MIT EECS ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

MIT STATISTICS + DATA SCIENCE CENTER

CSAIL

Automated Scalable Bayesian Inference via Data Summarization

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ITT Career Development
Assistant Professor,
MIT

With: Trevor Campbell, Jonathan H. Huggins

“Core” of the data set

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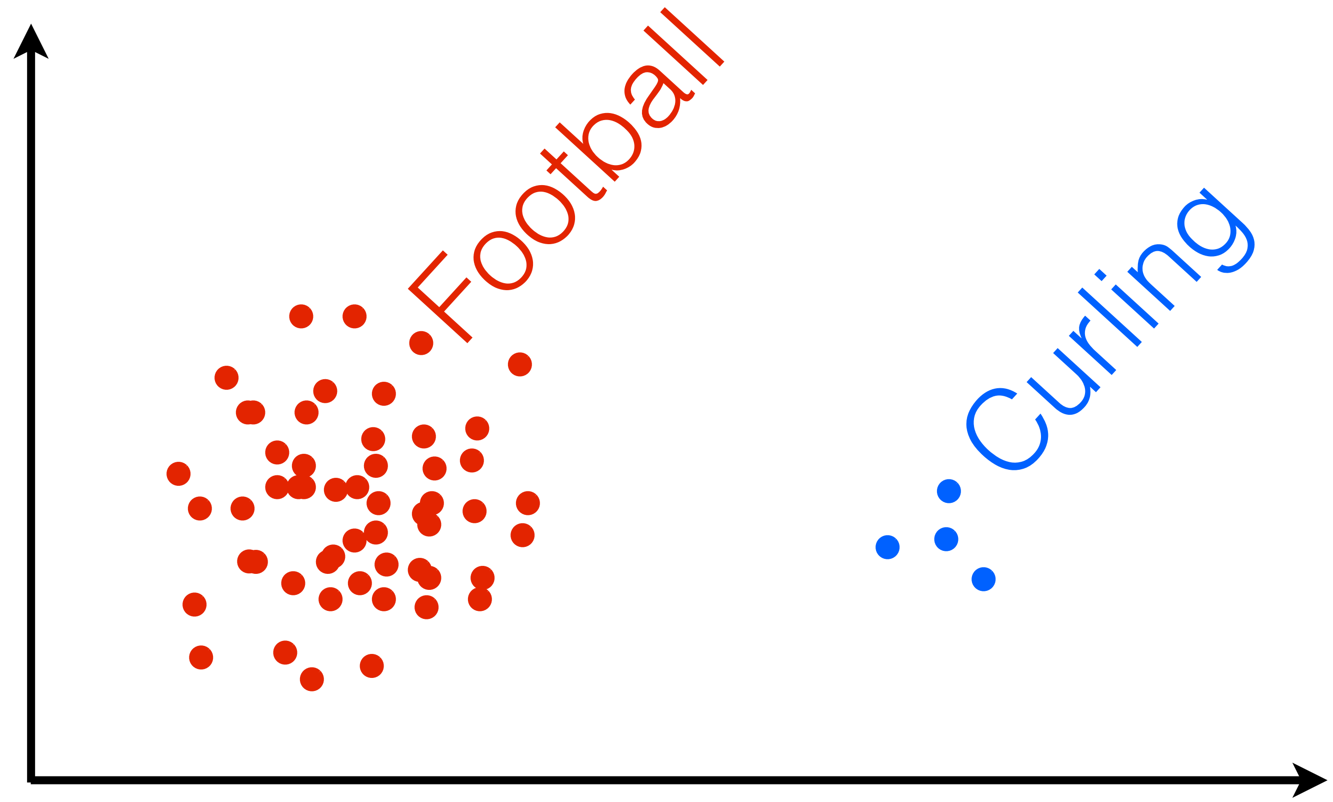

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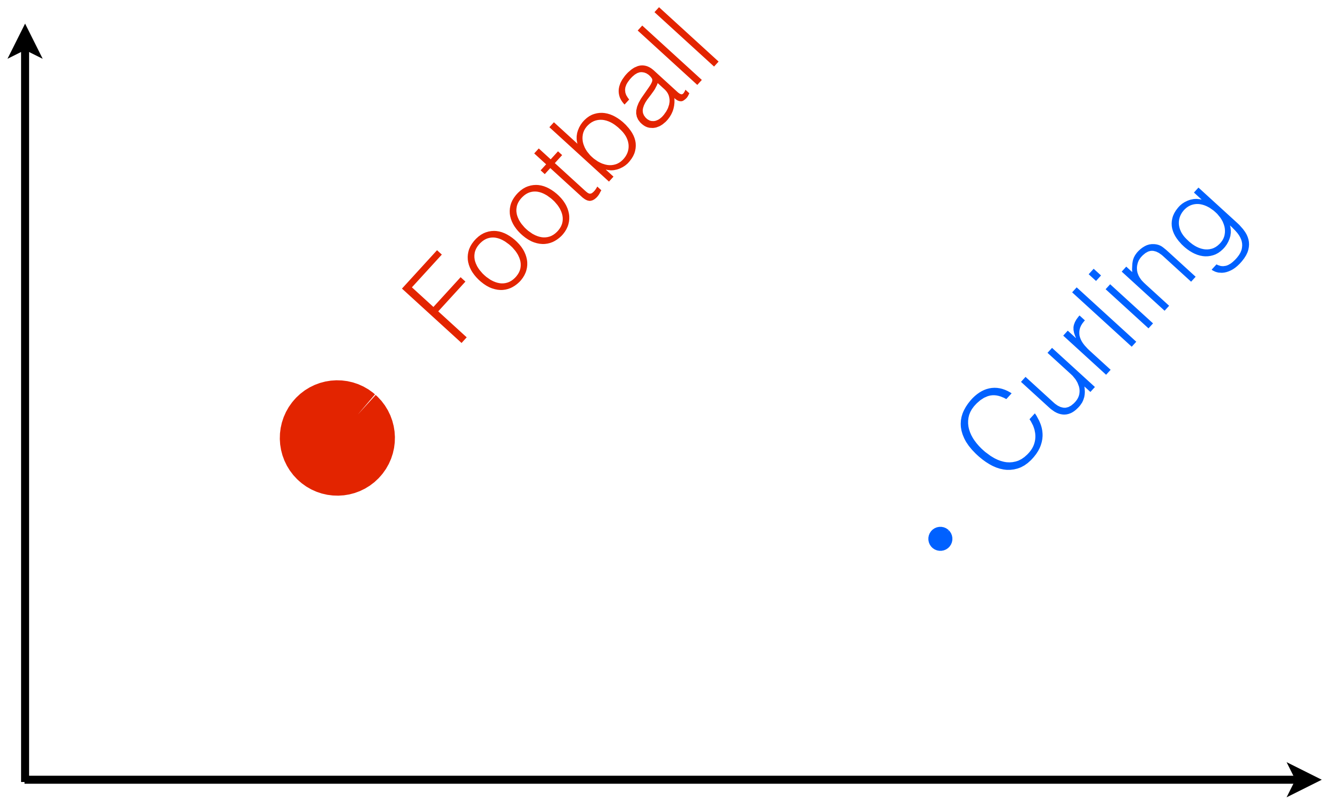

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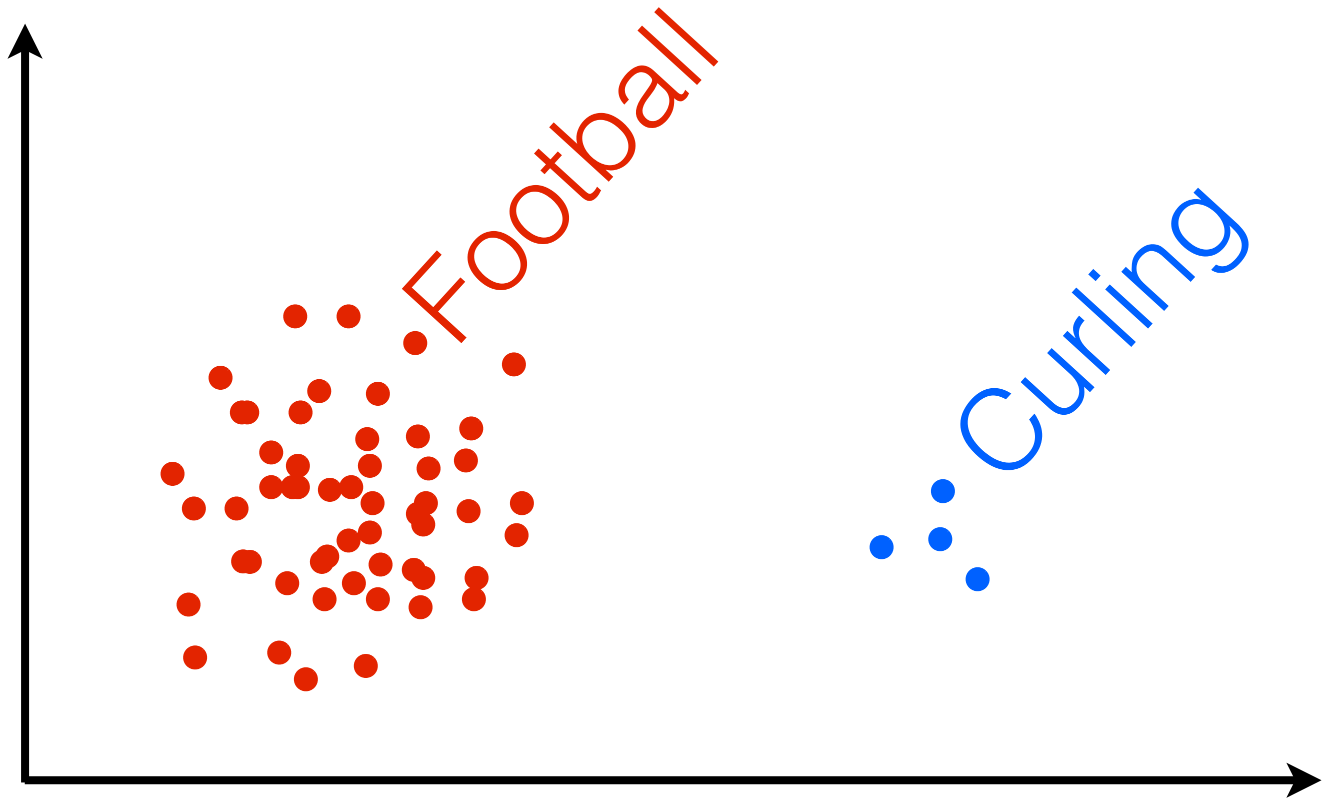

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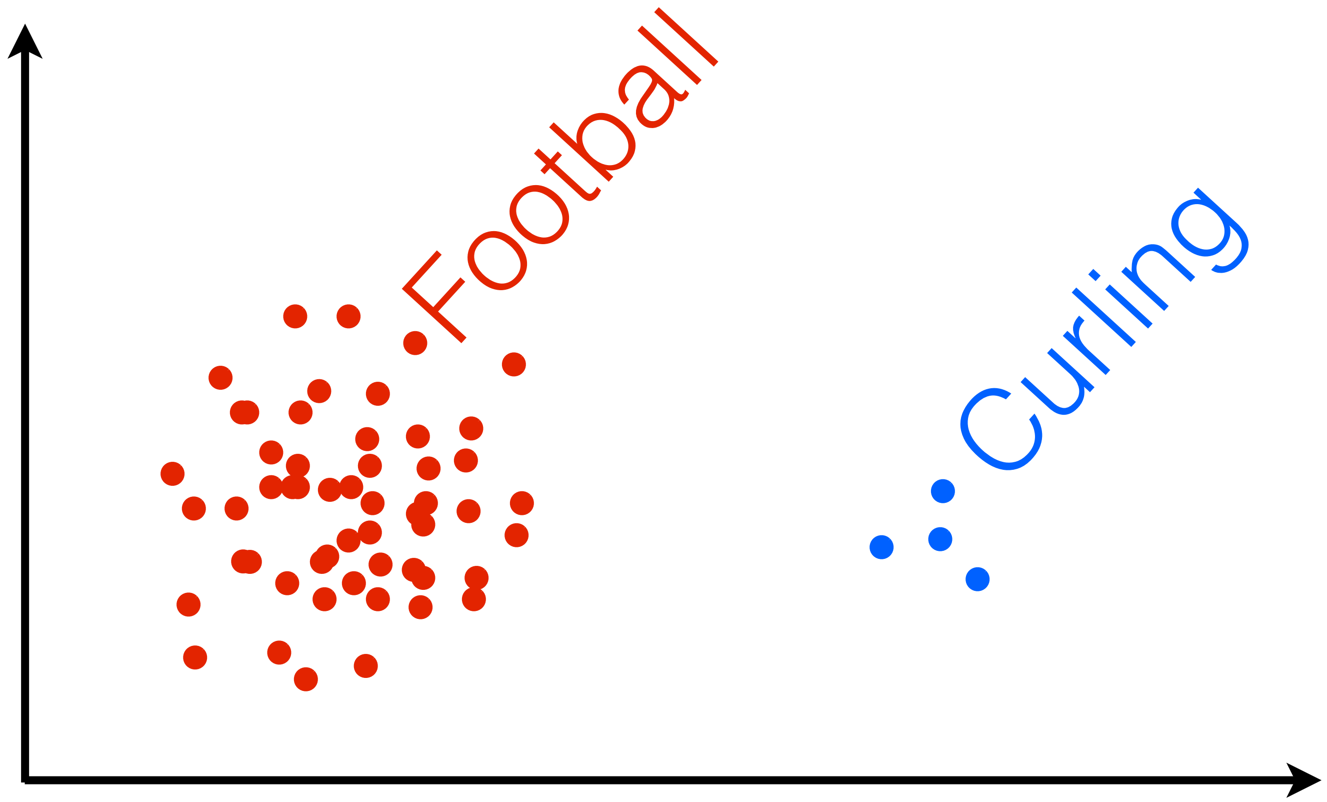

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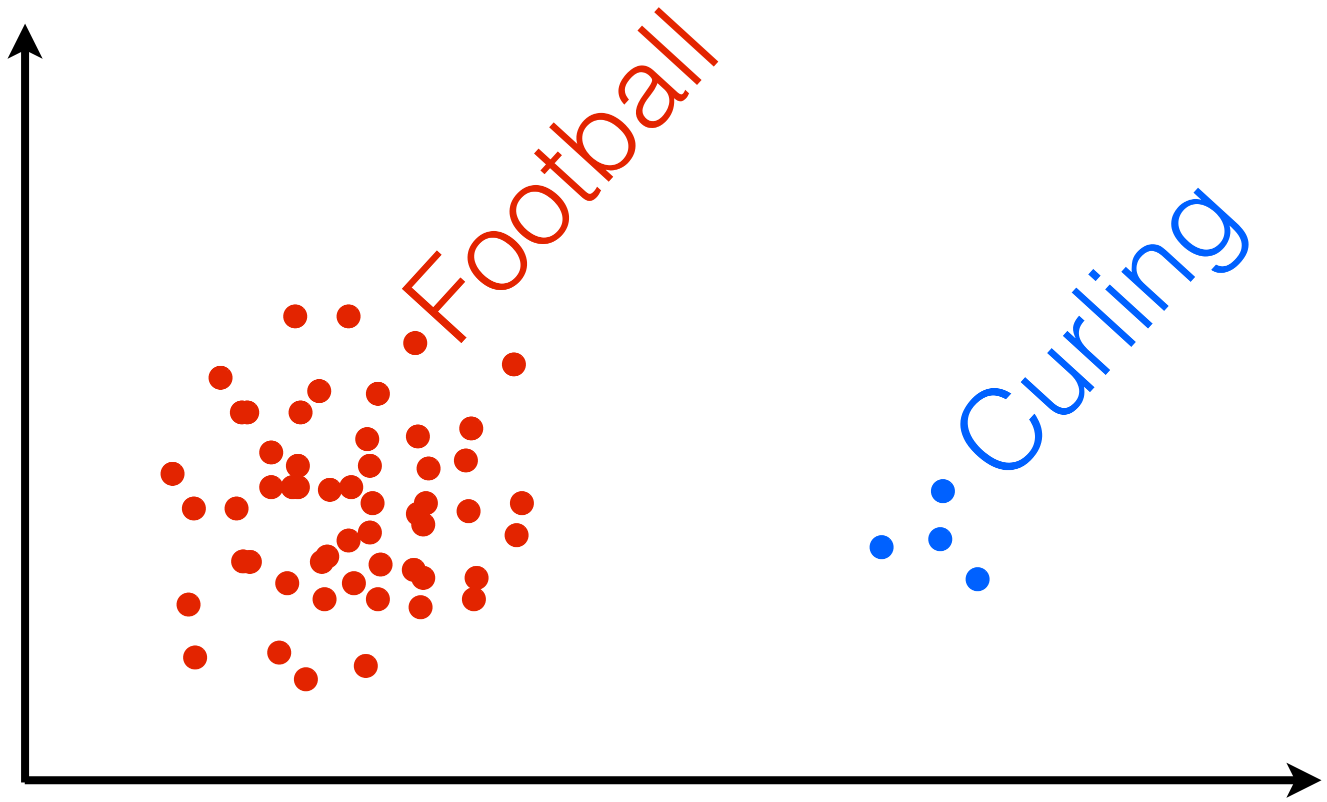

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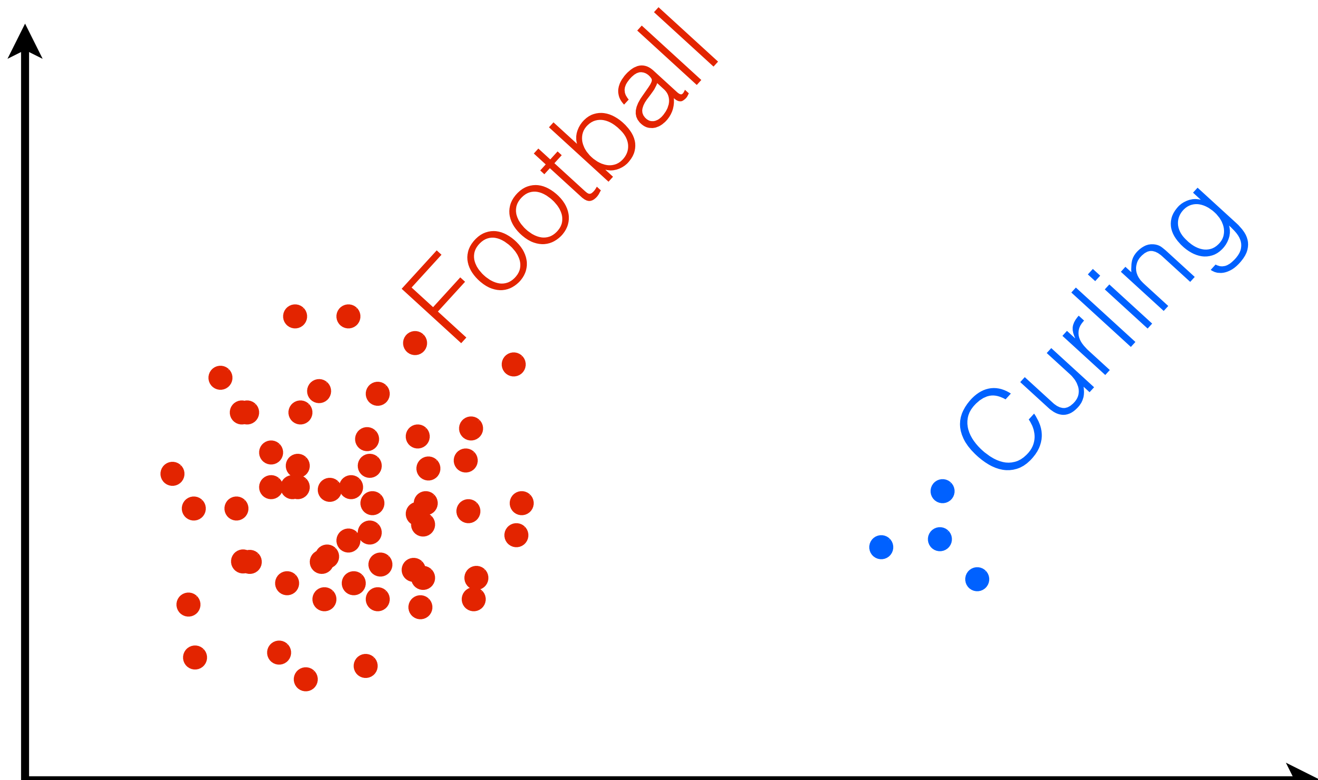

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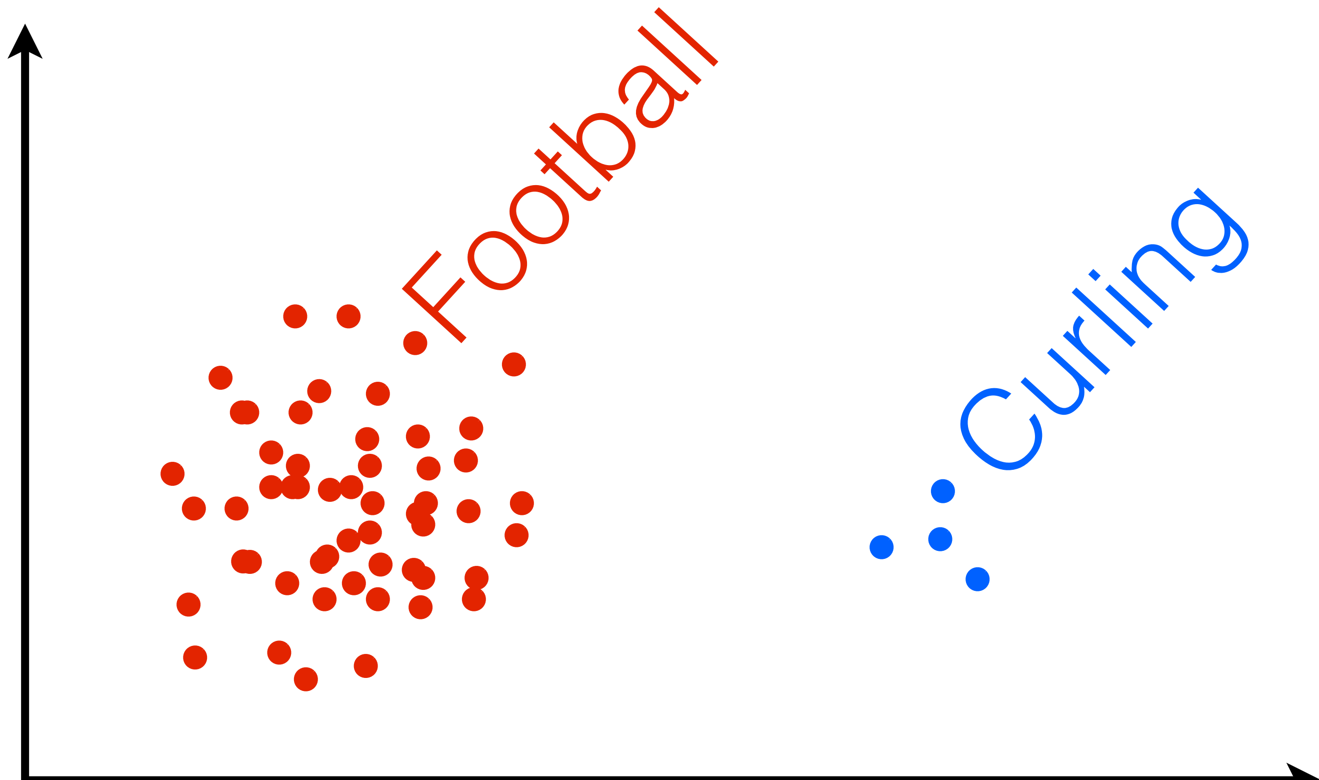

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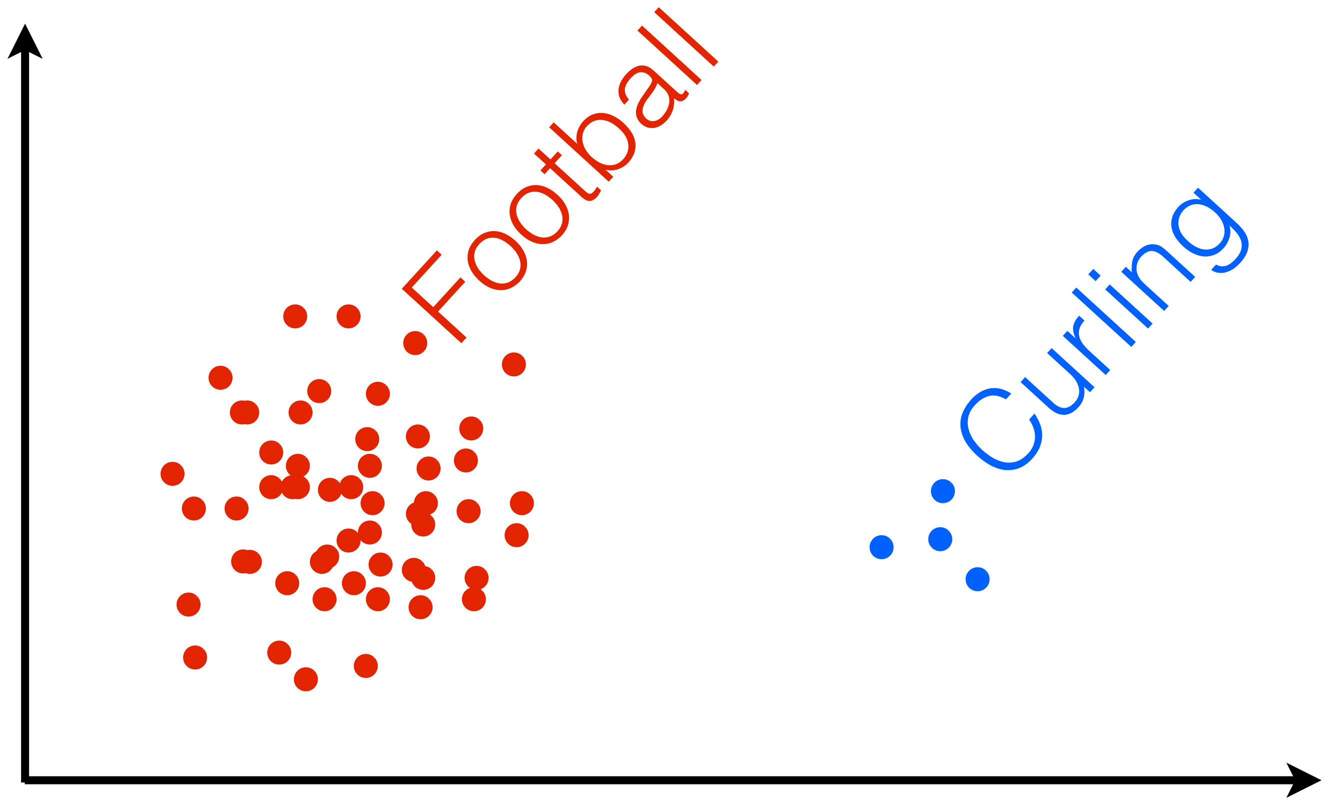

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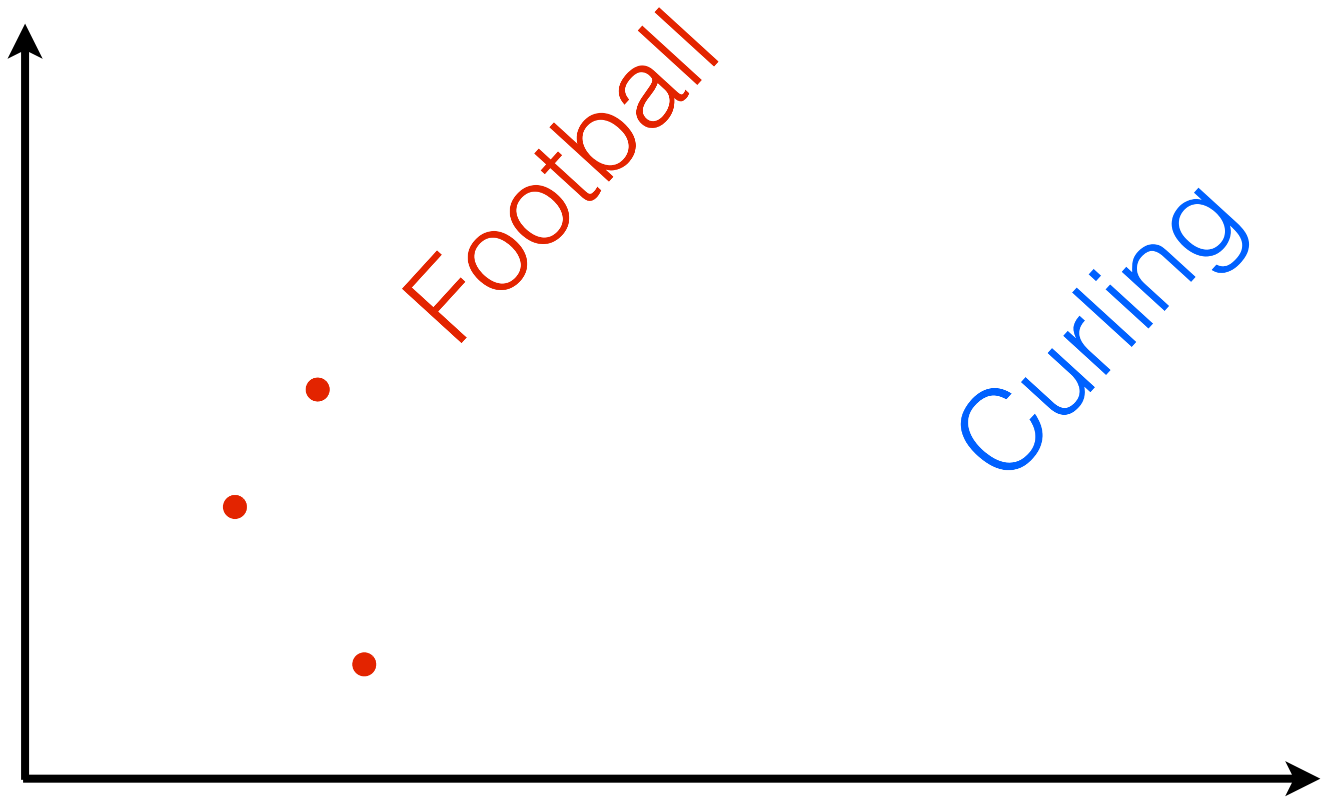

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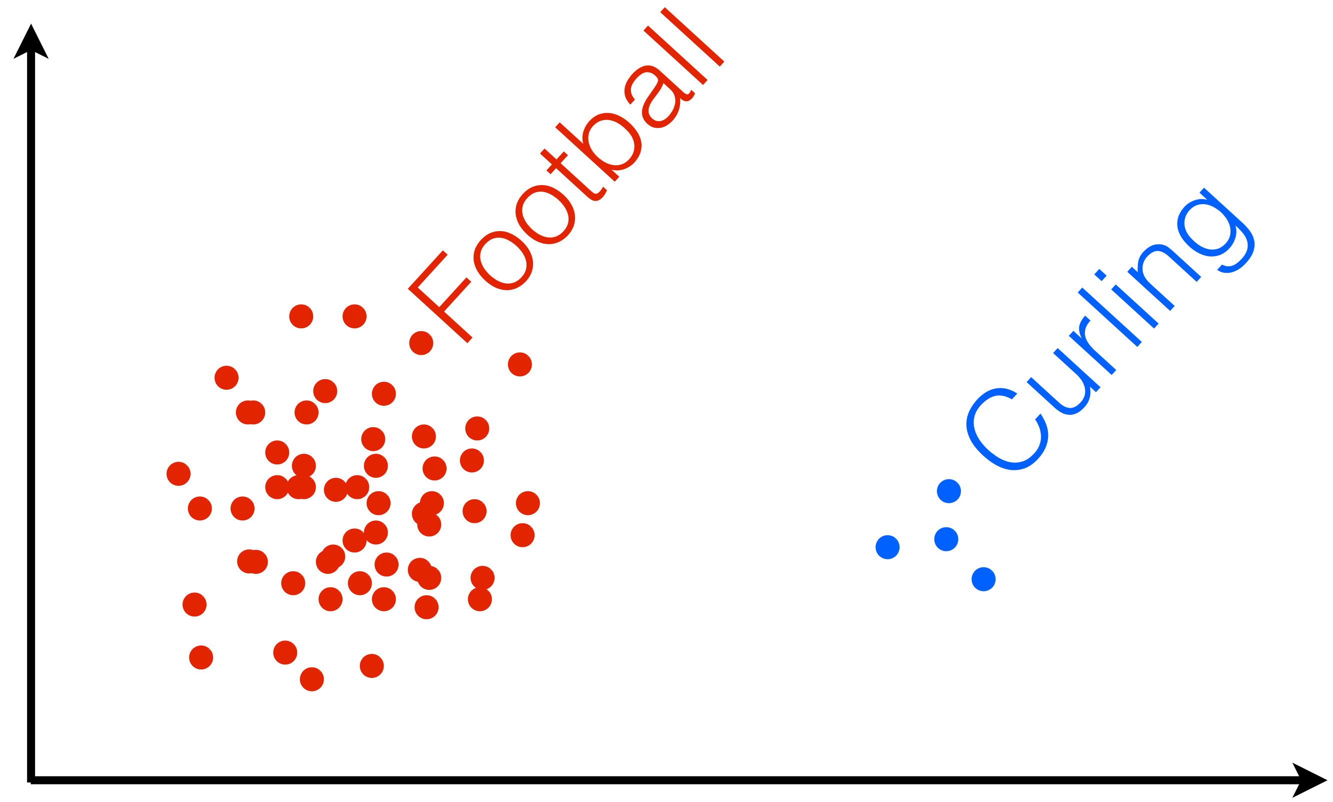

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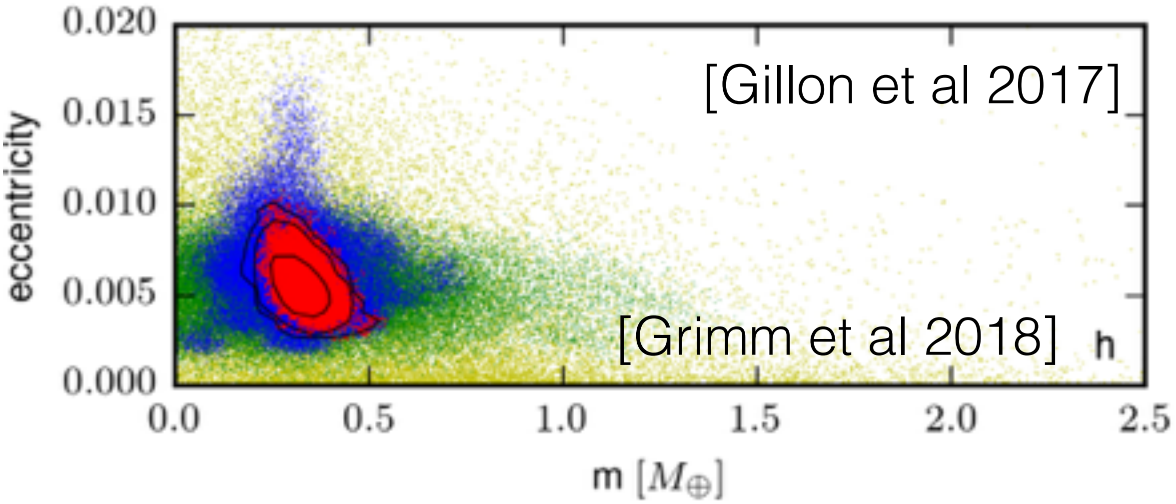

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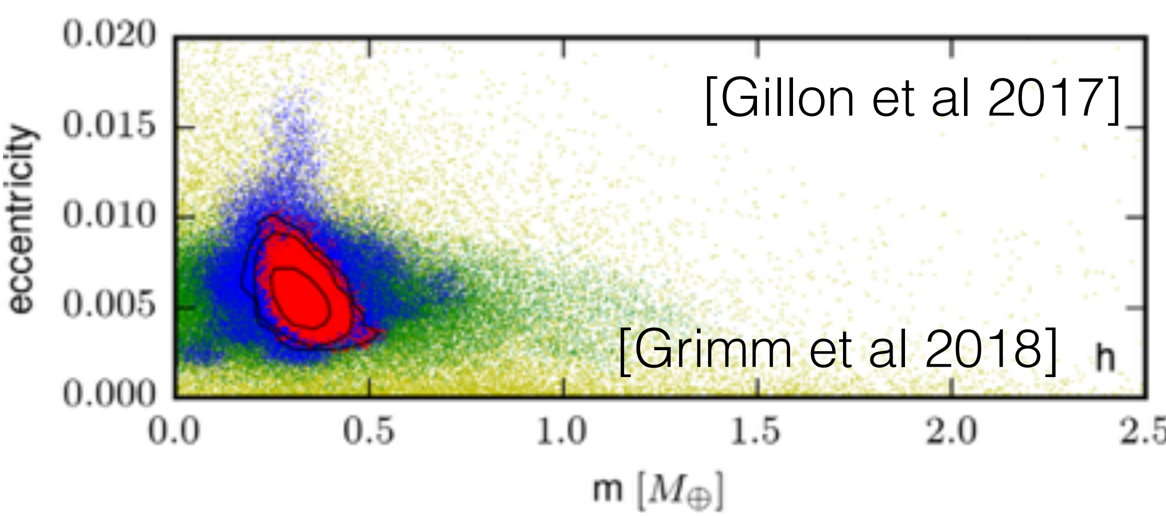

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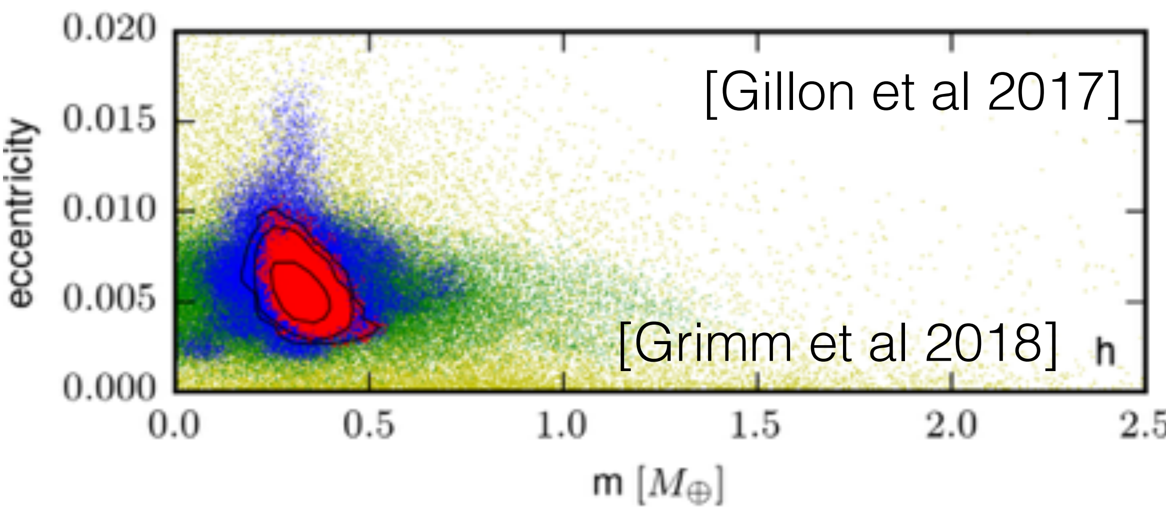

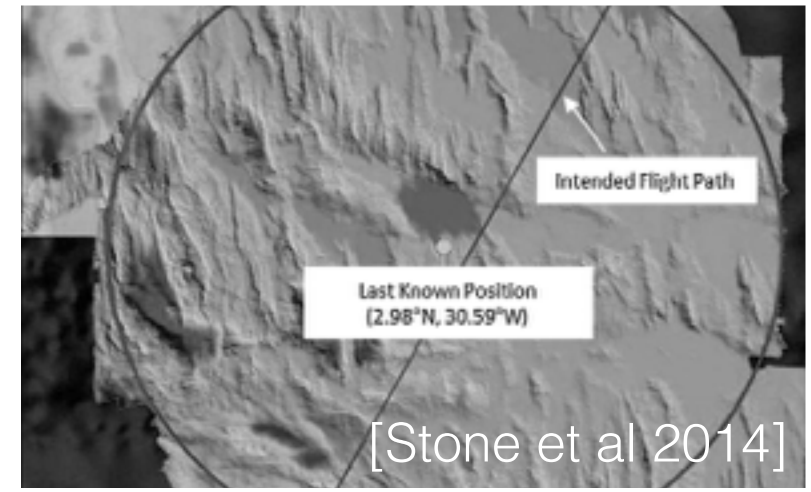

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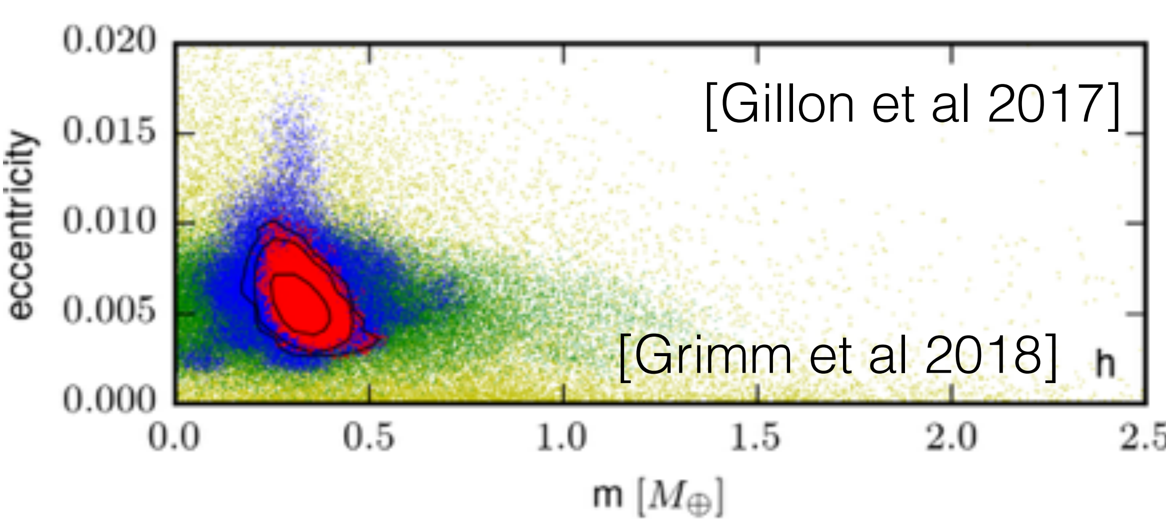

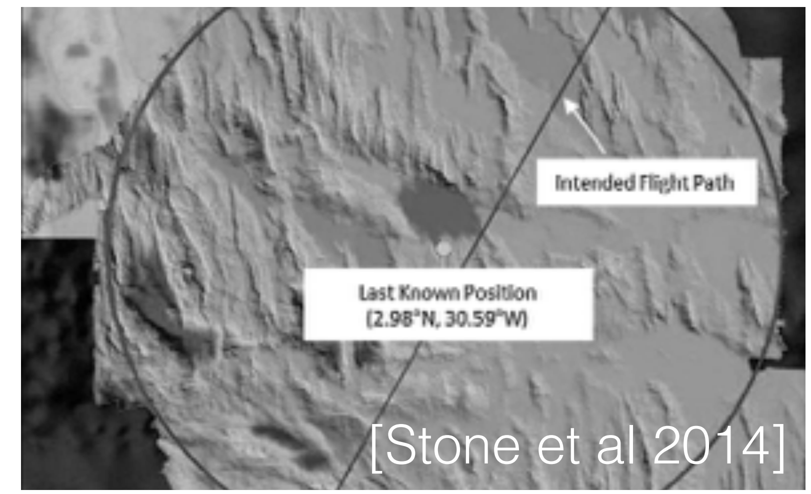

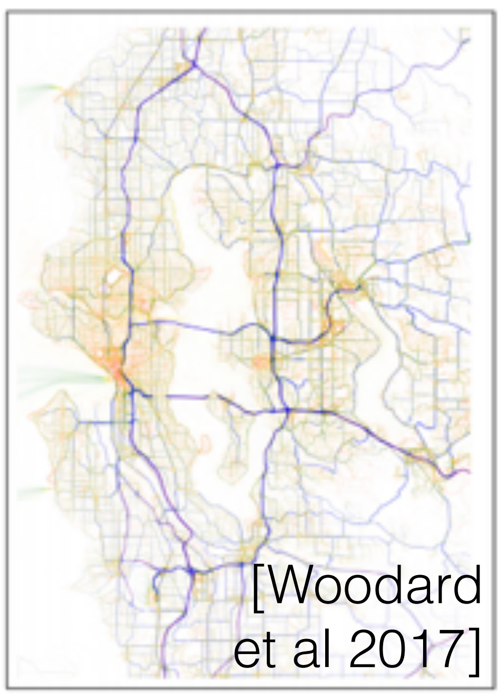

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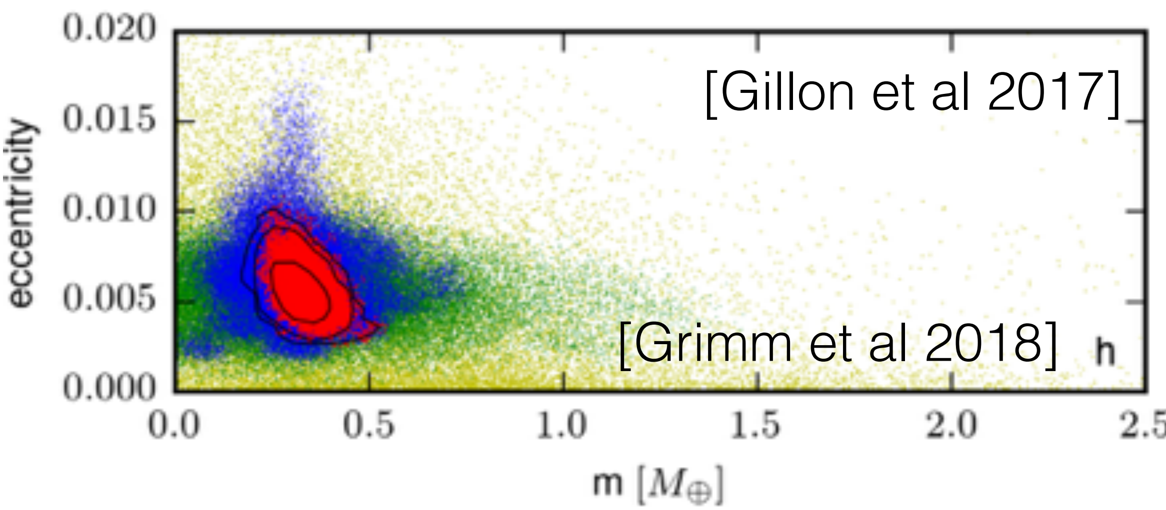

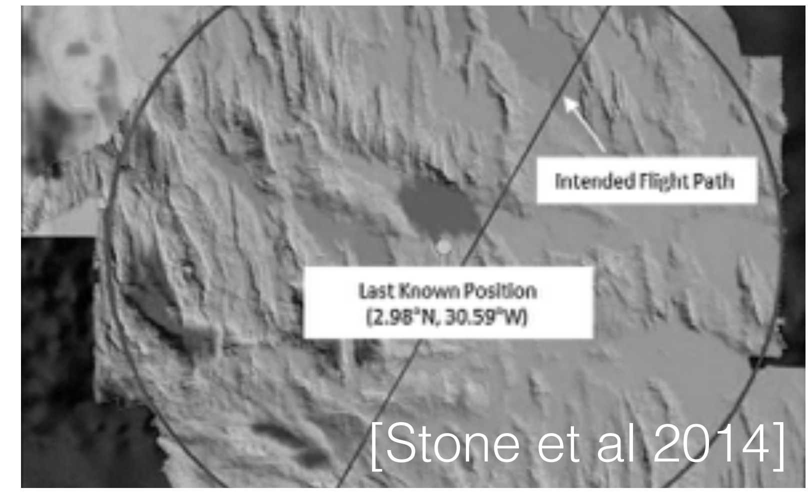

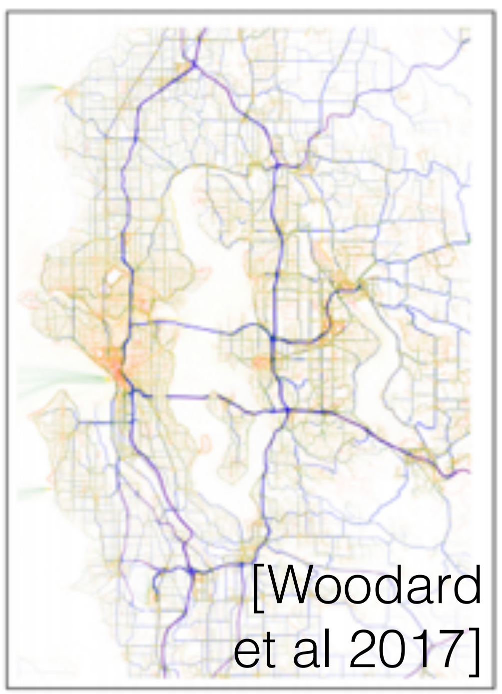

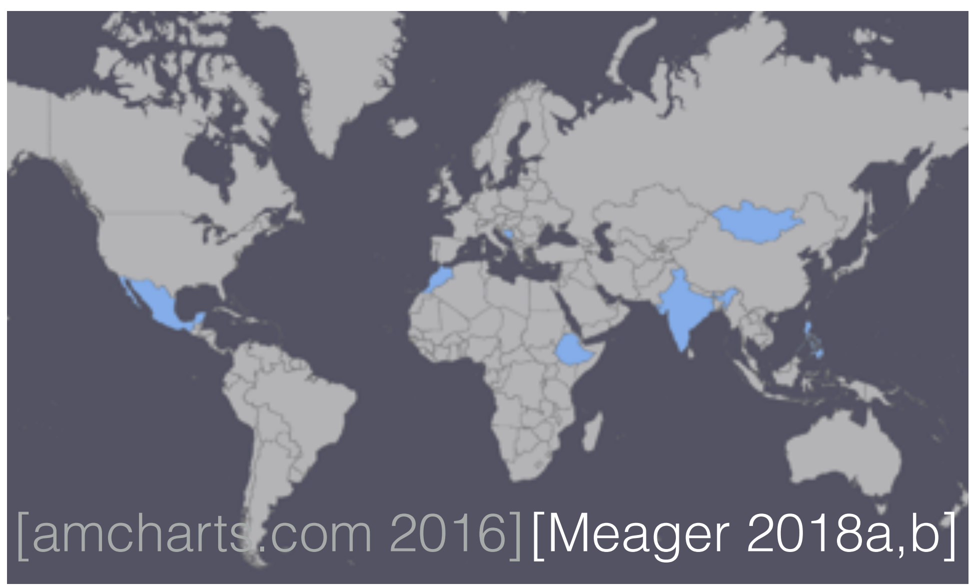

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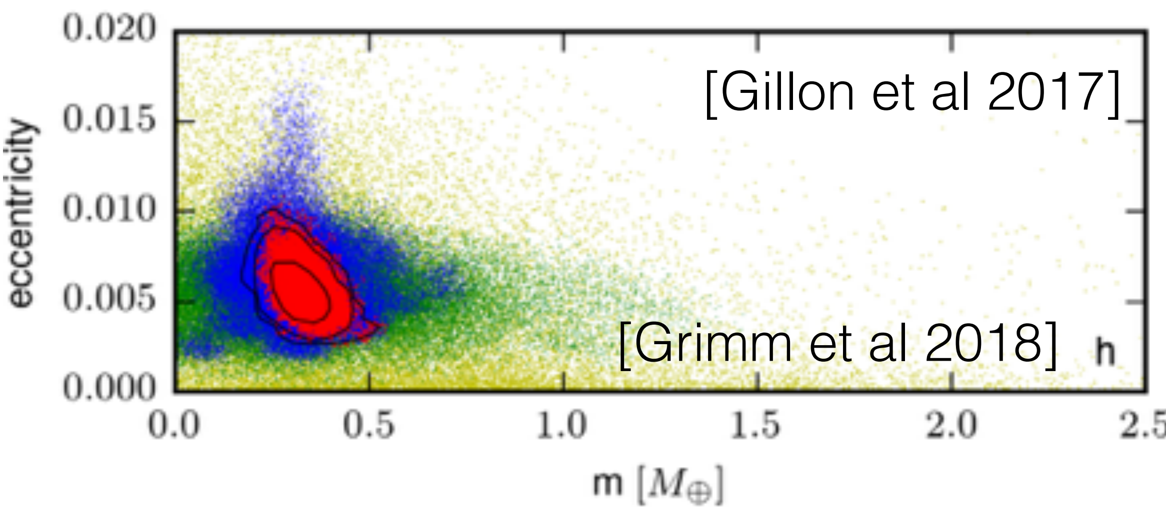

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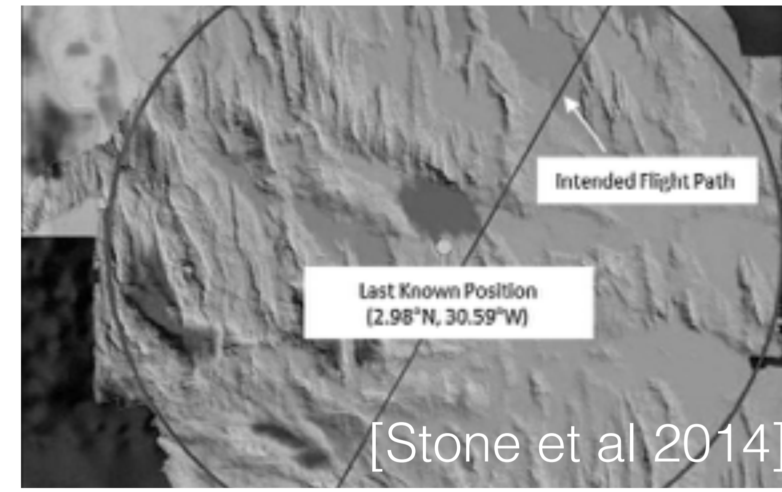

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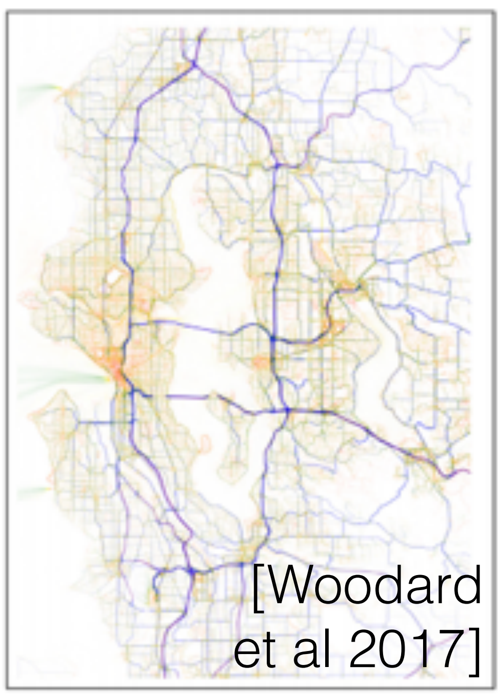
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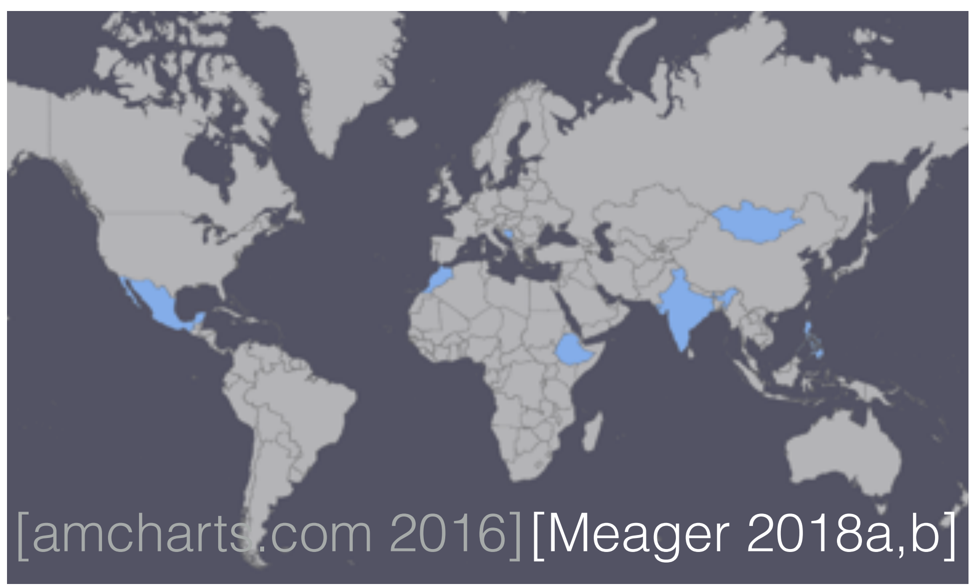
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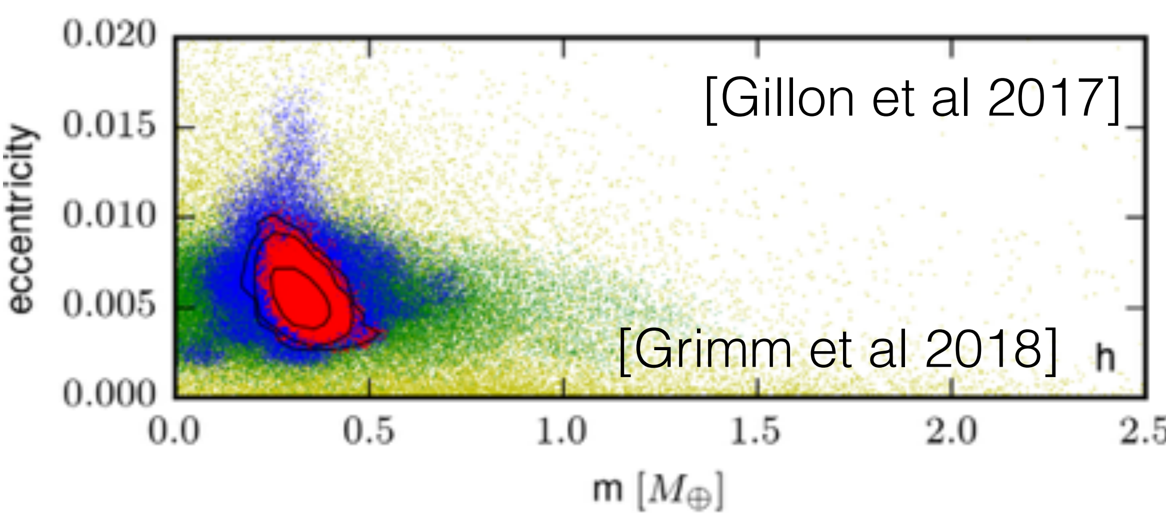
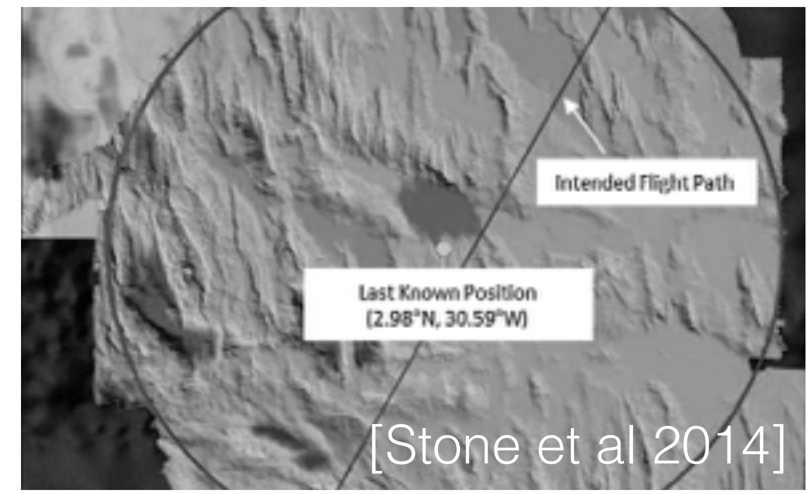
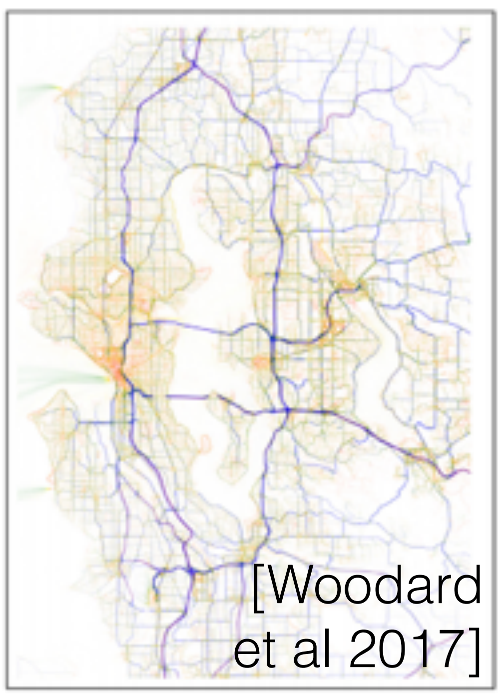
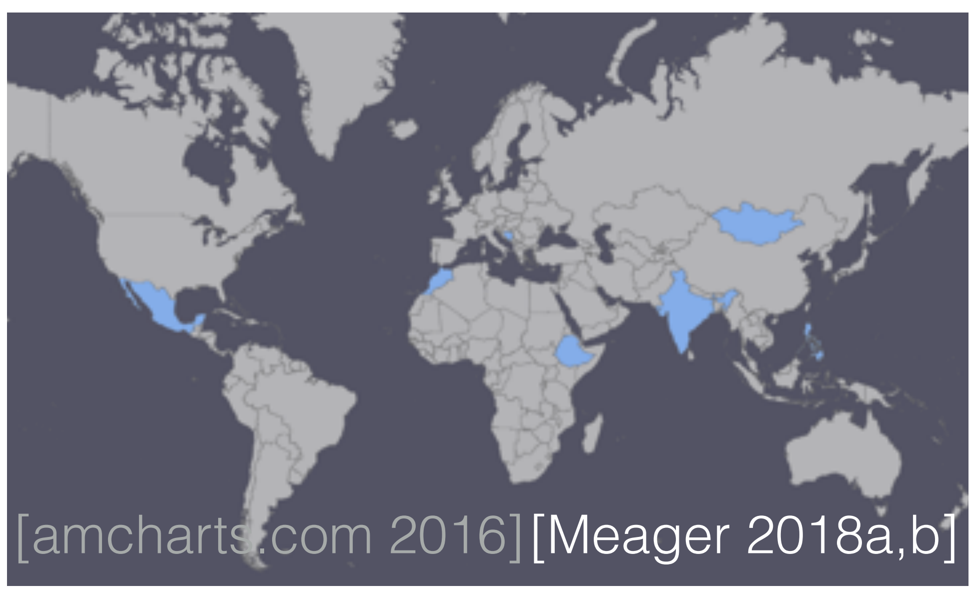


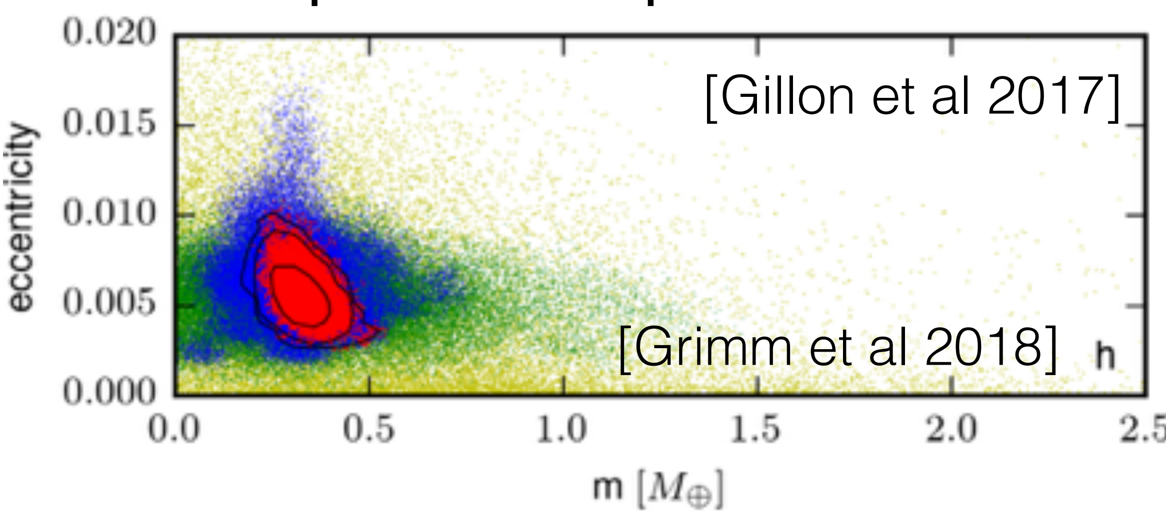






Bayesian inference

- Goals: good estimates, uncertainties; interpretable; complex; expert information



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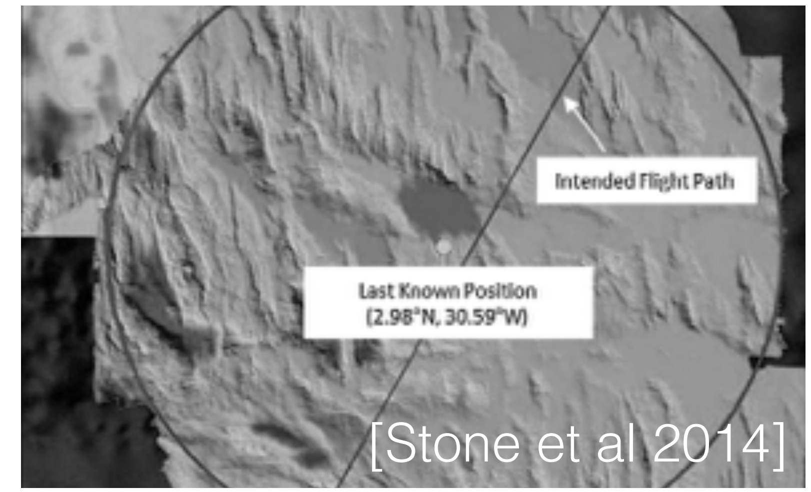

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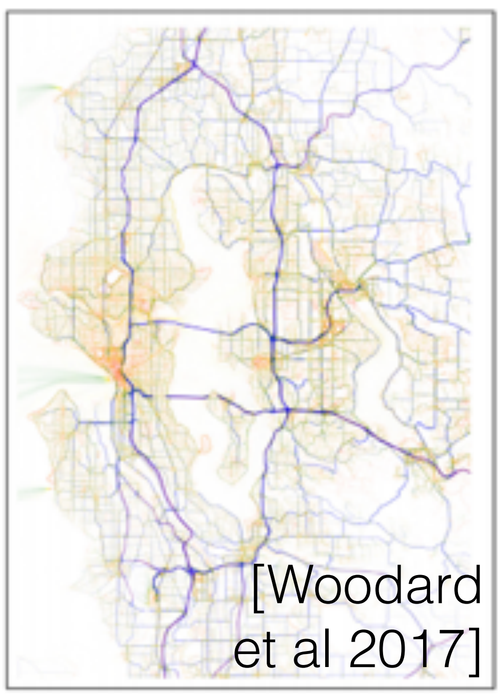
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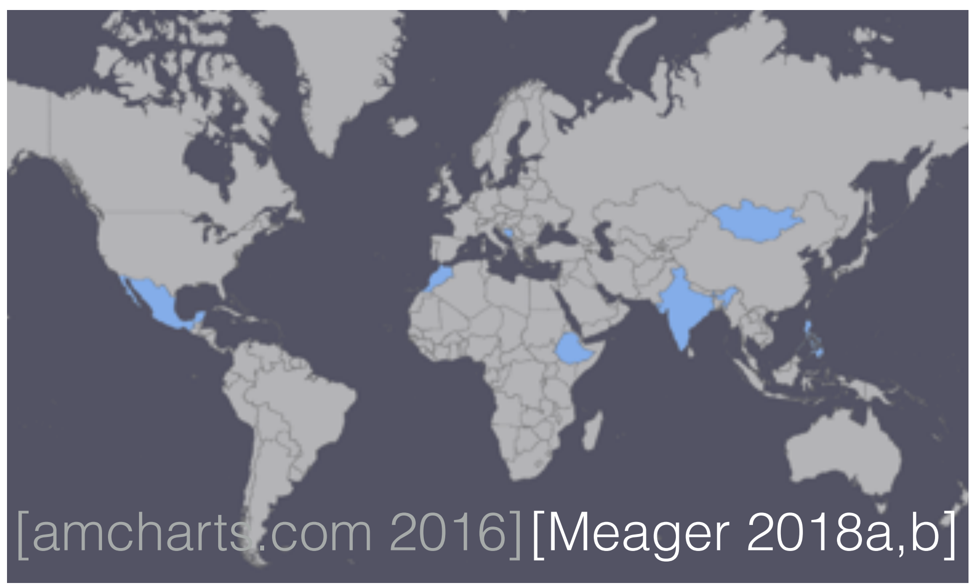
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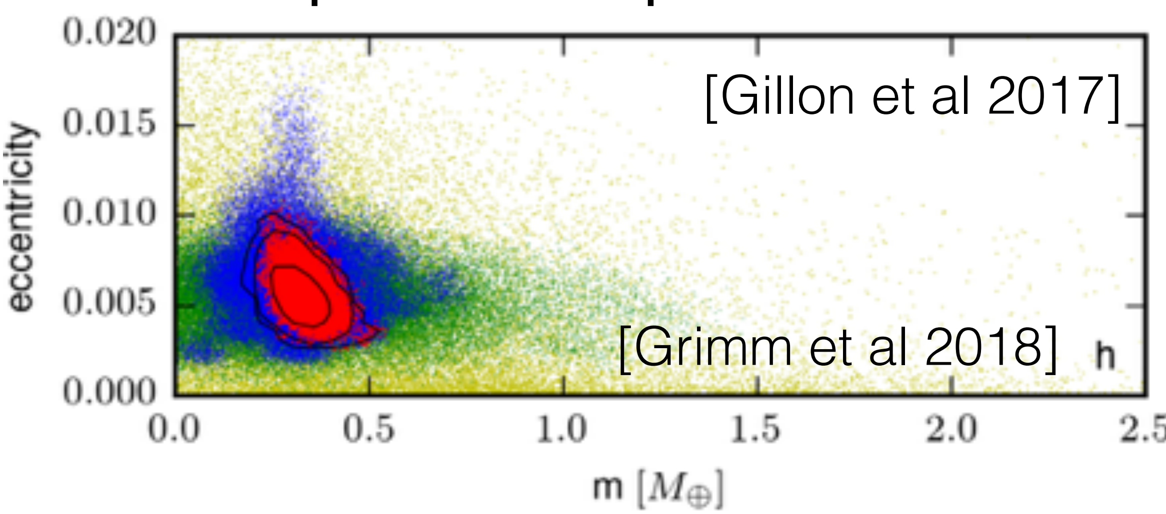

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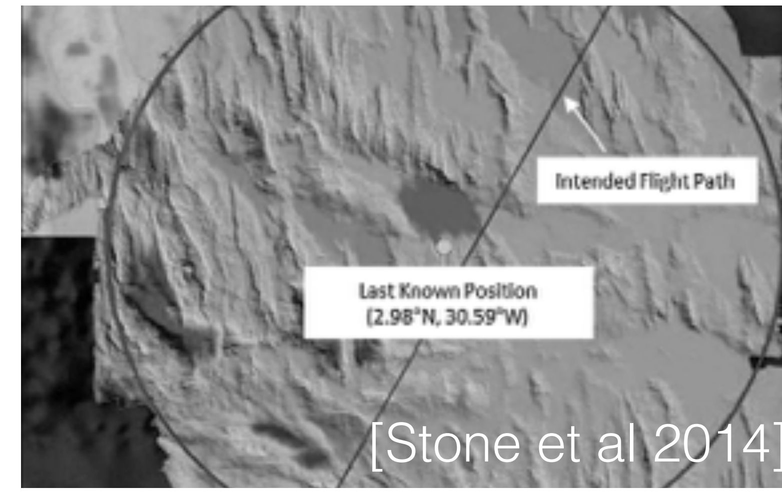

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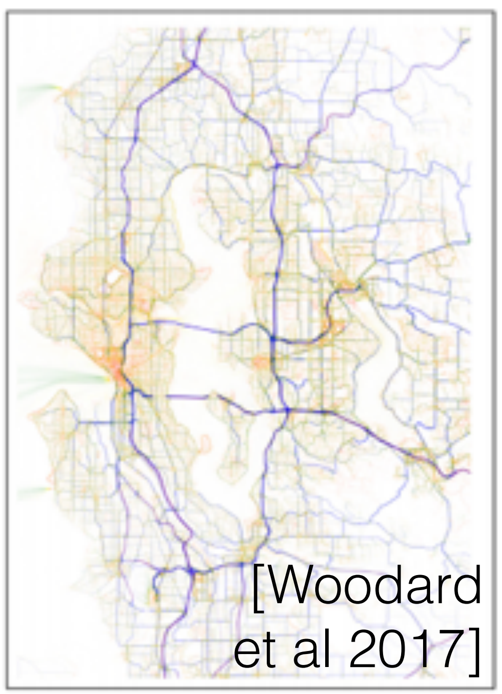
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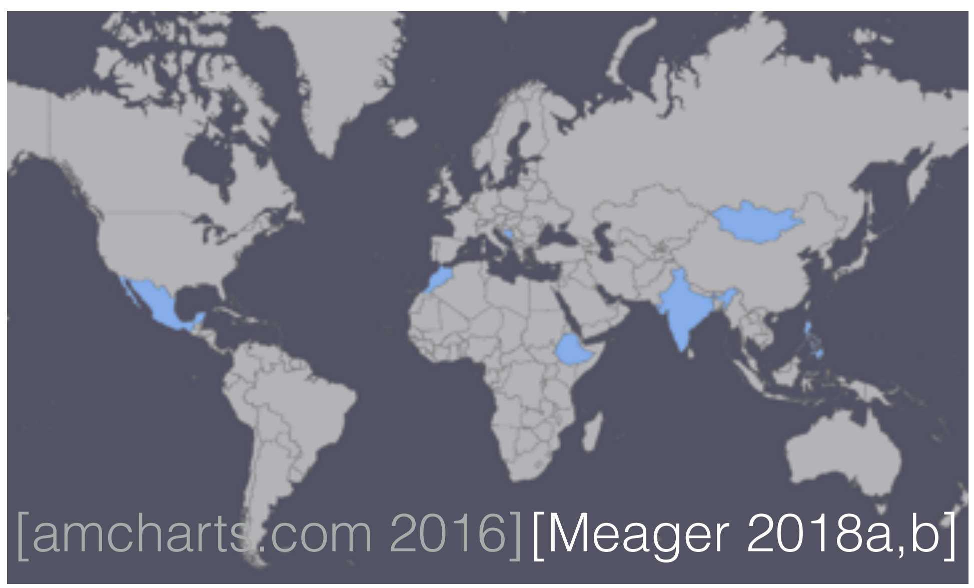
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[mc-stan.org]

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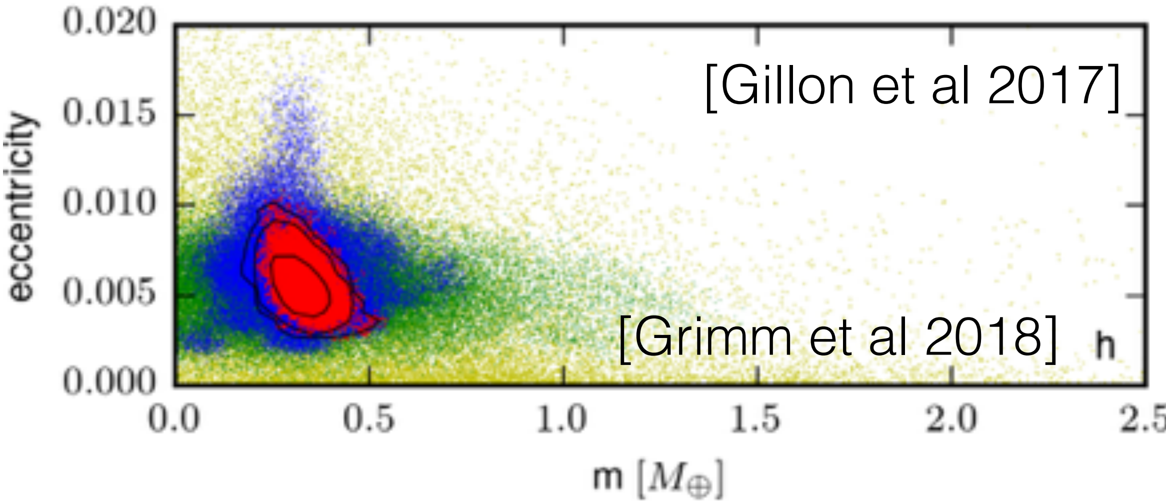

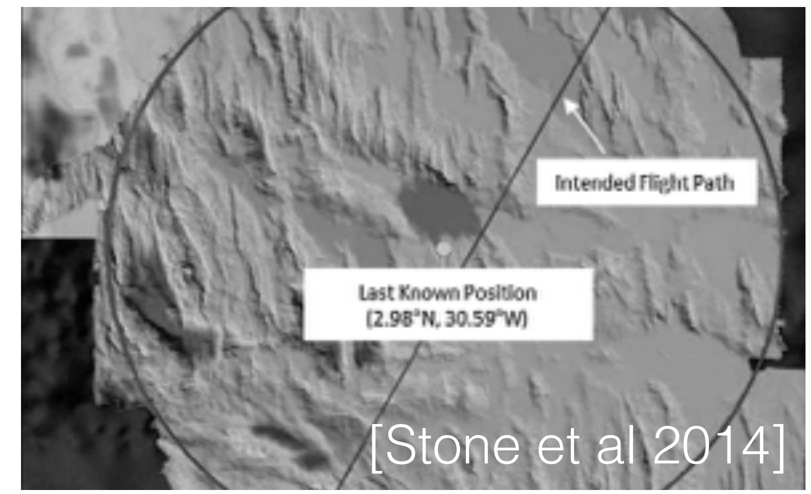

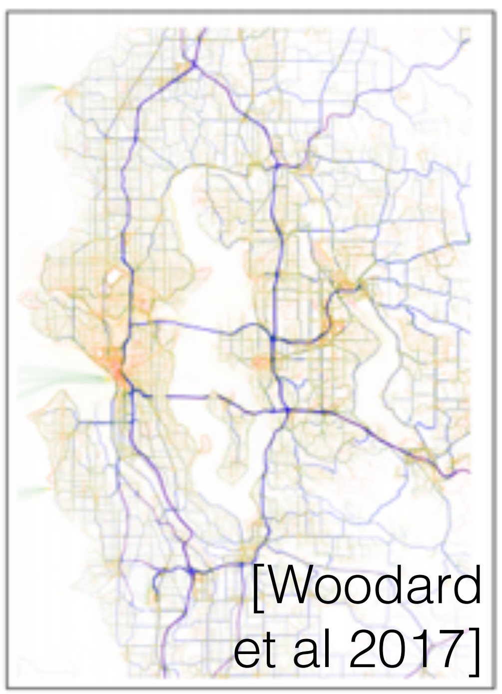

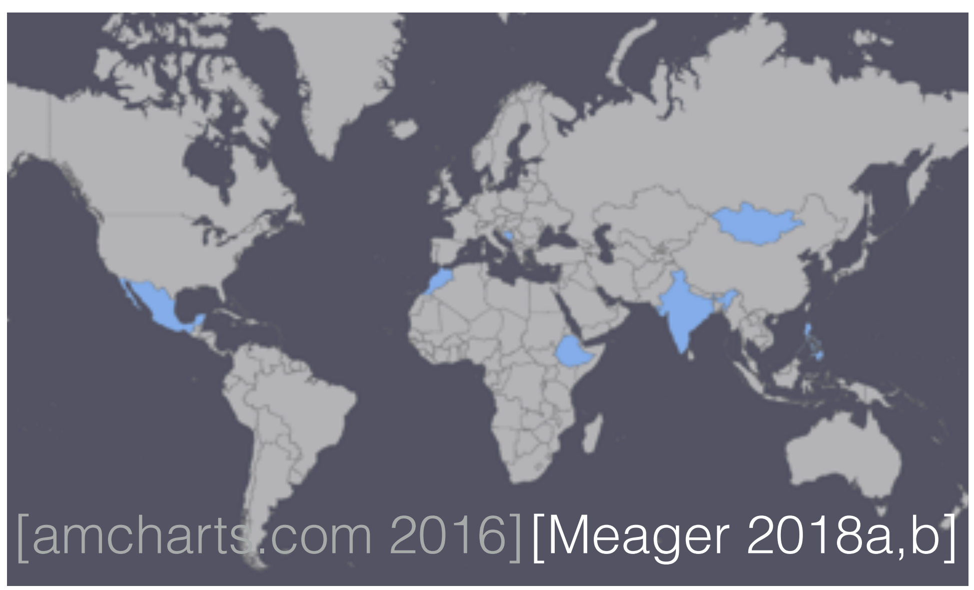

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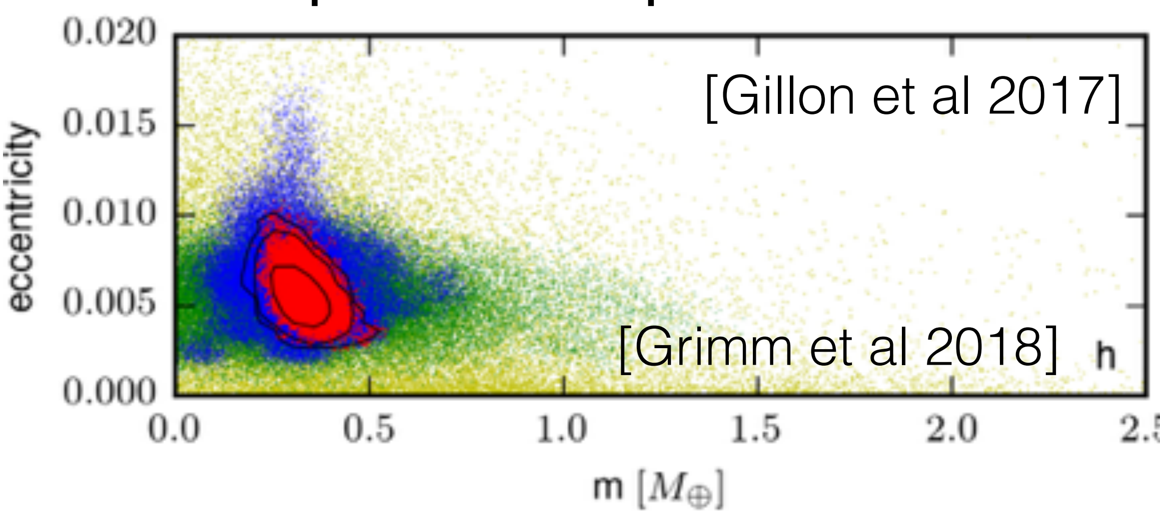

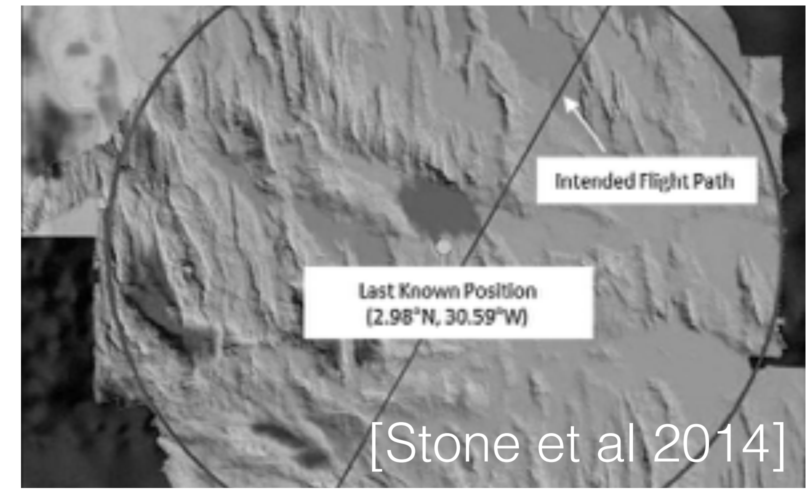

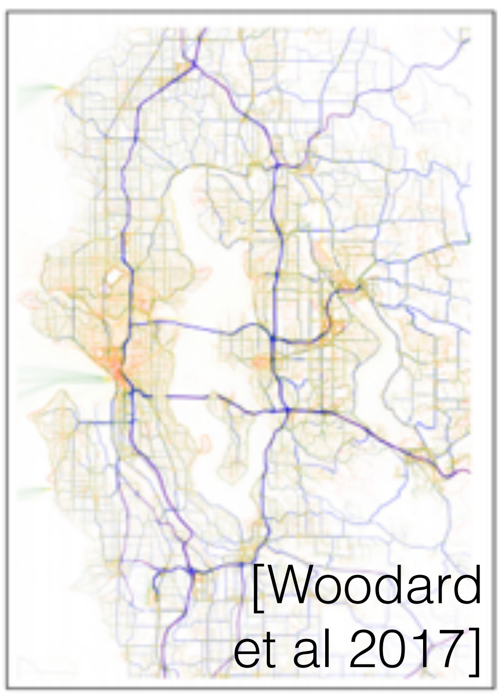

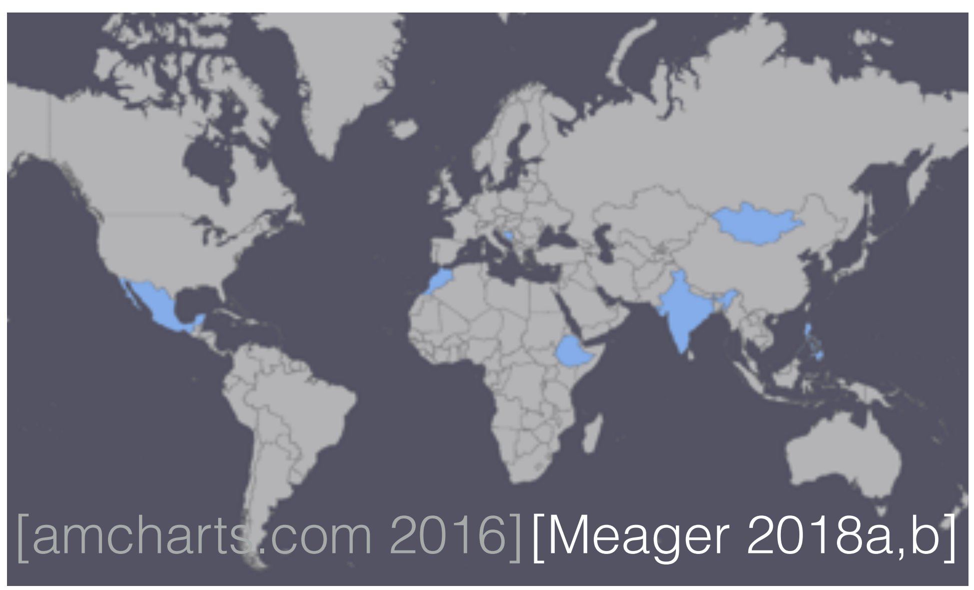

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- Challenge: existing methods can be slow, tedious, unreliable
- Our proposal: develop *coresets* for **scalable**, **automated** algorithms with **error bounds for output quality**

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$p(\theta)$

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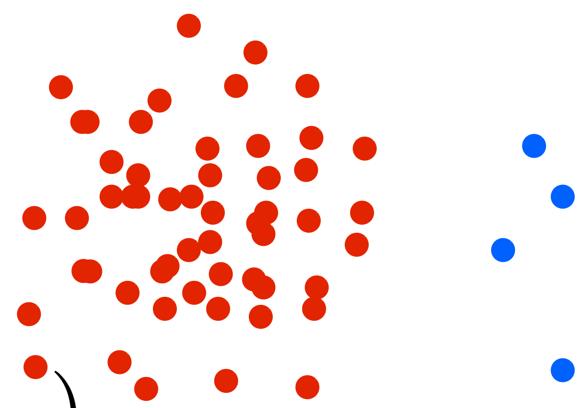

(x_n, y_n)

Bayesian inference

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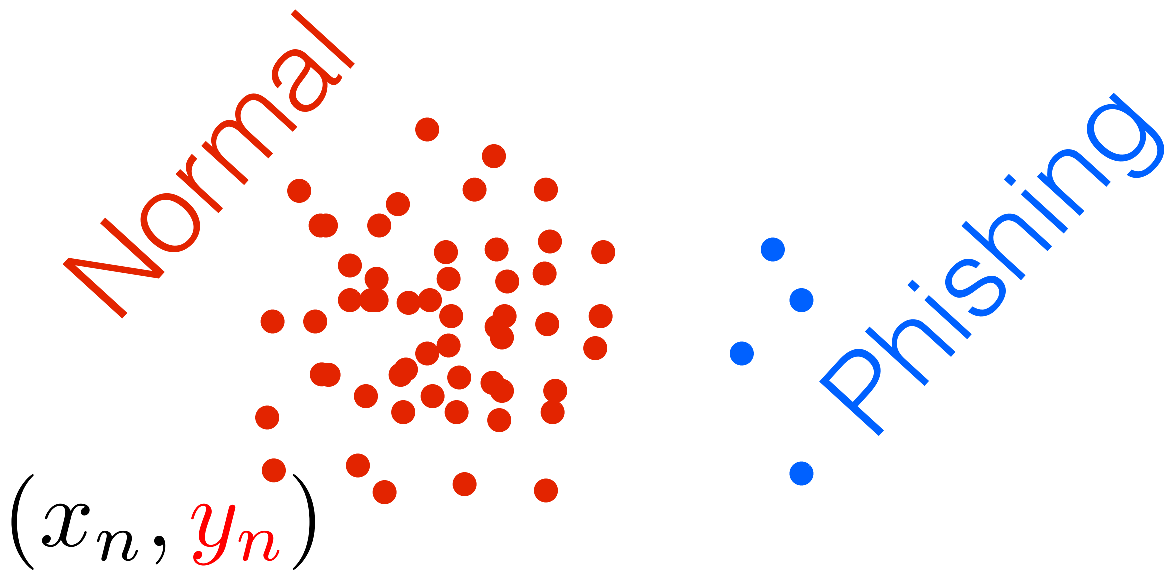

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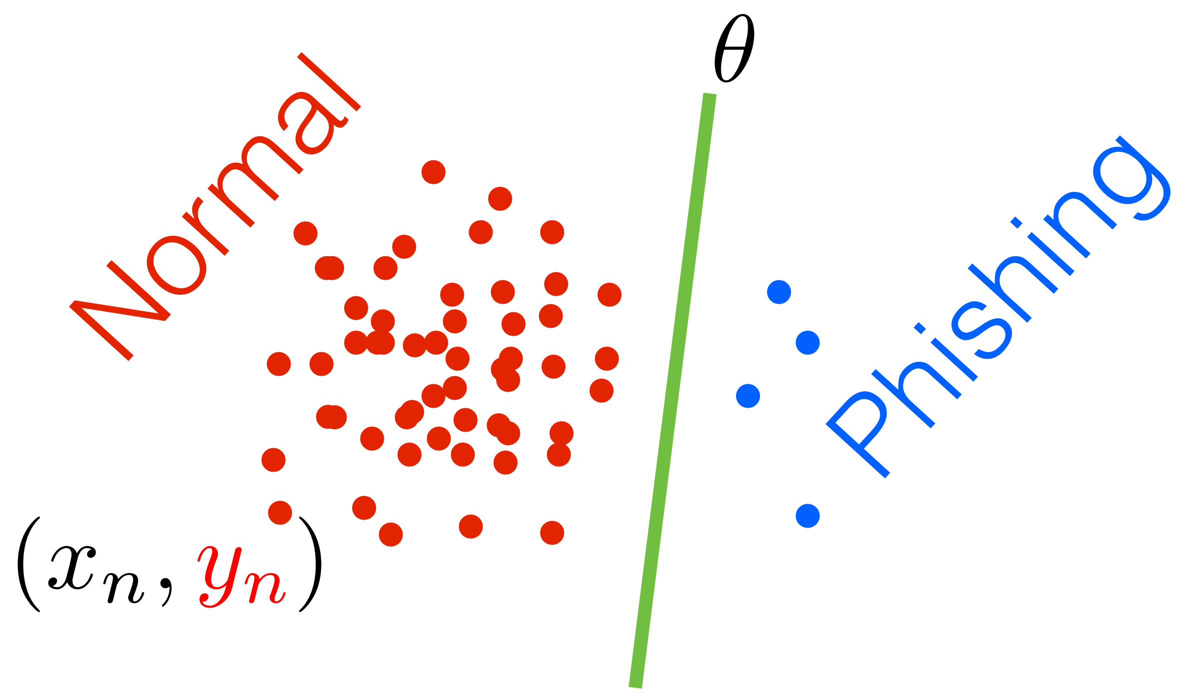

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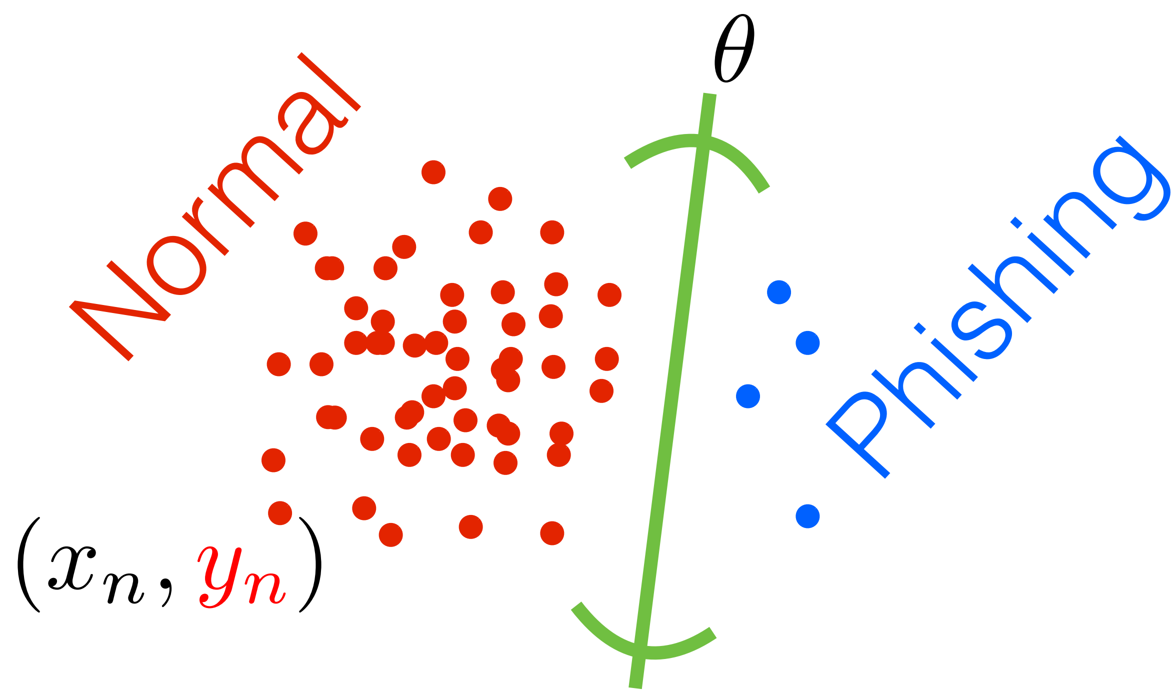

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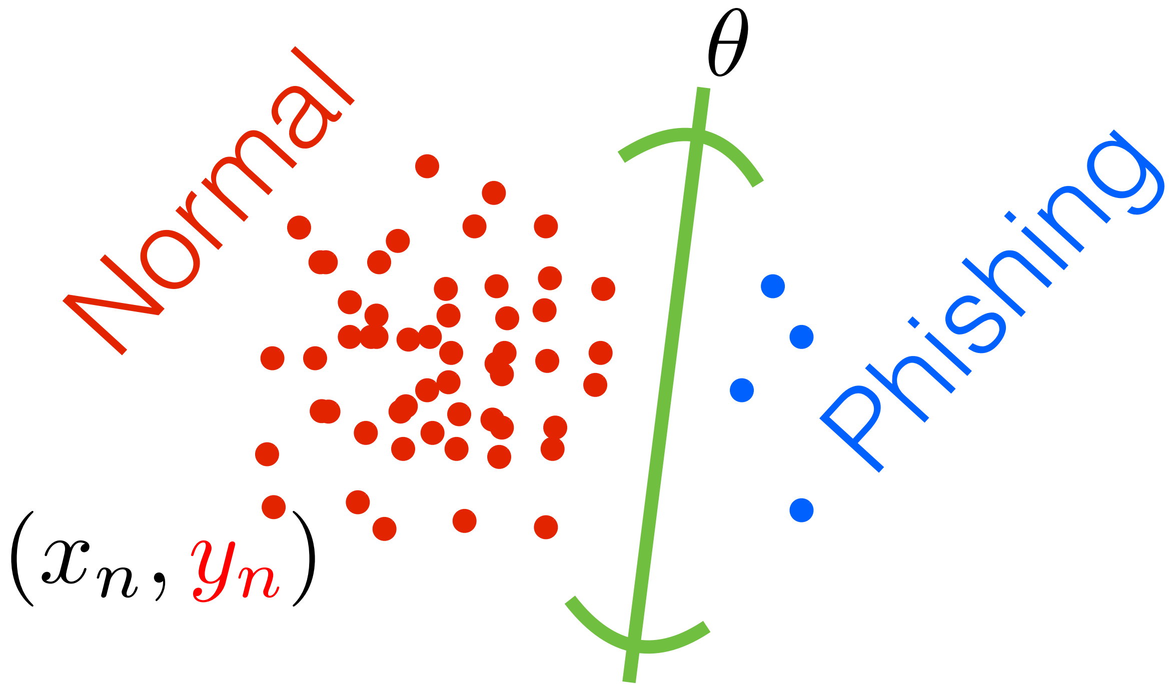

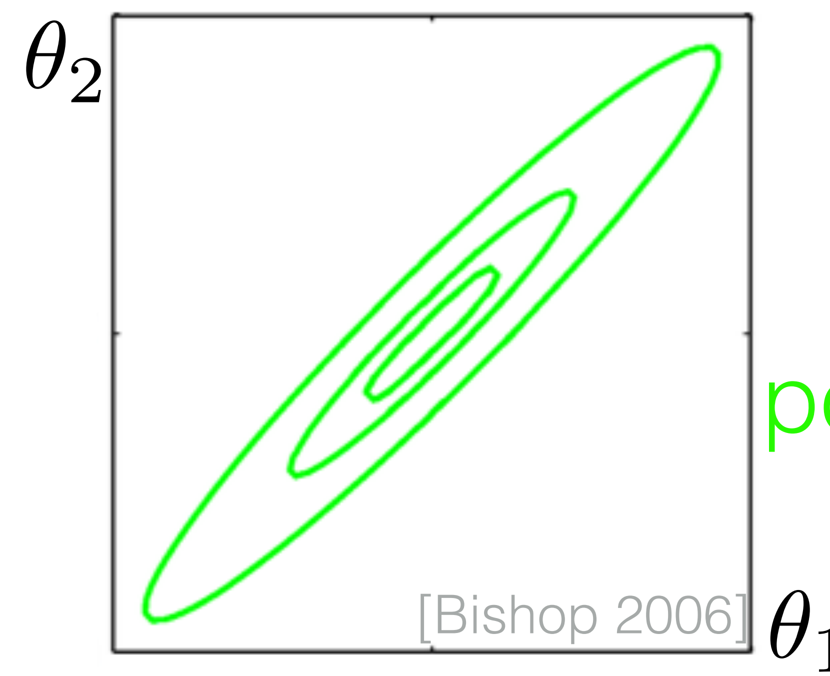

Exact posterior

Bayesian inference

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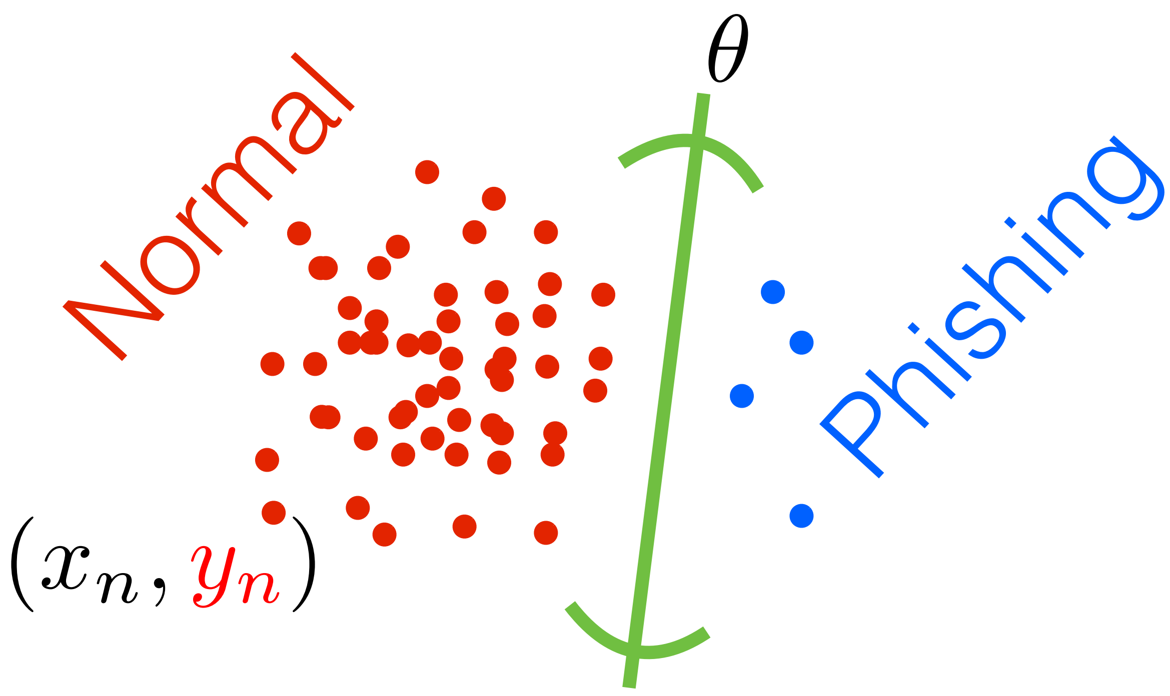

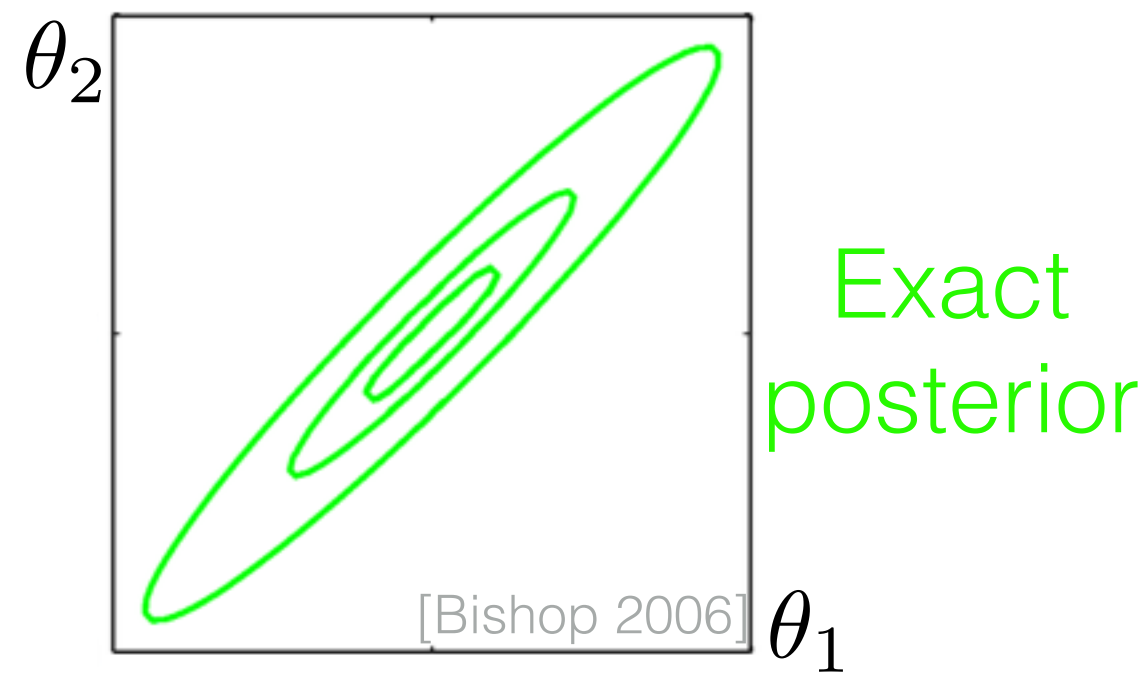

- MCMC: Eventually accurate but can be slow [Bardenet, Doucet, Holmes 2017]

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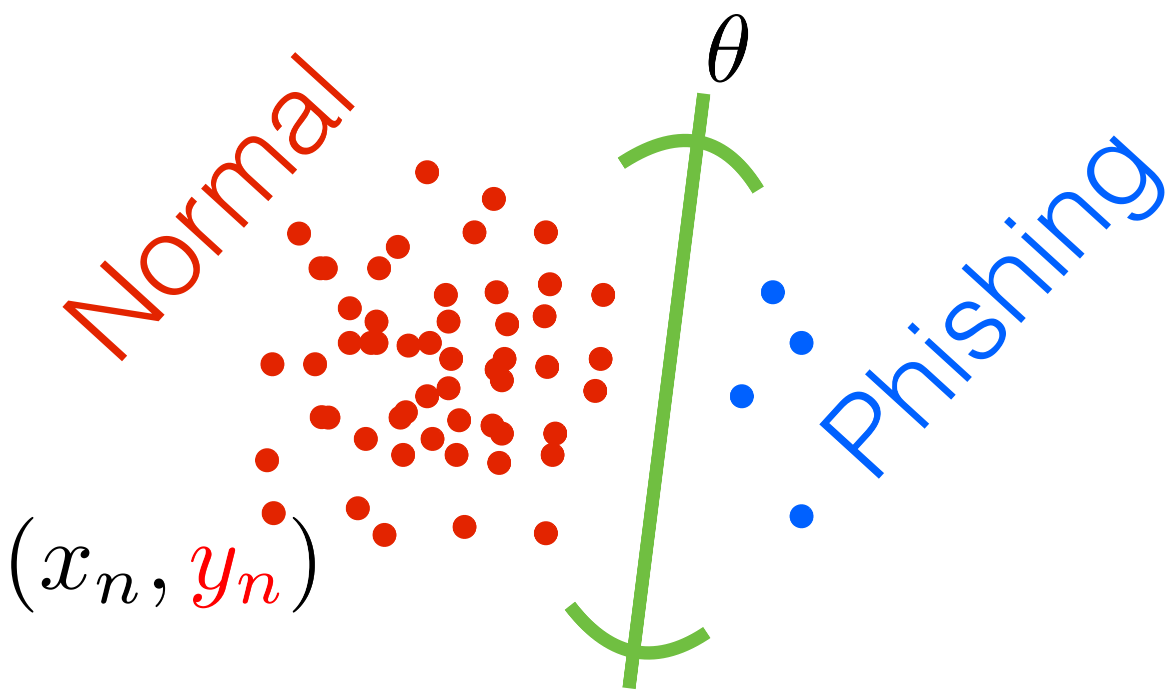

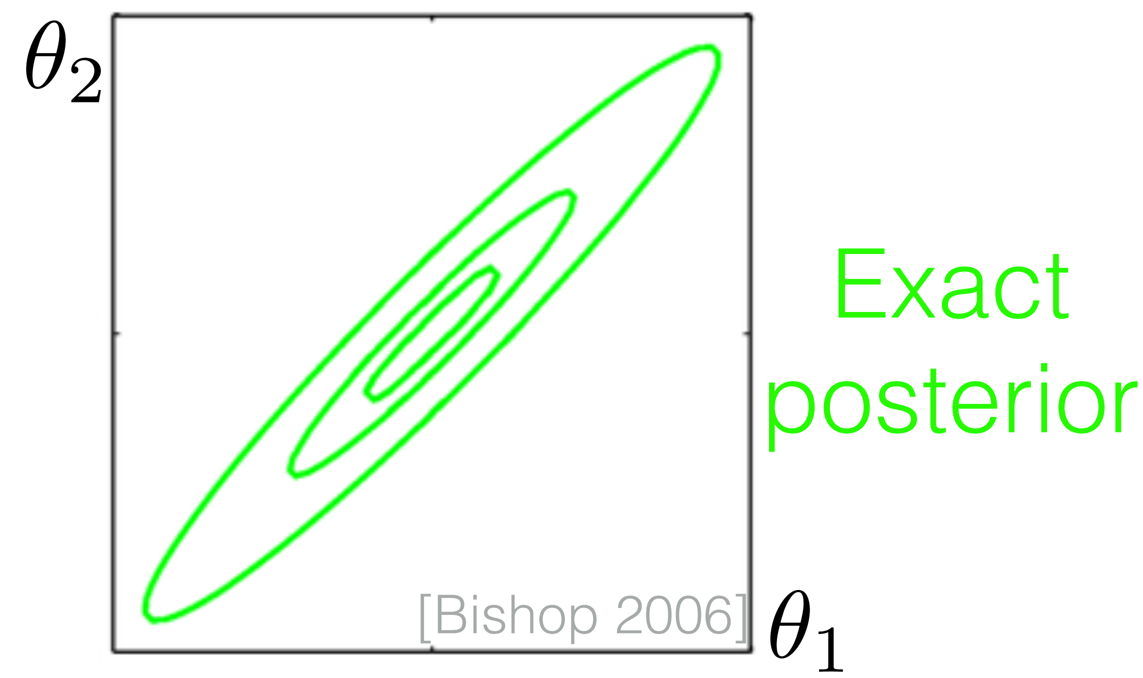

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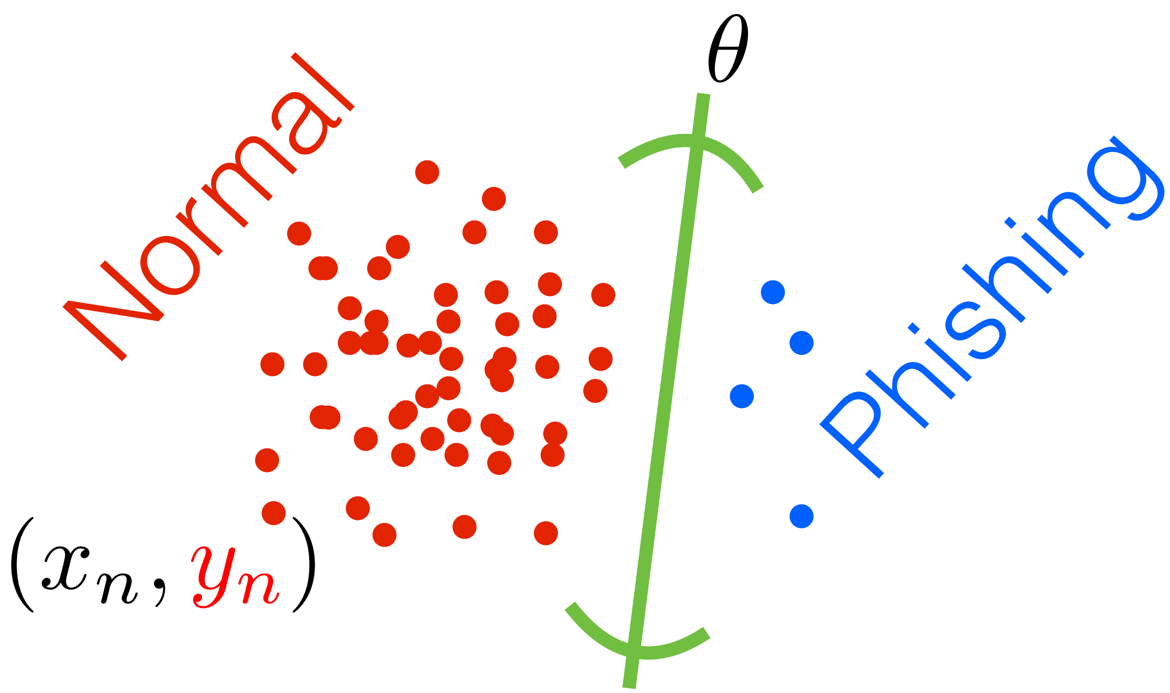

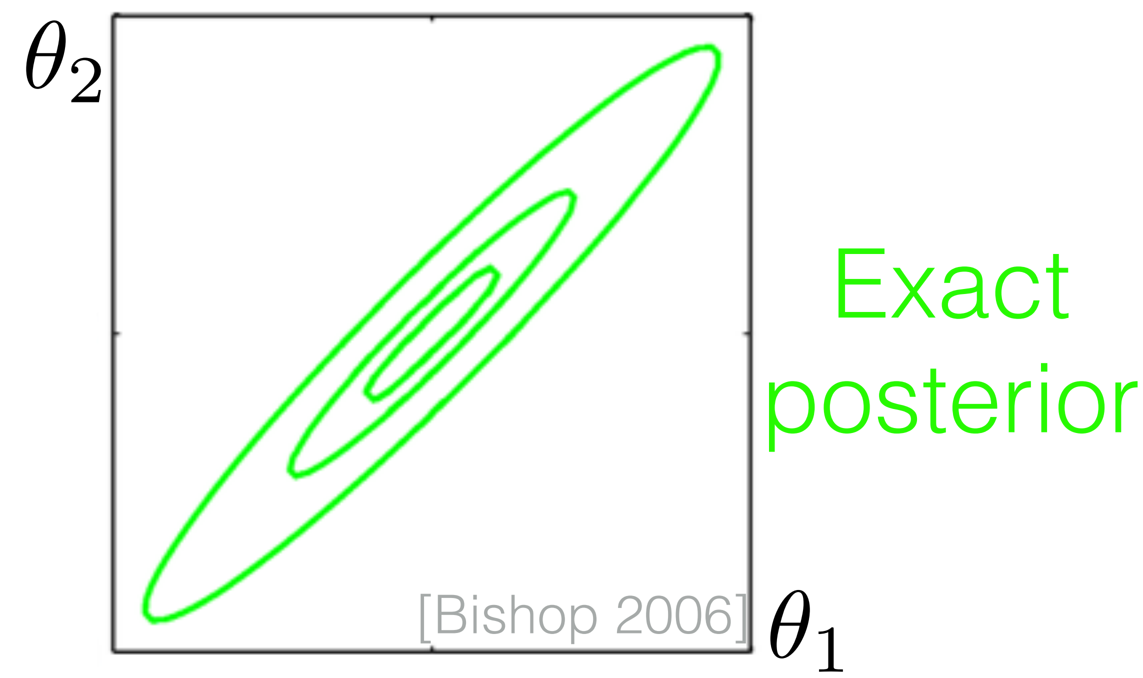

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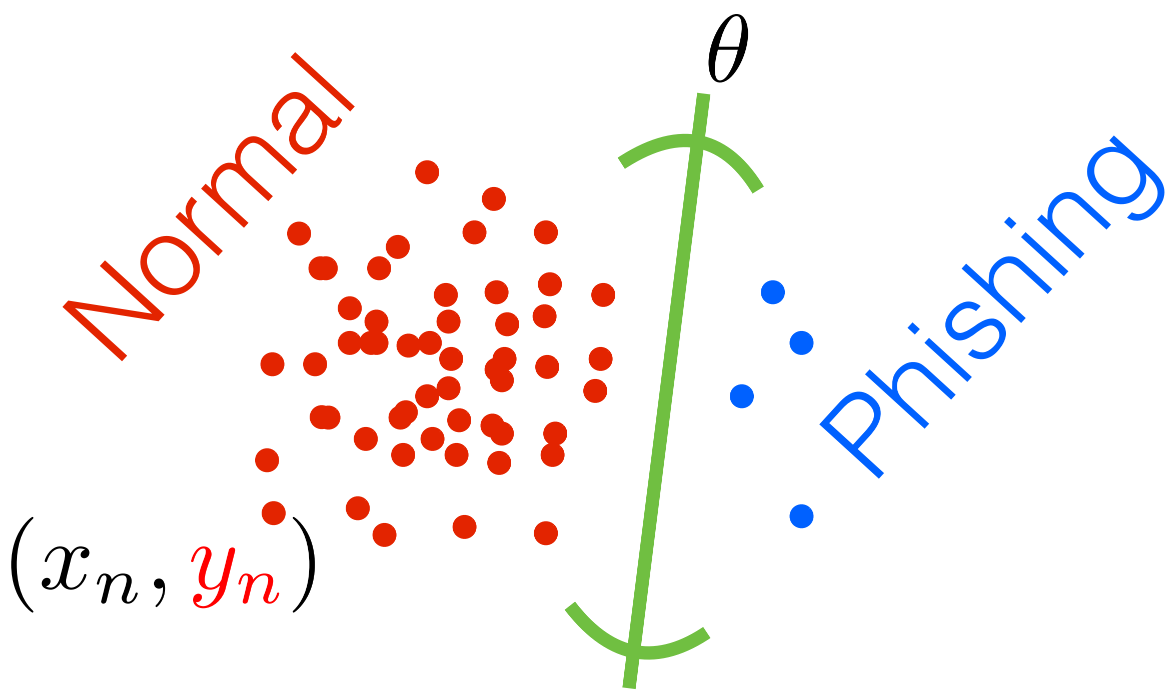
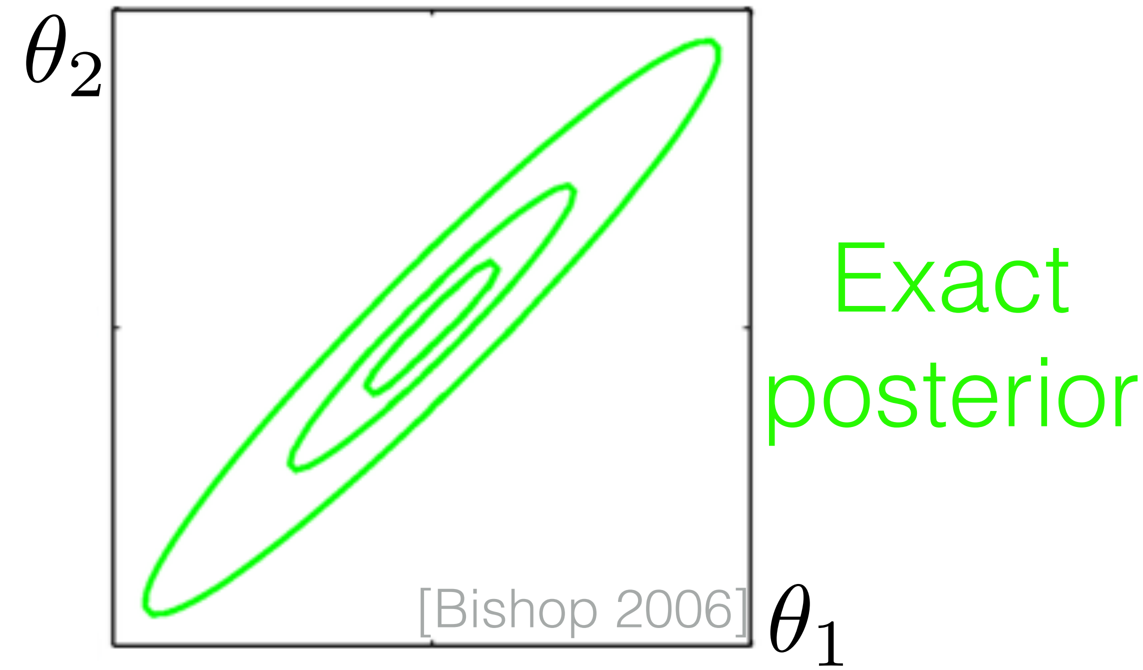
  - Fast

Bayesian inference

$p(\theta|y) \propto_{\theta} p(y|\theta)p(\theta)$





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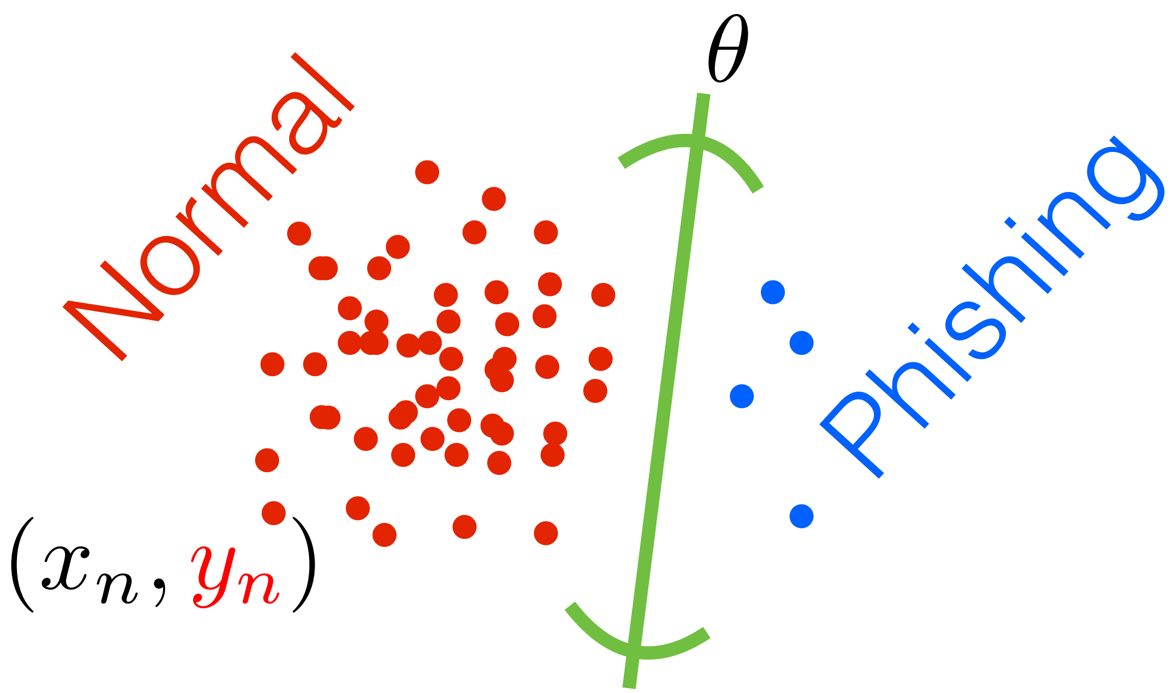

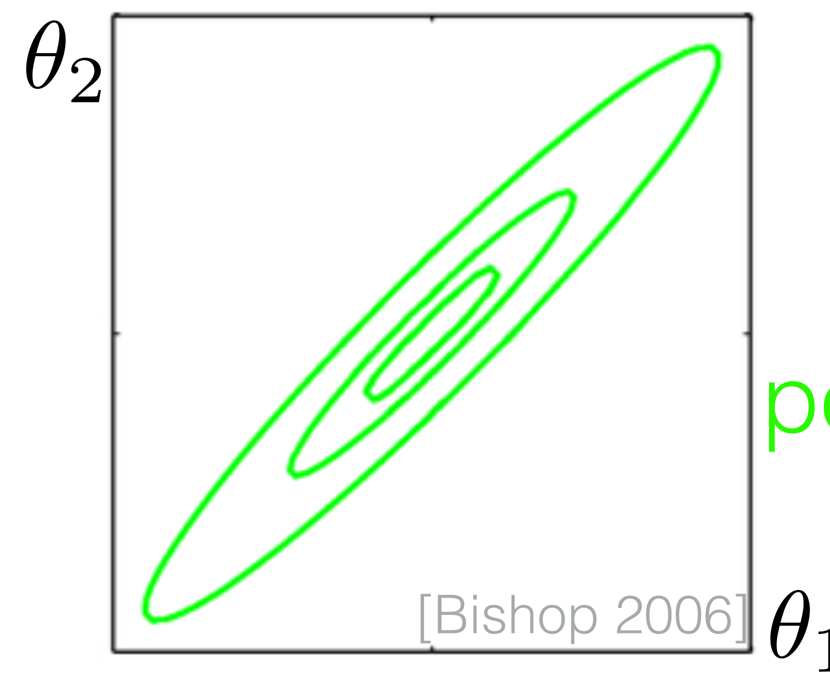

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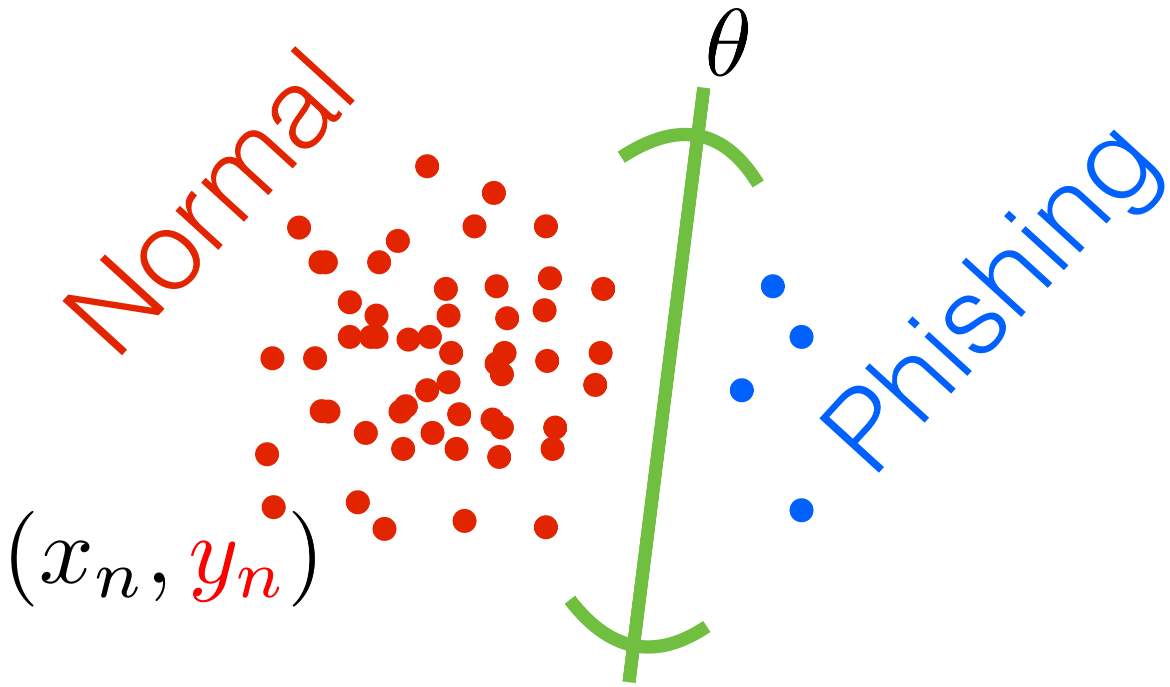

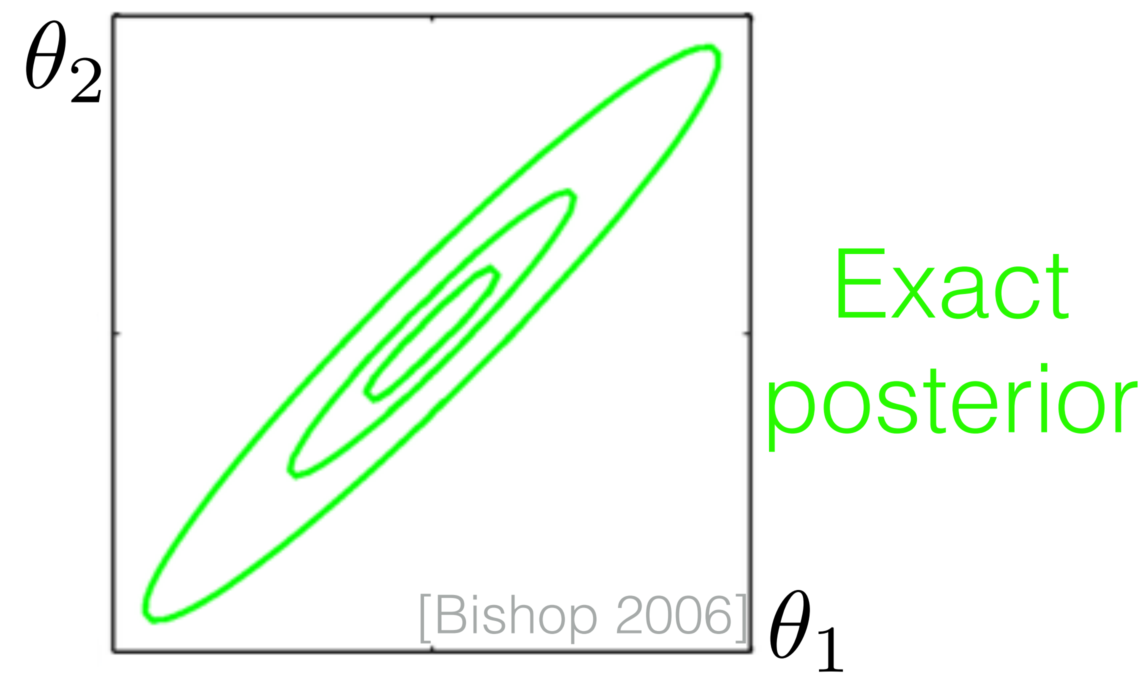

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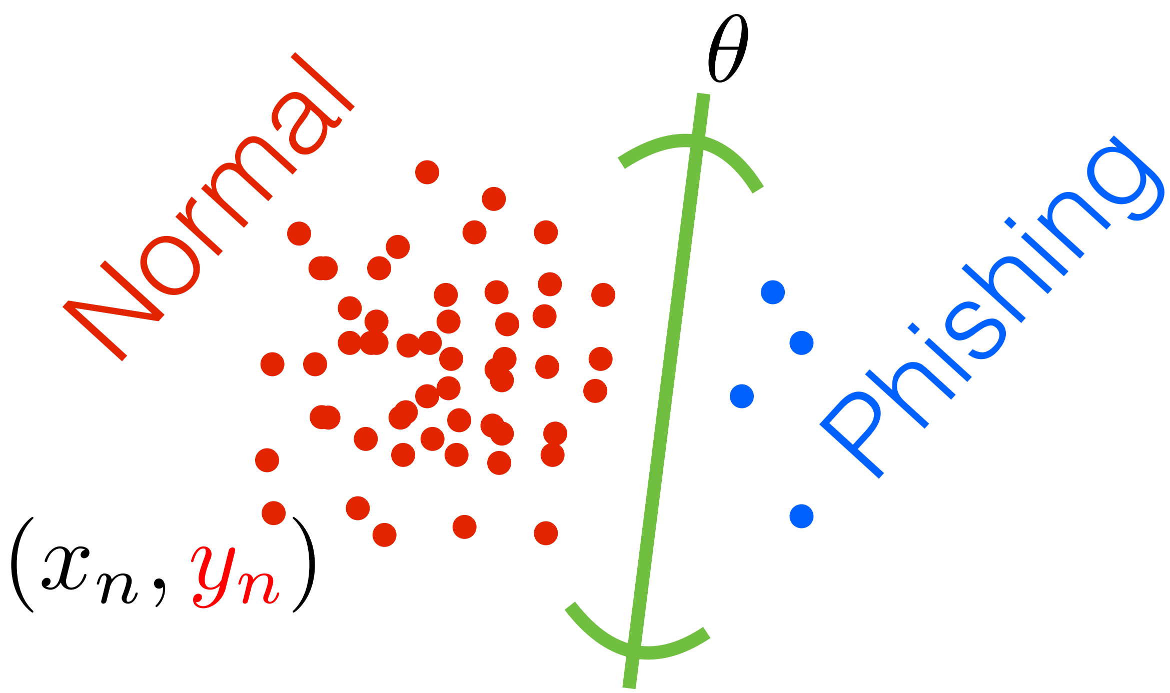

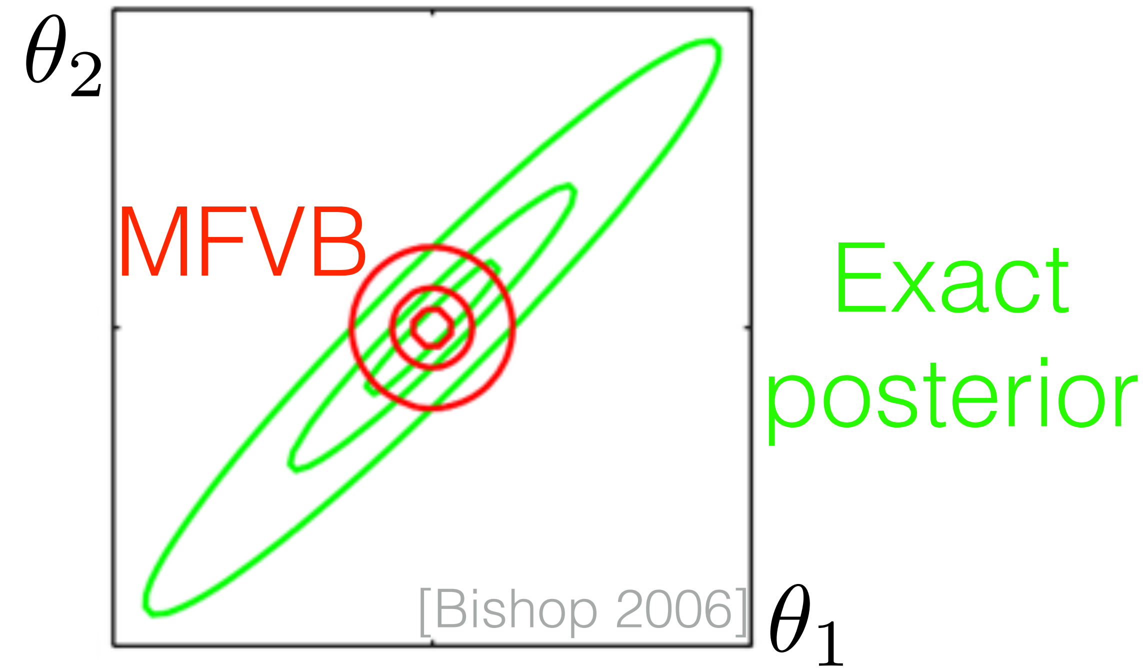

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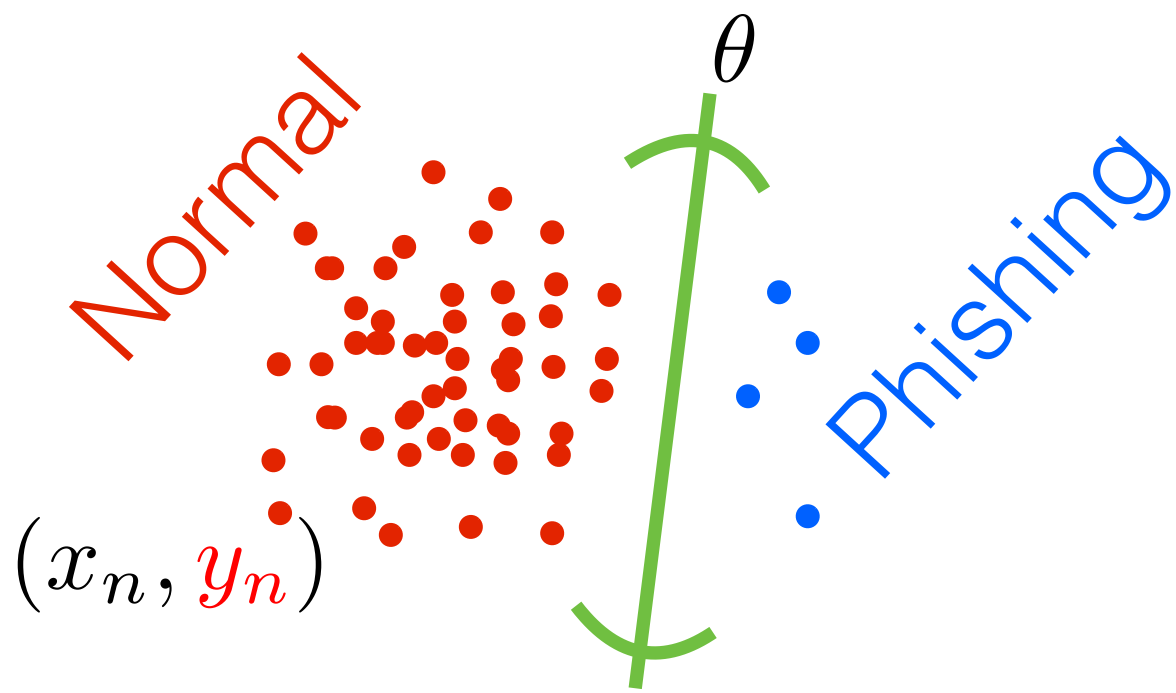

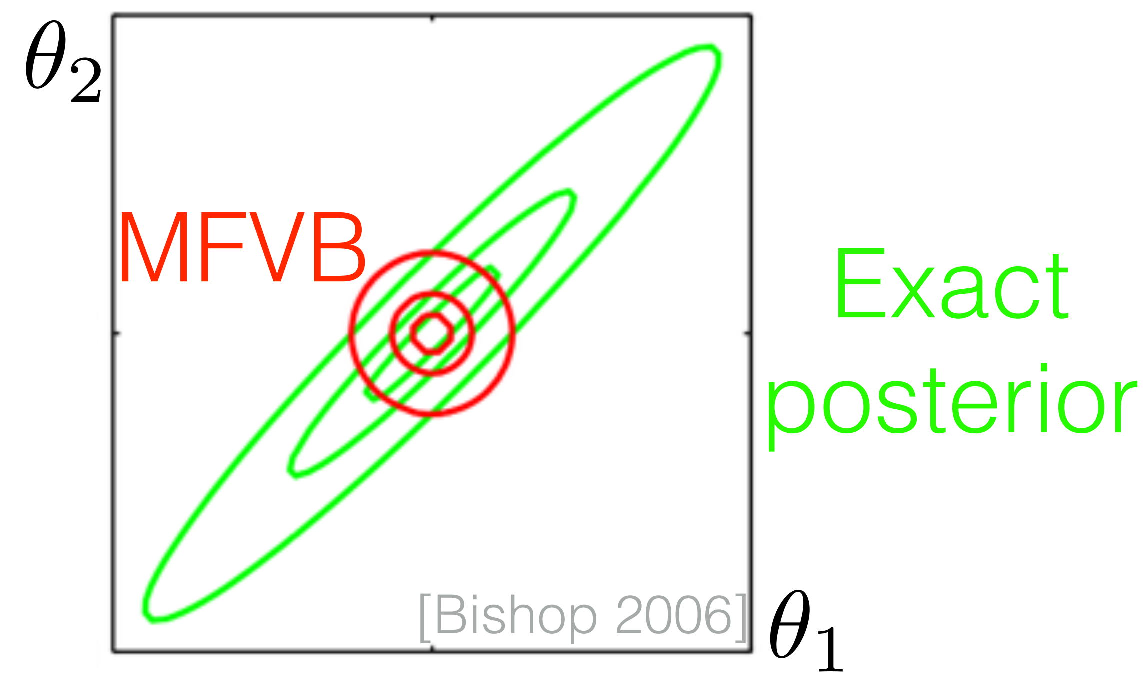

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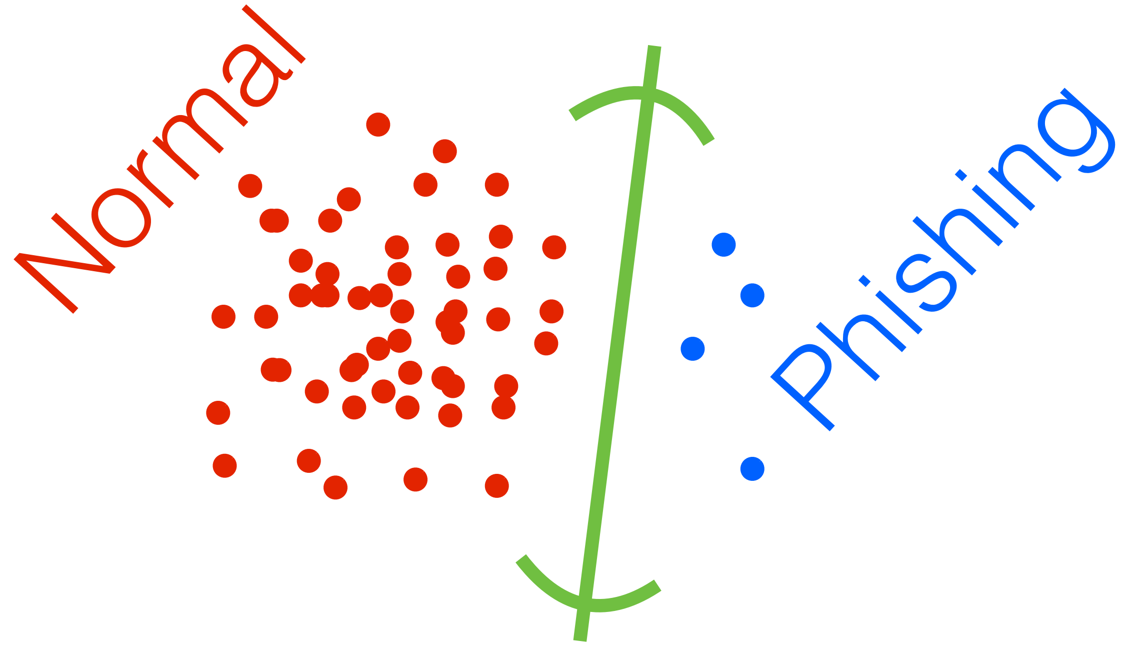
- Automation: e.g. Stan, NUTS, ADVI

[http://mc-stan.org/ ; Hoffman, Gelman 2014; Kucukelbir, Tran, Ranganath, Gelman, Blei 2017]

Bayesian coresets

- Posterior $p(\theta|y) \propto_\theta p(y|\theta)p(\theta)$



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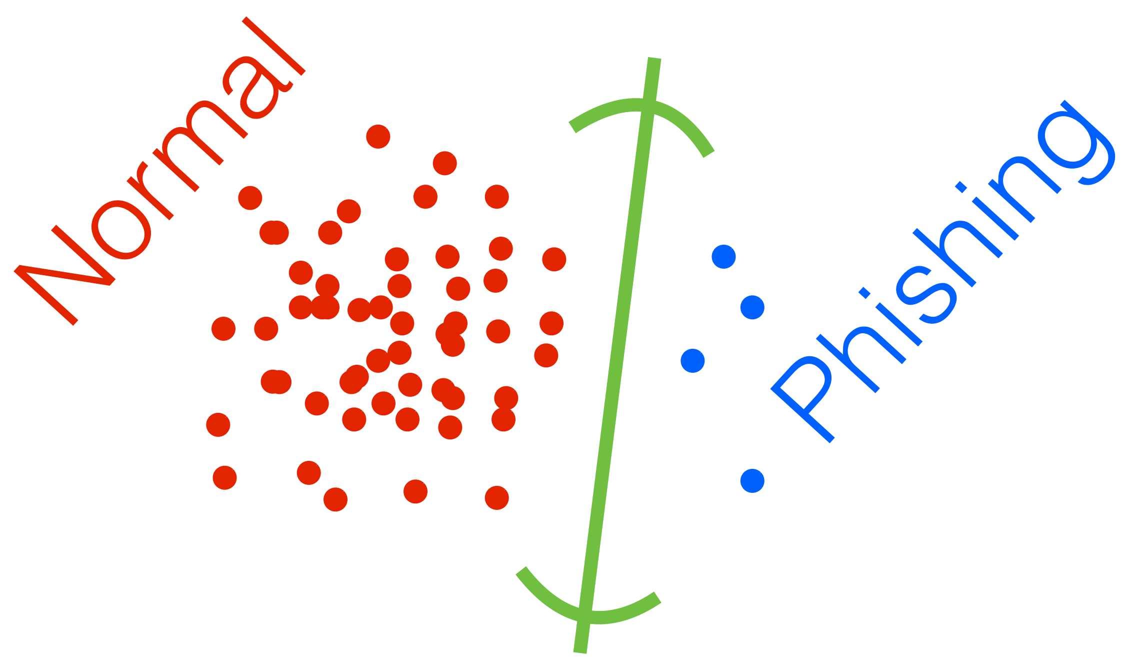

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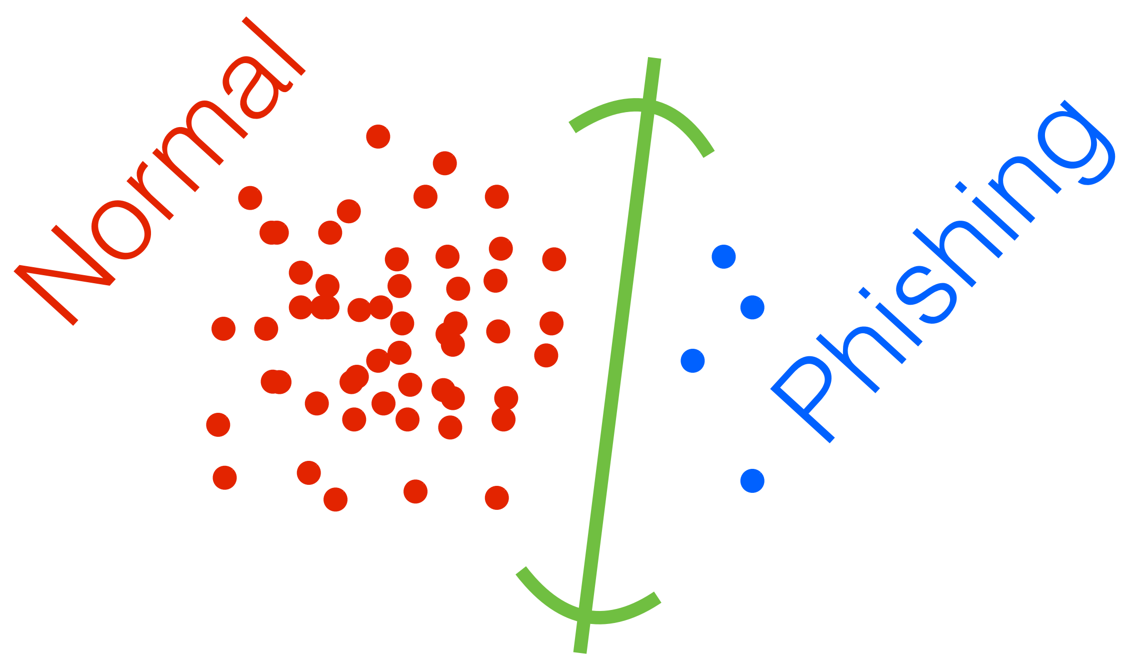

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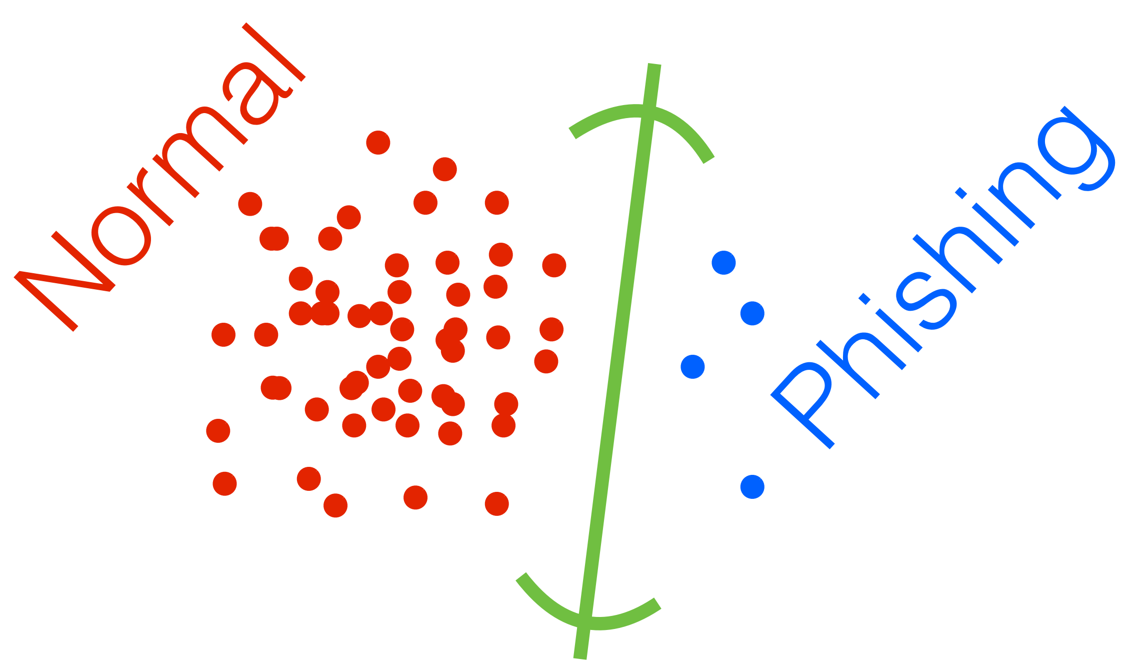
$$\|w\|_0 \ll N$$



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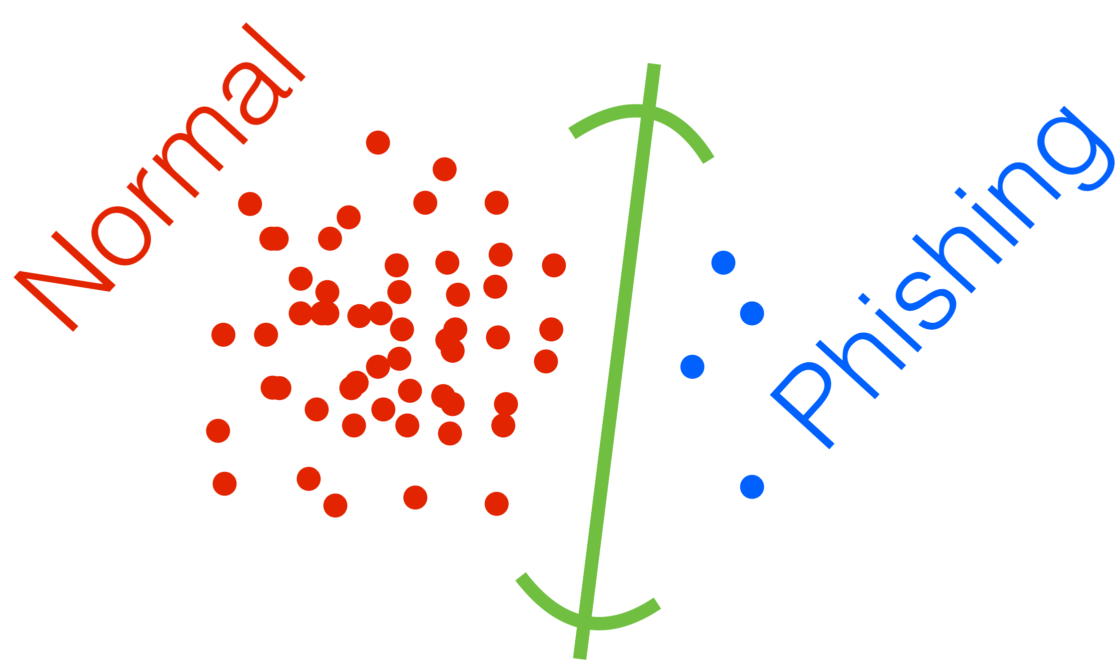

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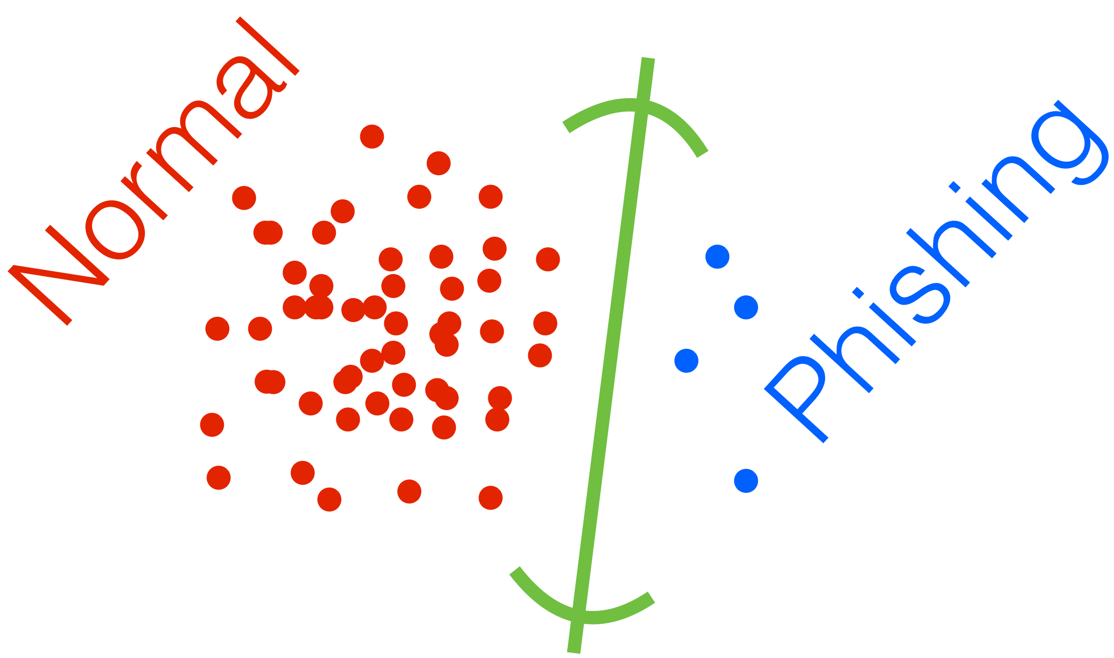

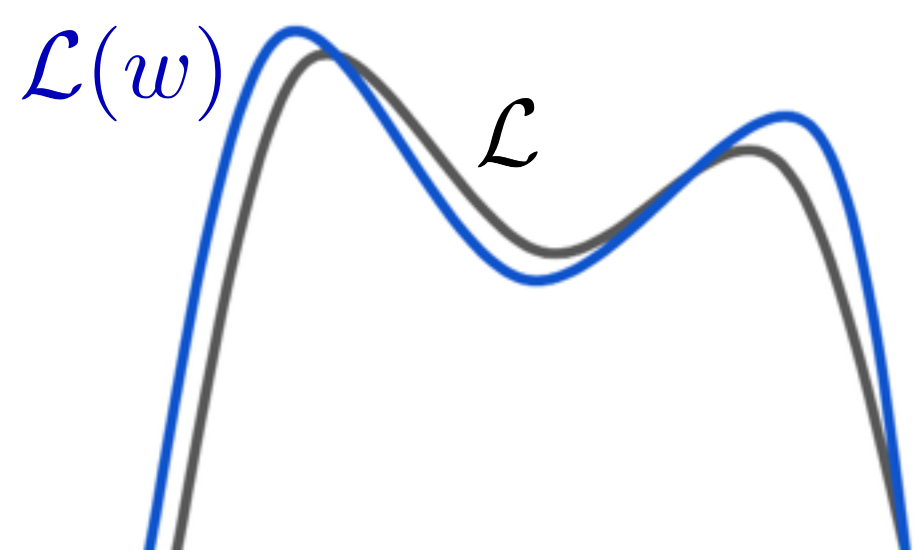

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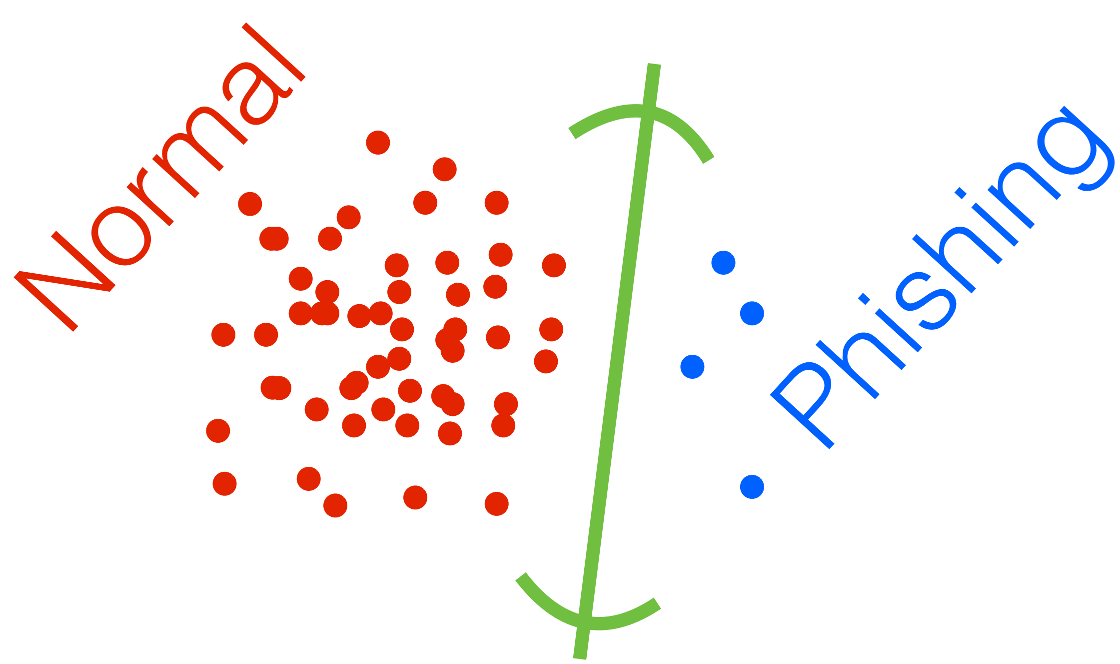

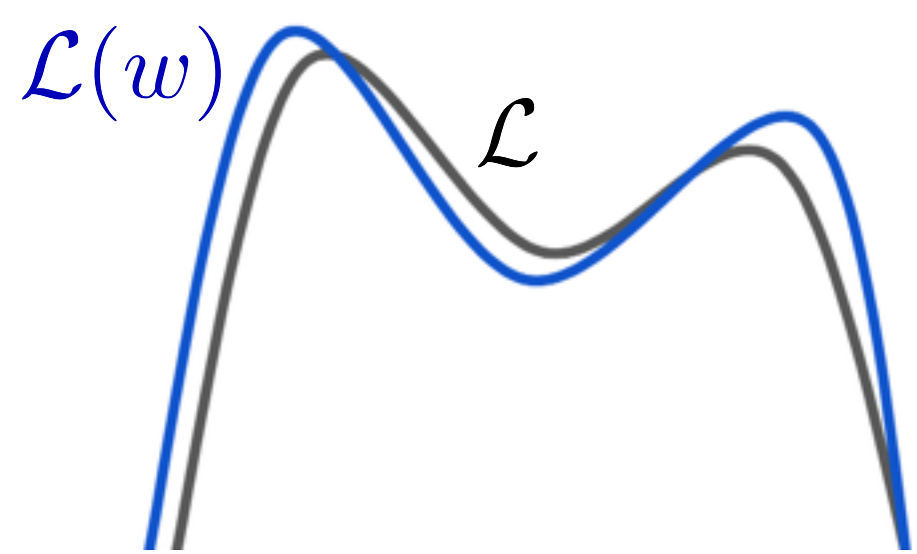

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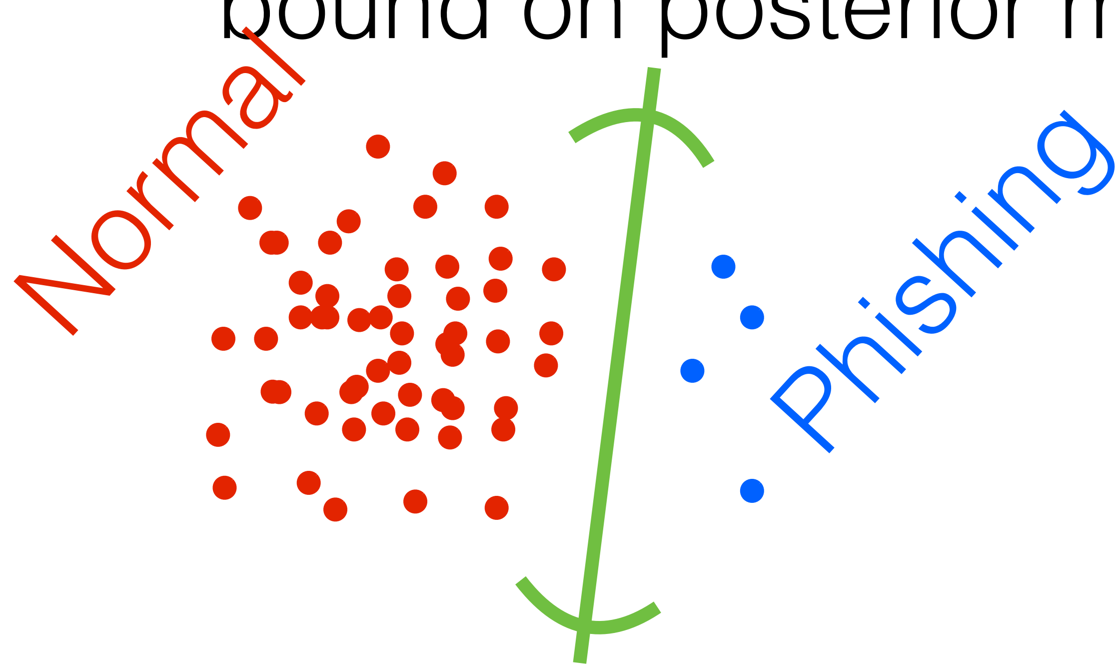
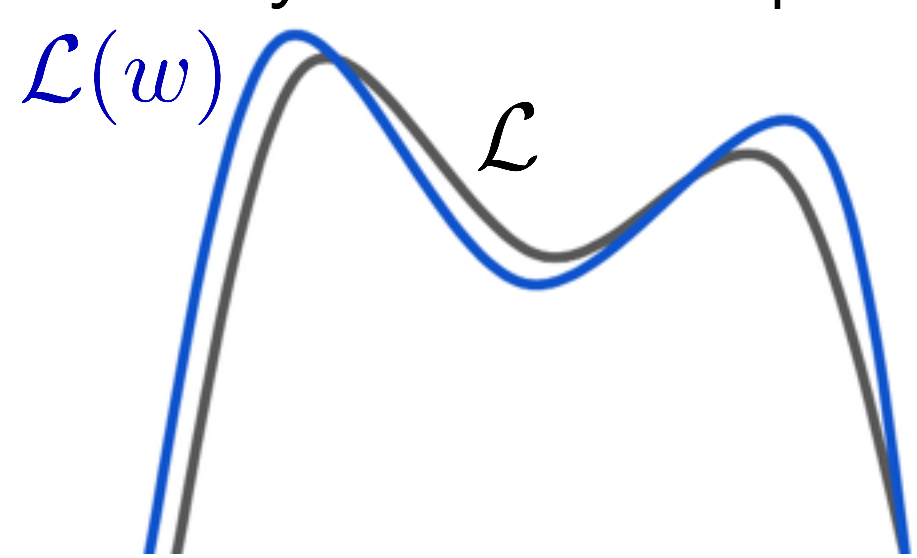
  - Bound on Wasserstein distance to exact posterior ➔ bound on posterior mean/uncertainty estimate quality





[Huggins, Campbell, Kasprzak, Broderick 2018]

Roadmap

- The “core” of the data set
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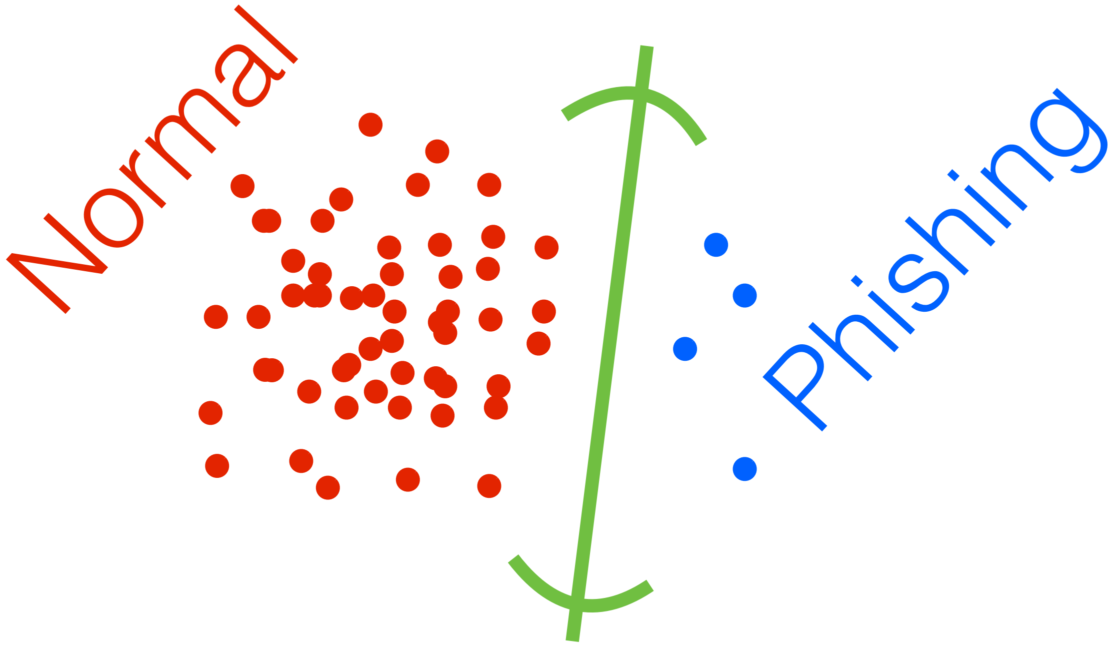

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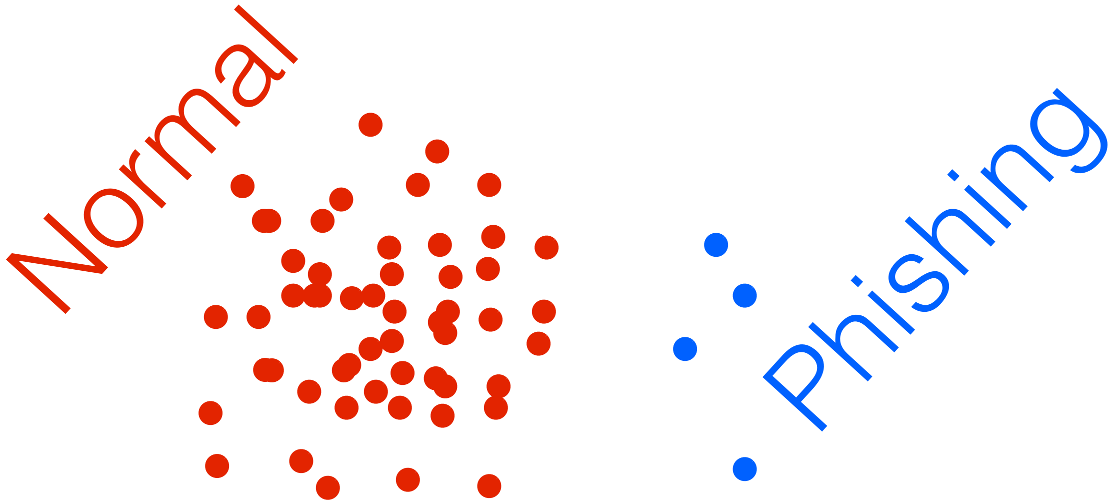

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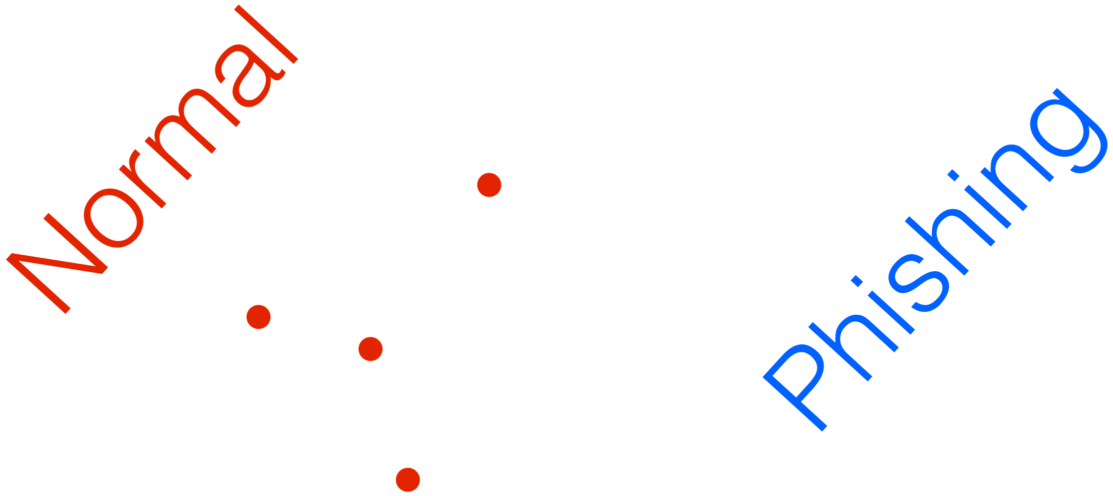

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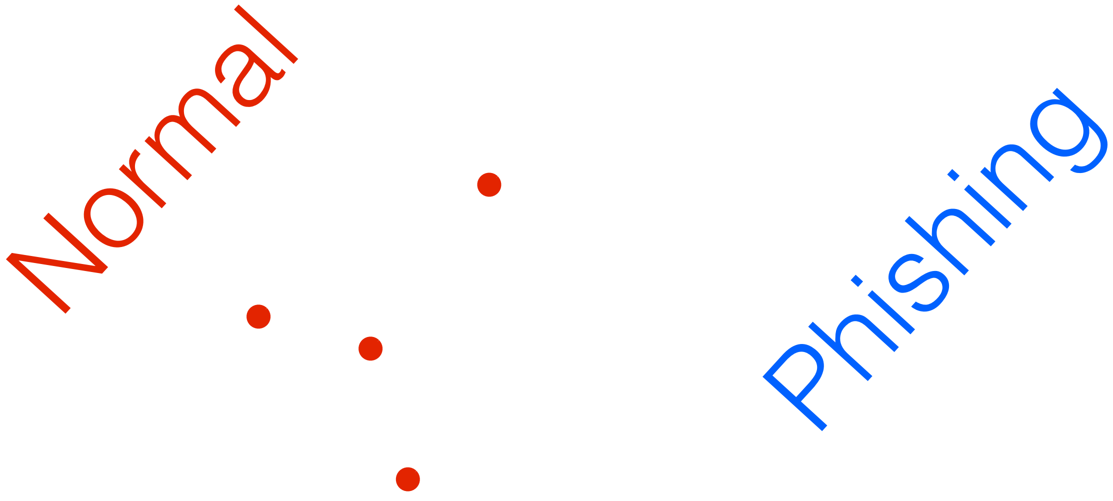

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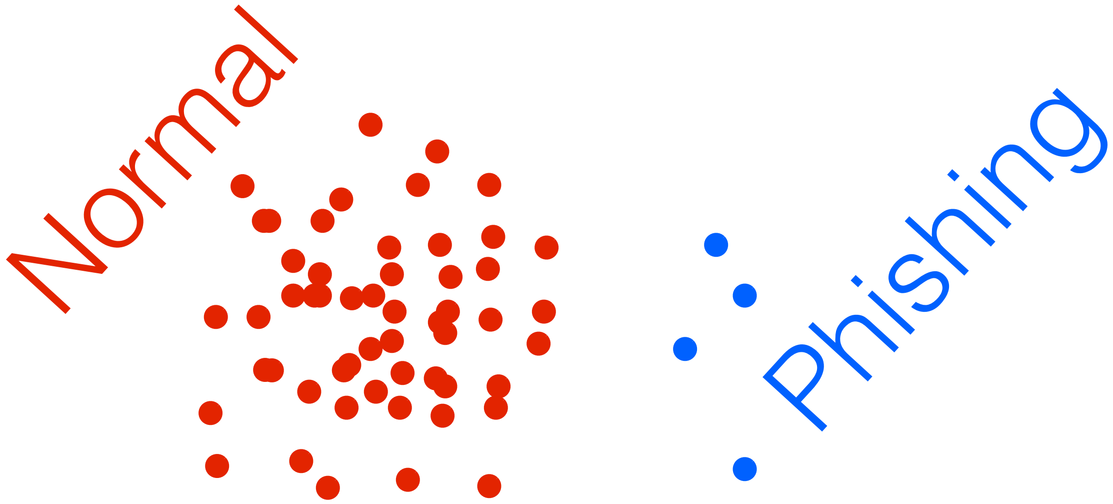

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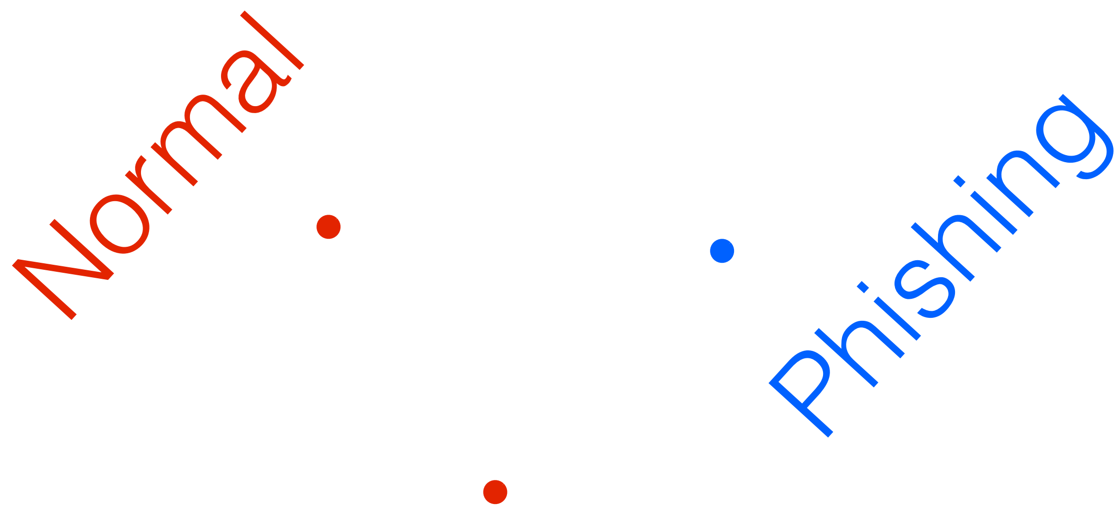

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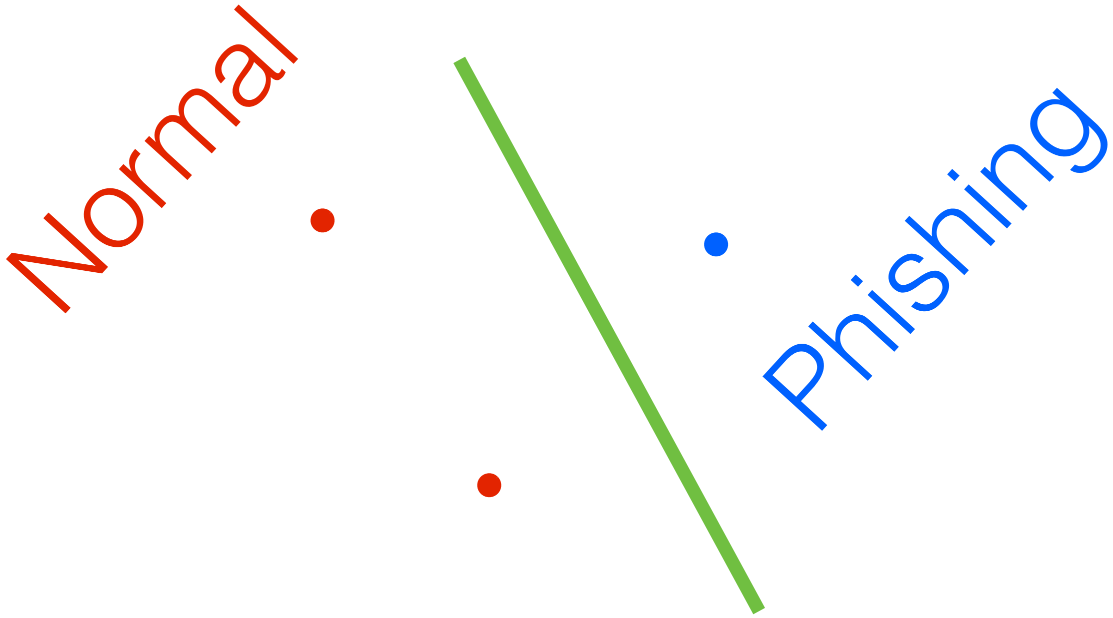

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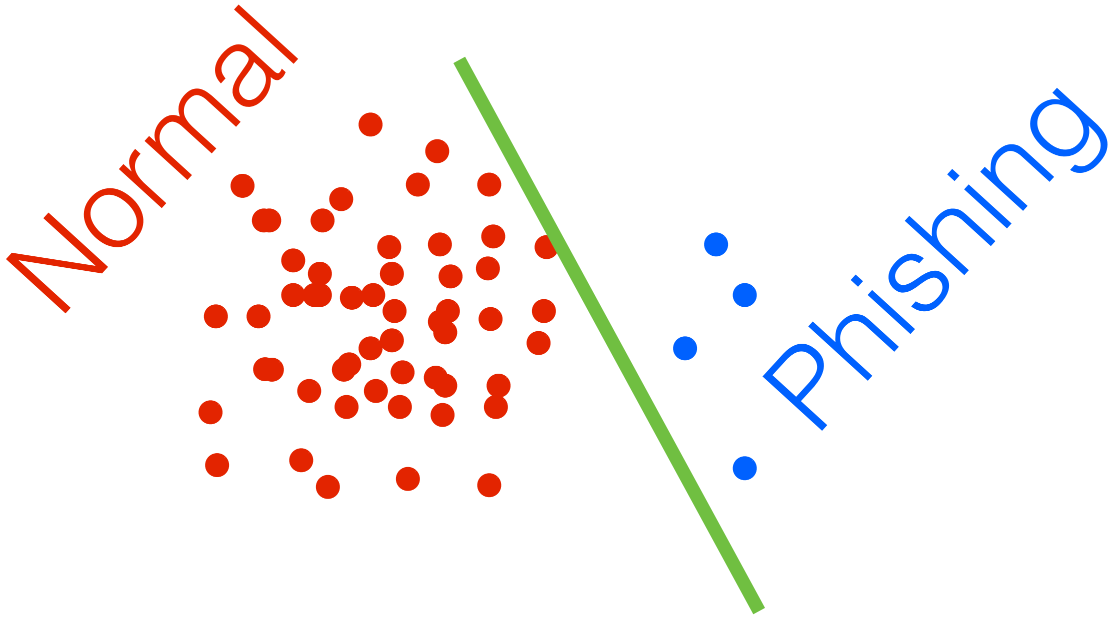

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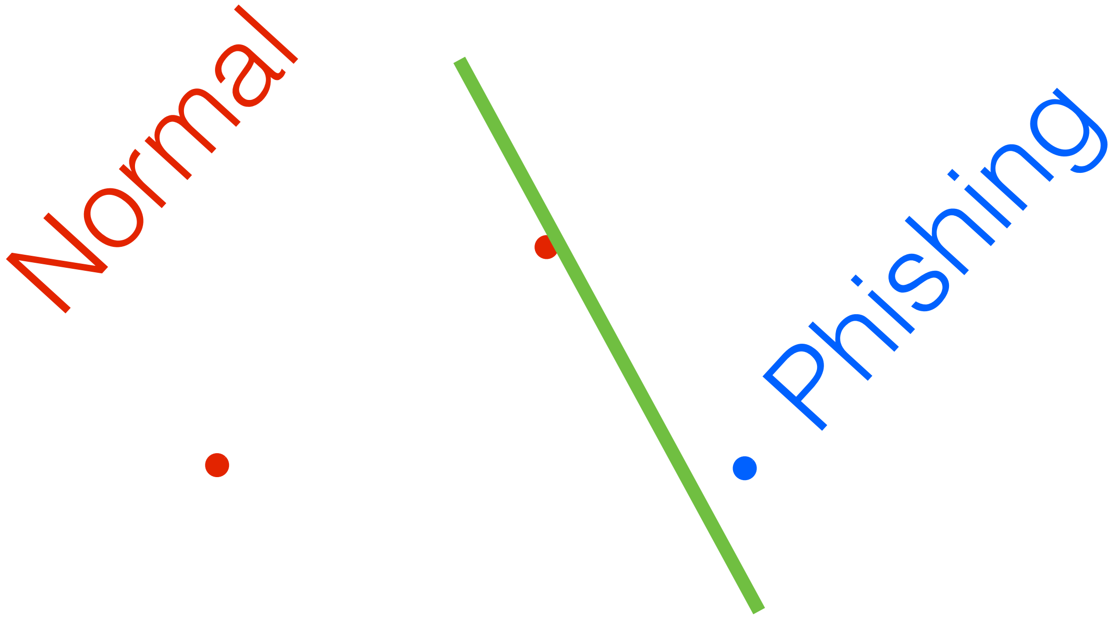

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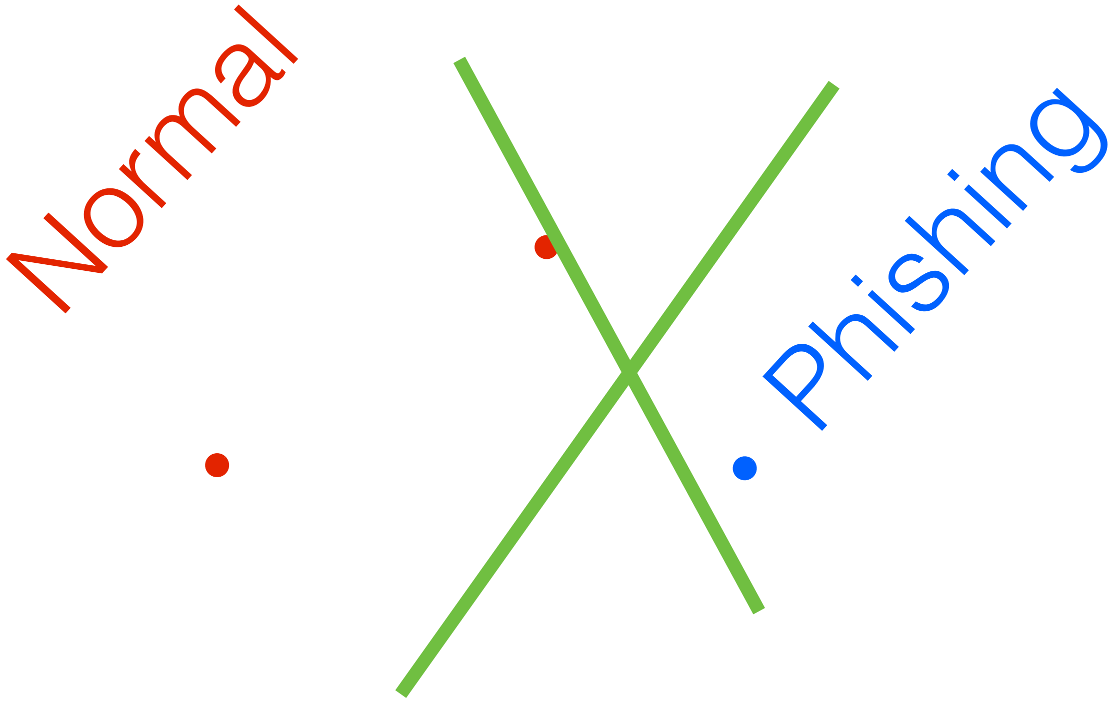

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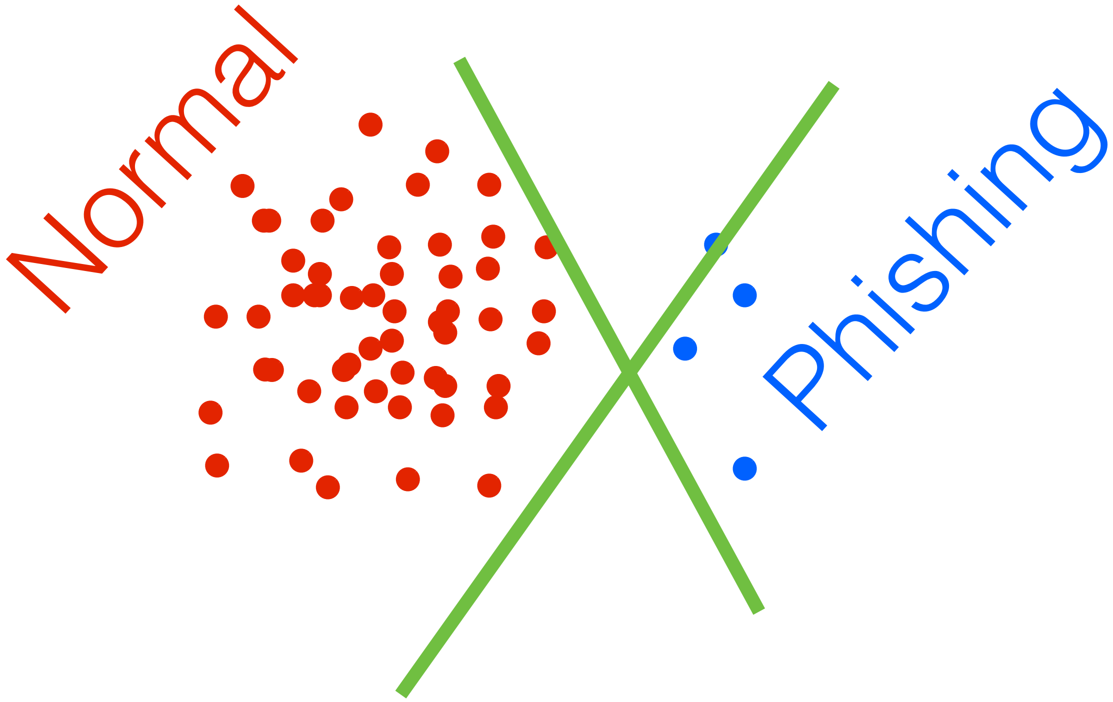

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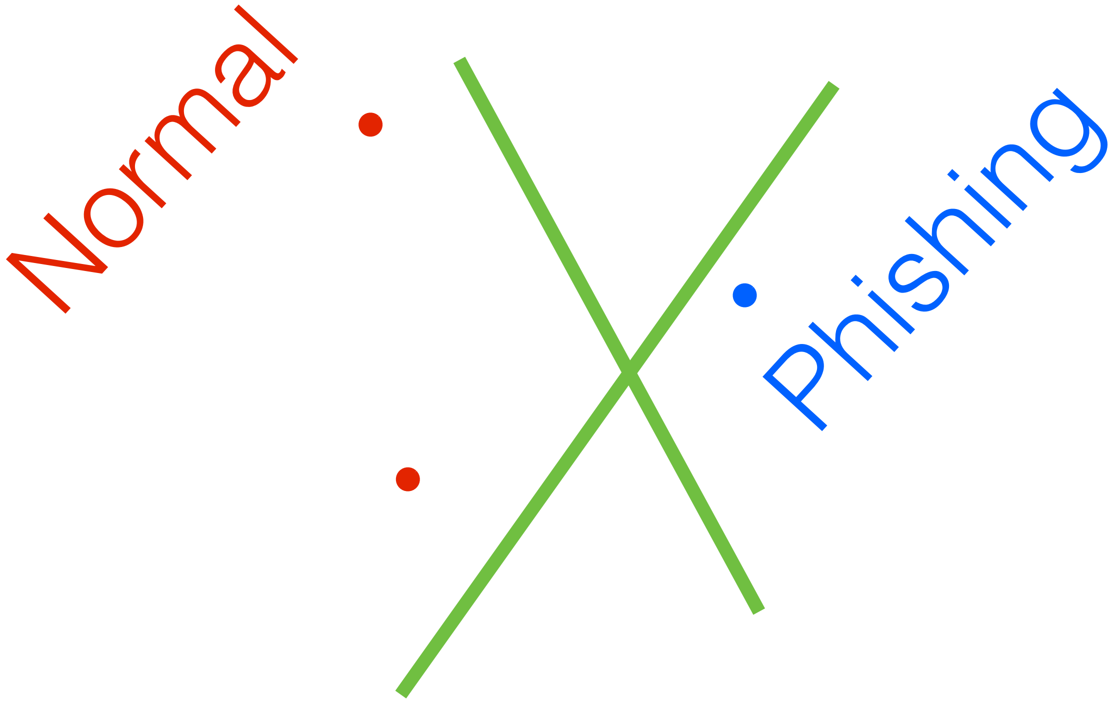

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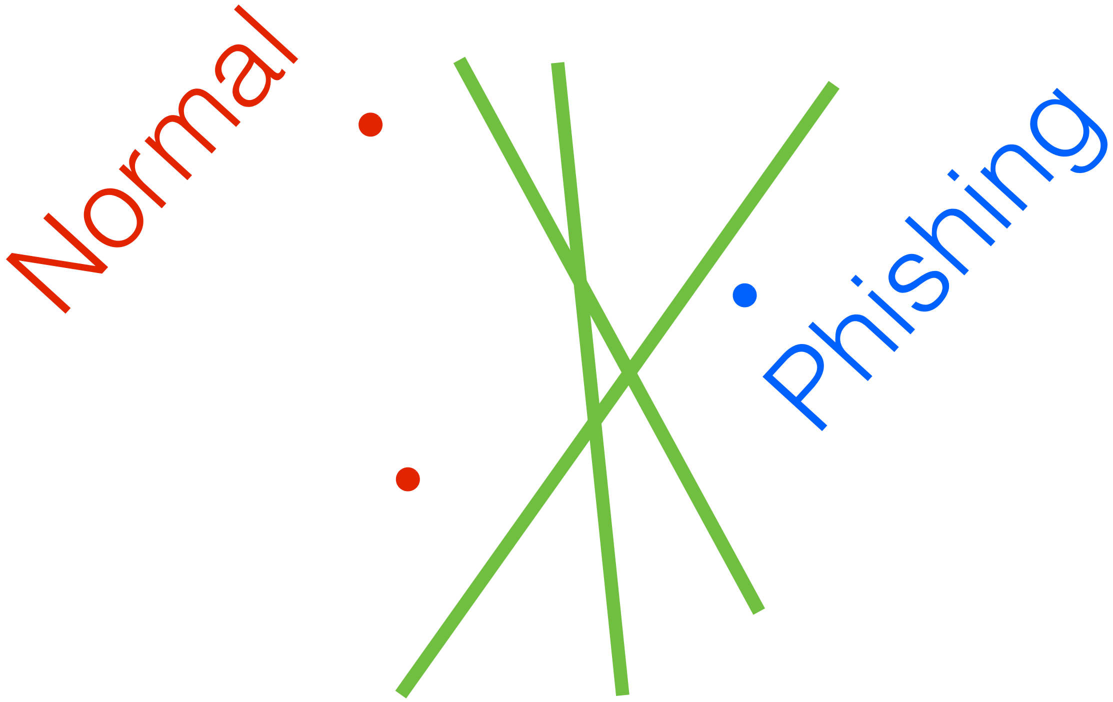

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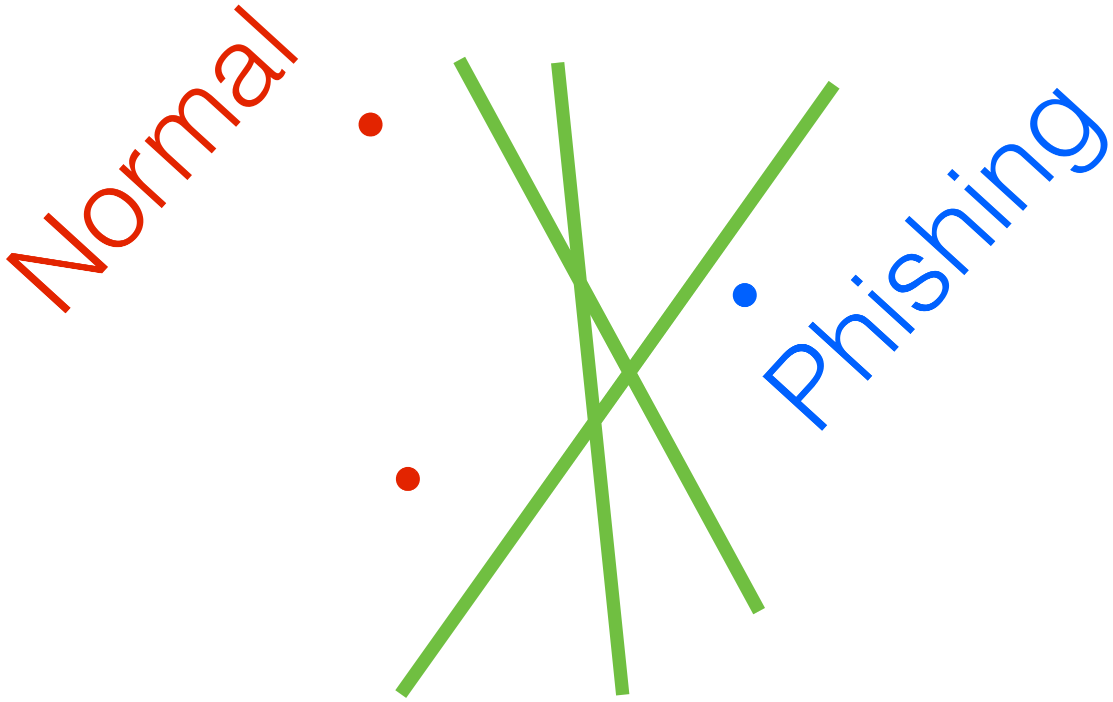

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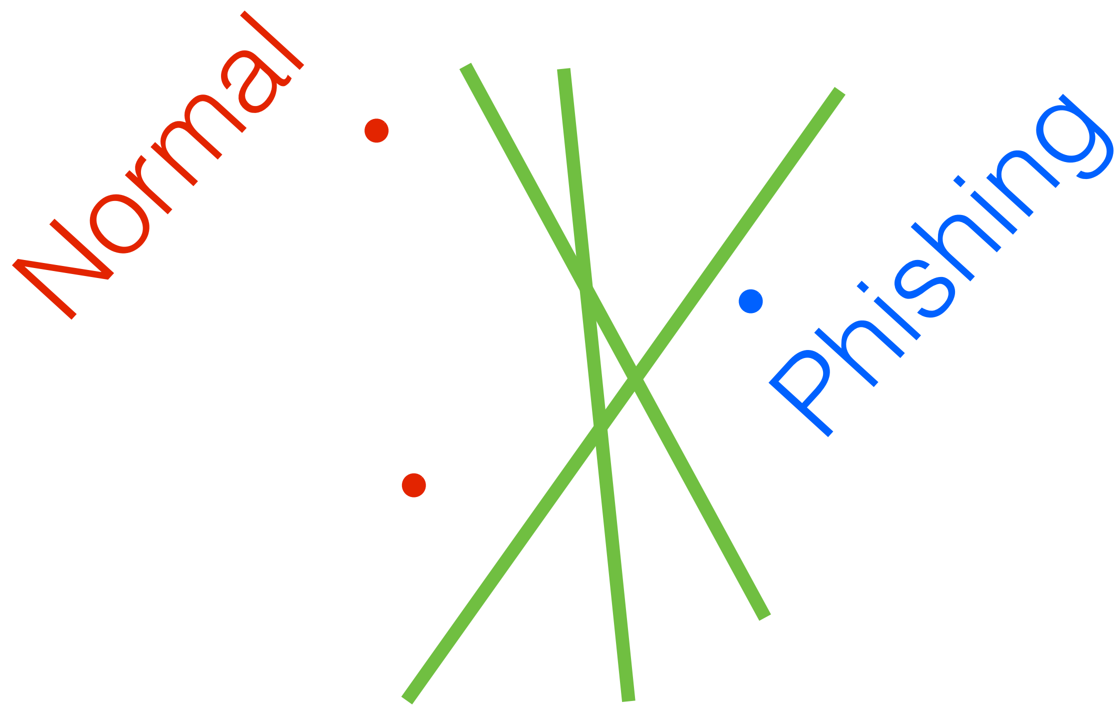
Uniform subsampling revisited



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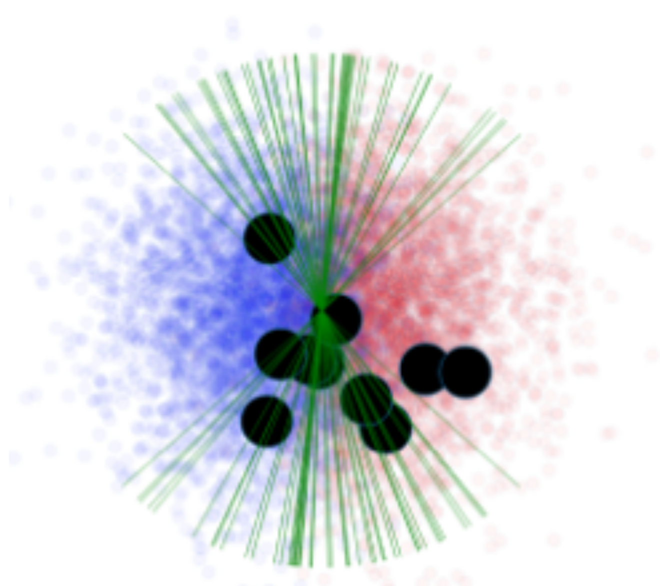

$M = 10$

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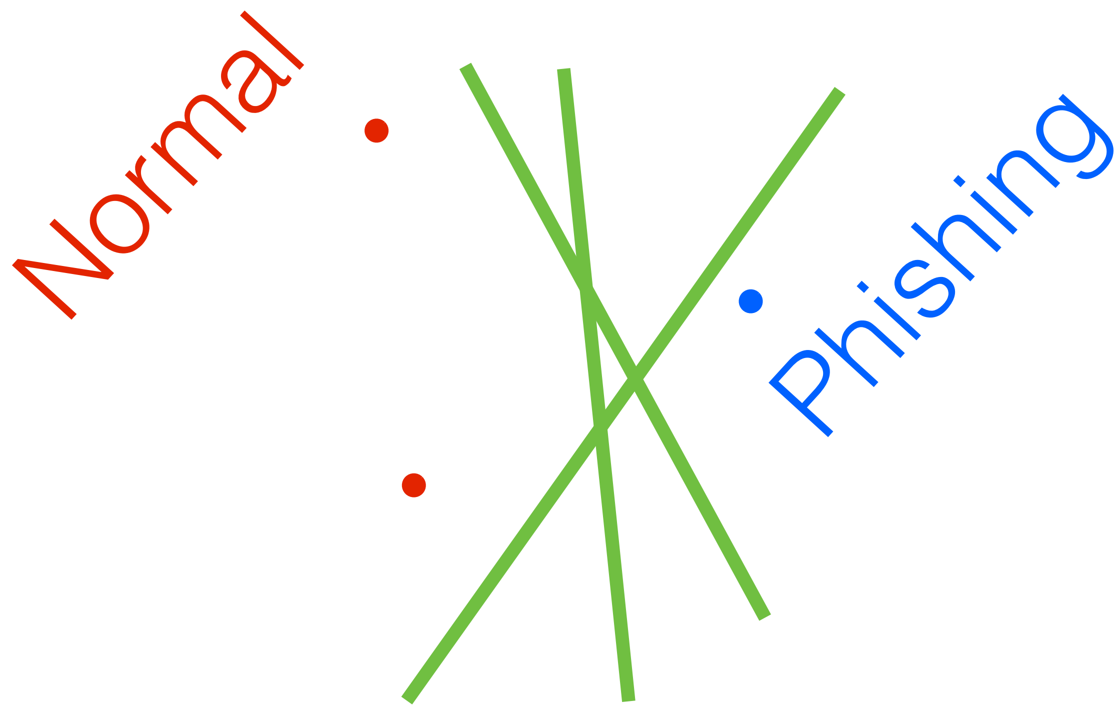

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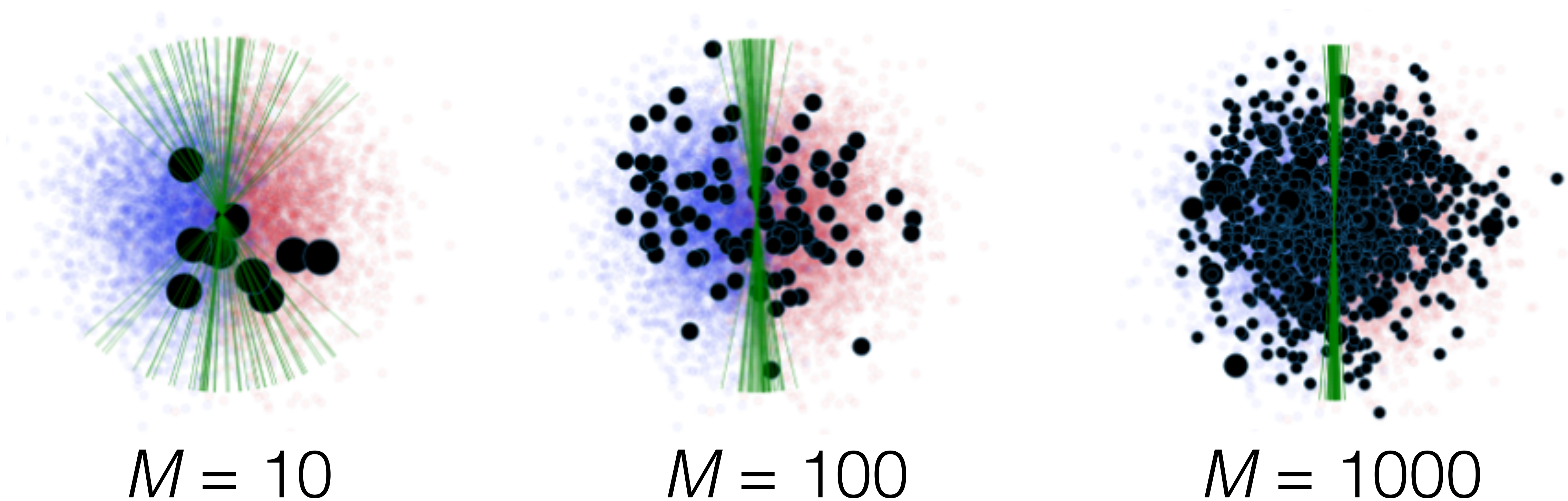

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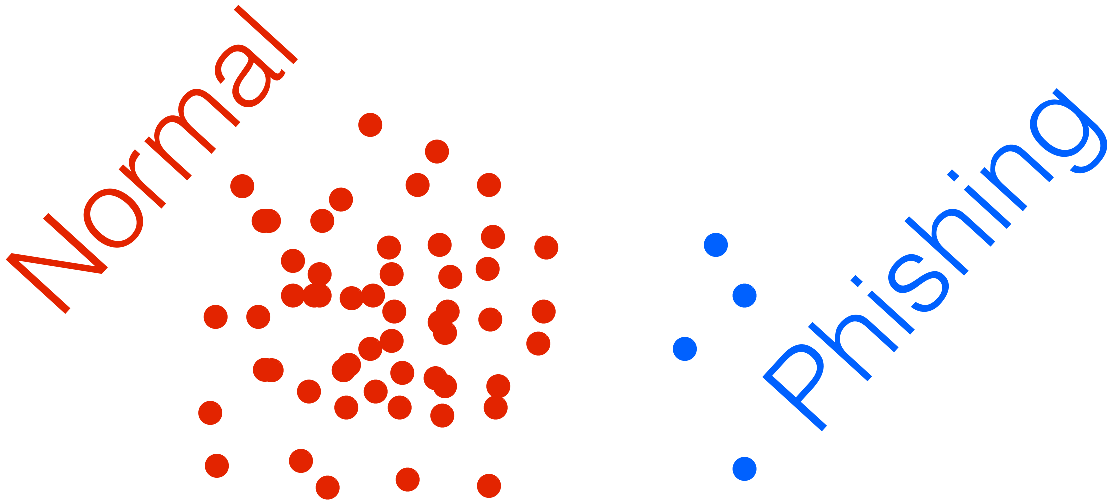

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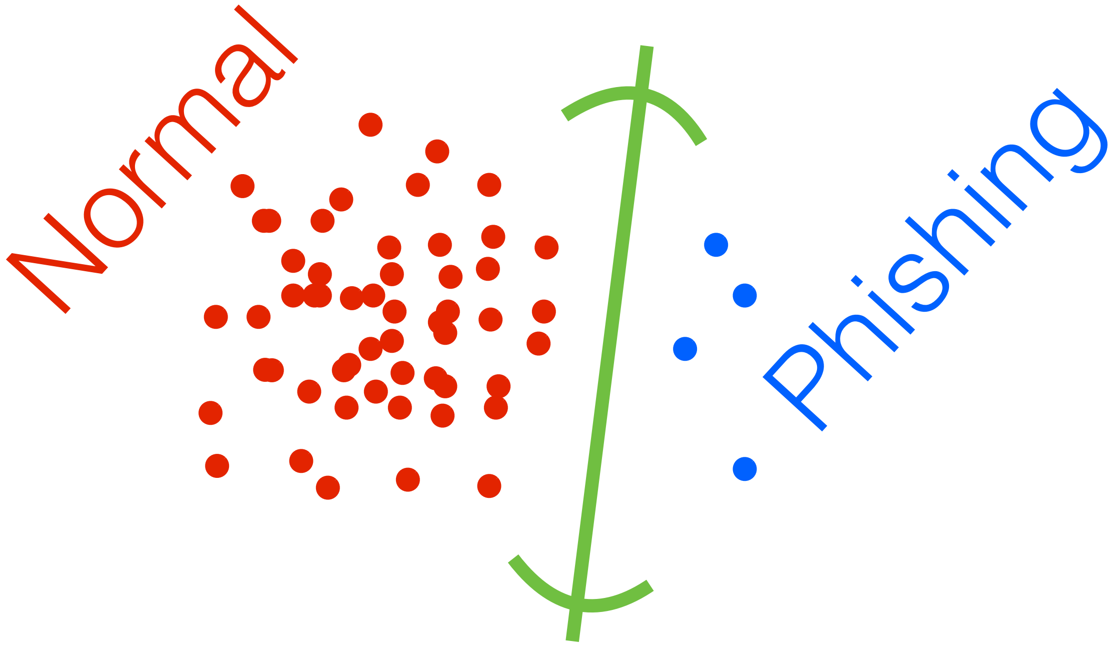

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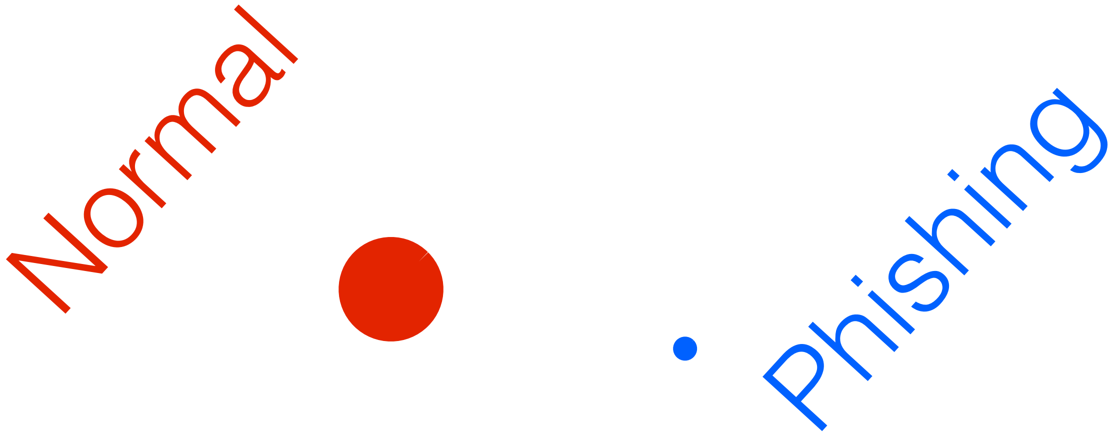

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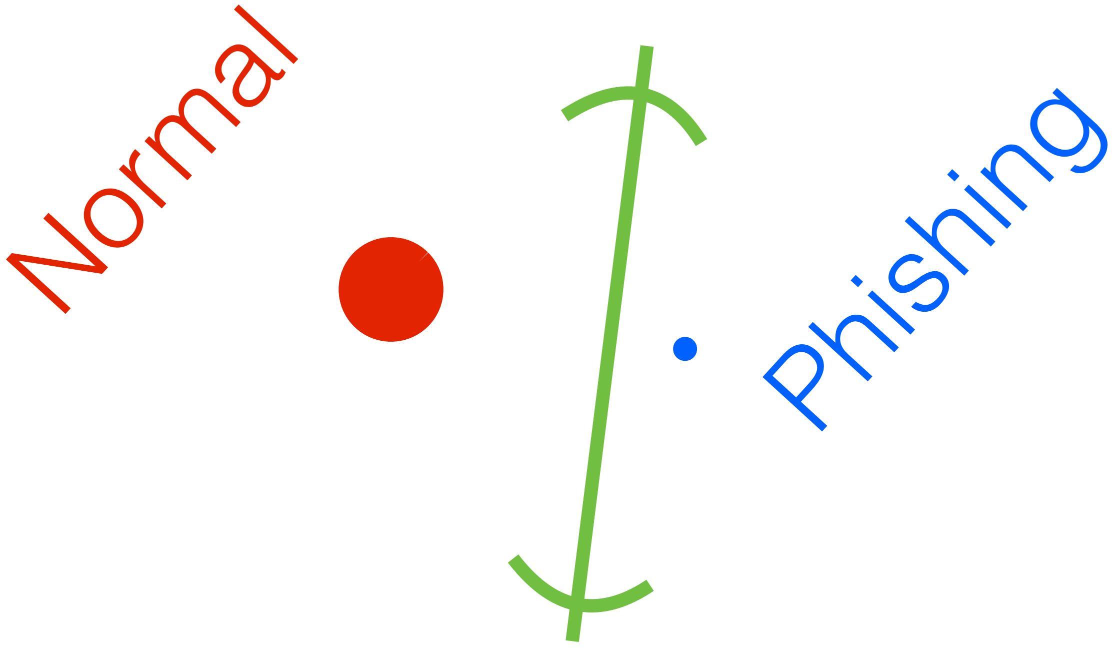

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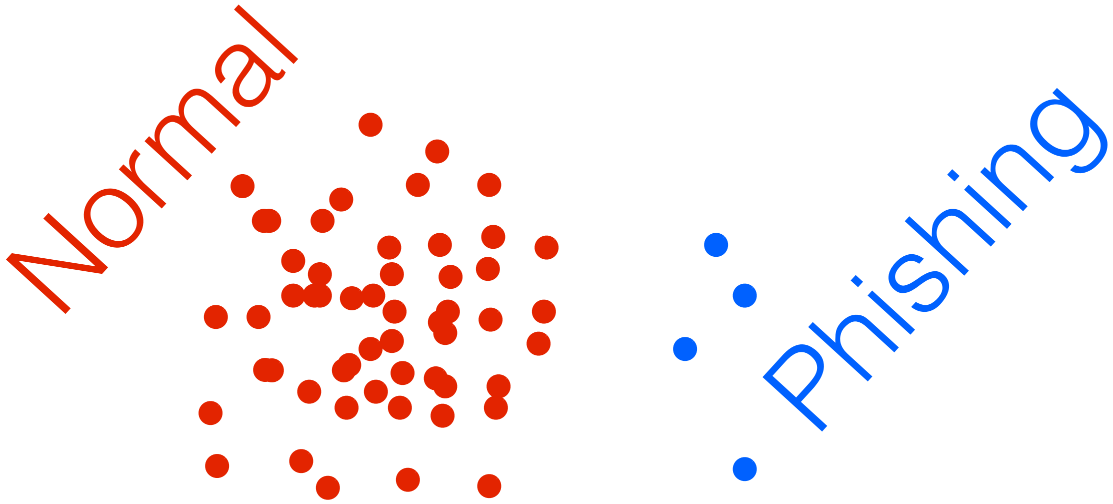

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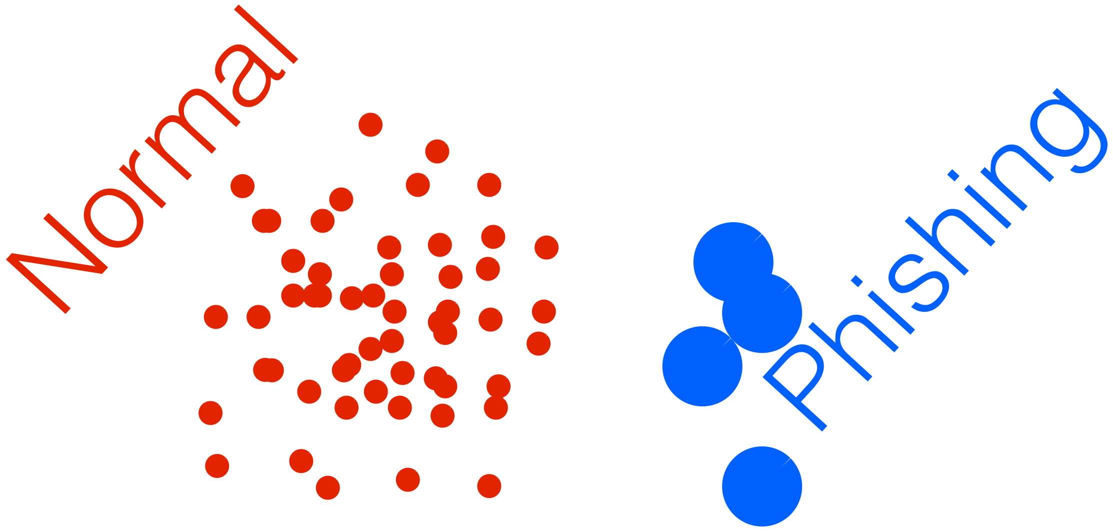

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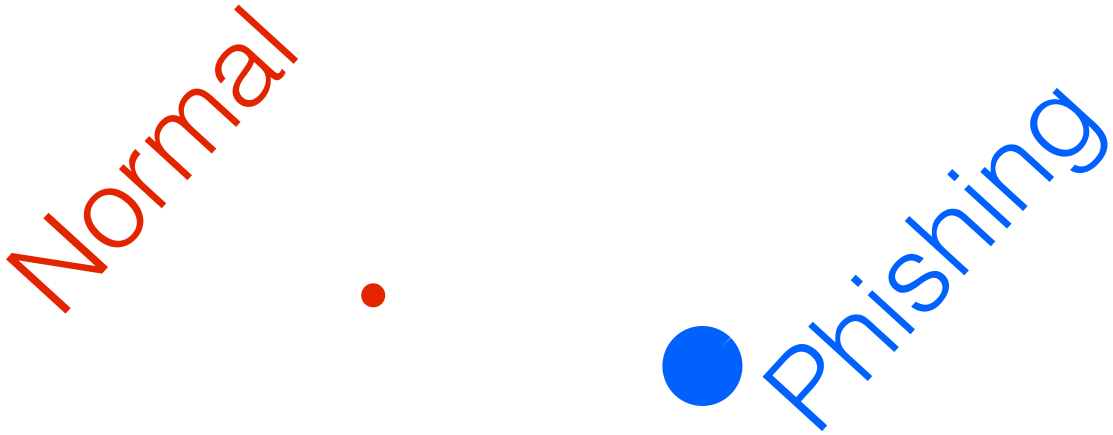

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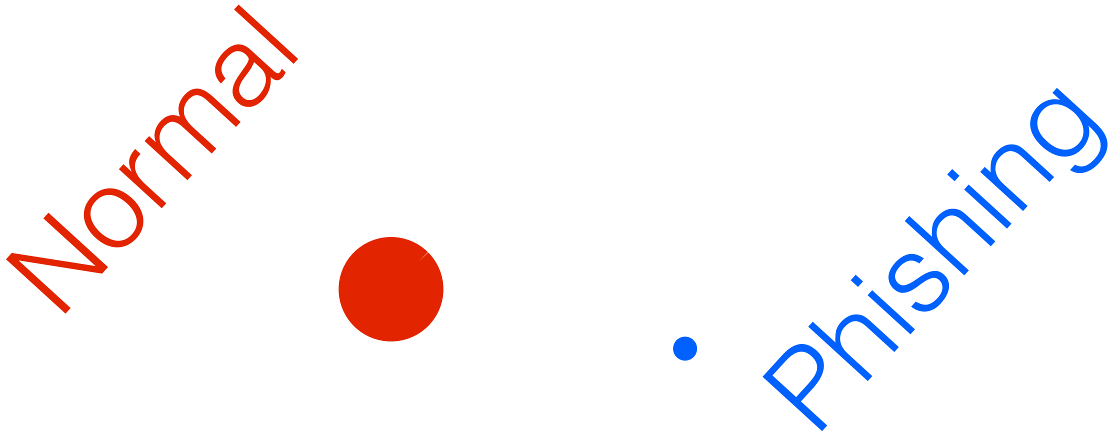

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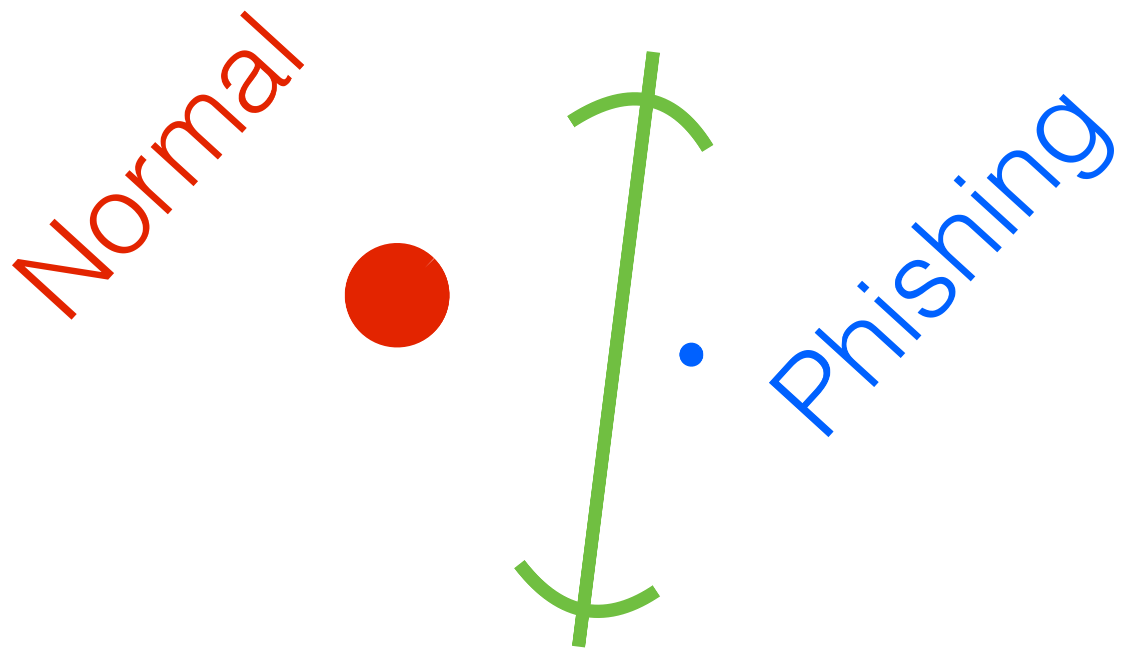

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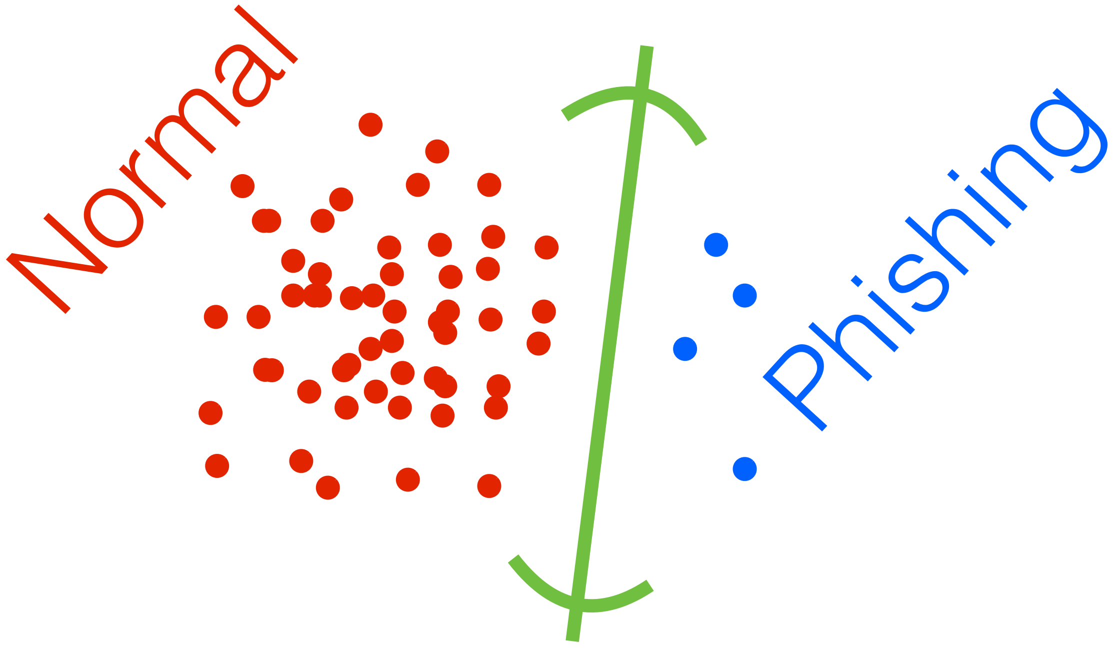

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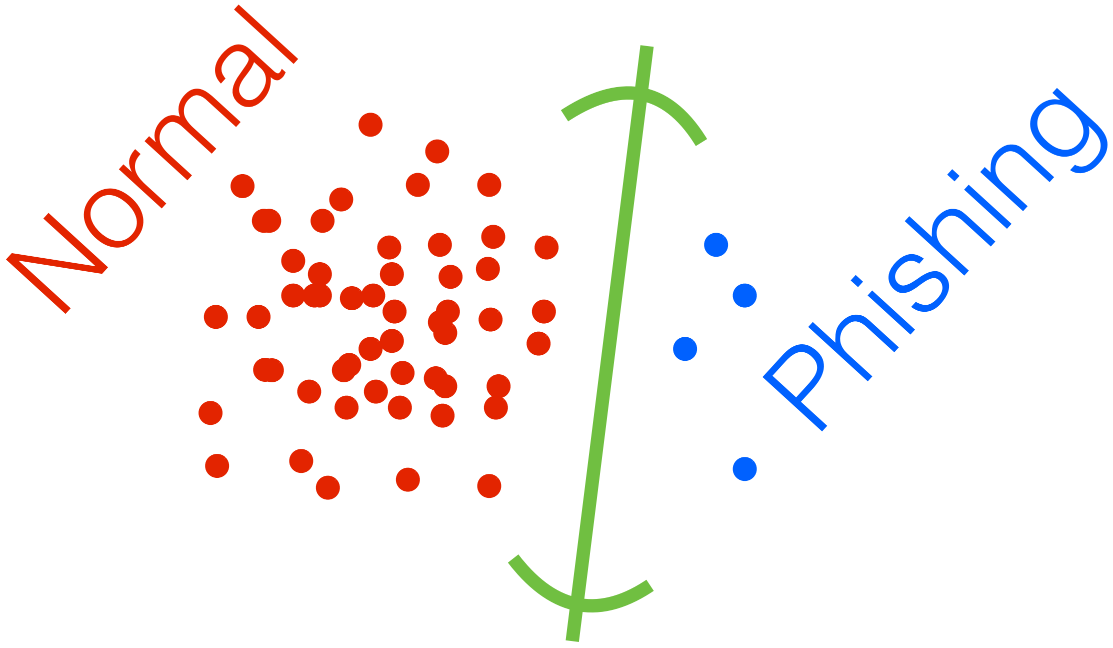
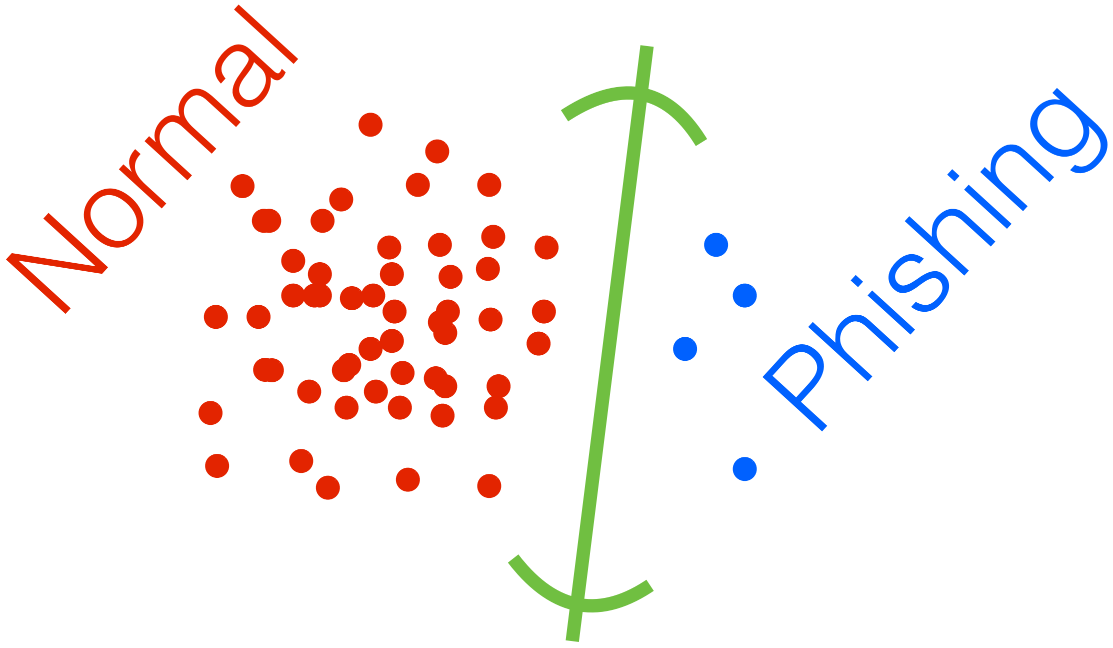


$$\sigma_n \propto \|\mathcal{L}_n\|$$

Importance sampling



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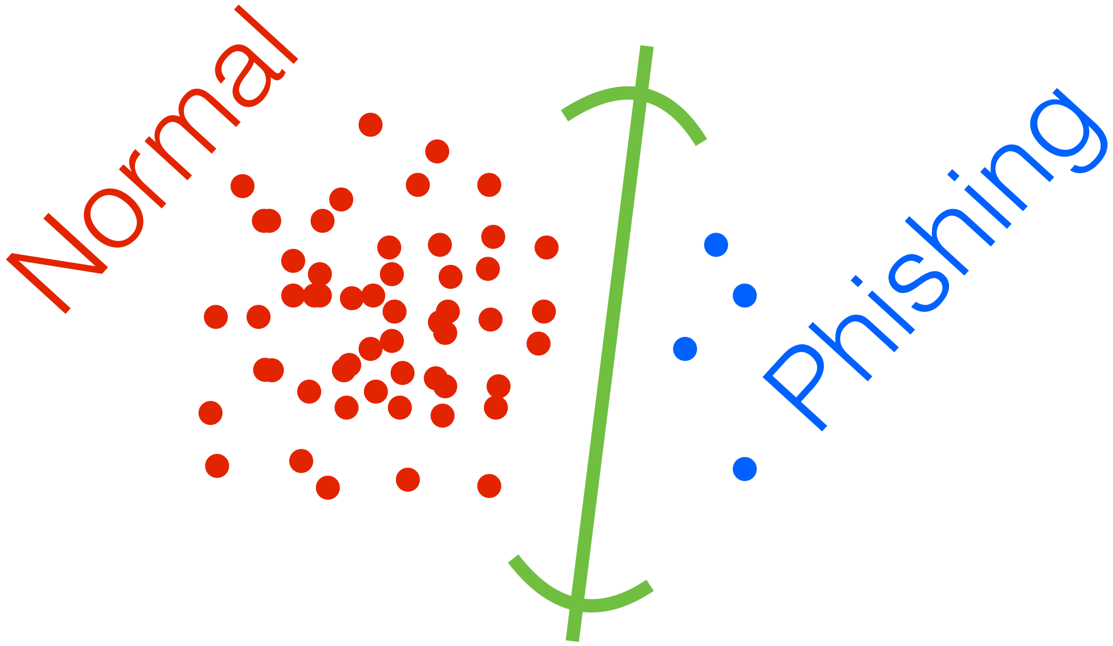

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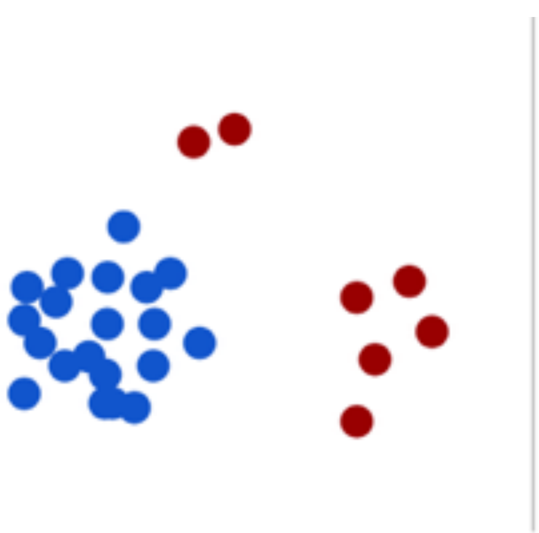
1. data



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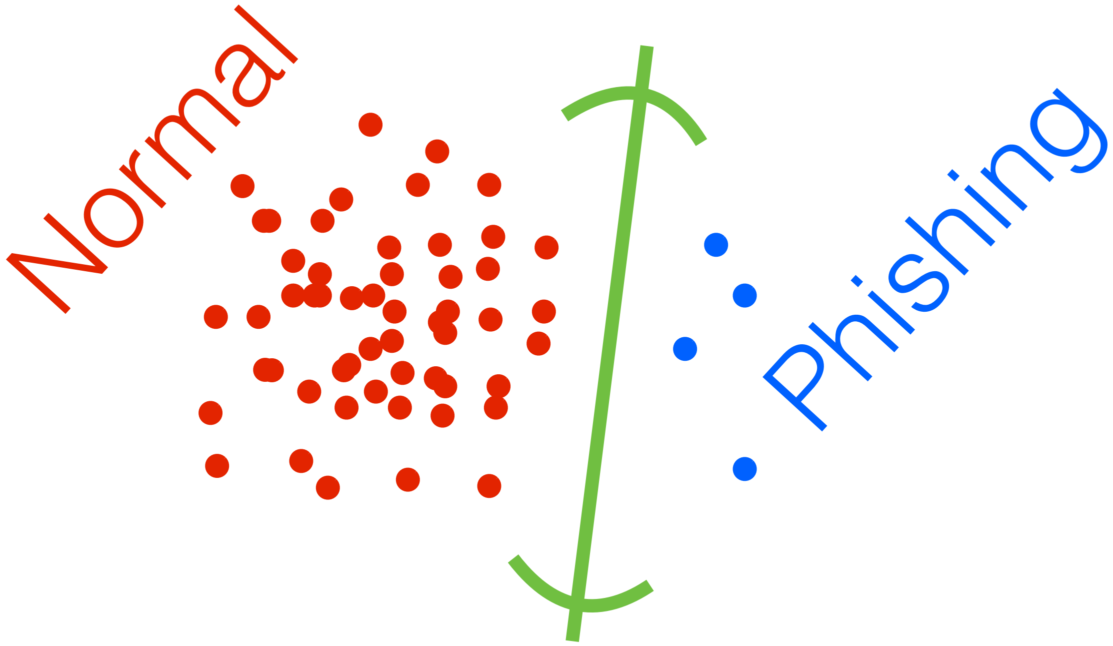

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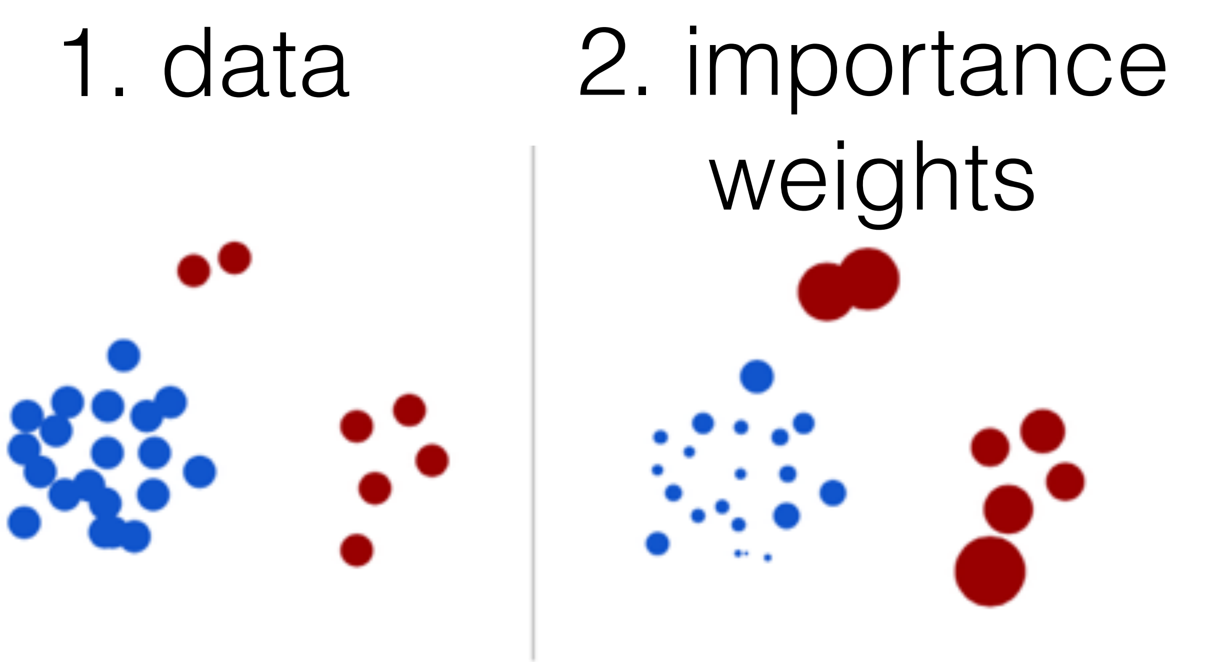
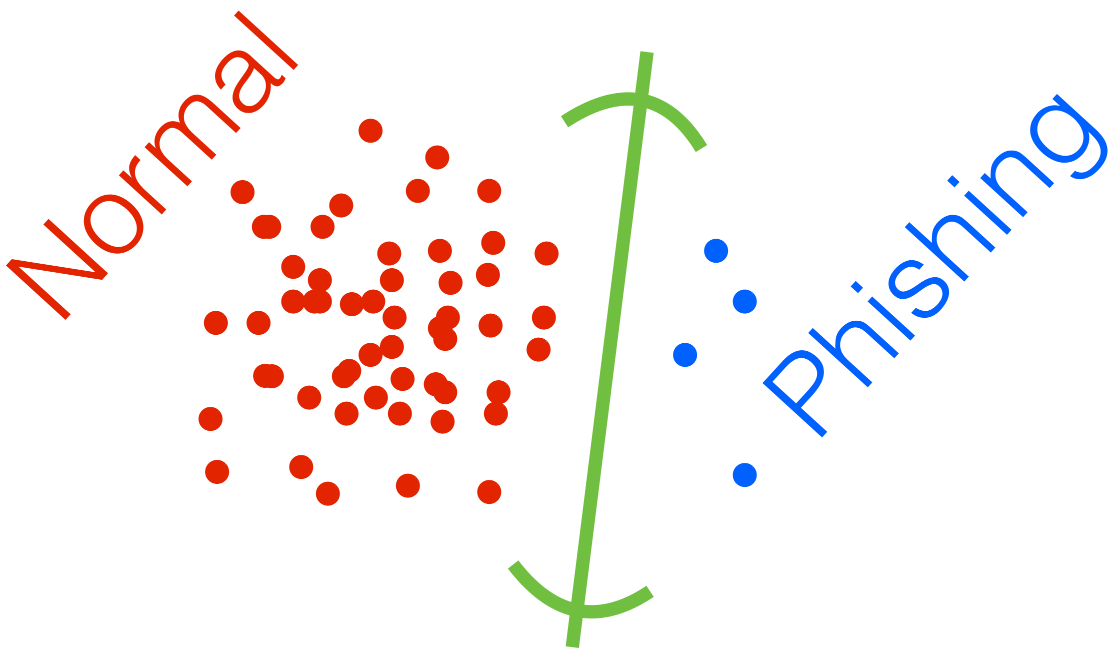


Importance sampling



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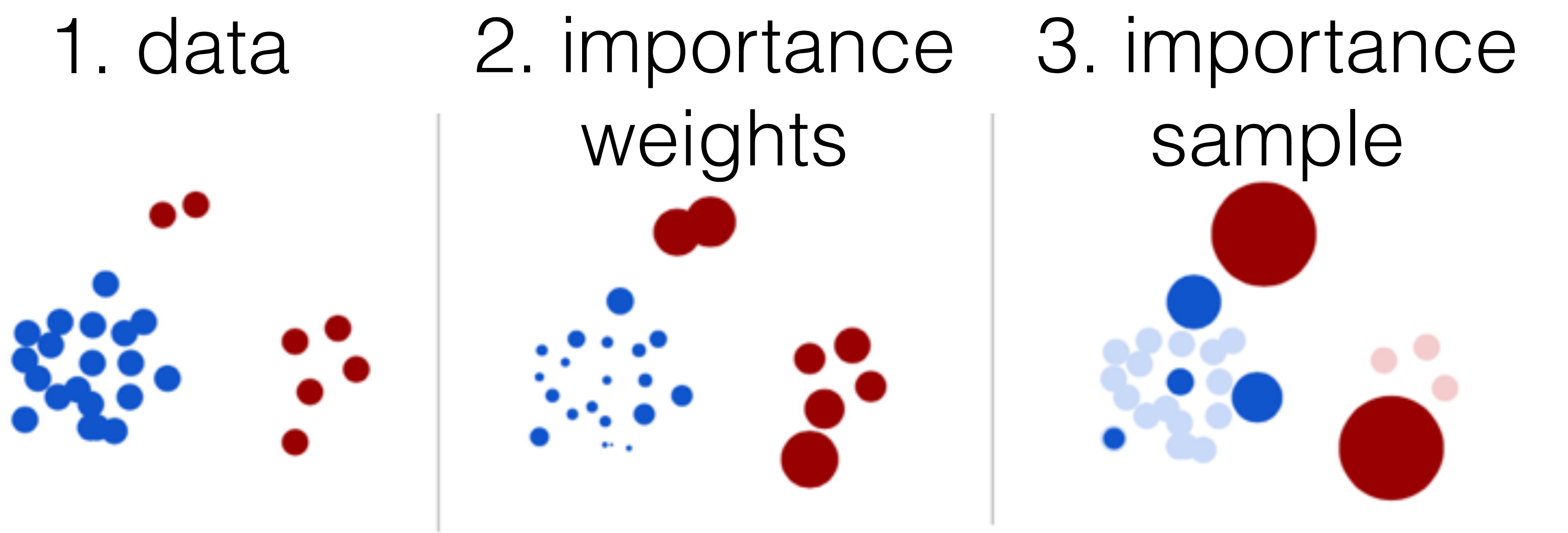

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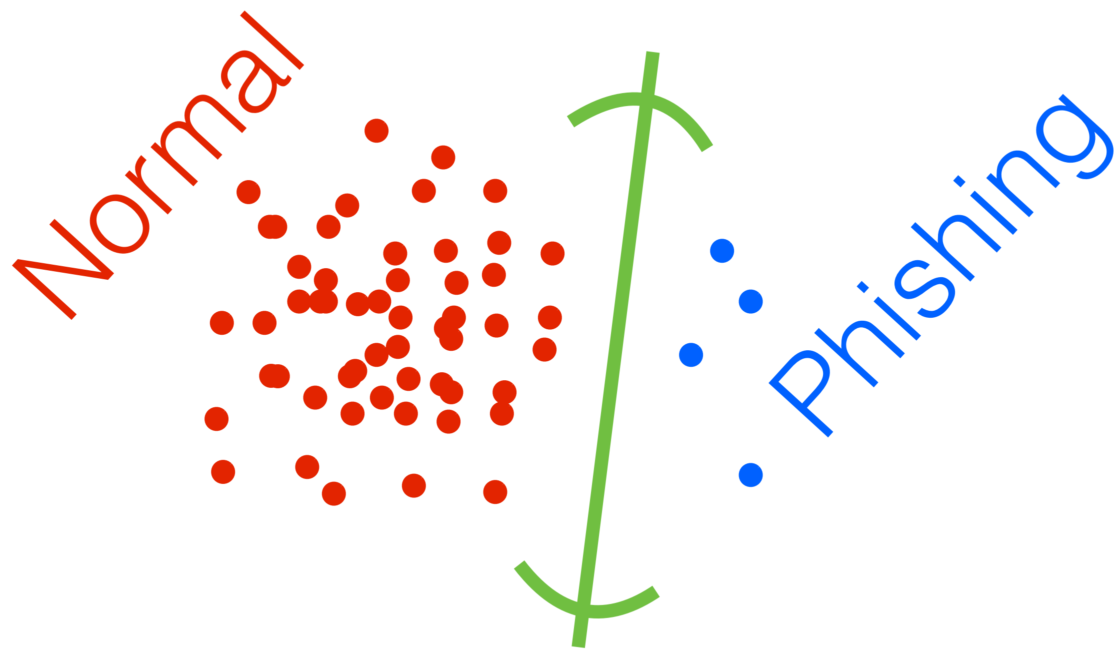

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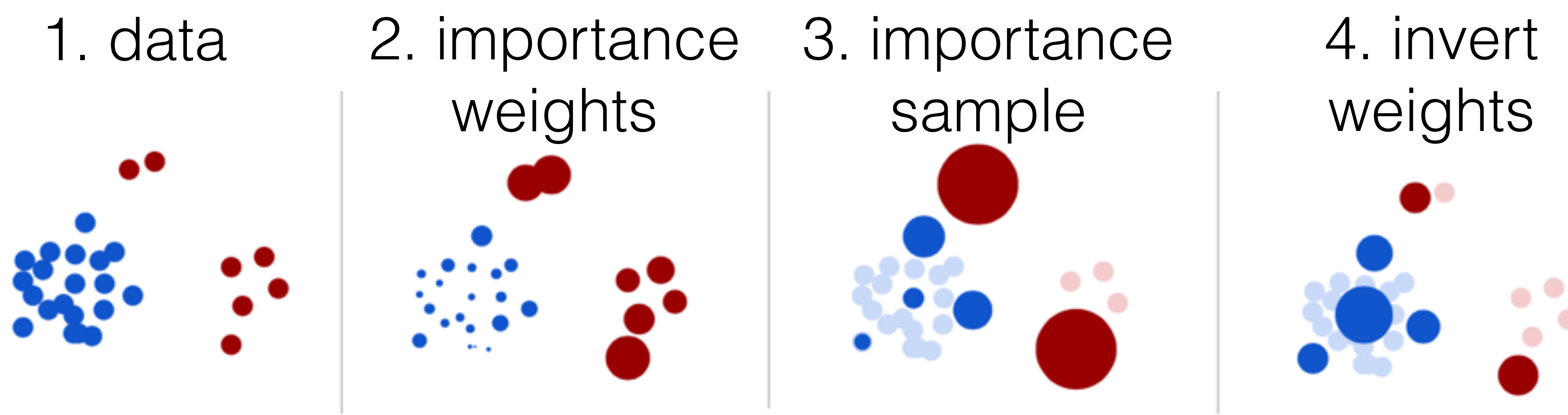

Importance sampling

Thm sketch (CB). $\delta \in (0,1)$. W.p. $\geq 1 - \delta$, after M iterations,

$$\|\mathcal{L}(w) - \mathcal{L}\| \leq \frac{\sigma \bar{\eta}}{\sqrt{M}} \left(1 + \sqrt{2 \log \frac{1}{\delta}}\right)$$

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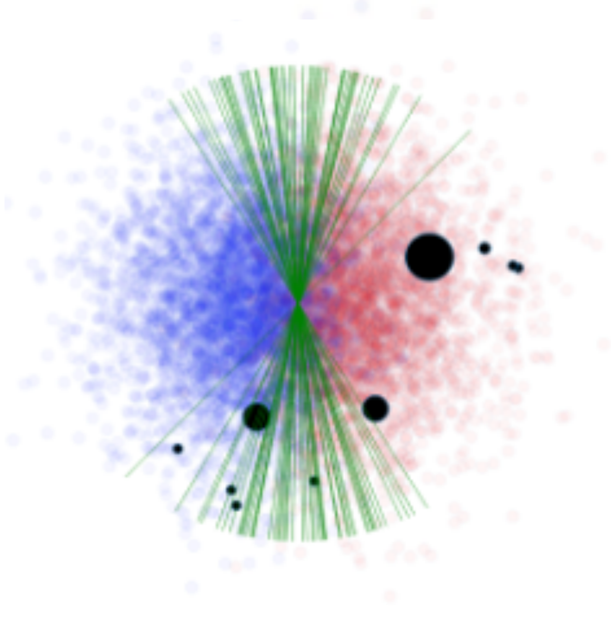

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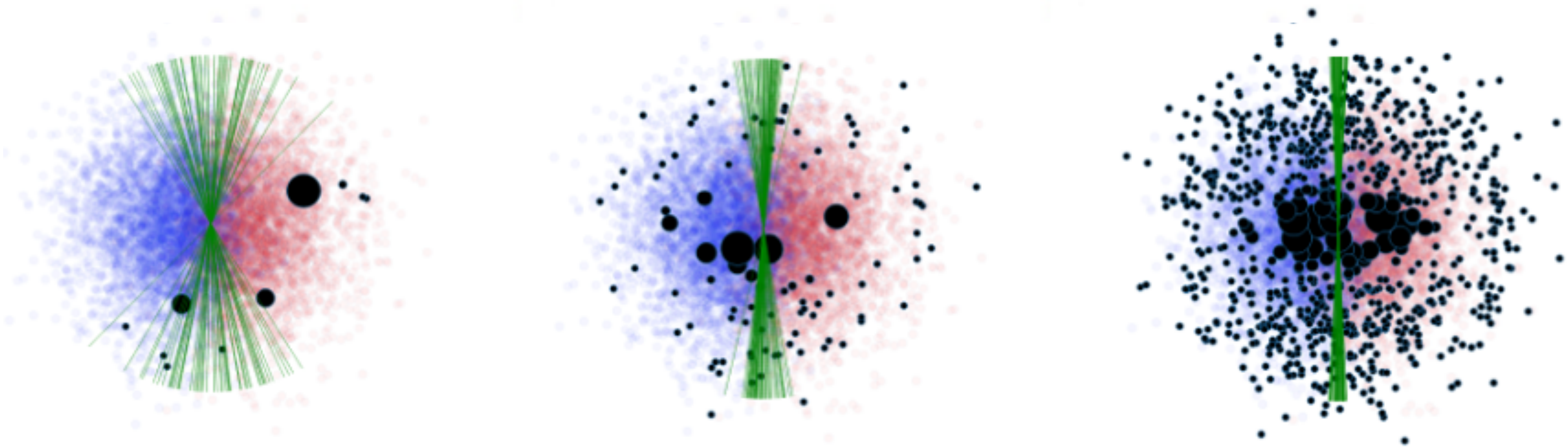

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Hilbert coresets

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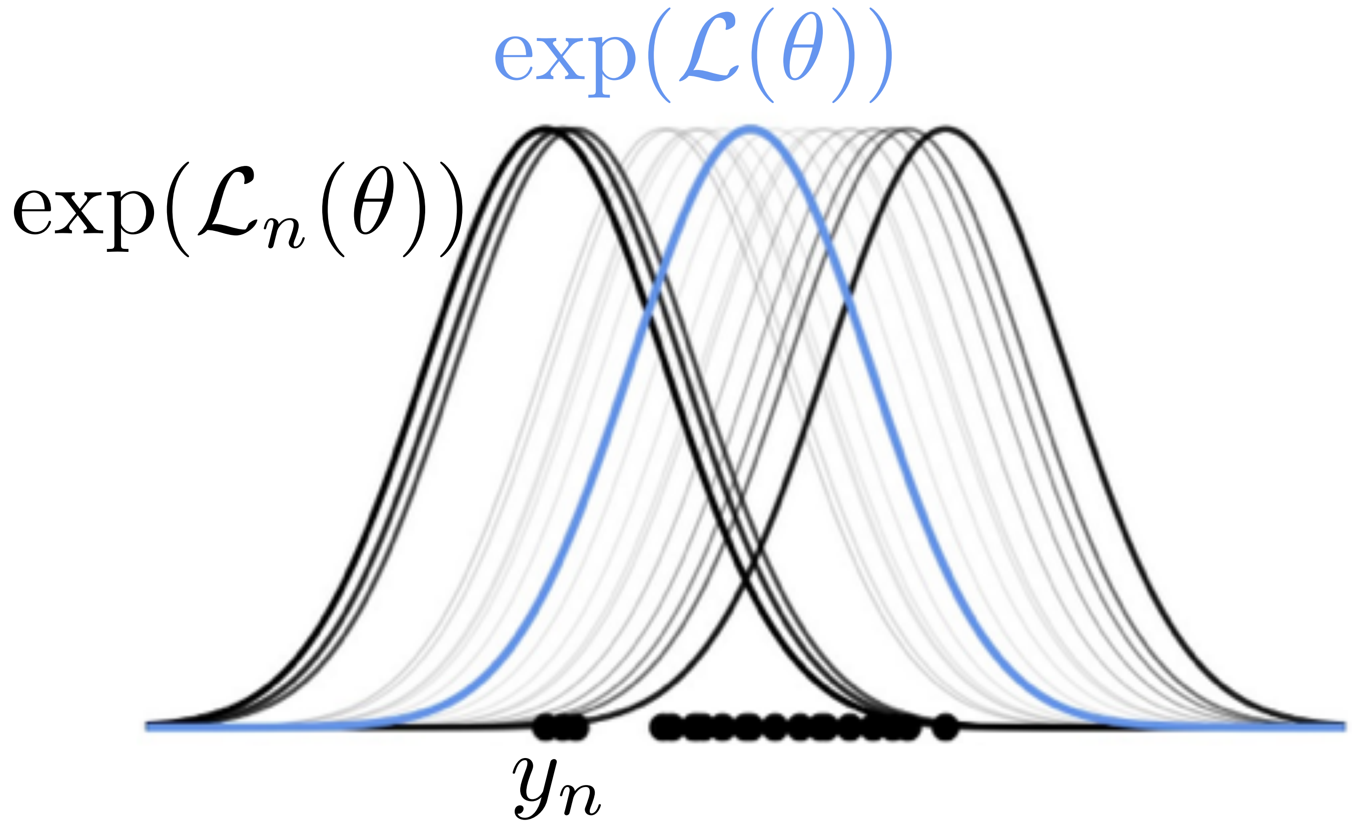

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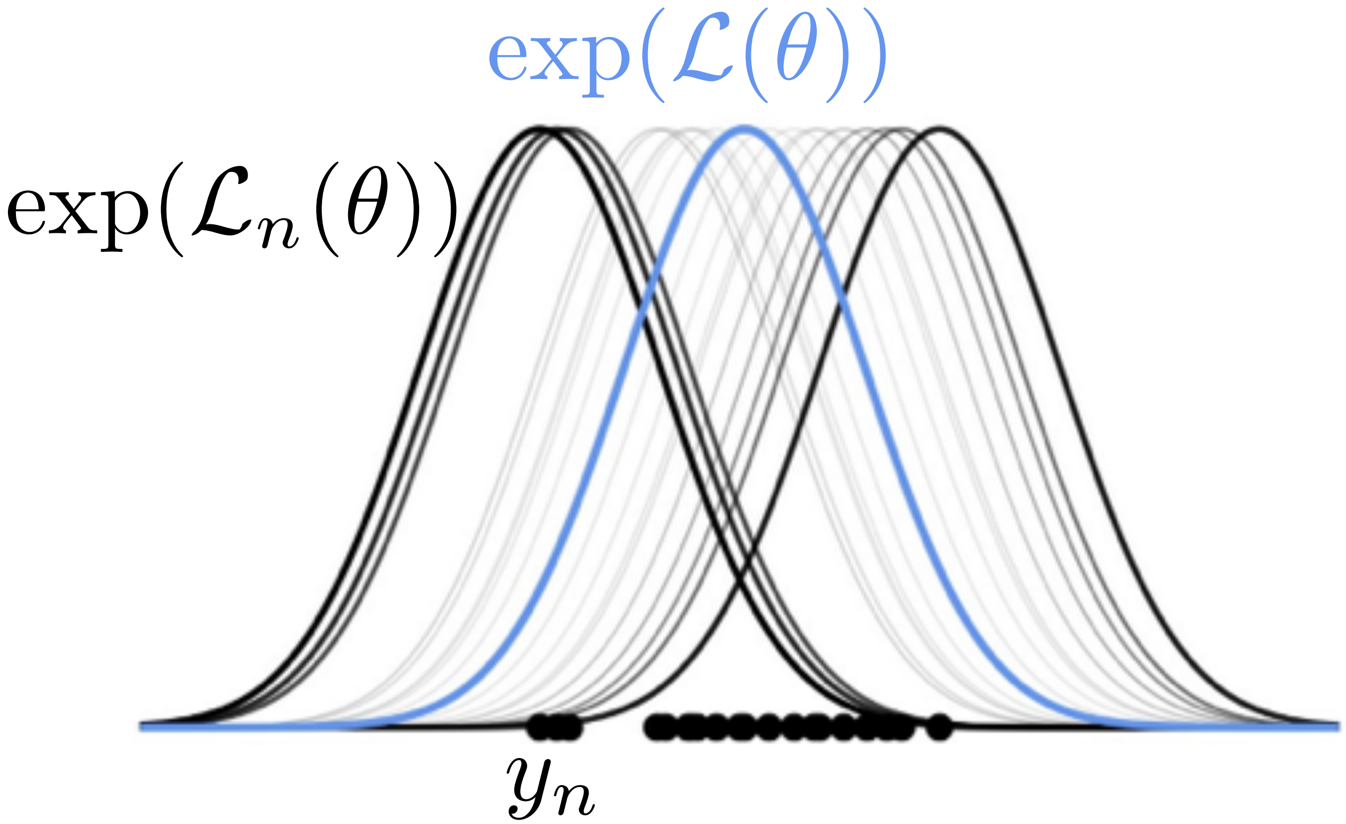

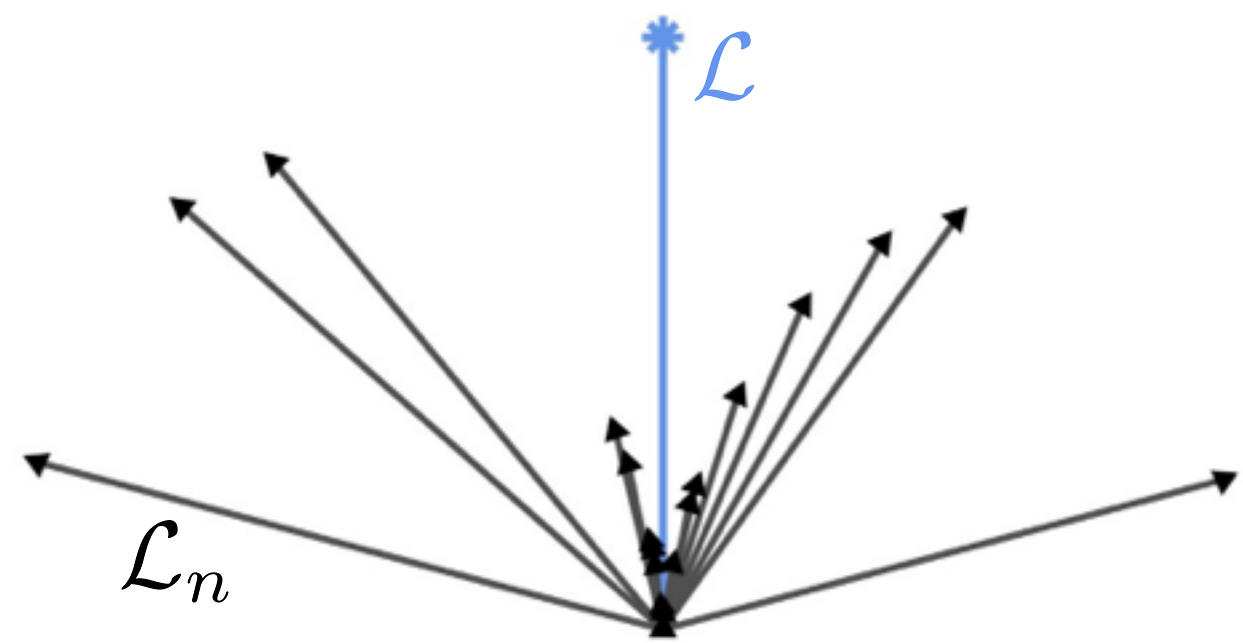

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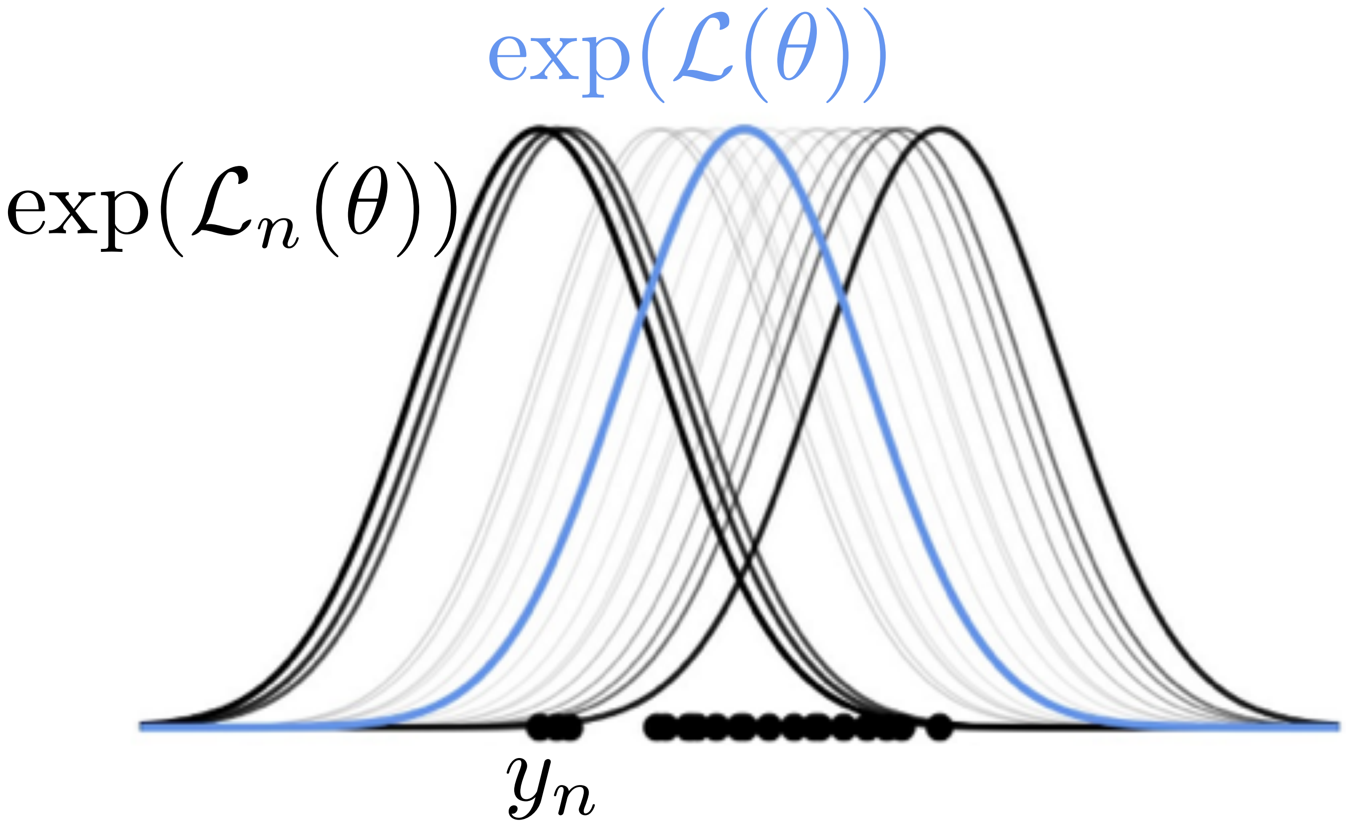

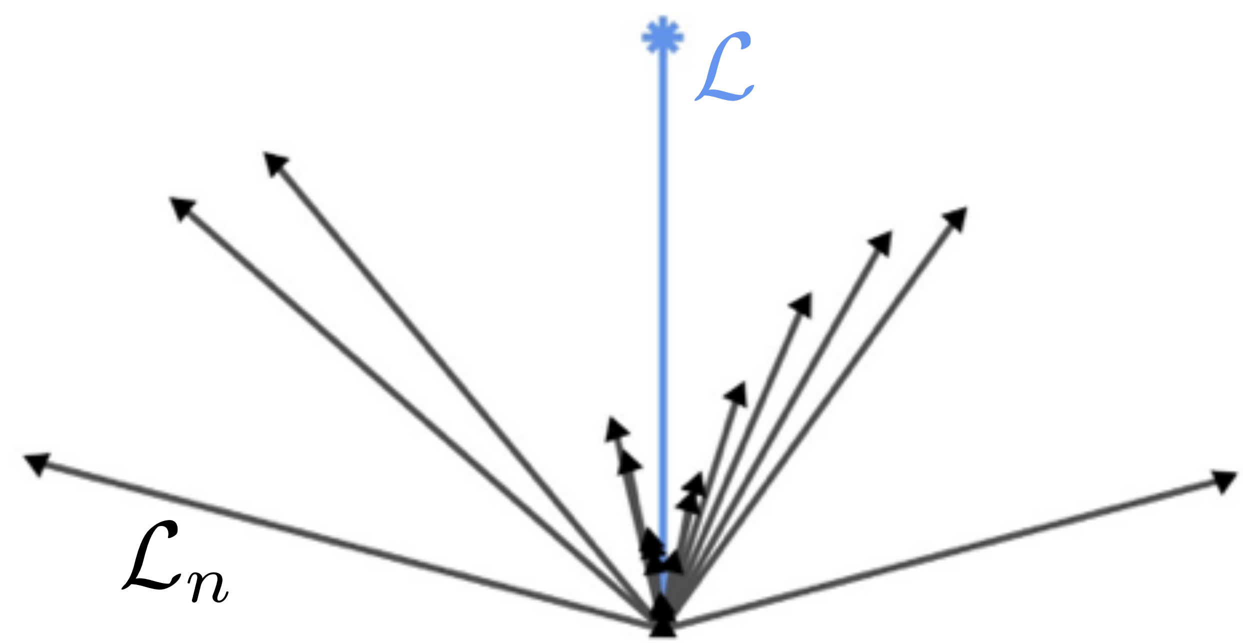

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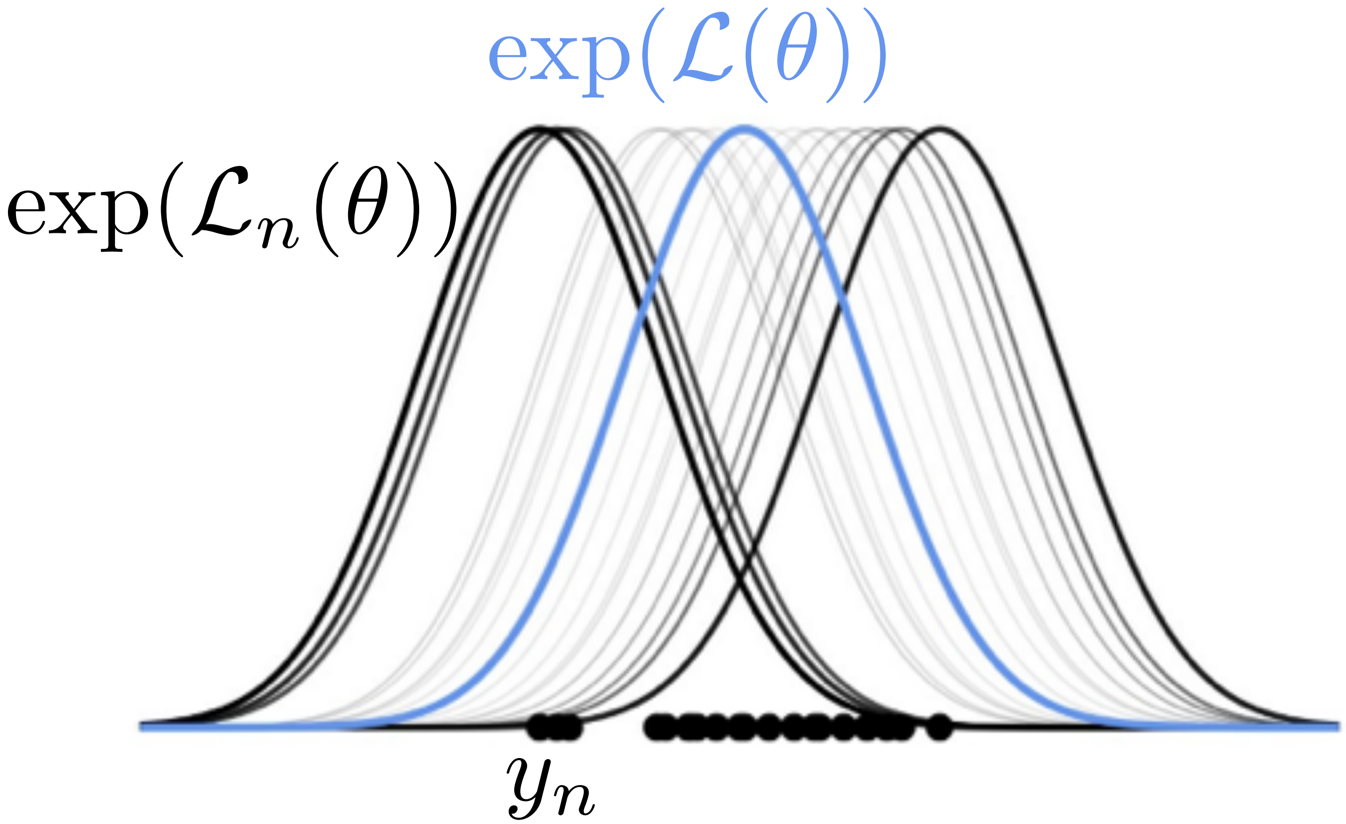

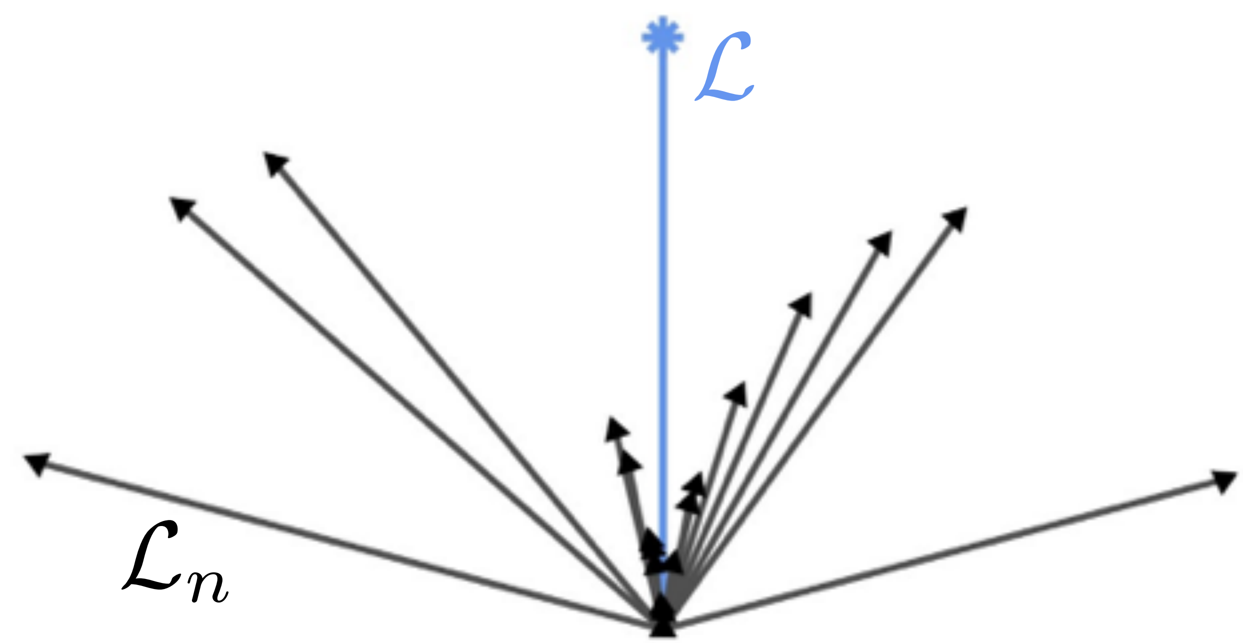

- need to consider (residual) error direction
- sparse optimization

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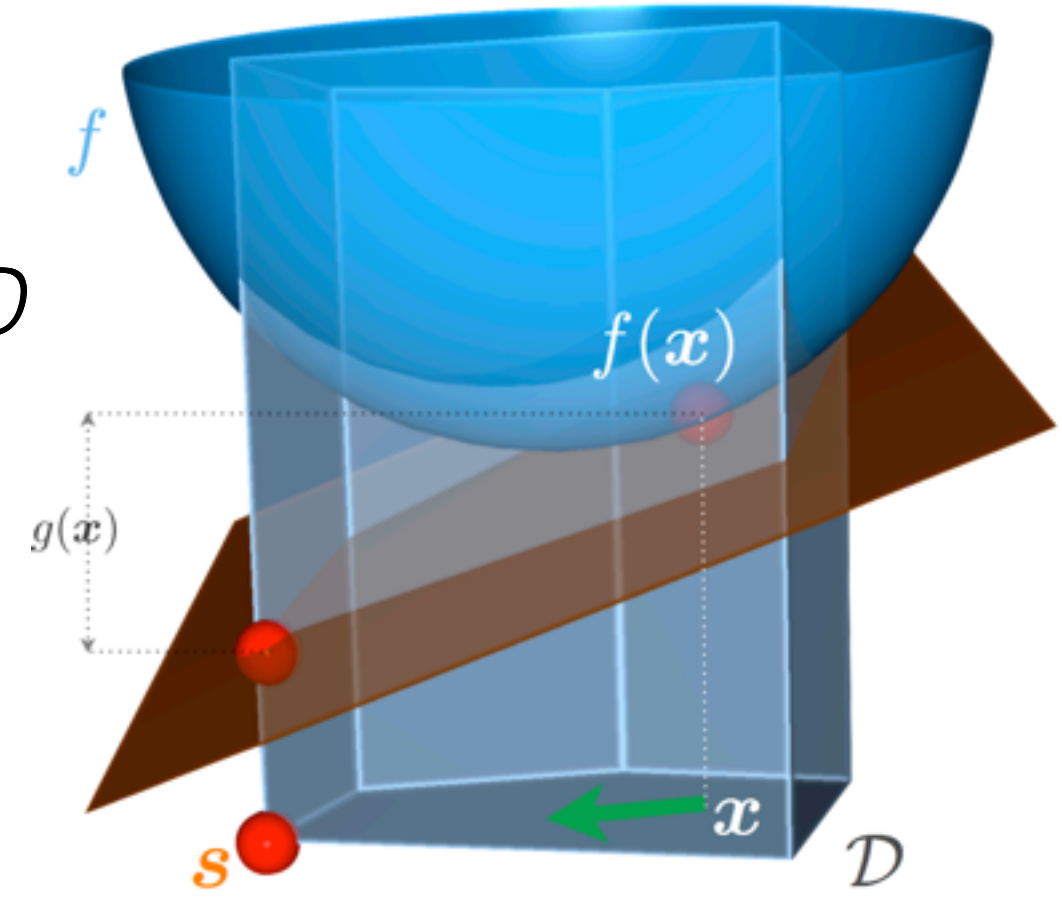
Convex optimization on a polytope *D*



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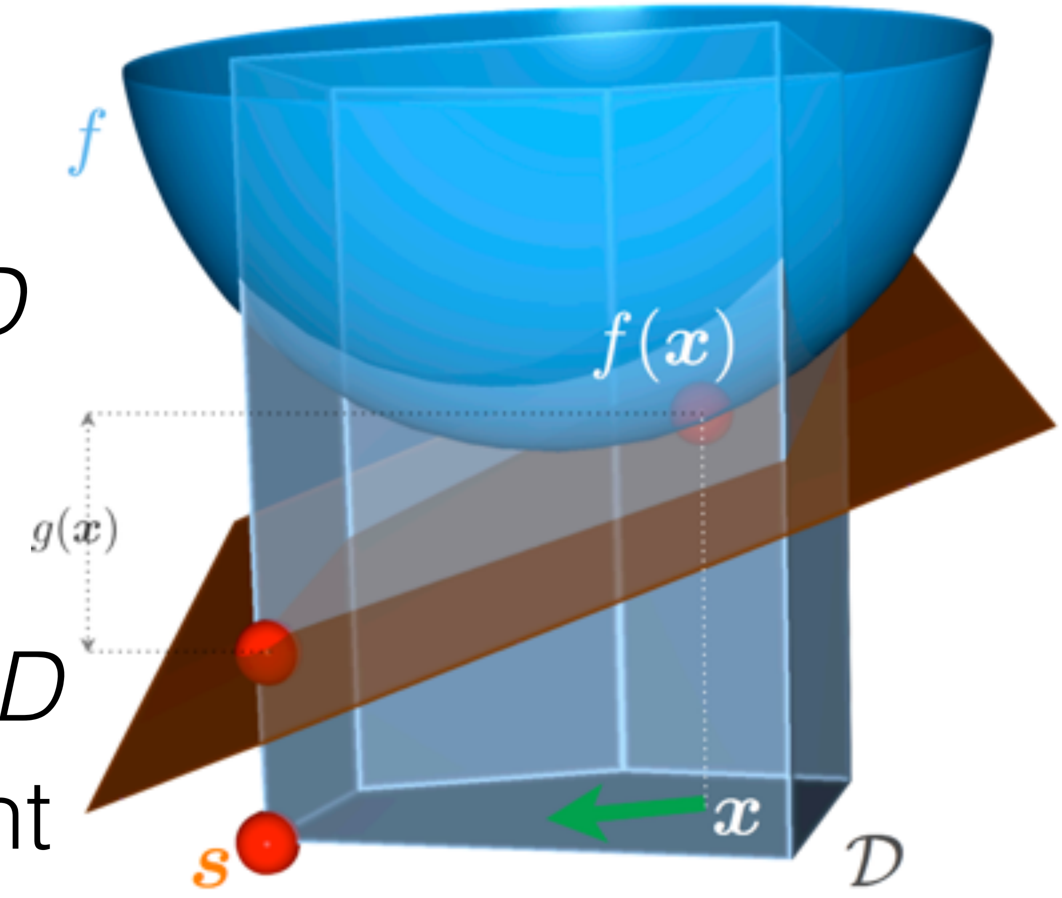

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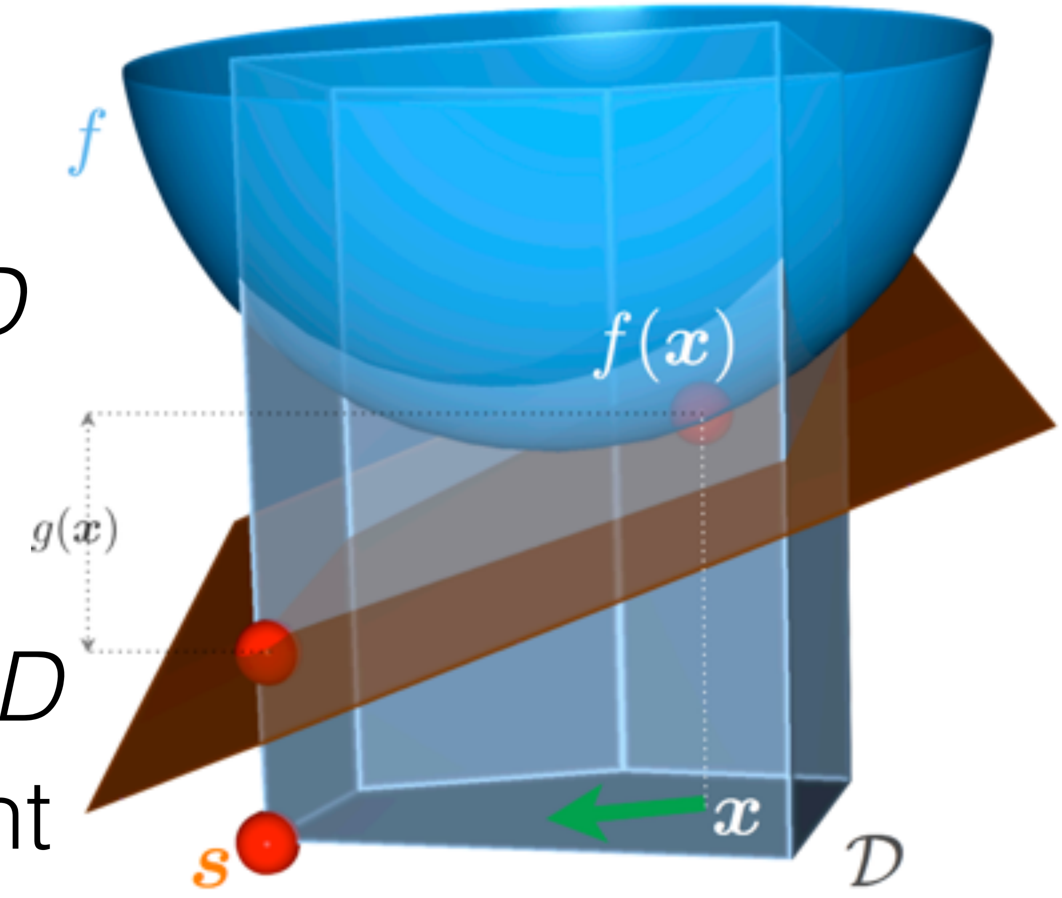

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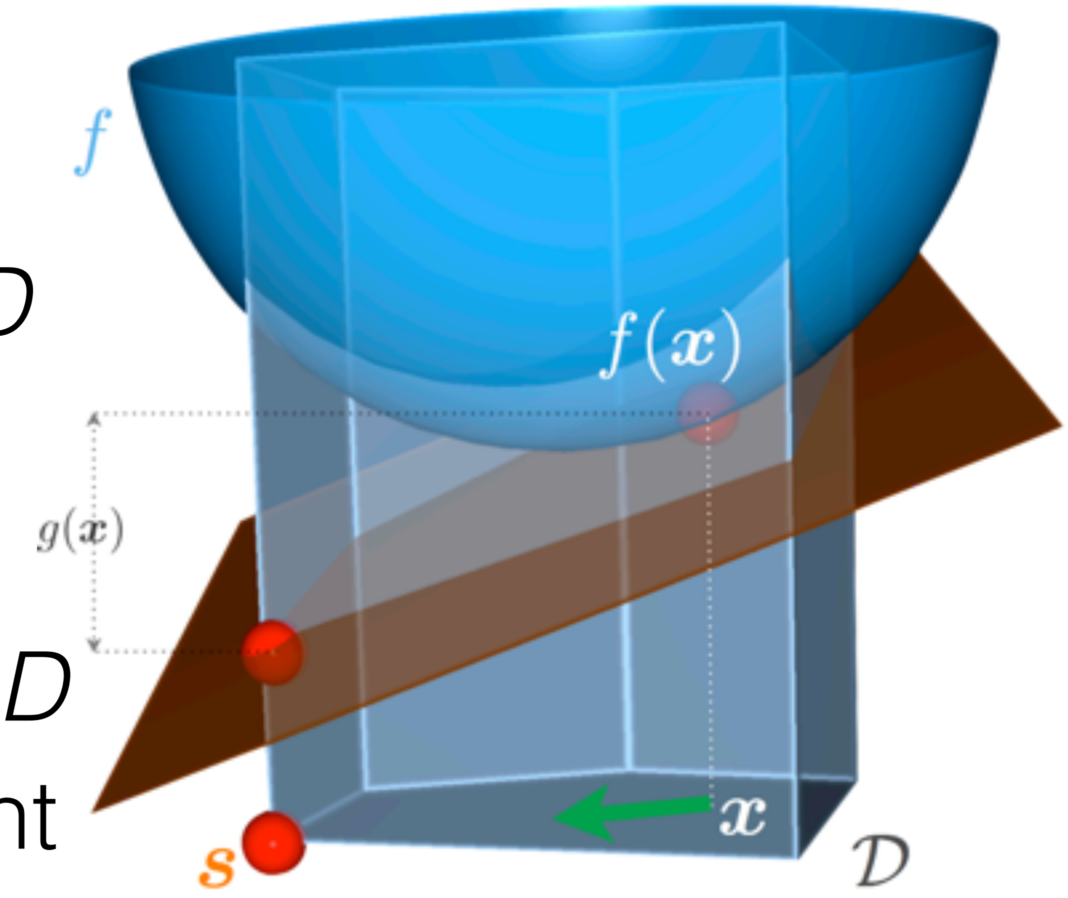

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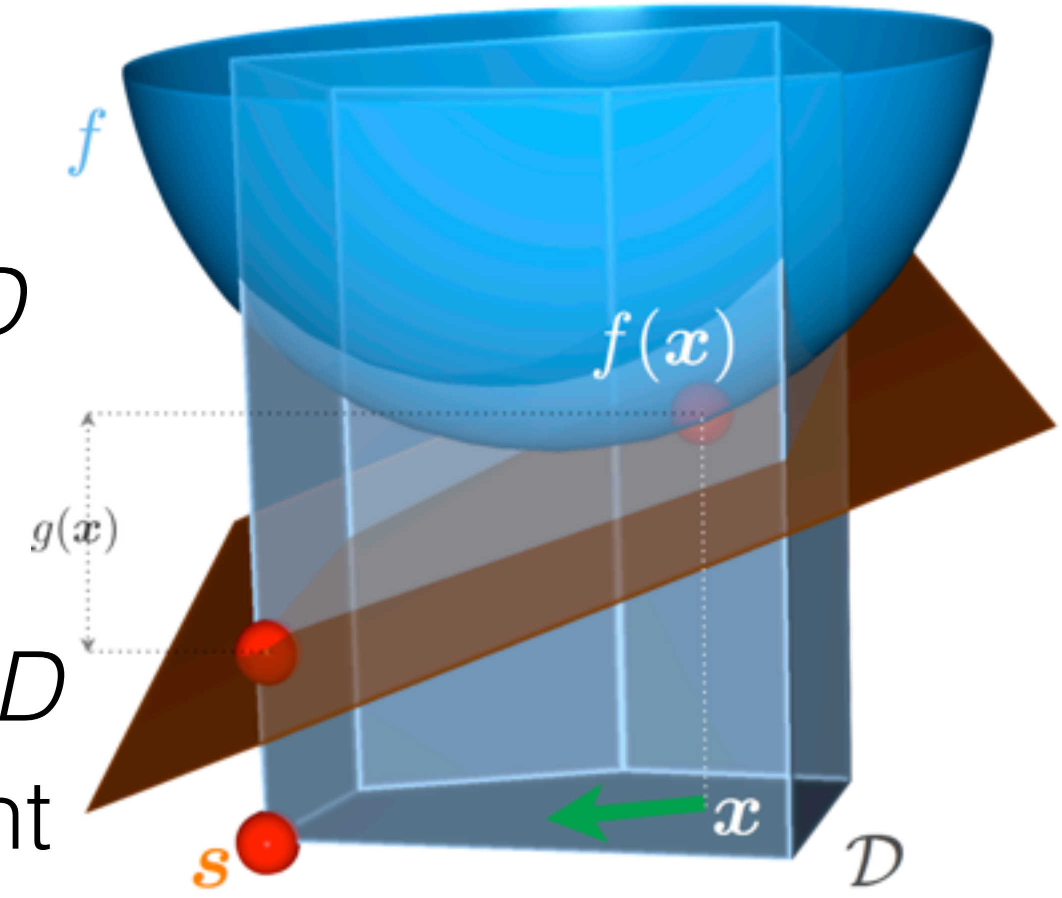

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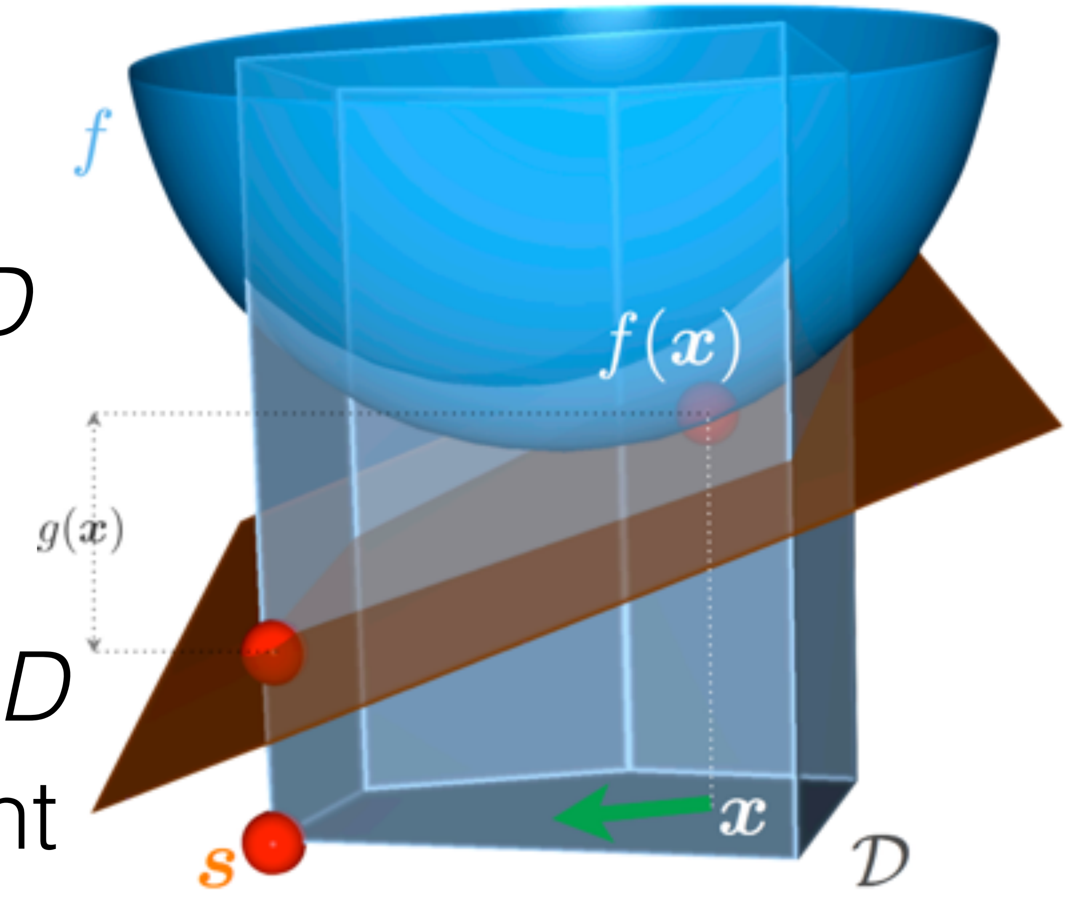

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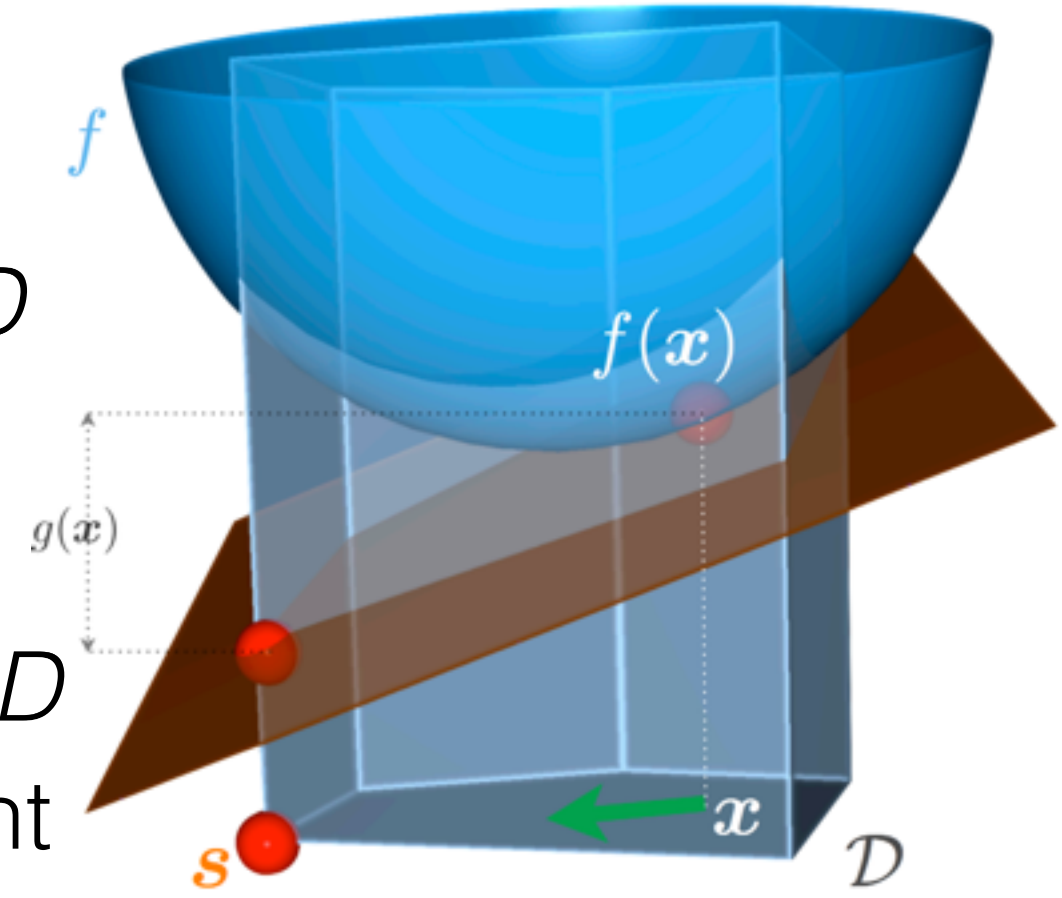

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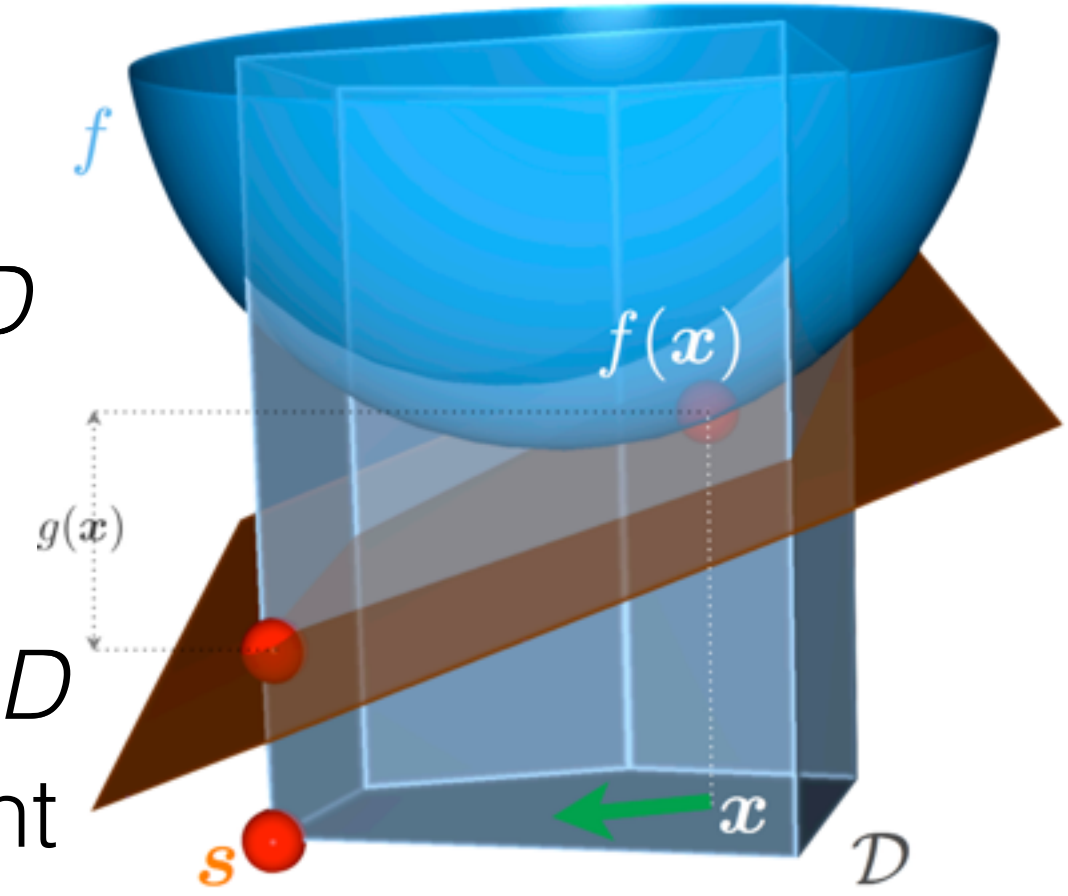

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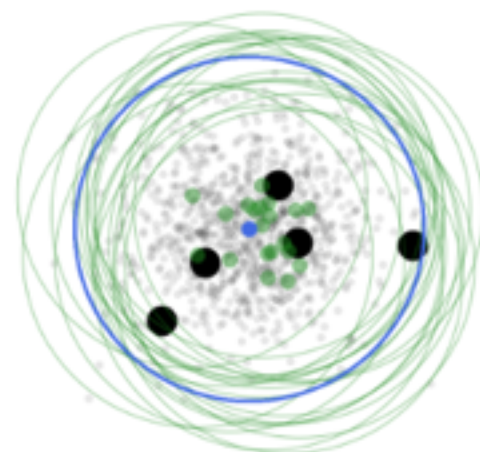
Gaussian model (simulated)

- 10K pts; norms, inference: closed-form

Uniform subsampling



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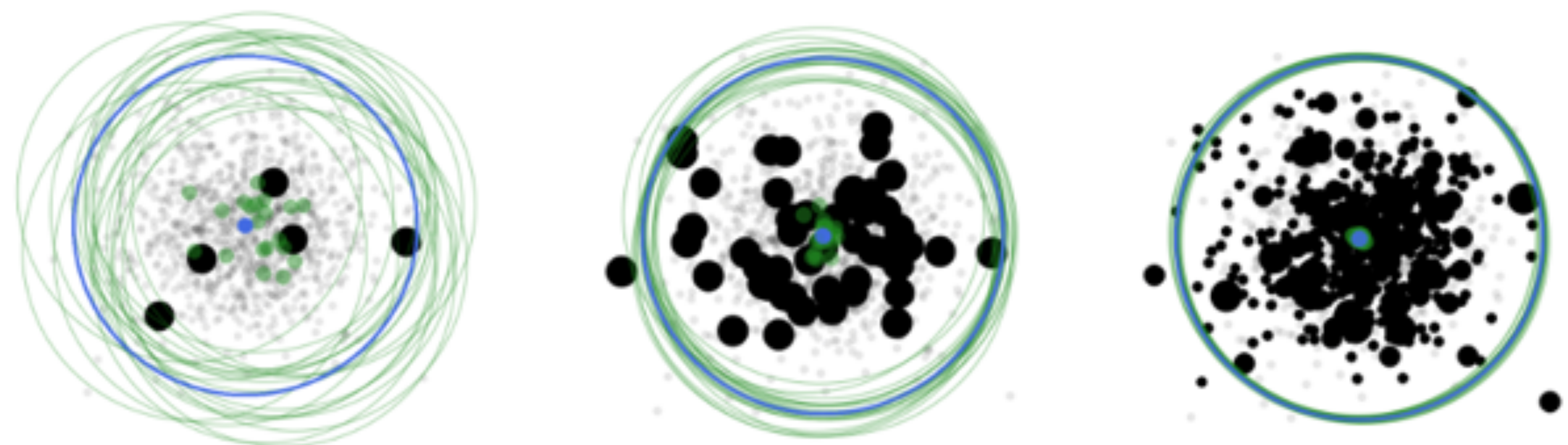

$M = 5$ $M = 50$ $M = 500$

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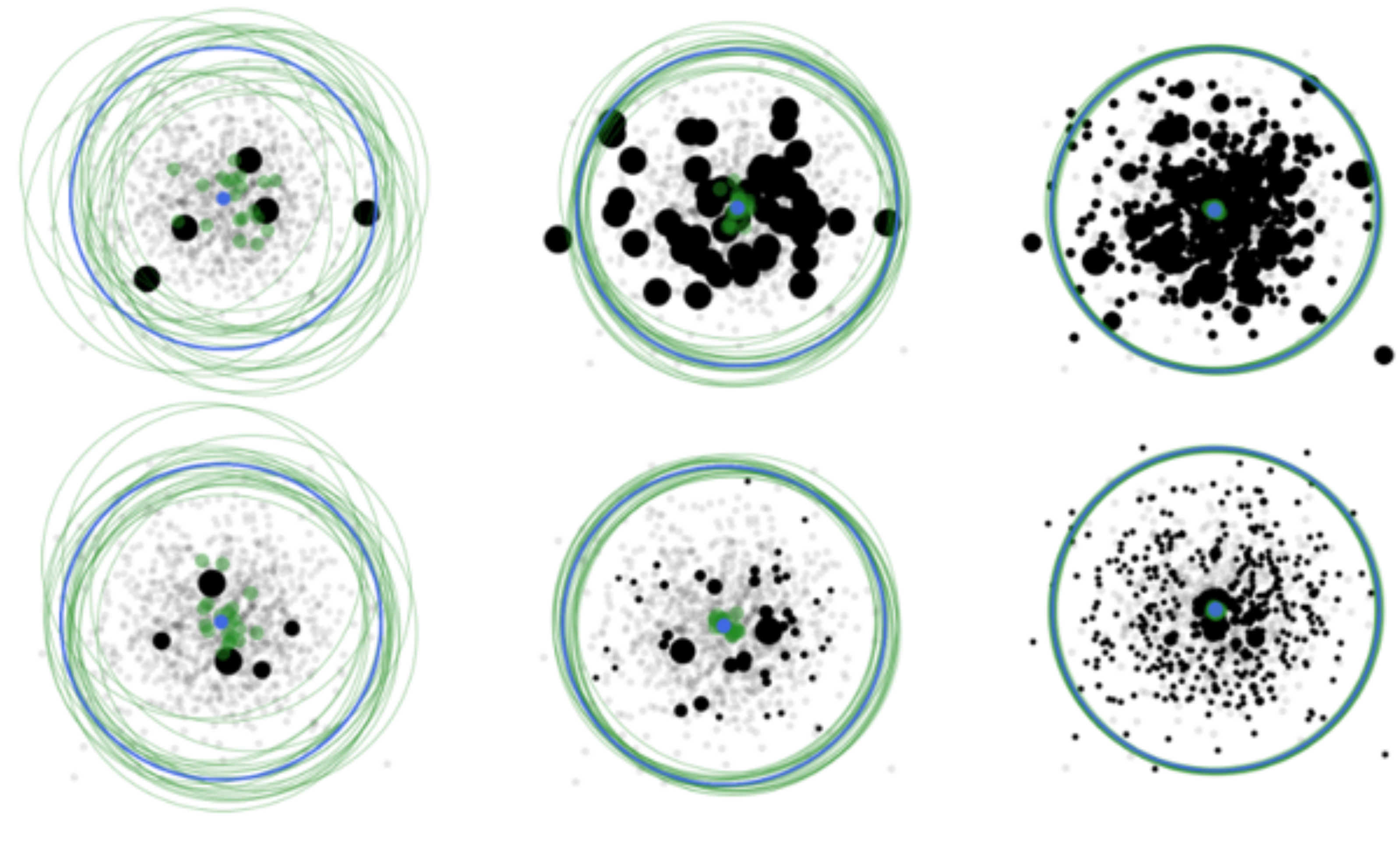

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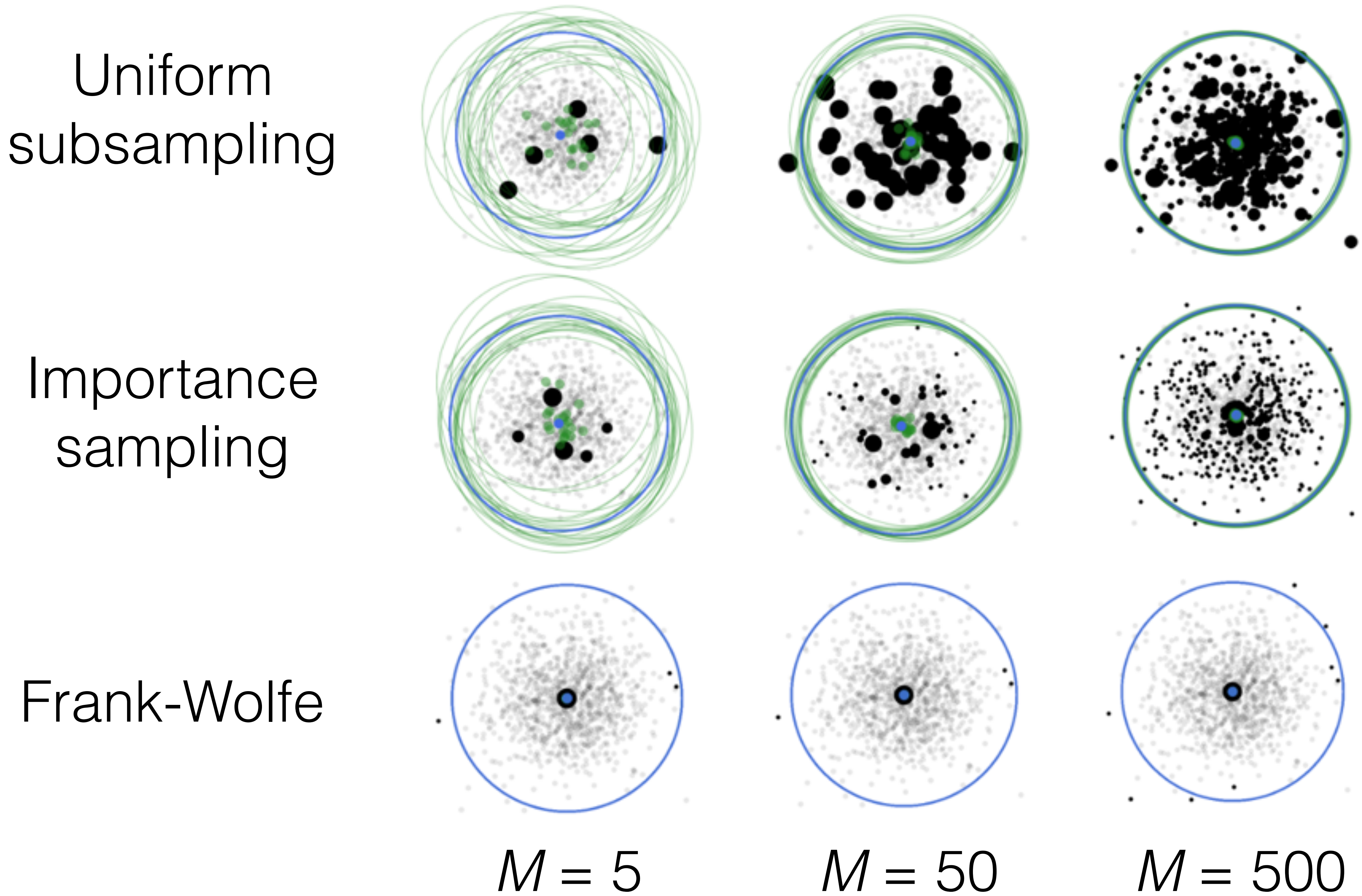

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lower
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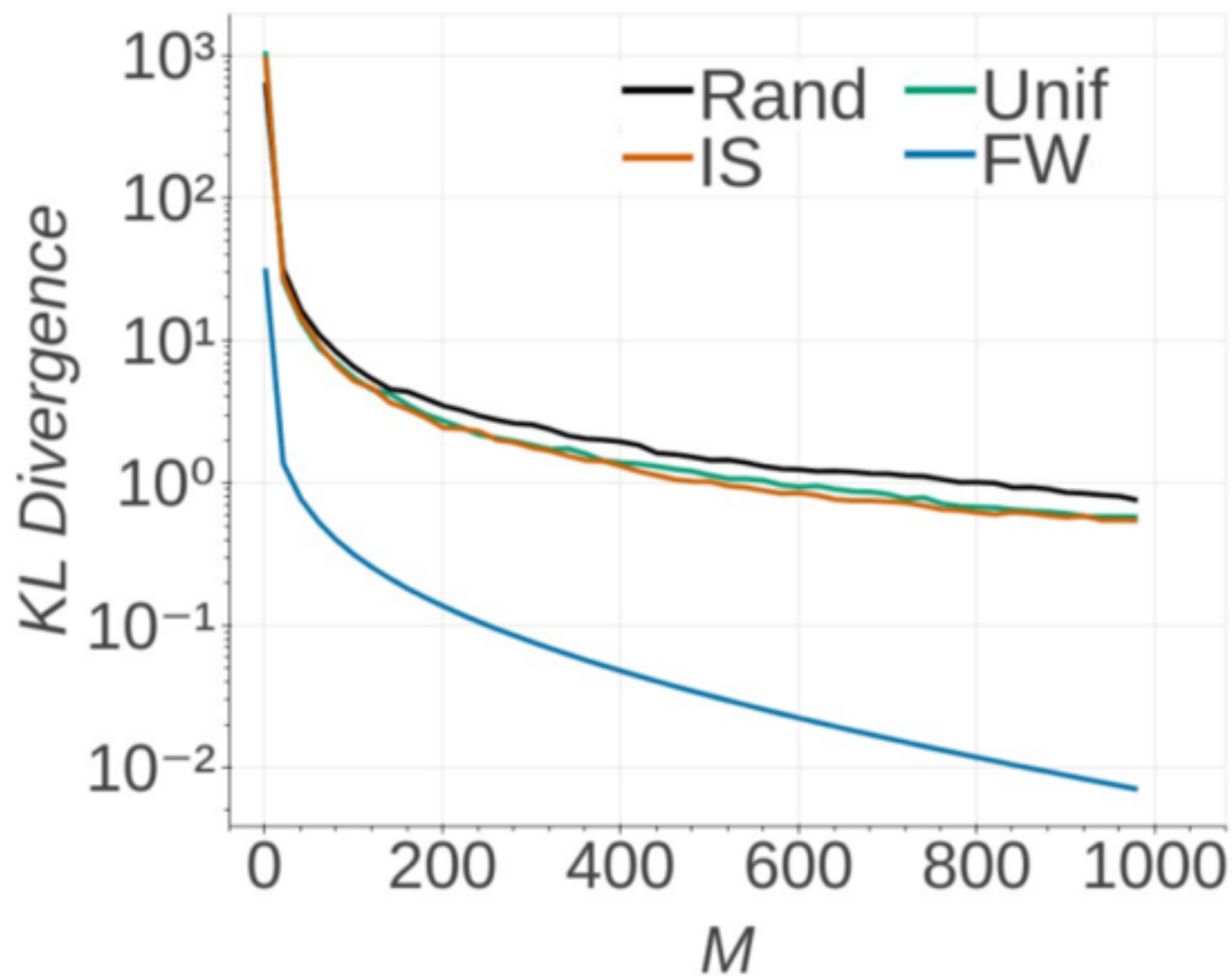

Logistic regression (simulated)

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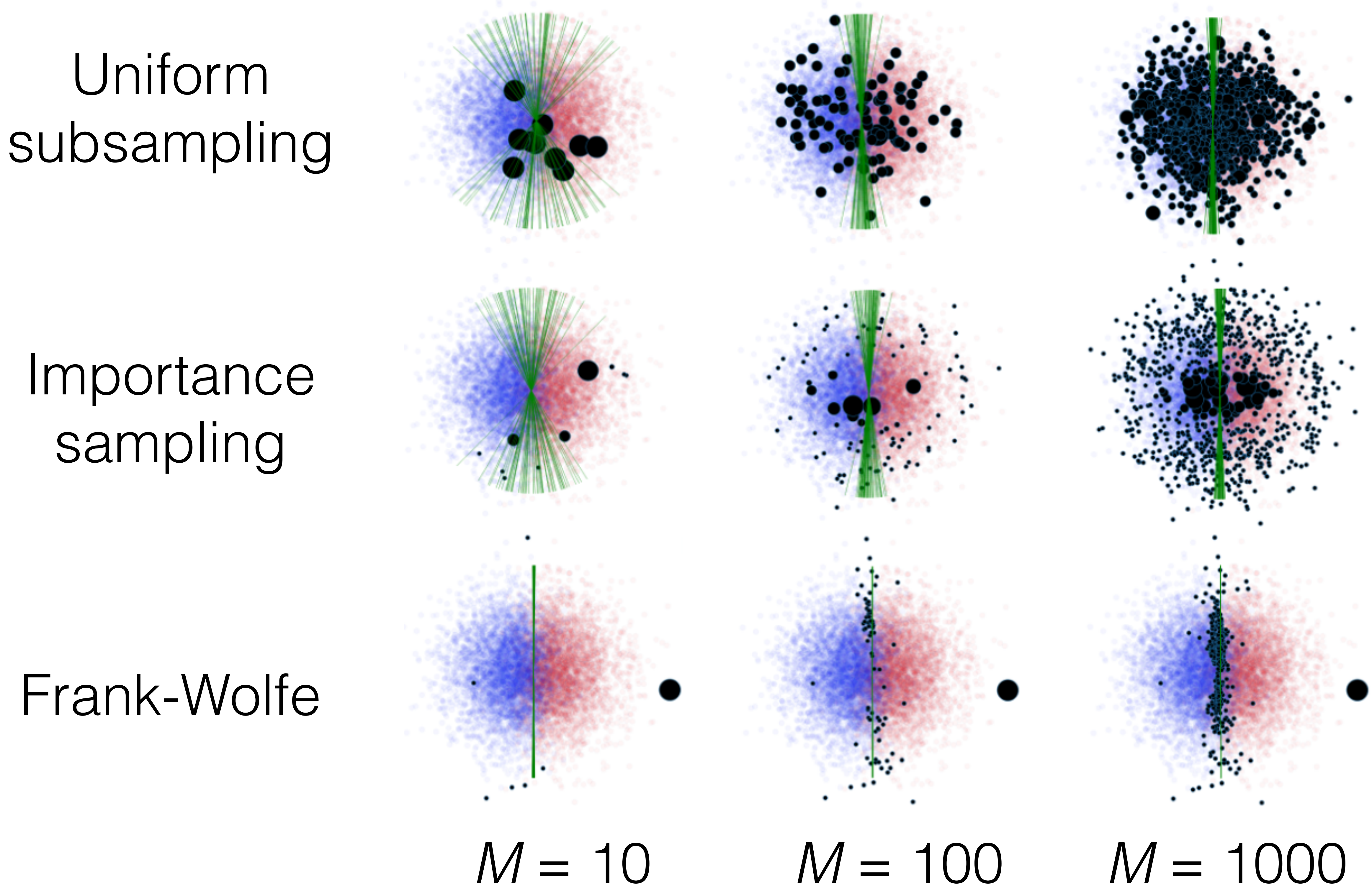

Poisson regression (simulated)

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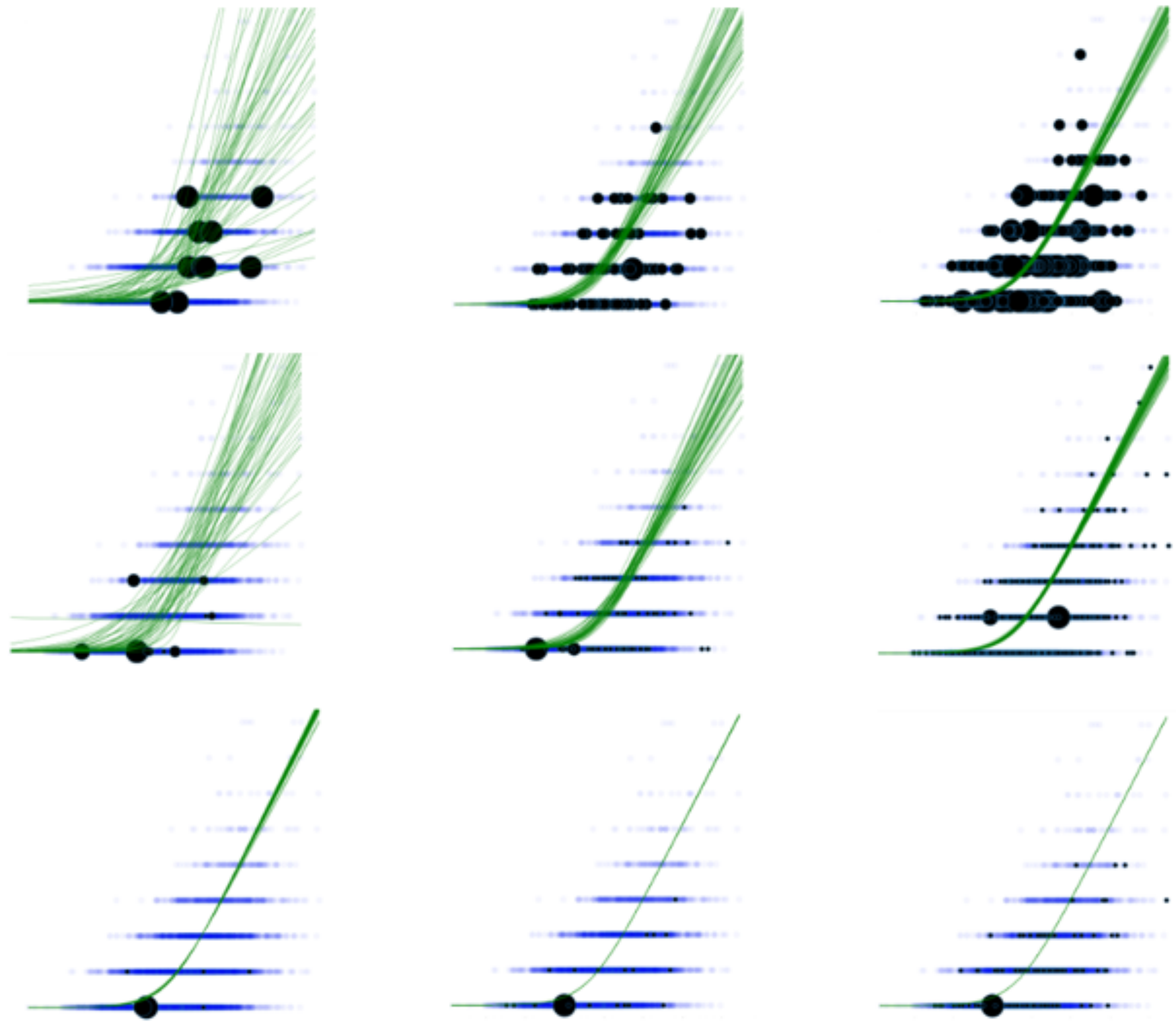
Uniform subsampling

Importance sampling

Frank-Wolfe



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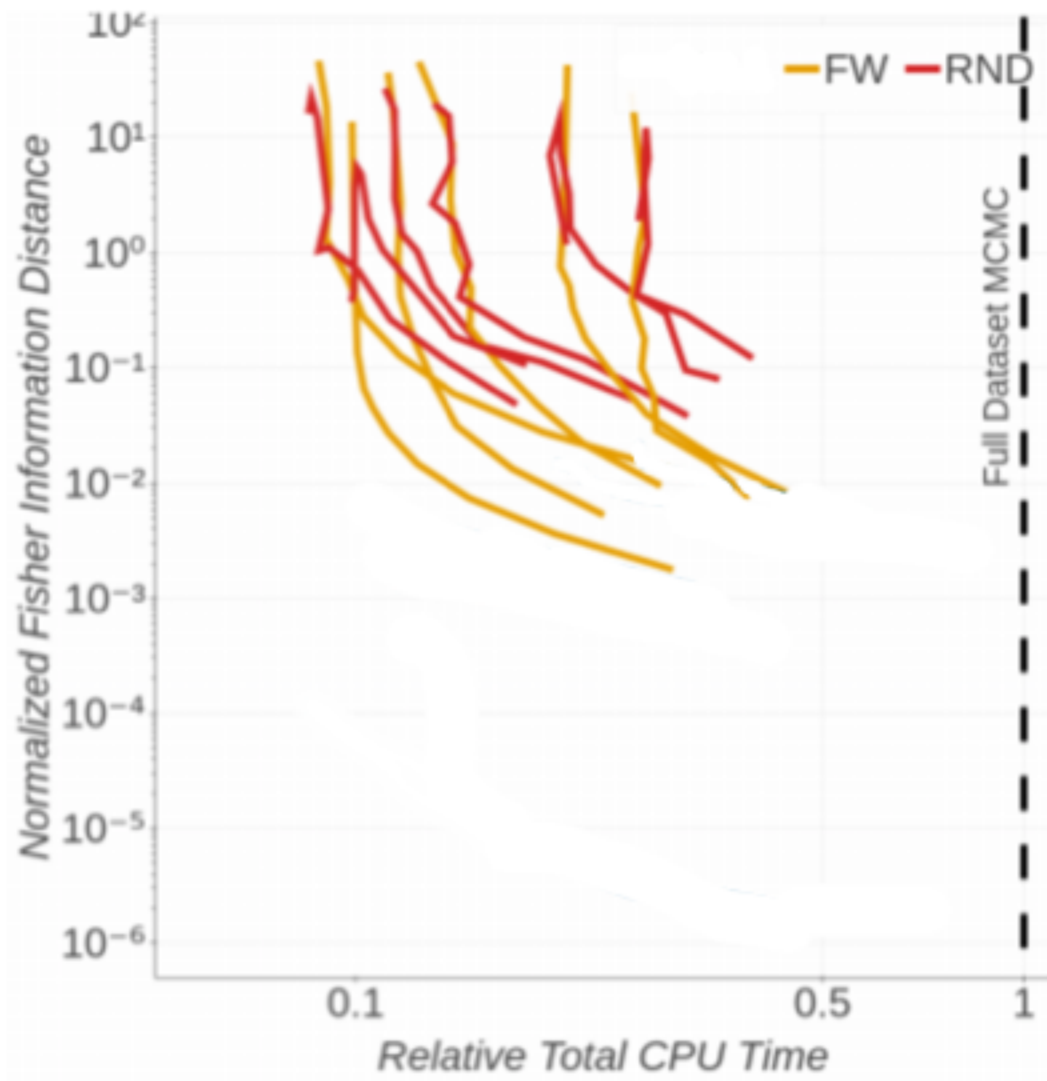
Real data experiments

lower error



less total time

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Frank Wolfe coresets

Data sets include:

- Phishing
- Chemical reactivity
- Bicycle trips
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[Campbell, Broderick 2017]

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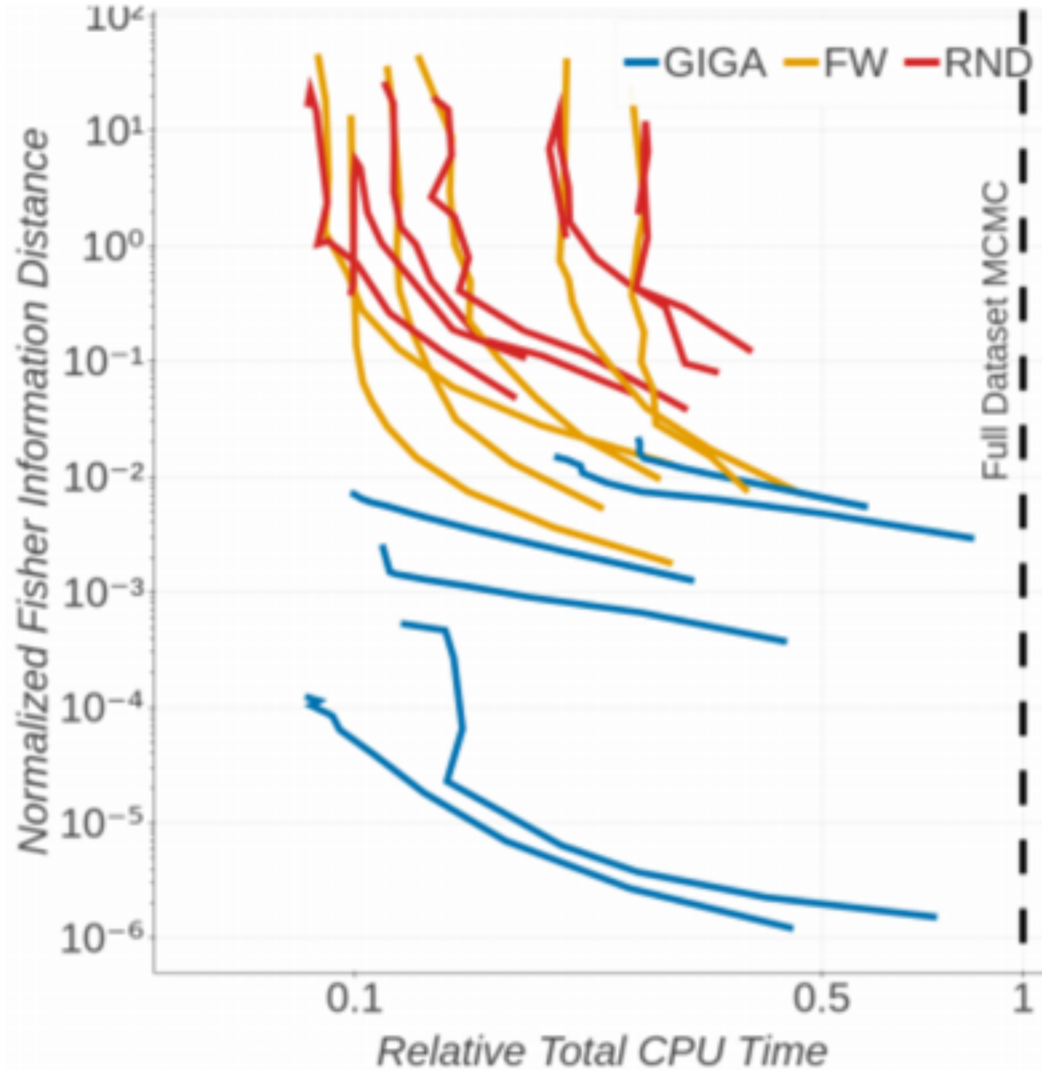

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GIGA coresets

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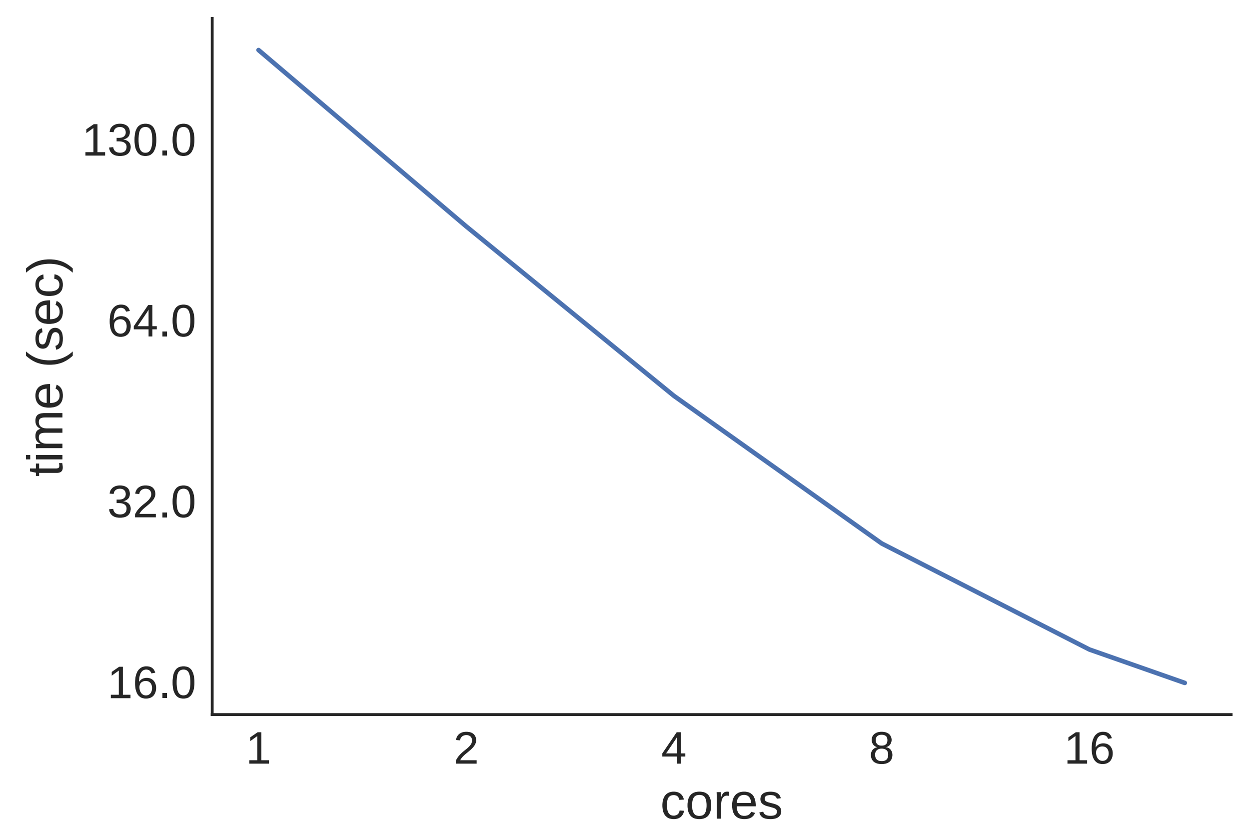
- Our proposal: (polynomial) *approximate* sufficient statistics

Data summarization

- 6M data points, 1000 features
- Streaming, distributed; minimal communication
- 22 cores, 16 sec
- Finite-data guarantees on Wasserstein distance to exact posterior



[Huggins, Adams, Broderick 2017]

Conclusions

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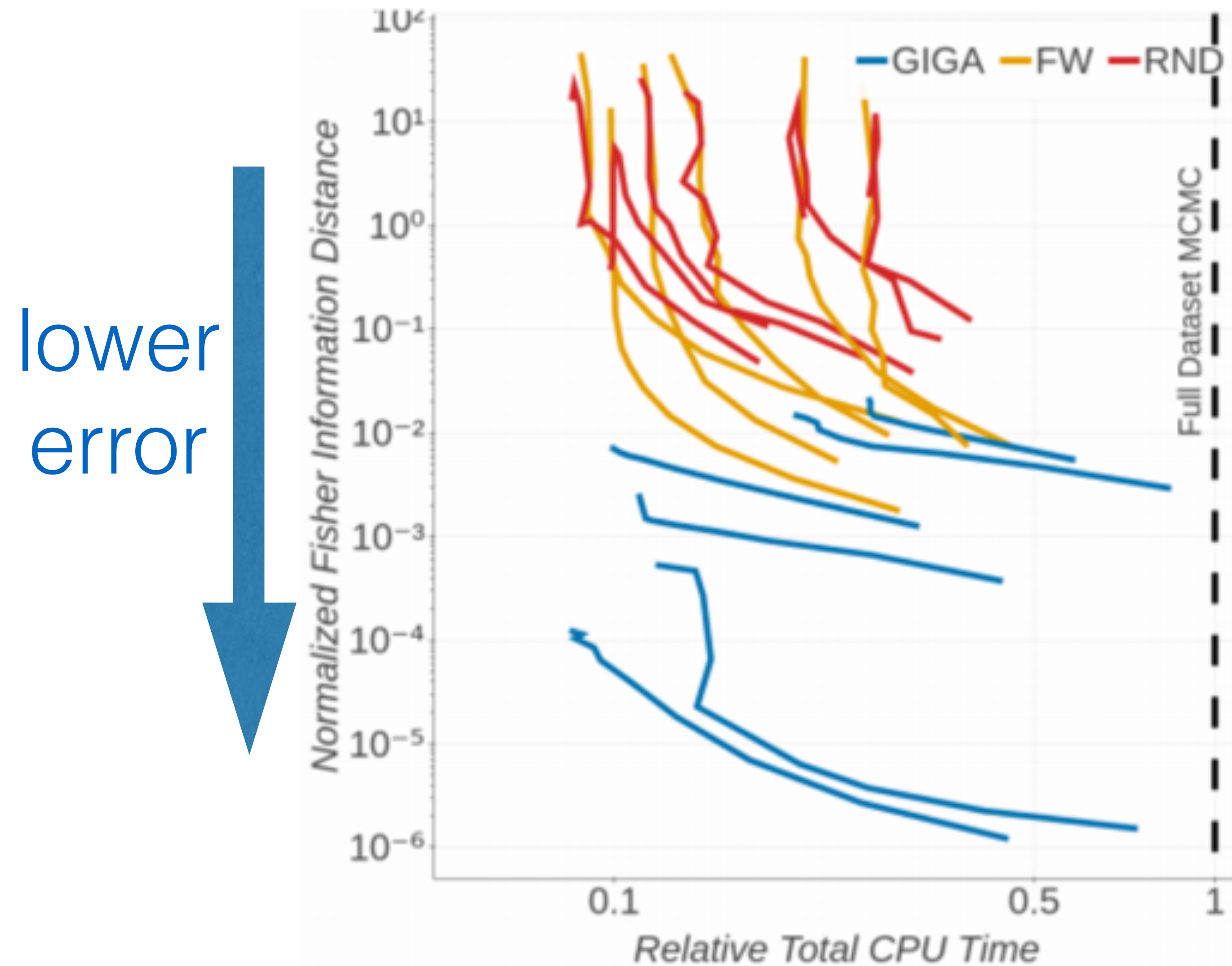

[Campbell, Broderick 2018]

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