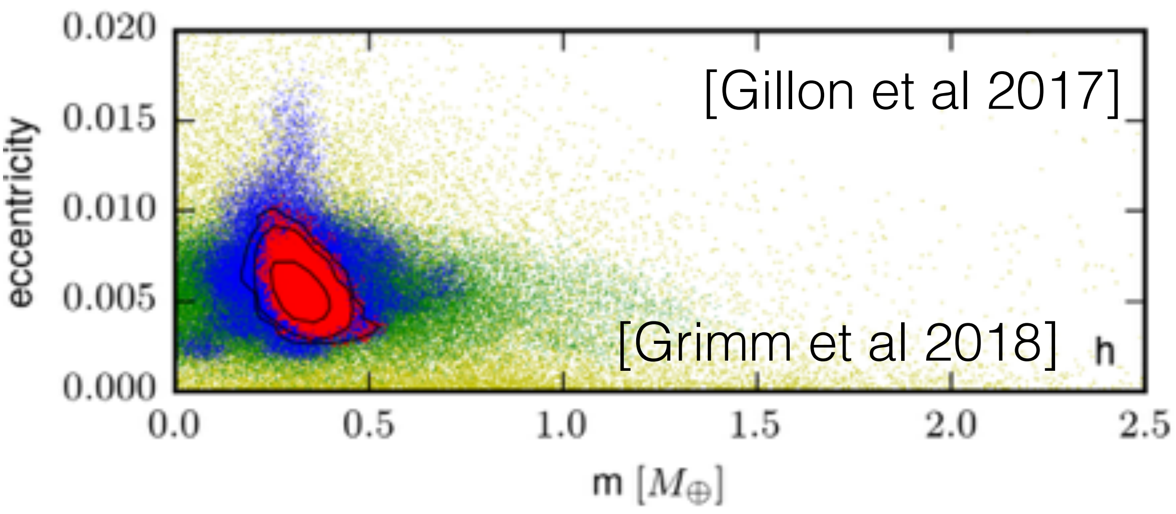
Variational Bayes and beyond: Bayesian inference for big data

Tamara Broderick
ITT Career Development
Assistant Professor,
MIT

Bayesian inference

Bayesian inference



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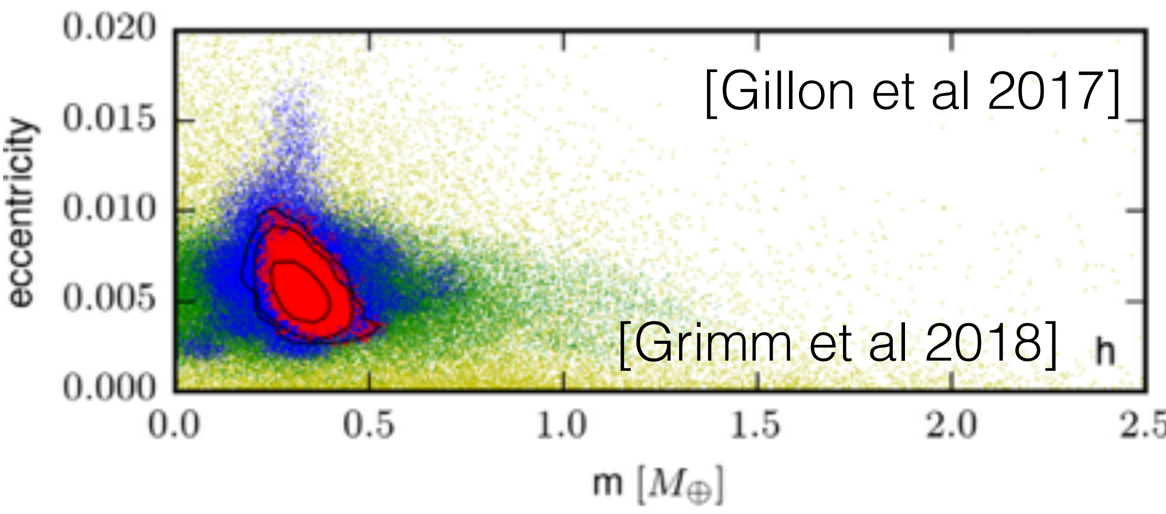

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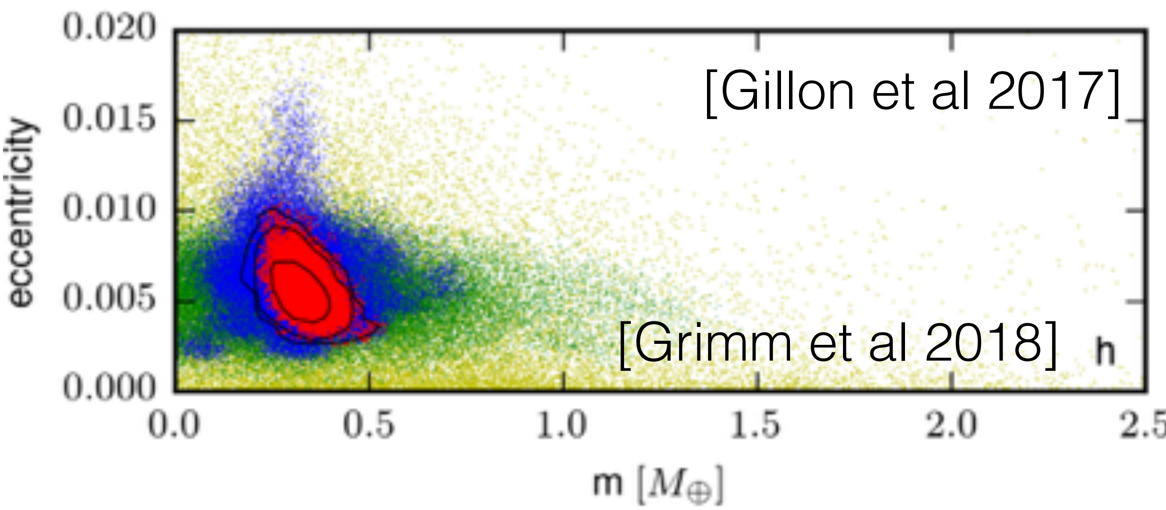

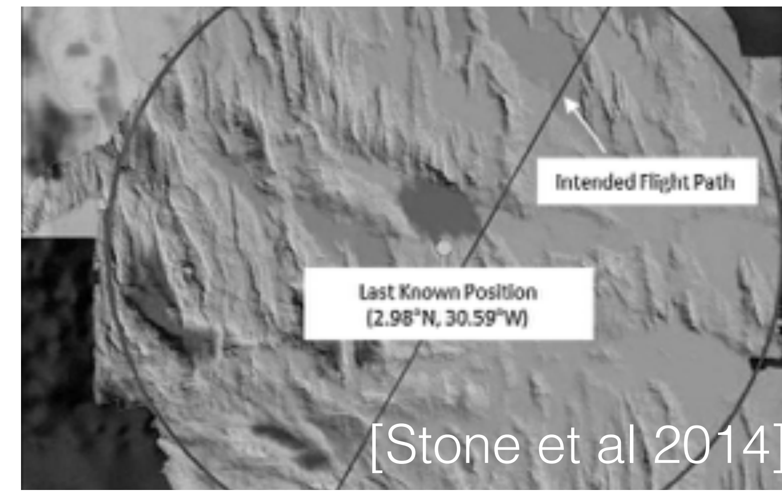

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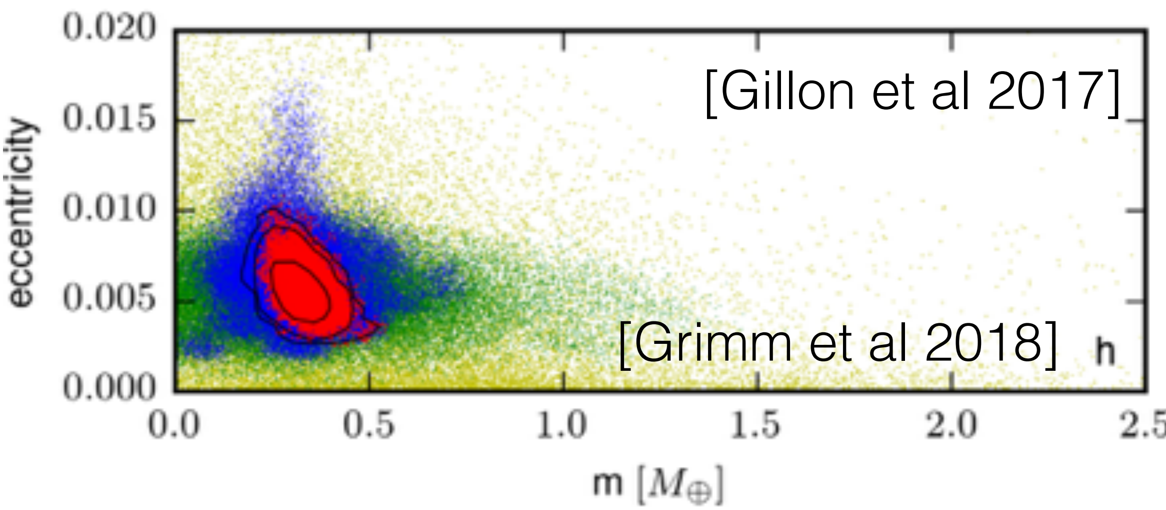

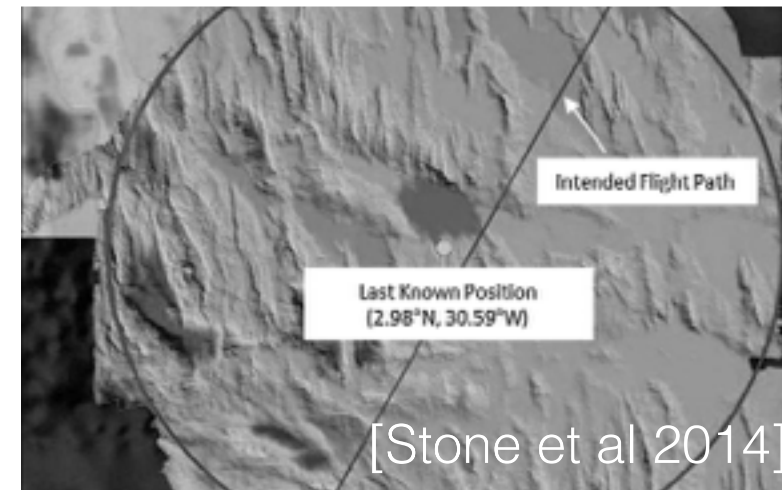

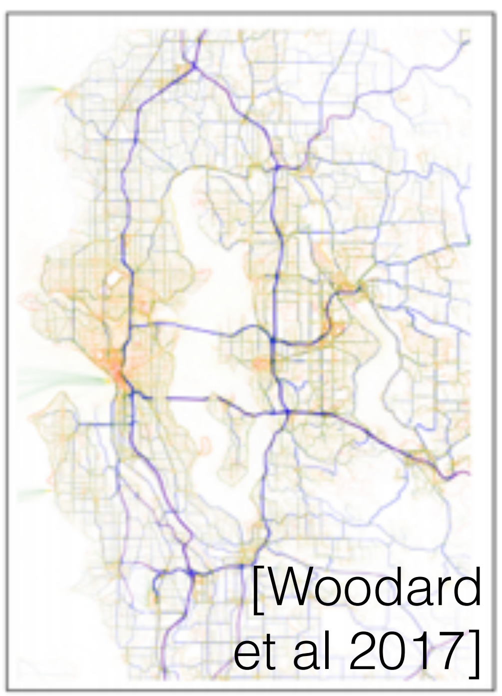

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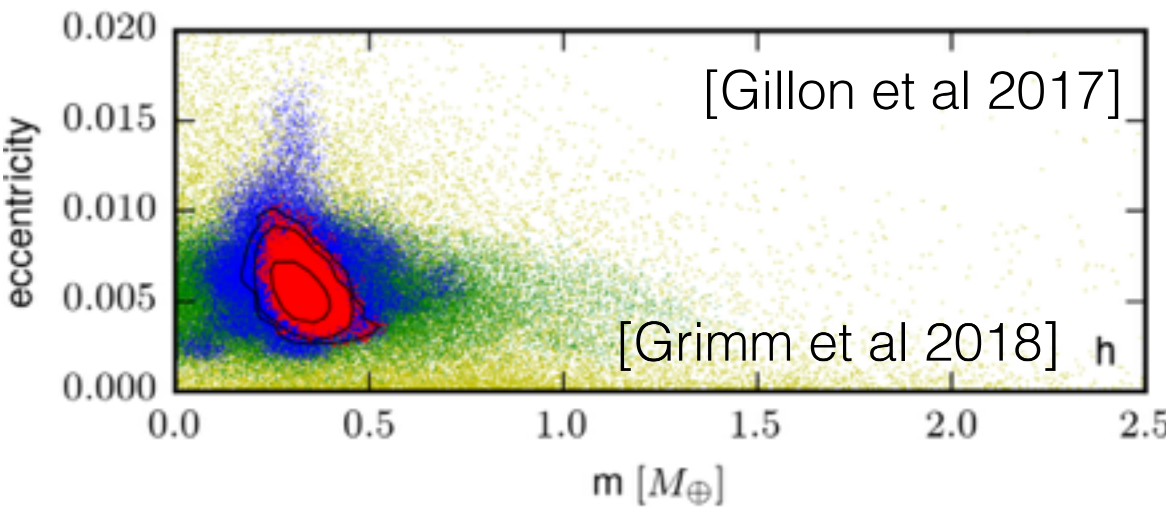

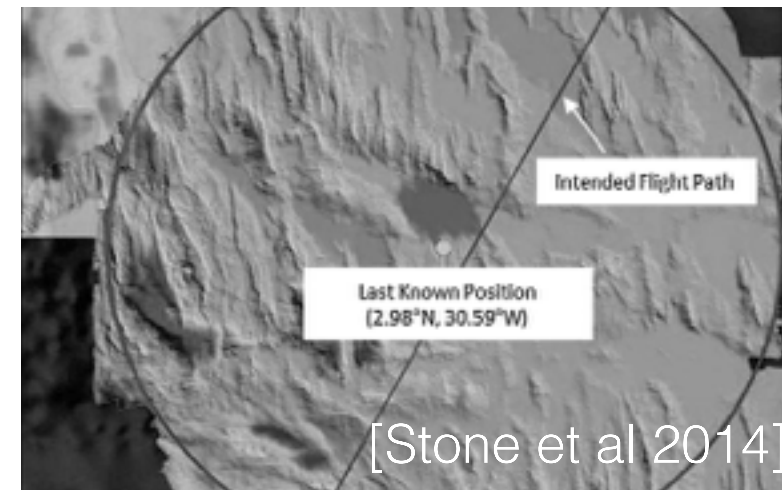

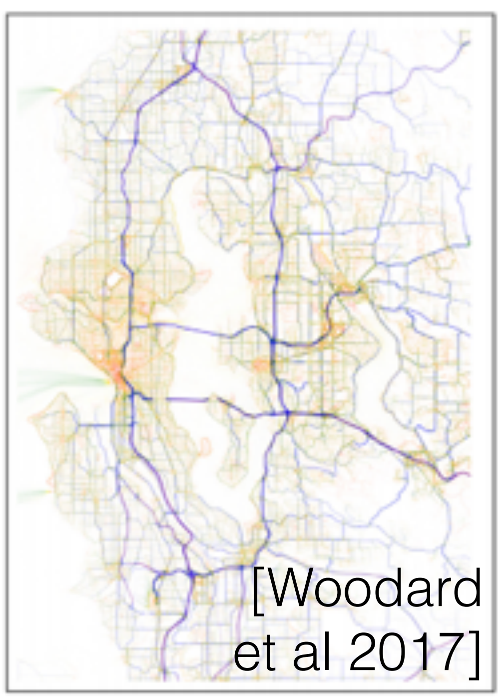

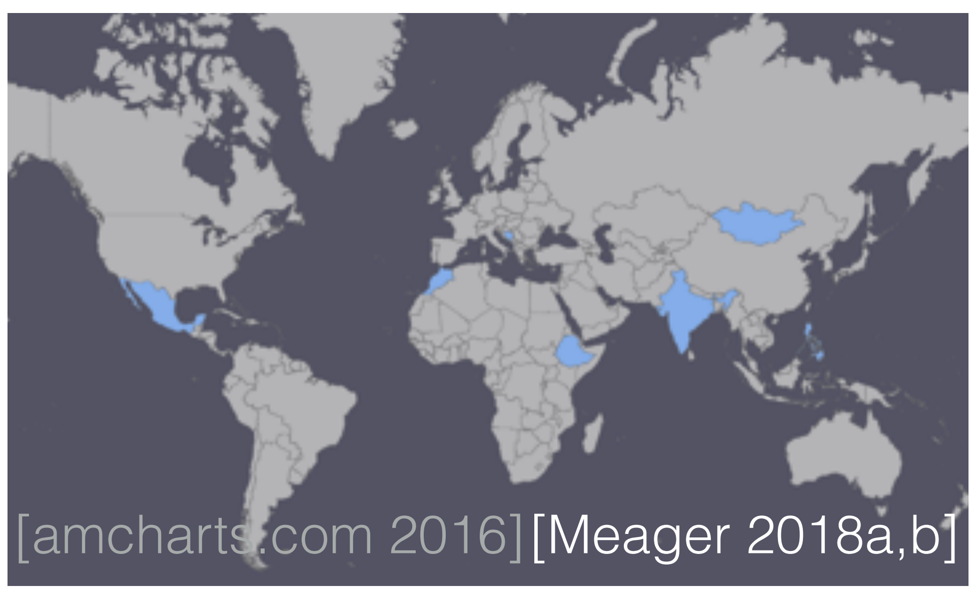

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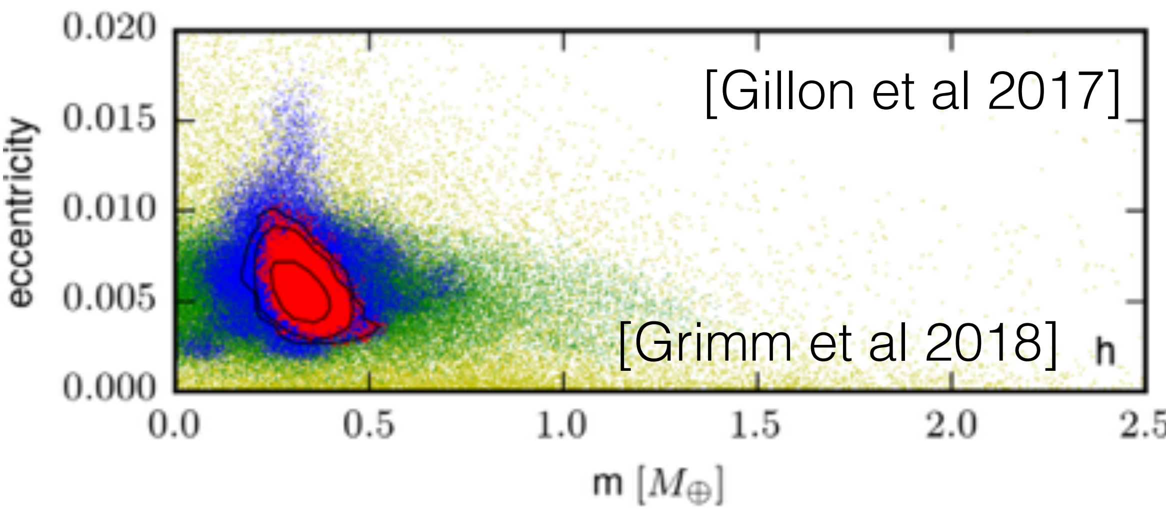

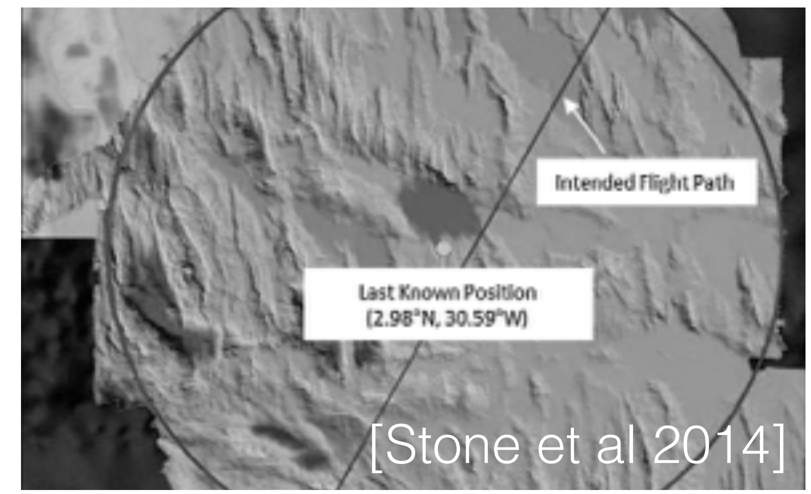

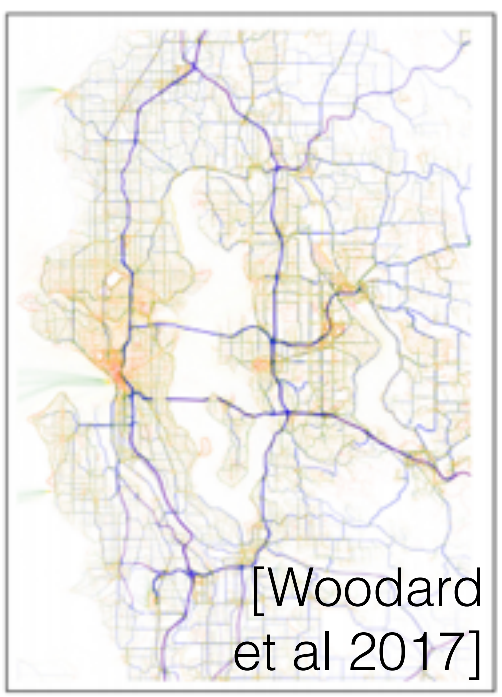

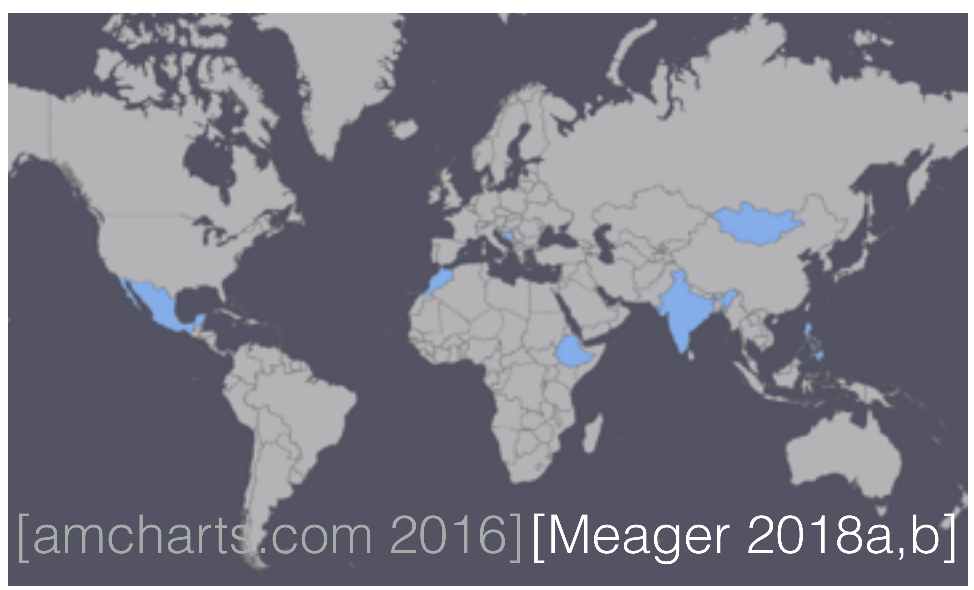

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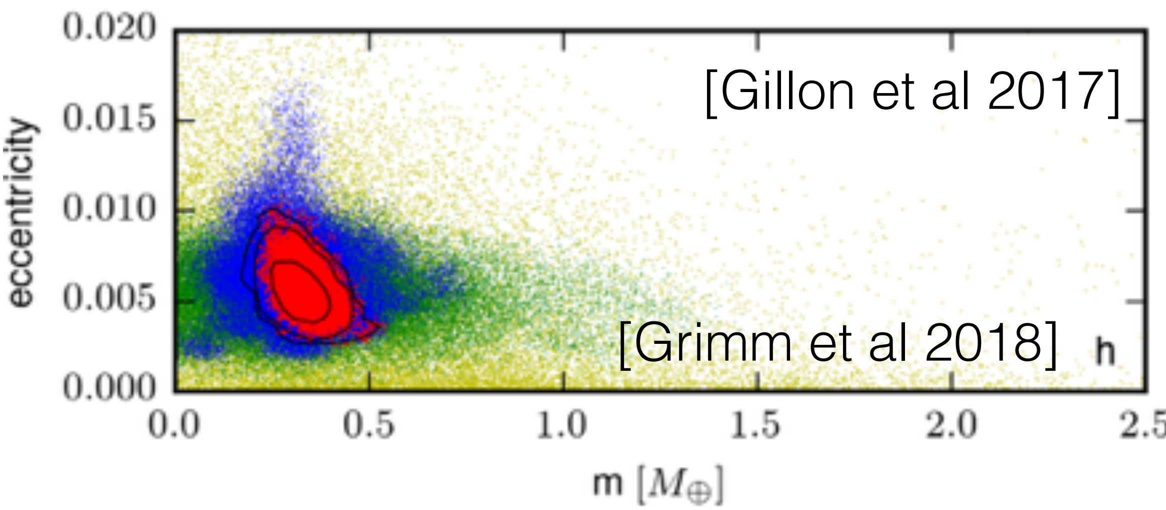
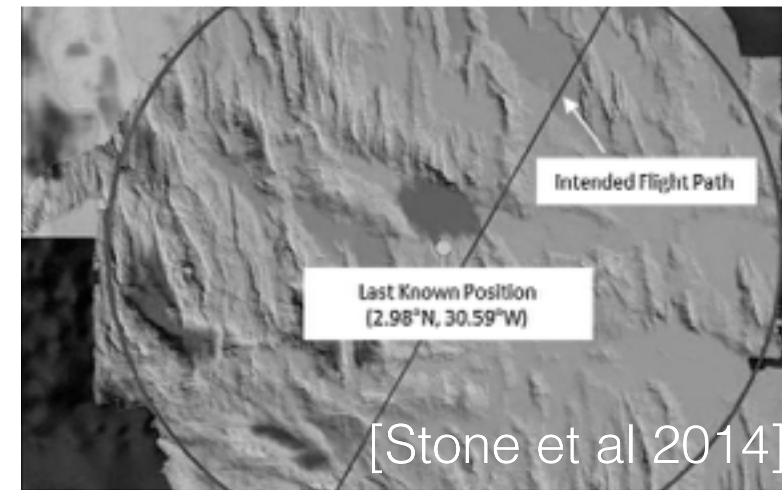
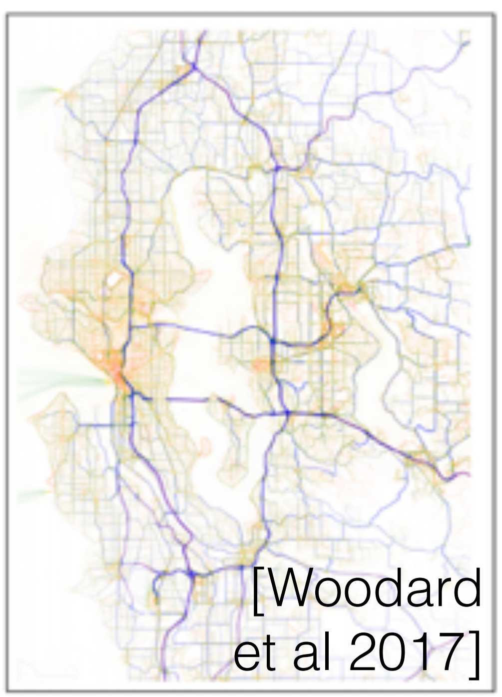
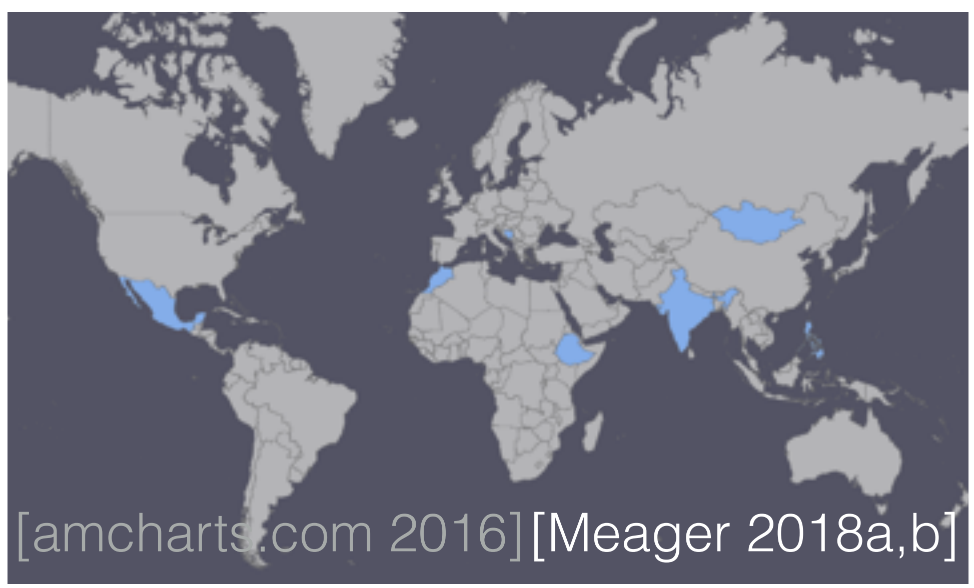


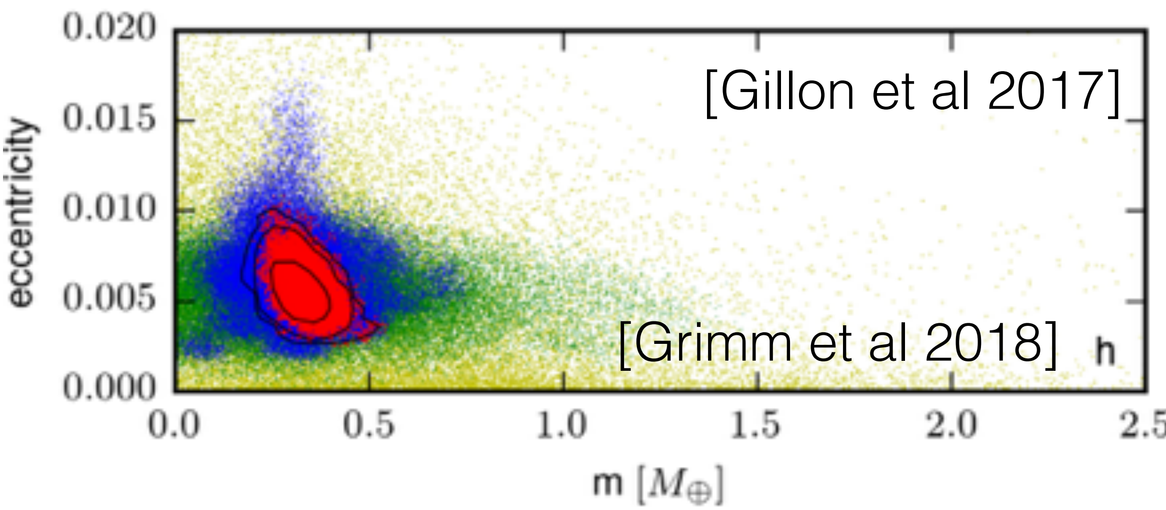


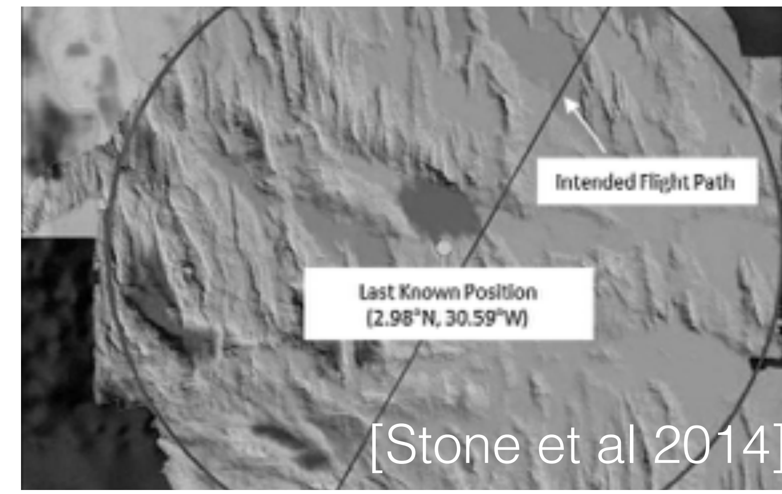


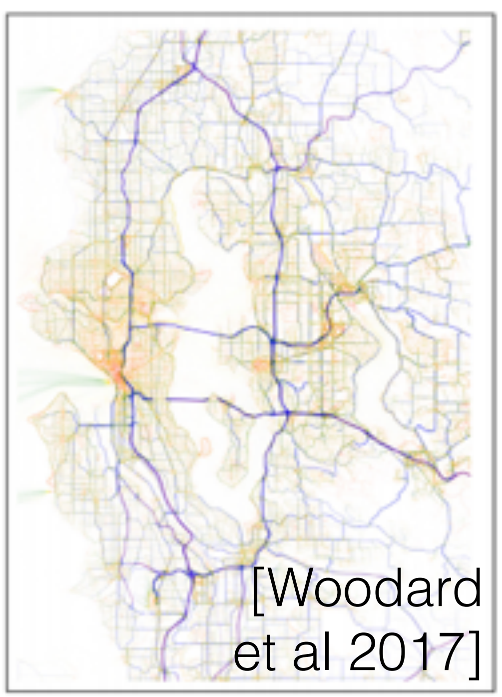


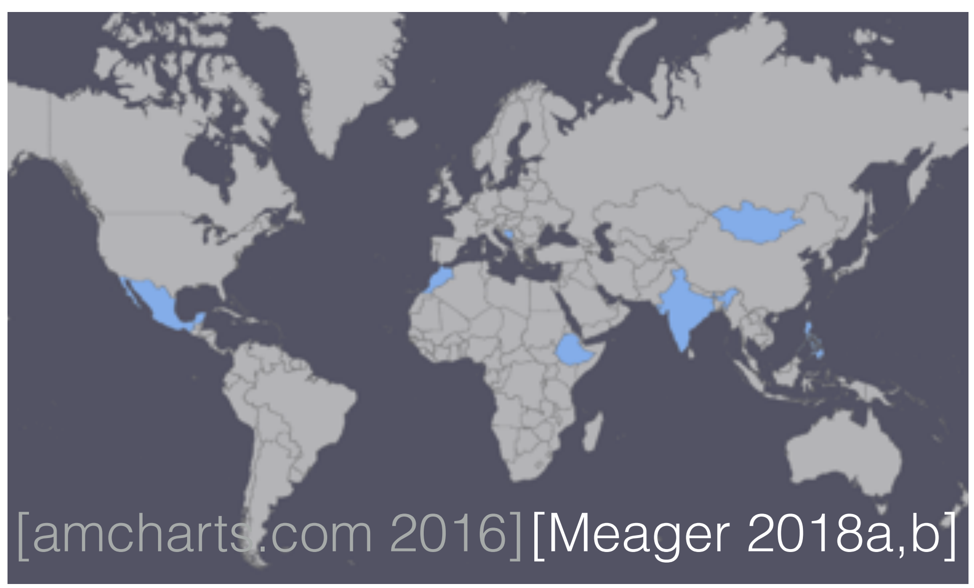
Bayesian inference

- Analysis goals: Point estimates, coherent uncertainties
  - Interpretable, complex, modular; expert information









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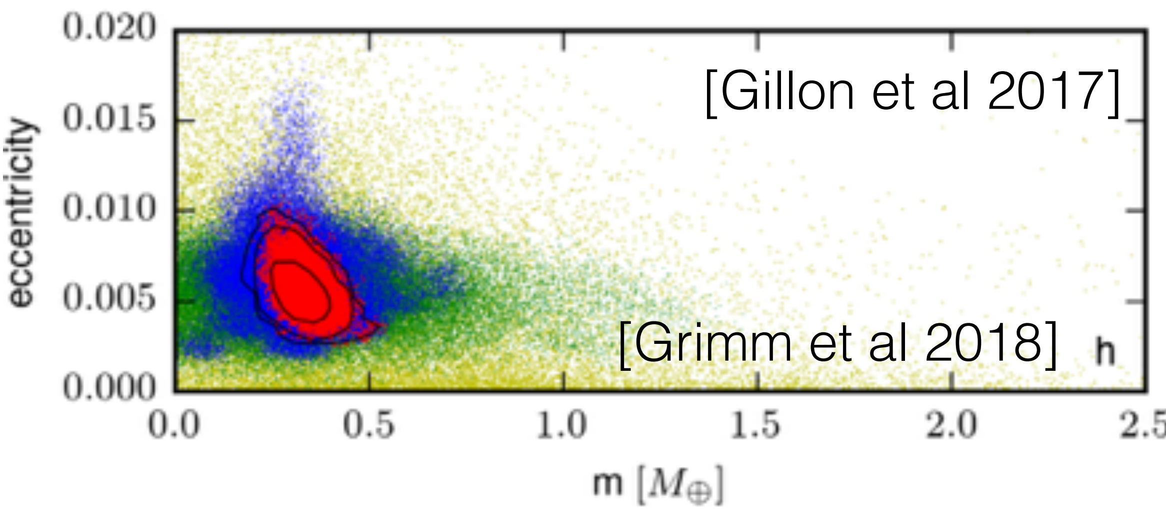

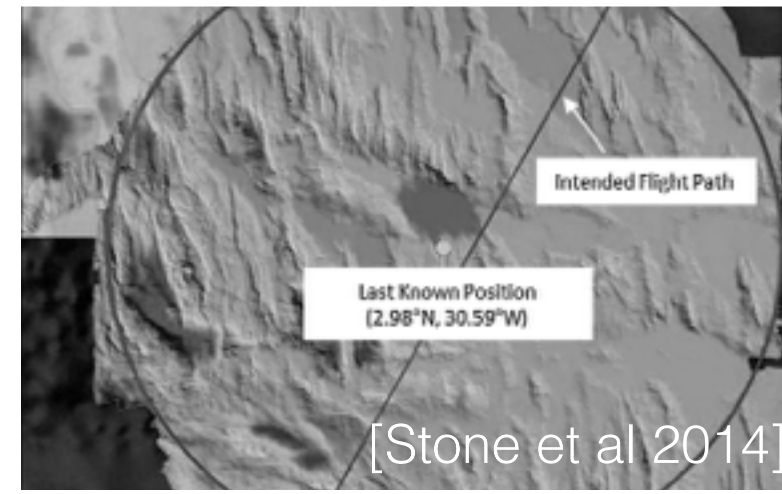

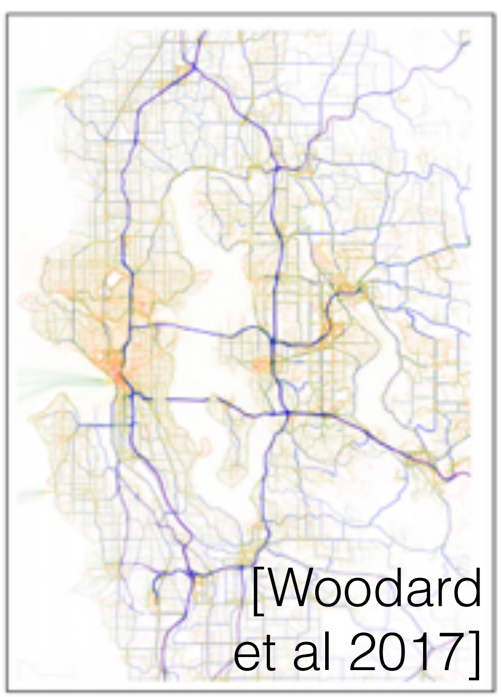

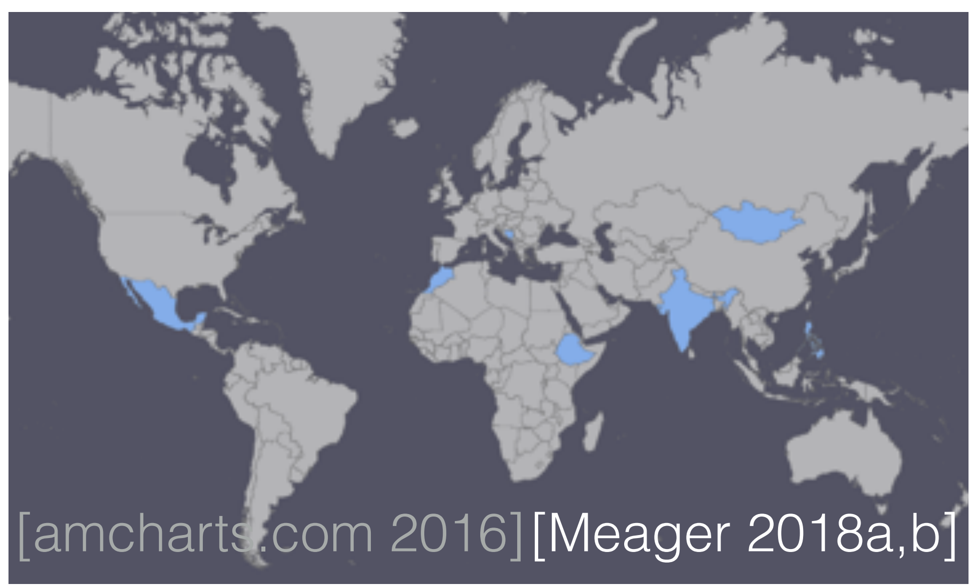

[mc-stan.org]

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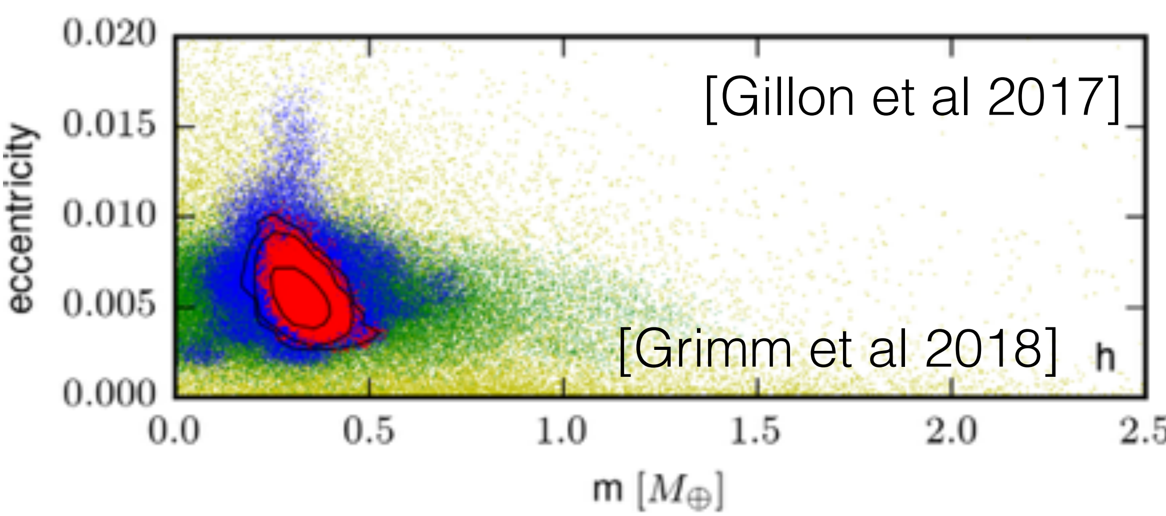

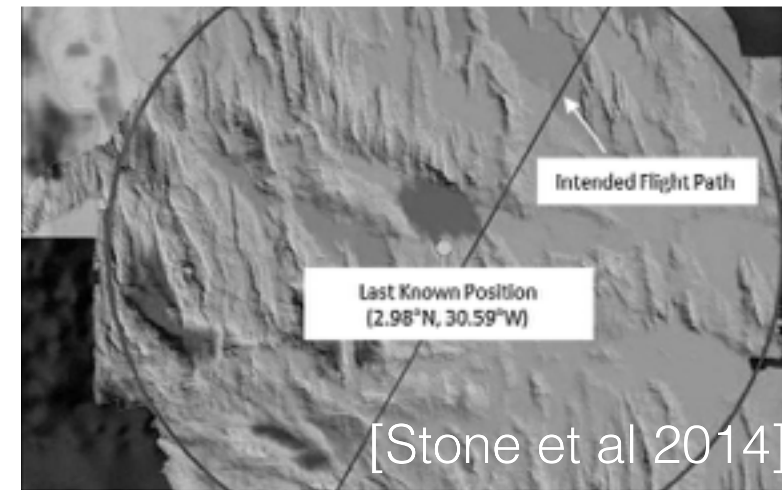

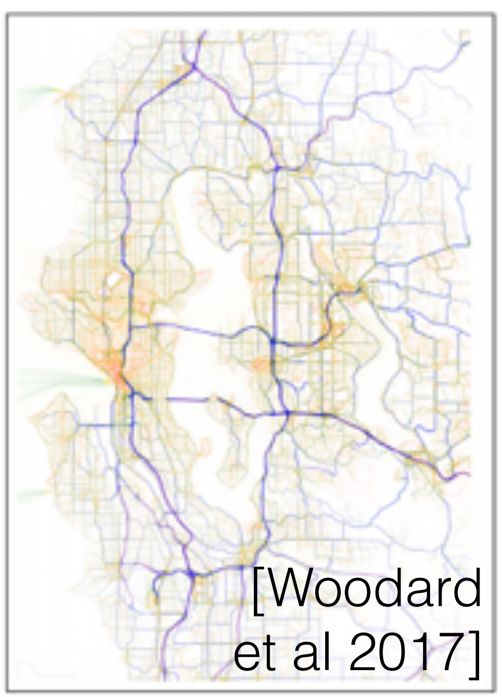

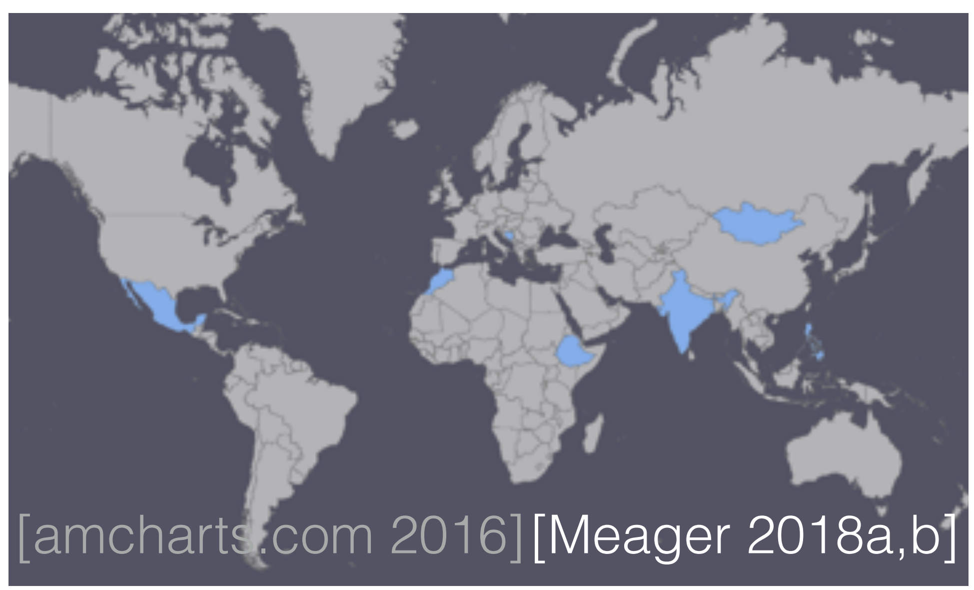

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- Challenge: fast (compute, user), reliable inference

Bayesian inference

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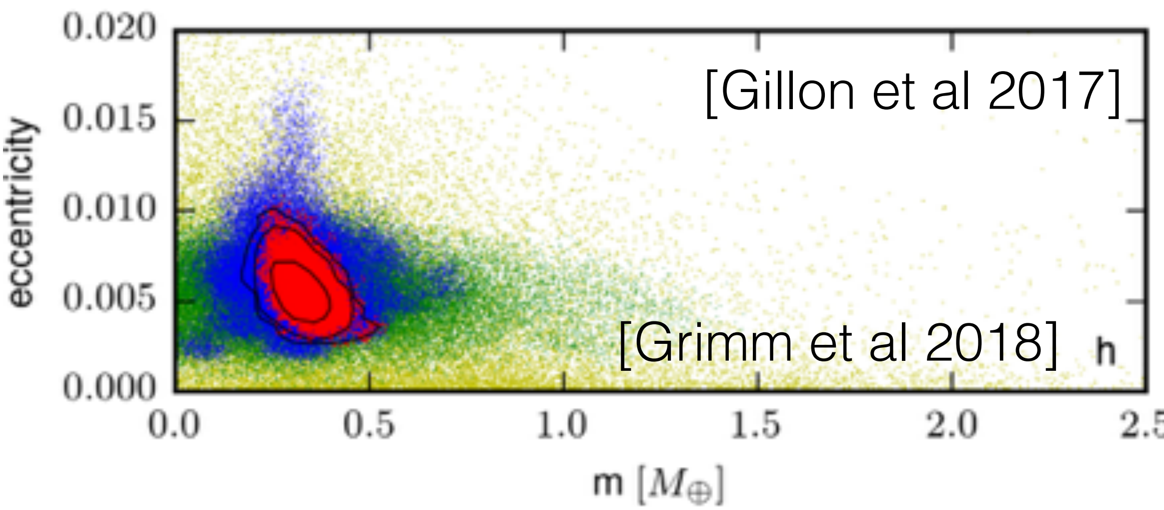

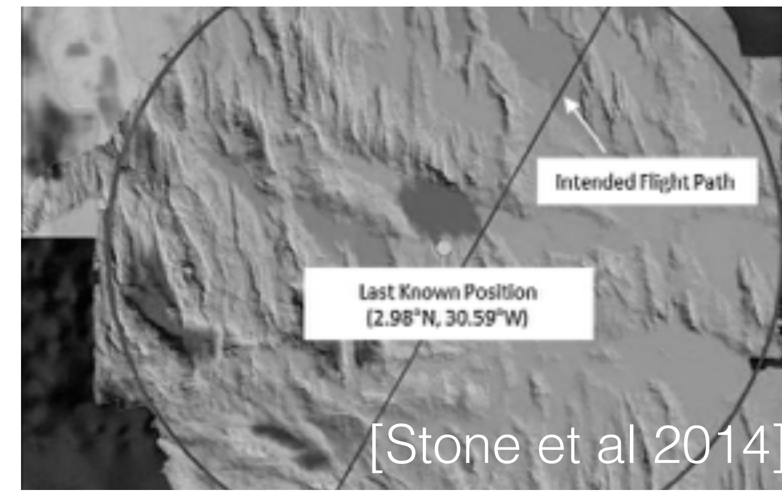

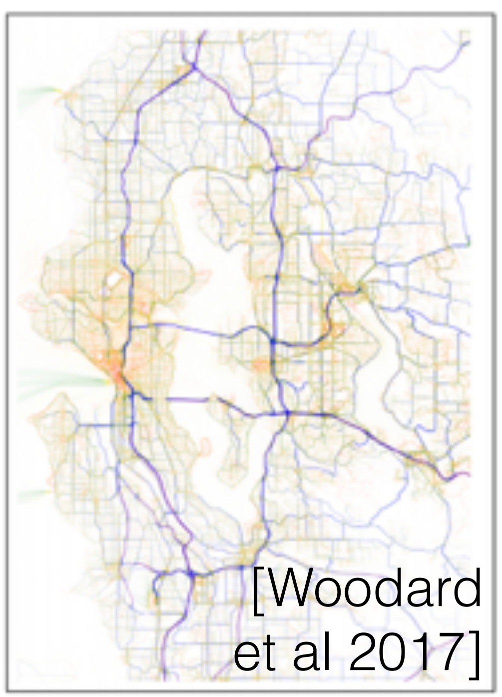

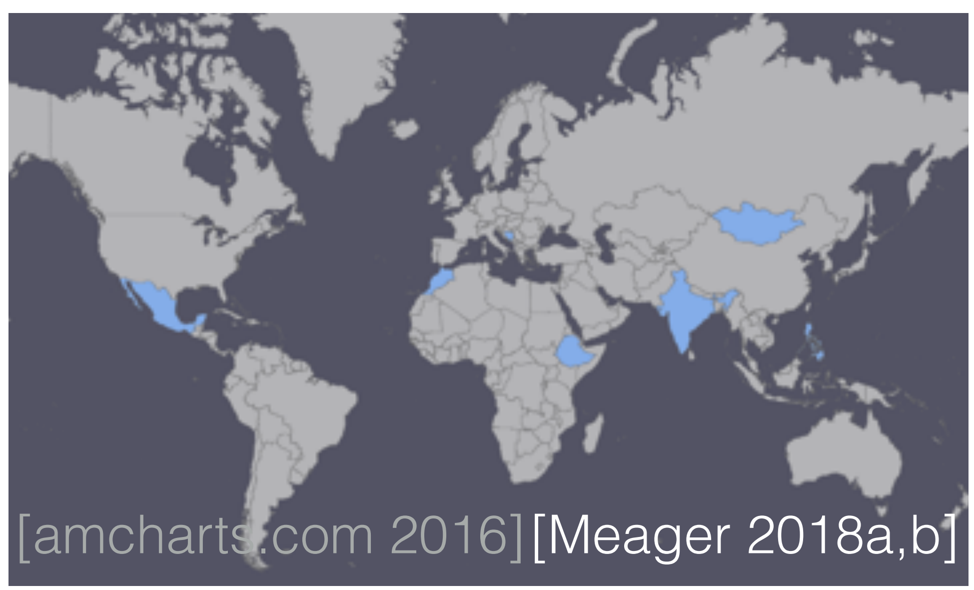

[mc-stan.org]

- Challenge: fast (compute, user), reliable inference
- Uncertainty doesn't have to disappear in large data sets

Variational Bayes

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- Modern problems: often large data, large dimensions

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"Arts"	"Budgets"	"Children"	"Education"
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FILM	TAX	WOMEN	STUDENTS
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[Blei et al 2003]

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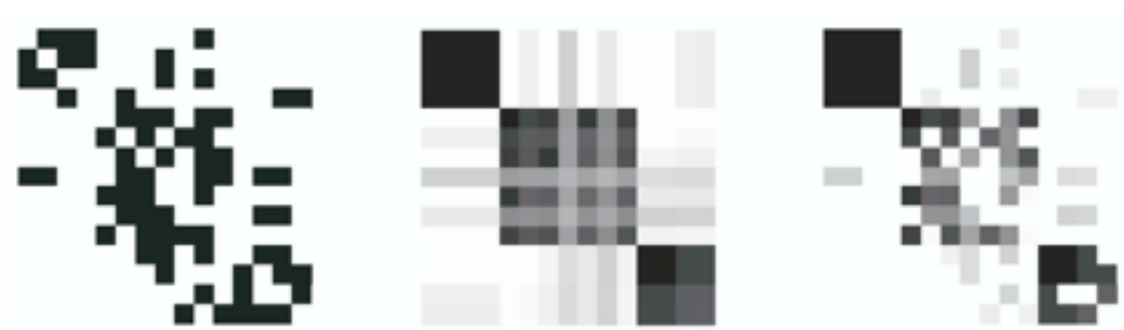

[Airoldi et al 2008]

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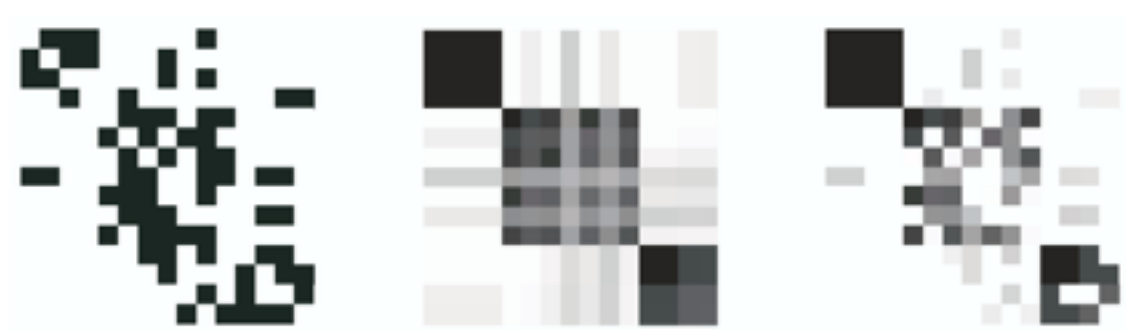

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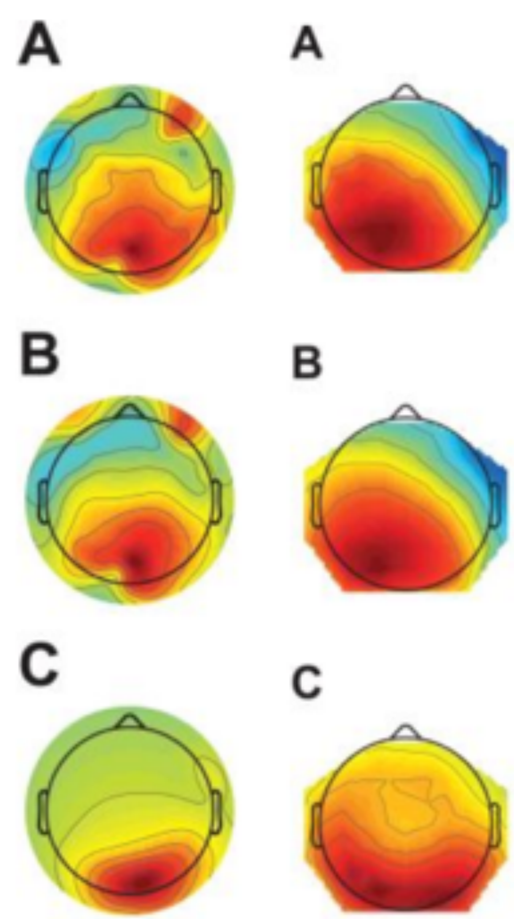

[Gershman et al 2014]

[Blei et al 2018]

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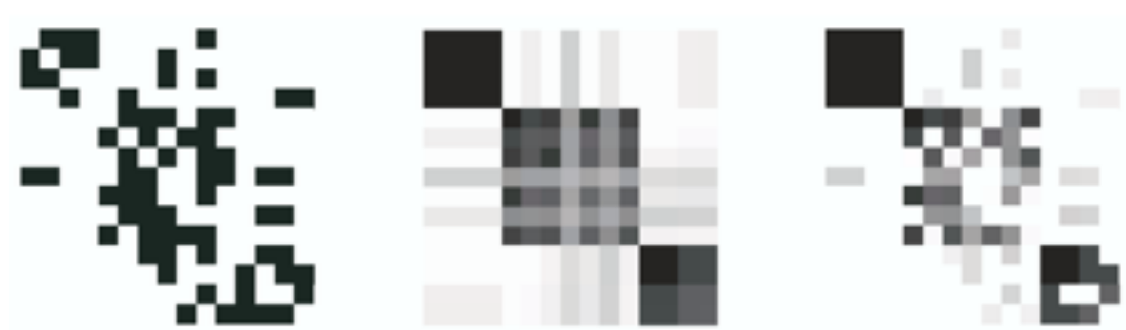

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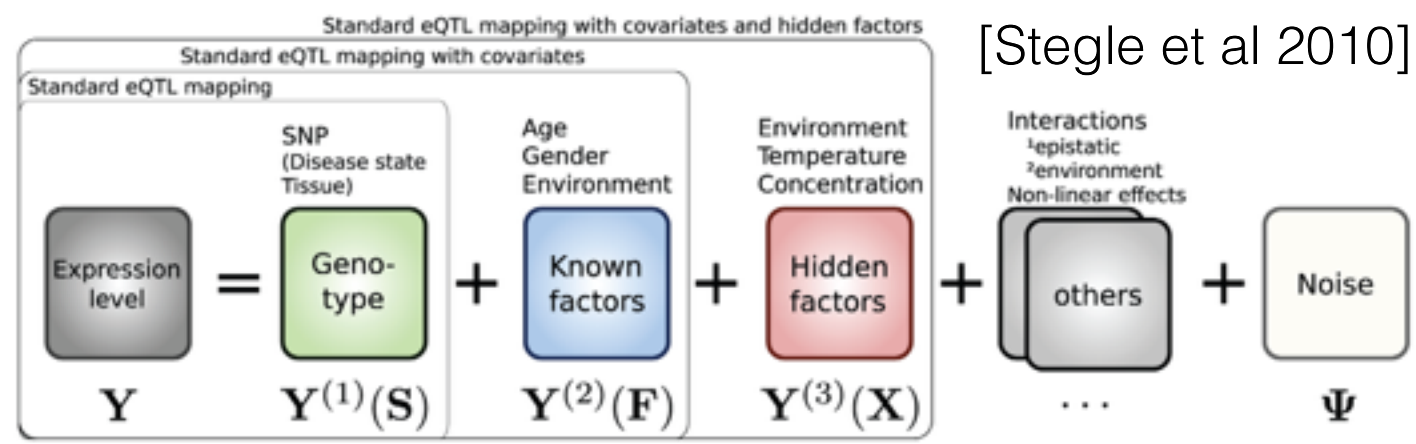
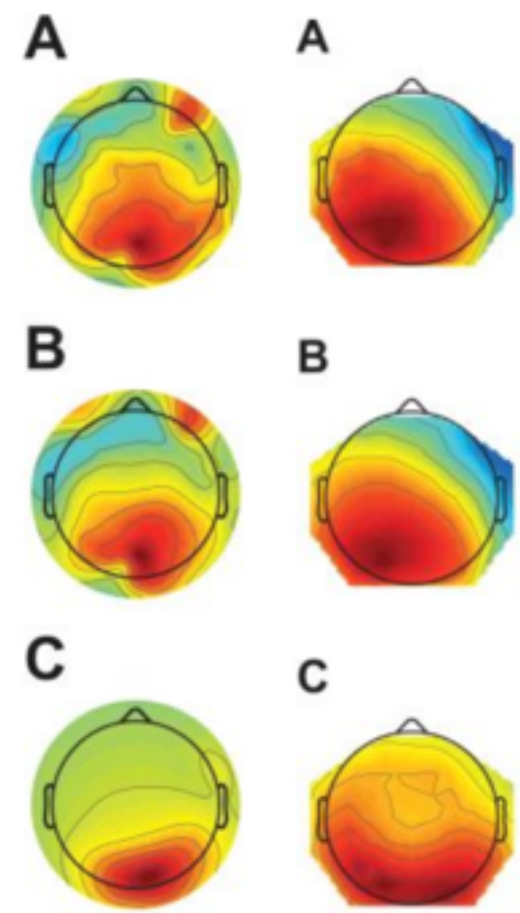


[Stegle et al 2010]



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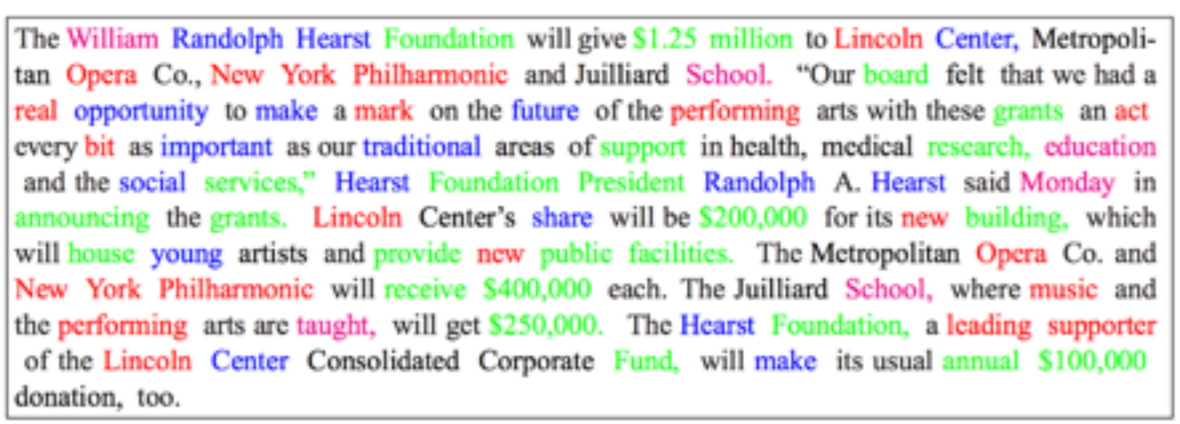

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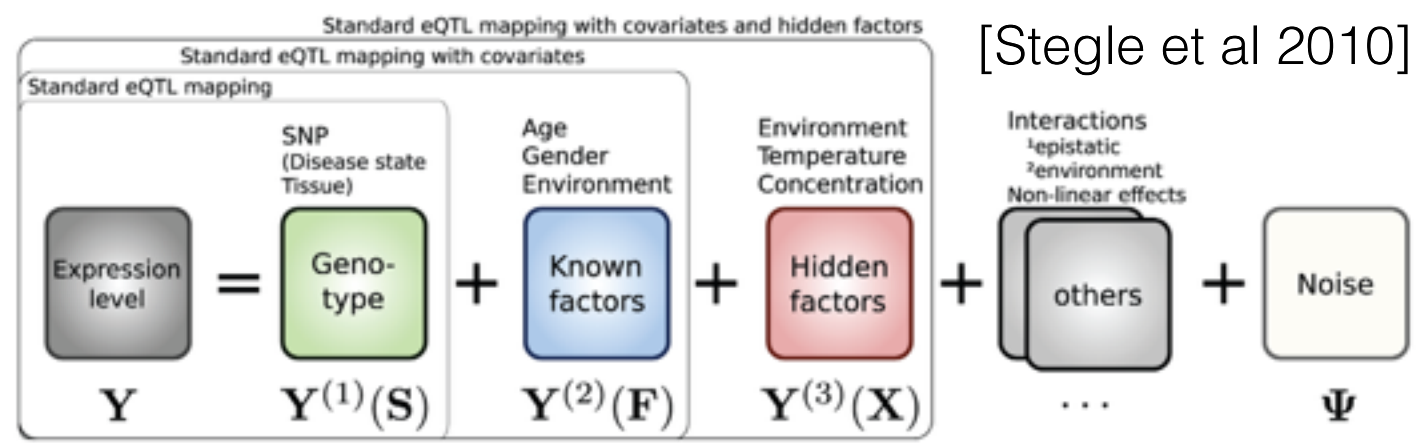

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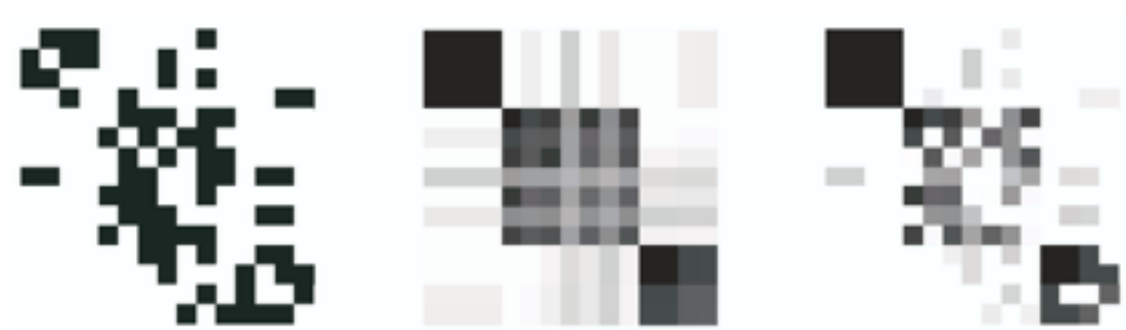

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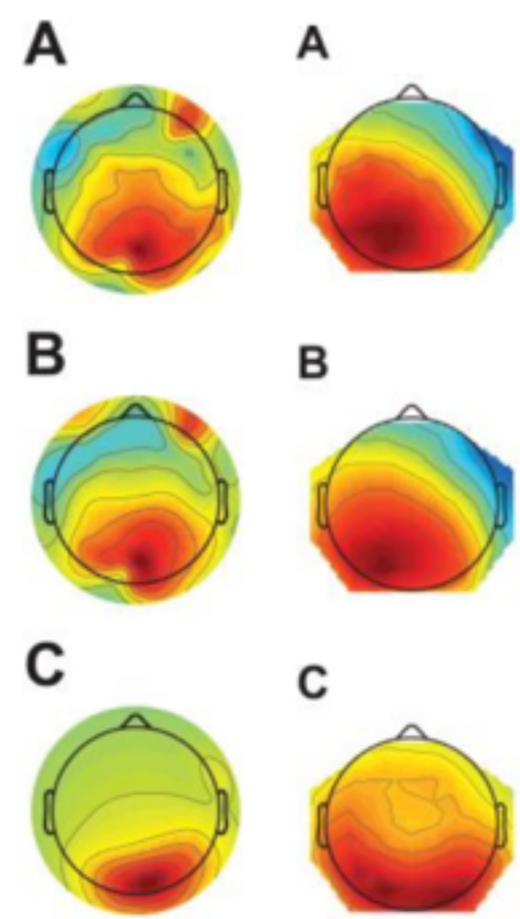

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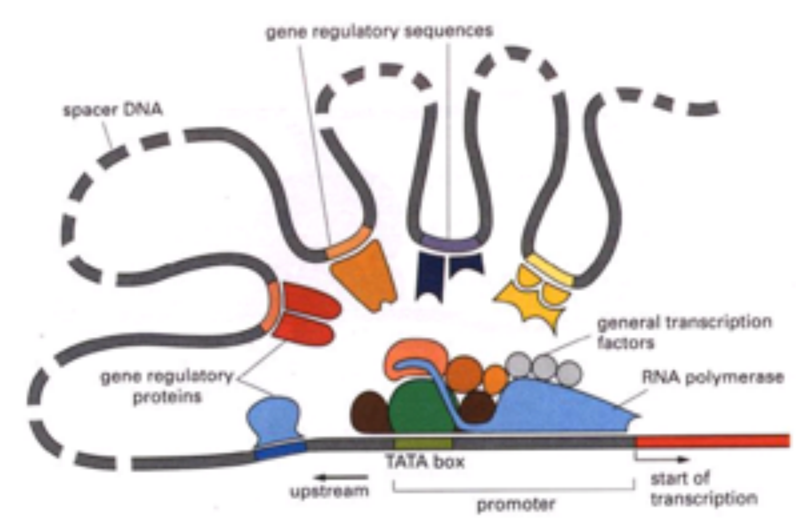

[Xing et al 2004]
[Xing 2003]

[Blei et al 2018]

Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use MFVB?
- When can we trust MFVB?
- Where do we go from here?

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parameters

$$p(\theta)$$

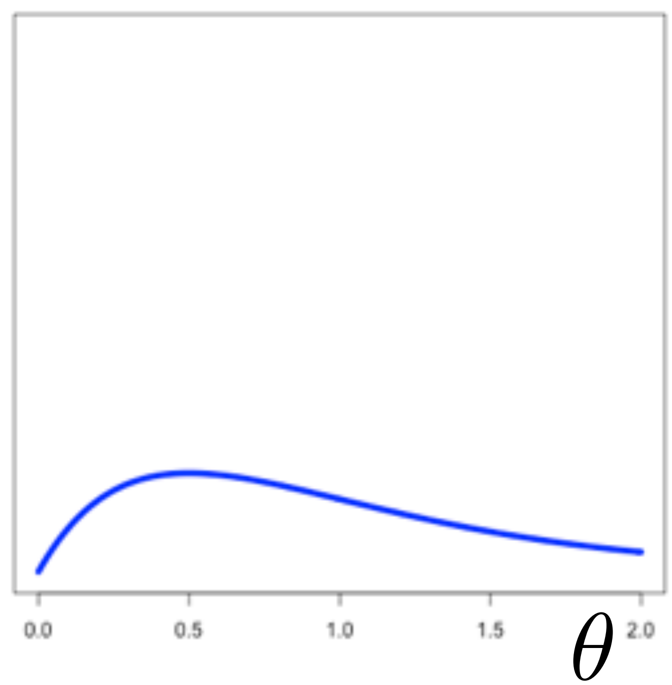
prior

Bayesian inference

parameters

$p(\theta)$

prior

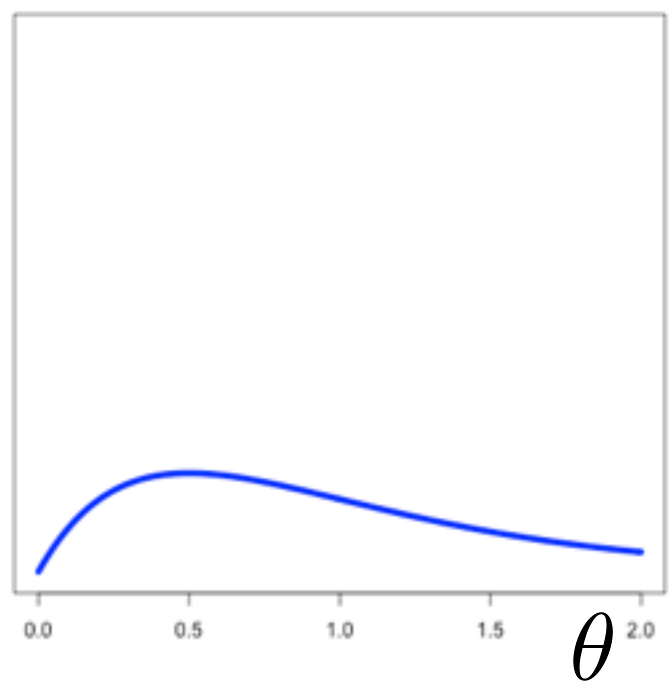


Bayesian inference

parameters

$$p(y_{1:N}|\theta)p(\theta)$$

likelihood prior



Bayesian inference

data parameters

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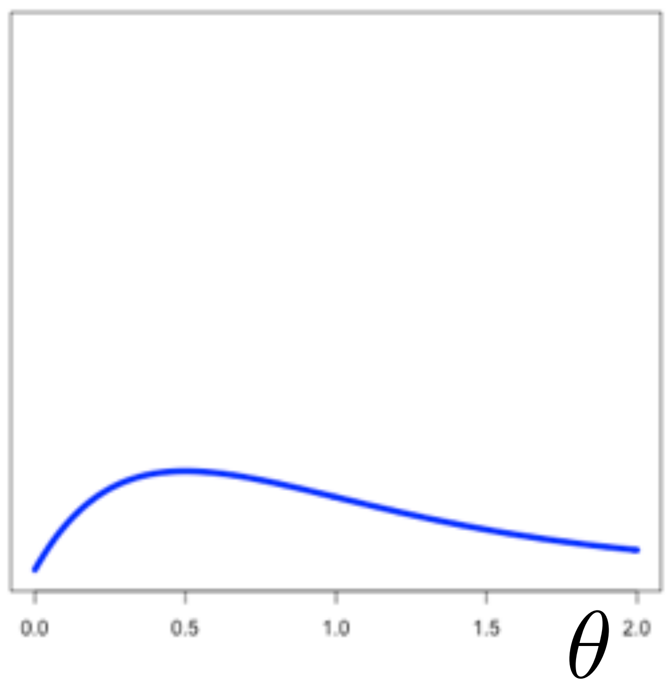
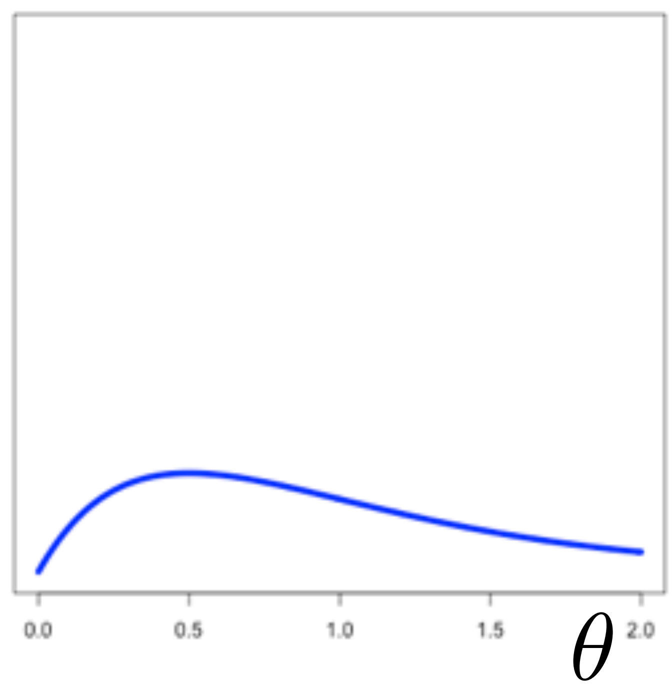


Bayesian inference

data parameters

$$p(\theta|y_{1:N}) \propto_\theta p(y_{1:N}|\theta)p(\theta)$$

posterior likelihood prior



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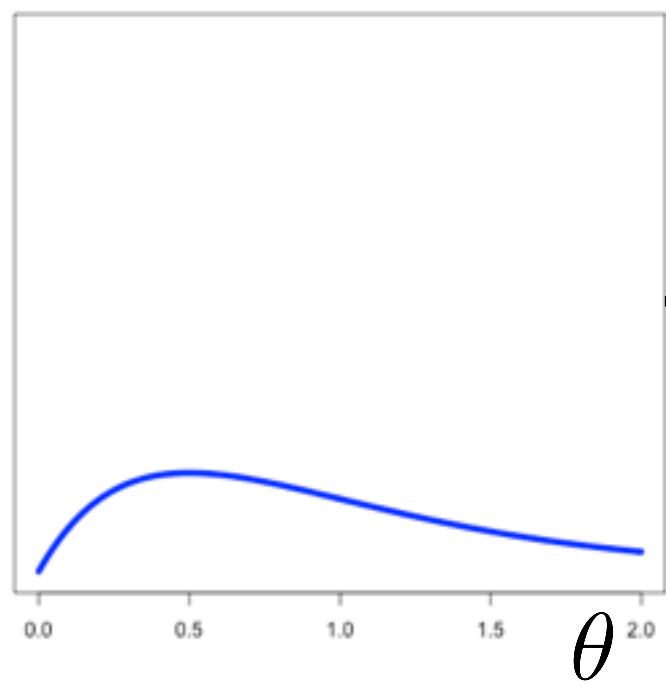

Bayes Theorem

Bayesian inference

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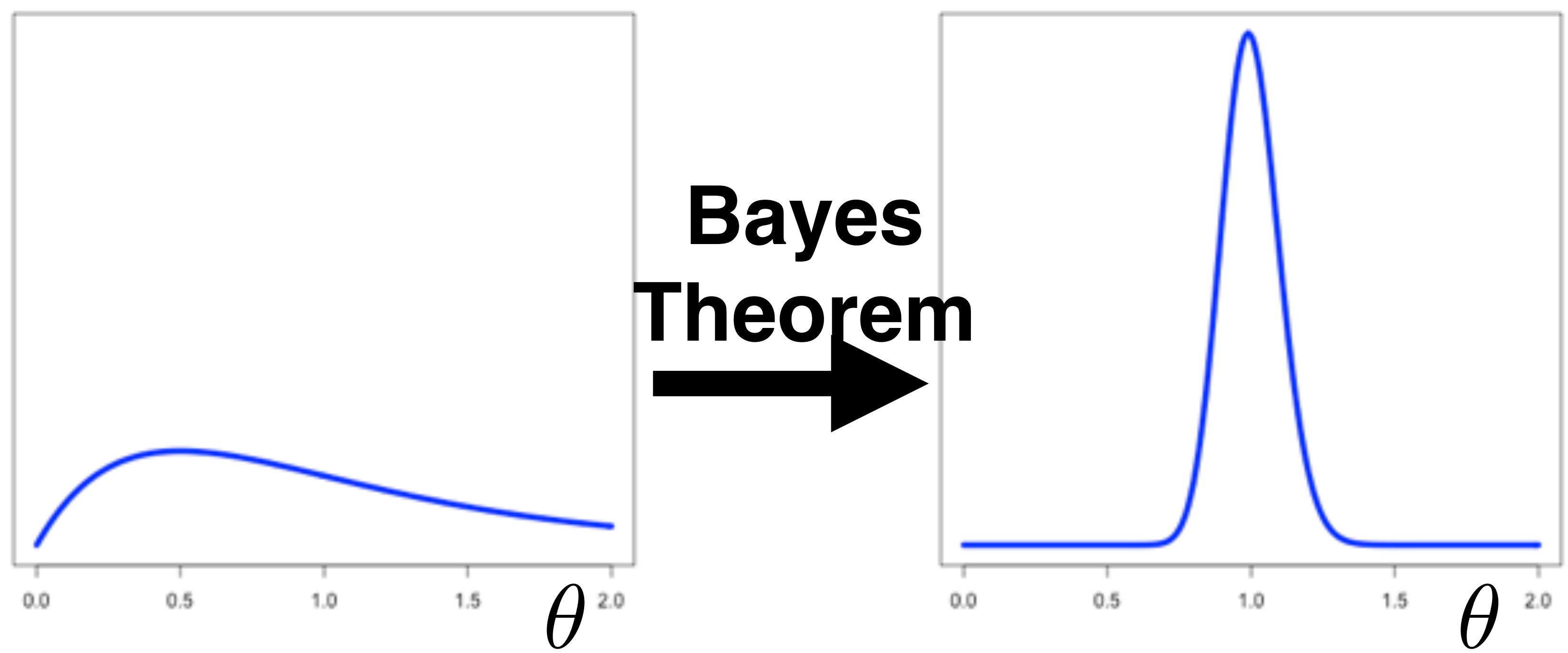

Bayesian inference

data parameters

$$p(\theta|y_{1:N}) \propto_\theta p(y_{1:N}|\theta)p(\theta)$$

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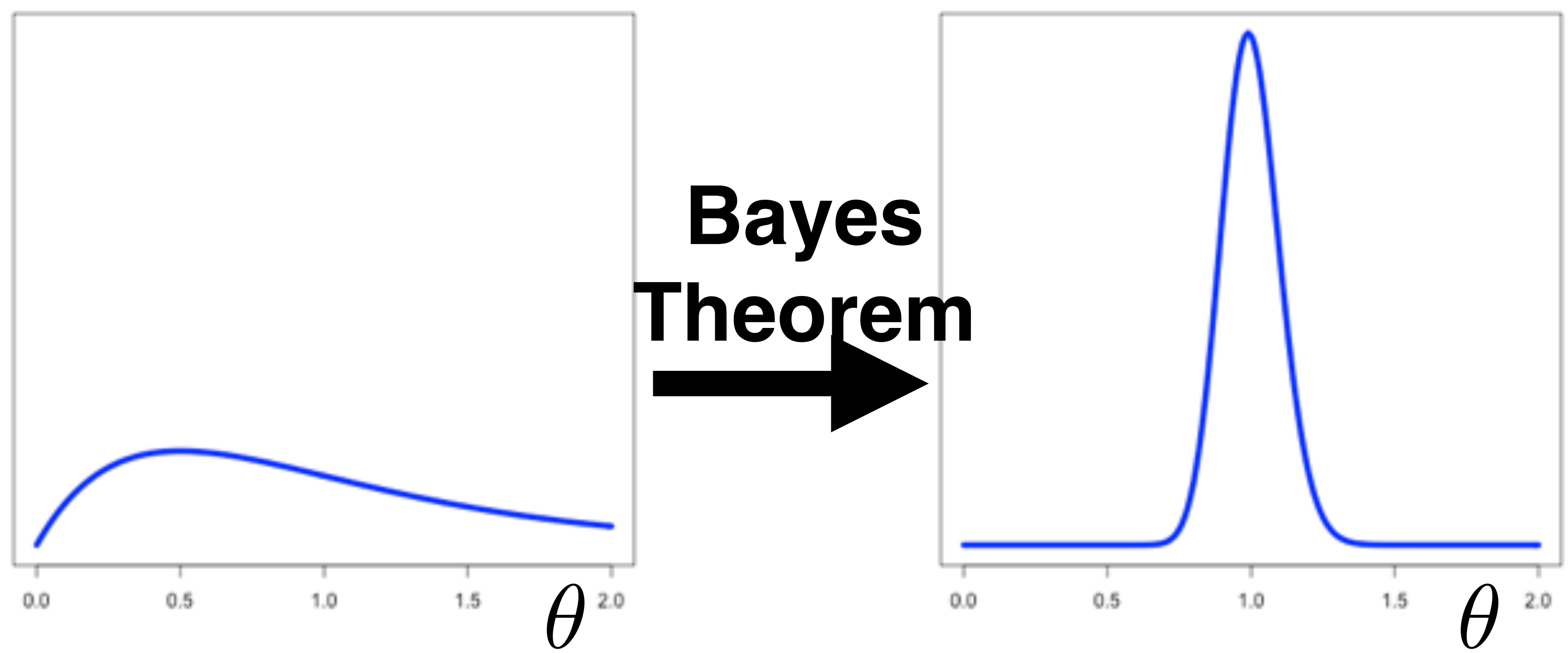

1. Build a model: choose prior, likelihood

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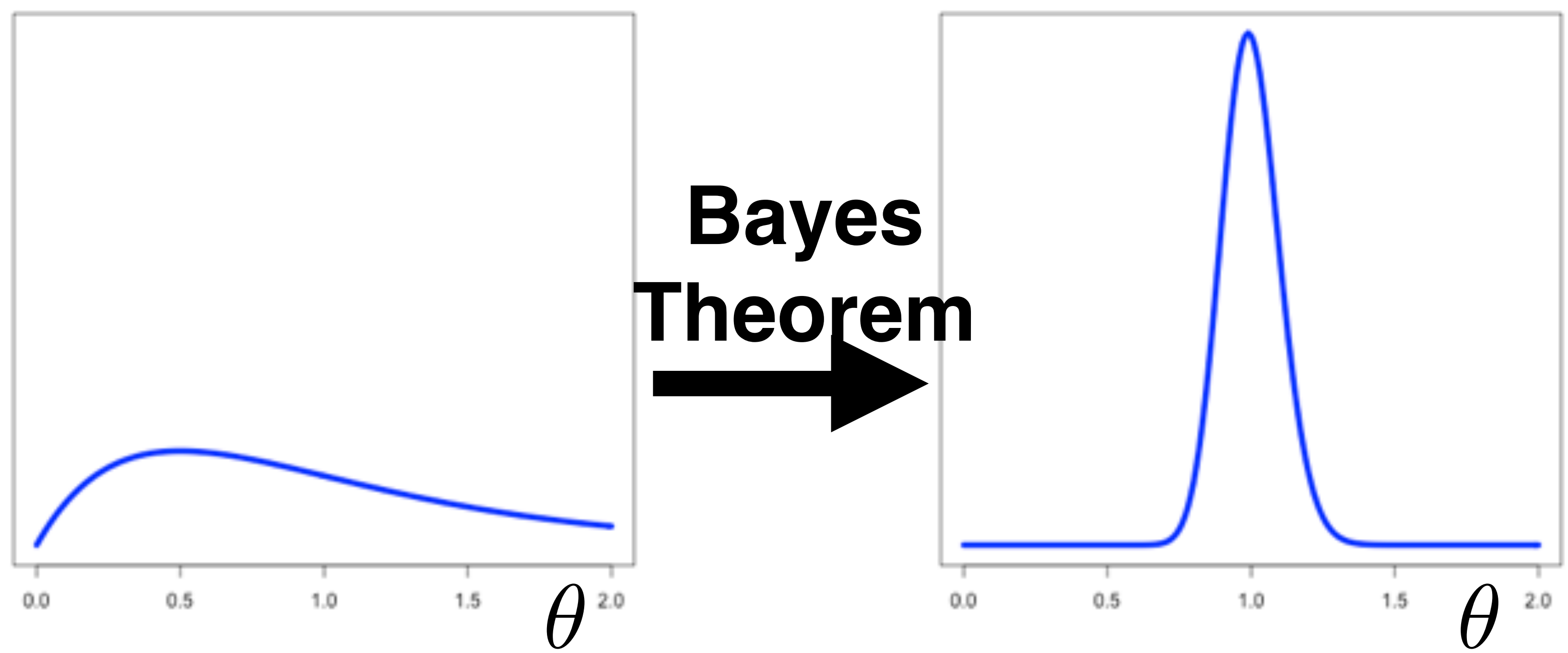

1. Build a model: choose prior, likelihood
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Bayesian inference

data → $y_{1:N}$; parameters → θ

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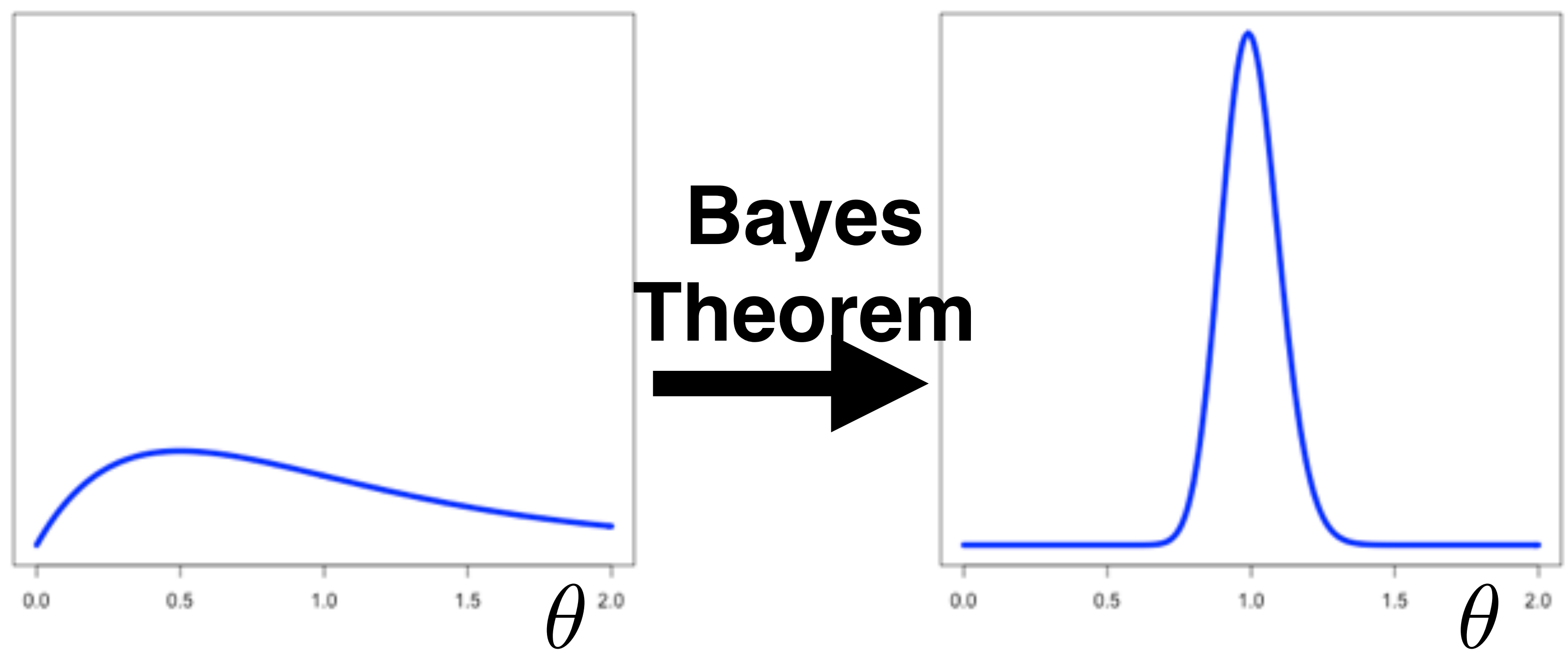

1. Build a model: choose prior, likelihood
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3. Report a summary, e.g. posterior means and (co)variances

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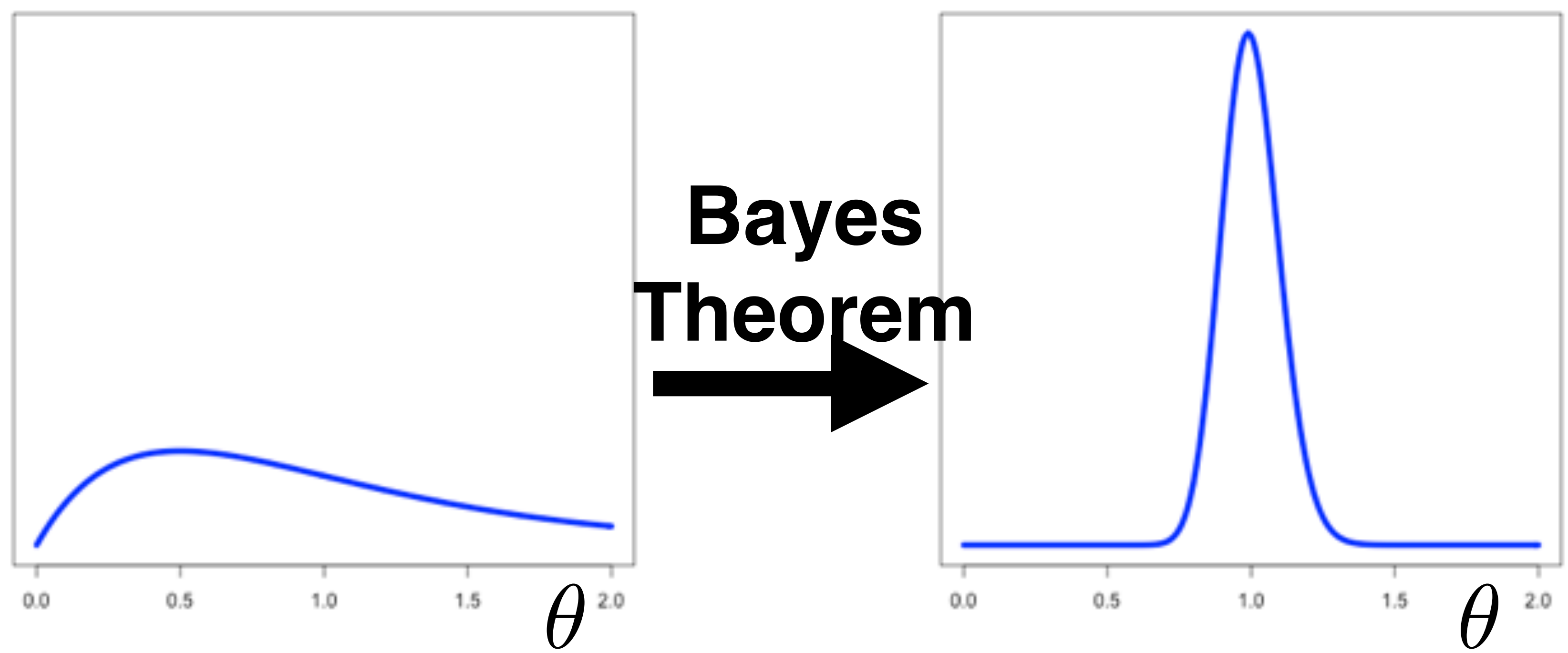

1. Build a model: choose prior, likelihood
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- Why are steps 2 and 3 hard?

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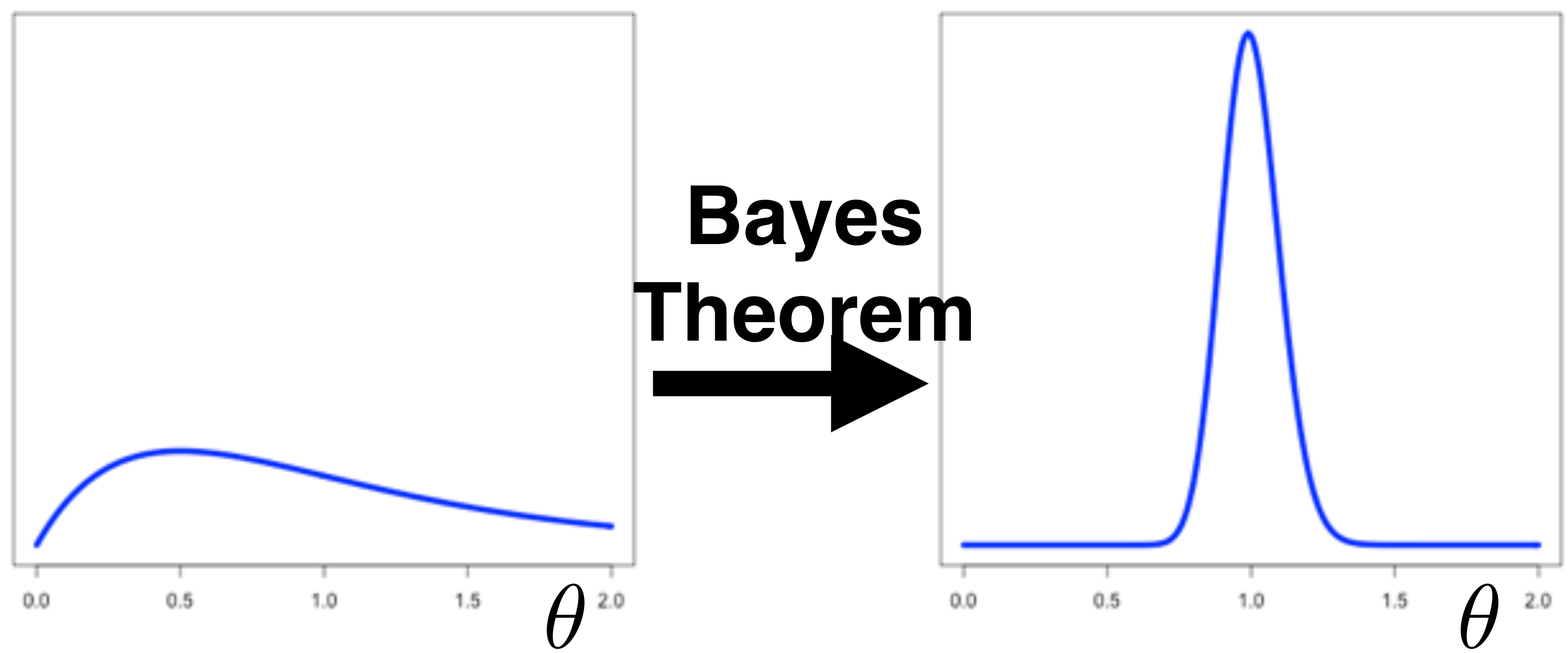

1. Build a model: choose prior, likelihood
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- Why are steps 2 and 3 hard? High-dimensional integration

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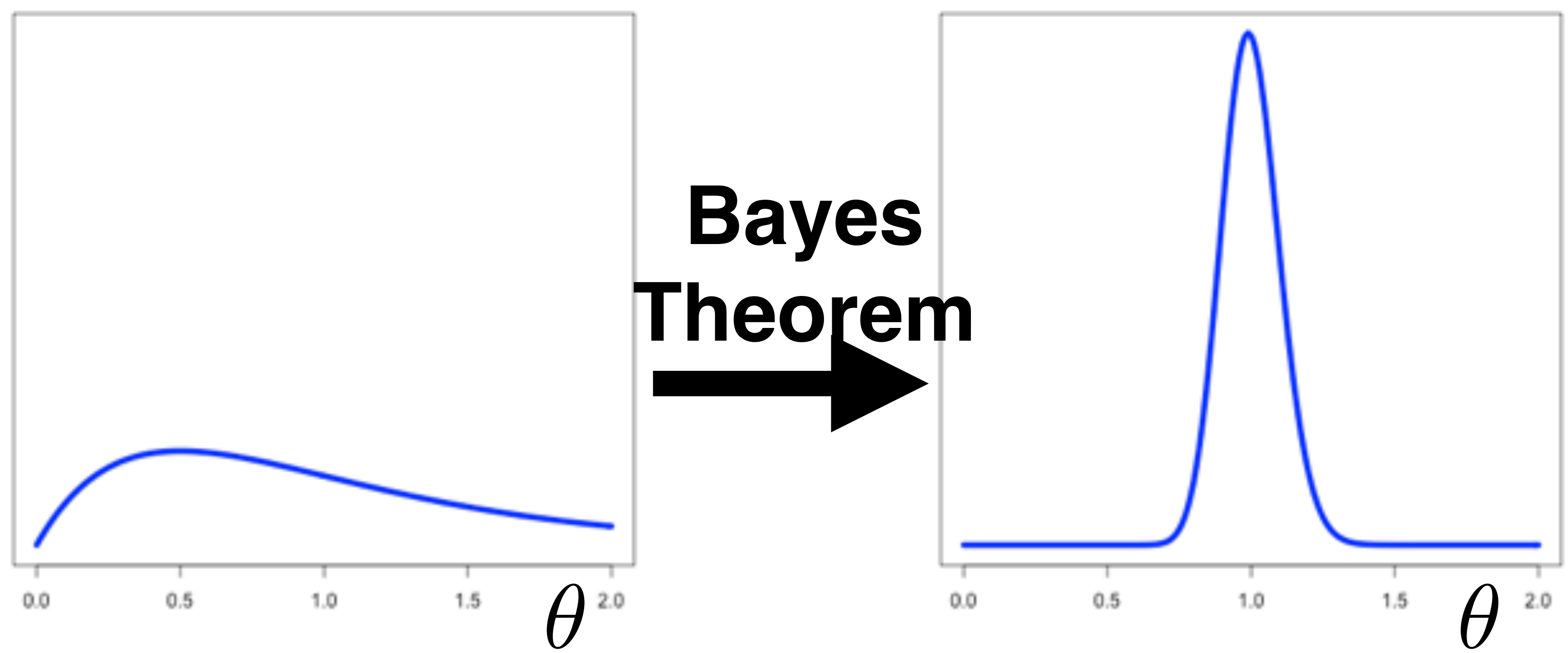

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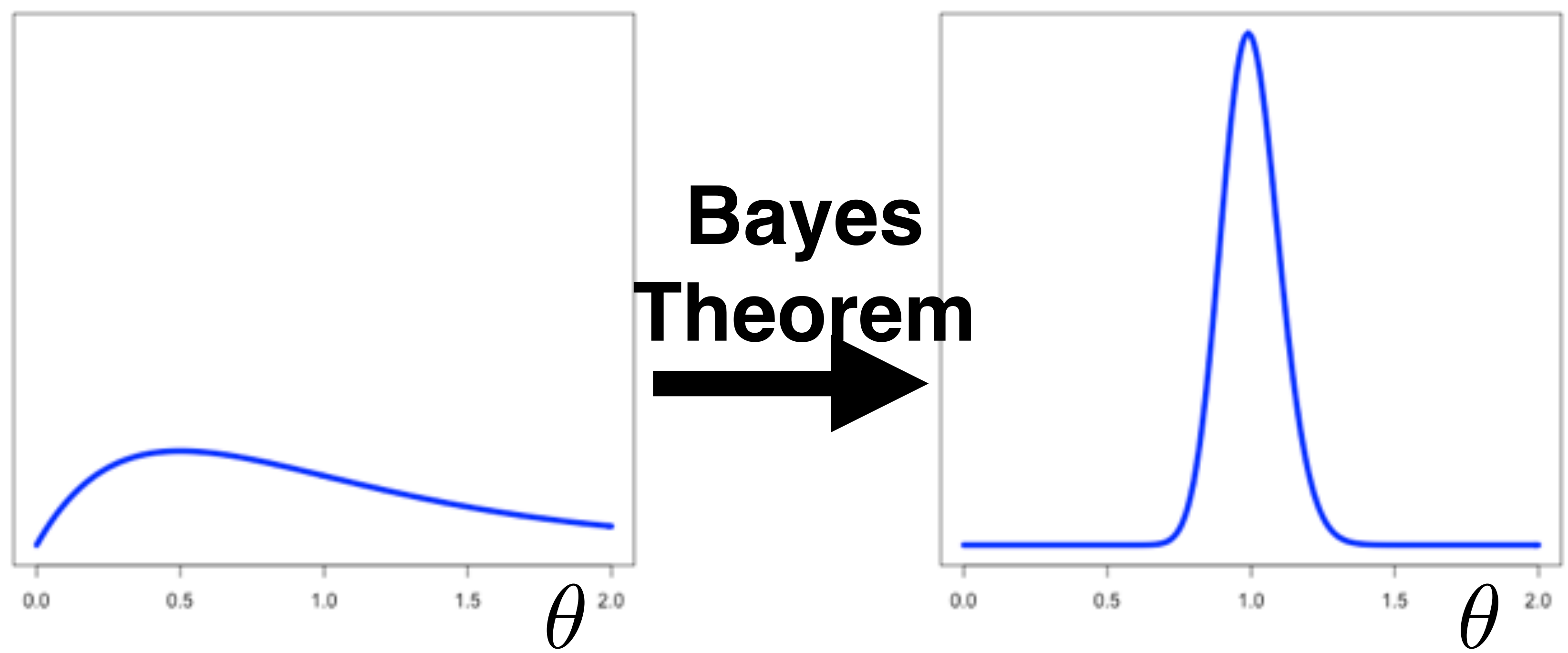

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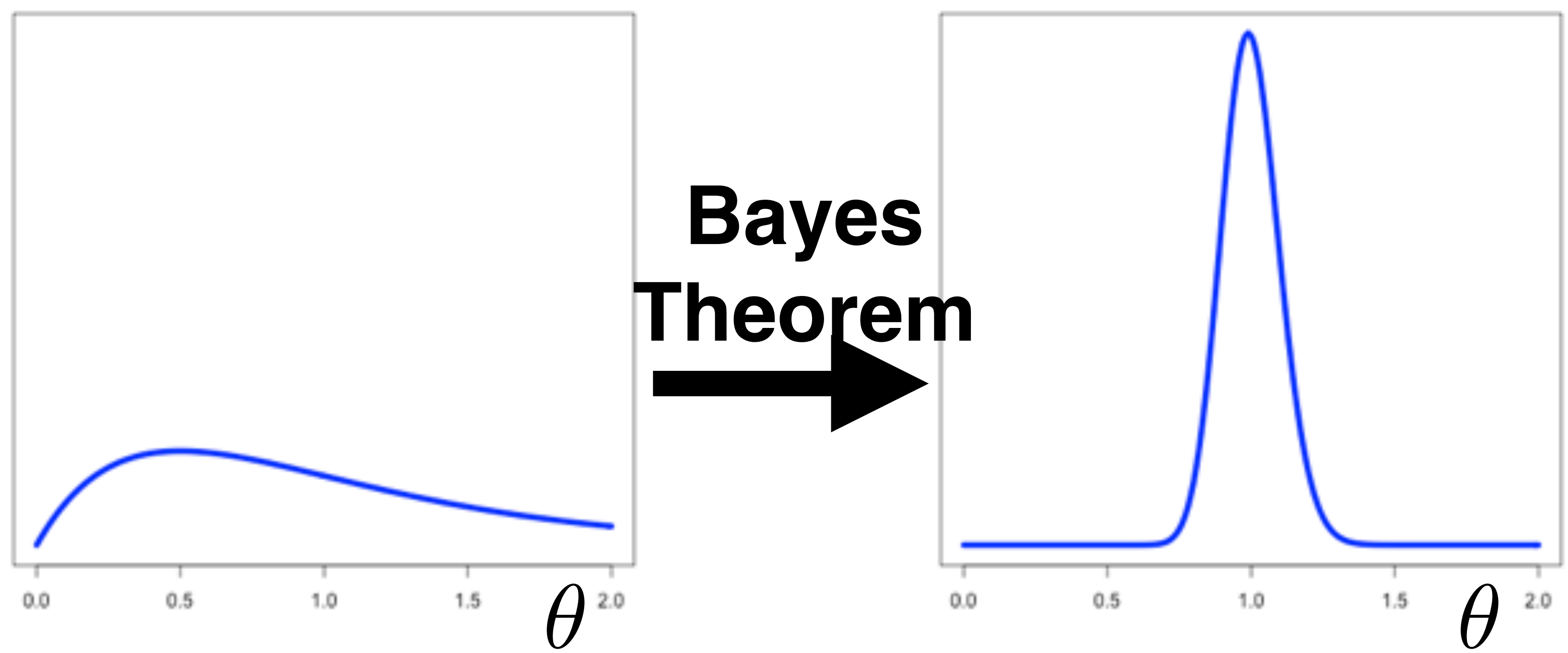

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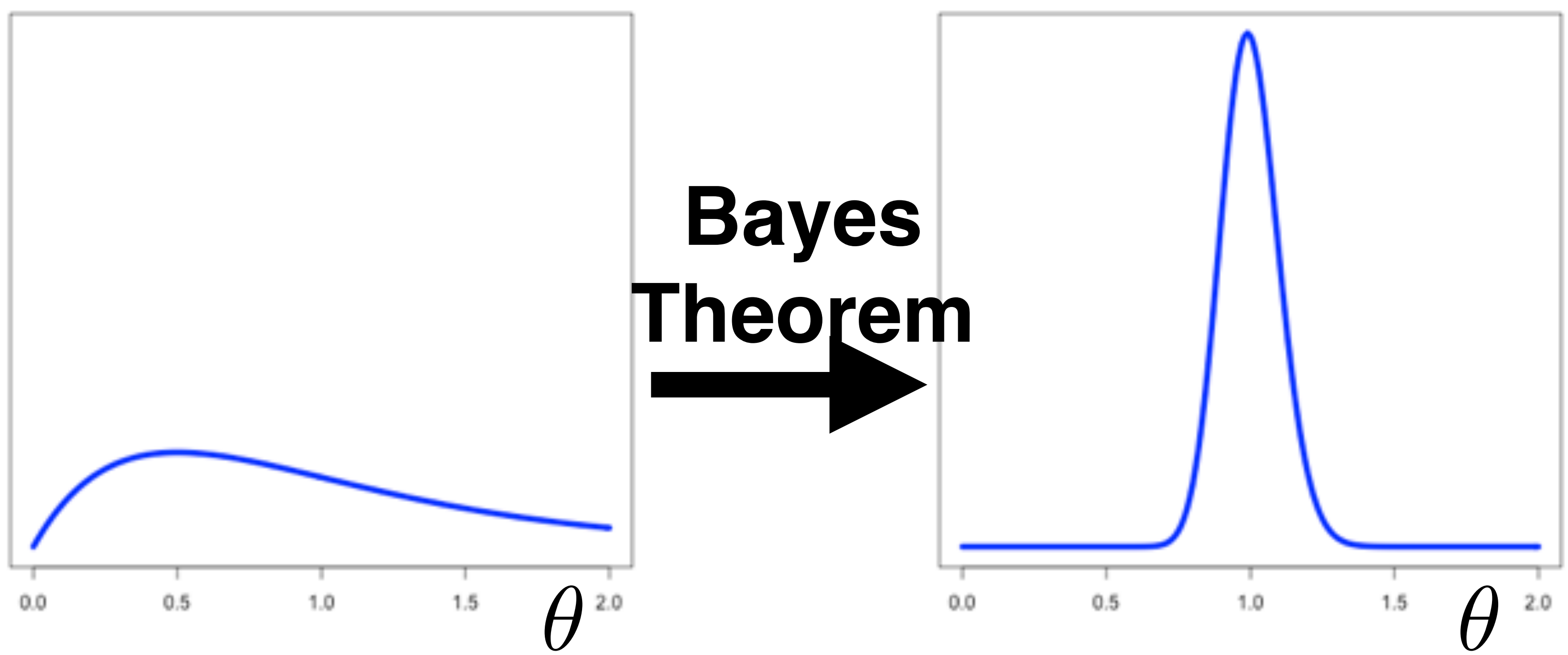

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- Why are steps 2 and 3 hard? High-dimensional integration
- Turn to approximation

Approximate Bayesian Inference

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[Bardenet, Doucet, Holmes 2017]

- Gold standard: Markov Chain Monte Carlo (MCMC)

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Instead: an optimization approach

- Approximate posterior with q^*

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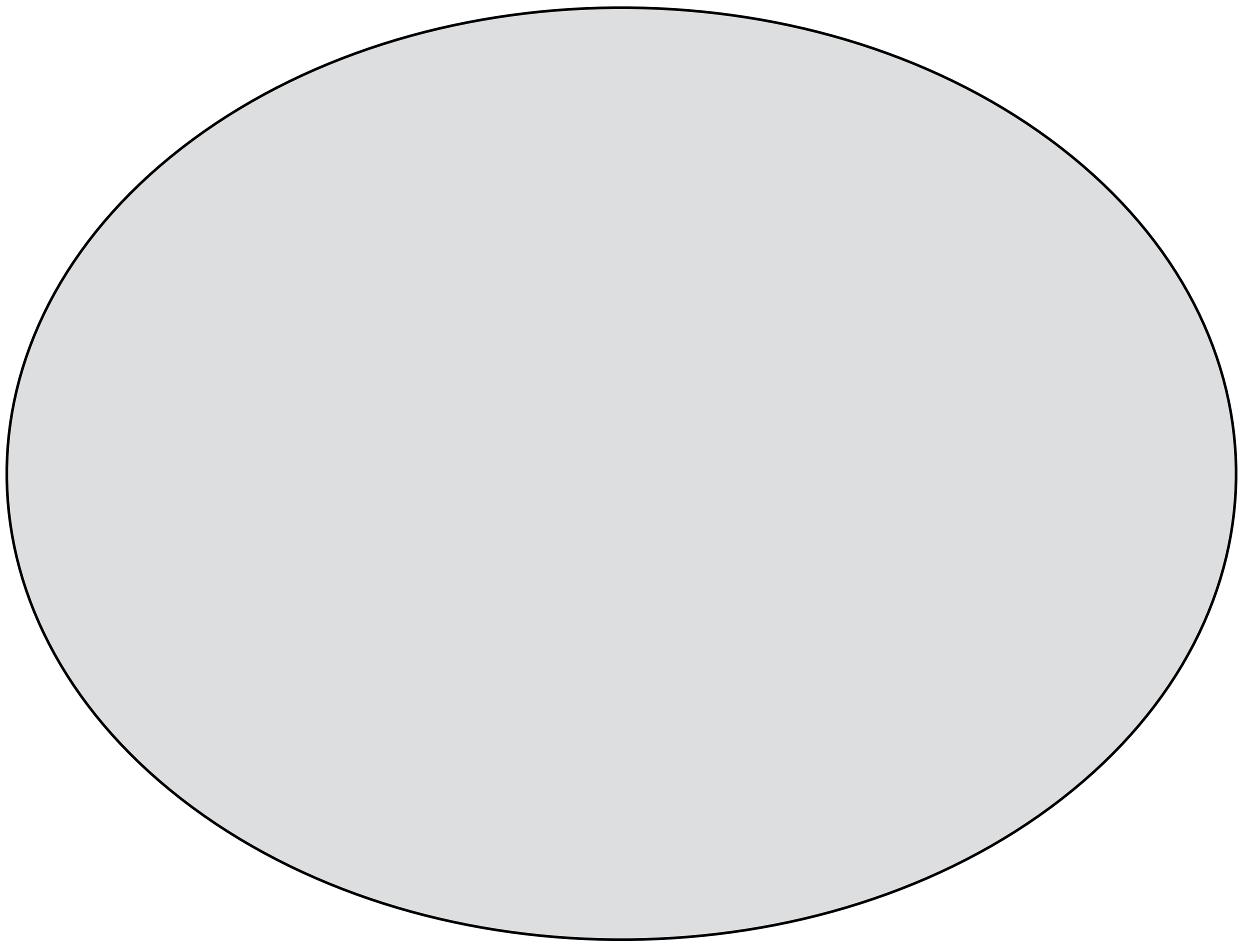

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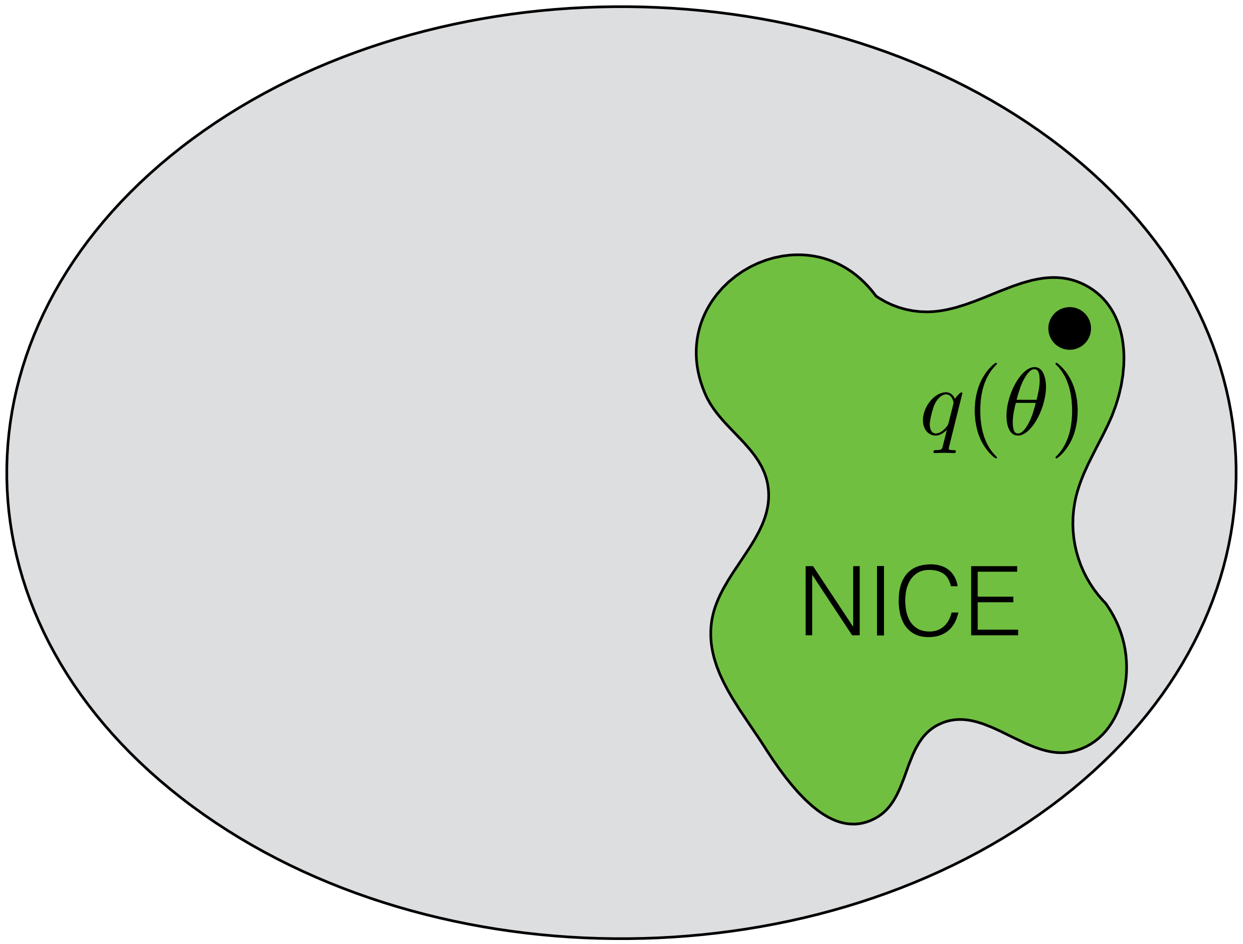

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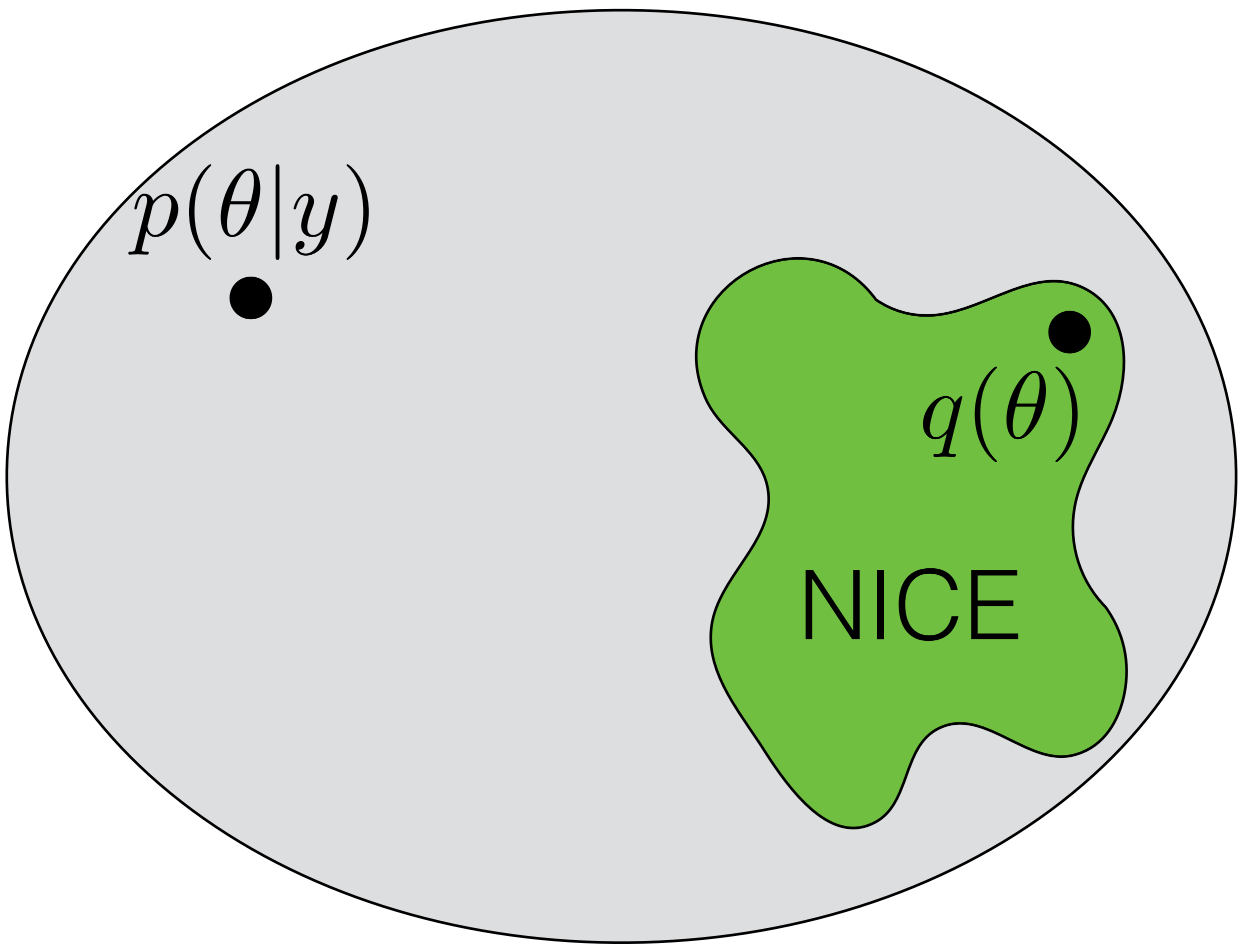

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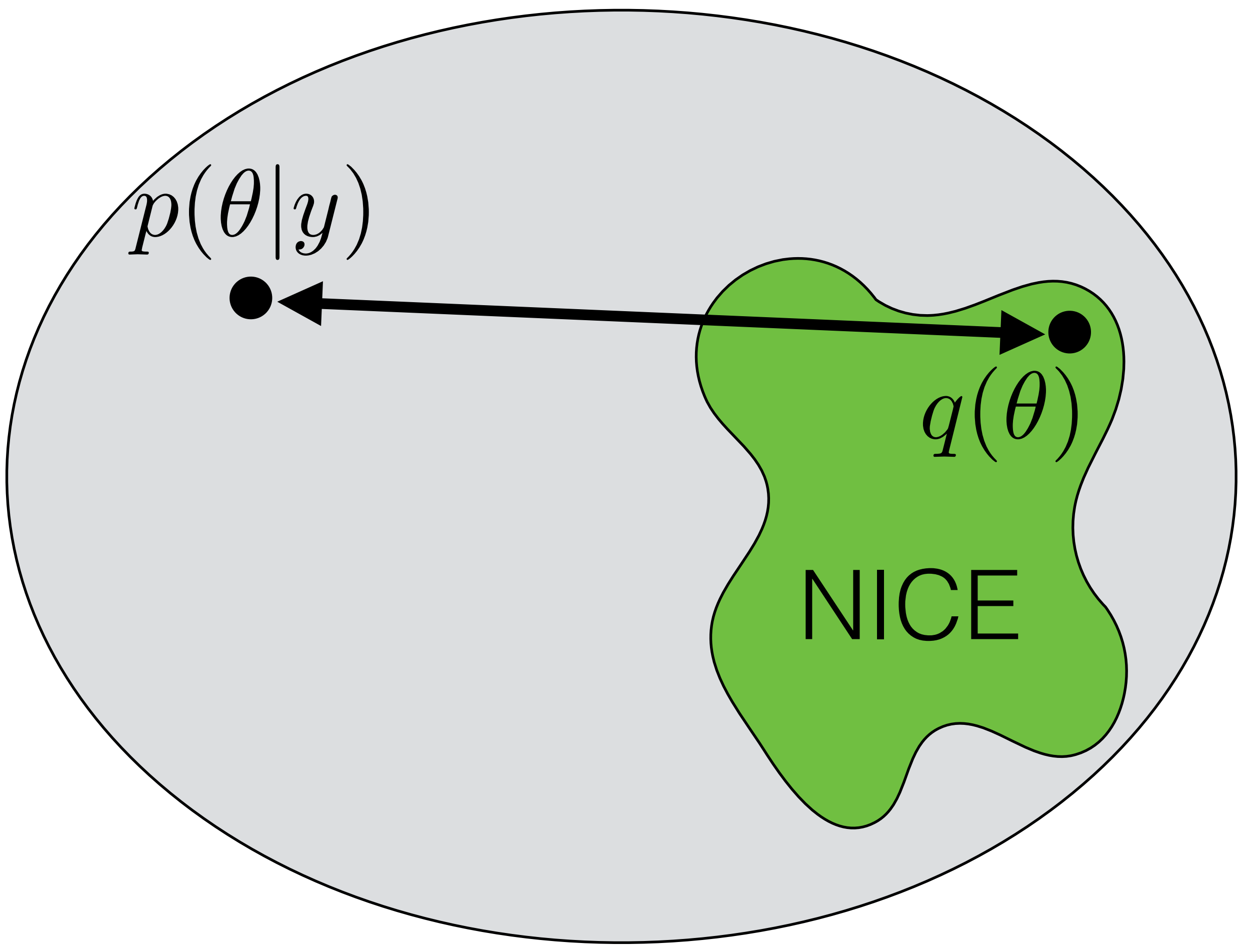

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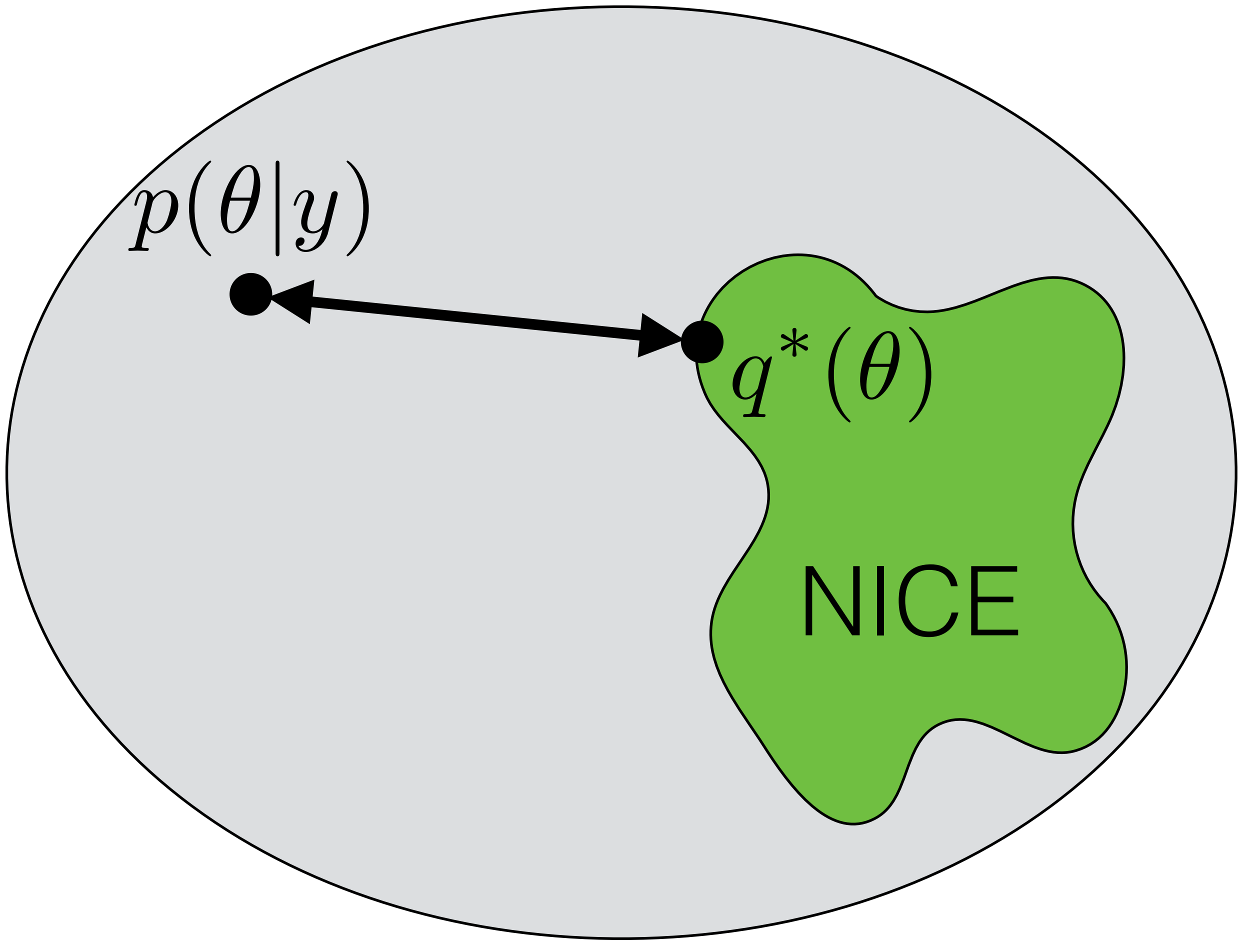

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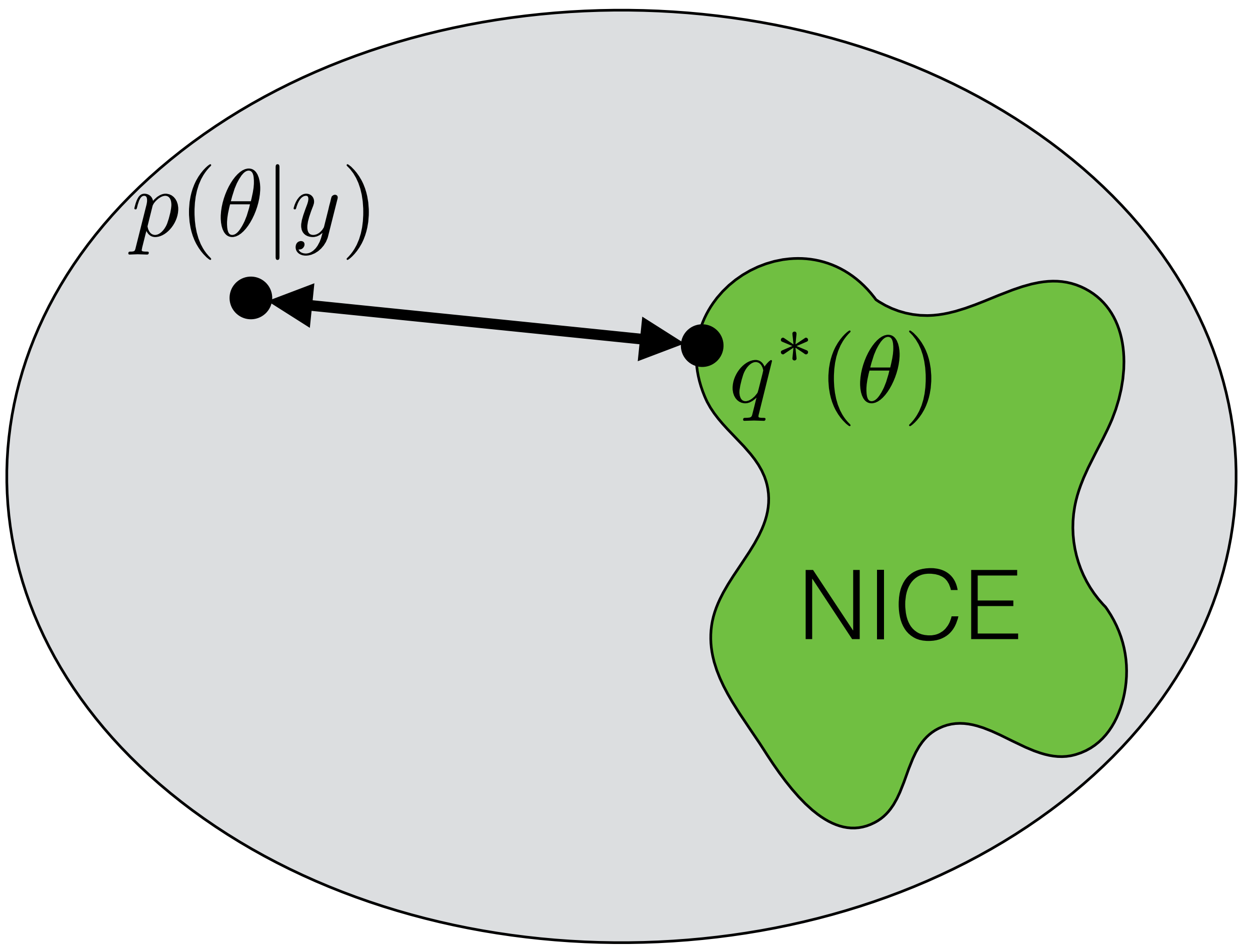

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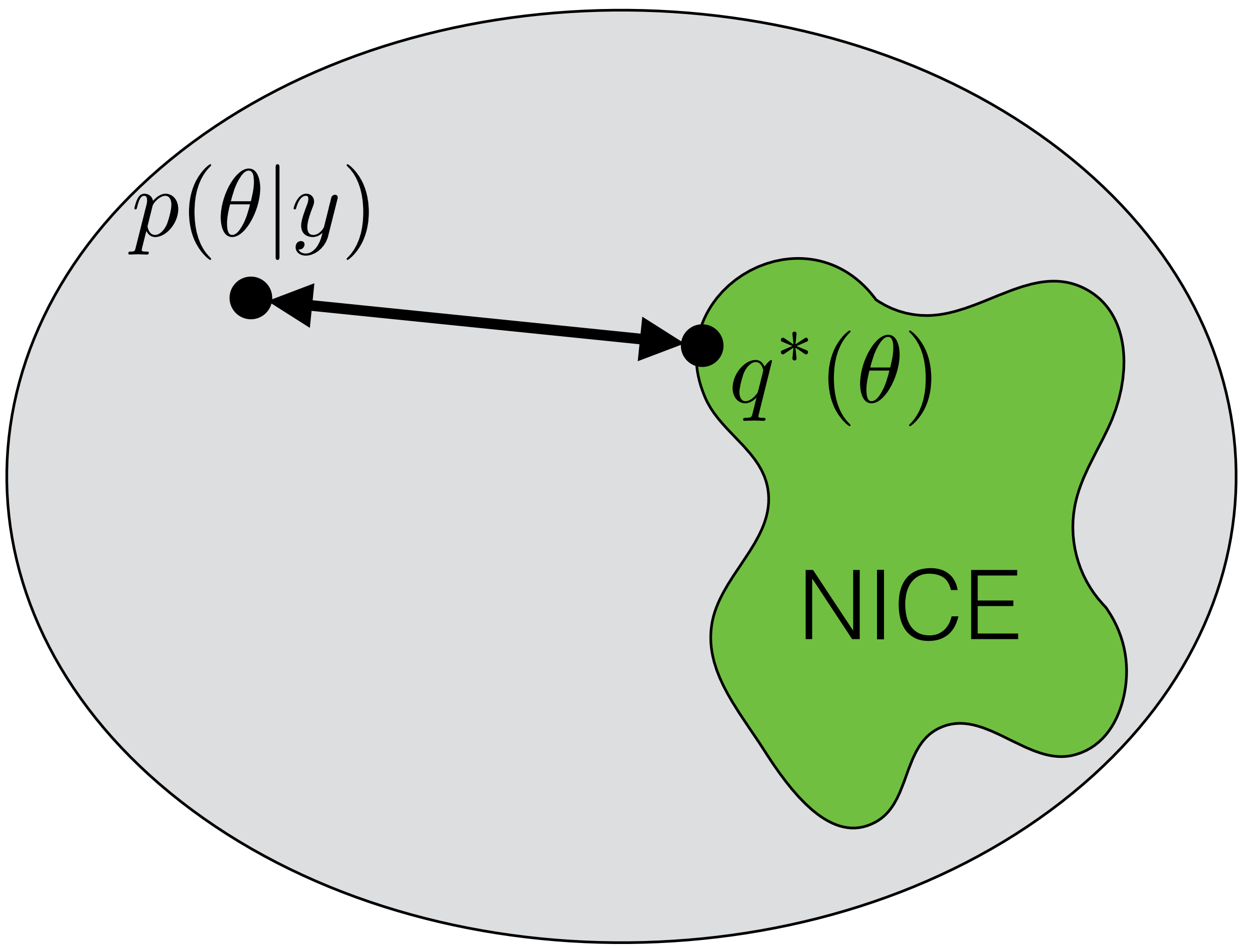

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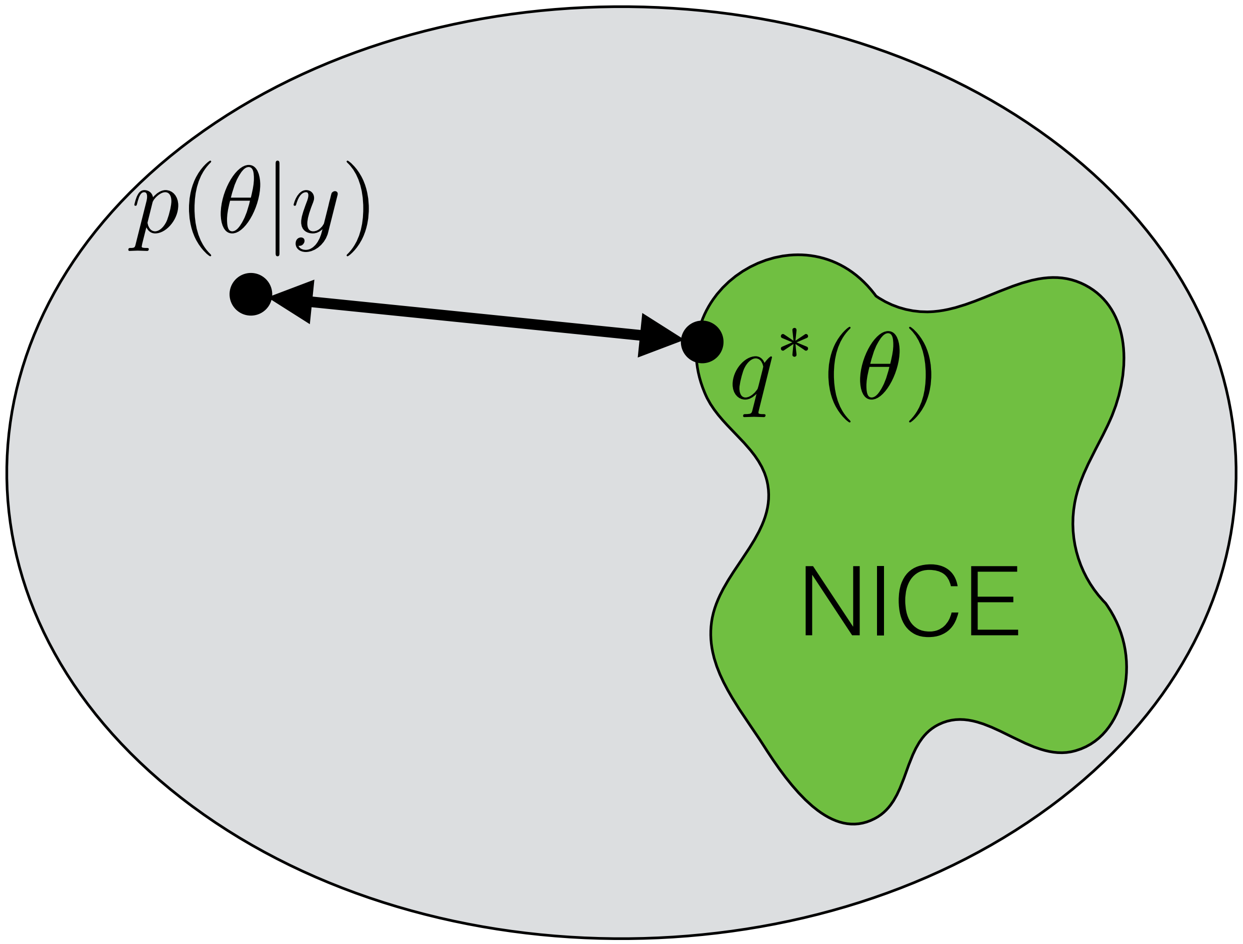

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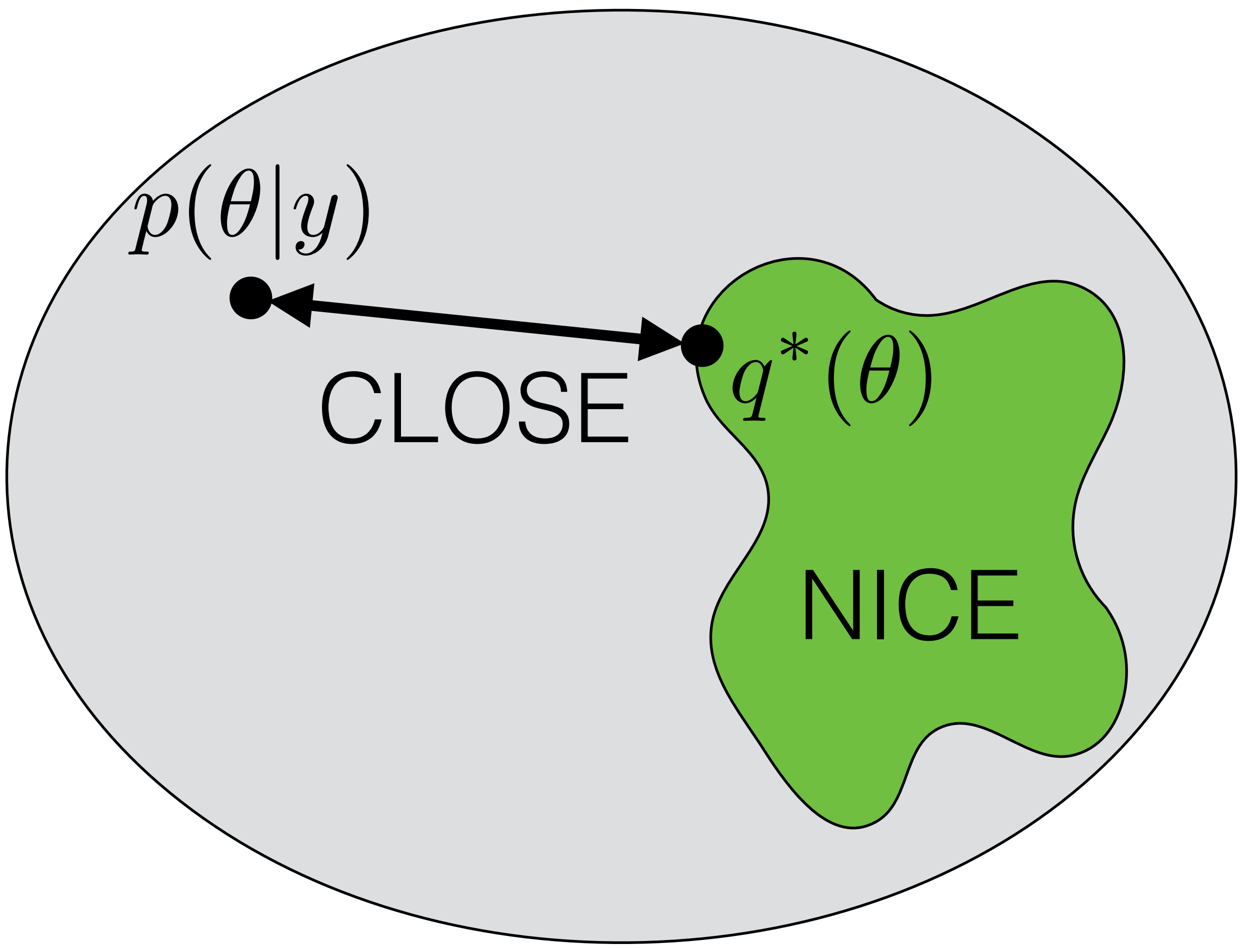

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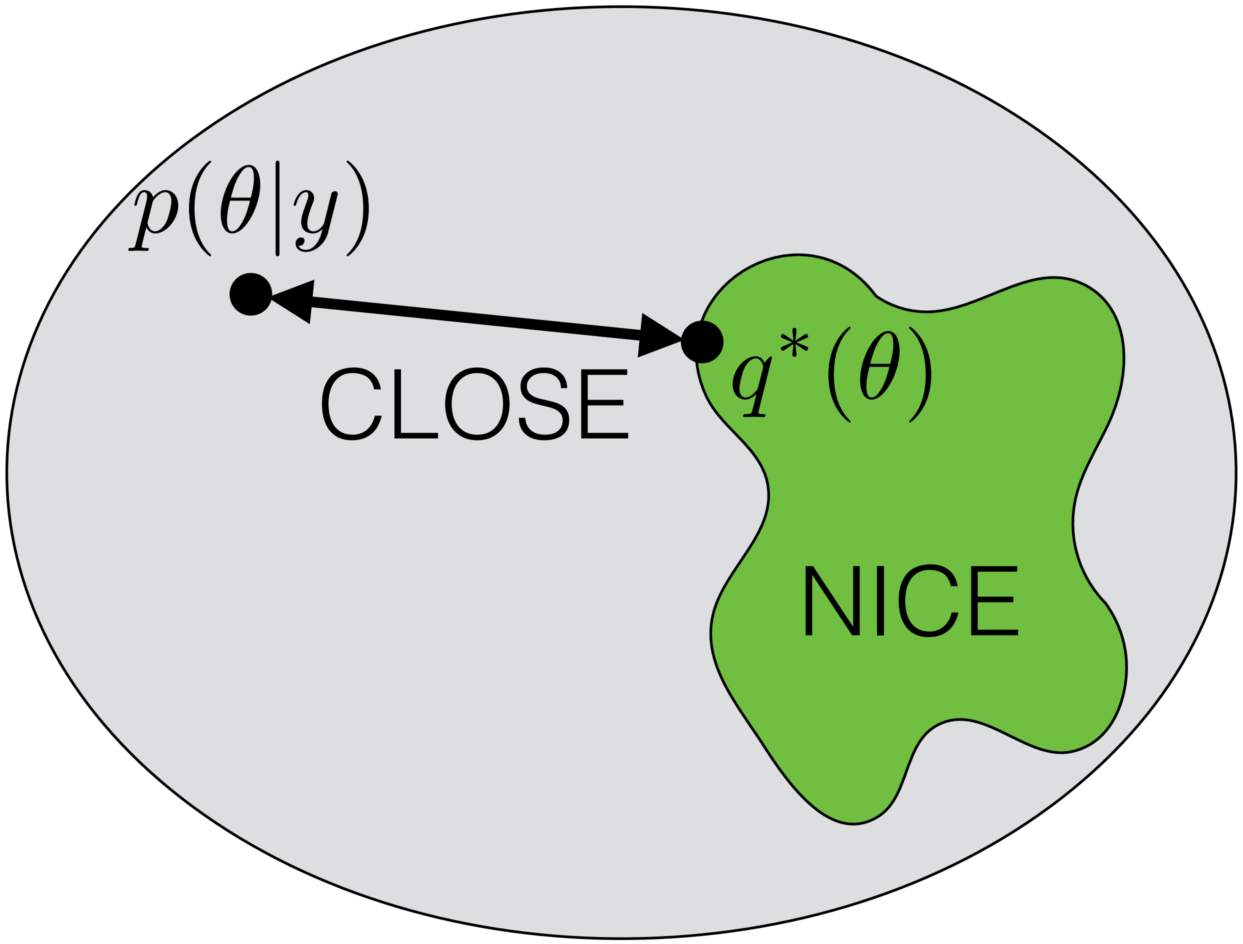

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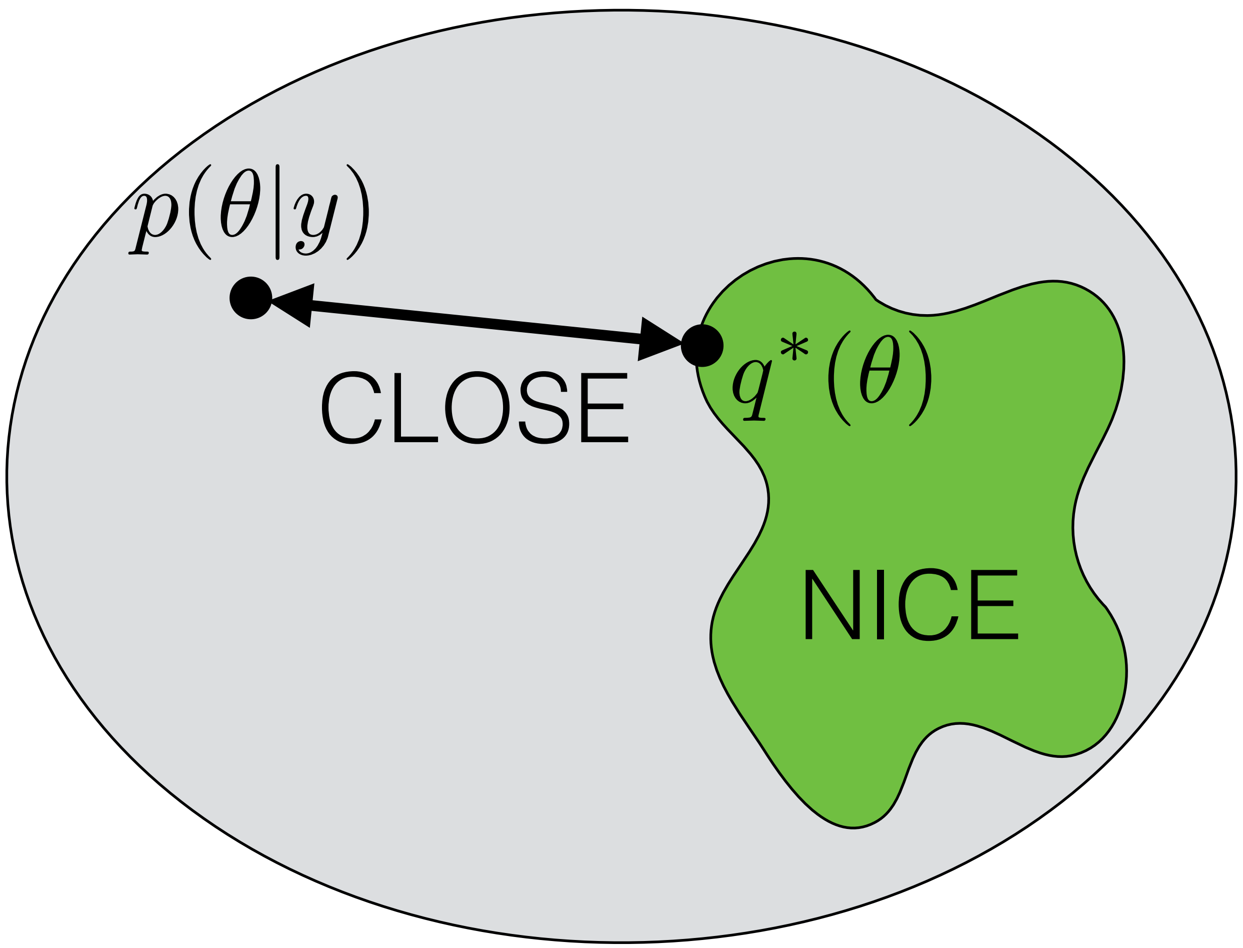

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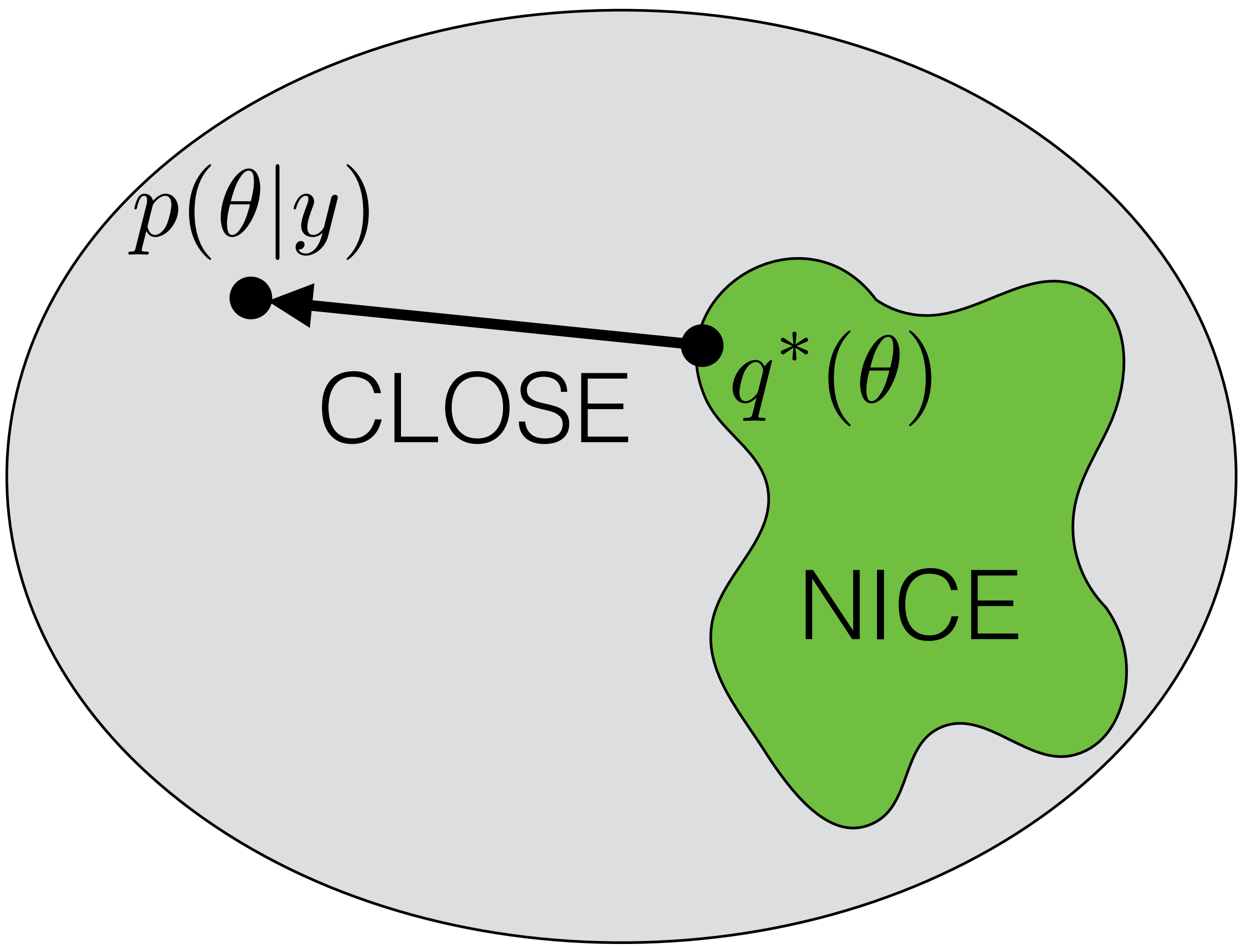

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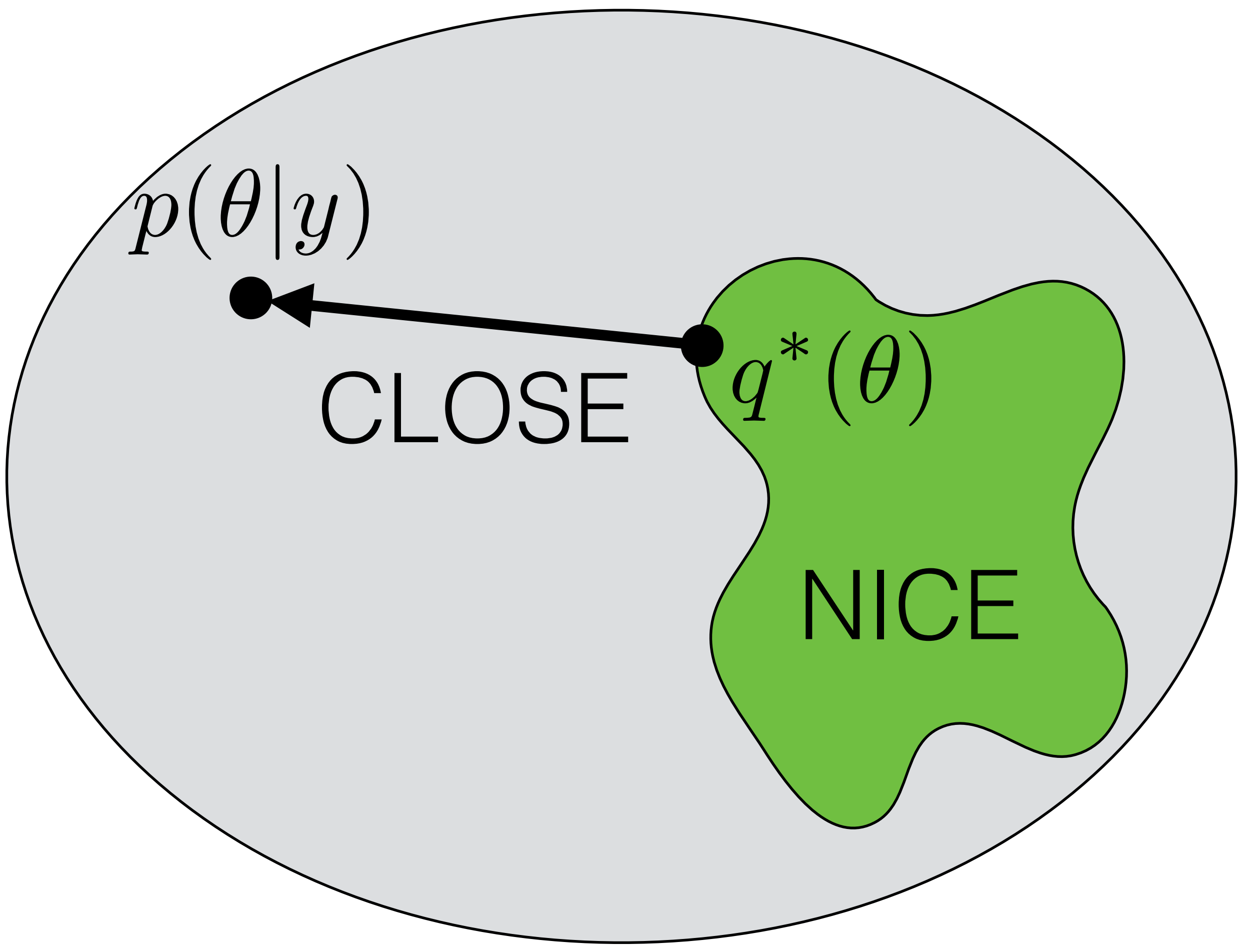

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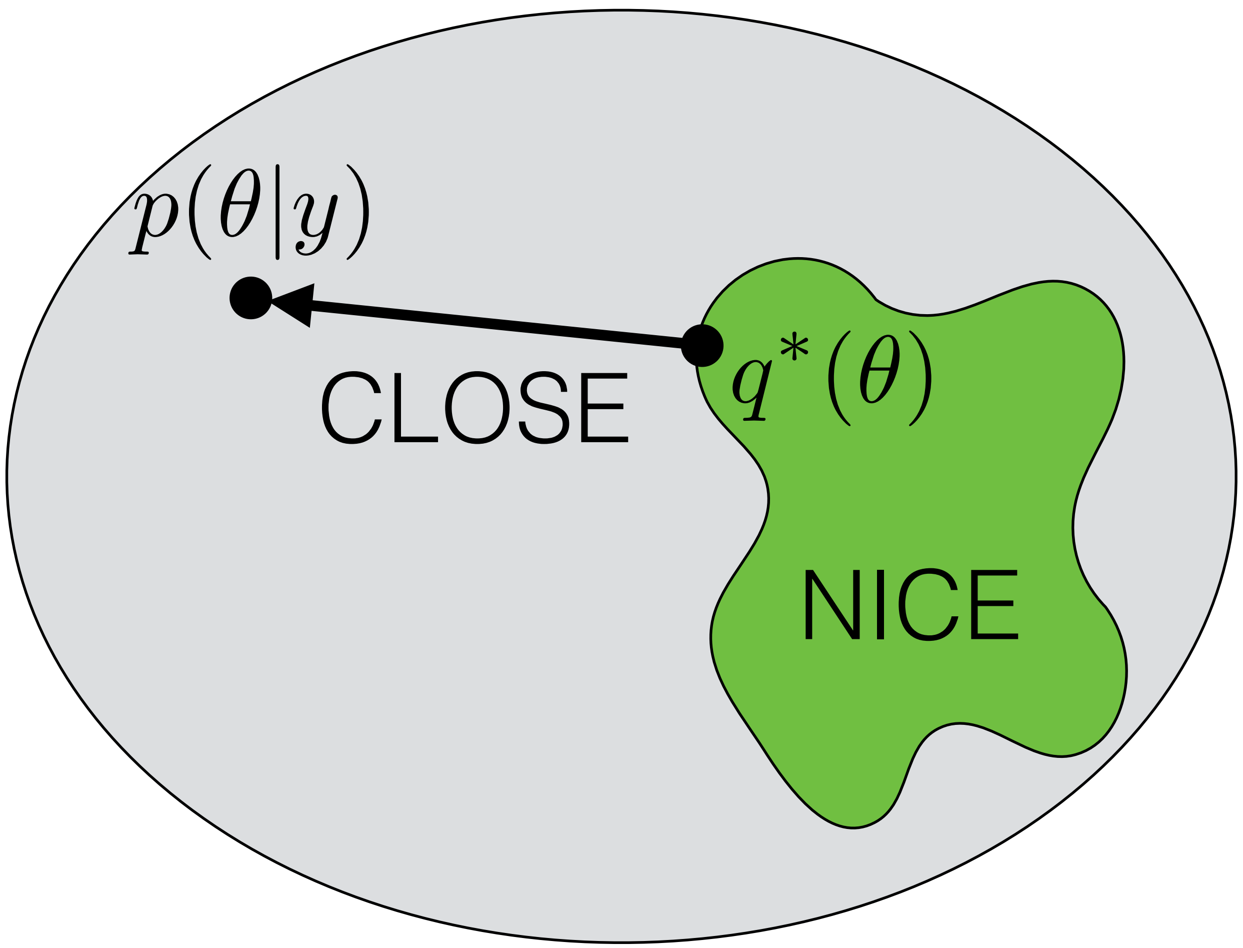

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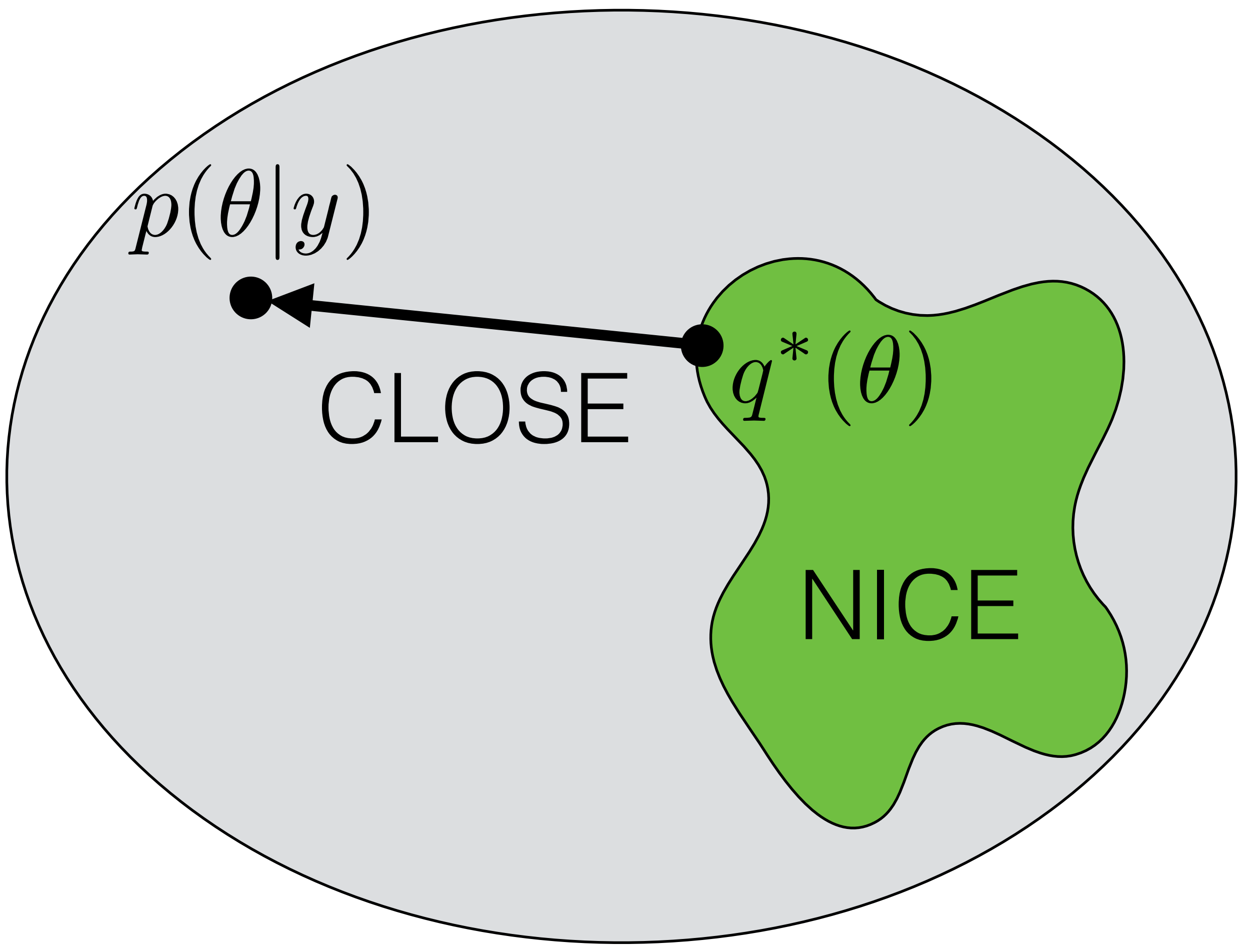

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[Bardenet, Doucet, Holmes 2017]

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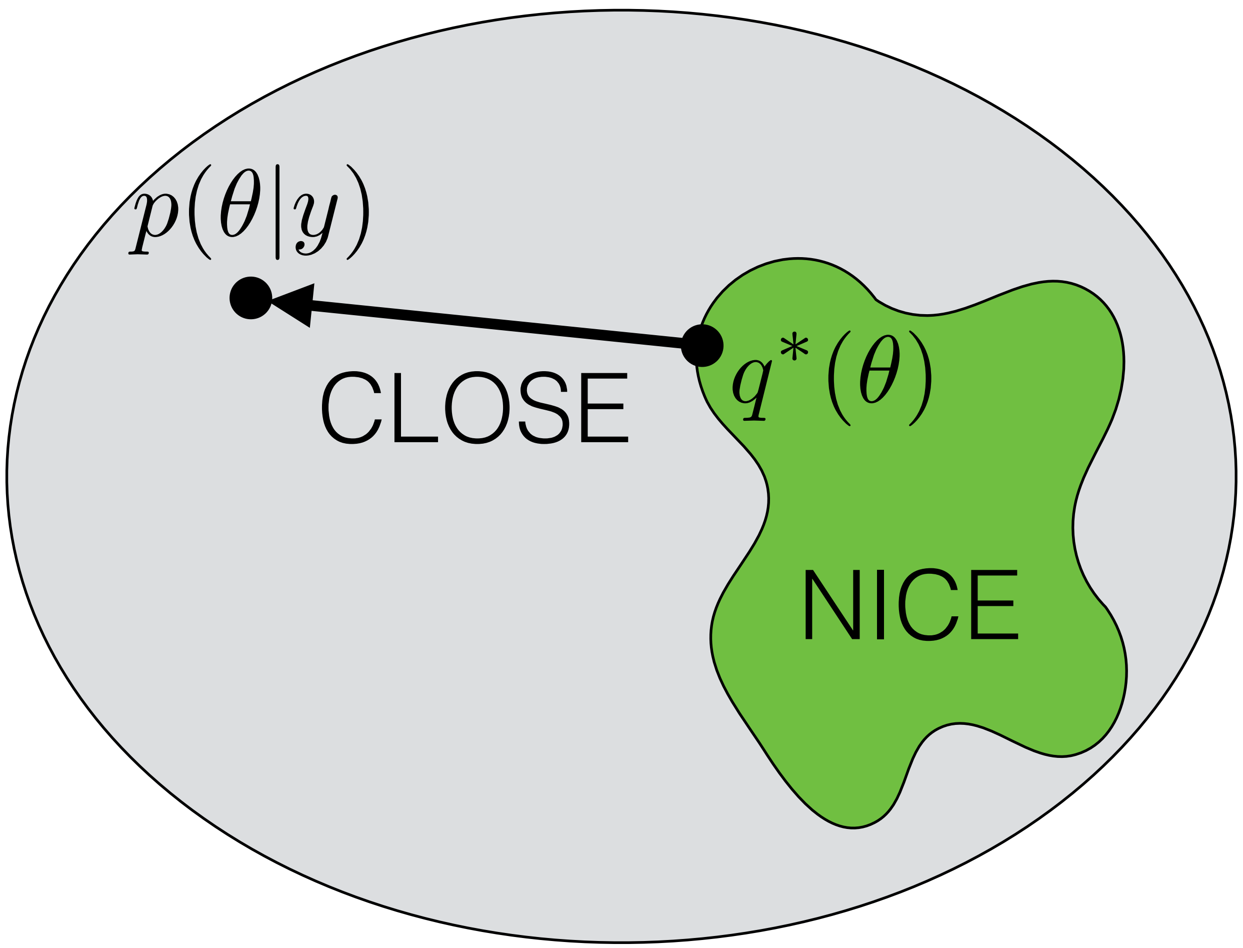

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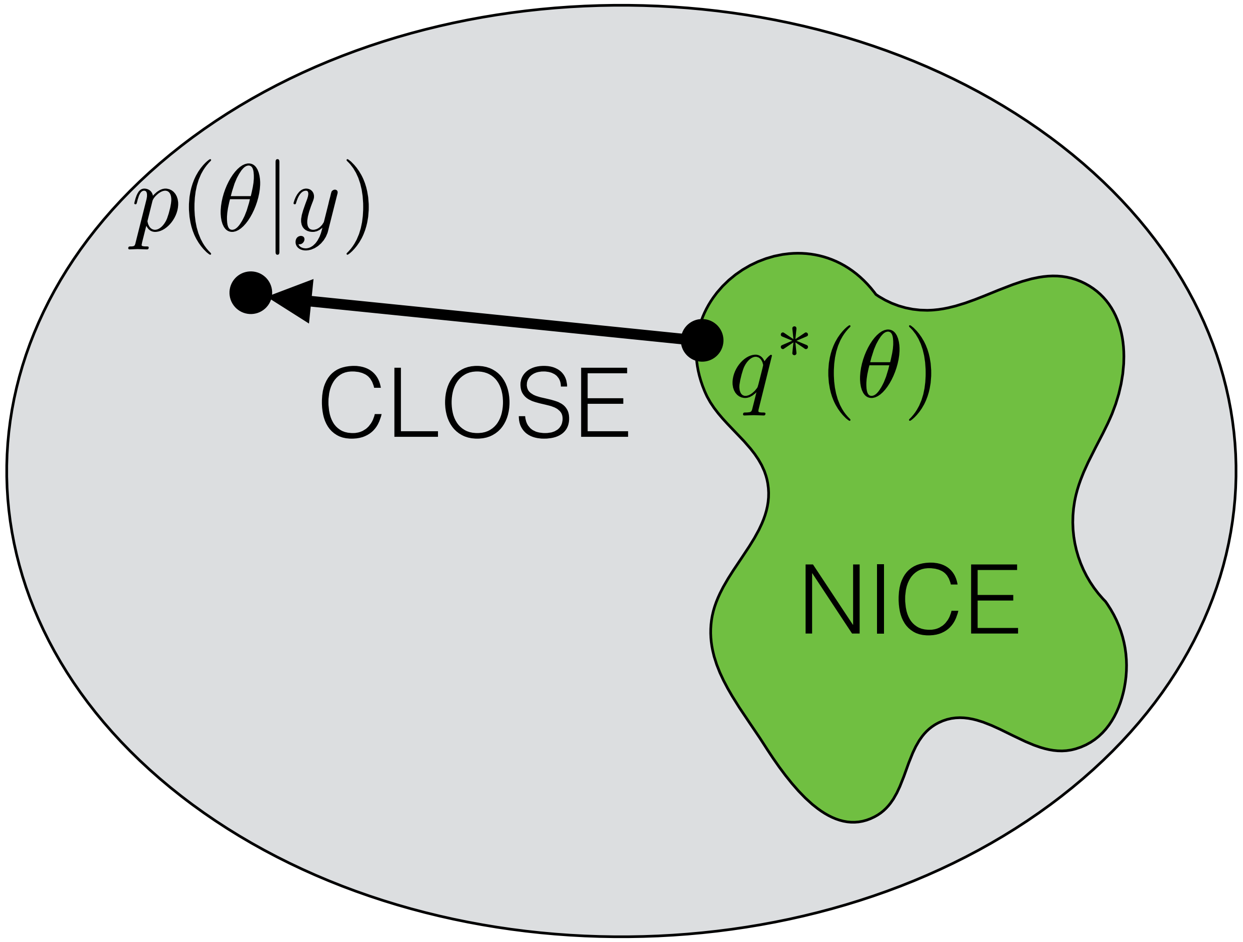
- VB practical success: point estimates and prediction, fast, streaming, distributed (3.6M Wikipedia, 350K Nature)

[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

Why KL?

- Variational Bayes

$$q^* = \text{argmin}_{q \in Q} \text{KL}\left(q(\cdot) || p(\cdot|y)\right)$$



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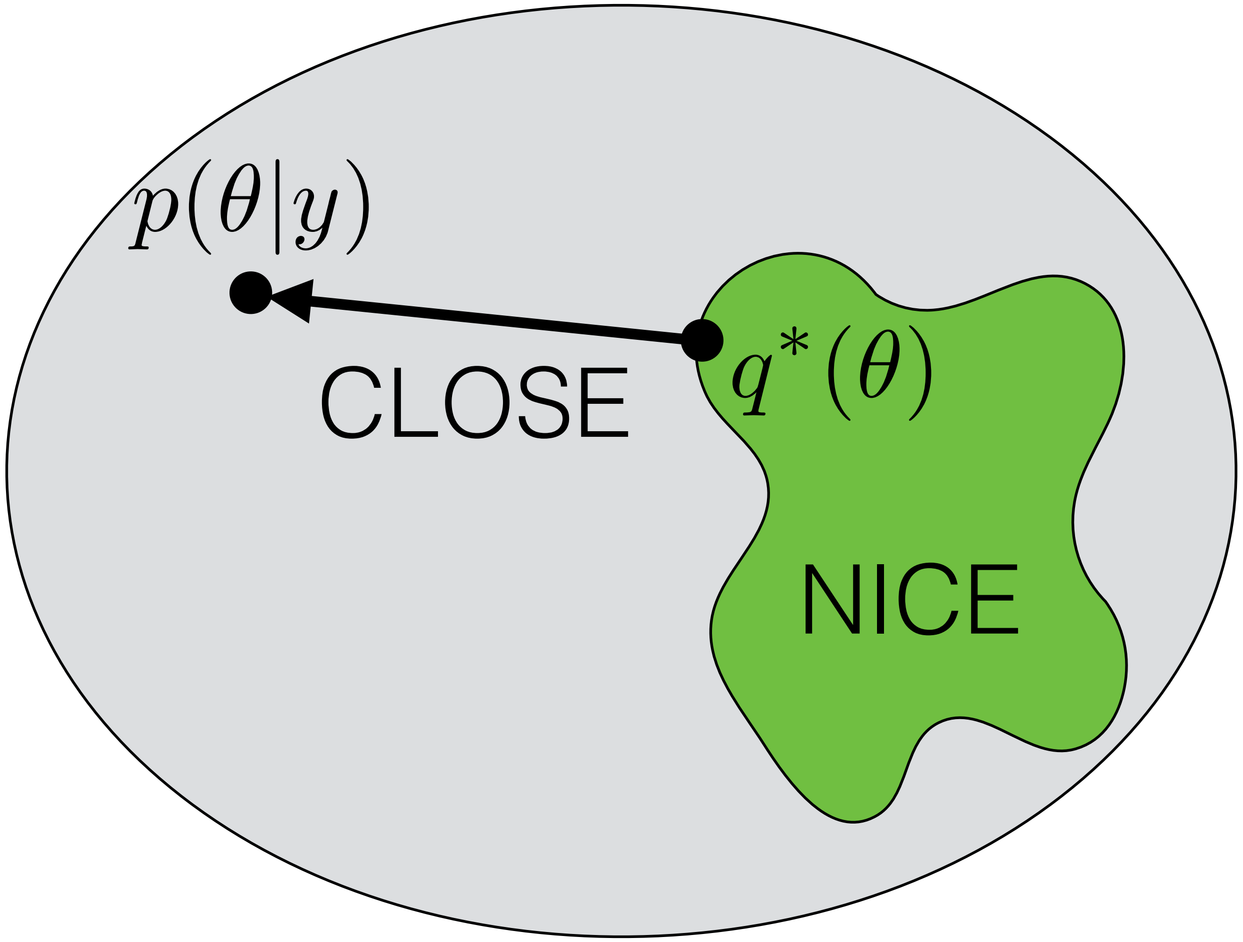

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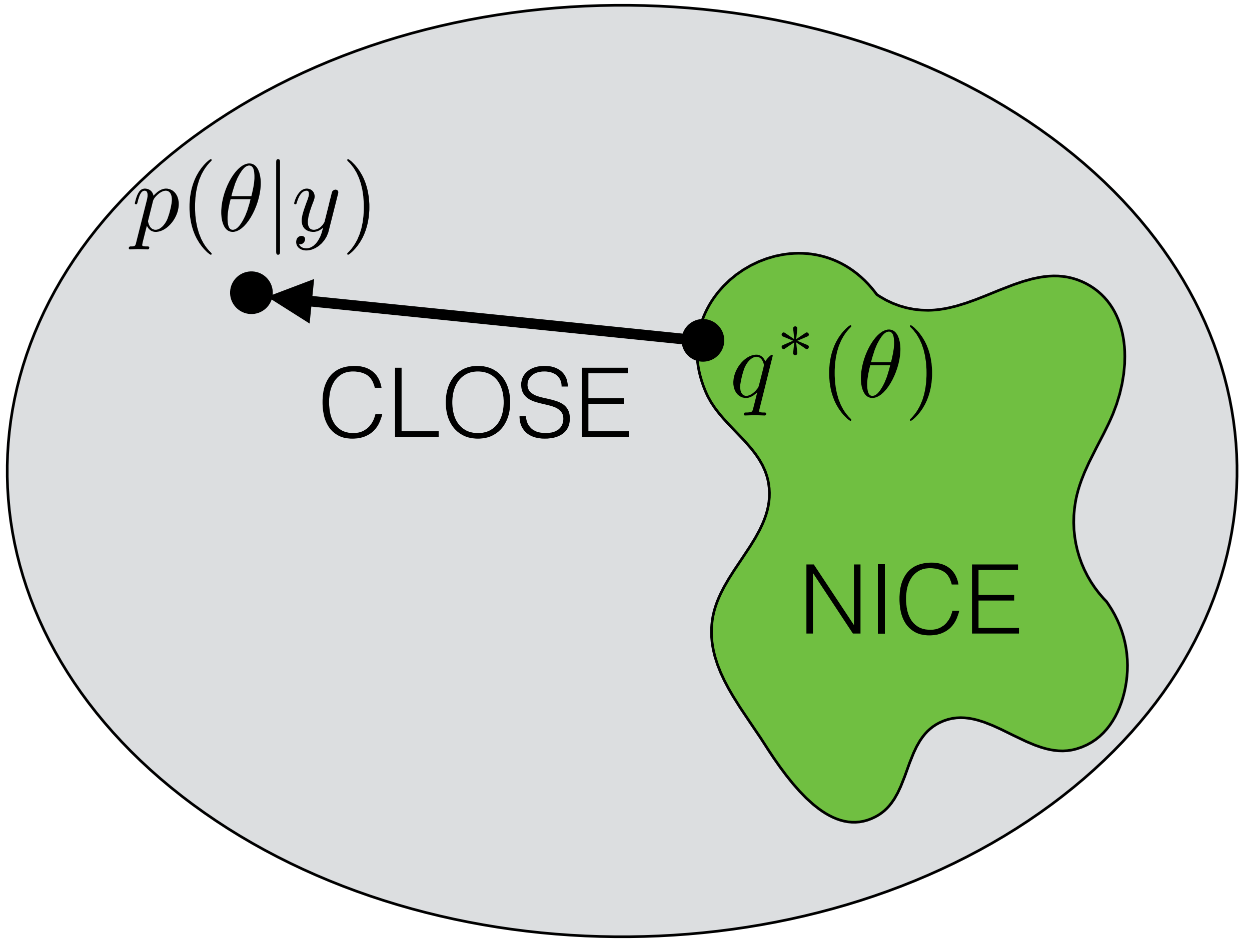

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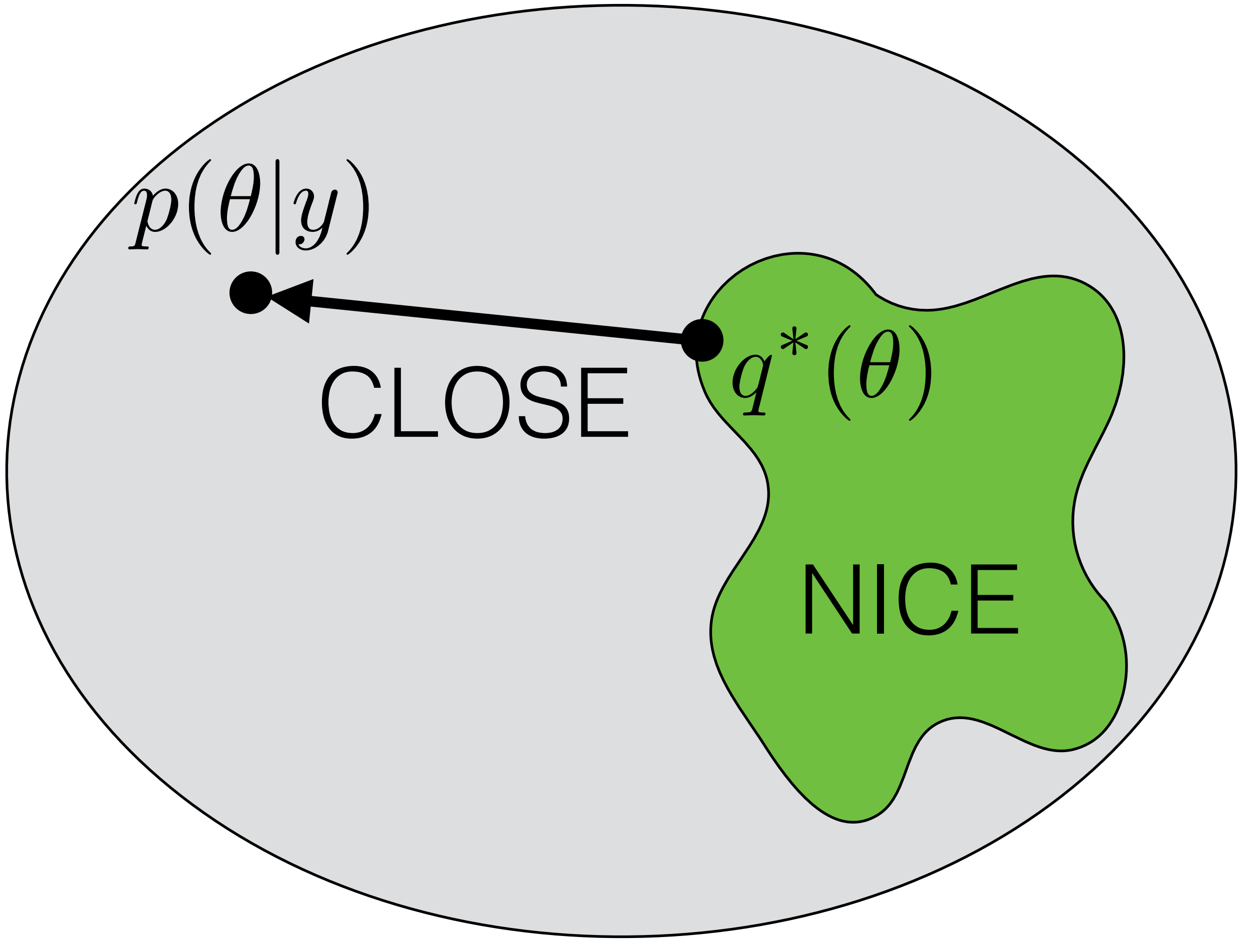

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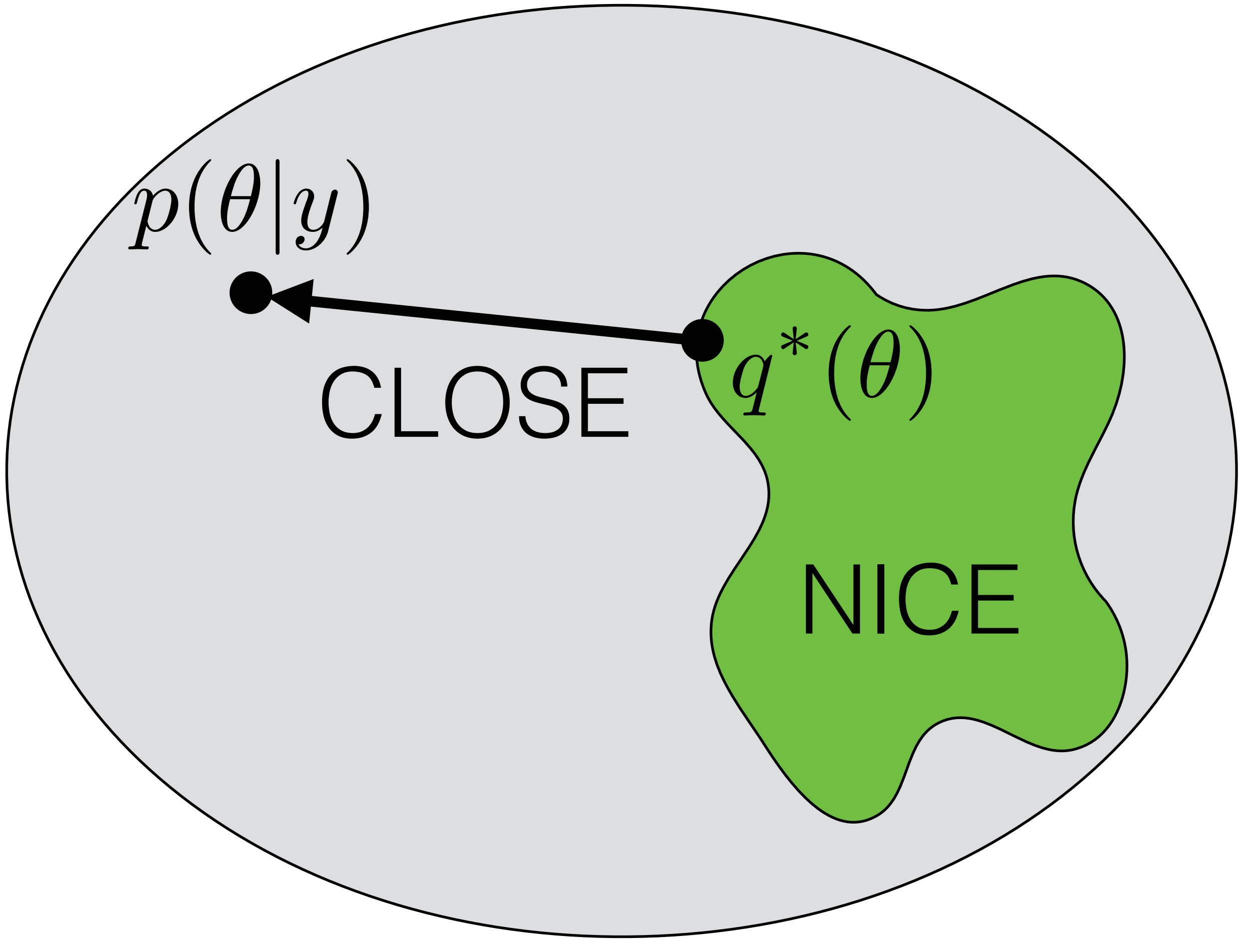

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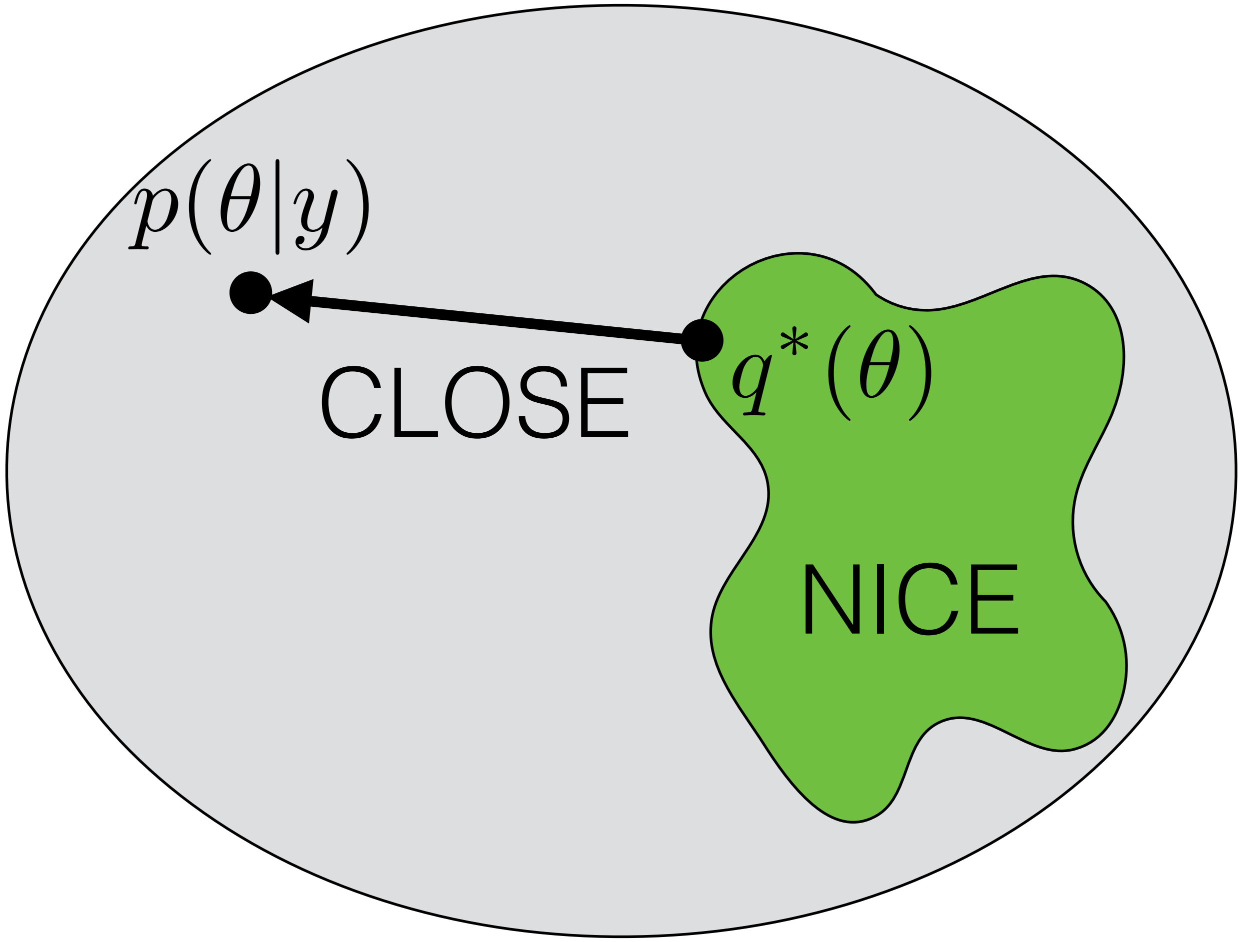

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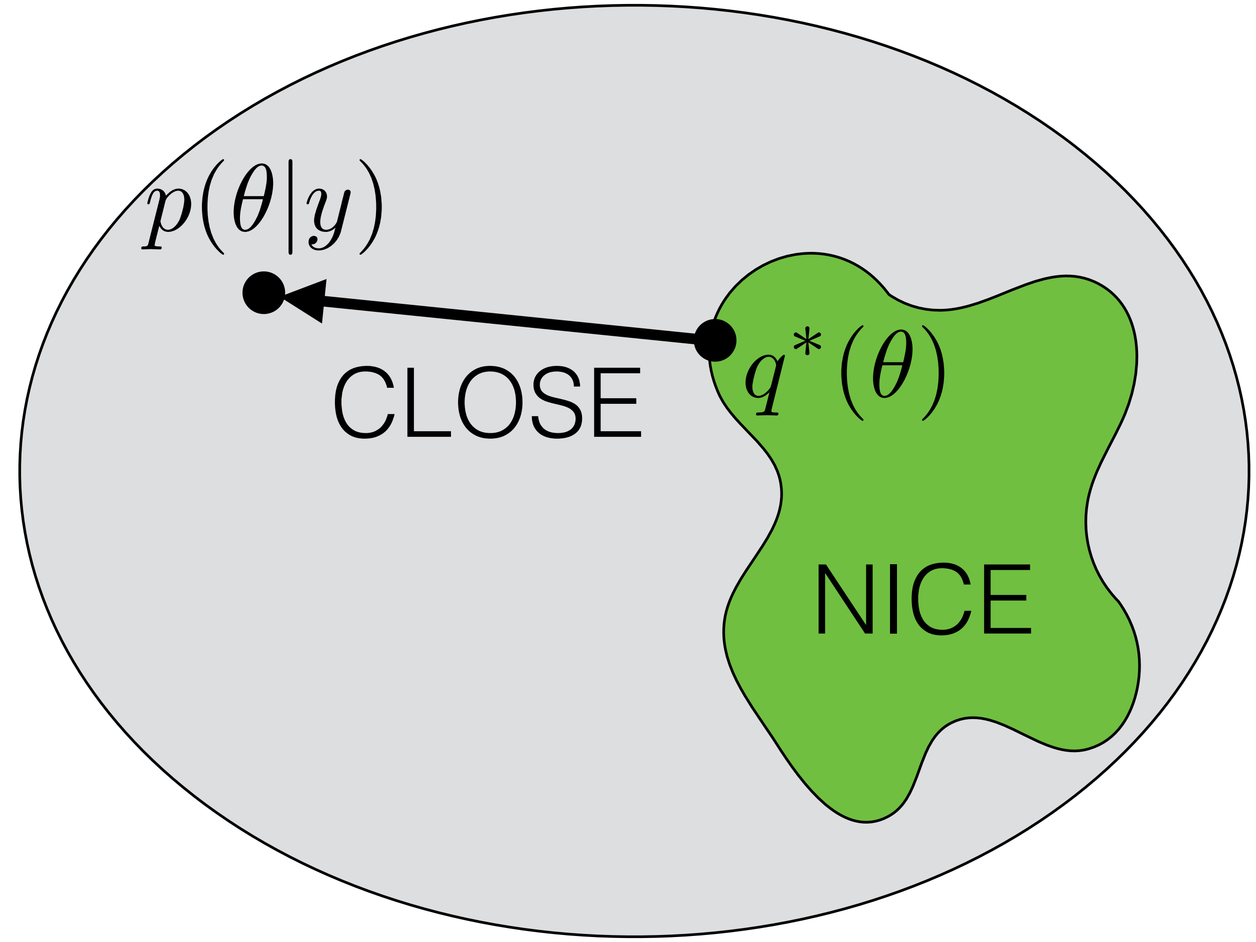

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"Evidence lower bound" (ELBO)

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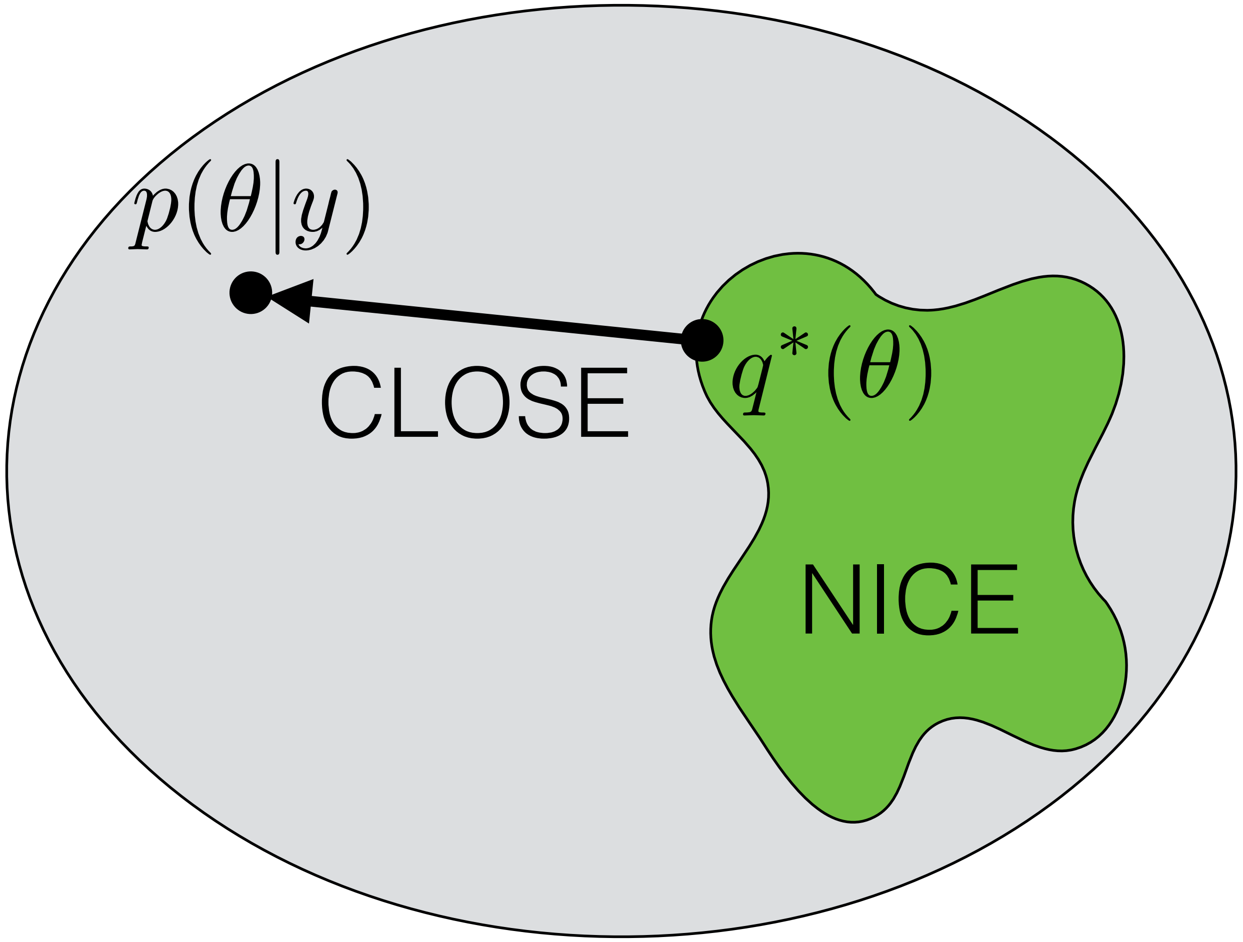

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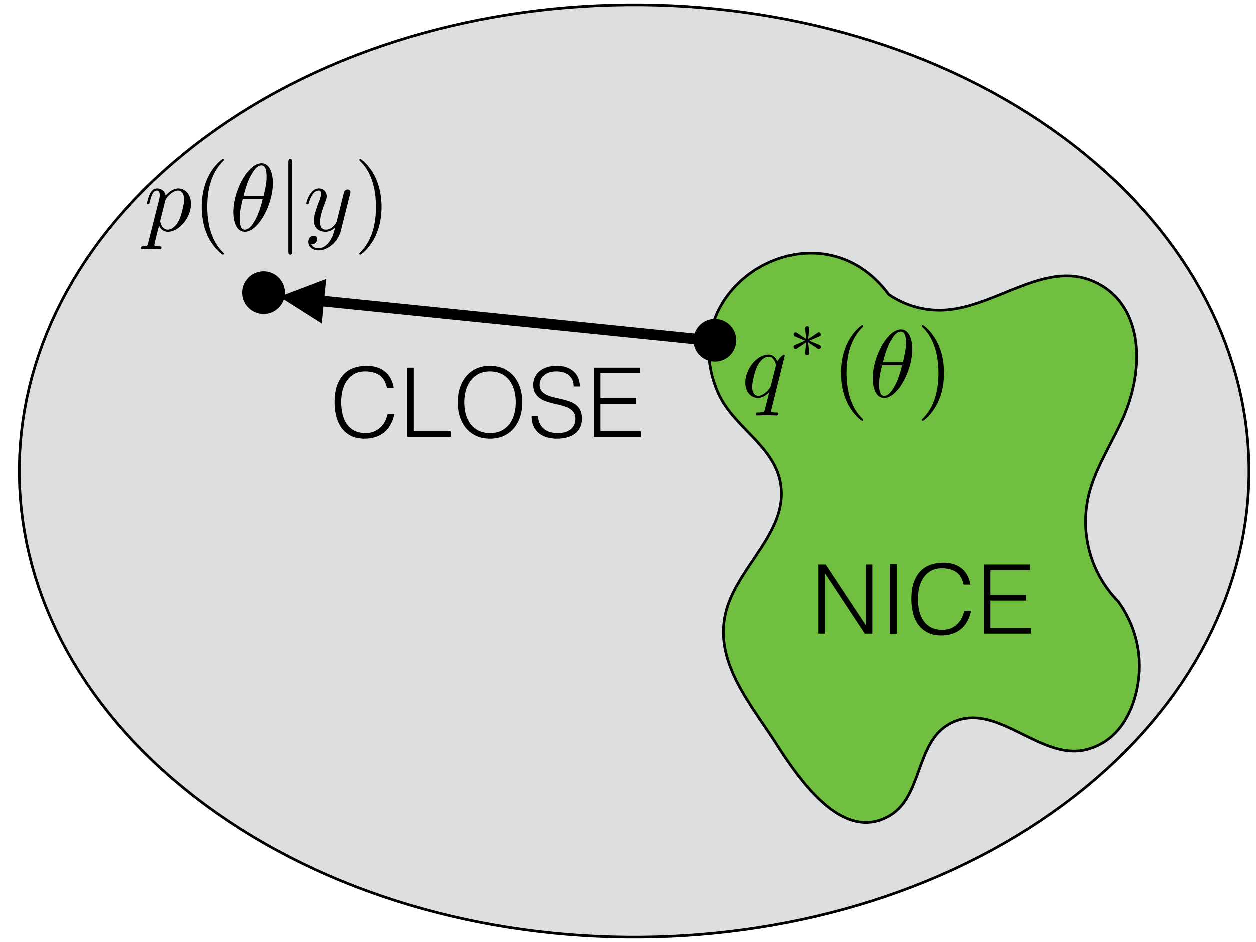

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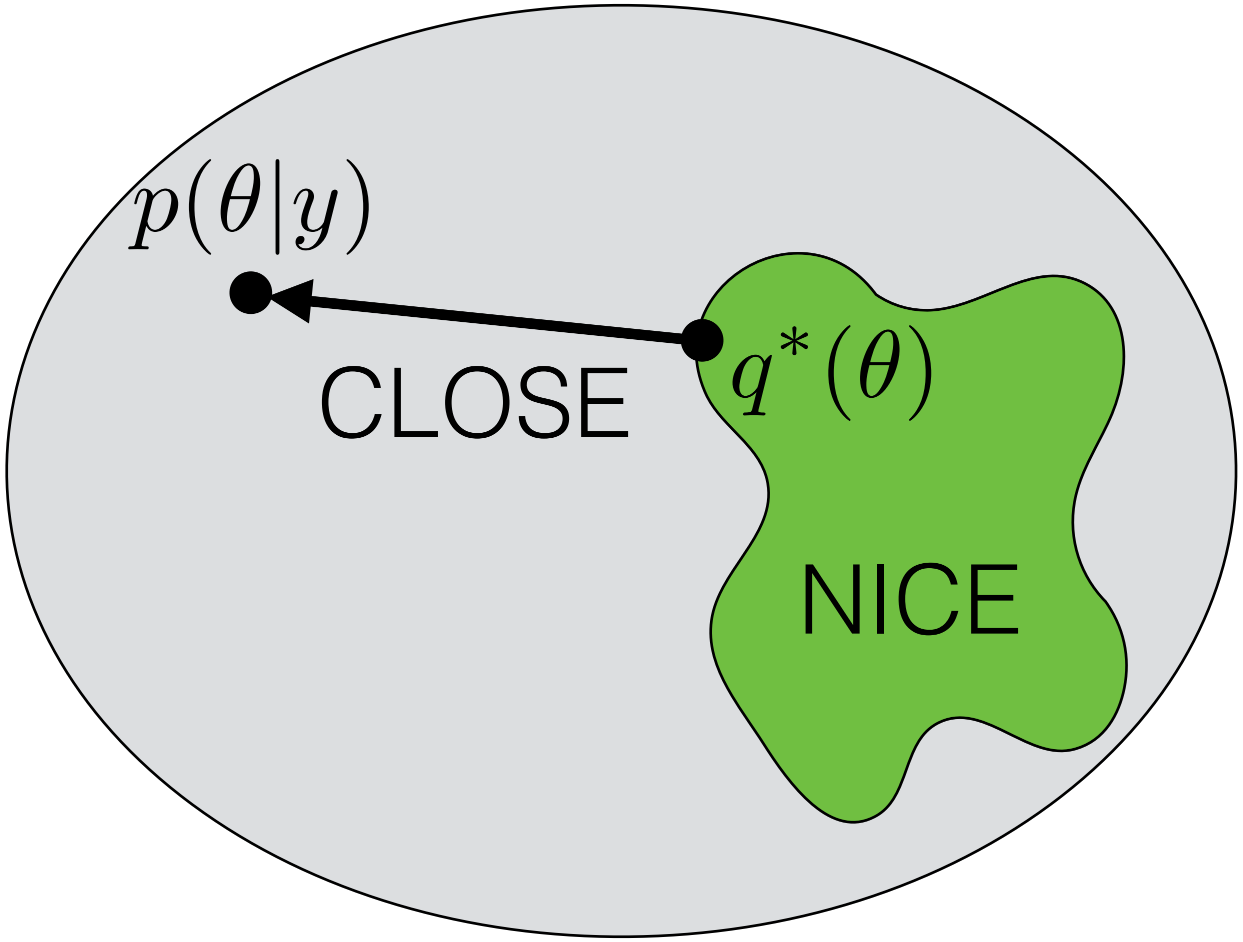

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- $\mathrm{KL} \geq 0 \Rightarrow \log p(y) \geq \mathrm{ELBO}$

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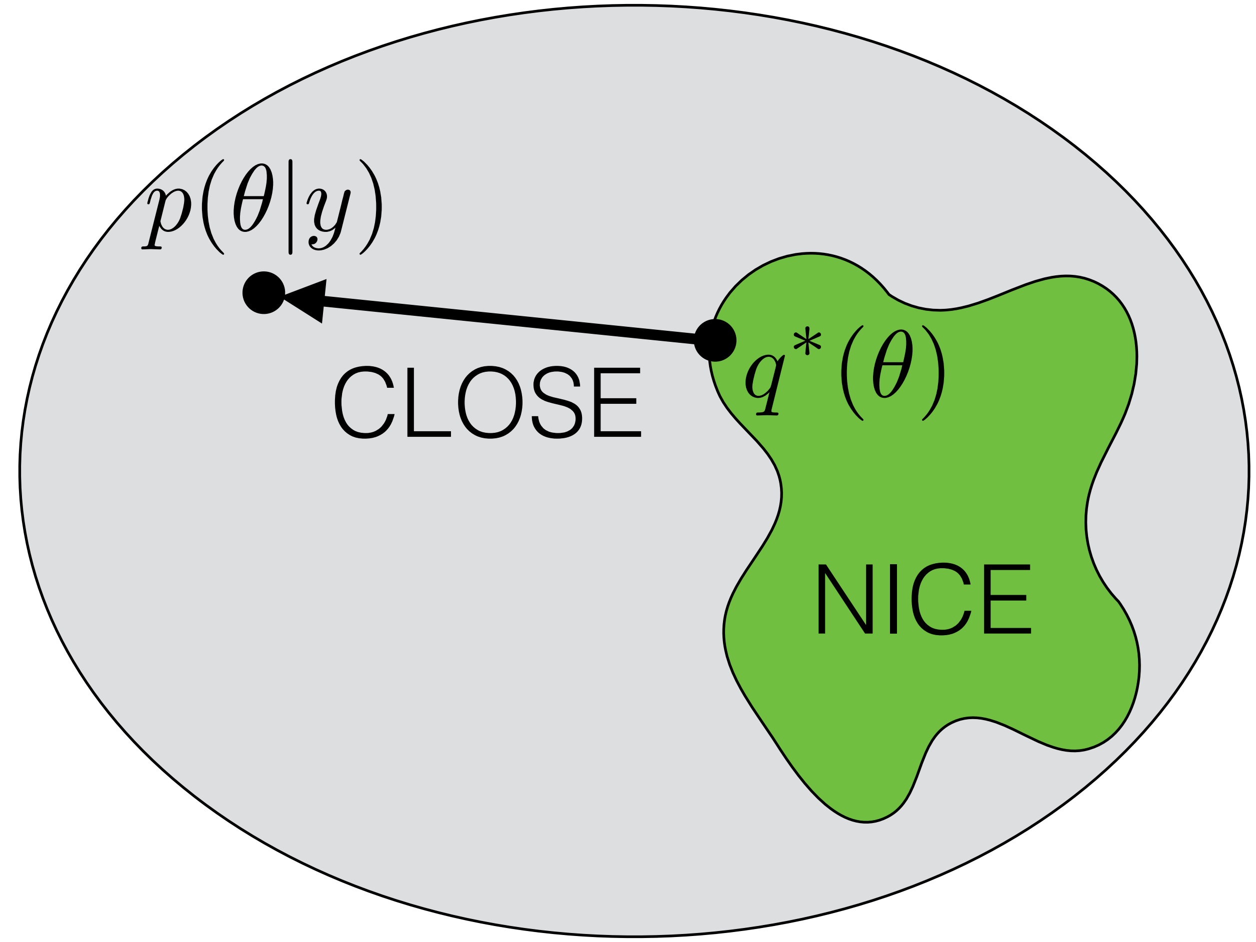

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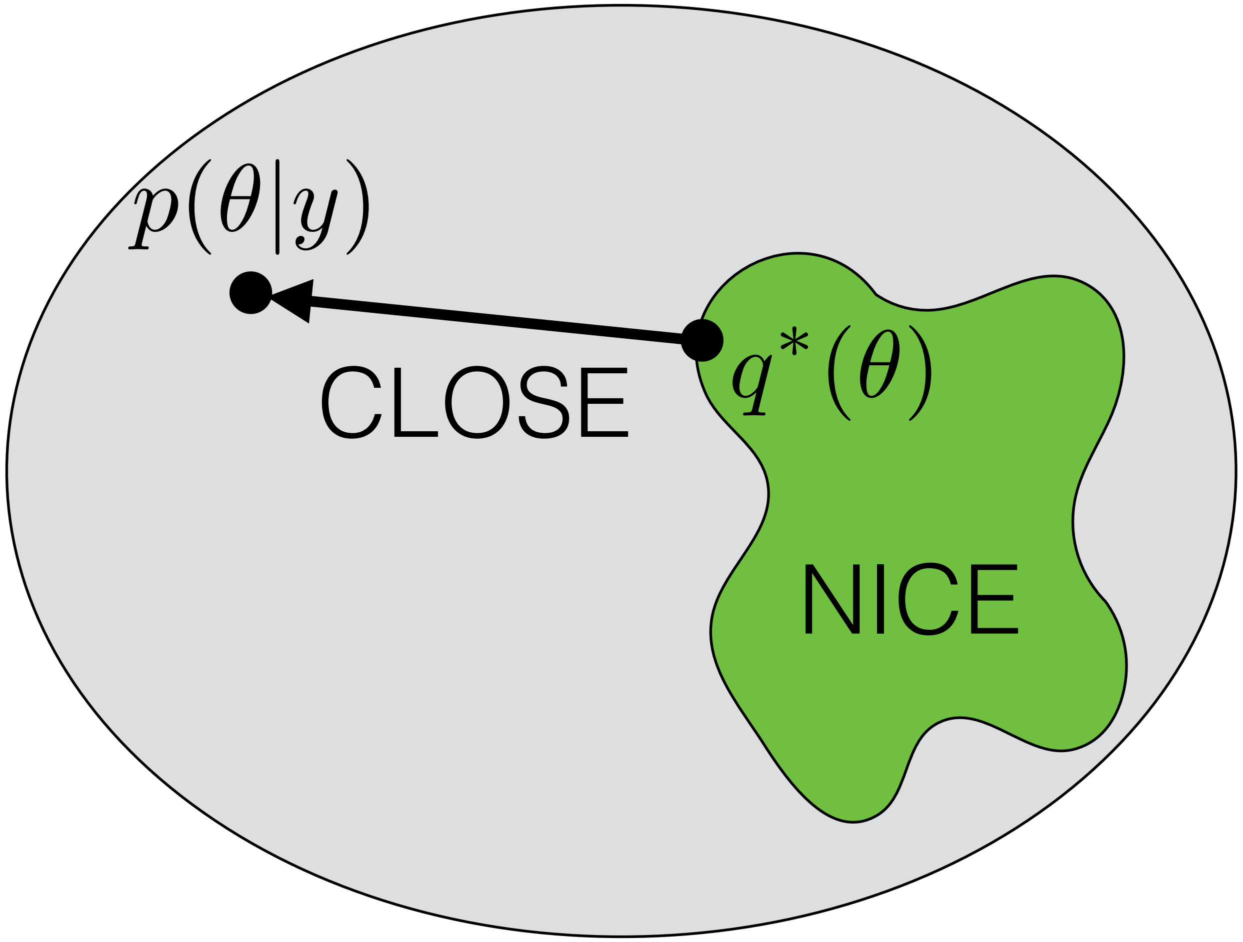
"Evidence lower bound" (ELBO)

- Exercise: Show $\text{KL} \geq 0$ [Bishop 2006, Sec 1.6.1]
- $\text{KL} \geq 0 \Rightarrow \log p(y) \geq \text{ELBO}$
- $q^* = \text{argmax}_{q \in Q} \text{ELBO}(q)$

Why KL?

- Variational Bayes

$$q^* = \operatorname{argmin}_{q \in Q} \mathrm{KL}\left(q(\cdot) || p(\cdot|y)\right)$$



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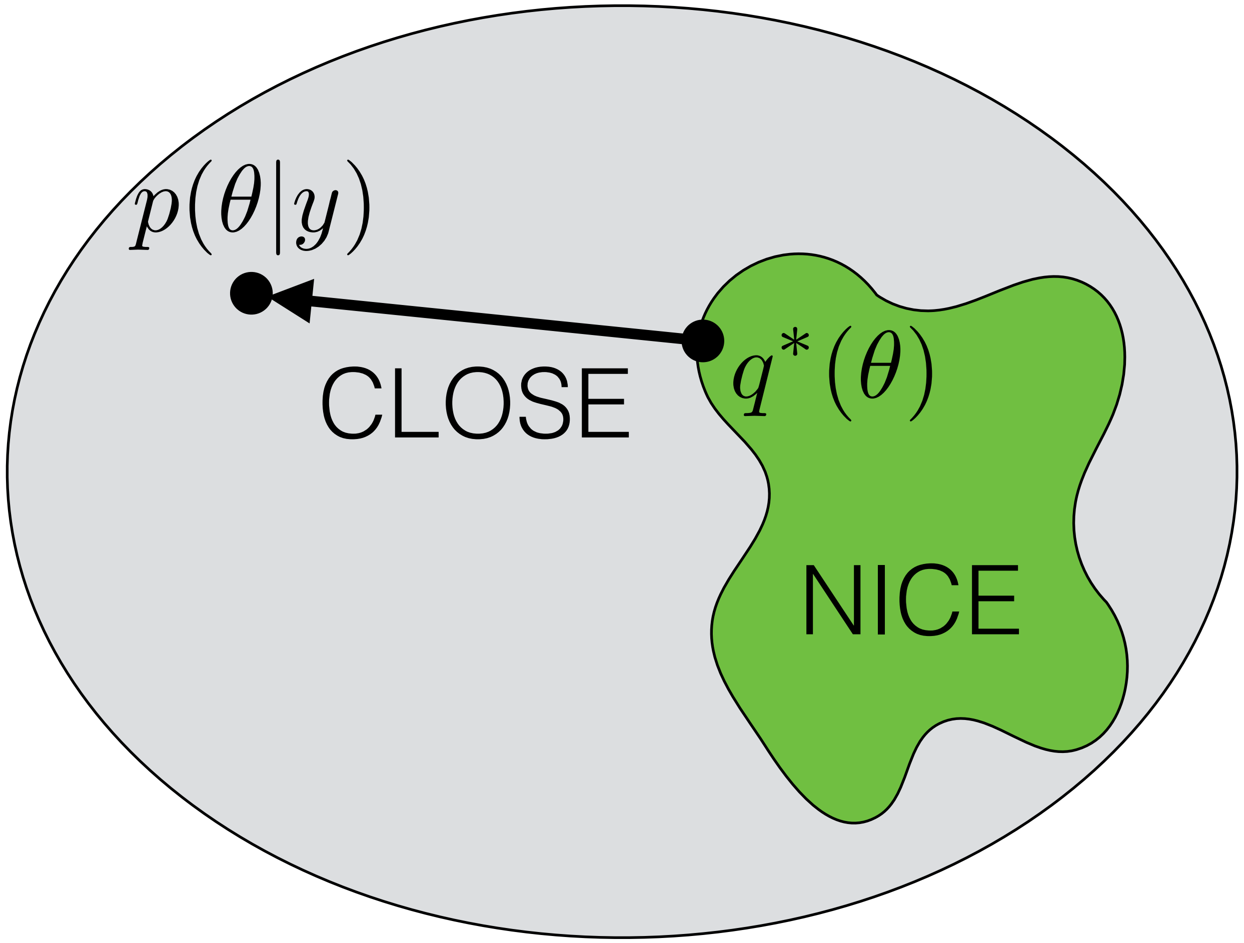

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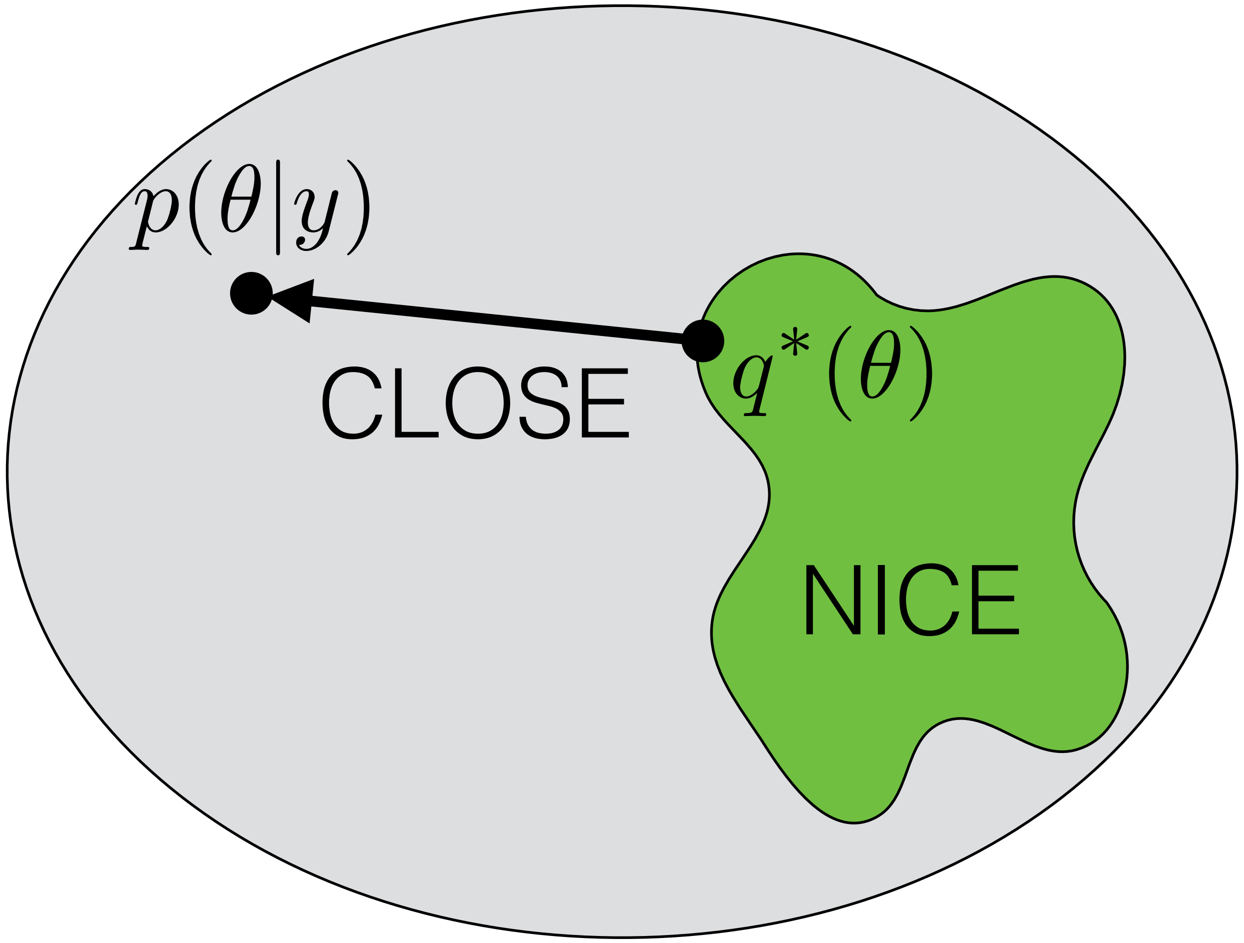

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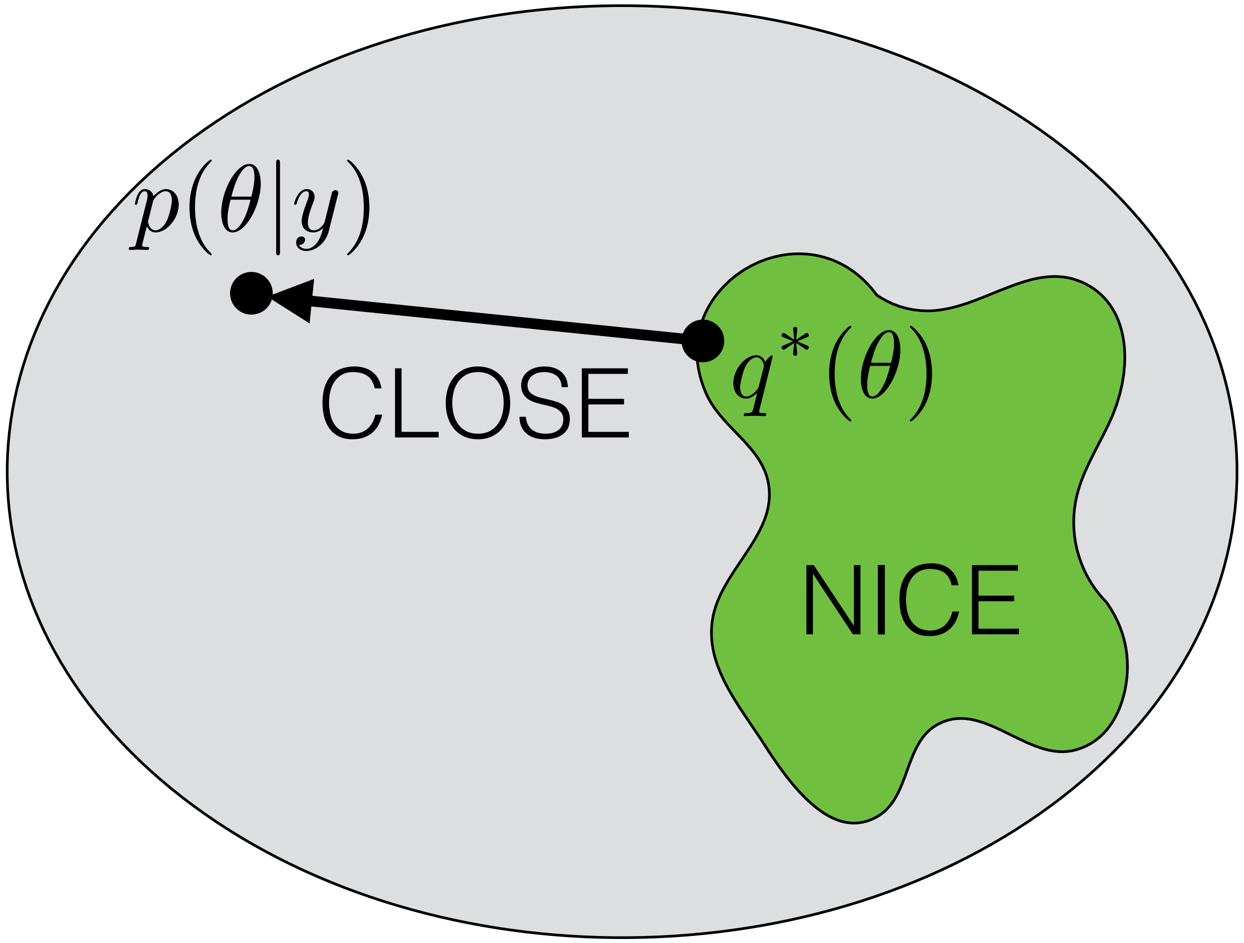

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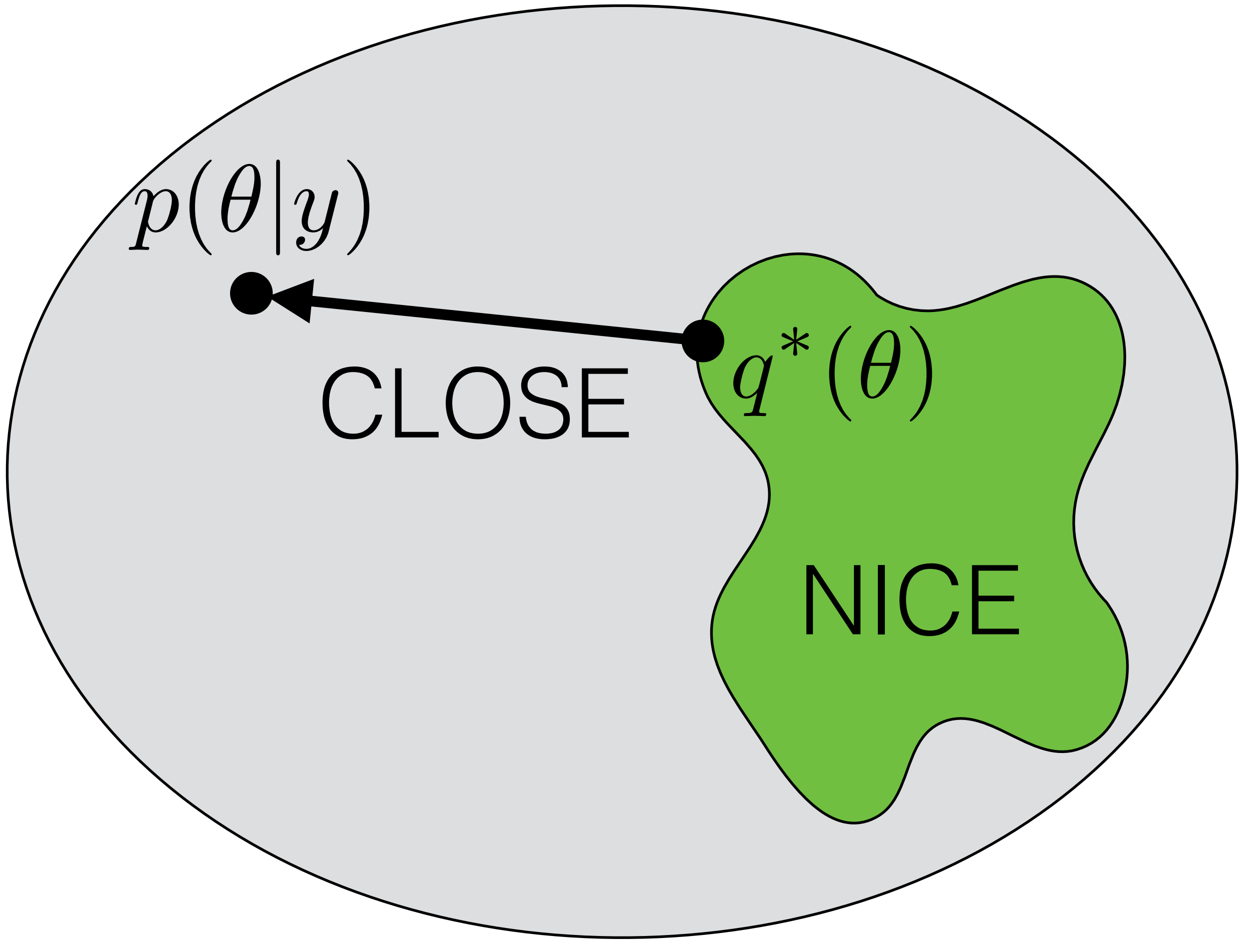

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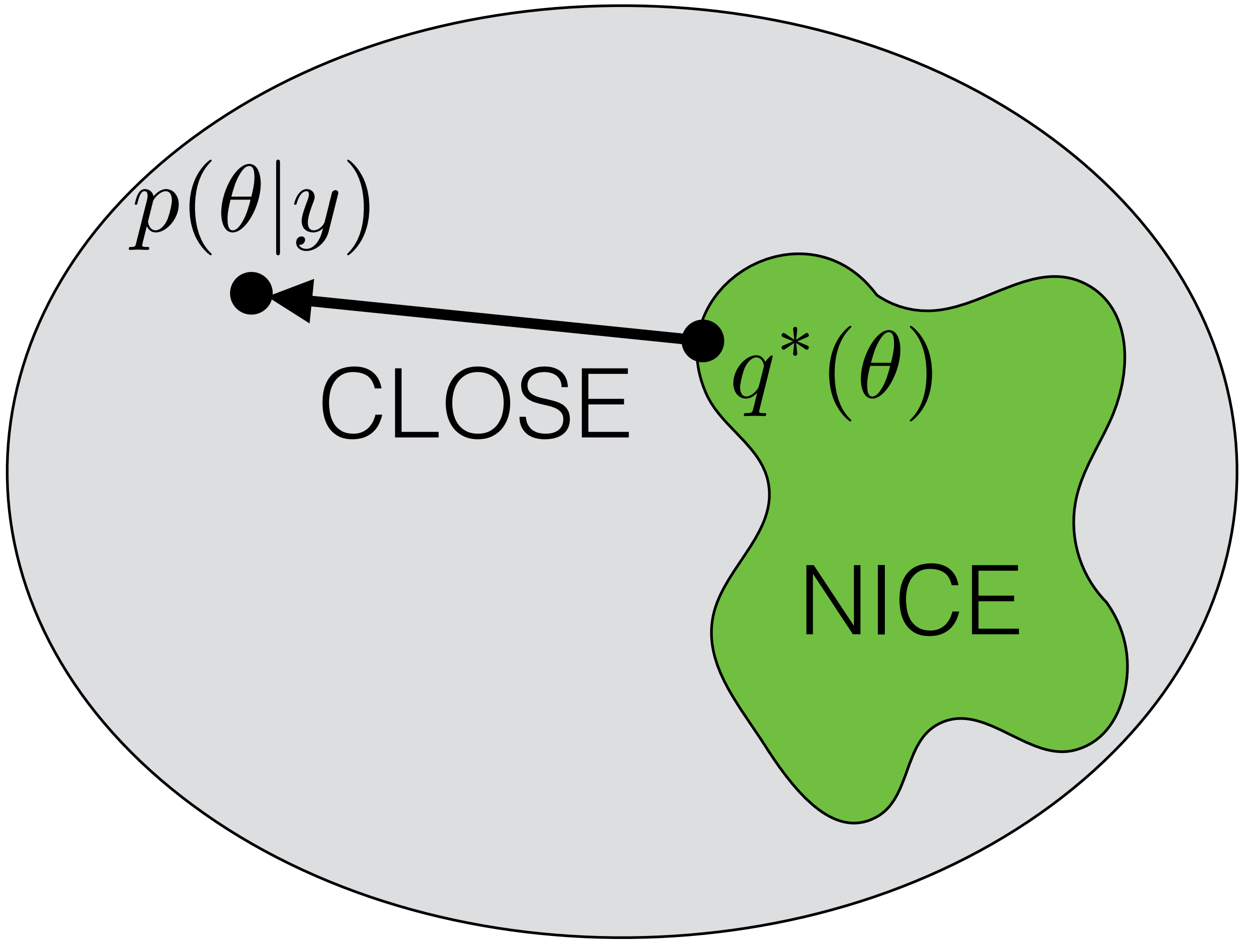

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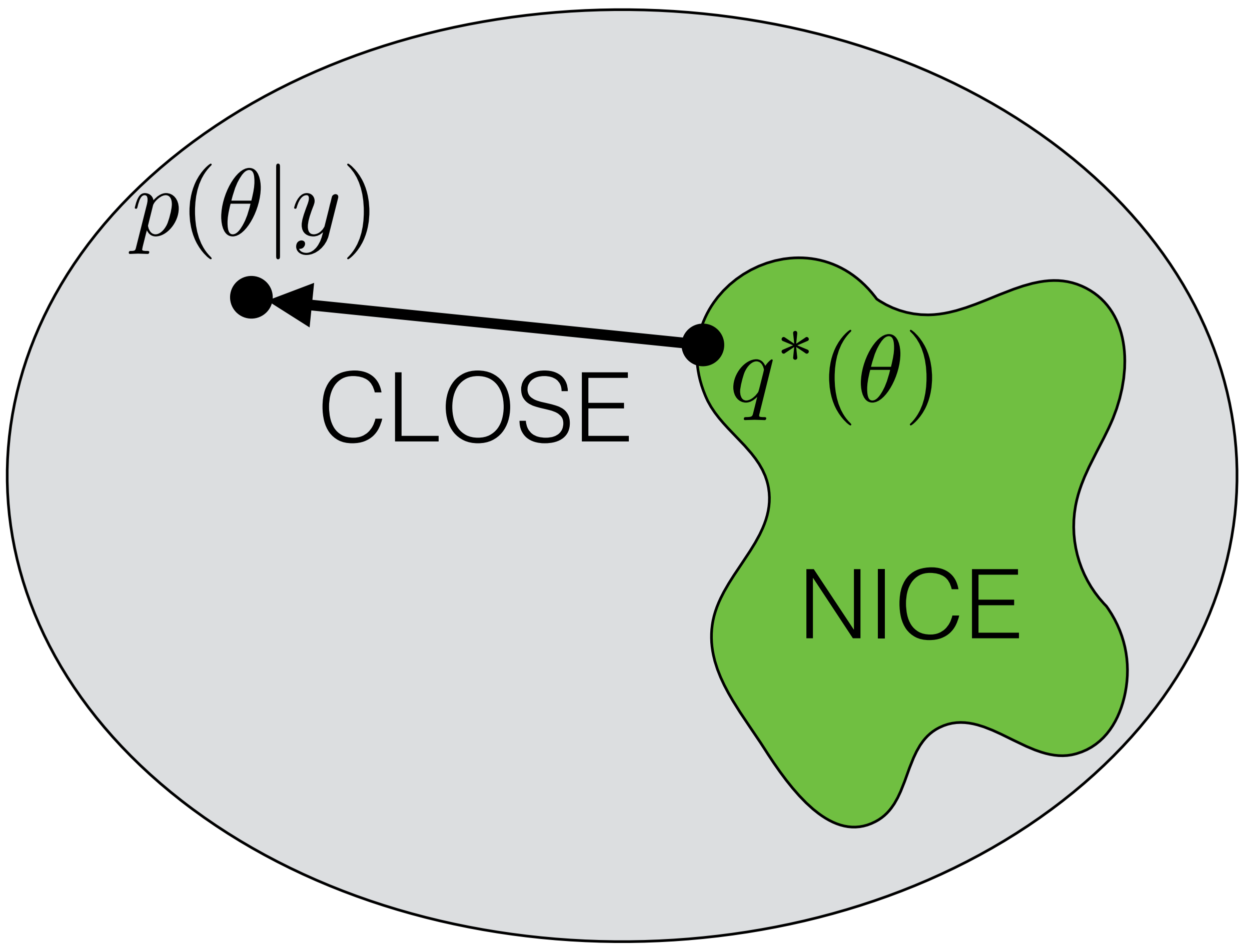

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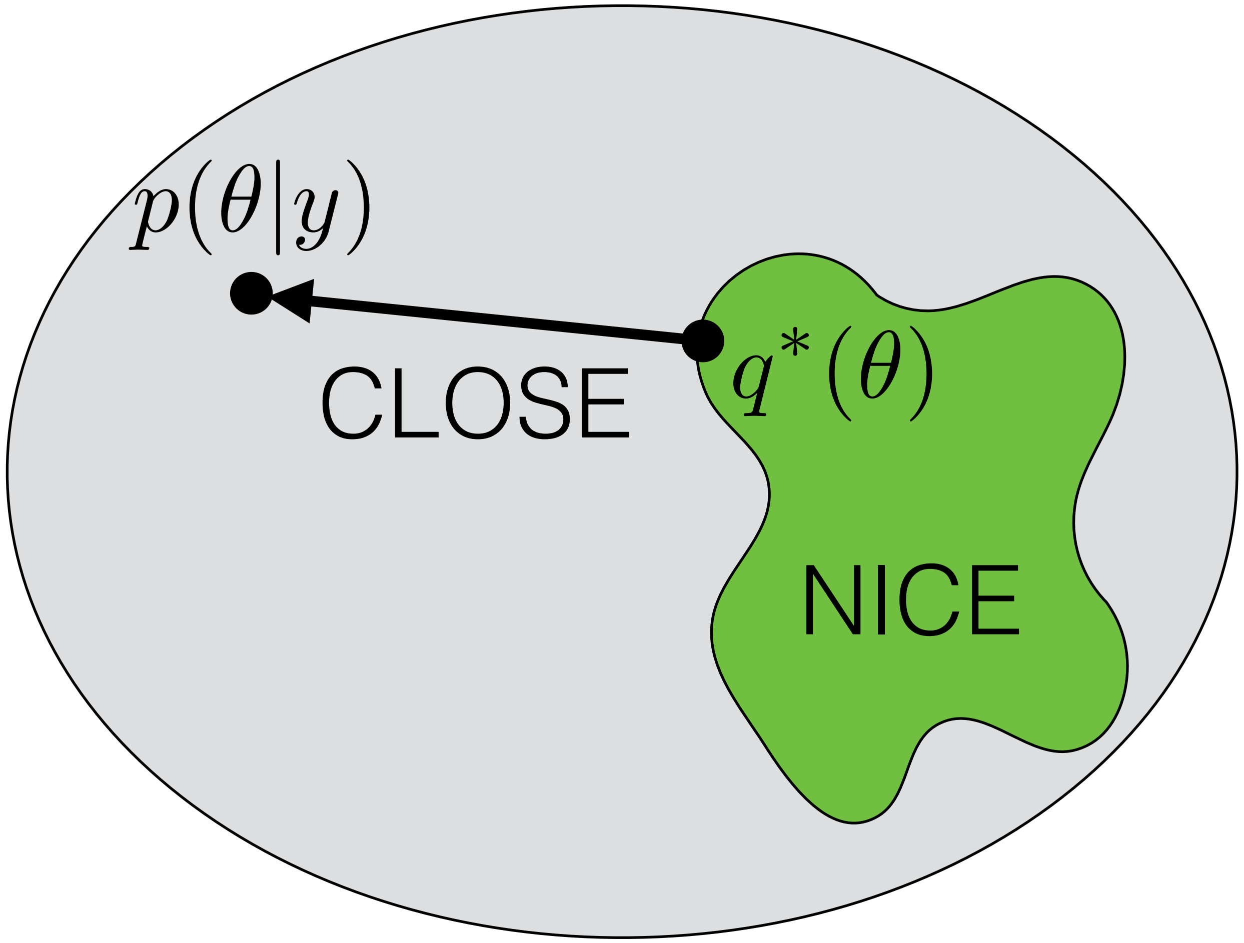

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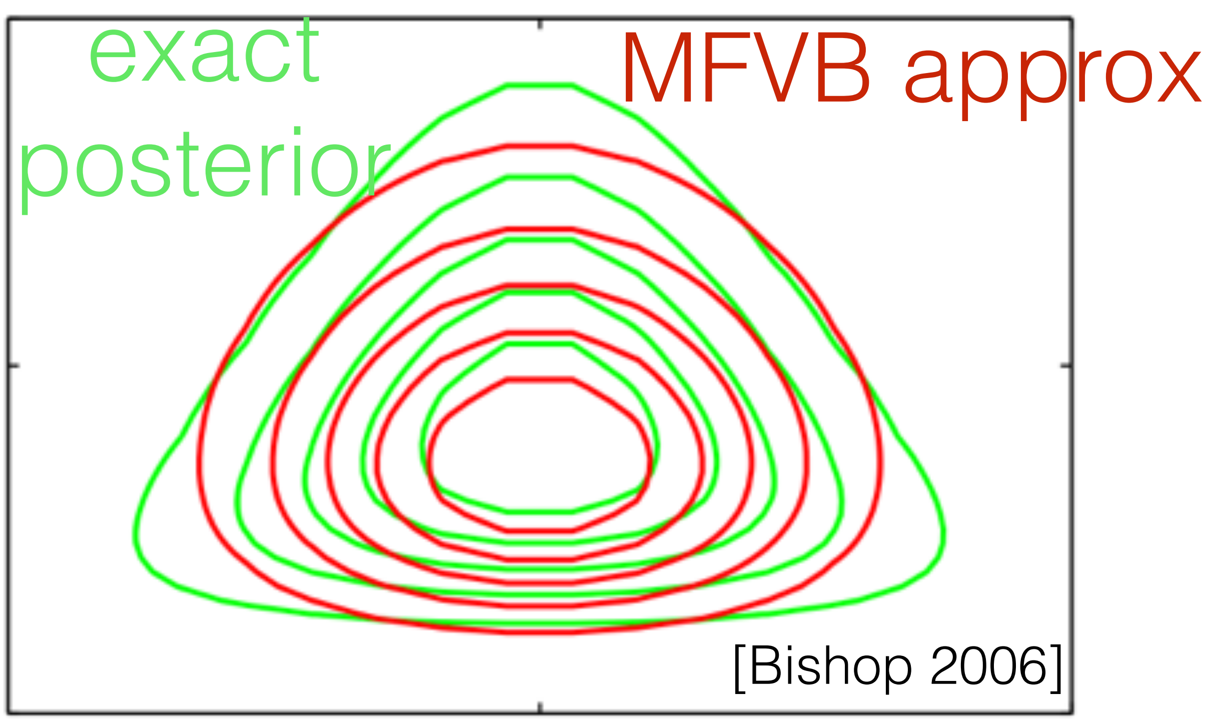

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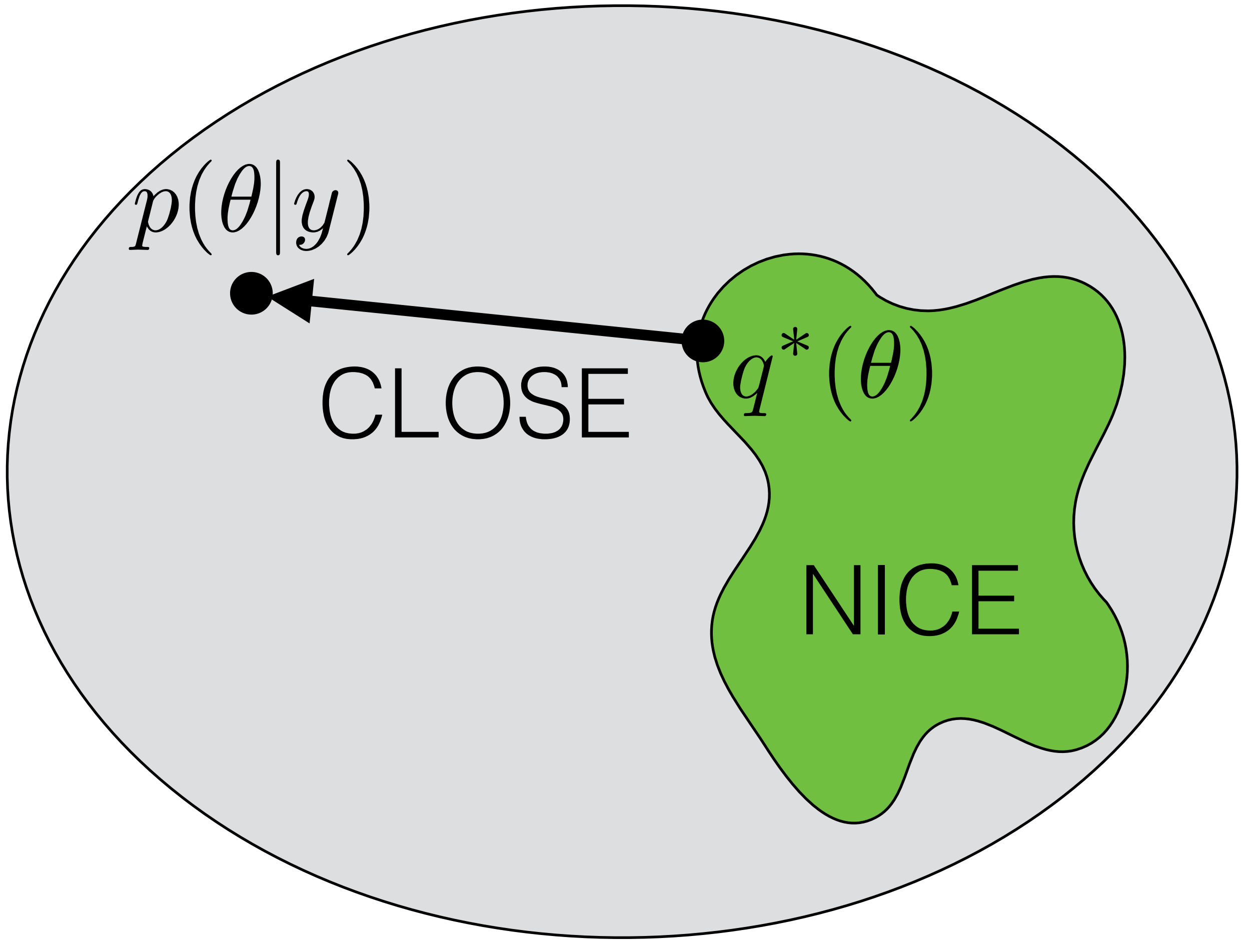

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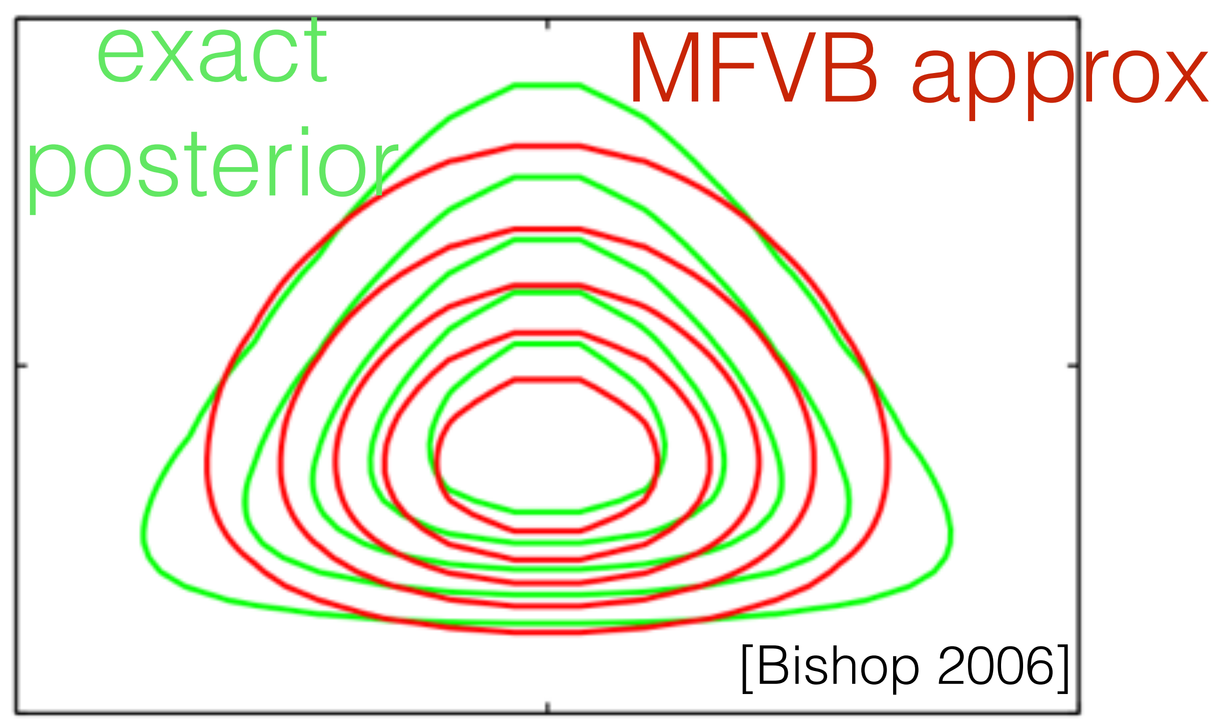

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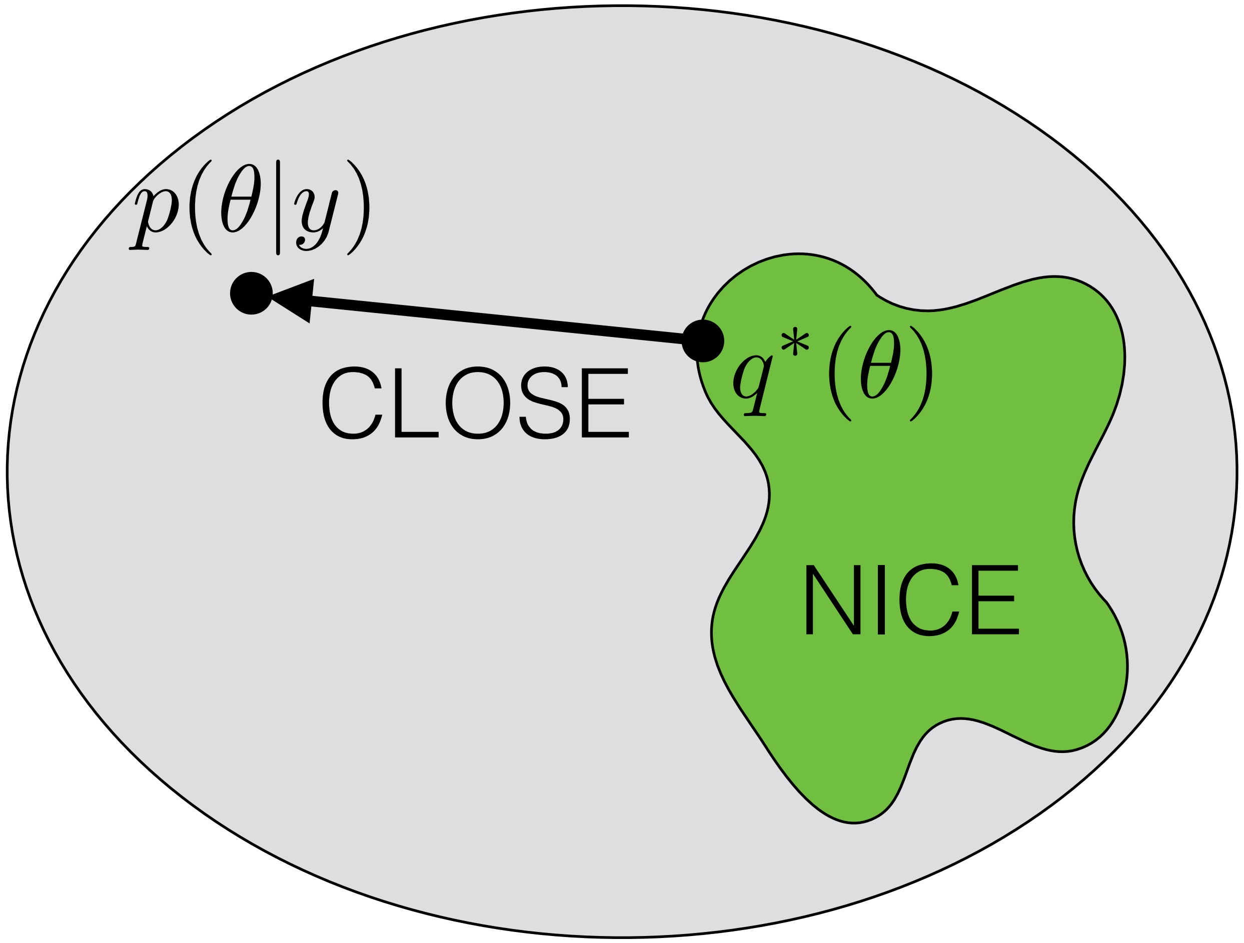

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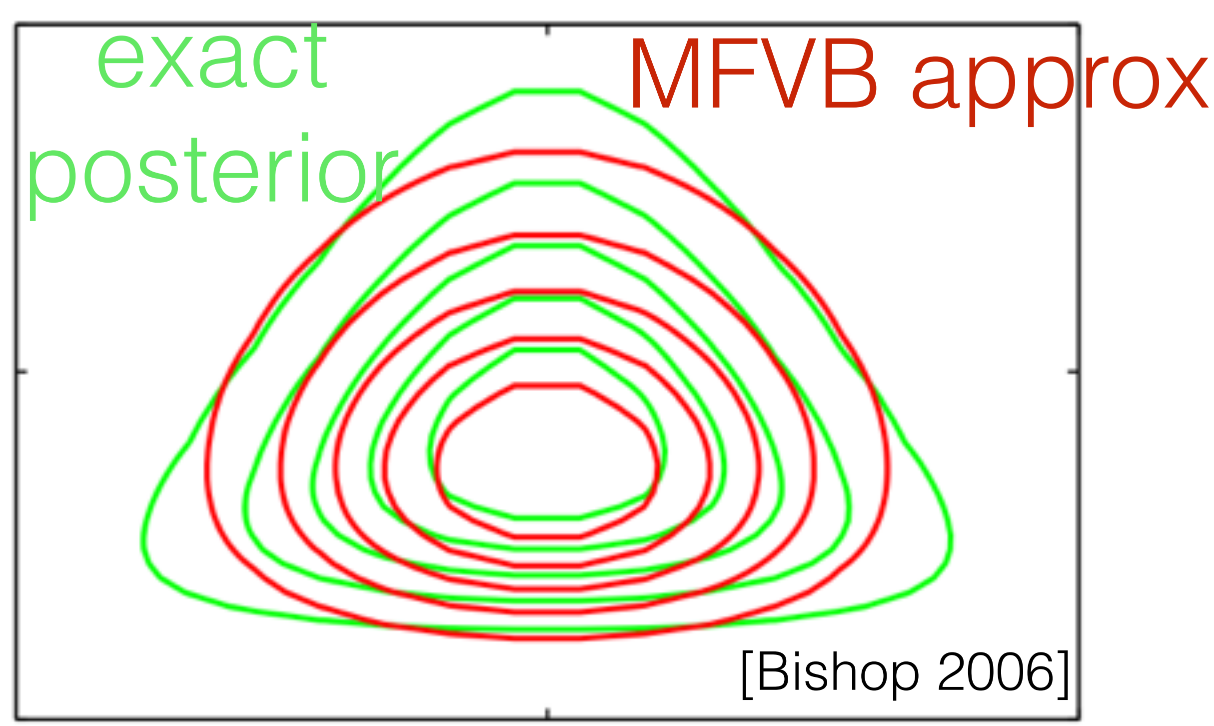

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- One option: Coordinate descent in $q_1, \ldots, q_J$

Approximate Bayesian inference

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Use q^* to approximate $p(\cdot|y)$

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Optimization

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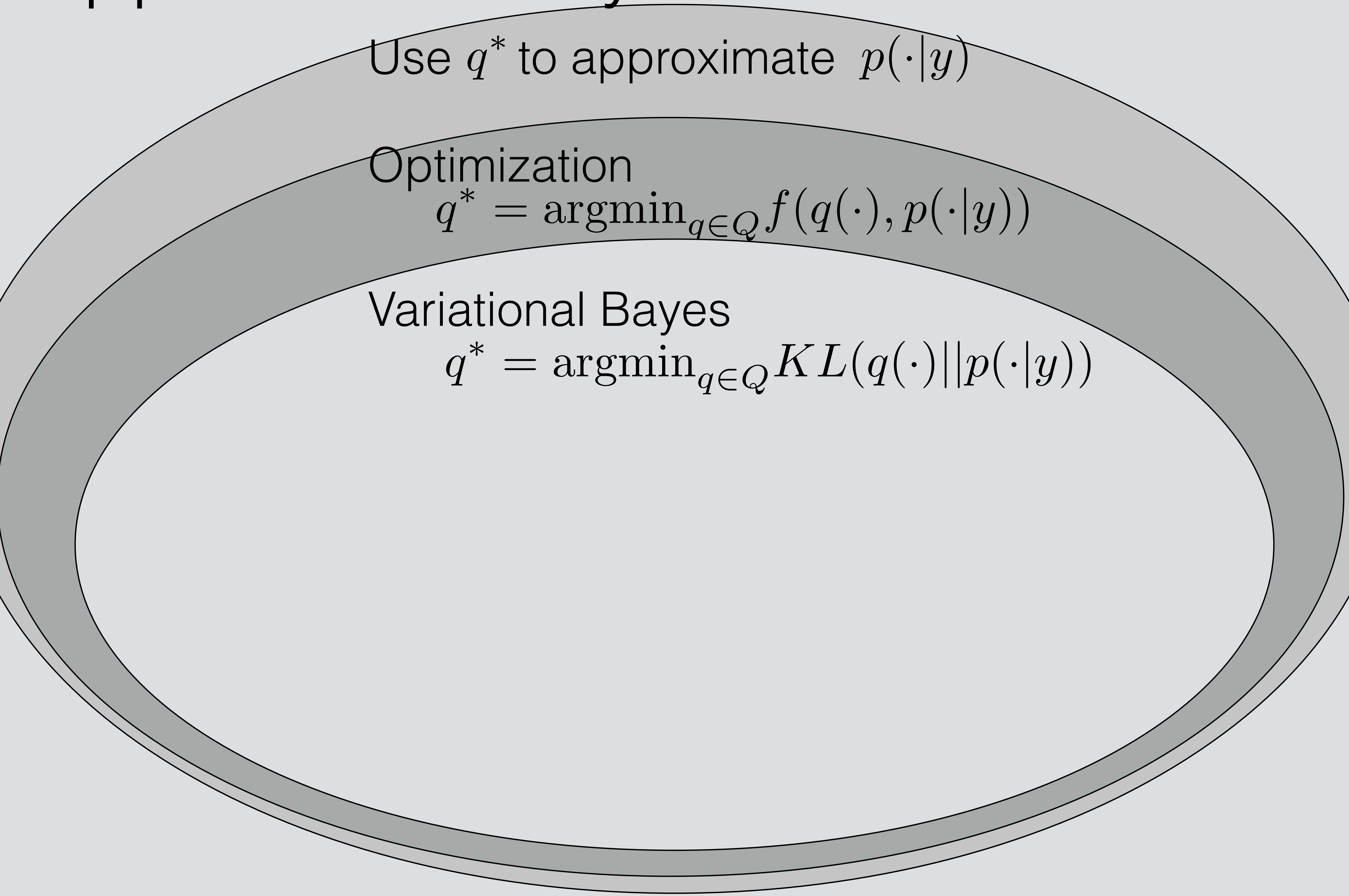

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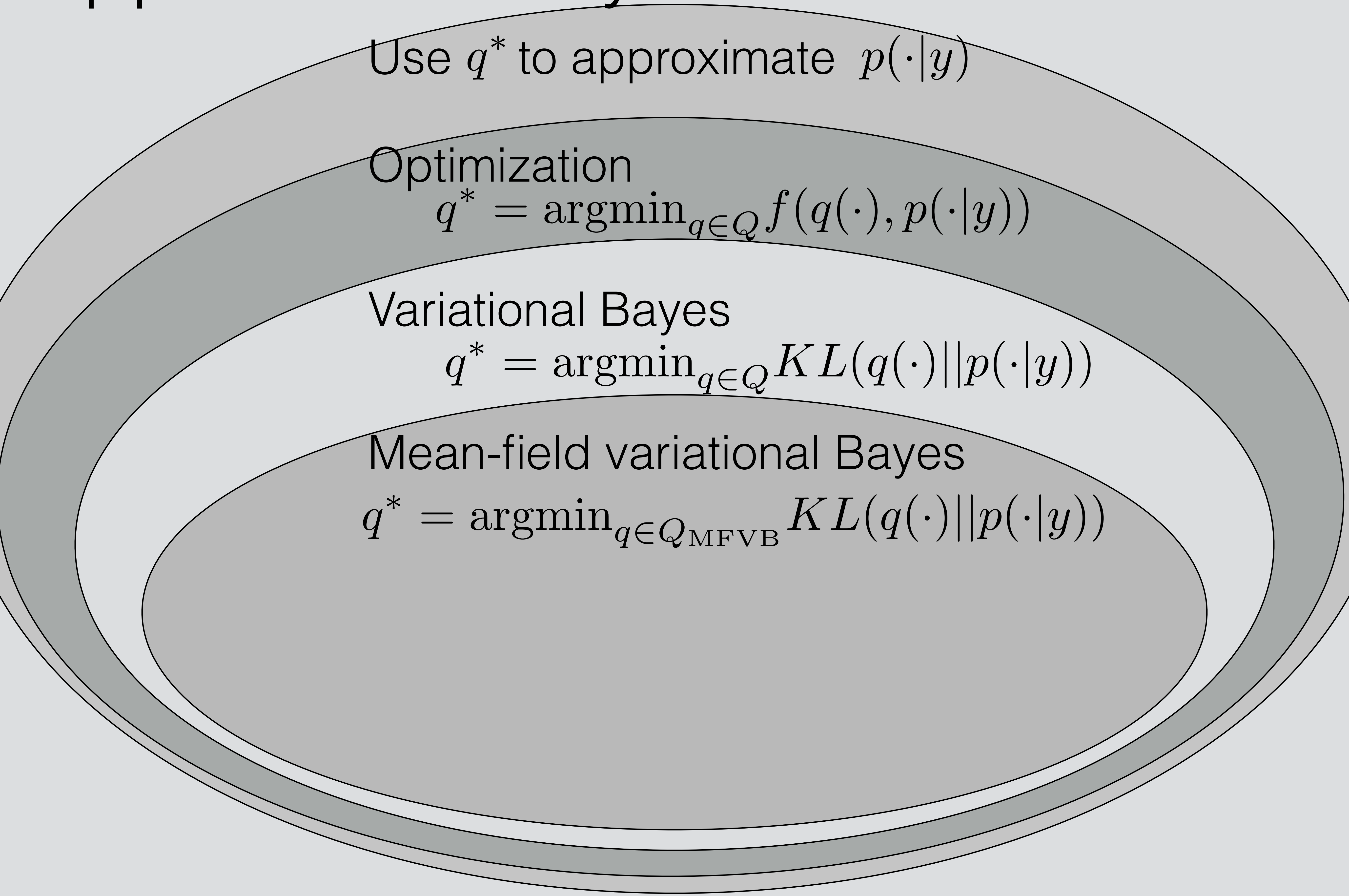

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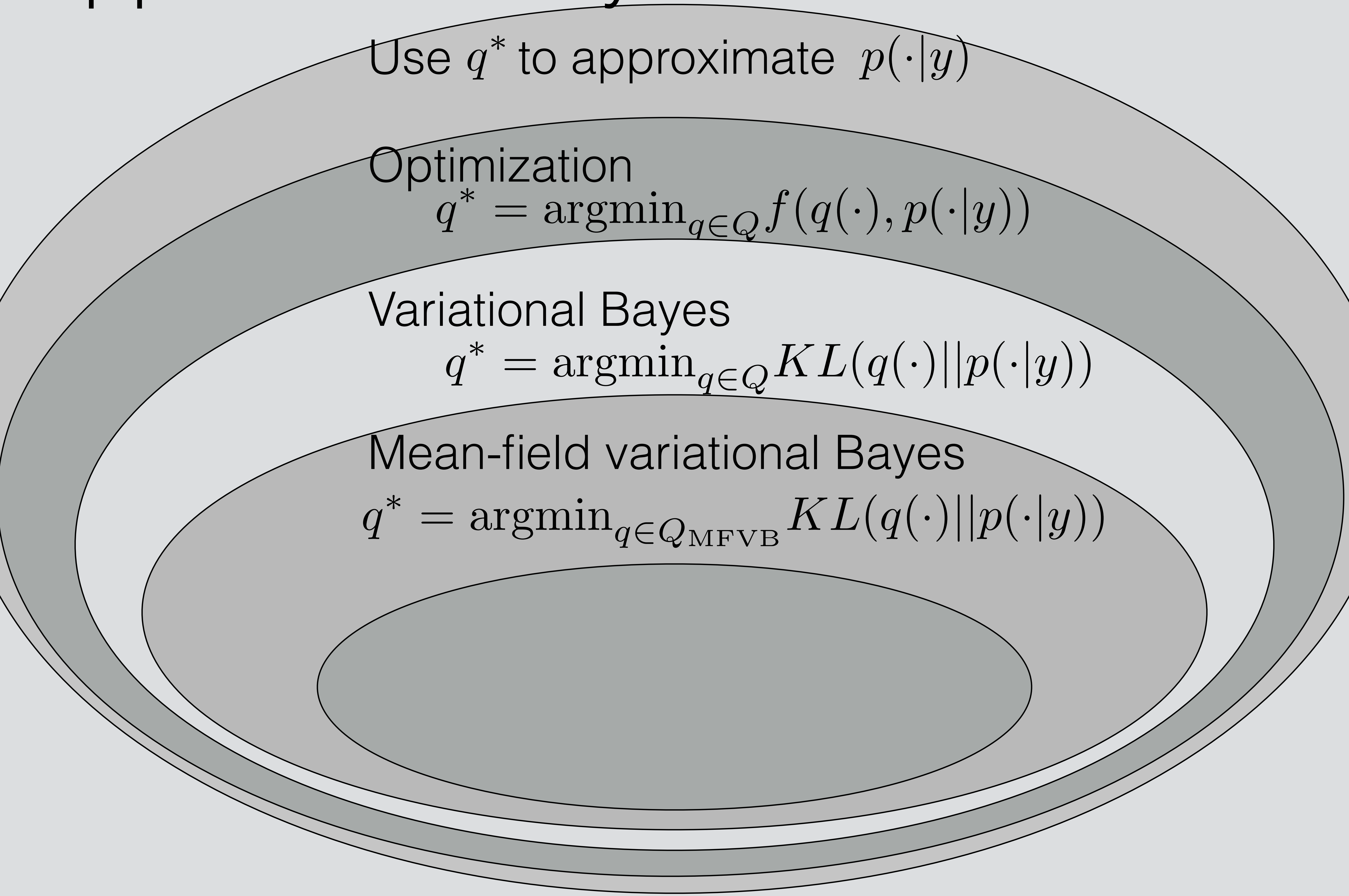

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- Automatic differentiation variational inference (ADVI) [Kucukelbir et al 2015, 2017]

Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use MFVB?
- When can we trust MFVB?
- Where do we go from here?

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Midge wing length

- Catalogued midge wing lengths (mm) $y = (y_1, \ldots, y_N)$

[Hoff 2009; Grogan, Wirth 1981; MacKay 2003; Bishop 2006]

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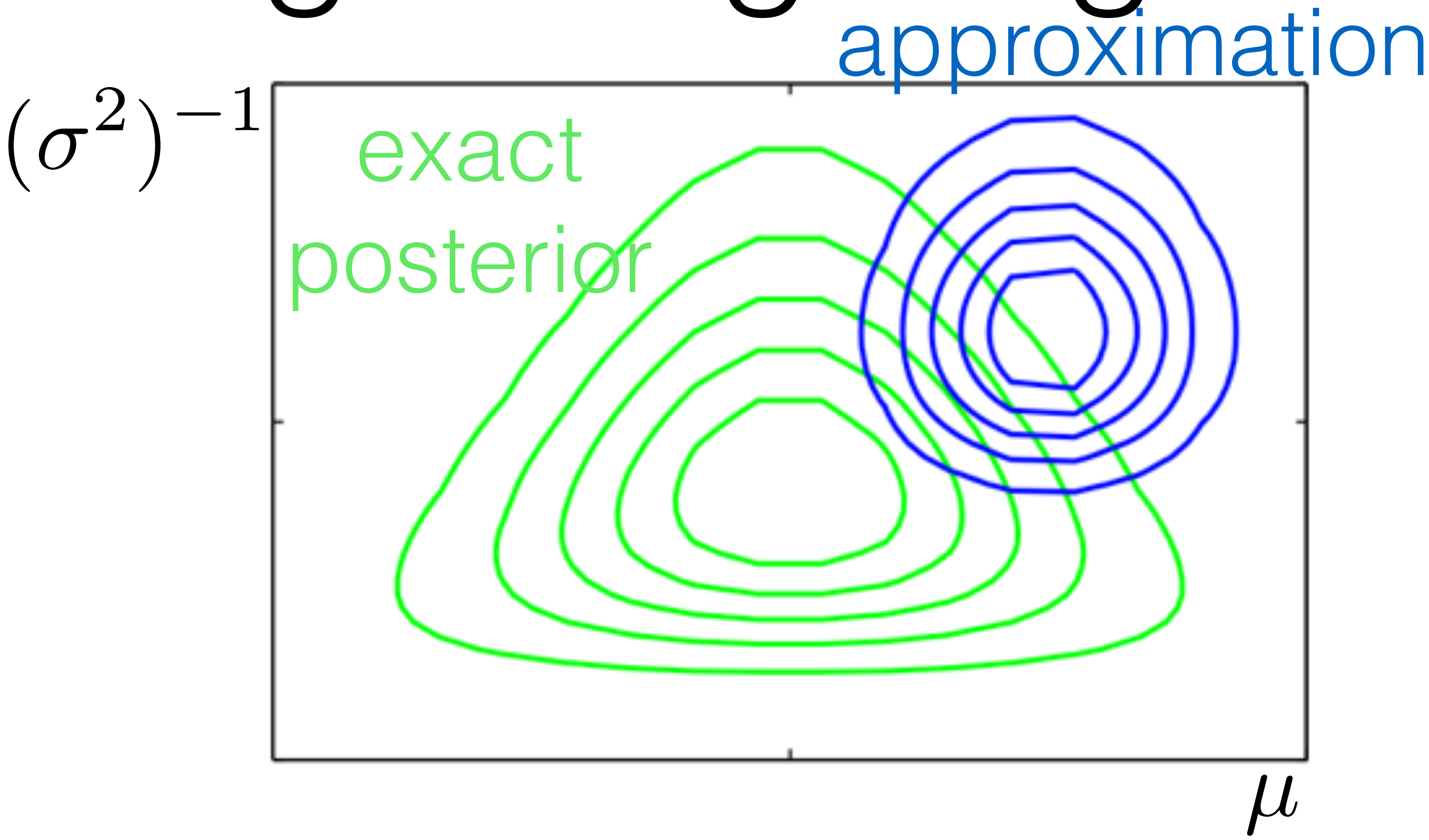

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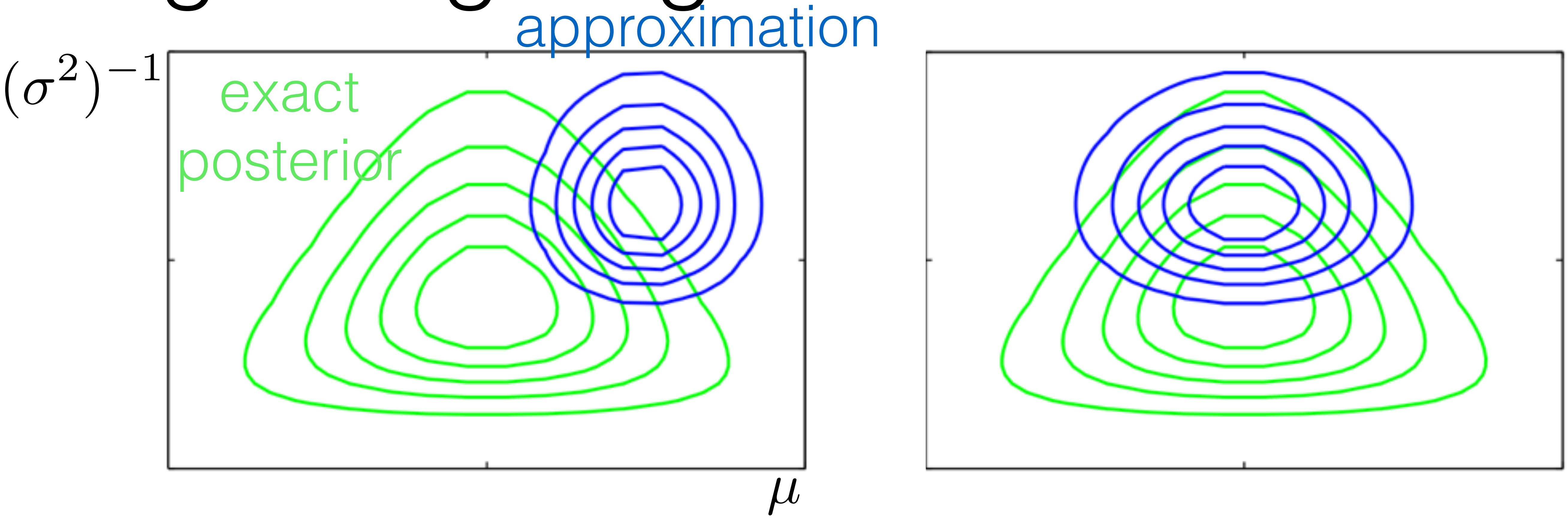

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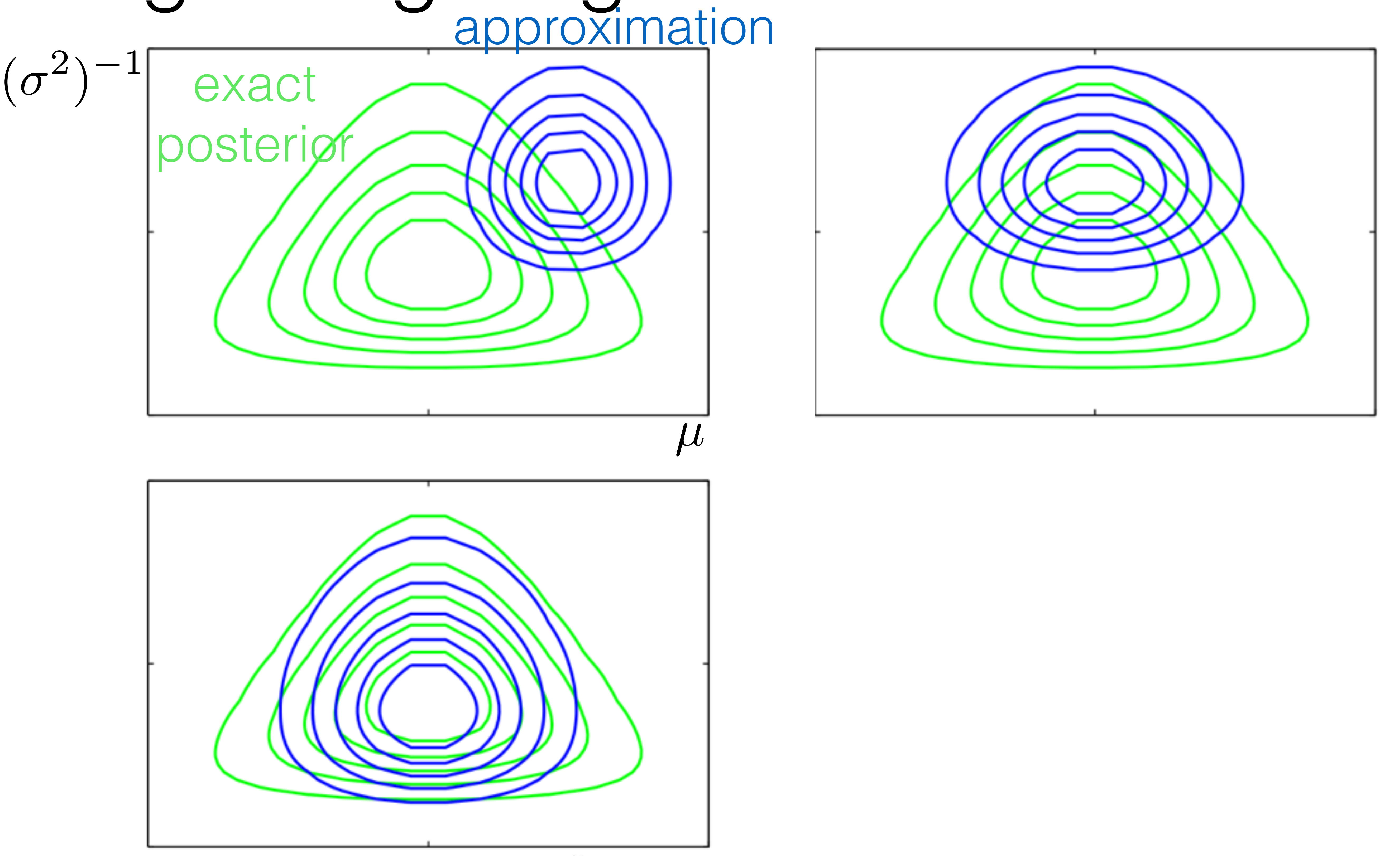

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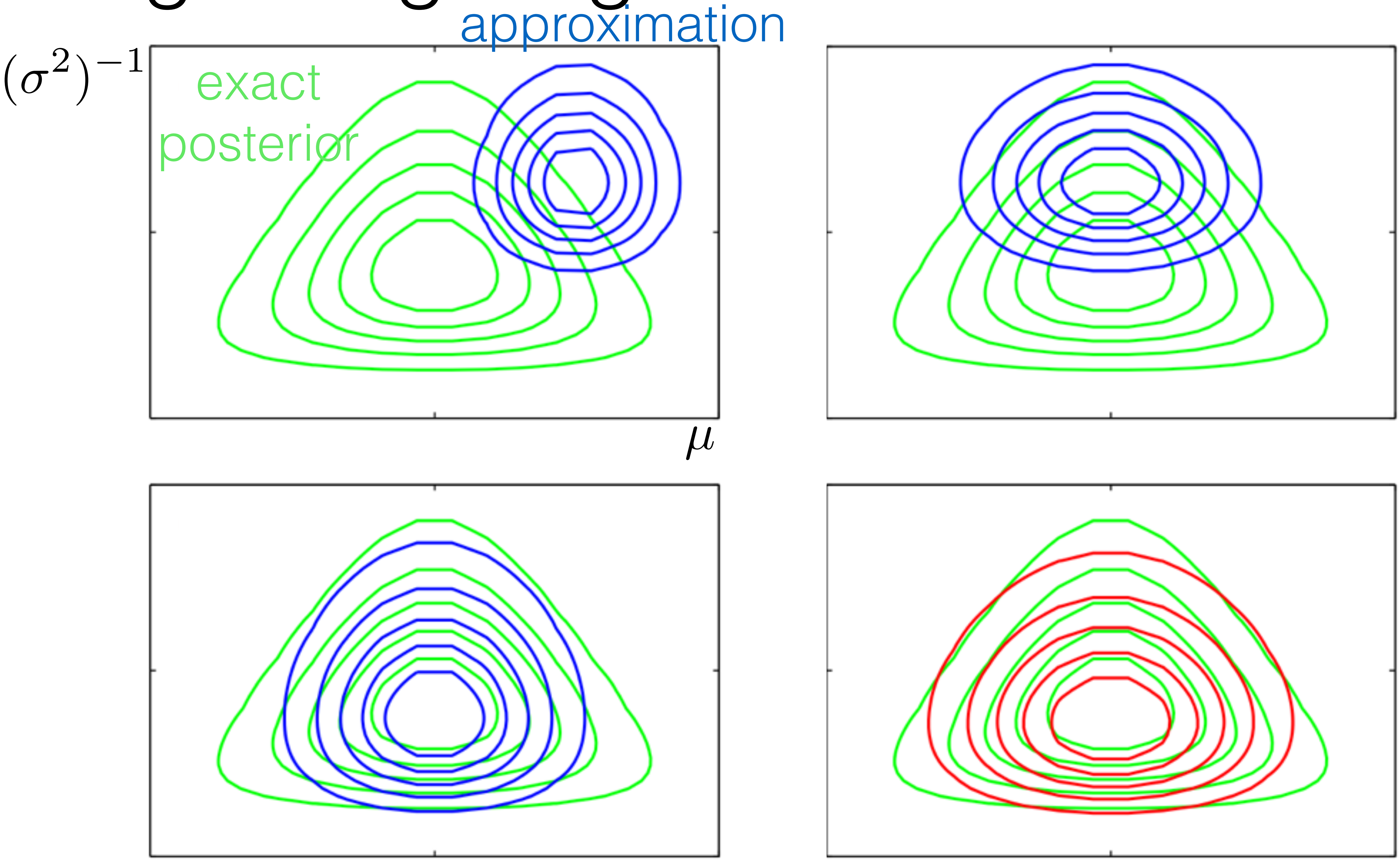

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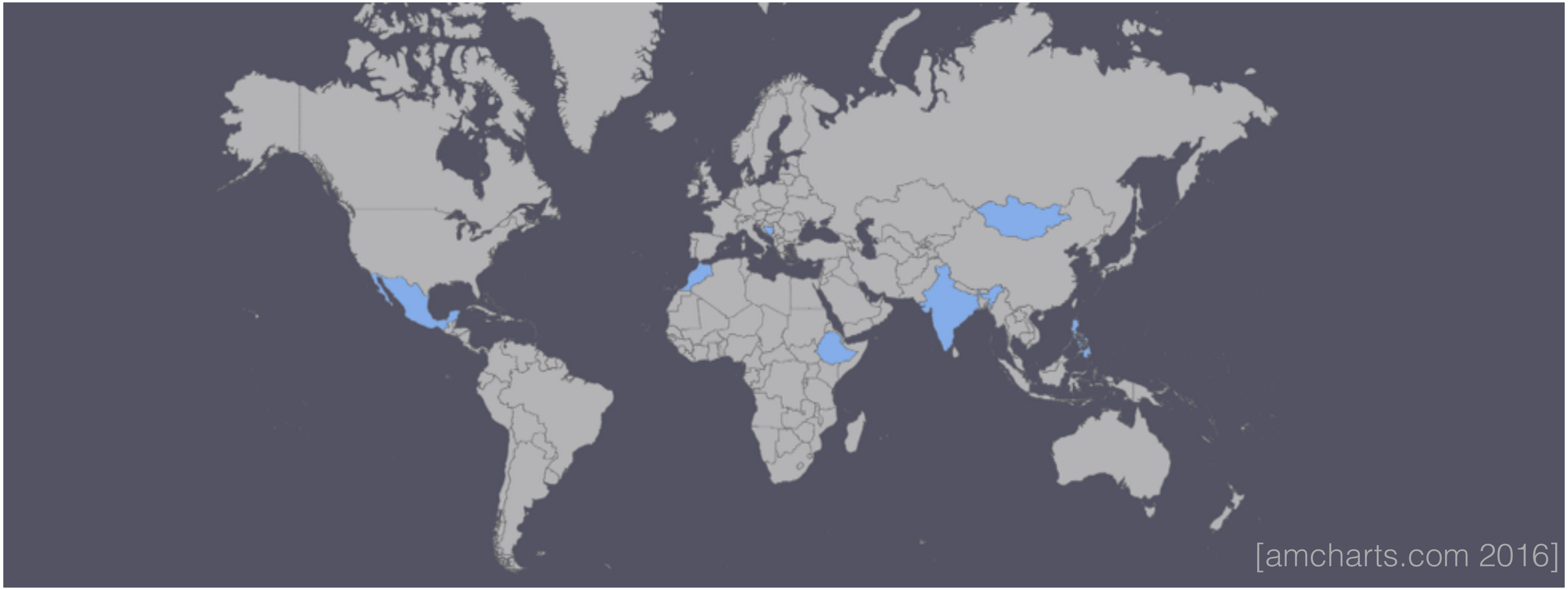
Microcredit Experiment



Microcredit Experiment

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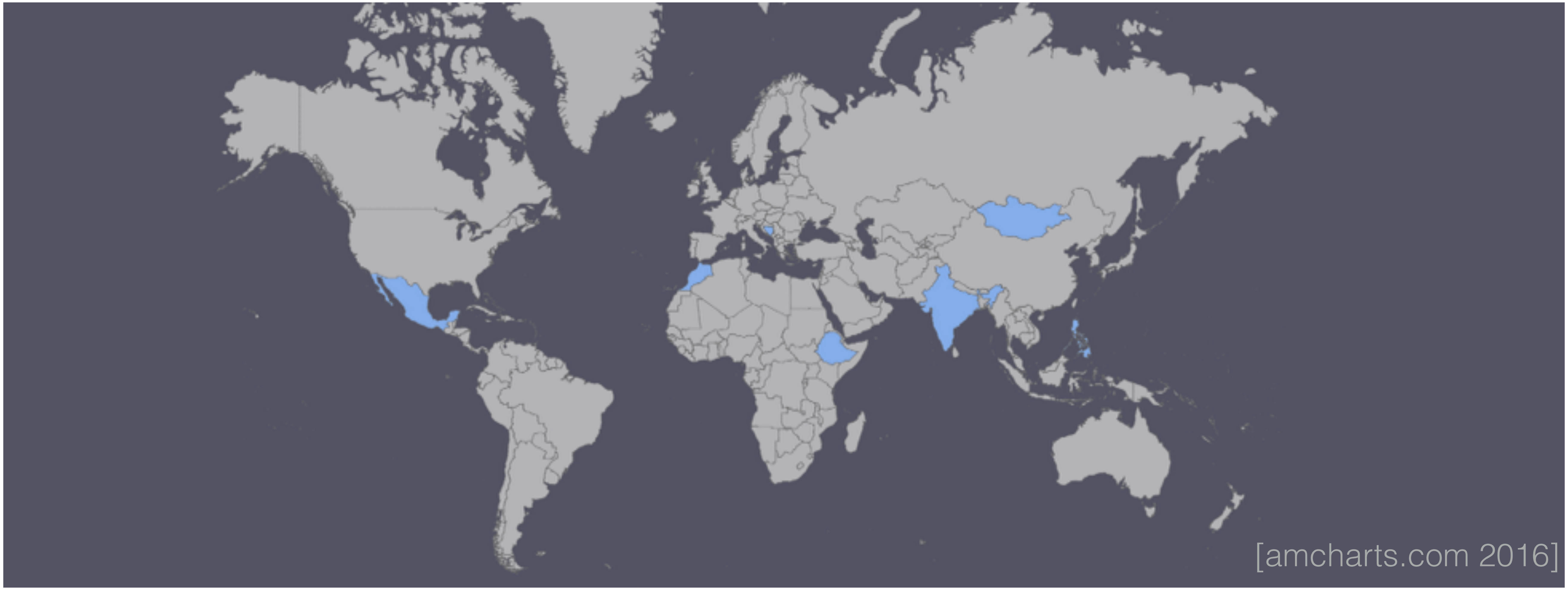
- *K* microcredit trials (Mexico, Mongolia, Bosnia, India, Morocco, Philippines, Ethiopia)



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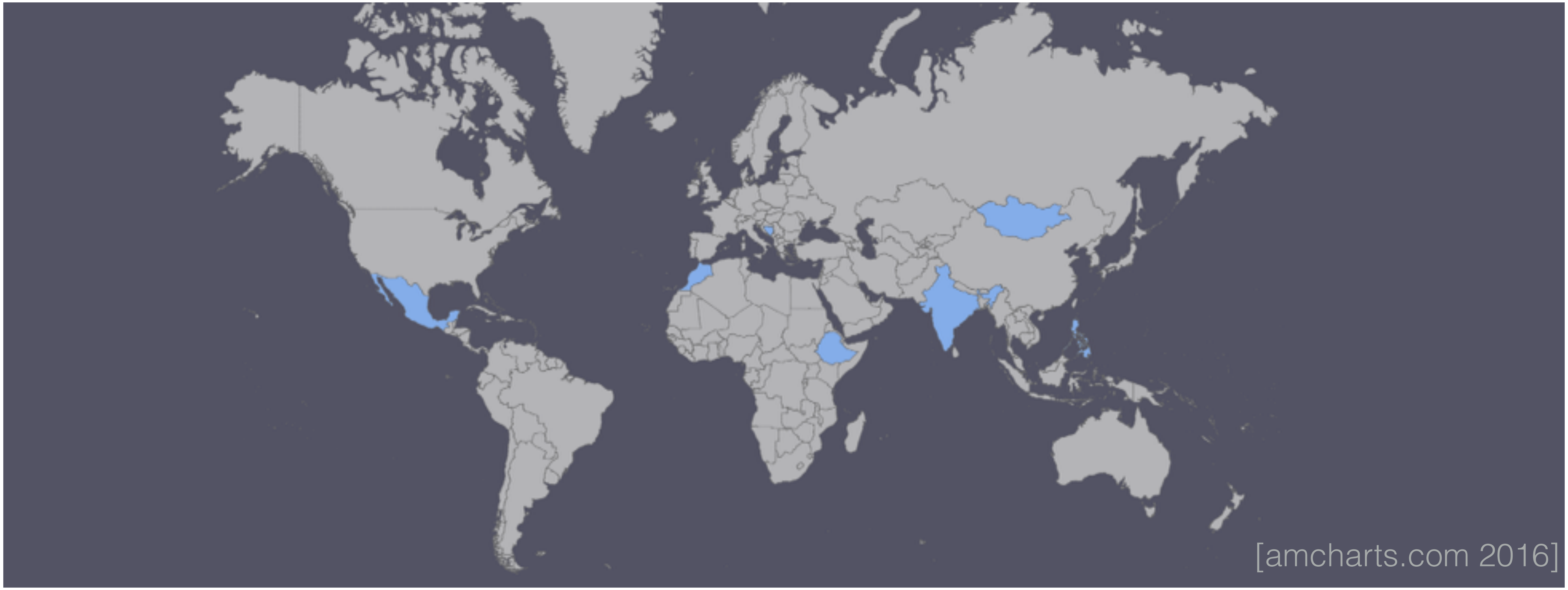

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- *K* microcredit trials (Mexico, Mongolia, Bosnia, India, Morocco, Philippines, Ethiopia)
- N_k businesses in *k*th site (~900 to ~17K)
- Profit of *n*th business at *k*th site:

1 if microcredit

profit

$$y_{kn} \overset{indep}{\sim} \mathcal{N}(\mu_k + T_{kn}\tau_k, \sigma_k^2)$$

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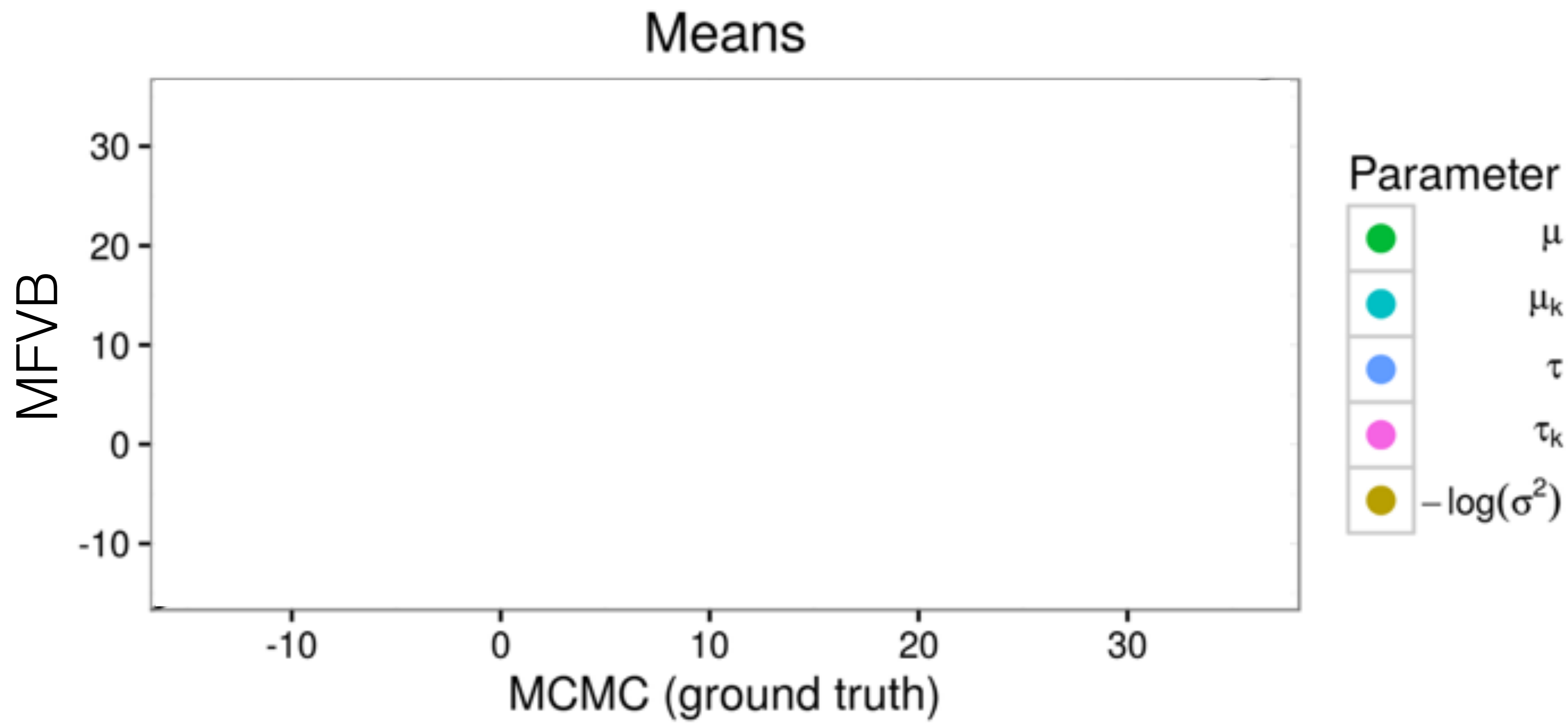
Microcredit



[Giordano, Broderick, Meager, Huggins, Jordan 2016]

Microcredit

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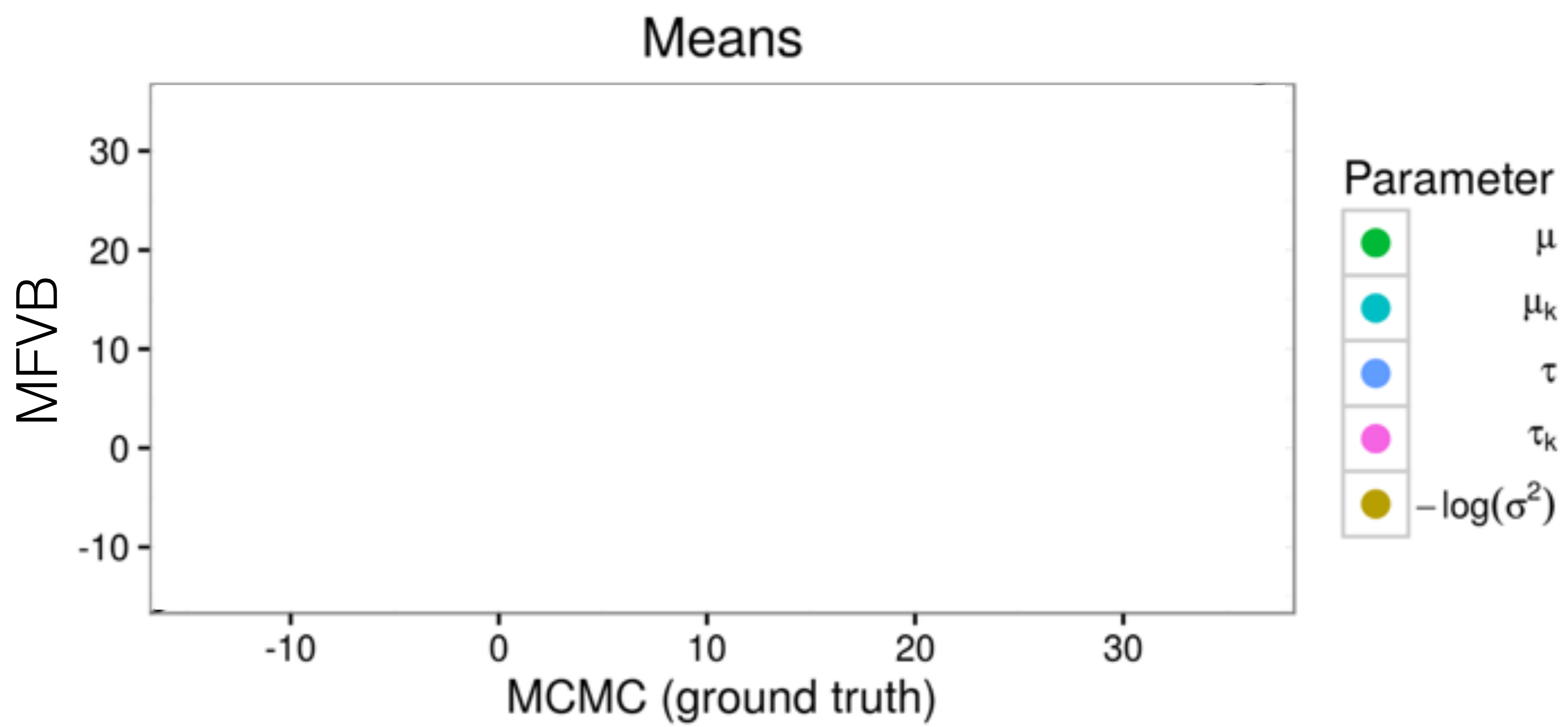

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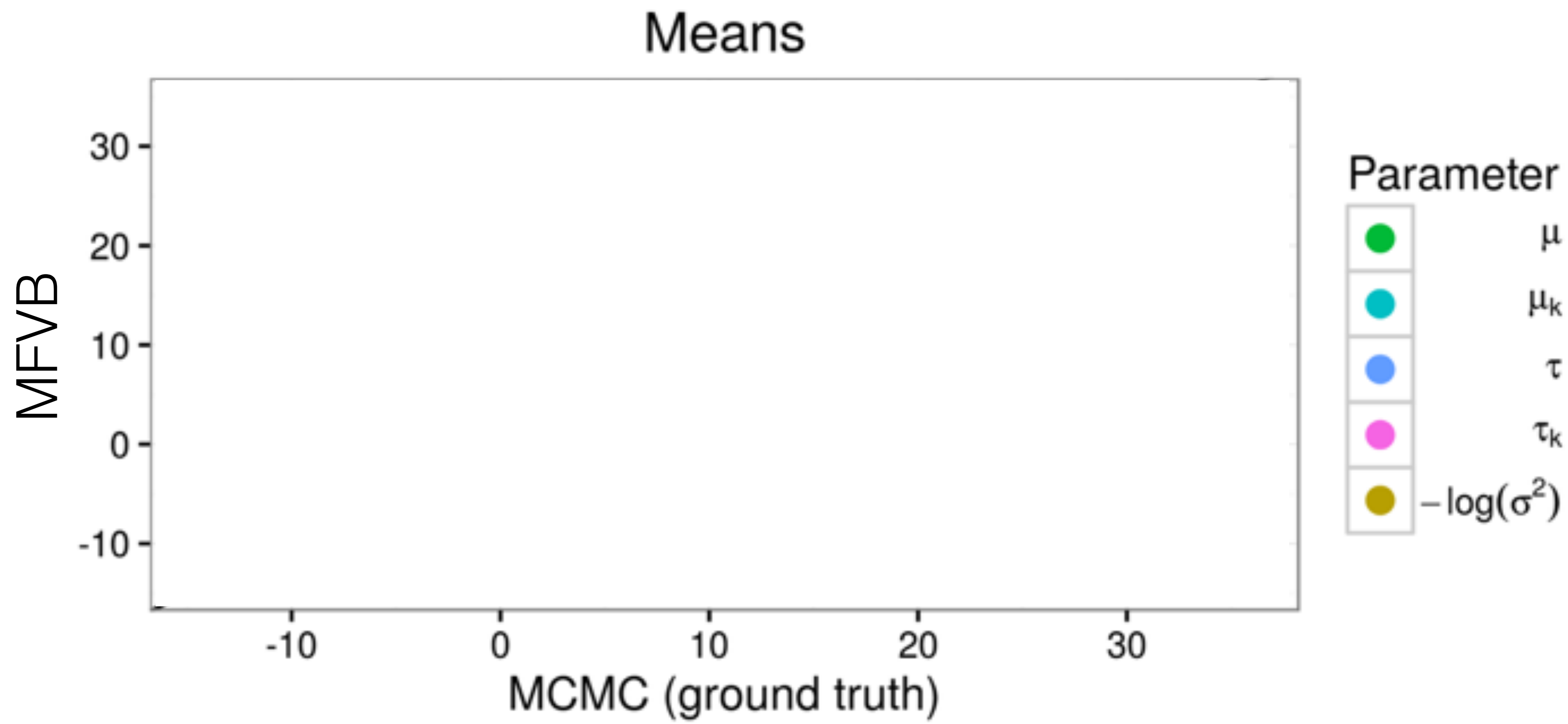

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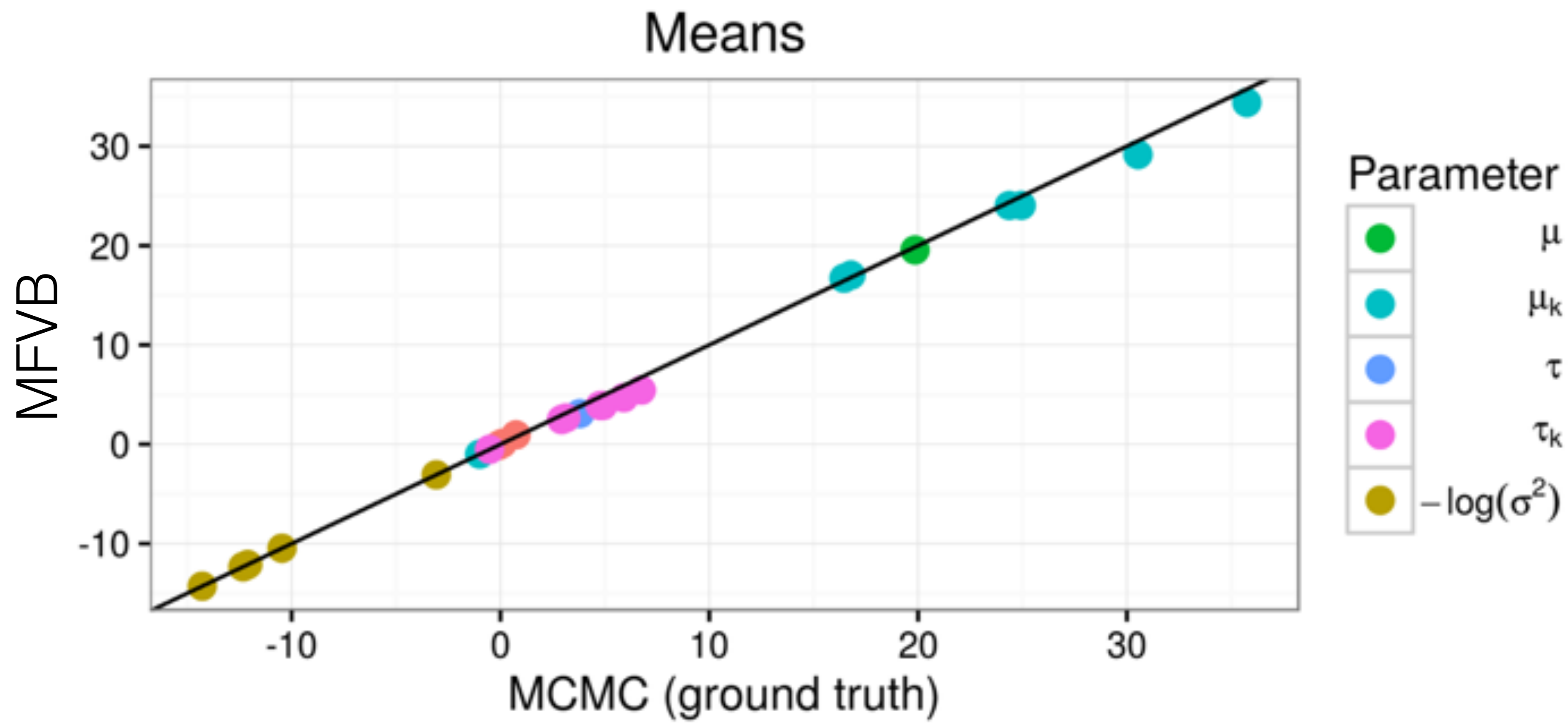

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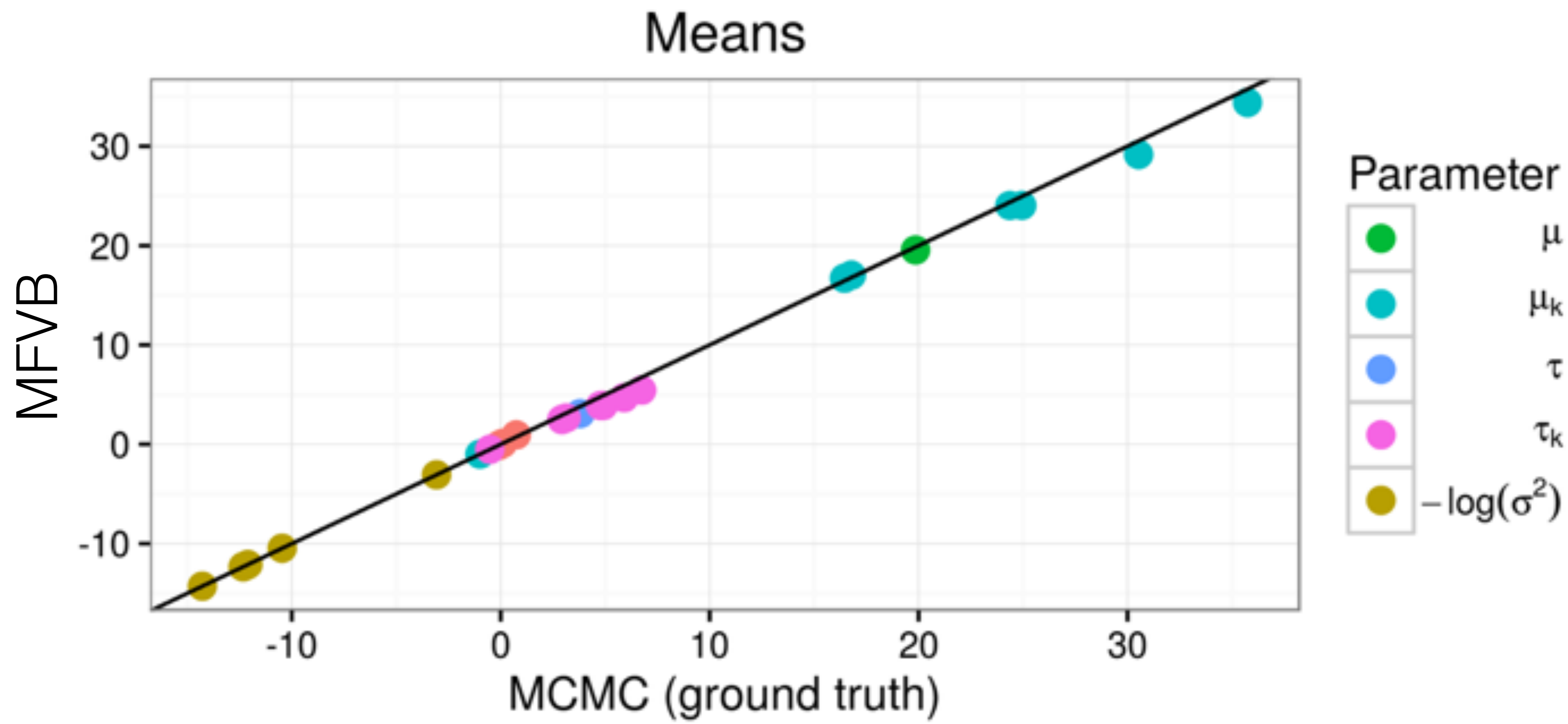

Criteo Online Ads Experiment

- Click-through conversion prediction
- Q: Will a customer (e.g.) buy a product after clicking?

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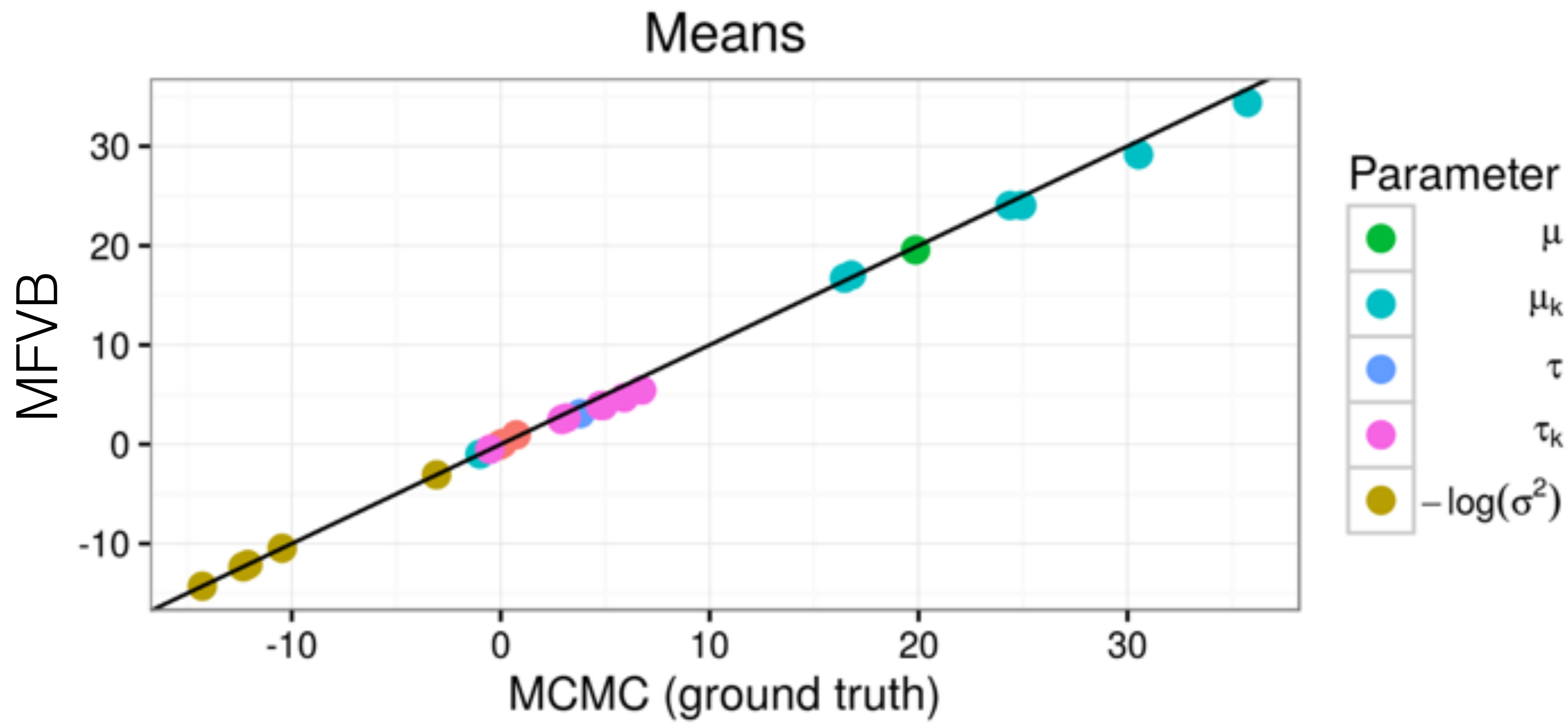

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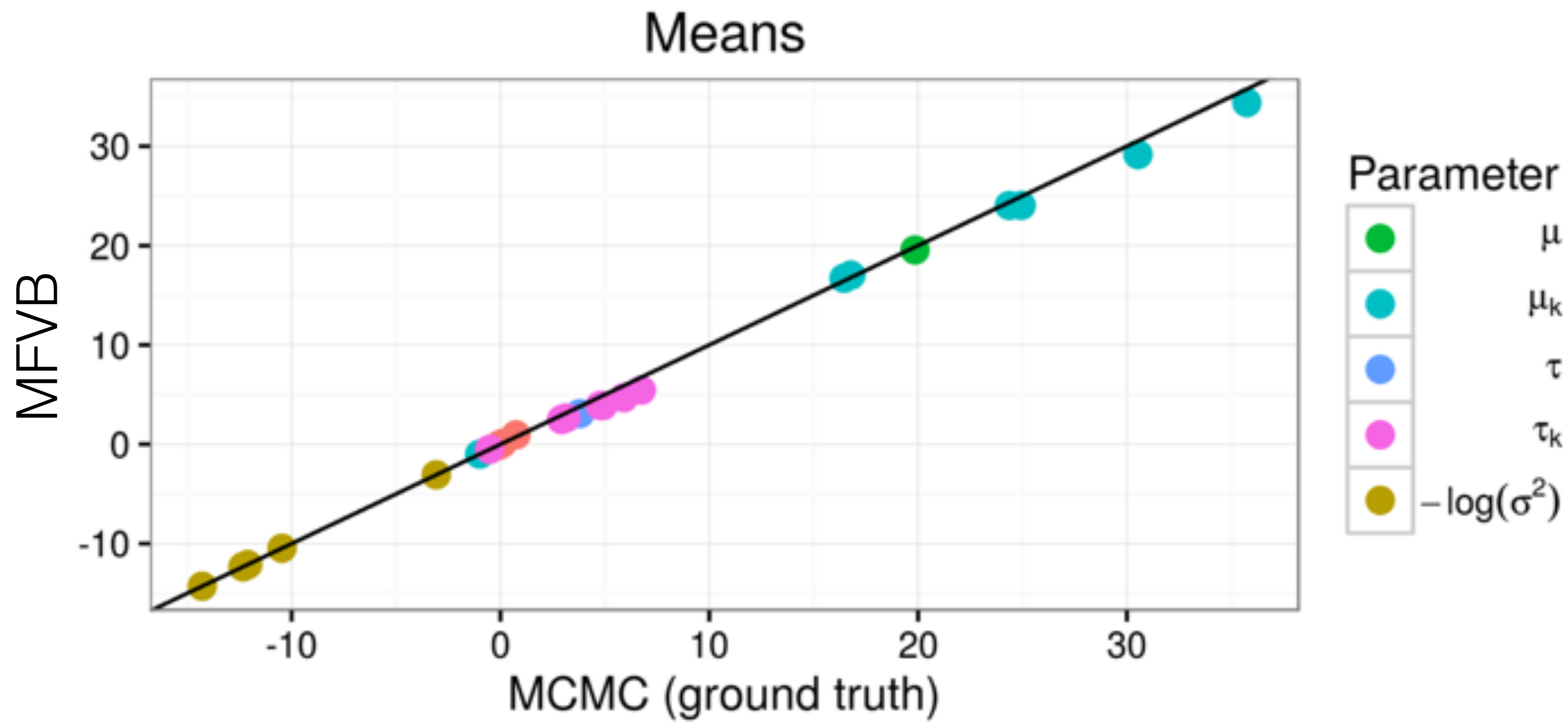

Criteo Online Ads Experiment

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- Logistic GLMM; N = 61,895 subset to compare to MCMC

[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2017]

Criteo Online Ads Experiment

[Giordano, Broderick, Jordan 2017]

Criteo Online Ads Experiment

- MAP: **12 s**

[Giordano, Broderick, Jordan 2017]

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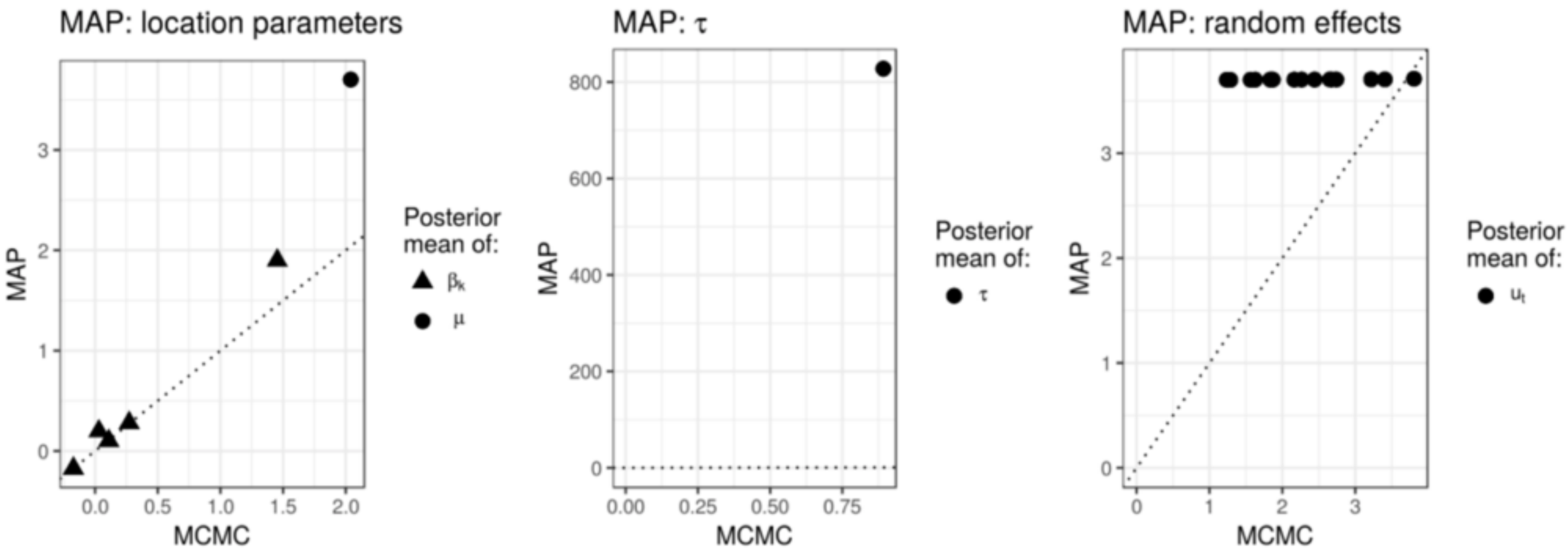

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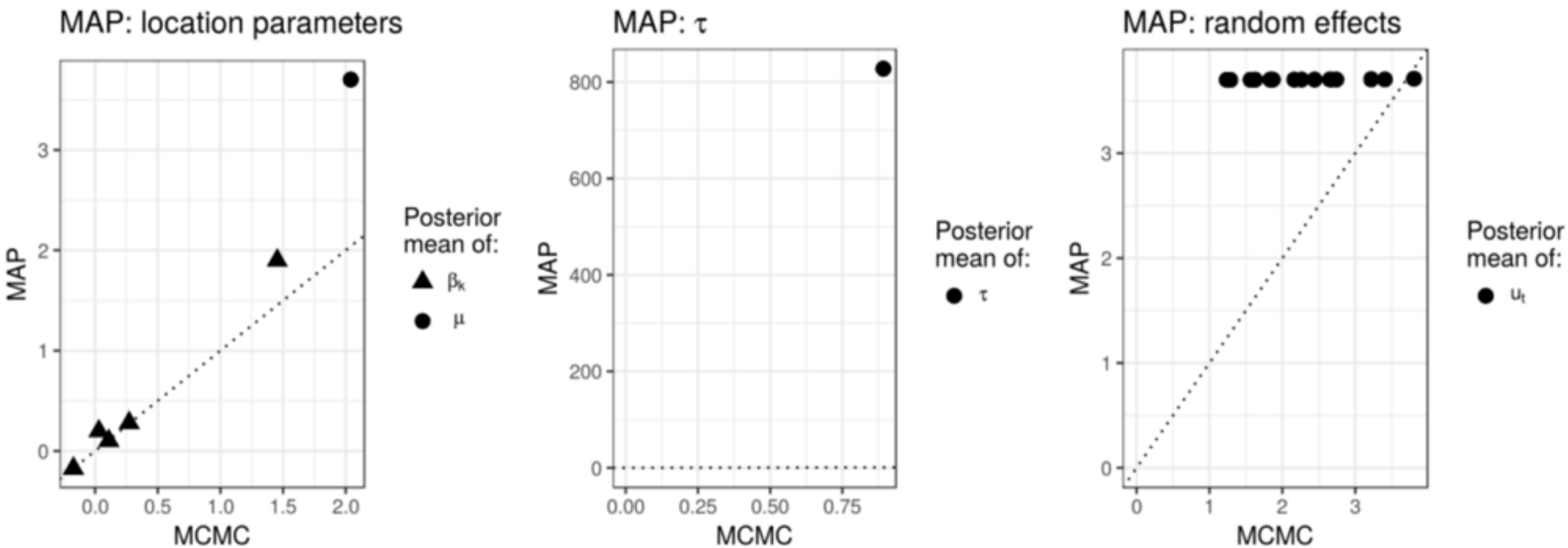

- MAP: **12 s**
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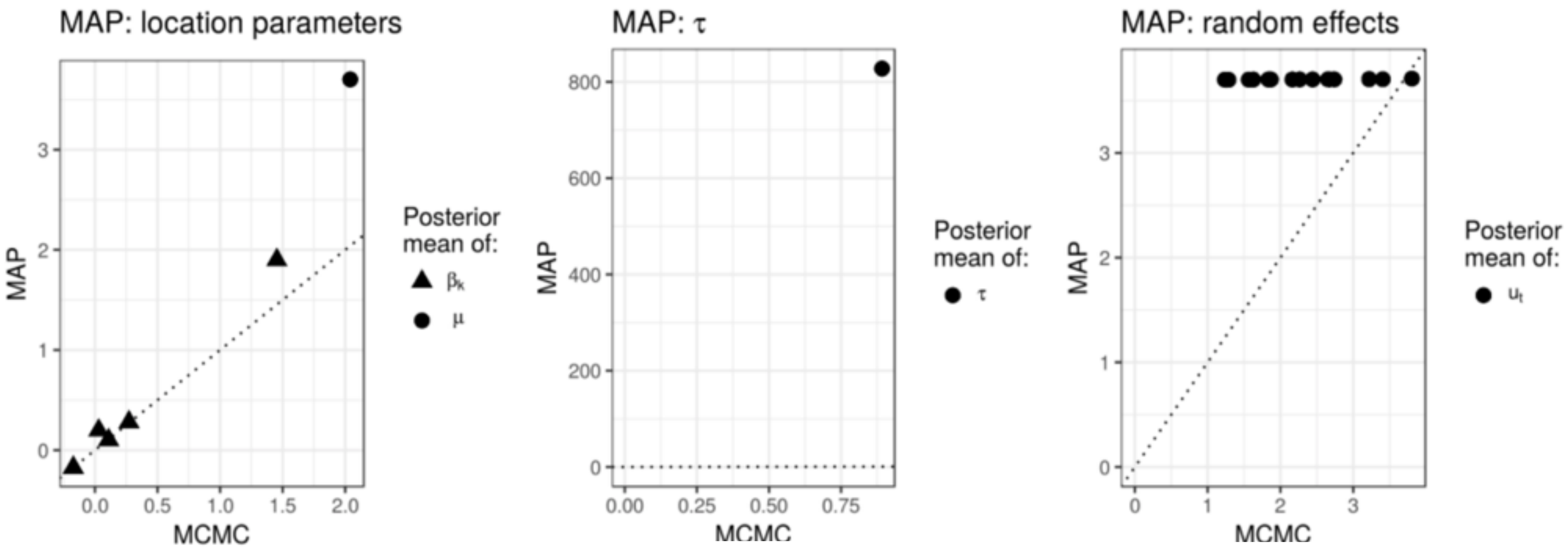

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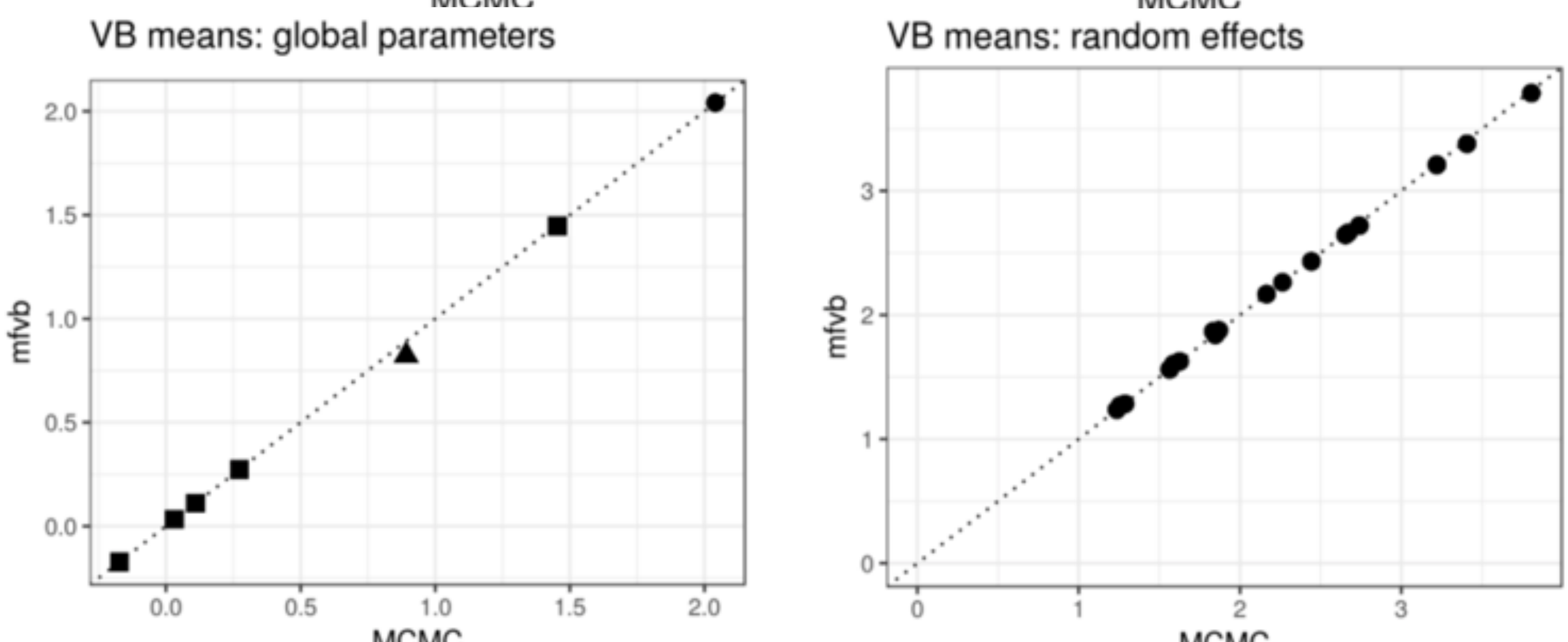

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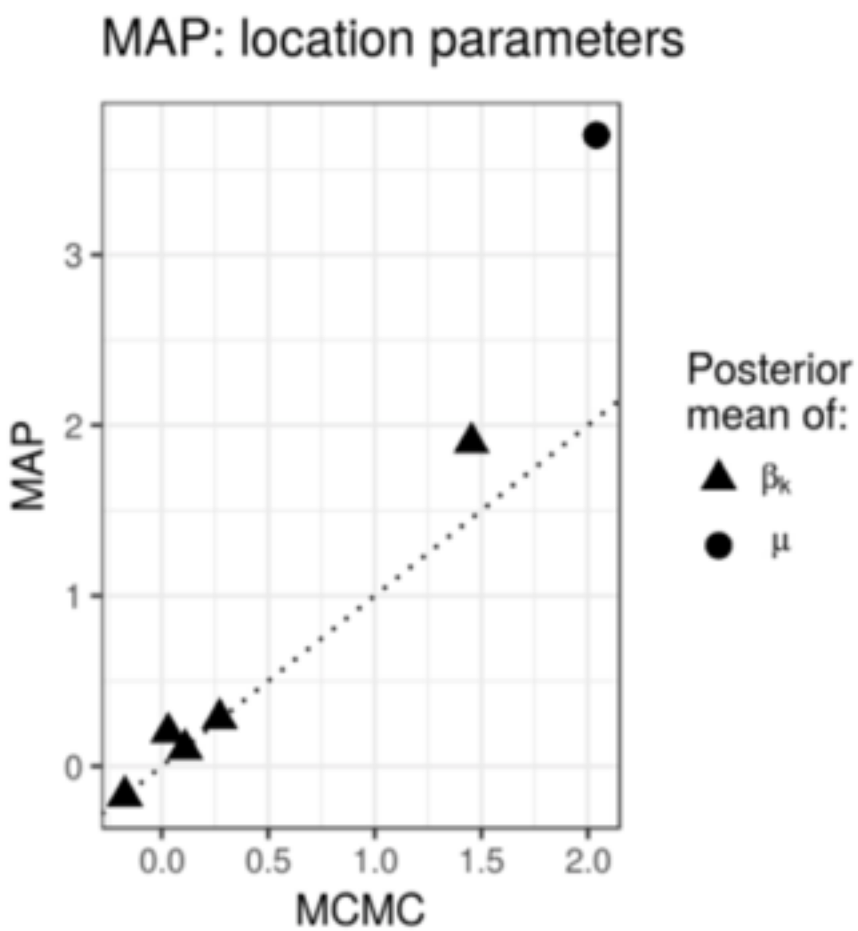

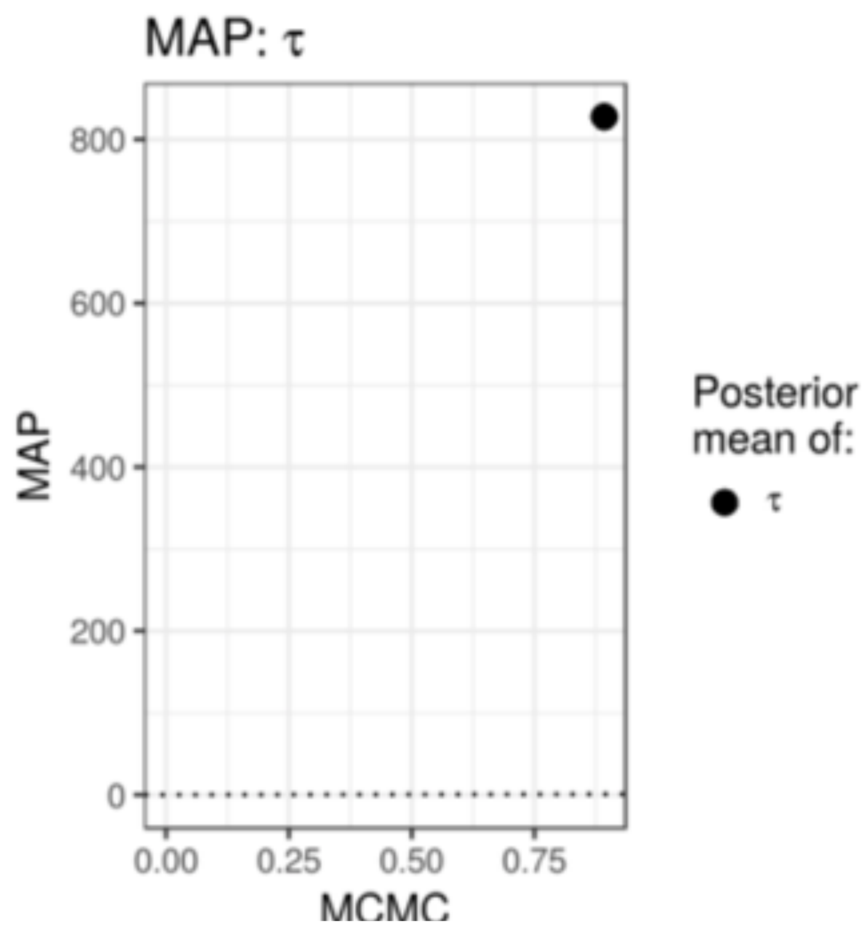

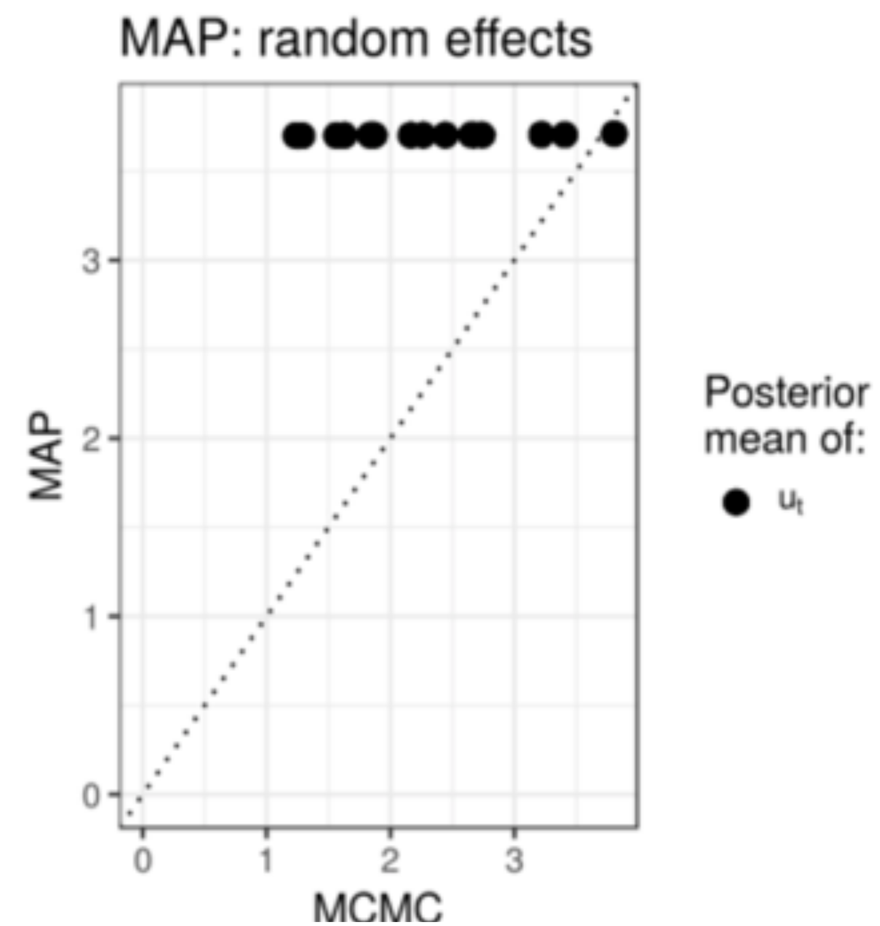

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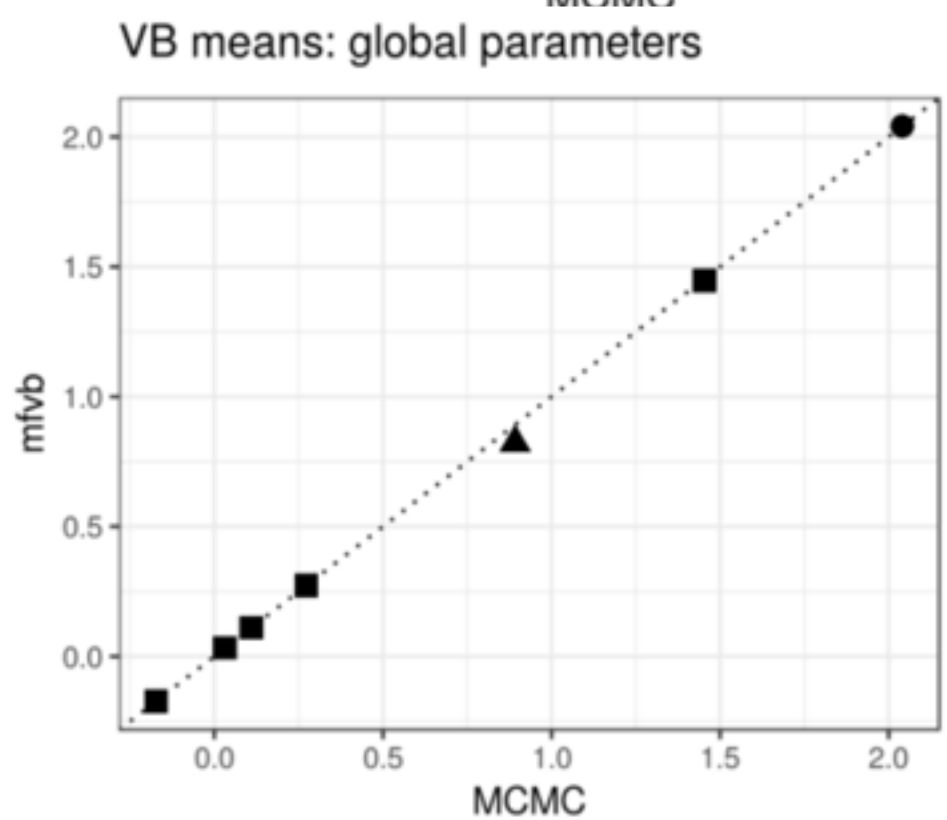
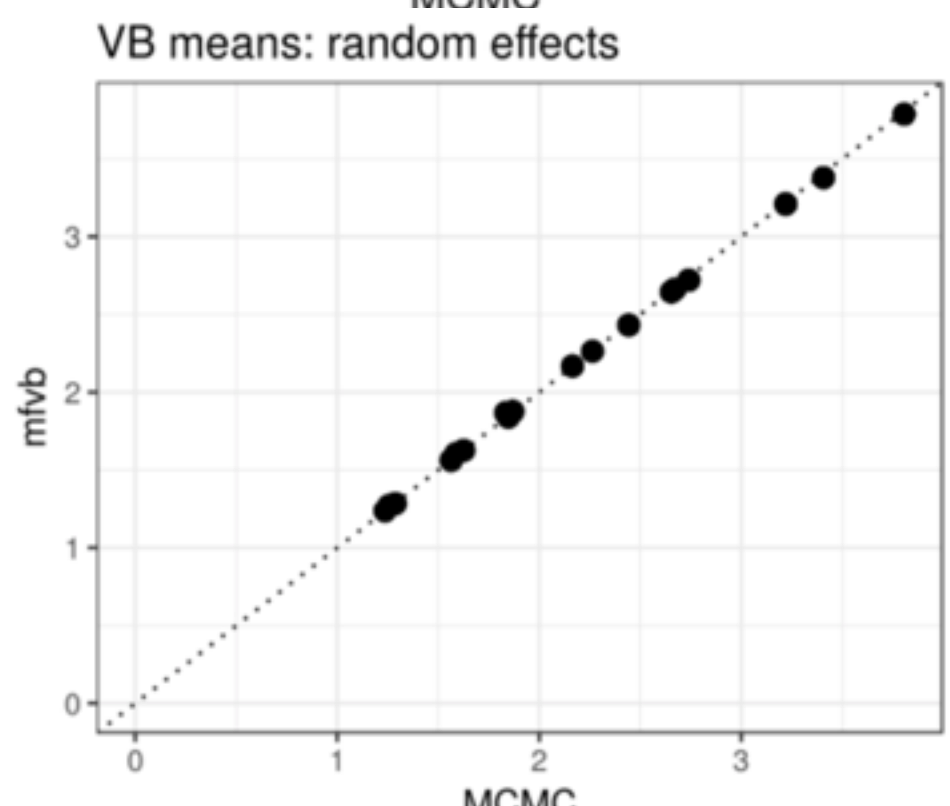
- MCMC (5K samples): 21,066 s (**5.85 h**)





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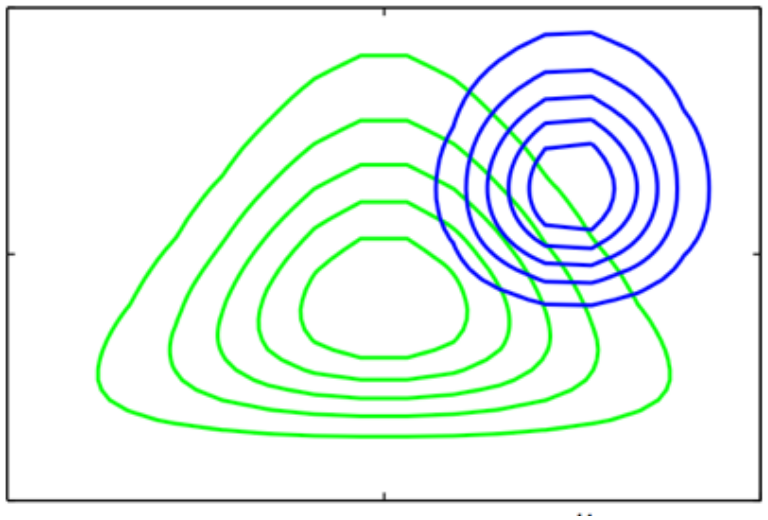
How to optimize: MFVB



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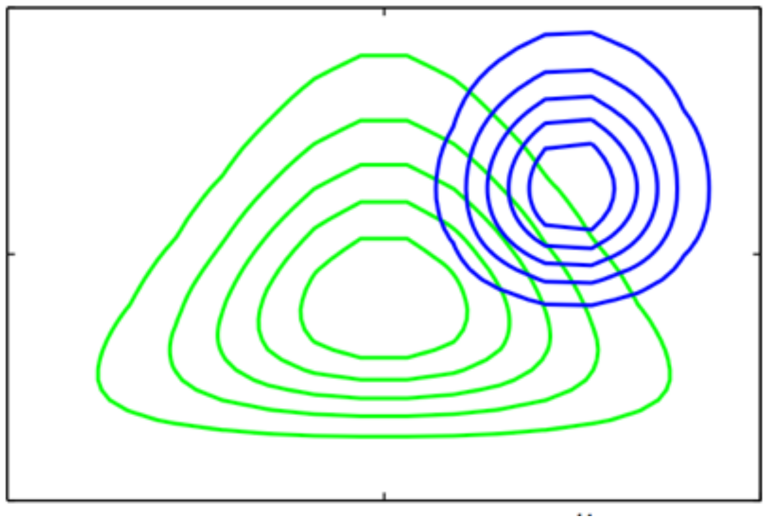

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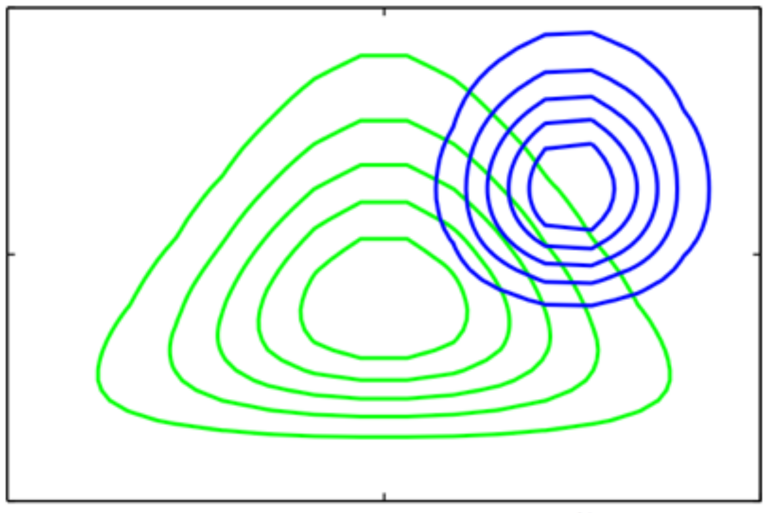

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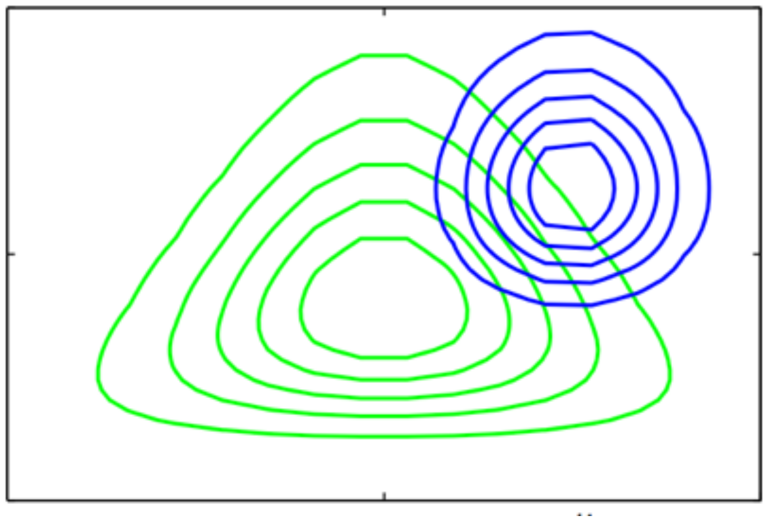

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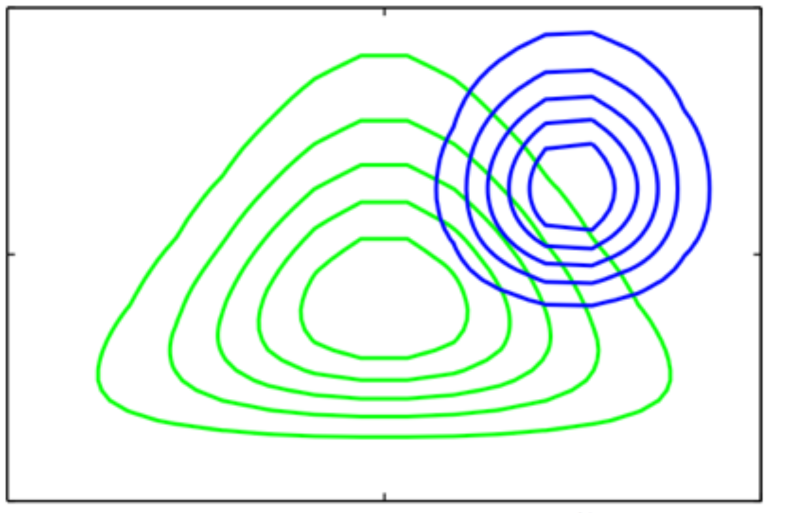

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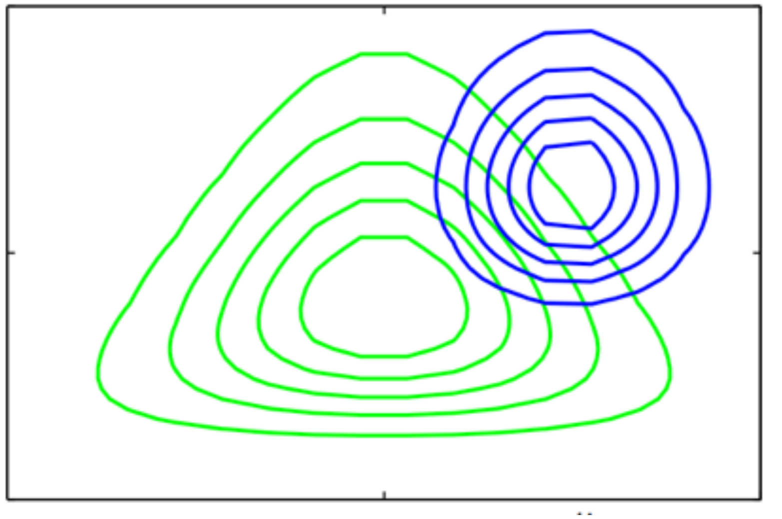

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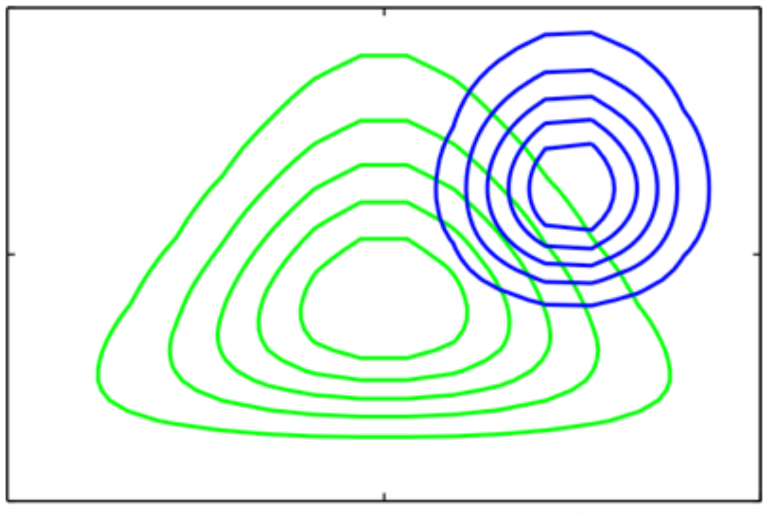

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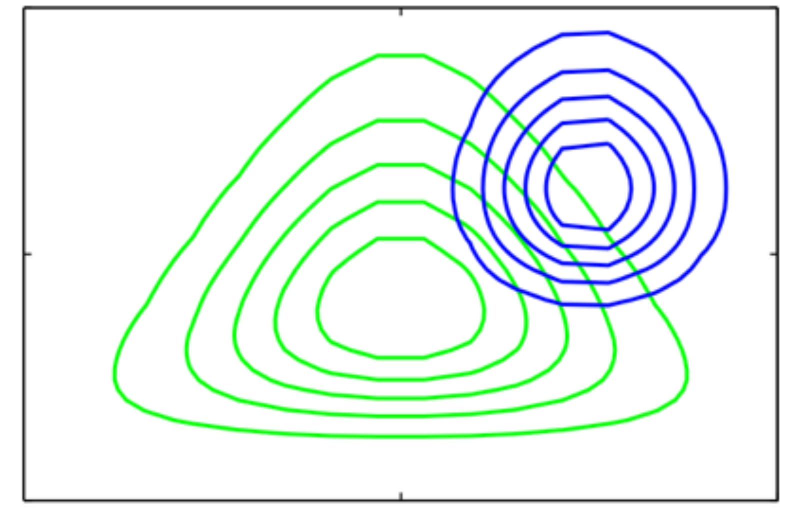

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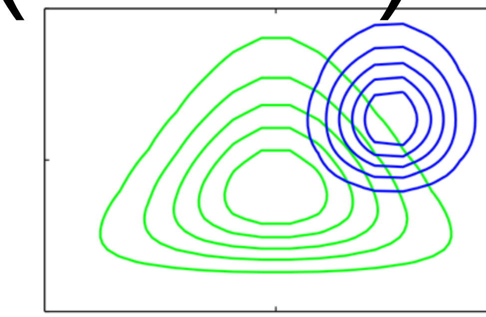
  - Automatic differentiation variational inference (ADVI)

[Kucukelbir et al 2015, 2017]

[Baydin et al 2018]

Stochastic gradient descent (SGD)



- MFVB: $\min_{\eta : q_\eta \in Q_{\mathrm{MFVB}}} -\mathbb{E}_{q_\eta} \log \frac{p(\theta, y_{1:N})}{q_\eta(\theta)} d\theta$

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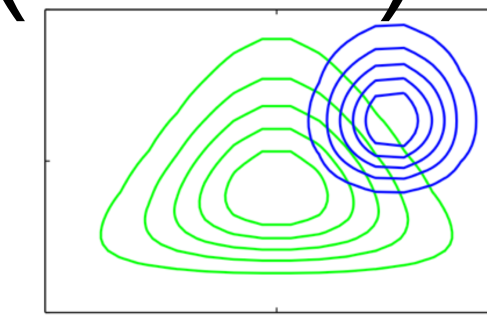

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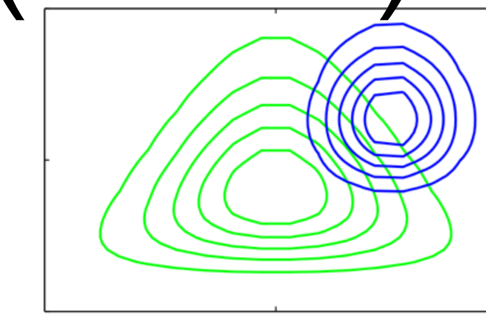

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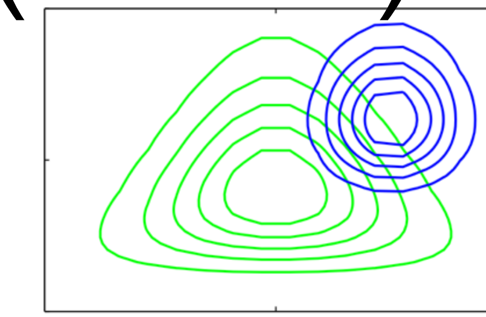

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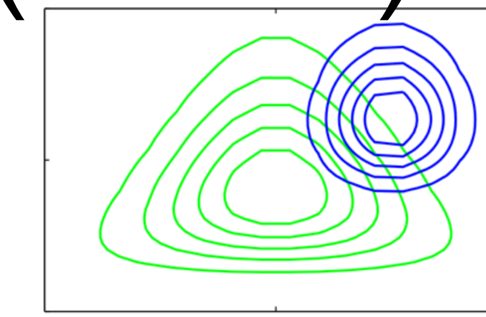

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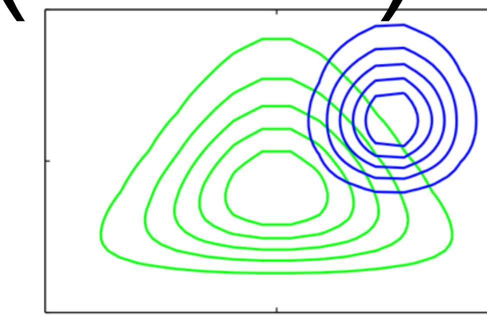

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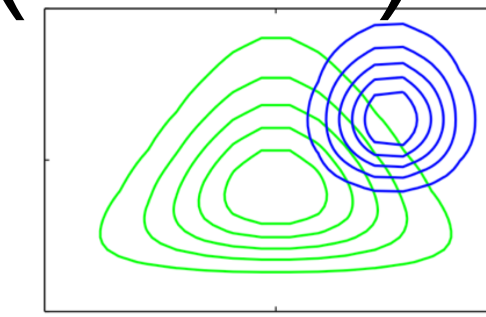

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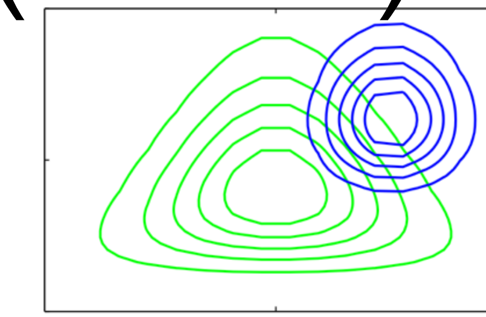

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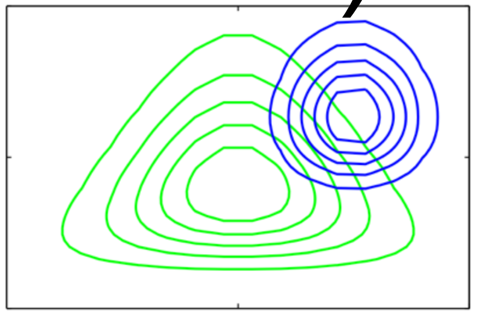

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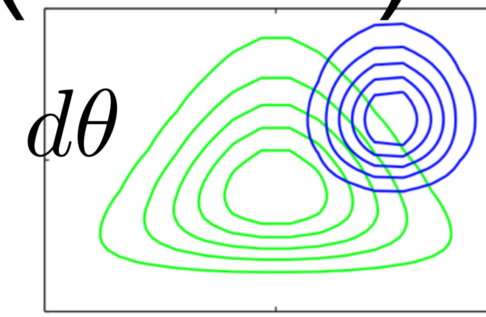

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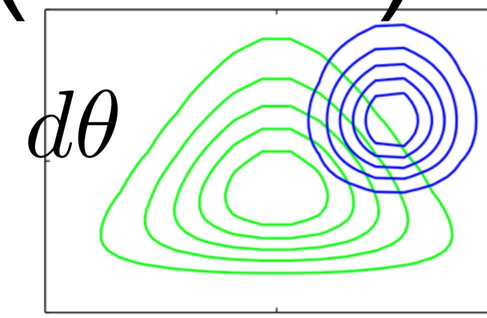

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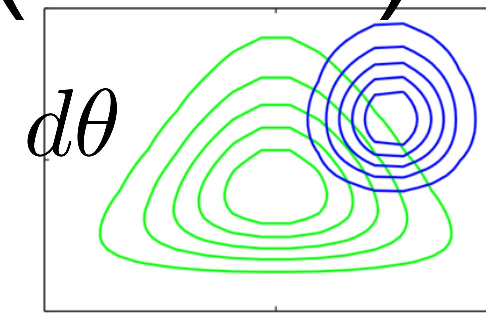

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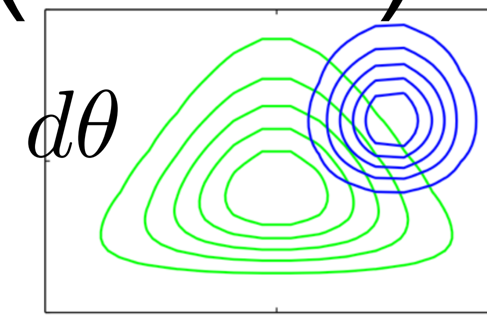

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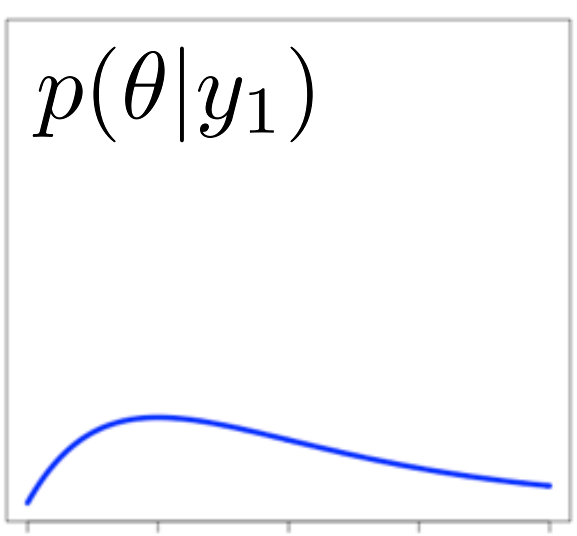

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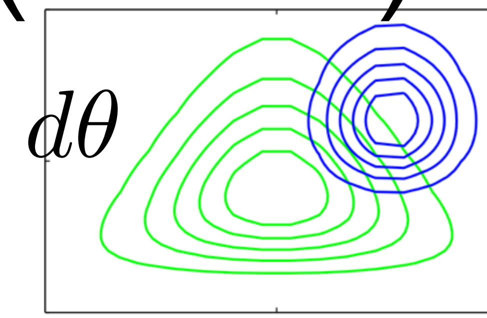

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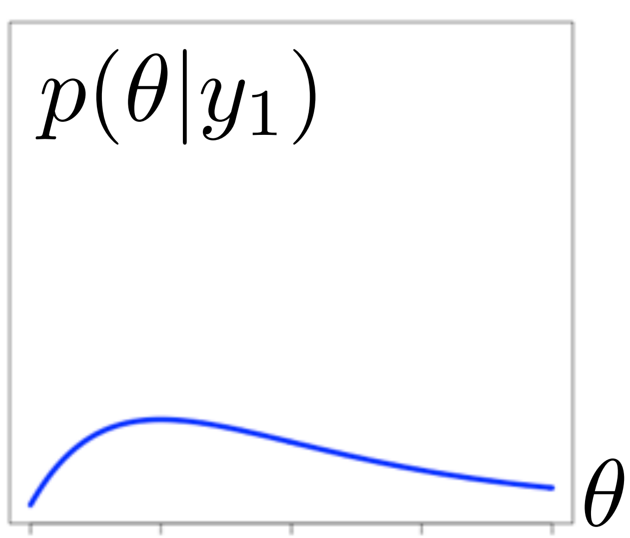

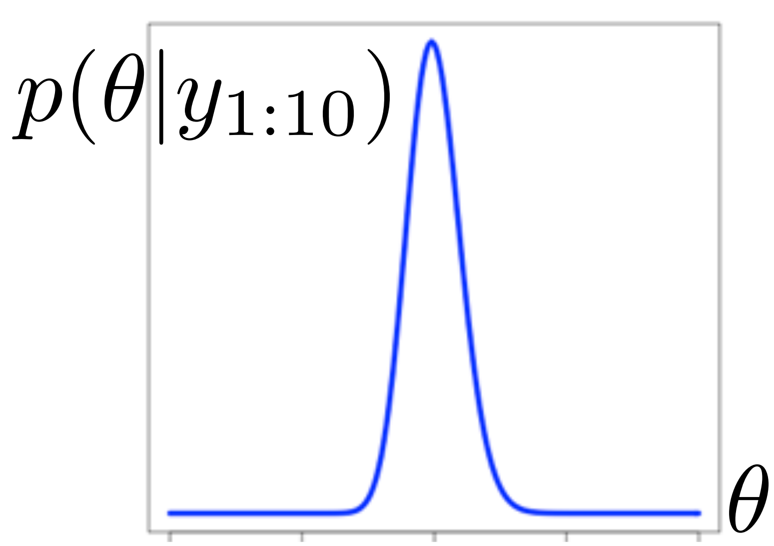

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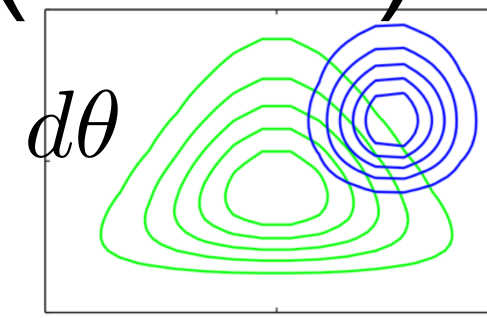

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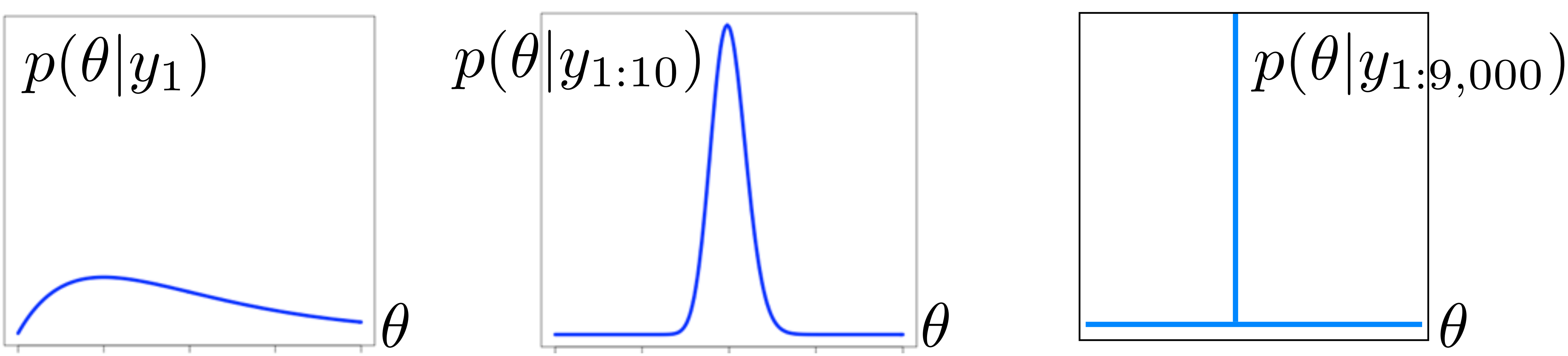

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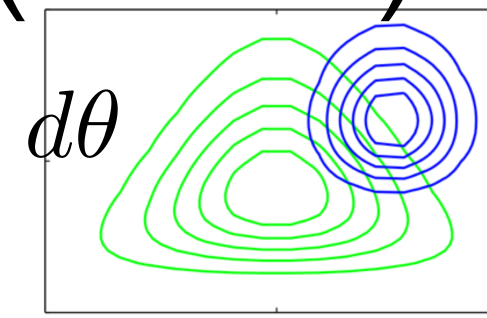

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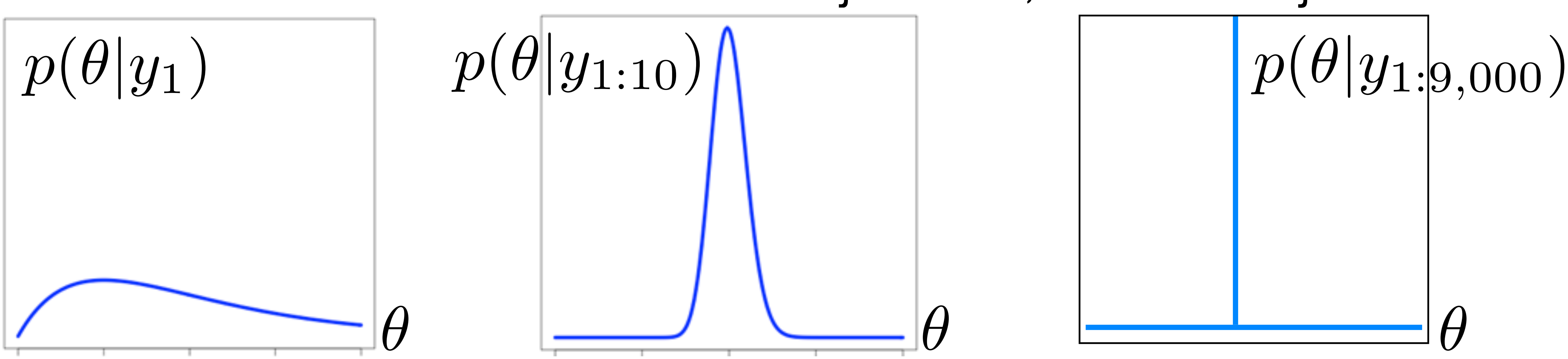

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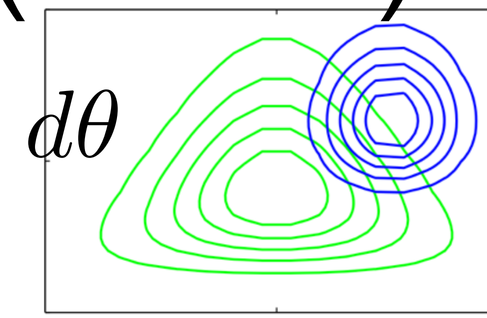

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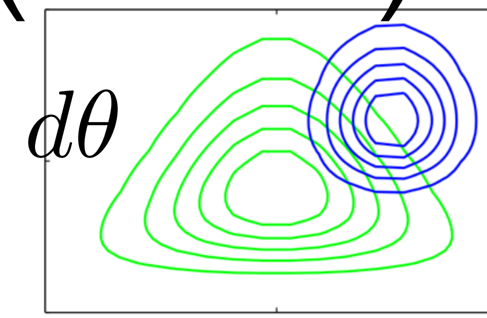

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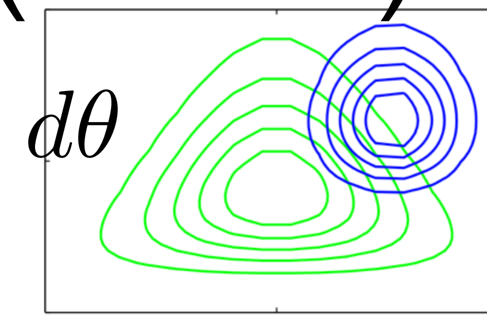

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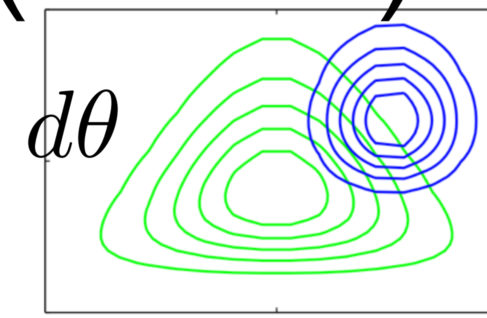

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- When can we trust MFVB?
- Where do we go from here?

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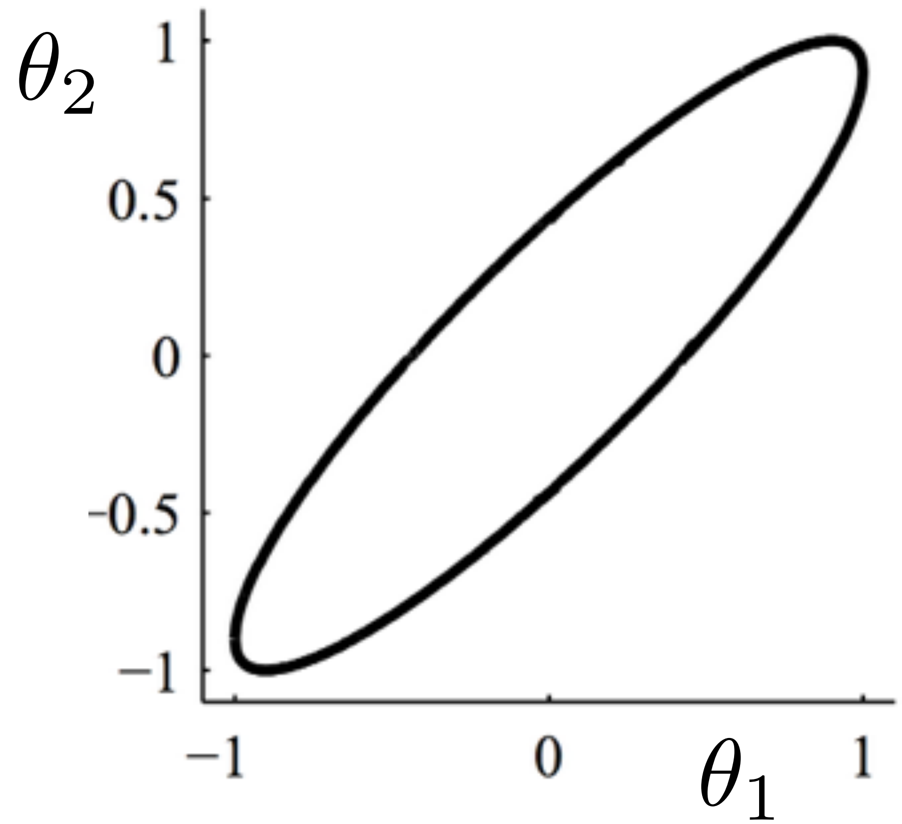

[Turner & Sahani 2011]

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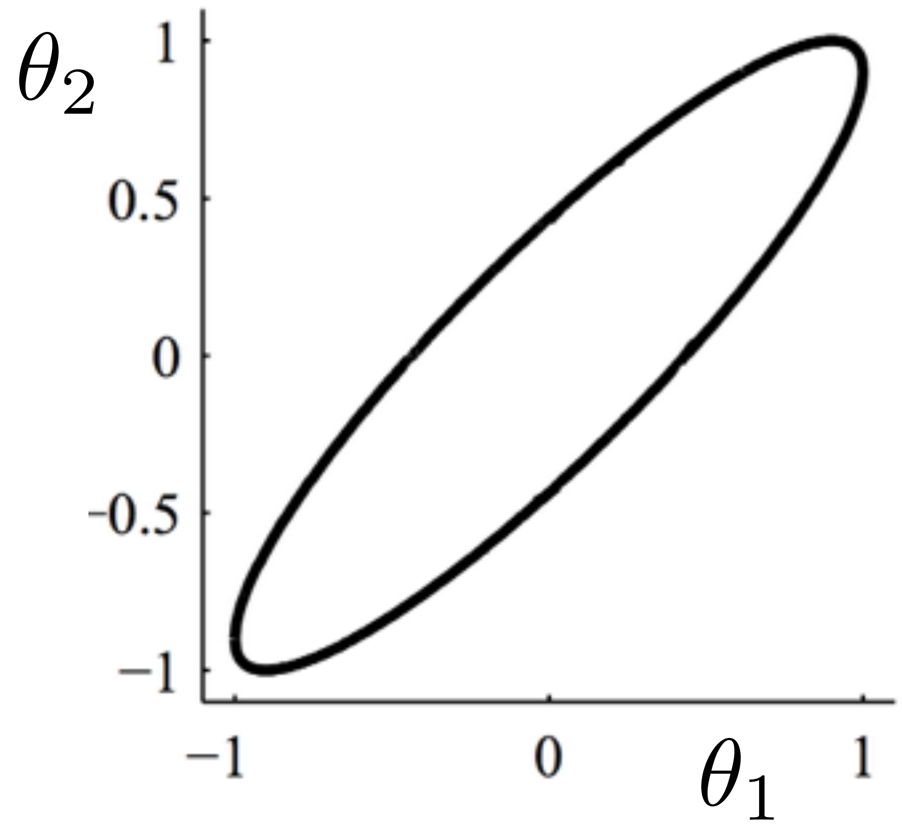

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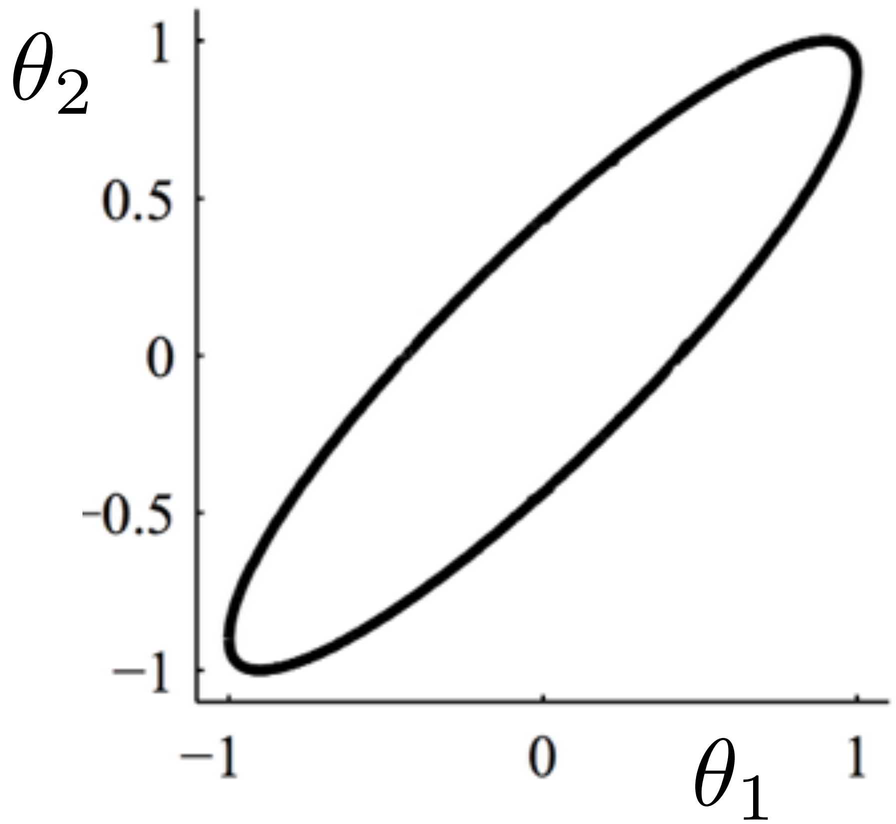

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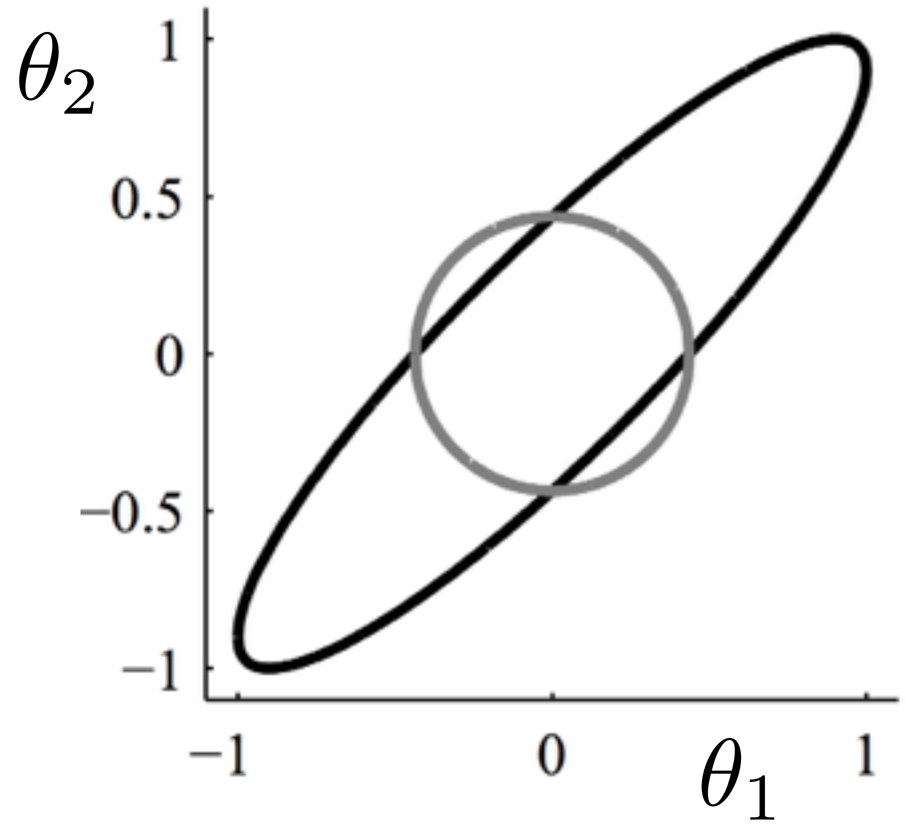

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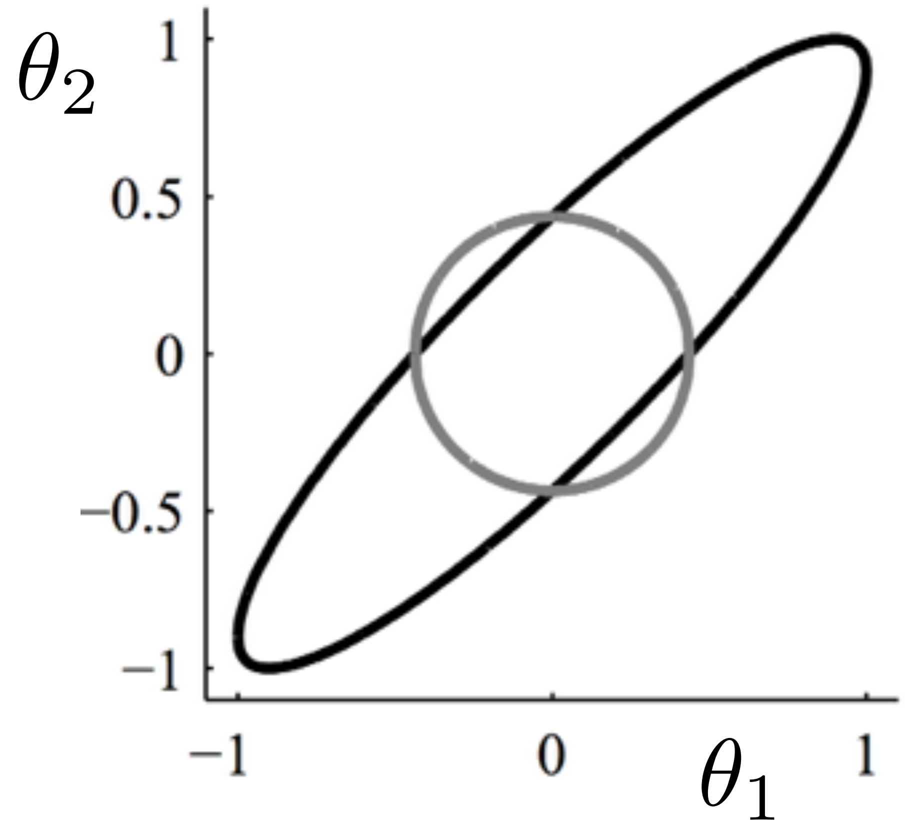

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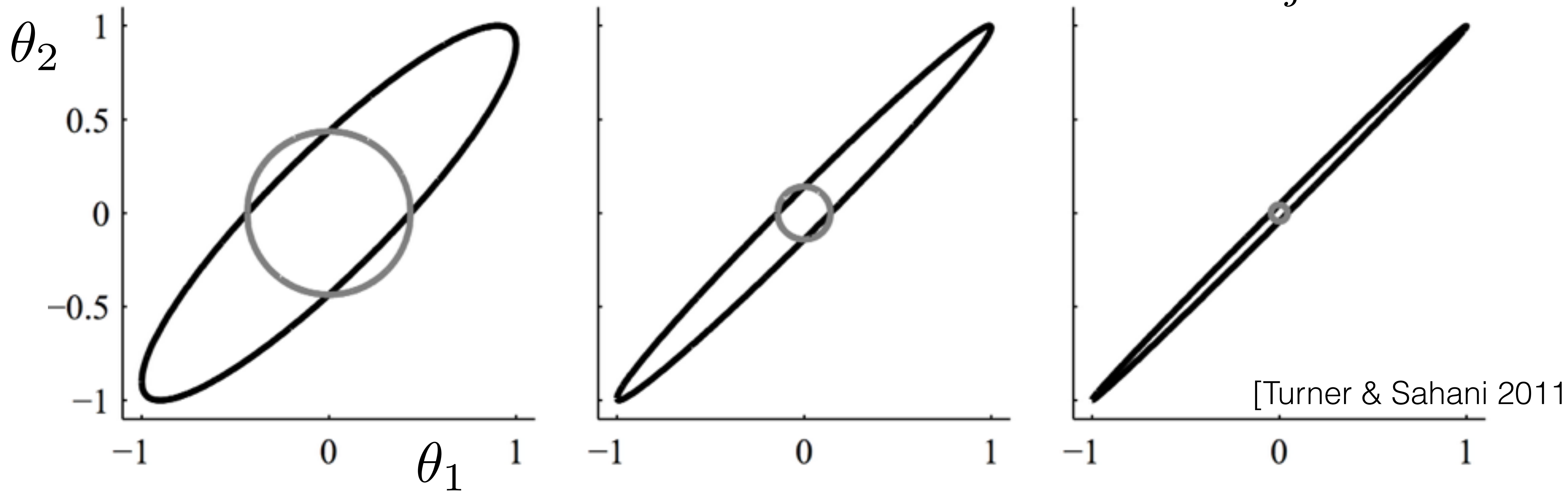

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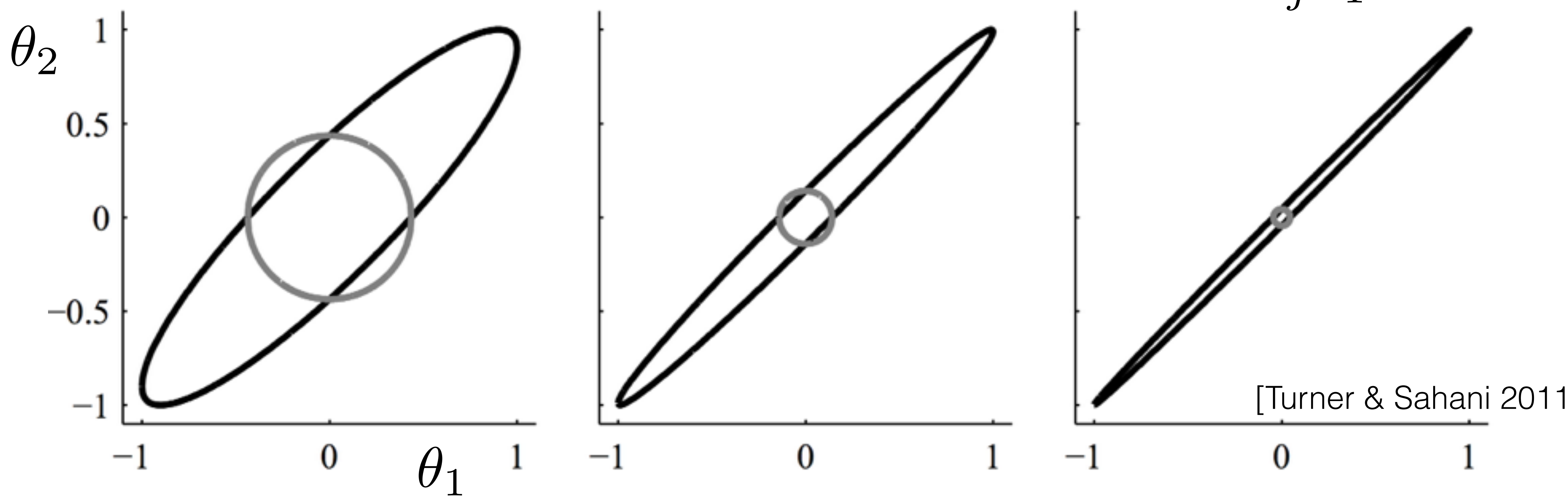

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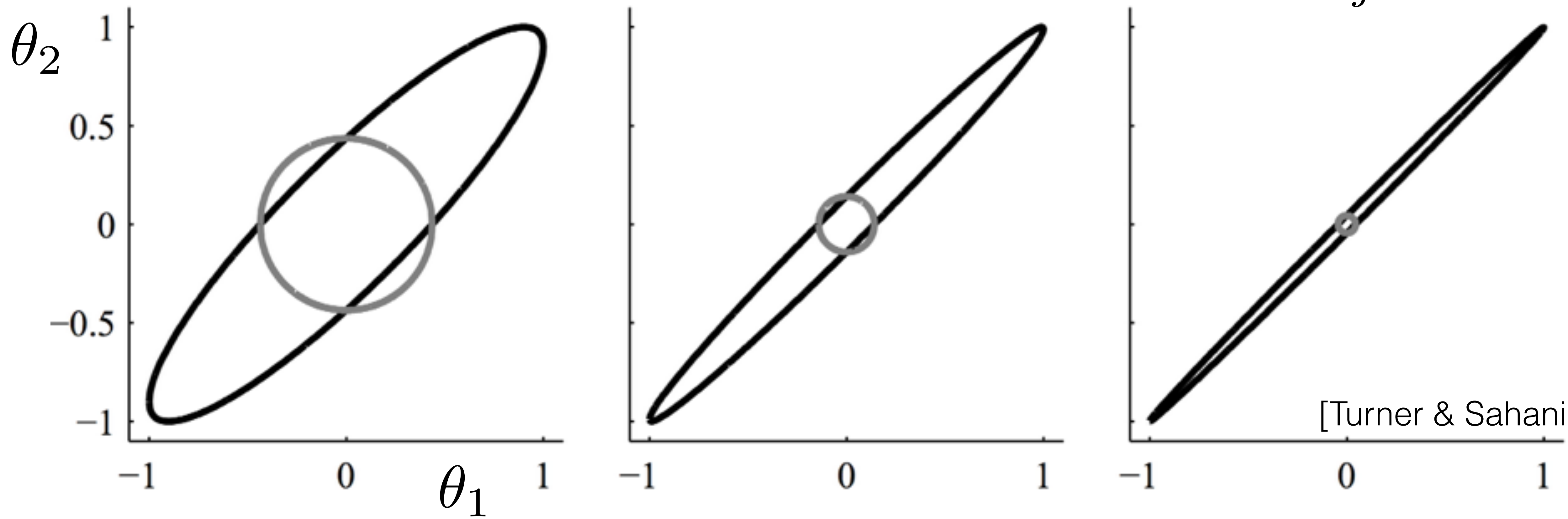

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- Microcredit

[Giordano, Broderick, Meager, Huggins, Jordan 2016]

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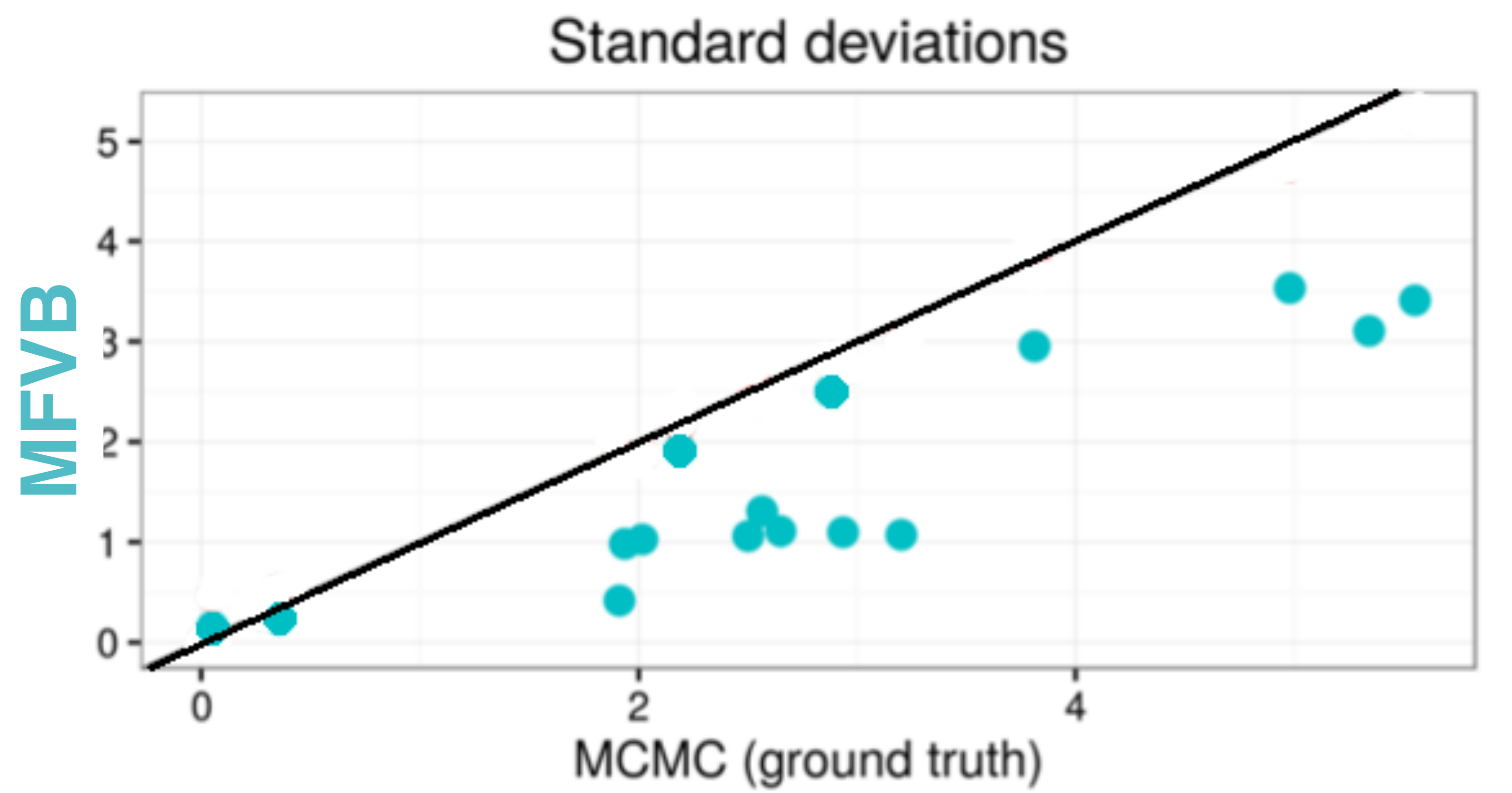

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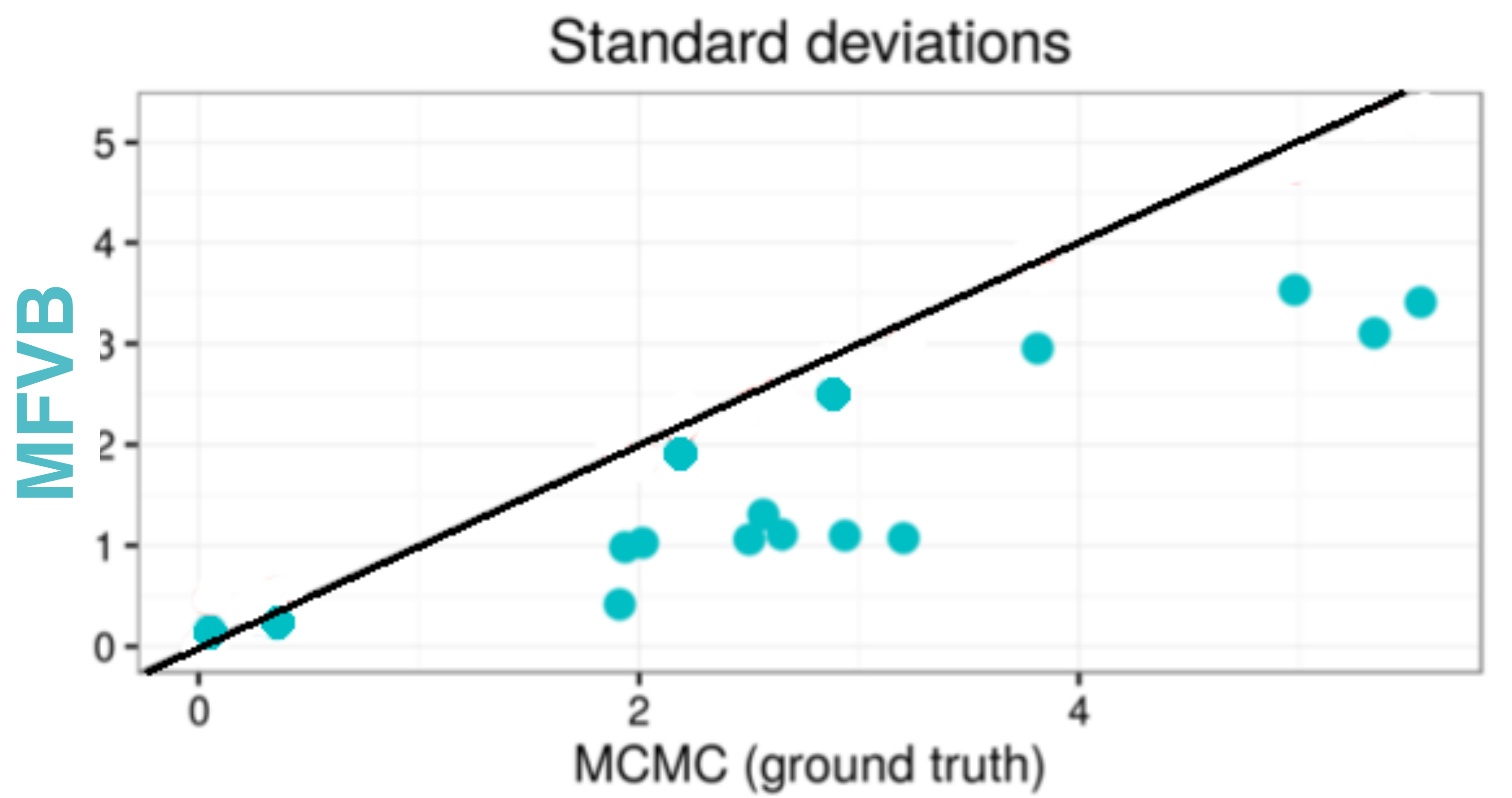

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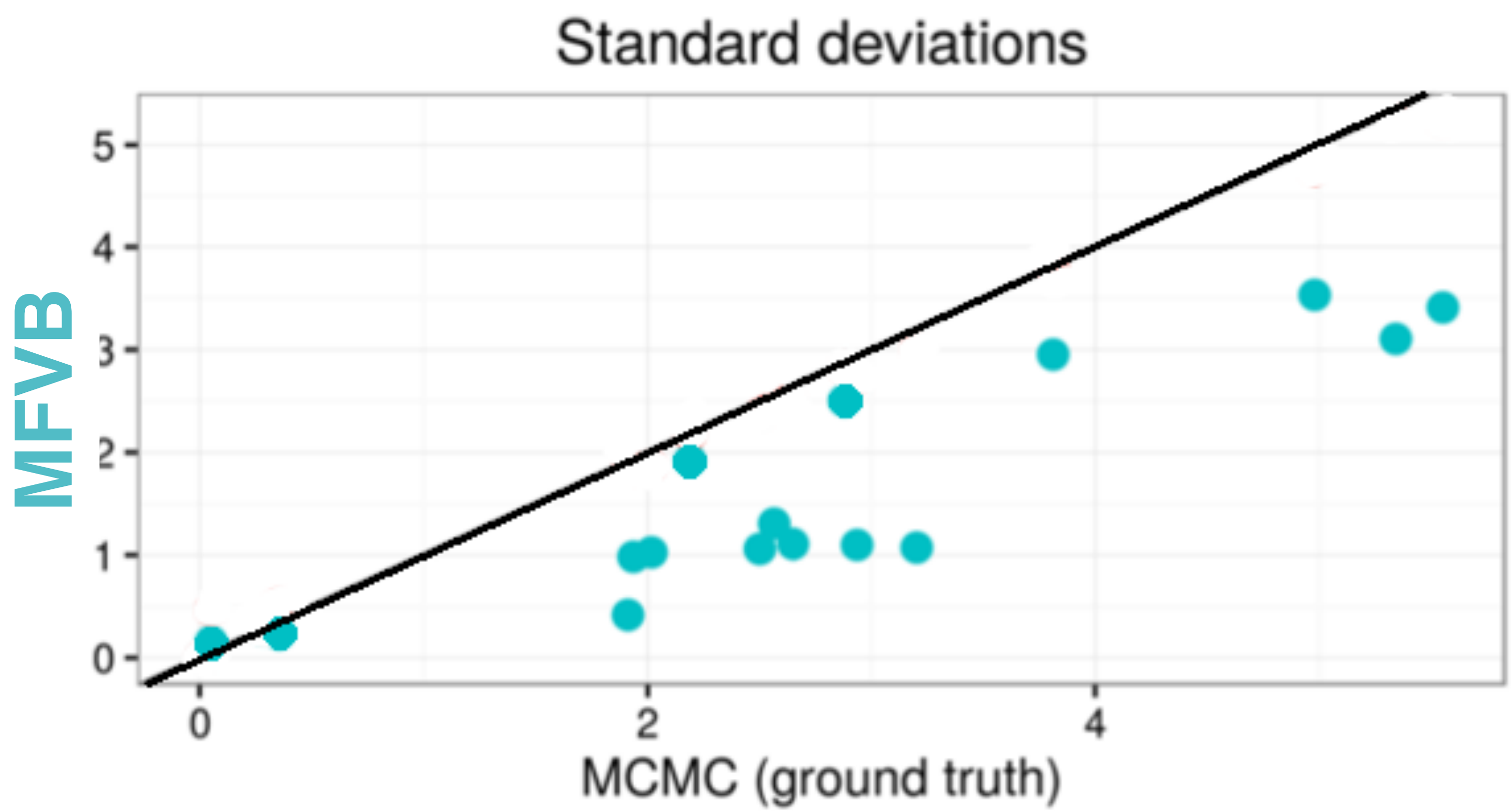

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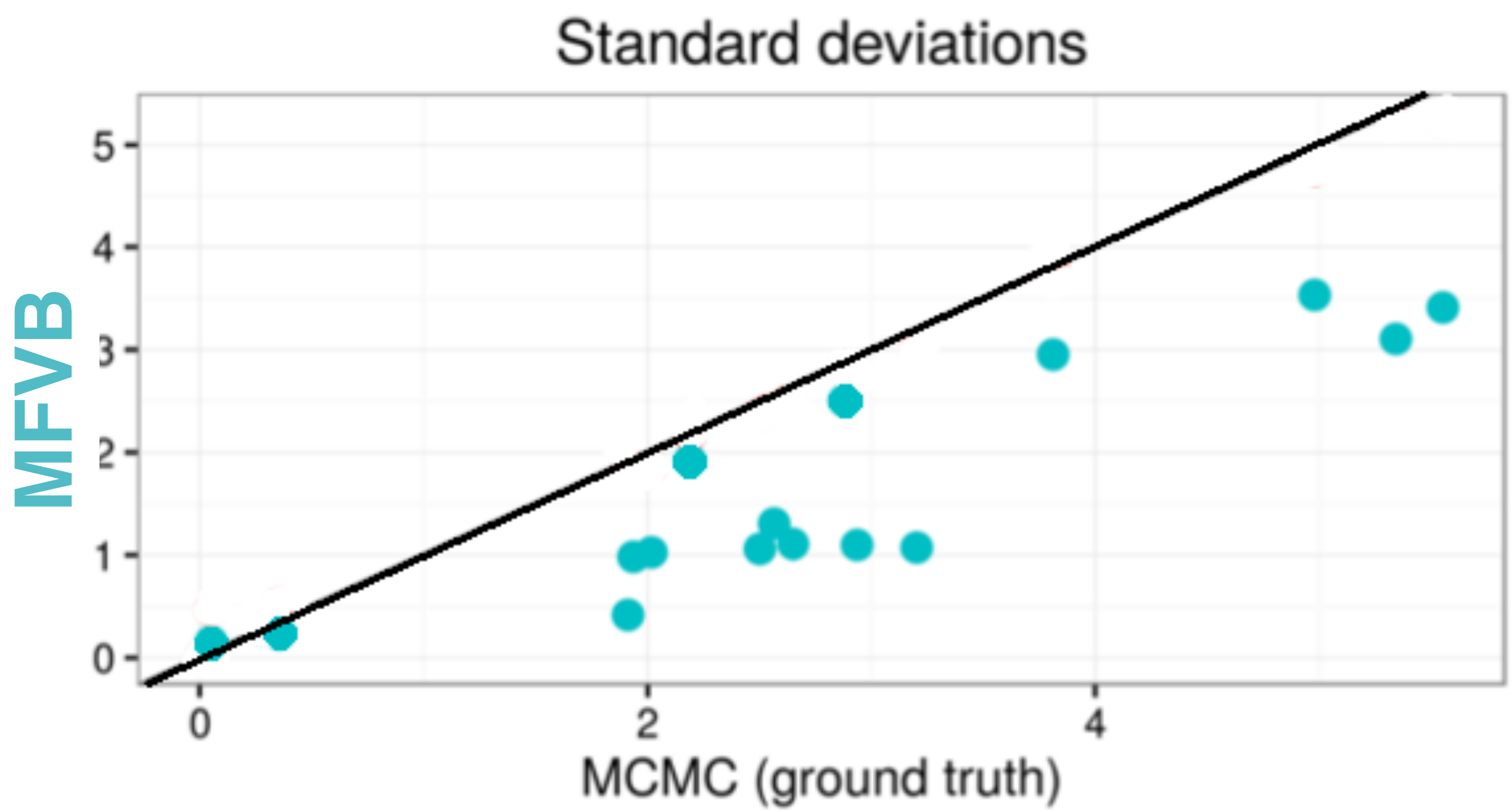

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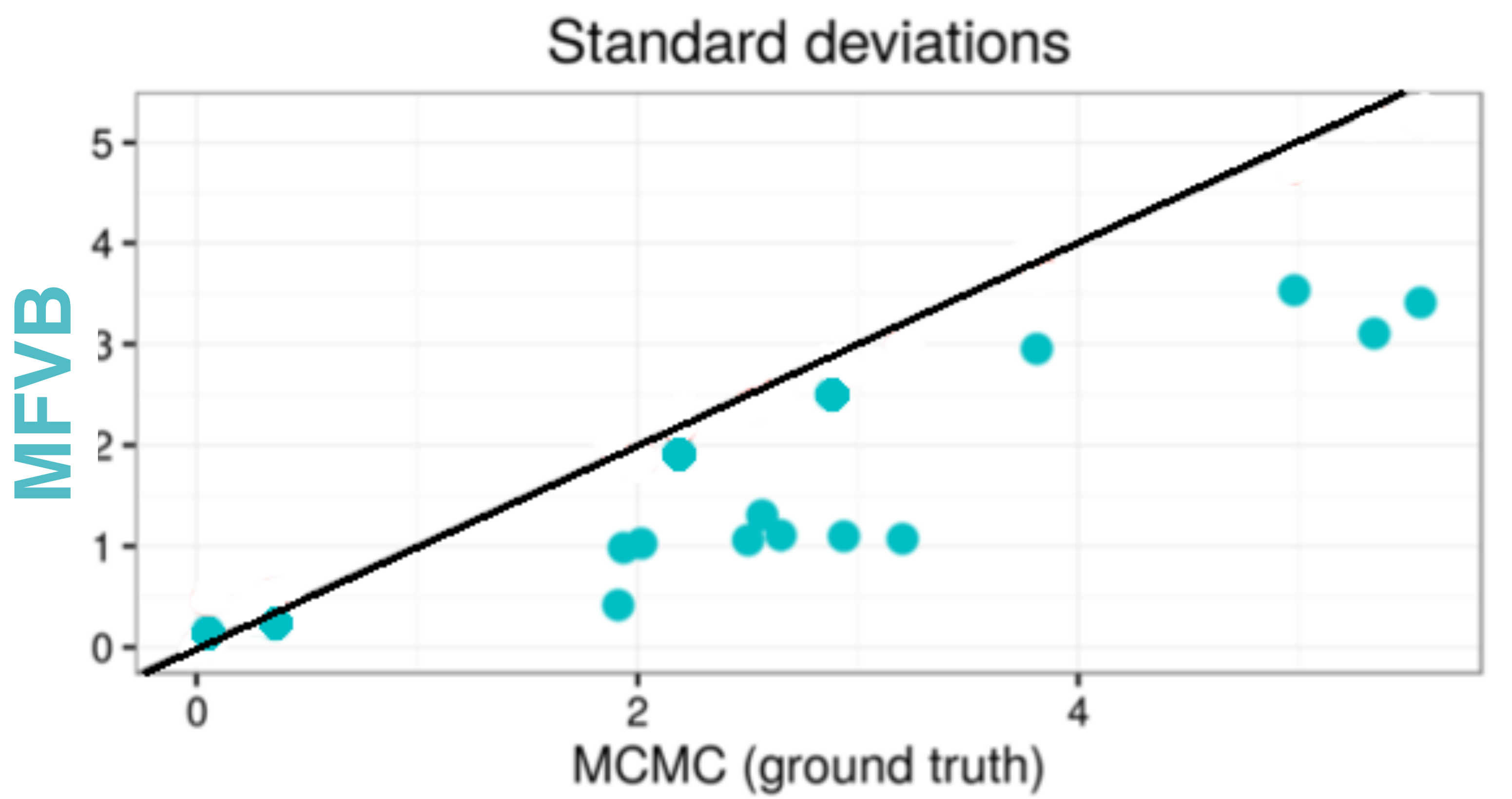
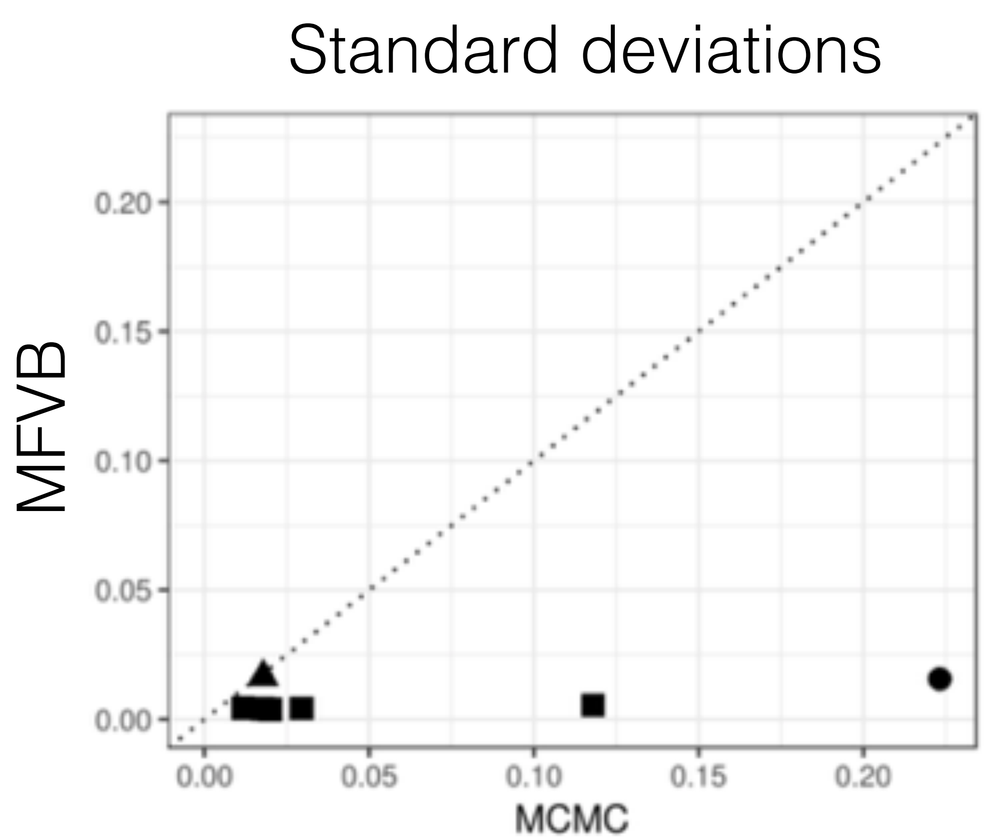


- Criteo online ads experiment



[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2017]

Posterior means: revisited

- Want to predict college GPA y_n

[Giordano, Broderick, Jordan 2015]

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- Model: $y_n | \beta, z, \sigma^2 \overset{indep}{\sim} \mathcal{N}(\beta^T x_n + z_{k(n)} r_n, \sigma^2)$

$$z_k | \rho^2 \overset{iid}{\sim} \mathcal{N}(0, \rho^2) \qquad (\sigma^2)^{-1} \sim \text{Gamma}(a_{\sigma^2}, b_{\sigma^2})$$

$$\beta \sim \mathcal{N}(0, \Sigma) \qquad (\rho^2)^{-1} \sim \text{Gamma}(a_{\rho^2}, b_{\rho^2})$$

[Giordano, Broderick, Jordan 2015]

Posterior means: revisited

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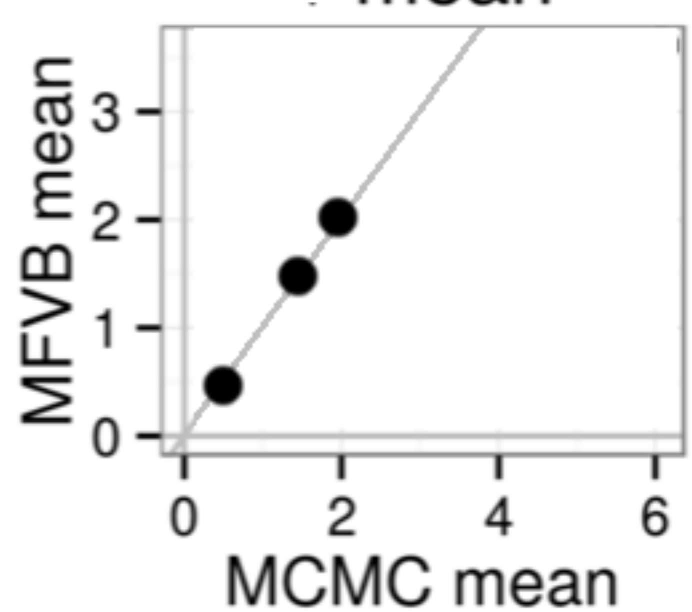
- Data simulated from model (3 data sets, 300 data points):

$(\rho^2)^{-1}$ mean



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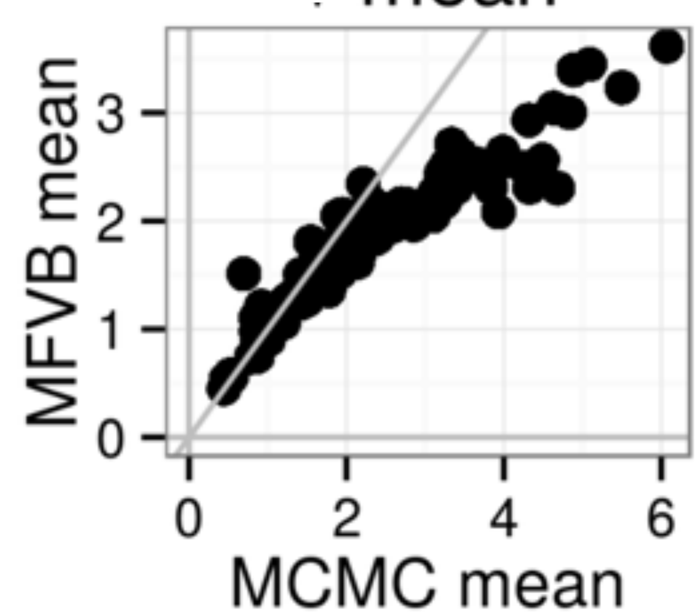
- Data simulated from model (100 data sets, 300 data points):

$(\rho^2)^{-1}$ mean



[Giordano, Broderick, Jordan 2015]

What can we do?

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- Data summarization for scalability (Part III)

  [Campbell, Broderick 2017, 2018]

  [Huggins, Campbell, Broderick 2016; Huggins, Adams, Broderick 2017]

References (1/6)

R Bardenet, A Doucet, and C Holmes. "On Markov chain Monte Carlo methods for tall data." *The Journal of Machine Learning Research* 18.1 (2017): 1515-1557.

AG Baydin, BA Pearlmutter, AA Radul, and JM Siskind. "Automatic differentiation in machine learning: a survey." ArXiv:1502.05767v4 (2018).

DM Blei, A Kucukelbir, and JD McAuliffe. "Variational inference: A review for statisticians." *Journal of the American Statistical Association* 112.518 (2017): 859-877.

T Broderick, N Boyd, A Wibisono, AC Wilson, and MI Jordan. Streaming variational Bayes. *NIPS* 2013.

CM Bishop. *Pattern Recognition and Machine Learning*. Springer-Verlag New York, 2006.

T Campbell and T Broderick. Automated scalable Bayesian inference via Hilbert coresets. Under review. ArXiv:1710.05053.

T Campbell and T Broderick. Bayesian Coreset Construction via Greedy Iterative Geodesic Ascent. *ICML* 2018, to appear.

RJ Giordano, T Broderick, and MI Jordan. "Linear response methods for accurate covariance estimates from mean field variational Bayes." *NIPS* 2015.

R Giordano, T Broderick, R Meager, J Huggins, and MI Jordan. "Fast robustness quantification with variational Bayes." *ICML 2016 Workshop on #Data4Good: Machine Learning in Social Good Applications*, 2016.

R Giordano, T Broderick, and MI Jordan. Covariances, robustness, and variational Bayes, 2017. Under review. ArXiv:1709.02536.

References (2/6)

J Gorham and L Mackey. "Measuring sample quality with Stein's method." *NIPS* 2015.

J Gorham, and L Mackey. "Measuring sample quality with kernels." ArXiv:1703.01717 (2017).

PD Hoff. *A first course in Bayesian statistical methods*. Springer Science & Business Media, 2009.

MD Hoffman, DM Blei, C Wang, and J Paisley. "Stochastic variational inference." *The Journal of Machine Learning Research* 14.1 (2013): 1303-1347.

JH Huggins, T Campbell, and T Broderick. Coresets for scalable Bayesian logistic regression. *NIPS* 2016.

JH Huggins, RP Adams, and T Broderick. PASS-GLM: Polynomial approximate sufficient statistics for scalable Bayesian GLM inference. *NIPS* 2017.

A Kucukelbir, R Ranganath, A Gelman, and D Blei. "Automatic variational inference in Stan." *NIPS* 2015.

A Kucukelbir, D Tran, R Ranganath, A Gelman, and DM Blei. "Automatic differentiation variational inference." *The Journal of Machine Learning Research* 18.1 (2017): 430-474.

DJC MacKay. *Information Theory, Inference, and Learning Algorithms*. Cambridge University Press, 2003.

Stan (open source software). http://mc-stan.org/ Accessed: 2018.

S Talts, M Betancourt, D Simpson, A Vehtari, and A Gelman. "Validating Bayesian Inference Algorithms with Simulation-Based Calibration." aArXiv:1804.06788 (2018).

References (3/6)

RE Turner and M Sahani. Two problems with variational expectation maximisation for time-series models. In D Barber, AT Cemgil, and S Chiappa, editors, *Bayesian Time Series Models*, 2011.

Y Yao, A Vehtari, D Simpson, and A Gelman. "Yes, but Did It Work?: Evaluating Variational Inference." ArXiv:1802.02538 (2018).

Application References (4/6)

Abbott, Benjamin P., et al. "Observation of gravitational waves from a binary black hole merger." *Physical Review Letters* 116.6 (2016): 061102.

Abbott, Benjamin P., et al. "The rate of binary black hole mergers inferred from advanced LIGO observations surrounding GW150914." *The Astrophysical Journal Letters* 833.1 (2016): L1.

Airoldi, Edoardo M., David M. Blei, Stephen E. Fienberg, and Eric P. Xing. "Mixed membership stochastic blockmodels." *Journal of Machine Learning Research* 9.Sep (2008): 1981-2014.

Blei, David M., Andrew Y. Ng, and Michael I. Jordan. "Latent Dirichlet allocation." *Journal of Machine Learning Research* 3.Jan (2003): 993-1022.

Chati, Yashovardhan Sushil, and Hamsa Balakrishnan. "A Gaussian process regression approach to model aircraft engine fuel flow rate." *Cyber-Physical Systems (ICCPS), 2017 ACM/IEEE 8th International Conference on*. IEEE, 2017.

Gershman, Samuel J., David M. Blei, Kenneth A. Norman, and Per B. Sederberg. "Decomposing spatiotemporal brain patterns into topographic latent sources." NeuroImage 98 (2014): 91-102.

Gillon, Michaël, et al. "Seven temperate terrestrial planets around the nearby ultracool dwarf star TRAPPIST-1." *Nature* 542.7642 (2017): 456.

Grimm, Simon L., et al. "The nature of the TRAPPIST-1 exoplanets." *Astronomy & Astrophysics* 613 (2018): A68.

Application References (5/6)

Grogan Jr, William L., and Willis W. Wirth. "A new American genus of predaceous midges related to Palpomyia and Bezzia (Diptera: Ceratopogonidae). Un nuevo género Americano de purrujas depredadoras relacionadas con Palpomyia y Bezzia (Diptera: Ceratopogonidae)." *Proceedings of the Biological Society of Washington*. 94.4 (1981): 1279-1305.

Kuikka, Sakari, Jarno Vanhatalo, Henni Pulkkinen, Samu Mäntyniemi, and Jukka Corander. "Experiences in Bayesian inference in Baltic salmon management." *Statistical Science* 29.1 (2014): 42-49.

Meager, Rachael. "Understanding the impact of microcredit expansions: A Bayesian hierarchical analysis of 7 randomized experiments." *AEJ: Applied*, to appear, 2018a.

Meager, Rachael. "Aggregating Distributional Treatment Effects: A Bayesian Hierarchical Analysis of the Microcredit Literature." Working paper, 2018b.

Stegle, Oliver, Leopold Parts, Richard Durbin, and John Winn. "A Bayesian framework to account for complex non-genetic factors in gene expression levels greatly increases power in eQTL studies." PLoS computational biology 6.5 (2010): e1000770.

Stone, Lawrence D., Colleen M. Keller, Thomas M. Kratzke, and Johan P. Strumpfer. "Search for the wreckage of Air France Flight AF 447." *Statistical Science* (2014): 69-80.

Woodard, Dawn, Galina Nogin, Paul Koch, David Racz, Moises Goldszmidt, and Eric Horvitz. "Predicting travel time reliability using mobile phone GPS data." *Transportation Research Part C: Emerging Technologies* 75 (2017): 30-44.

Xing, Eric P., Wei Wu, Michael I. Jordan, and Richard M. Karp. "LOGOS: a modular Bayesian model for de novo motif detection." Journal of Bioinformatics and Computational Biology 2.01 (2004): 127-154.

Additional image references (6/6)

amCharts. Visited Countries Map. https://www.amcharts.com/visited_countries/ Accessed: 2016.

Baltic Salmon Fund. https://www.en.balticsalmonfund.org/about_us Accessed: 2018.

ESO/L. Calçada/M. Kornmesser. 16 October 2017, 16:00:00. Obtained from: https://commons.wikimedia.org/wiki/File:Artist%E2%80%99s_impression_of_merging_neutron_stars.jpg || Source: https://www.eso.org/public/images/eso1733a/ (Creative Commons Attribution 4.0 International License)

J. Herzog. 3 June 2016, 17:17:30. Obtained from: https://commons.wikimedia.org/wiki/File:Airbus_A350-941_F-WWCF_MSN002_ILA_Berlin_2016_17.jpg (Creative Commons Attribution 4.0 International License)

E. Xing. 2003. Slides "LOGOS: a modular Bayesian model for de novo motif detection." Obtained from: https://www.cs.cmu.edu/~epxing/papers/Old_papers/slide_CSB03/CSB1.pdf Accessed: 2018.