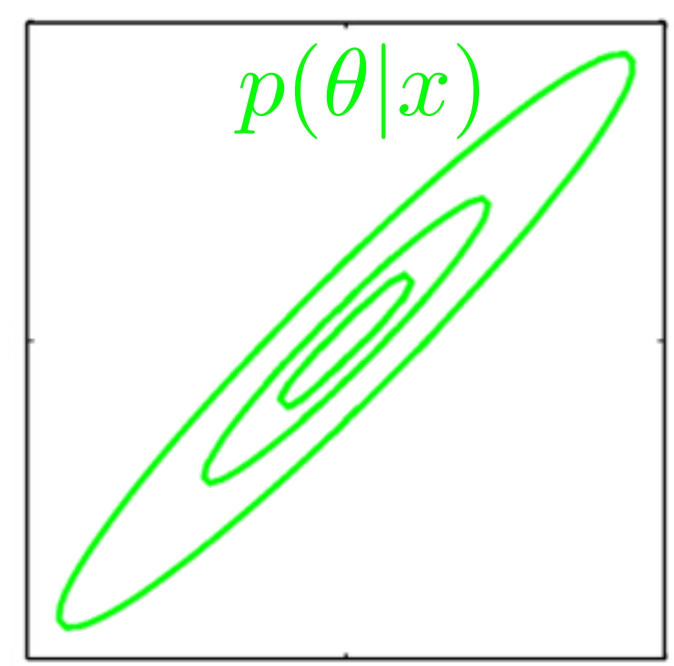

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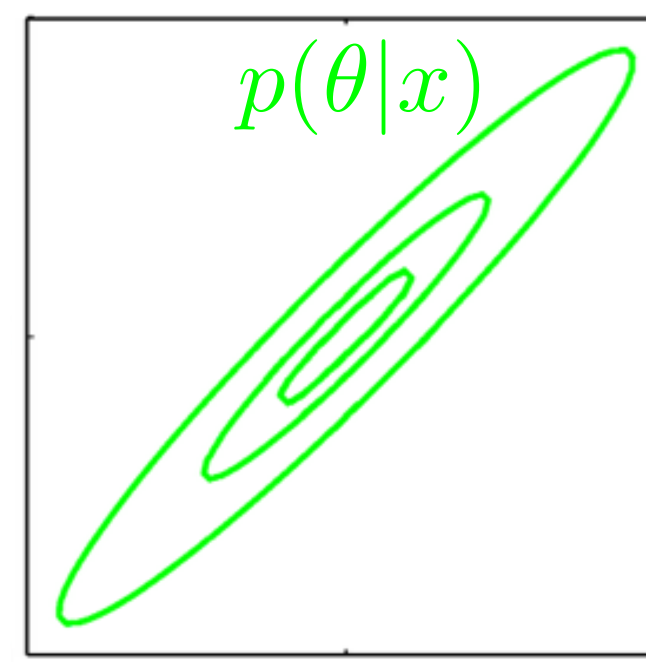

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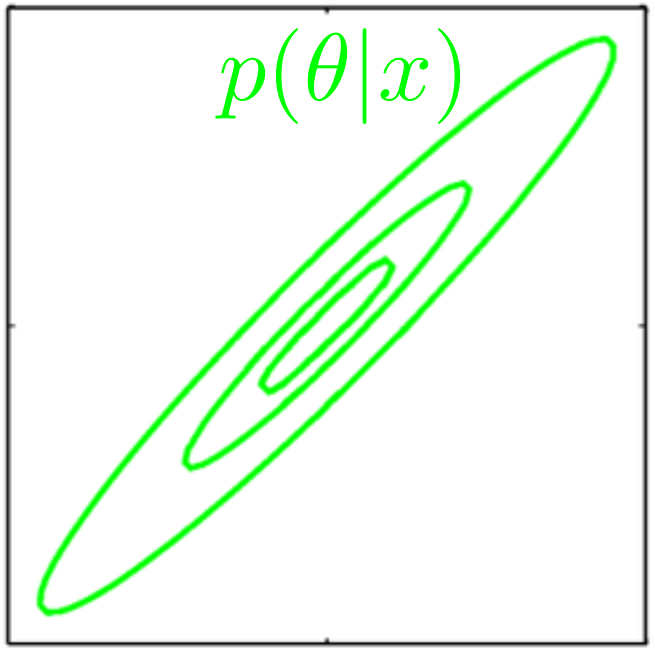

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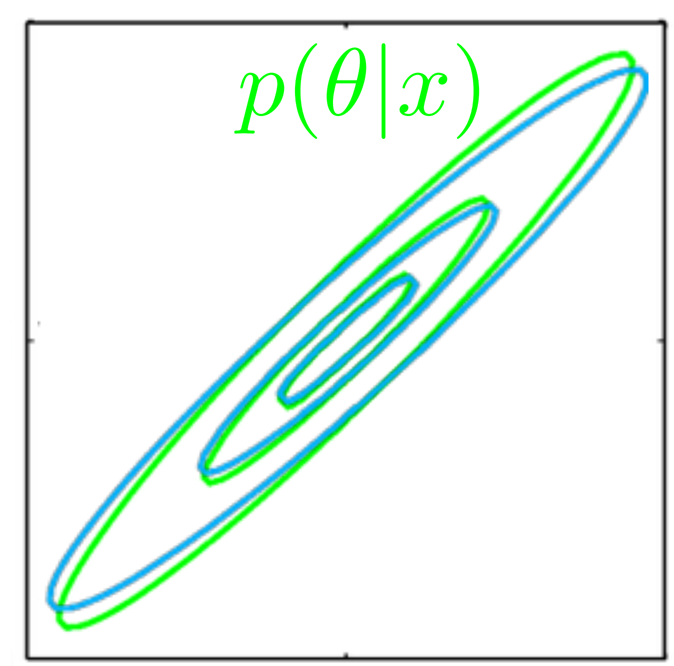

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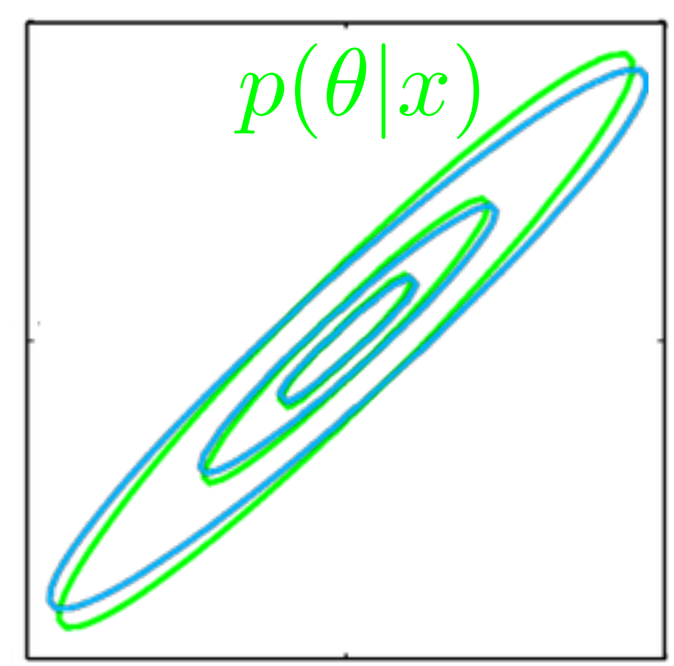

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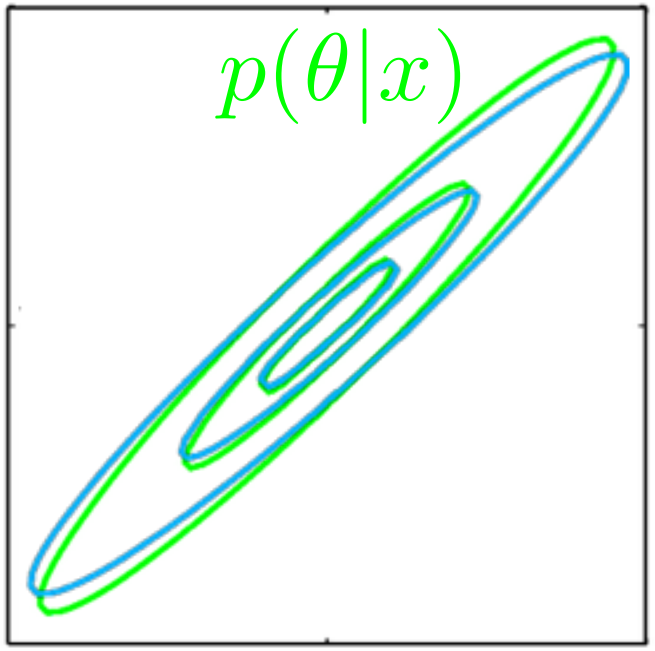

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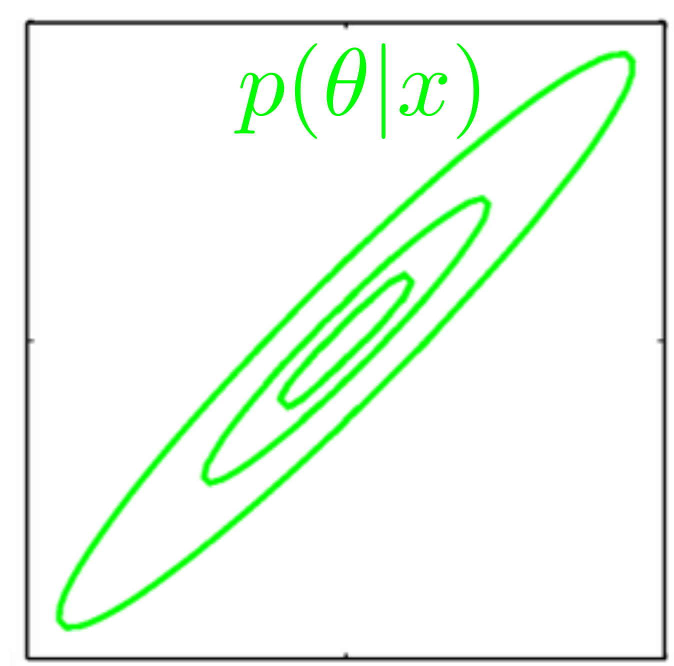

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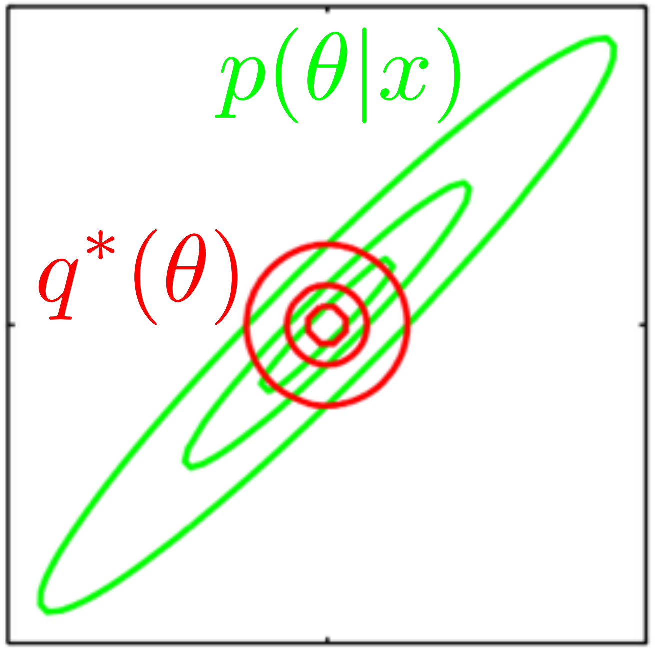

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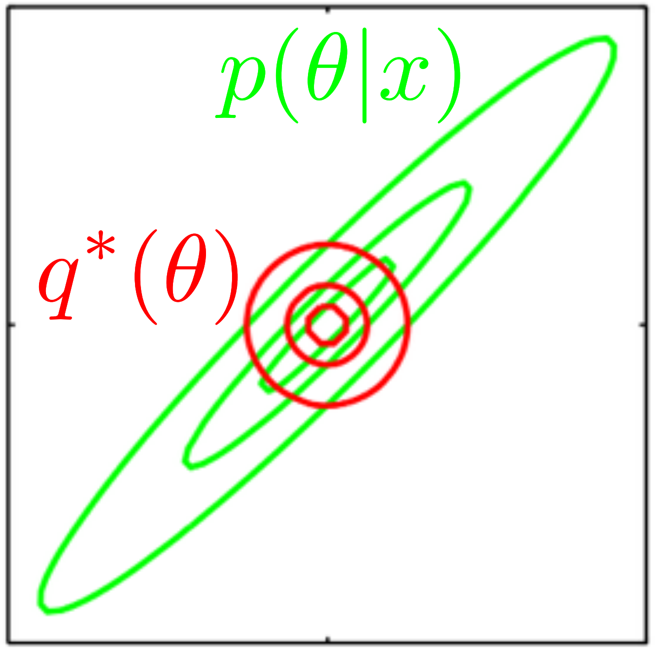

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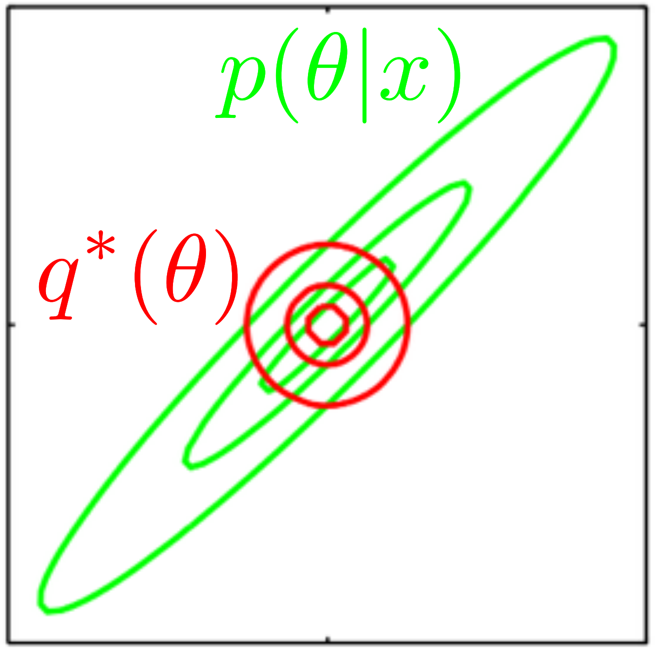

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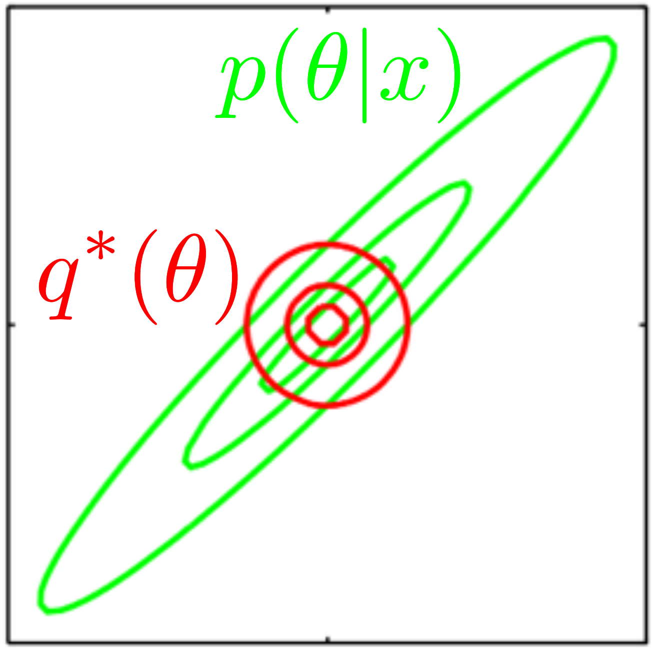

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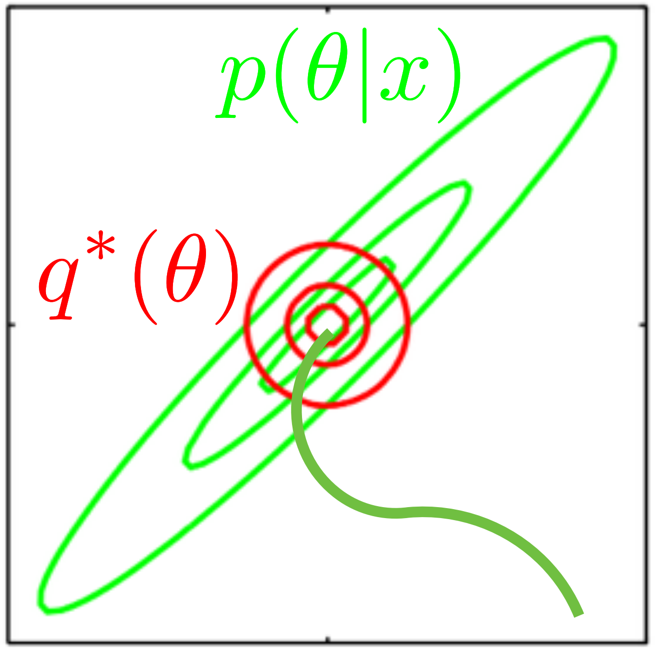

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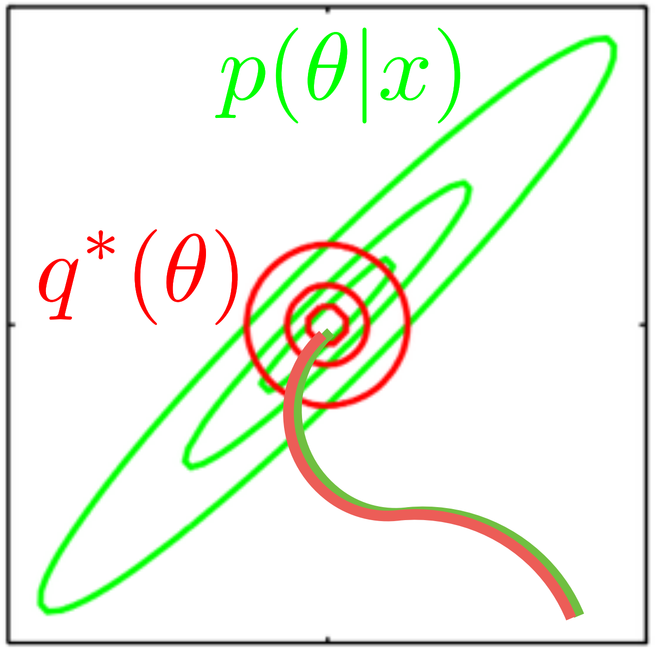

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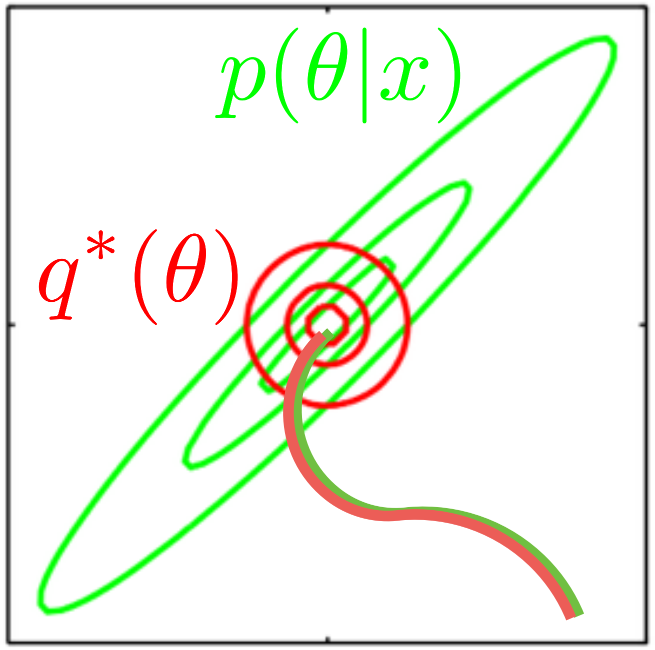

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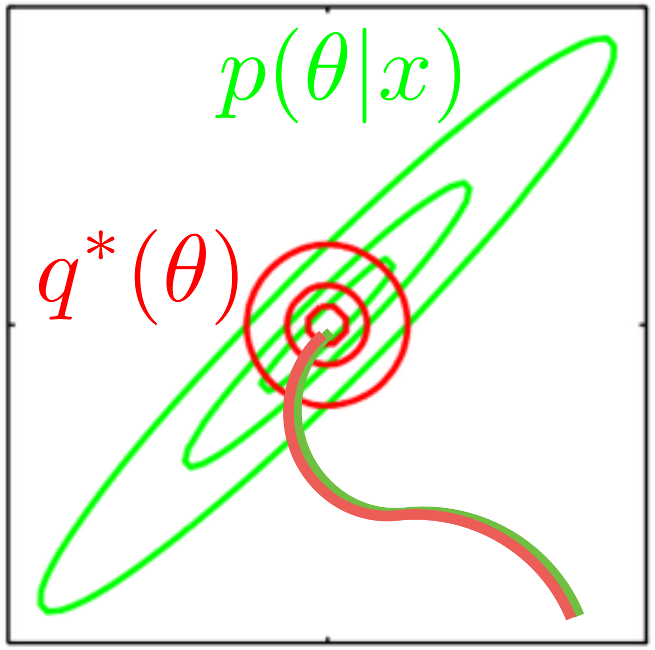

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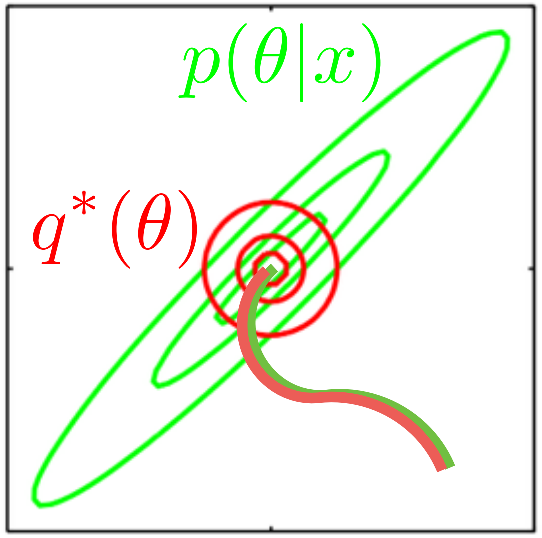

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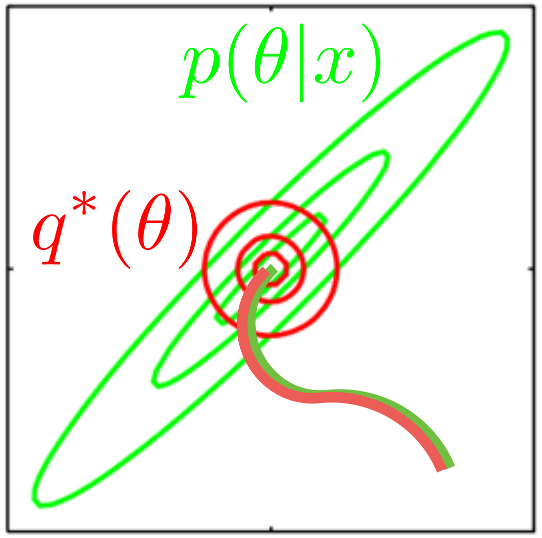

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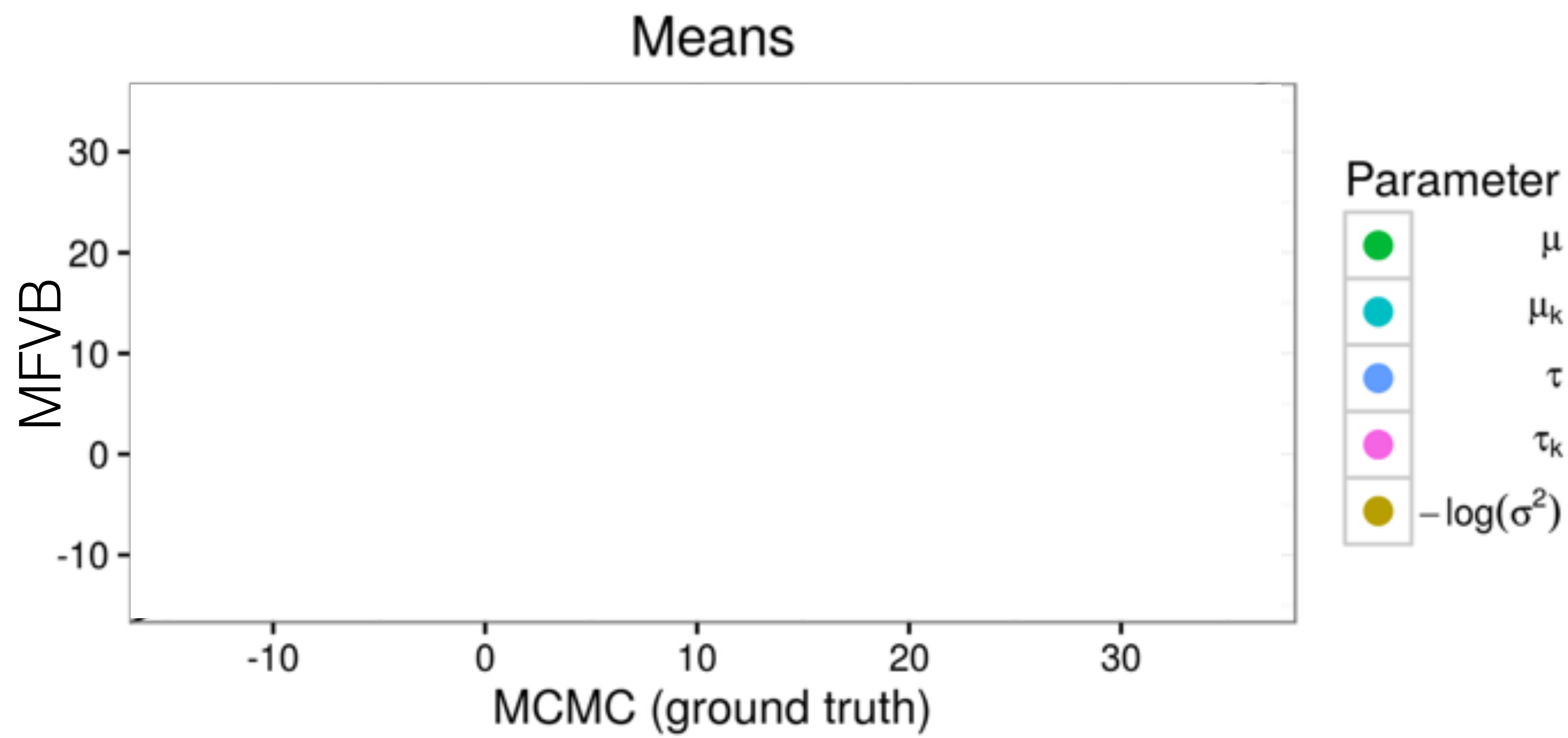
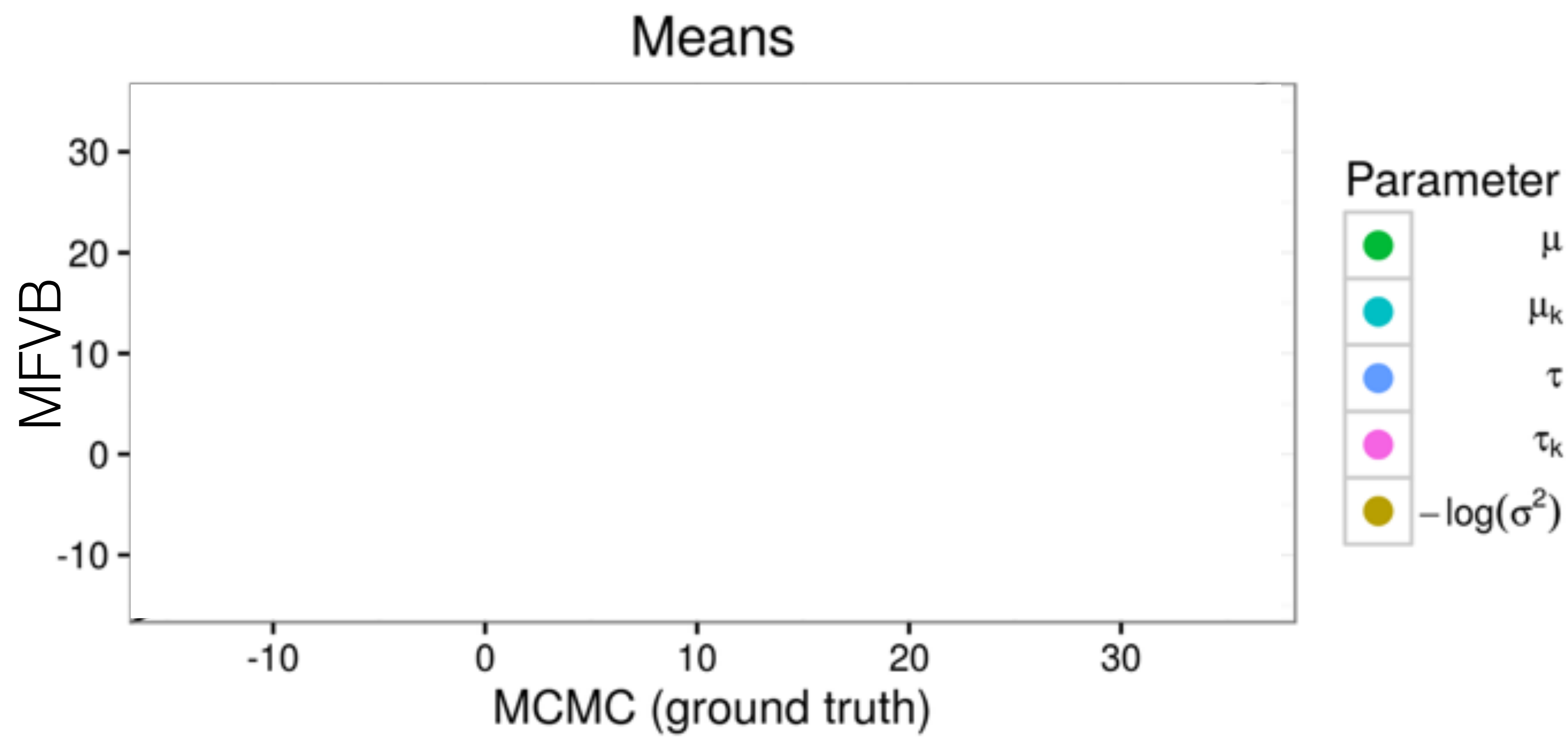


Microcredit Experiment

- *One set* of 2500 MCMC draws: **45 minutes**



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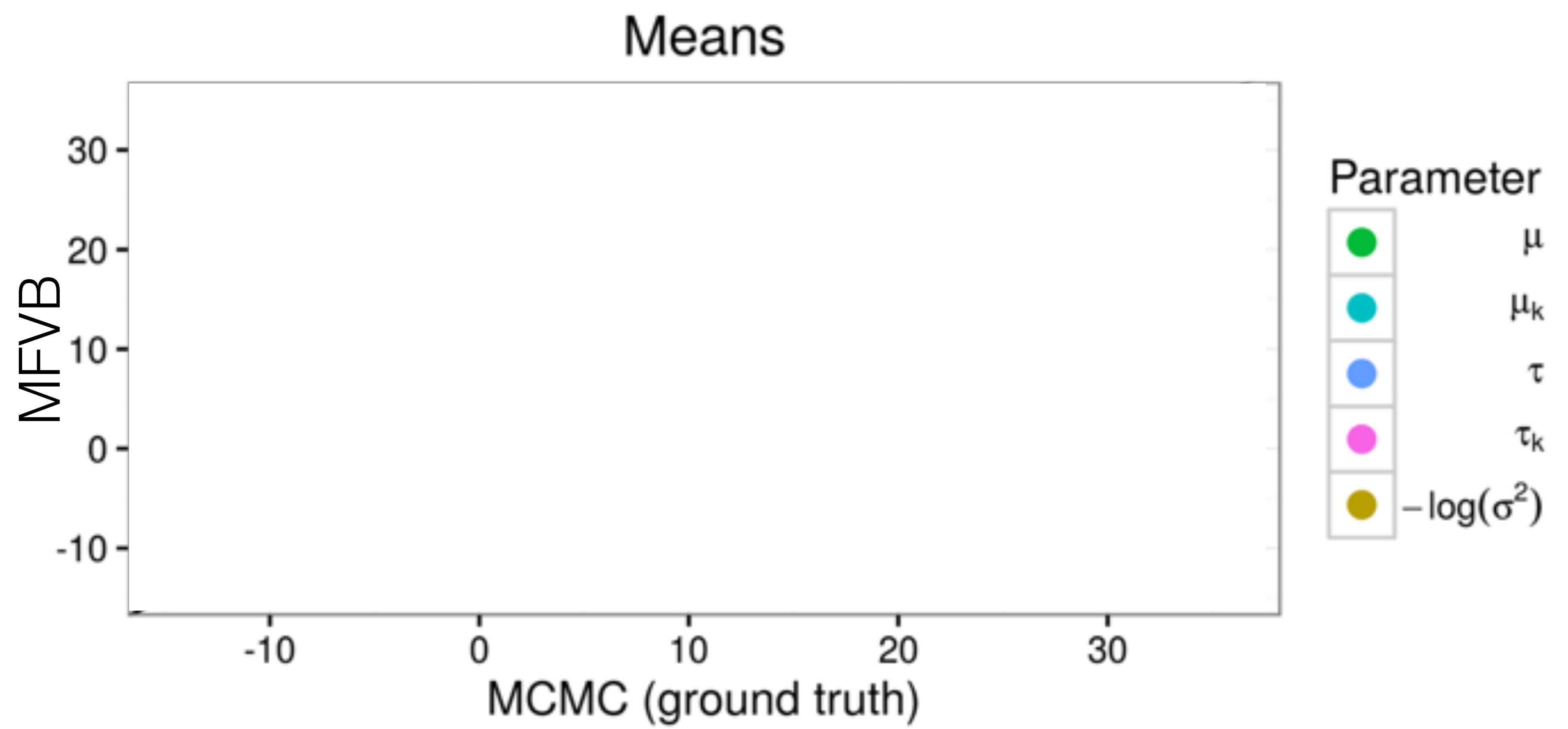

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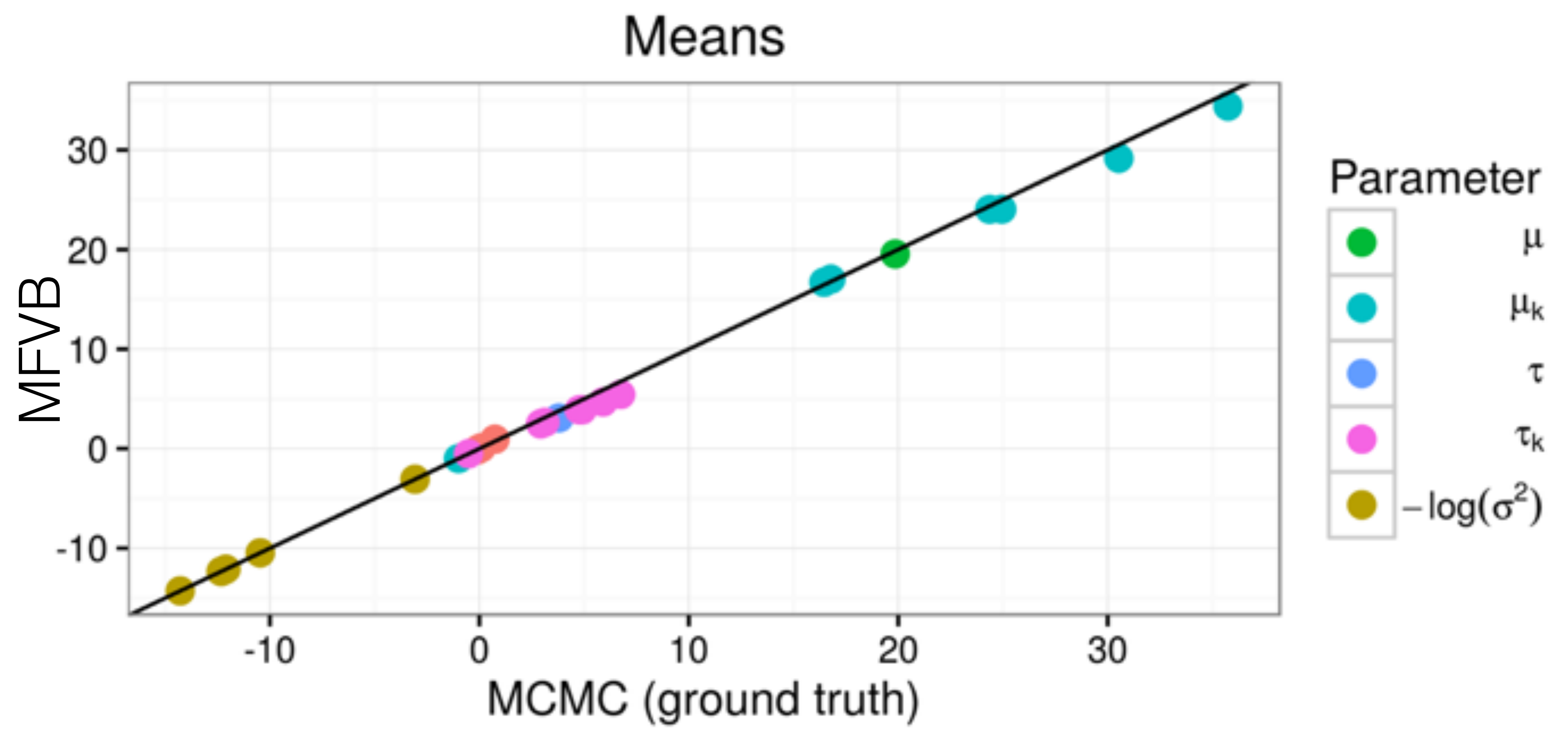

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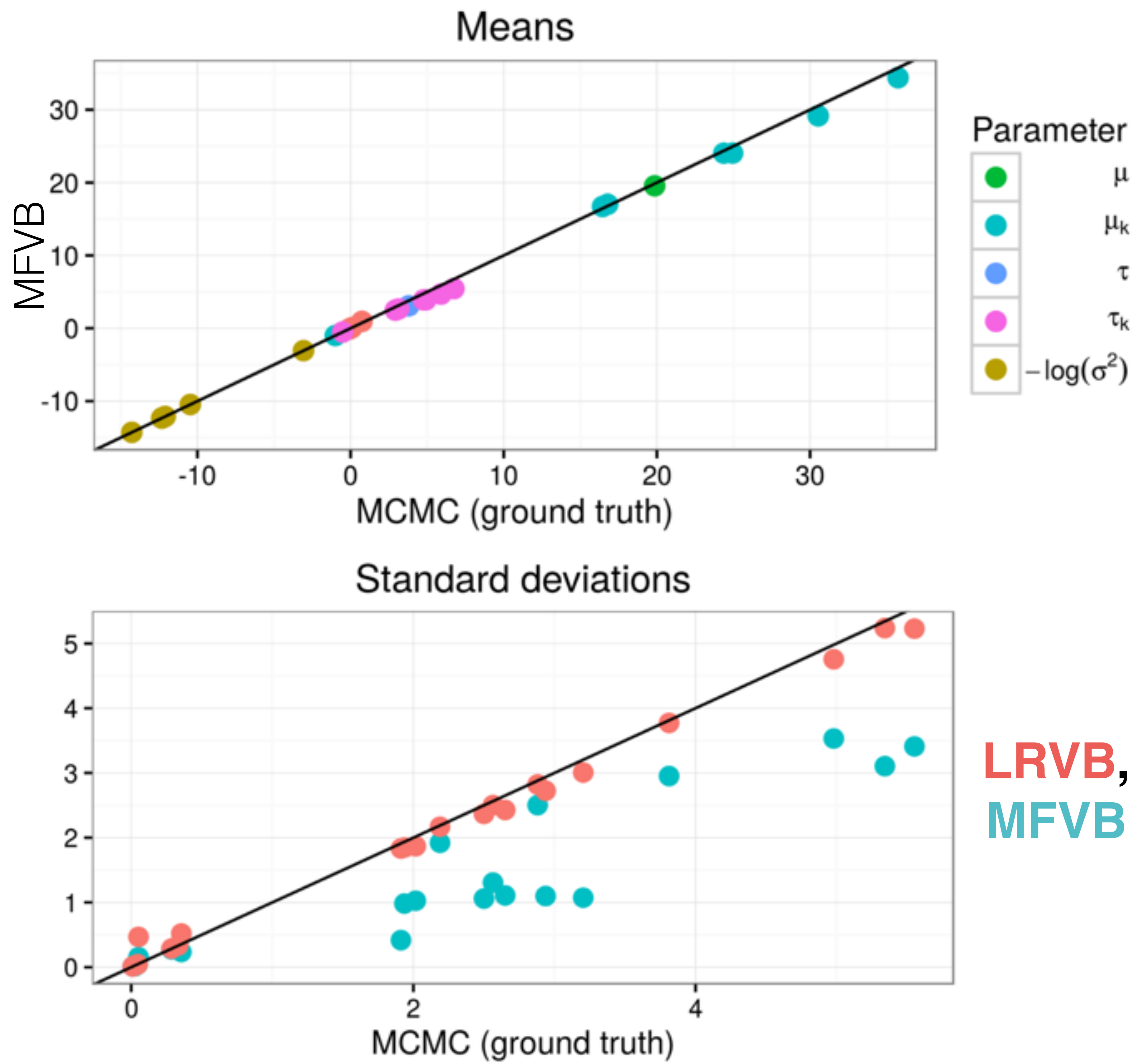

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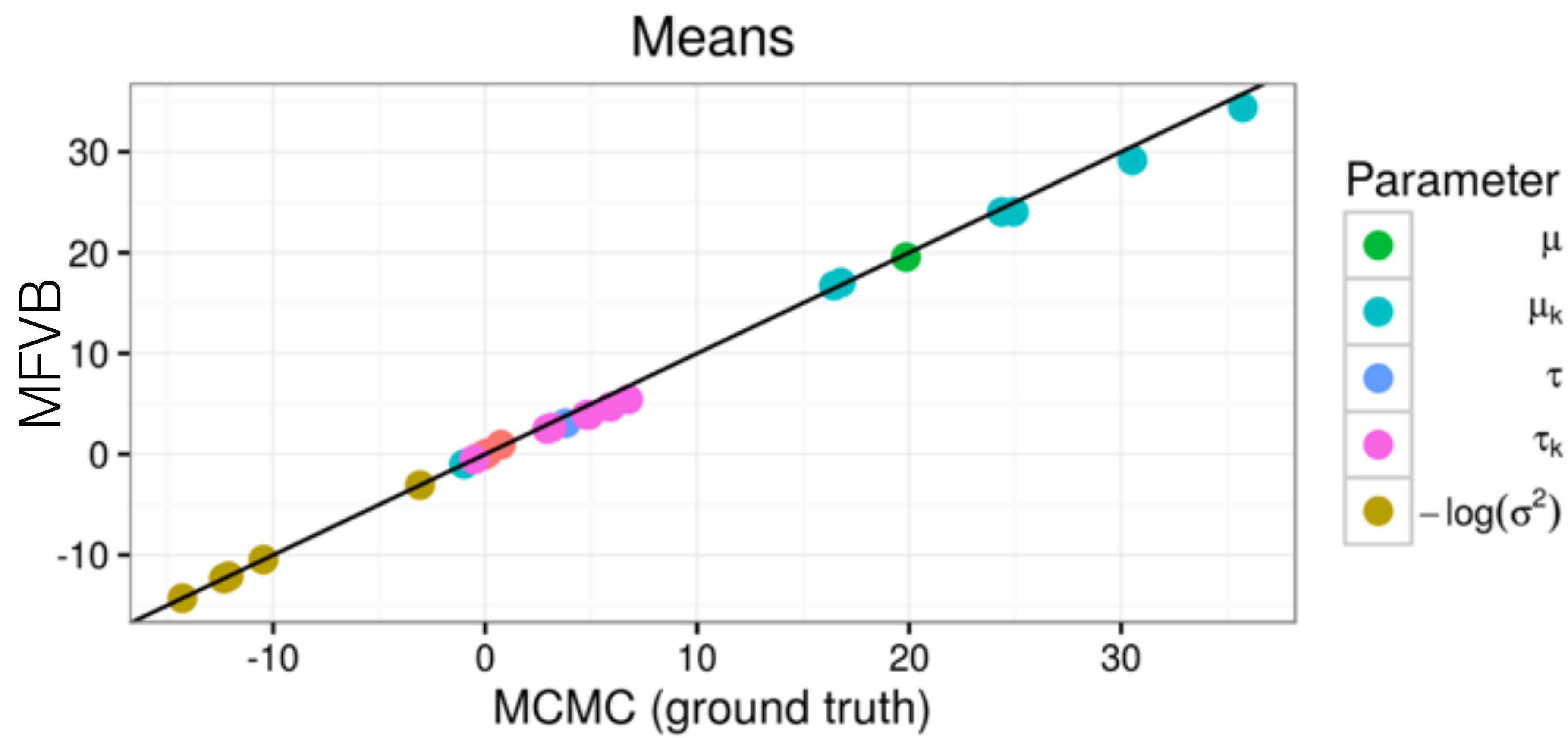

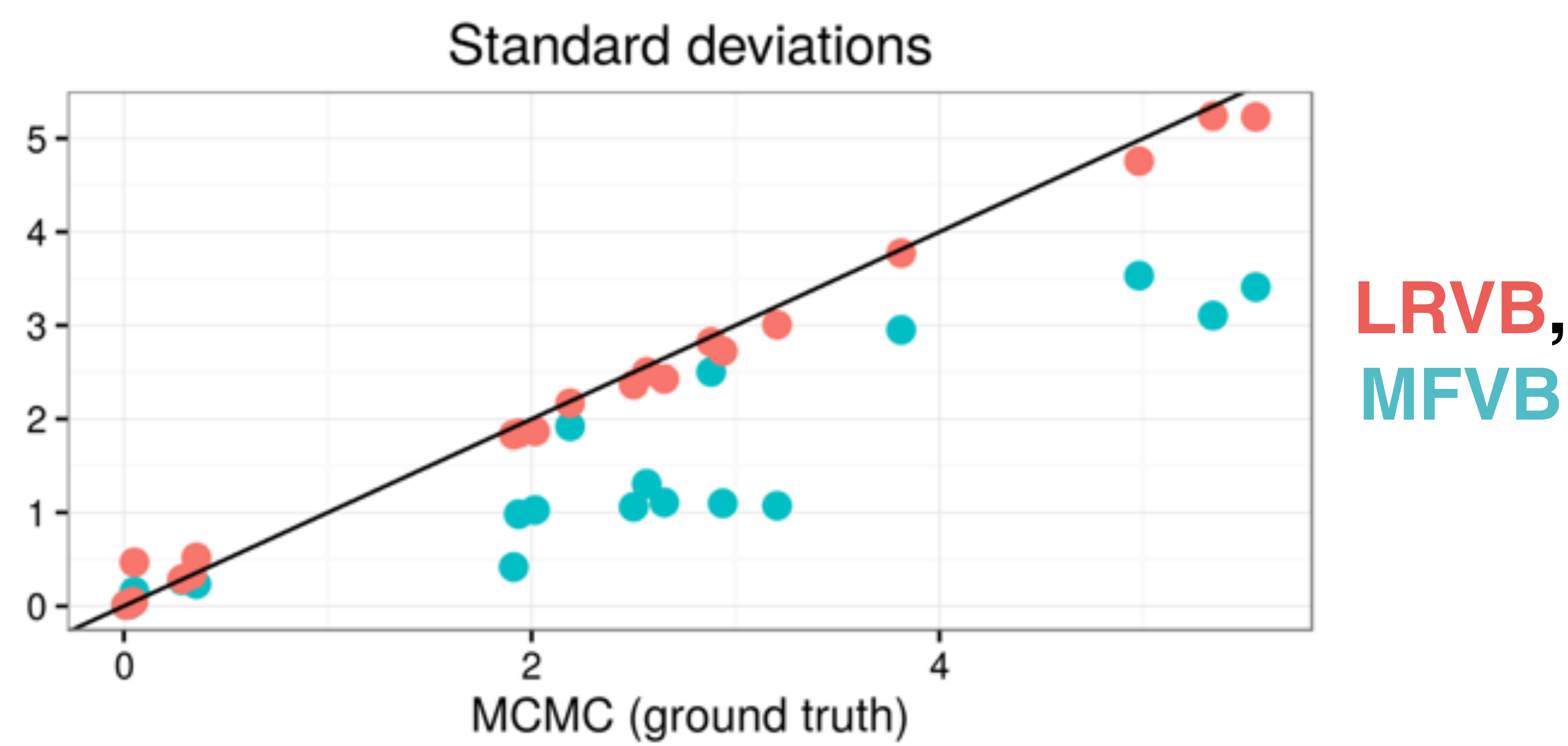

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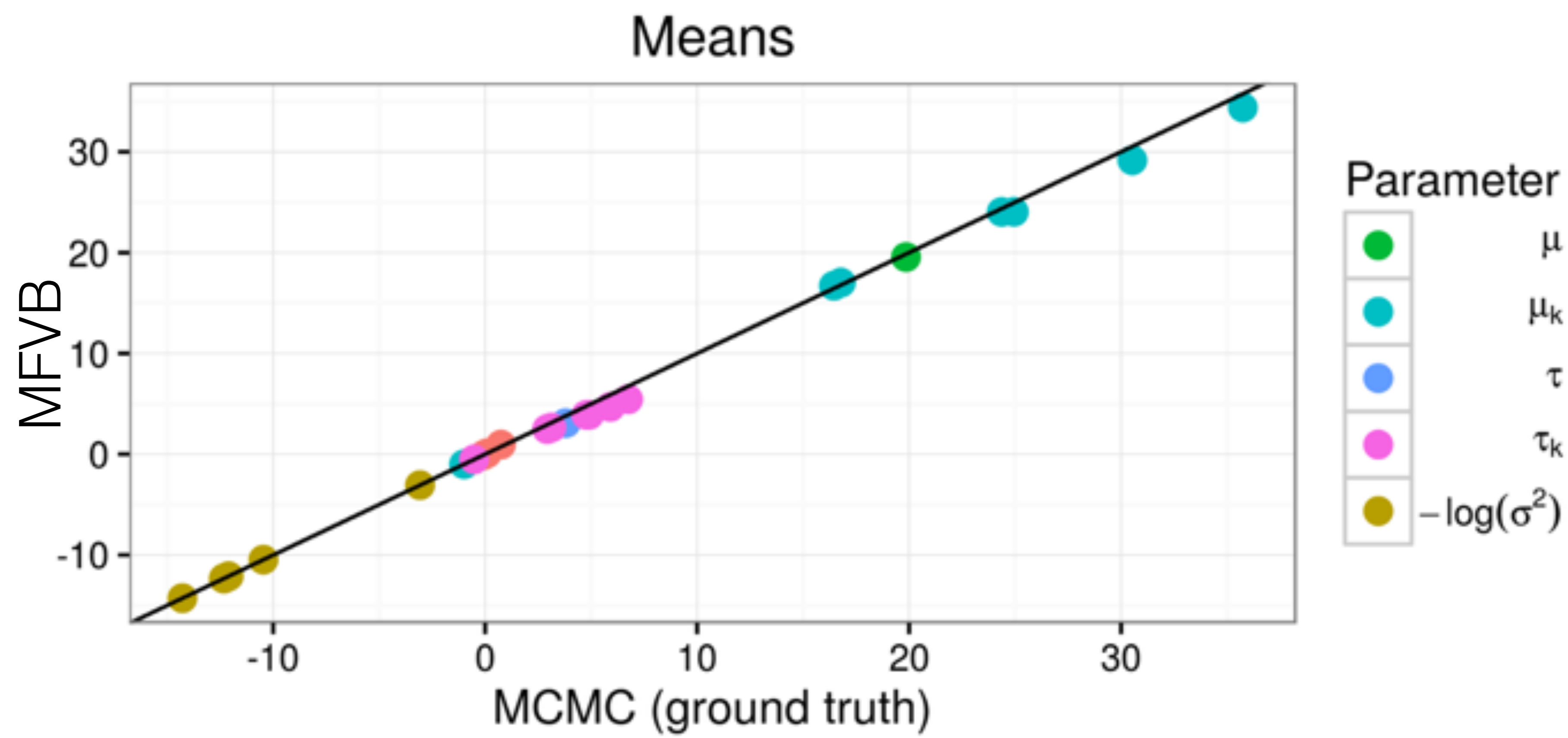

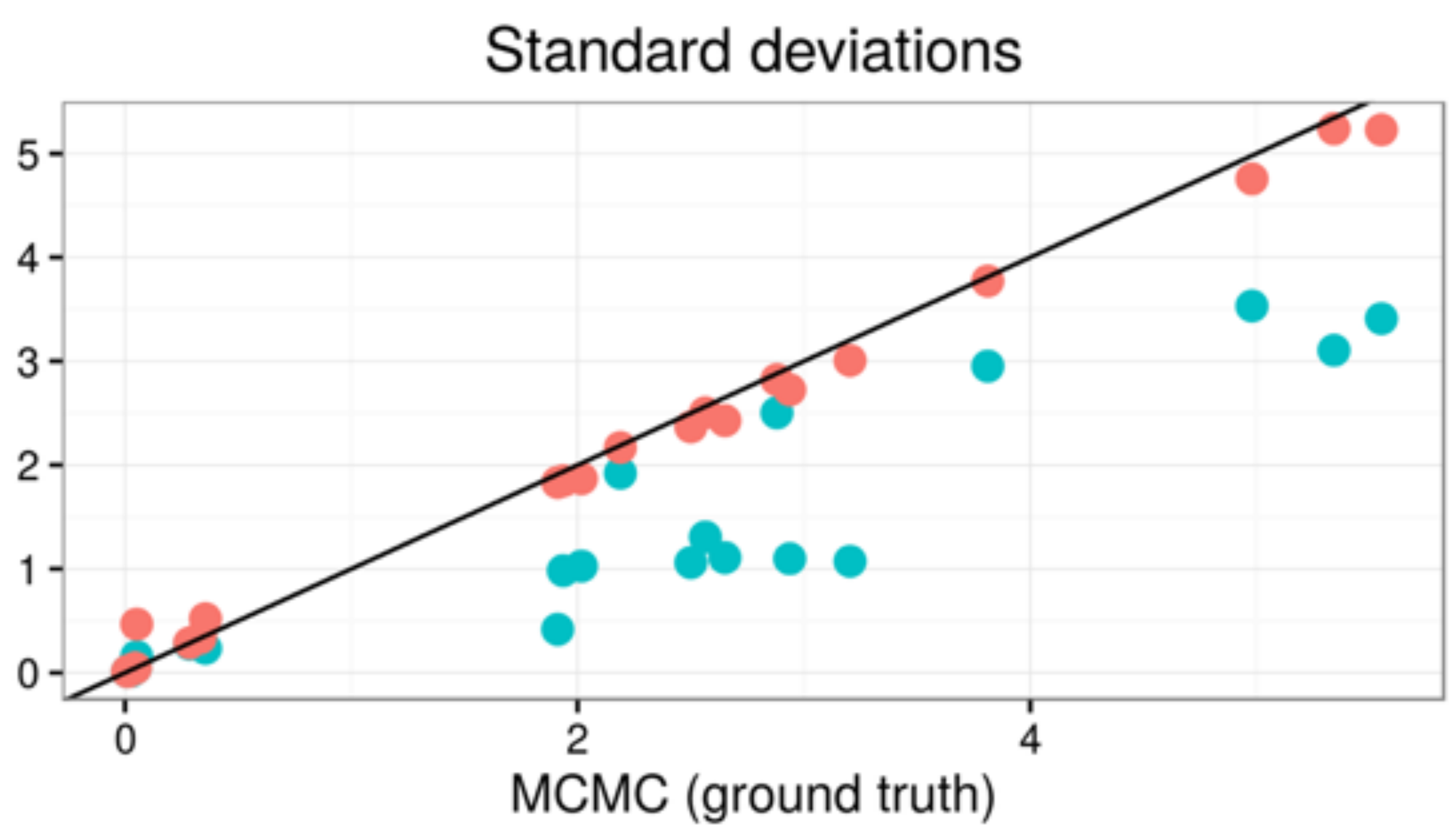

LRVB, **MFVB**

Microcredit Experiment

- *One set* of 2500 MCMC draws: **45 minutes**
- All of MFVB optimization, LRVB uncertainties, all sensitivity measures: **58 seconds**
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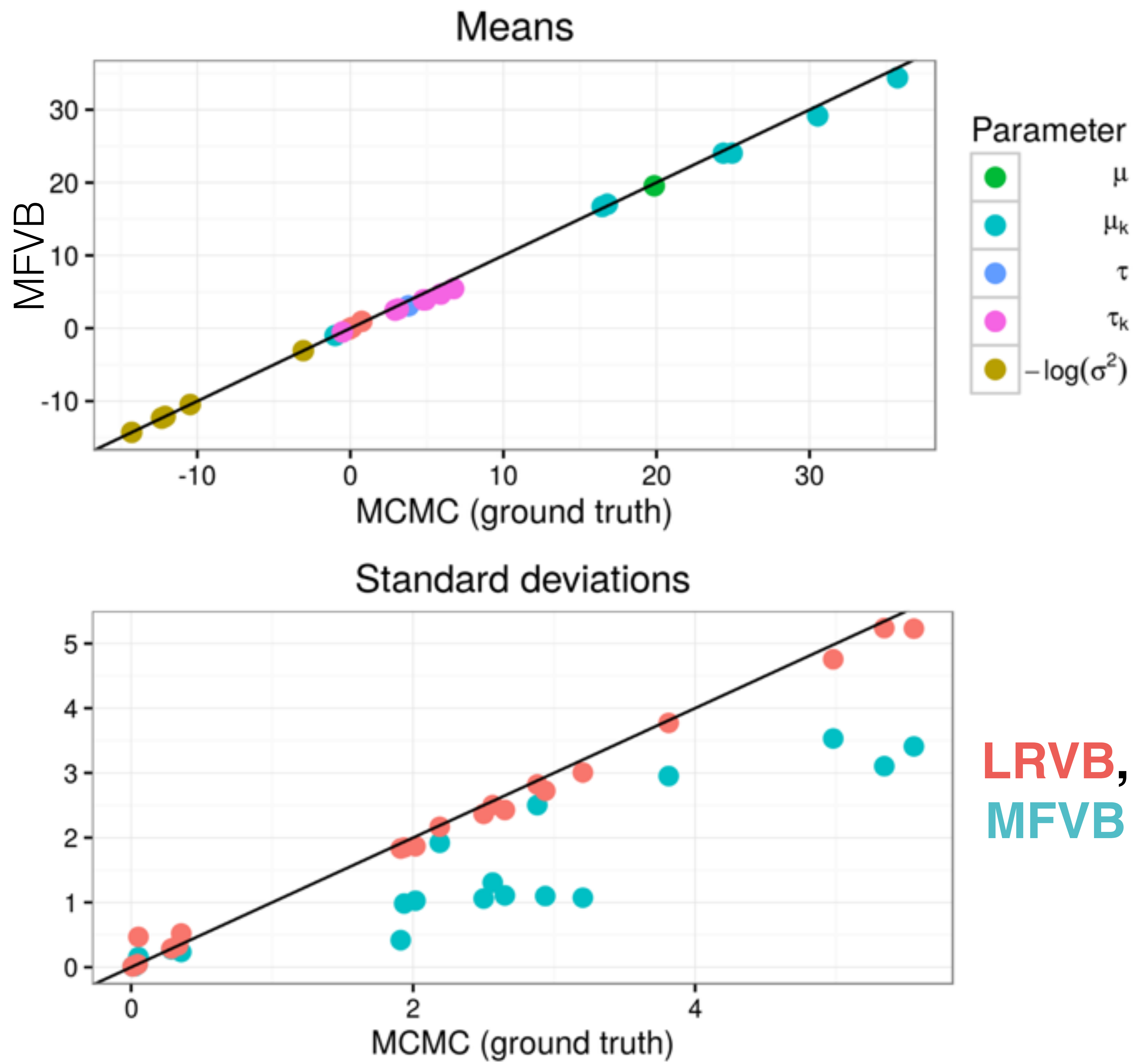

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$$P(z_{nk} = 1) = \pi_k, \quad p(x|\pi, \mu, \Lambda, z) = \prod_{n=1:N} \prod_{k=1:K} \mathcal{N}(x_n|\mu_k, \Lambda_k^{-1})^{z_{nk}}$$

with conjugate priors on π, μ, Λ

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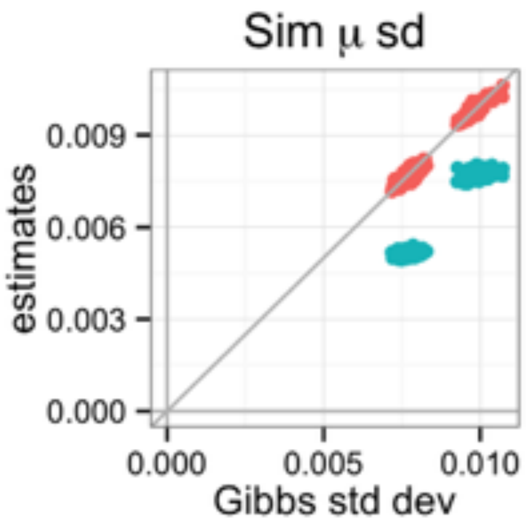

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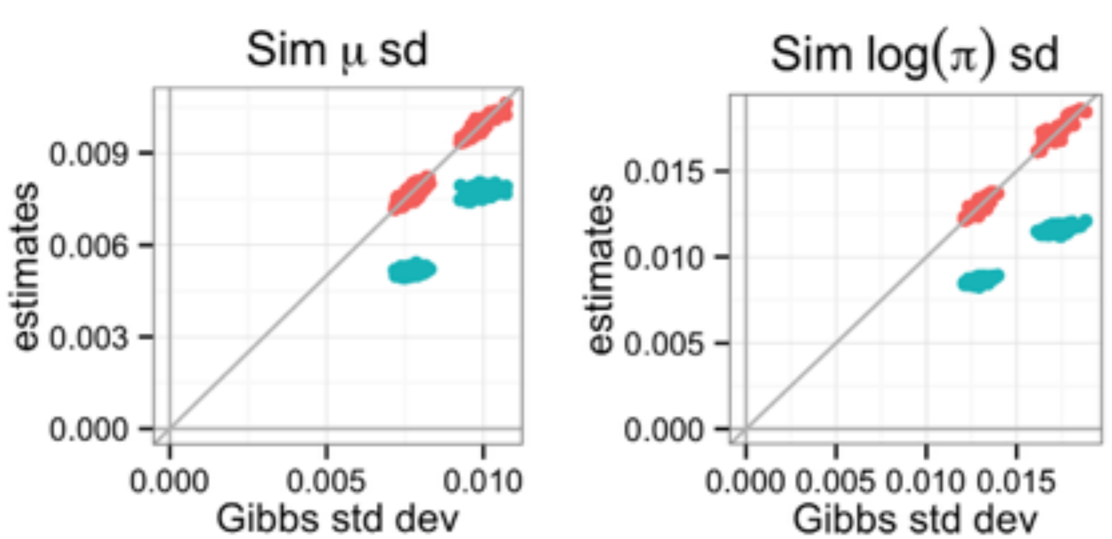

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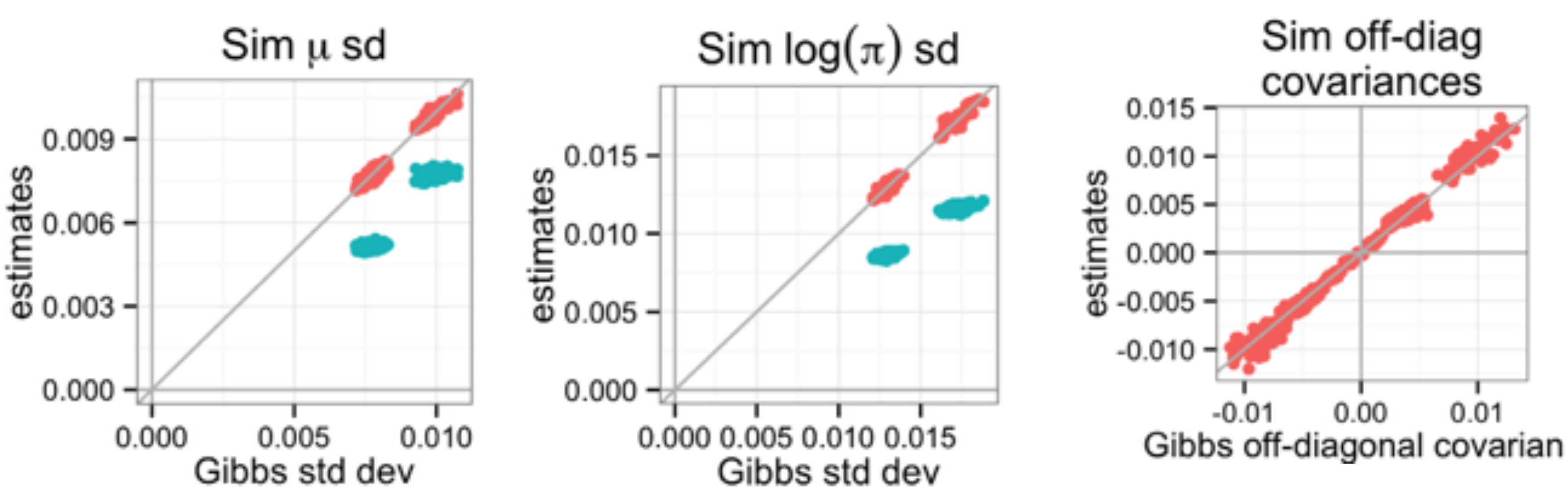

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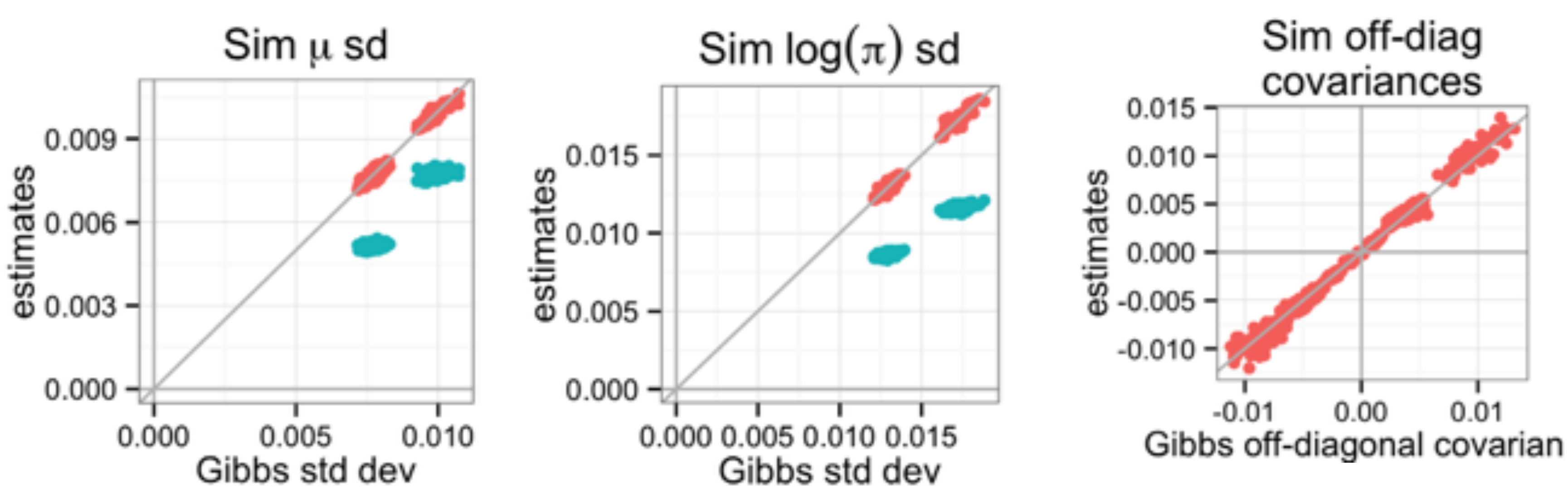

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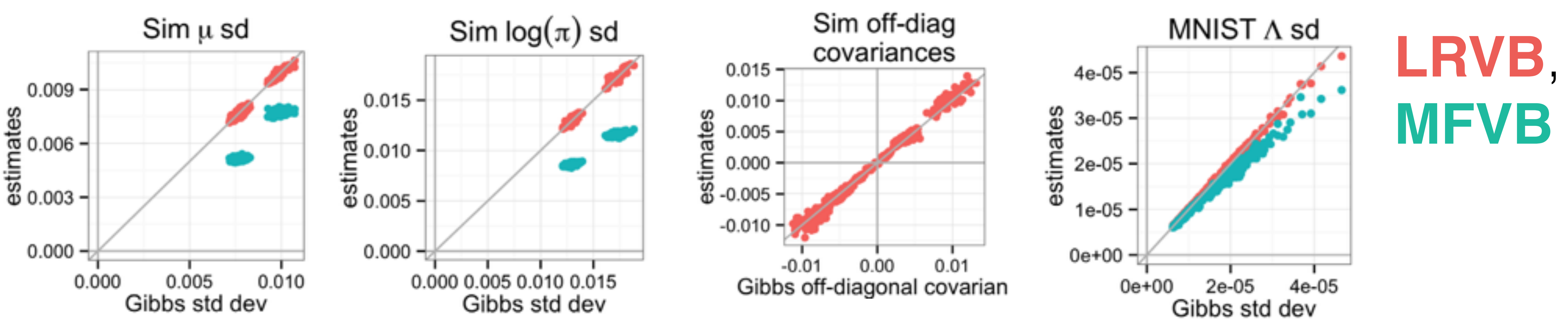

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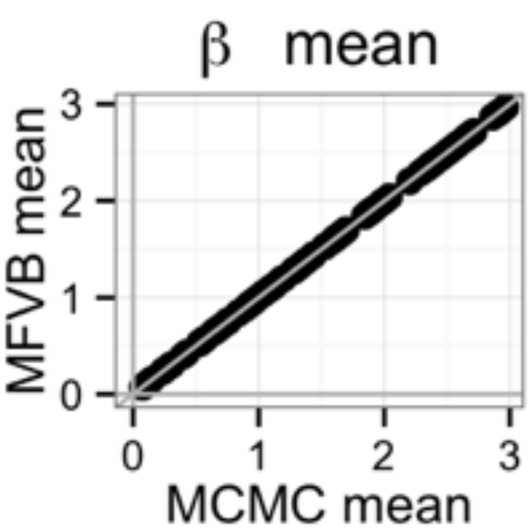

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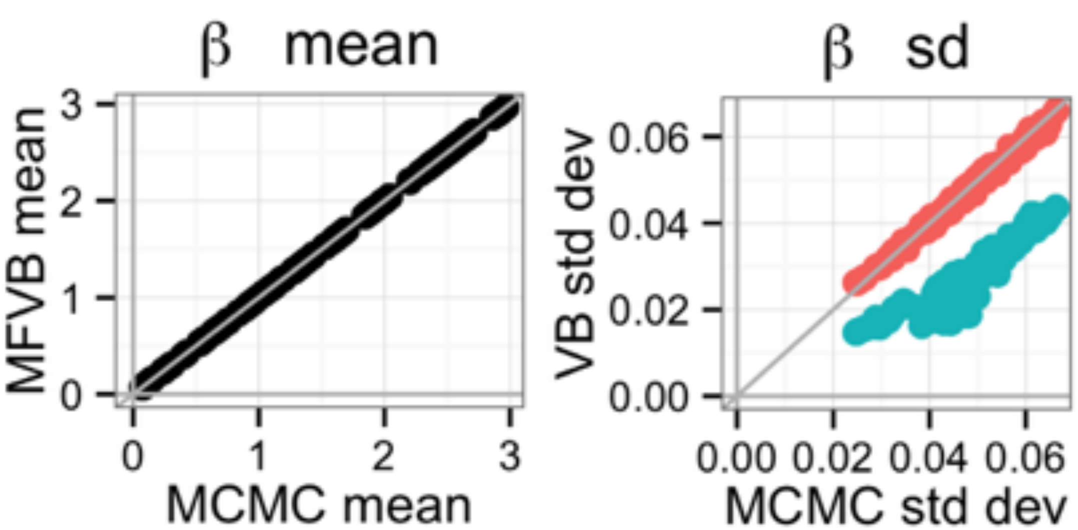

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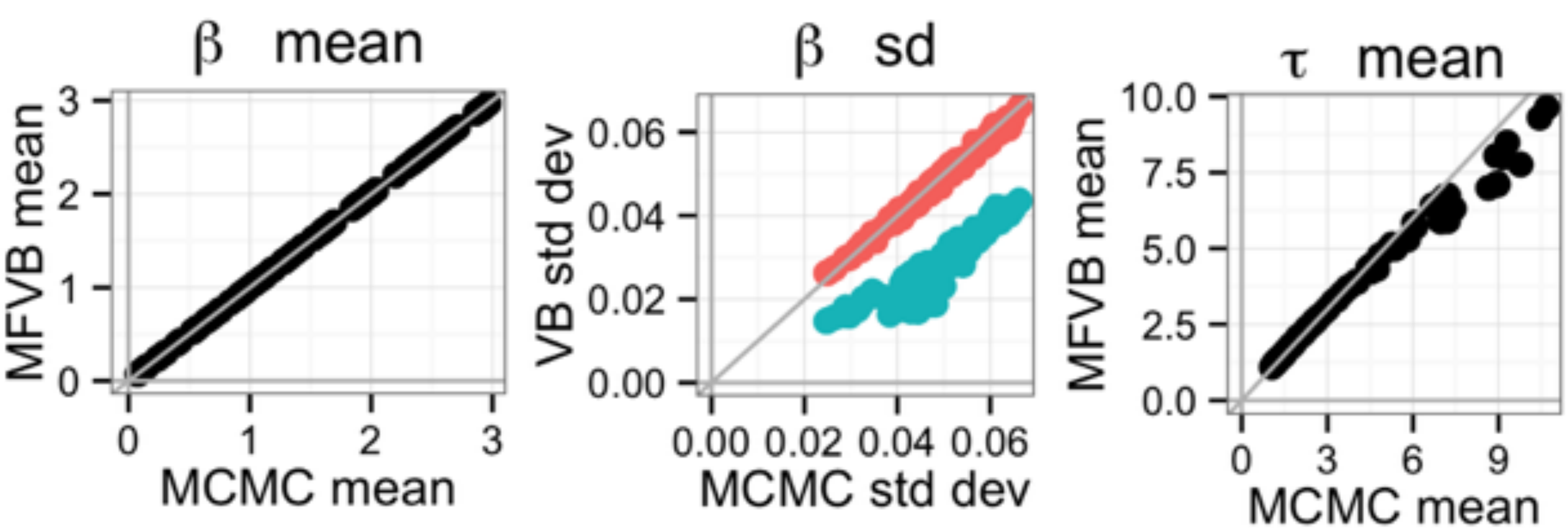

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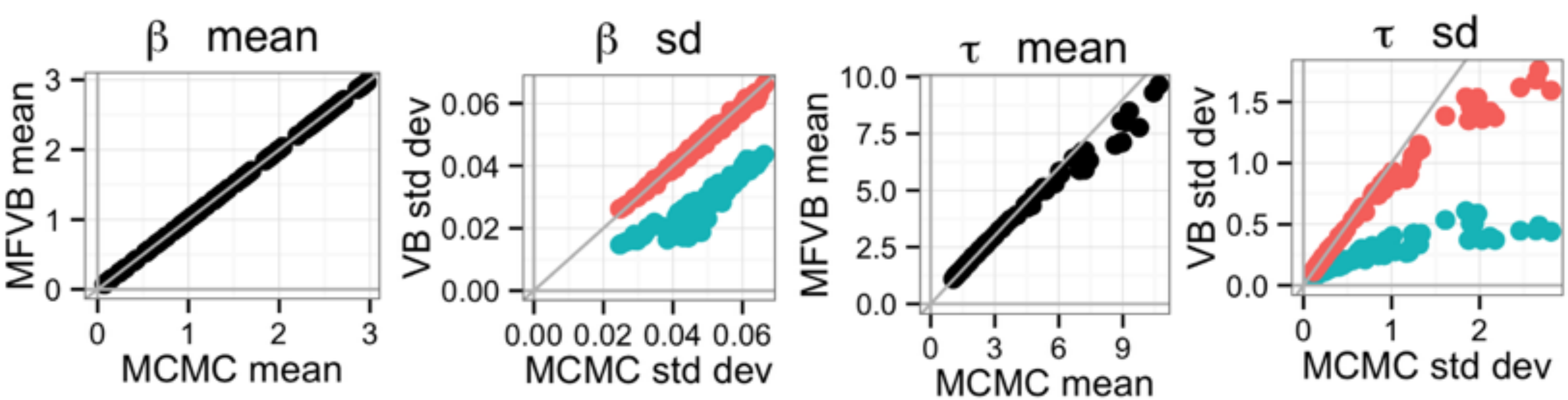

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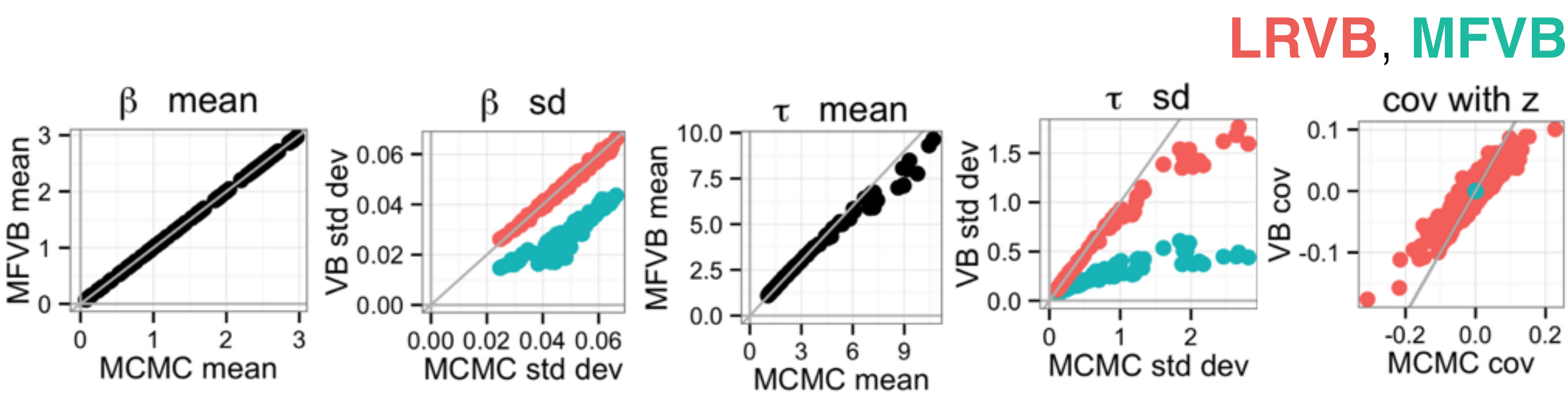

Scaling the matrix inverse

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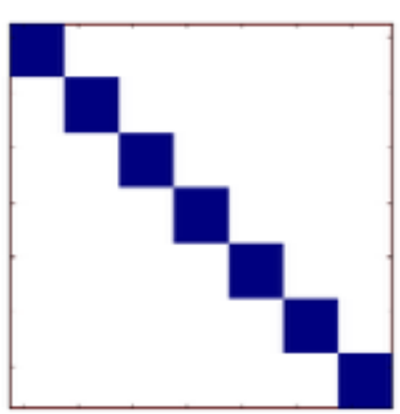

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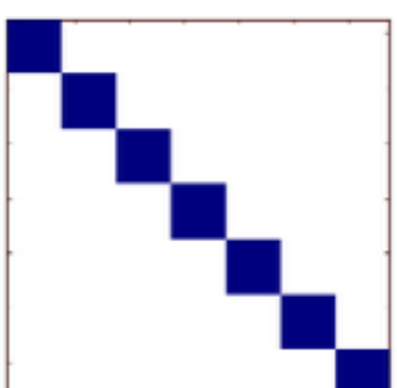
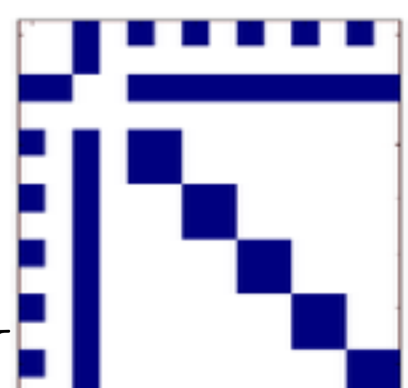
V 

H 

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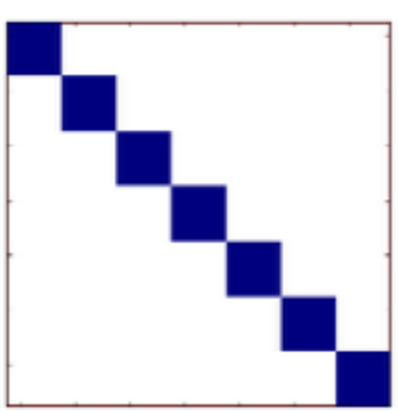
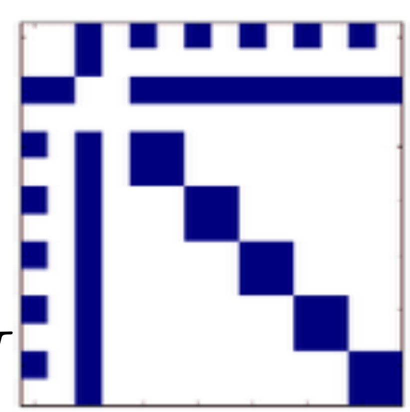
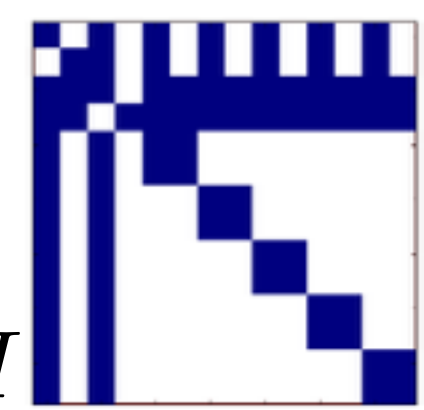
V 

H 

$I - VH$ 

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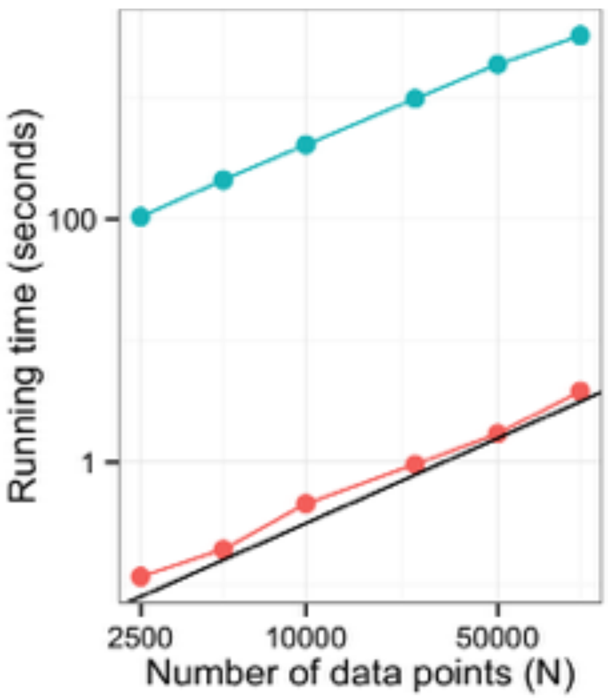

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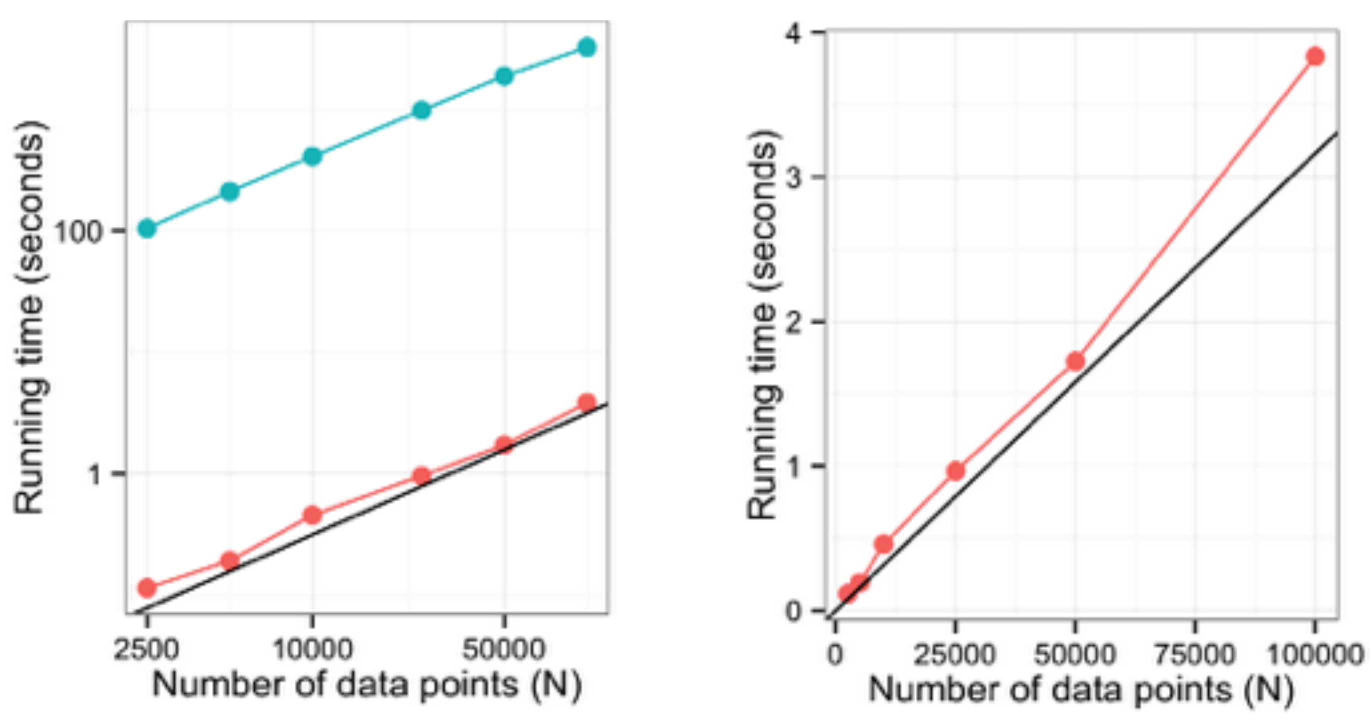

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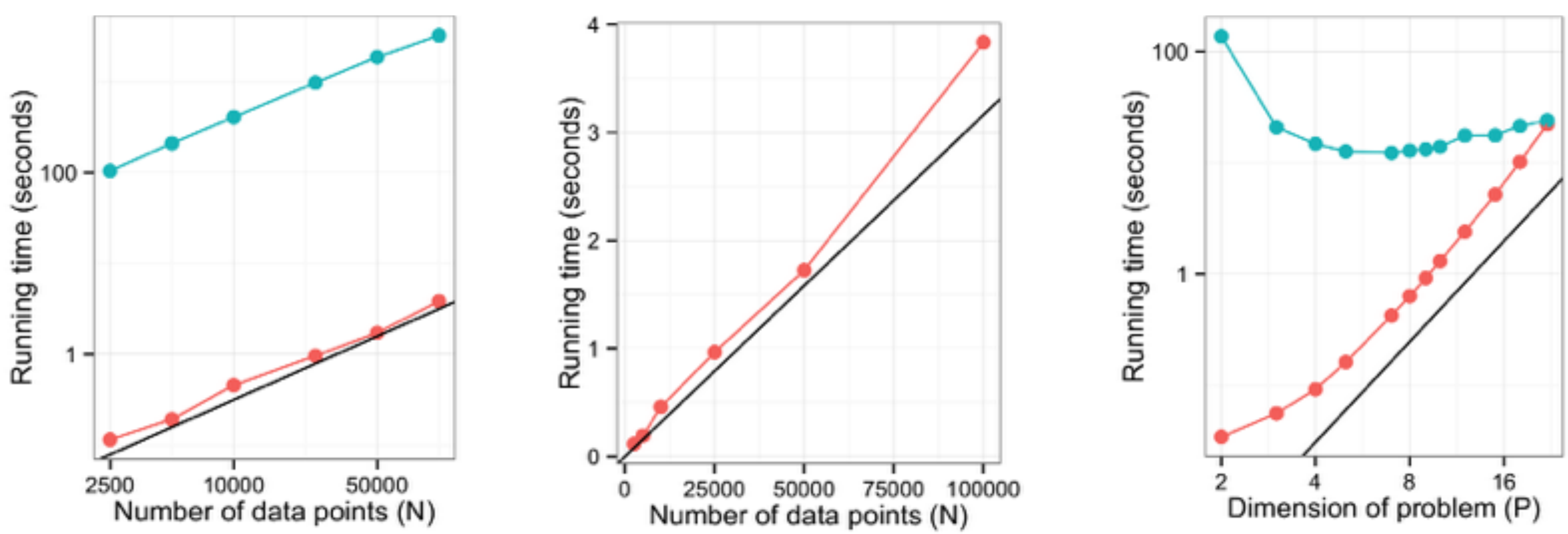

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- Variational Bayes as an alternative to MCMC
- Challenges of VB
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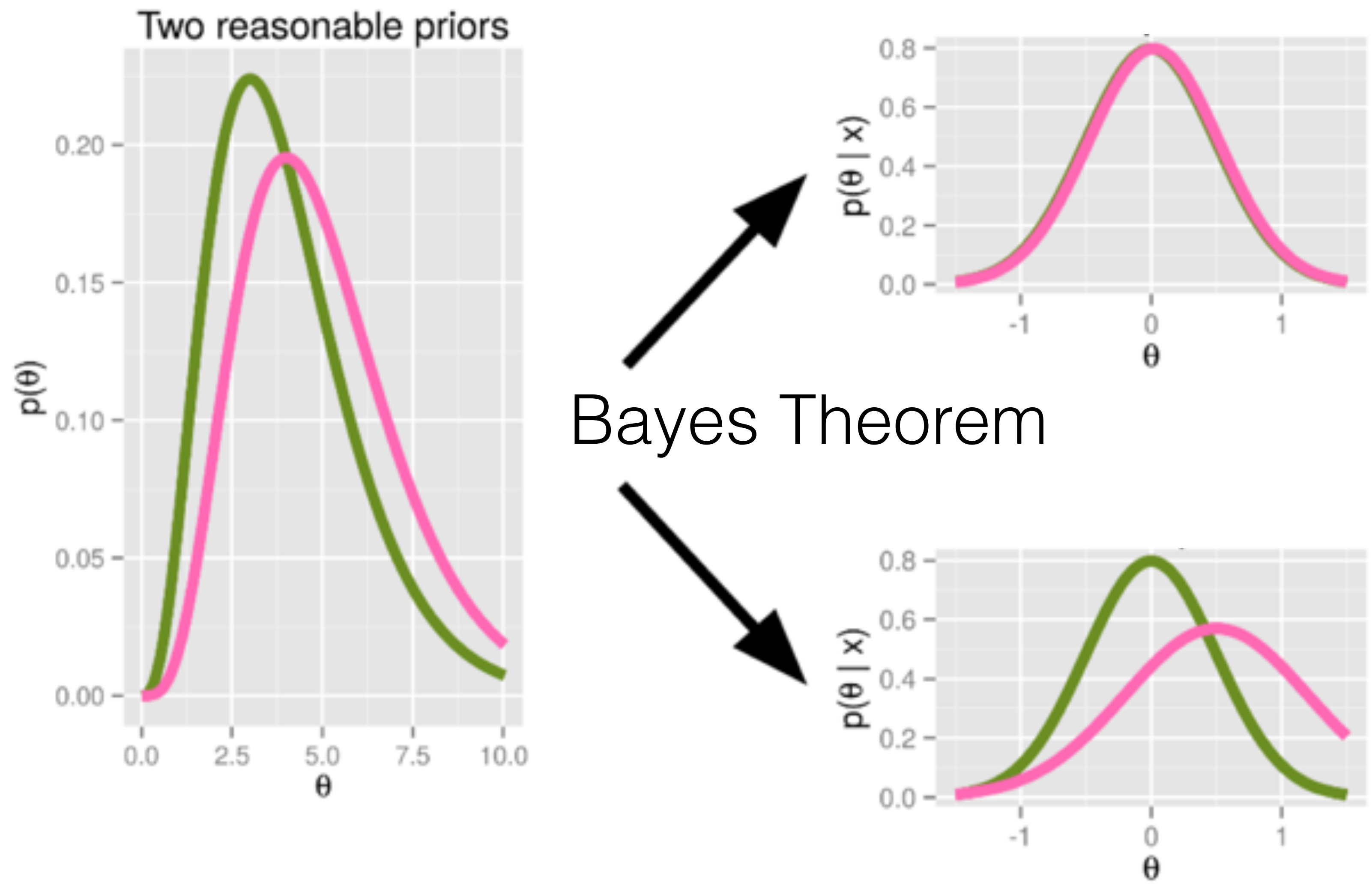

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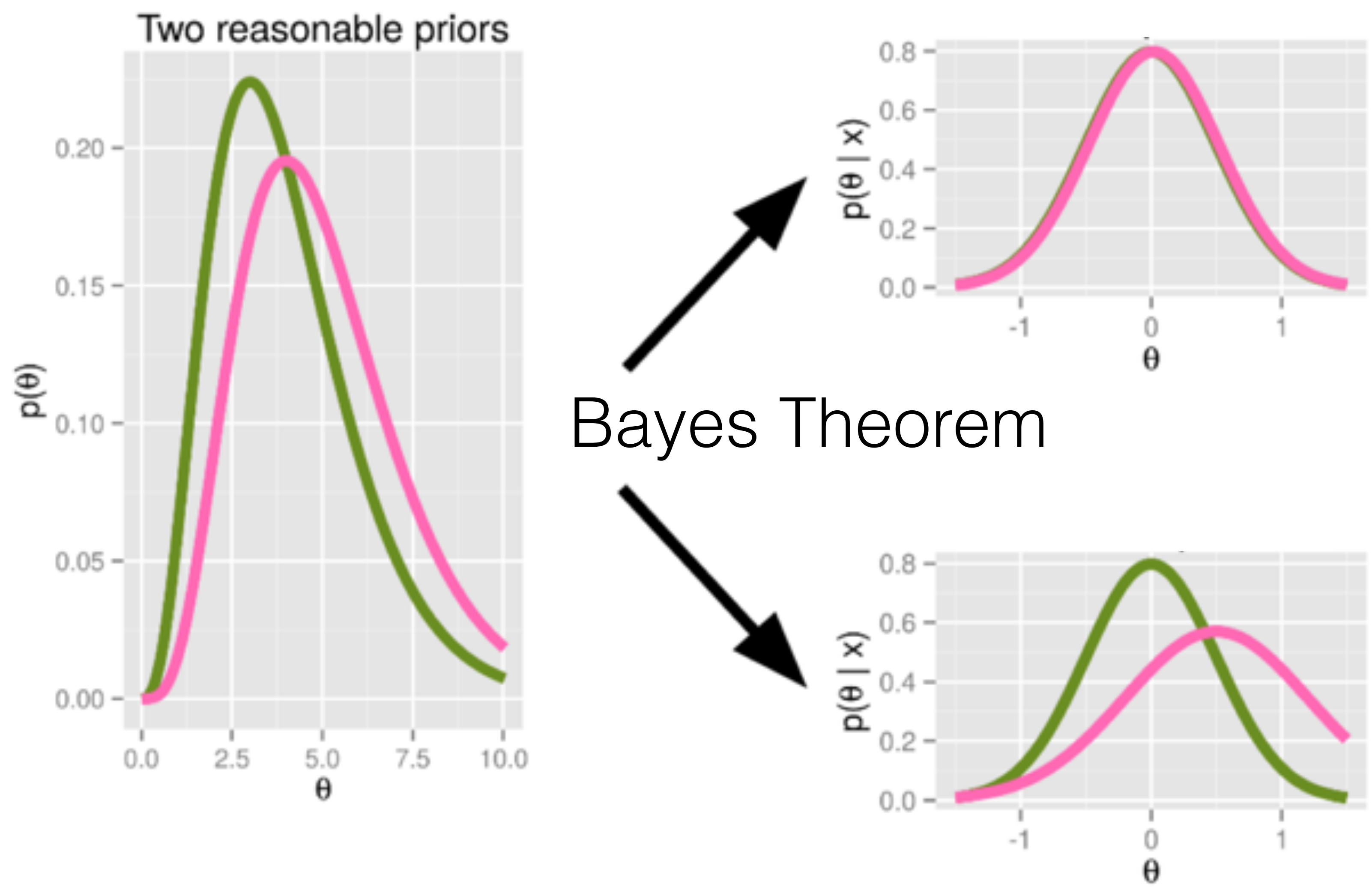

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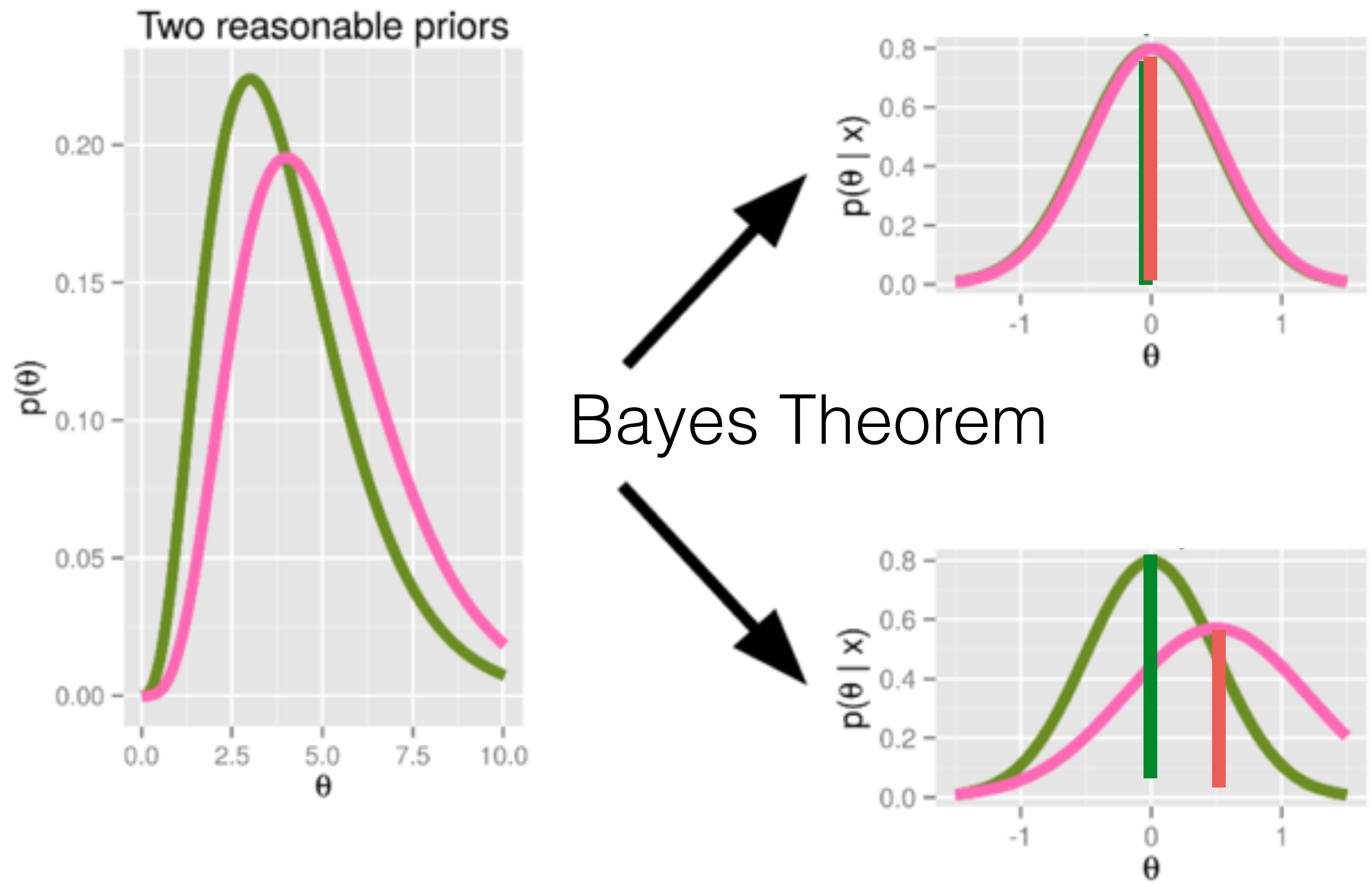

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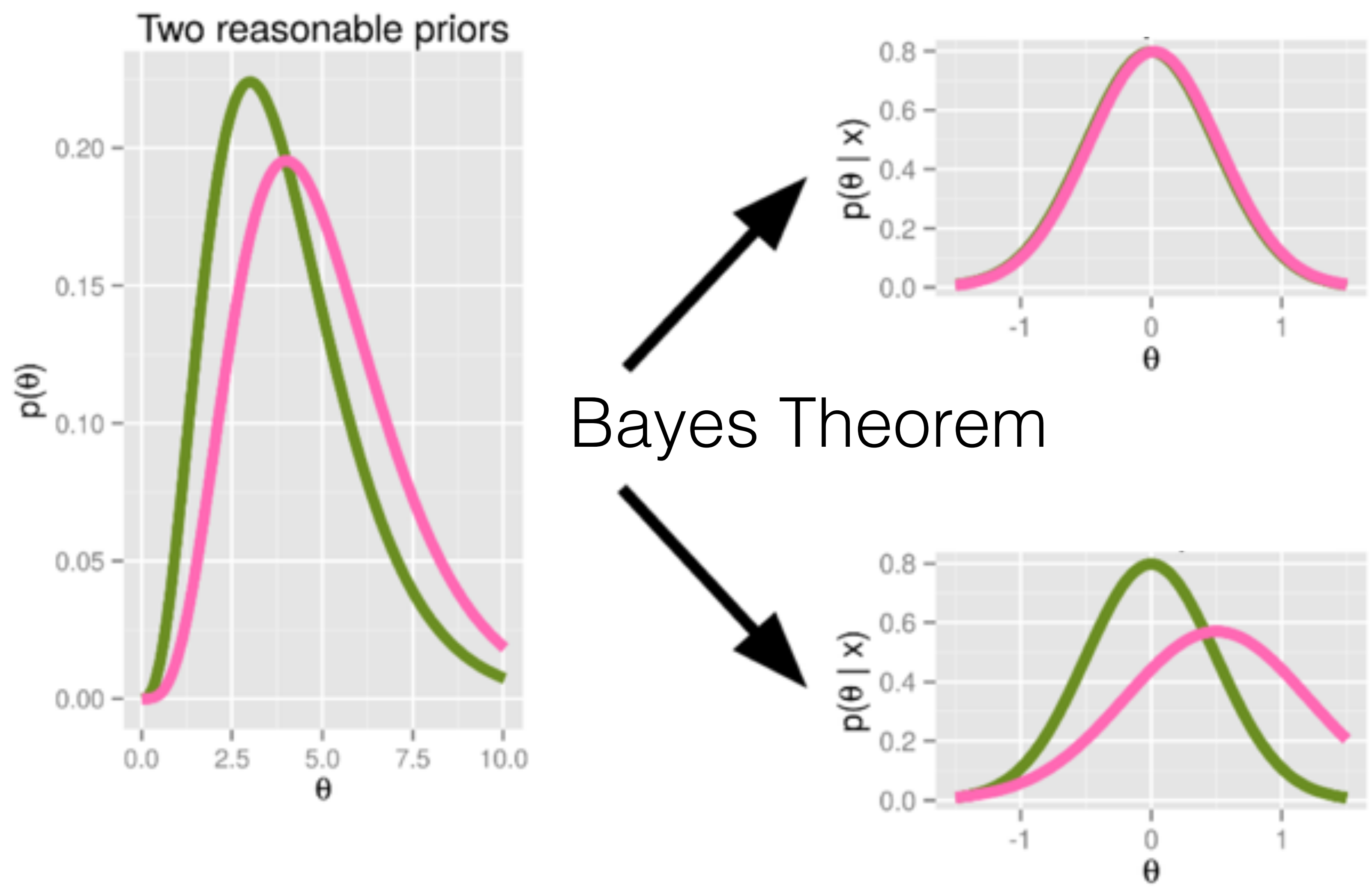

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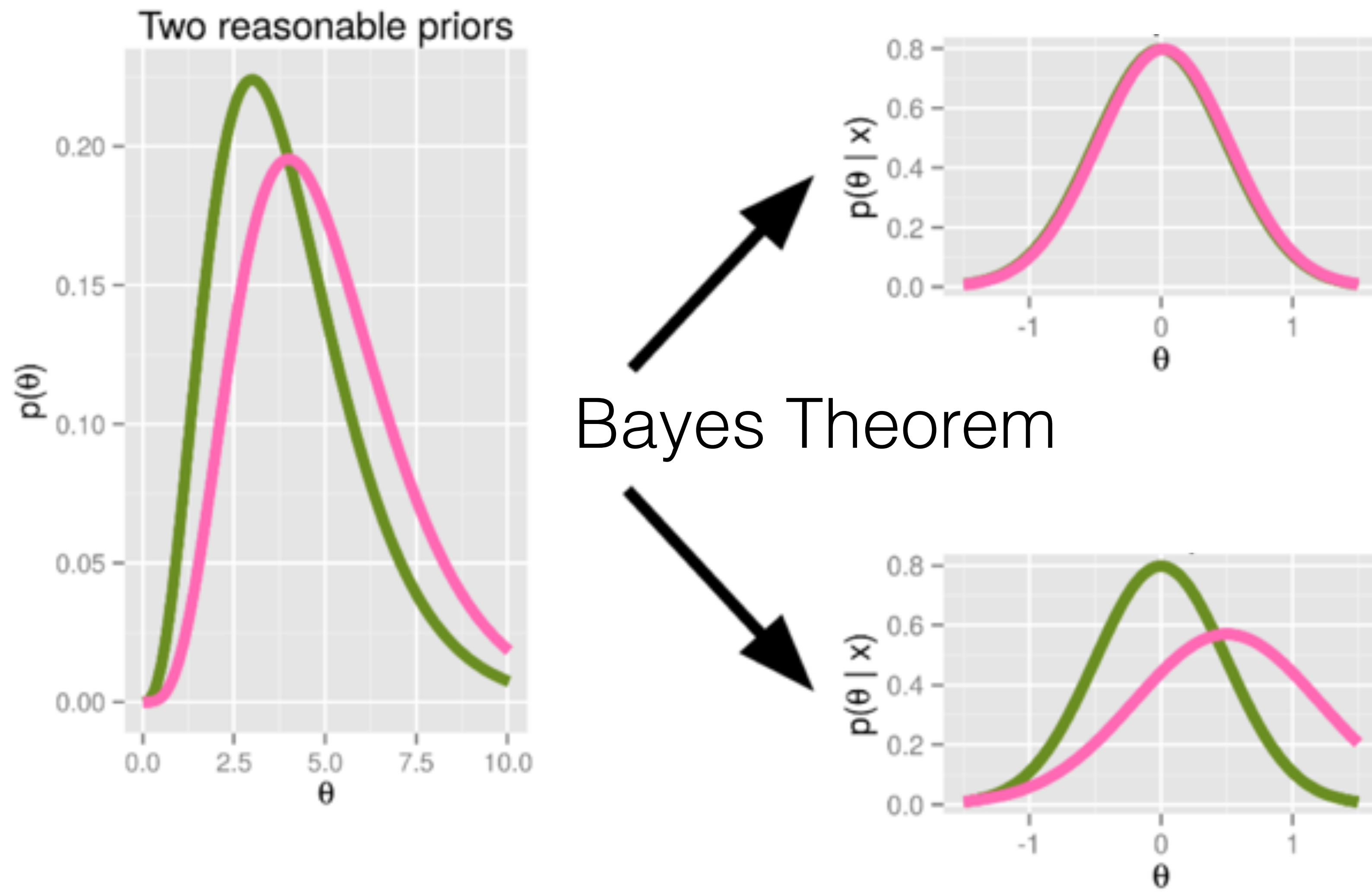

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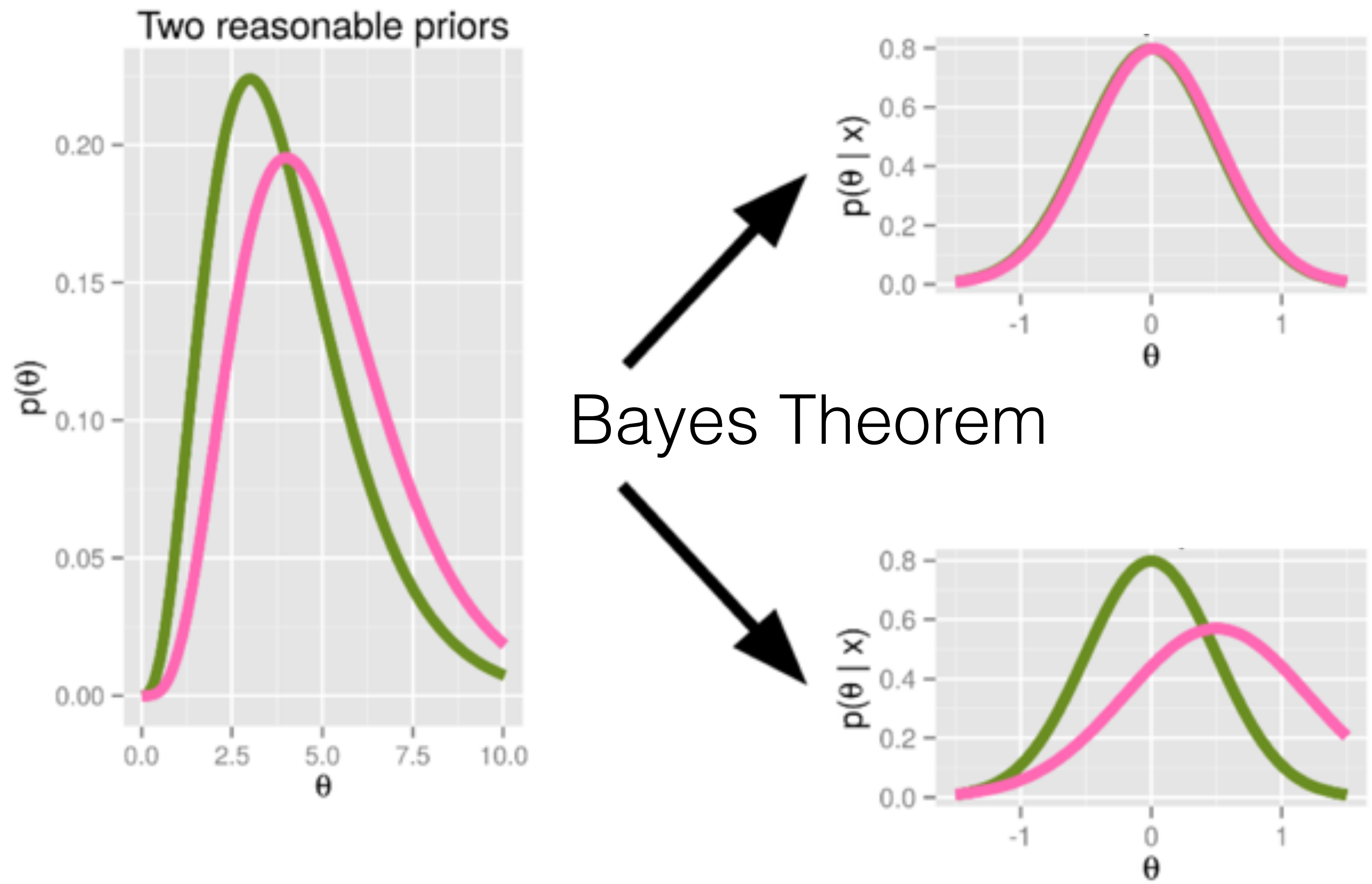

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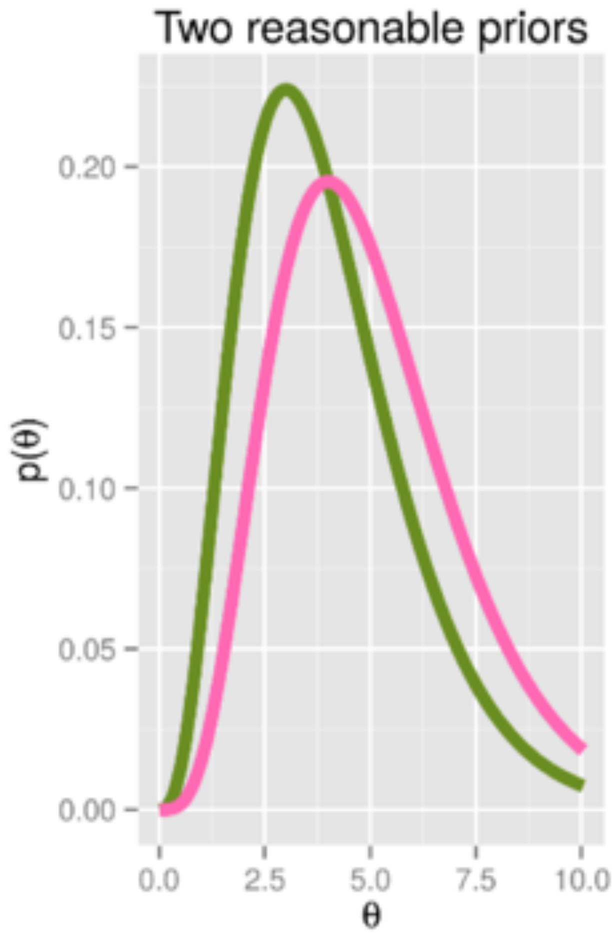

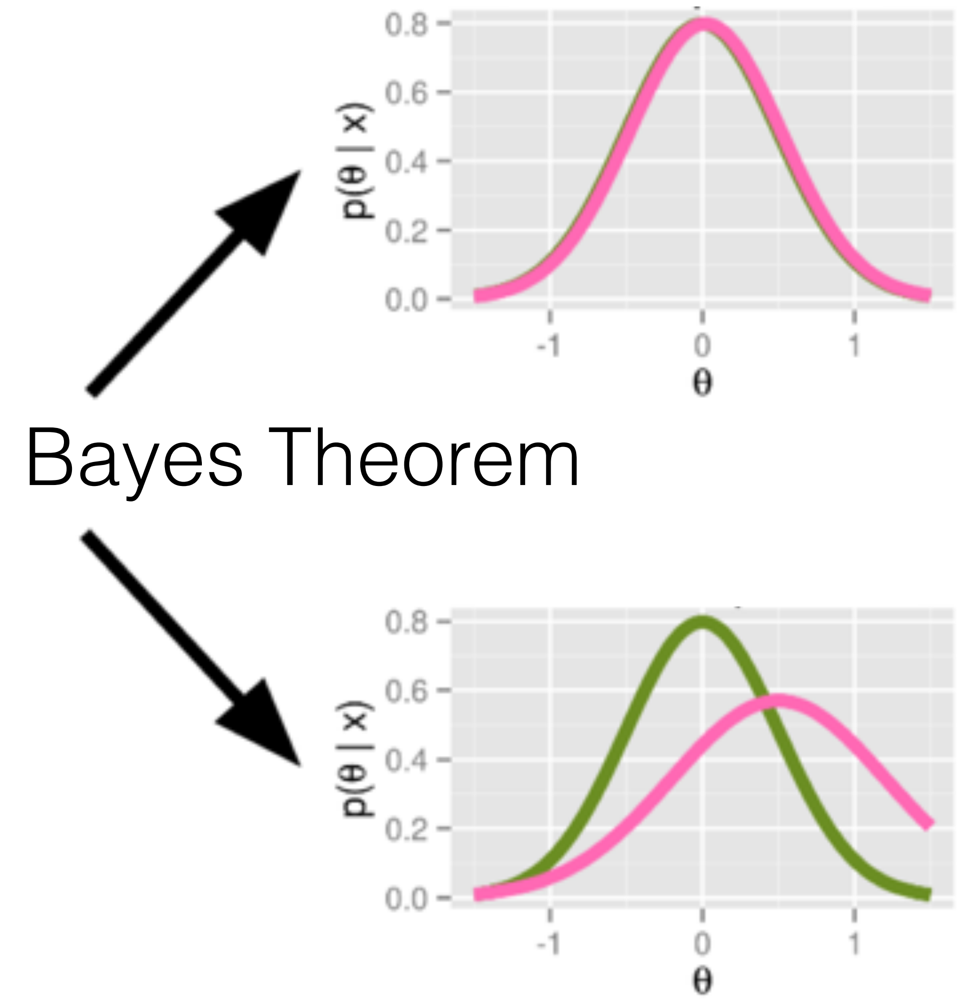

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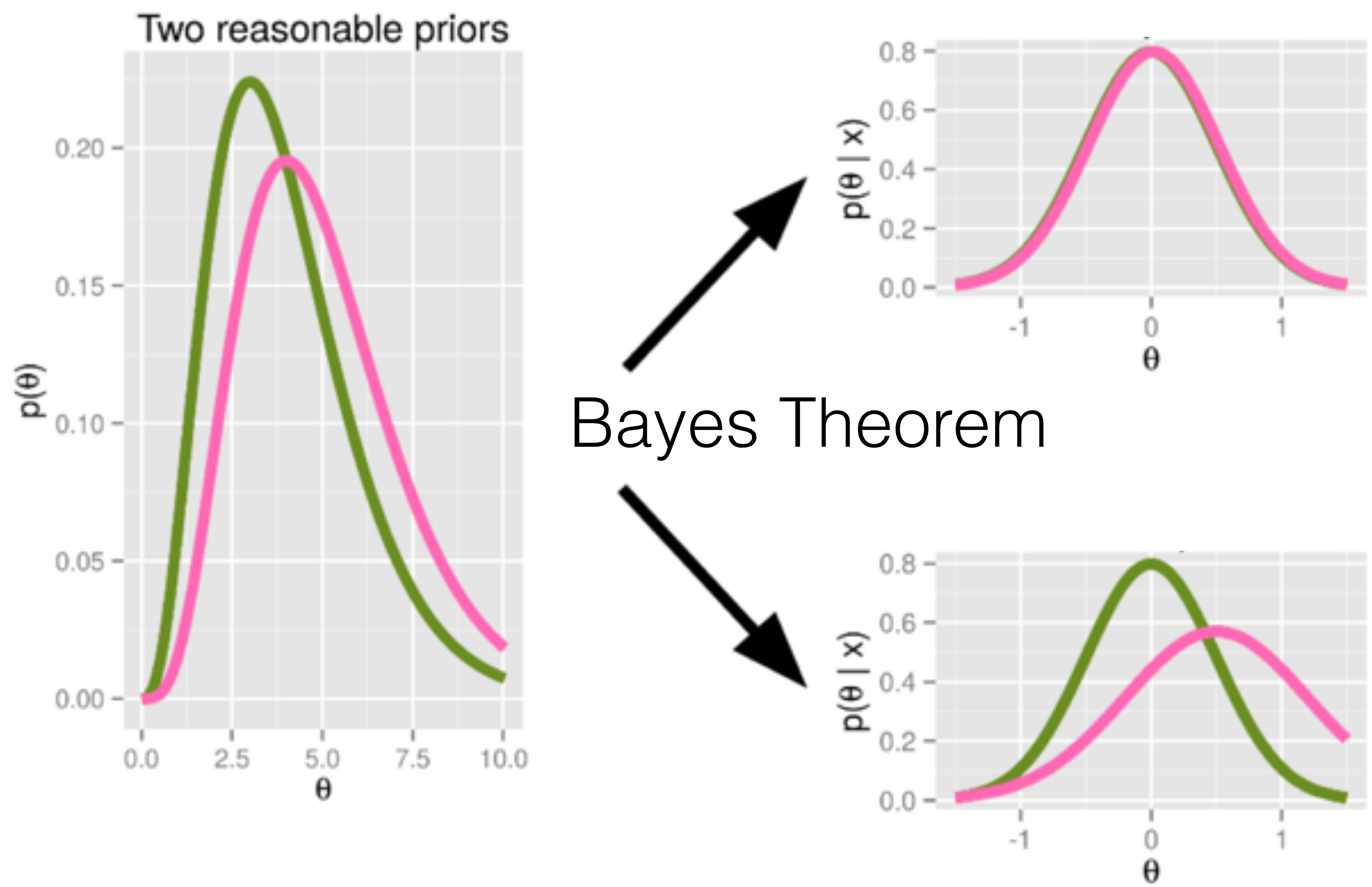
$$\propto_\theta p(x|\theta)p(\theta|\alpha)$$

- Sensitivity

$$S := \left. \frac{d\mathbb{E}_{p_\alpha}[g(\theta)]}{d\alpha} \right|_\alpha \Delta\alpha$$
$$\approx \left. \frac{d\mathbb{E}_{q^*_\alpha}[g(\theta)]}{d\alpha} \right|_\alpha \Delta\alpha =: \hat{S}$$

← LRVB estimator



Robustness quantification

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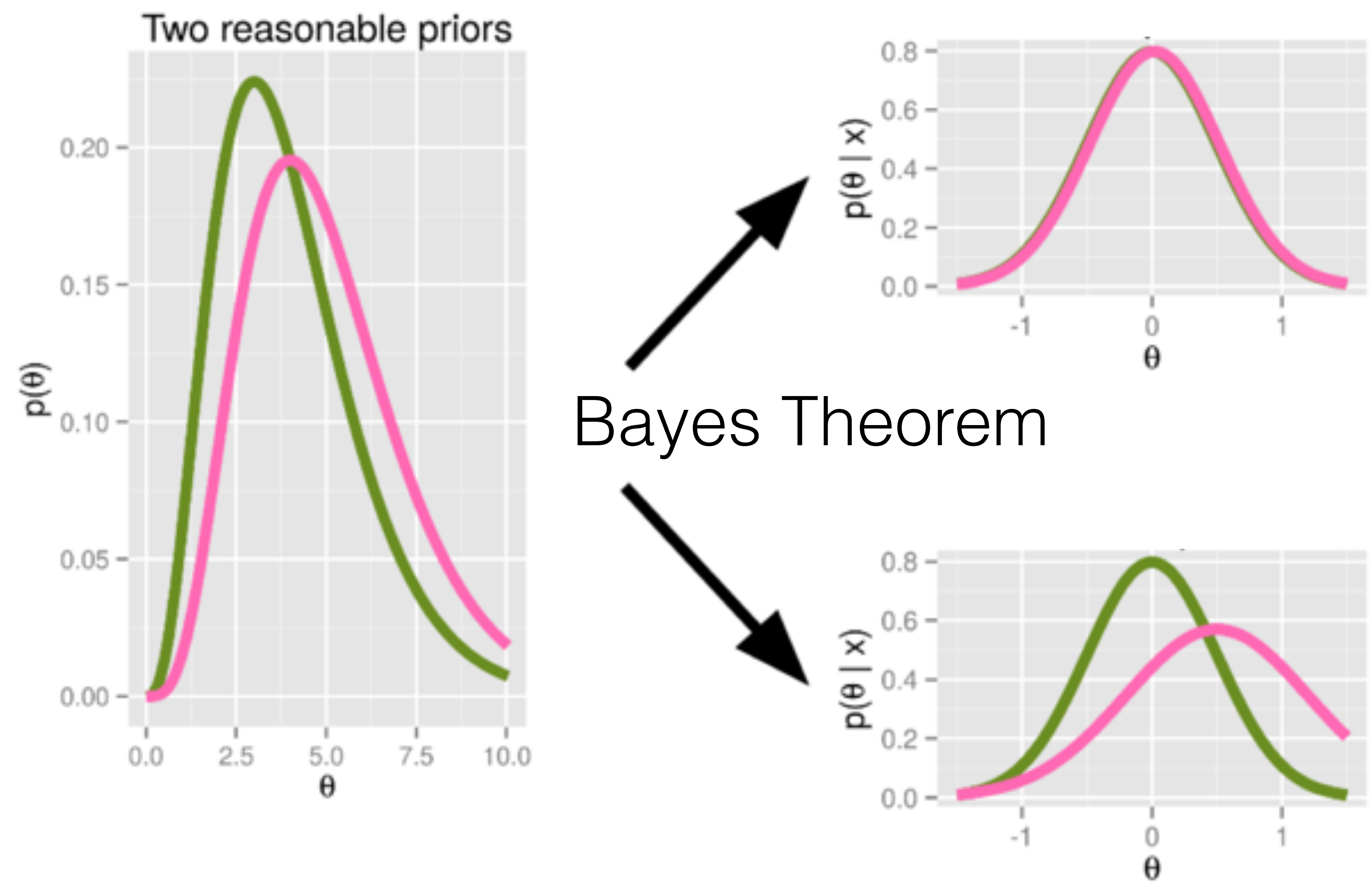

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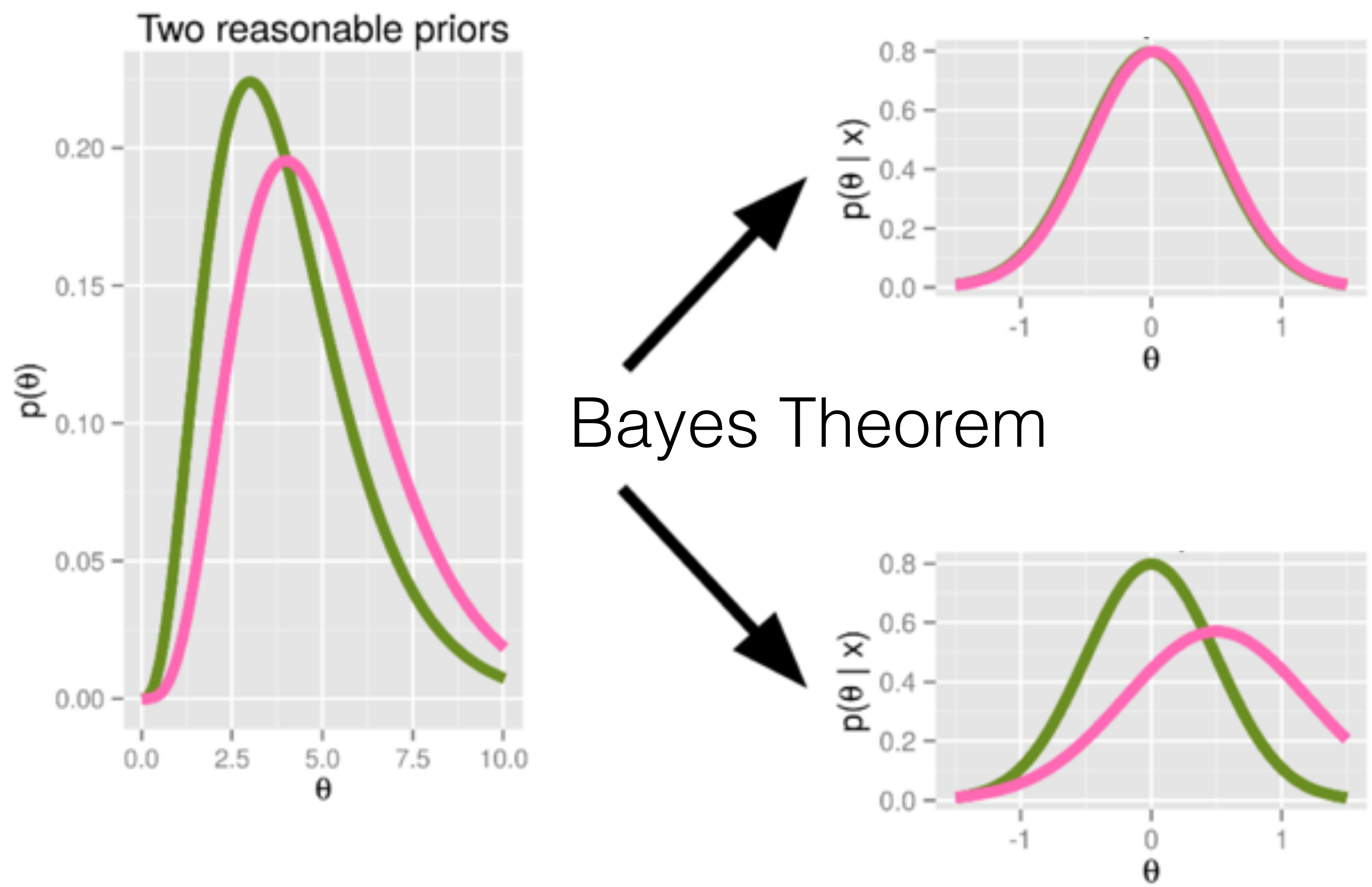

← LRVB estimator

- When q^*_α in exponential family

$$\hat{S} = A\left(\left.\frac{\partial^2 KL}{\partial m \partial m^T}\right|_{m=m^*}\right)^{-1} B$$

Microcredit Experiment

- Simplified from Meager (2015)
- *K* microcredit trials (Mexico, Mongolia, Bosnia, India, Morocco, Philippines, Ethiopia)
- N_k businesses in *k*th site (~900 to ~17K)
- Profit of *n*th business at *k*th site:

profit → 1 if microcredit →

$$y_{kn} \overset{indep}{\sim} \mathcal{N}(\mu_k + T_{kn}\tau_k, \sigma_k^2)$$

- Priors and hyperpriors:

$$\begin{pmatrix} \mu_k \\ \tau_k \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu \\ \tau \end{pmatrix}, C\right) \qquad \begin{pmatrix} \mu \\ \tau \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu_0 \\ \tau_0 \end{pmatrix}, \Lambda^{-1}\right)$$

$$\sigma_k^{-2} \overset{iid}{\sim} \Gamma(a, b) \qquad C \sim \mathrm{Sep\&LKJ}(\eta, c, d)$$

Microcredit Experiment

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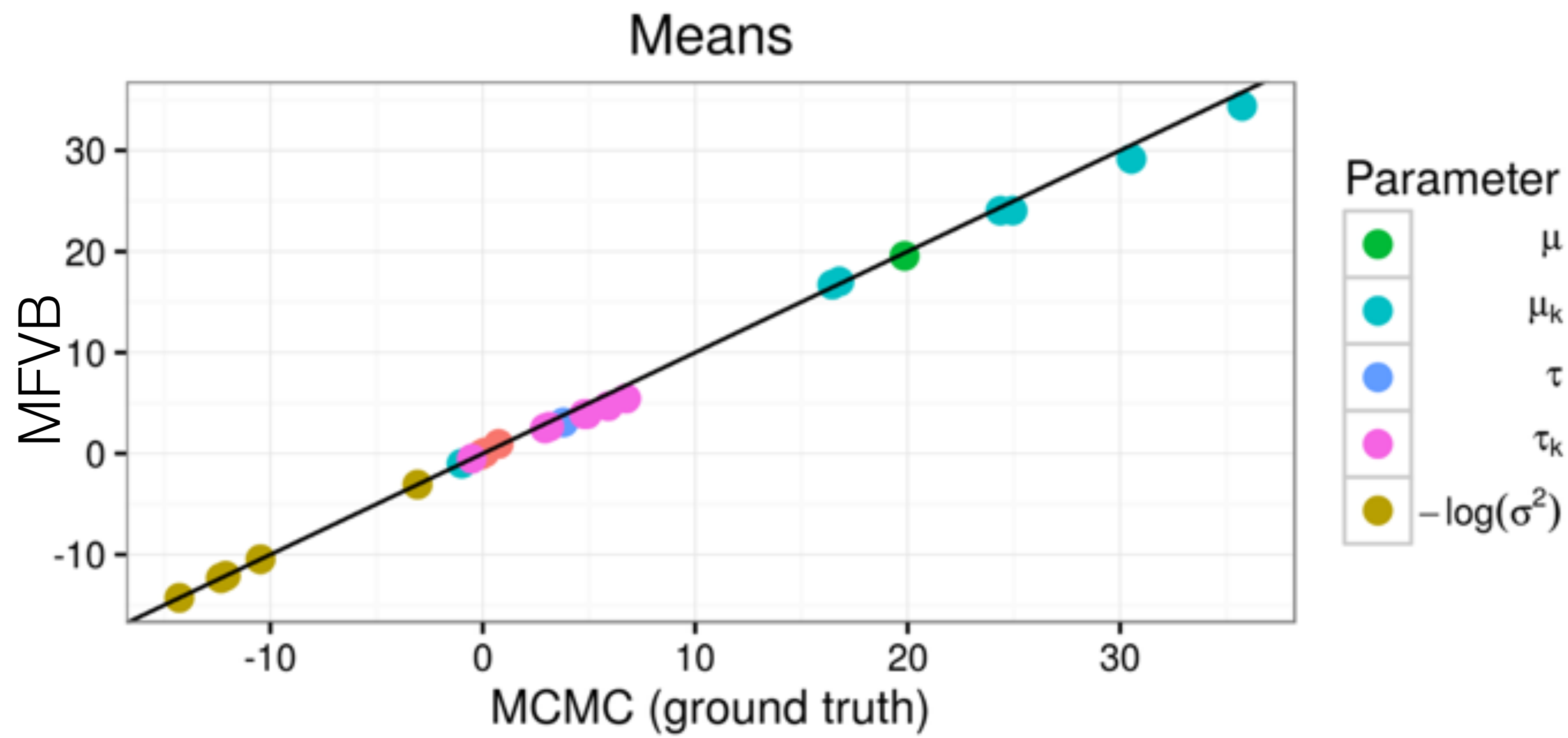

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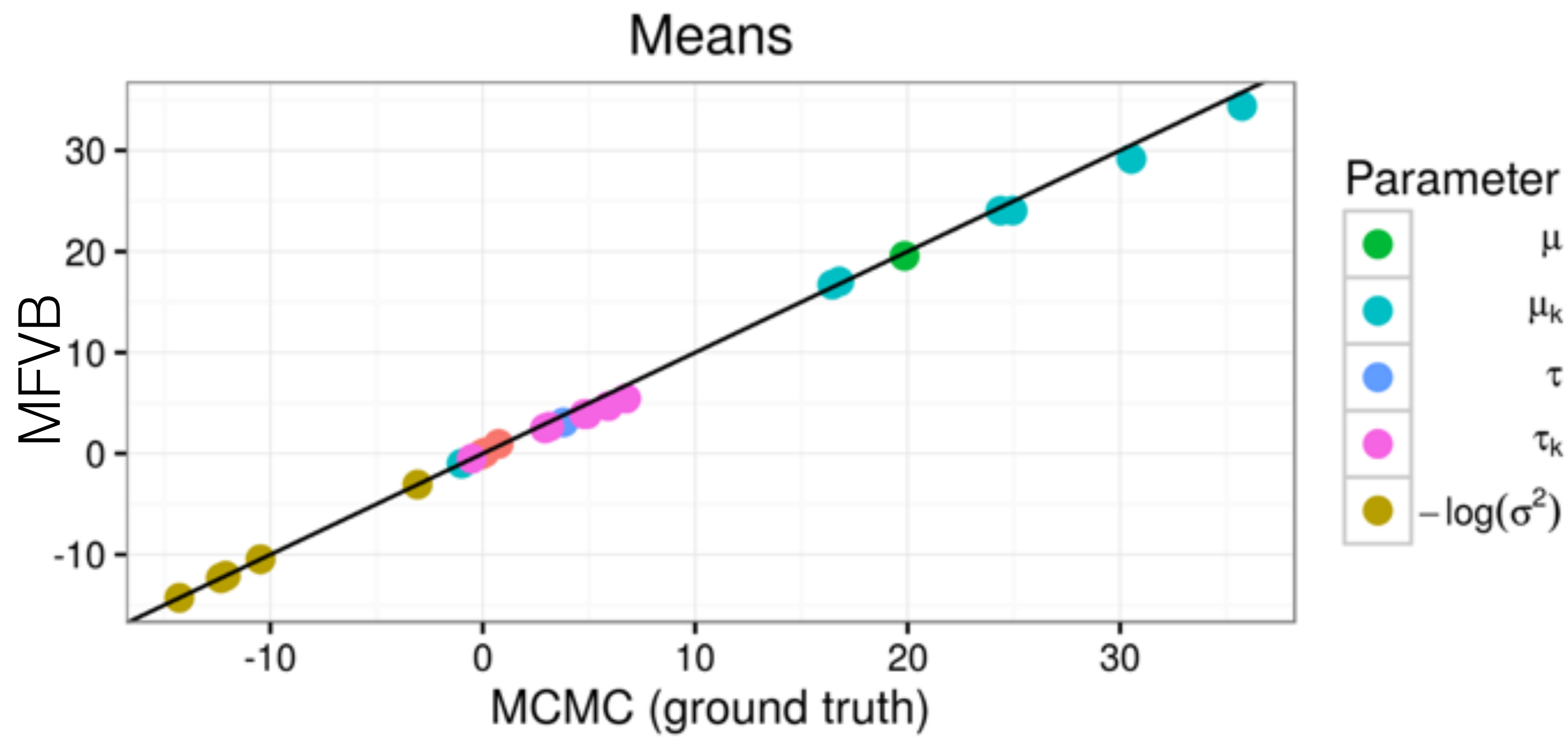

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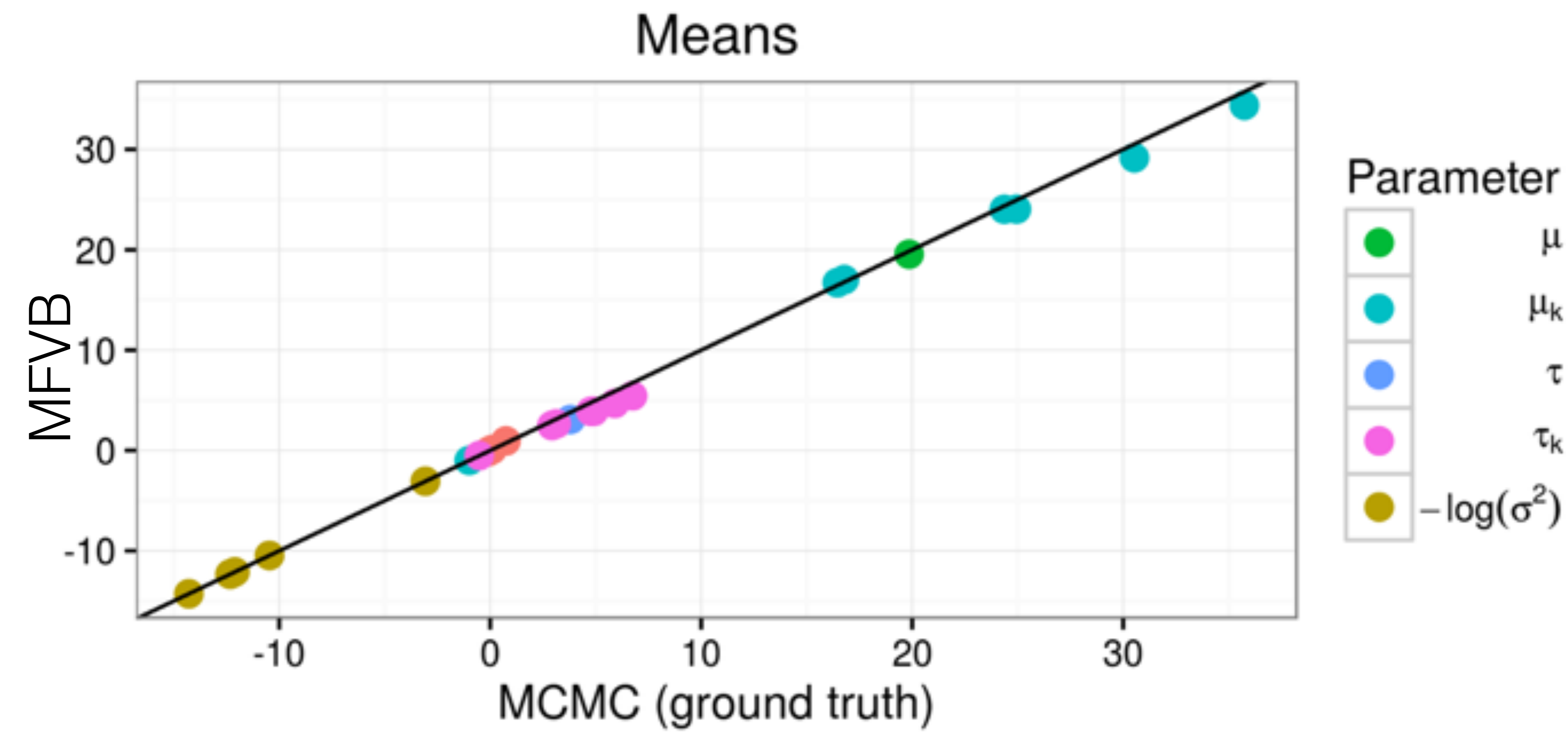

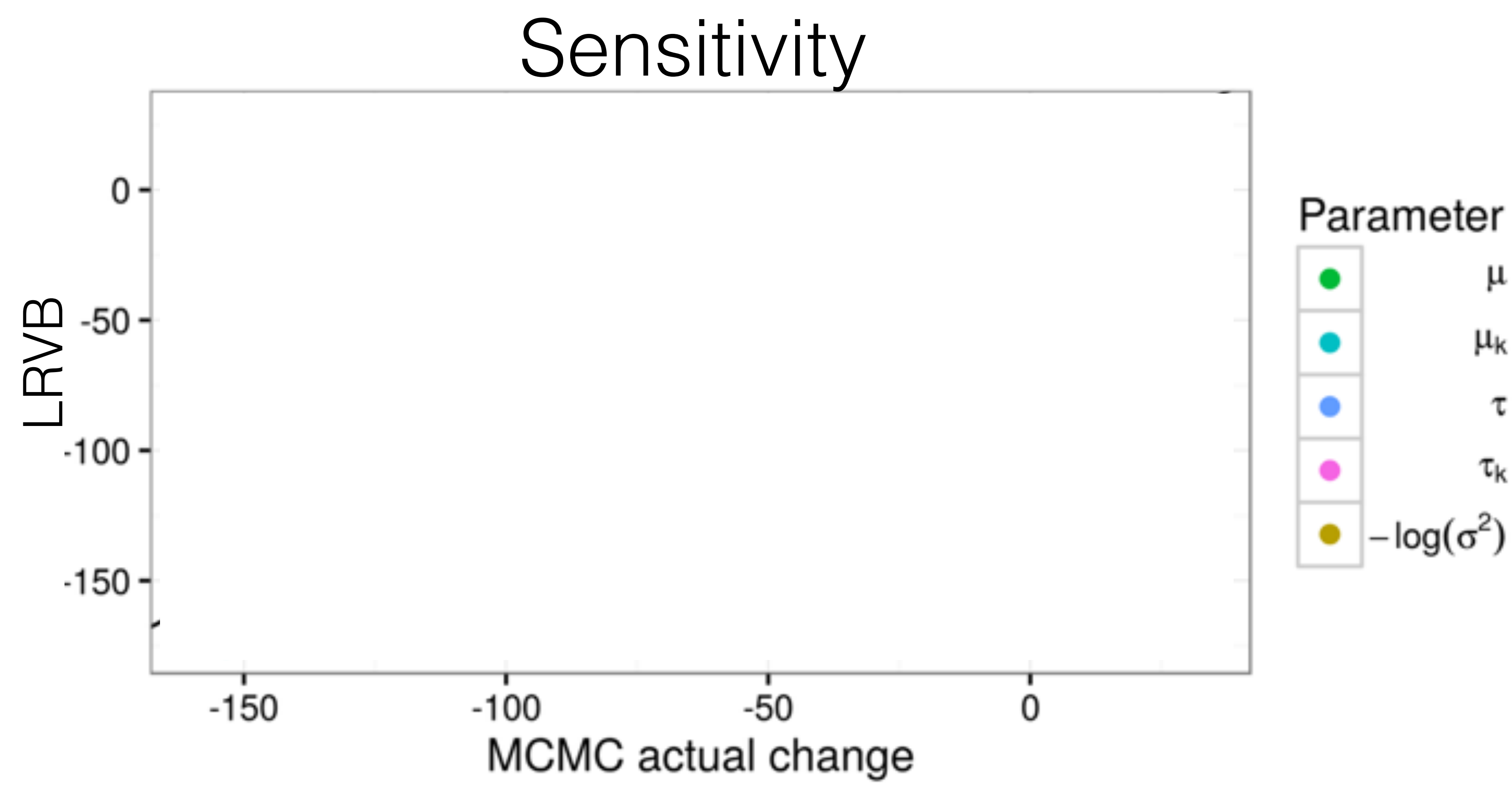

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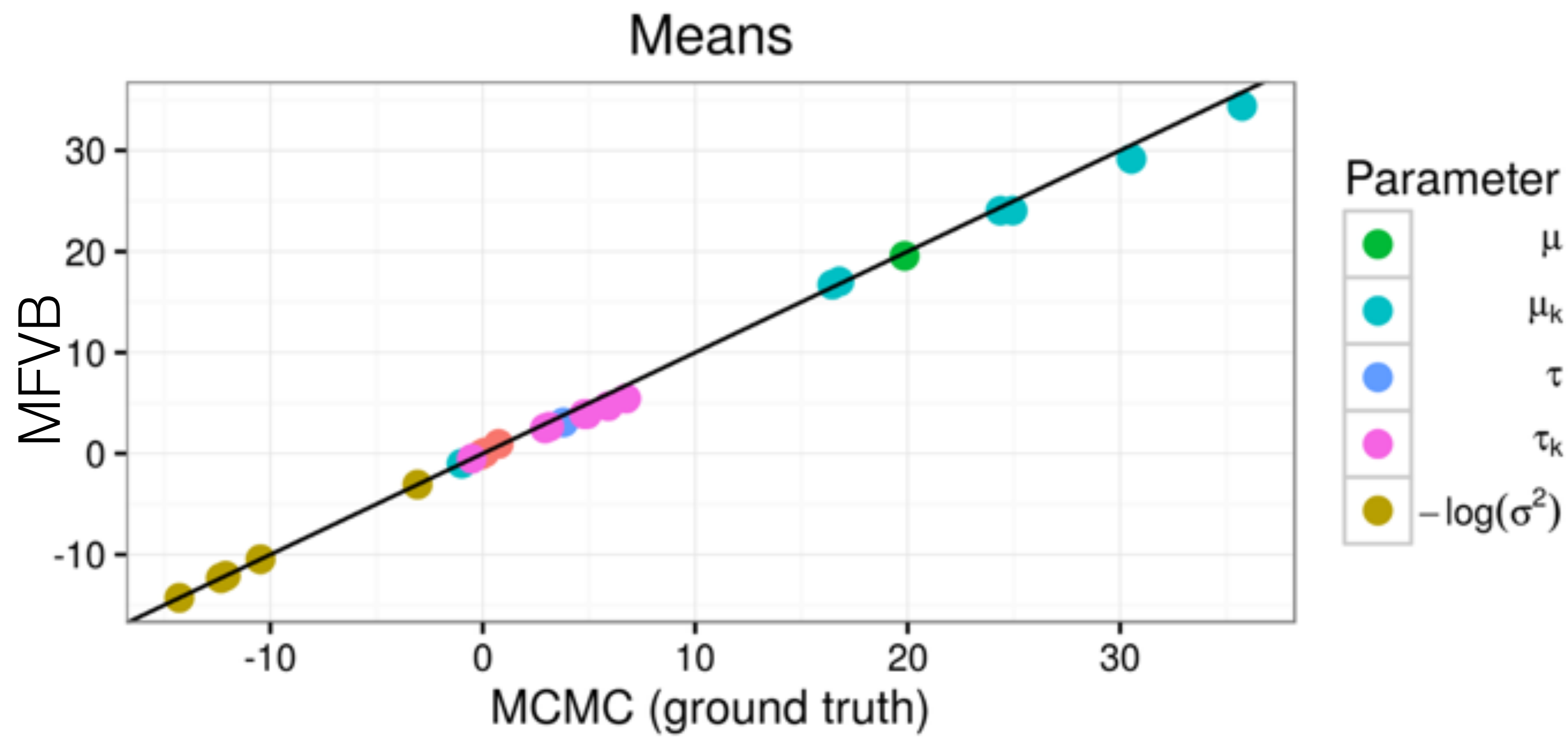

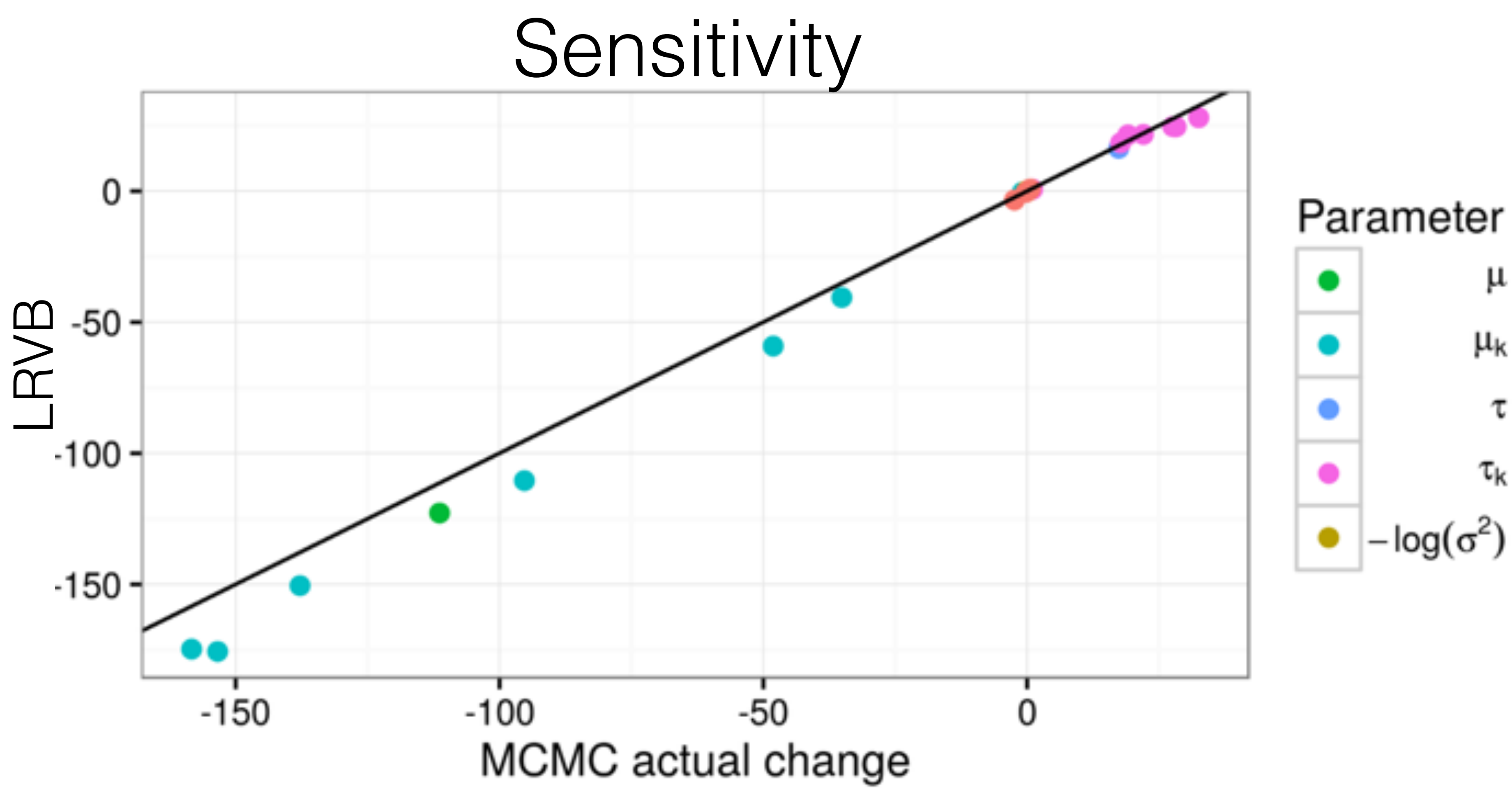

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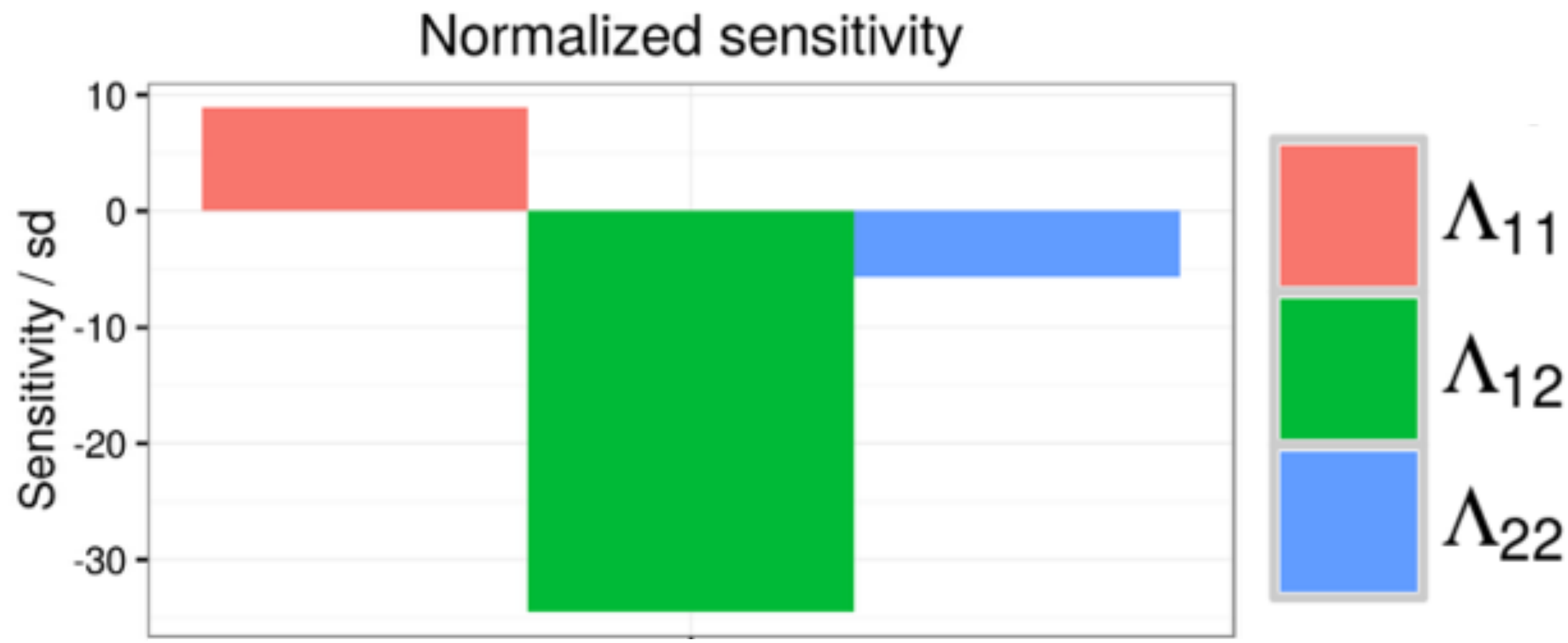

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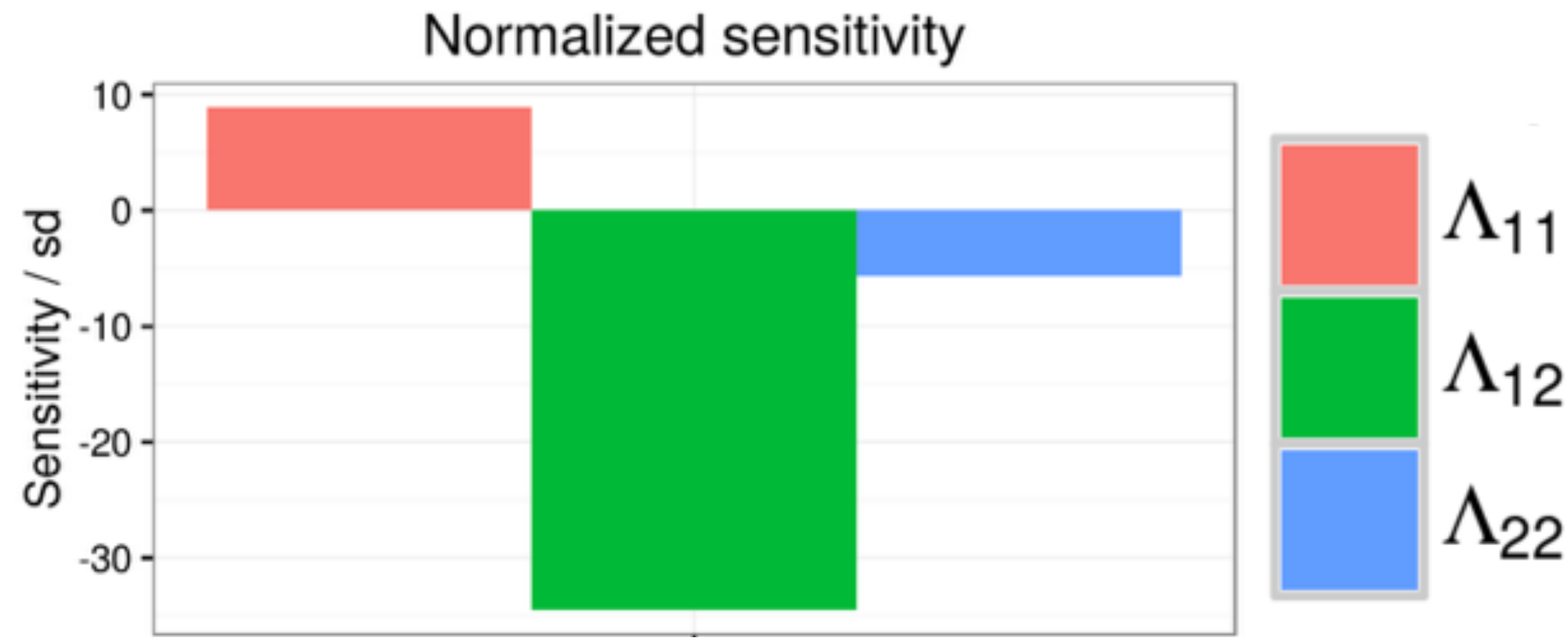
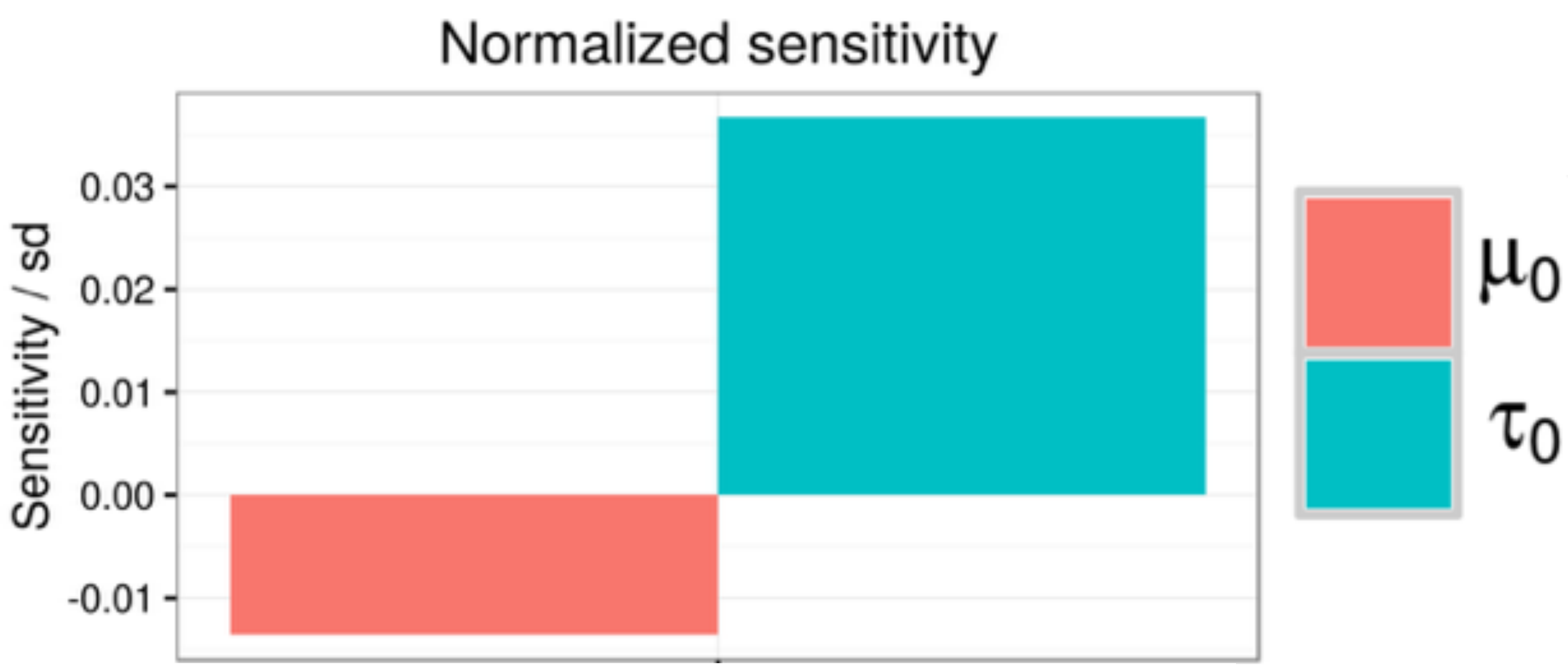
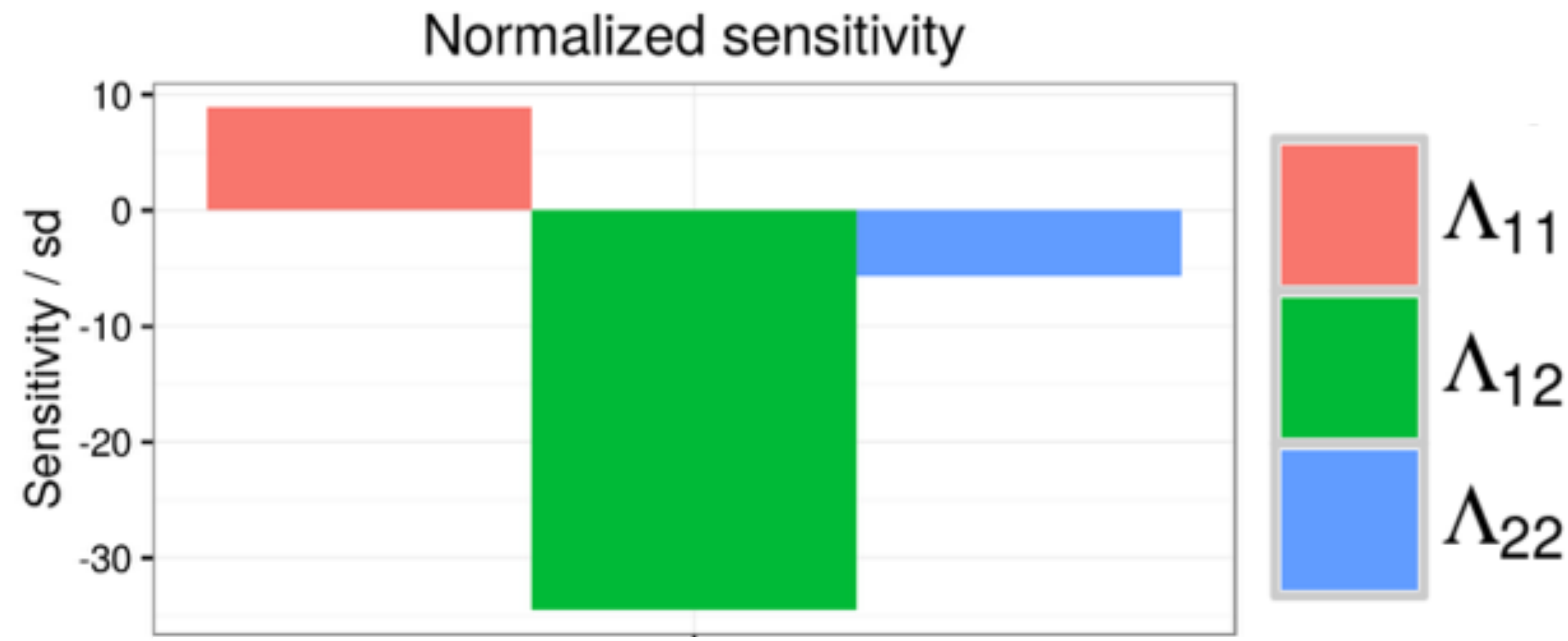
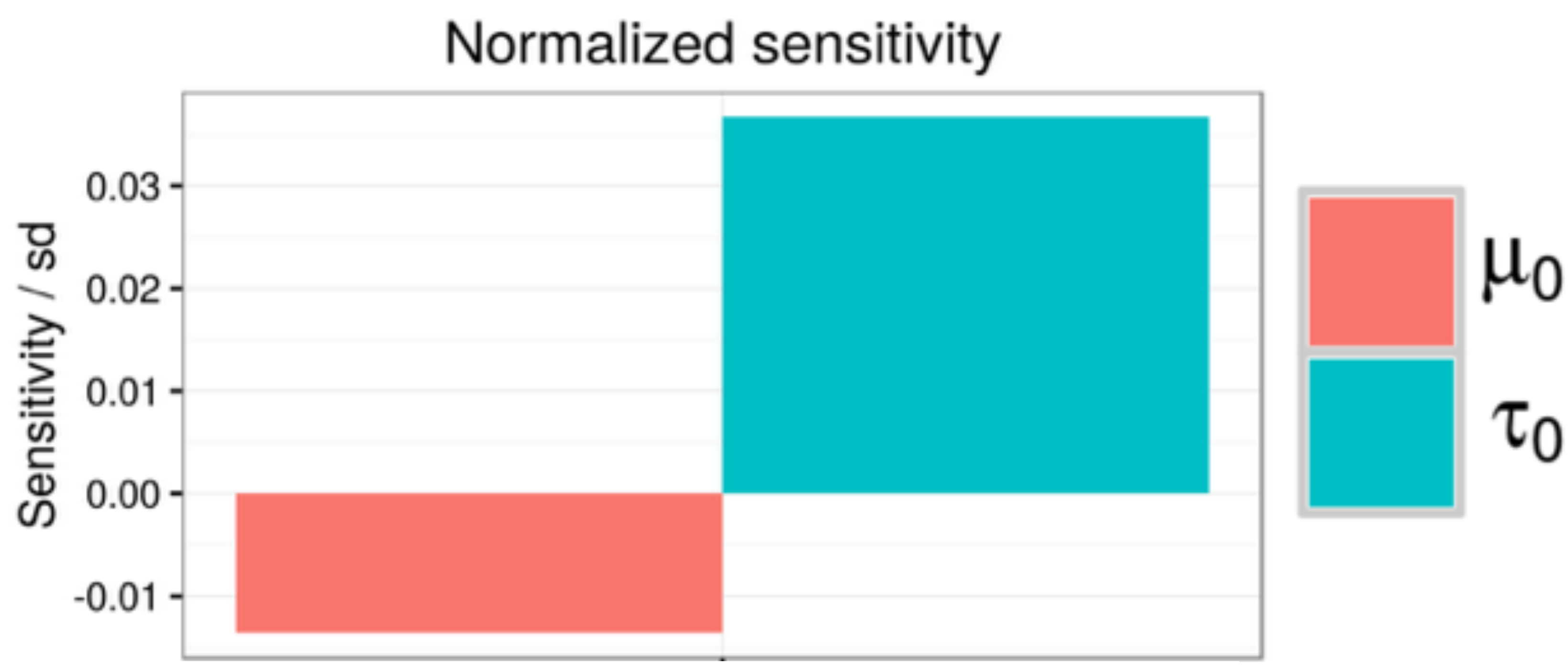




Microcredit Experiment

- Sensitivity of the expected microcredit effect (τ)
- Normalized to be on scale of τ std devs
- τ mean (MFVB): 3.08 USD PPP
- τ std dev (LRVB): 1.83 USD PPP
- Mean is 1.68 std dev from 0





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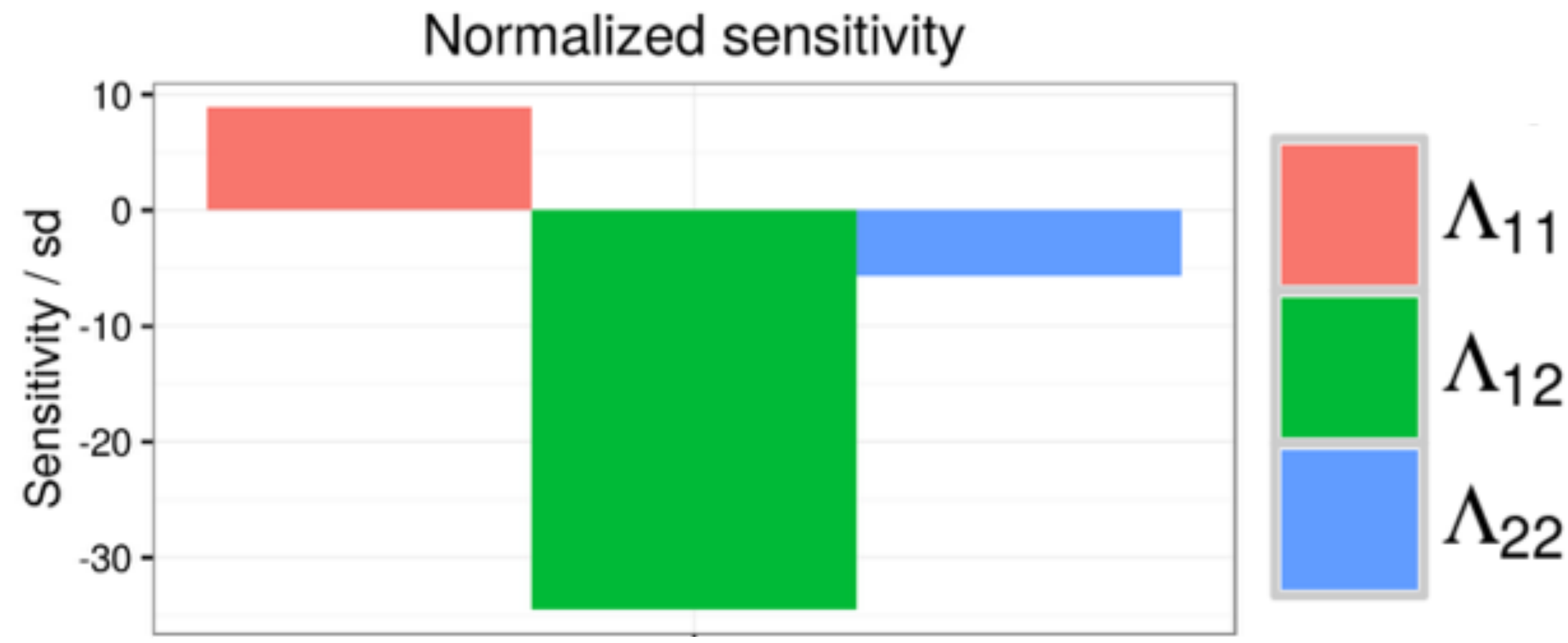

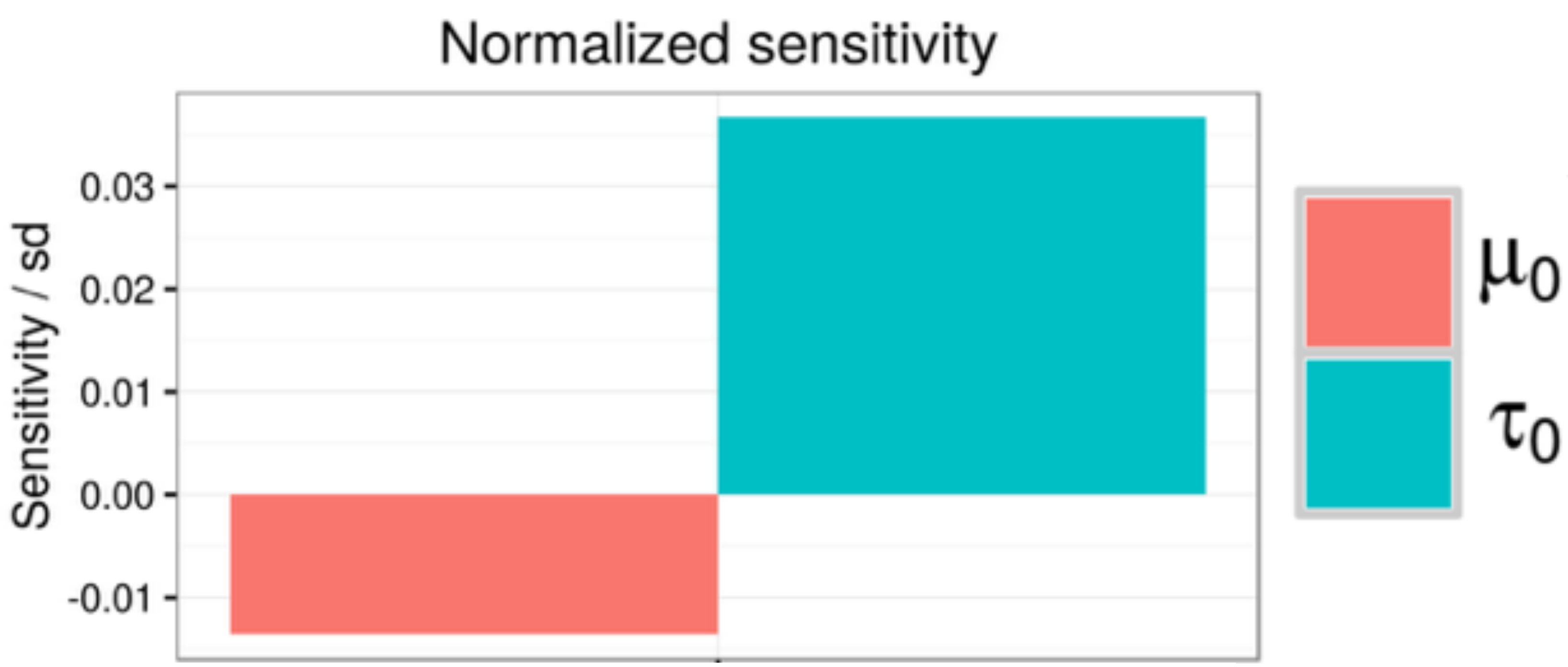

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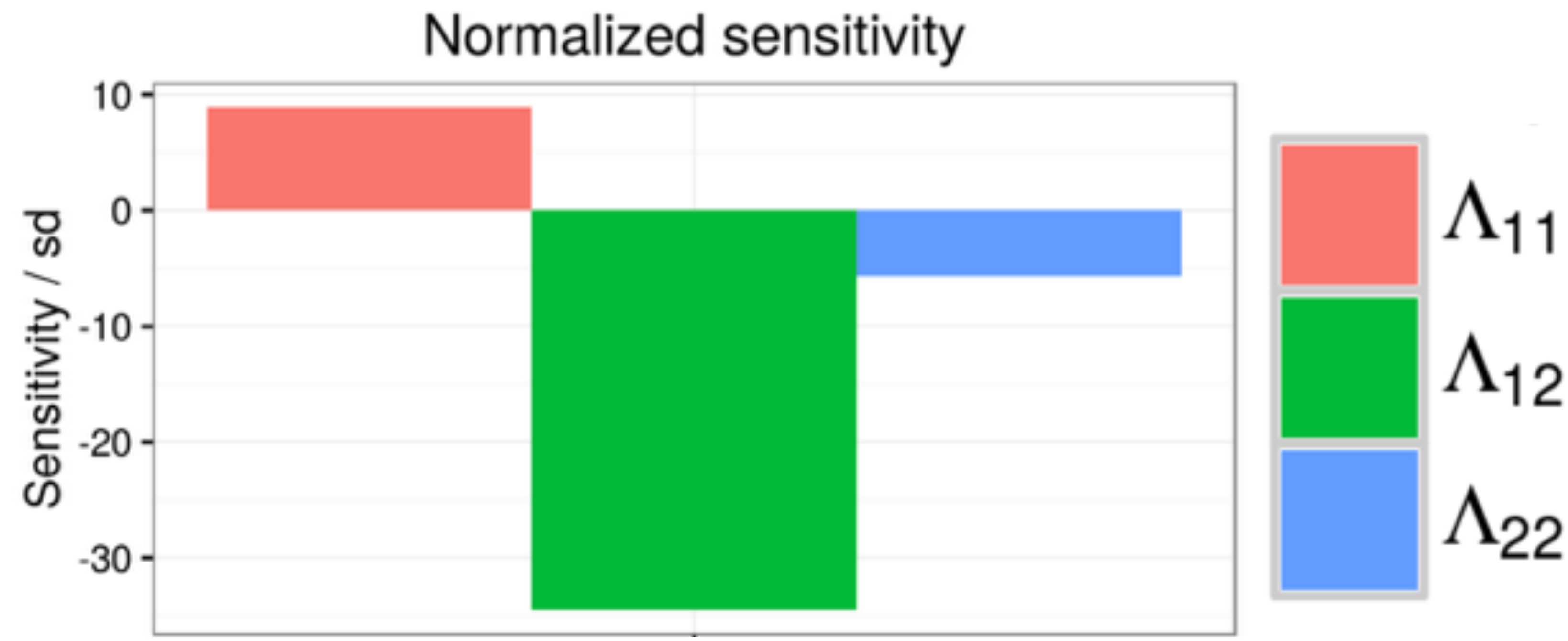
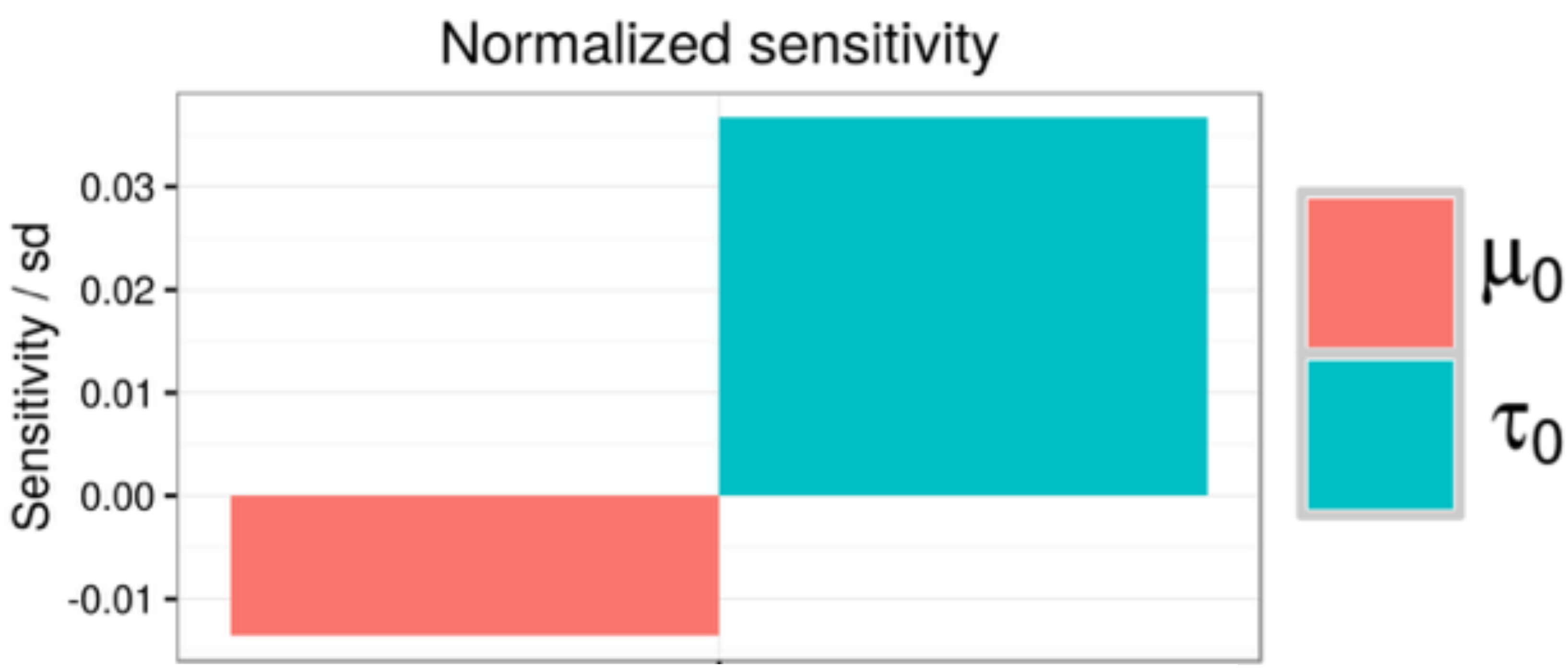
$\Rightarrow$ Mean > 2 std dev





Conclusions

- We provide *linear response variational Bayes*: supplements MFVB for fast & accurate **covariance** estimate
- More from LRVB: fast & accurate **robustness** quantification
- Interested in your data and models:
  - Sensitivity to prior perturbations
  - Sensitivity to likelihood, data perturbations

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