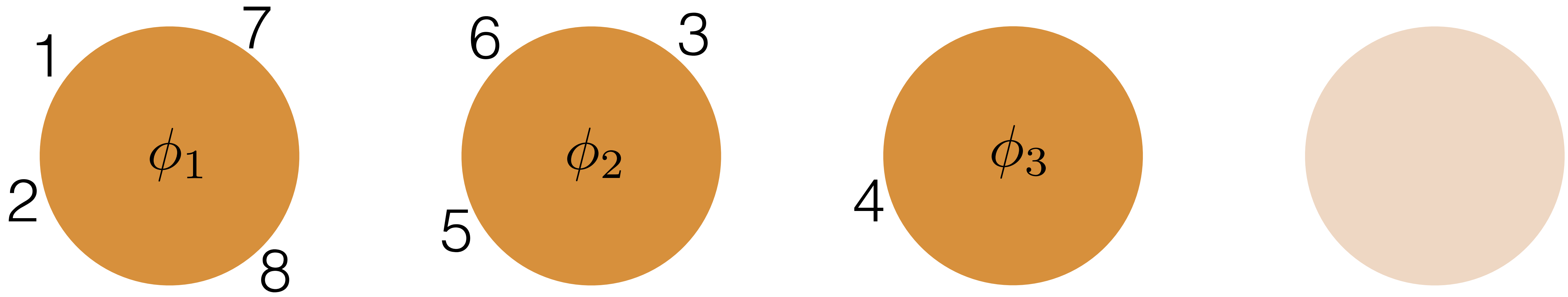
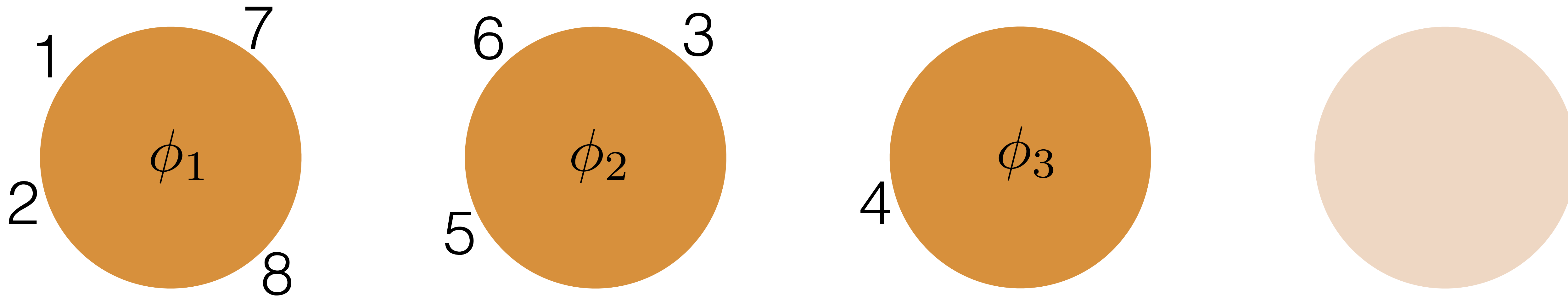
Nonparametric Bayesian Statistics: Part III

Tamara Broderick
ITT Career Development Assistant Professor
Electrical Engineering & Computer Science
MIT

Chinese restaurant process

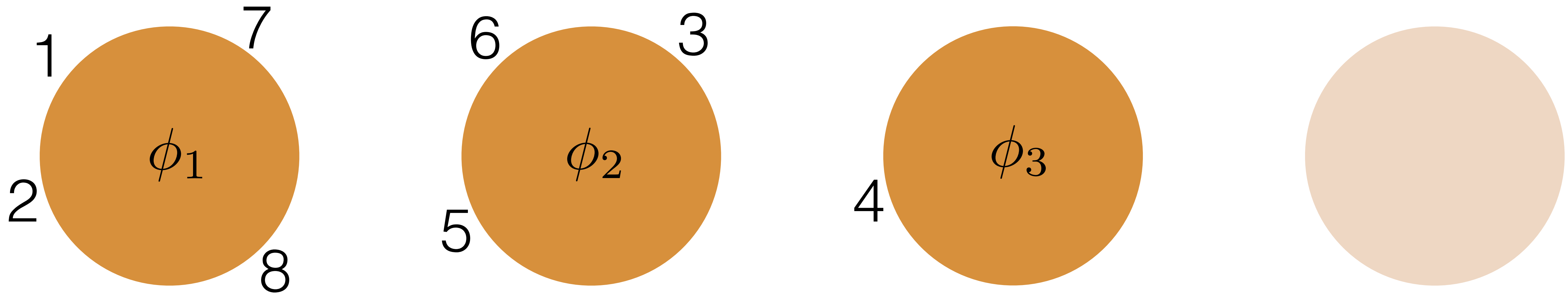


Chinese restaurant process



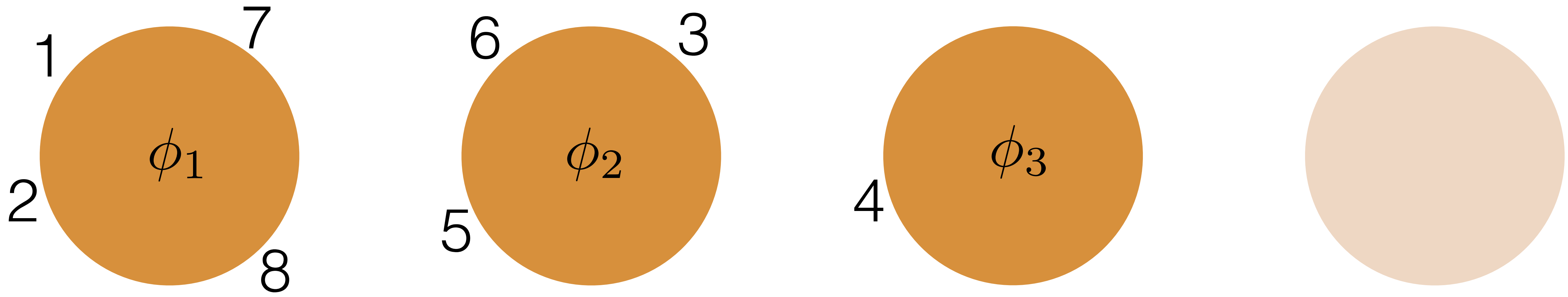
- Each customer walks into the restaurant

Chinese restaurant process



- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there

Chinese restaurant process



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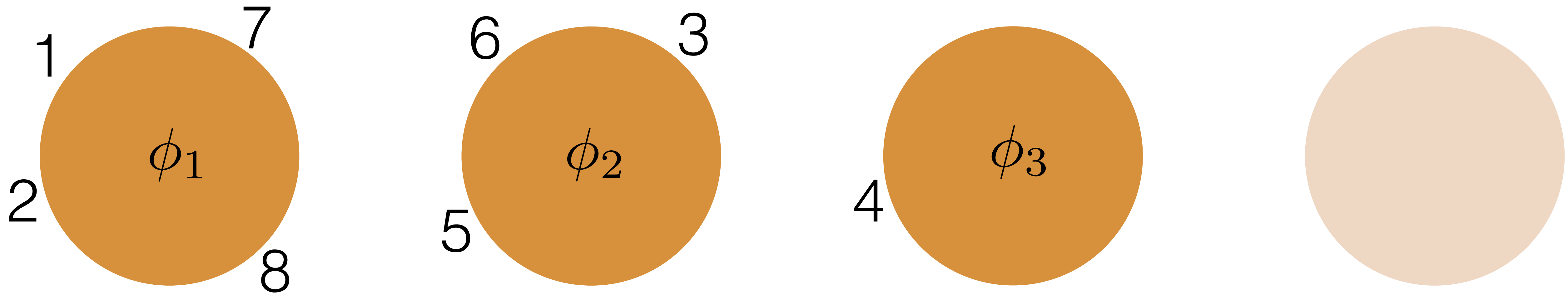

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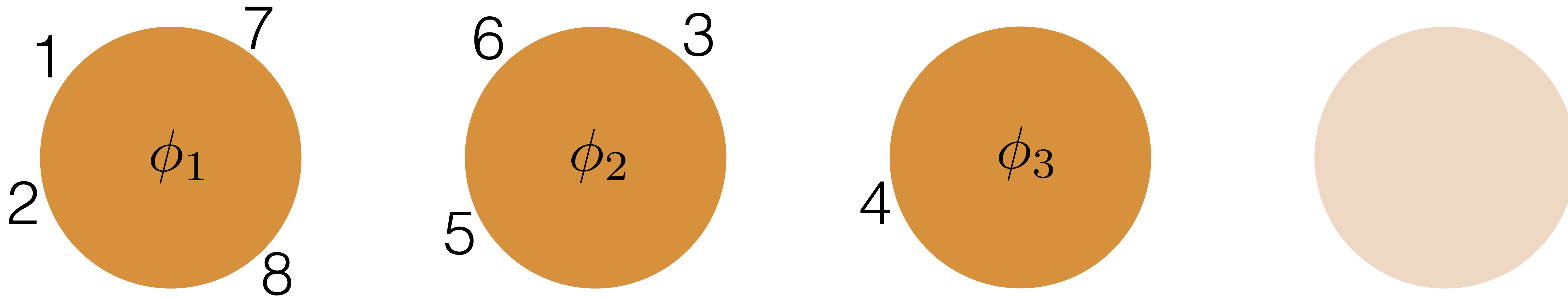
- Marginal for the Categorical likelihood with GEM prior

Chinese restaurant process



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$$z_1 = z_2 = z_7 = z_8 = 1, z_3 = z_5 = z_6 = 2, z_4 = 3$$

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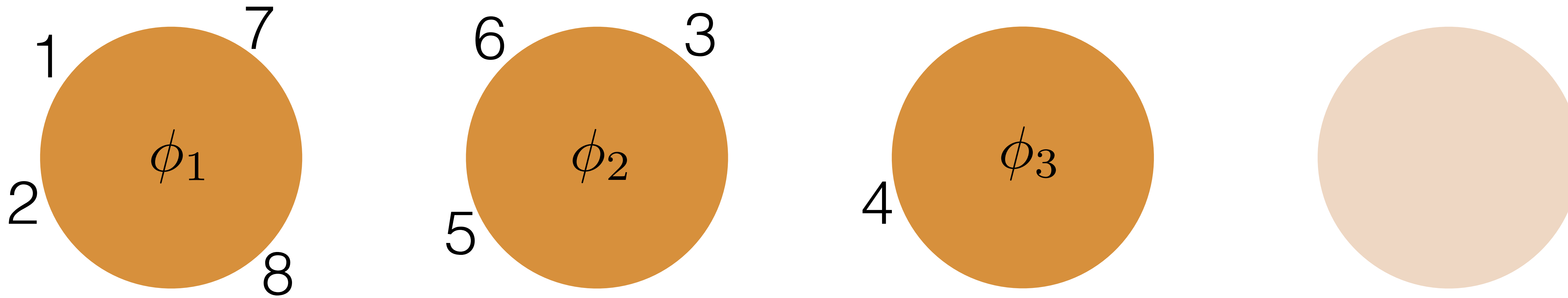

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$$\Rightarrow \Pi_8 = \{\{1, 2, 7, 8\}, \{3, 5, 6\}, \{4\}\}$$

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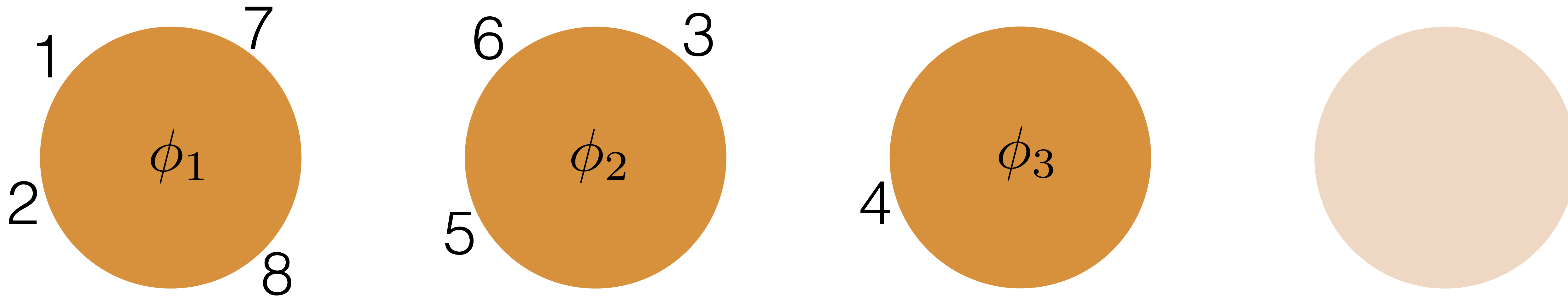

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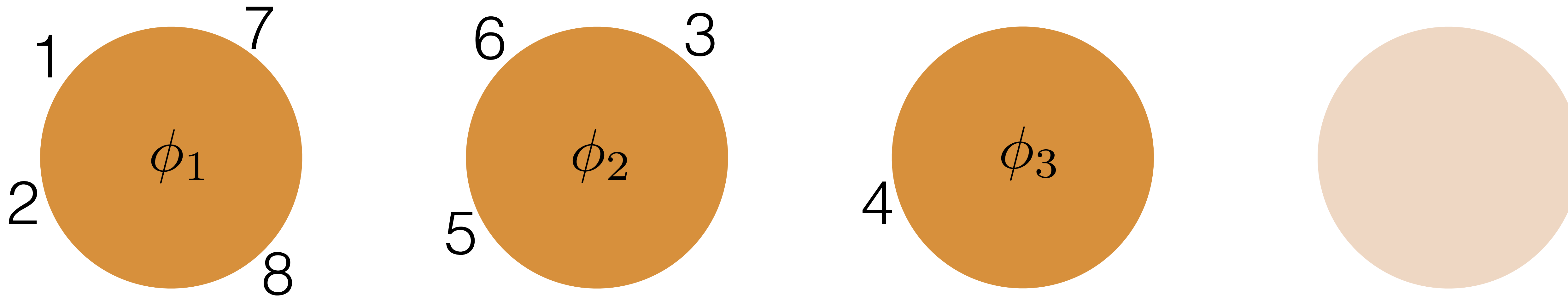
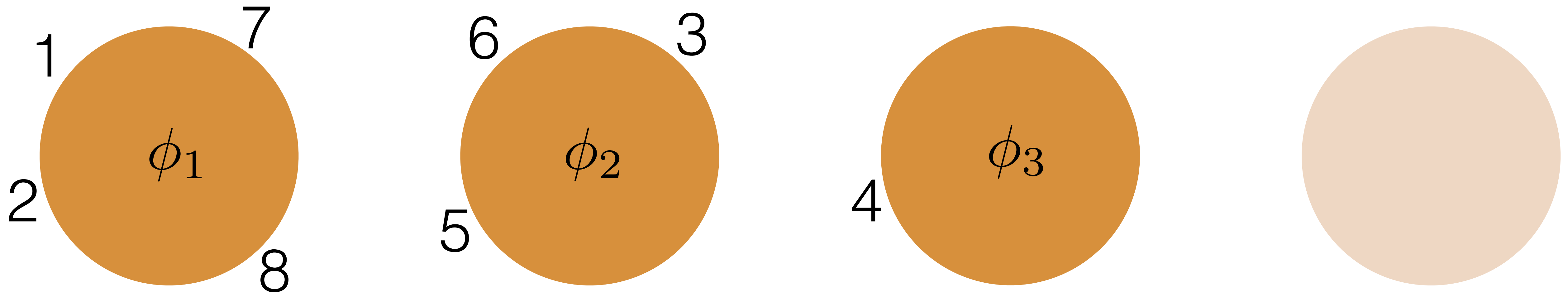
- *Partition of [8]*: set of mutually exclusive & exhaustive sets of $[8] := \{1, \ldots, 8\}$

Chinese restaurant process



- Probability of this seating:

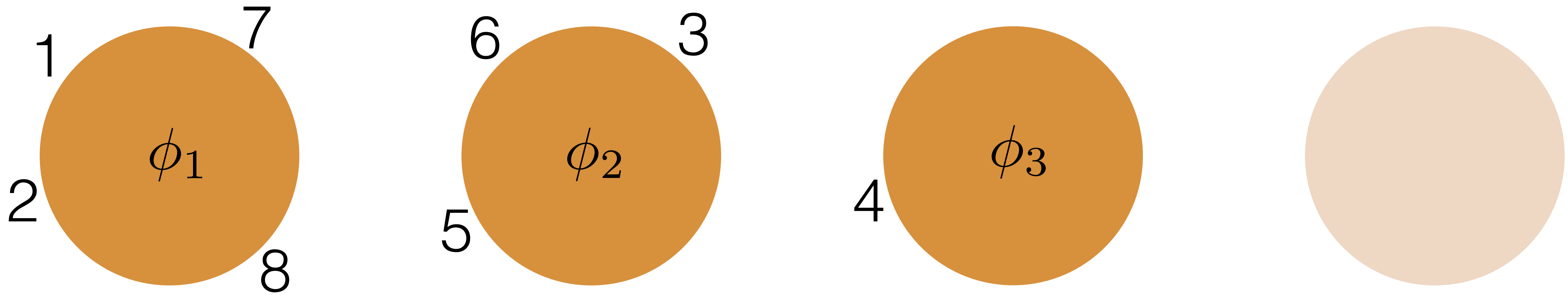
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha}$$

Chinese restaurant process



- Probability of this seating:

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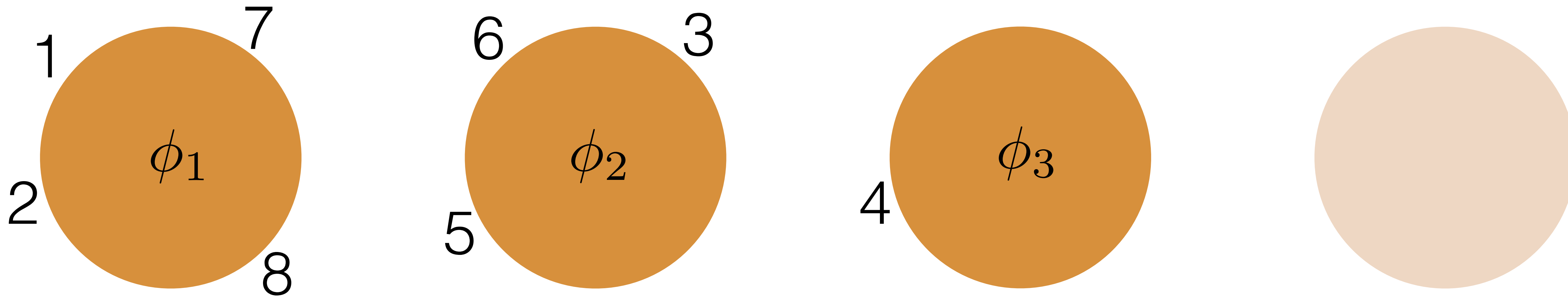
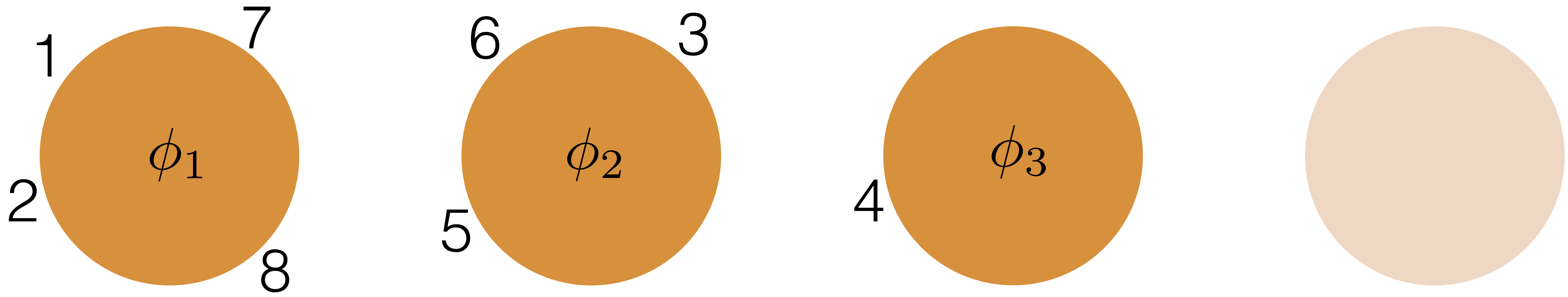
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha + 1} \cdot \frac{\alpha}{\alpha + 2}$$

Chinese restaurant process



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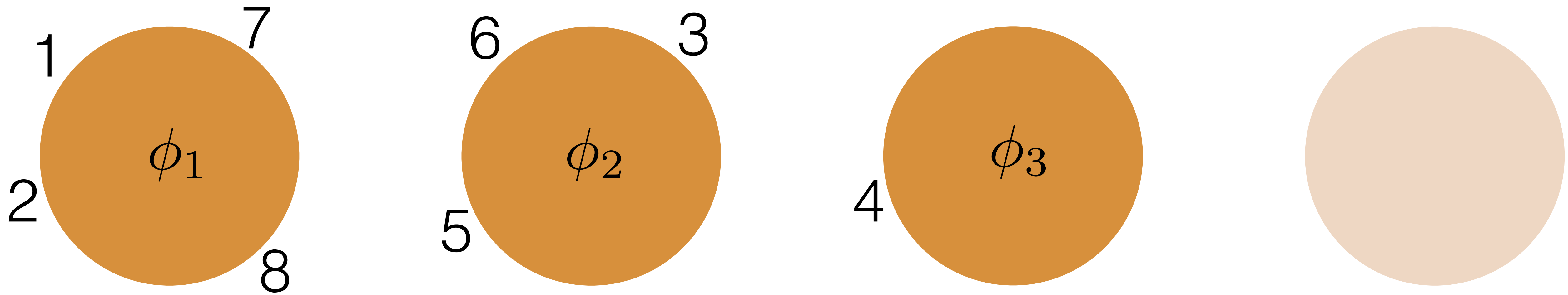
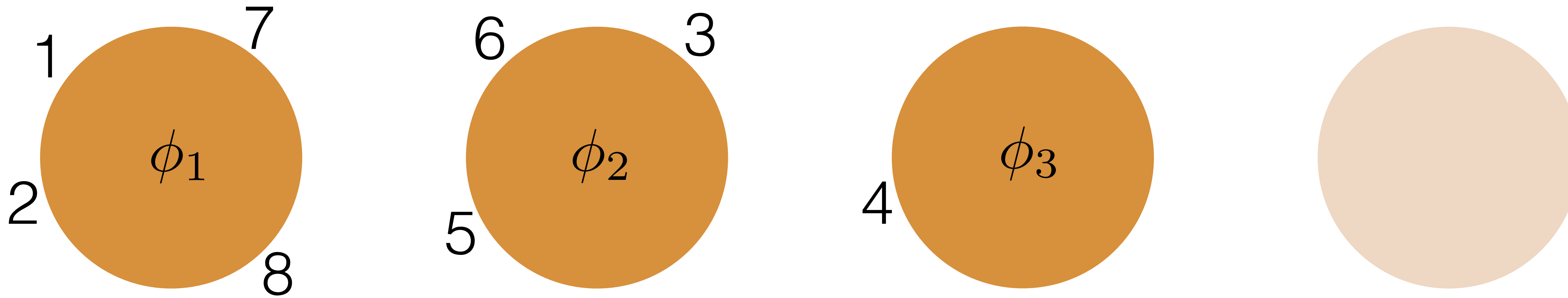


- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4}$$

Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5}$$

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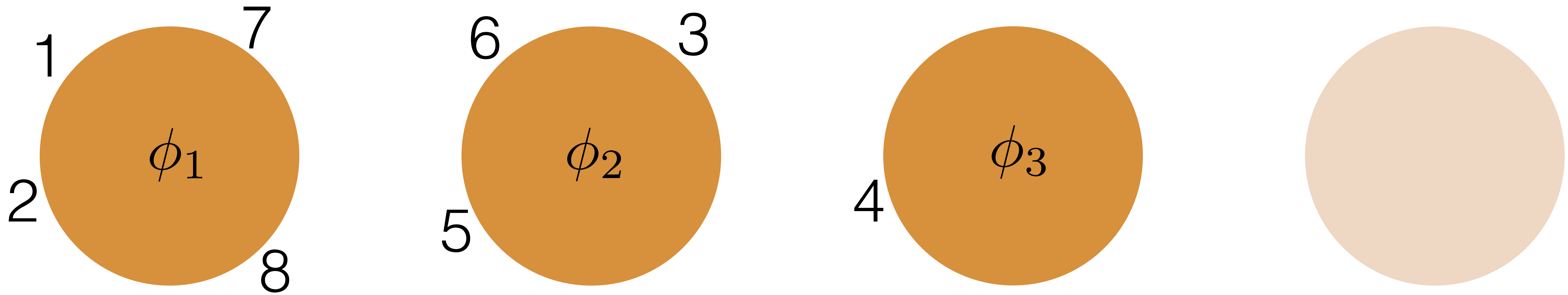

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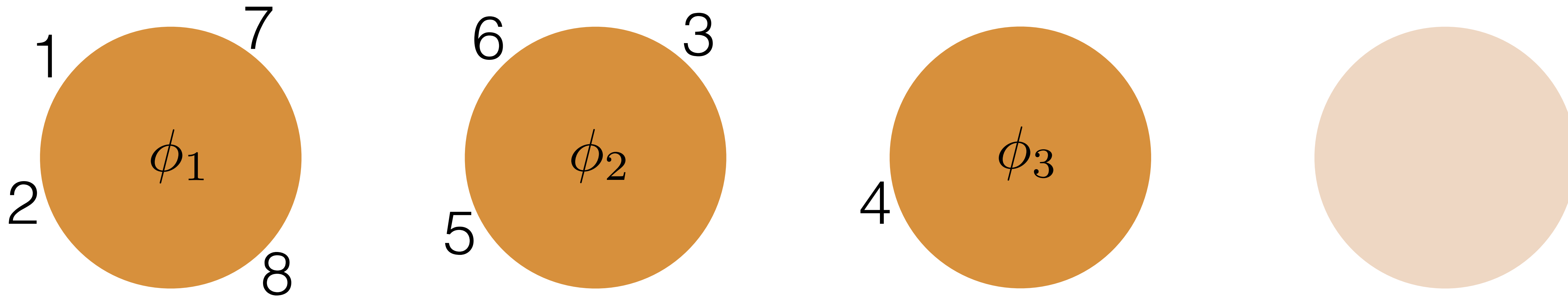

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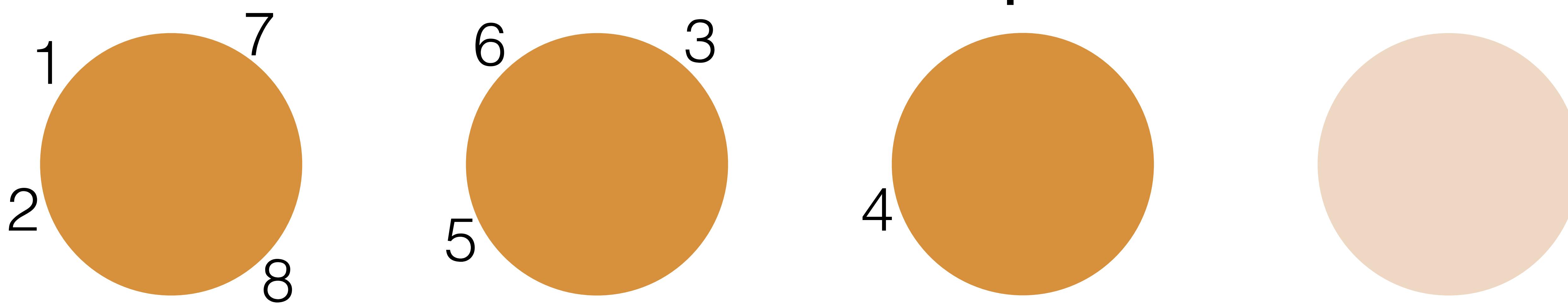

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- Probability of *N* customers (K_N tables, n_k at table *k*):

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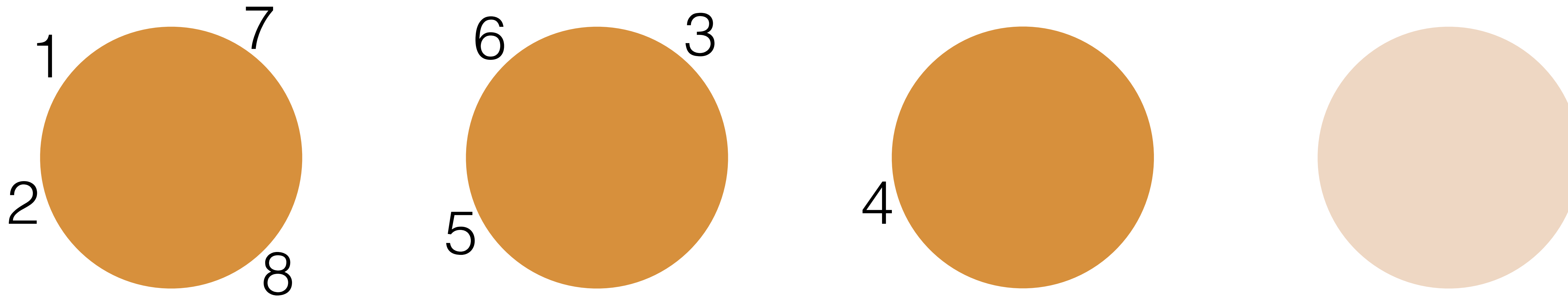

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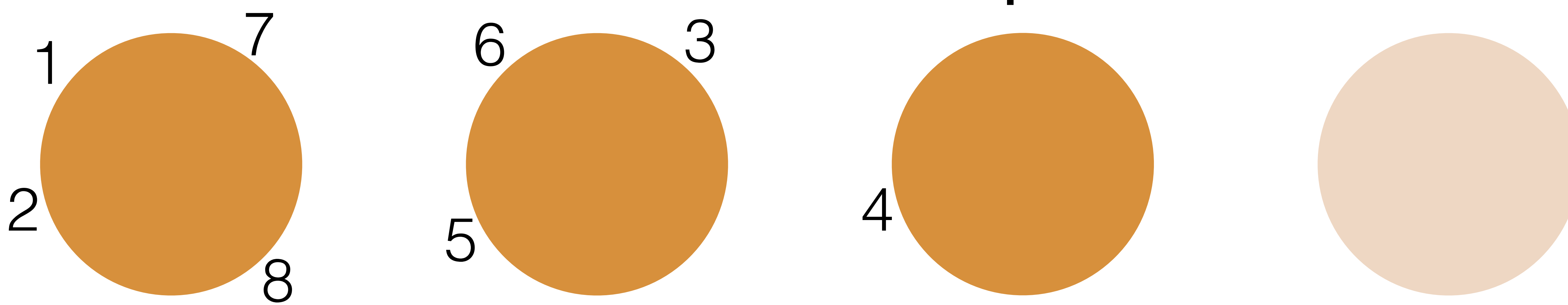

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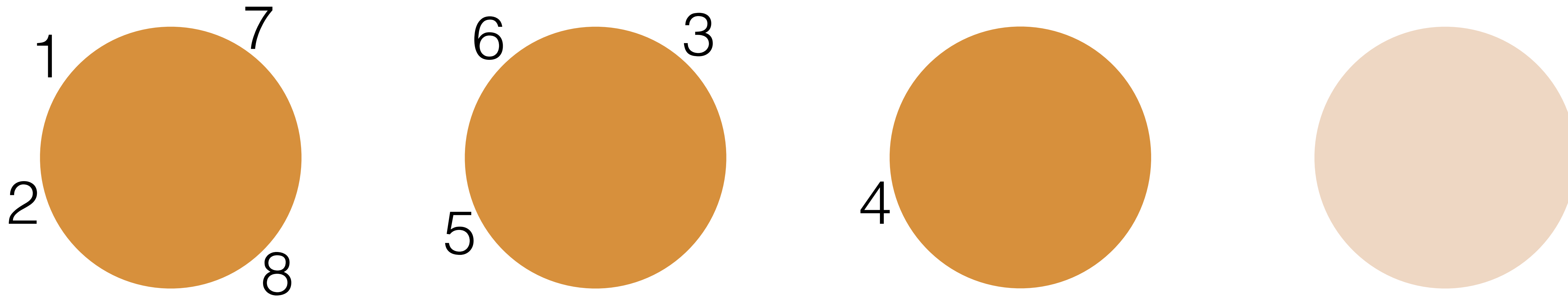

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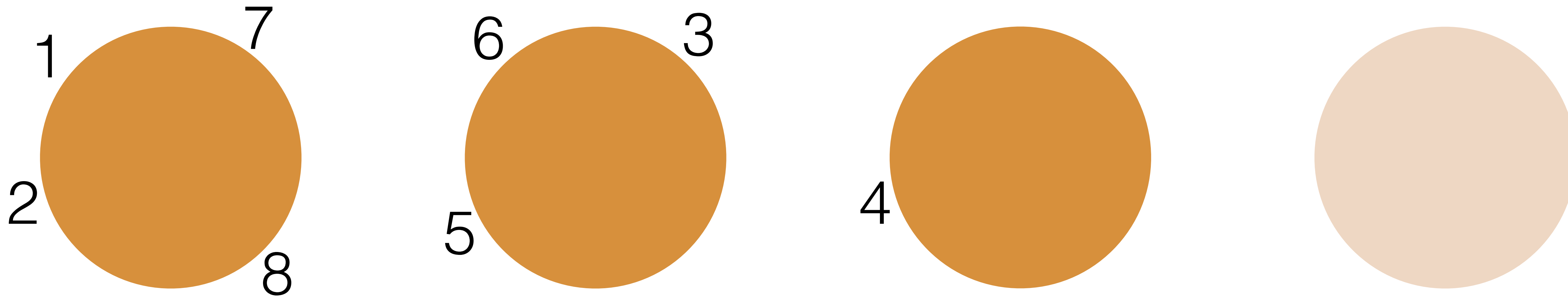

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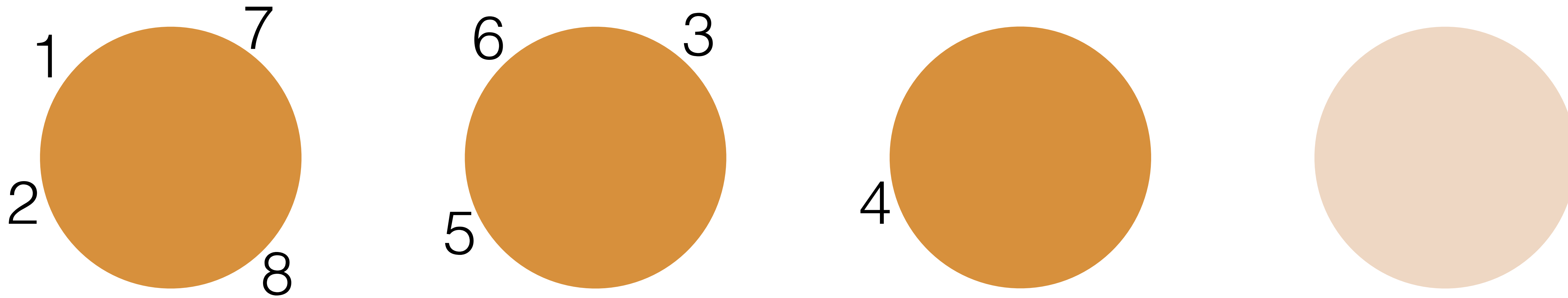

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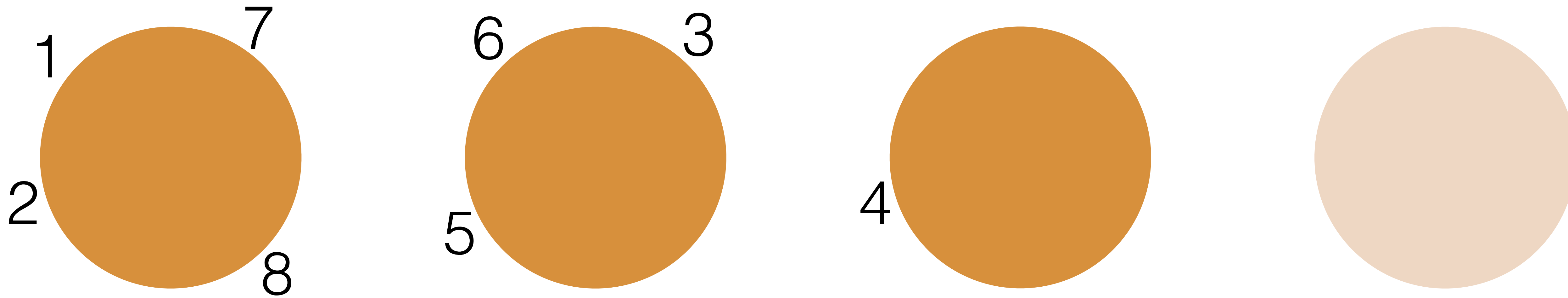

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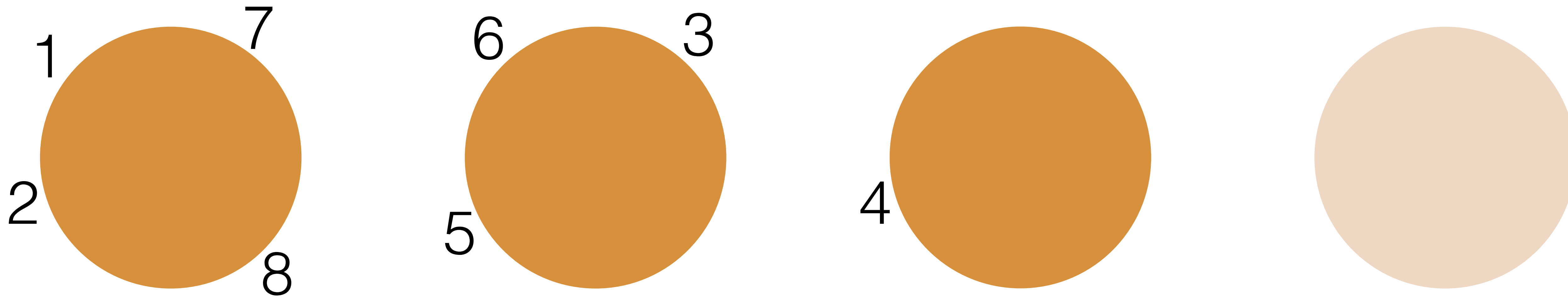

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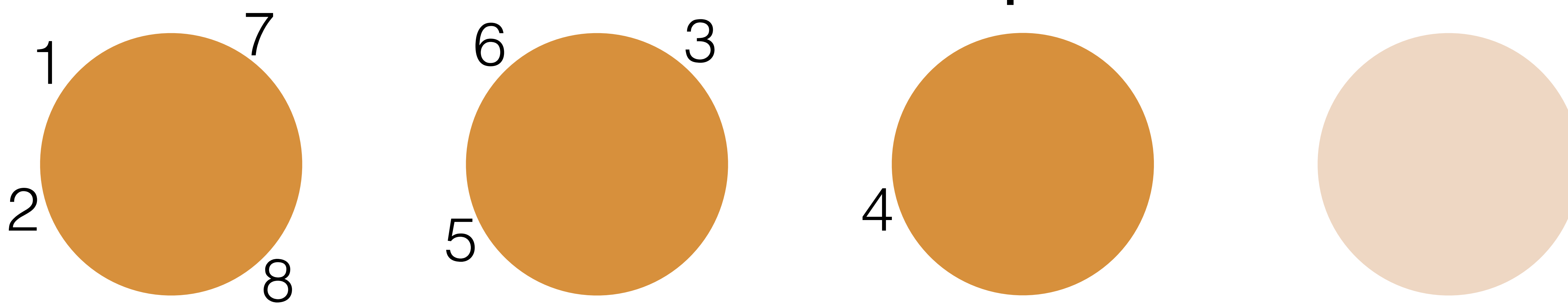

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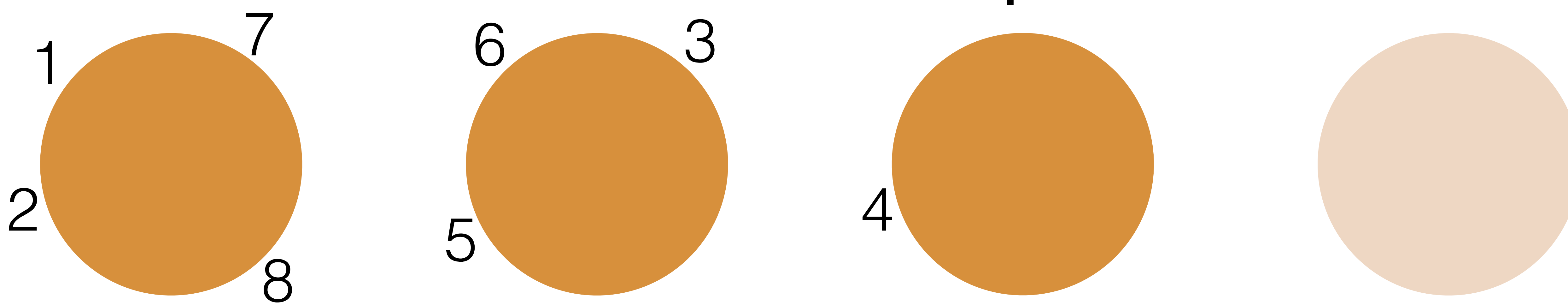

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- Probability of *N* customers (K_N tables, #*C* at table *C*):

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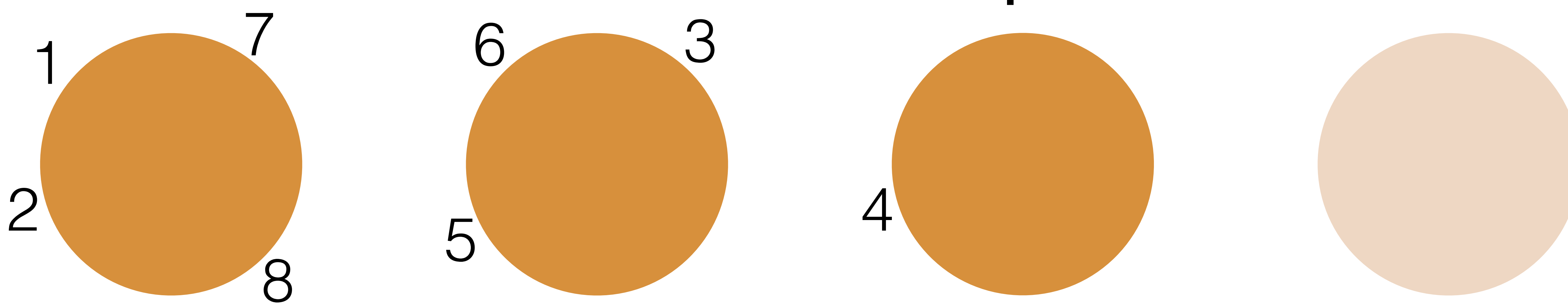

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- Probability of N customers (K_N tables, $\#C$ at table C):

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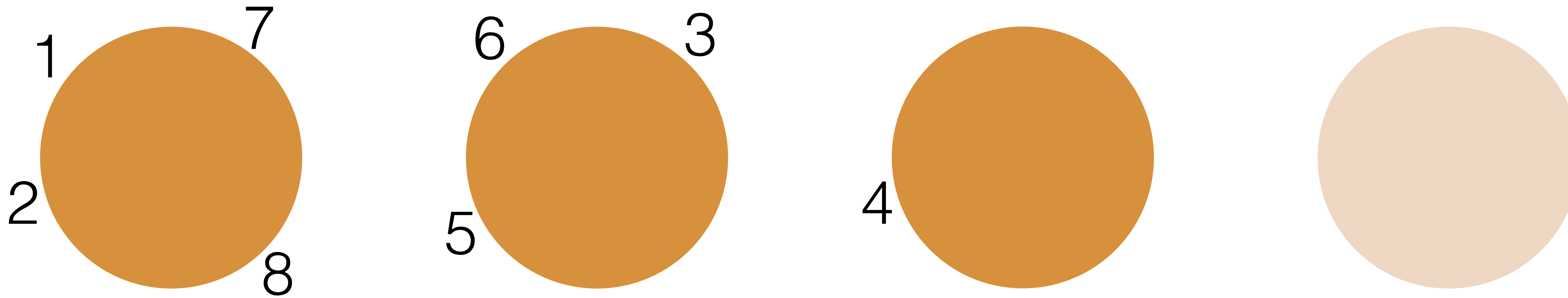

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- Prob doesn't depend on customer order: *exchangeable*

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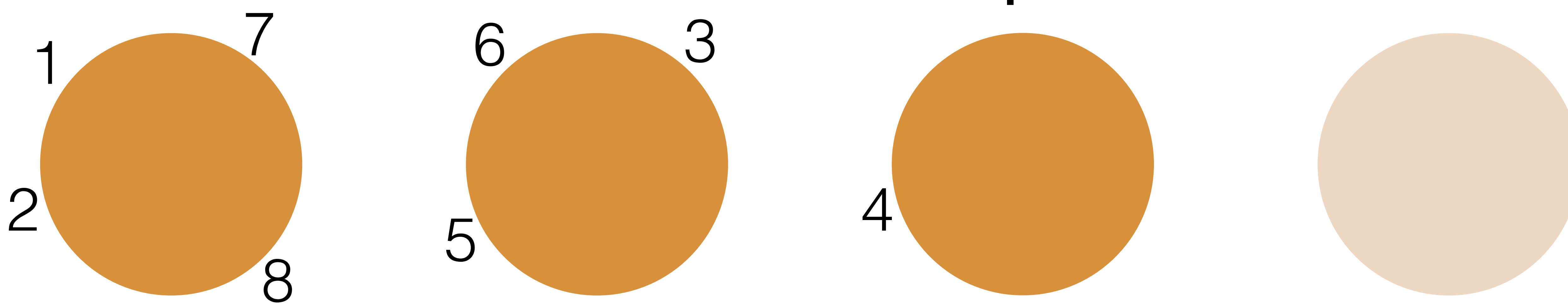

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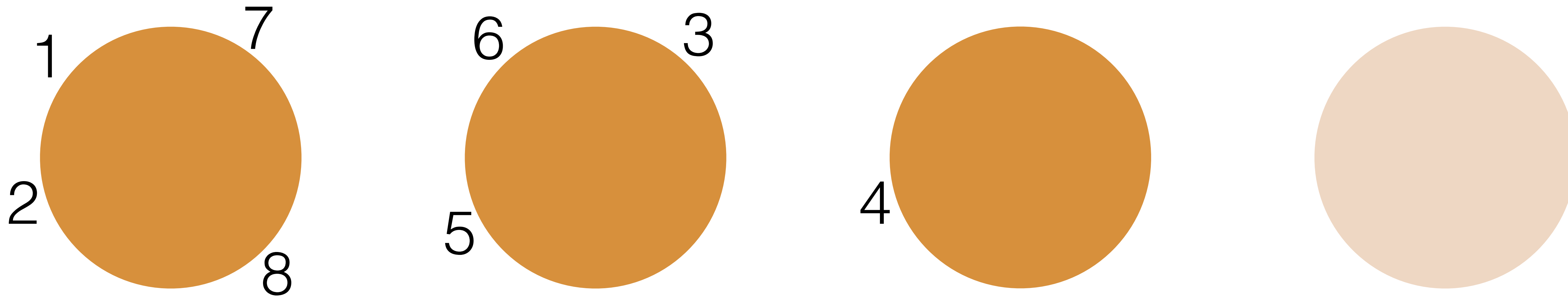
$$\mathbb{P}(\Pi_8 = \{\{1,2,7,8\},\{3,5,6\},\{4\}\}) = \mathbb{P}(\Pi_8 = \{\{2,3,8,1\},\{4,6,7\},\{5\}\})$$

Chinese restaurant process



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$$\mathbb{P}(\Pi_8 = \{\{1,2,7,8\},\{3,5,6\},\{4\}\}) = \mathbb{P}(\Pi_8 = \{\{2,3,8,1\},\{4,6,7\},\{5\}\})$$

- Can always pretend n is the last customer and calculate $p(\Pi_N | \Pi_{N,\text{-}n})$

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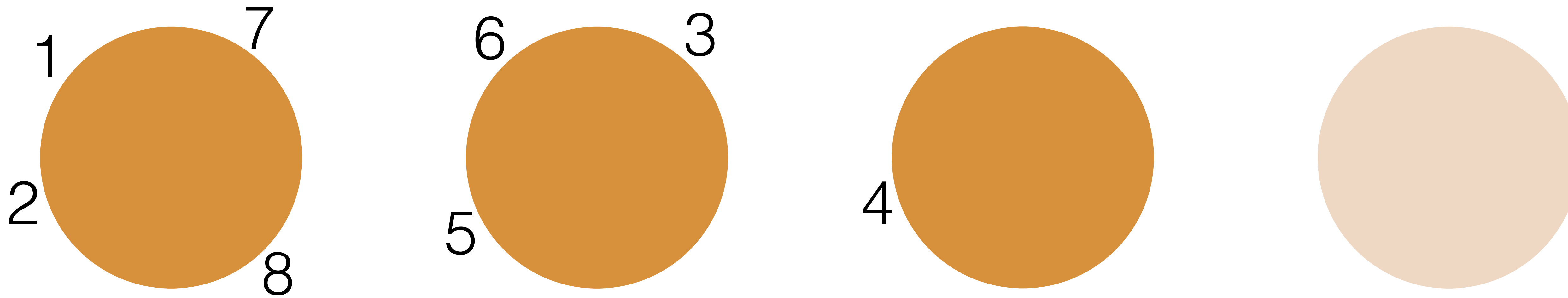

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- Can always pretend *n* is the last customer and calculate $p(\Pi_N | \Pi_{N,\text{-}n})$
  - e.g. $\Pi_{8,\text{-}5} = \{\{1,2,7,8\},\{3,6\},\{4\}\}$

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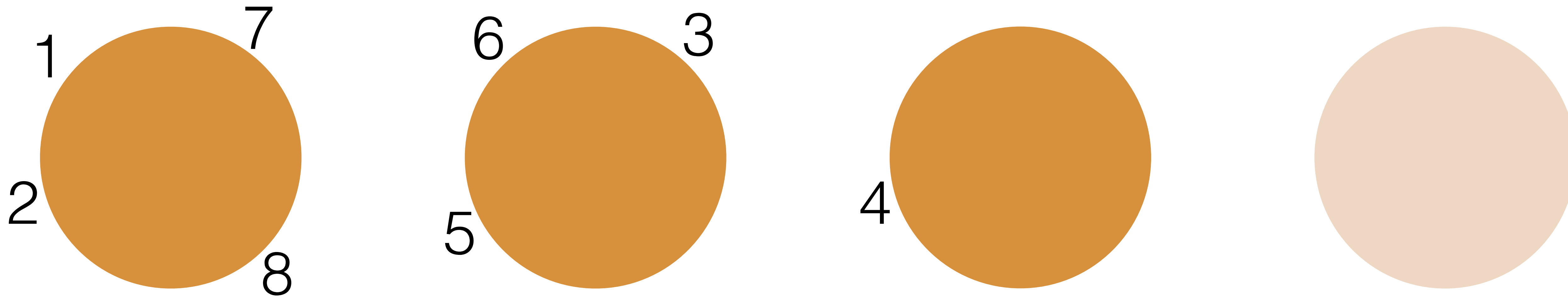


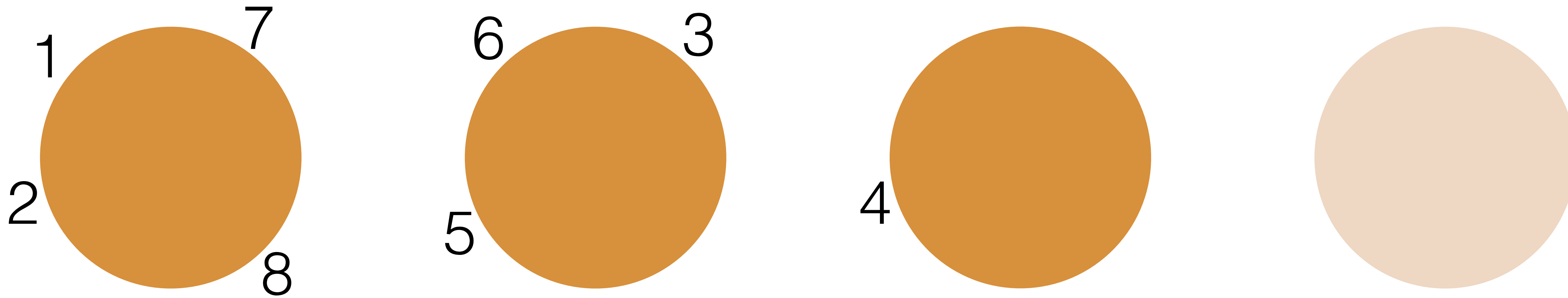
- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N}\prod_{C\in\Pi_N}(\#C-1)!}{\alpha\cdots(\alpha+N-1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N|\Pi_{N,-n}) =$$

Chinese restaurant process



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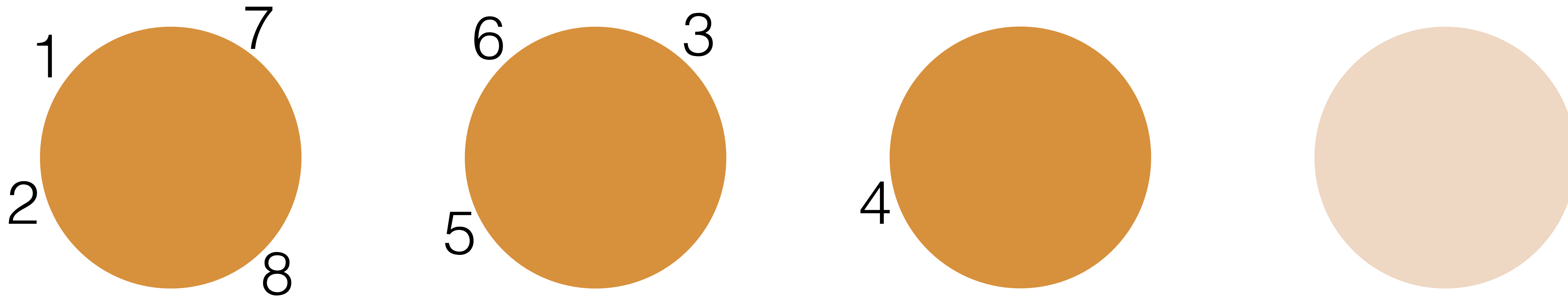

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- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} & \text{if } n \text{ joins cluster } C \\ & \text{if } n \text{ starts a new cluster} \end{cases}$$

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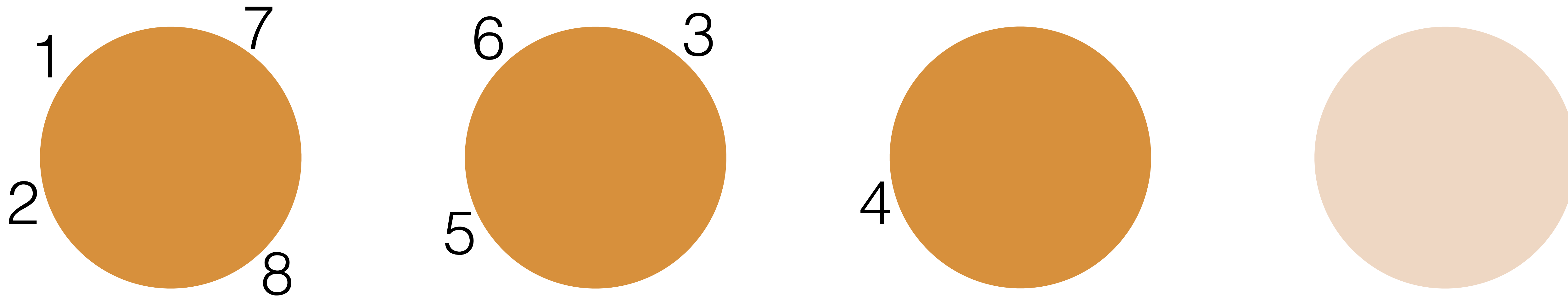

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- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha + N - 1} & \text{if } n \text{ joins cluster } C \\ & \text{if } n \text{ starts a new cluster} \end{cases}$$

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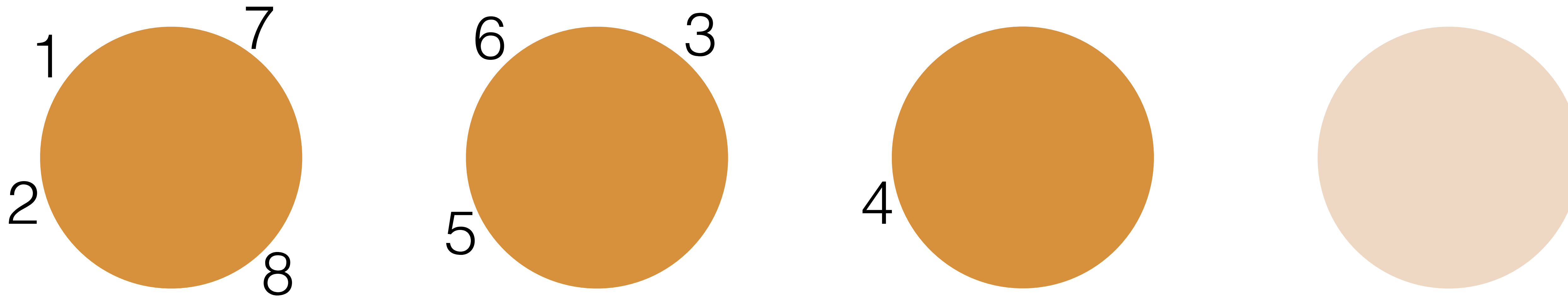

- Probability of N customers (K_N tables, #C at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

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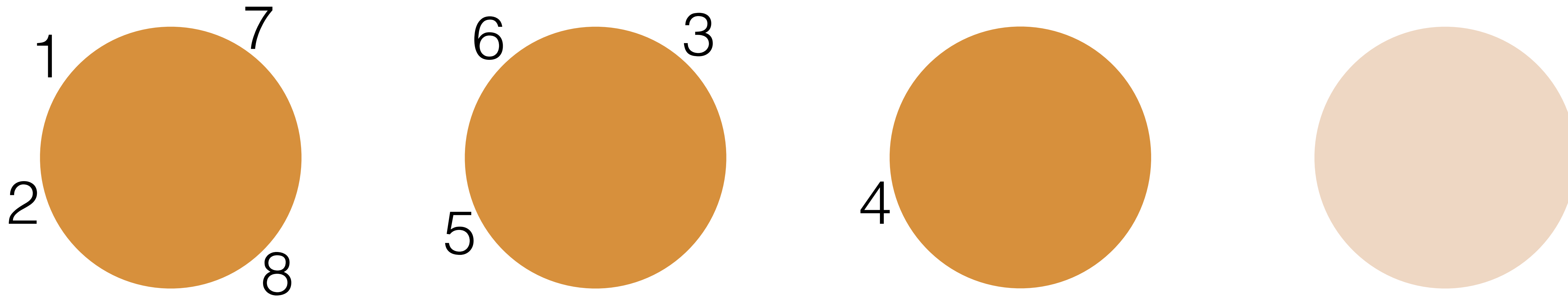
$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha + N - 1} & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha + N - 1} & \text{if } n \text{ starts a new cluster} \end{cases}$$

Chinese restaurant process



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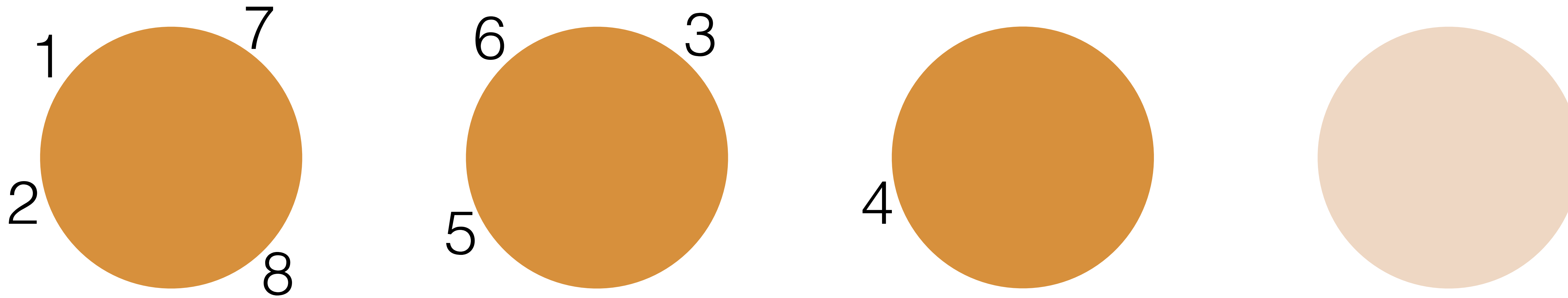
Chinese restaurant process



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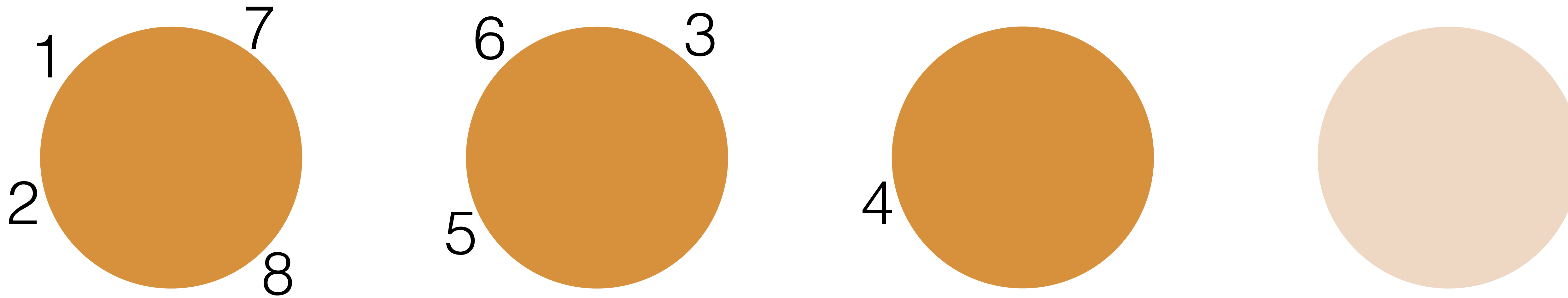
Chinese restaurant process



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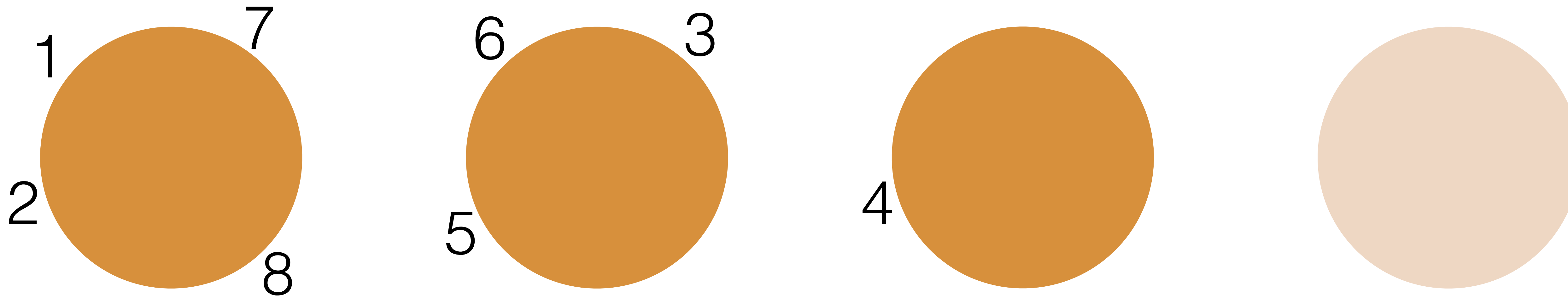
Chinese restaurant process



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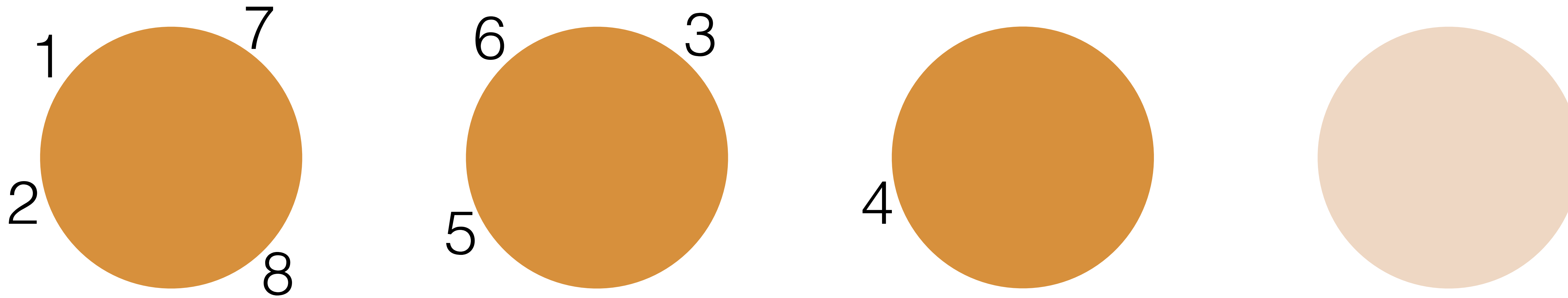

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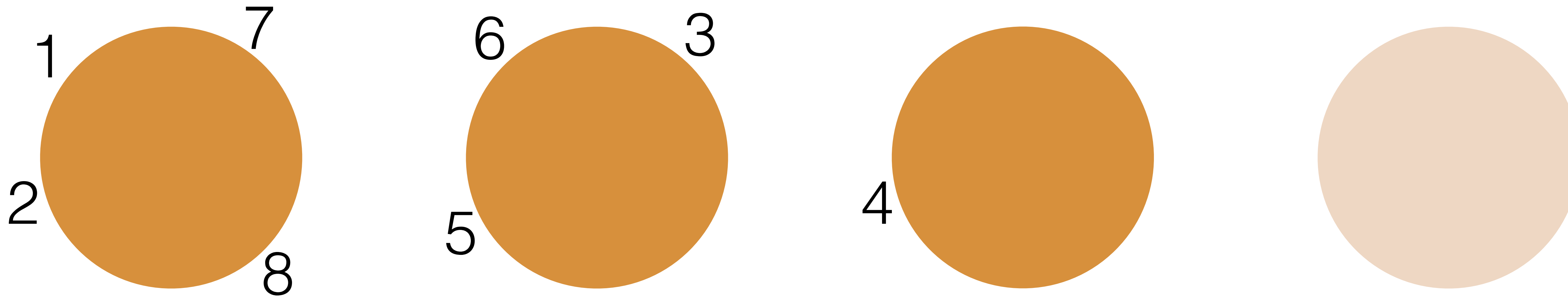

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CRP mixture model: inference

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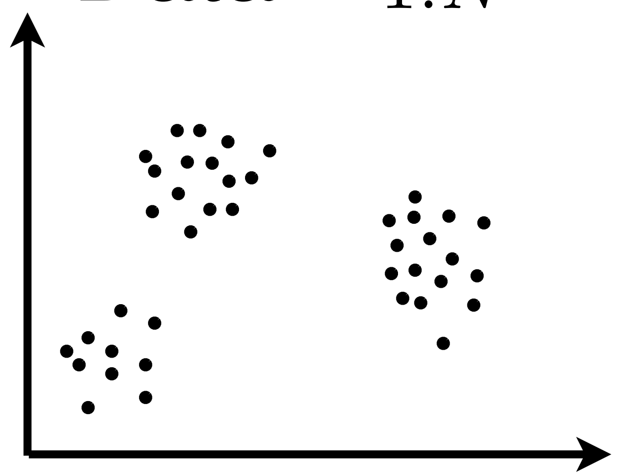
- Data $x_{1:N}$



CRP mixture model: inference

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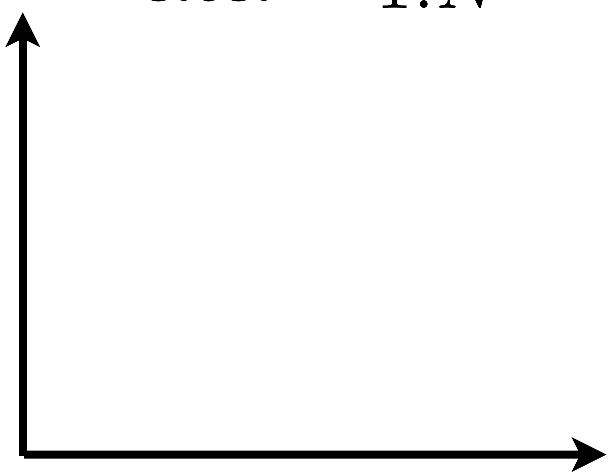

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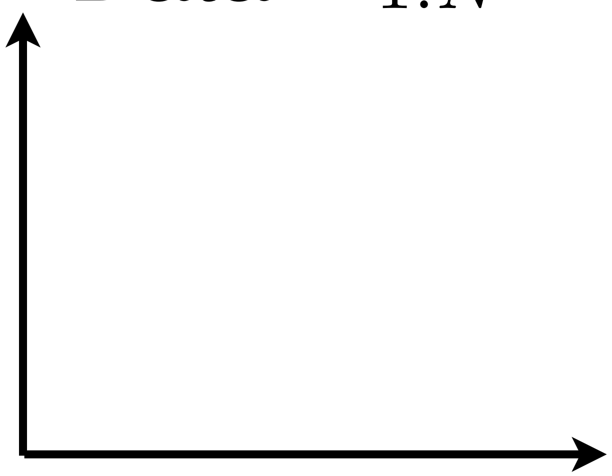

- Generative model

CRP mixture model: inference

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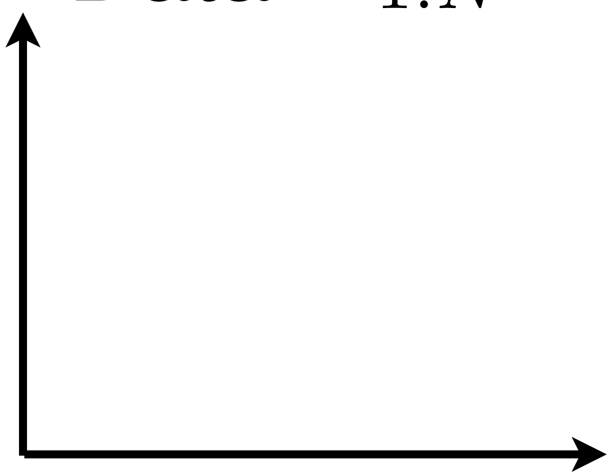

- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$

CRP mixture model: inference

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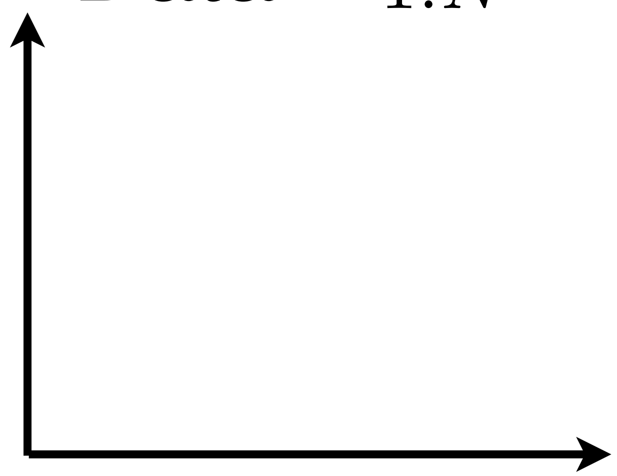

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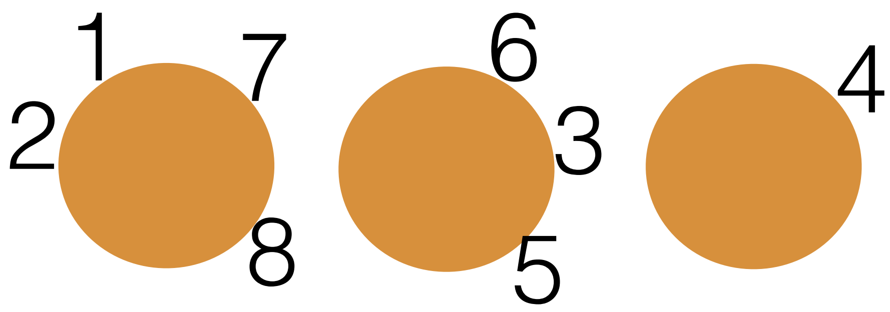

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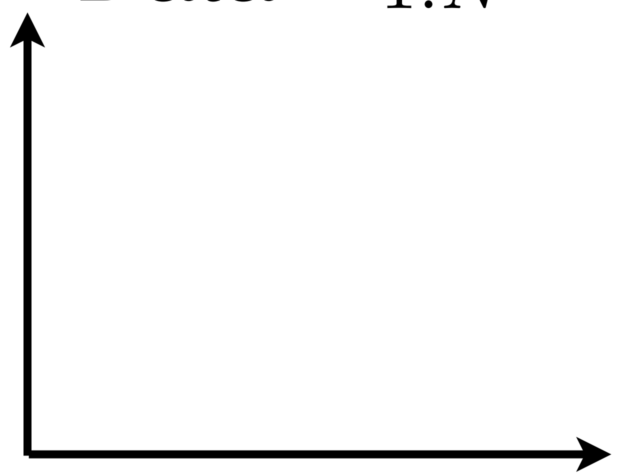

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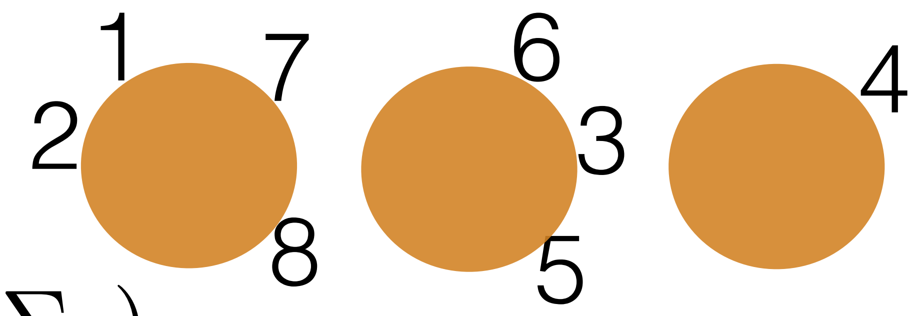

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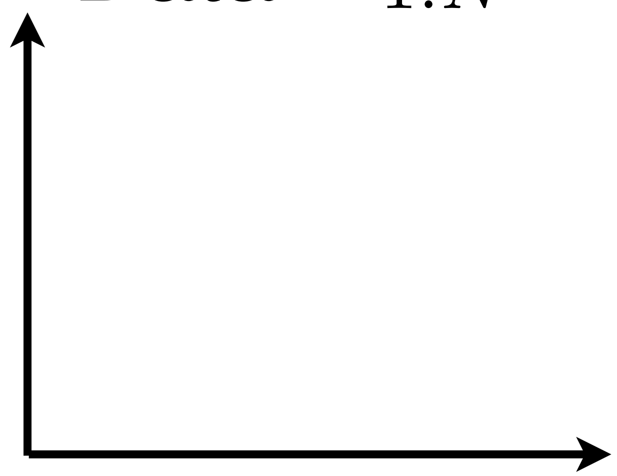

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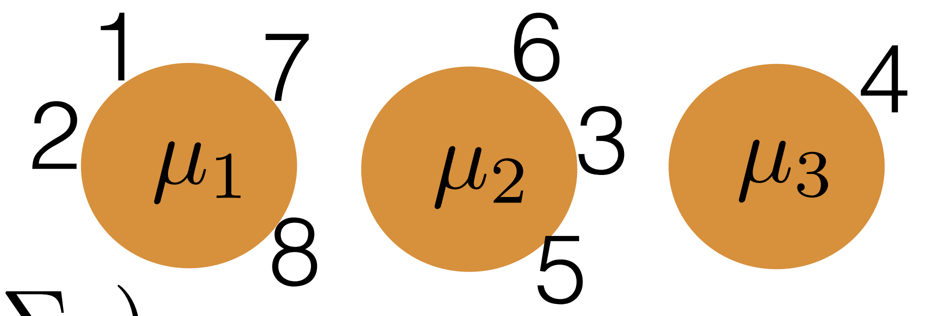

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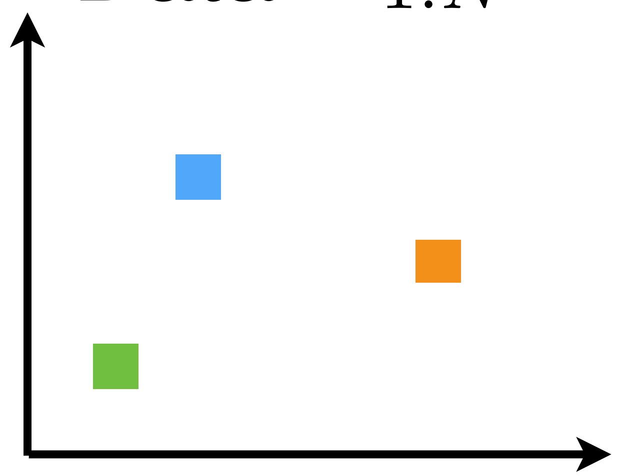

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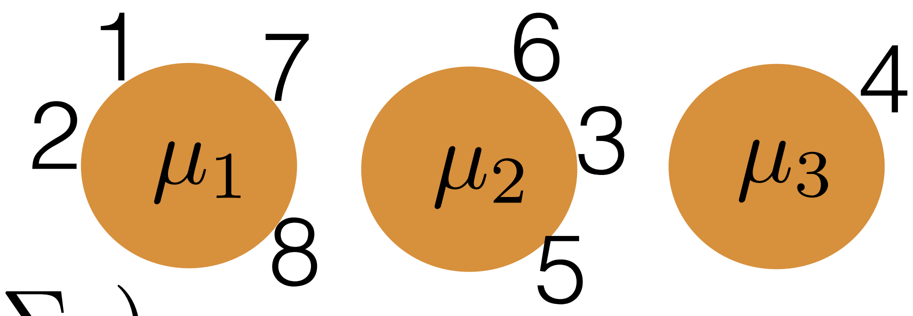

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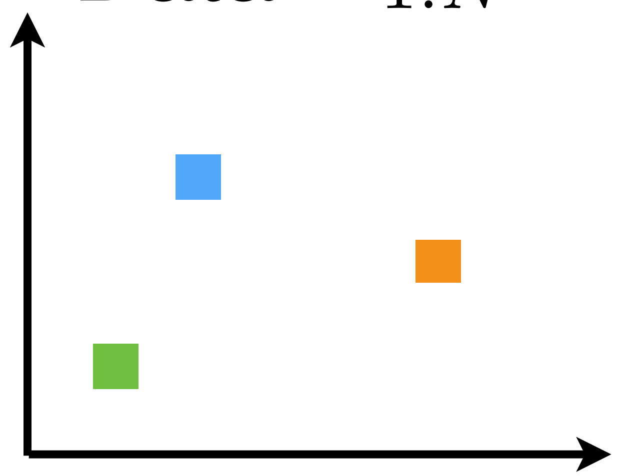

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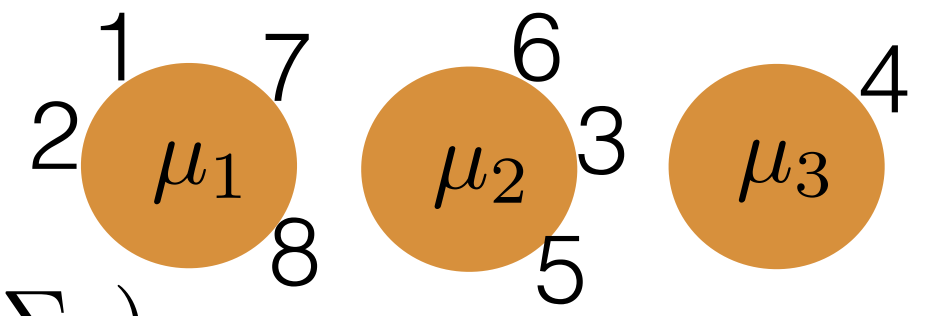

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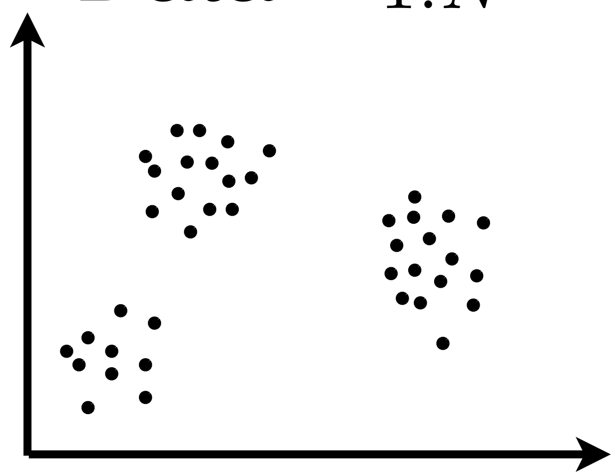

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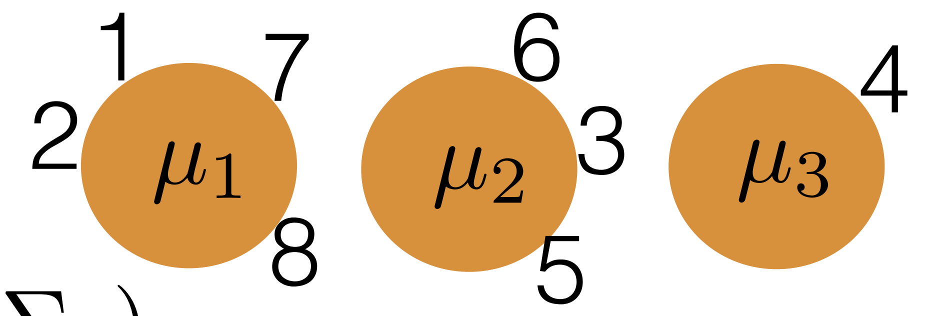

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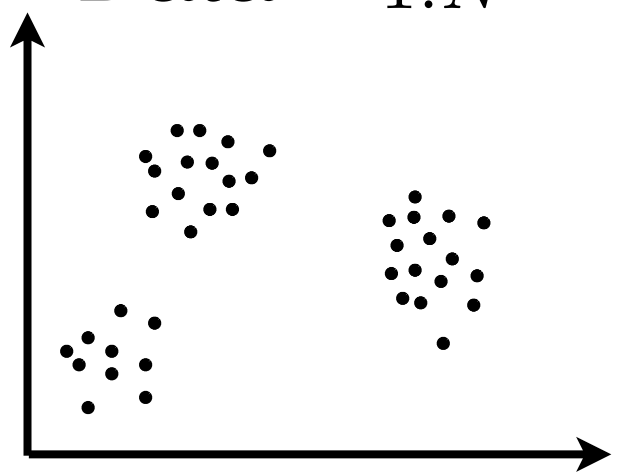

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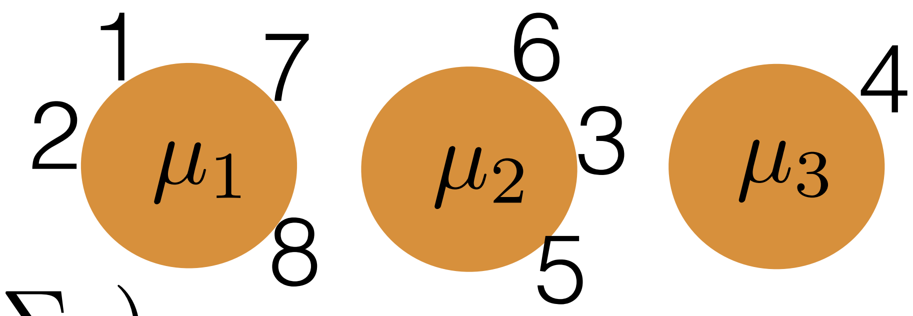
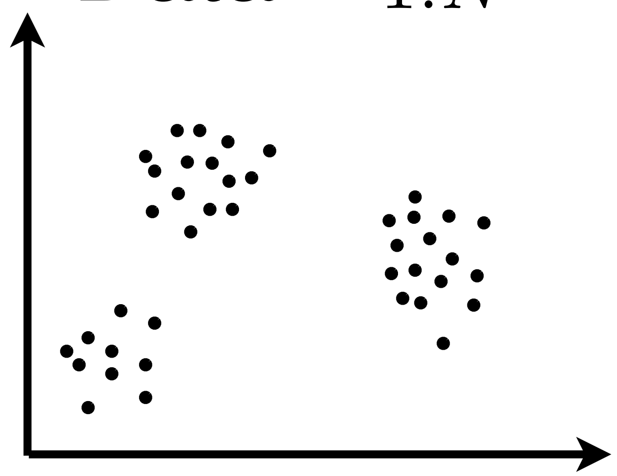


- Want: posterior

CRP mixture model: inference

- Data $x_{1:N}$



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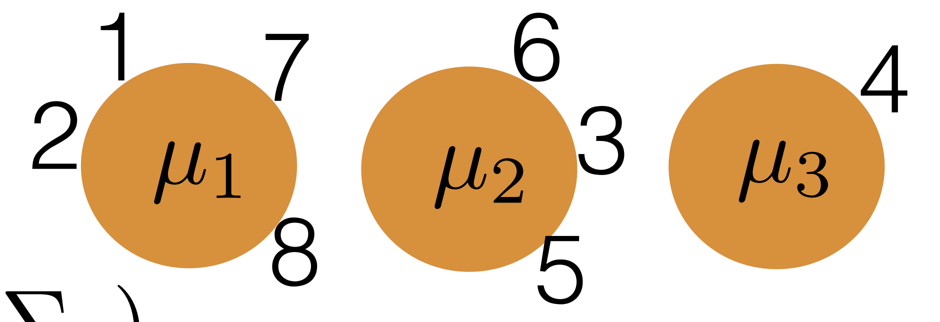
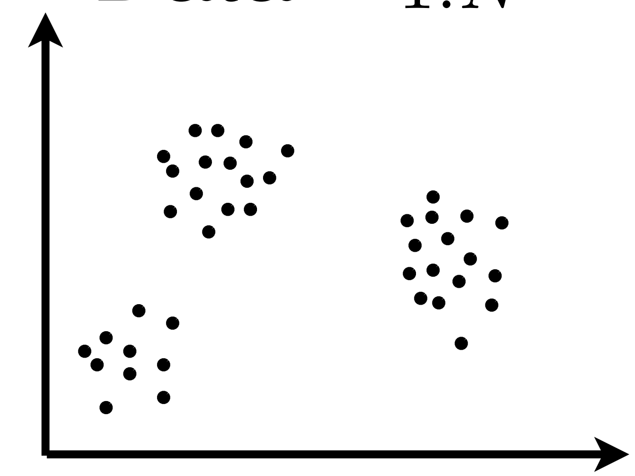


- Want: posterior $p(\Pi_N | x_{1:N})$

CRP mixture model: inference

- Data $x_{1:N}$



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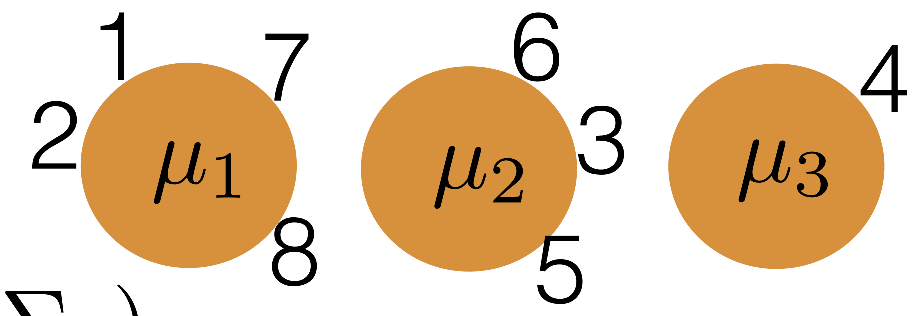

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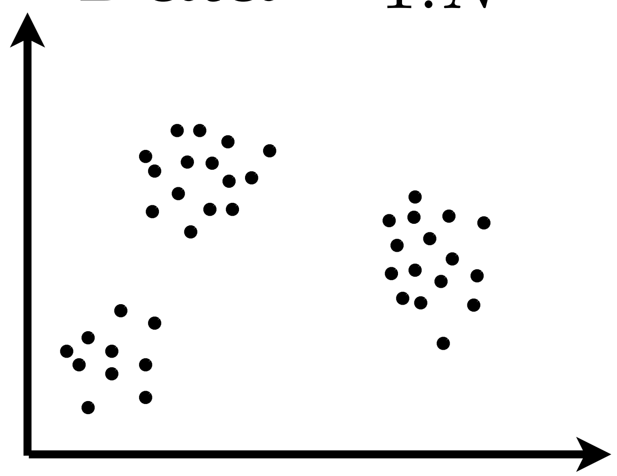
- Gibbs sampler:

CRP mixture model: inference

- Data $x_{1:N}$



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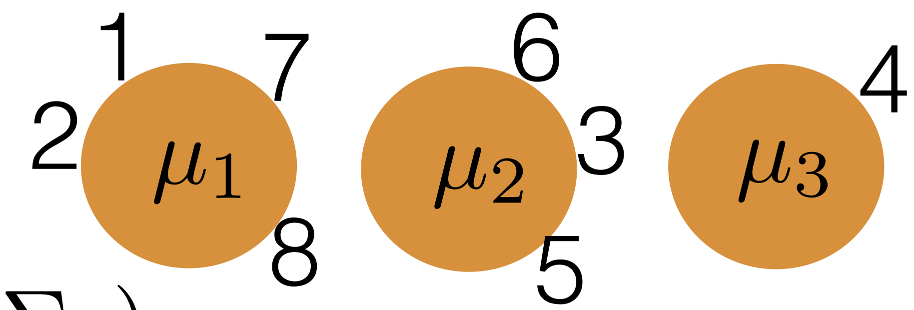

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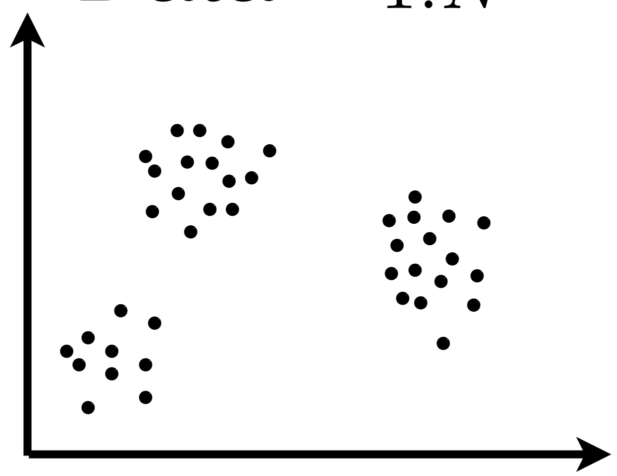
CRP mixture model: inference

- Data $x_{1:N}$



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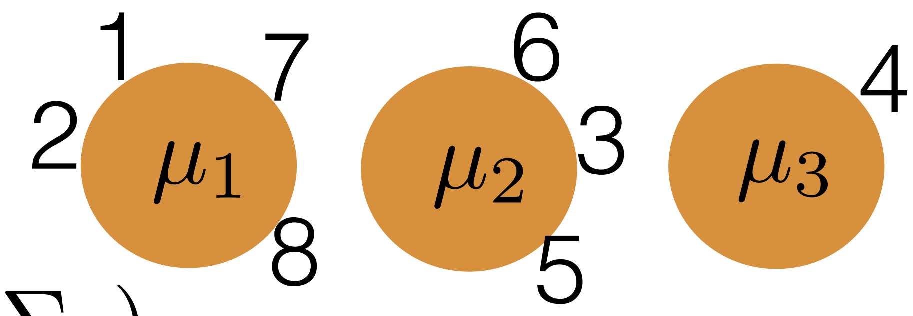

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CRP mixture model: inference

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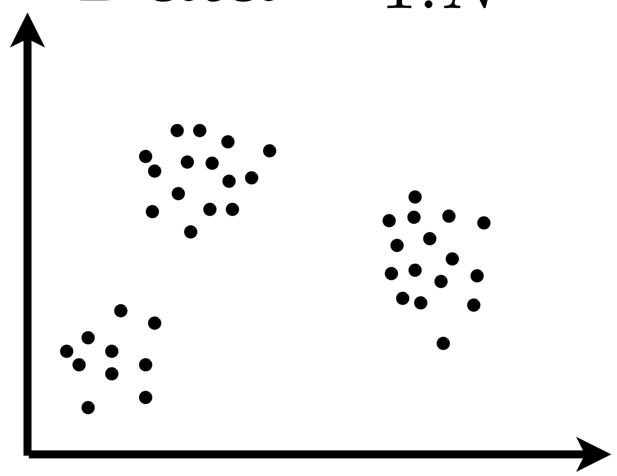

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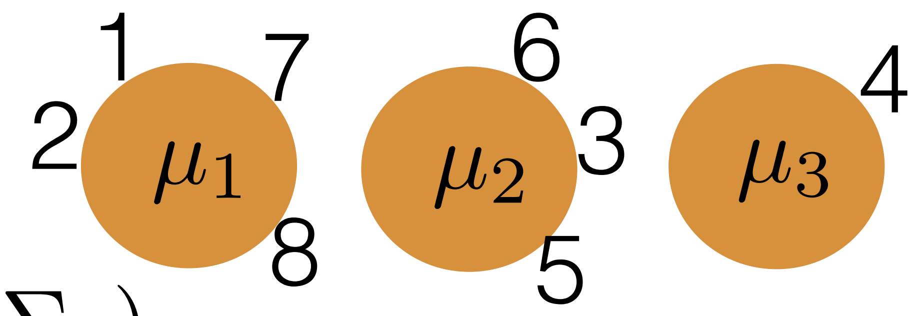

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if *n* joins cluster *C*

CRP mixture model: inference

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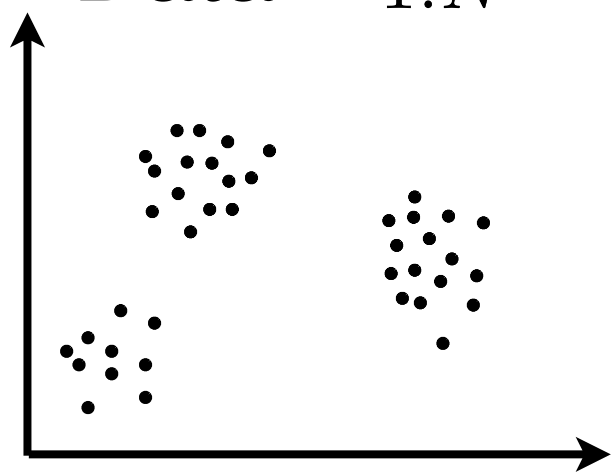
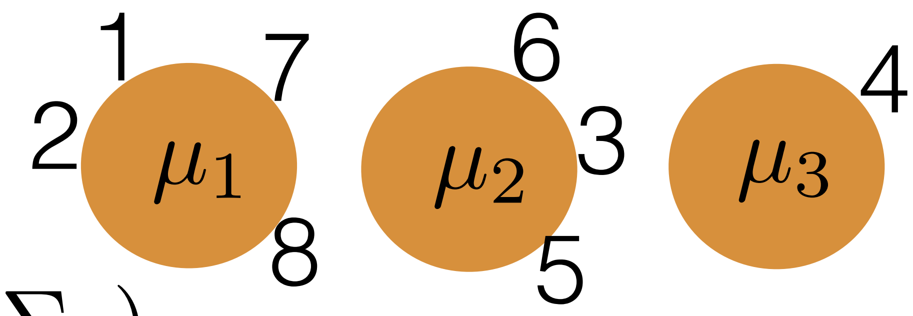


- Generative model



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CRP mixture model: inference

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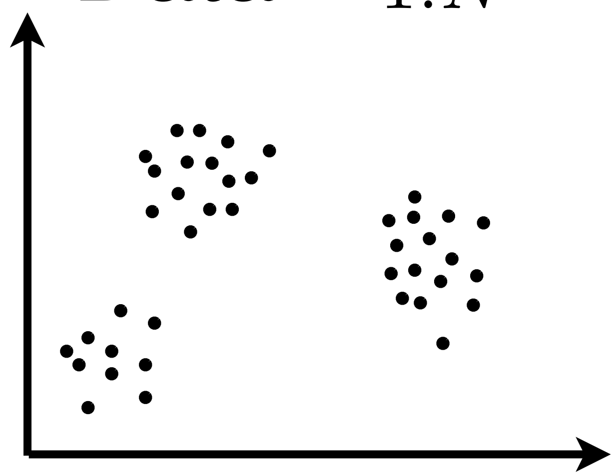

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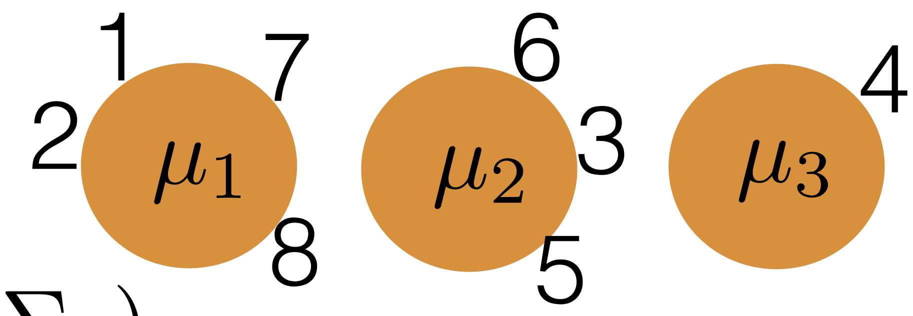

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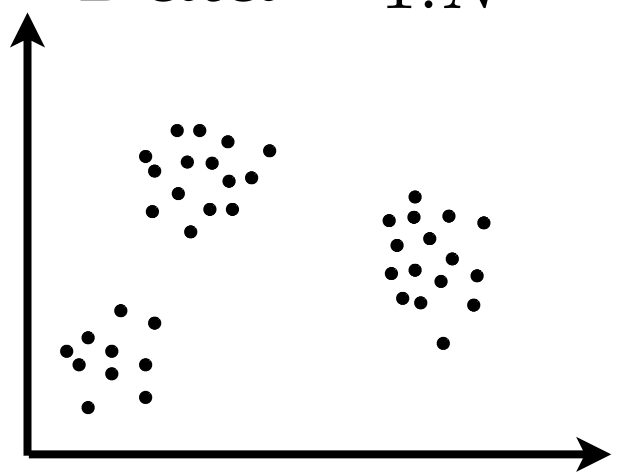

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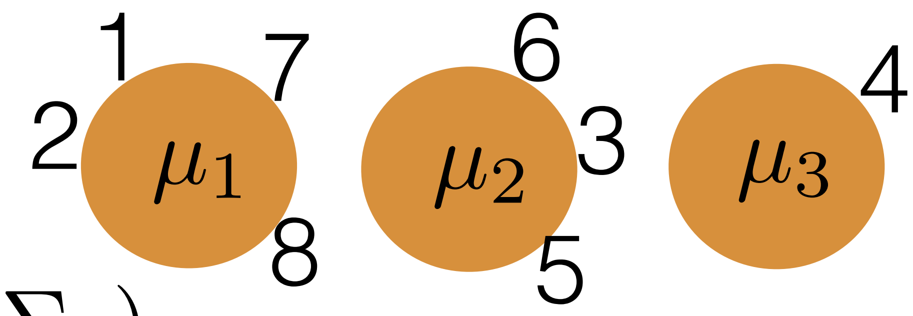

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CRP mixture model: inference

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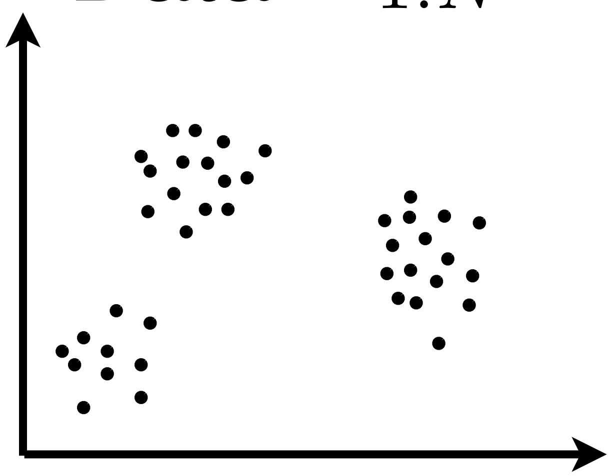

- Generative model

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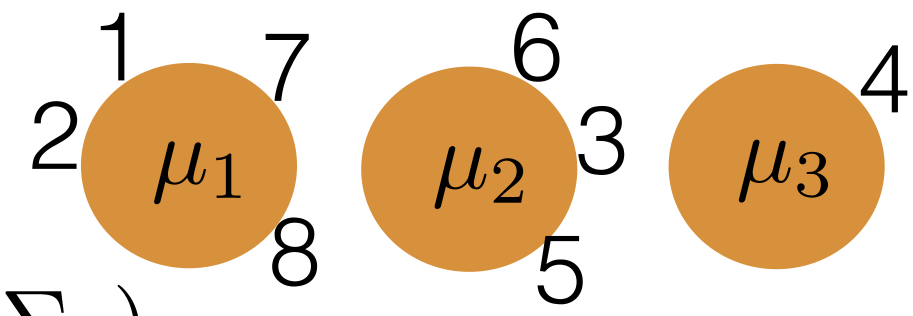

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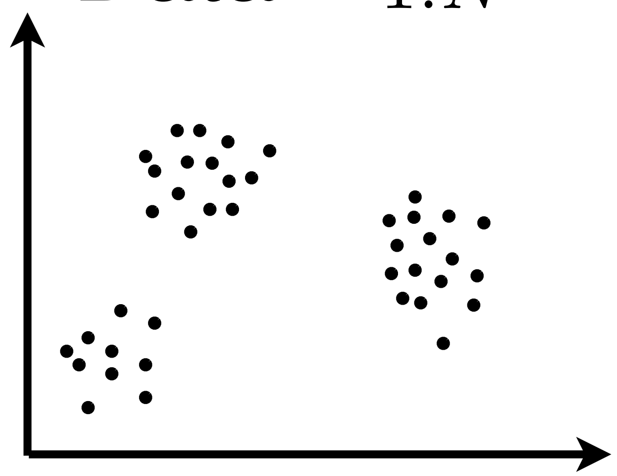

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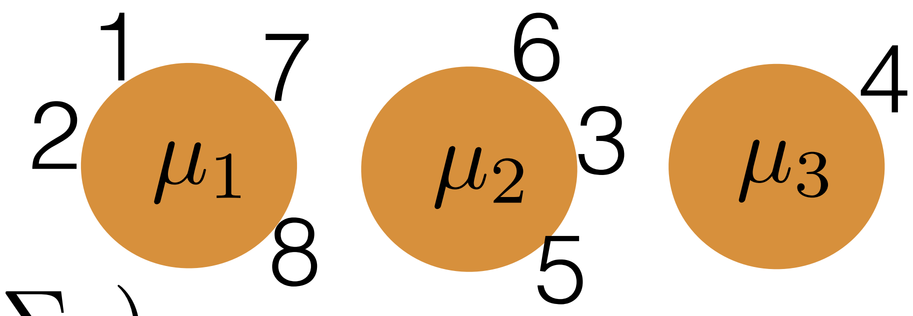

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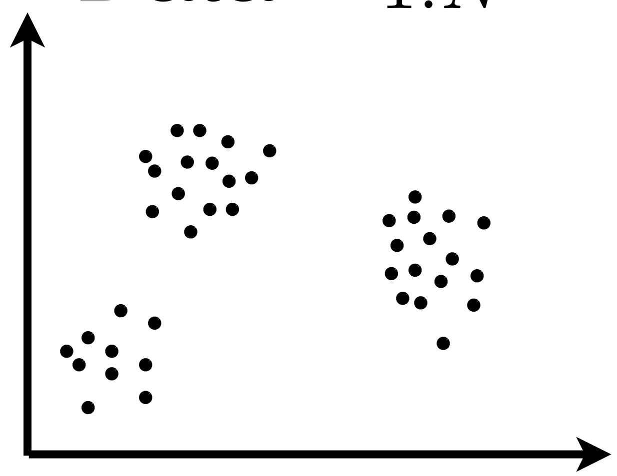

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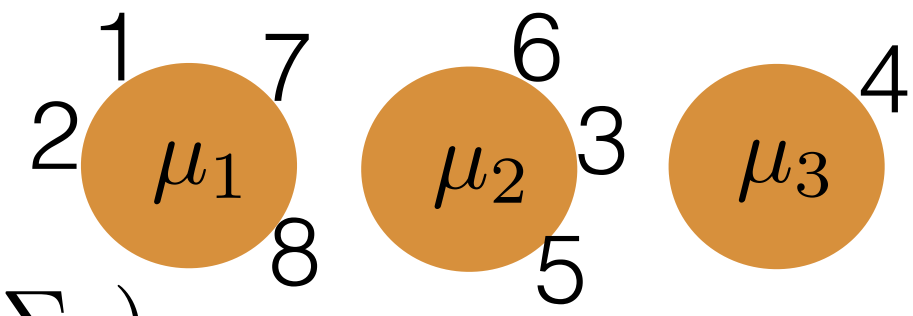

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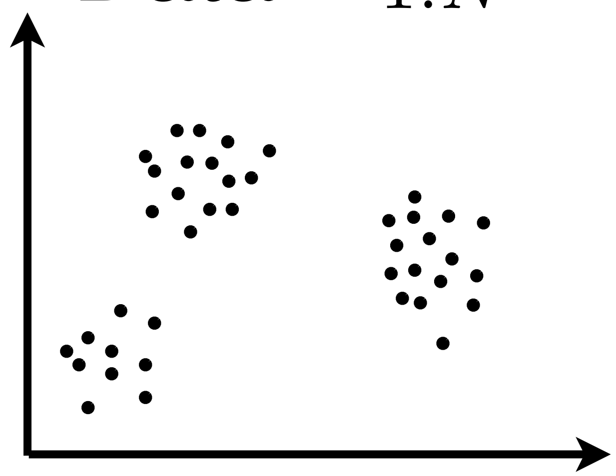

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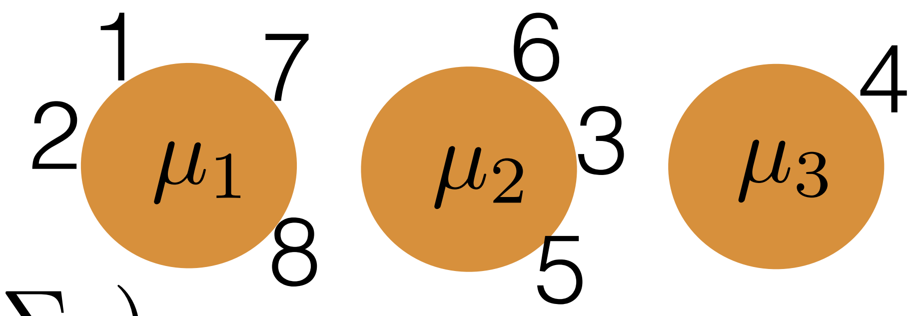

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[MacEachern 1994; Neal 1992; Neal 2000]

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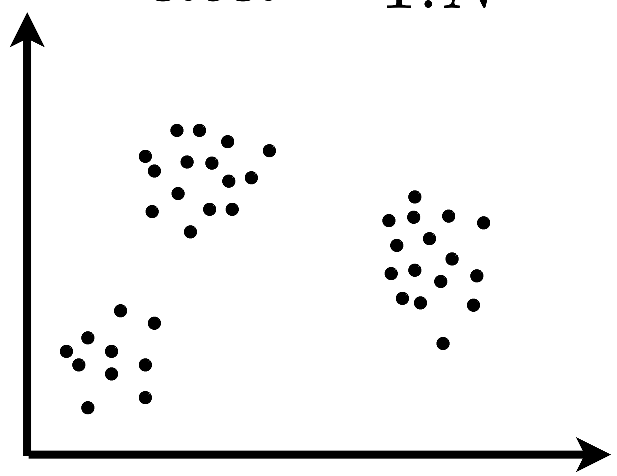

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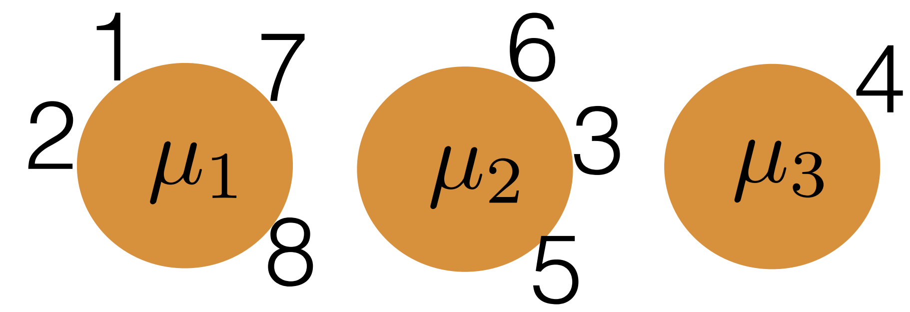

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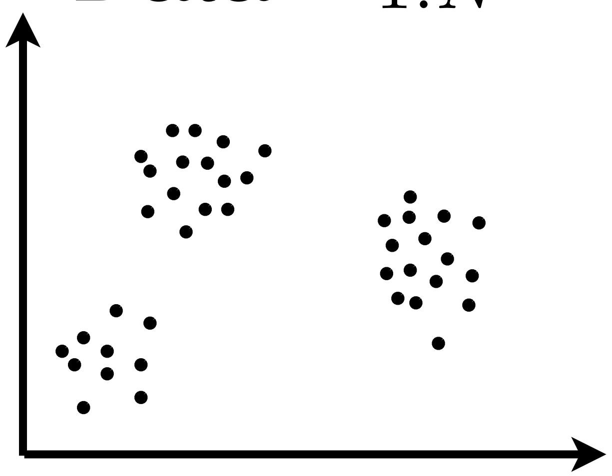

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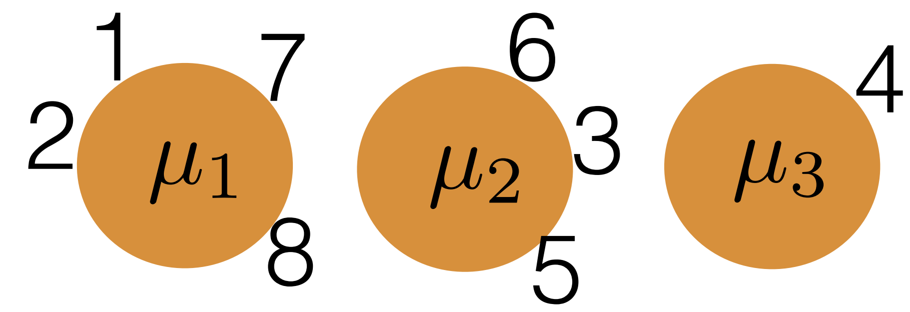

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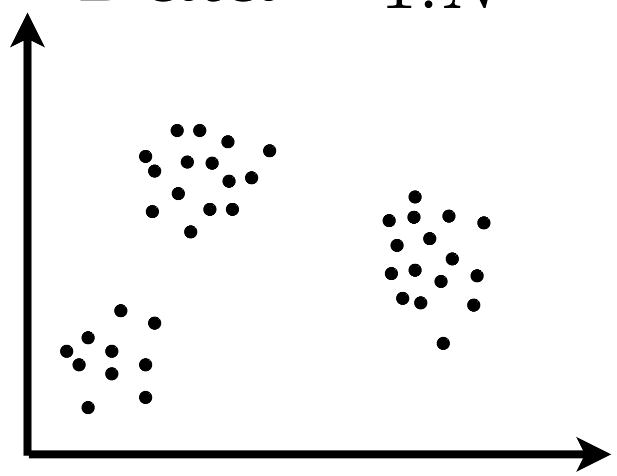

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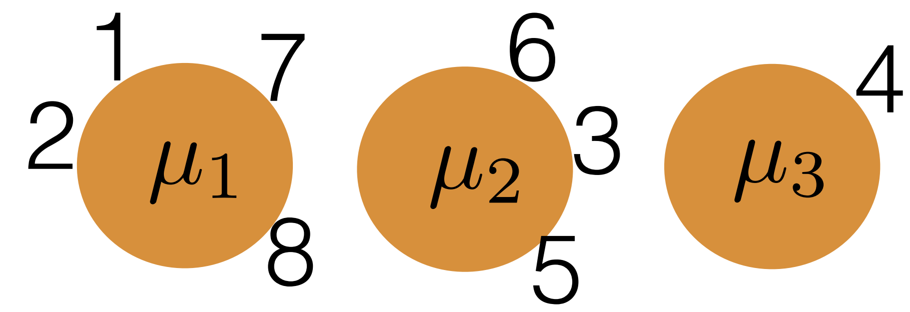

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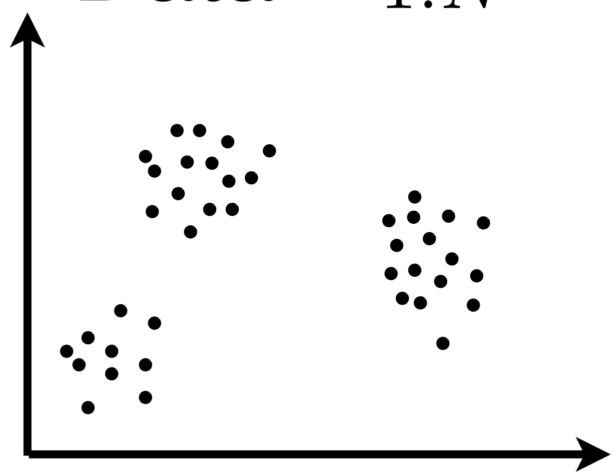

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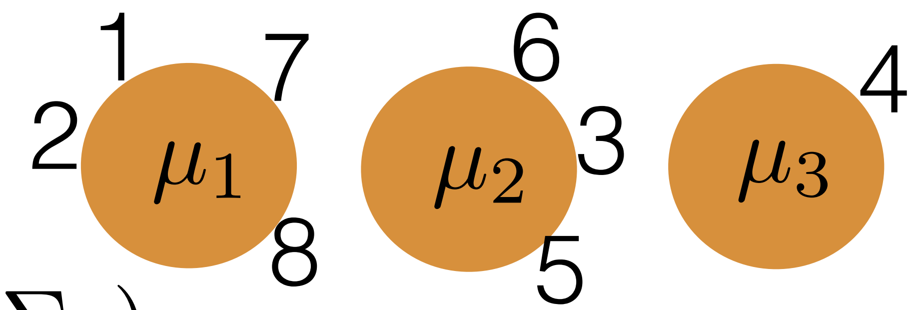

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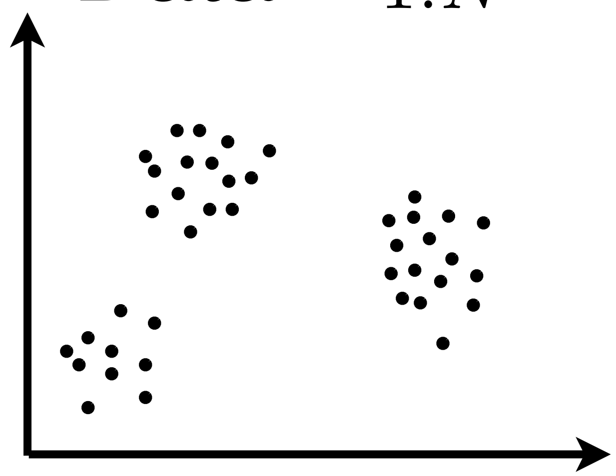

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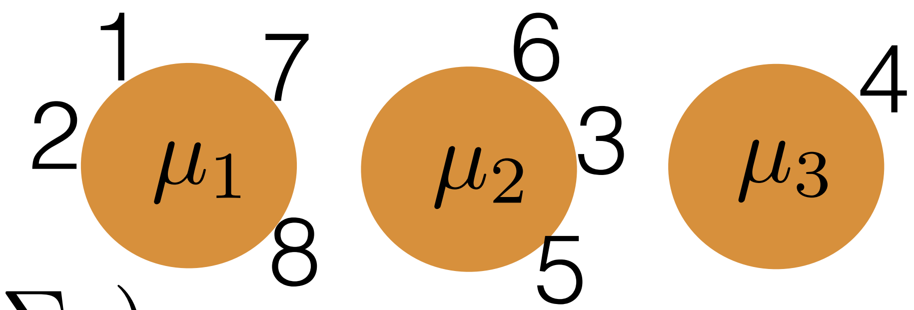

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[demo]

[MacEachern 1994; Neal 1992; Neal 2000]

More Markov Chain Monte Carlo

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- Slice sampling

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  - auxiliary variable ➔ finite conditionals

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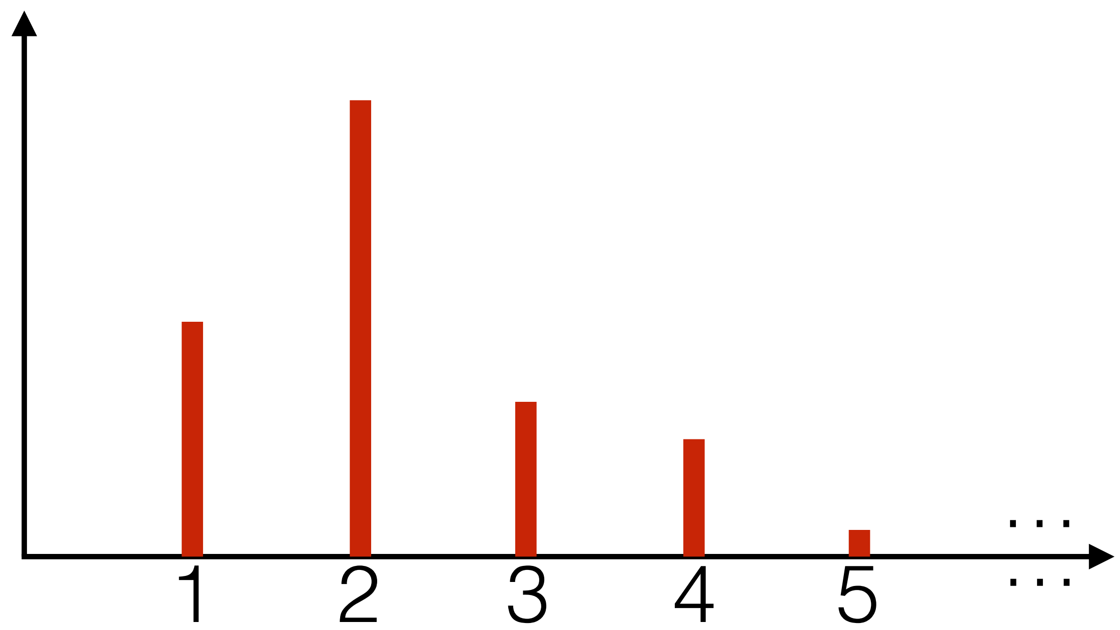

More Markov Chain Monte Carlo

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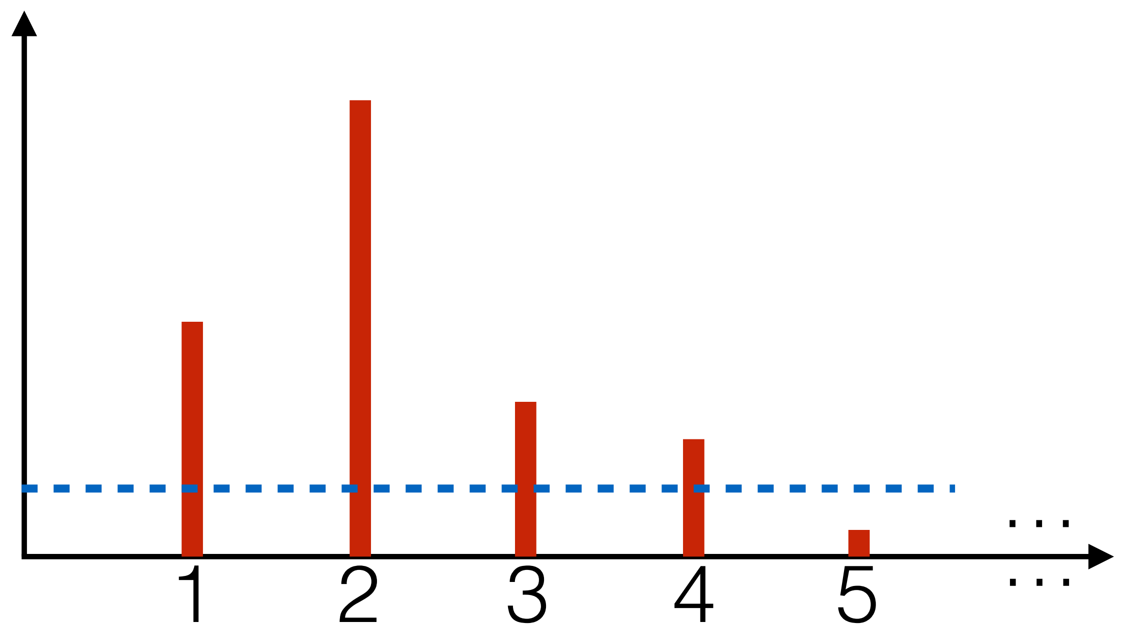

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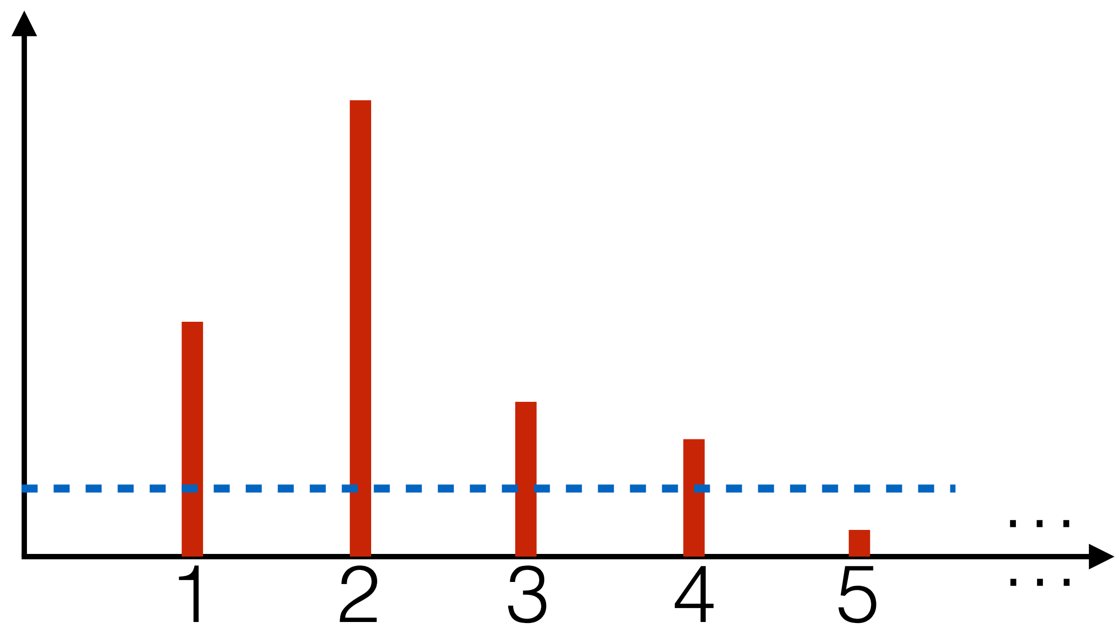

- Approximate with truncated distribution

[Ishwaran, James 2001; Campbell*, Huggins*, Broderick 2016]

More Markov Chain Monte Carlo

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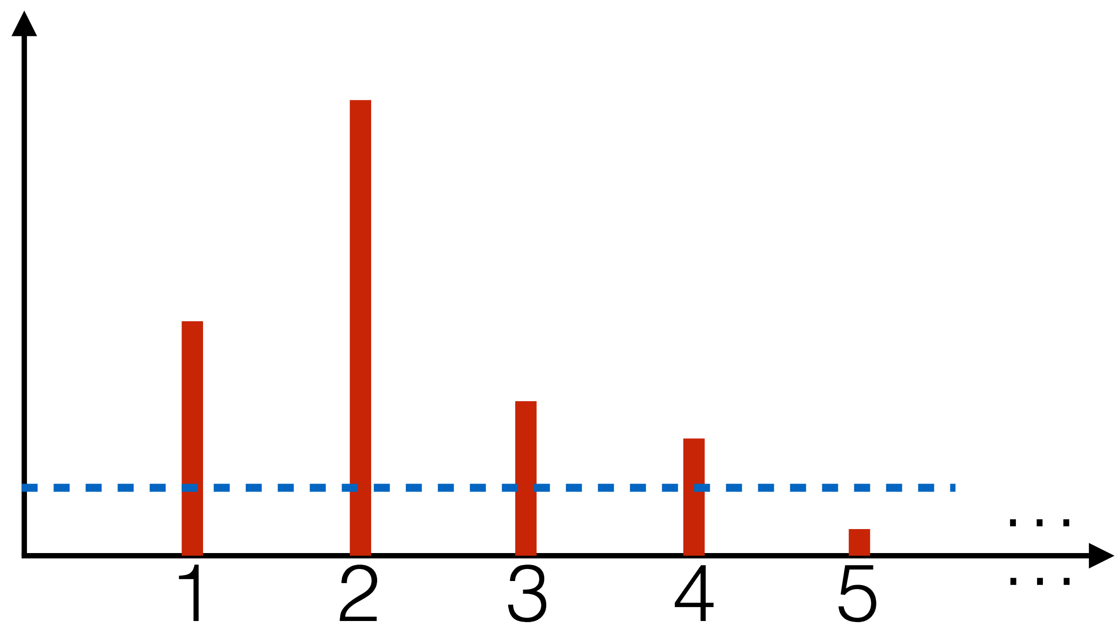

- Approximate with truncated distribution
  - E.g., Hamiltonian Monte Carlo

[Ishwaran, James 2001; Campbell*, Huggins*, Broderick 2016]

Variational Bayes

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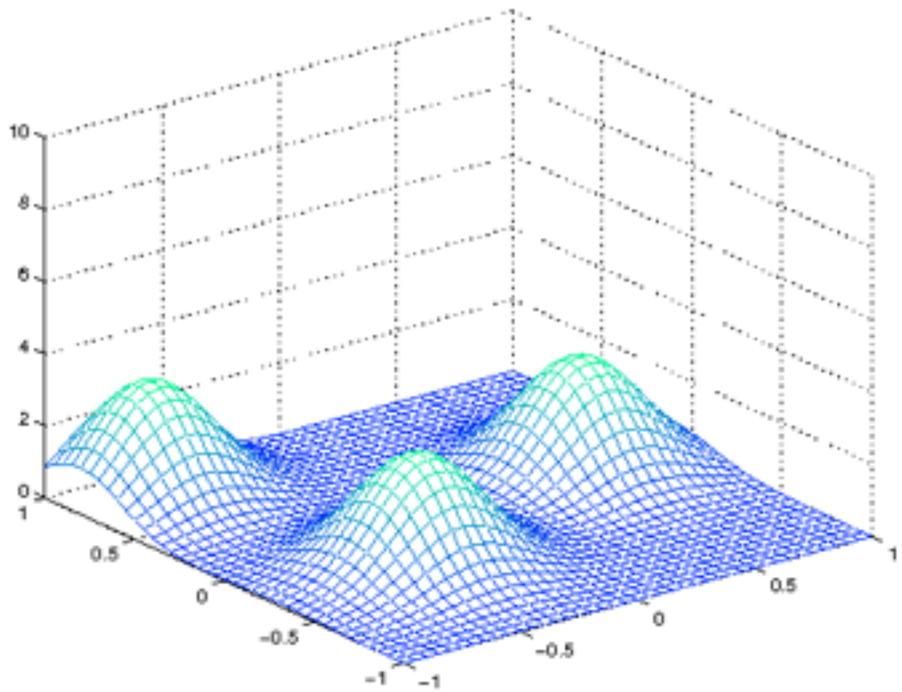
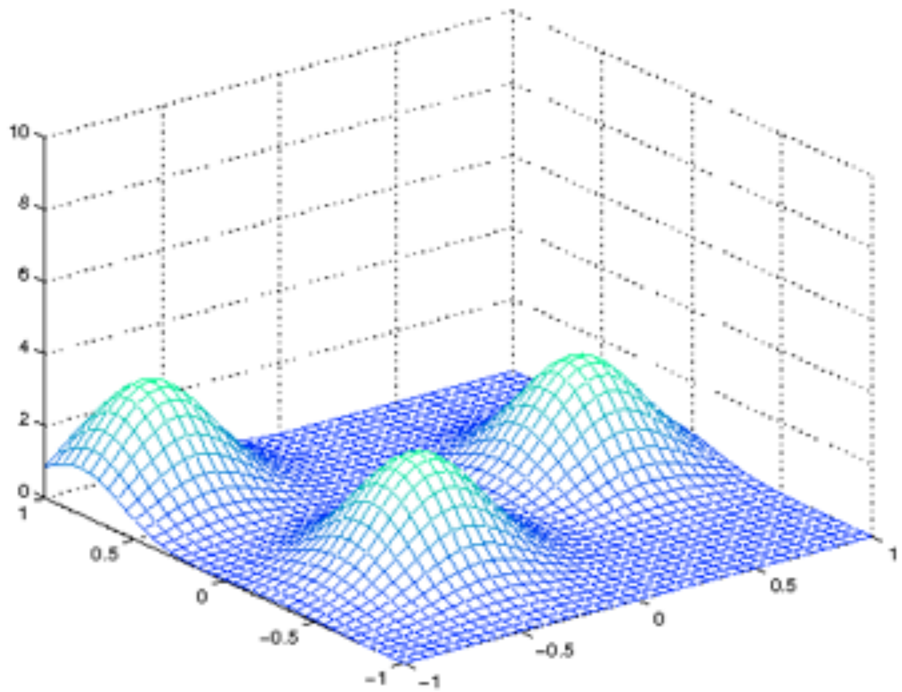


- Variational Bayes (VB)

Variational Bayes



- Variational Bayes (VB)
  - Approximation $q^*(\theta)$ for posterior $p(\theta|x)$

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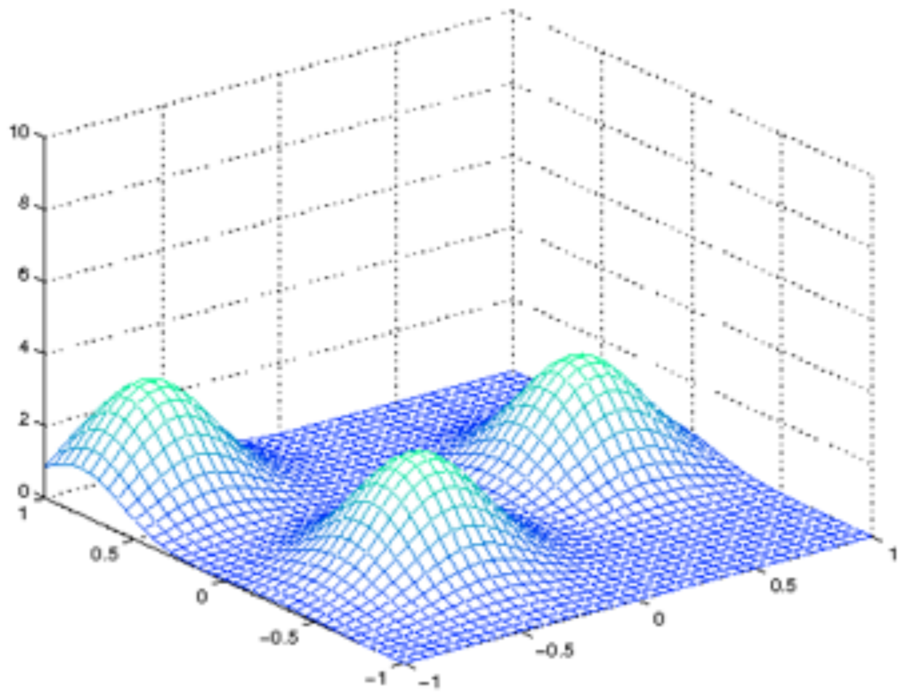

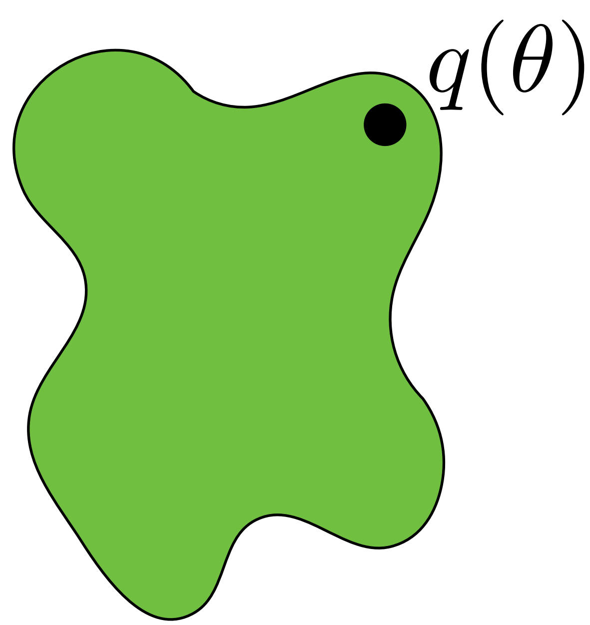

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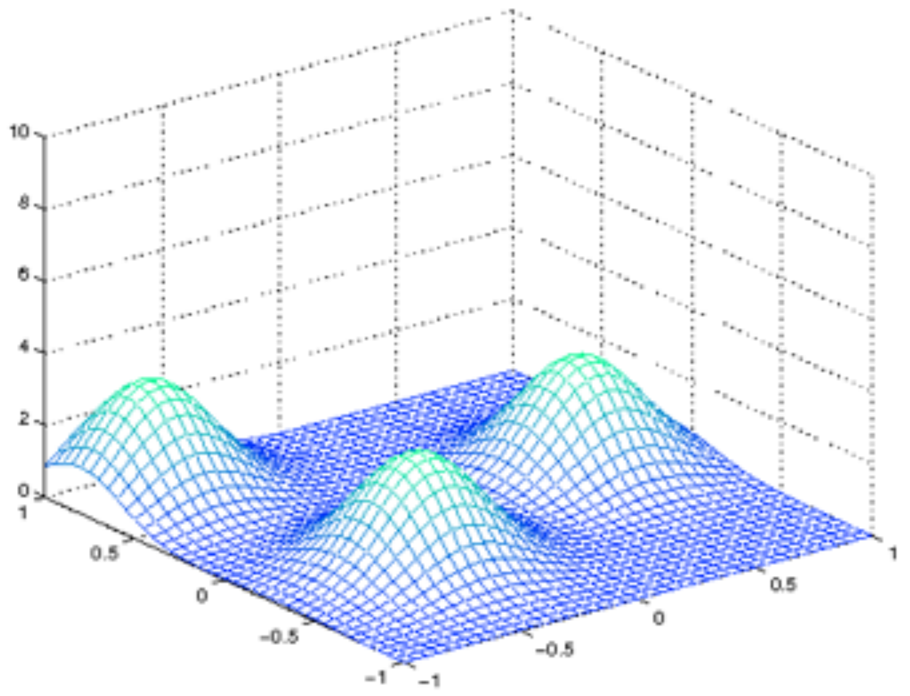

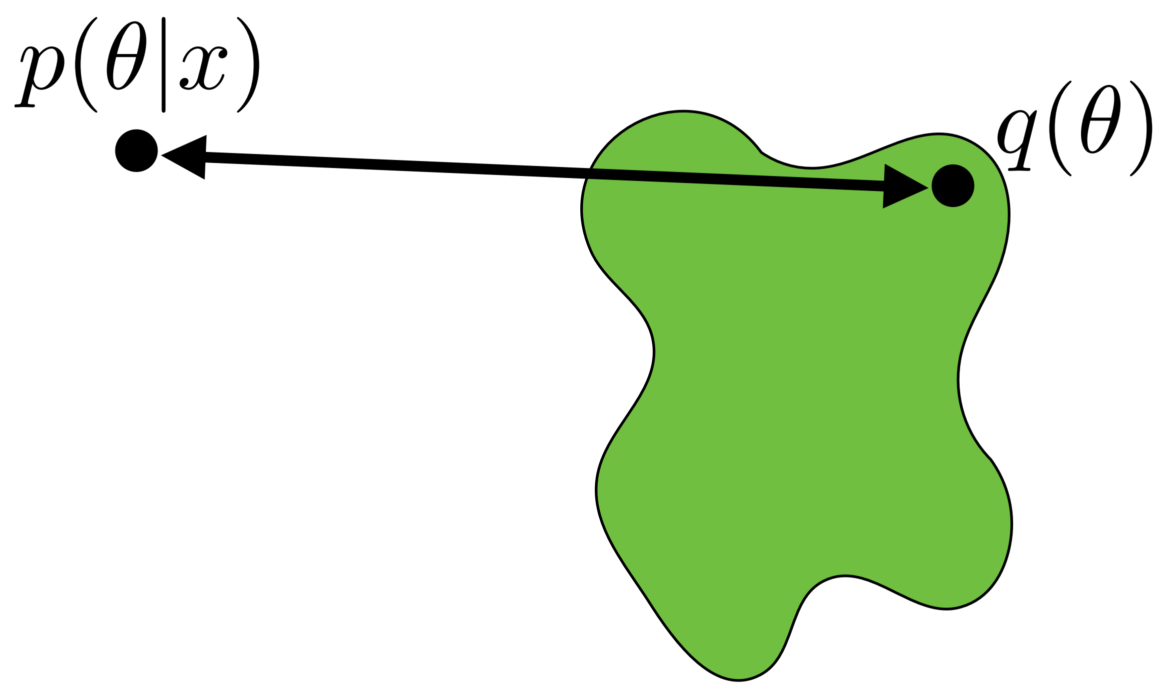

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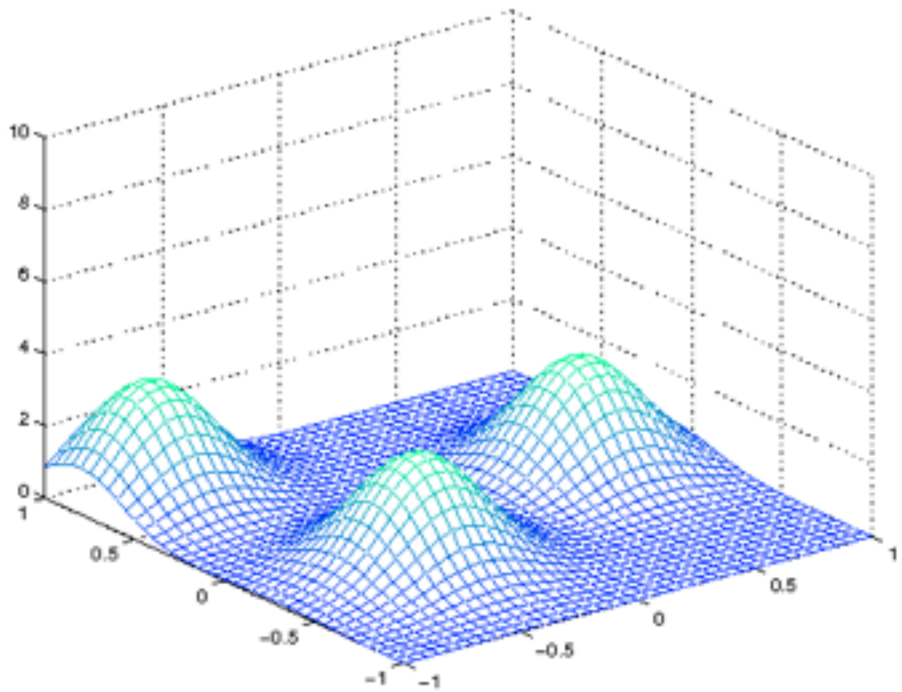

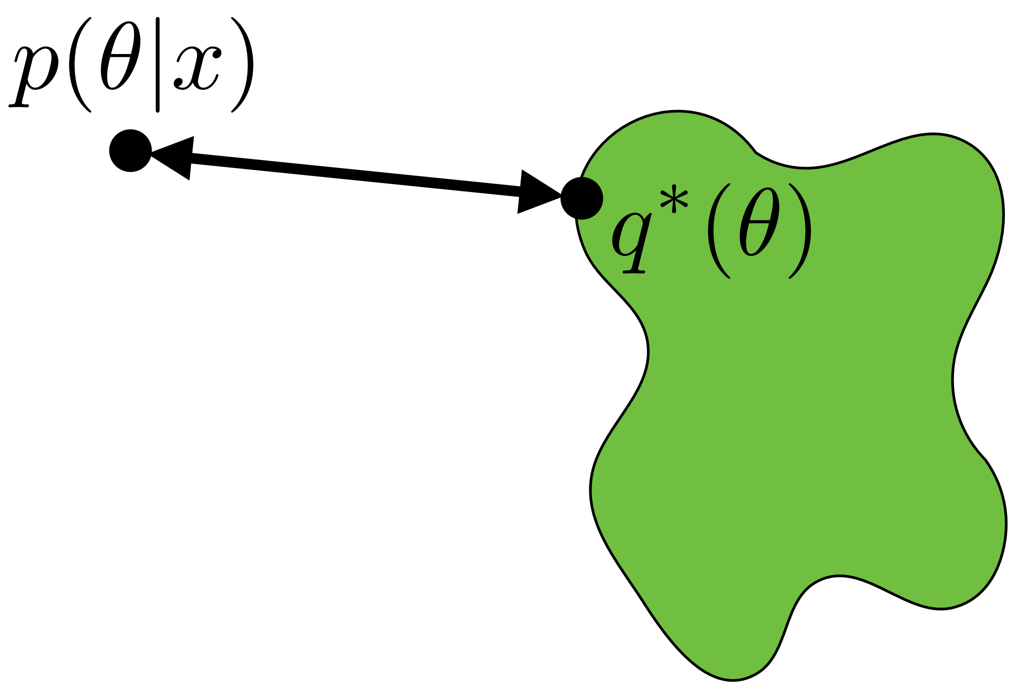

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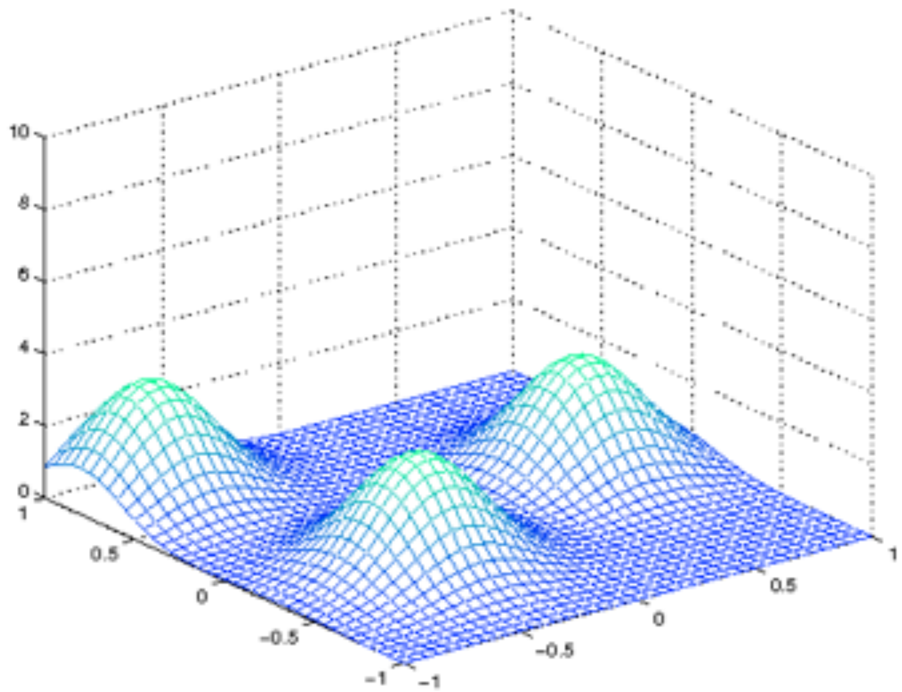

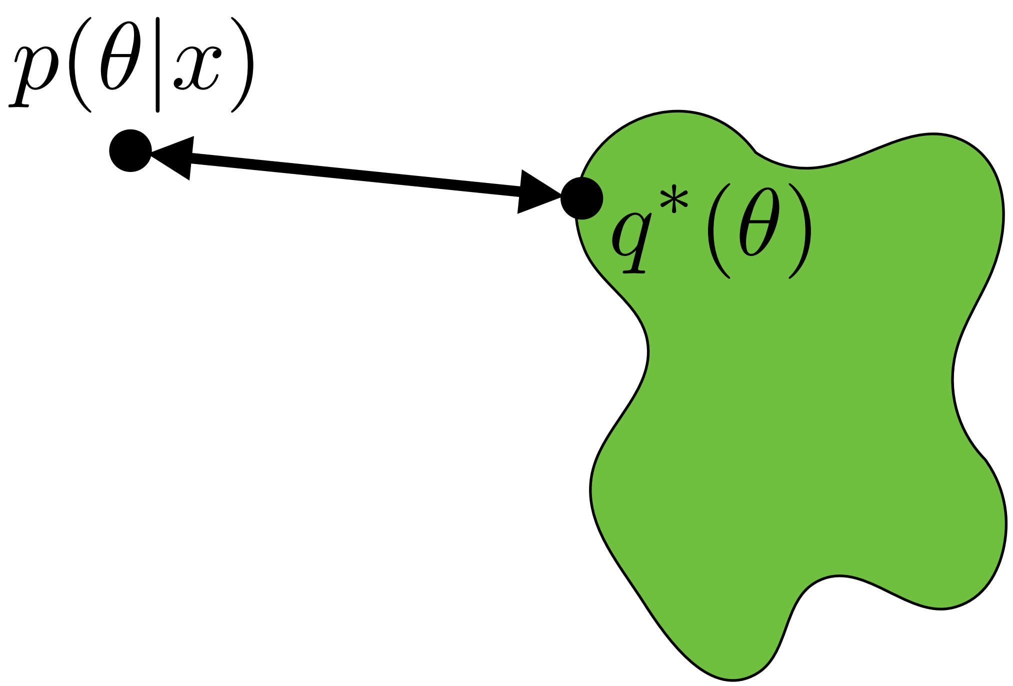

- Variational Bayes (VB)
  - Approximation $q^*(\theta)$ for posterior $p(\theta|x)$
  - "Close": Minimize Kullback-Liebler (KL) divergence:

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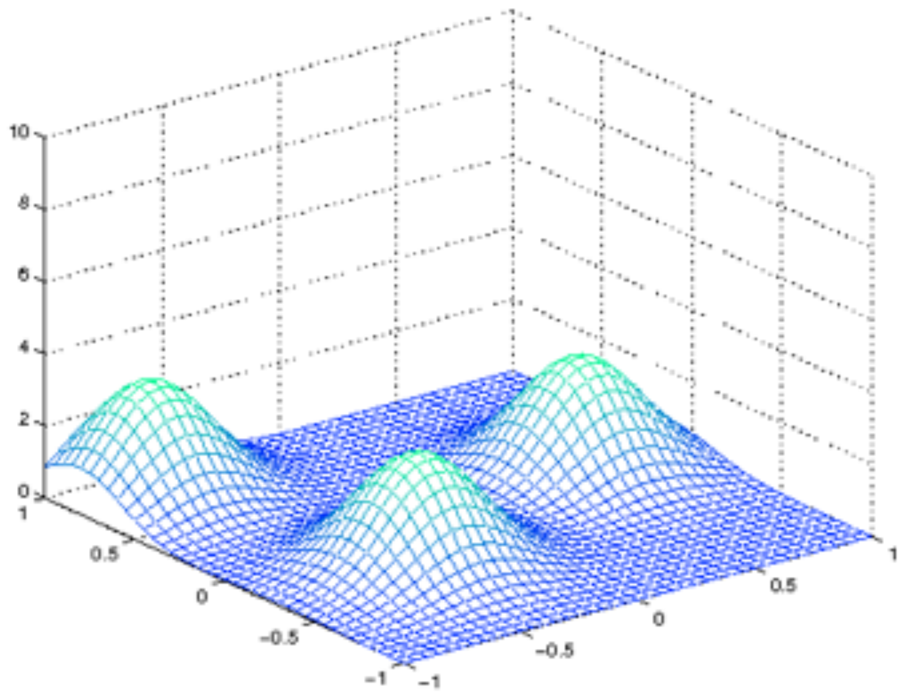
Variational Bayes



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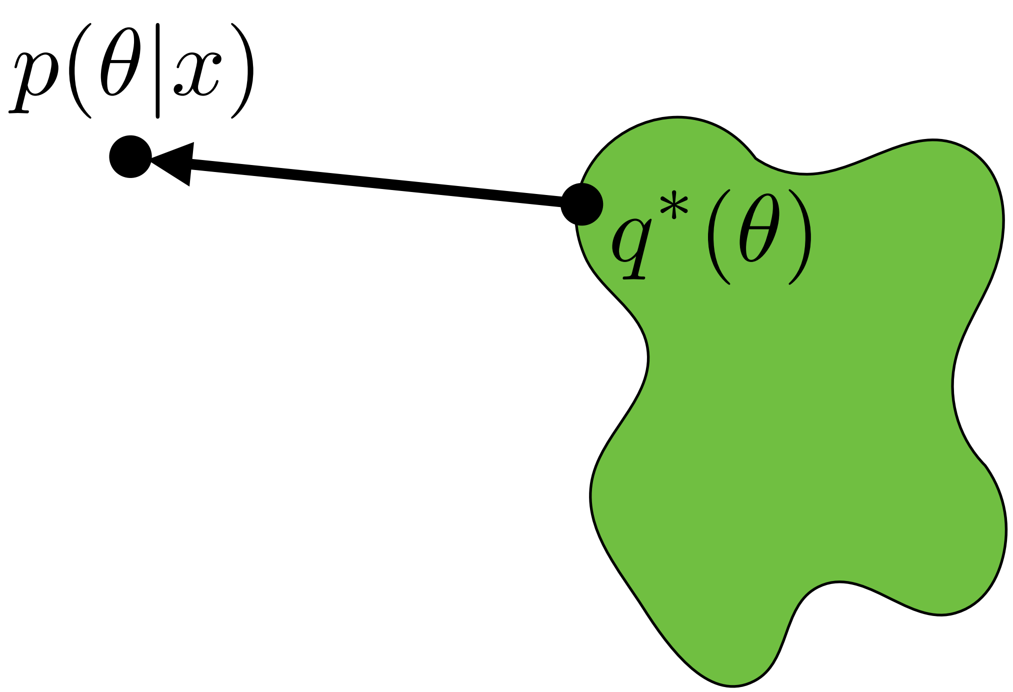

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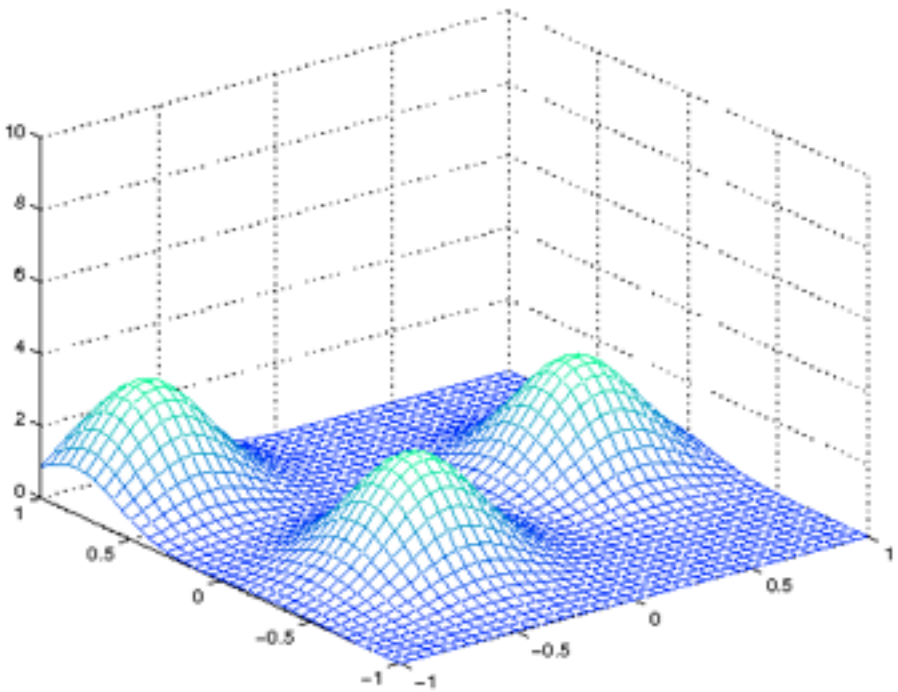

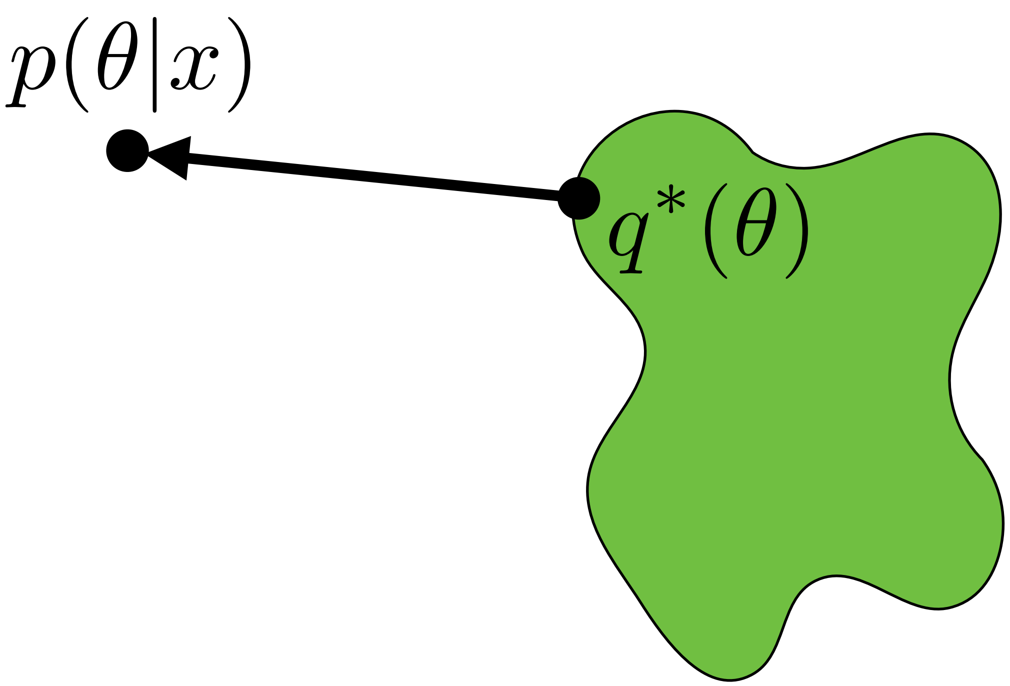

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  - "Close": Minimize Kullback-Liebler (KL) divergence:
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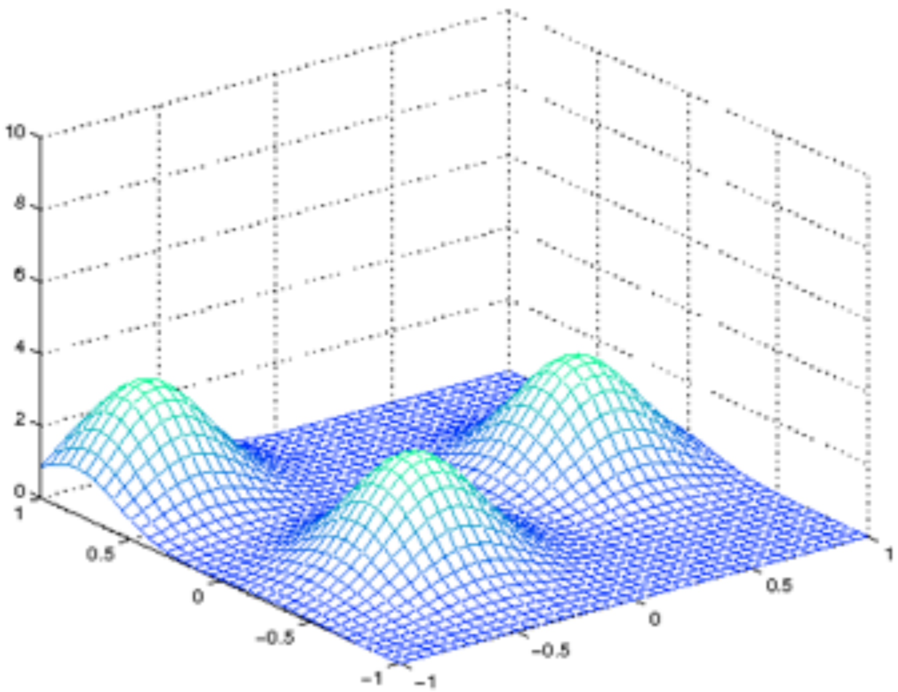
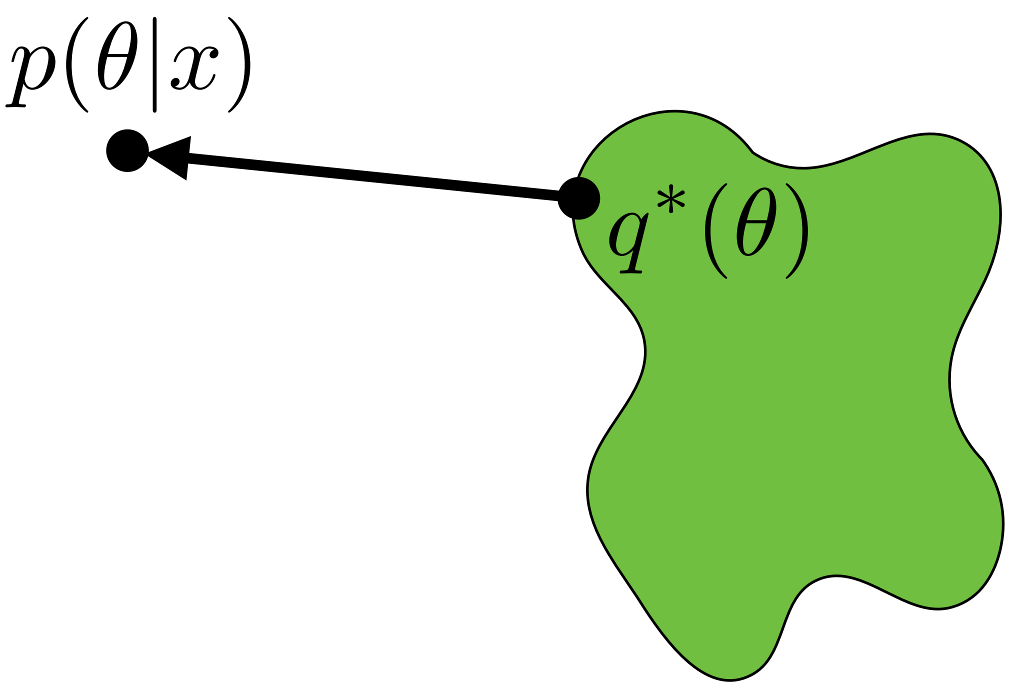
  - "Nice": factorizes, exponential family, truncation

Variational Bayes





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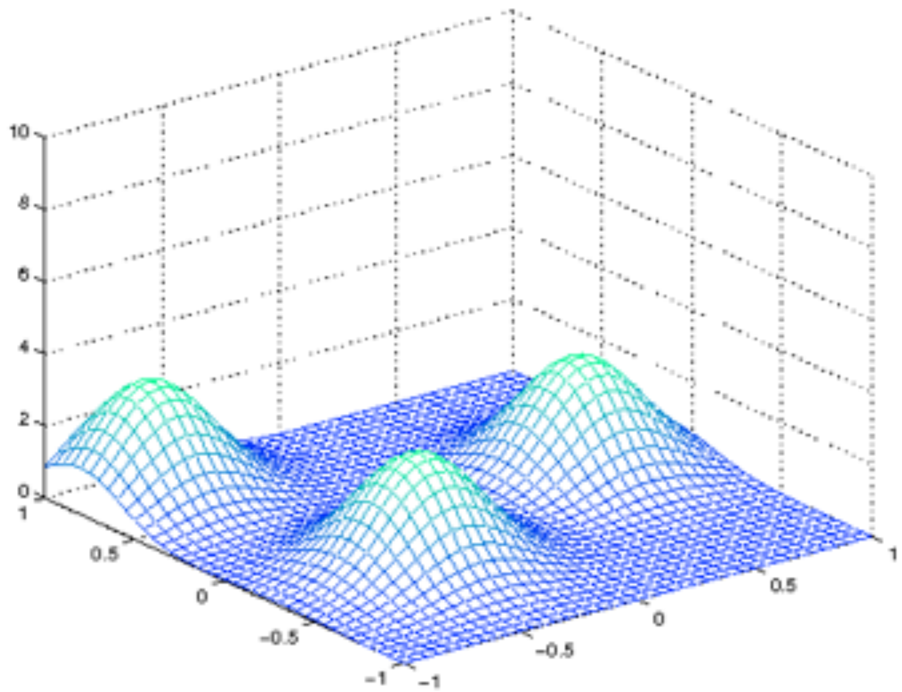

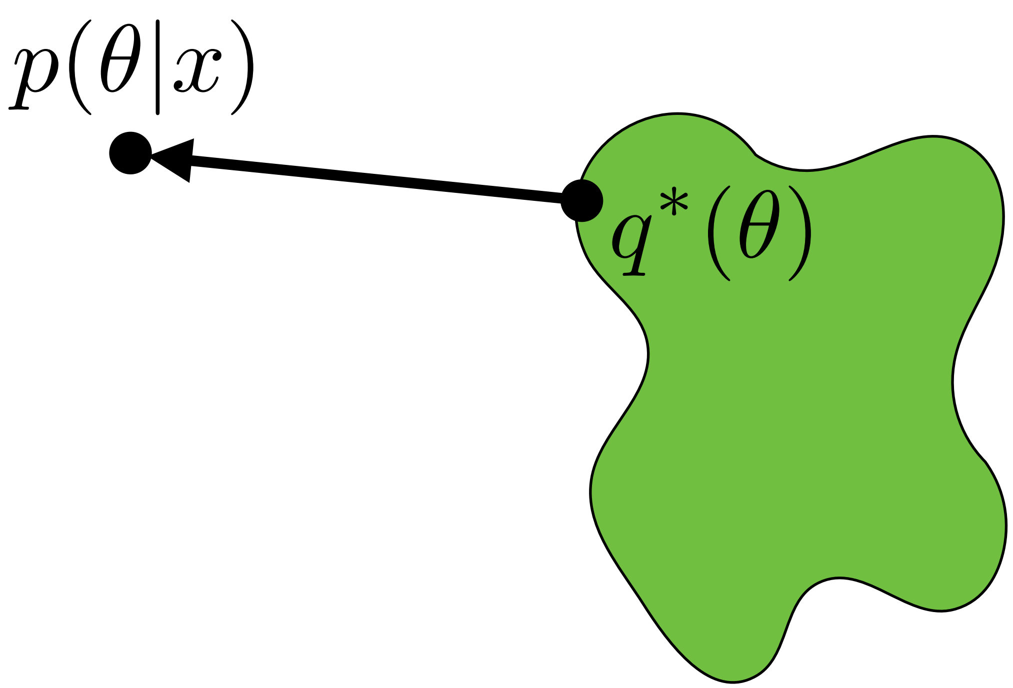

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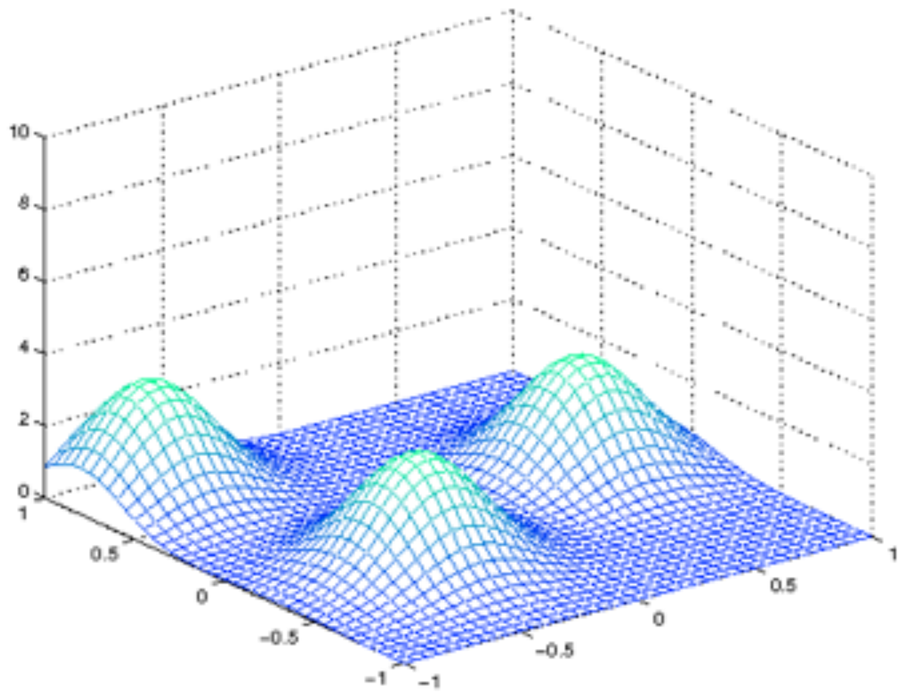
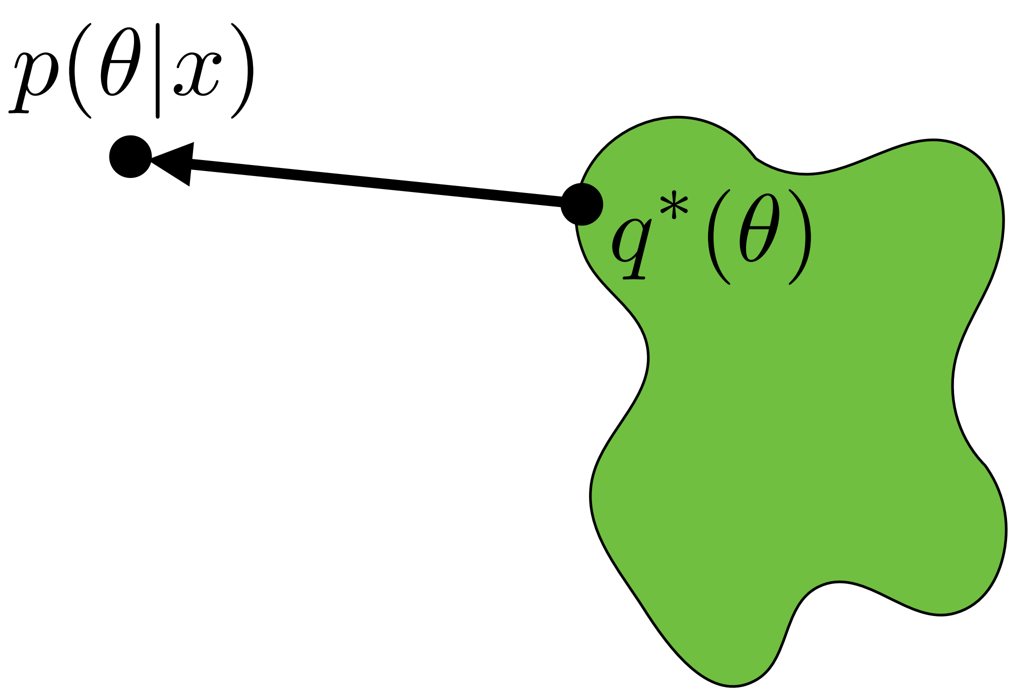
Variational Bayes



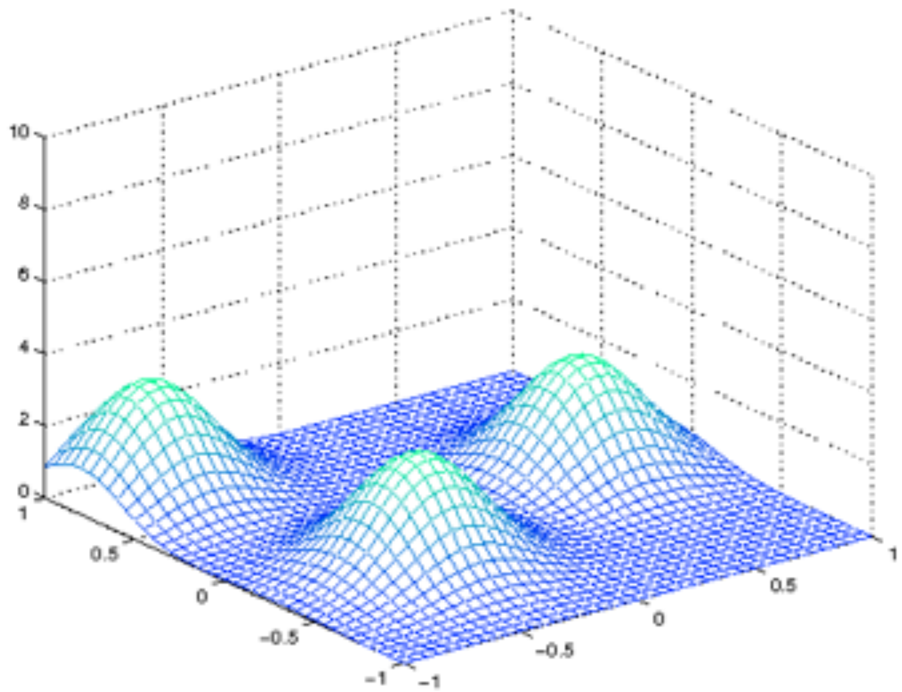


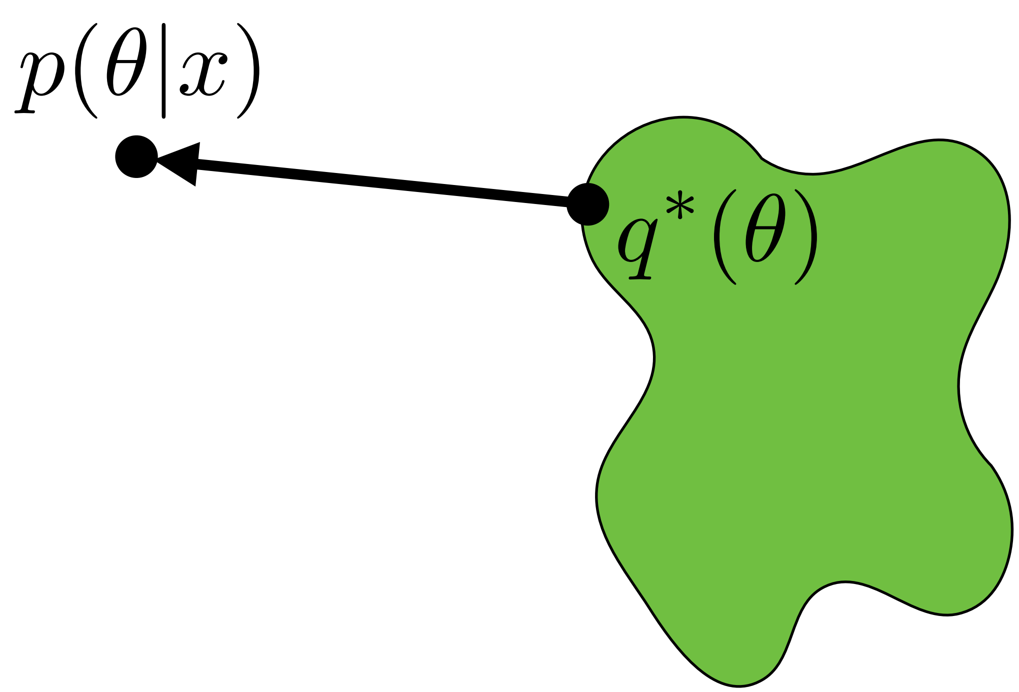
- Variational Bayes (VB)
  - Approximation $q^*(\theta)$ for posterior $p(\theta|x)$
  - "Close": Minimize Kullback-Liebler (KL) divergence:
    $$KL(q\|p(\cdot|x))$$
  - "Nice": factorizes, exponential family, truncation

- VB practical success
  - point estimates and prediction
  - fast, streaming, distributed

[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

Variational Bayes





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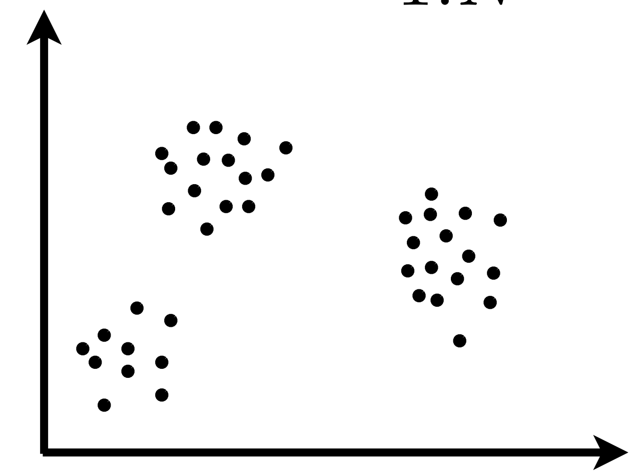
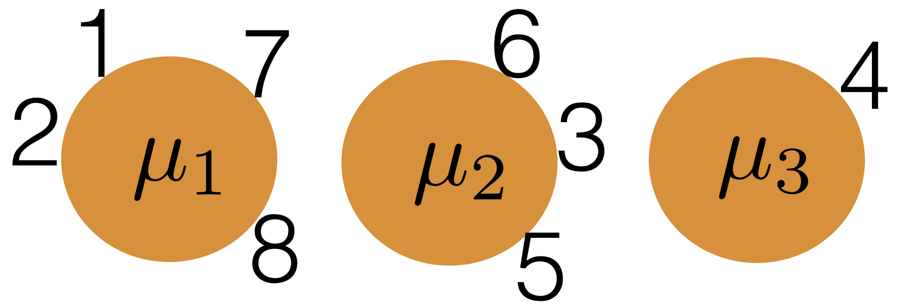
- VB practical success
  - point estimates and prediction
  - fast, streaming, distributed
  - Linear response VB (LRVB) for accurate covariance

[Broderick, Boyd, Wibisono, Wilson, Jordan 2013; Giordano, Broderick, Jordan 2015]

Exercises

- Data $x_{1:N}$





Exercises

- Code a CRP mixture model simulator

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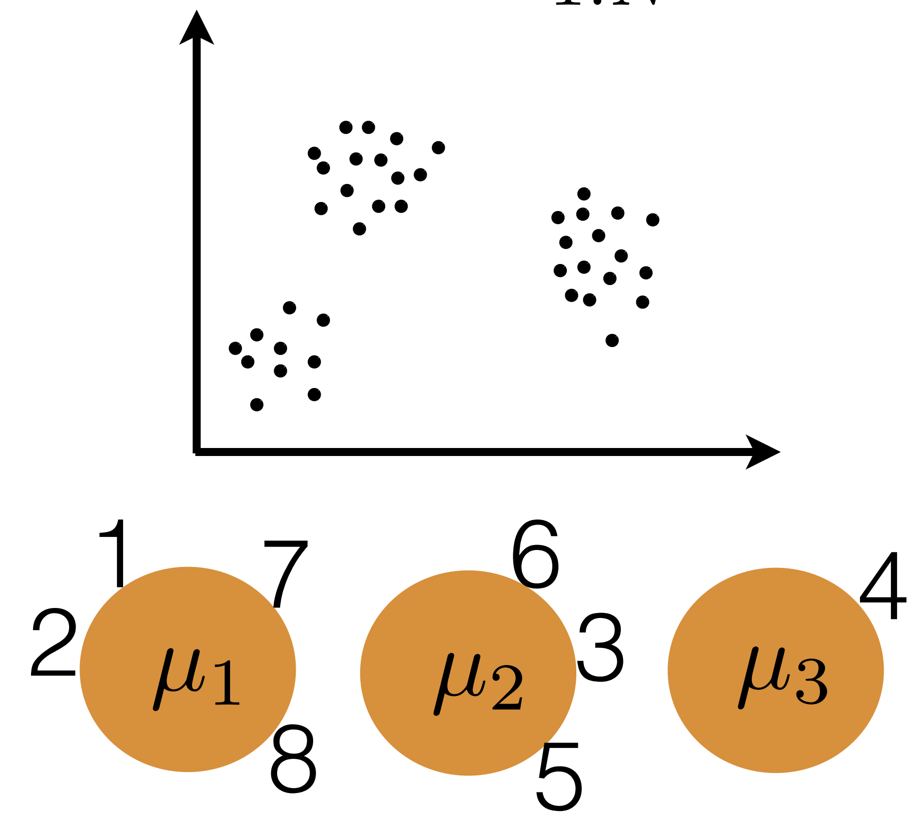

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- Code a CRP mixture model simulator
- Derive the CRP mixture model Gibbs sampler in the slides; derive $p(x_{C \cup \{n\}} | x_C)$ explicitly for a Gaussian mixture

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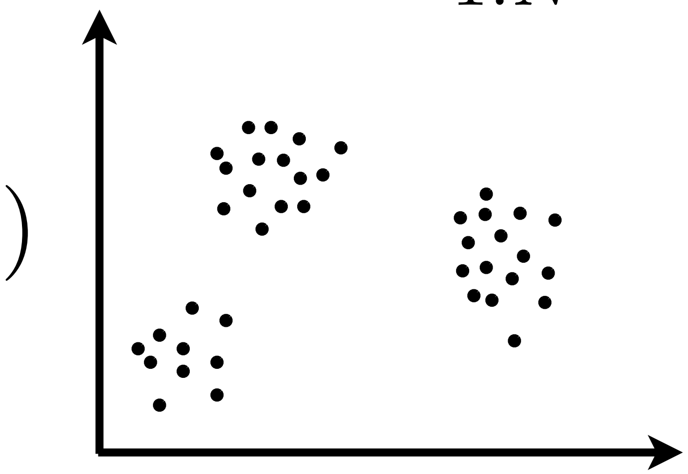

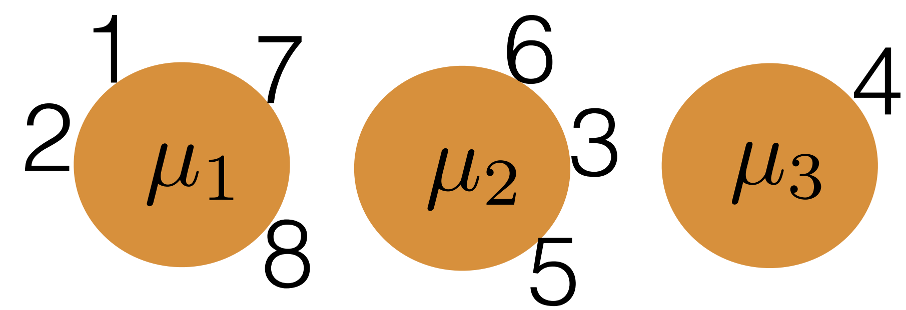

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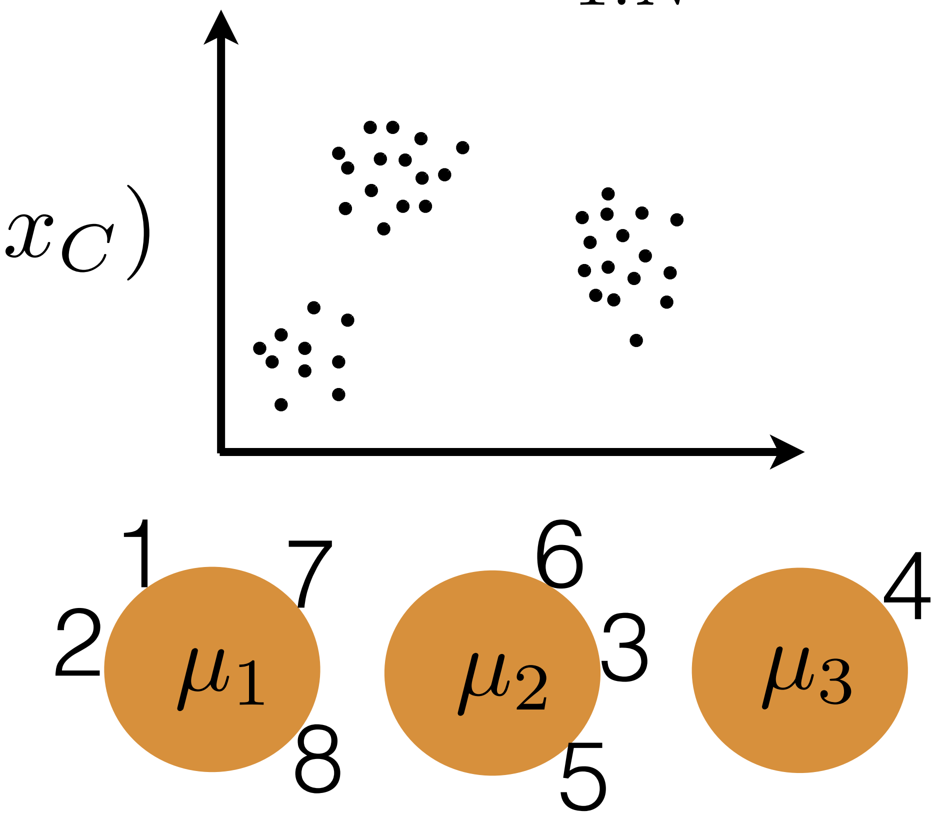

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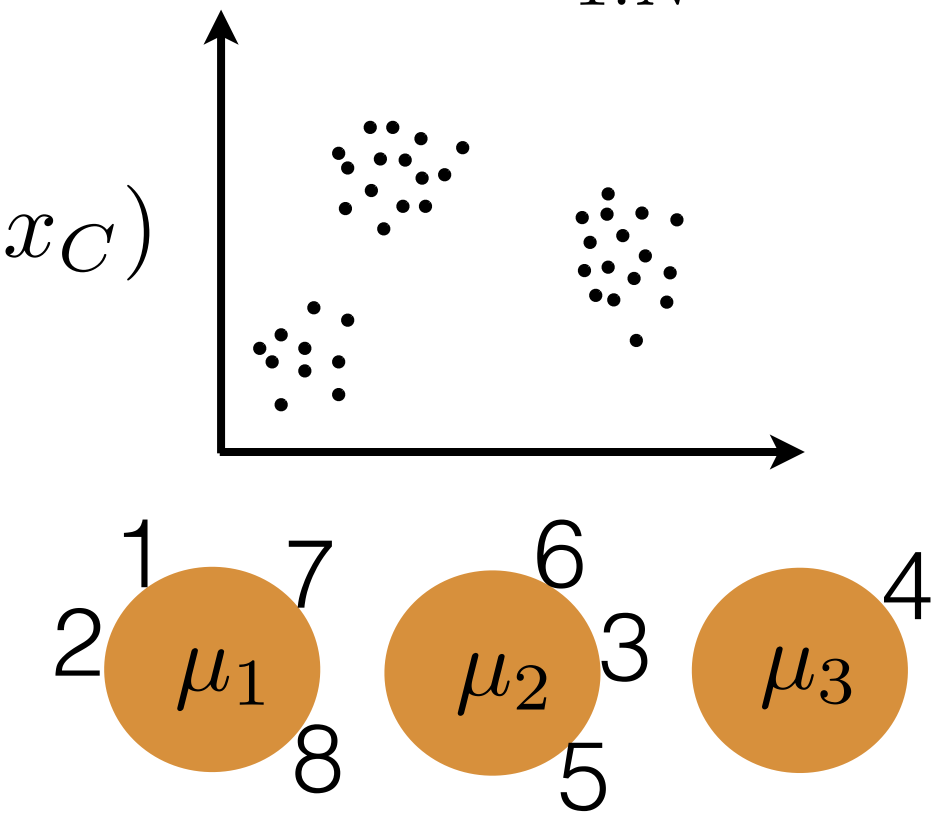

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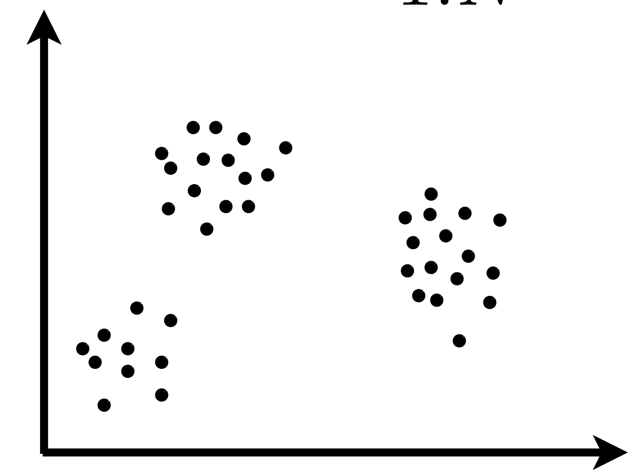

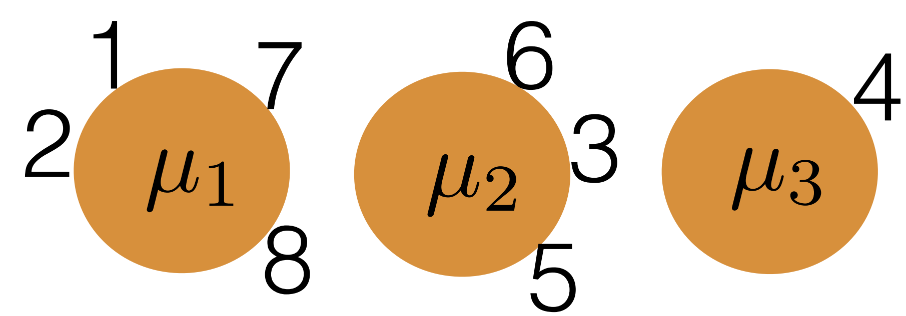

- Review Gibbs sampling, slice sampling [Neal 2003], variational Bayes [Bishop 2006]
- Read [Neal 2000] and code a DPMM Gibbs sampler

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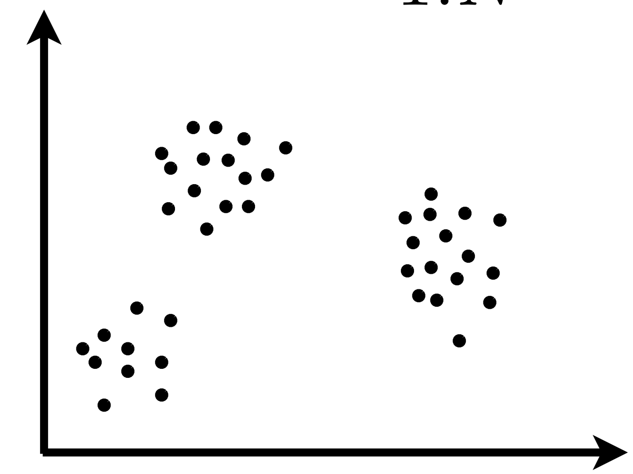

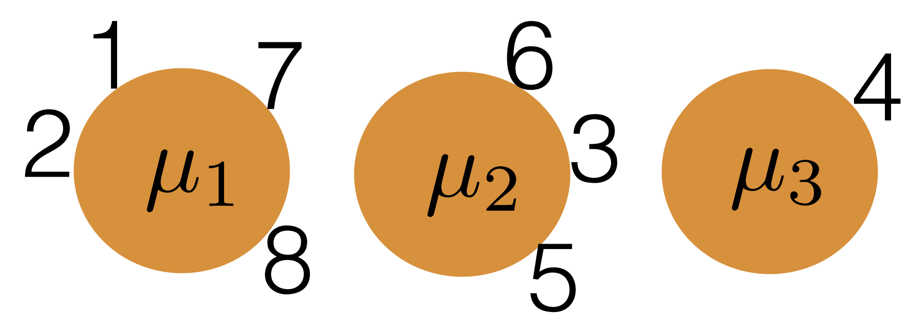

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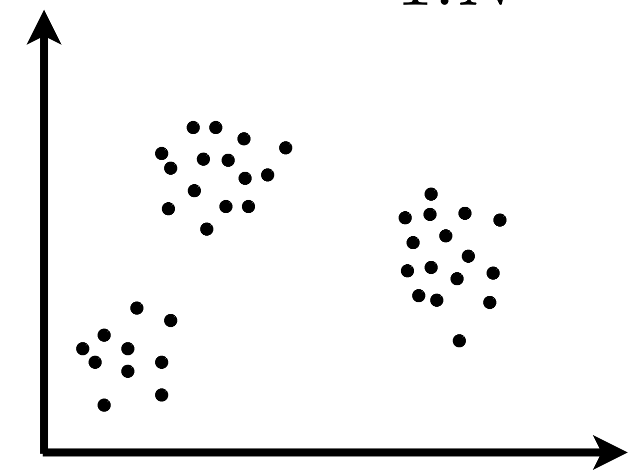
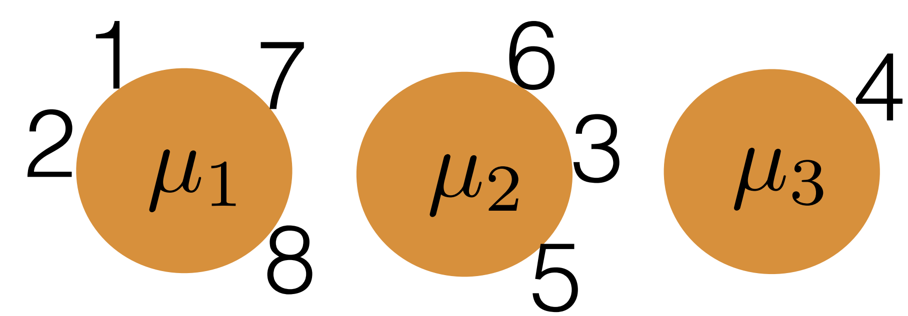
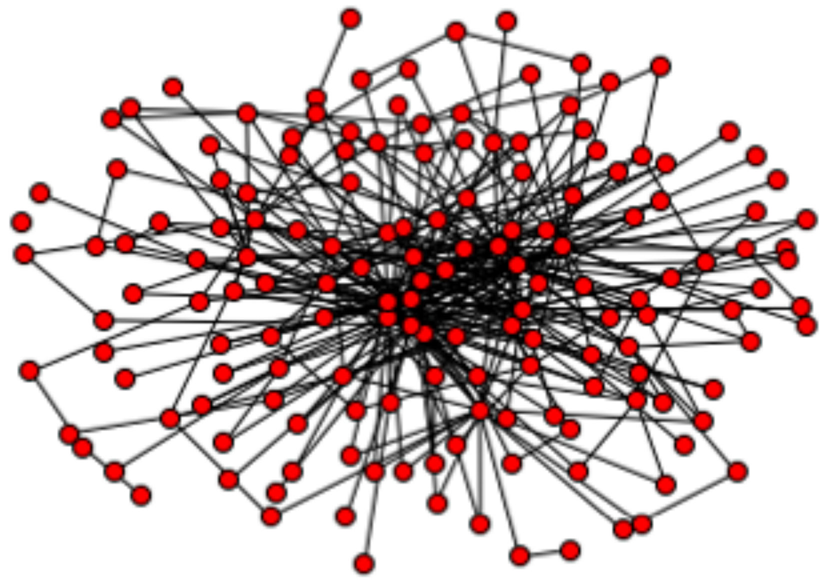
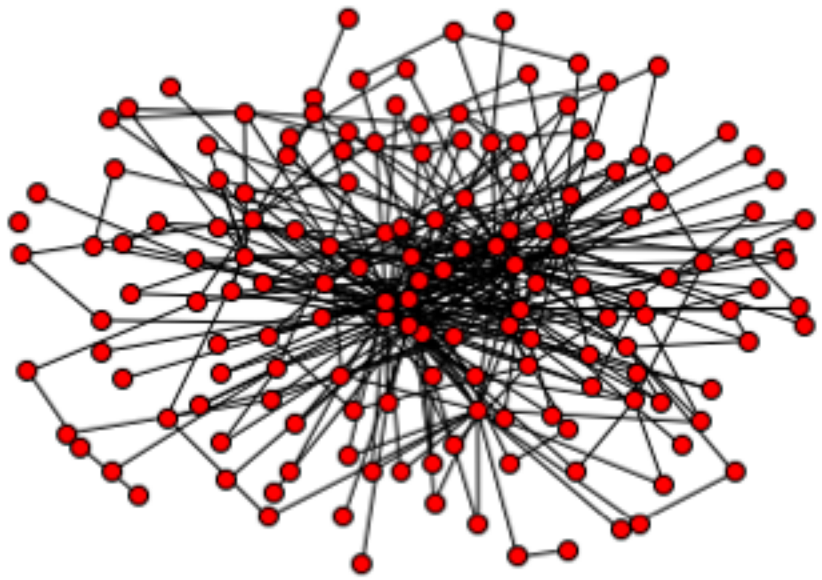




[Tiia Monto, https://commons.wikimedia.org/wiki/File:Trail_and_mountain.jpg]



[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008; Lloyd et al 2012]



social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

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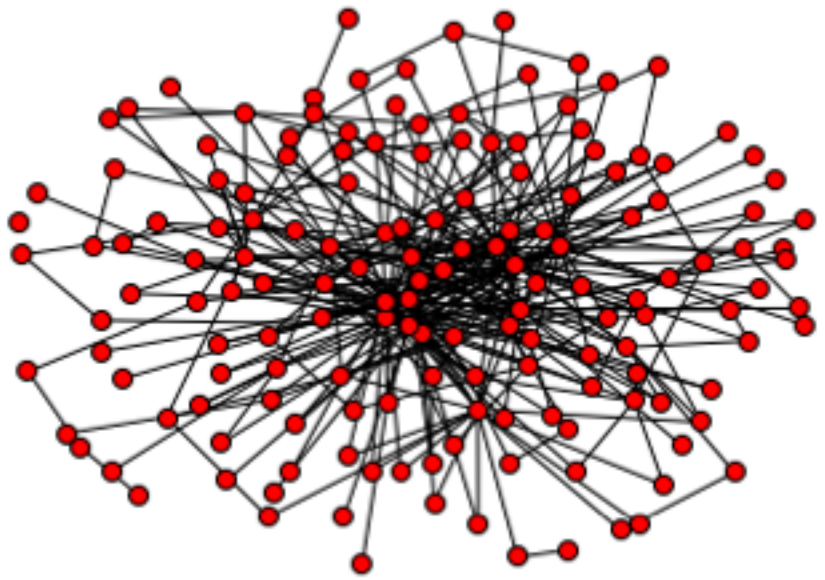
Probabilistic models for graphs

$p($  $)$

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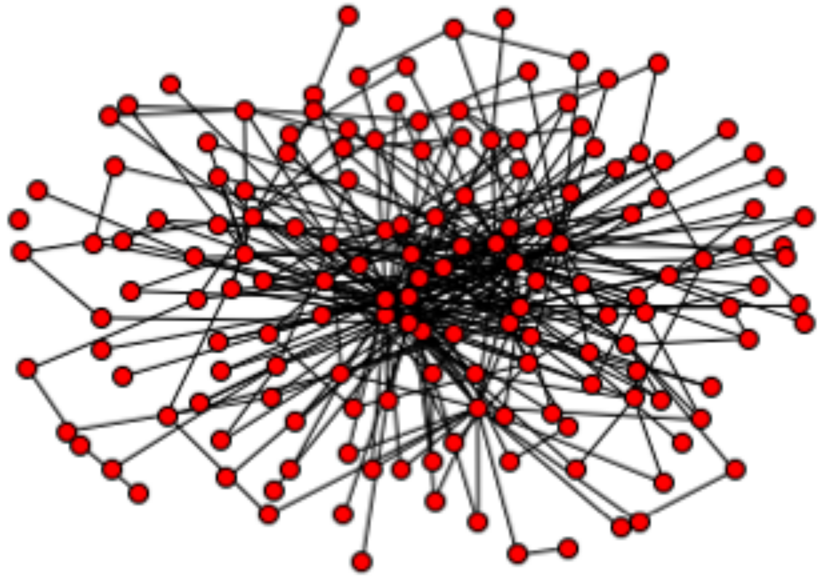
$p($  $)$

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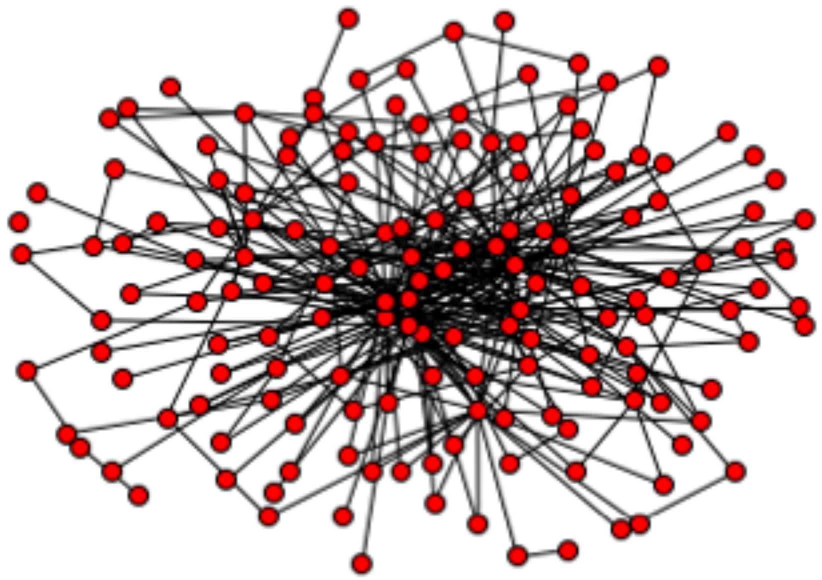
$p($  $)$

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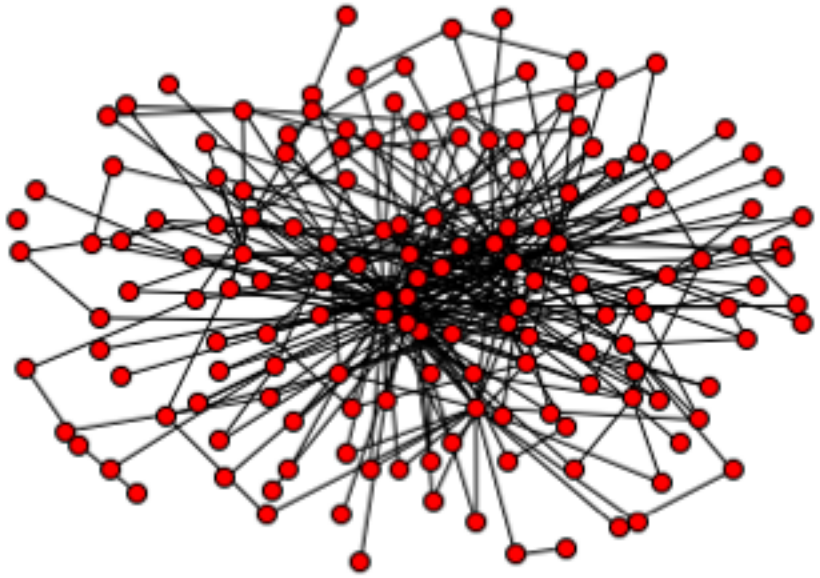
$p($  $)$

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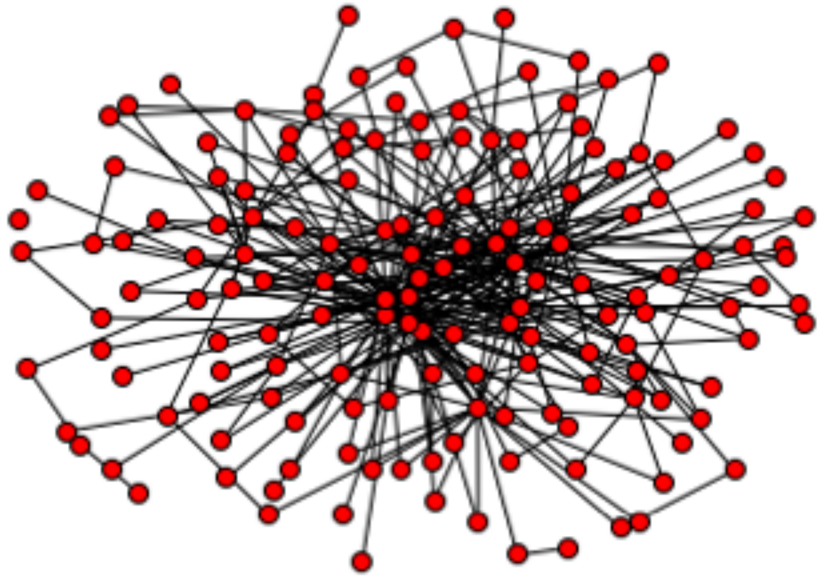
$p($  $)$

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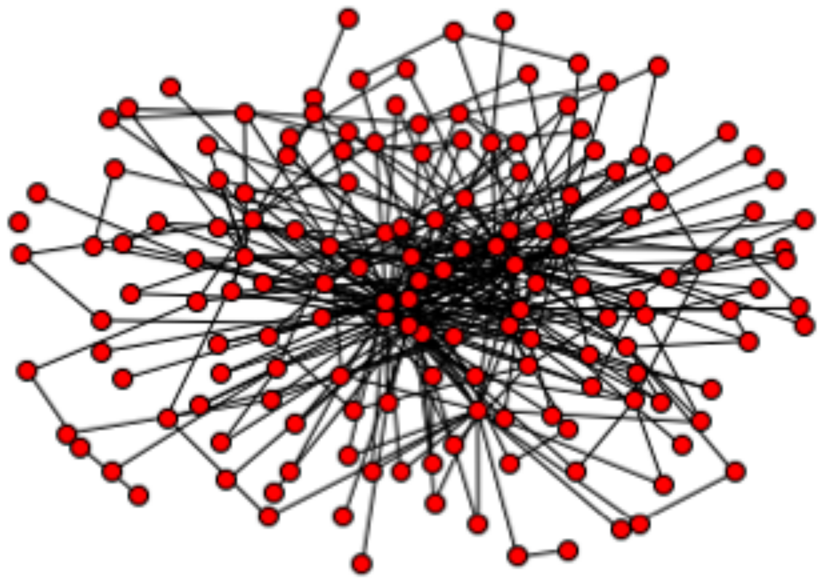
$p($  $)$

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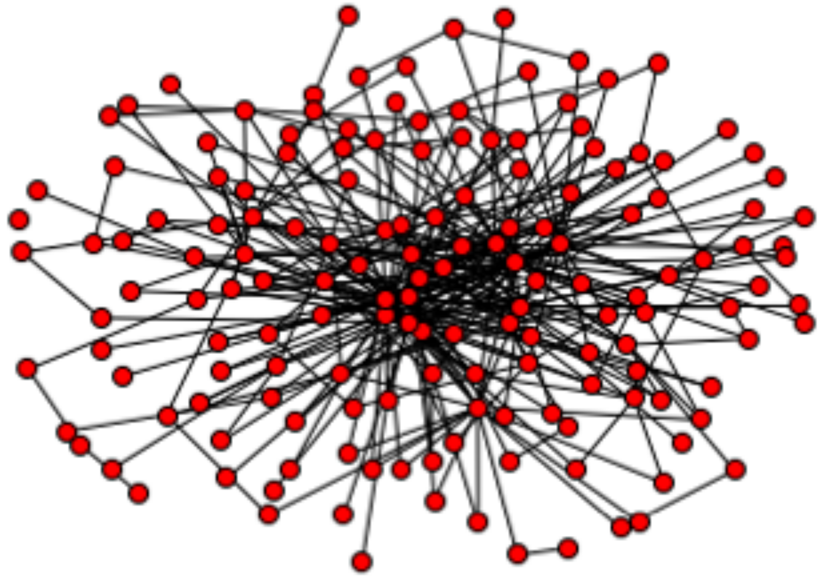
$p($  $)$

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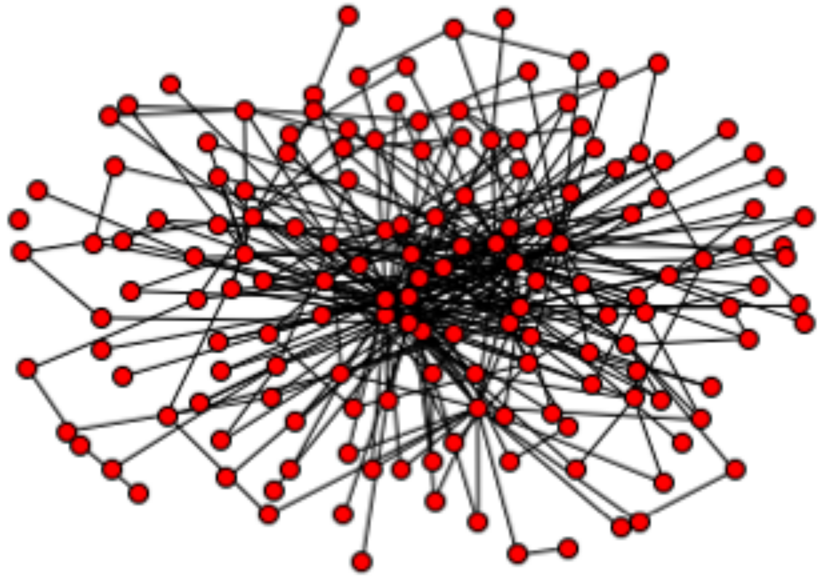
$p($  $)$

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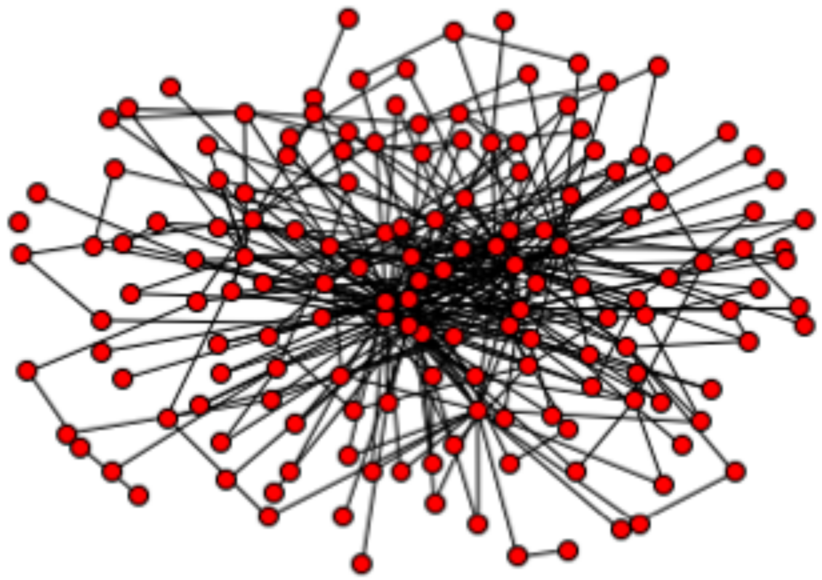
$p($  $)$

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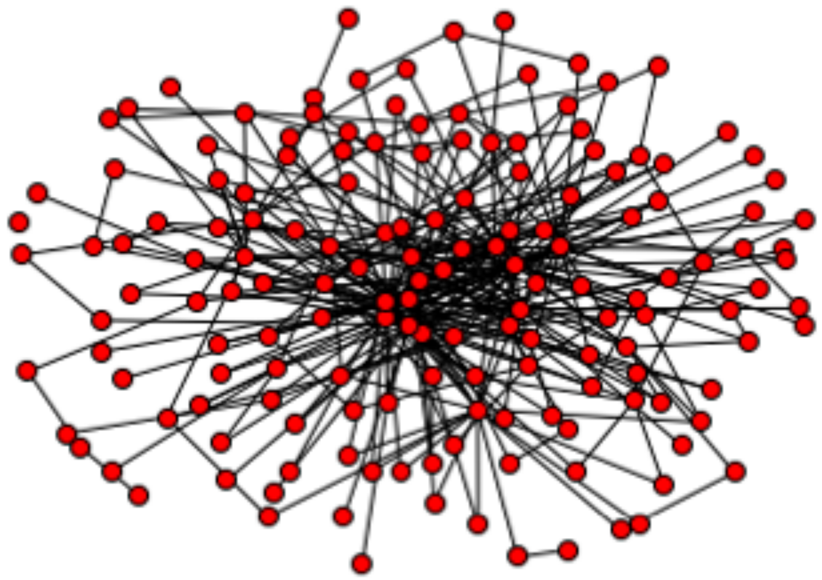
$p($  $)$

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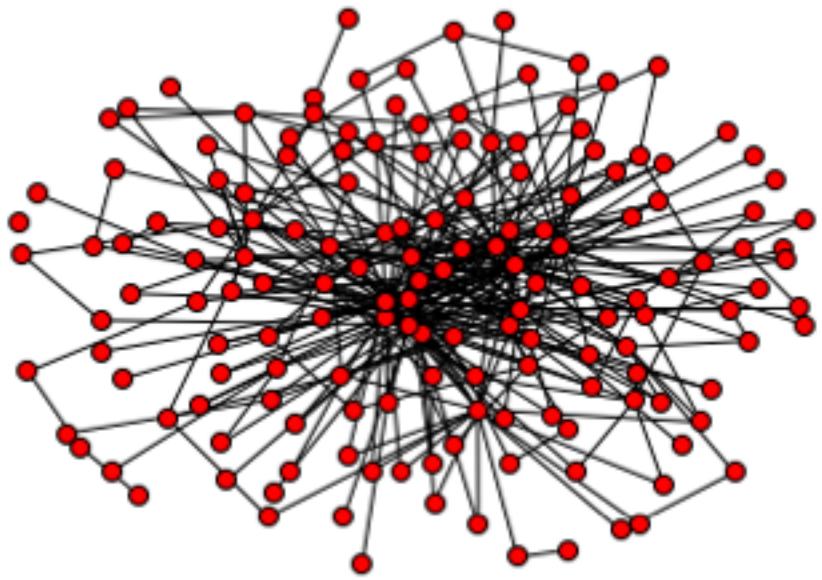
$p($  $)$

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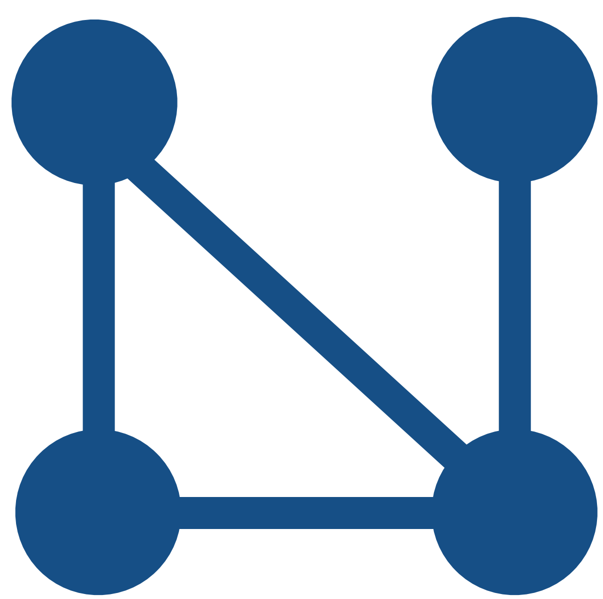
  - Concurrent & independent graphs work by Crane & Dempsey

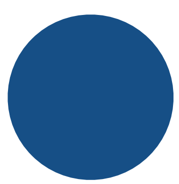
 [B, Cai 2015; Cai, B 2015a,b; Crane, Dempsey 2015a,b,16a,b; Cai, Campbell, B 2016; Campbell, Cai, B 2016]

Sequence of graphs

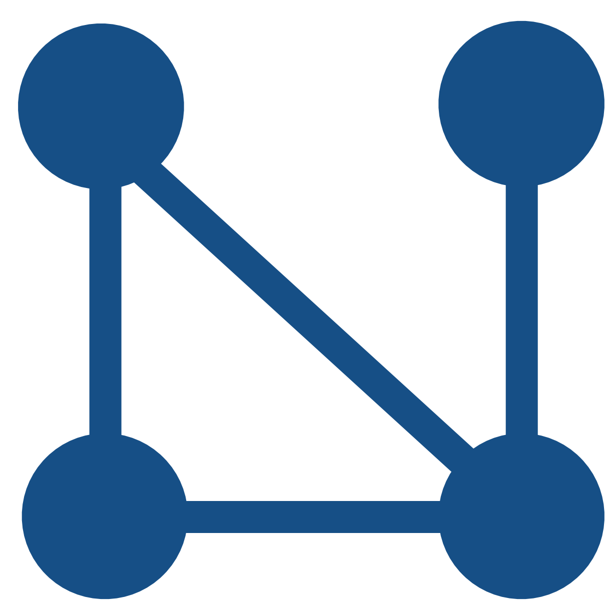


G

Sequence of graphs



G_1



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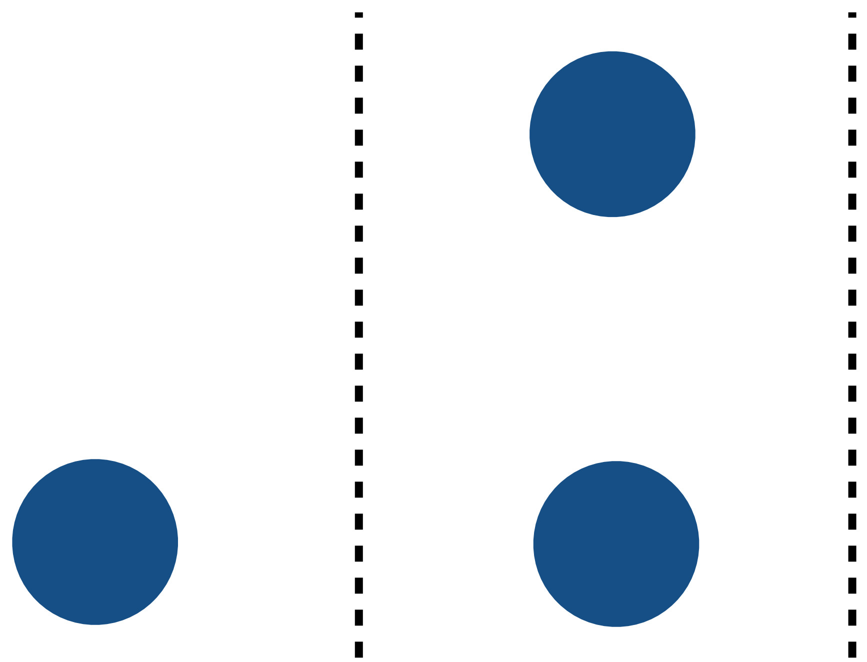
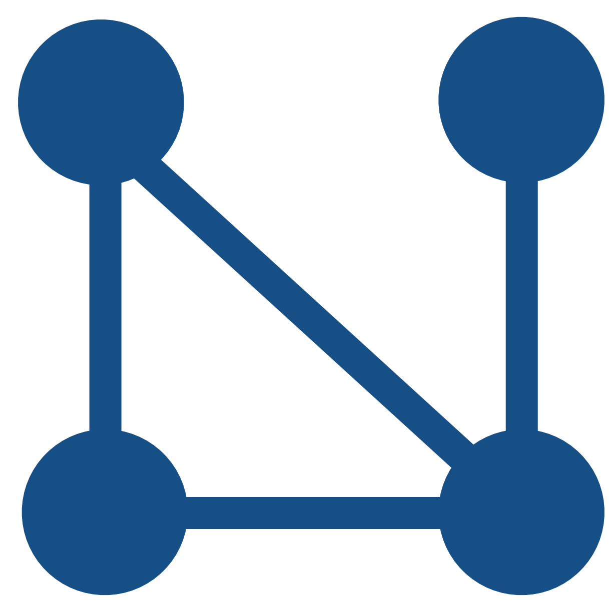


G_1 G_2



G

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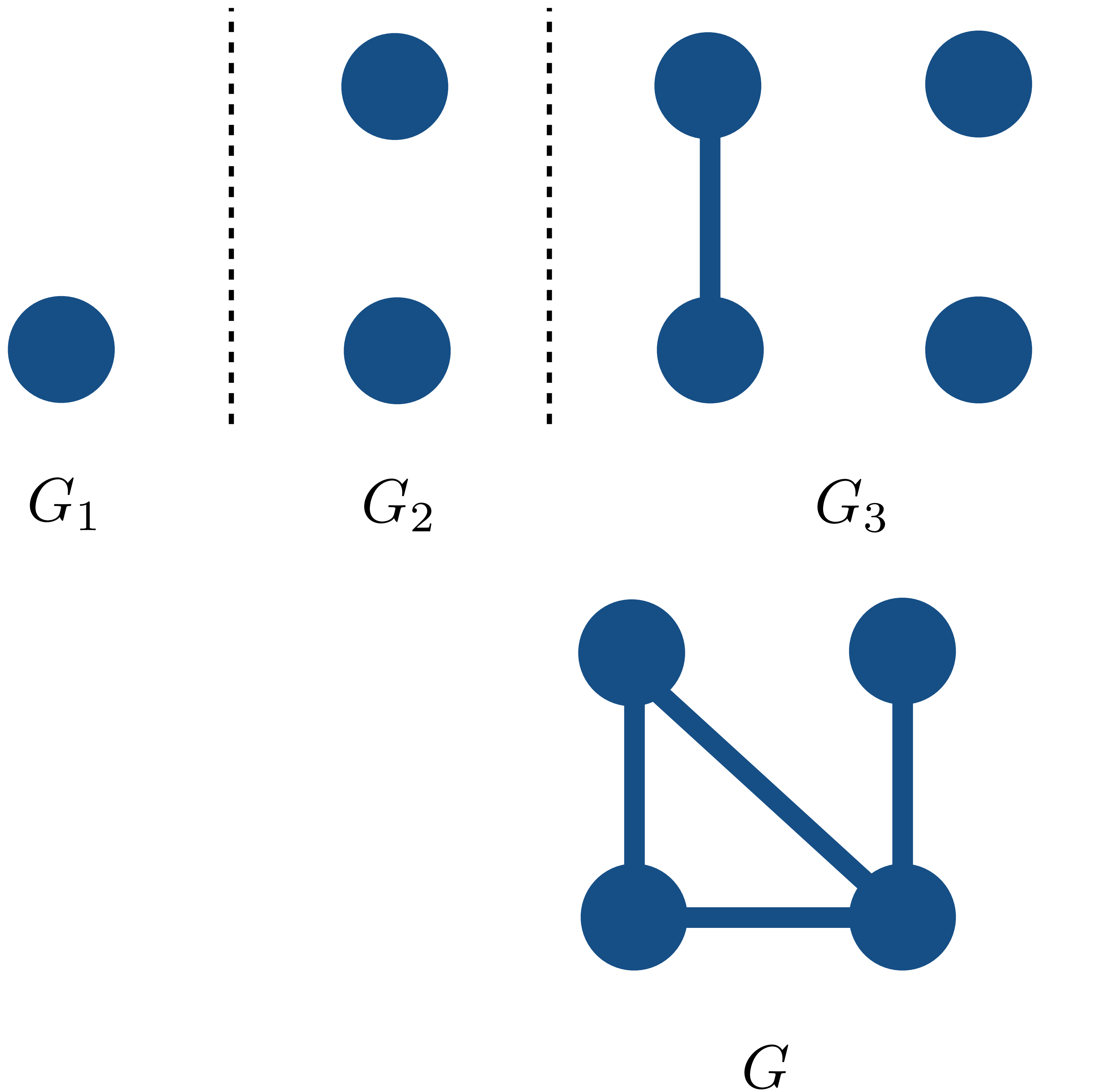

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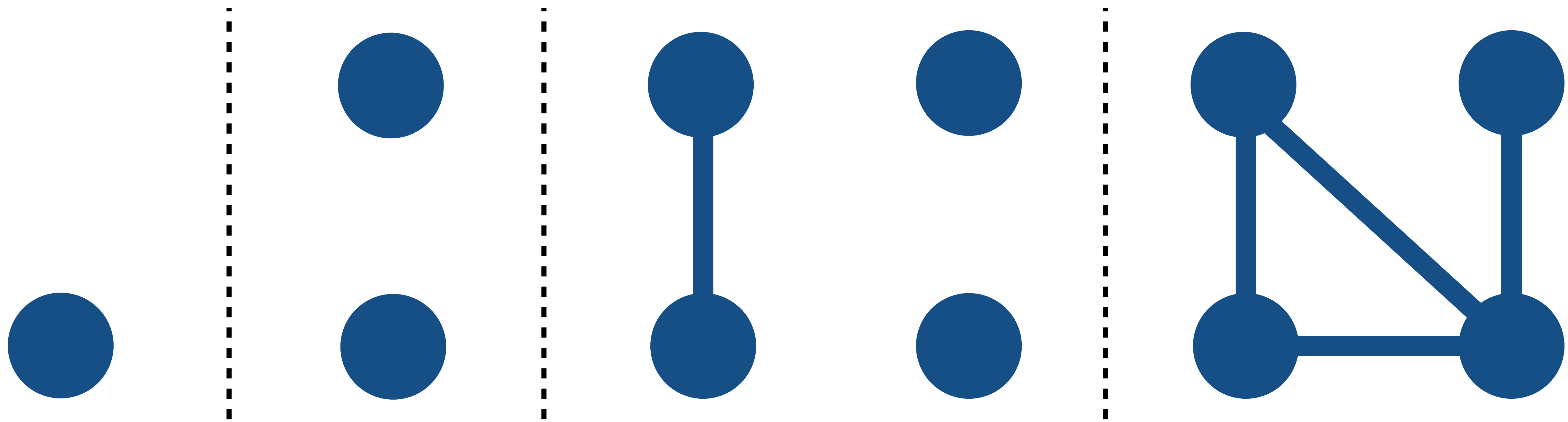
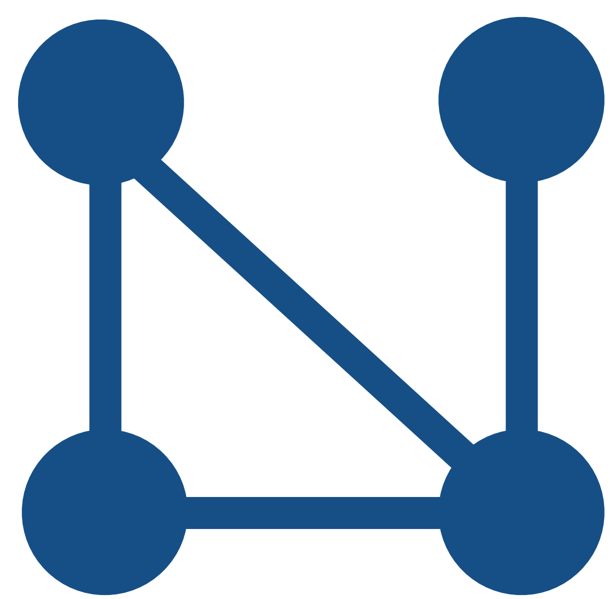


G_1 G_2 G_3 G_4



G

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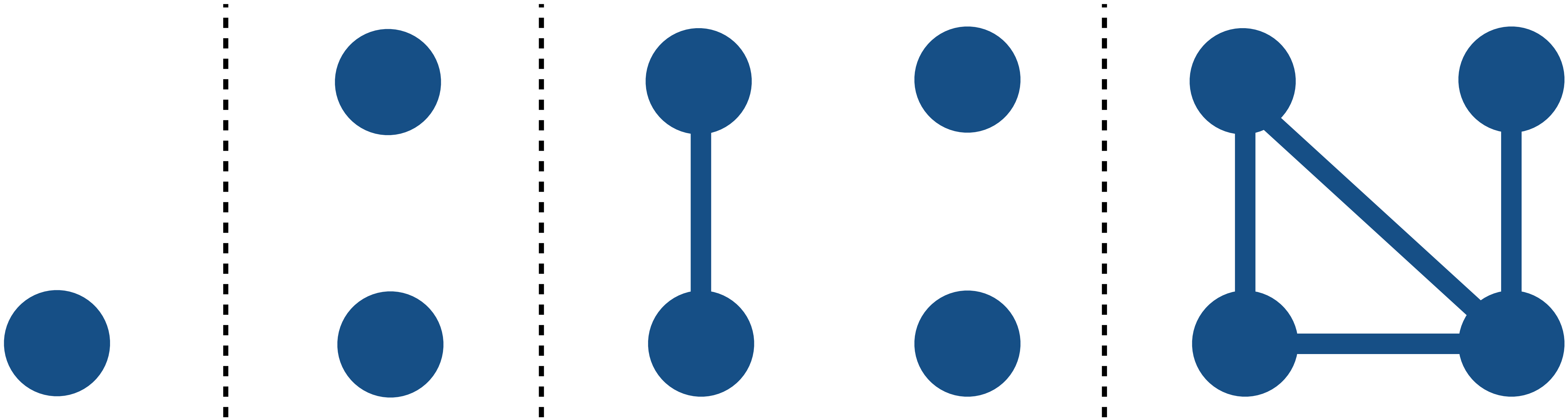
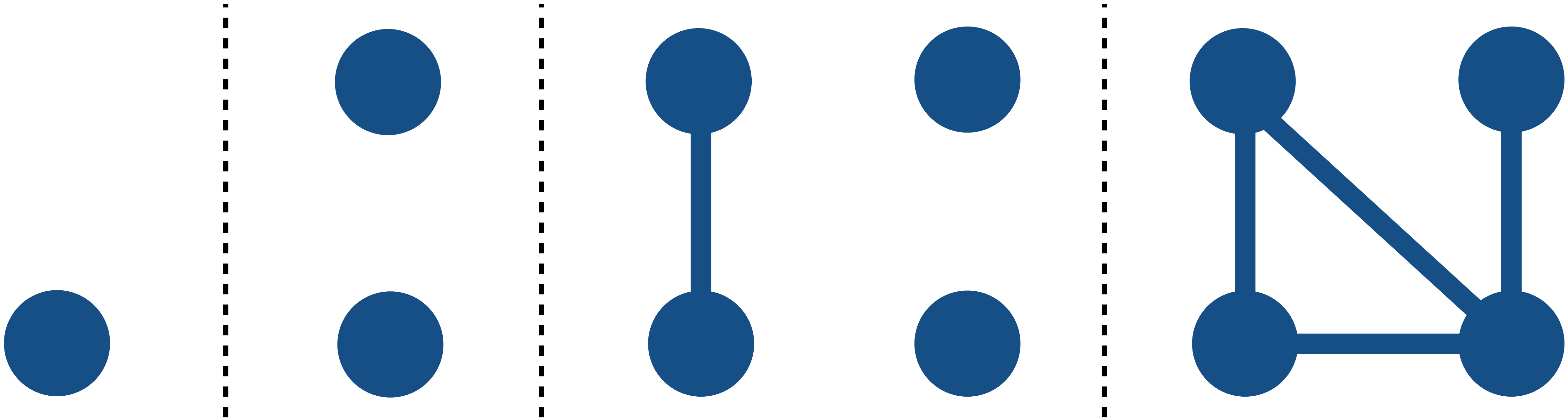


G_1 G_2 G_3 G_4

...

Sequence of graphs



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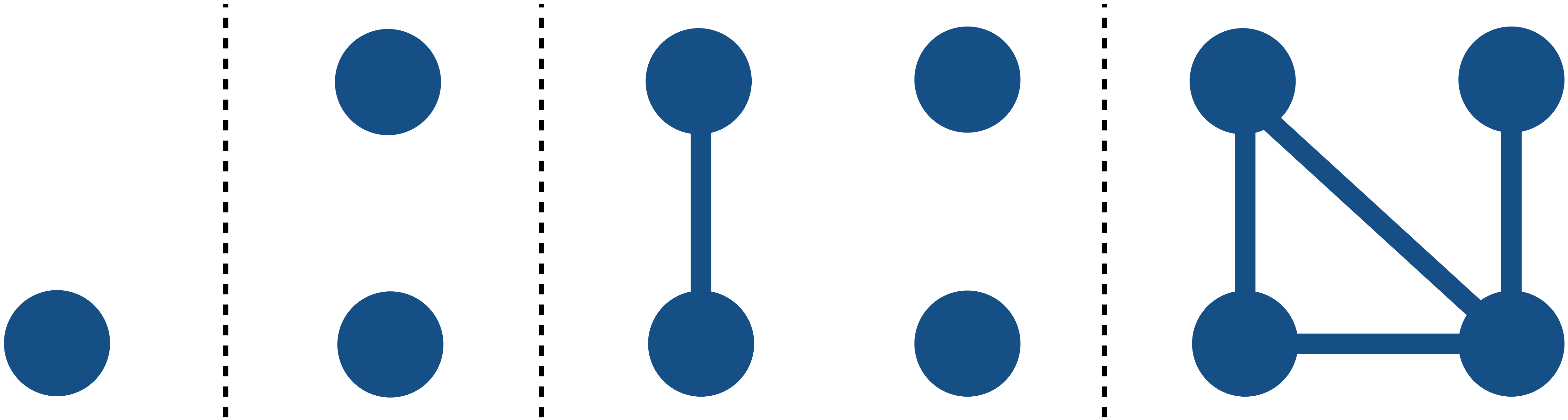
If $\#\mathrm{nodes}(G_n) \to \infty$,

Sequence of graphs



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If $\#\text{nodes}(G_n) \to \infty$,

- *Dense* graph sequence $\#\text{edges}(G_n) \geq c \cdot [\#\text{nodes}(G_n)]^2$

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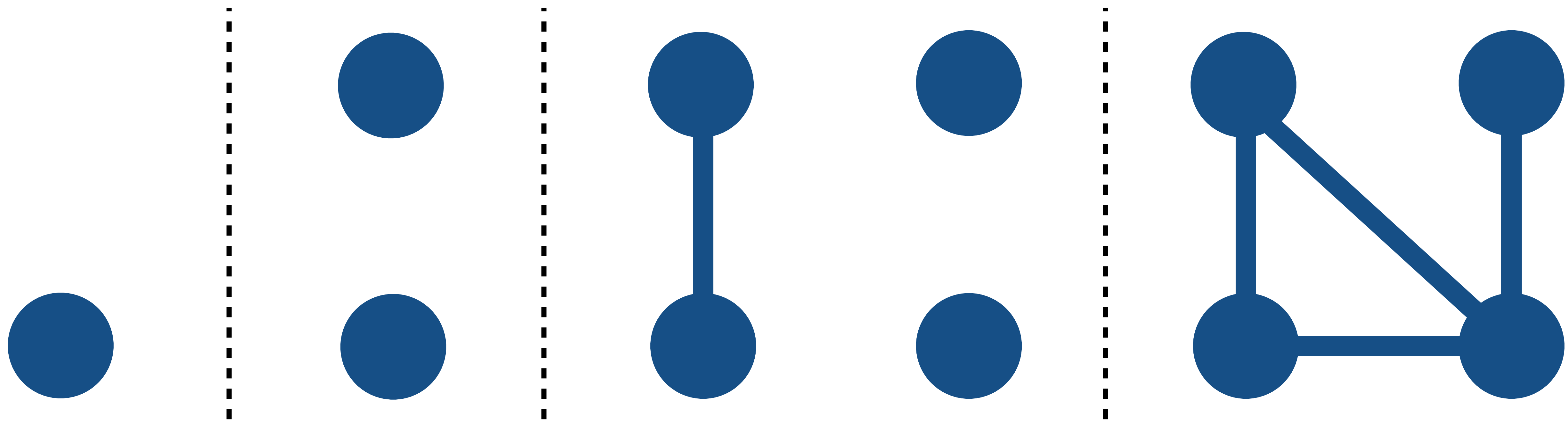

G_1 G_2 G_3 G_4

If $\#\text{nodes}(G_n) \to \infty$,

- *Dense* graph sequence $\#\text{edges}(G_n) \geq c \cdot [\#\text{nodes}(G_n)]^2$
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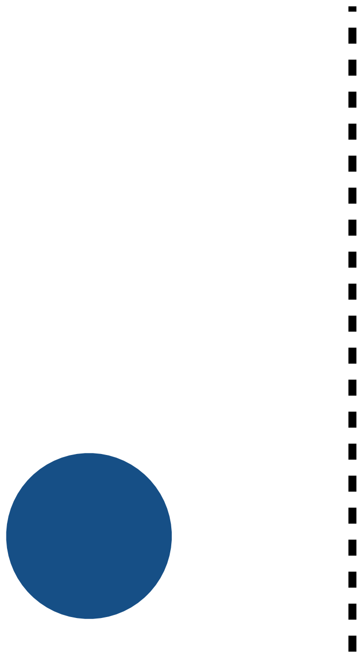
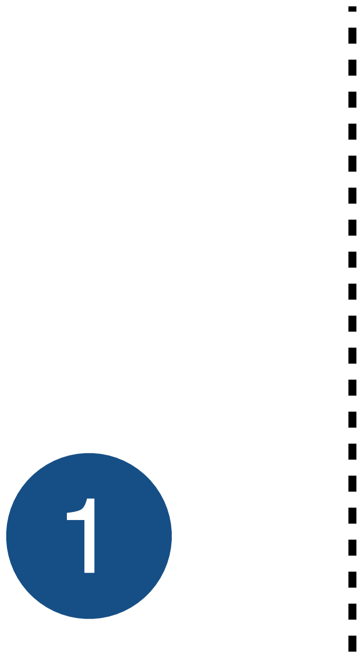
The Old Way: Nodes



G_1

[Hoover 1979, Aldous 1981]

The Old Way: Nodes



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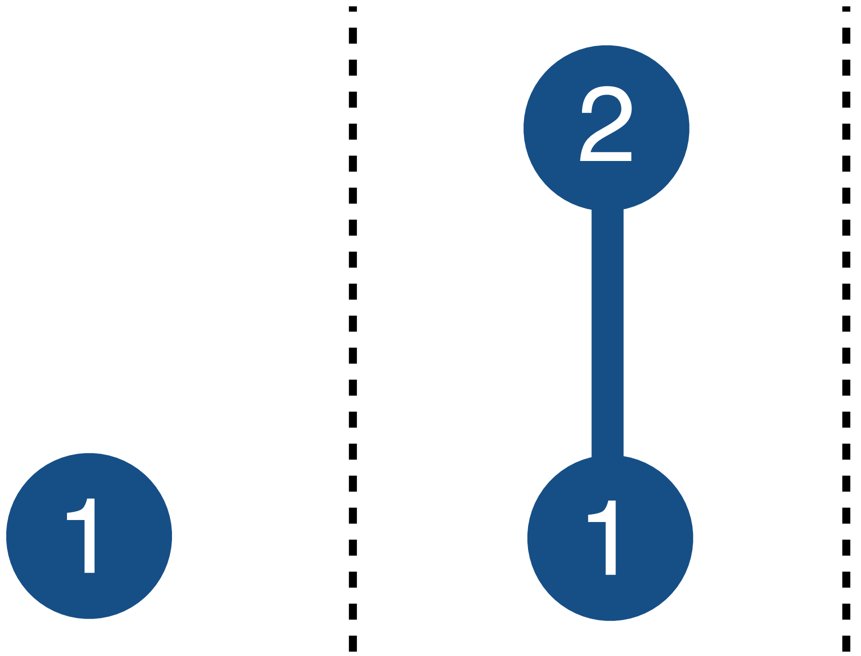

G_1 G_2

[Hoover 1979, Aldous 1981]

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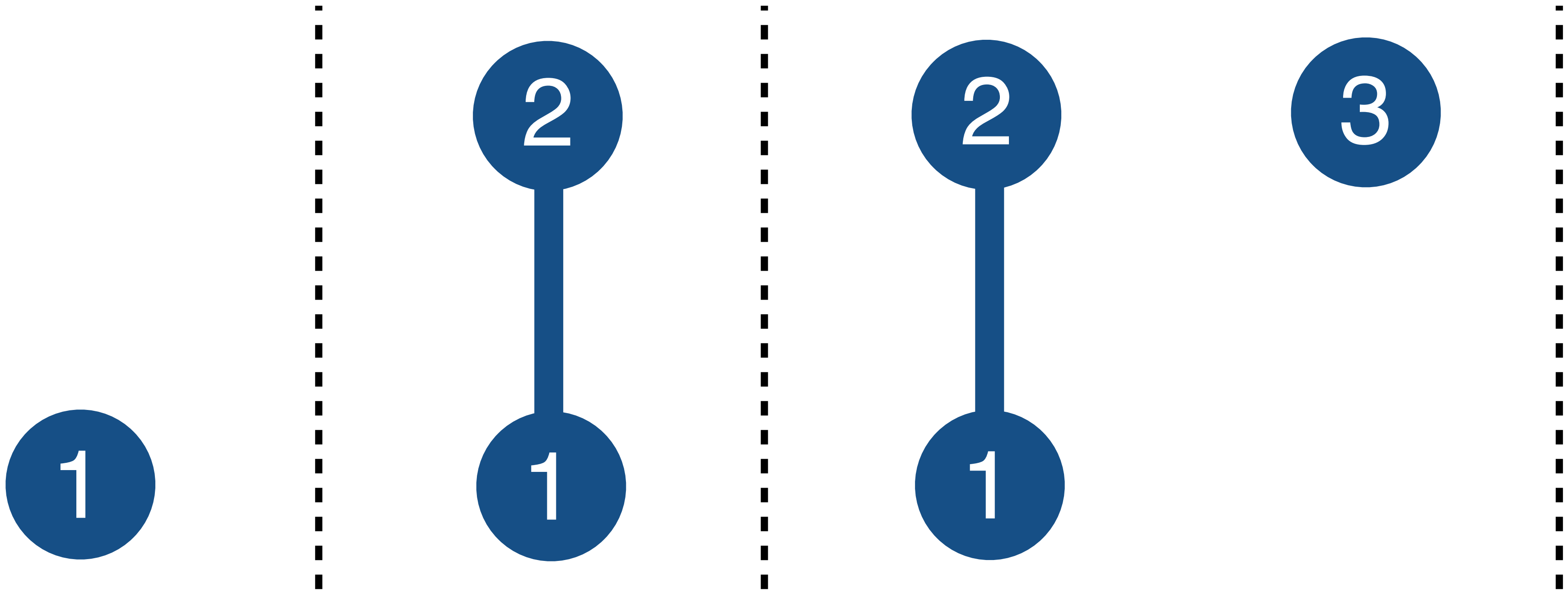

G_1 G_2 G_3

[Hoover 1979, Aldous 1981]

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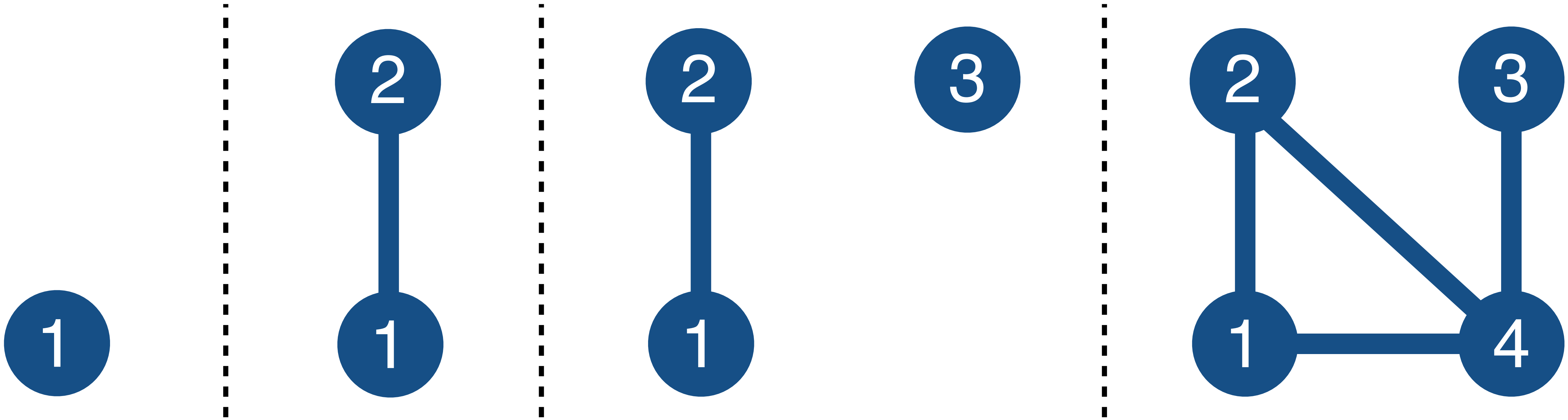
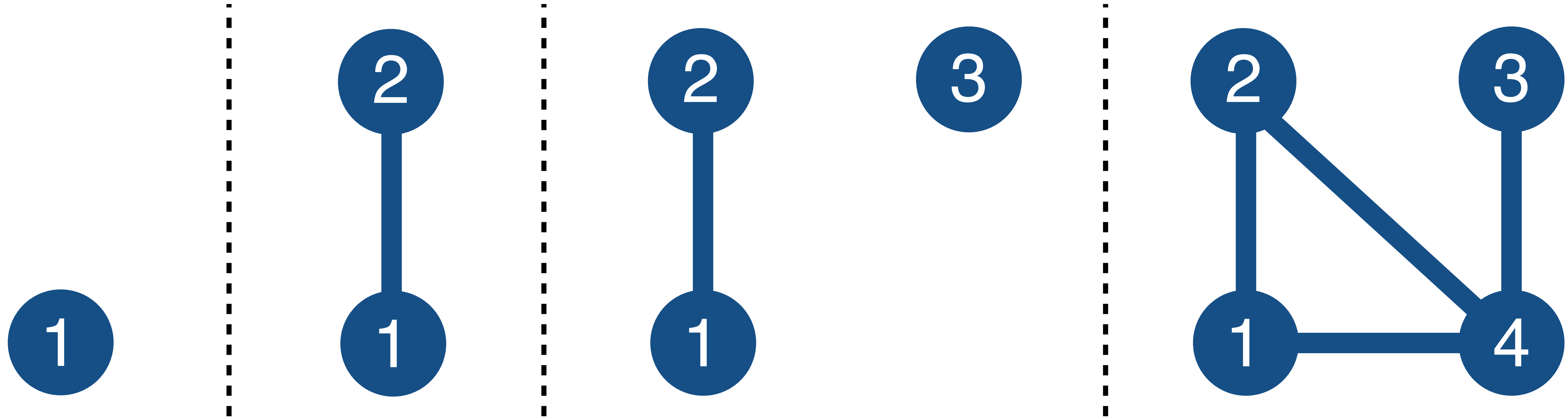


G_1 G_2 G_3 G_4

[Hoover 1979, Aldous 1981]

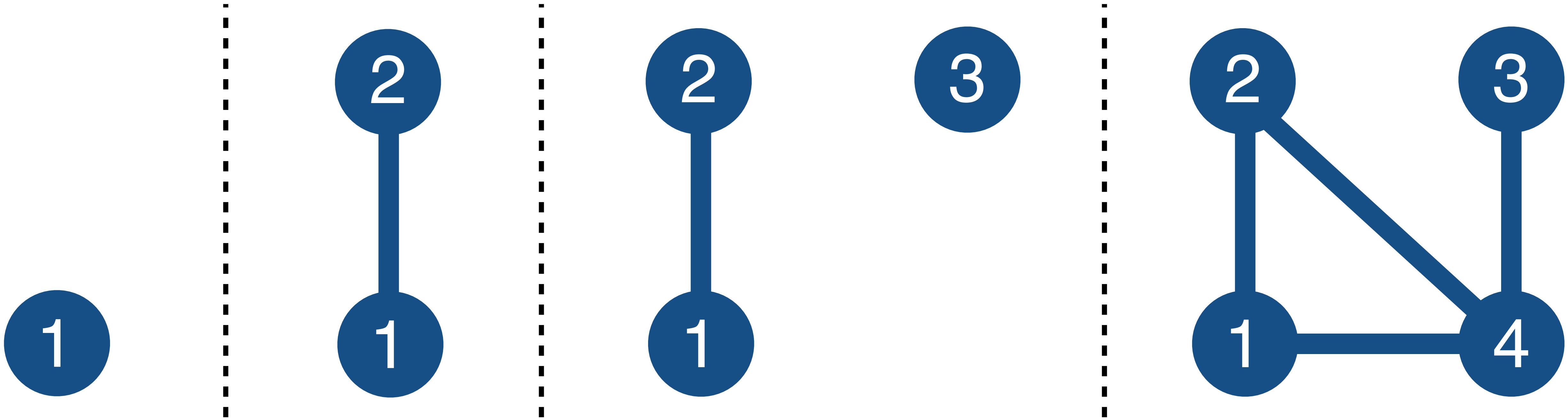
The Old Way: Exchangeability



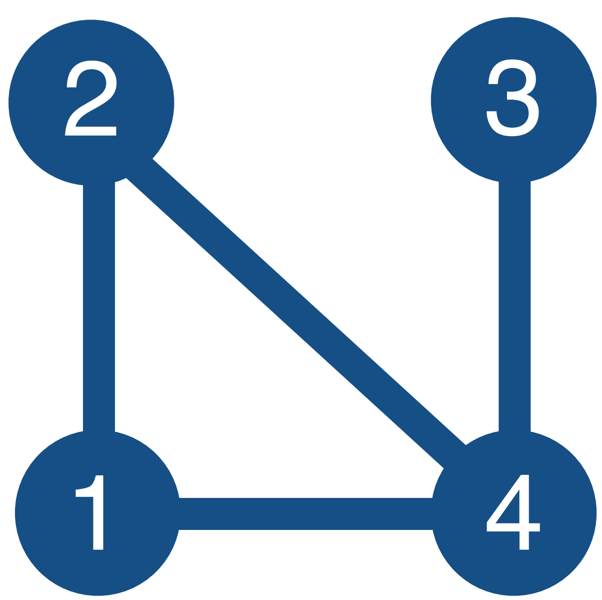
G_1 G_2 G_3 G_4

[Hoover 1979, Aldous 1981]

The Old Way: Exchangeability

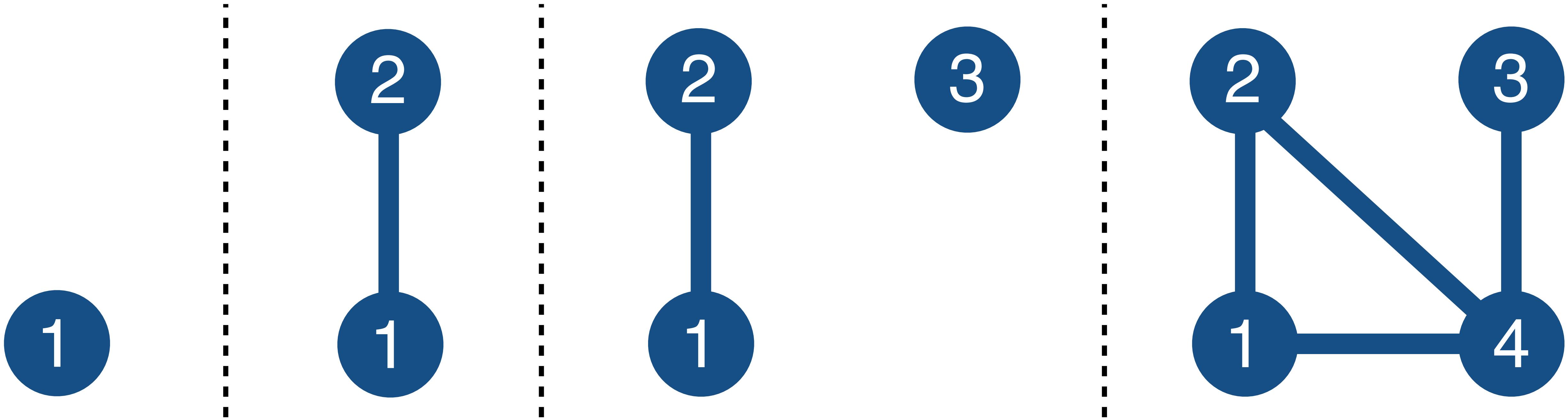


G_1 G_2 G_3 G_4

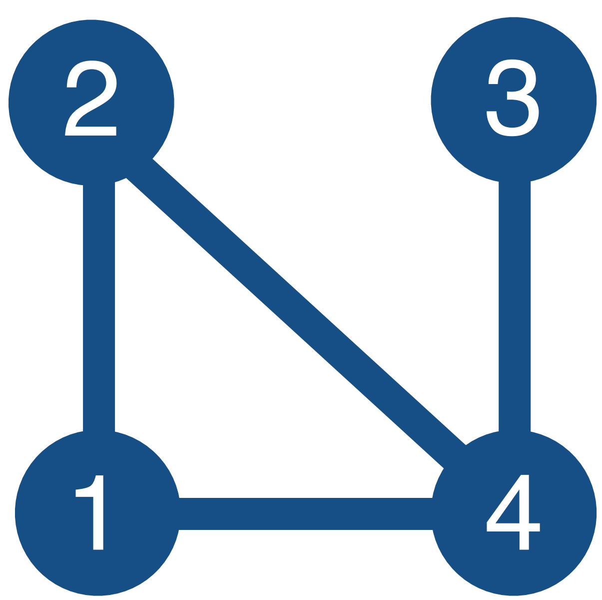


[Hoover 1979, Aldous 1981]

The Old Way: Exchangeability

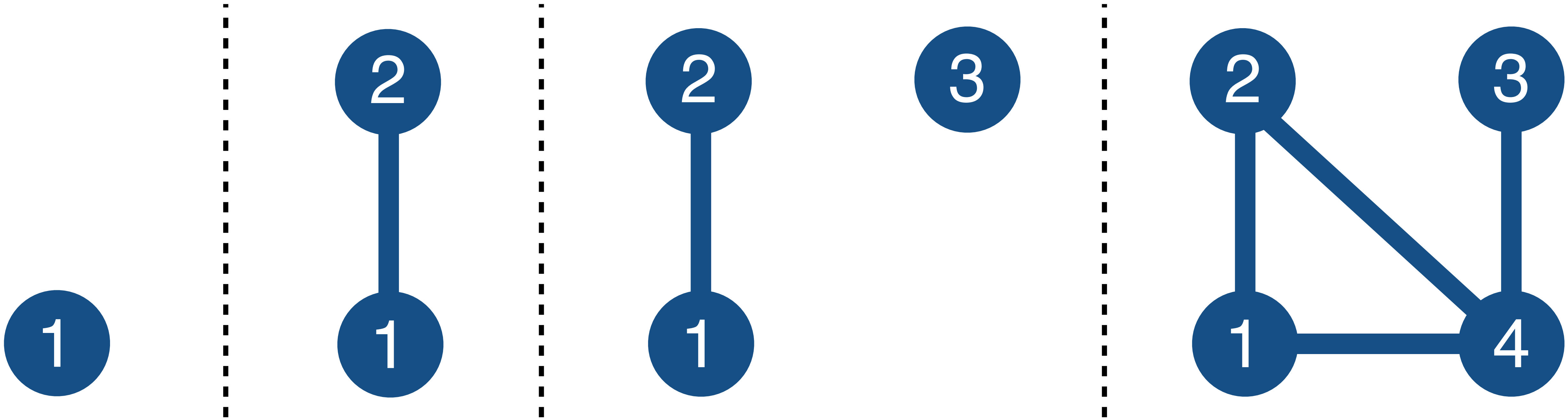


G_1 G_2 G_3 G_4

$p($  $)$

[Hoover 1979, Aldous 1981]

The Old Way: Exchangeability



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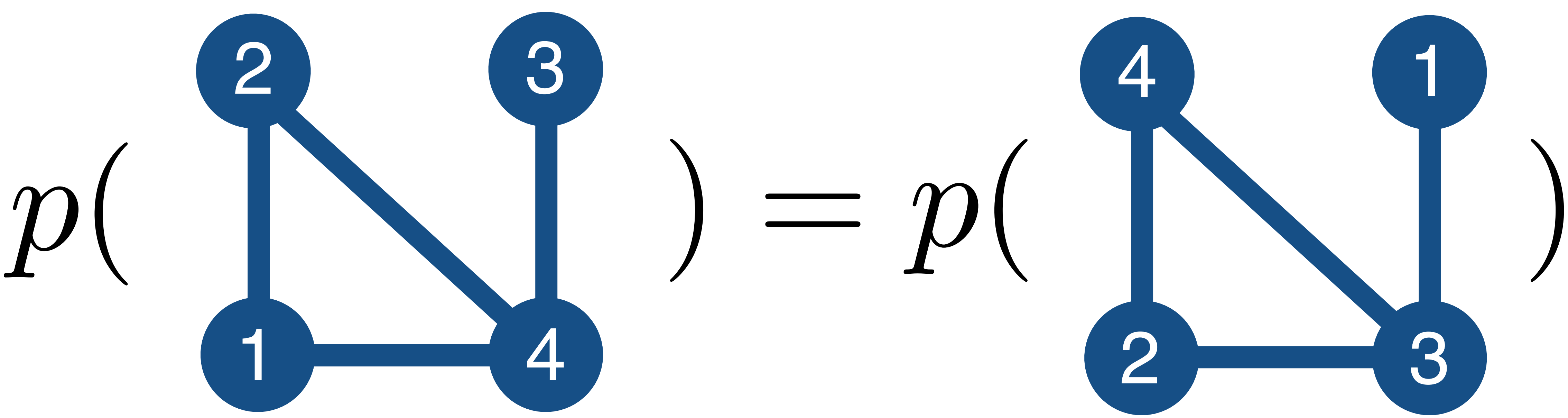
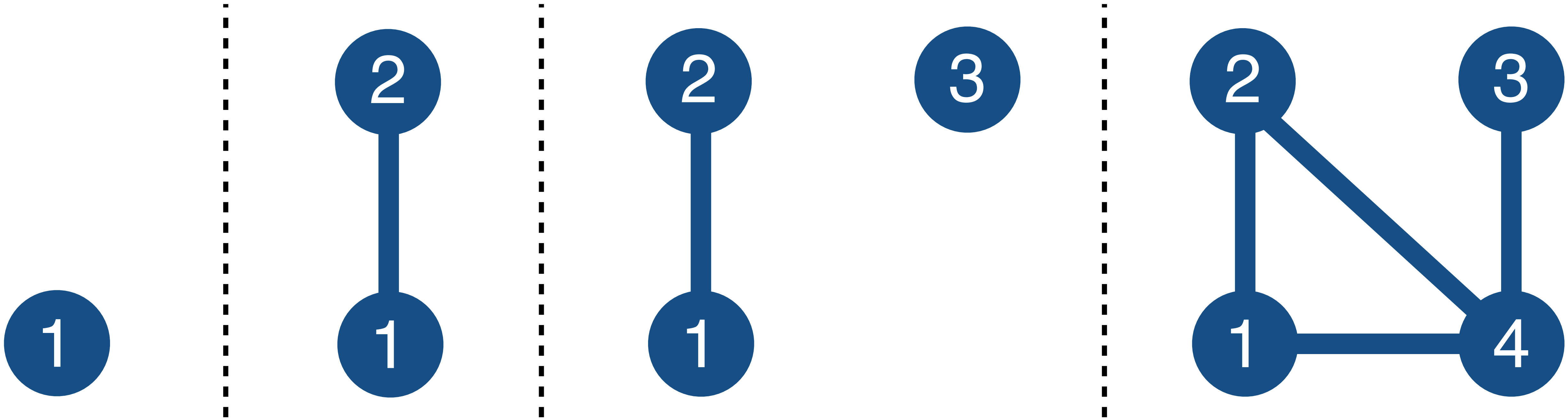
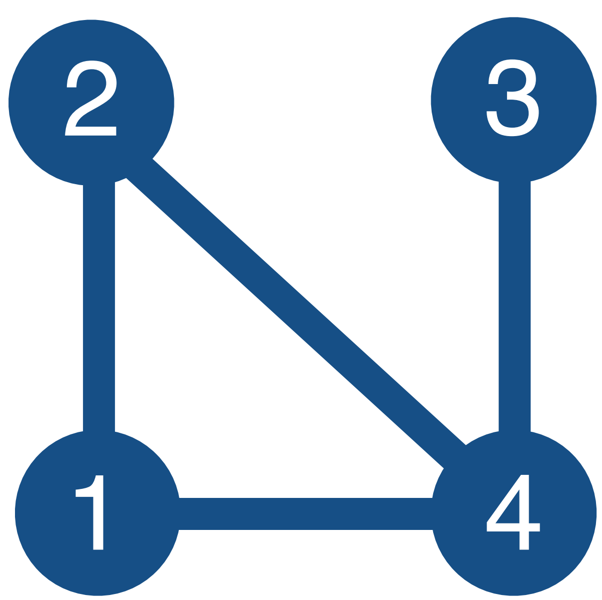


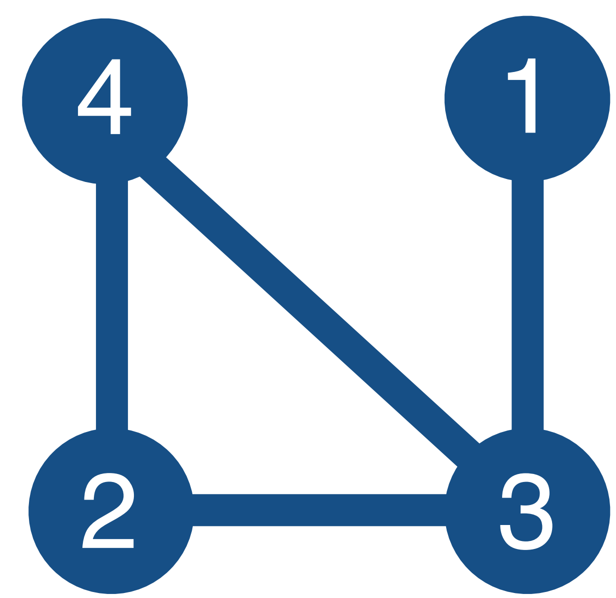
[Hoover 1979, Aldous 1981]

The Old Way: Node exchangeability



G_1 G_2 G_3 G_4

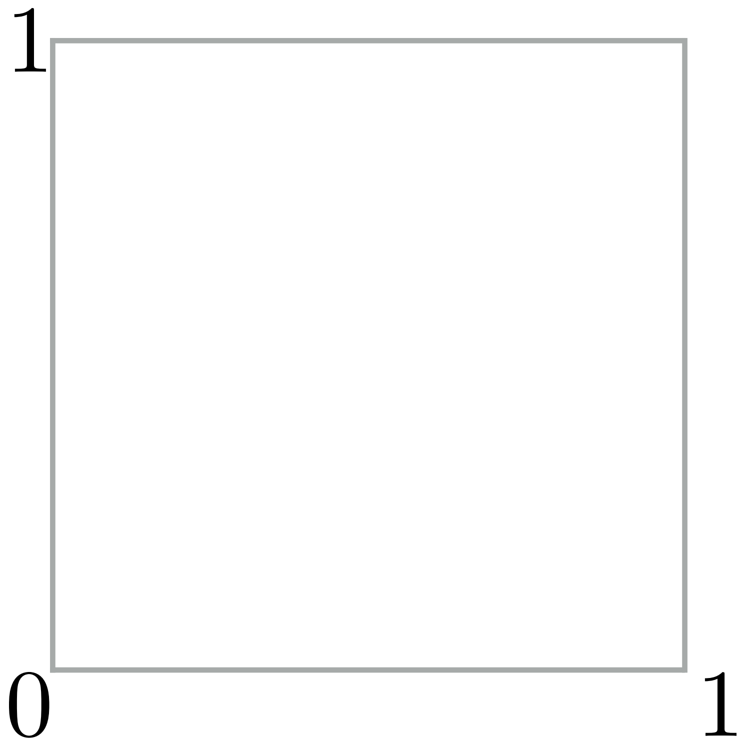
$p($ 

$) = p($ 

$)$

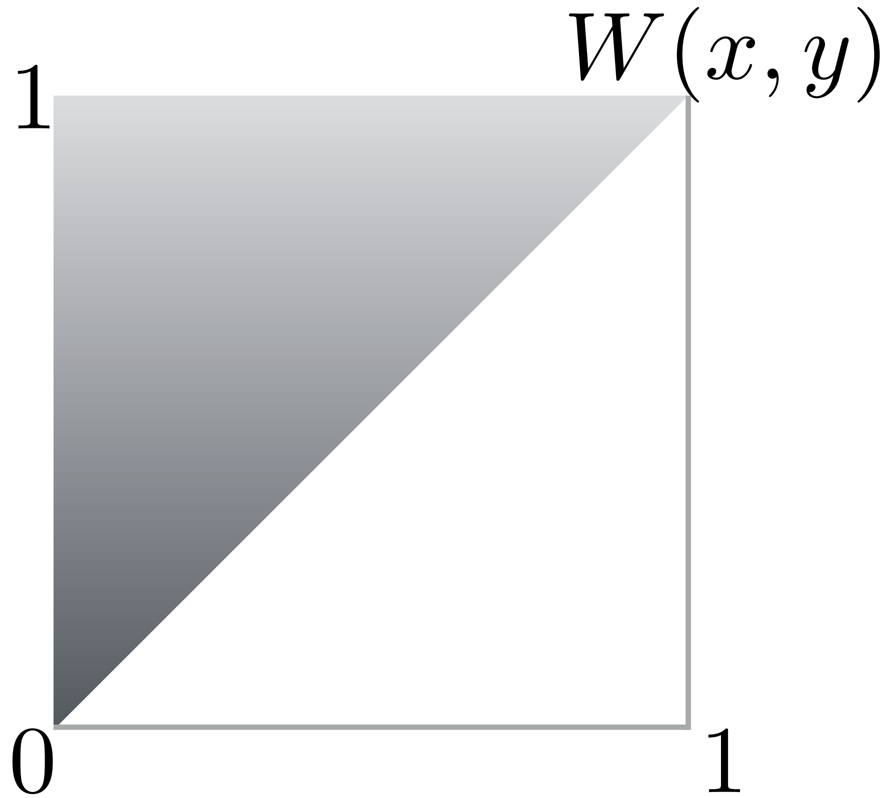
[Hoover 1979, Aldous 1981]

Aldous-Hoover



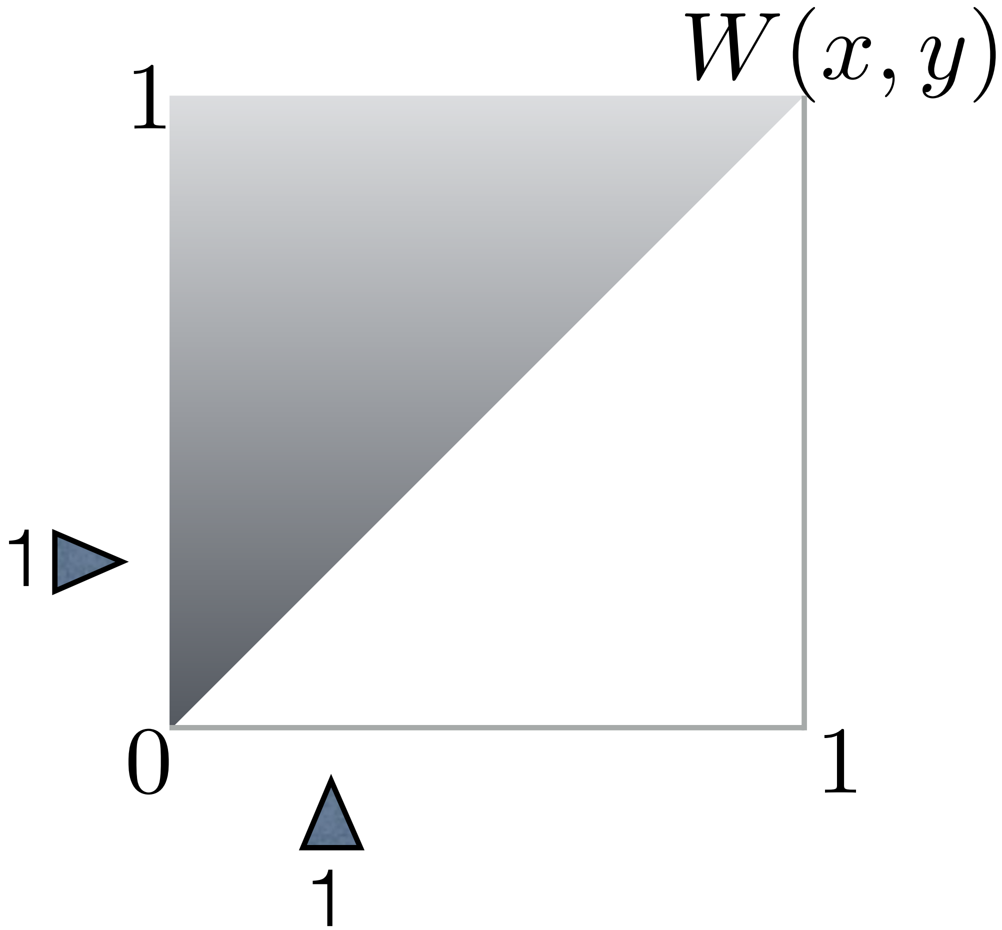
[Hoover 1979, Aldous 1981]

Aldous-Hoover



[Hoover 1979, Aldous 1981]

Aldous-Hoover



[Hoover 1979, Aldous 1981]

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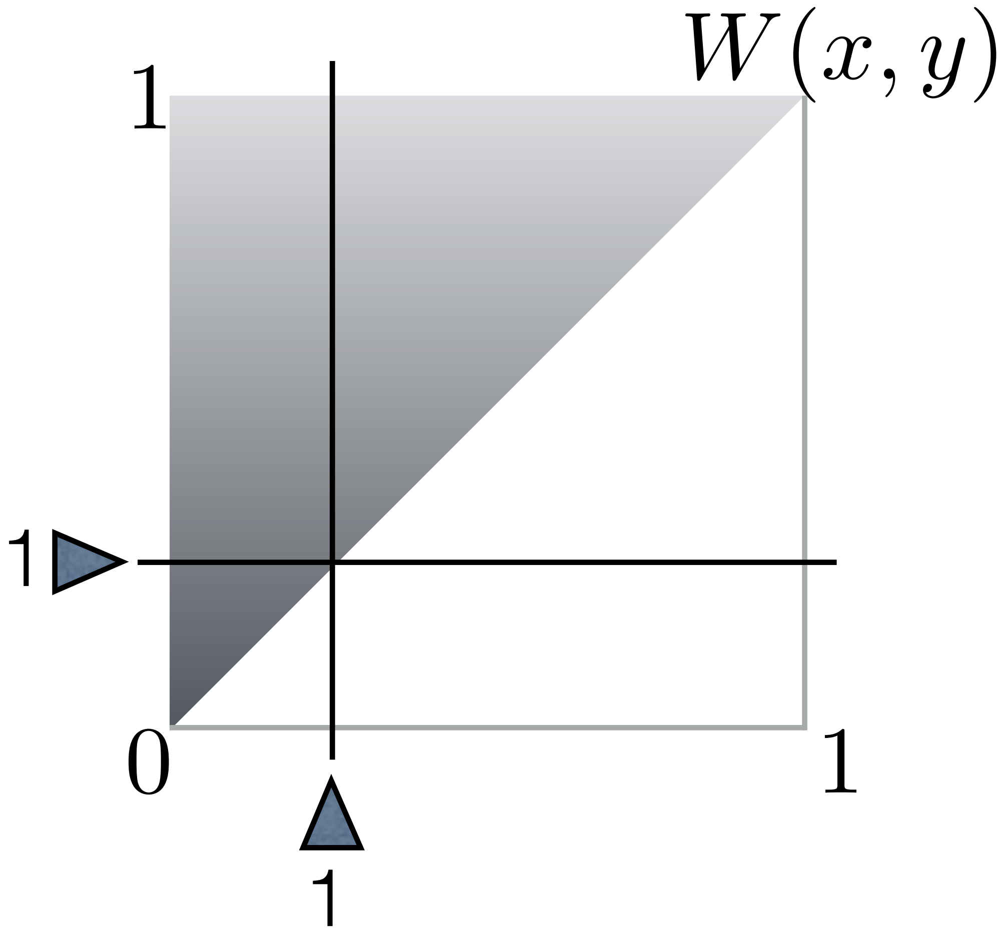

[Hoover 1979, Aldous 1981]

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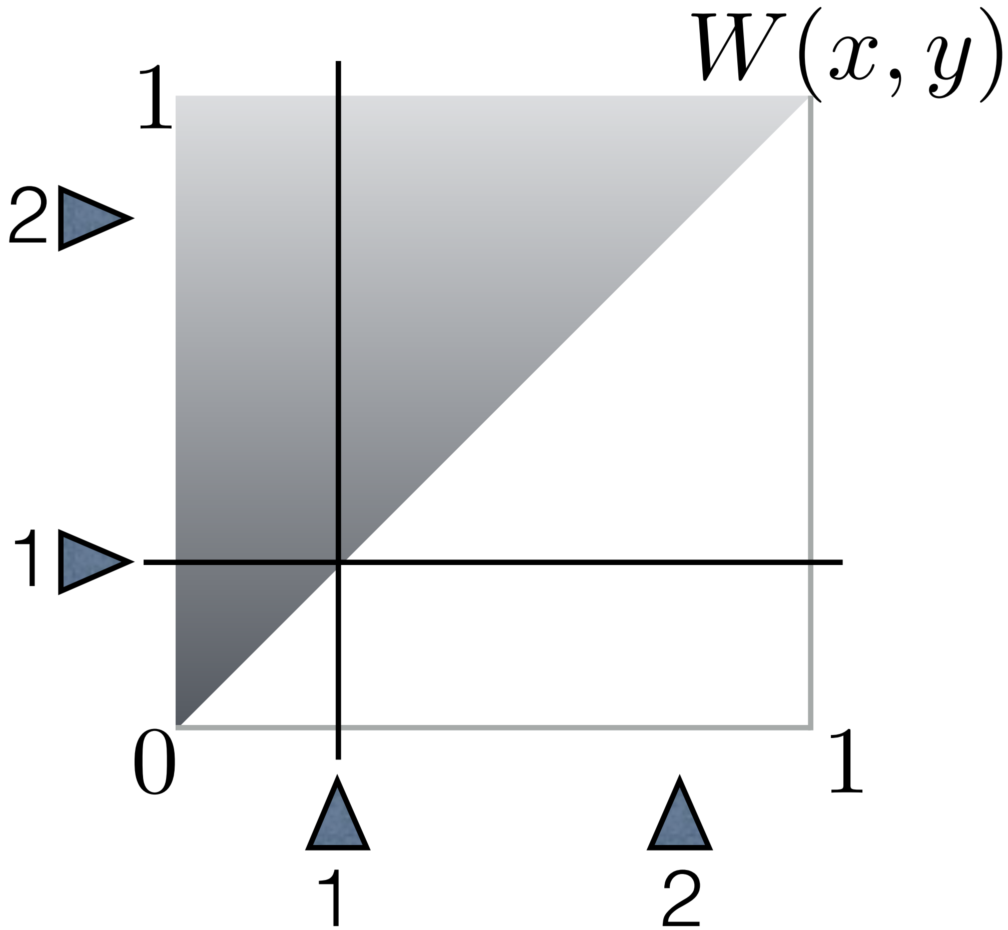
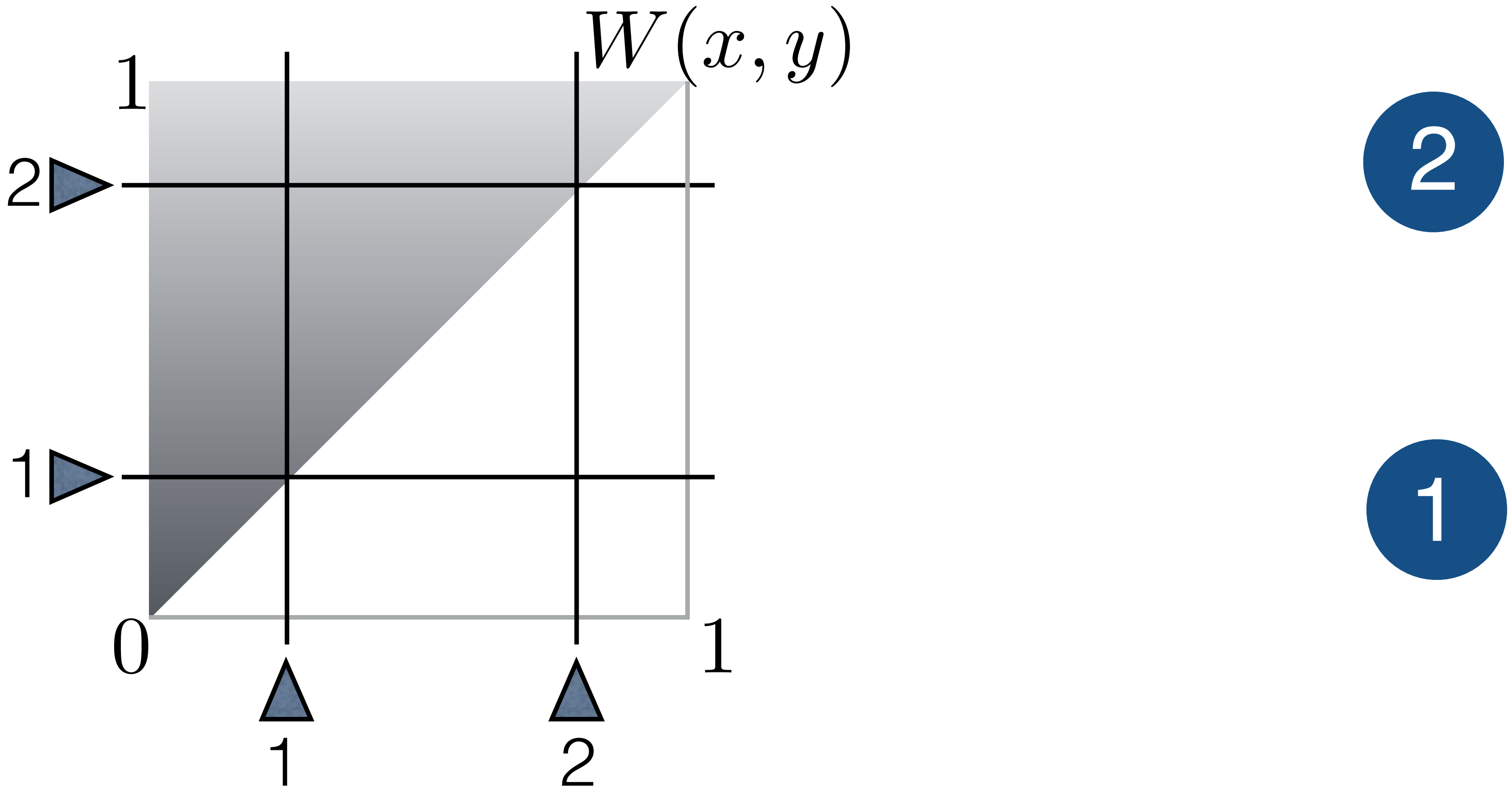


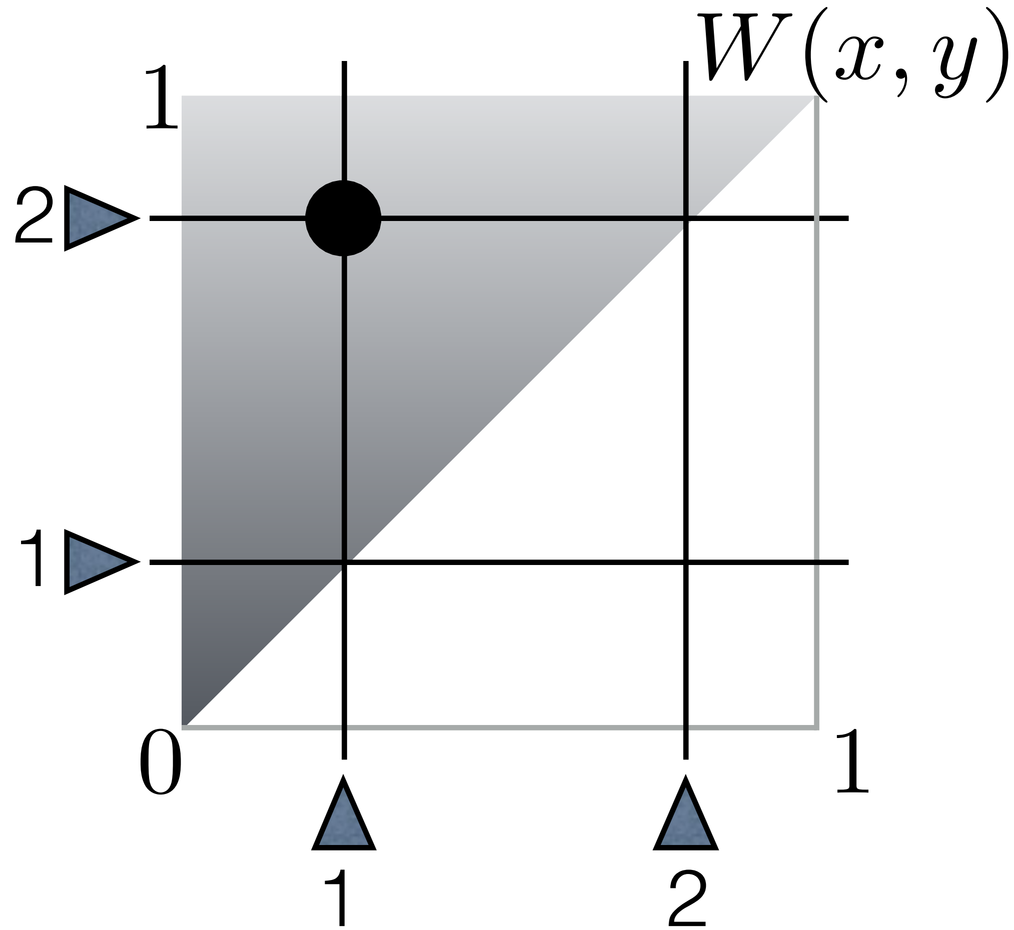
[Hoover 1979, Aldous 1981]

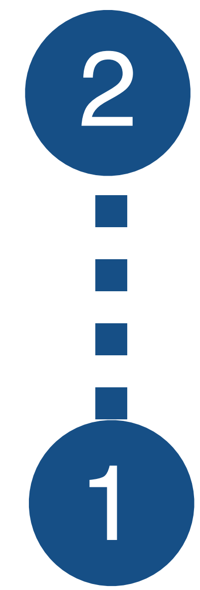
Aldous-Hoover



[Hoover 1979, Aldous 1981]

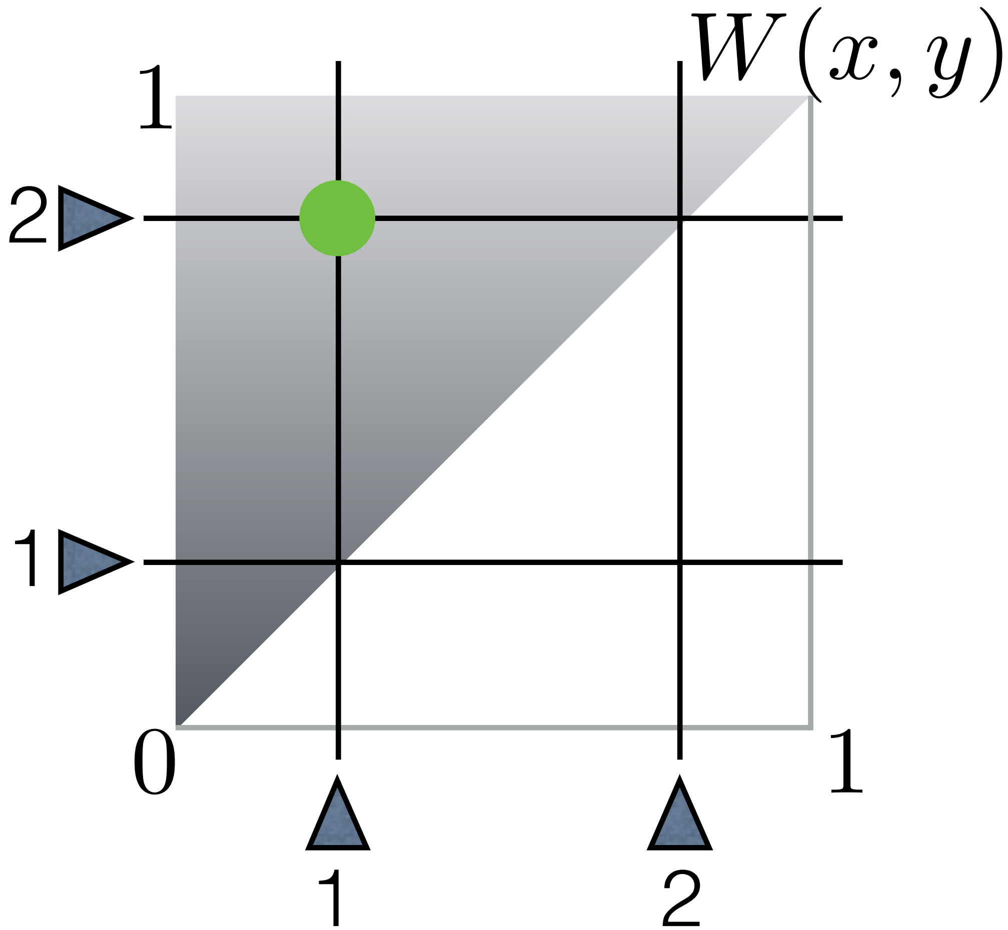
Aldous-Hoover

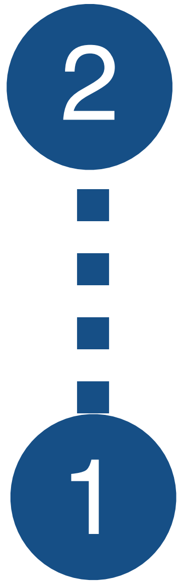




[Hoover 1979, Aldous 1981]

Aldous-Hoover





[Hoover 1979, Aldous 1981]

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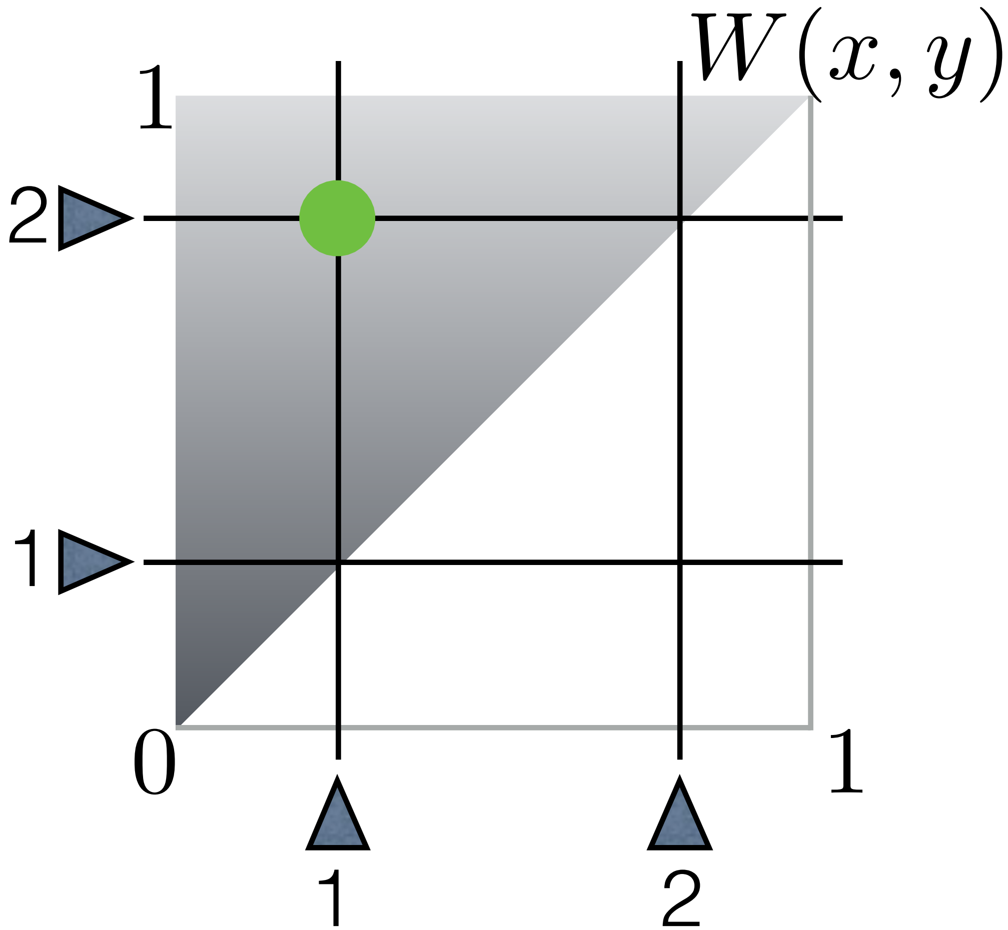

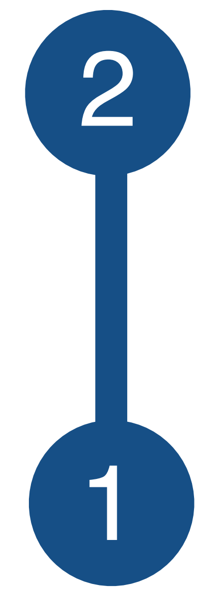

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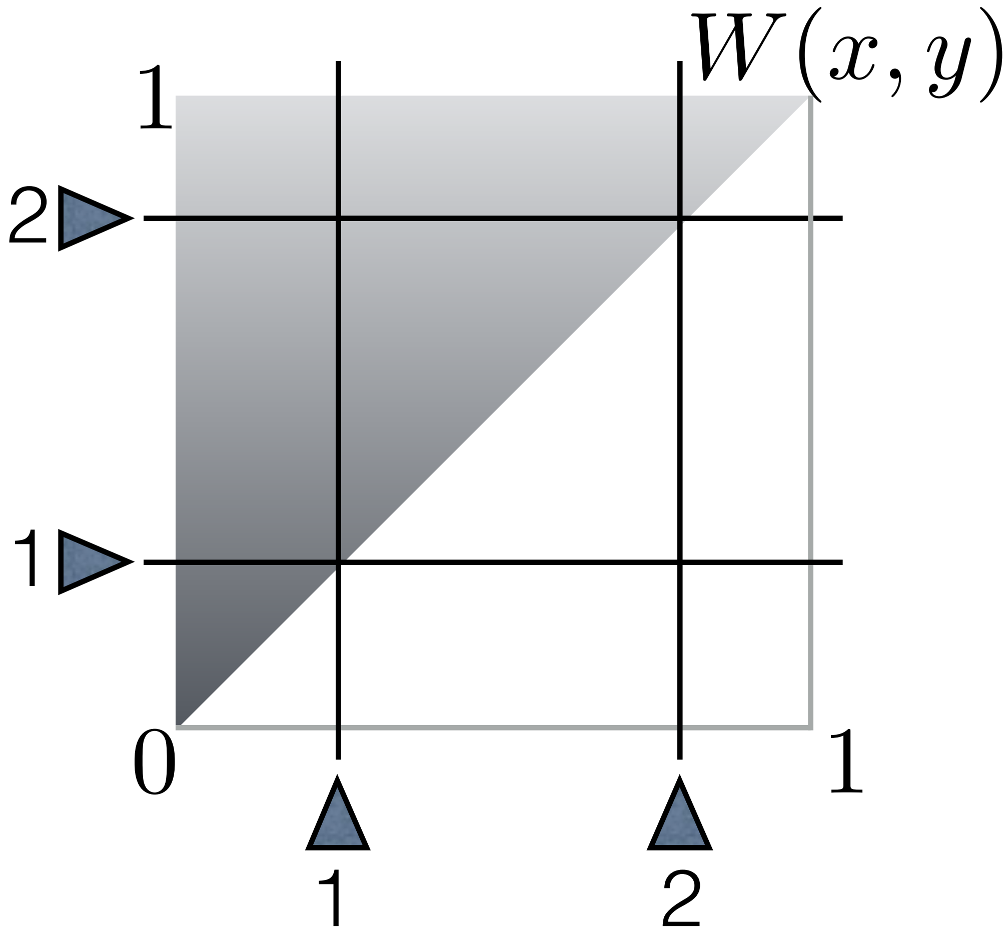

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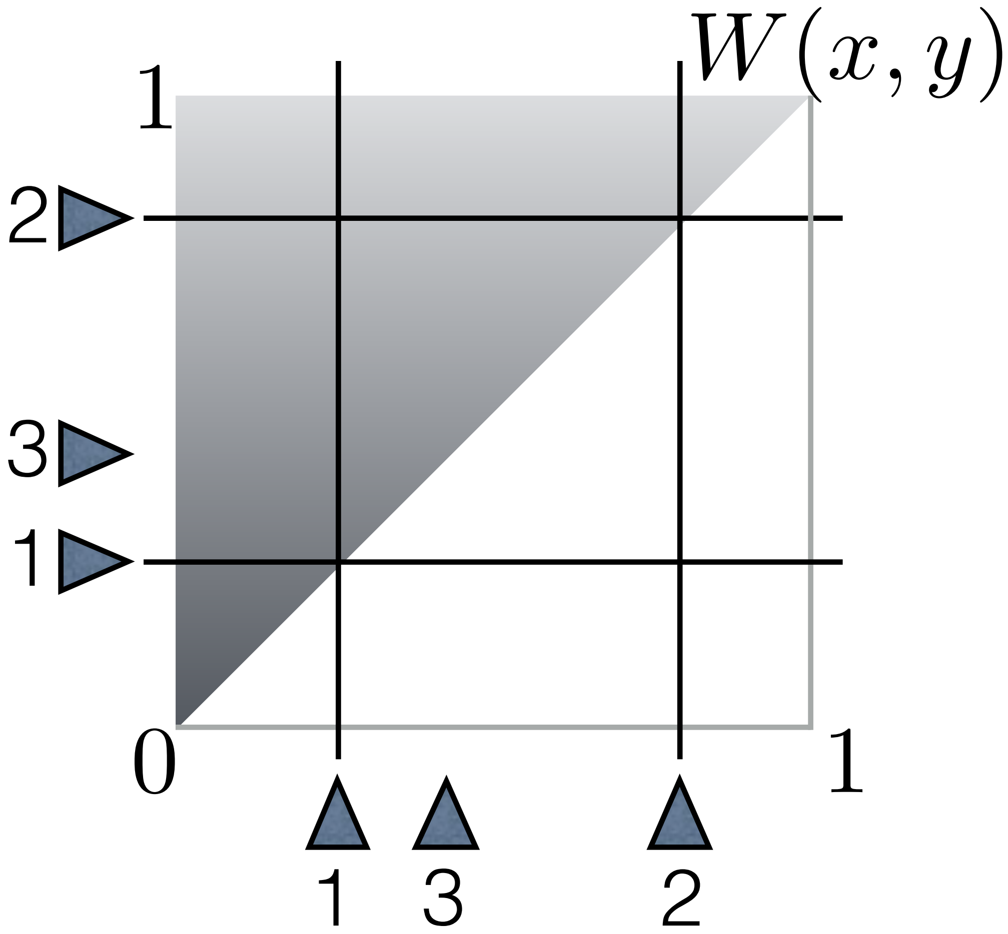

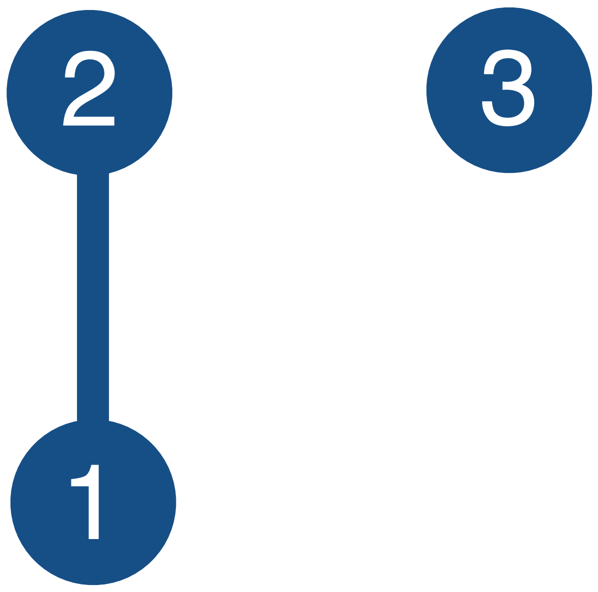

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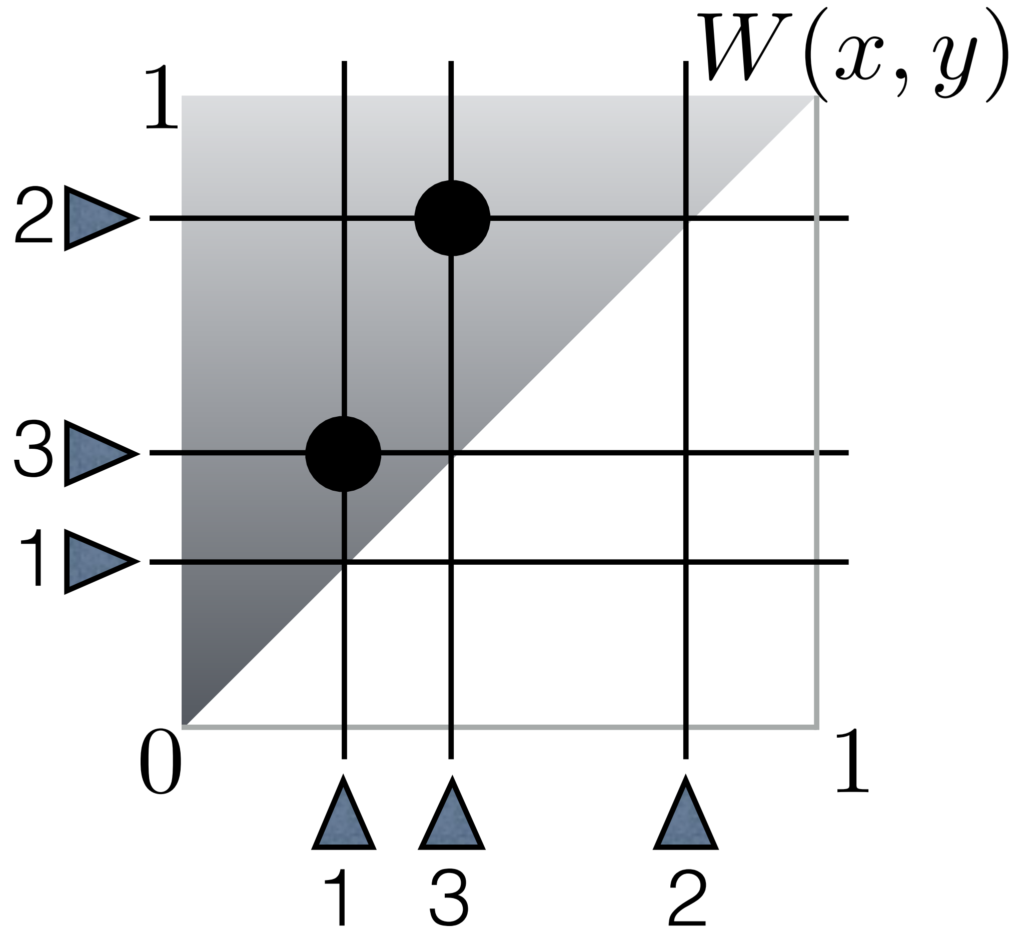

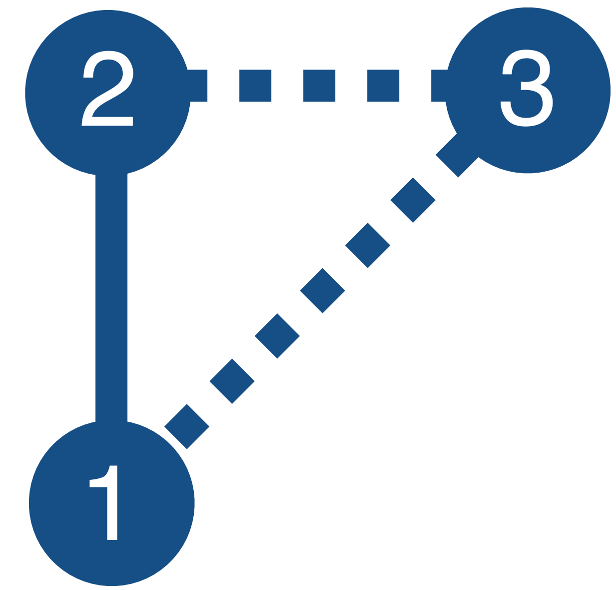

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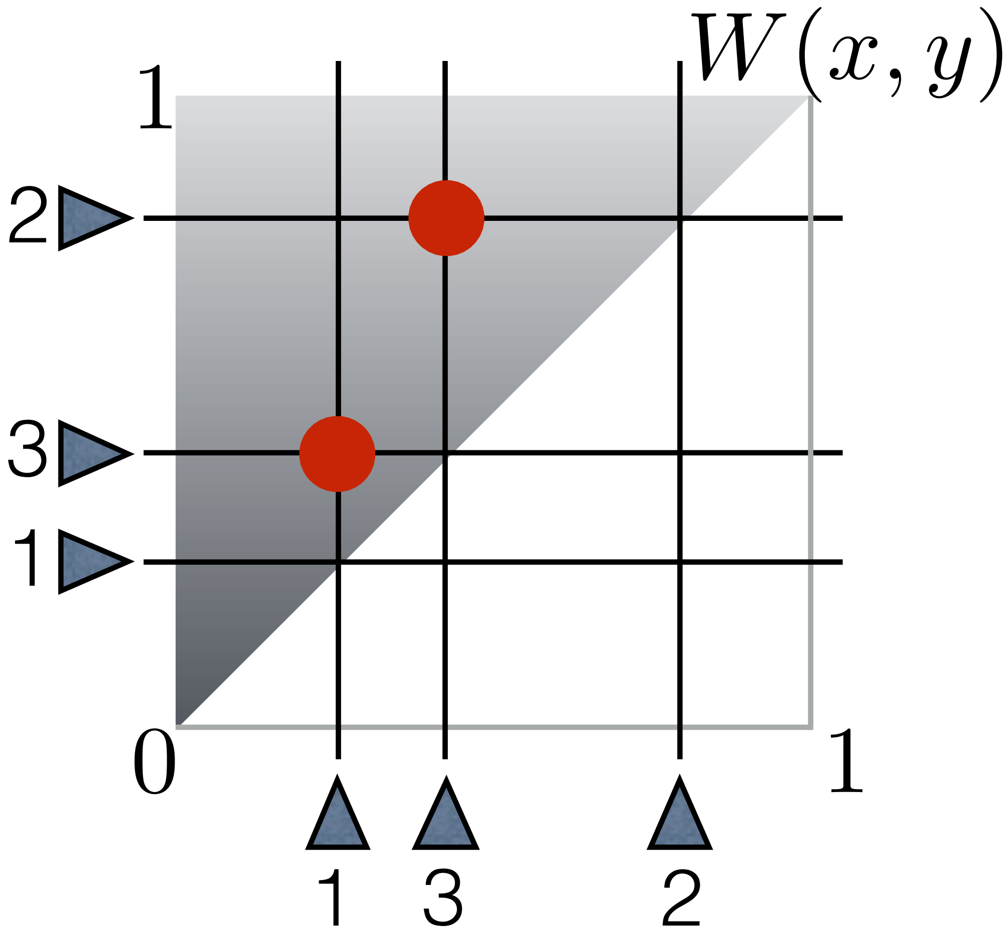
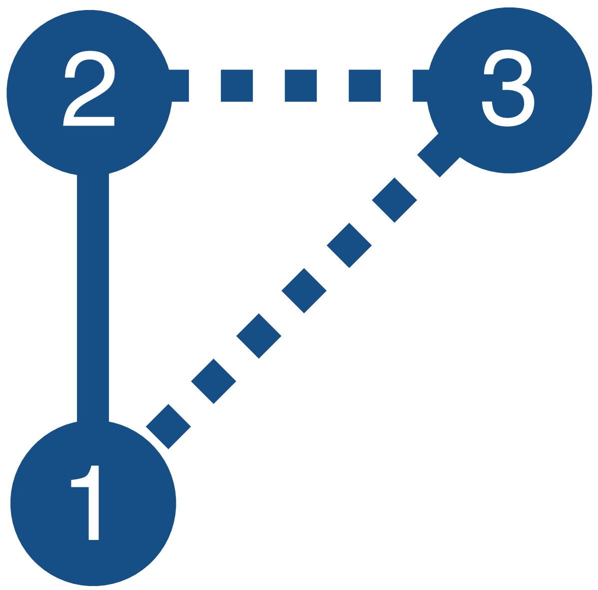
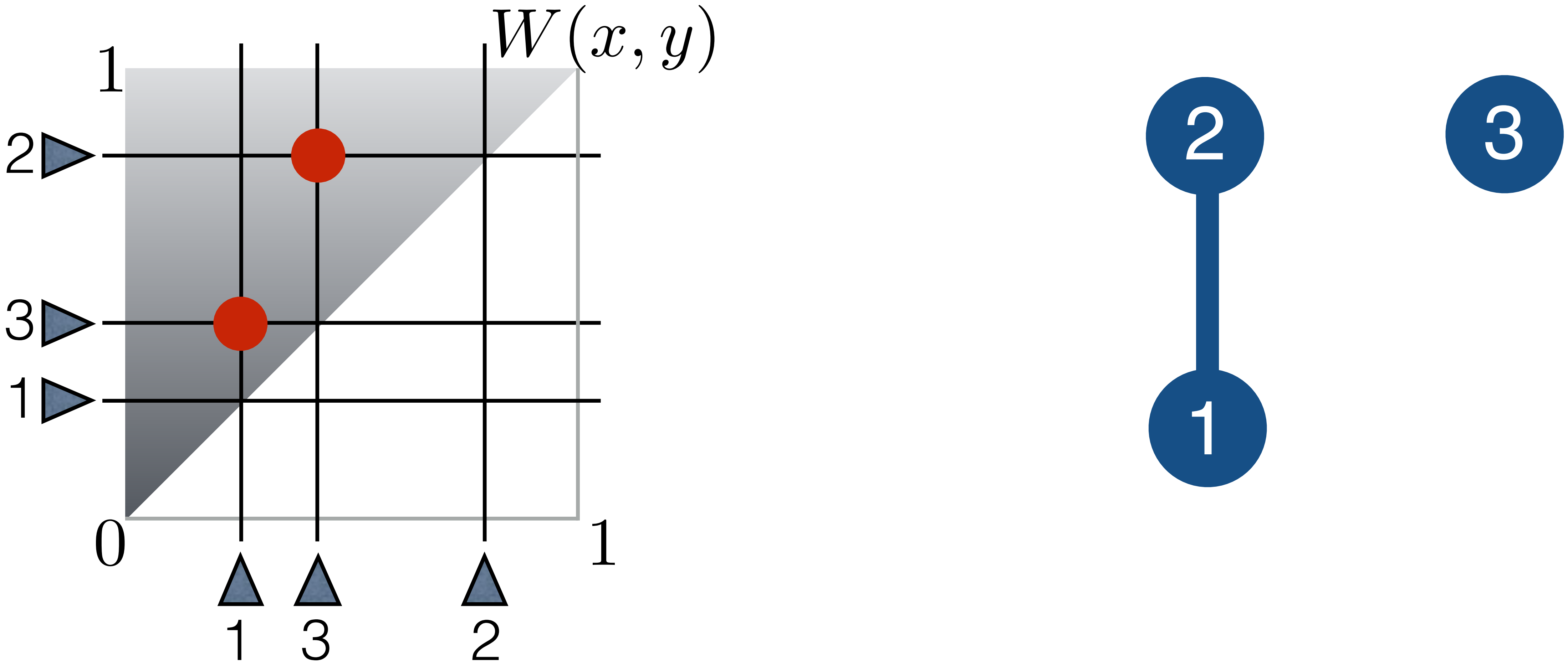




[Hoover 1979, Aldous 1981]

Aldous-Hoover



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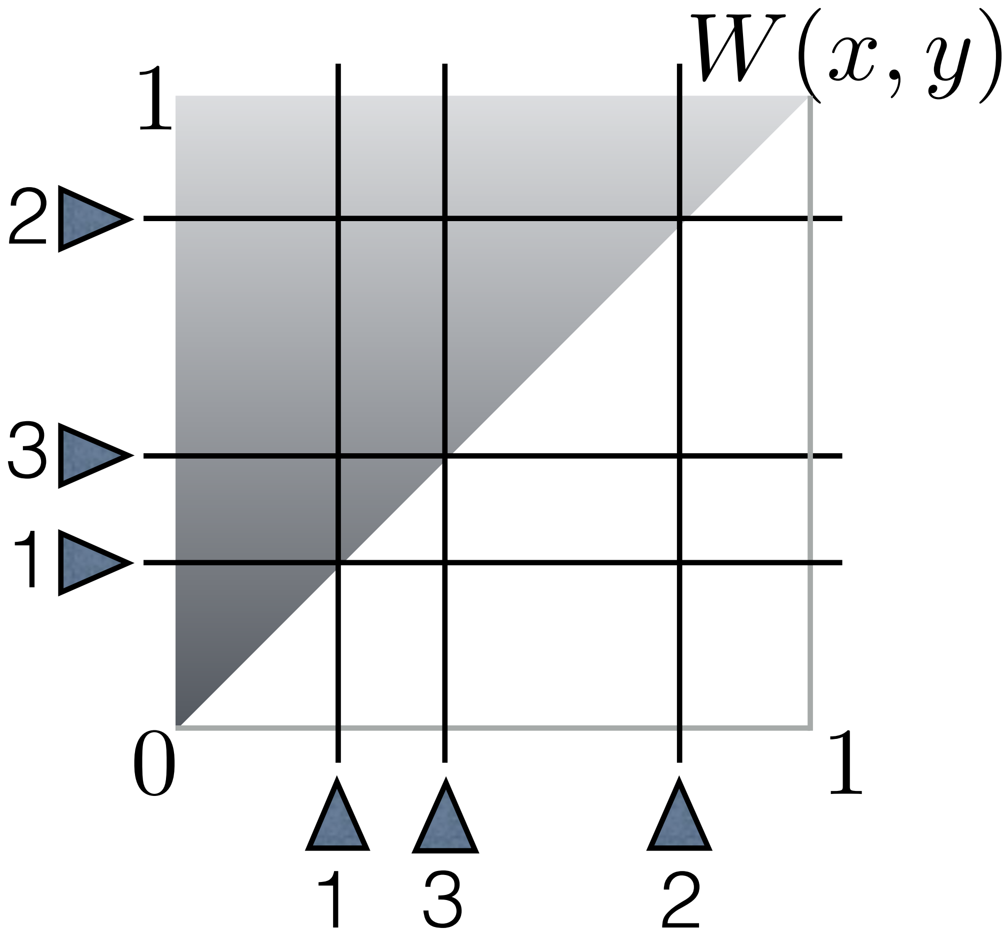

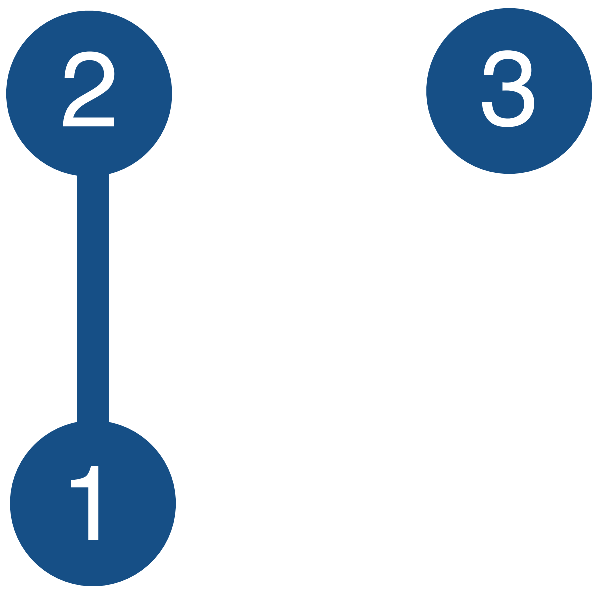

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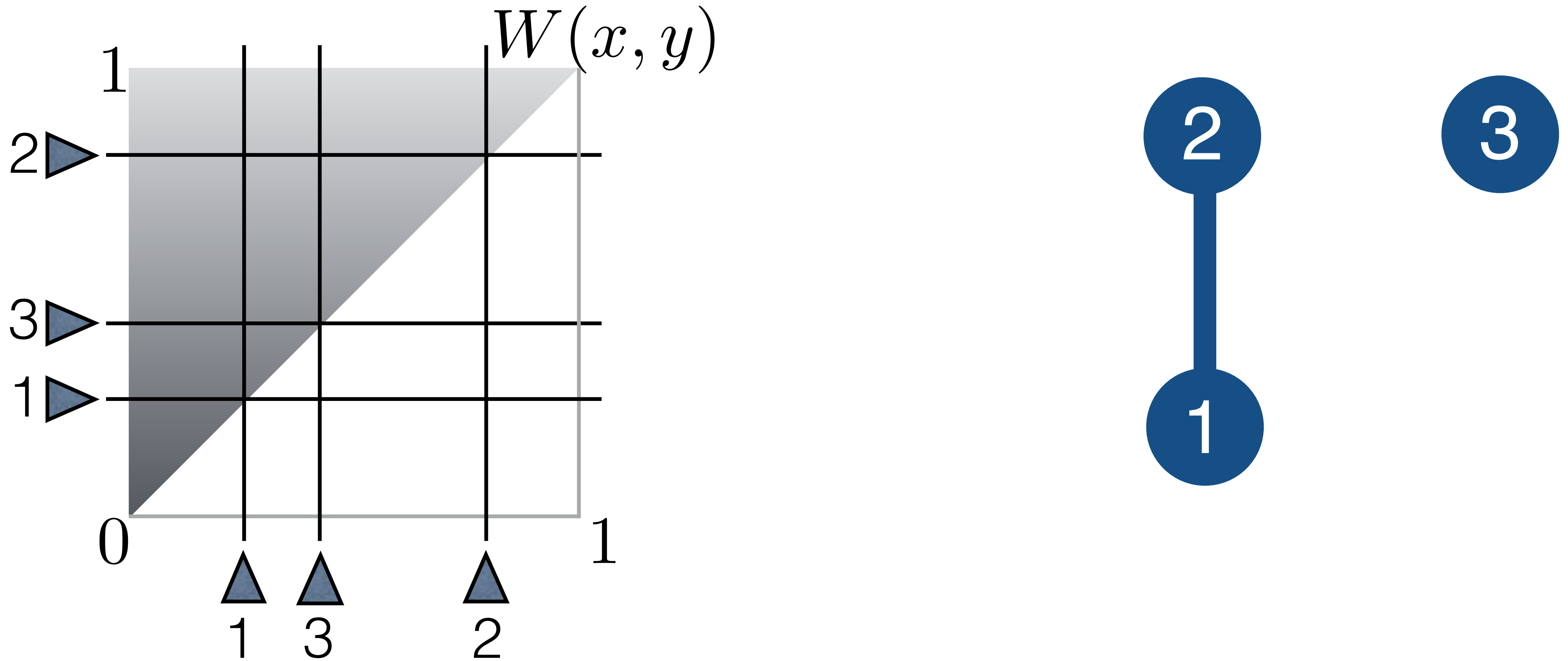
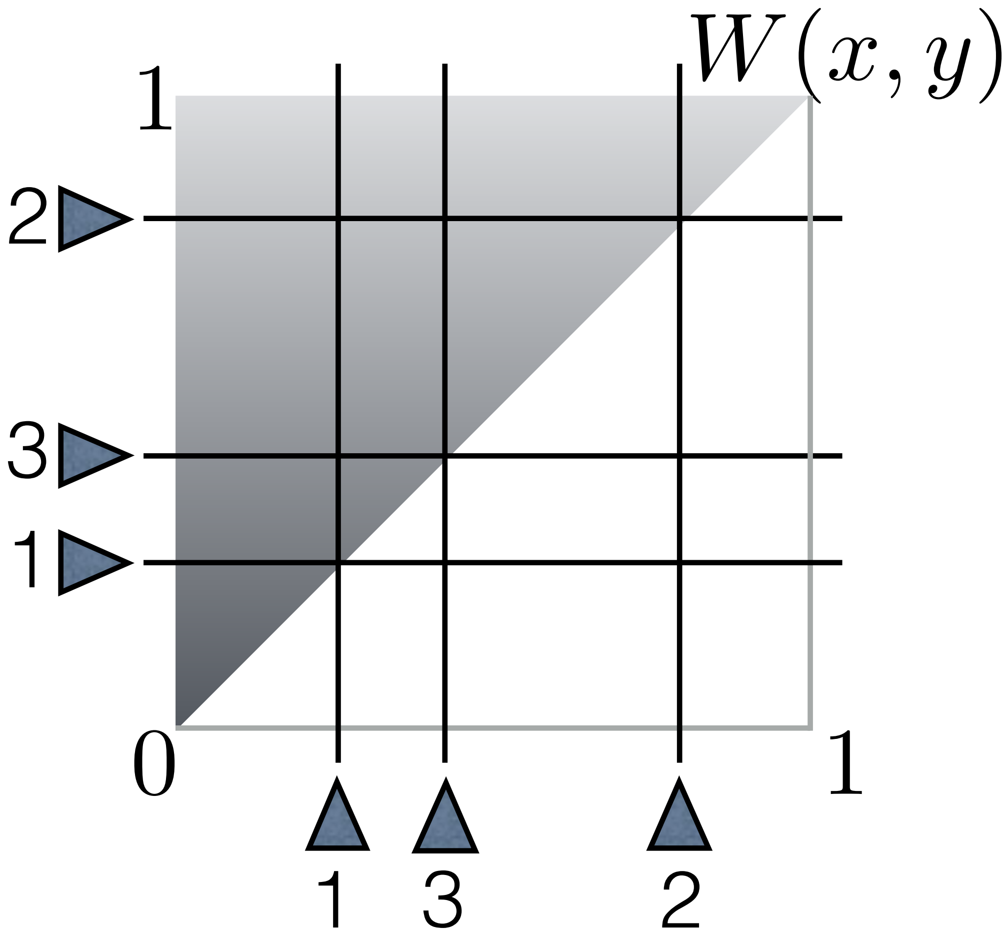
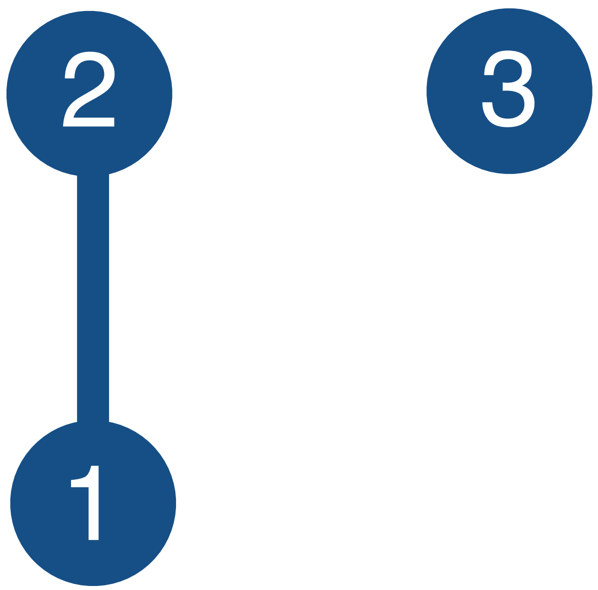


Thm (AH). Every node-exchangeable graph has a *graphon* rep

[Hoover 1979, Aldous 1981]

Aldous-Hoover



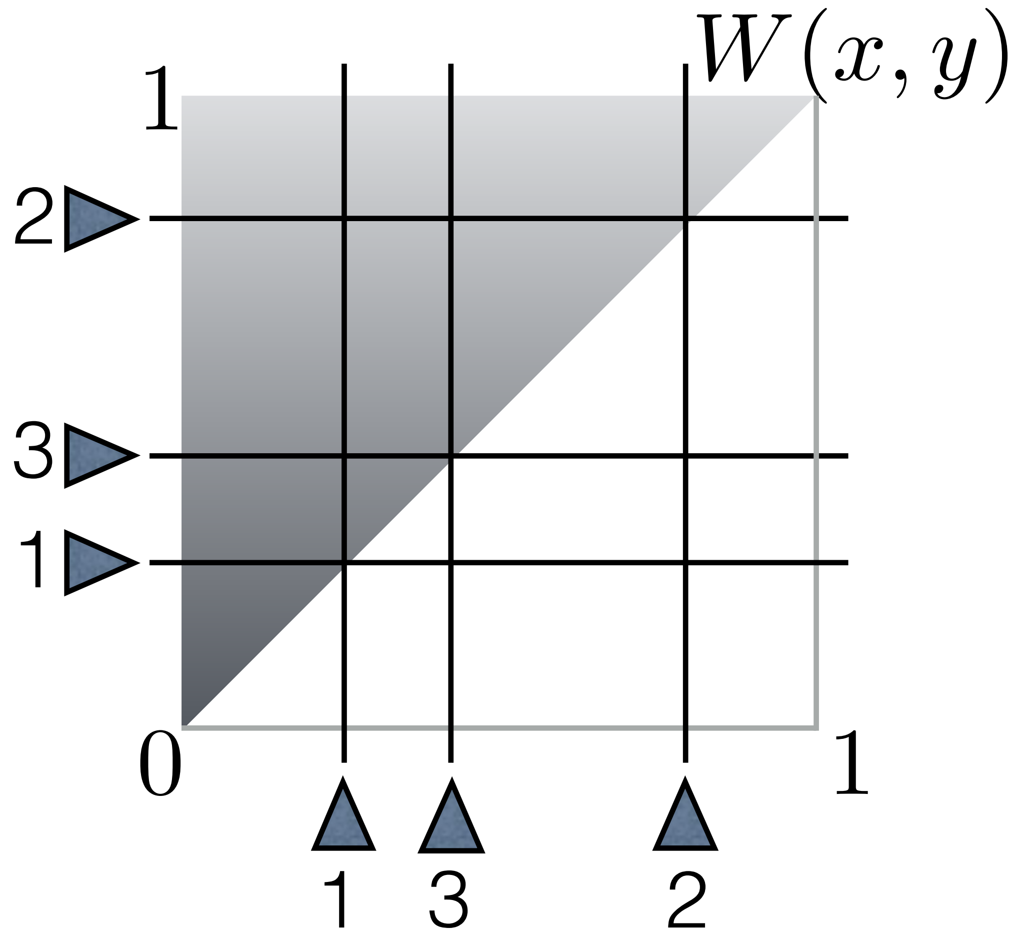


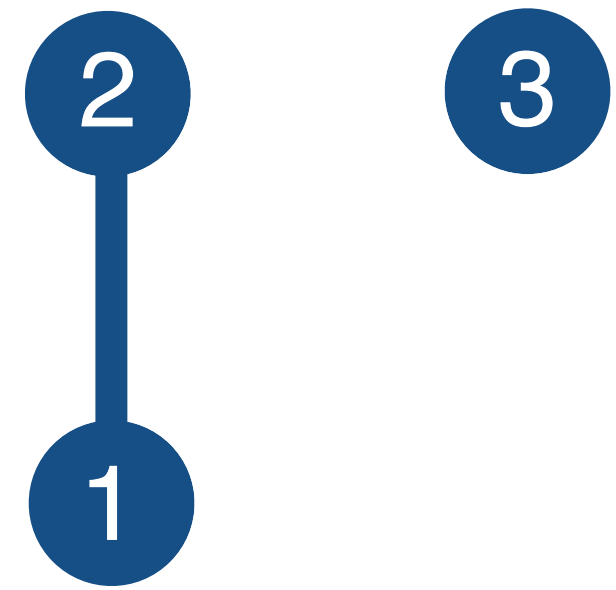
Thm (AH). Every node-exchangeable graph has a *graphon* rep

$$\mathbb{E}[\#\text{edges}(G_n)]$$

[Hoover 1979, Aldous 1981, Orbanz, Roy 2015]

Aldous-Hoover



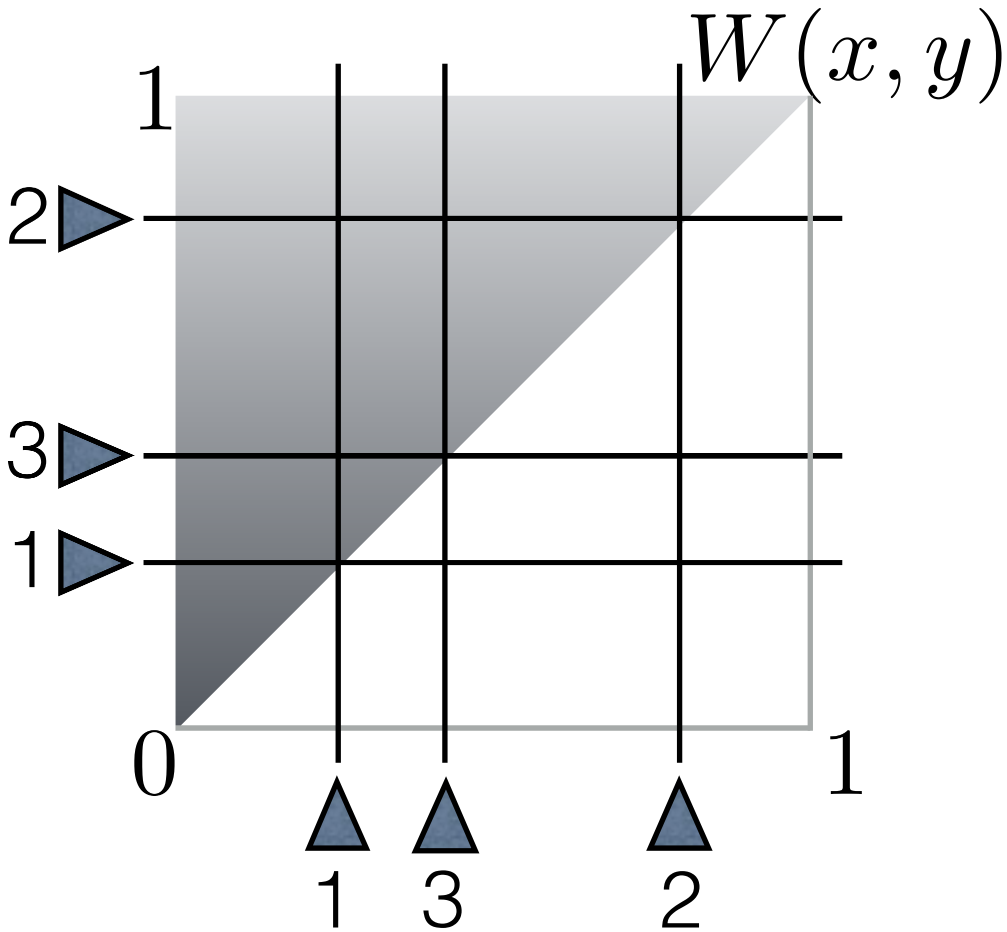


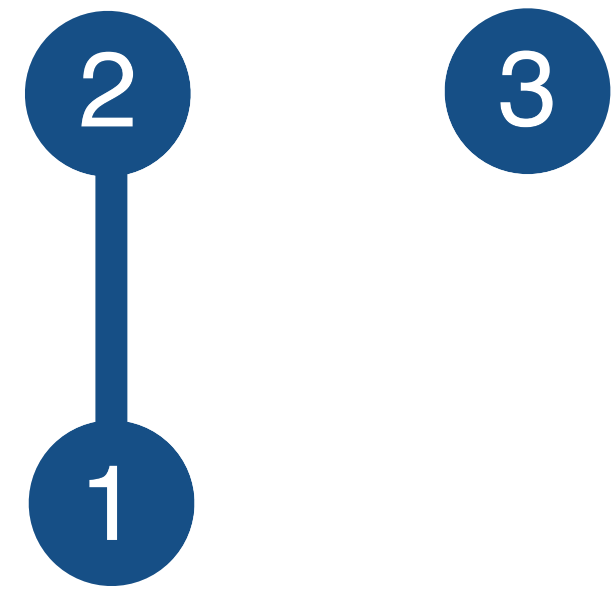
Thm (AH). Every node-exchangeable graph has a *graphon* rep

$$\mathbb{E}[\#\text{edges}(G_n)] = \mathbb{E}\left[\binom{n}{2}\frac{1}{2}\int_0^1\int_0^1 W(x,y)\,dx\,dy\right]$$

[Hoover 1979, Aldous 1981, Orbanz, Roy 2015]

Aldous-Hoover





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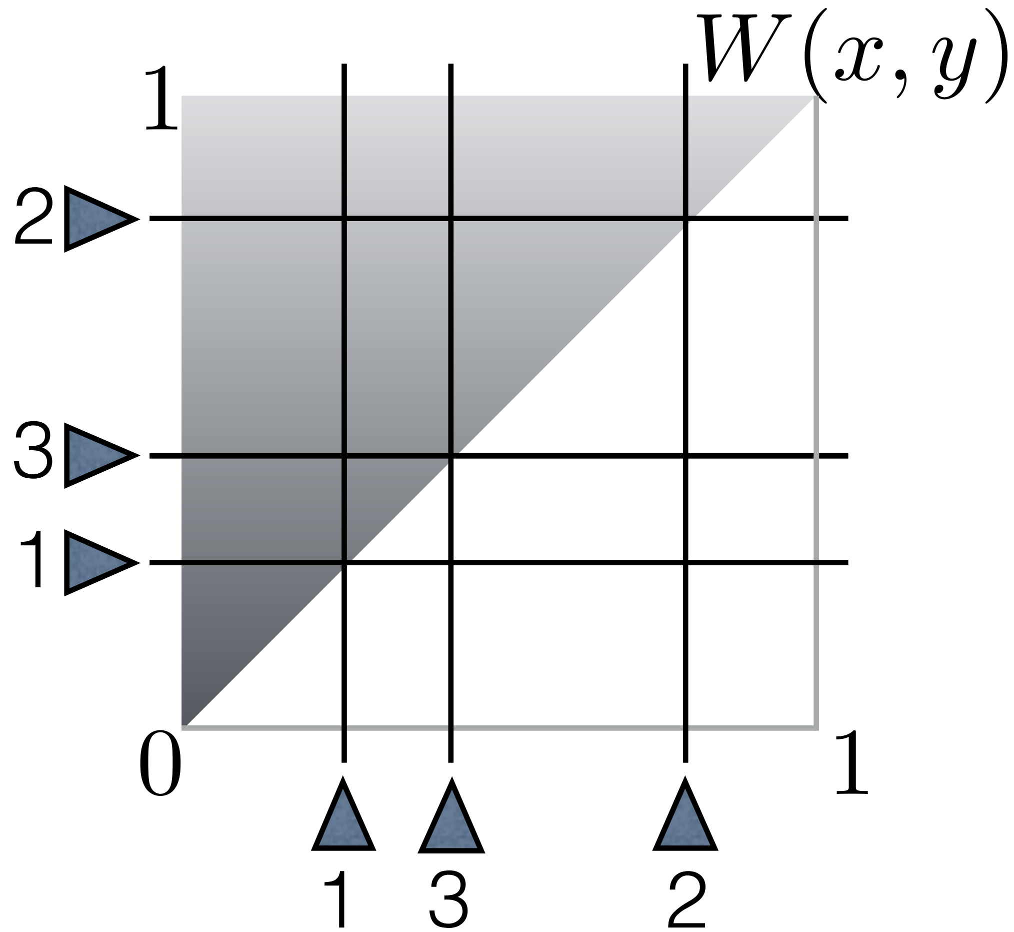
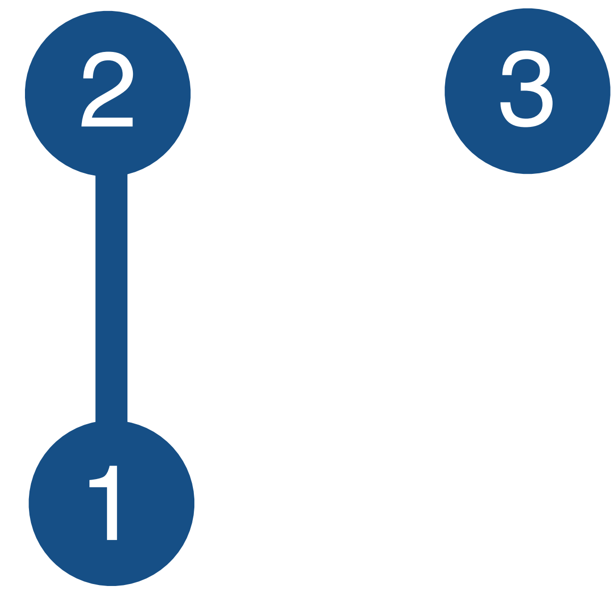
Aldous-Hoover





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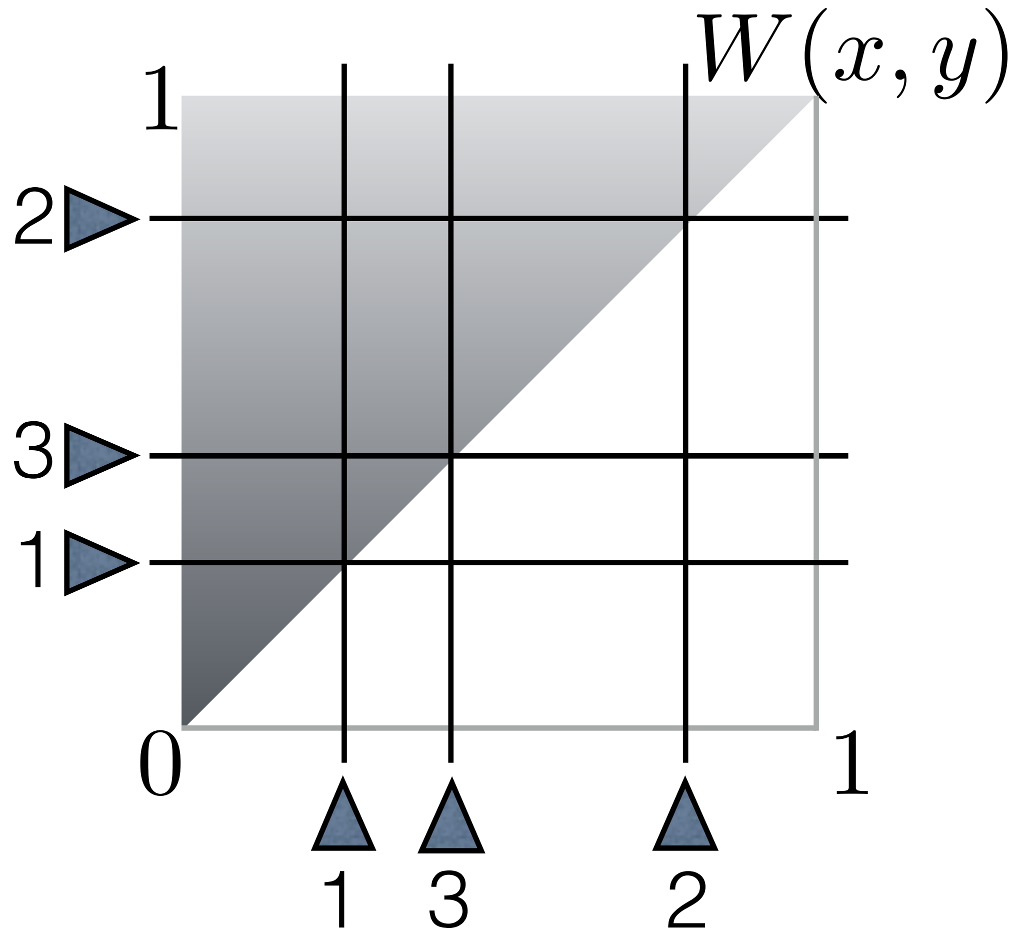
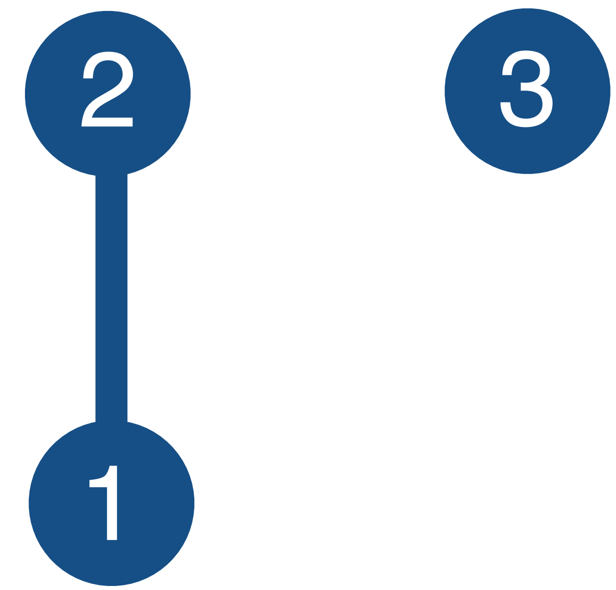
[Hoover 1979, Aldous 1981, Orbanz, Roy 2015]

Aldous-Hoover





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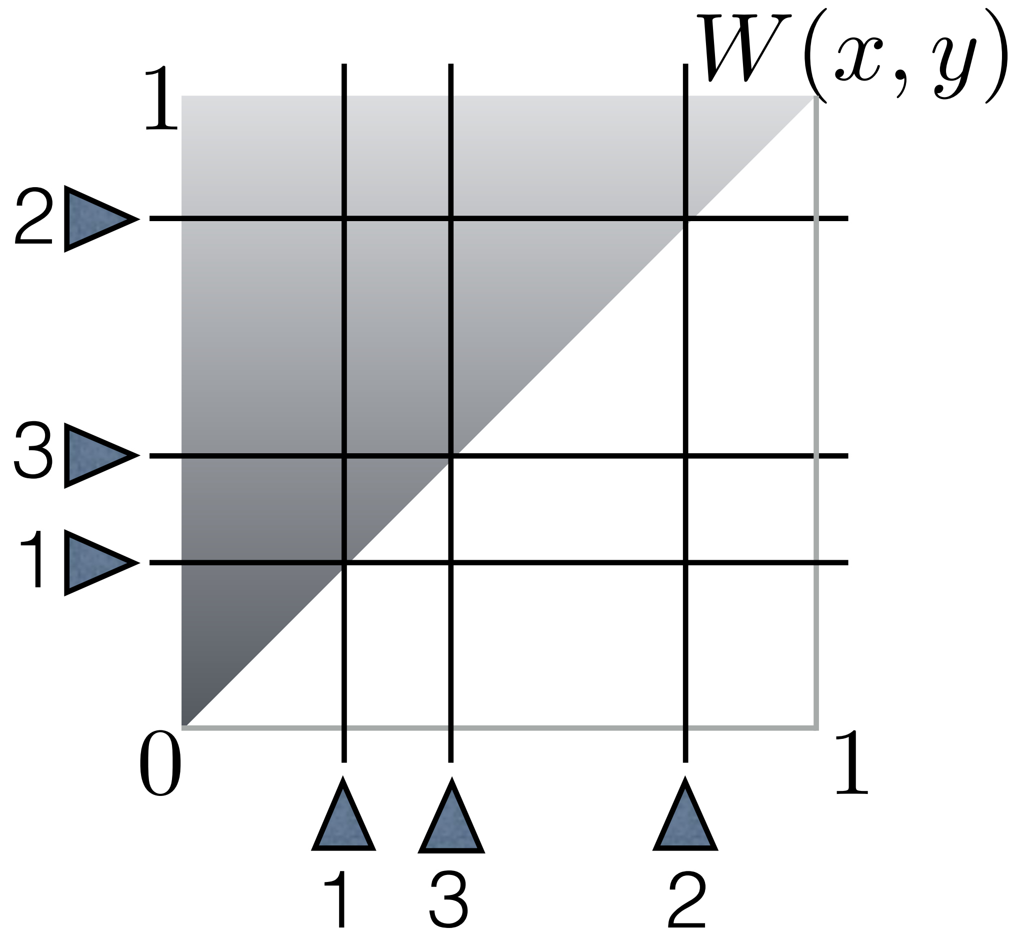
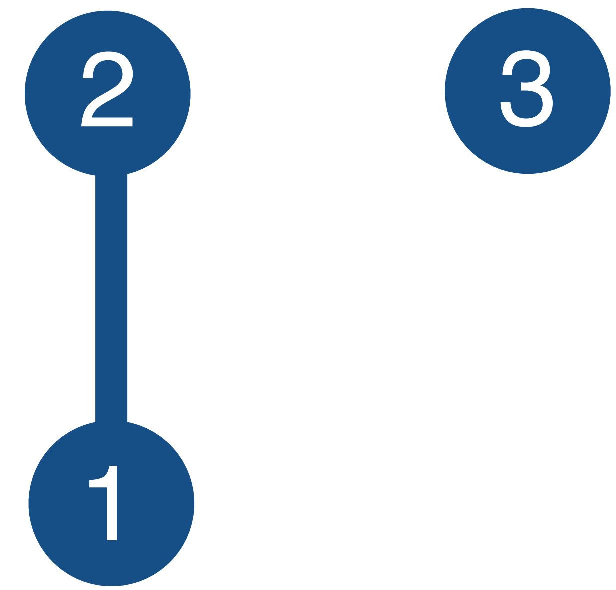
Cor. Every node-exch graph sequence is dense (or empty) a.s.

[Hoover 1979, Aldous 1981, Orbanz, Roy 2015]

Aldous-Hoover





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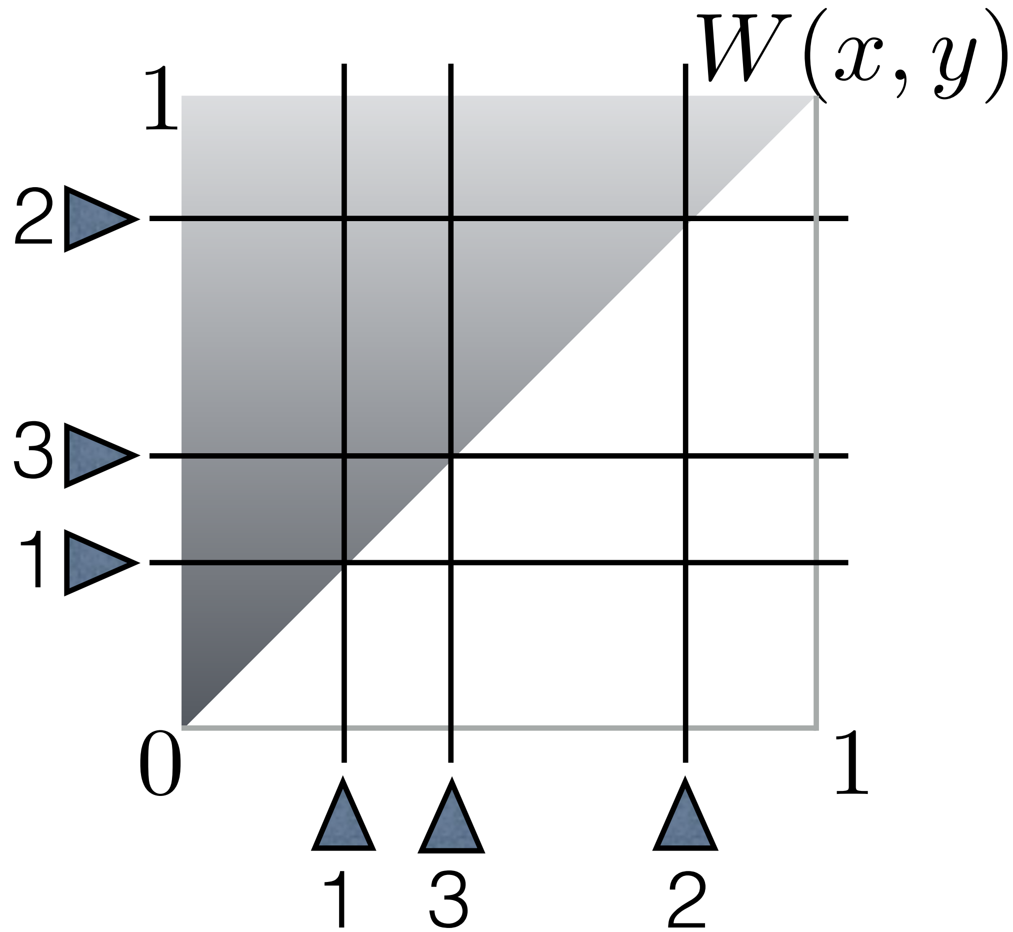
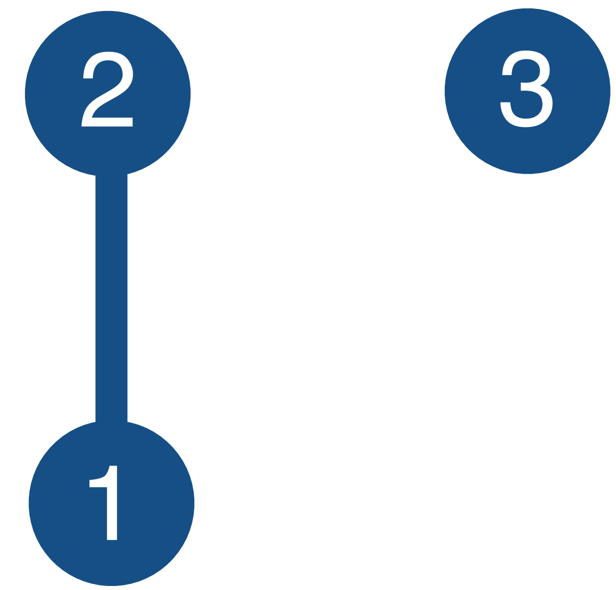
Cor. Every node-exch graph sequence is dense (or empty) a.s.

Intuition: To a given node, all other nodes look the same.

[Hoover 1979, Aldous 1981, Orbanz, Roy 2015]

Aldous-Hoover





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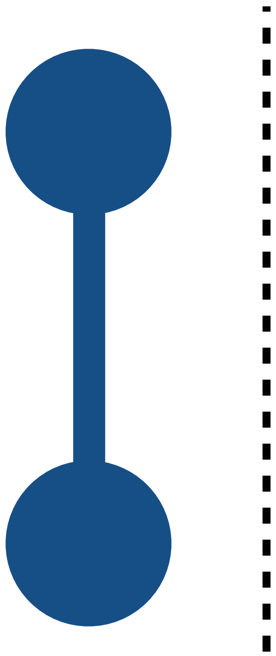
[Caron, Fox 2014; Veitch, Roy 2015; Borgs, Chayes, Cohn, Holden 2016;
Broderick, Cai 2015; Cai, Broderick 2015a,b; Crane, Dempsey 2015a,b, 2016a; Cai, Campbell, Broderick 2016]

A New Way: Edges



G_1

 [Broderick, Cai 2015; Cai, Broderick 2015a,b; Crane, Dempsey 2015a,b,16a; Cai, Campbell, Broderick 2016]

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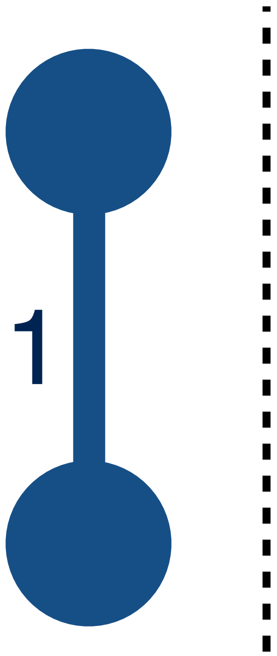

G_1

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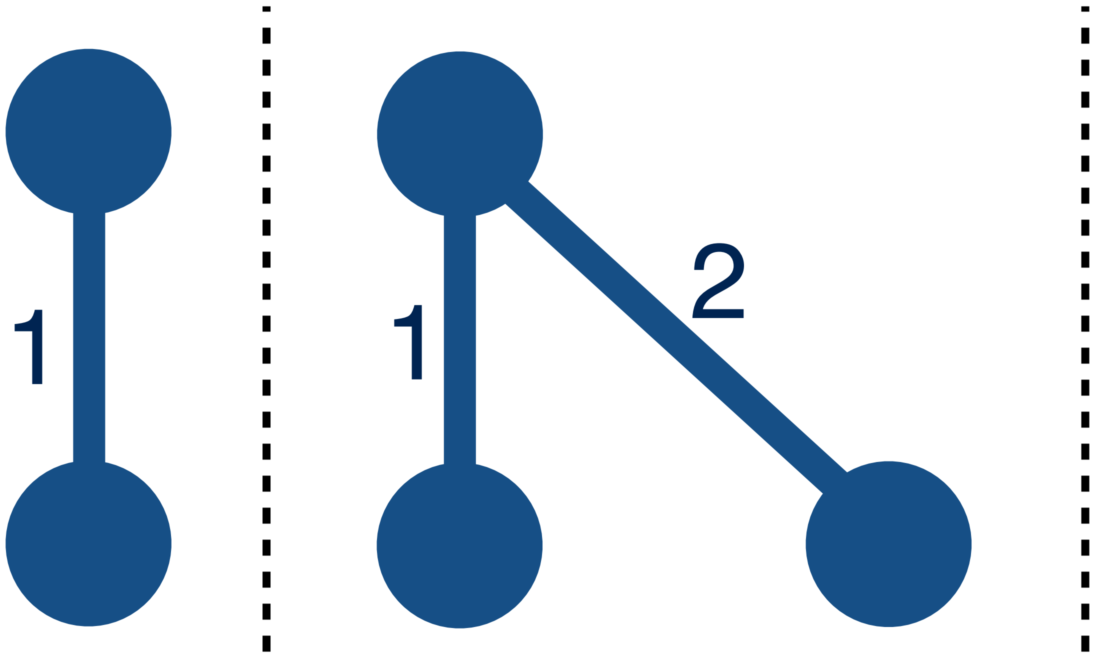

G_1 G_2

 [Broderick, Cai 2015; Cai, Broderick 2015a,b; Crane, Dempsey 2015a,b,16a; Cai, Campbell, Broderick 2016]

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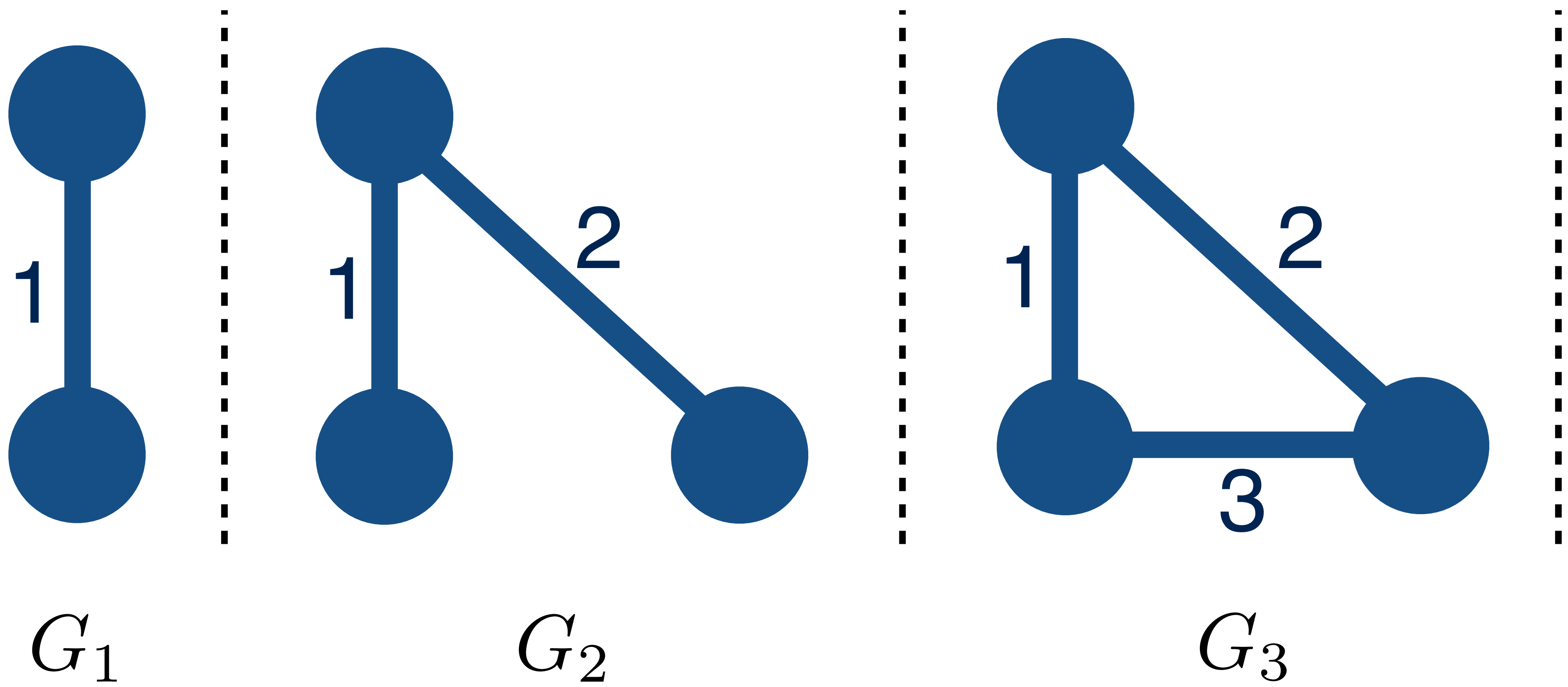

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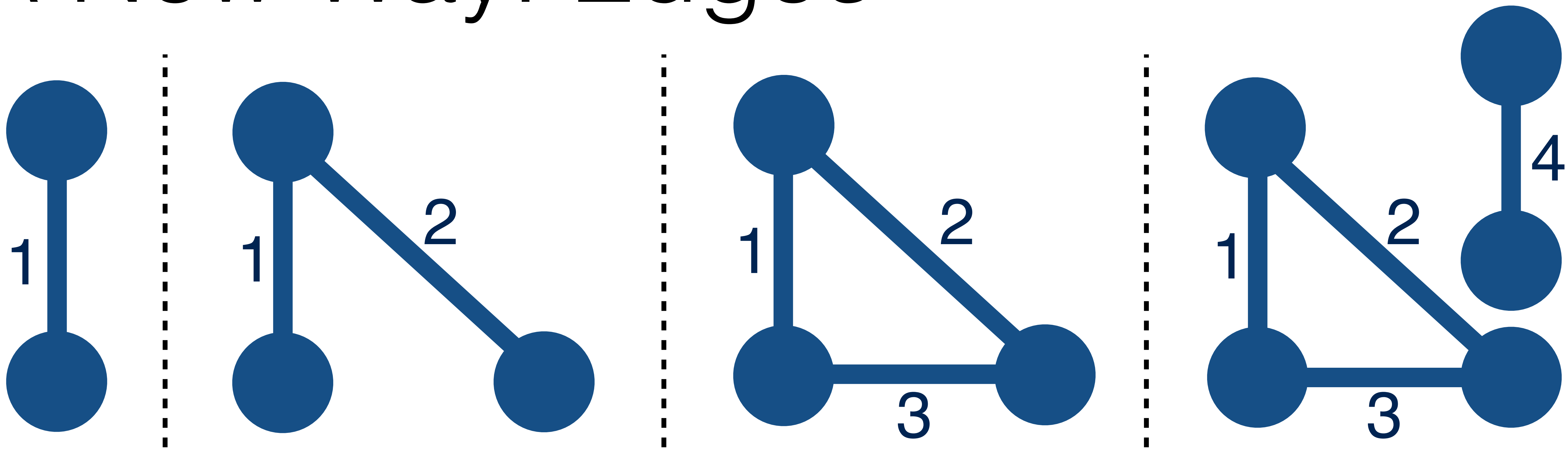
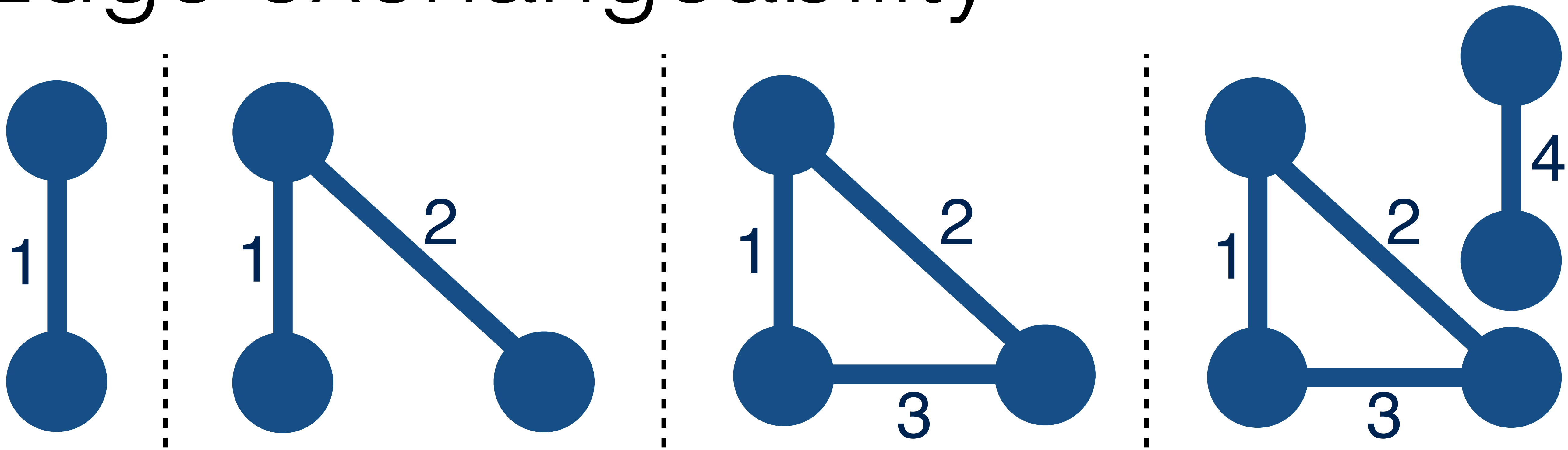


G_1 G_2 G_3 G_4

[Broderick, Cai 2015; Cai, Broderick 2015a,b; Crane, Dempsey 2015a,b,16a; Cai, Campbell, Broderick 2016]

Edge exchangeability



G_1 G_2 G_3 G_4

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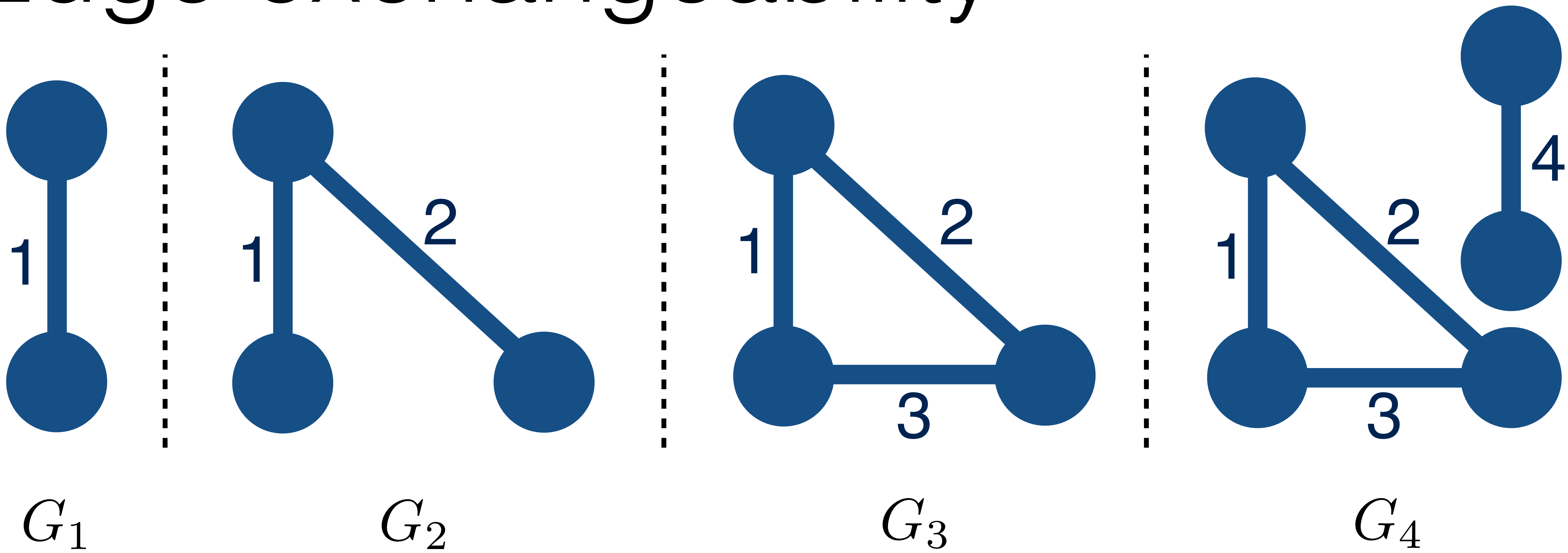

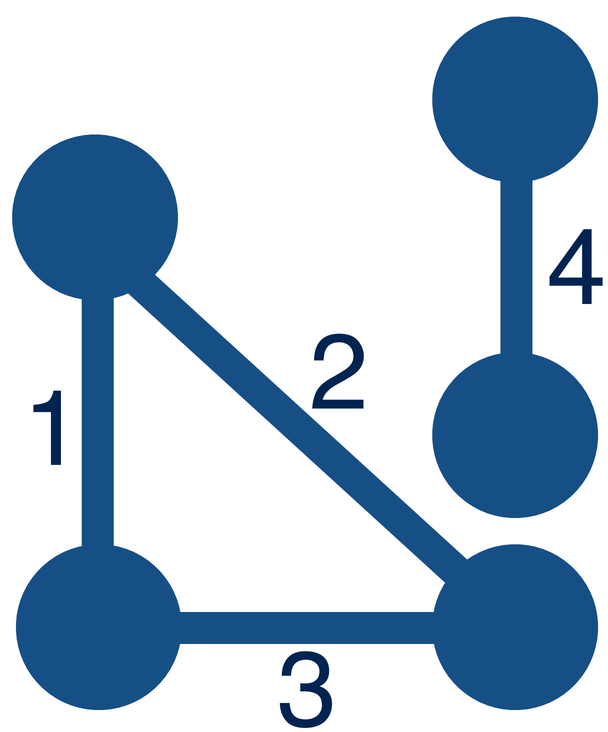

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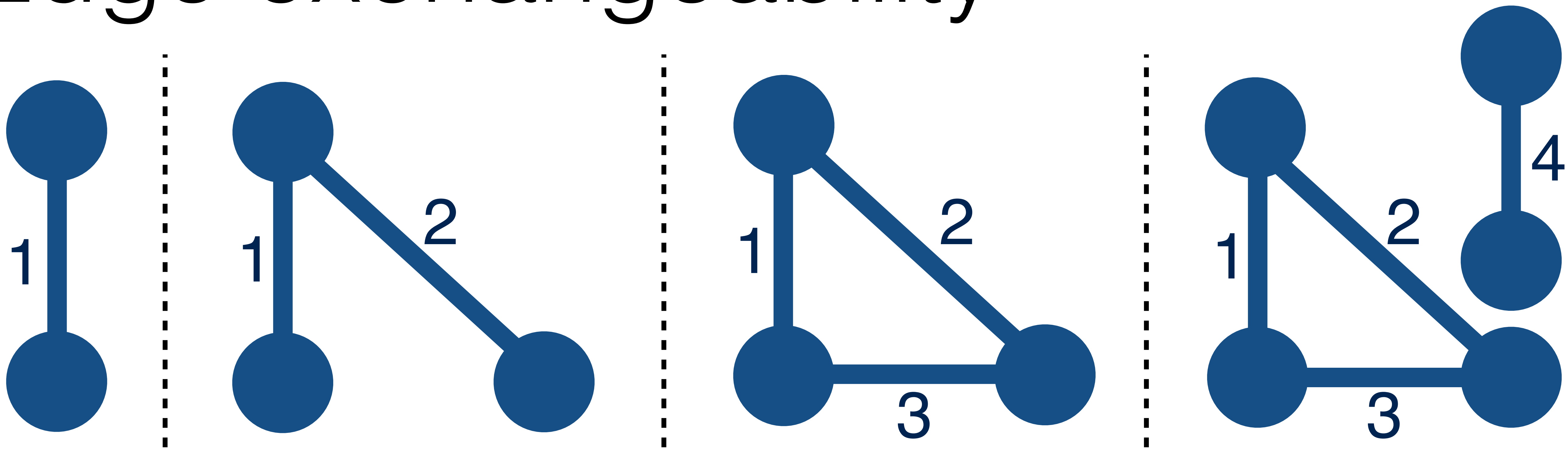
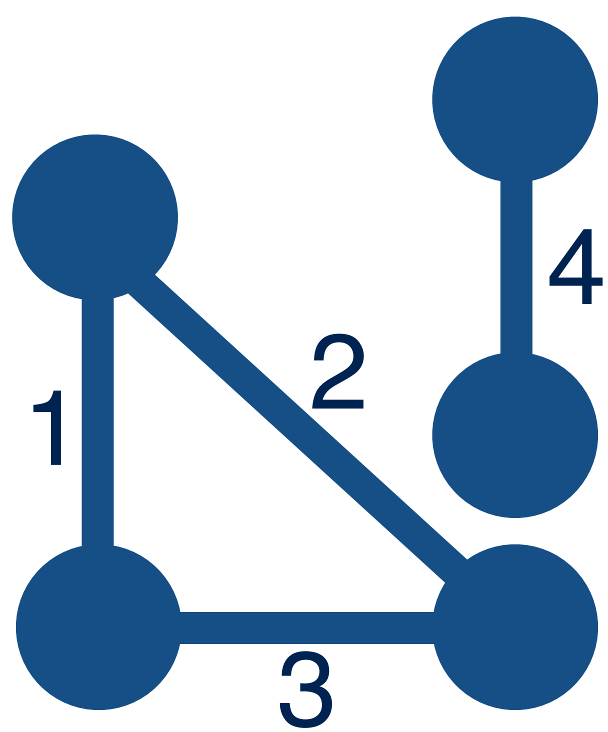


G_1 G_2 G_3 G_4

$p($  $)$

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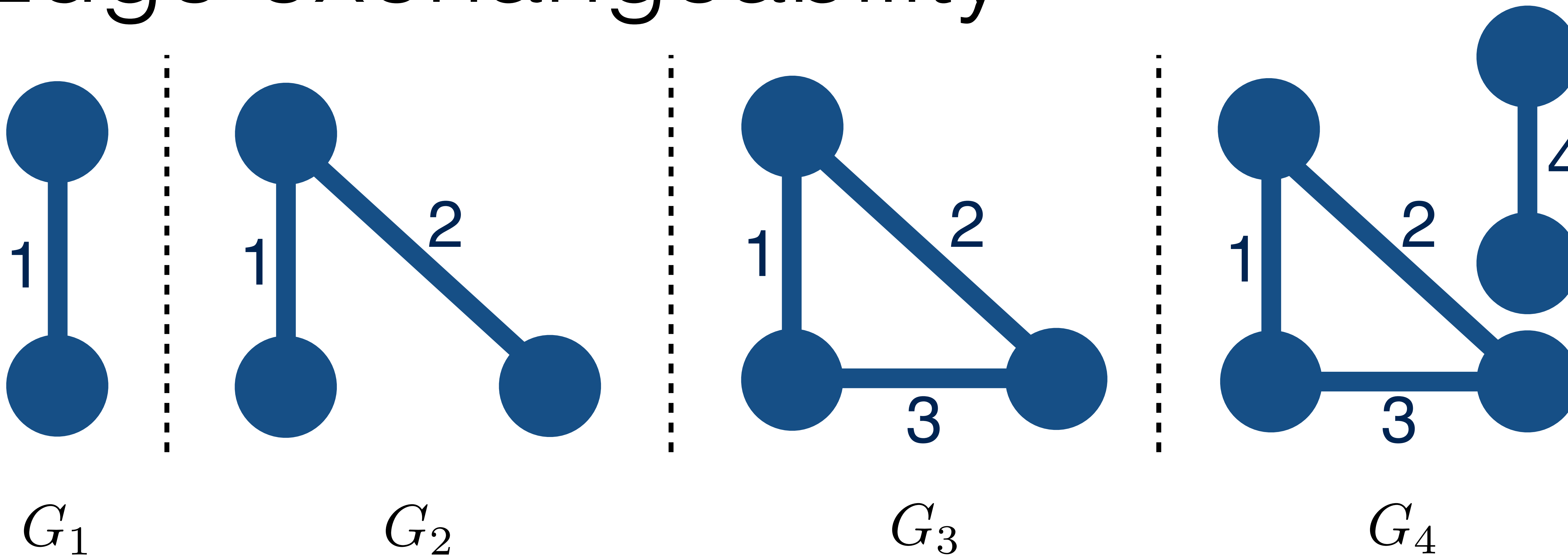

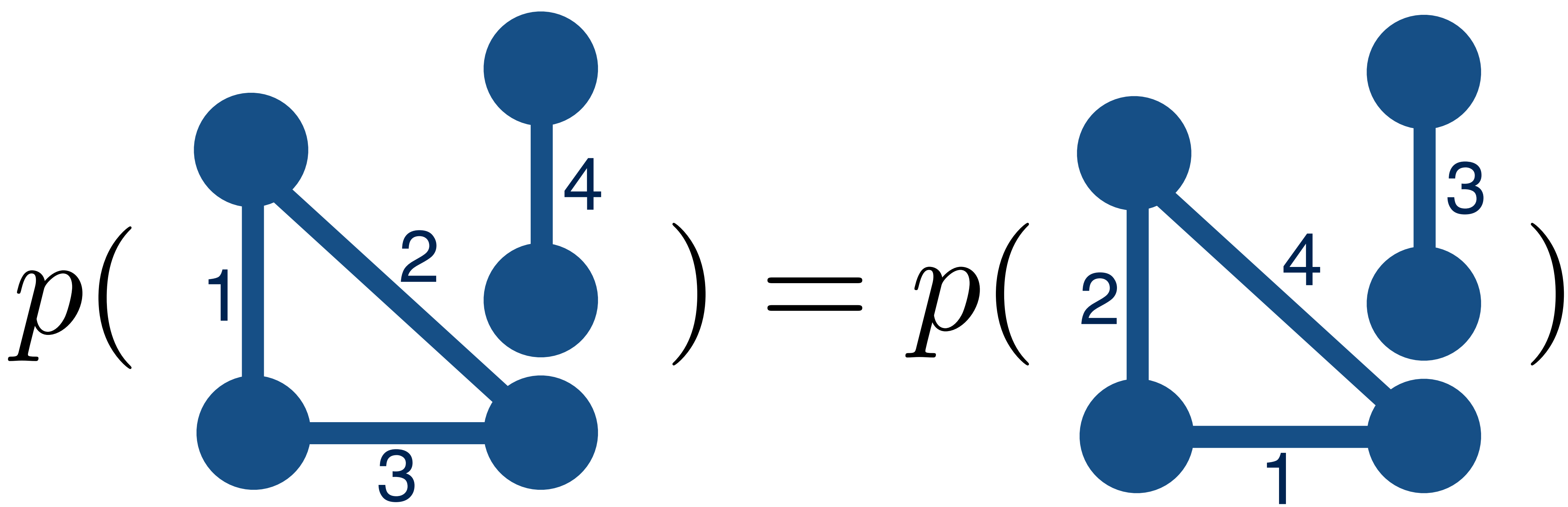

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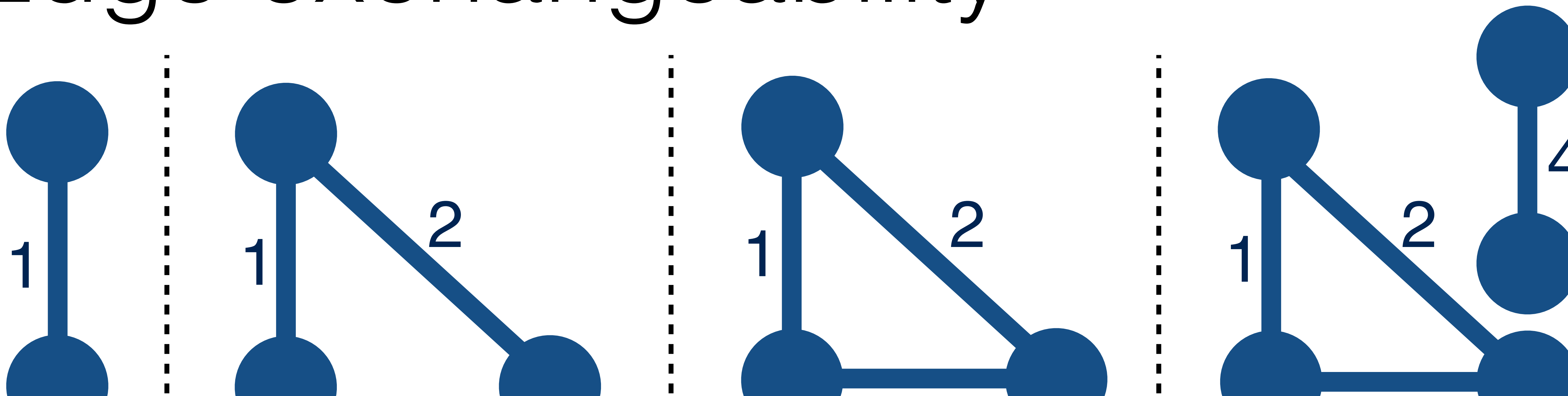
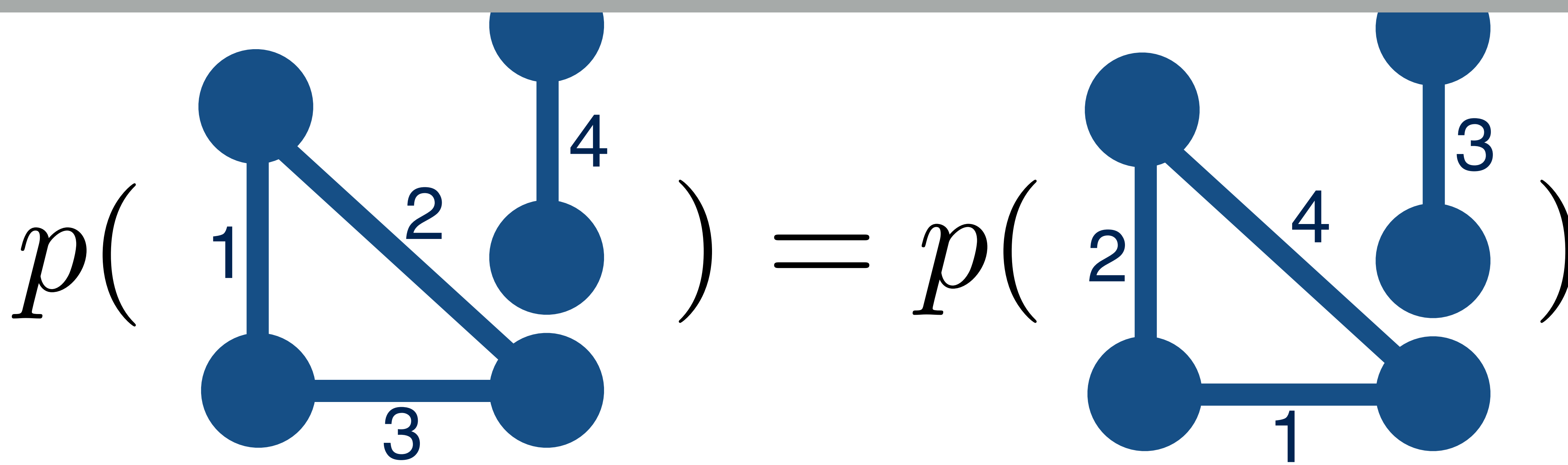
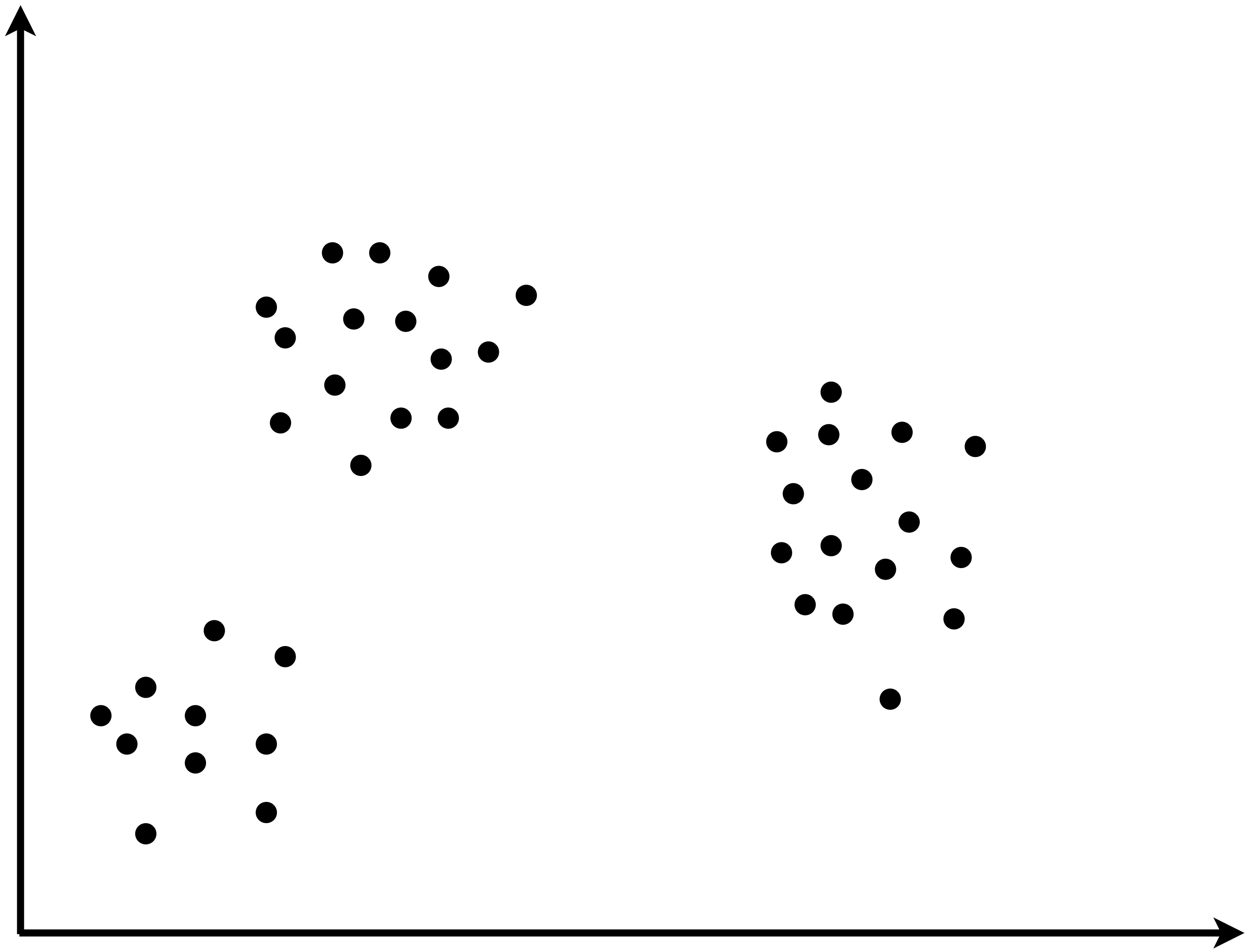


Thm (CCB). A wide class of edge-exchangeable graph models yields sparse graph sequences.

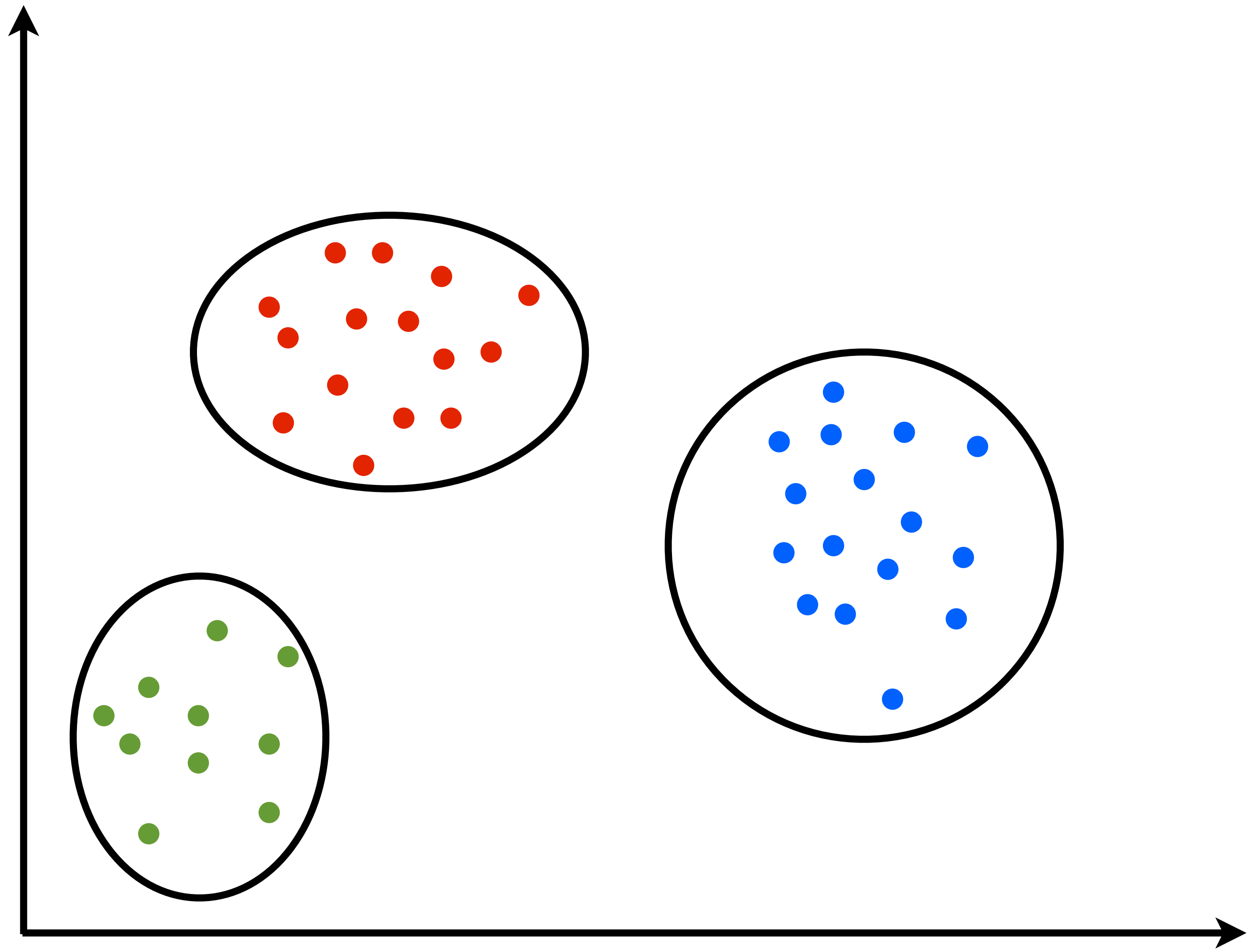


 [Broderick, Cai 2015; Cai, Broderick 2015a,b; Crane, Dempsey 2015a,b,16a; Cai, Campbell, Broderick 2016]

Clustering

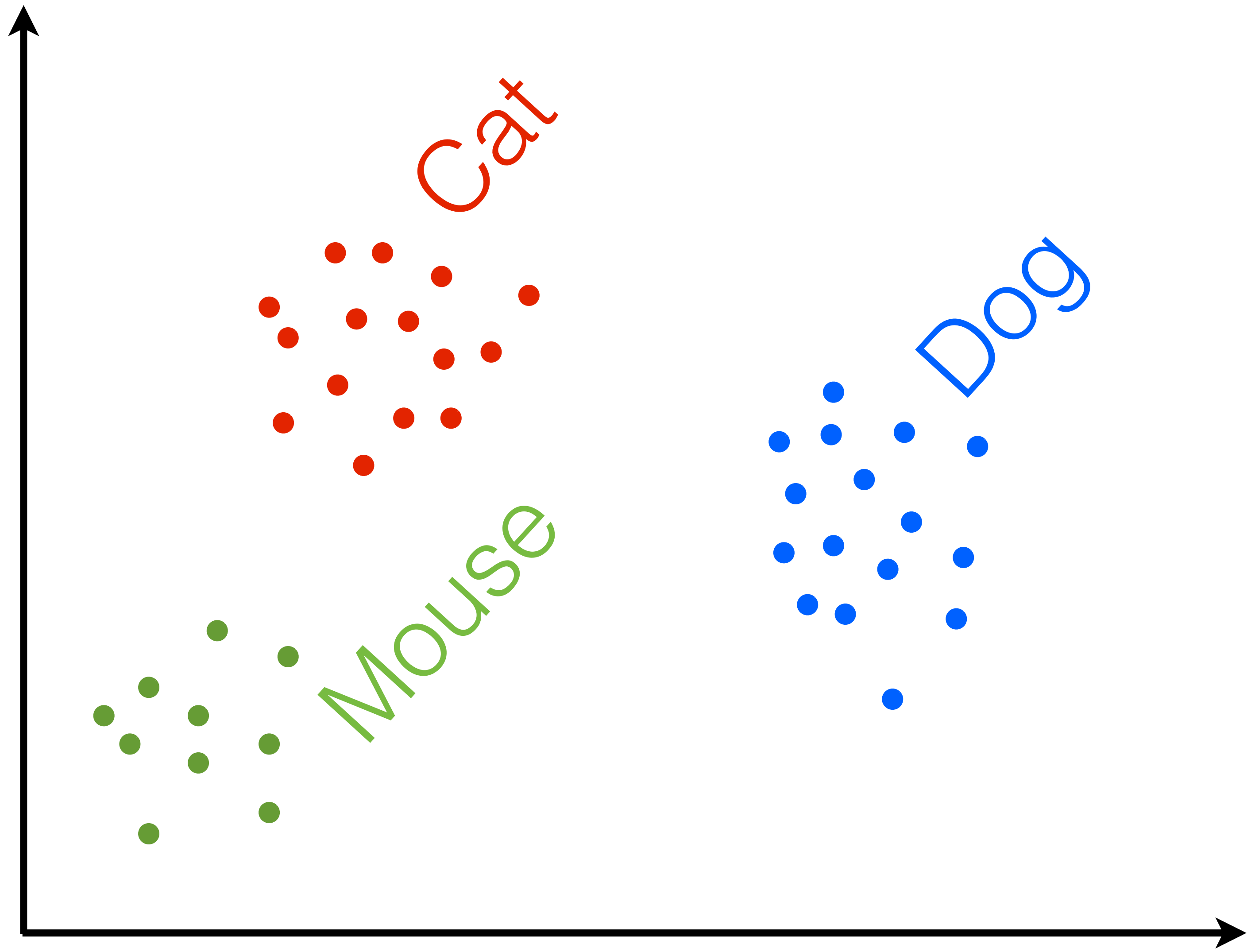


Clustering



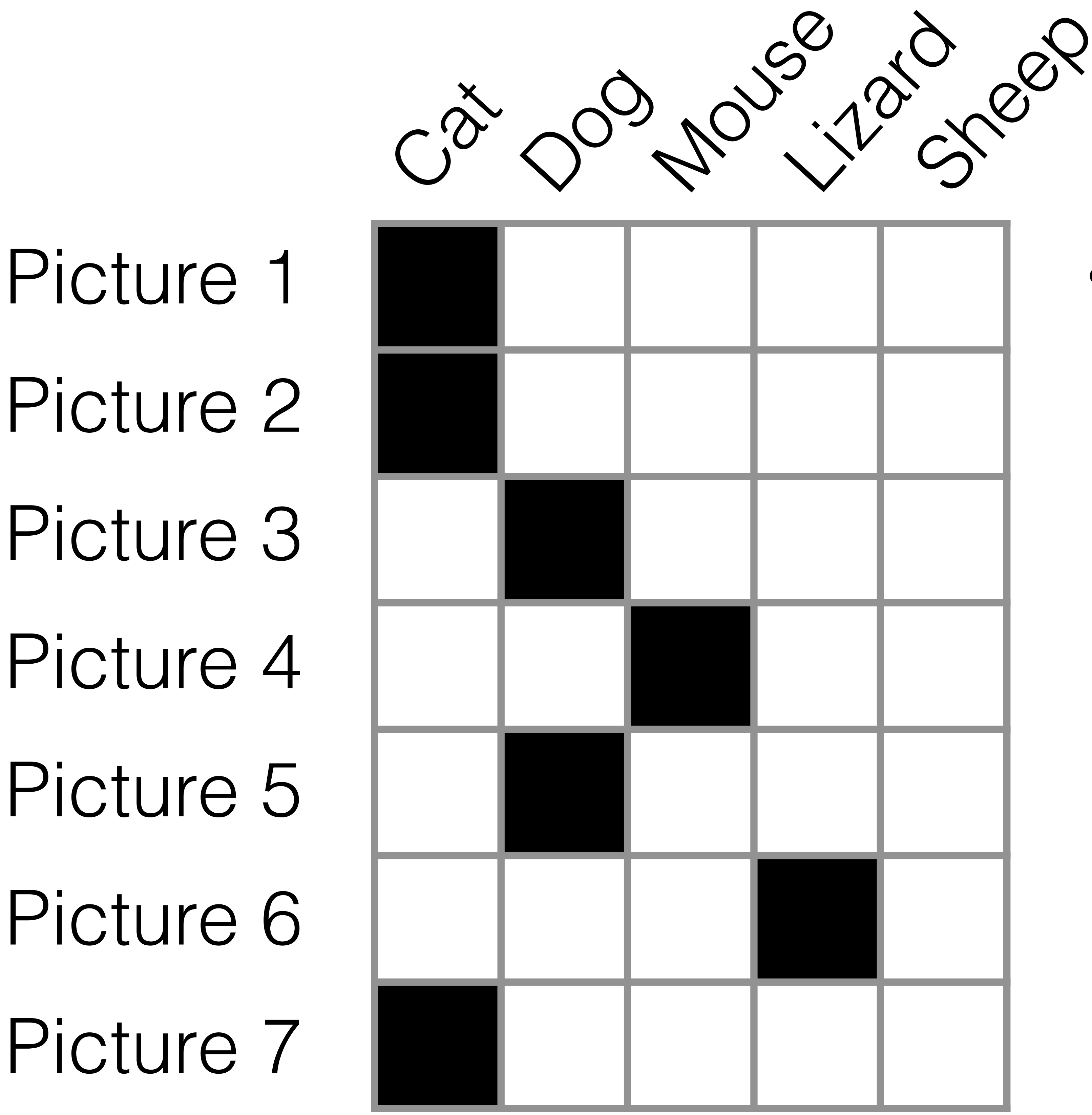
“Clusters”

Clustering



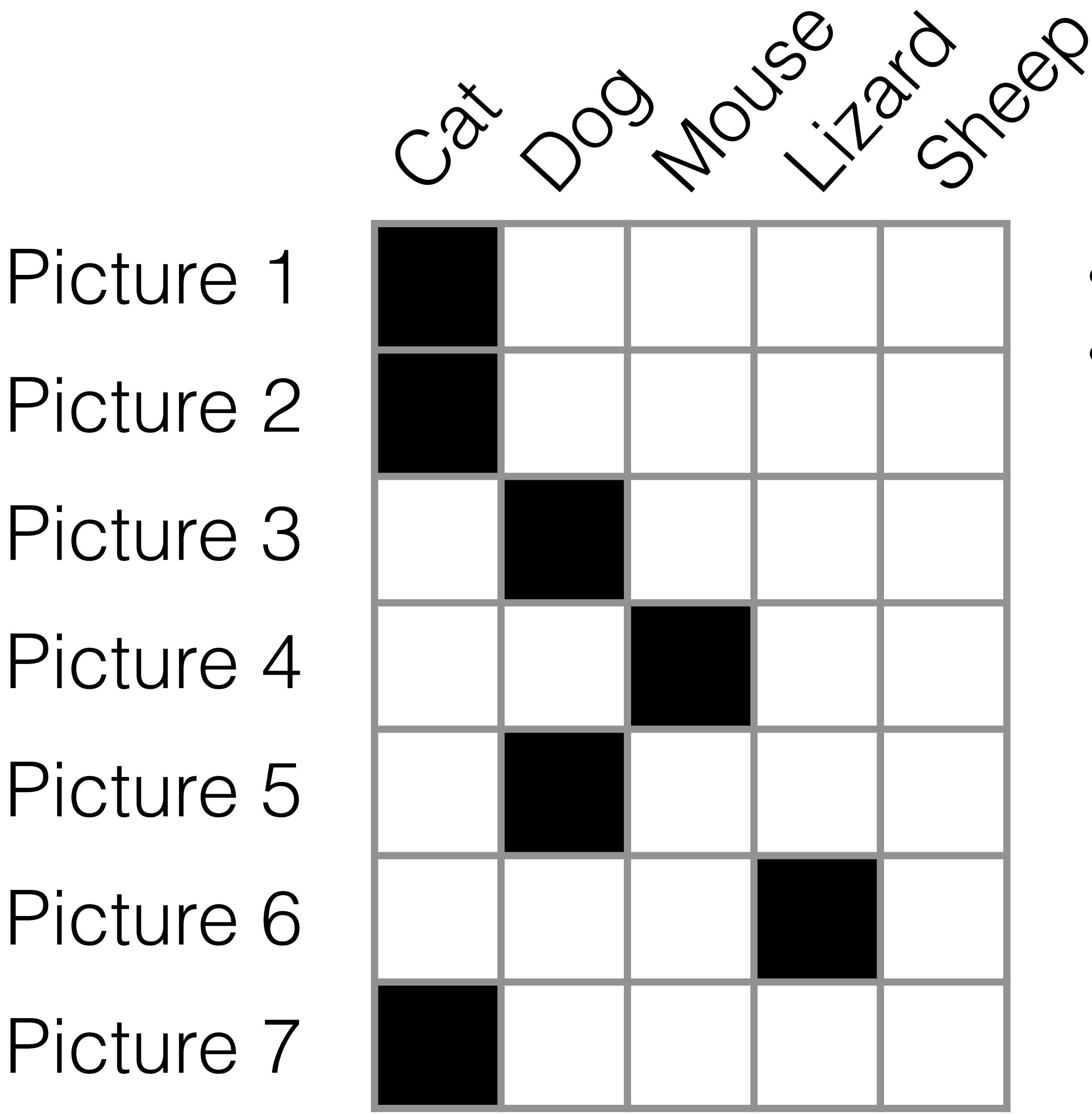
“Clusters”

Clustering



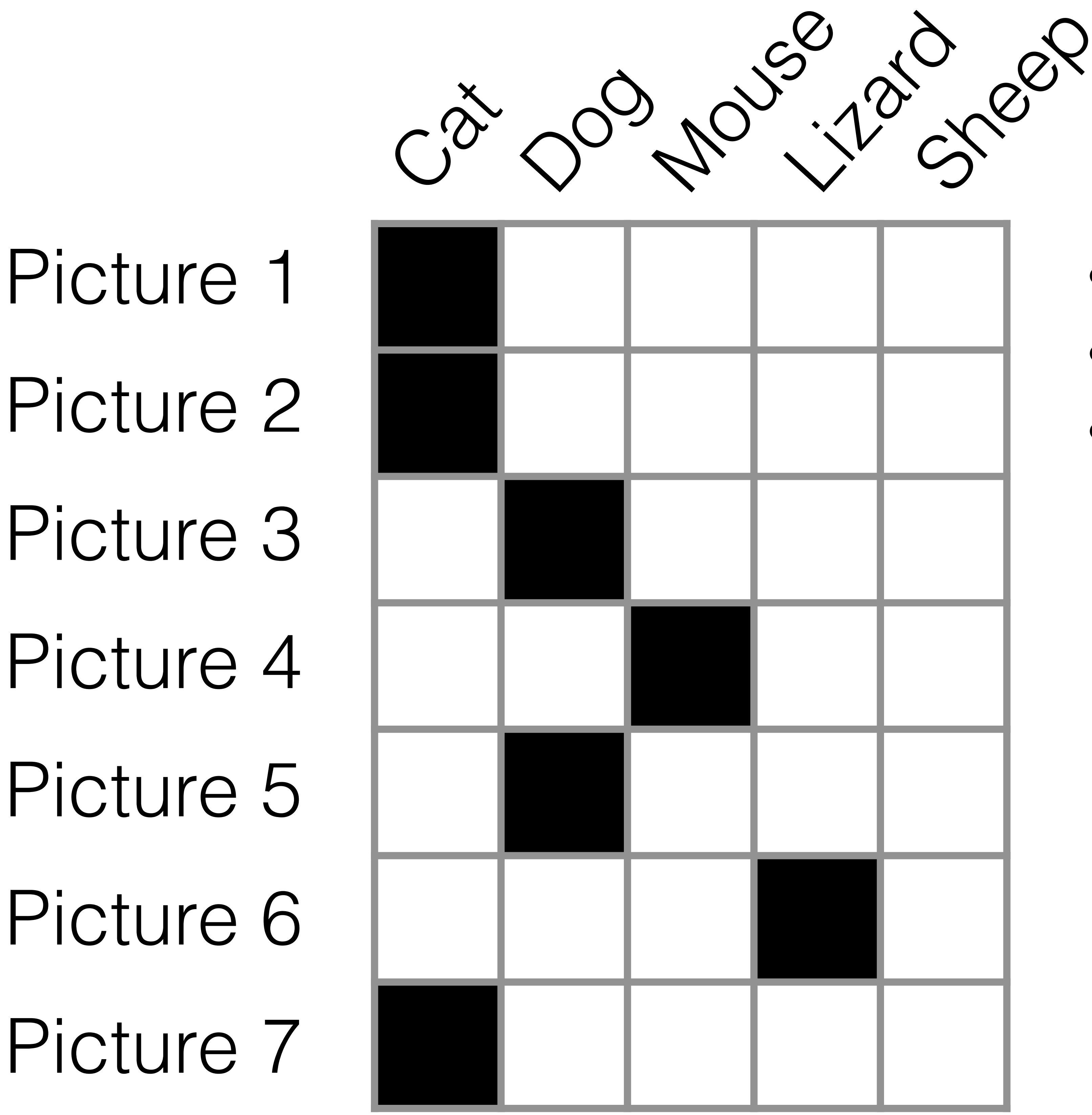
- Groups: clusters

Clustering



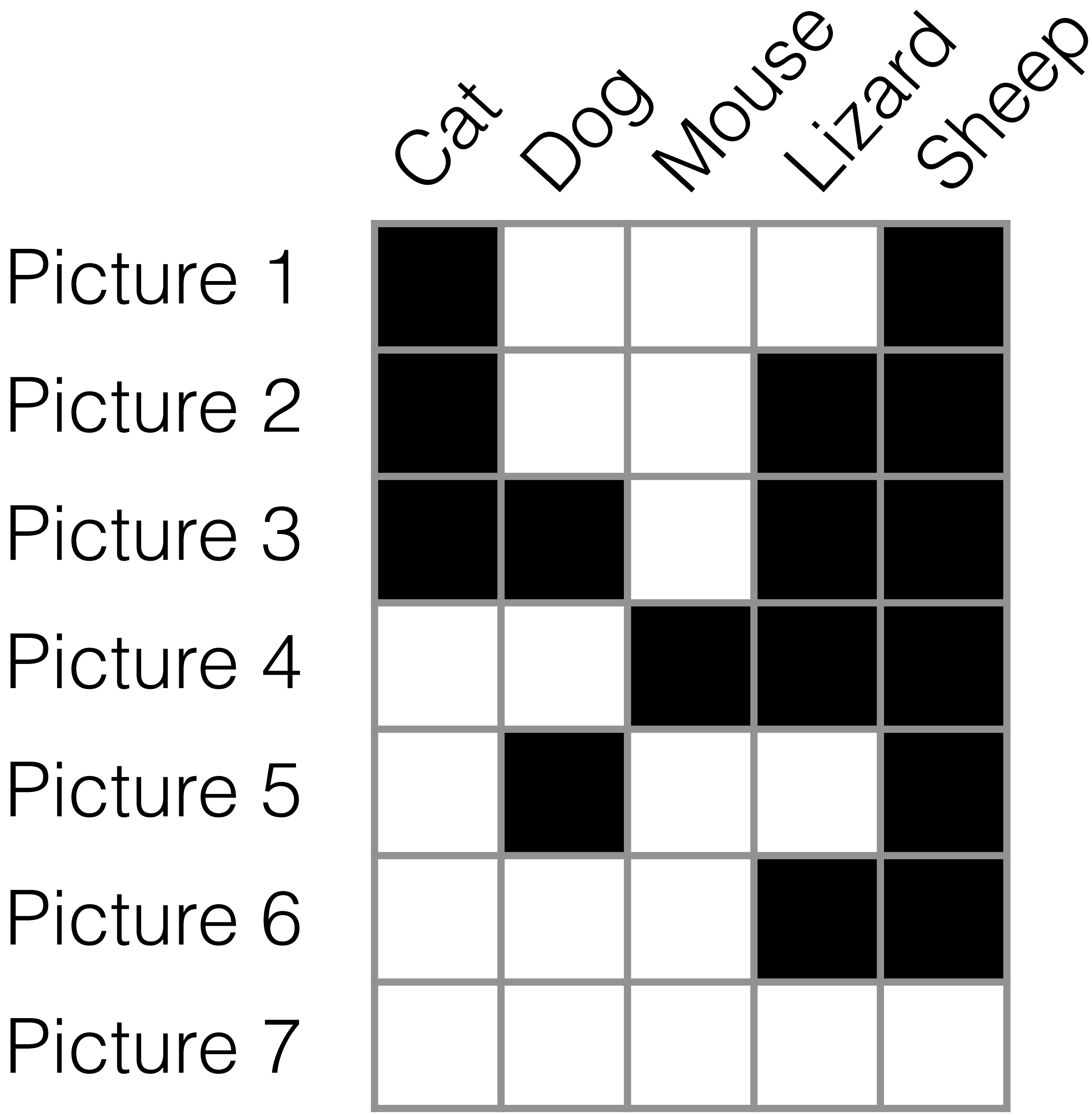
- Groups: clusters
- Exchangeable

Clustering

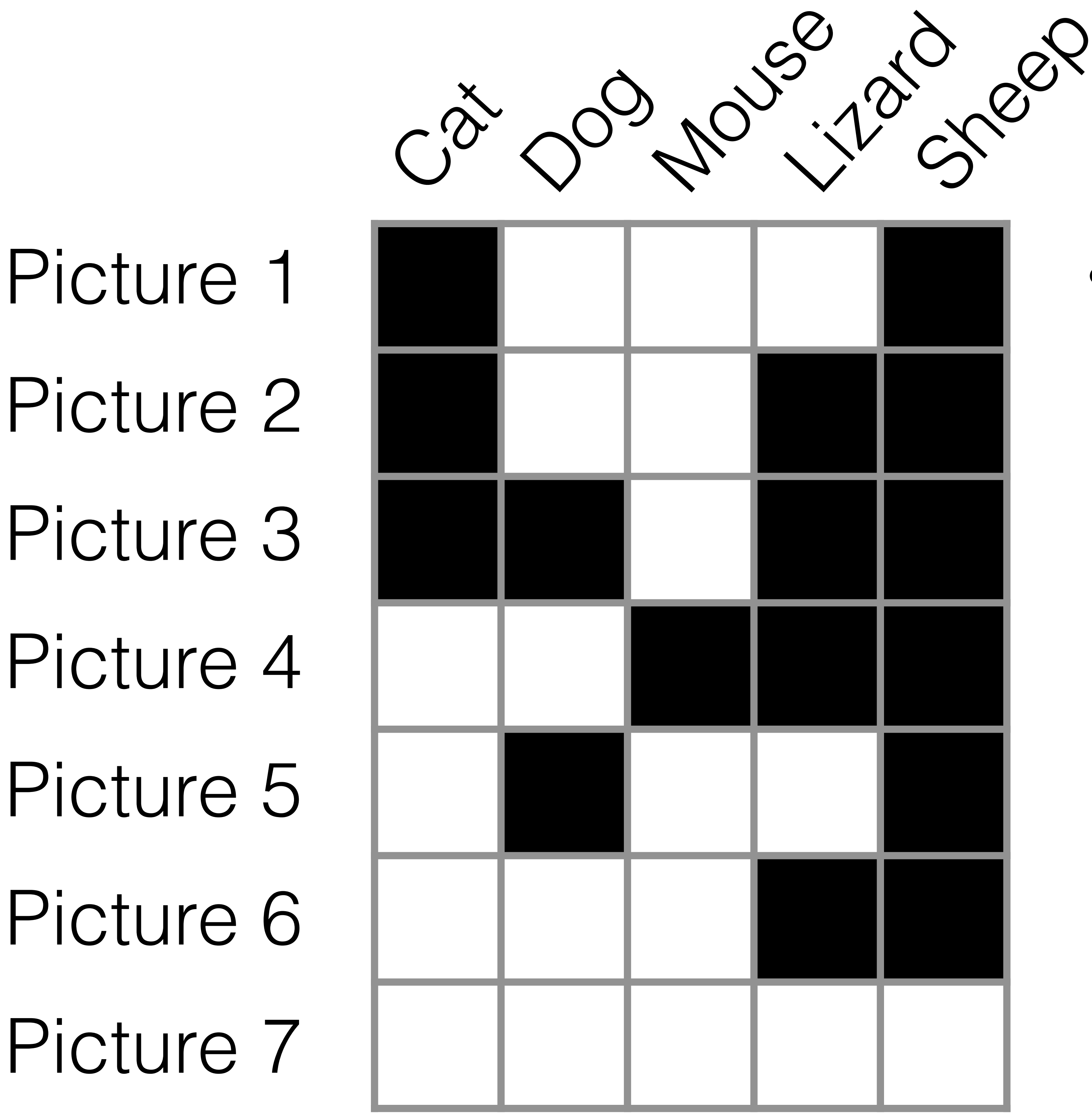


- Groups: clusters
- Exchangeable
- Chinese restaurant process/Dirichlet process

Feature allocation

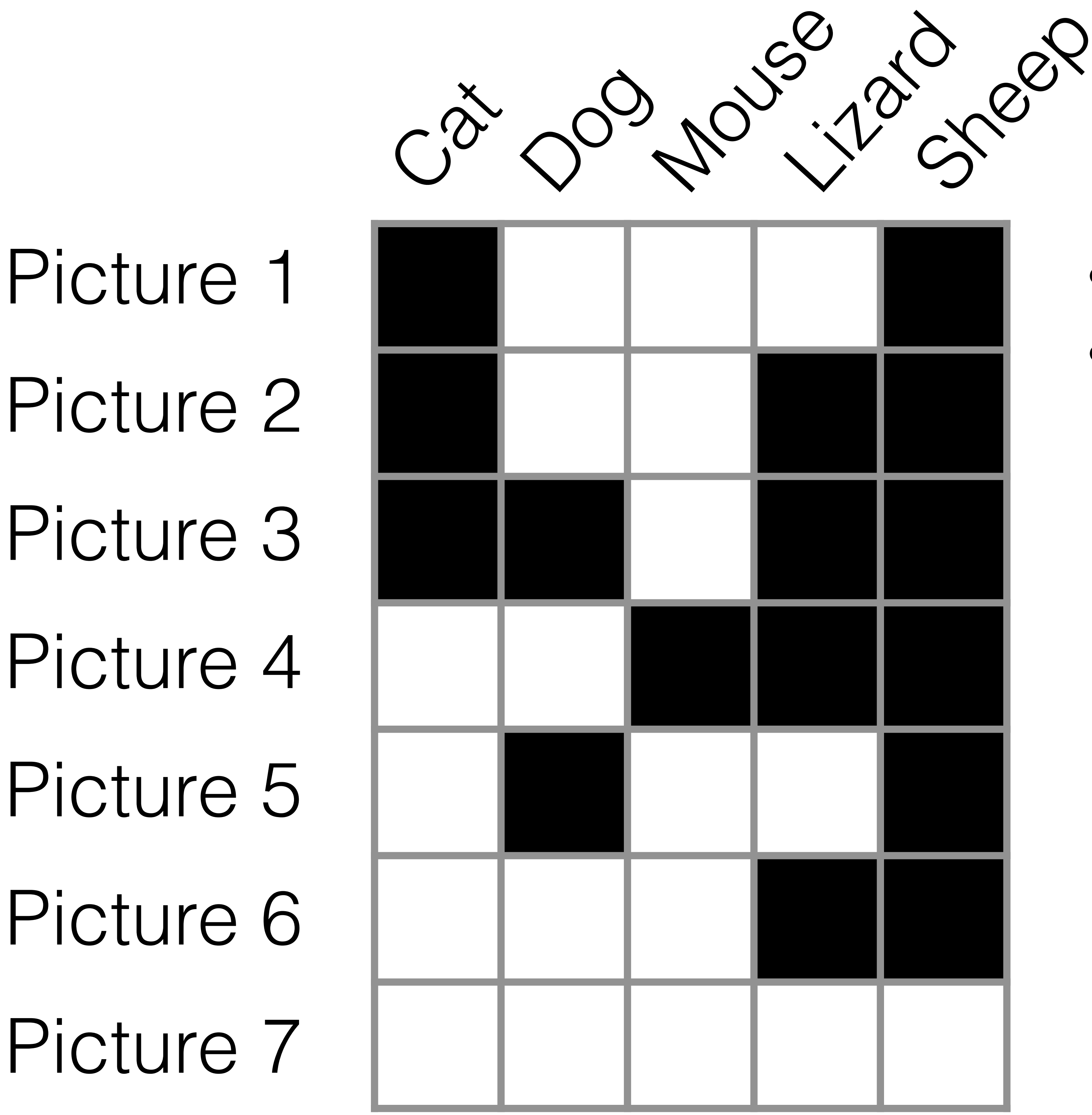


Feature allocation



- Groups: features

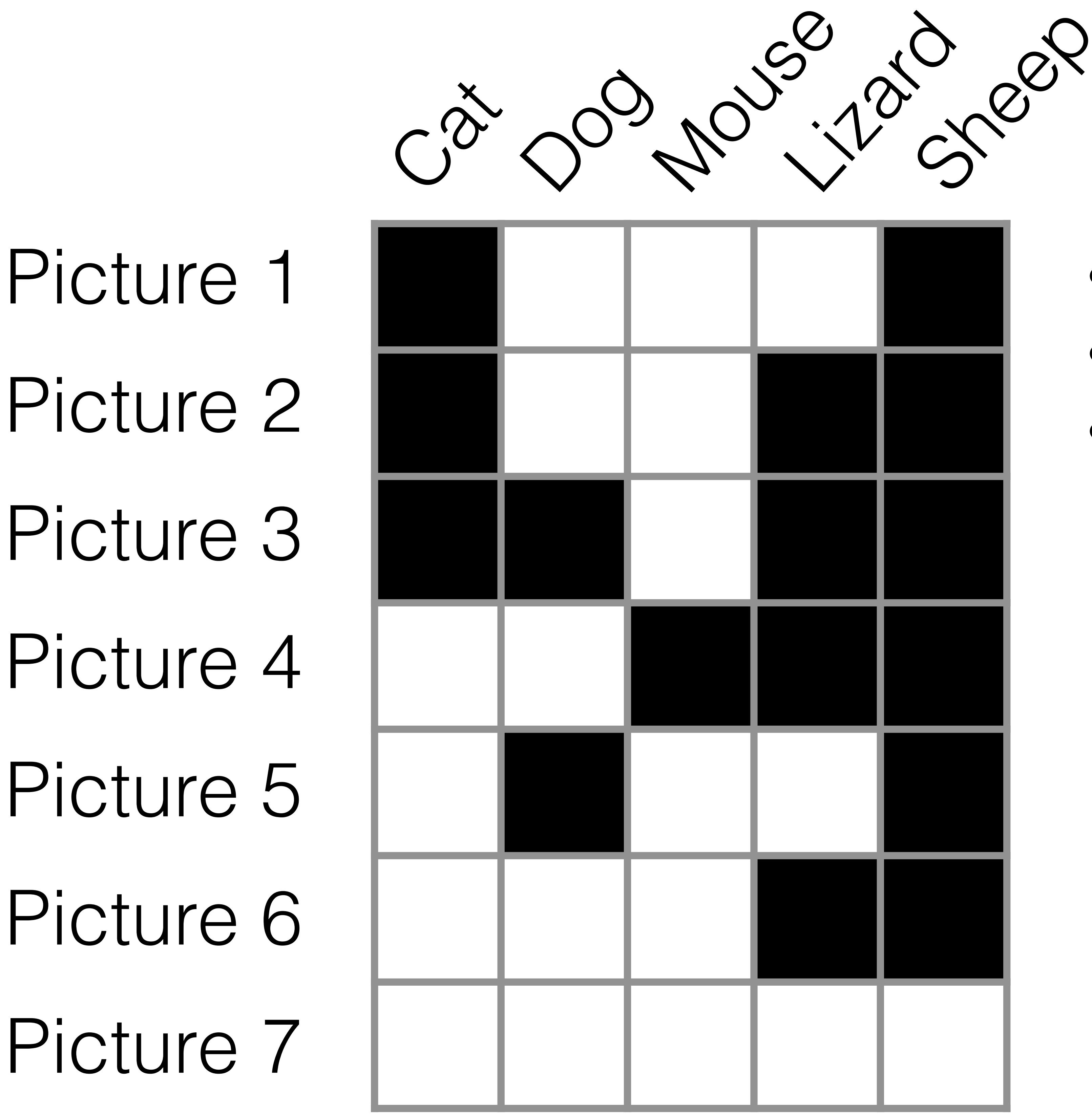
Feature allocation



- Groups: features
- Exchangeable

[Broderick, Jordan, Pitman 2013; Broderick, Pitman, Jordan 2013]

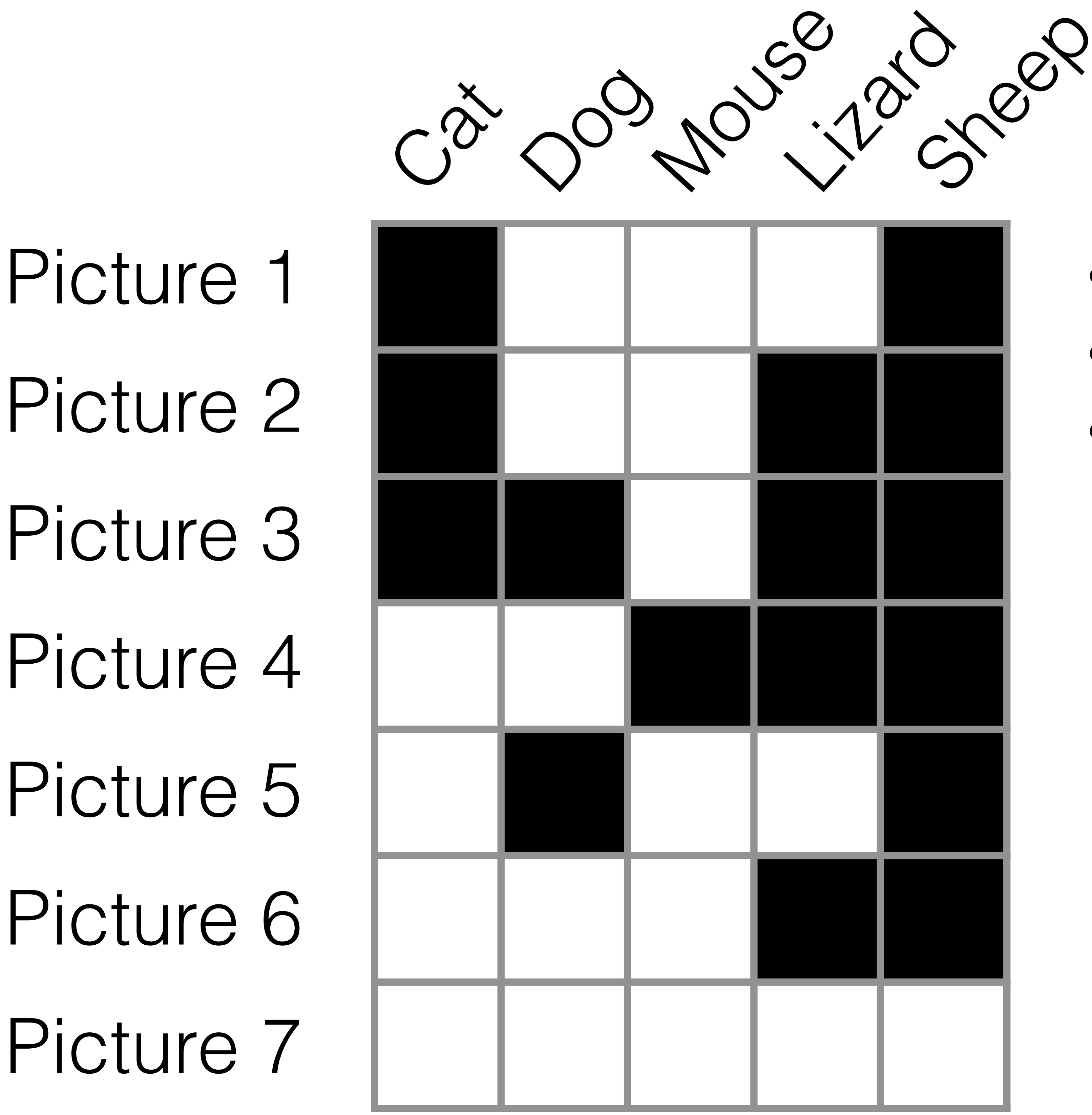
Feature allocation



- Groups: features
- Exchangeable
- Indian buffet process/beta process

[Broderick, Jordan, Pitman 2013; Broderick, Pitman, Jordan 2013]

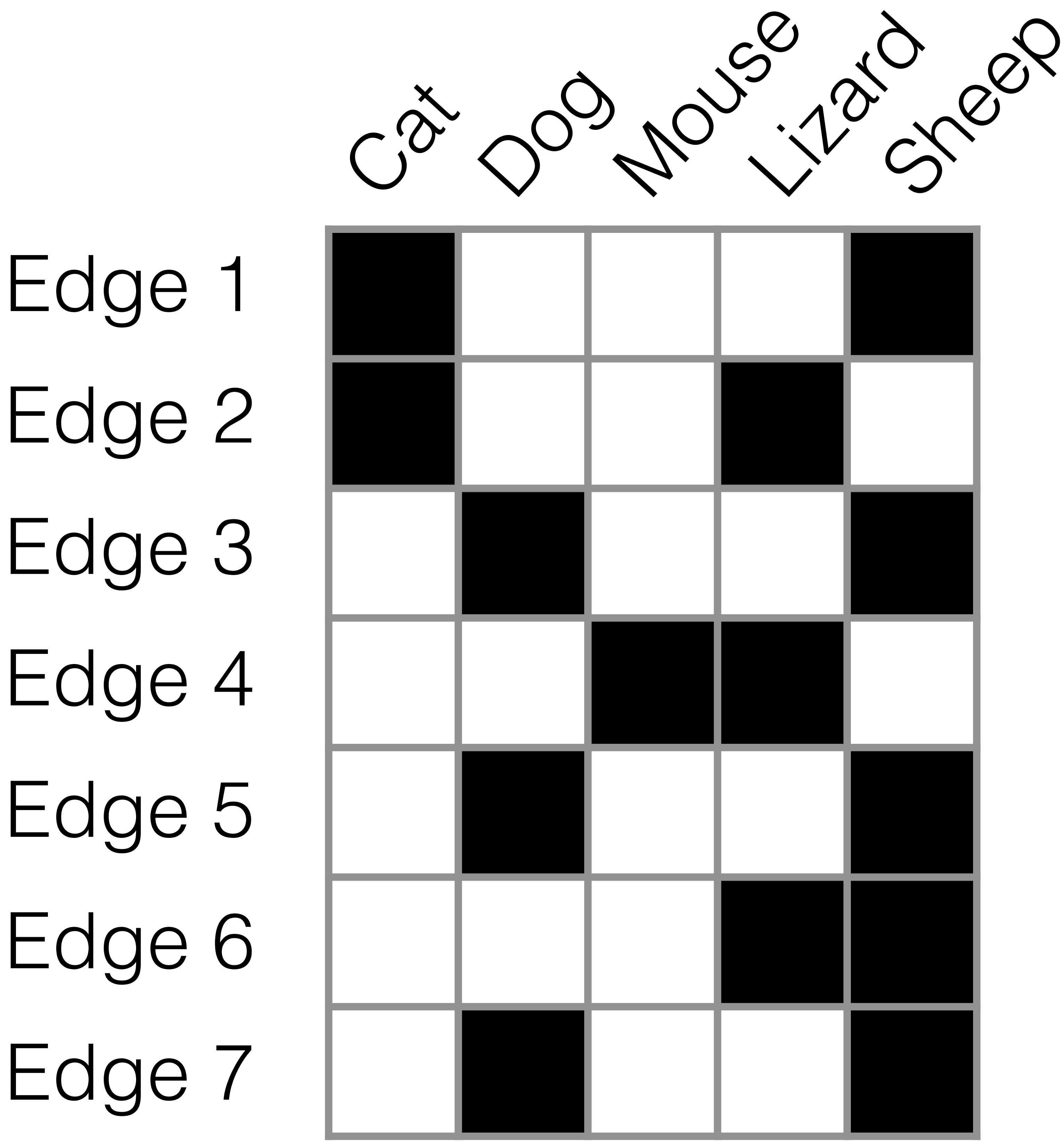
Feature allocation



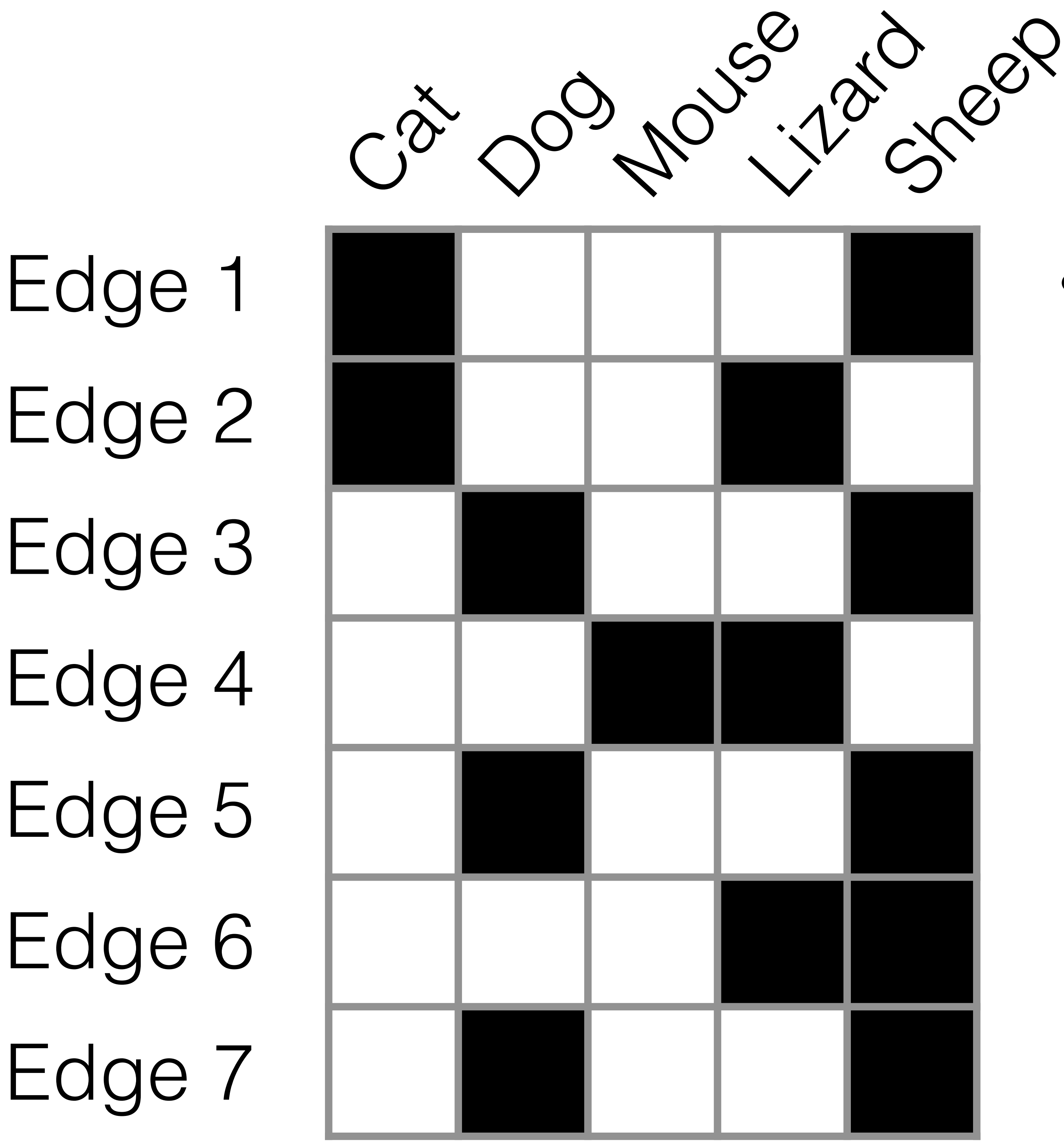
- Groups: features
- Exchangeable
- Indian buffet process/beta process

[Broderick, Jordan, Pitman 2013; Broderick, Pitman, Jordan 2013; Campbell, Cai, Broderick 2016]

Graph

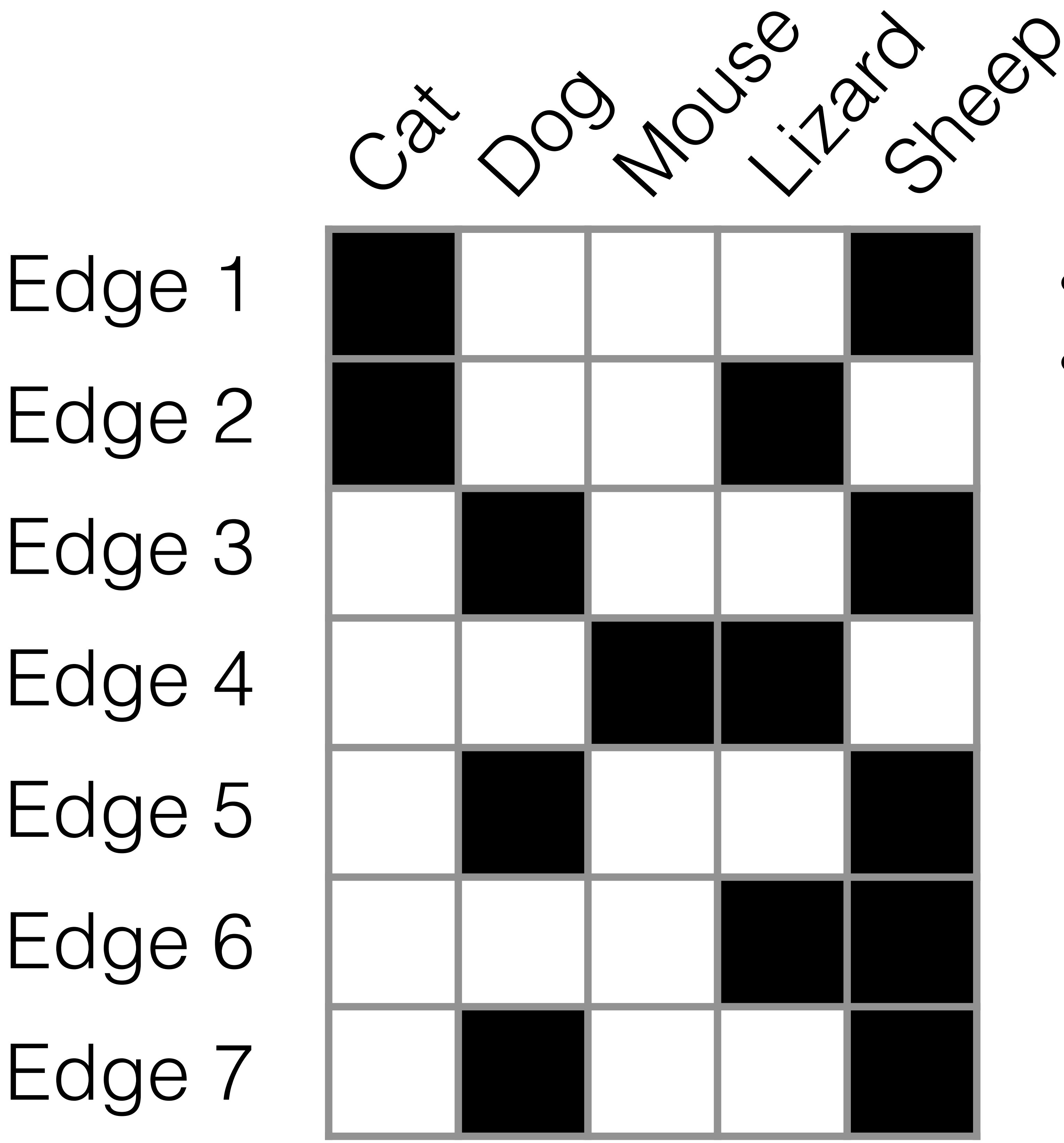

Cat
Dog
Mouse
Lizard
Sheep
Edge 1
Edge 2
Edge 3
Edge 4
Edge 5
Edge 6
Edge 7

Graph



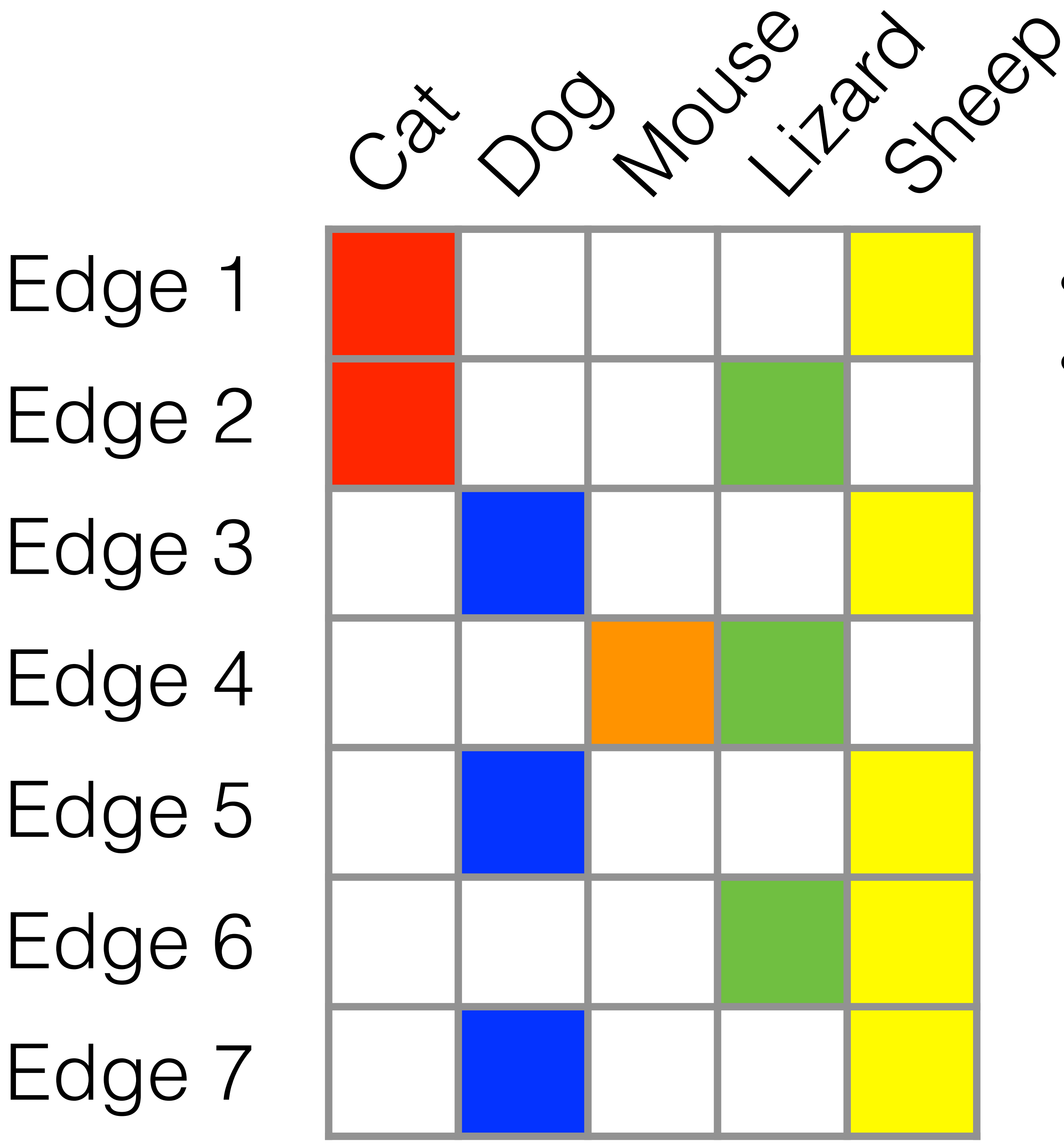
- Groups: vertices

Graph



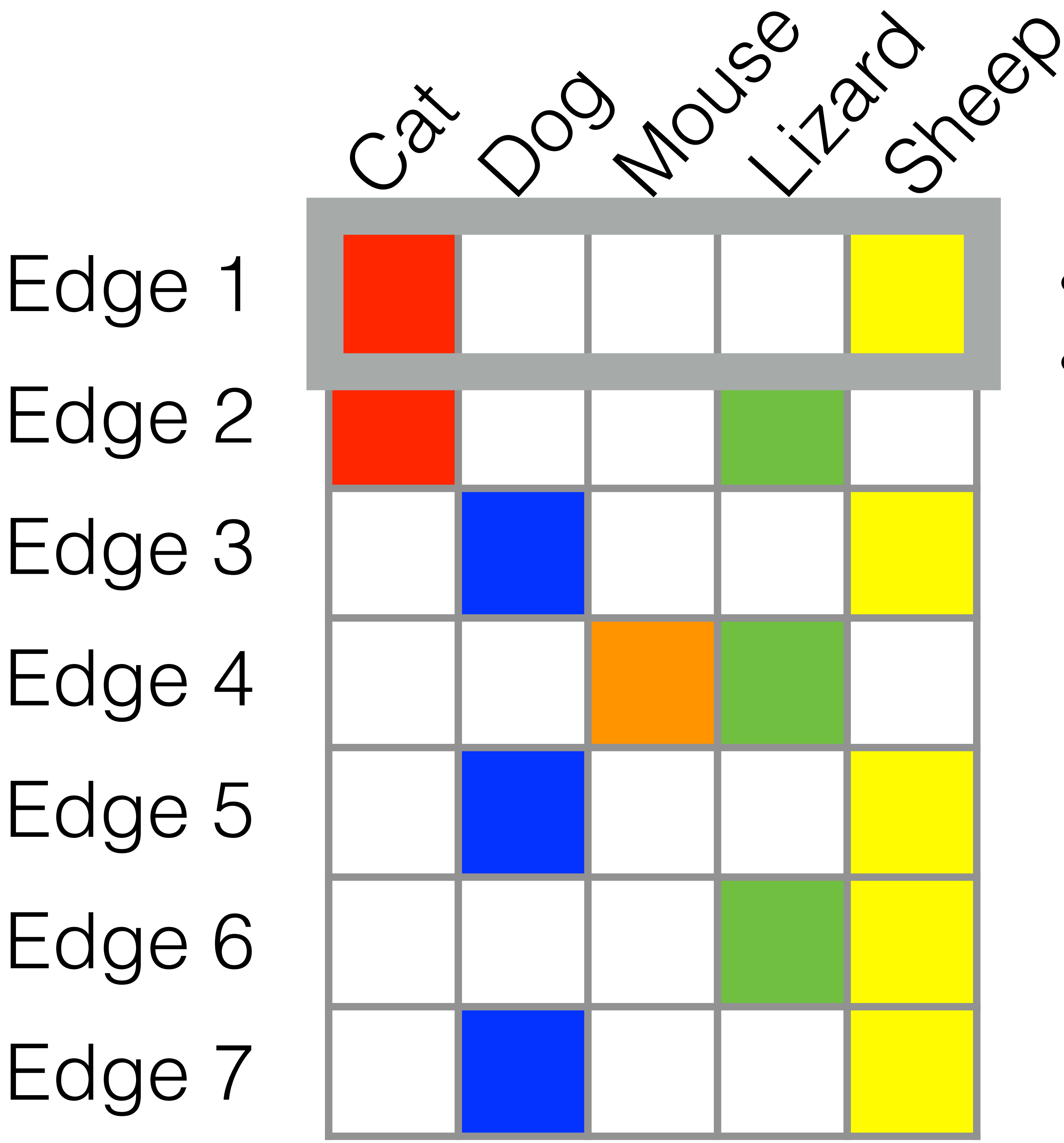
- Groups: vertices
- Edge-exchangeable

Graph



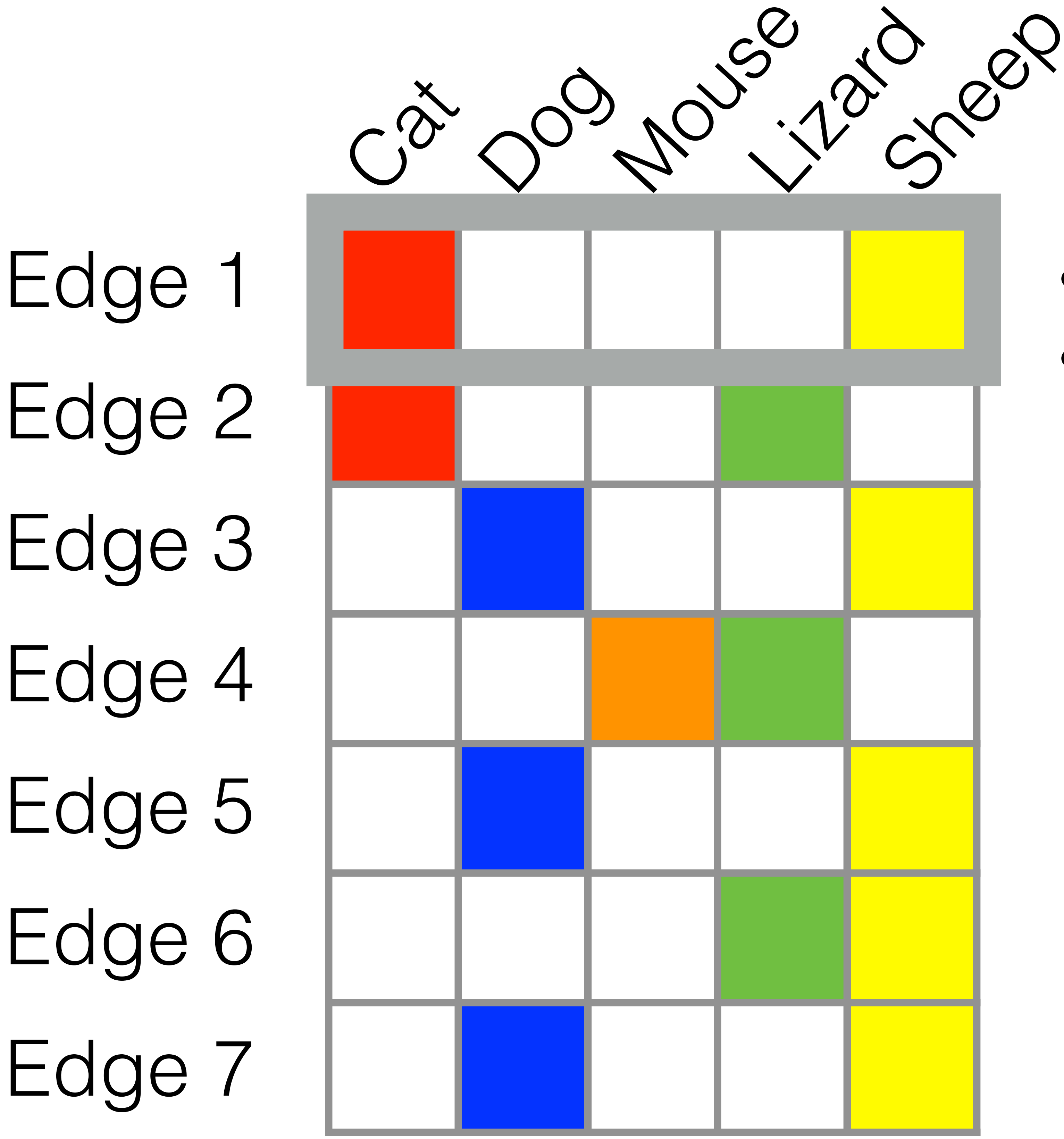
- Groups: vertices
- Edge-exchangeable

Graph

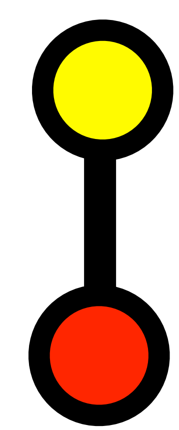


- Groups: vertices
- Edge-exchangeable

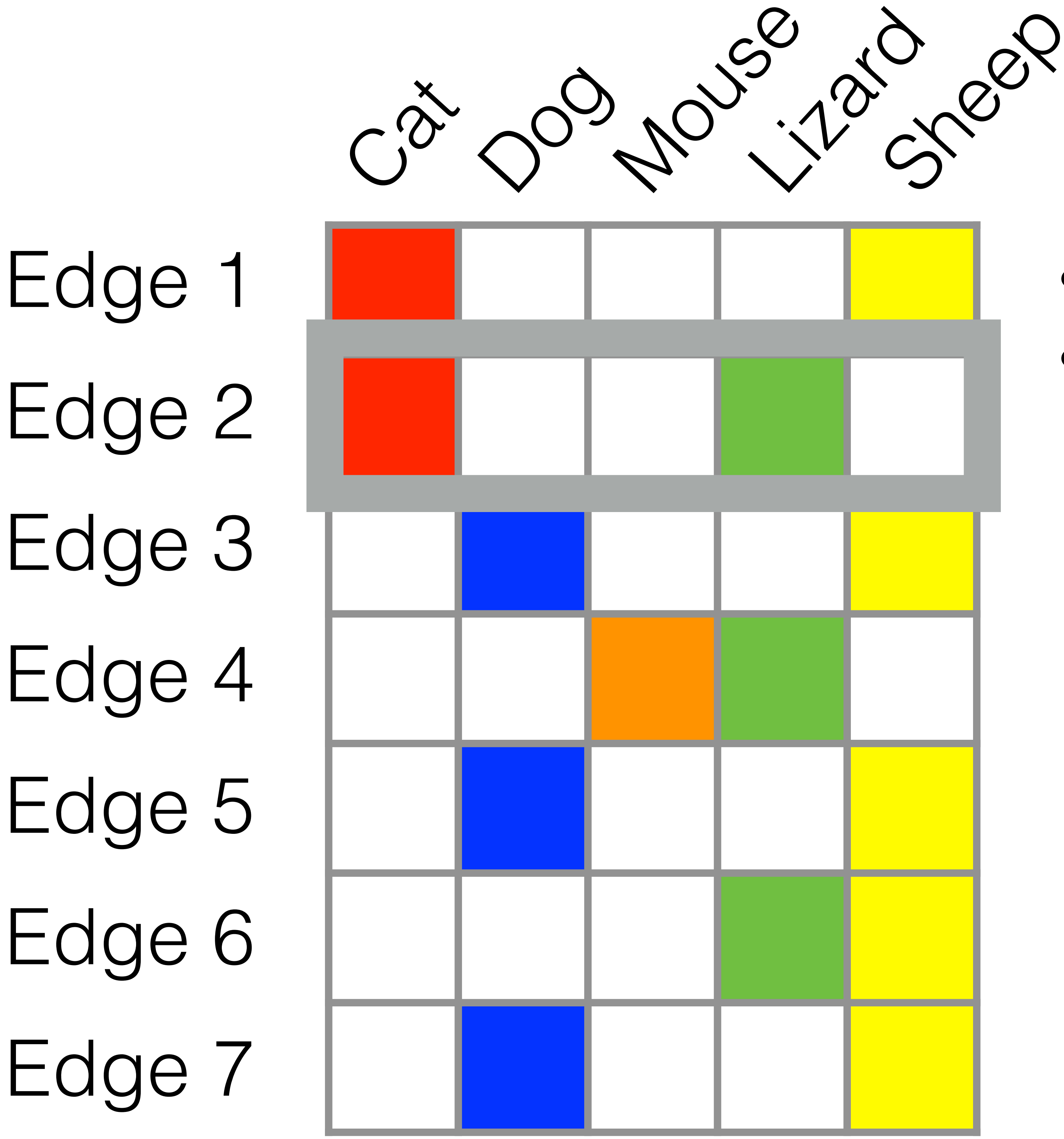
Graph



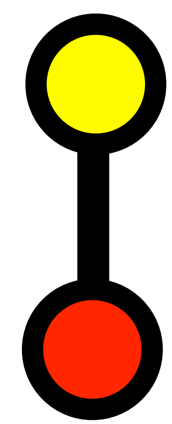
- Groups: vertices
- Edge-exchangeable



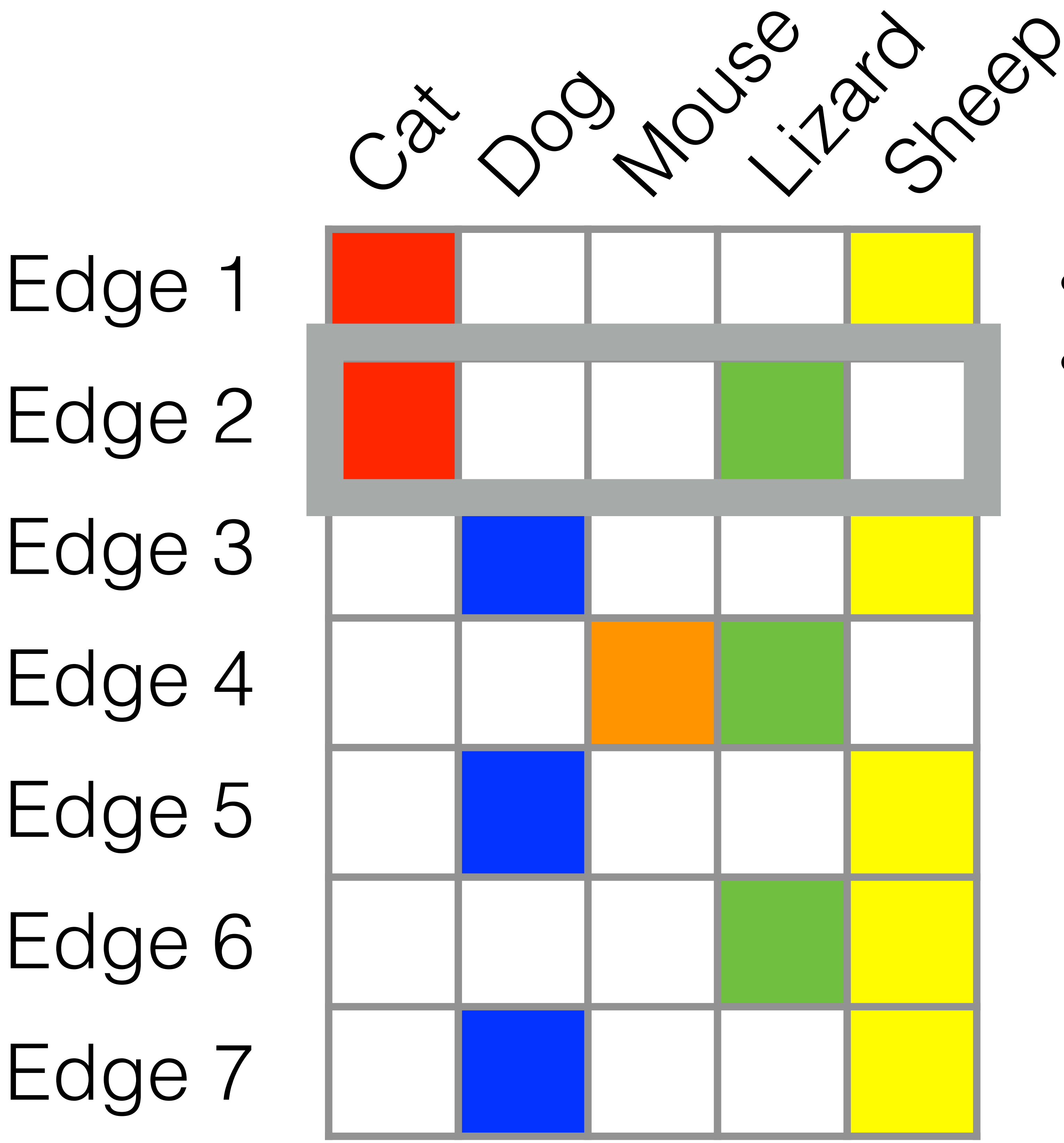
Graph



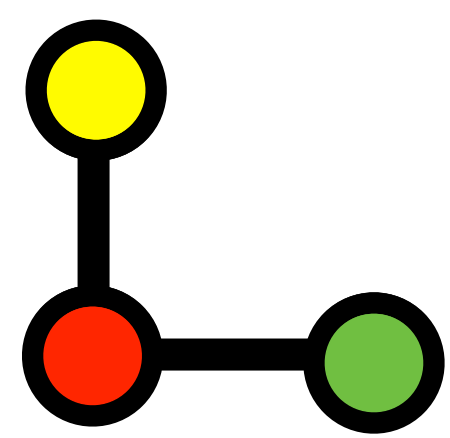
- Groups: vertices
- Edge-exchangeable



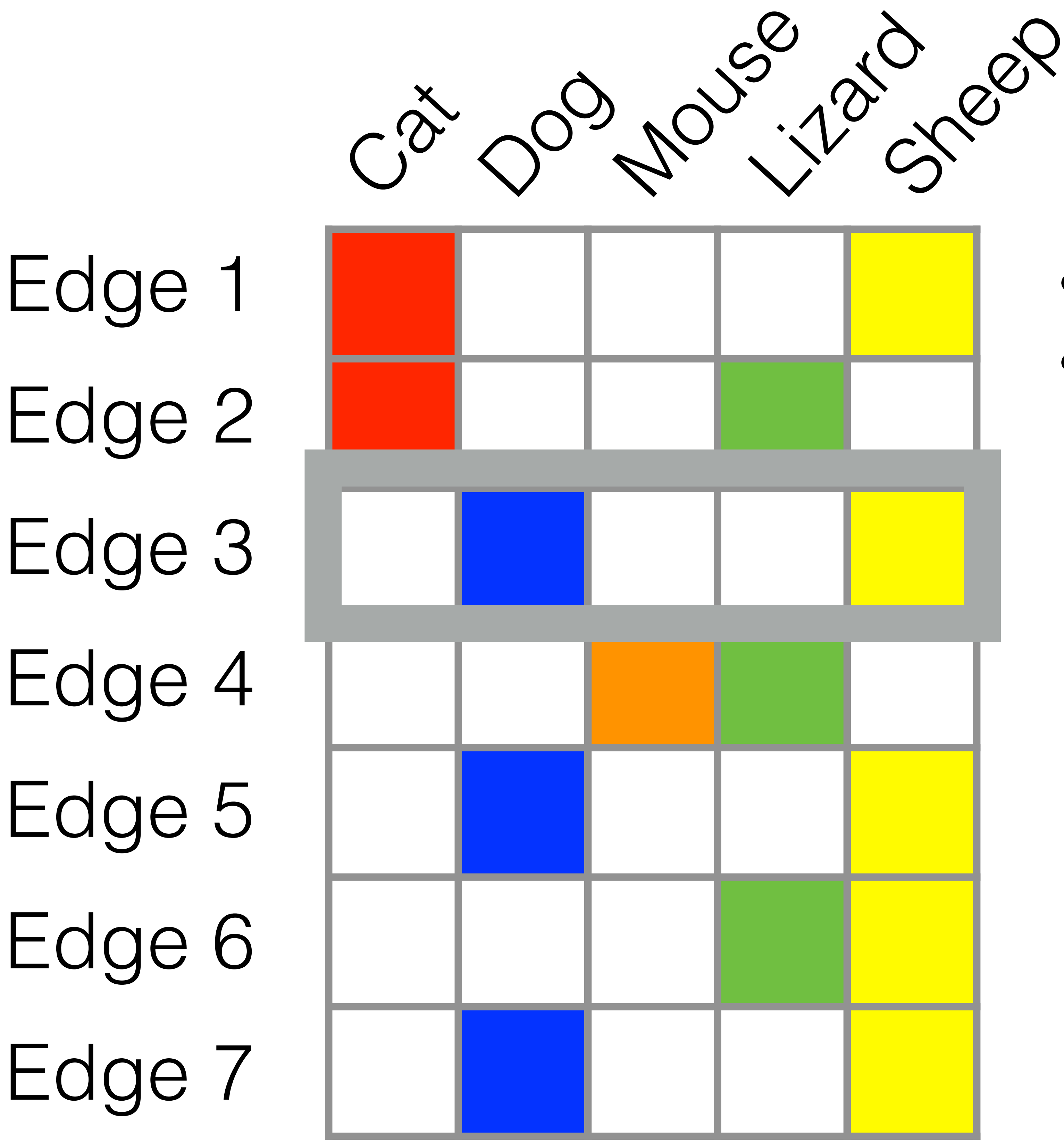
Graph



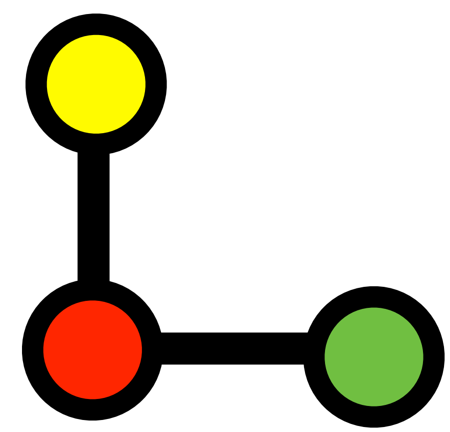
- Groups: vertices
- Edge-exchangeable



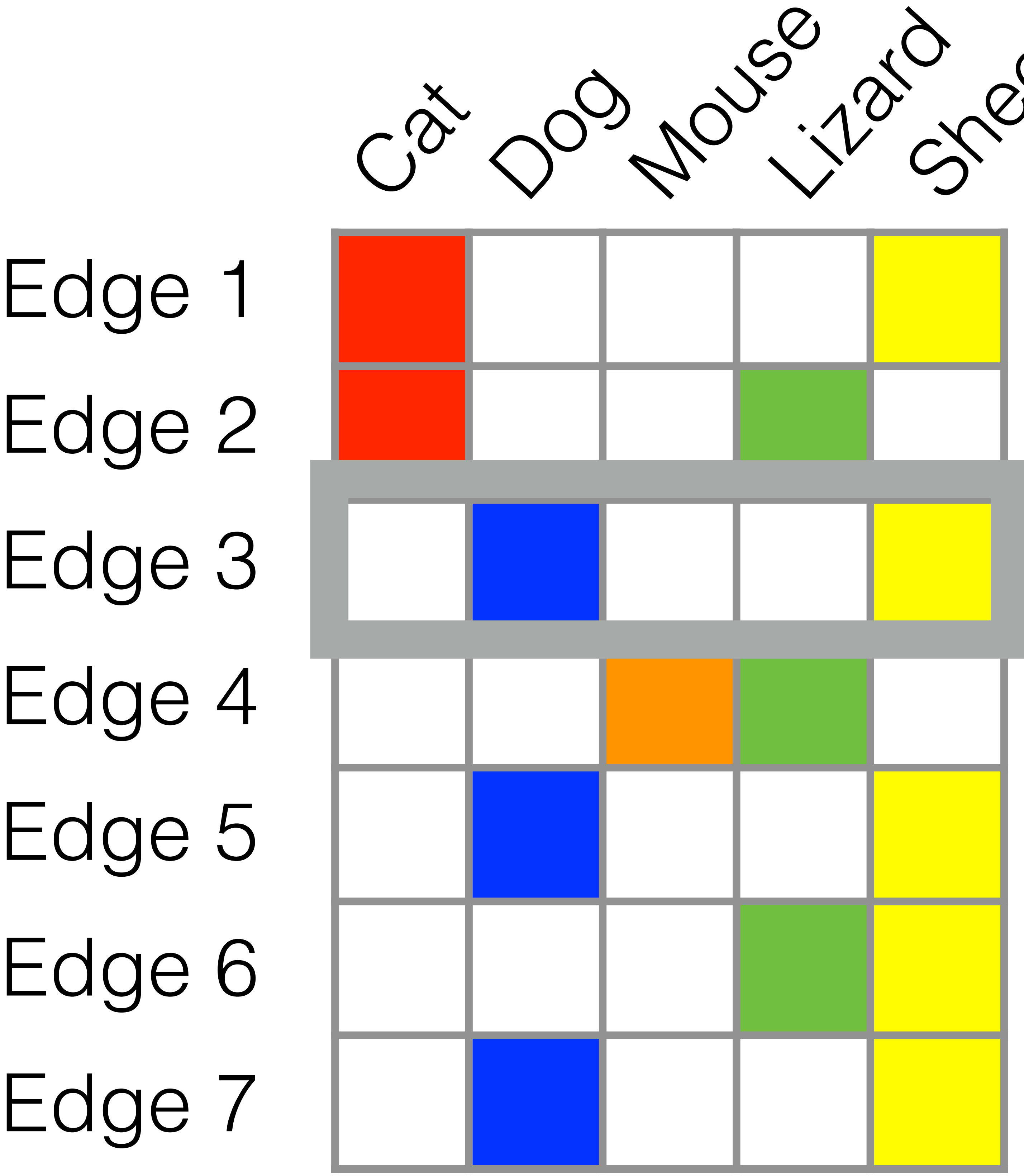
Graph



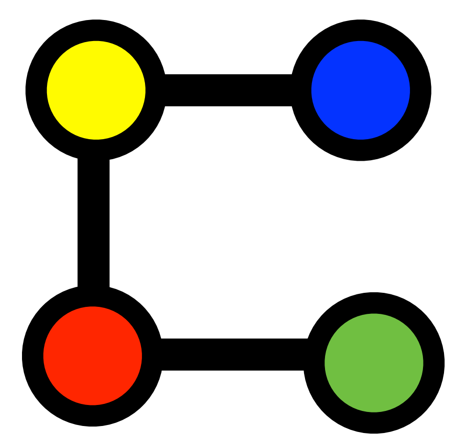
- Groups: vertices
- Edge-exchangeable



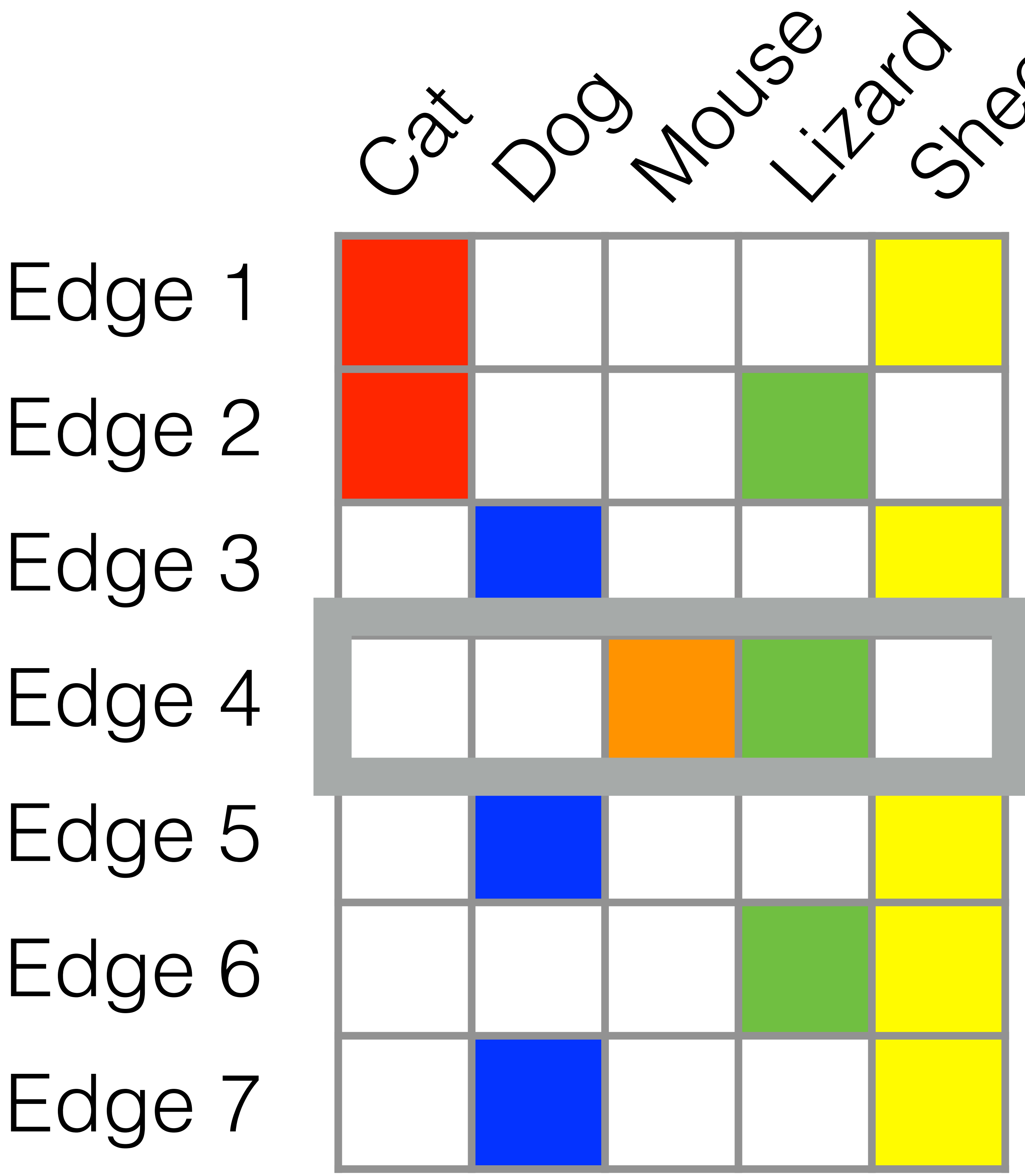
Graph



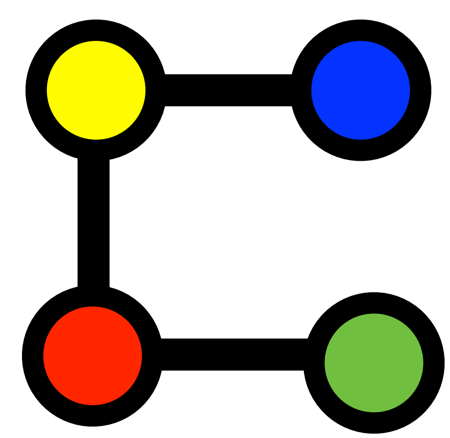
- Groups: vertices
- Edge-exchangeable



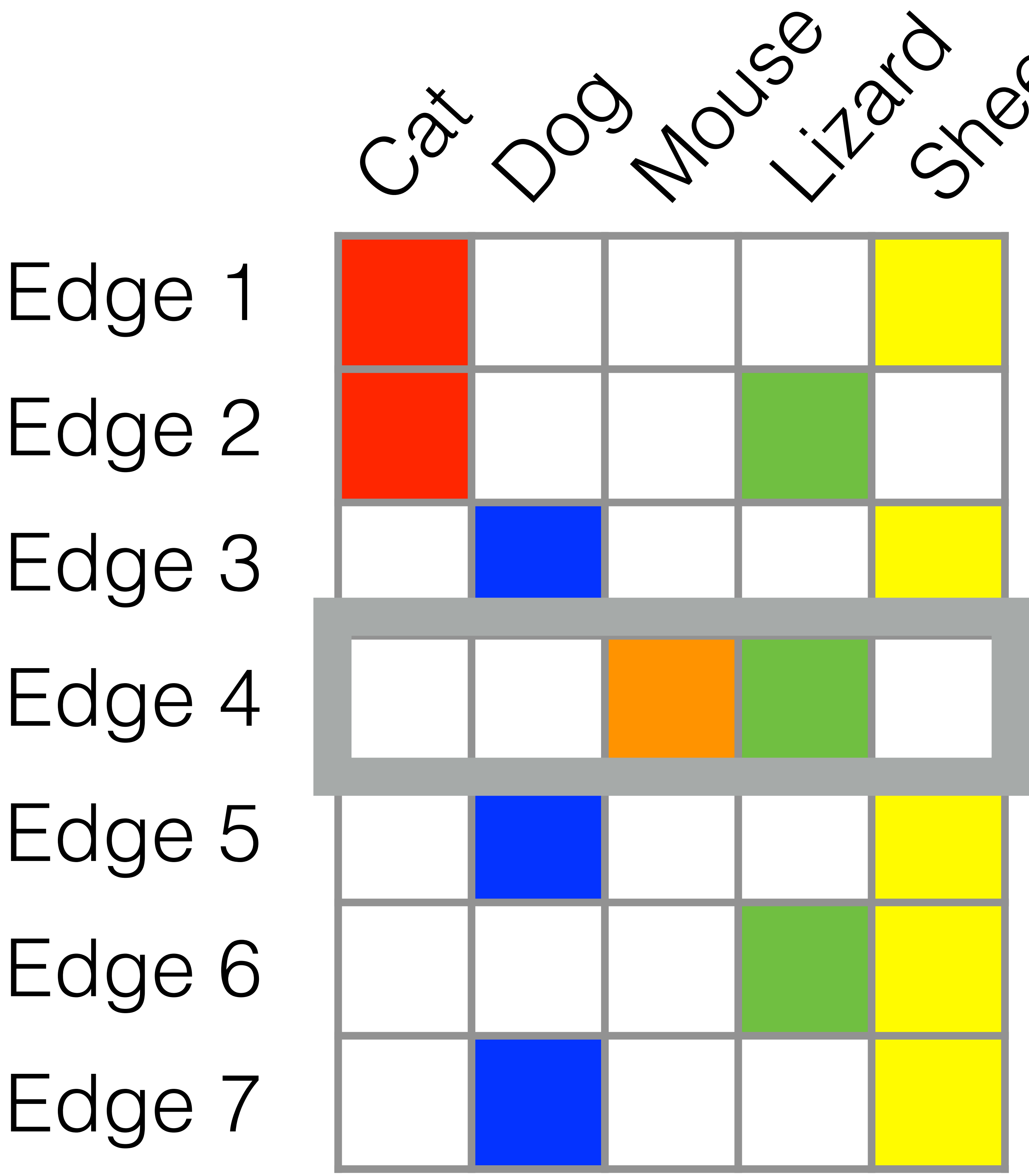
Graph



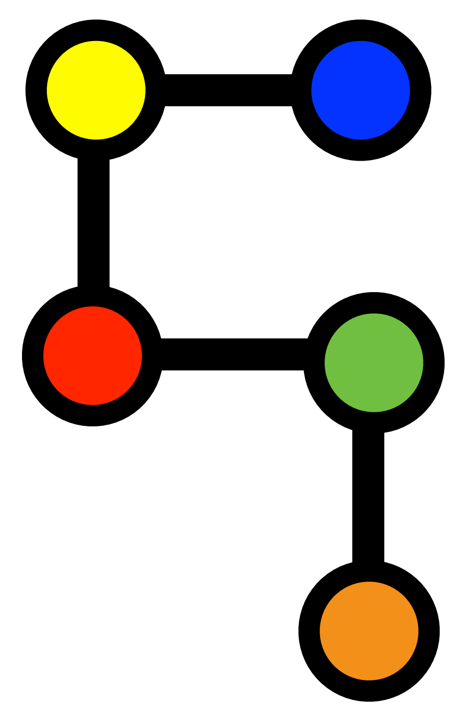
- Groups: vertices
- Edge-exchangeable



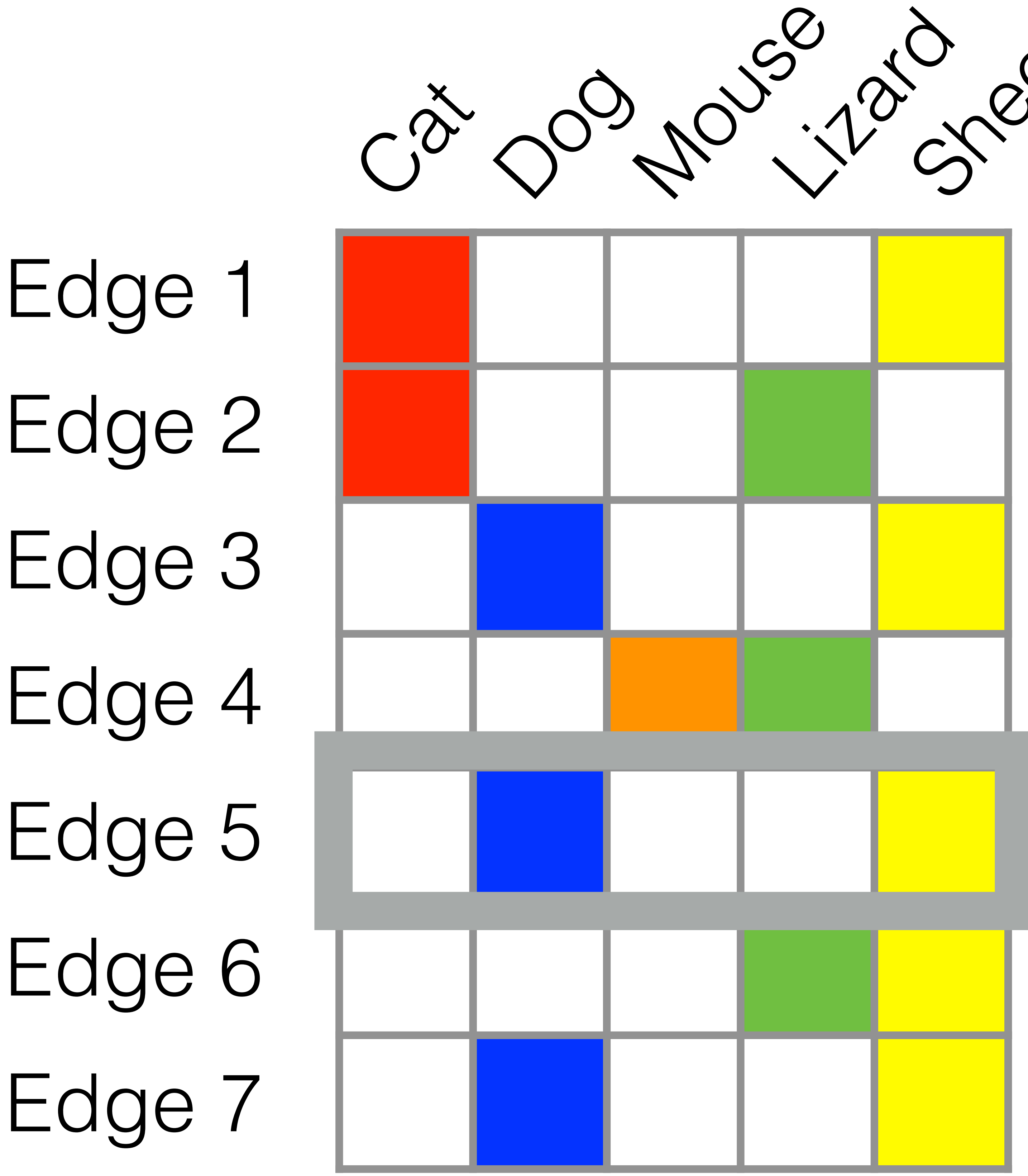
Graph



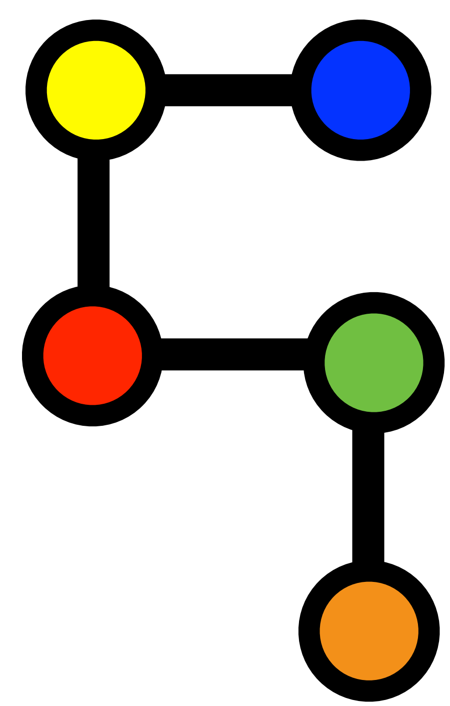
- Groups: vertices
- Edge-exchangeable



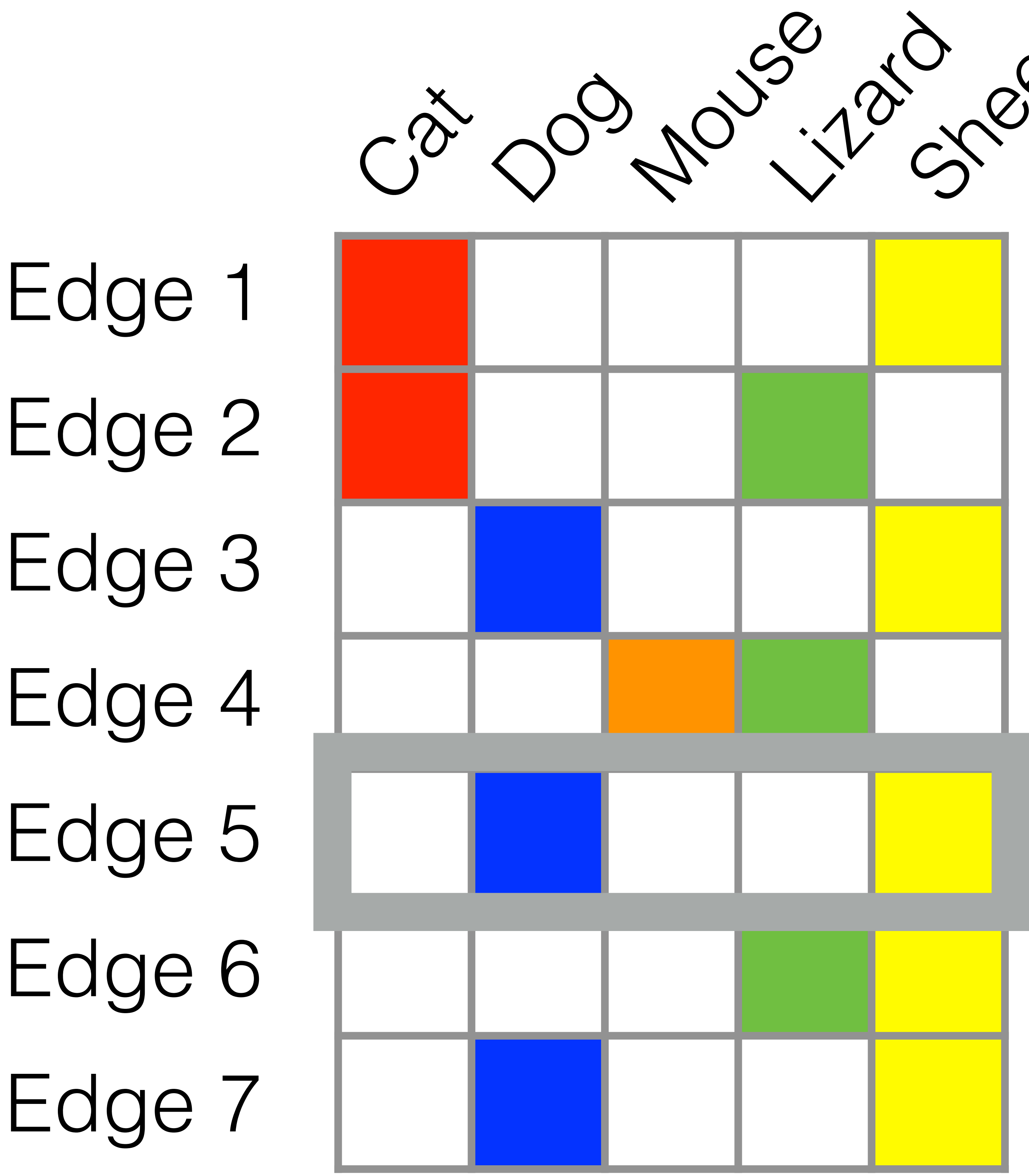
Graph



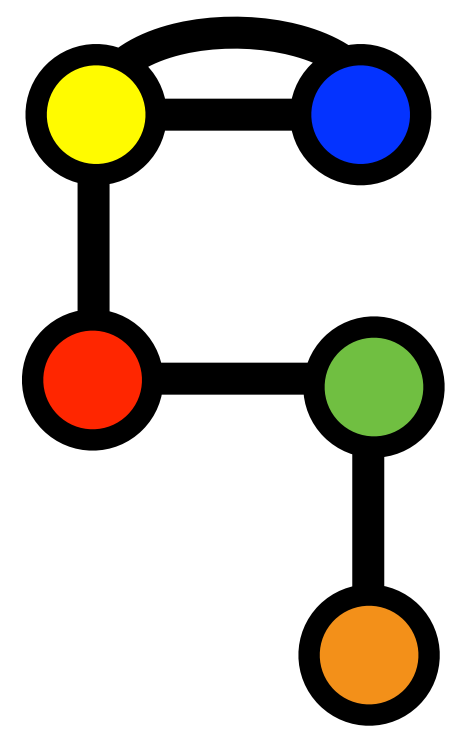
- Groups: vertices
- Edge-exchangeable



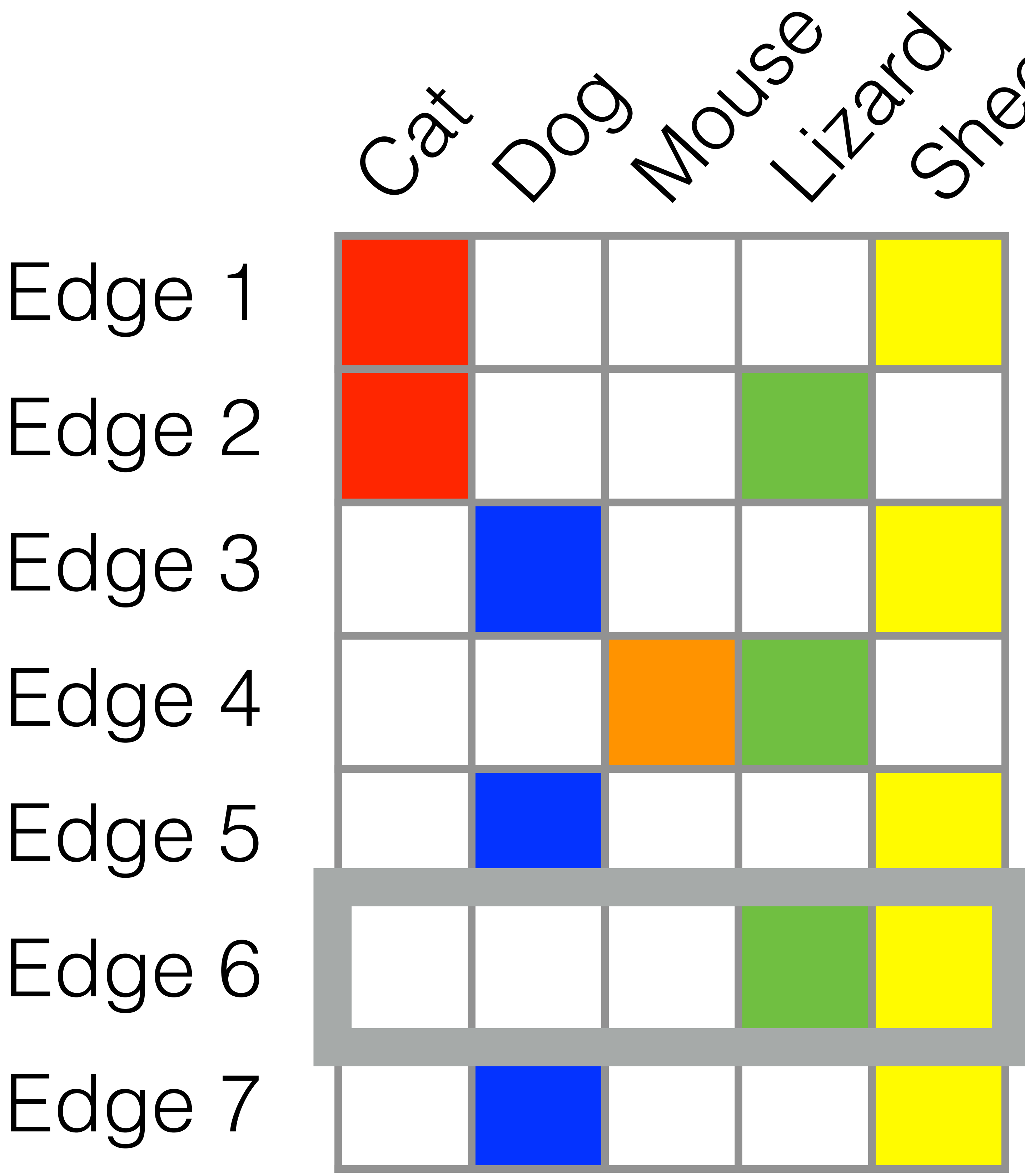
Graph



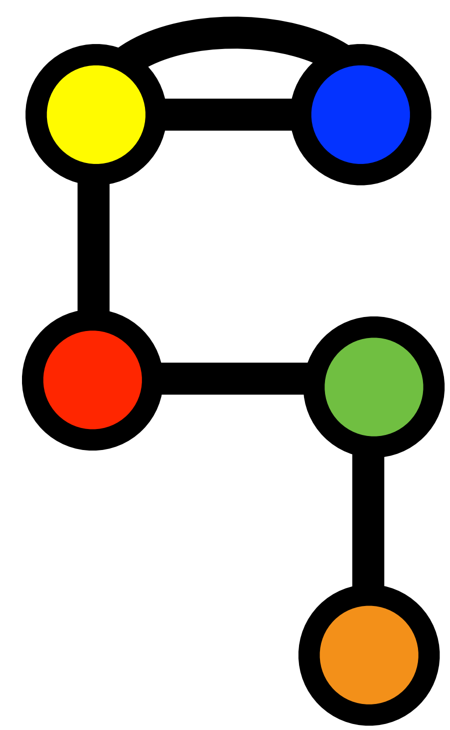
- Groups: vertices
- Edge-exchangeable



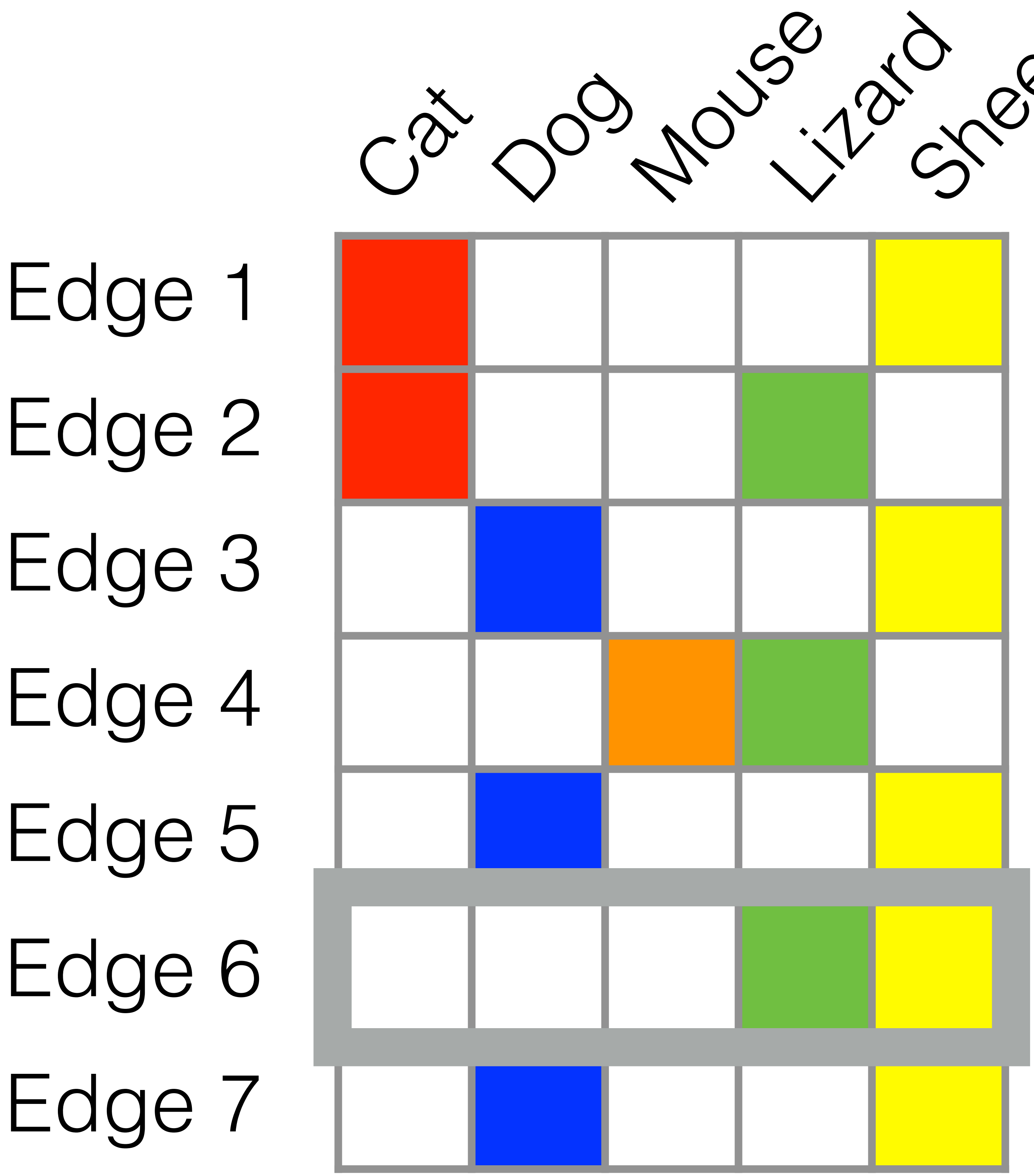
Graph



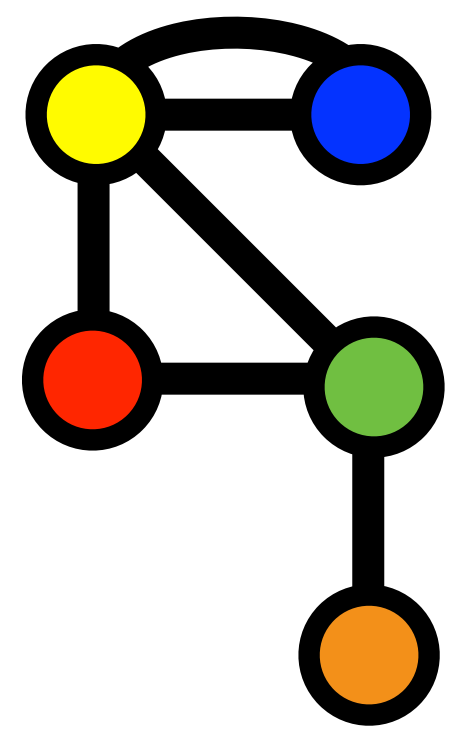
- Groups: vertices
- Edge-exchangeable



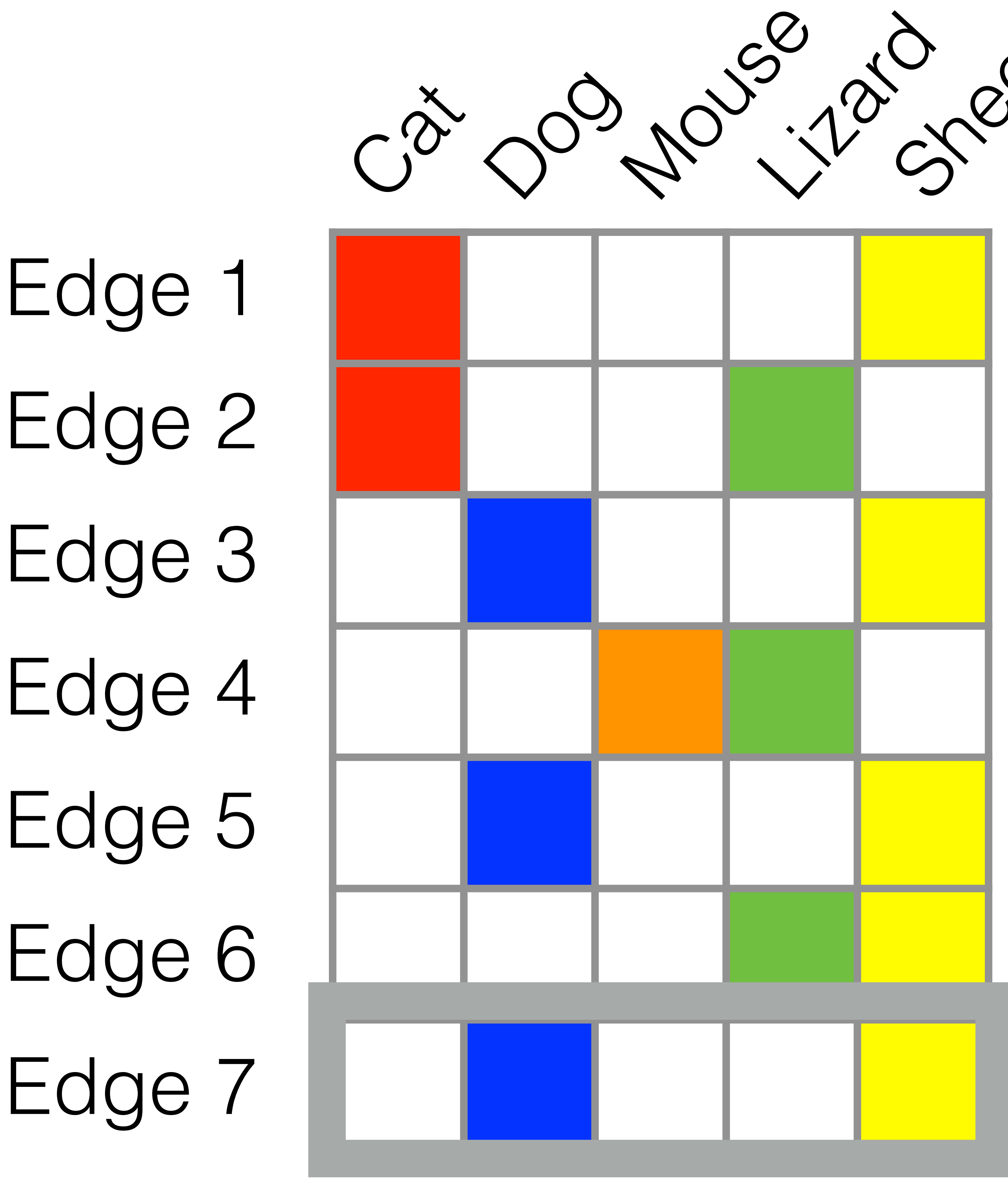
Graph



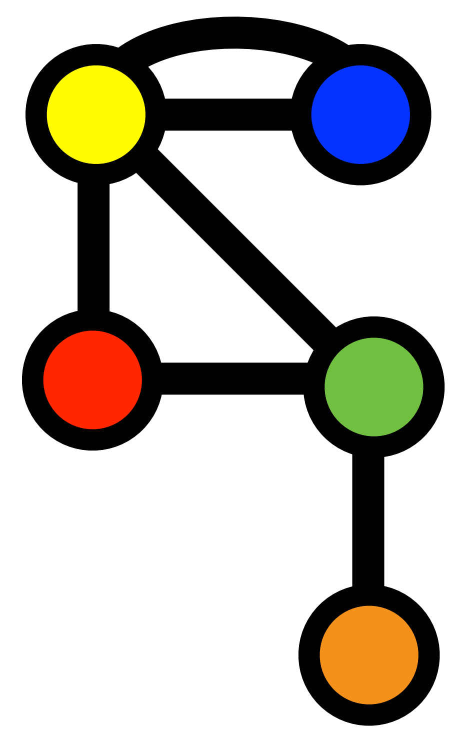
- Groups: vertices
- Edge-exchangeable



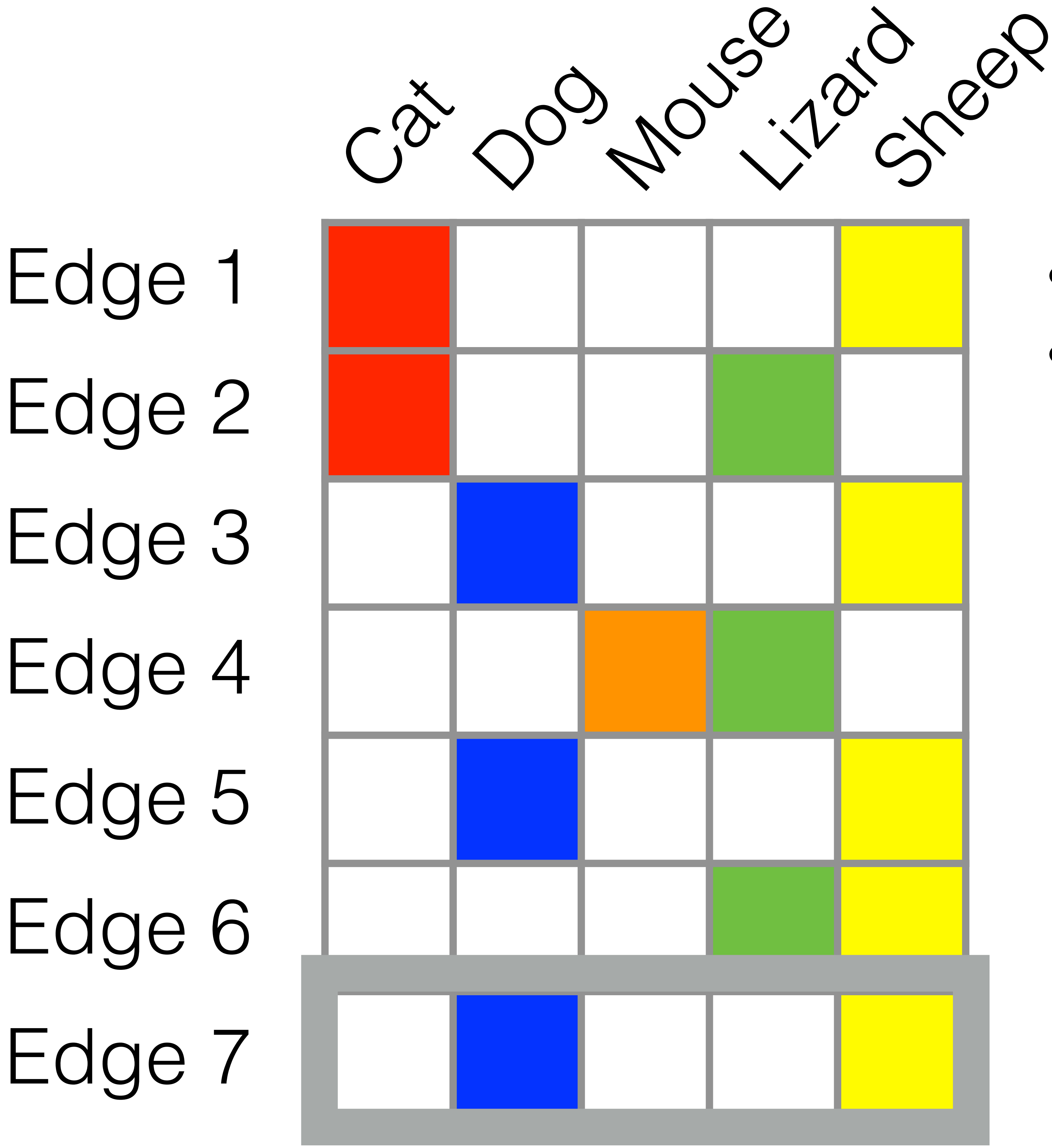
Graph



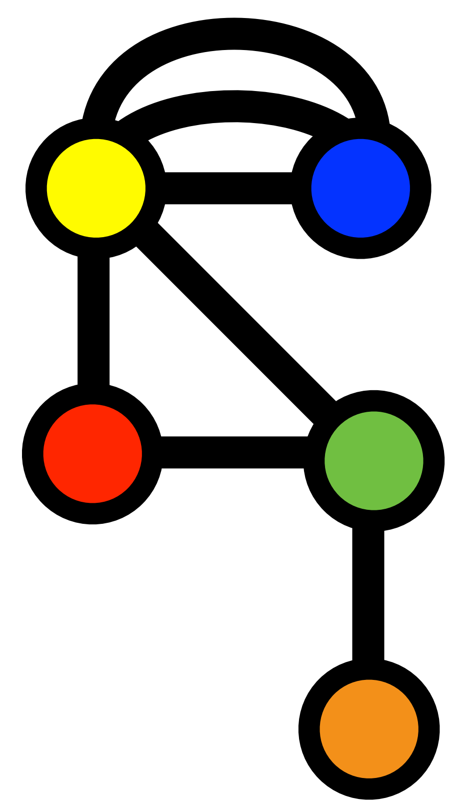
- Groups: vertices
- Edge-exchangeable



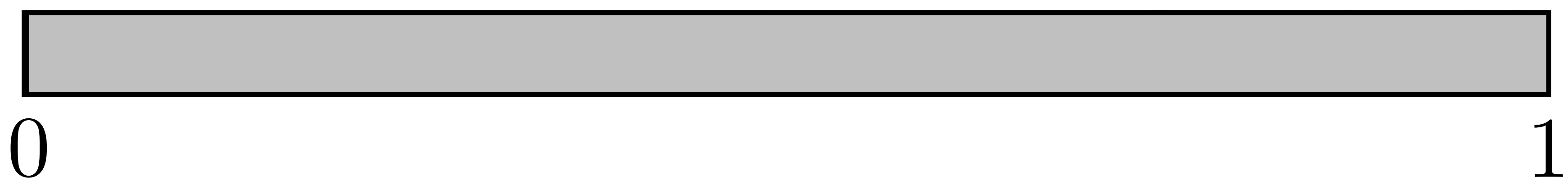
Graph



- Groups: vertices
- Edge-exchangeable

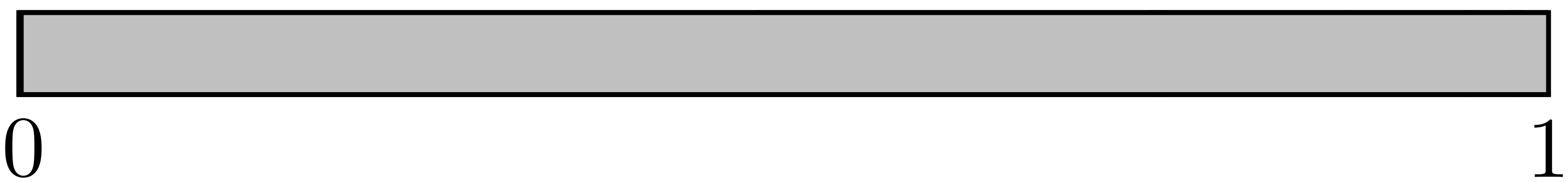


De Finetti Theorems



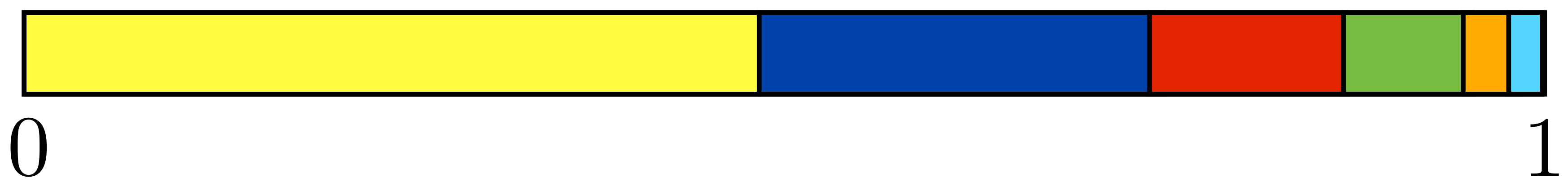
[Kingman 1978]

Clustering



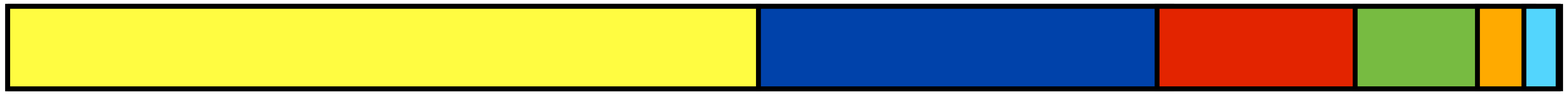
[Kingman 1978]

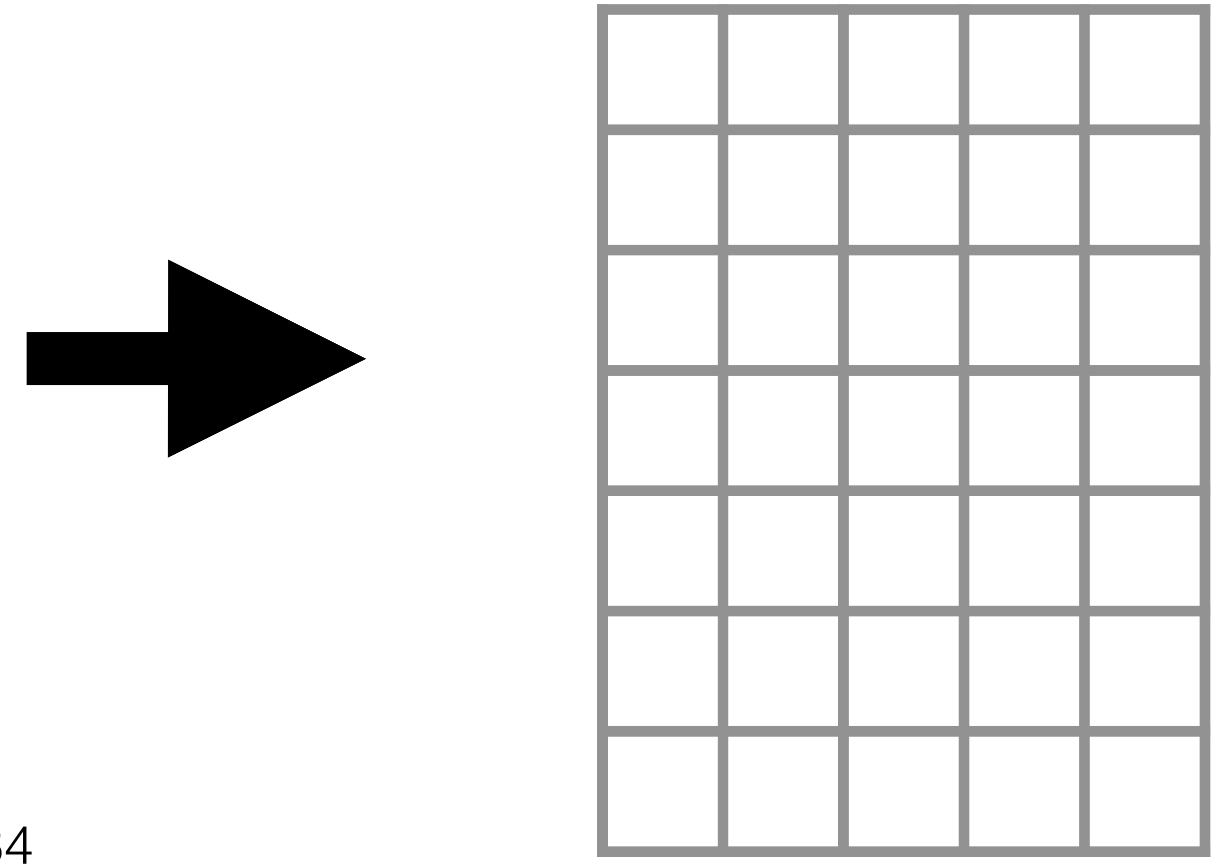
Clustering



[Kingman 1978]

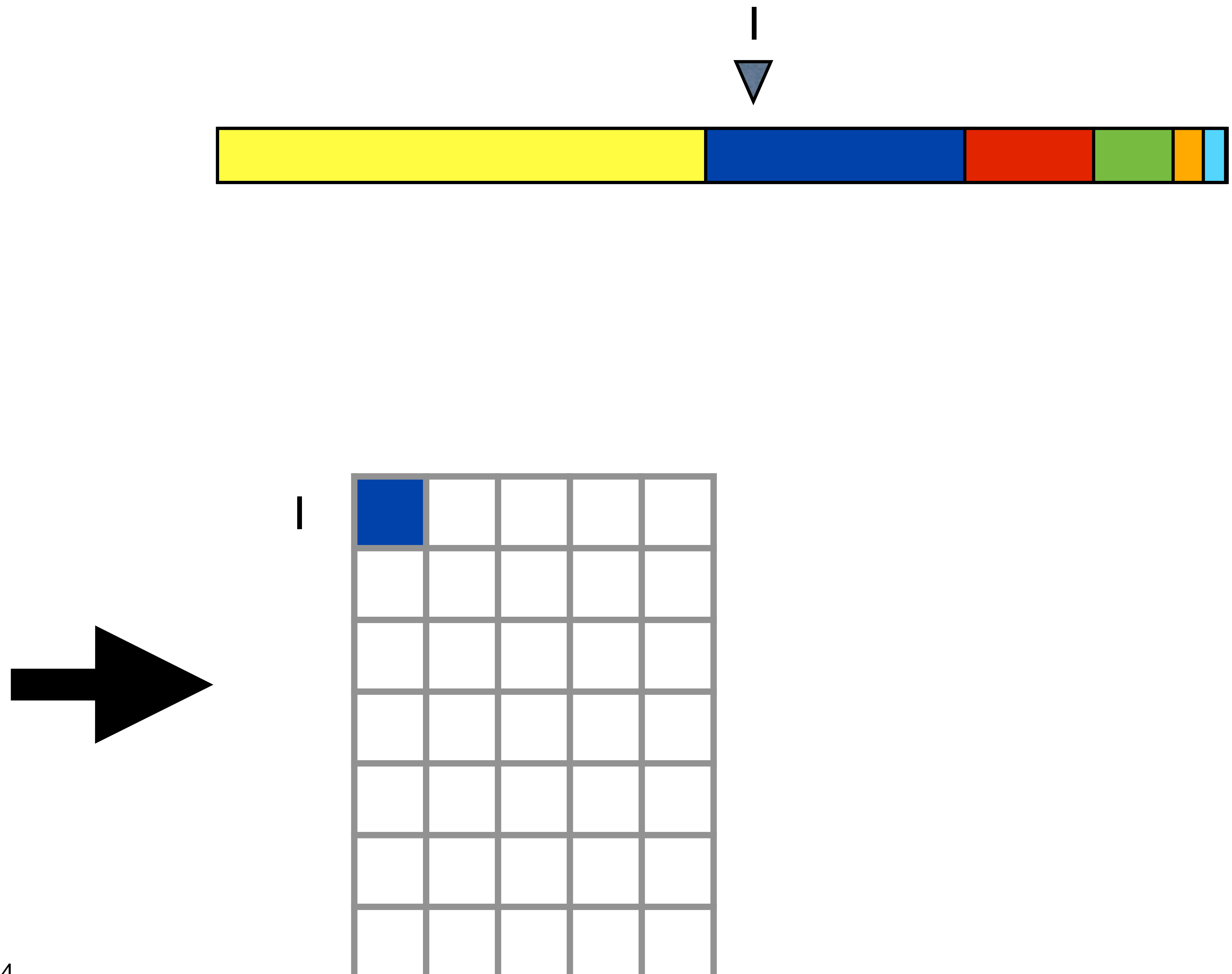
Clustering





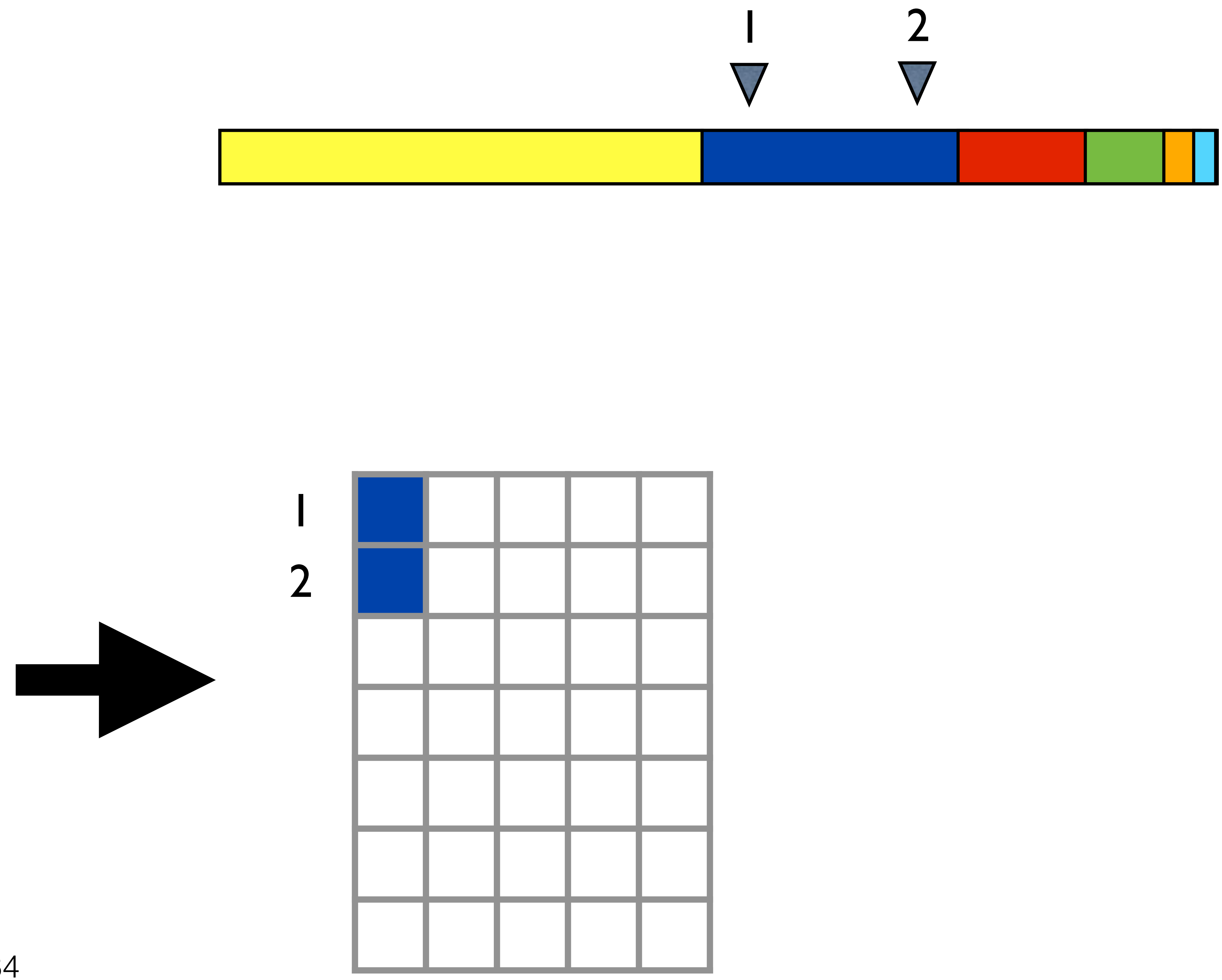
[Kingman 1978]

Clustering



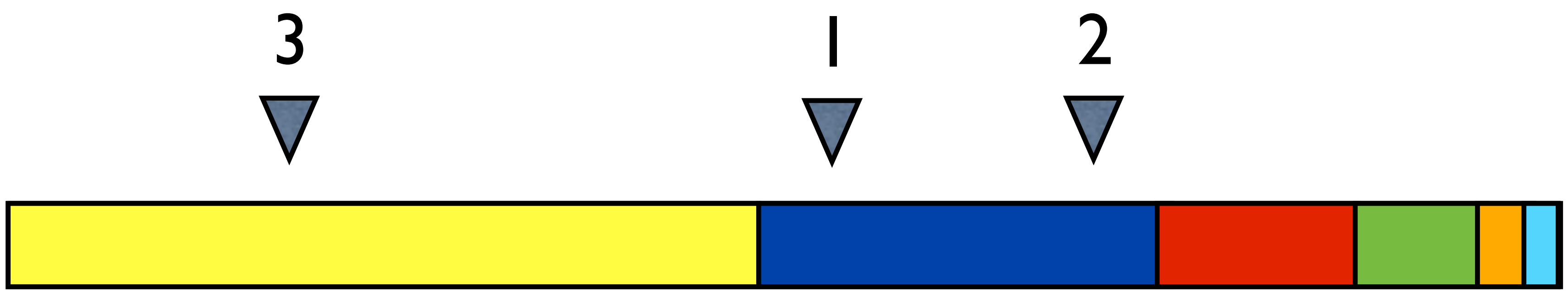
[Kingman 1978]

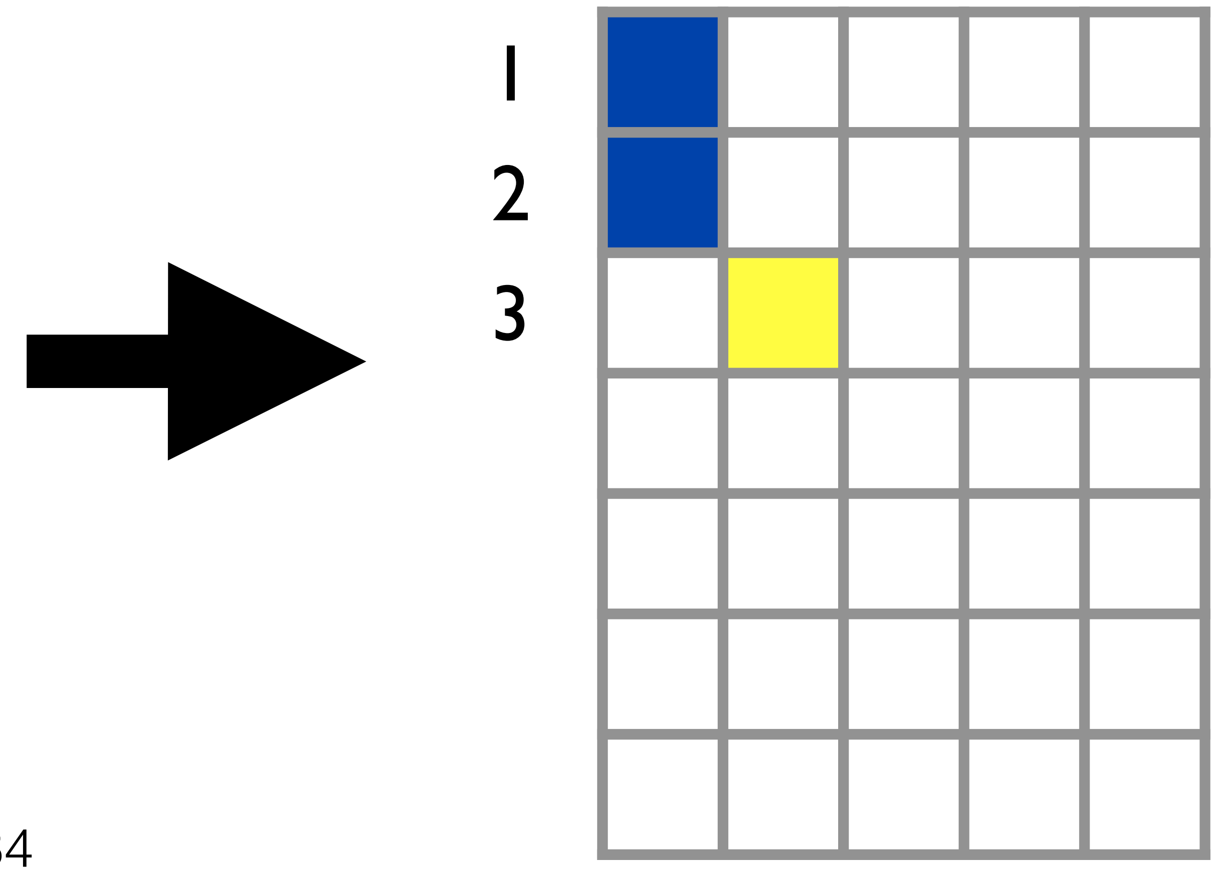
Clustering



[Kingman 1978]

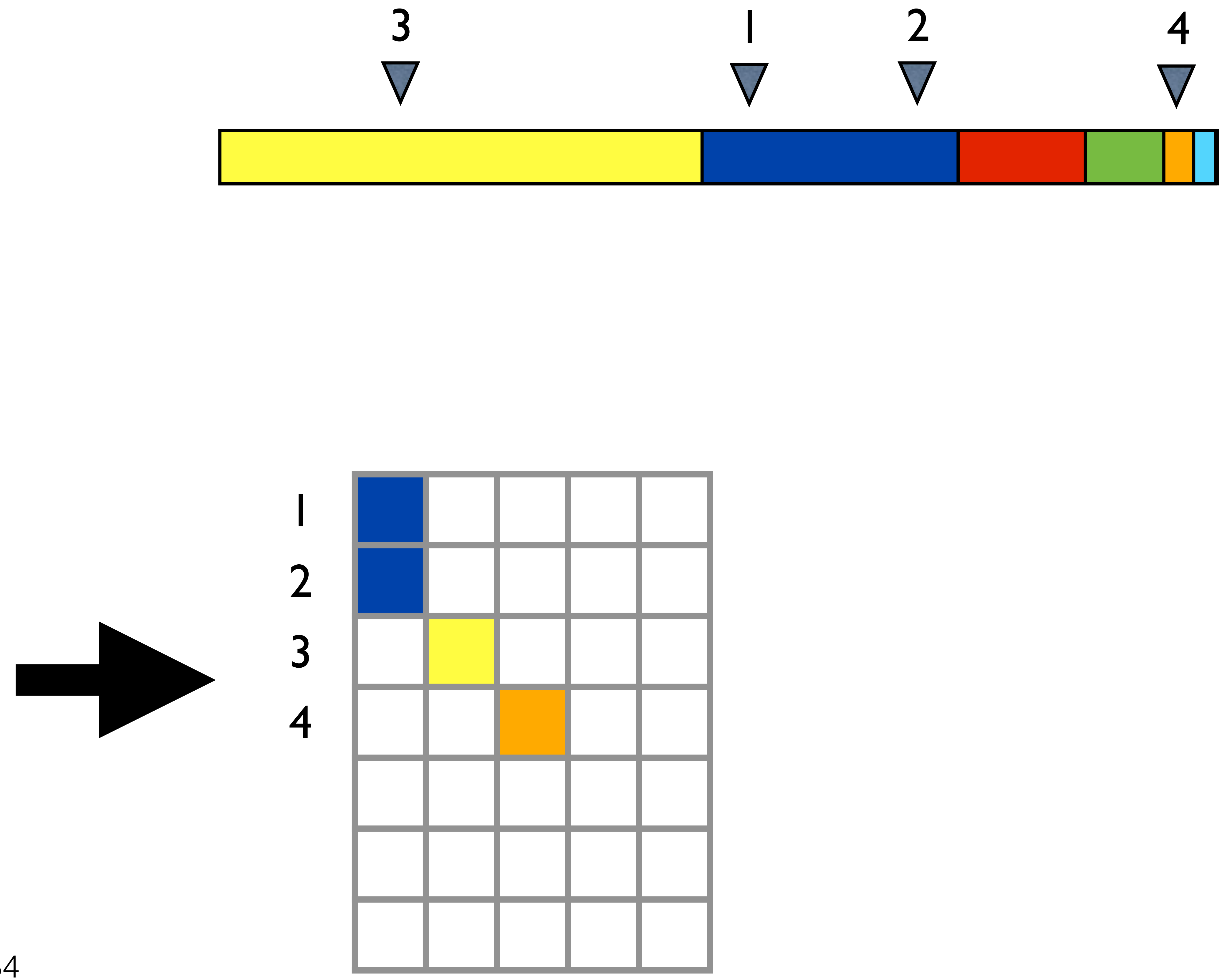
Clustering





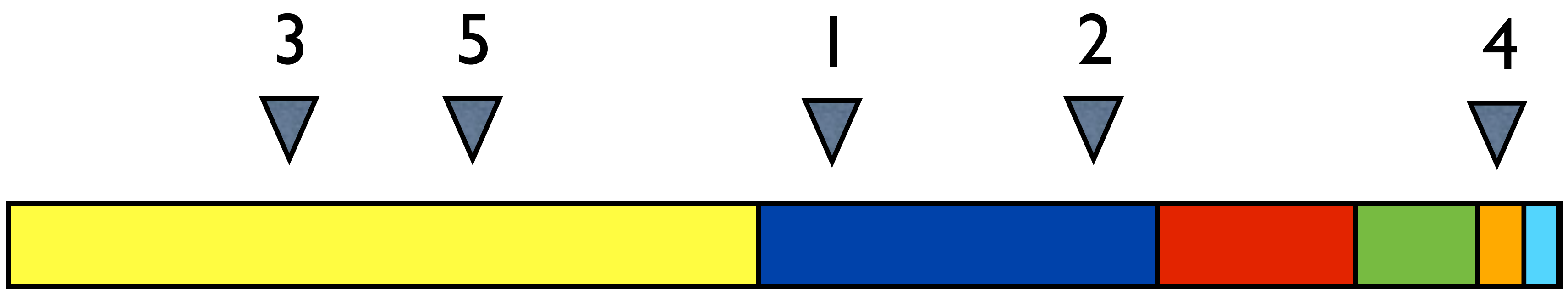
[Kingman 1978]

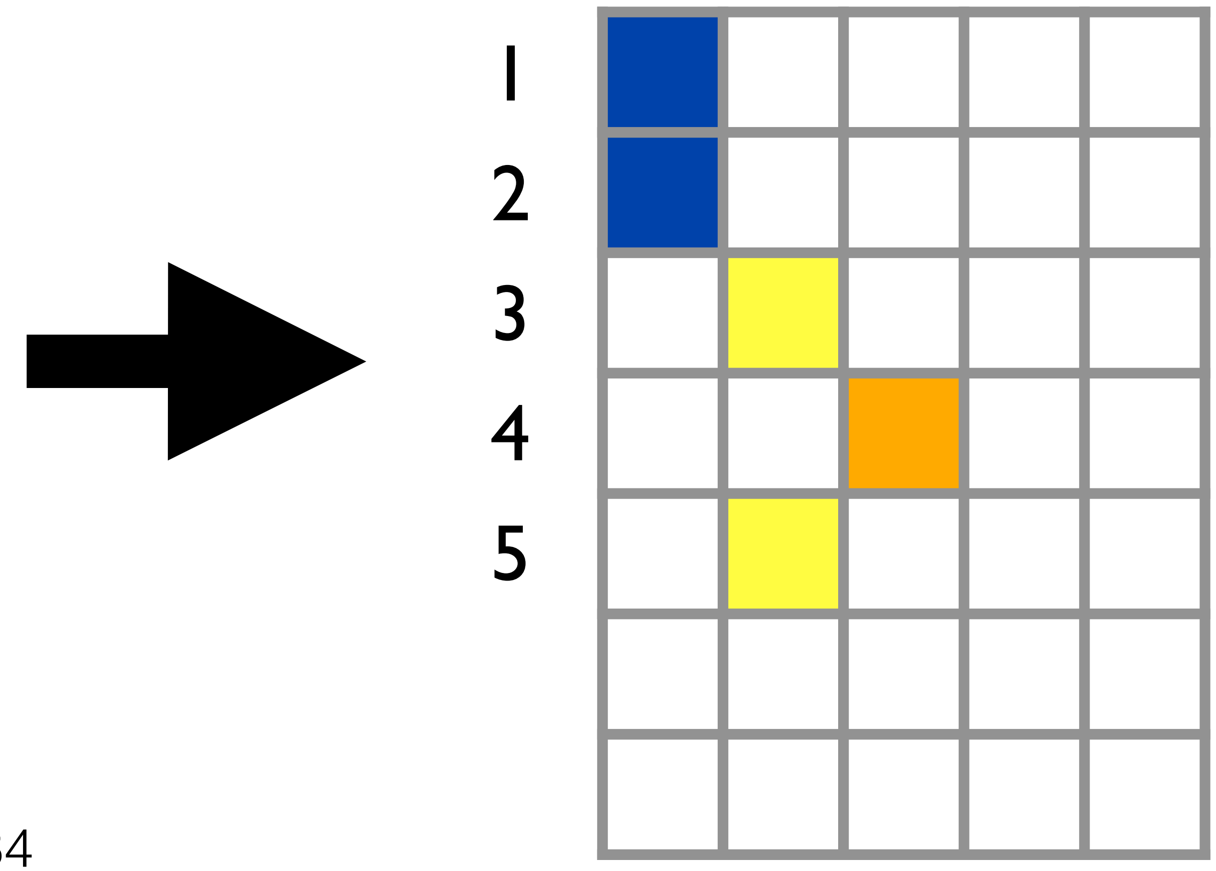
Clustering



[Kingman 1978]

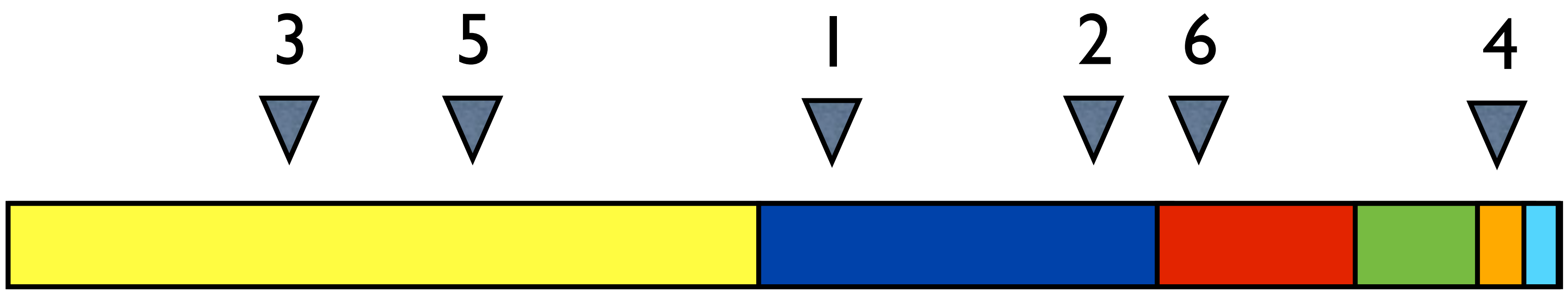
Clustering

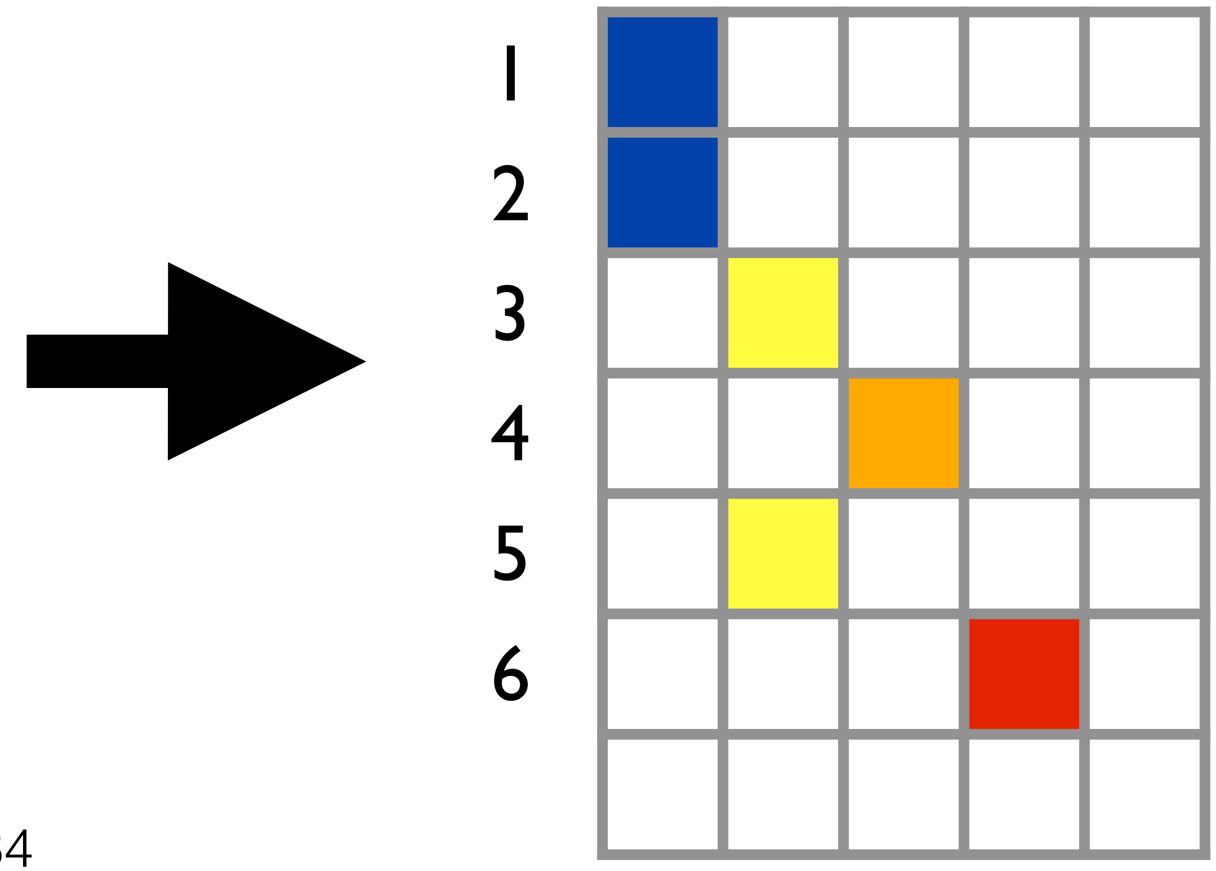




[Kingman 1978]

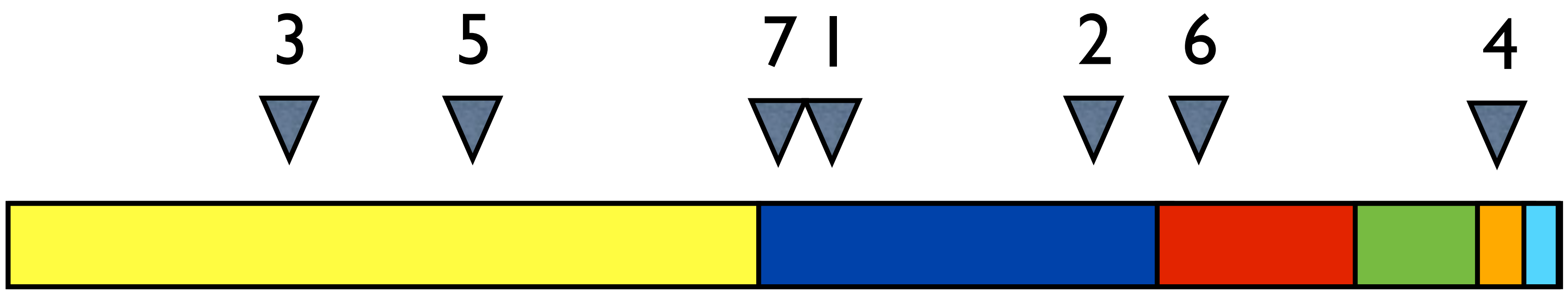
Clustering

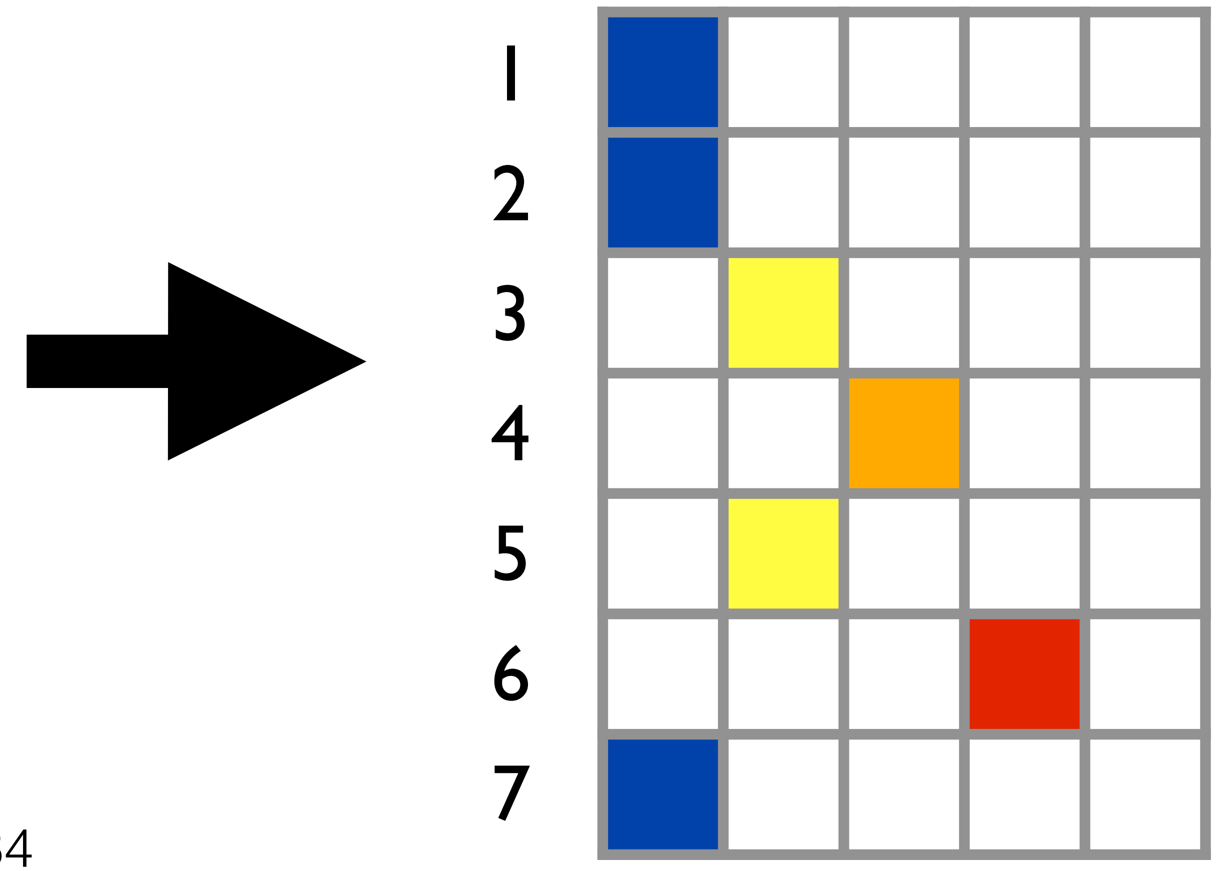




[Kingman 1978]

Clustering

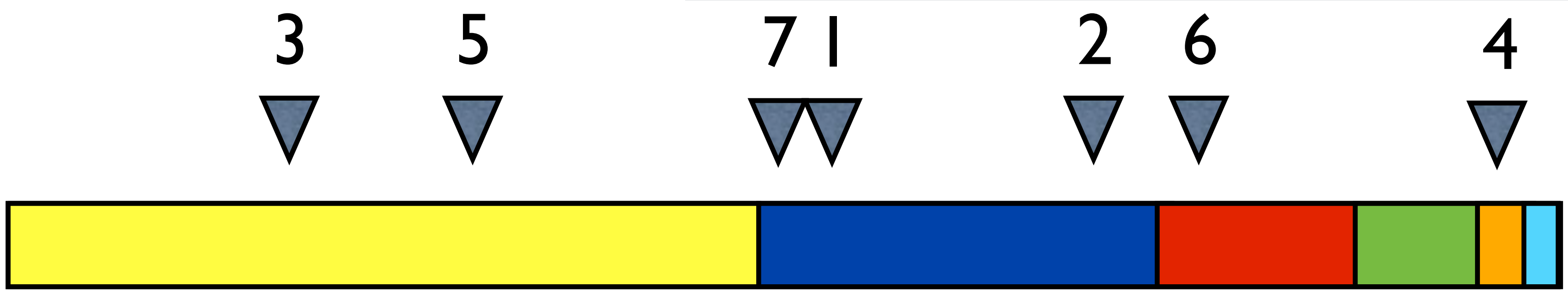


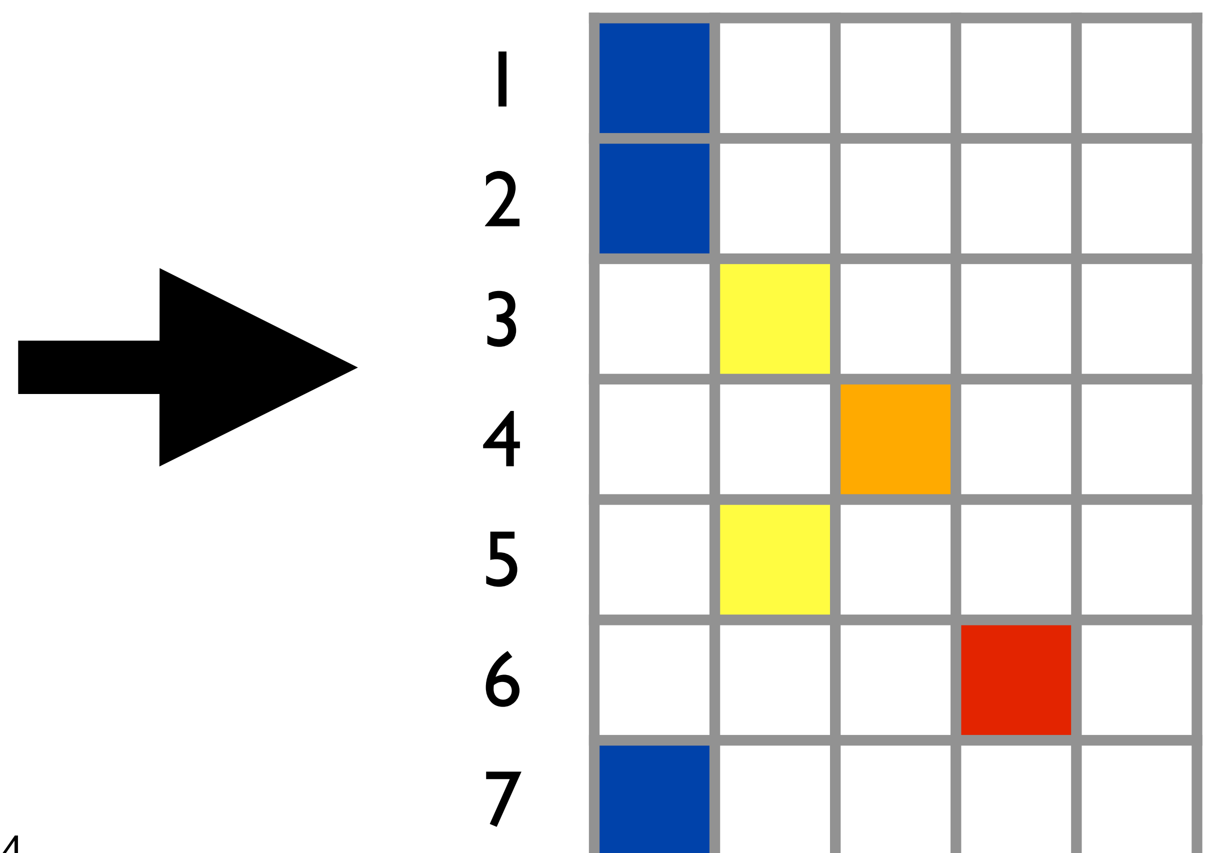


[Kingman 1978]

Clustering

Thm (Kingman). A clustering is exchangeable iff it has a Kingman paintbox representation





[Kingman 1978]

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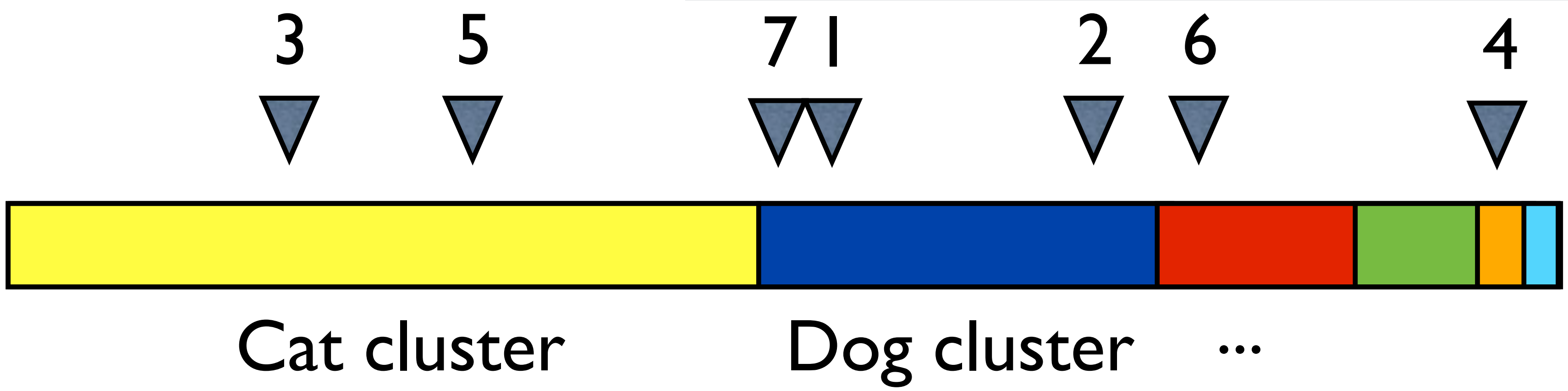

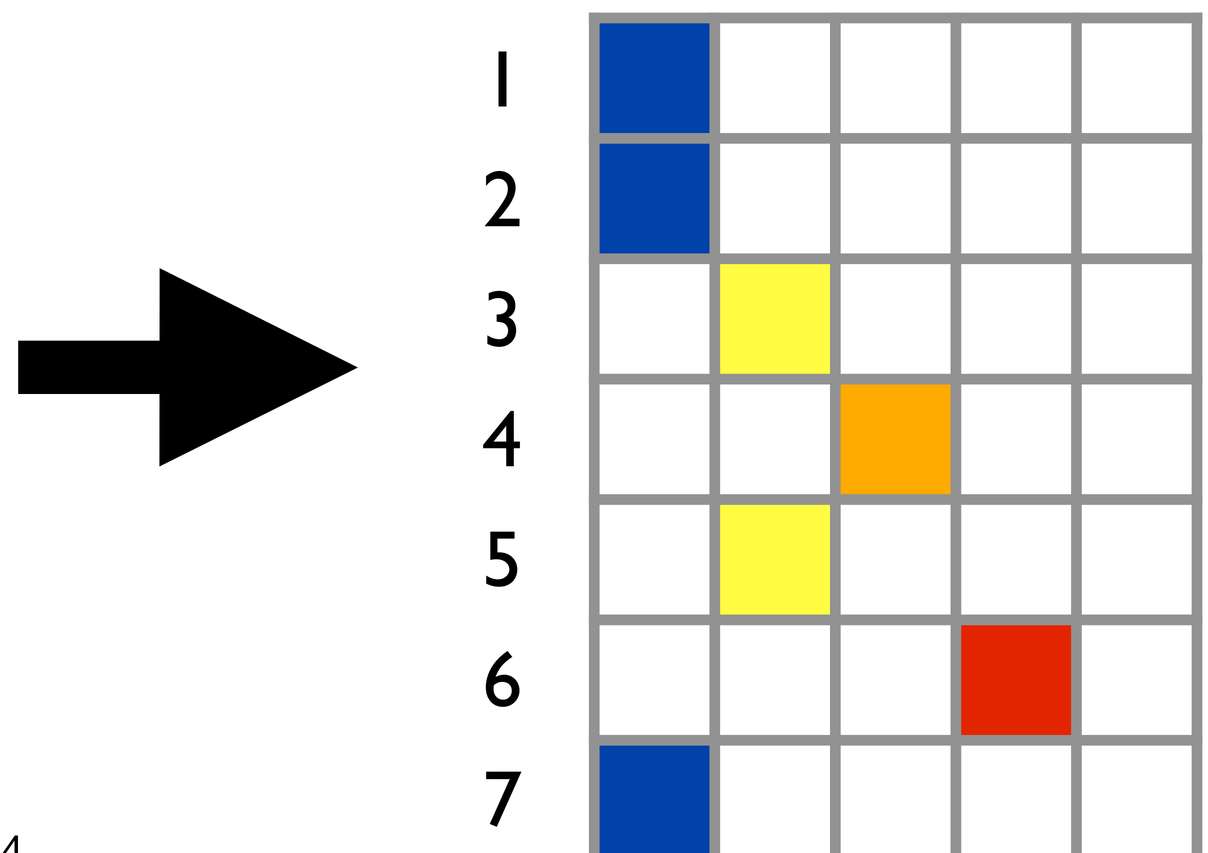

[Kingman 1978]

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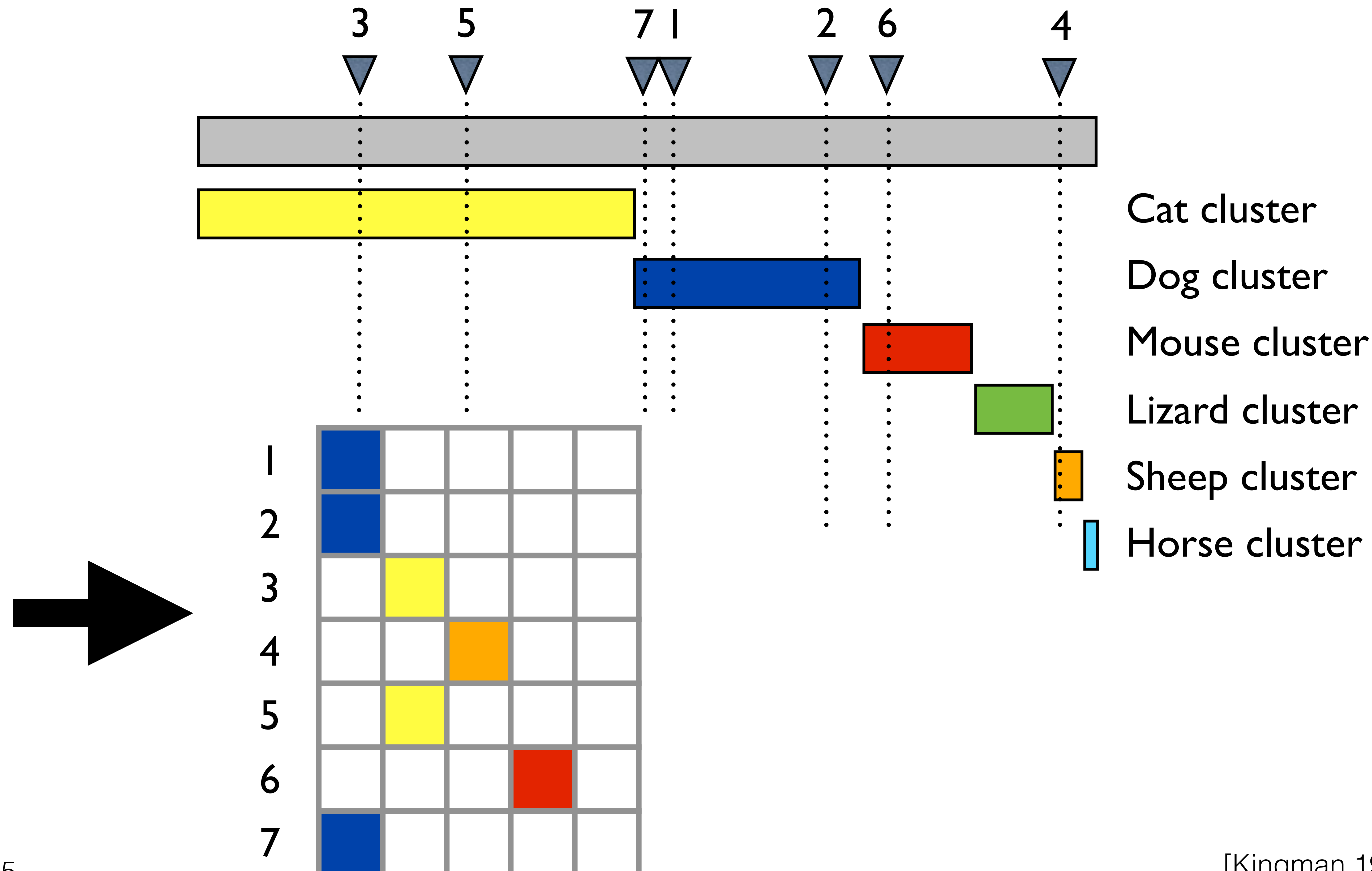
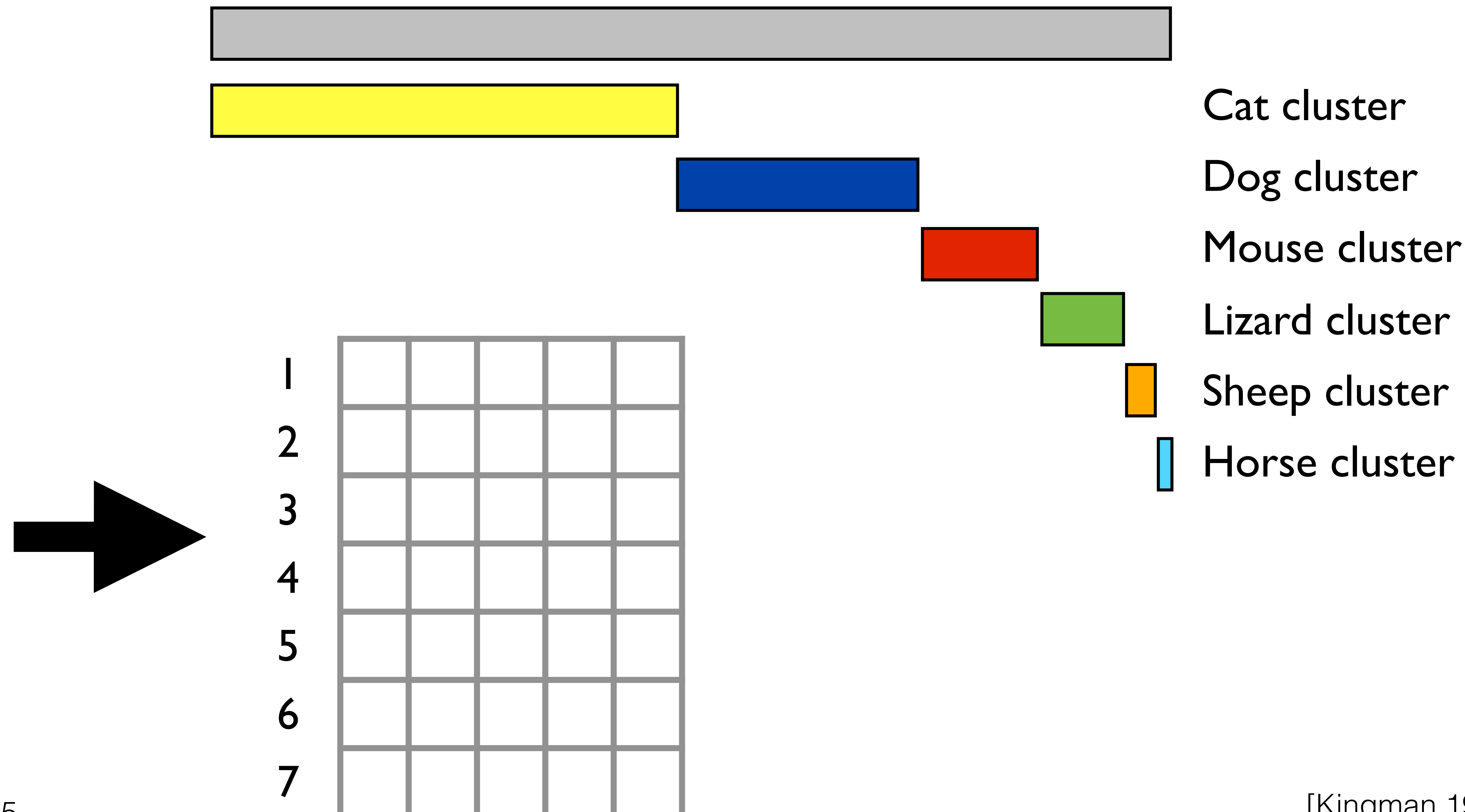
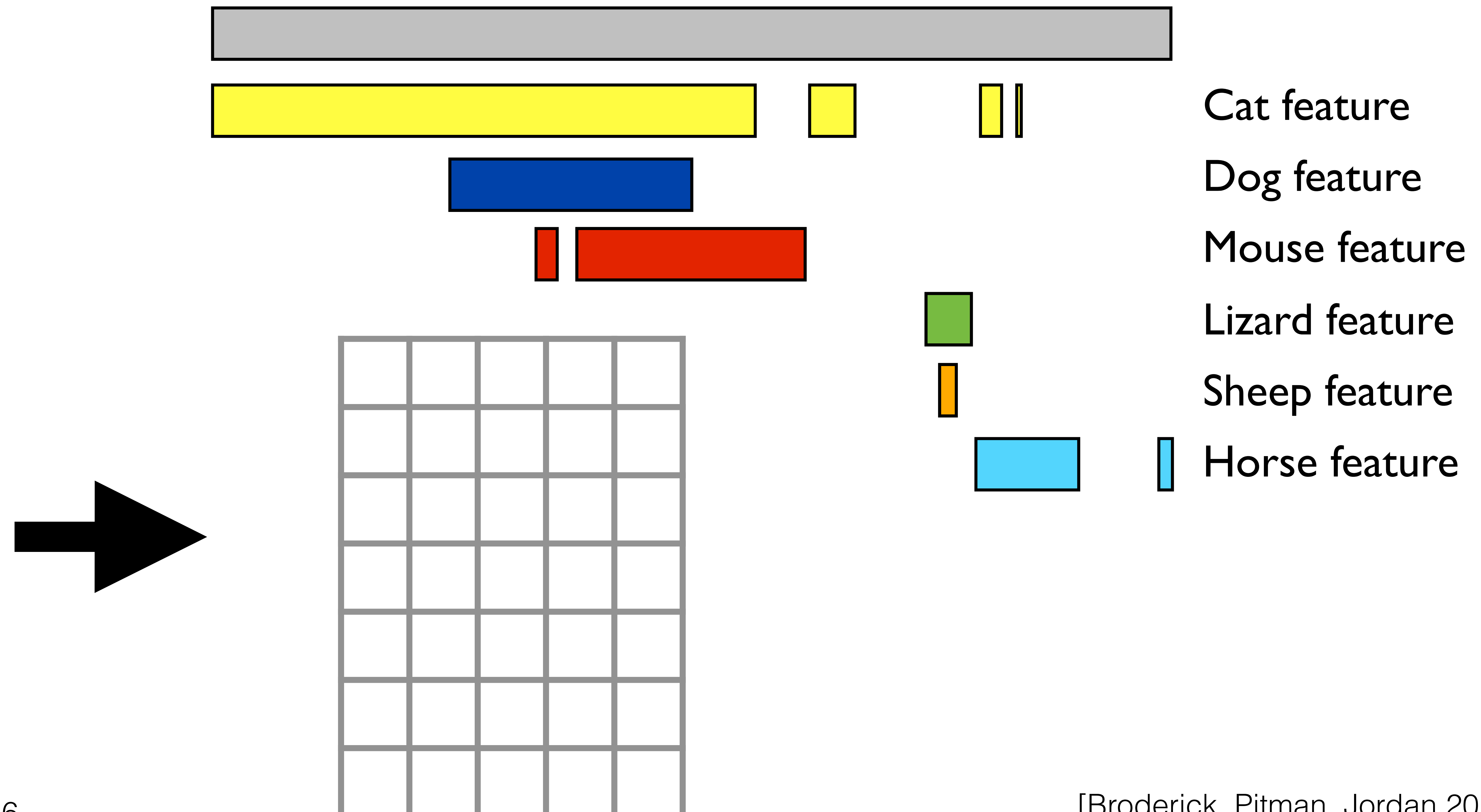


[Kingman 1978]

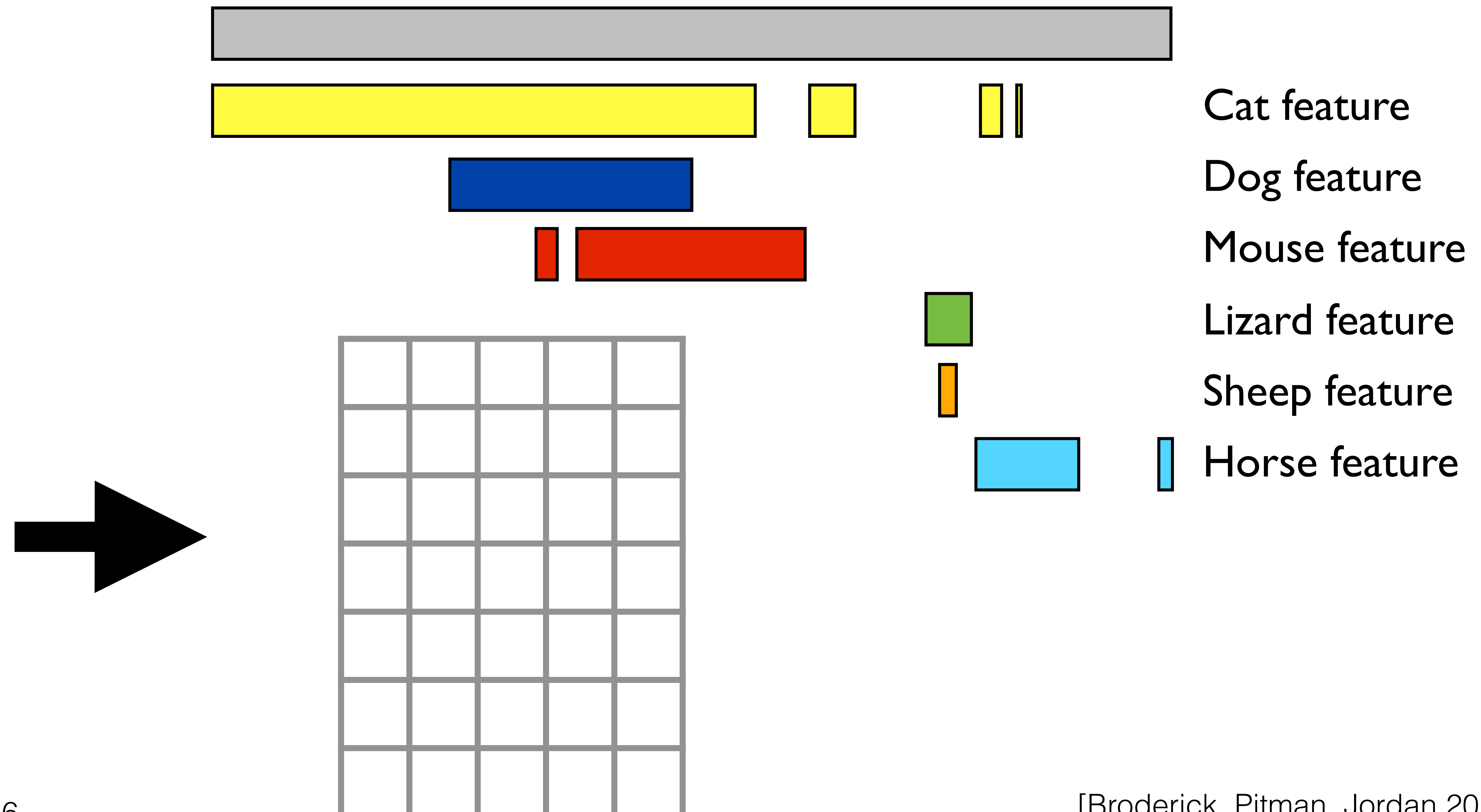


[Kingman 1978]



[Broderick, Pitman, Jordan 2013]

Feature allocation



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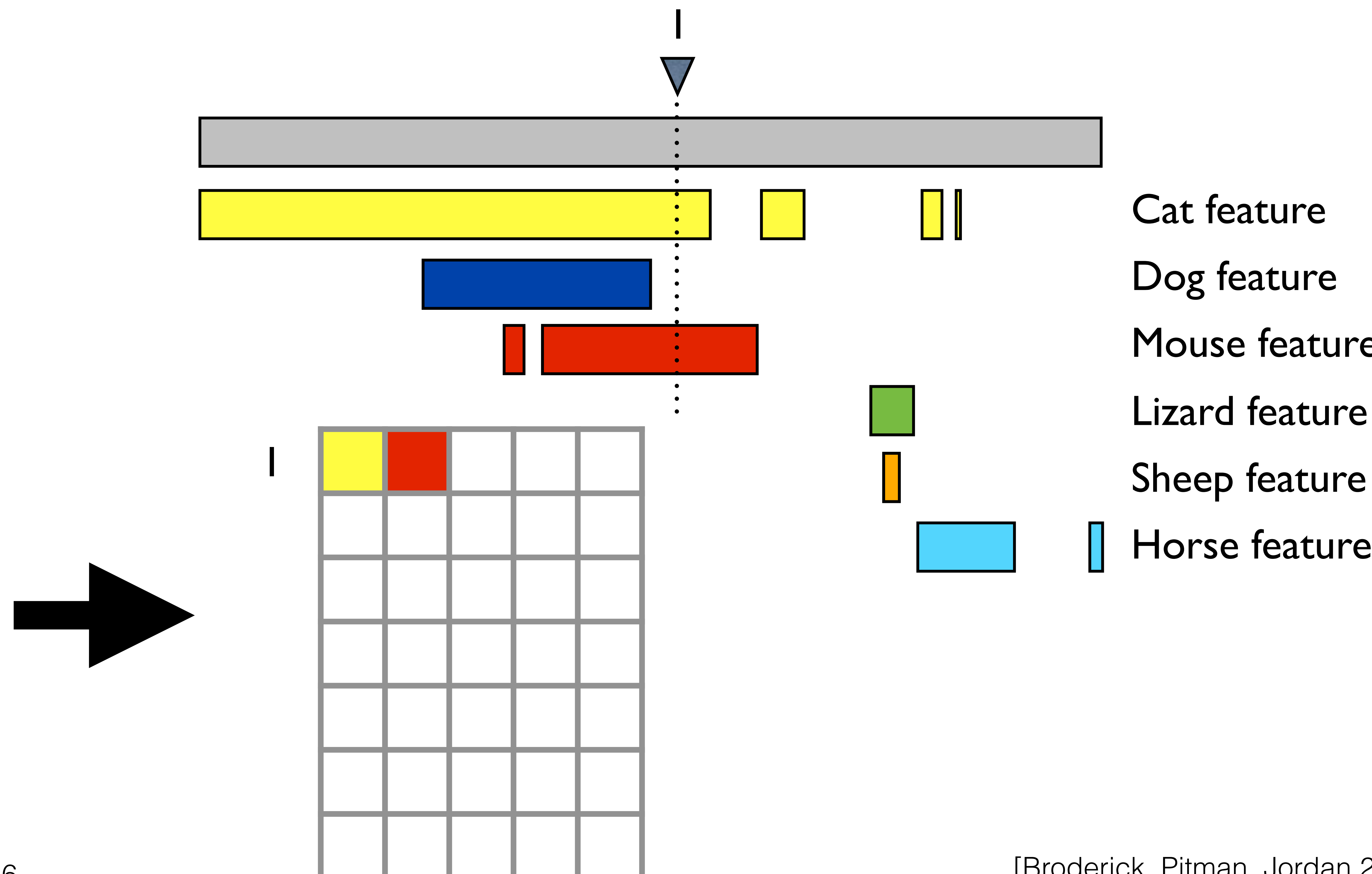
Feature allocation



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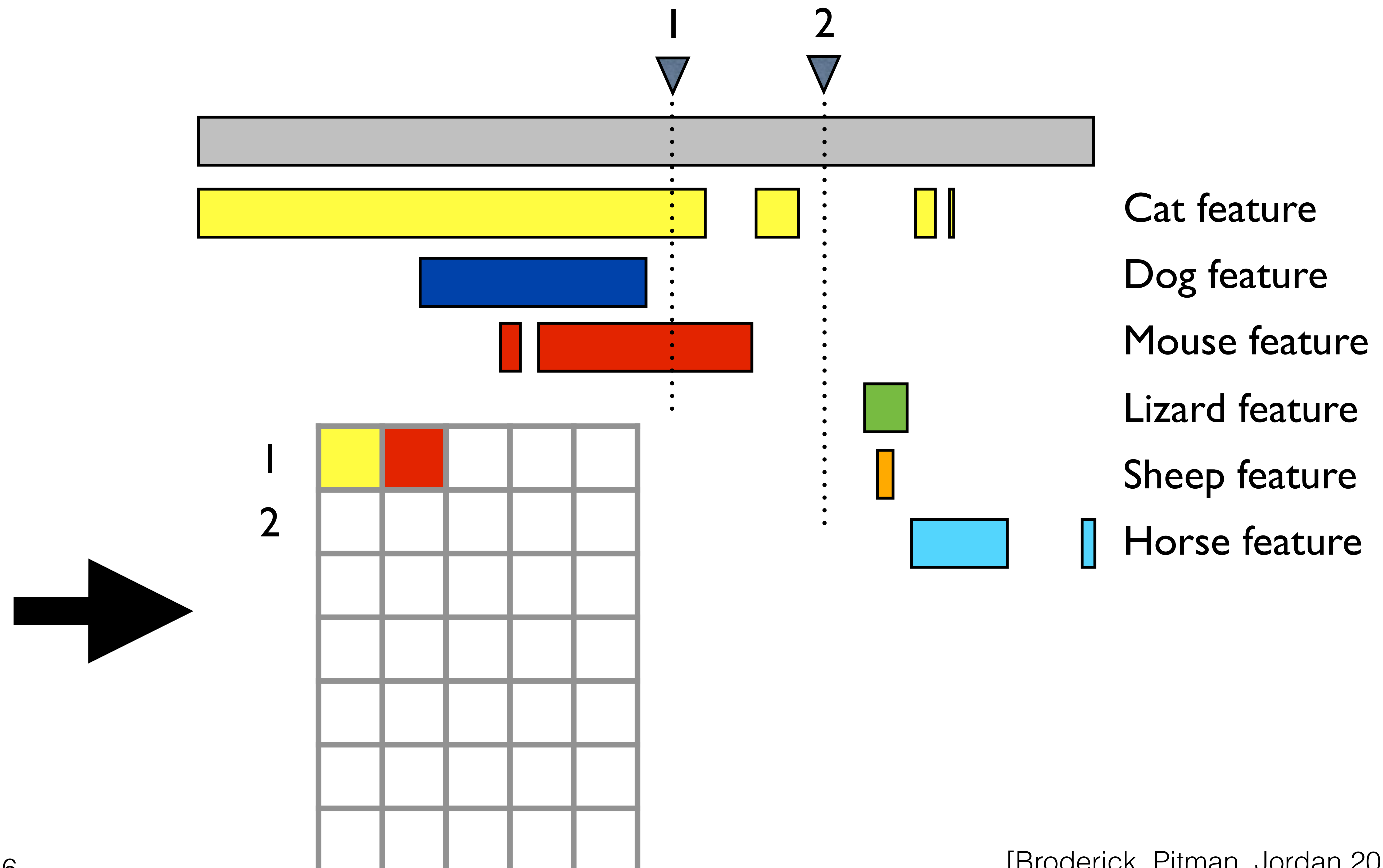
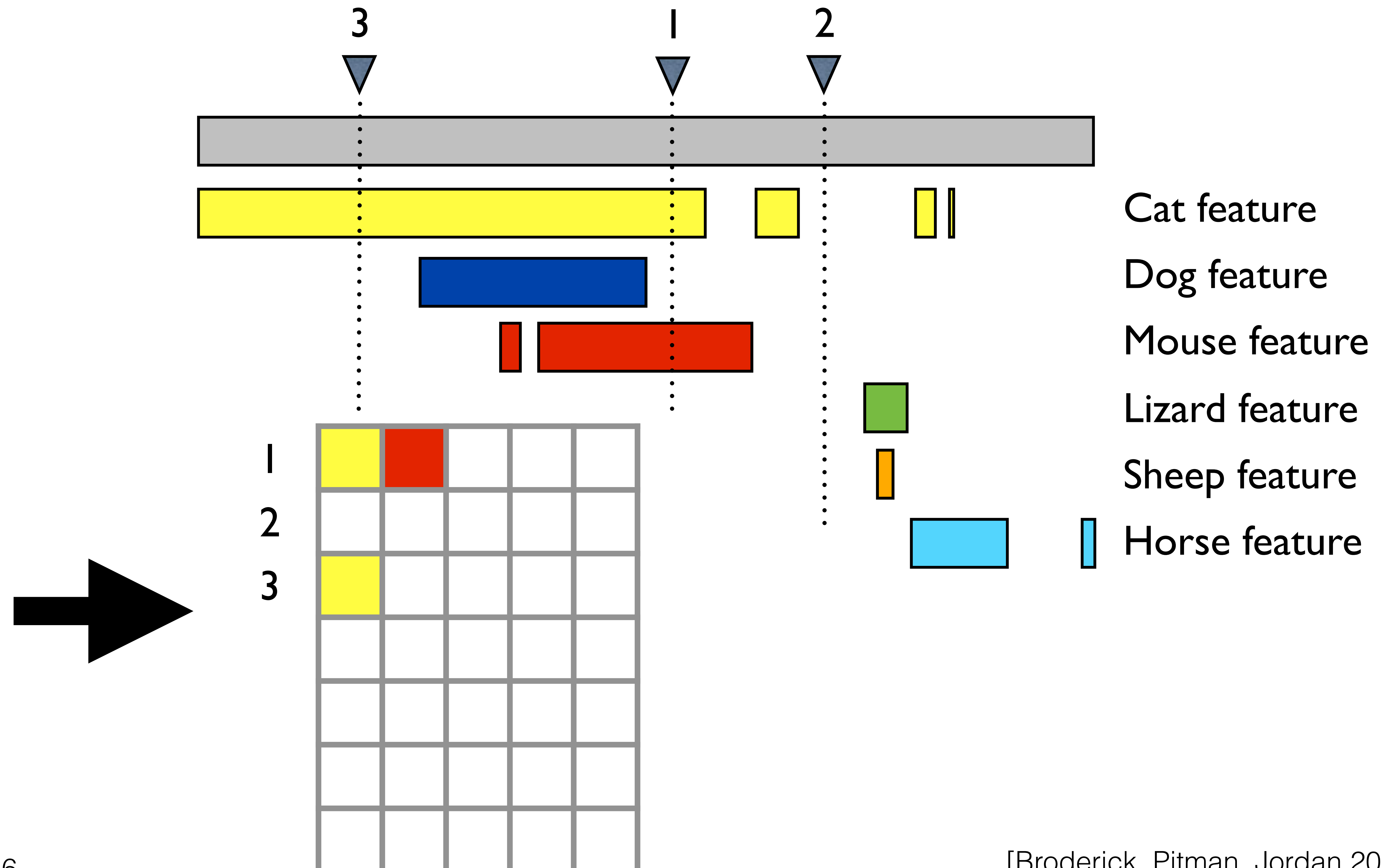
Feature allocation



[Broderick, Pitman, Jordan 2013]

Feature allocation



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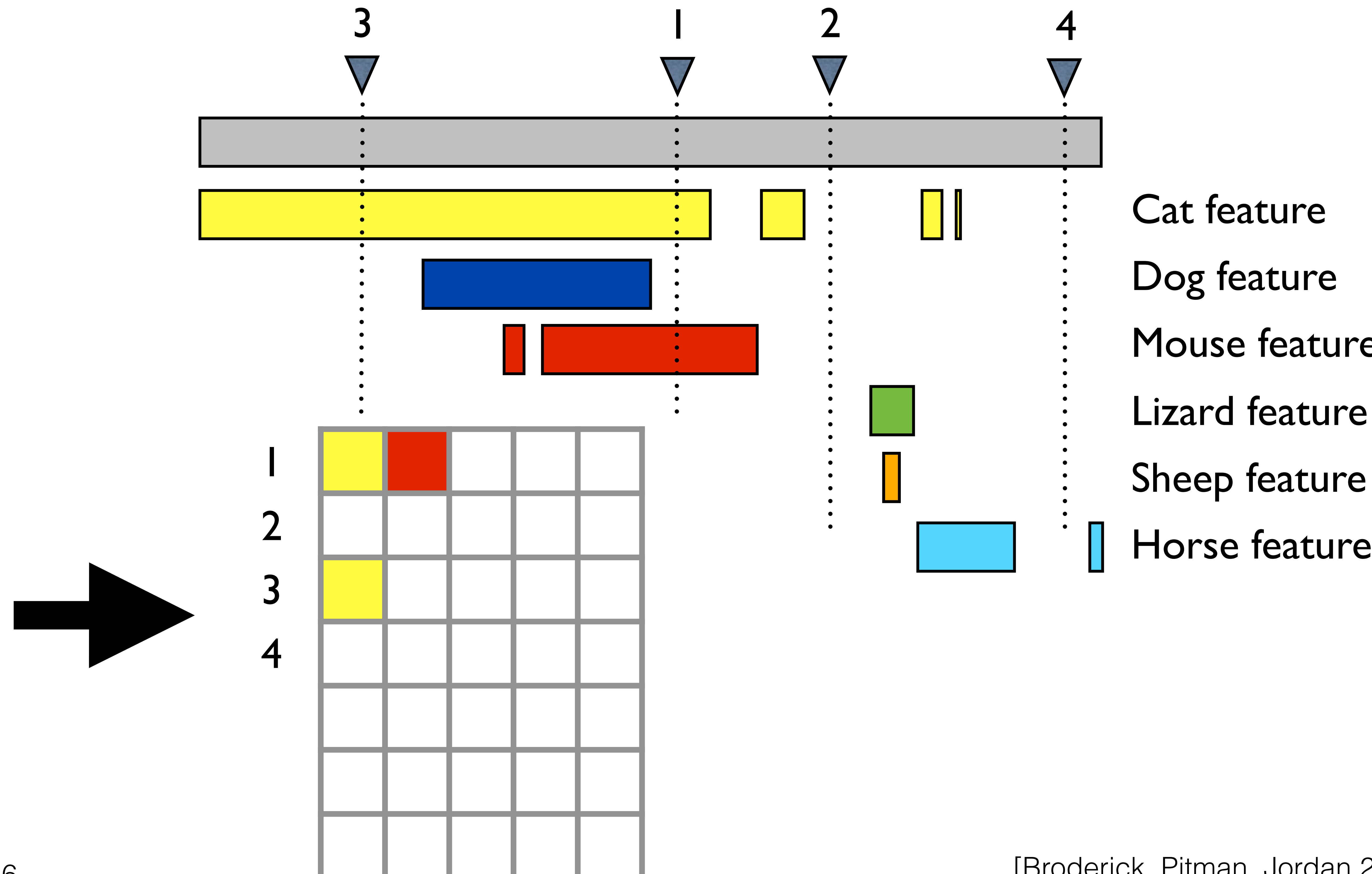

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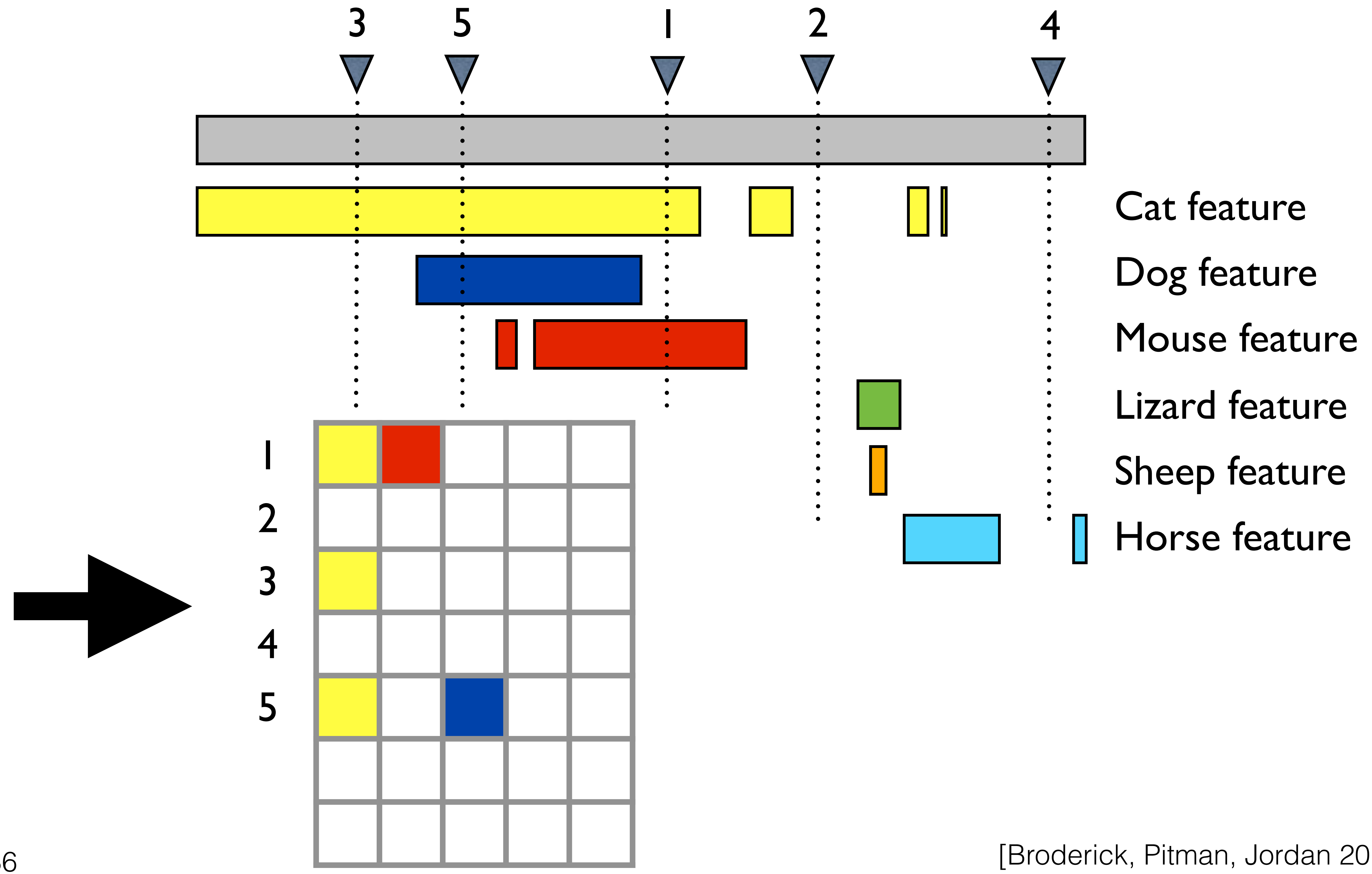

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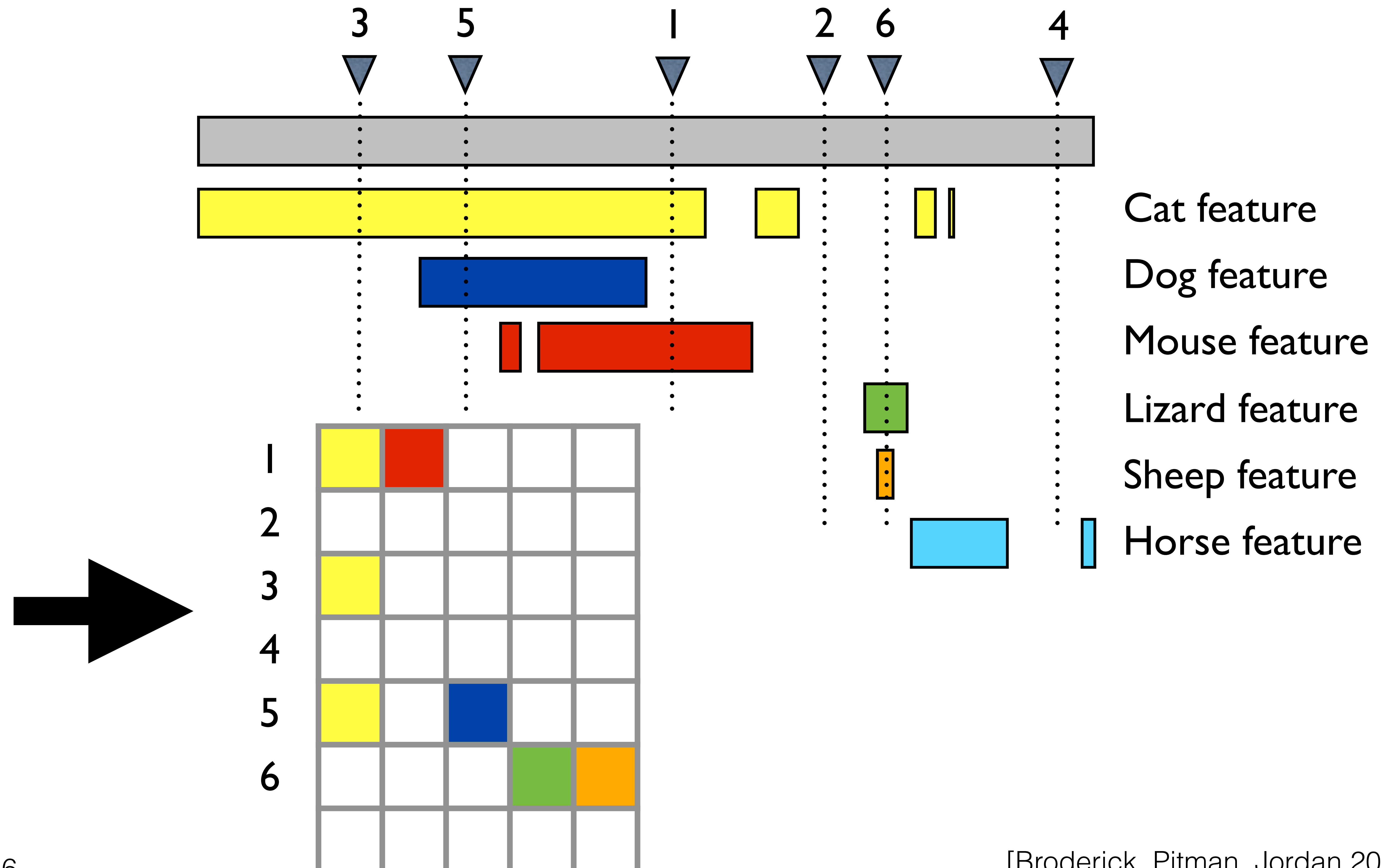

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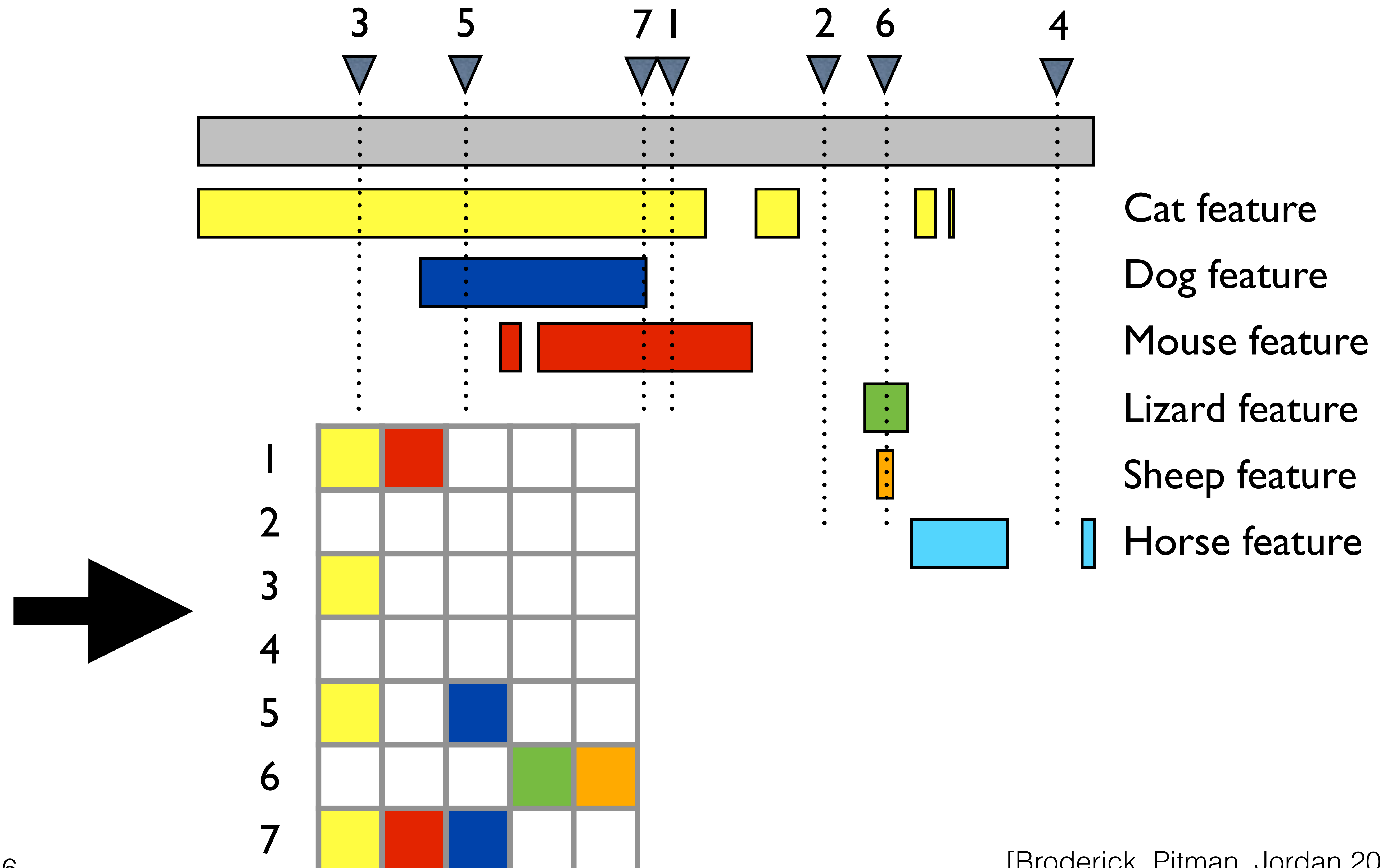

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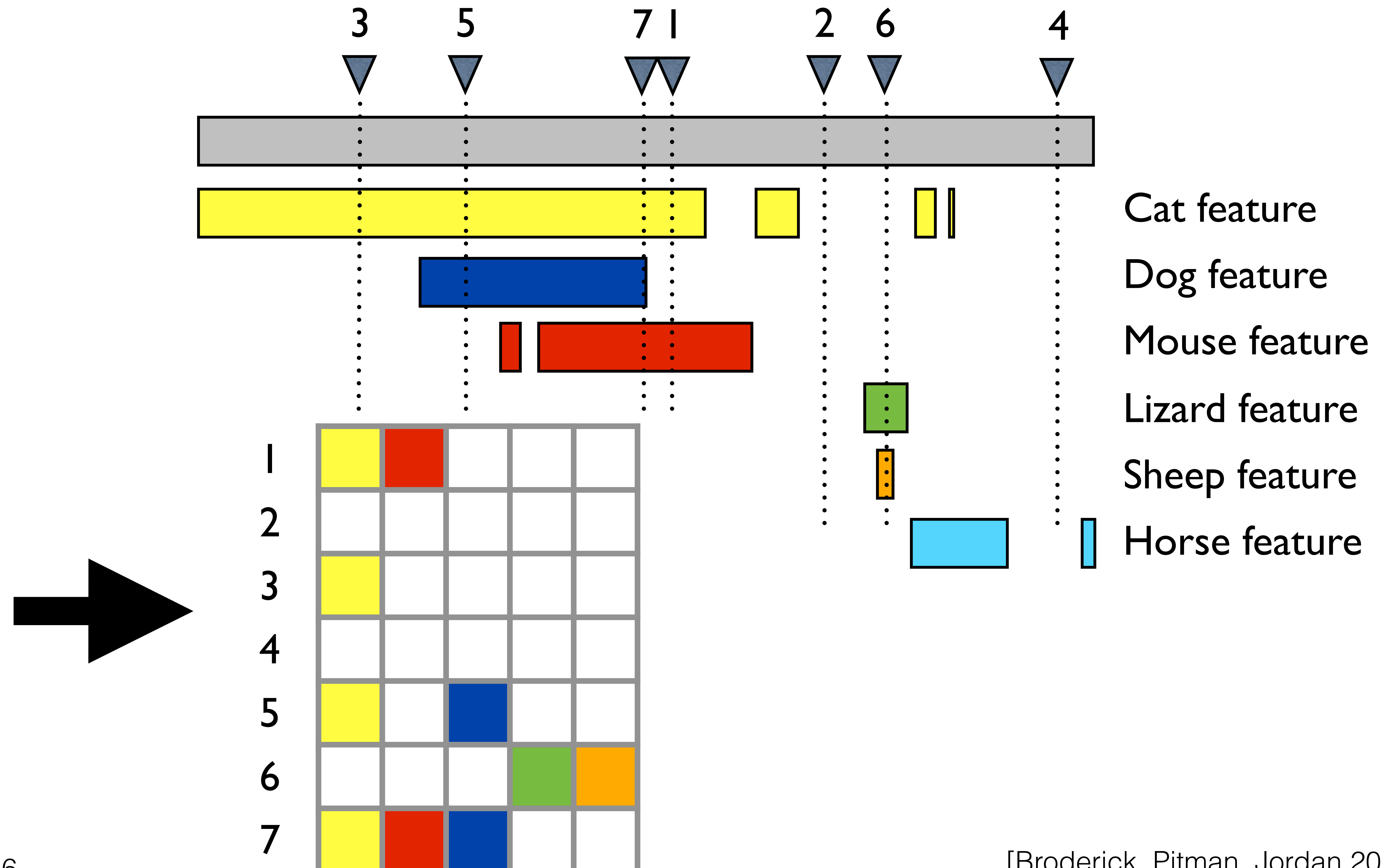
Feature allocation

Thm (BPJ). A feature allocation is exchangeable iff it has a feature paintbox representation



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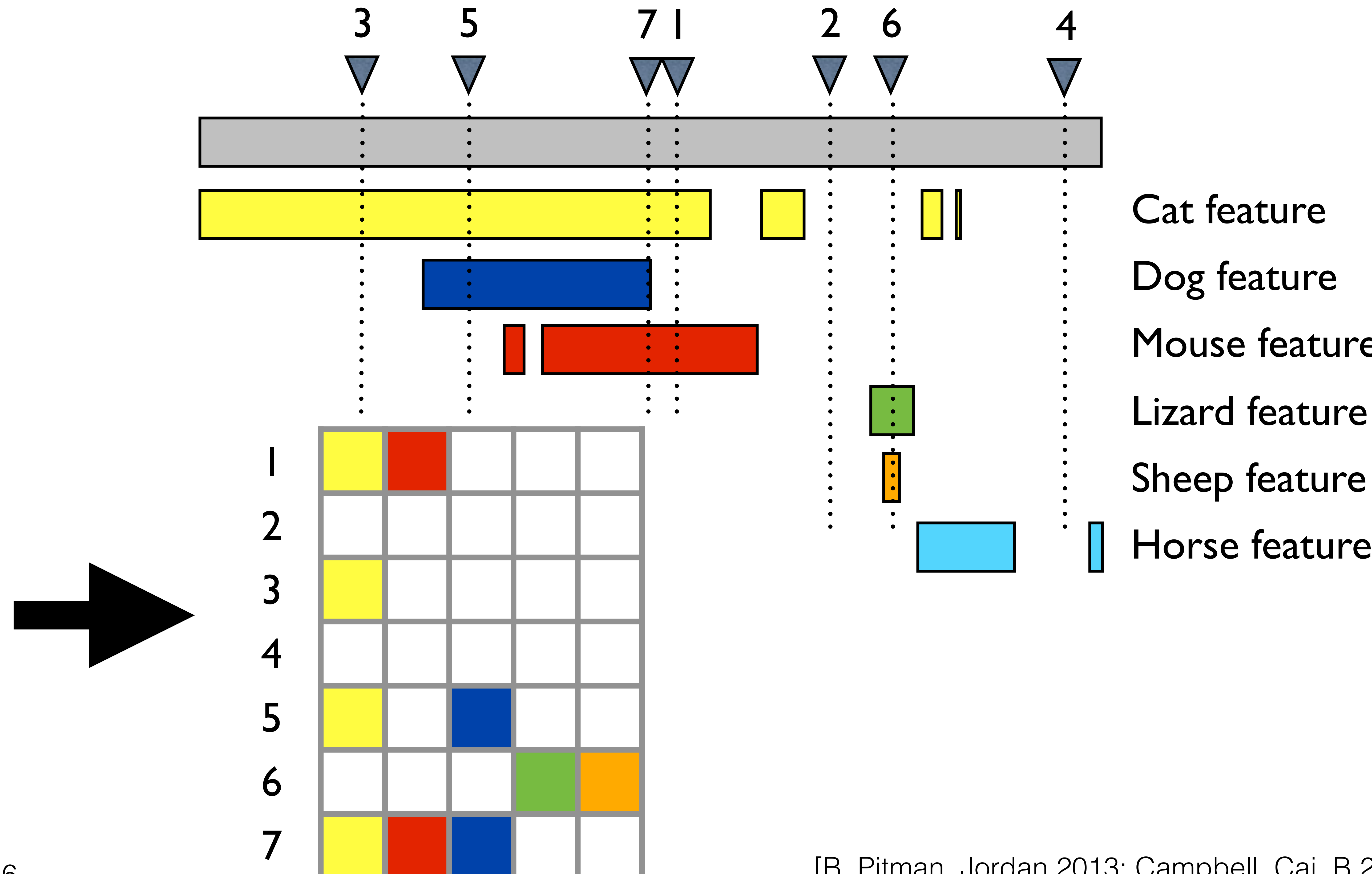
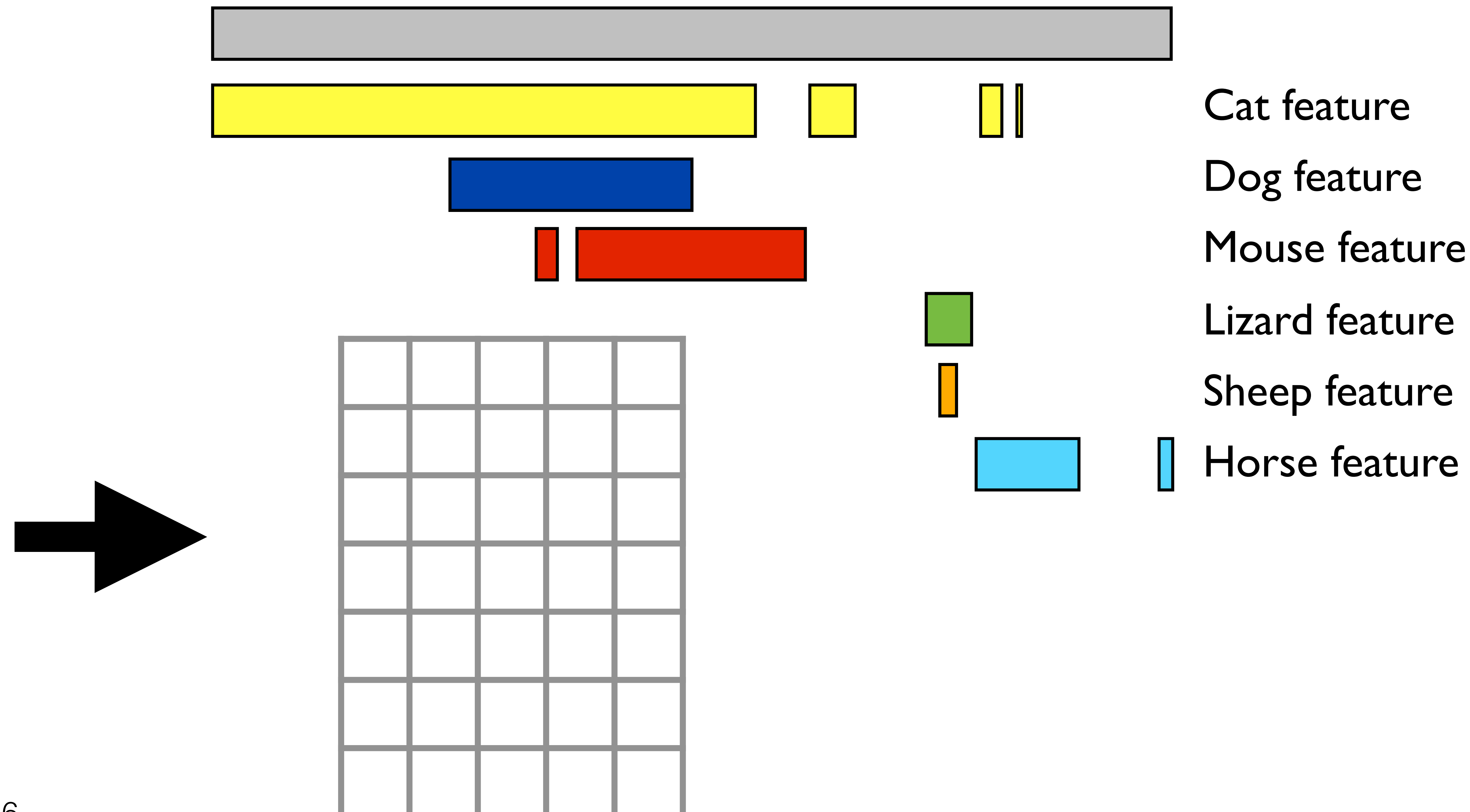
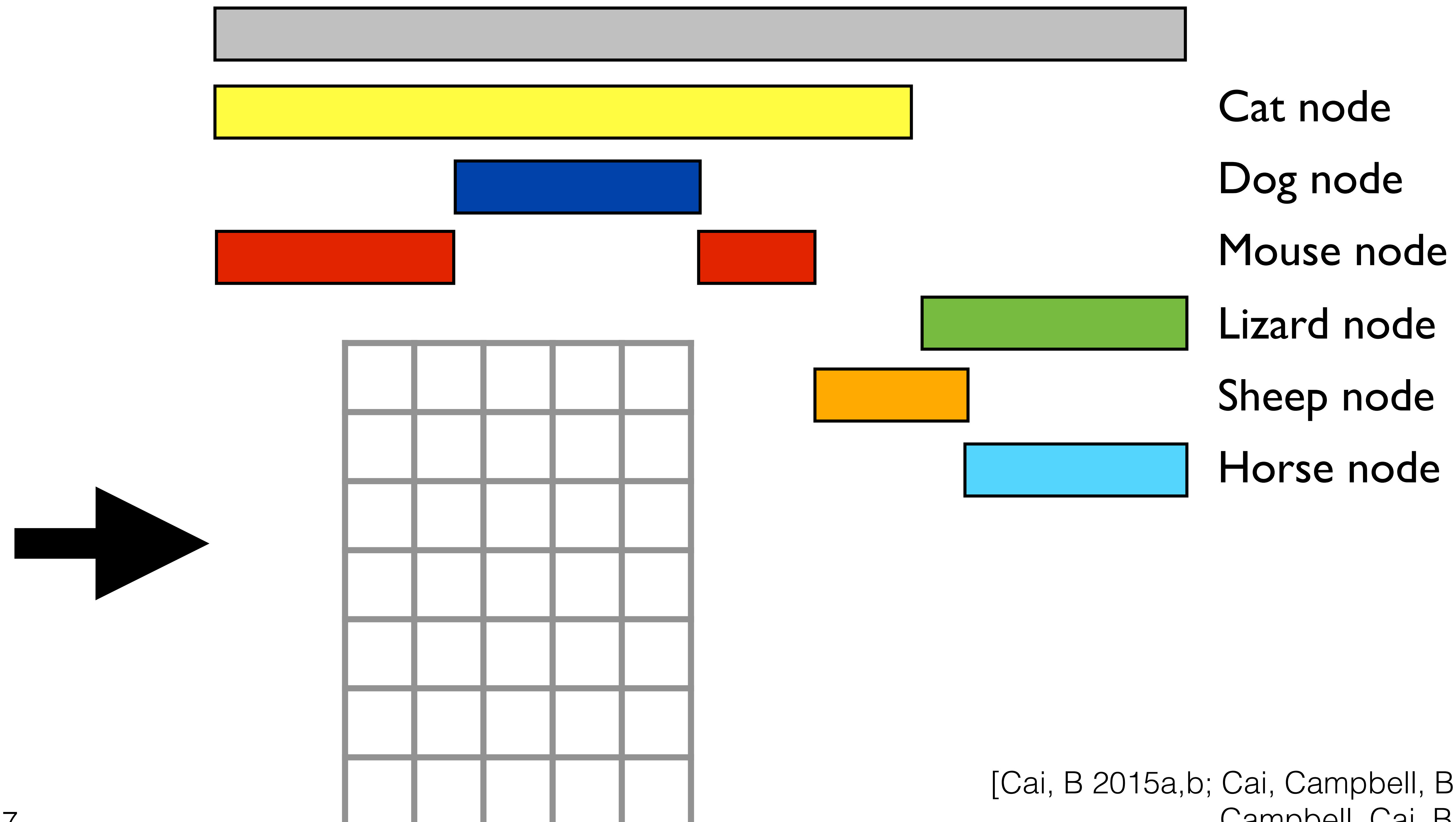


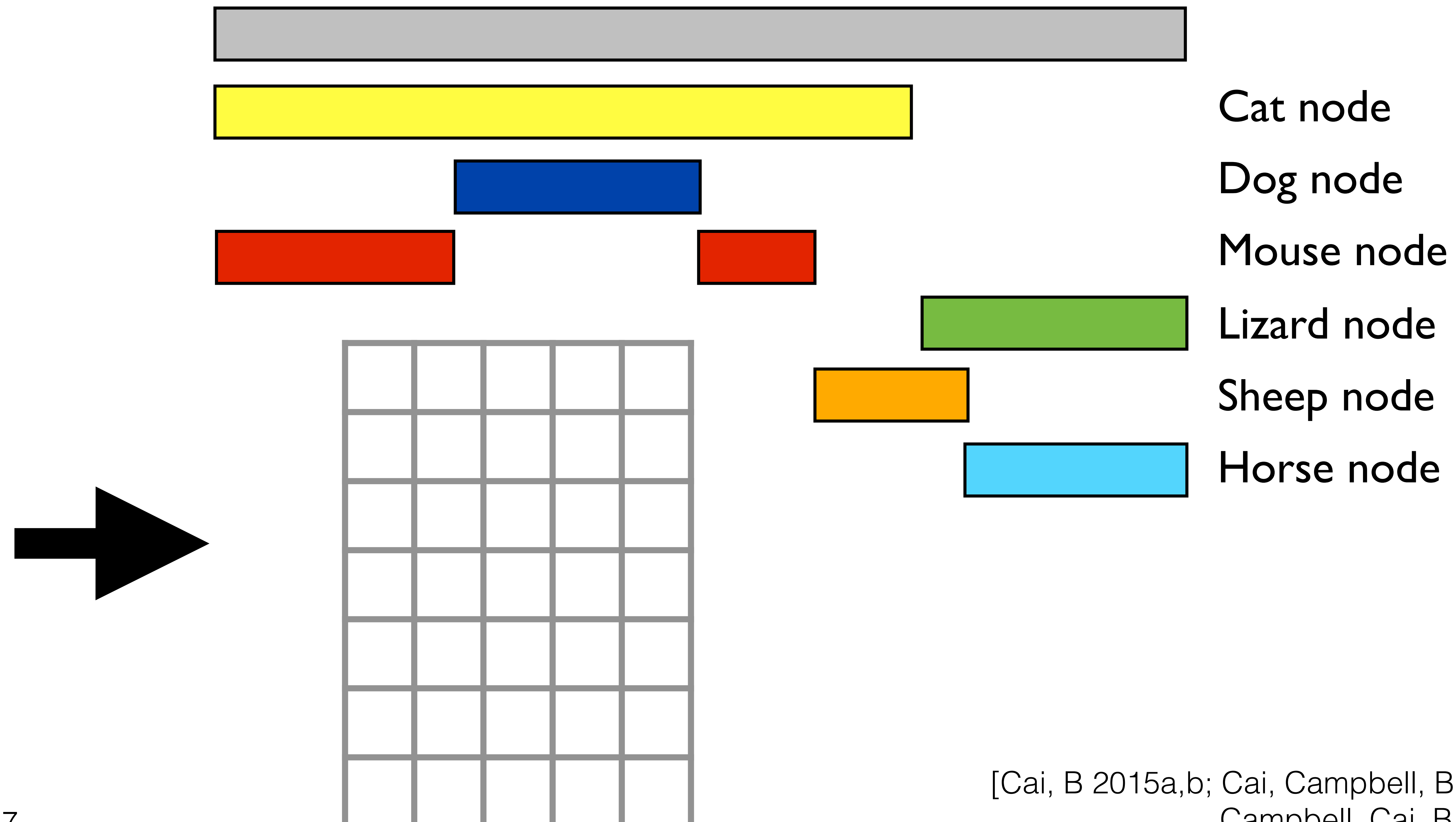
[B, Pitman, Jordan 2013; Campbell, Cai, B 2016]


Cat feature
Dog feature
Mouse feature
Lizard feature
Sheep feature
Horse feature



[Cai, B 2015a,b; Cai, Campbell, B, 2016; Campbell, Cai, B, 2016]

Edge-exchangeable graph



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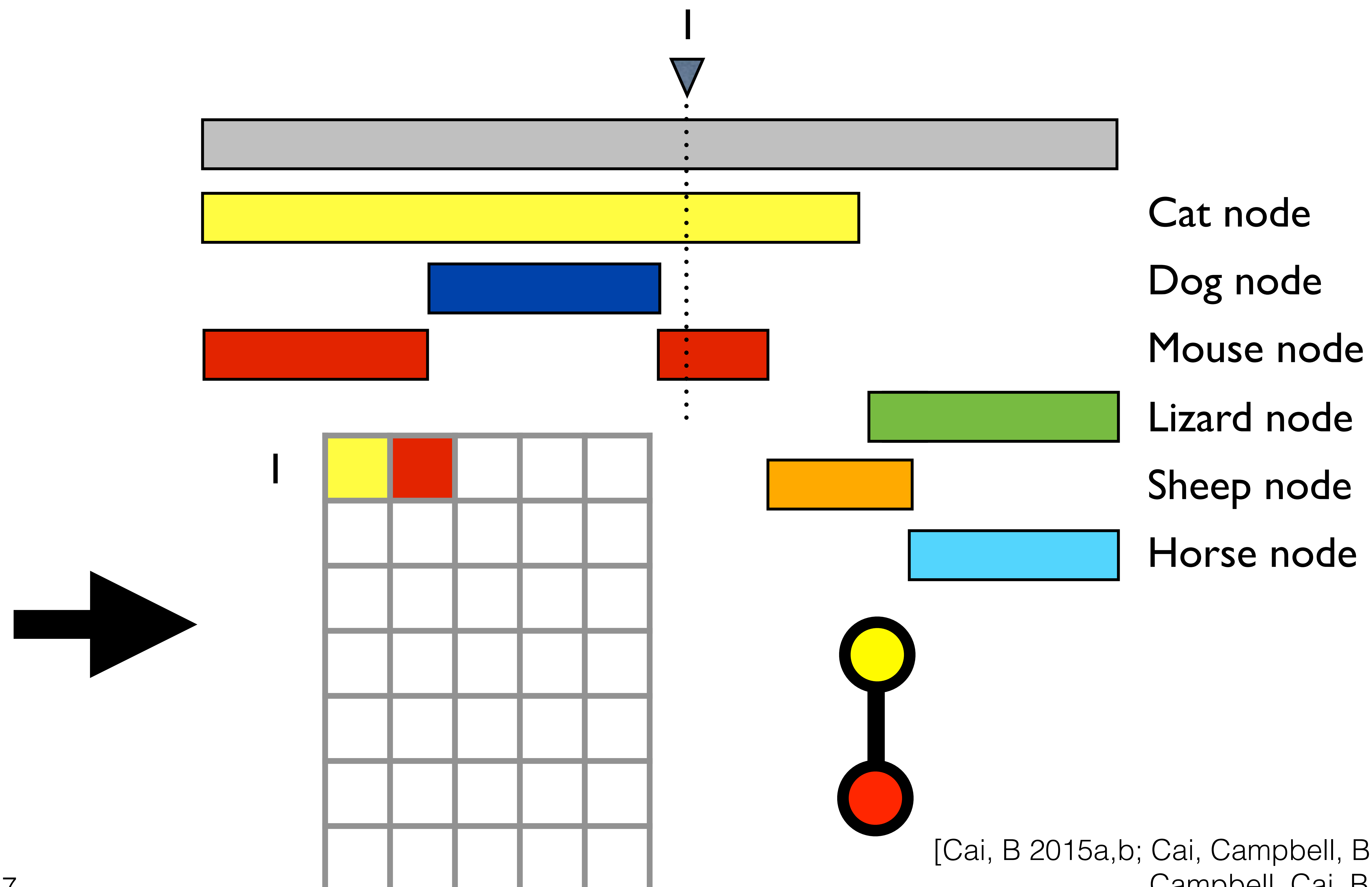

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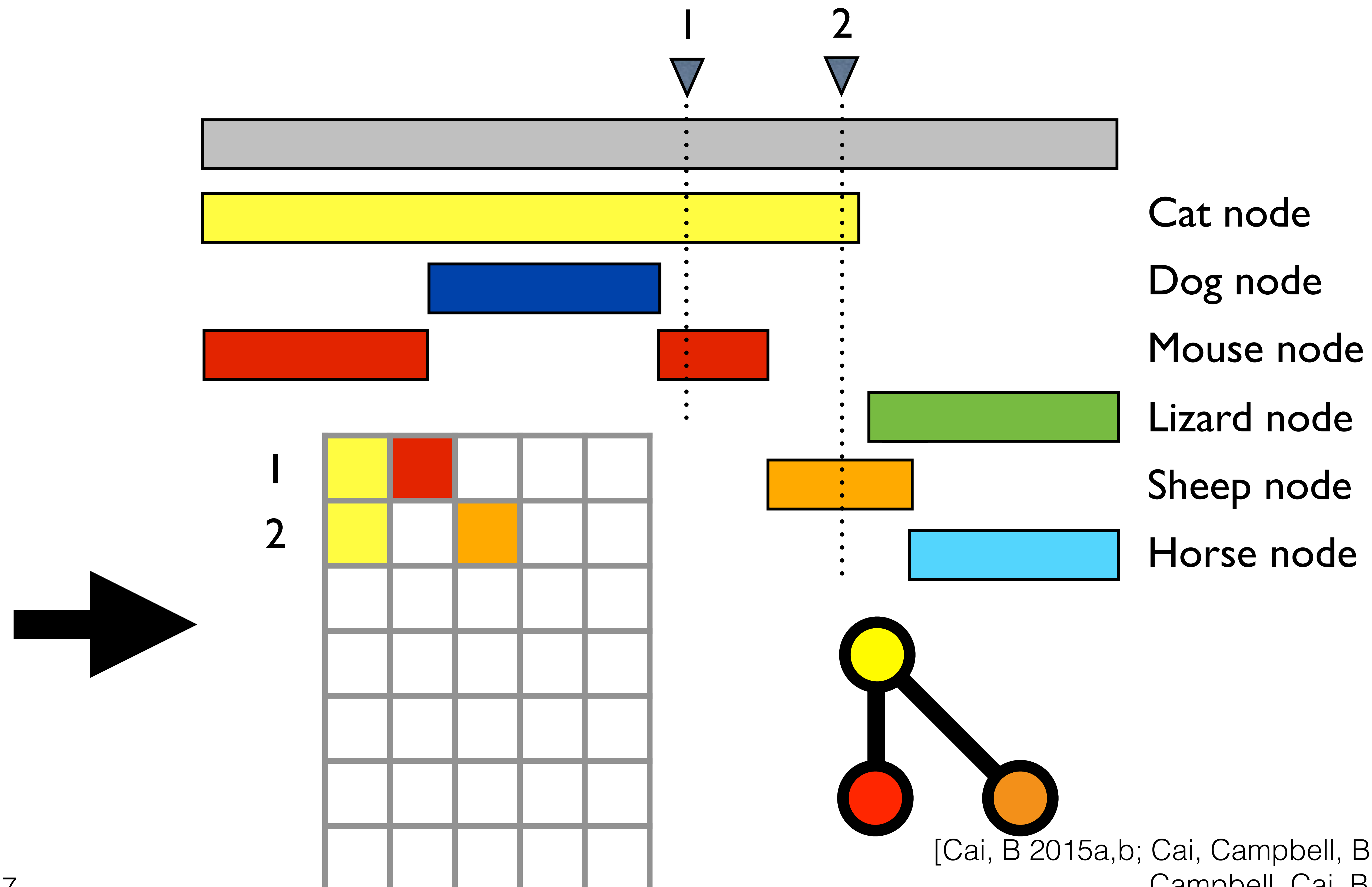

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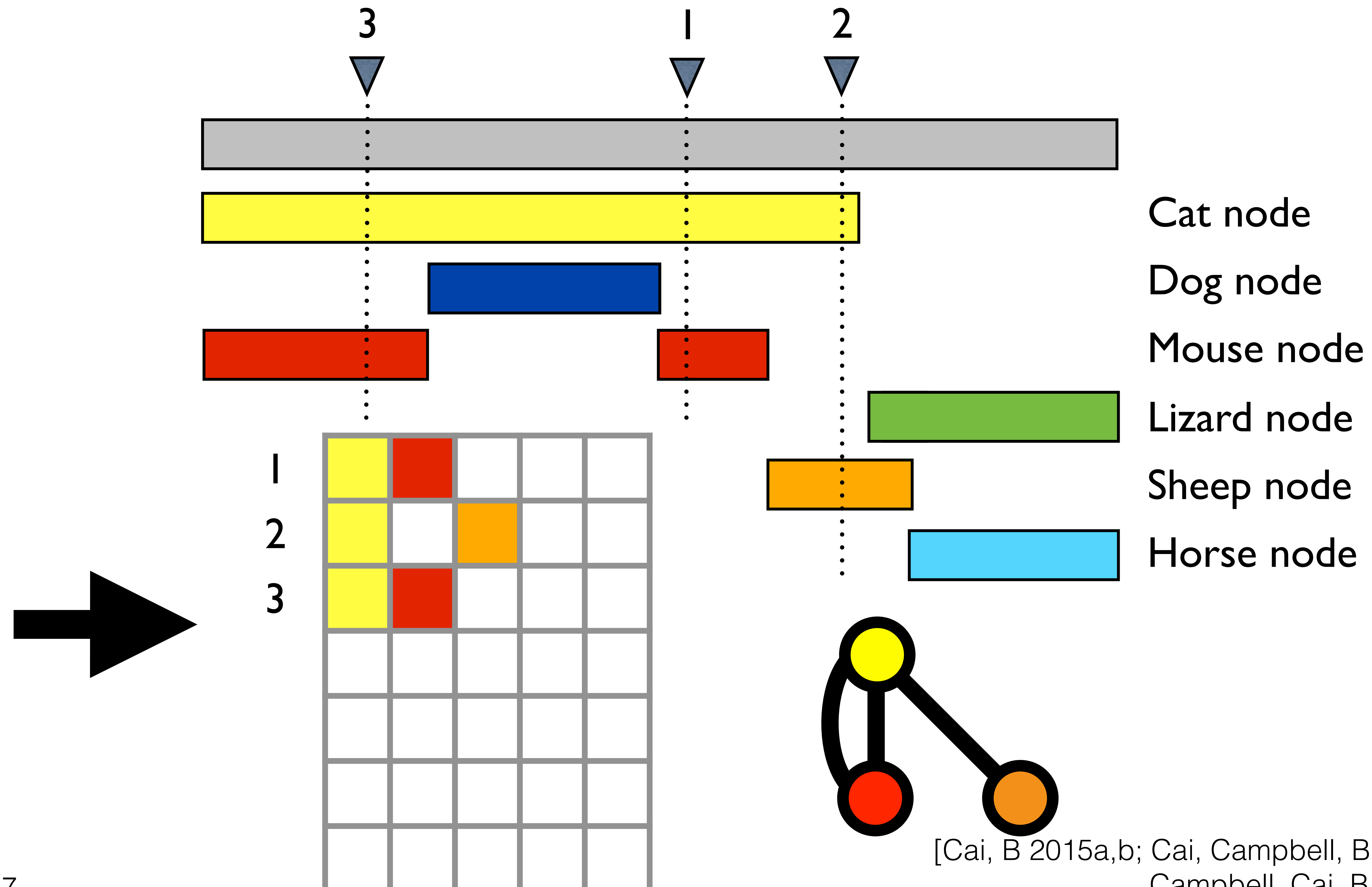

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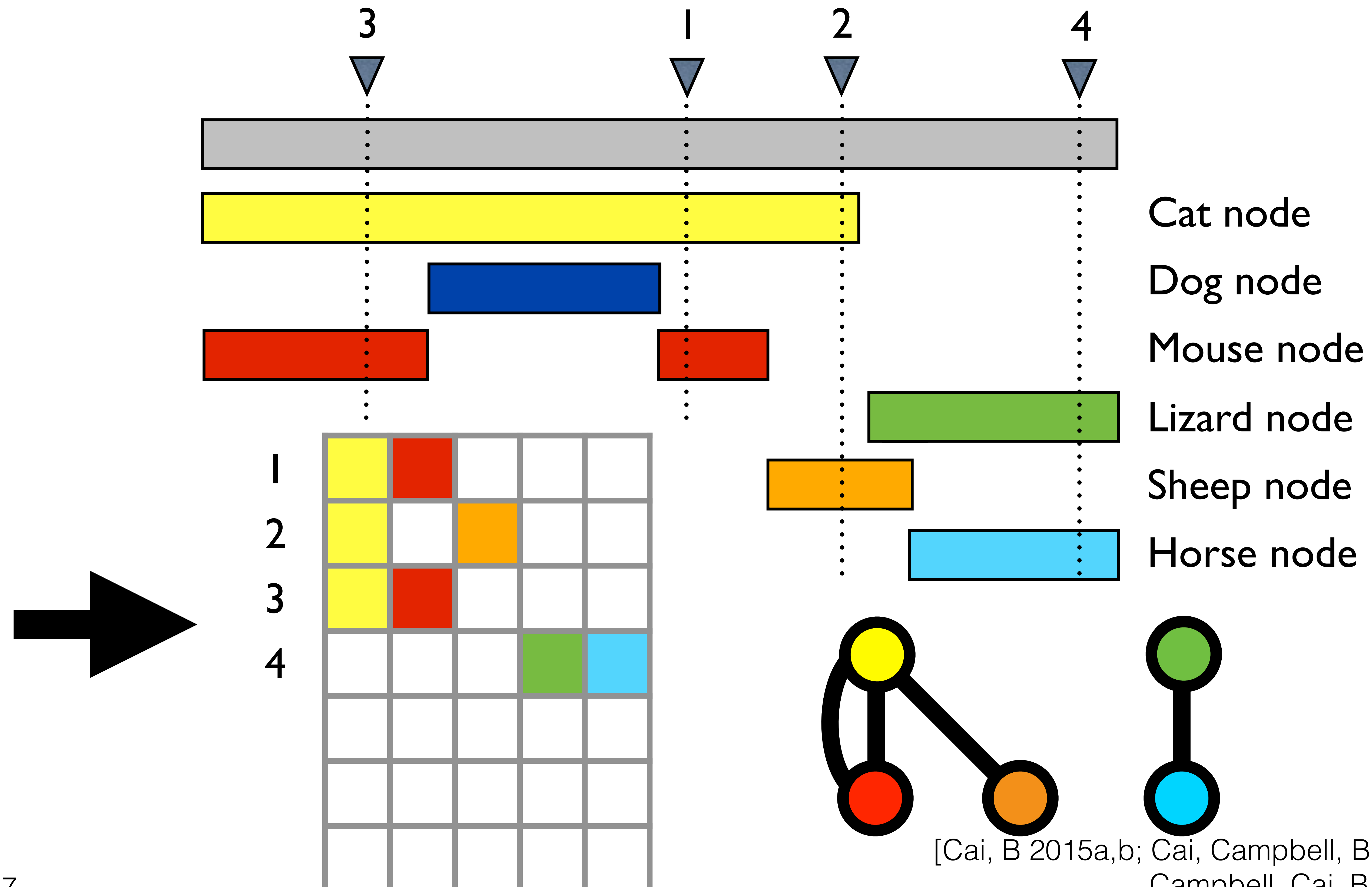

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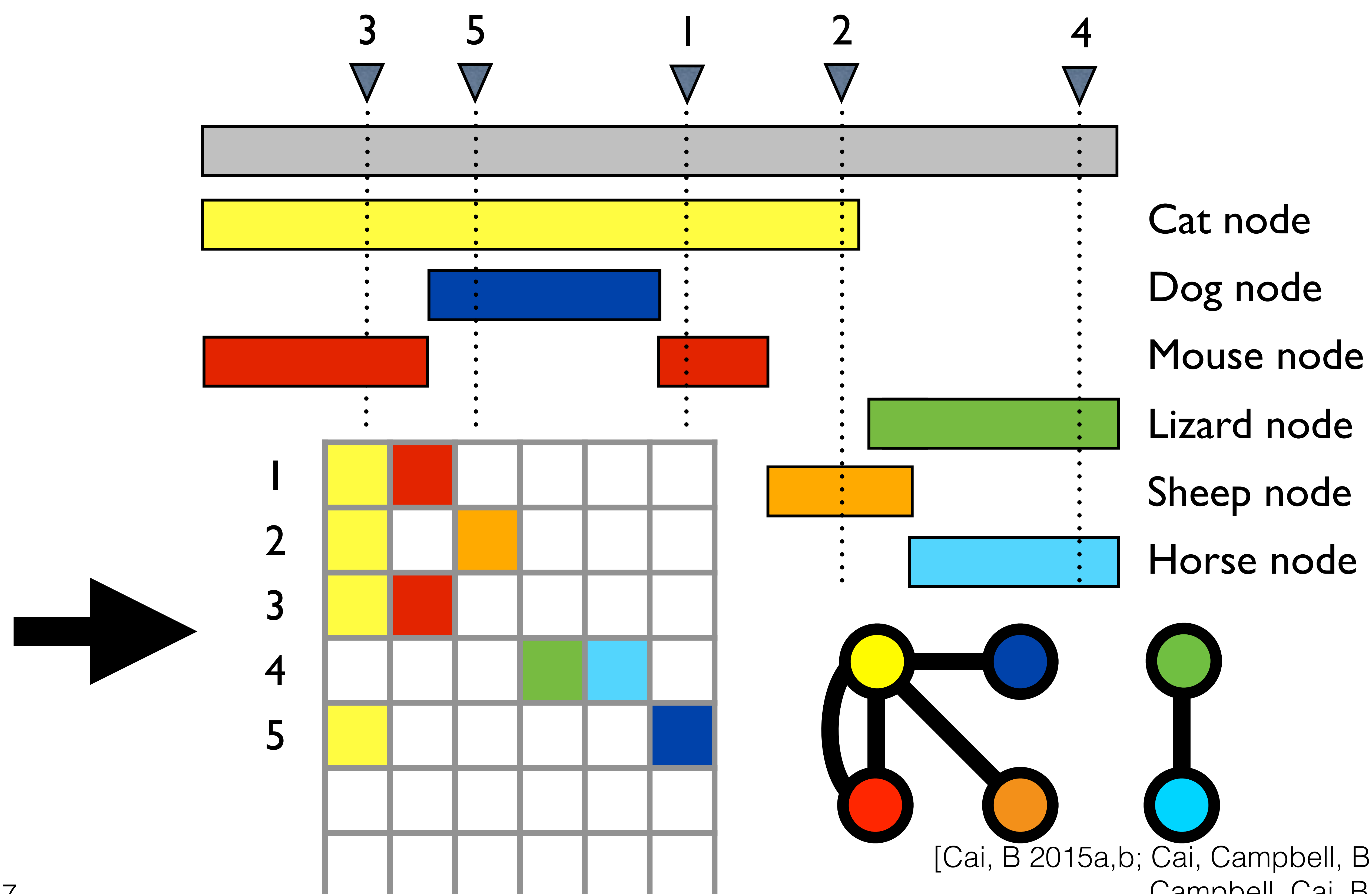

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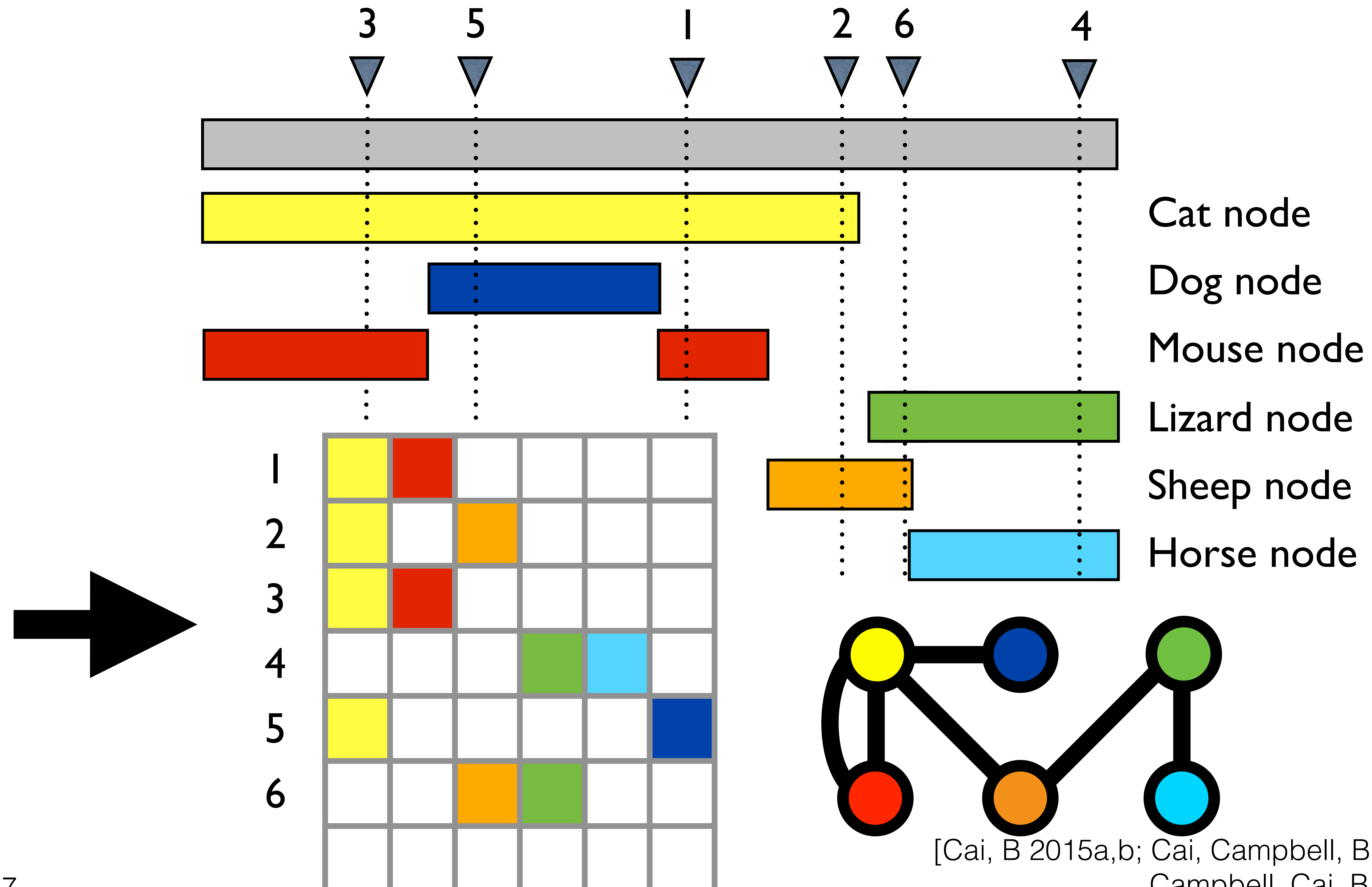

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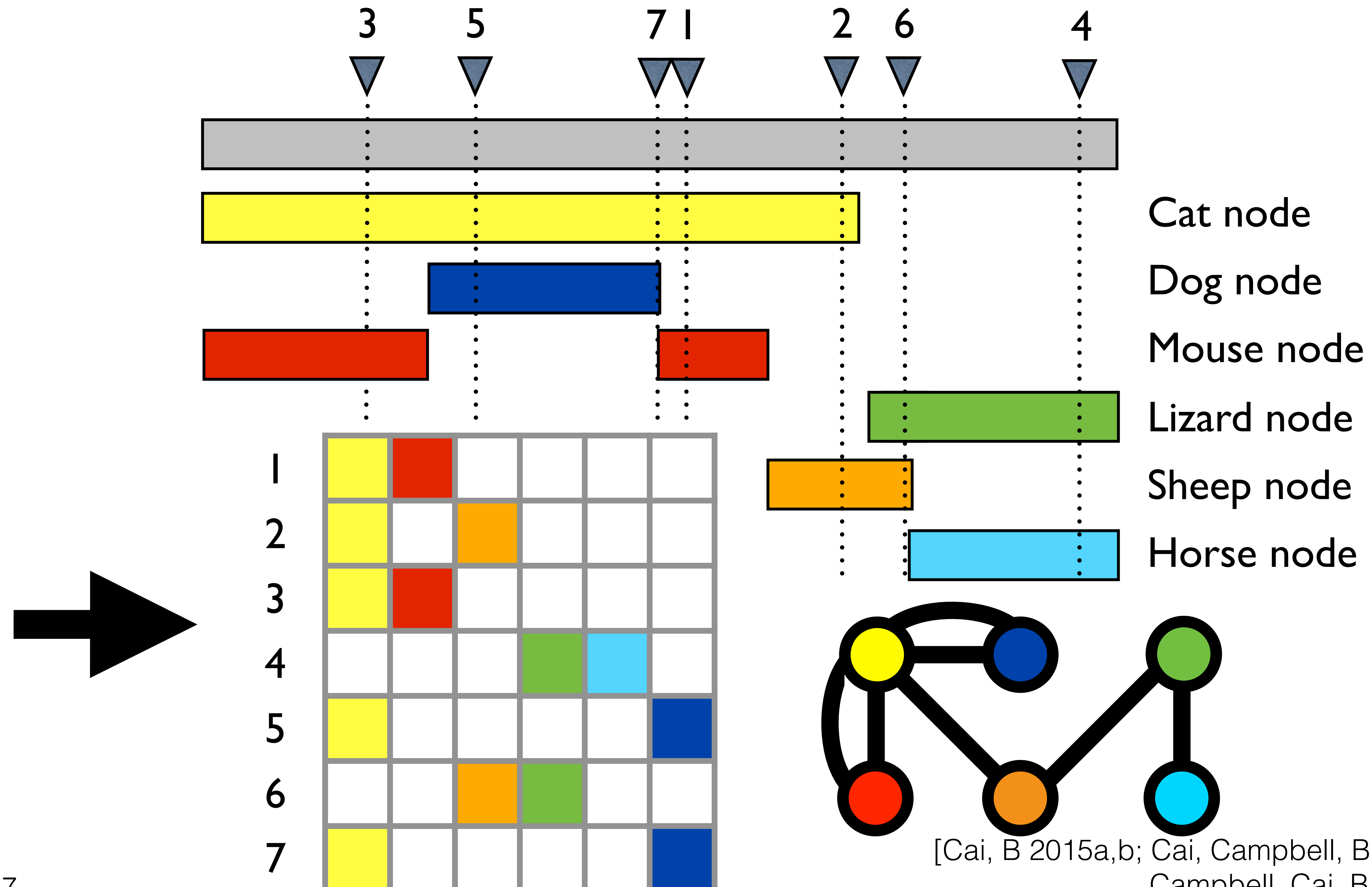

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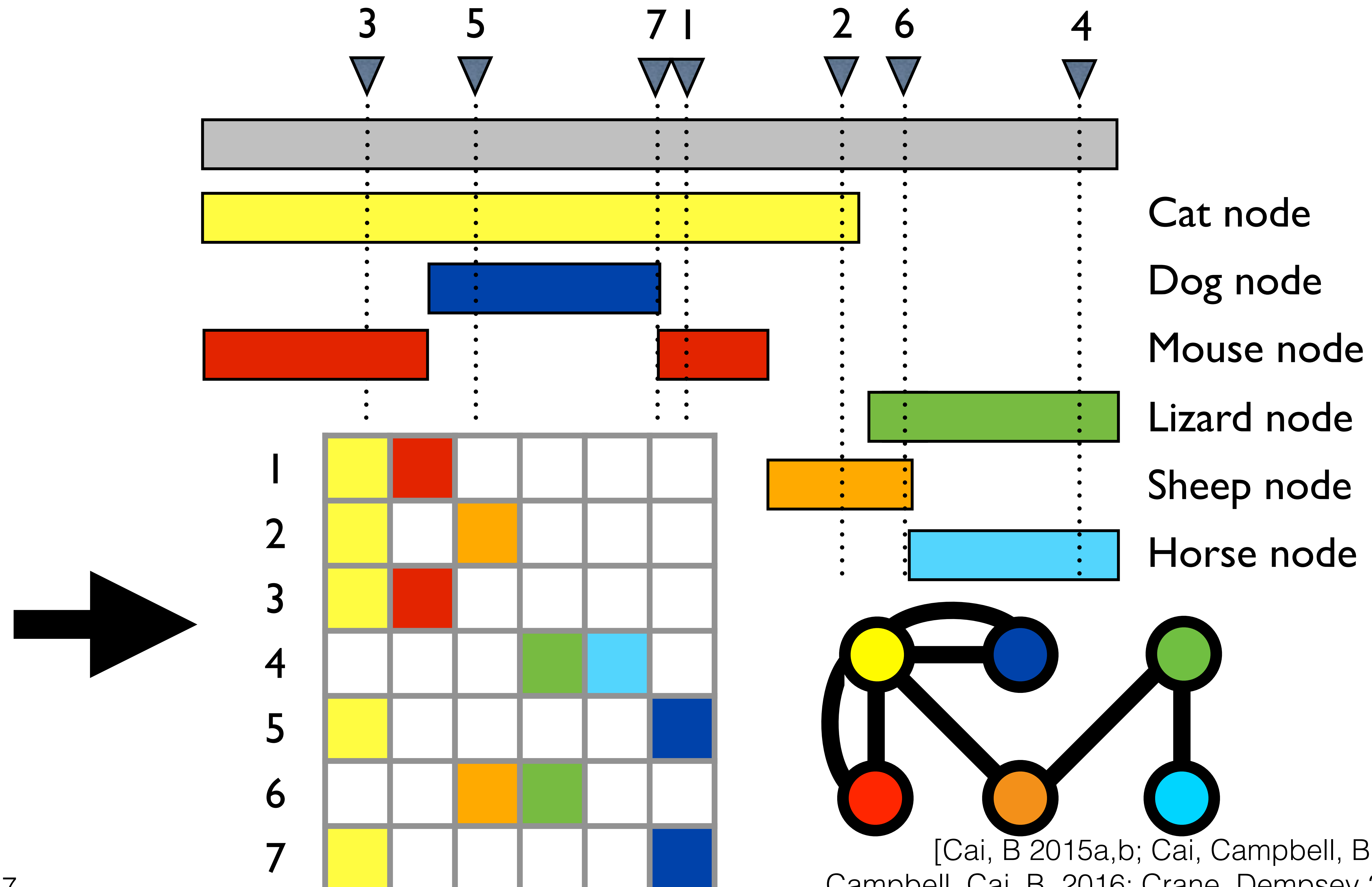

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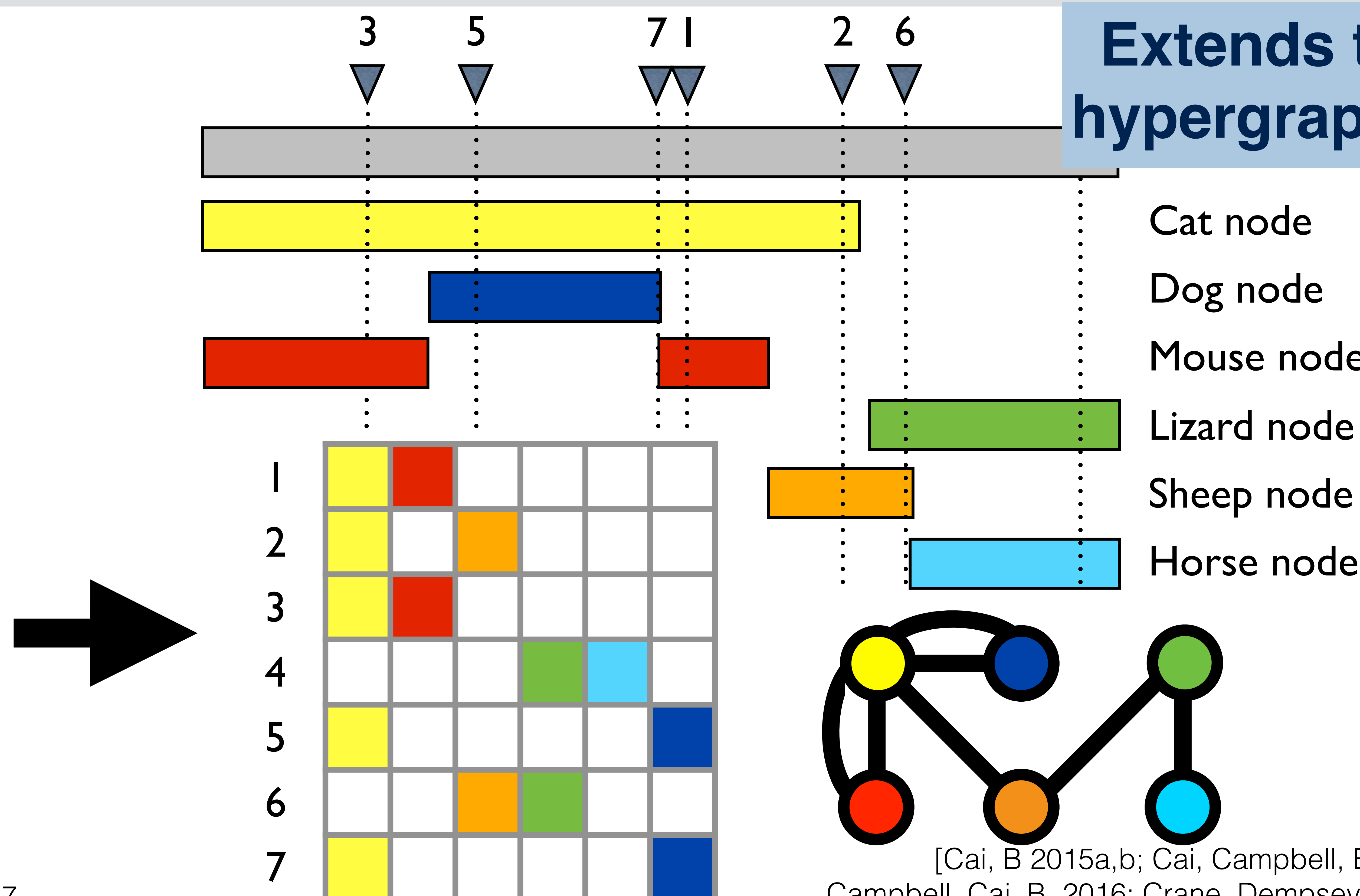
Extends to hypergraphs



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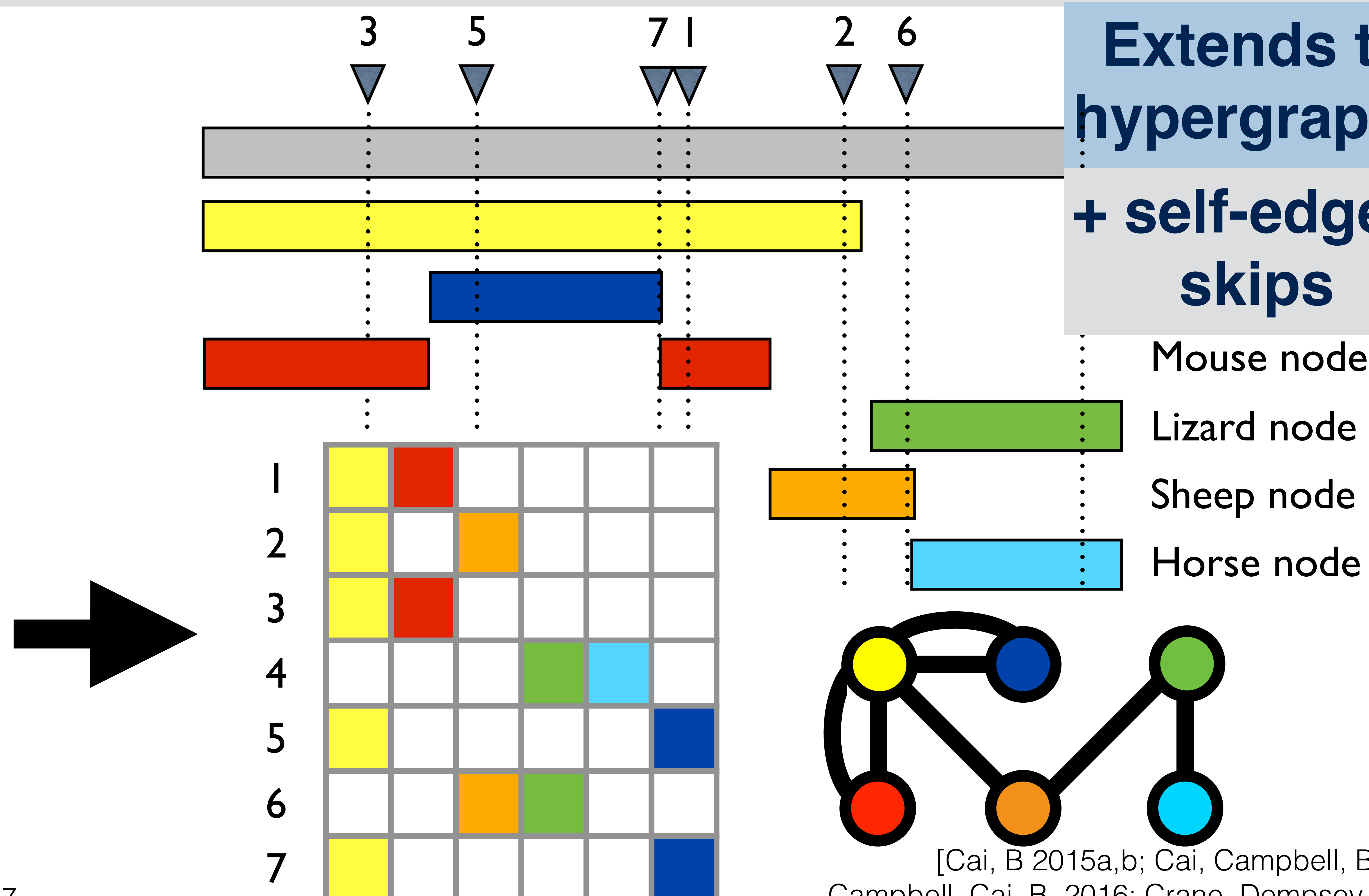
Extends to hypergraphs

+ self-edges, skips



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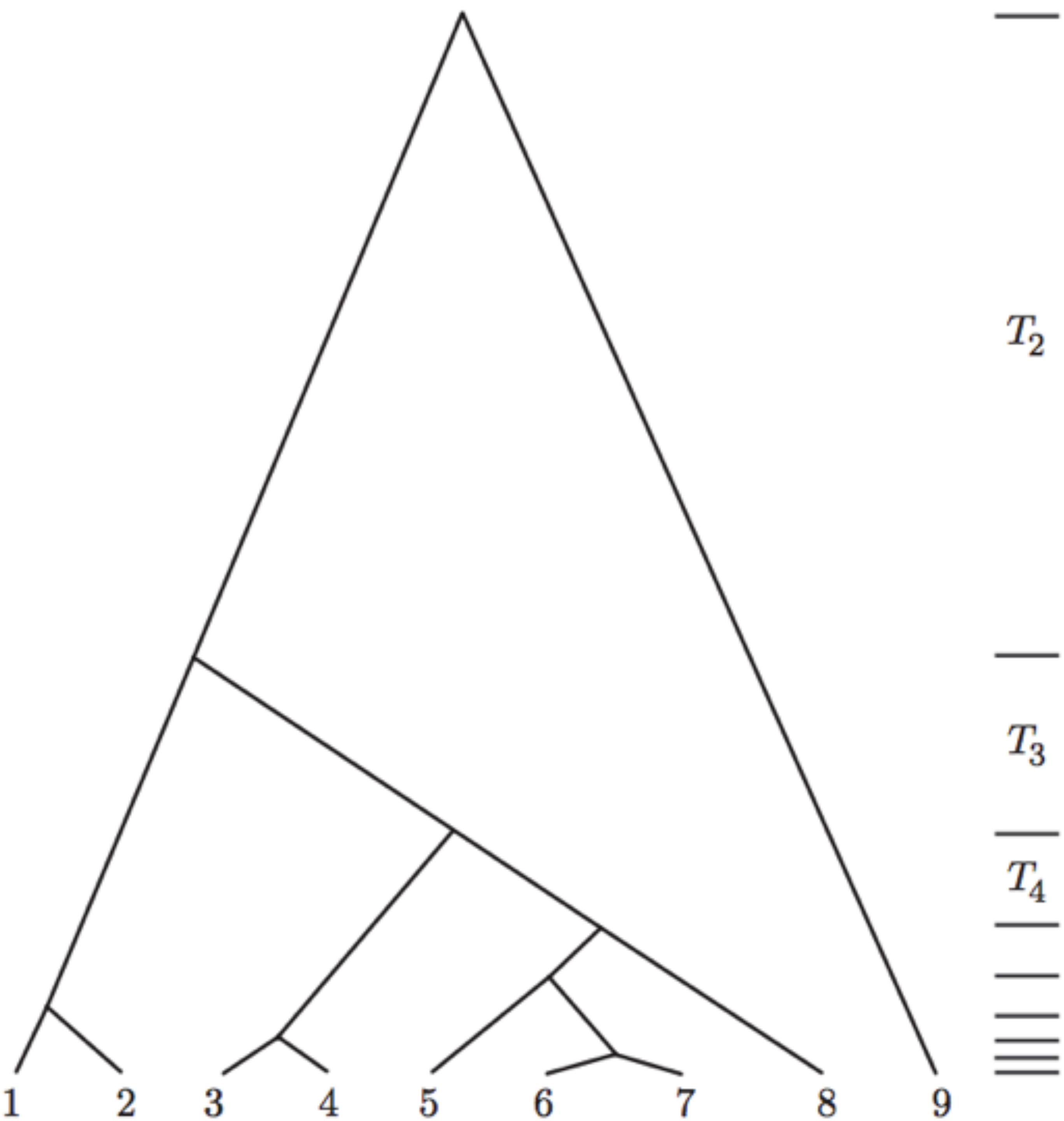
Genealogy, trees, beyond trees



[Wakeley 2008]

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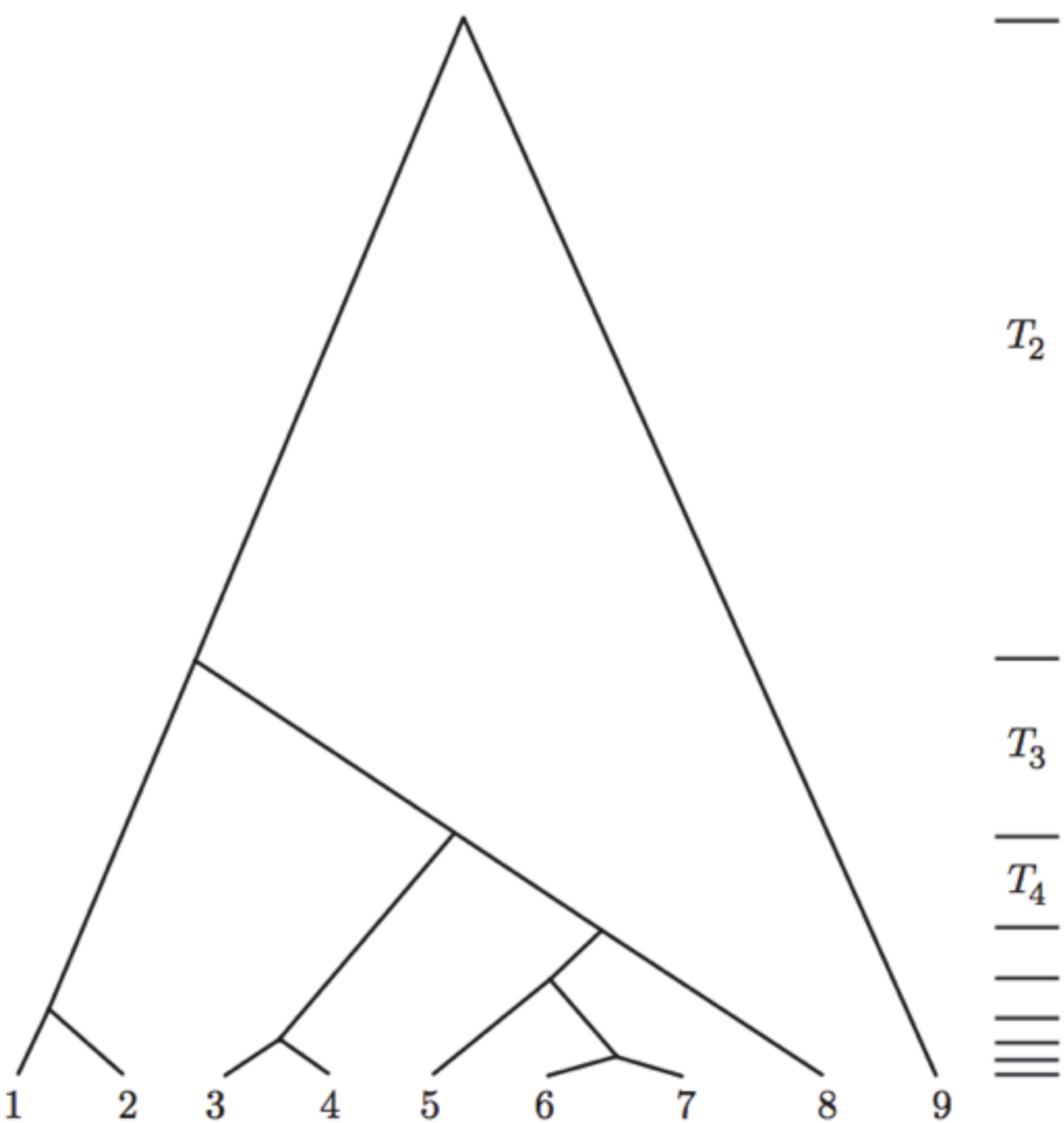

[Wakeley 2008]

- Kingman coalescent

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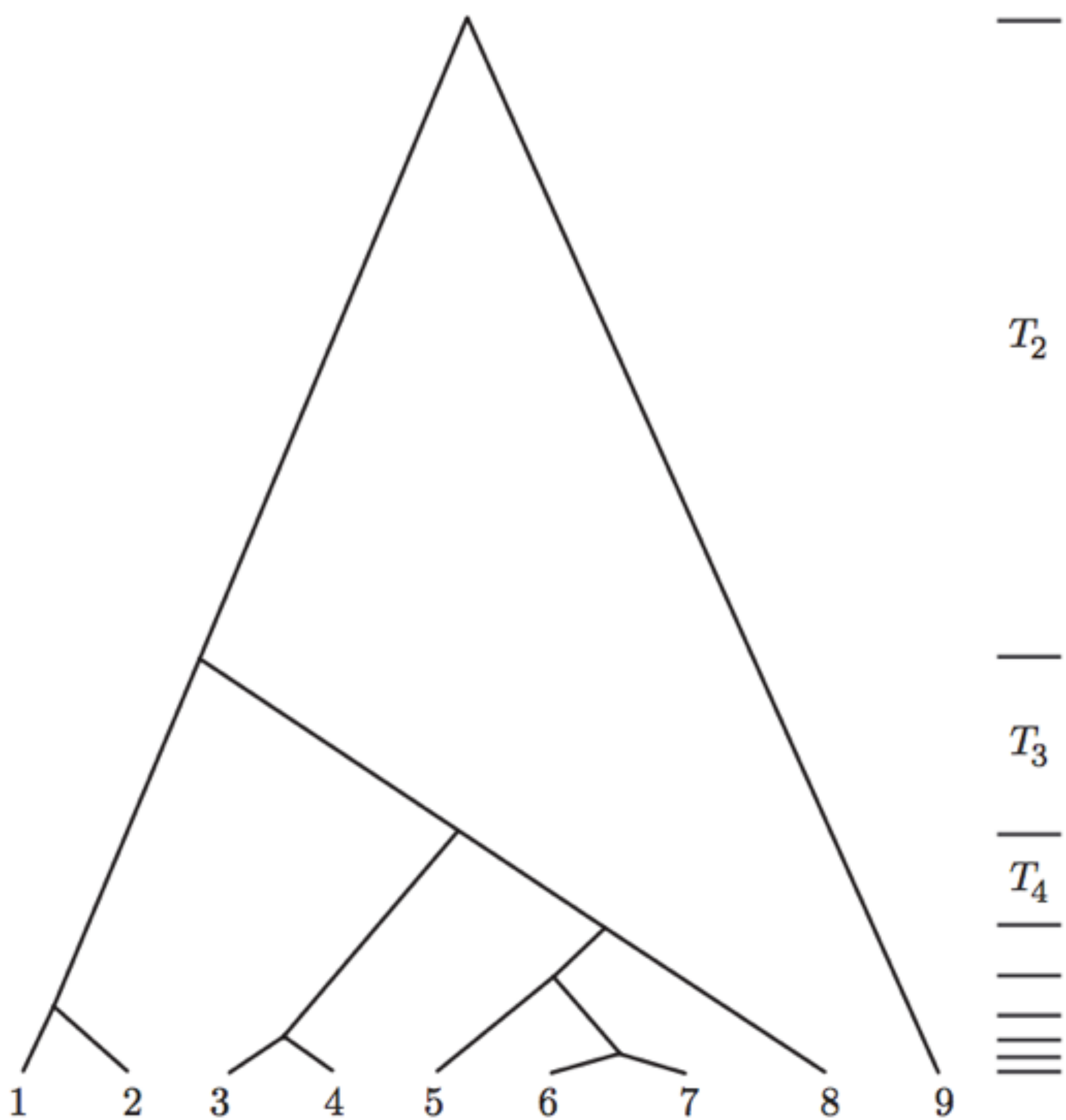

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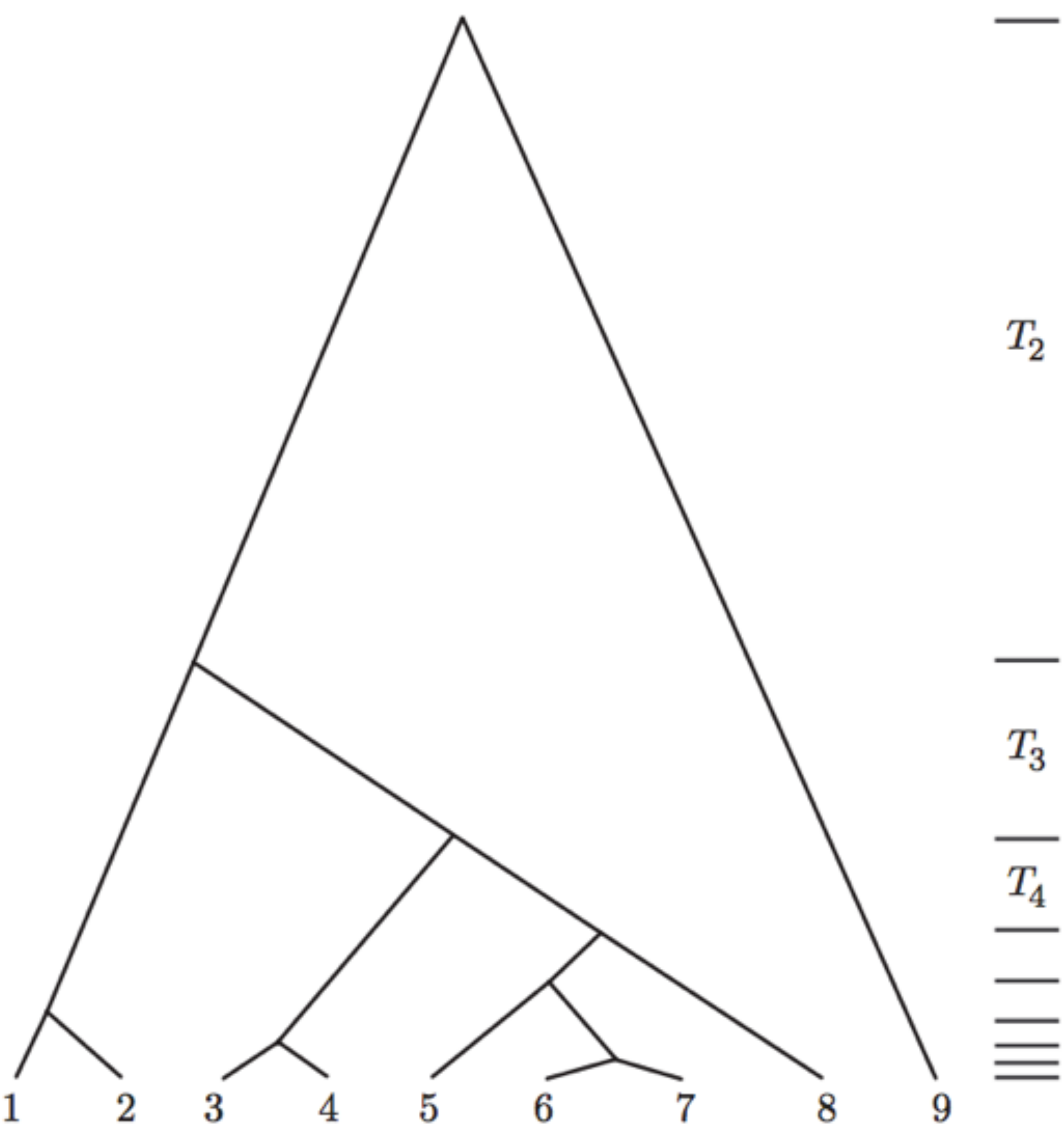

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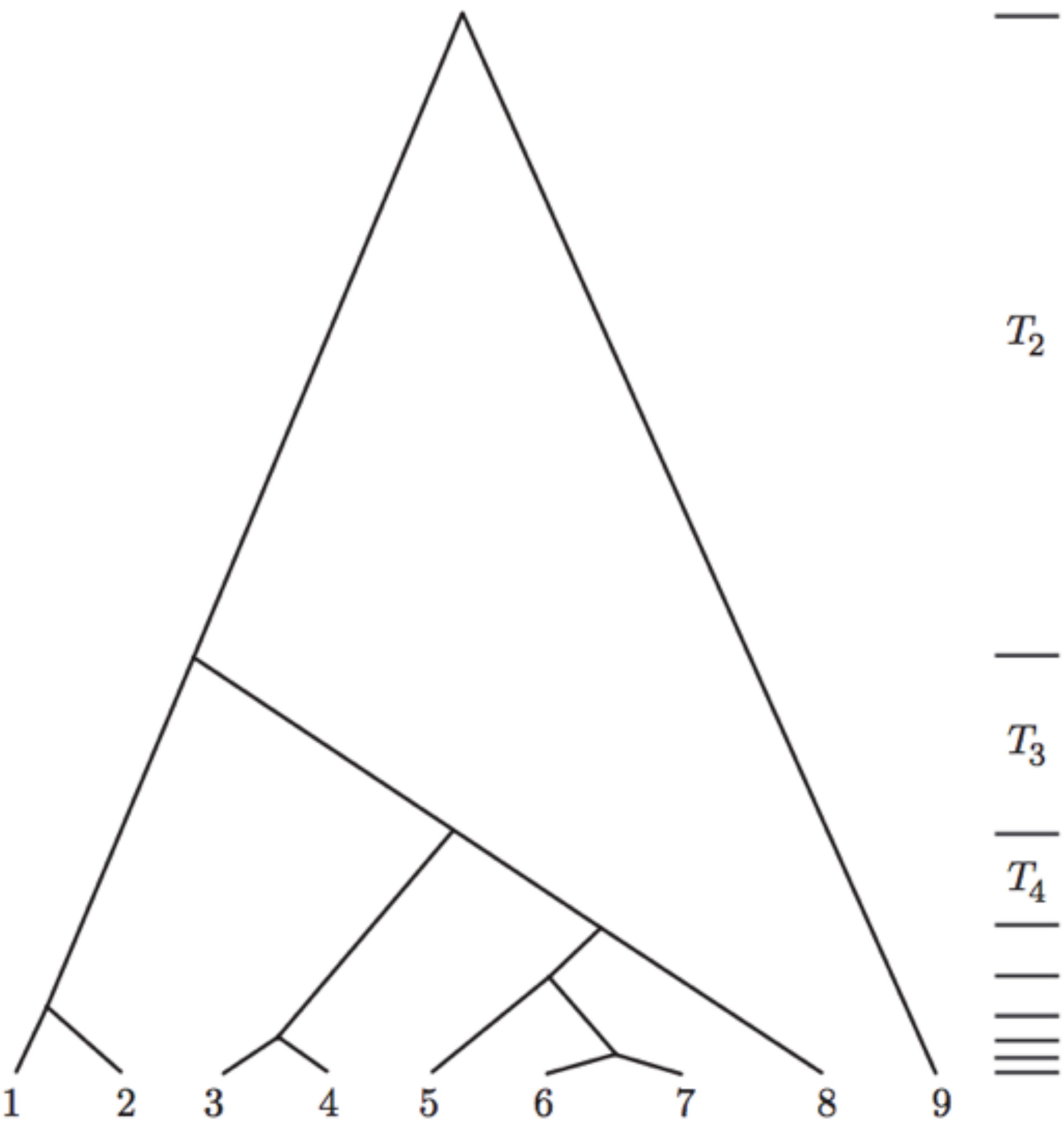

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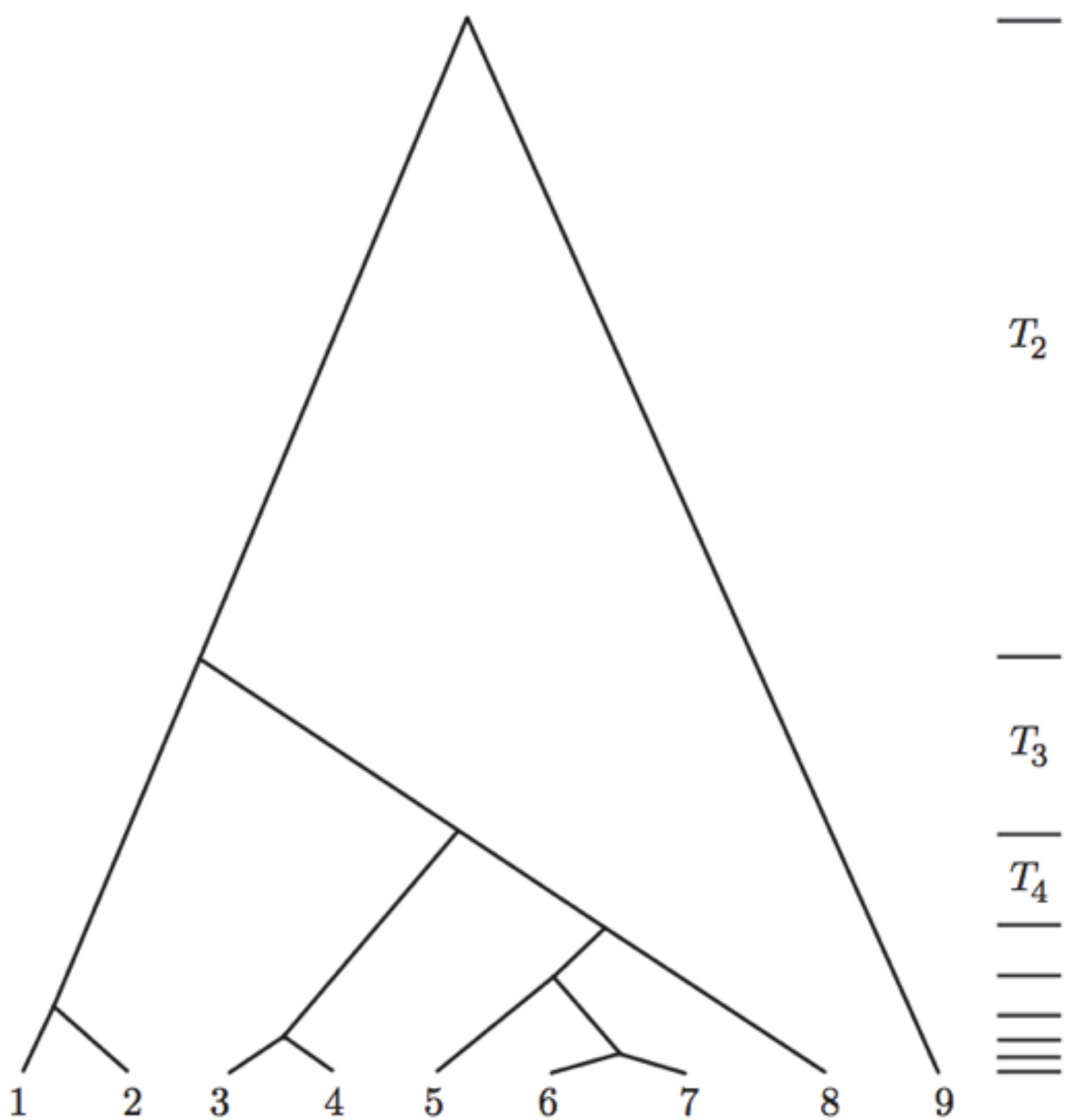

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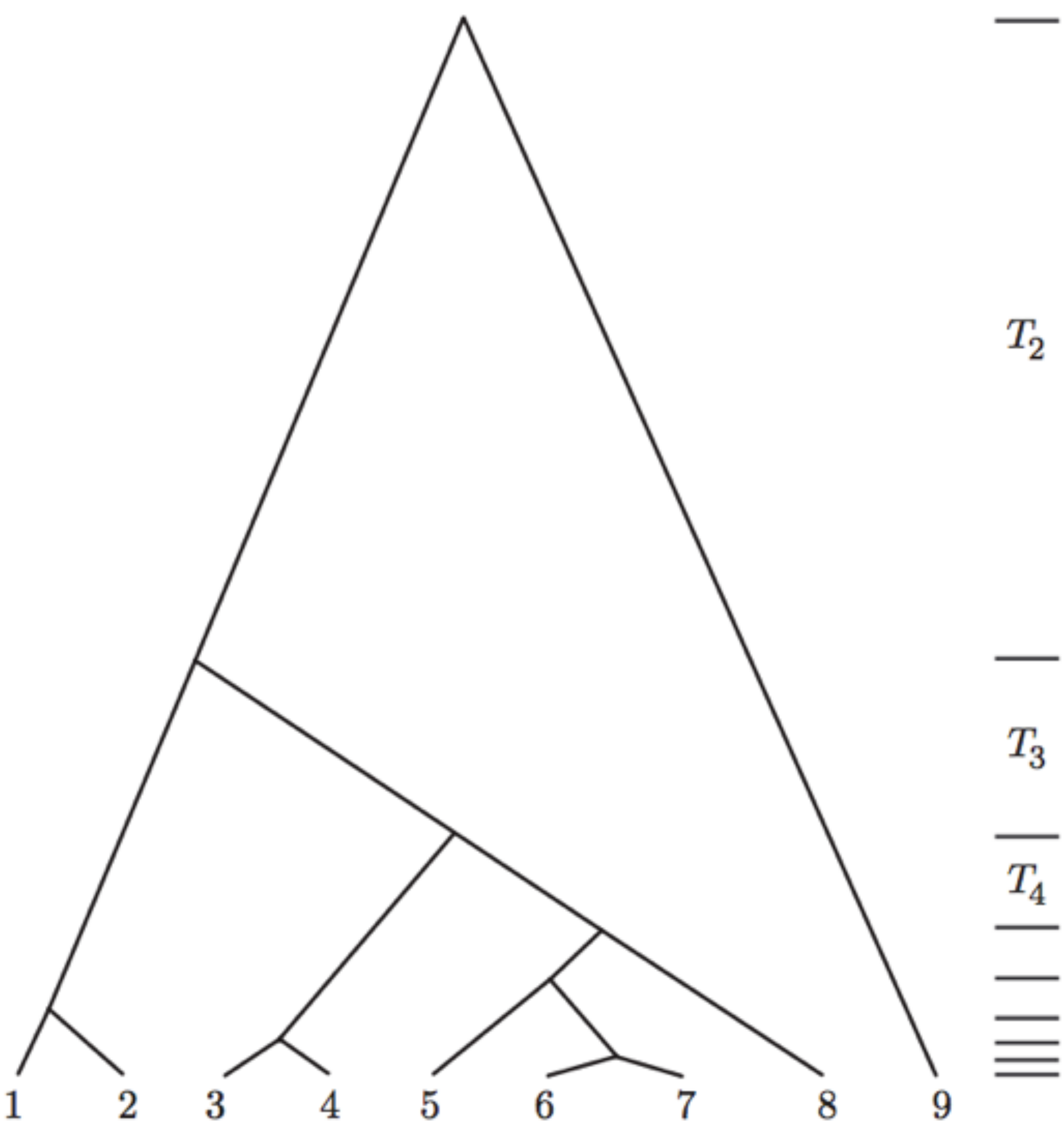

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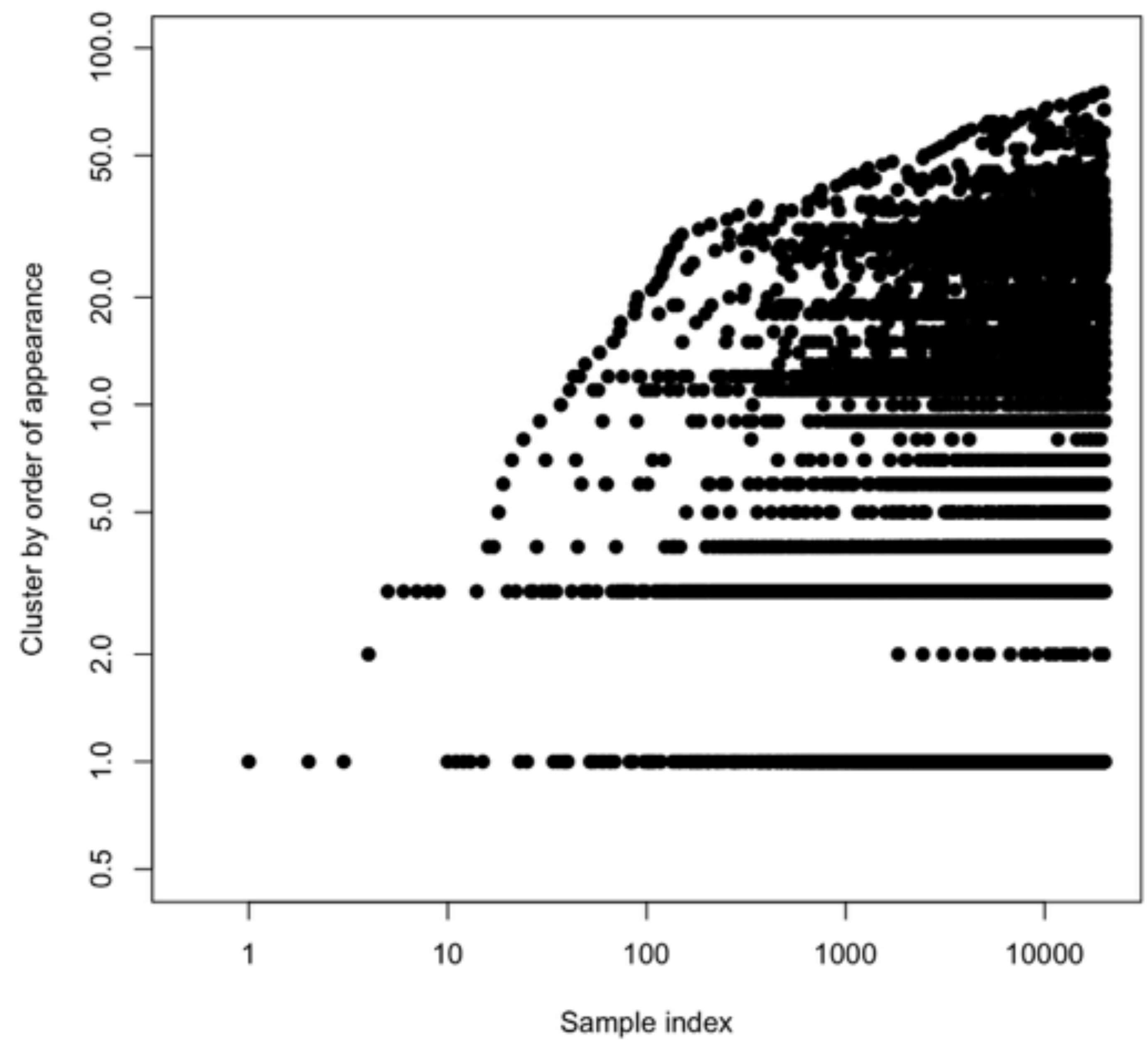
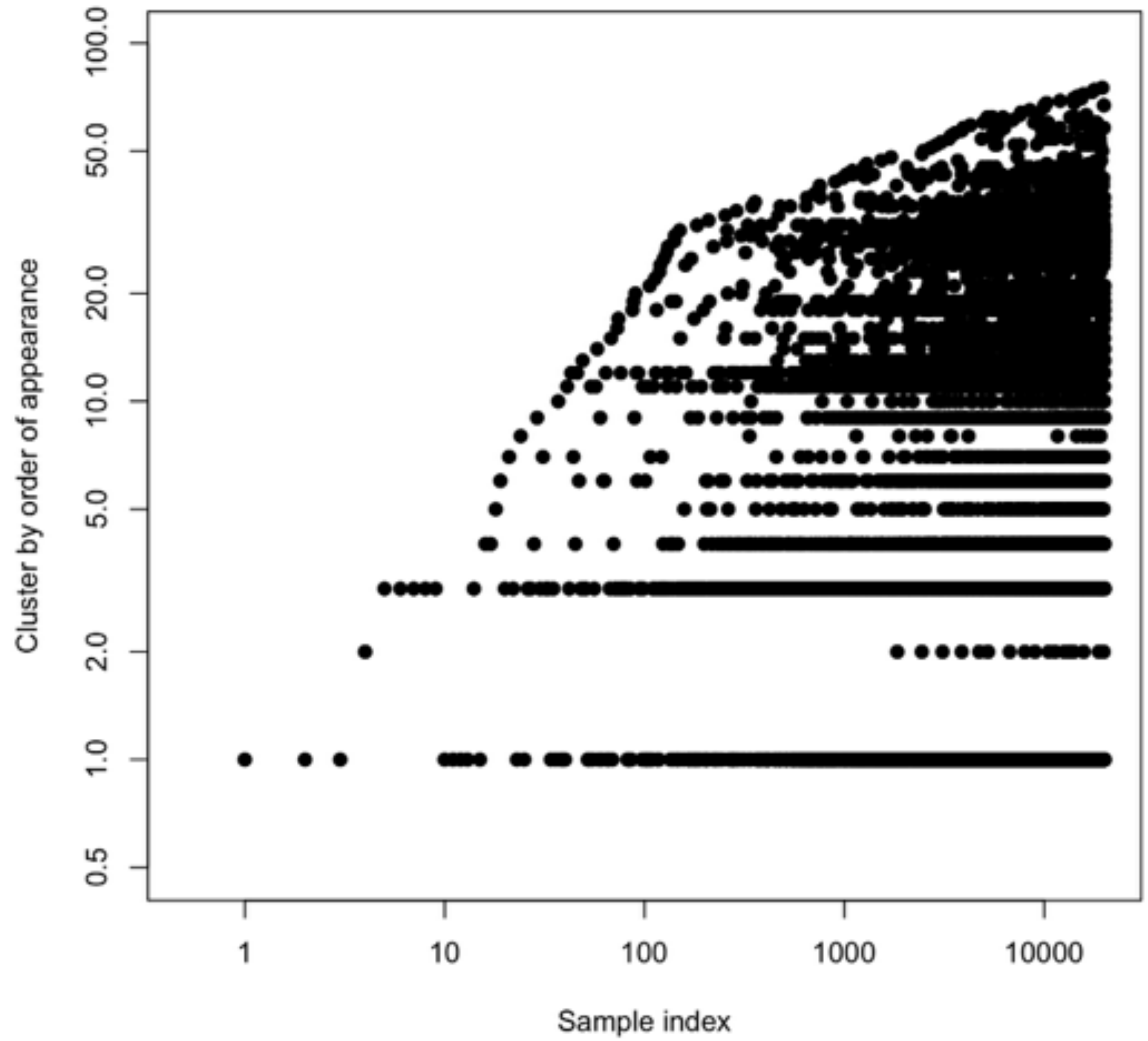
Power laws



Cluster by order of appearance
Sample index
100.0
50.0
20.0
10.0
5.0
2.0
1.0
0.5
1
10
100
1000
10000

Power laws

- K_N := # clusters occupied by N data points



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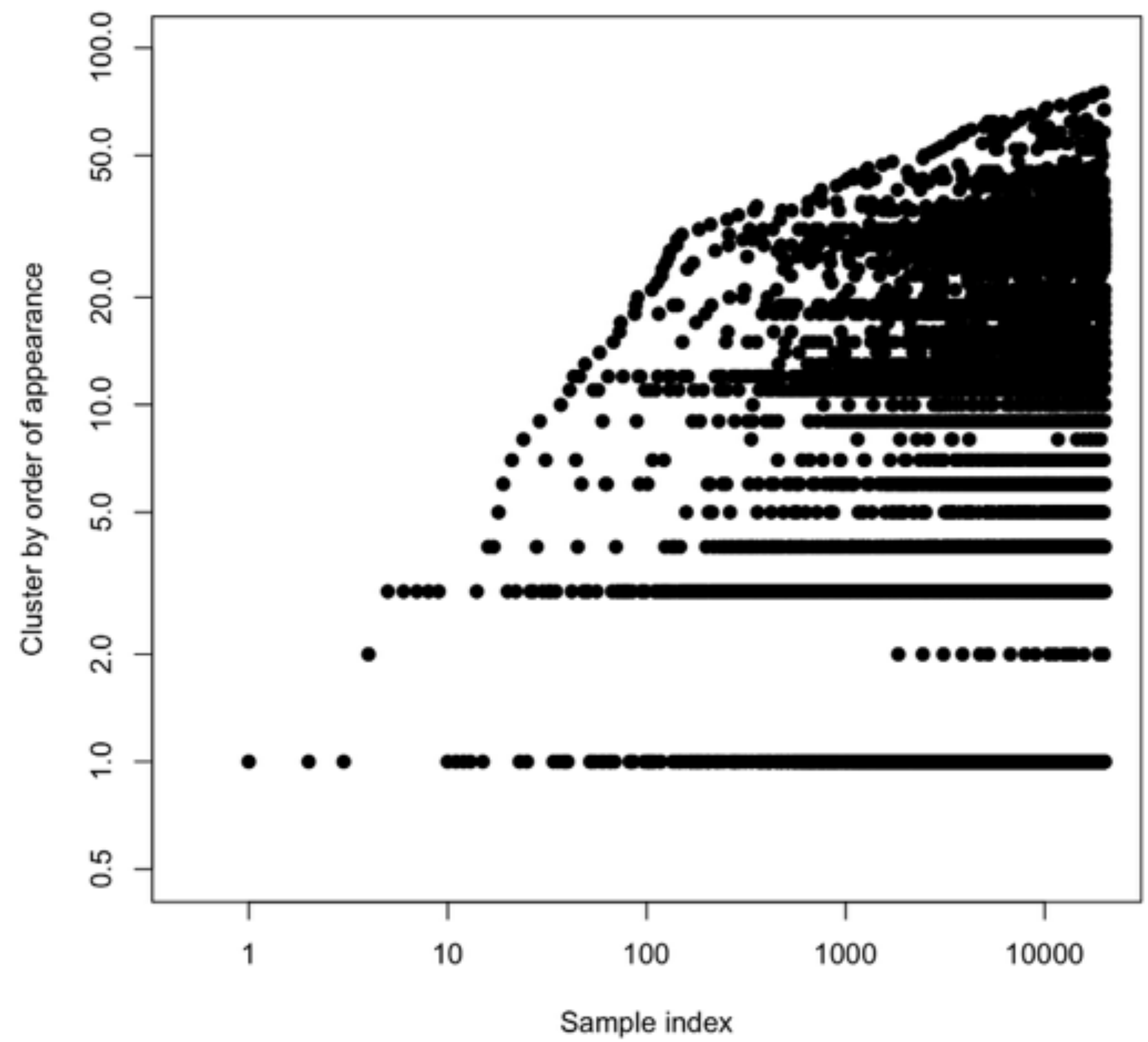

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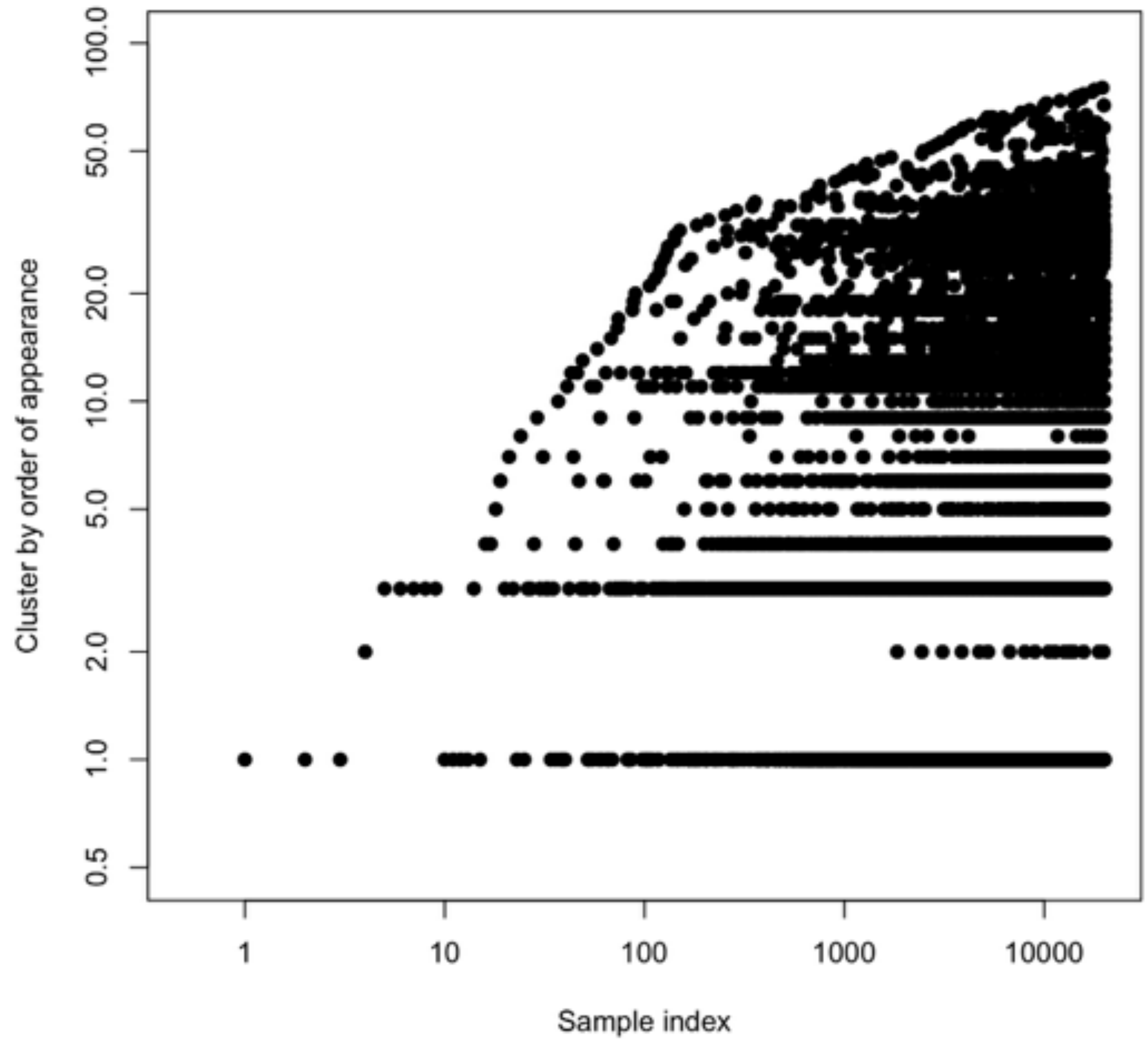

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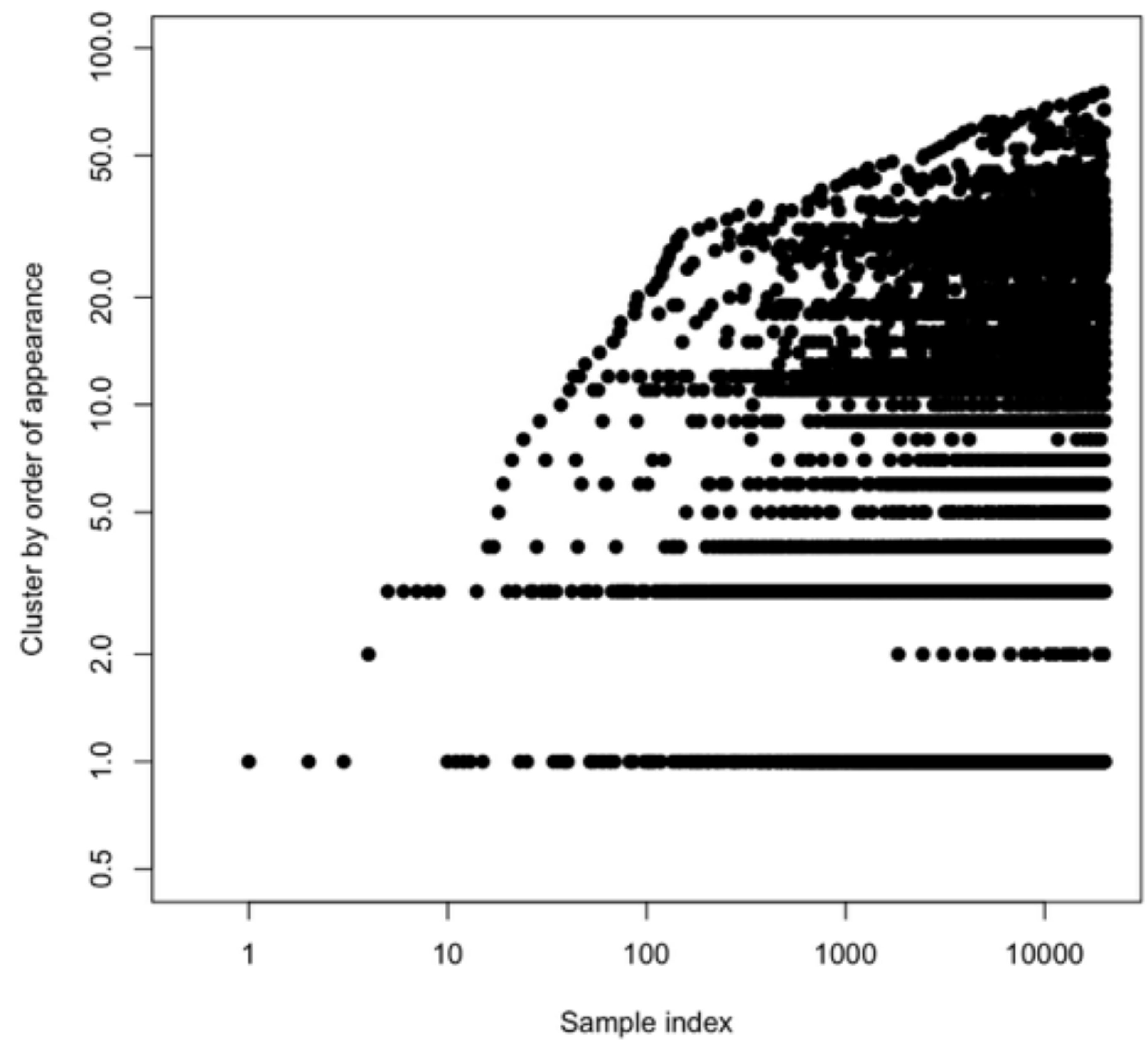

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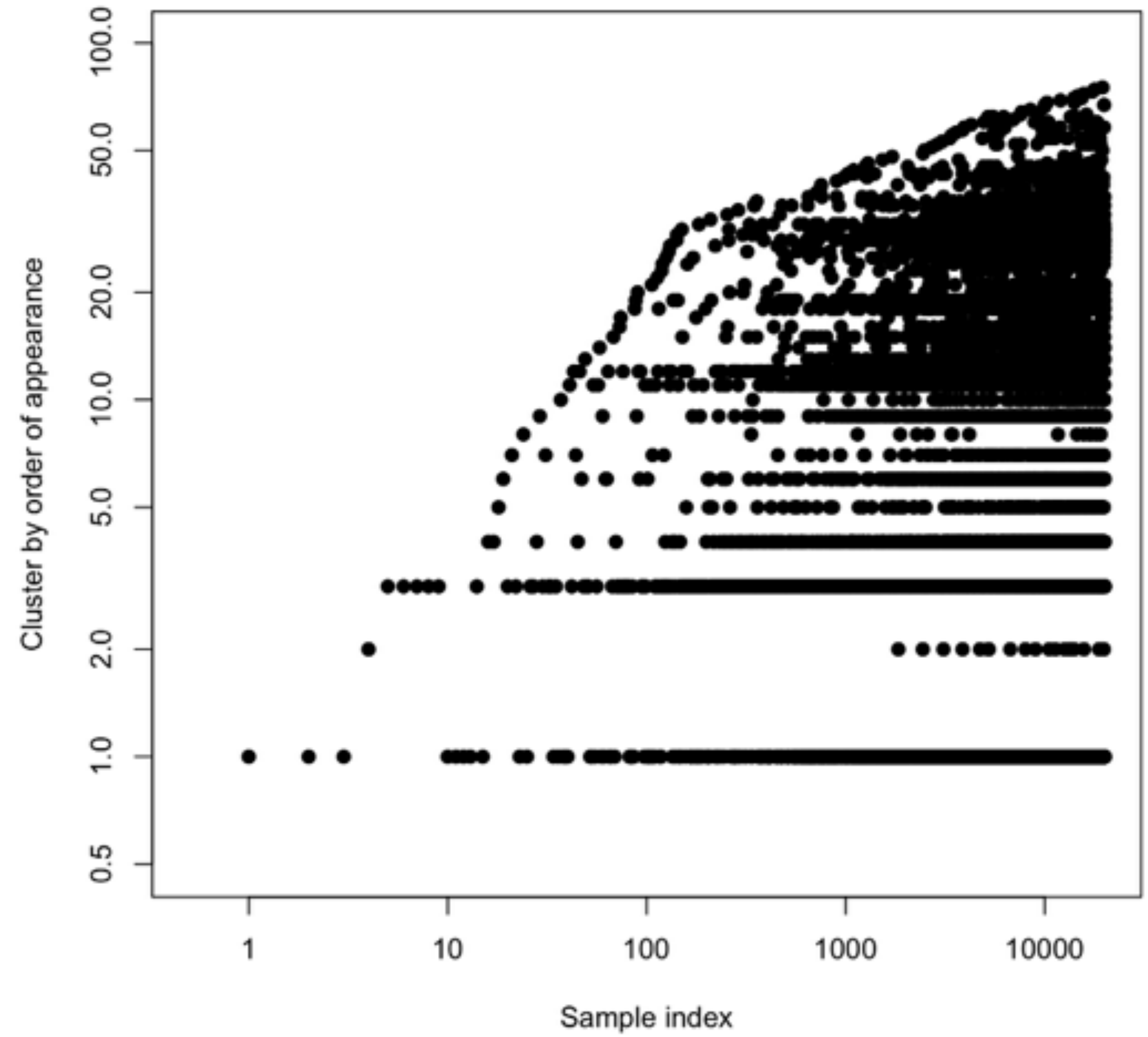

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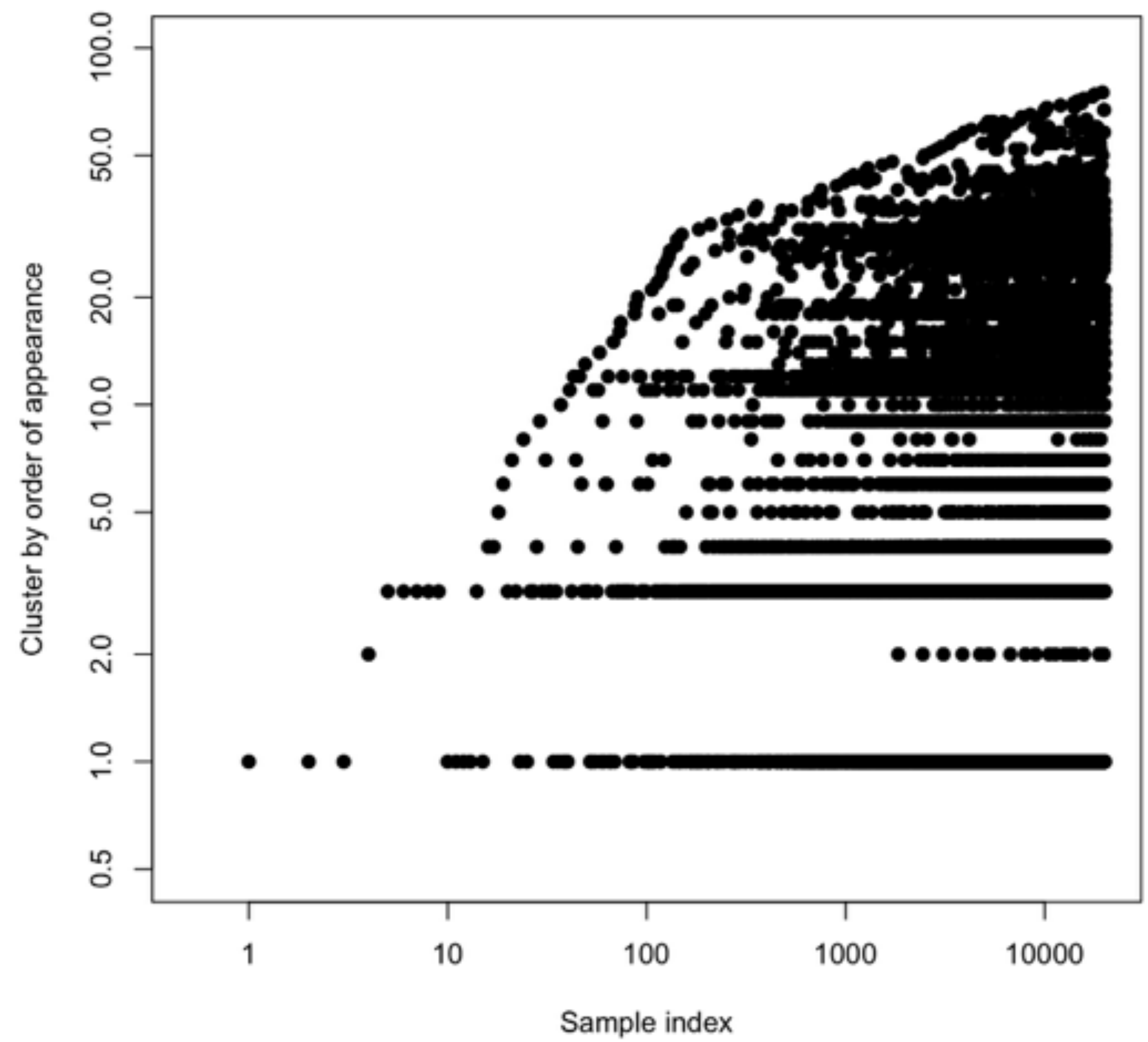

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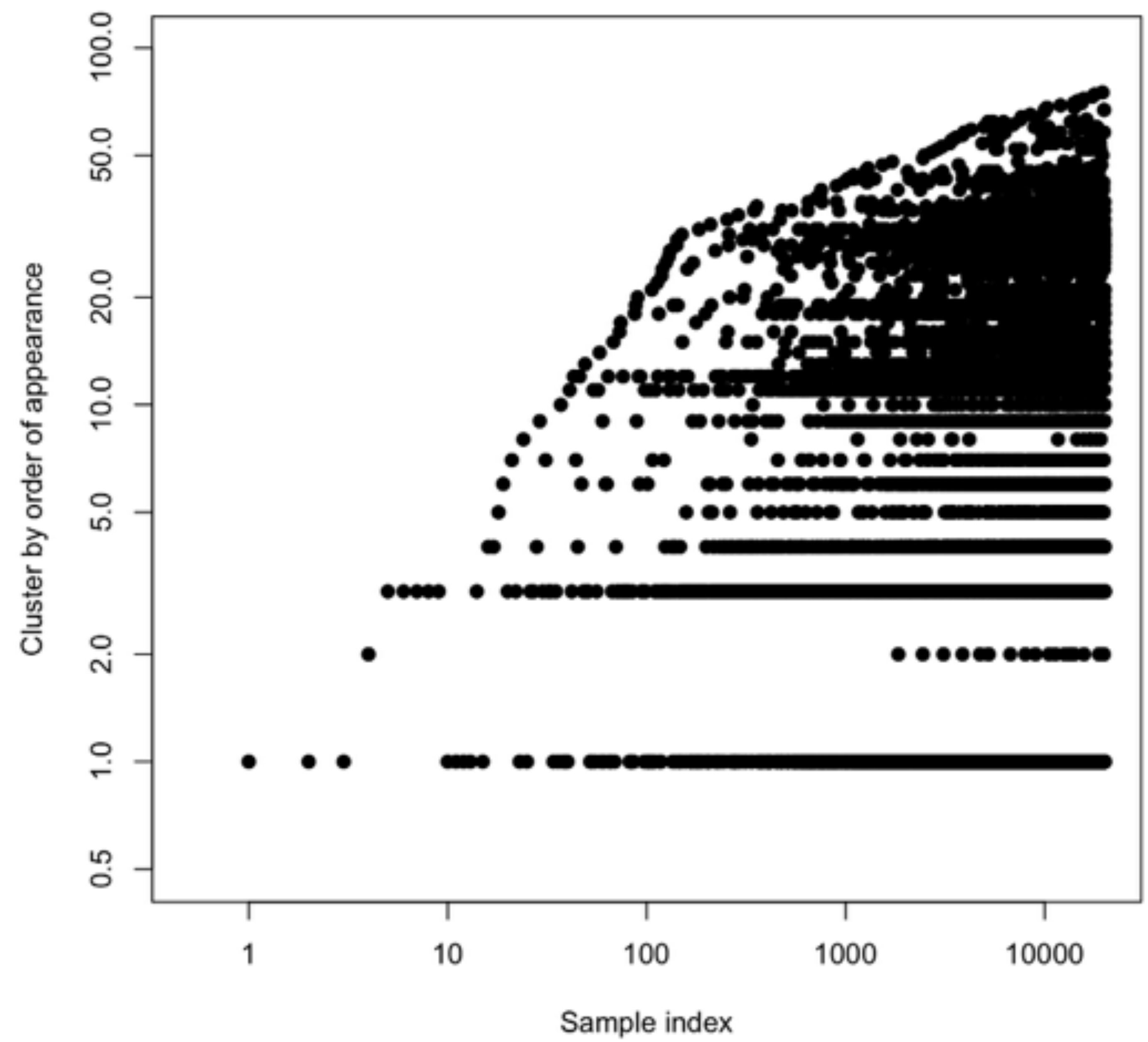

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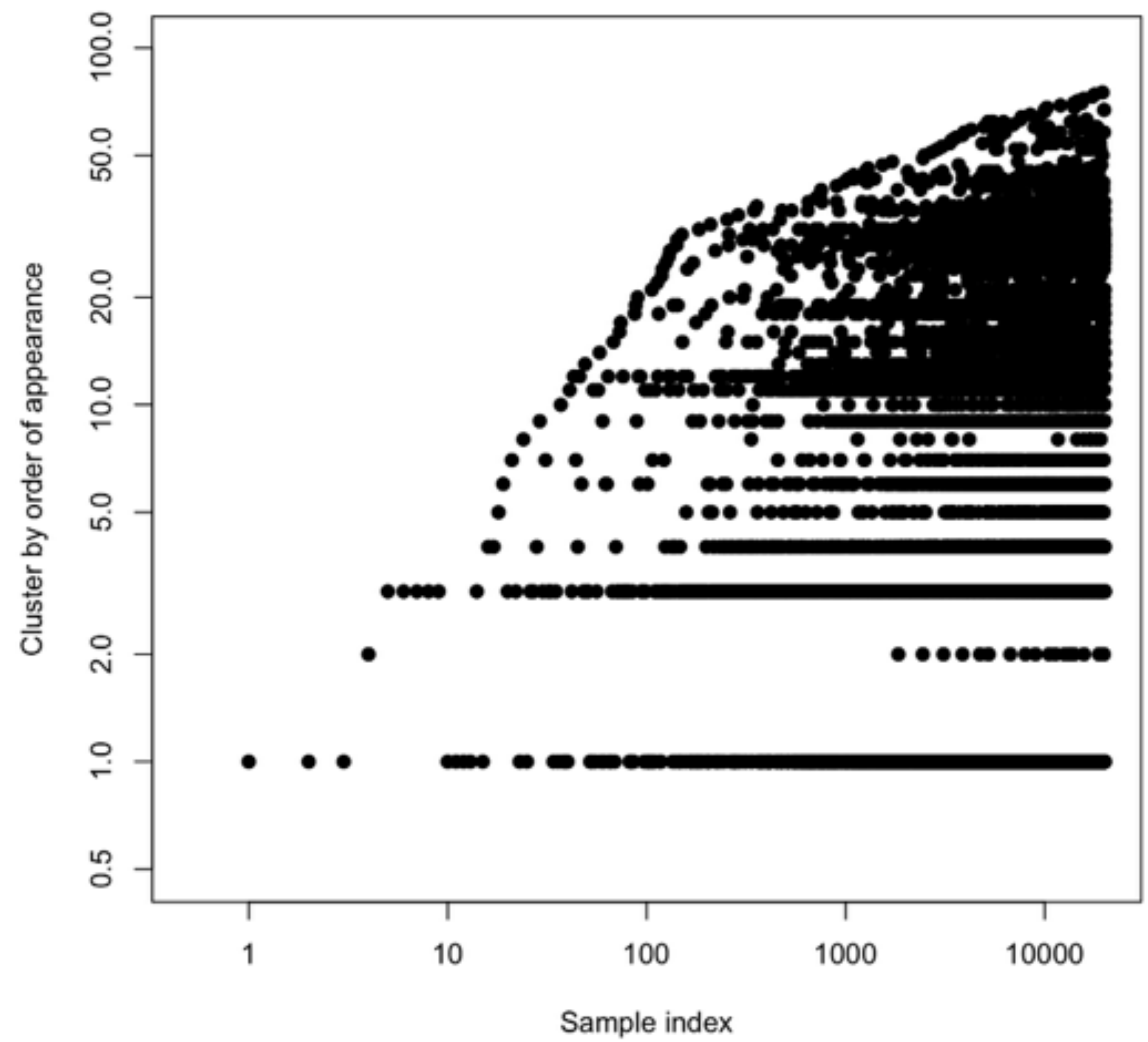

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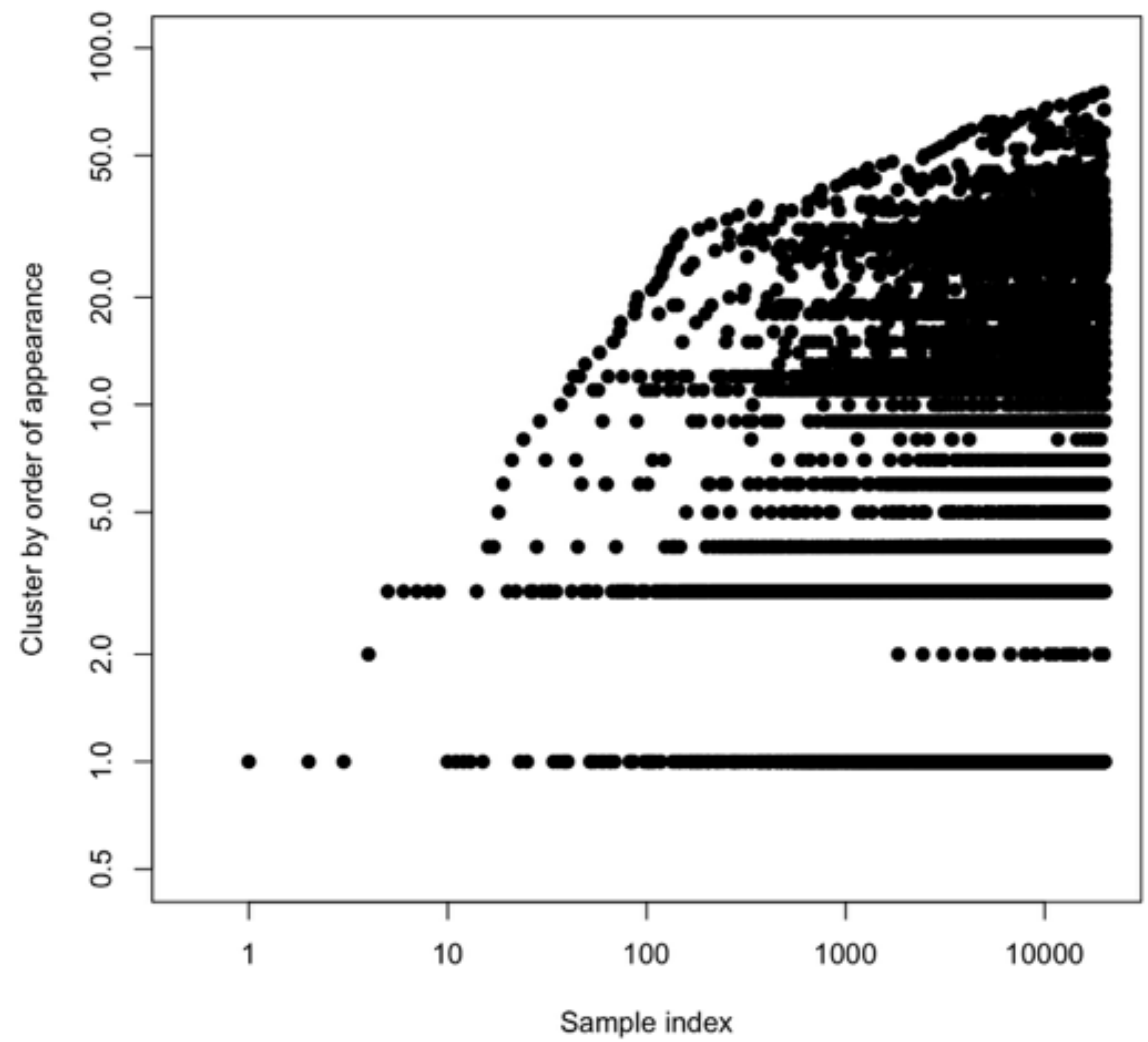
  - related to Zipf's law (ranked frequencies)



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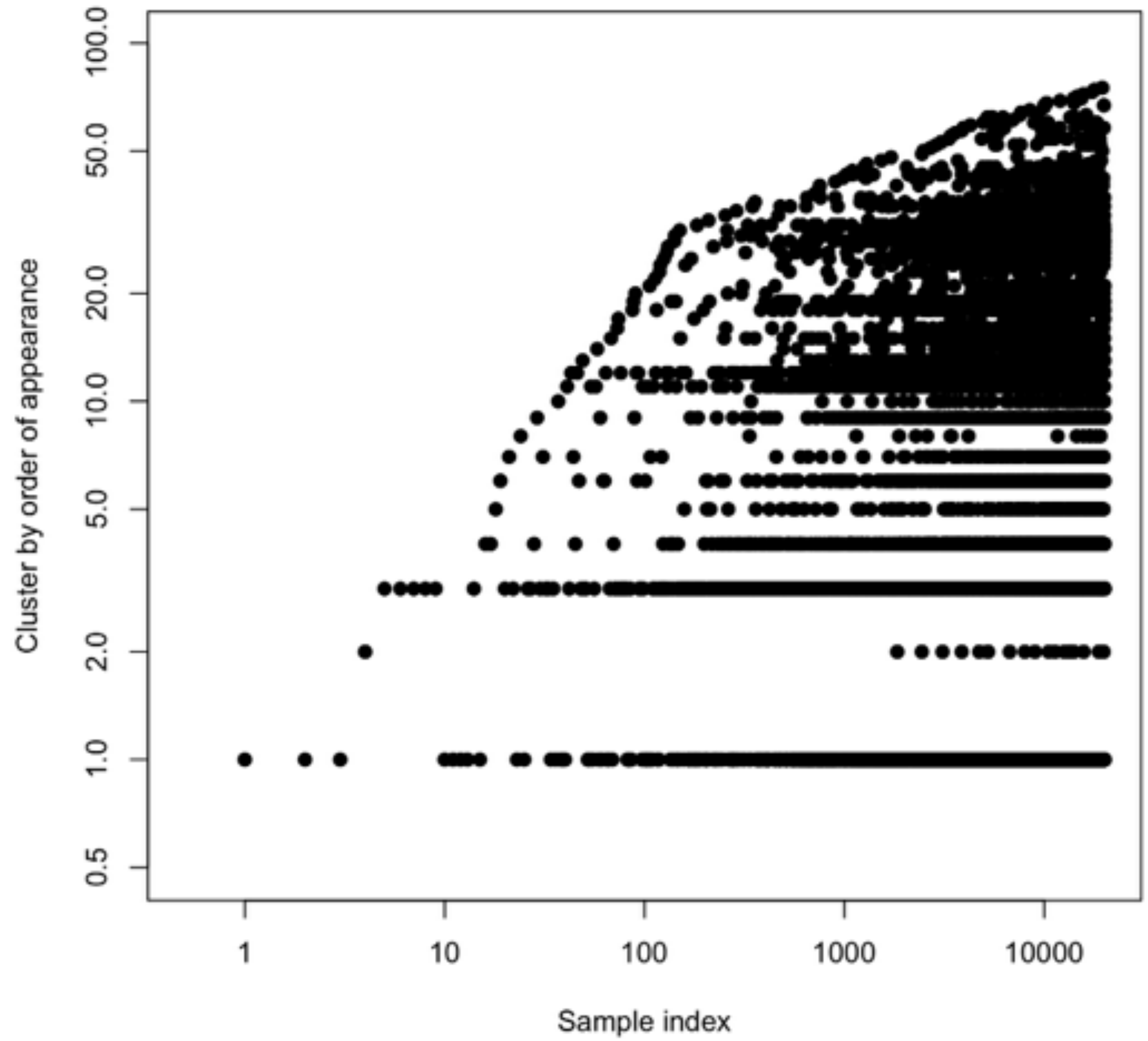

[Gnedin, et al 2007, Pitman, Yor 1997, Goldwater et al 2005, Teh 2006, Broderick et al 2012]

Hierarchies

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[Teh et al 2006, Rodríguez et al 2008

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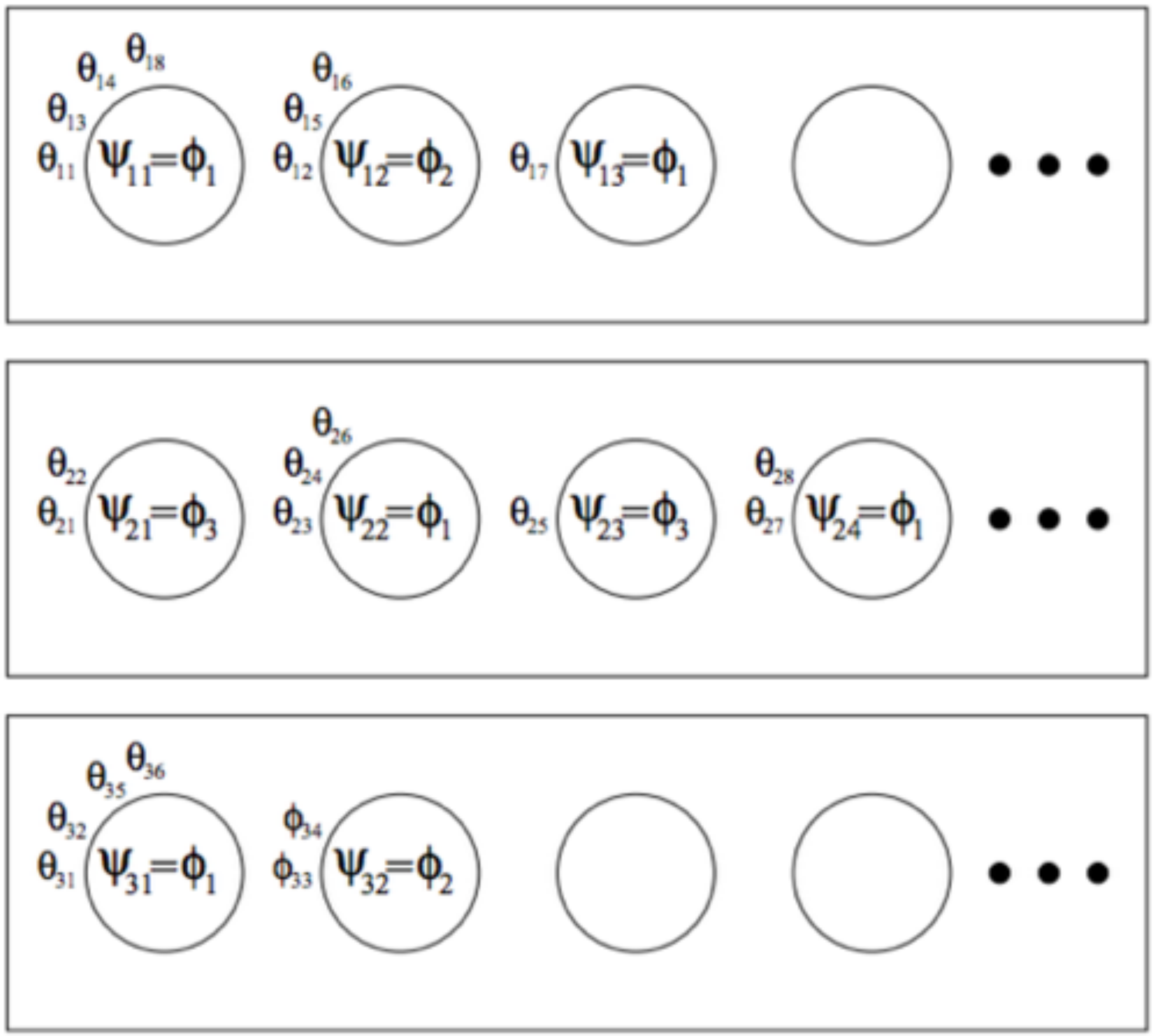

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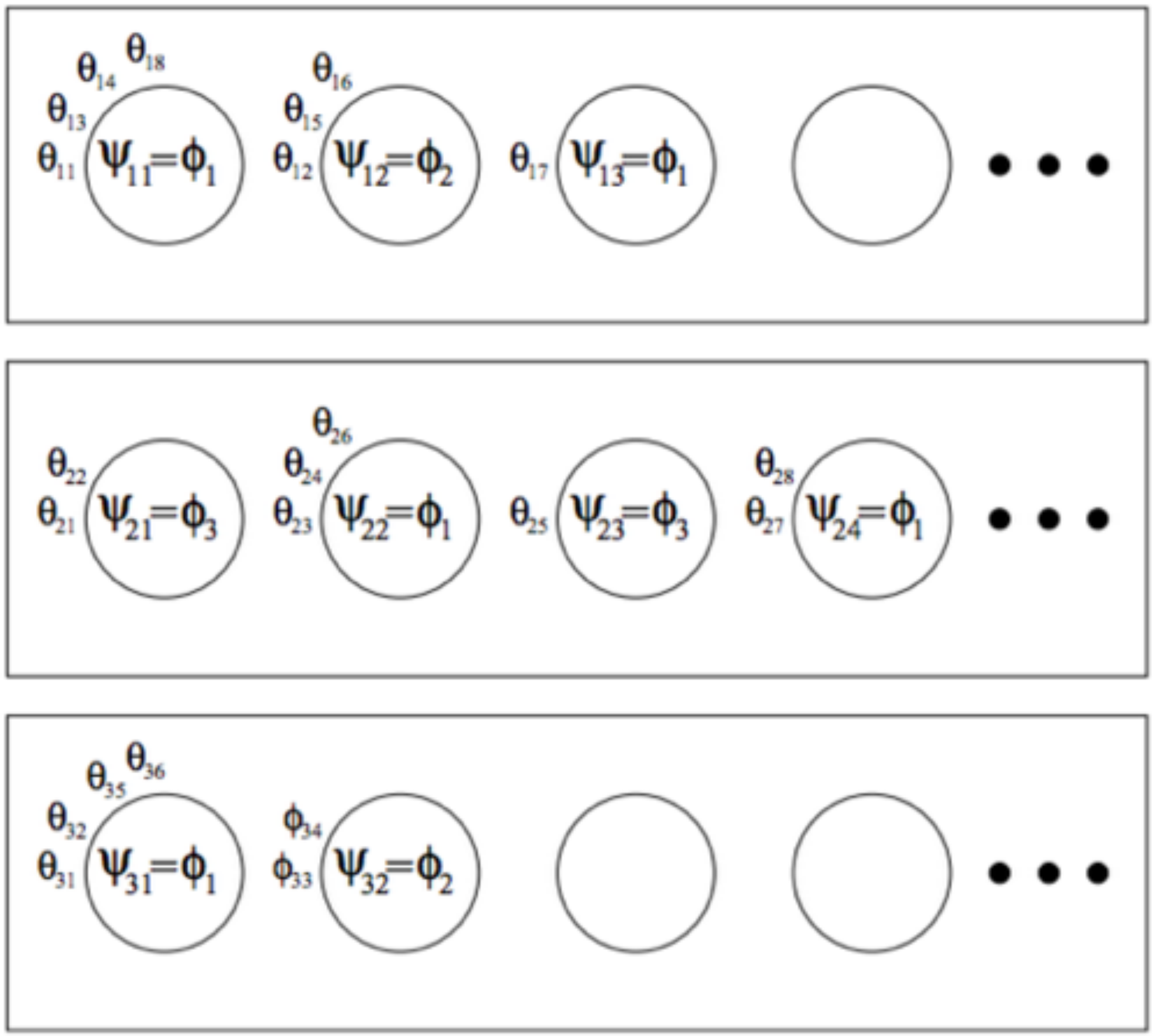

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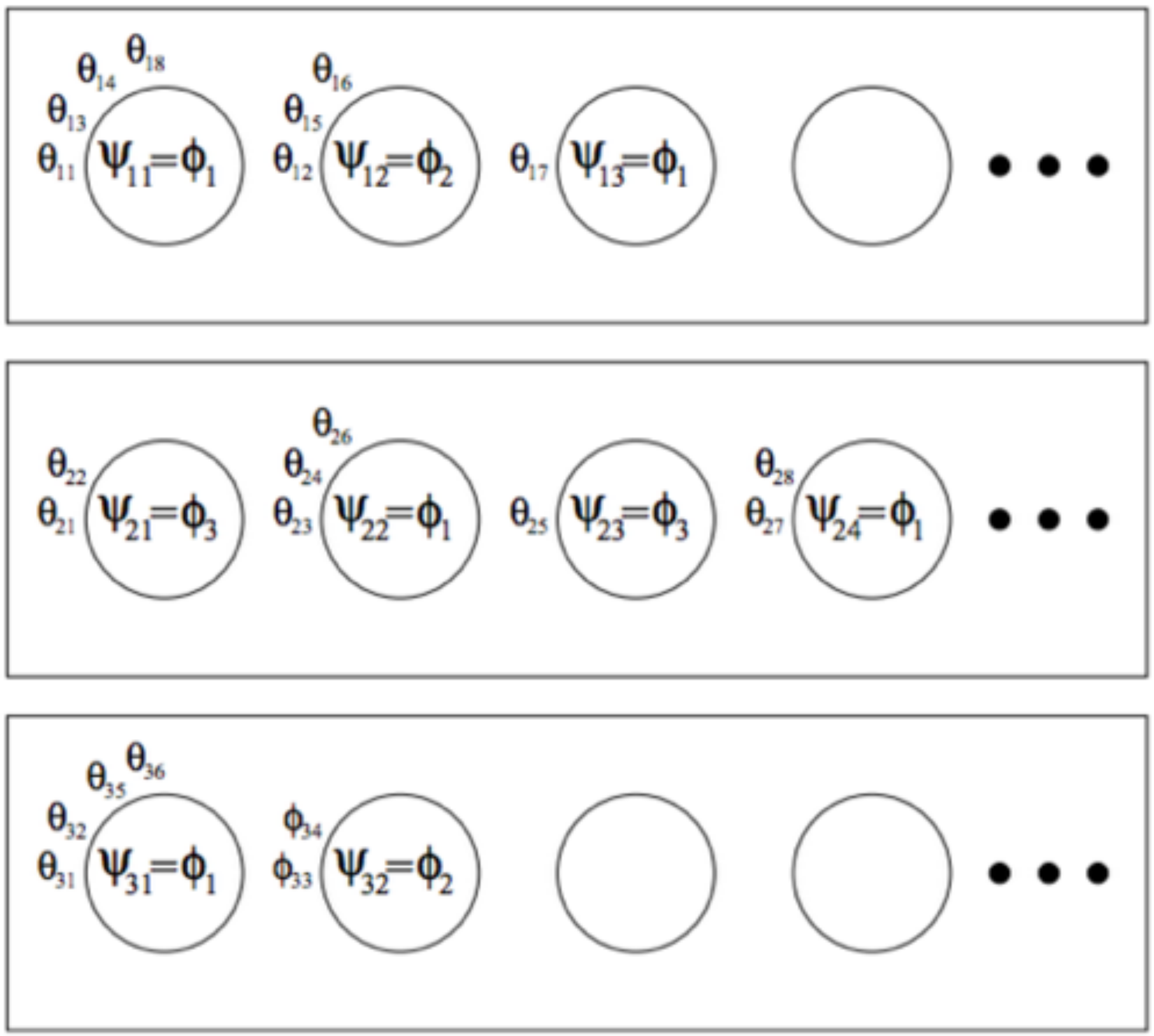

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Nonparametric Bayes

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- Bayesian statistics that is not parametric

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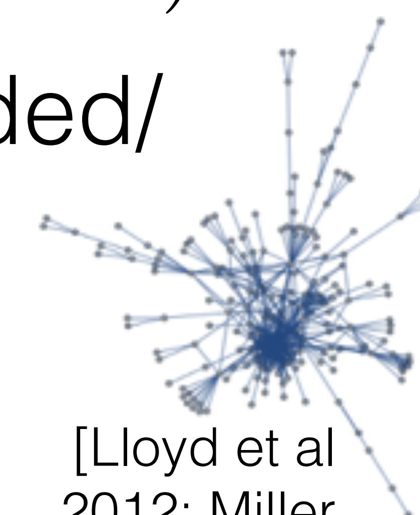
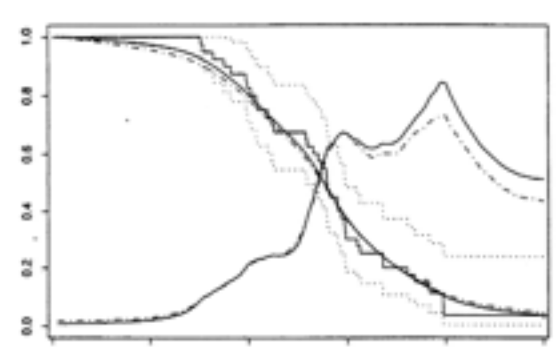


[Lloyd et al 2012; Miller et al, 2010]

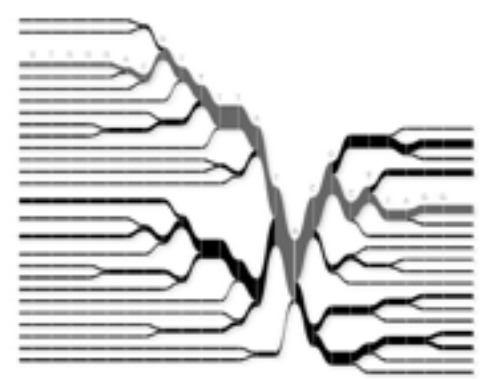
[wikipedia.org]

[Ed Bowlby, NOAA]

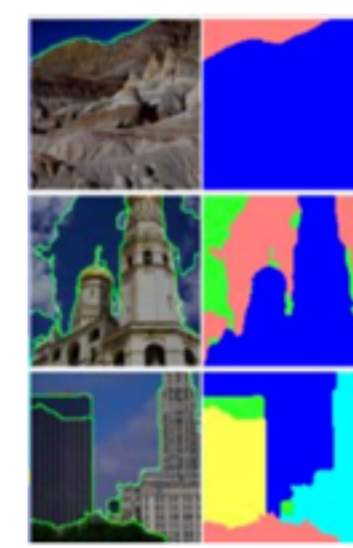
[Fox, et al 2014]



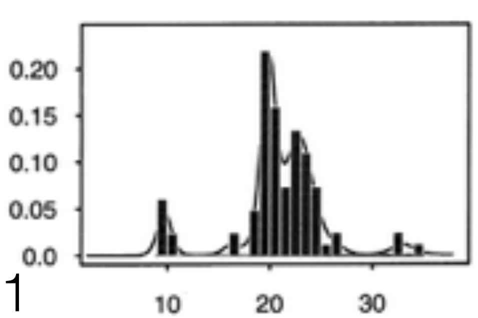
[Arjas, Gasbarra 1994]



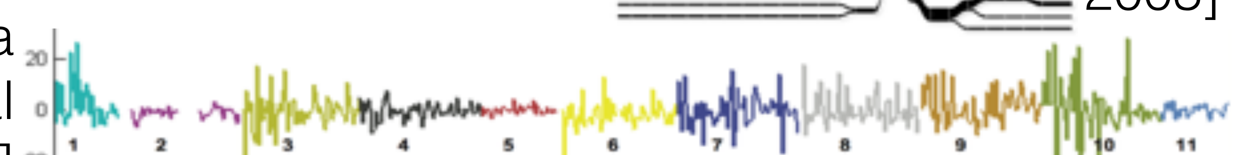
[Ewens, 1972; Hartl, Clark 2003]



[Sudderth, Jordan 2009]



[Escobar, West 1995; Ghosal, et al 1999]



[Saria et al 2010]

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