Nonparametric Bayesian Statistics: Part III

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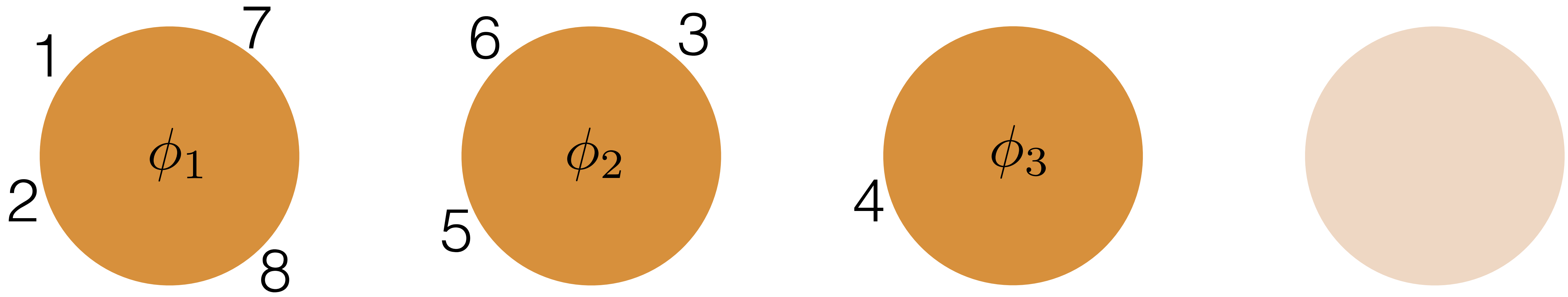
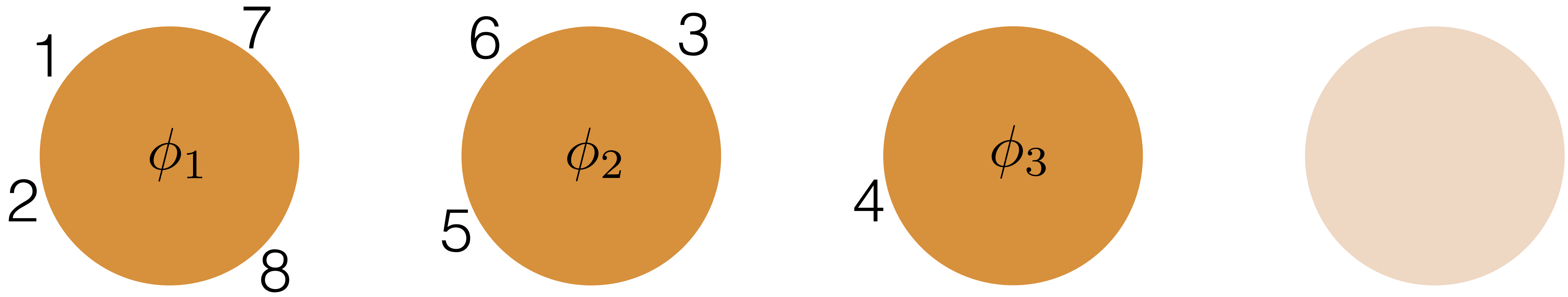
Electrical Engineering & Computer Science
MIT

Chinese restaurant process

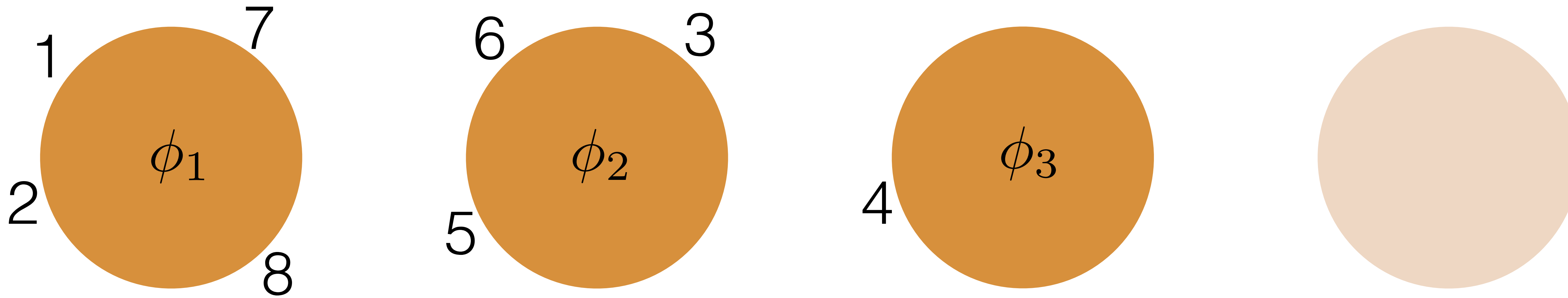


Chinese restaurant process



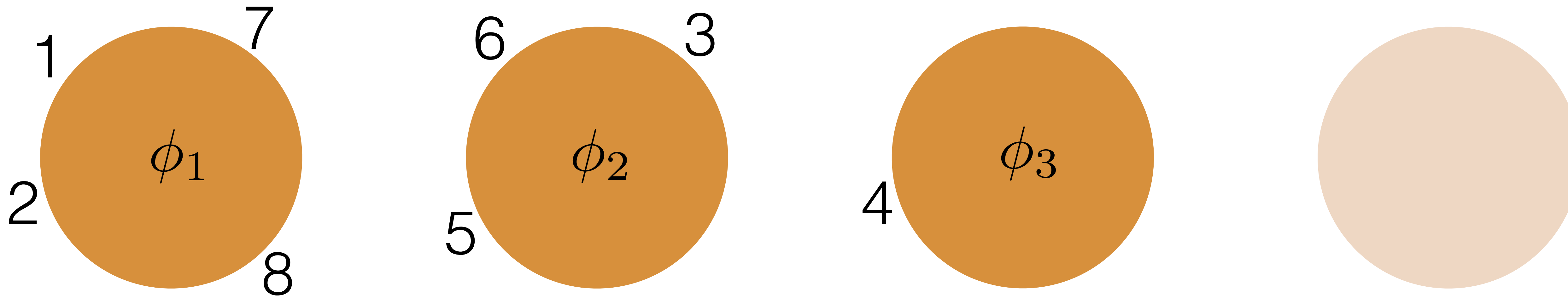
- Each customer walks into the restaurant

Chinese restaurant process



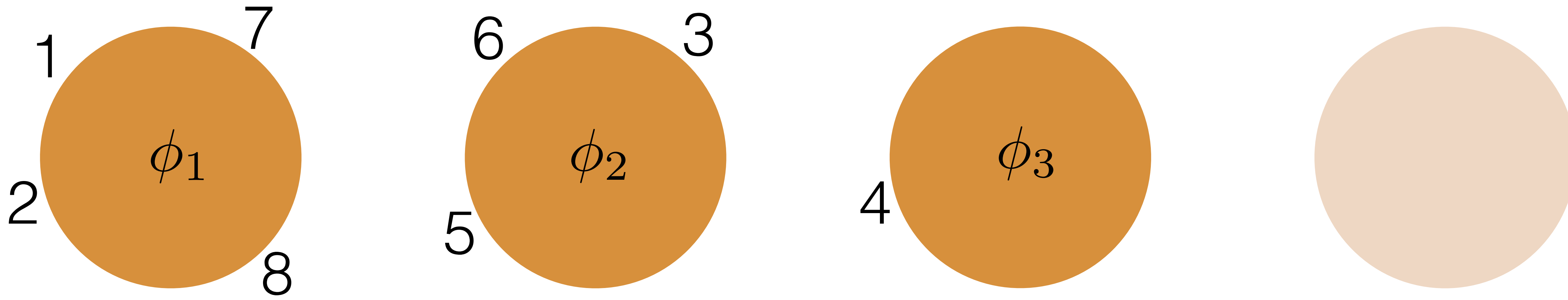
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there

Chinese restaurant process



- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



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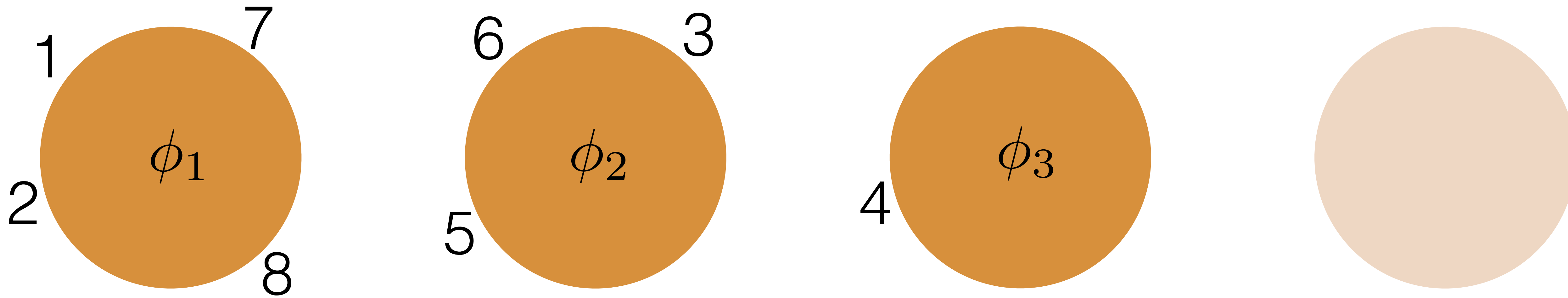
- Marginal for the Categorical likelihood with GEM prior

Chinese restaurant process



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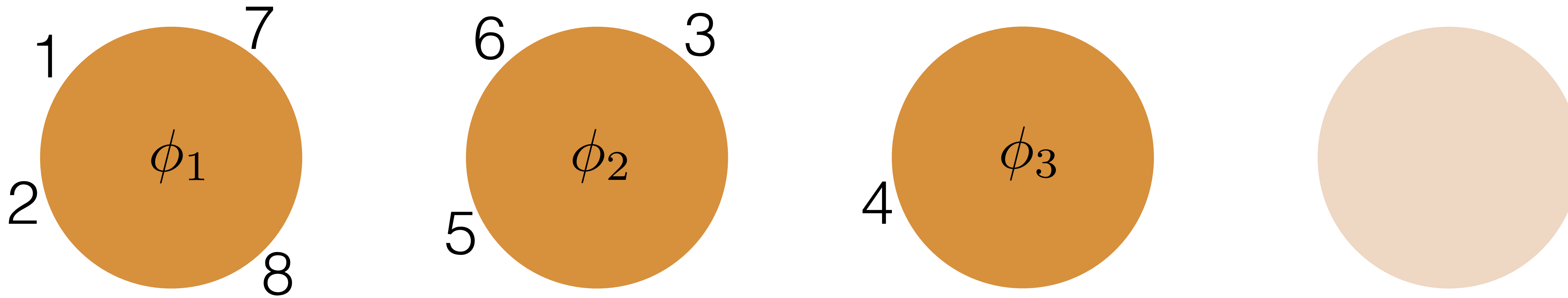
$$z_1 = z_2 = z_7 = z_8 = 1, z_3 = z_5 = z_6 = 2, z_4 = 3$$

Chinese restaurant process



- Same thing we just did
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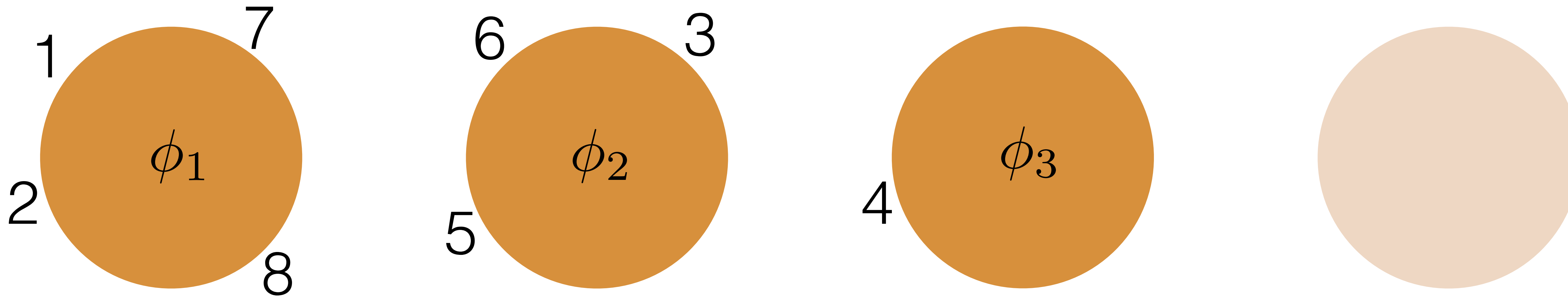
$$z_1 = z_2 = z_7 = z_8 = 1, z_3 = z_5 = z_6 = 2, z_4 = 3$$
$$\Rightarrow \Pi_8 = \{\{1, 2, 7, 8\}, \{3, 5, 6\}, \{4\}\}$$

Chinese restaurant process



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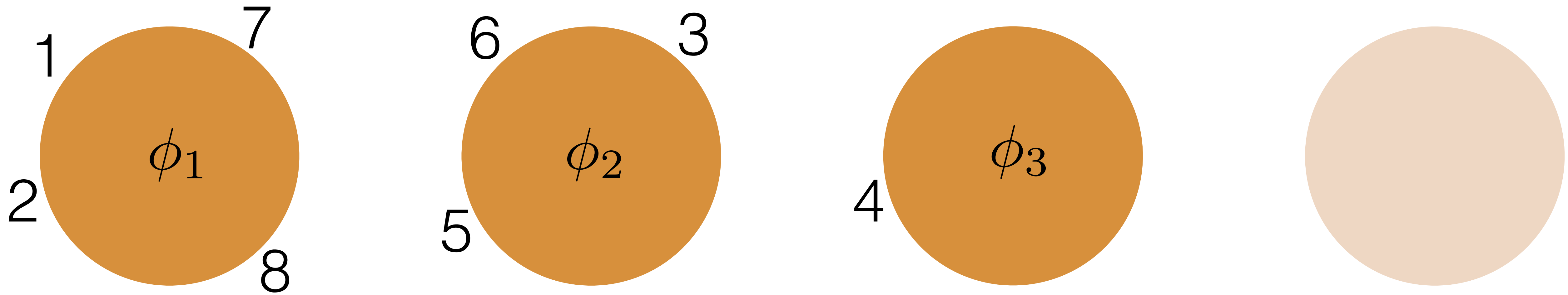
$$\Rightarrow \Pi_8 = \{\{1,2,7,8\},\{3,5,6\},\{4\}\}$$

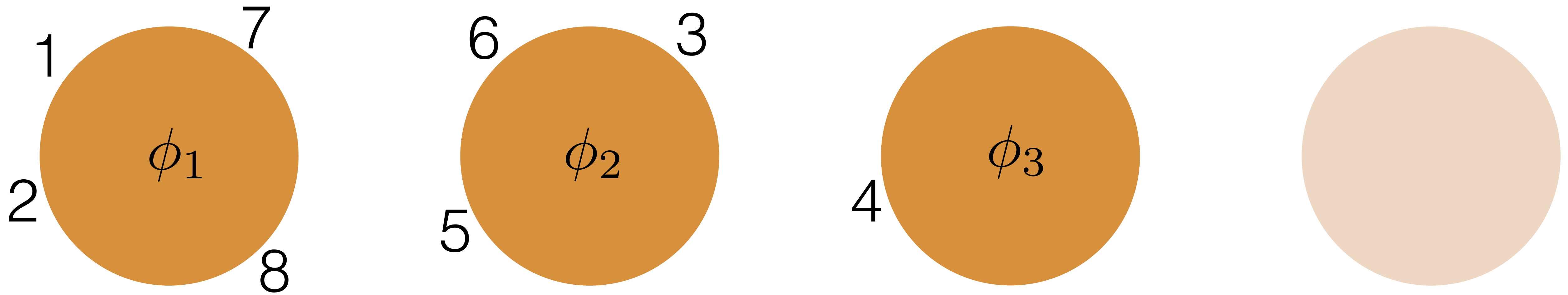
- *Partition of [8]*: set of mutually exclusive & exhaustive sets of $[8] := \{1, \ldots, 8\}$

Chinese restaurant process



- Probability of this seating:

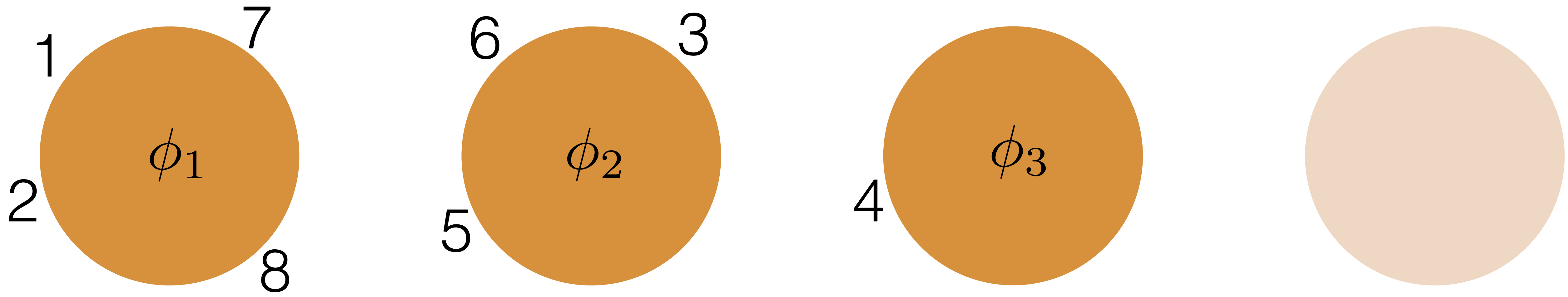
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha}$$

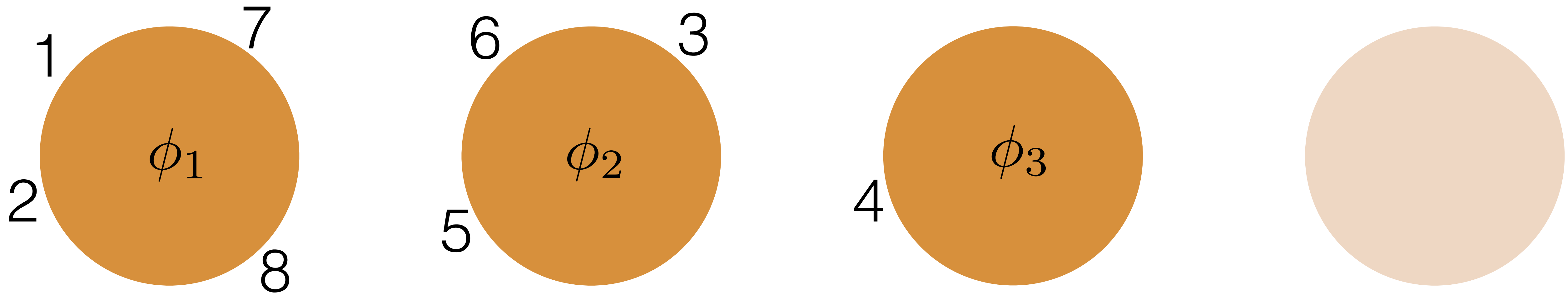
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha + 1}$$

Chinese restaurant process



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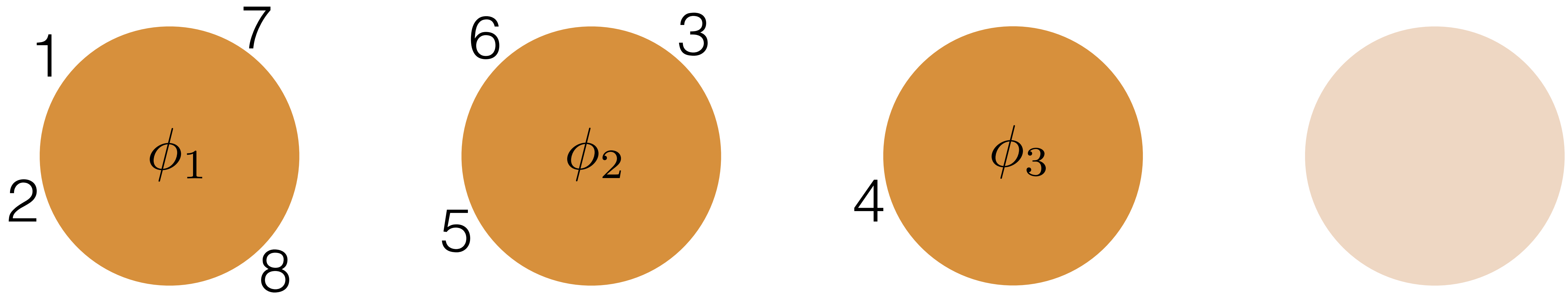
Chinese restaurant process



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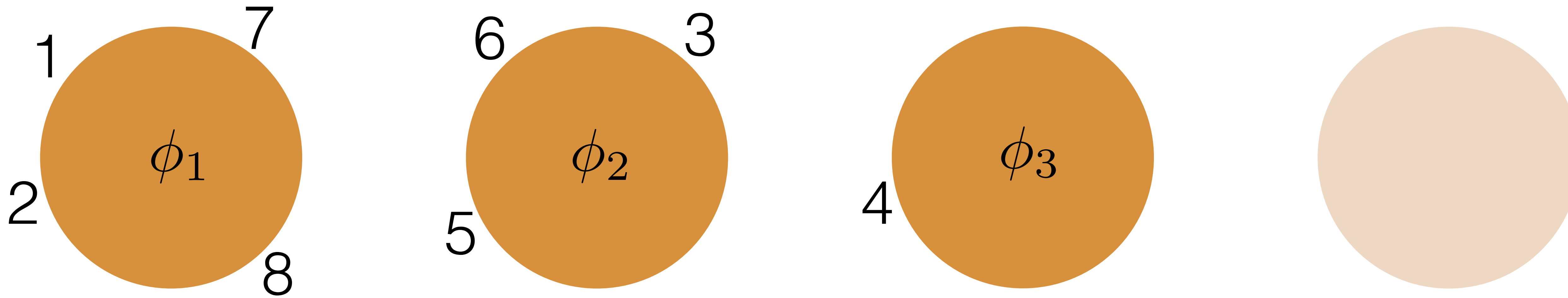
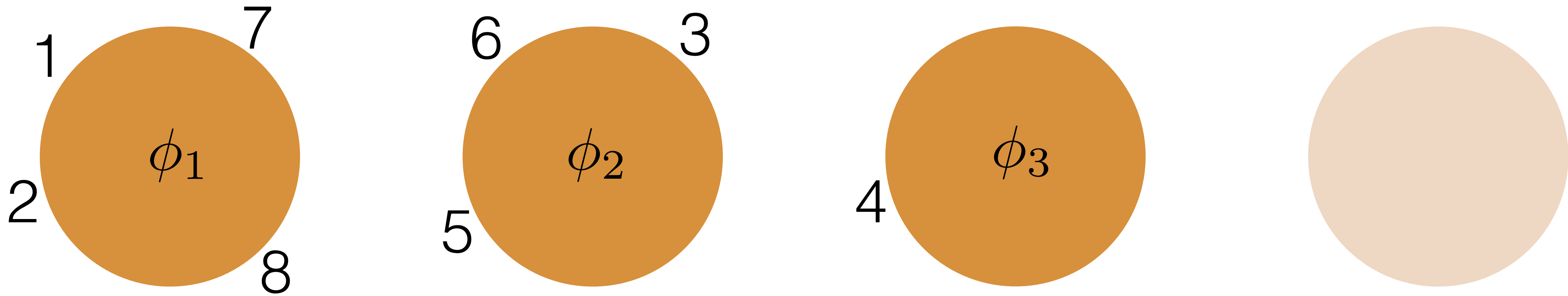
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4}$$

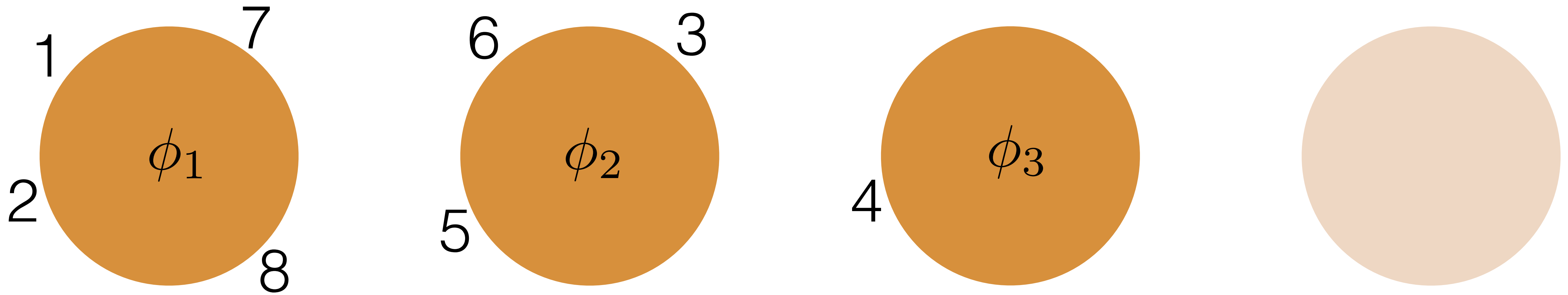
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5}$$

Chinese restaurant process



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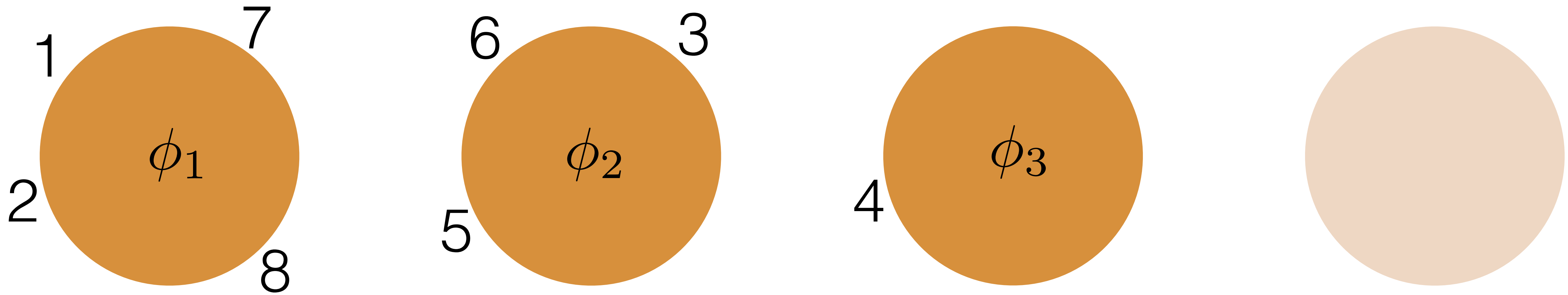

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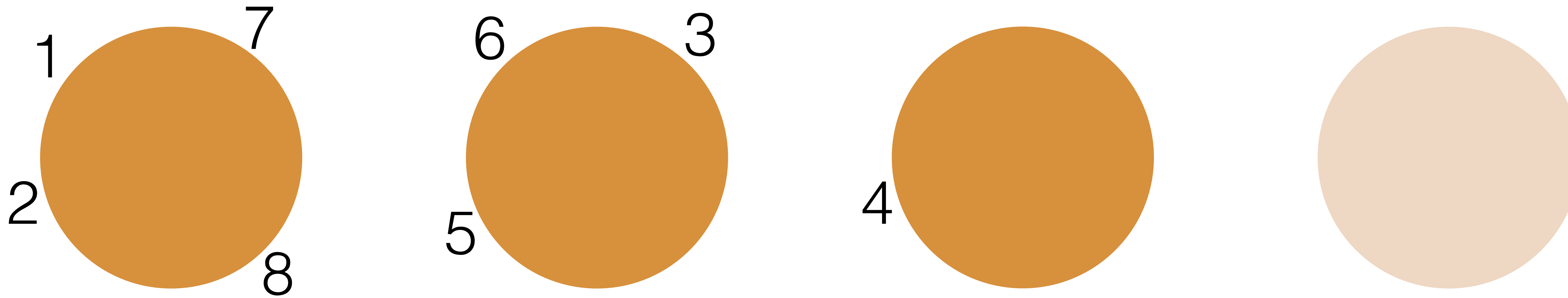
Chinese restaurant process



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- Probability of *N* customers (K_N tables, n_k at table *k*):

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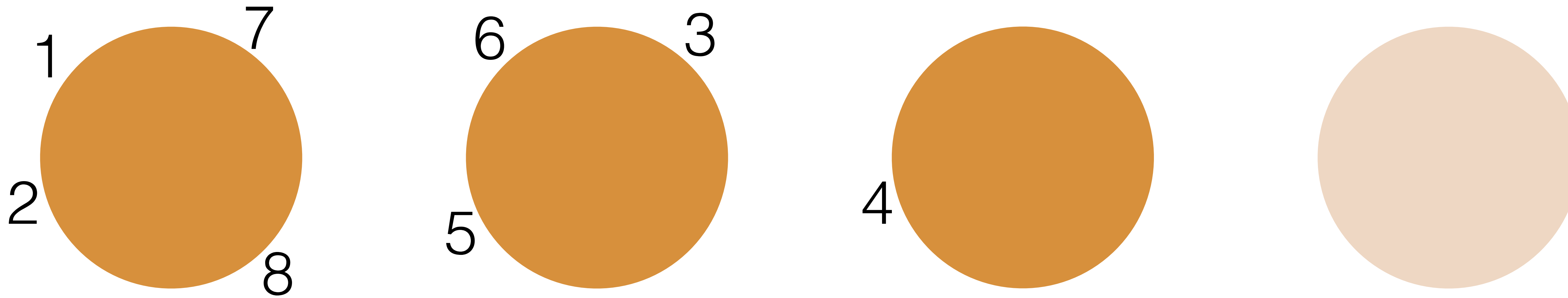

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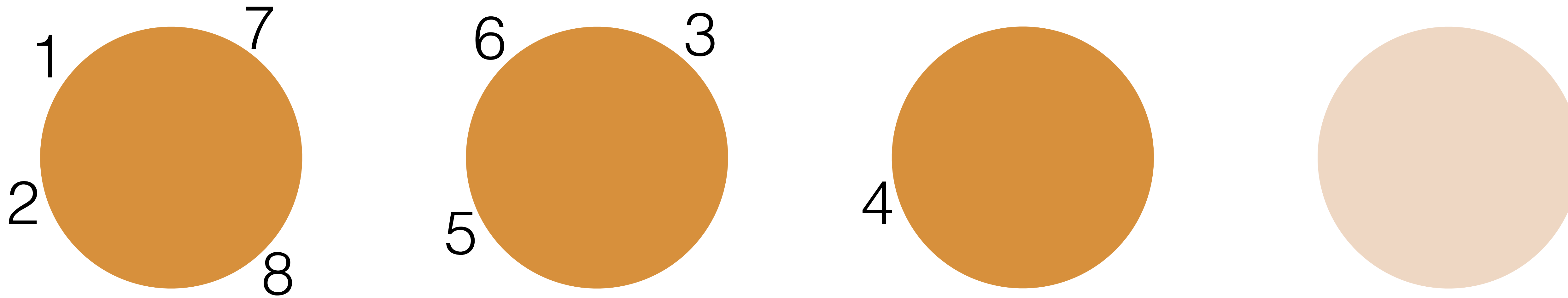

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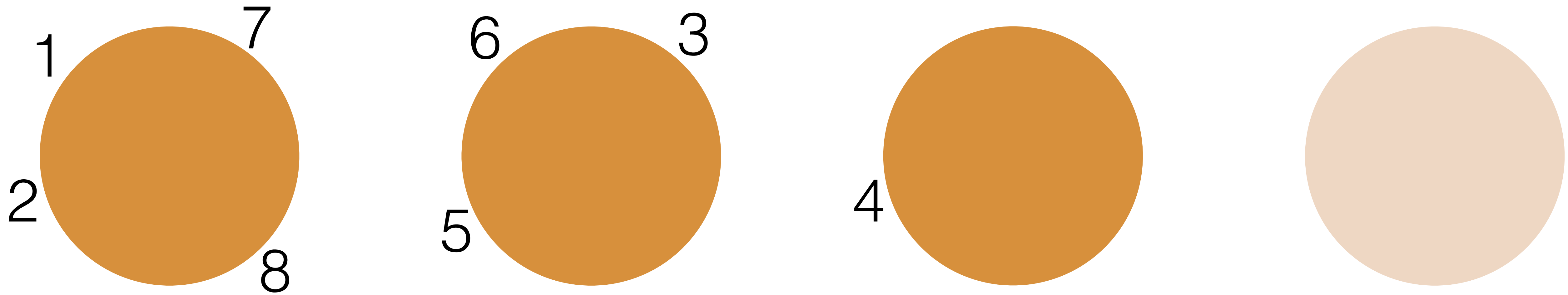
$$\frac{}{\alpha \cdots (\alpha + N - 1)}$$

Chinese restaurant process



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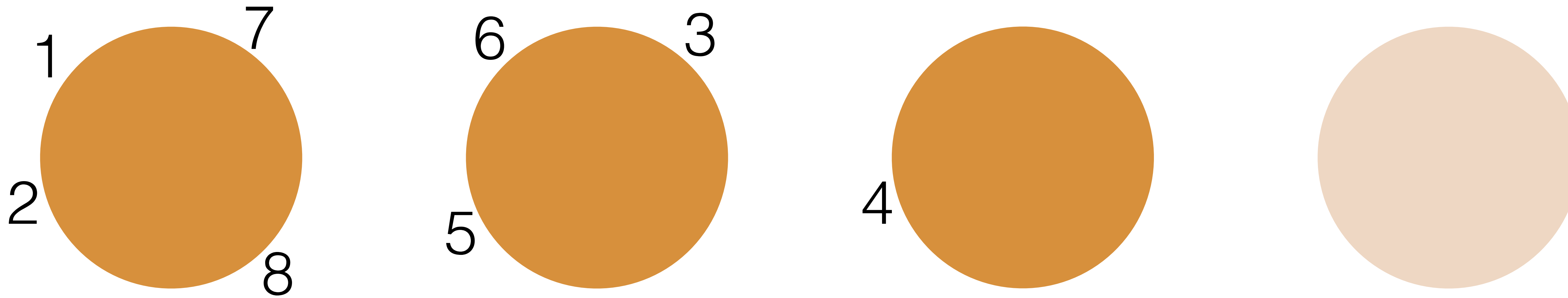
- Probability of N customers (K_N tables, n_k at table k):

$$\frac{\alpha^{K_N}}{\alpha \cdots (\alpha+N-1)}$$

Chinese restaurant process



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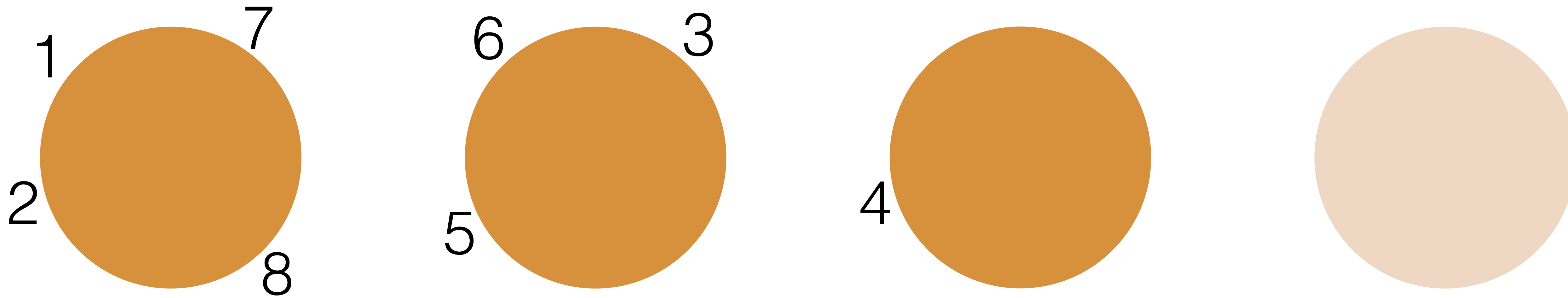

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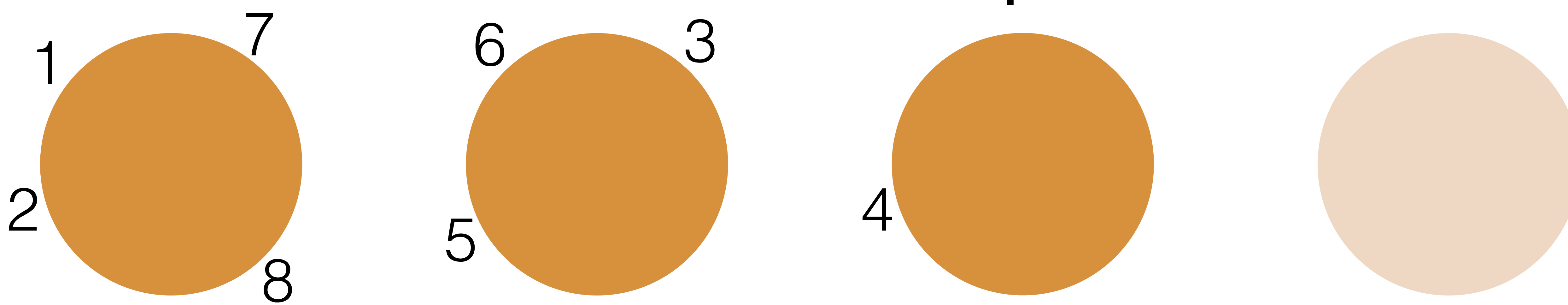

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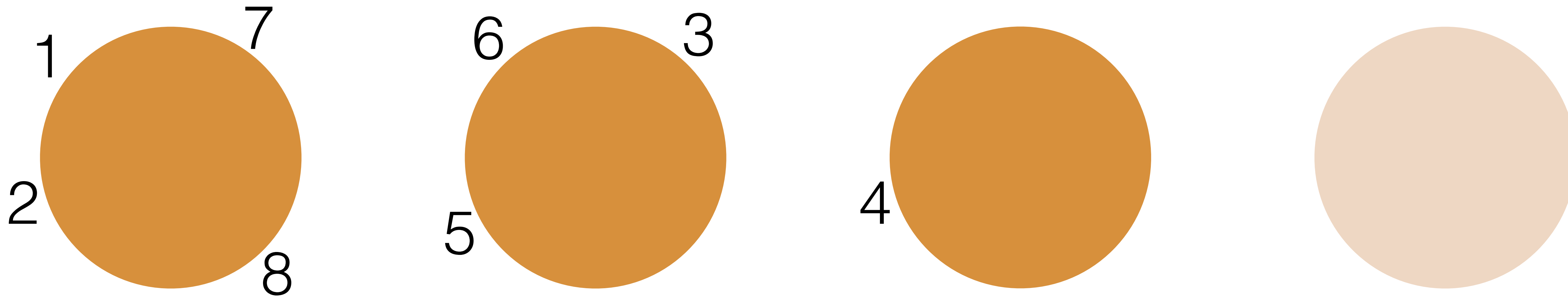

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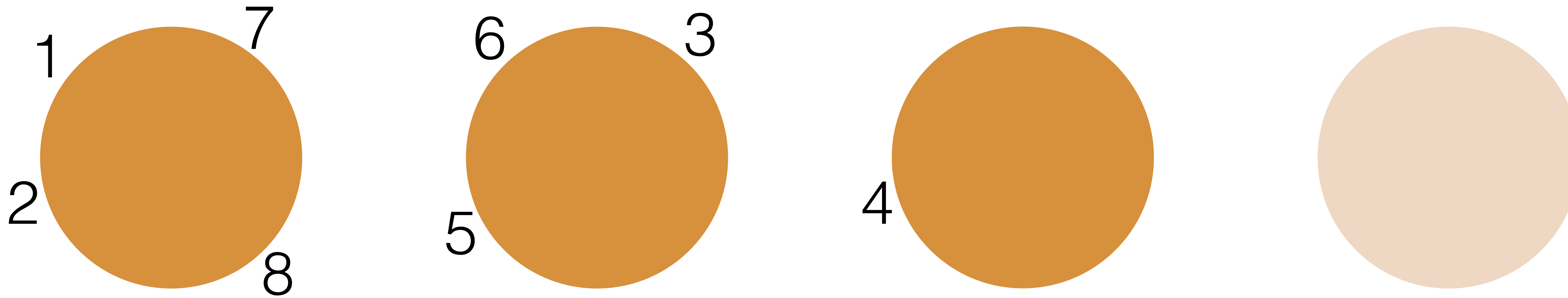

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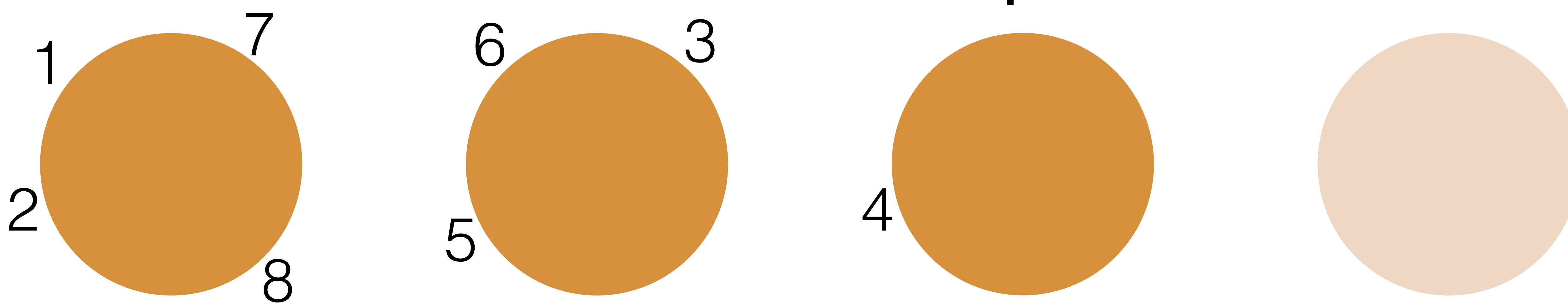
- Probability of *N* customers (K_N tables, n_k at table *k*):

$$\frac{\alpha^{K_N} \prod_{k=1}^{K_N} (n_k - 1)!}{\alpha \cdots (\alpha + N - 1)}$$

Chinese restaurant process



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- Probability of N customers (K_N tables, $\#C$ at table C):

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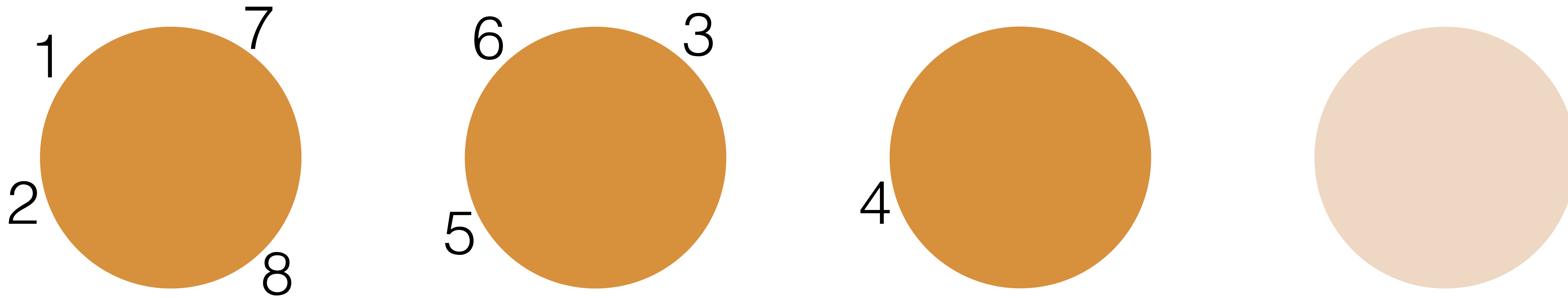

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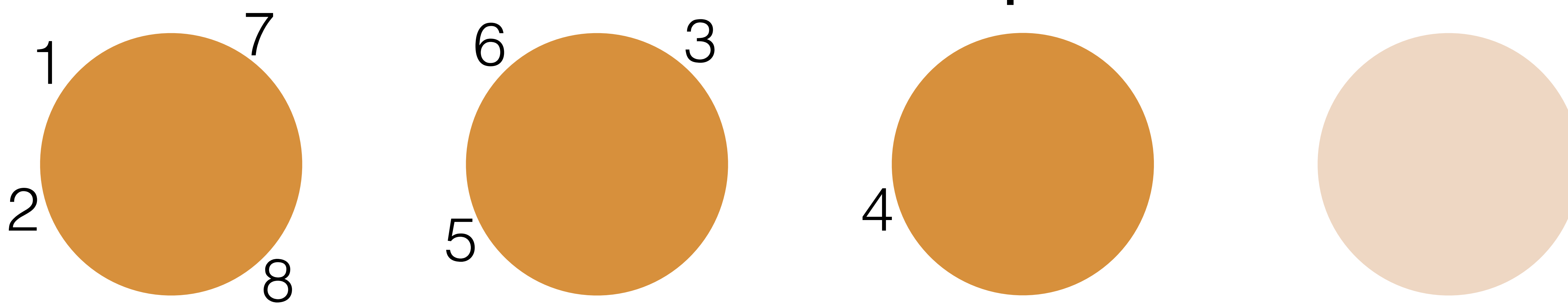
- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

Chinese restaurant process



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- Probability of *N* customers (K_N tables, #*C* at table *C*):

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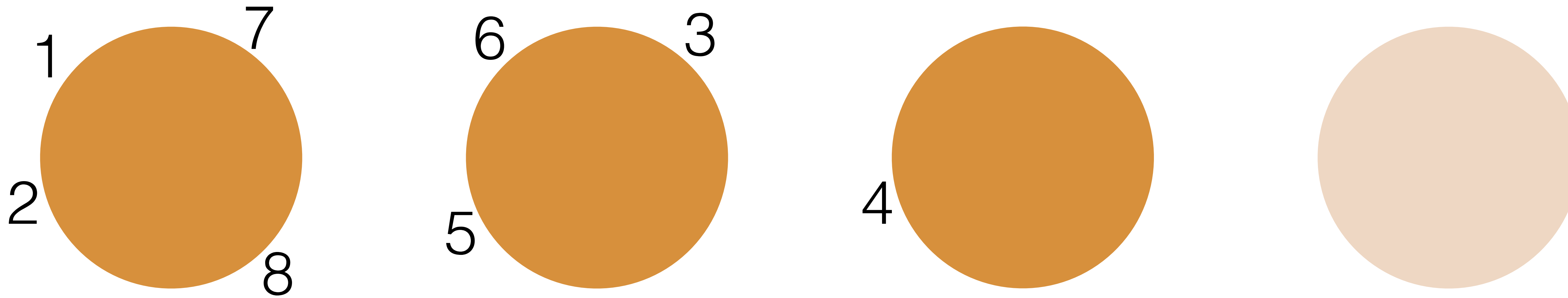
- Prob doesn't depend on customer order: *exchangeable*

Chinese restaurant process



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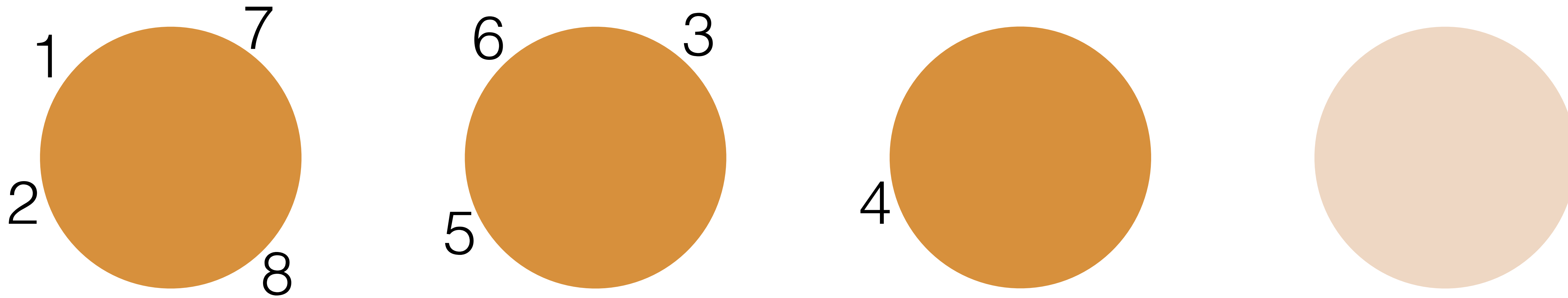
- Prob doesn't depend on customer order: *exchangeable*

$$\mathbb{P}(\Pi_8 = \{\{1,2,7,8\},\{3,5,6\},\{4\}\}) = \mathbb{P}(\Pi_8 = \{\{2,3,8,1\},\{4,6,7\},\{5\}\})$$

Chinese restaurant process



- Probability of this seating:

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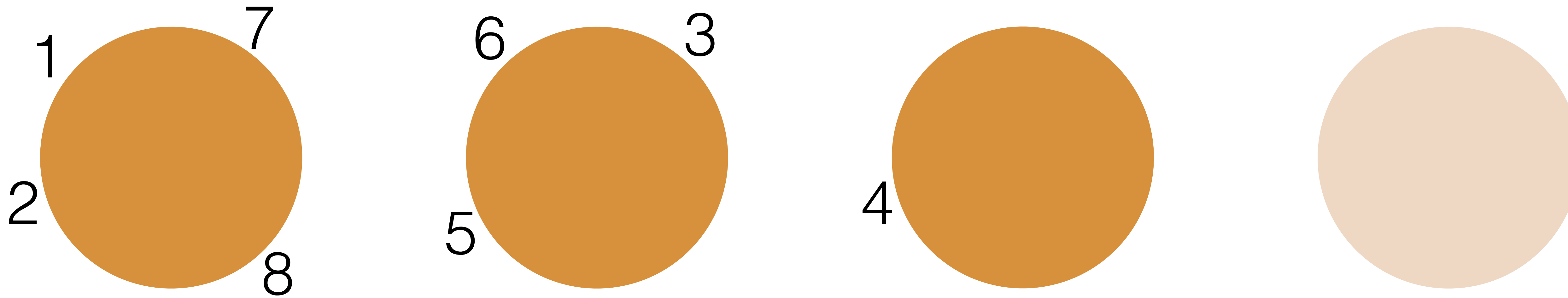
- Prob doesn't depend on customer order: *exchangeable*

$$\mathbb{P}(\Pi_8 = \{\{1,2,7,8\},\{3,5,6\},\{4\}\}) = \mathbb{P}(\Pi_8 = \{\{2,3,8,1\},\{4,6,7\},\{5\}\})$$

- Can always pretend *n* is the last customer and calculate $p(\Pi_N | \Pi_{N,\text{-}n})$

Chinese restaurant process



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- Probability of *N* customers (K_N tables, #*C* at table *C*):

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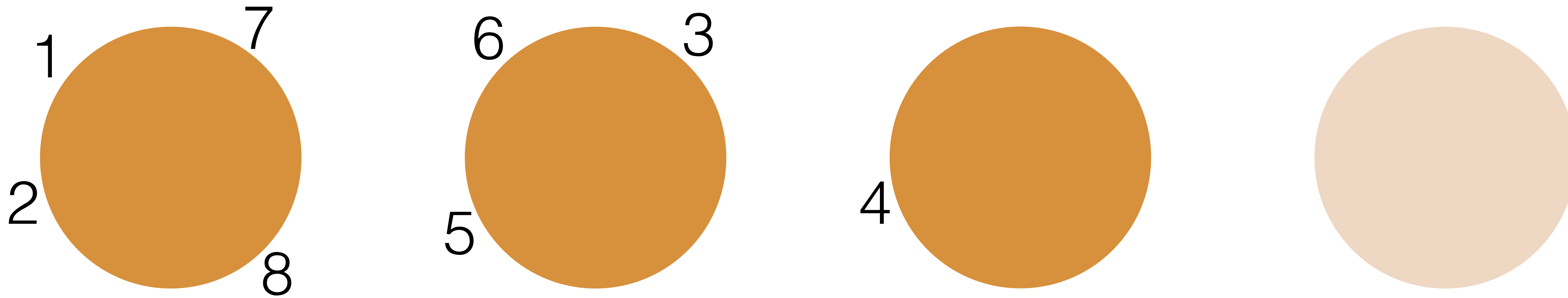
- Prob doesn't depend on customer order: *exchangeable*

$$\mathbb{P}(\Pi_8 = \{\{1,2,7,8\},\{3,5,6\},\{4\}\}) = \mathbb{P}(\Pi_8 = \{\{2,3,8,1\},\{4,6,7\},\{5\}\})$$

- Can always pretend *n* is the last customer and calculate $p(\Pi_N | \Pi_{N,\text{-}n})$
  - e.g. $\Pi_{8,\text{-}5} = \{\{1,2,7,8\},\{3,6\},\{4\}\}$

Chinese restaurant process



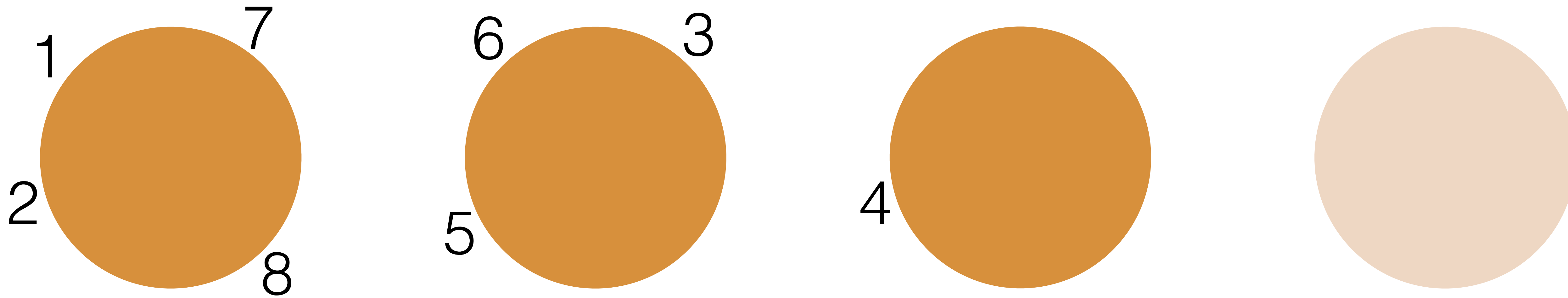
- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) =$$

Chinese restaurant process



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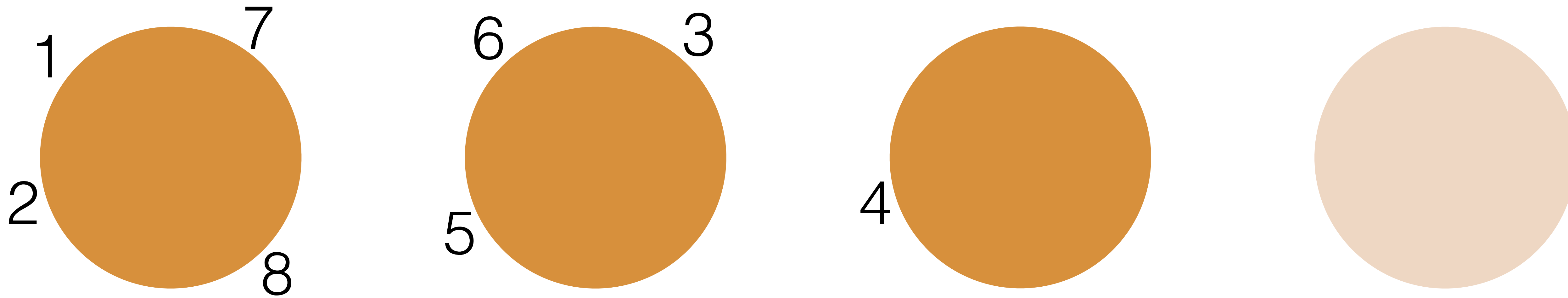

- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} & \text{if } n \text{ joins cluster } C \\ & \text{if } n \text{ starts a new cluster} \end{cases}$$

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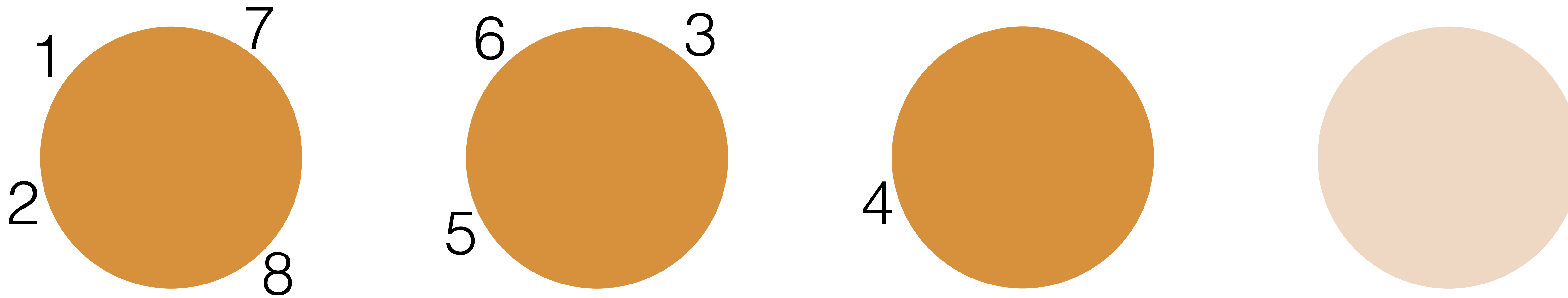


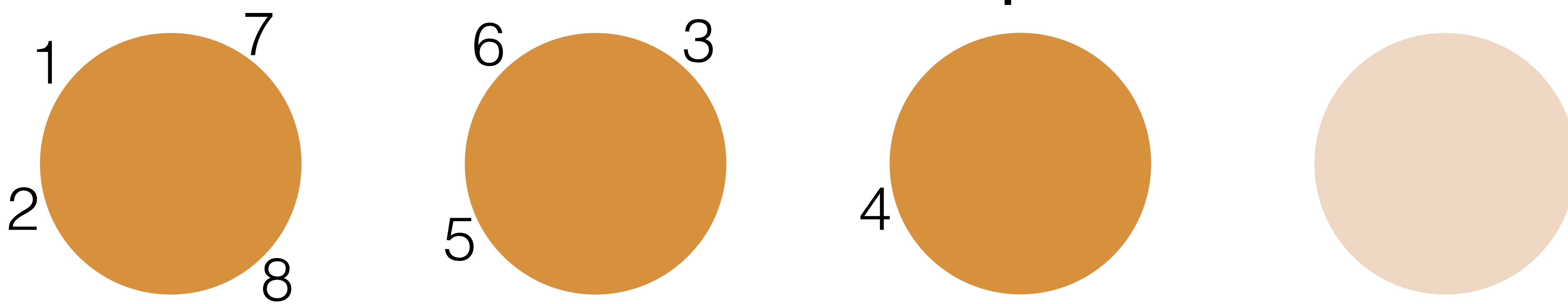
- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha + N - 1} & \text{if } n \text{ joins cluster } C \\ & \text{if } n \text{ starts a new cluster} \end{cases}$$

Chinese restaurant process



- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

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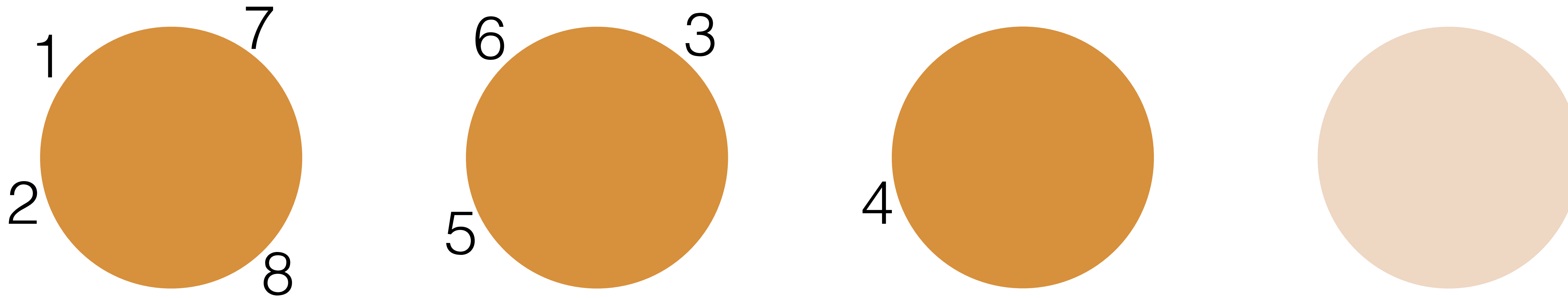

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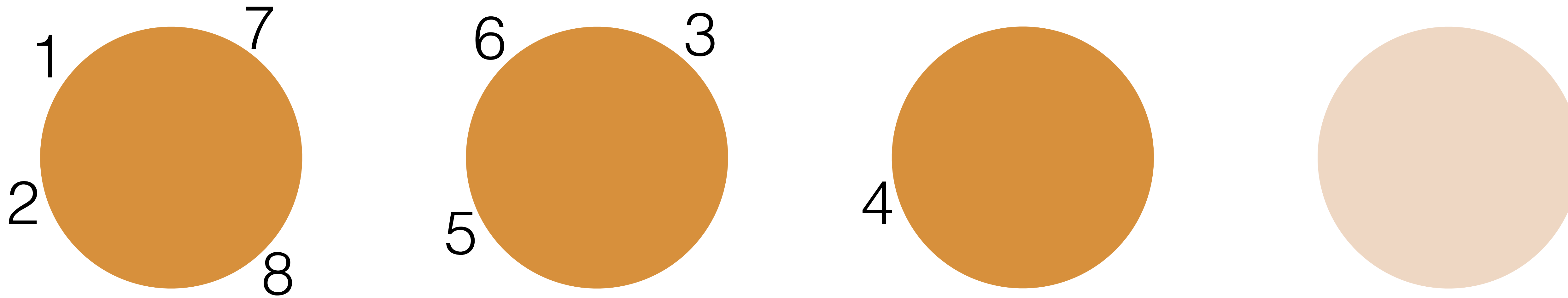
- Gibbs sampling review:

Chinese restaurant process



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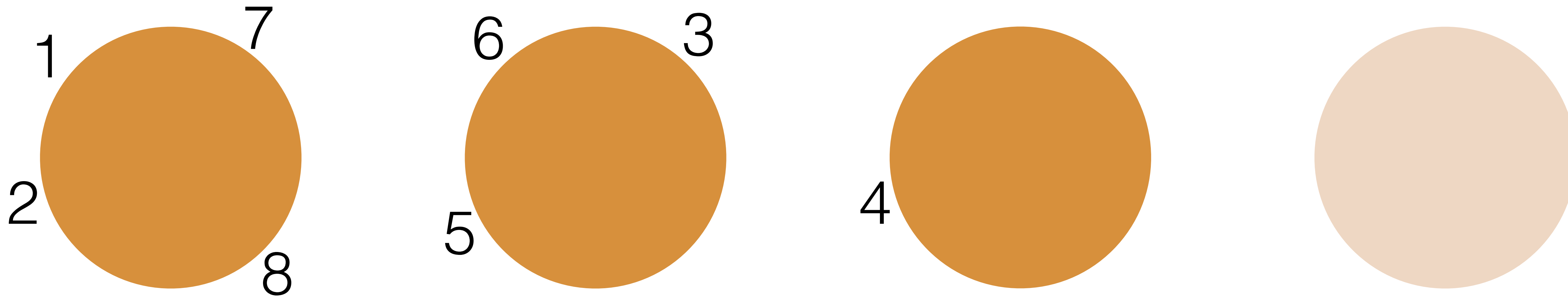
- Gibbs sampling review: target distribution $p(v_1, v_2, v_3)$

Chinese restaurant process



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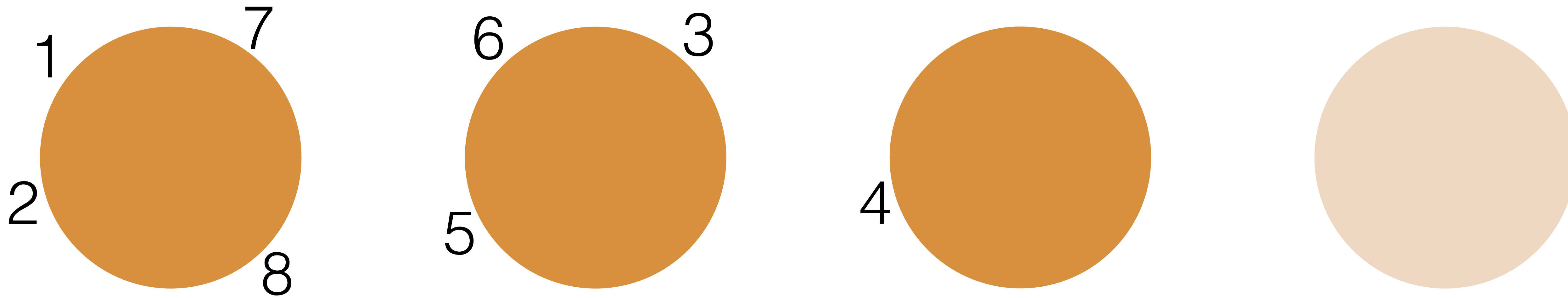
  - Start: $v_1^{(0)}, v_2^{(0)}, v_3^{(0)}$

Chinese restaurant process



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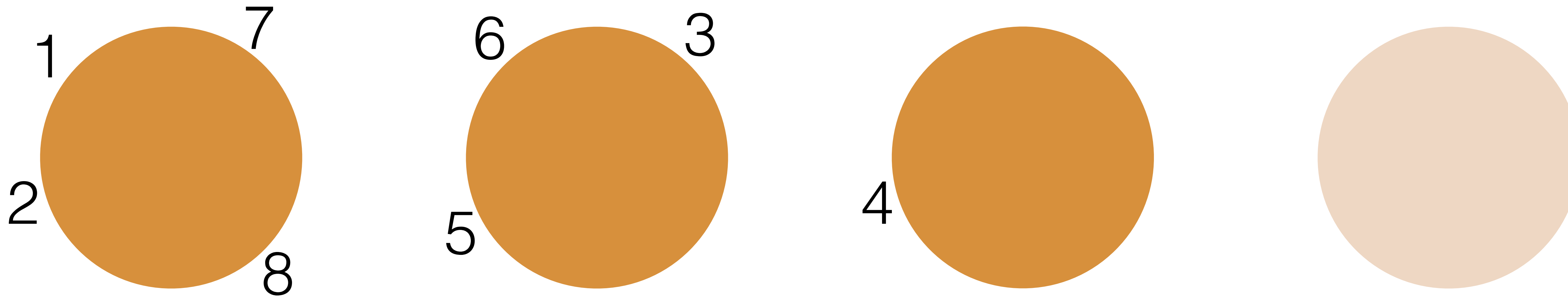
  - Start: $v_1^{(0)}, v_2^{(0)}, v_3^{(0)}$
  - t th step: $v_1^{(t)} \sim p(v_1 | v_2^{(t-1)}, v_3^{(t-1)})$

Chinese restaurant process



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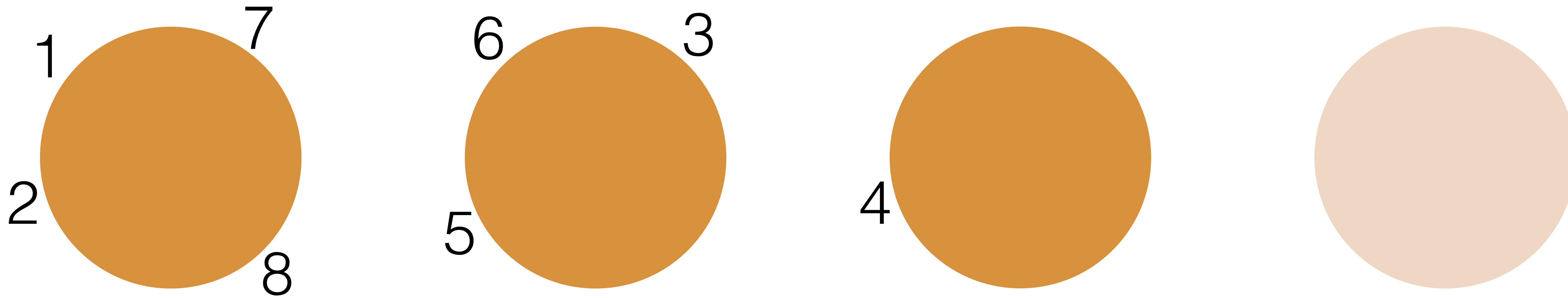
Chinese restaurant process



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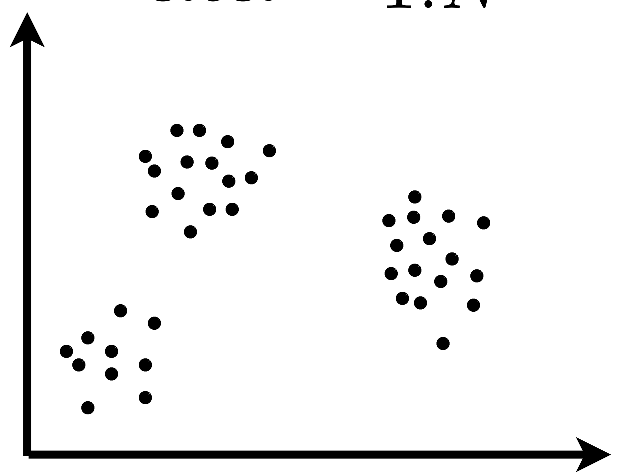
CRP mixture model: inference

CRP mixture model: inference

- Data $x_{1:N}$



CRP mixture model: inference

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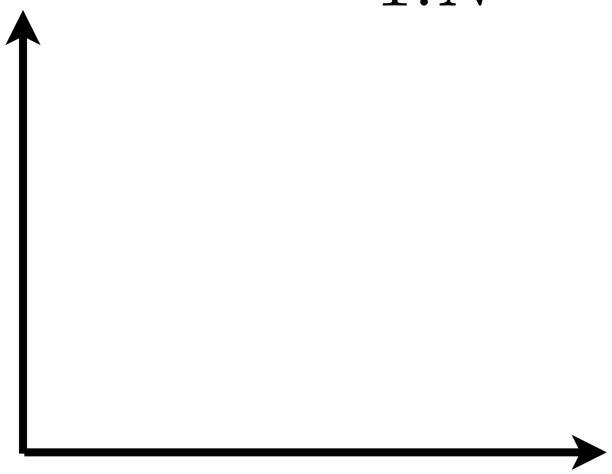
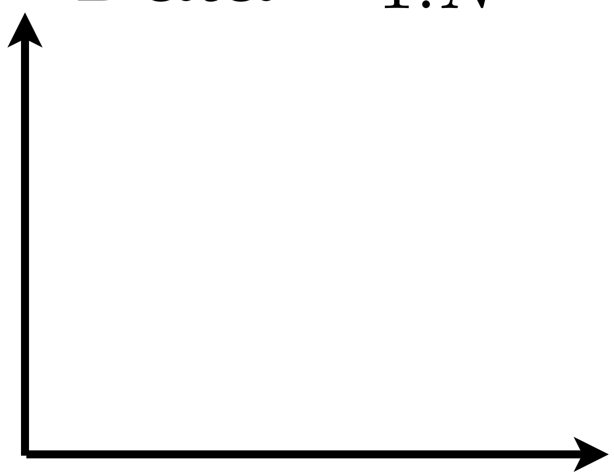


CRP mixture model: inference

- Data $x_{1:N}$
- Generative model



CRP mixture model: inference

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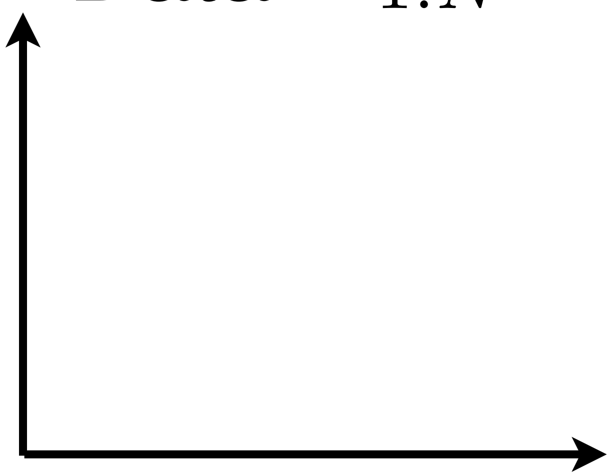
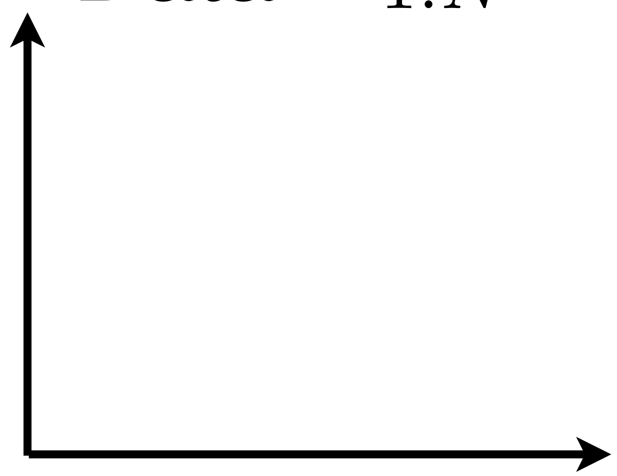


- Generative model

  $\Pi_N \sim \mathrm{CRP}(N, \alpha)$

CRP mixture model: inference

- Data $x_{1:N}$



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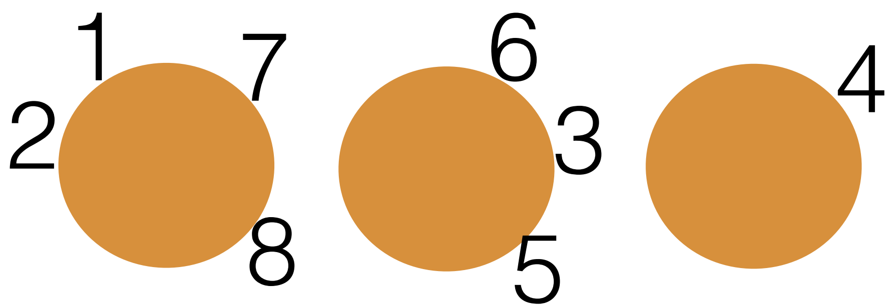

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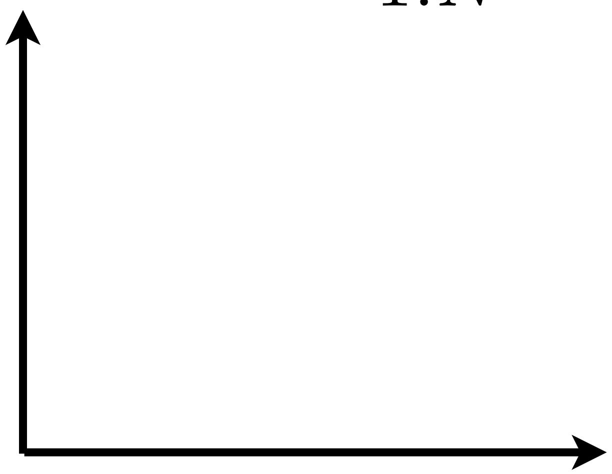

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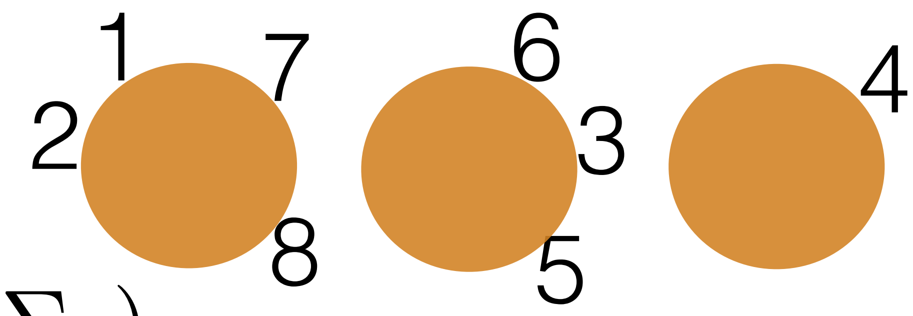

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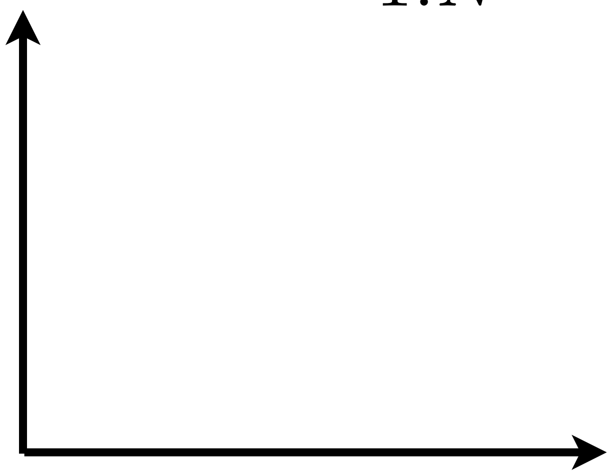

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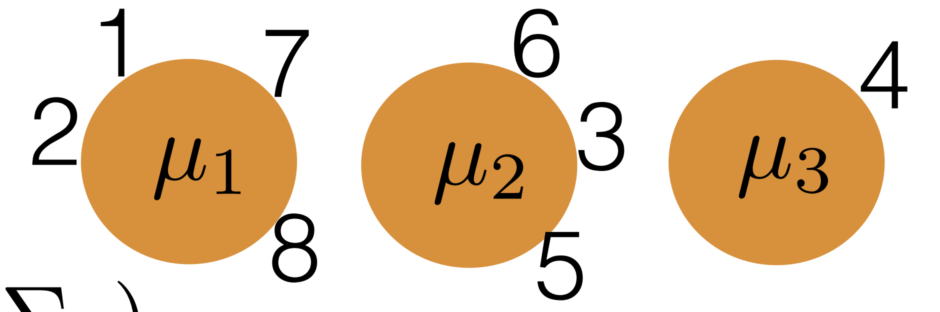

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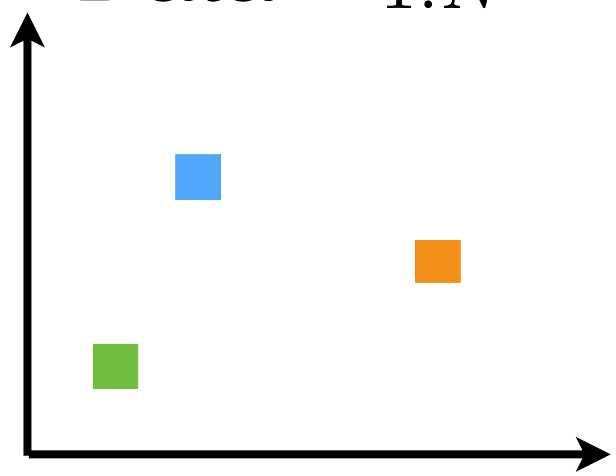

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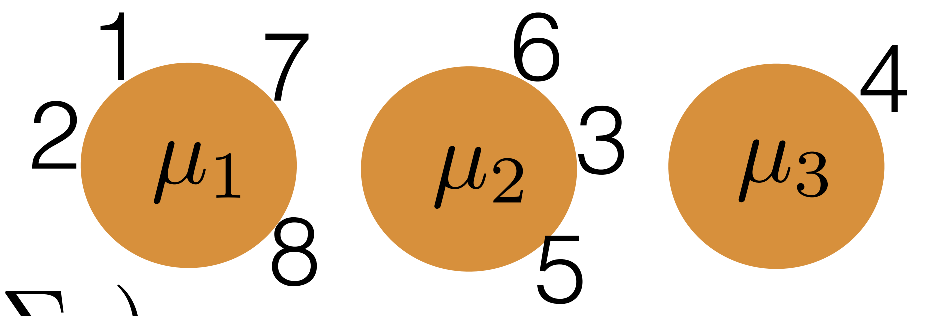

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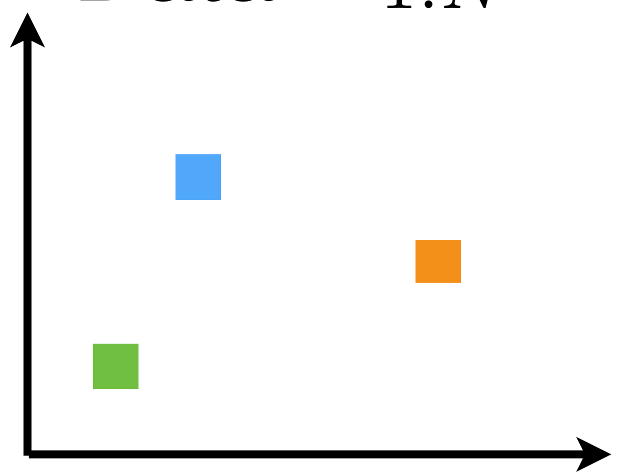

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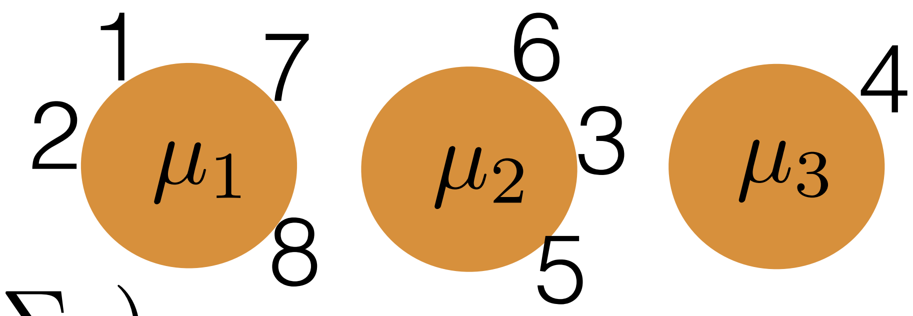

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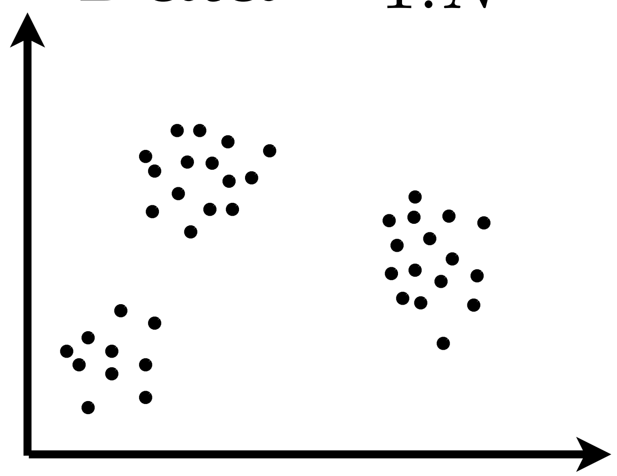

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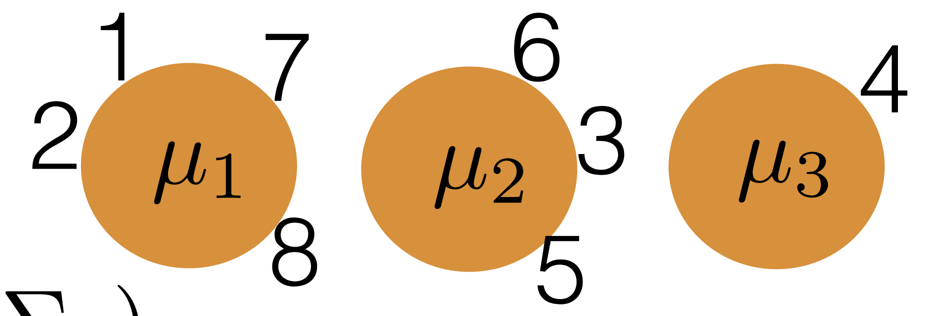

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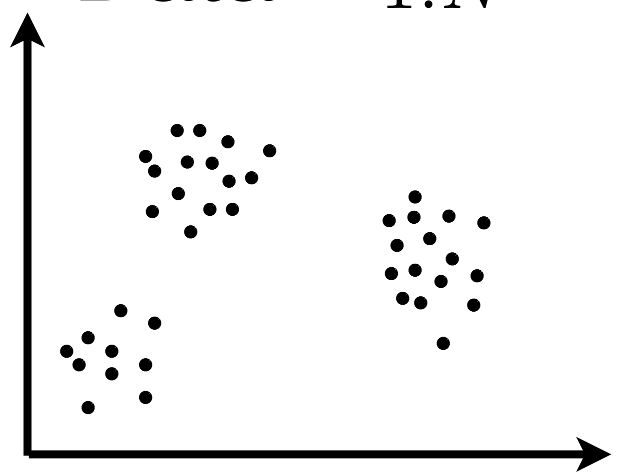

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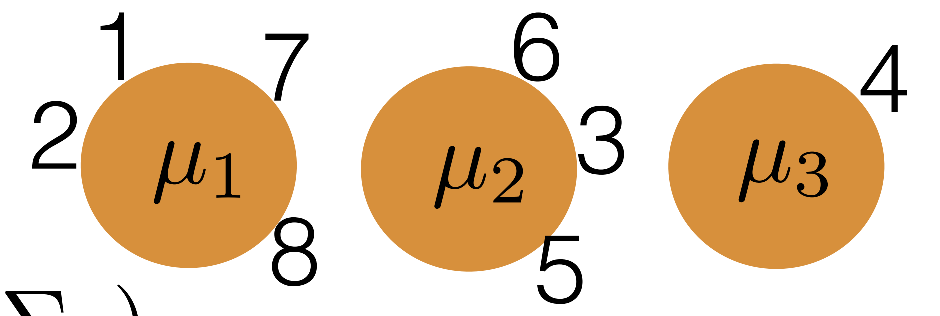
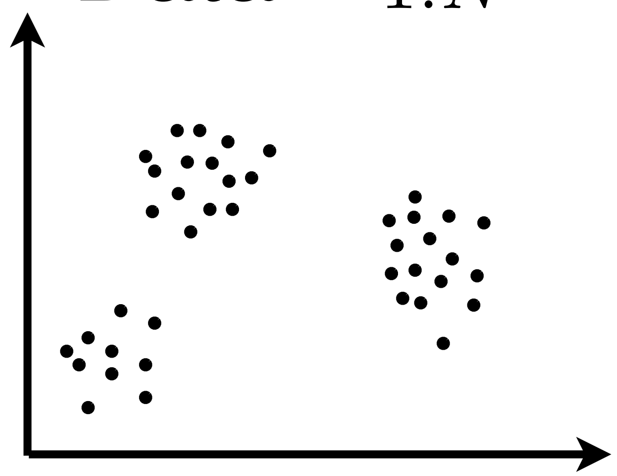


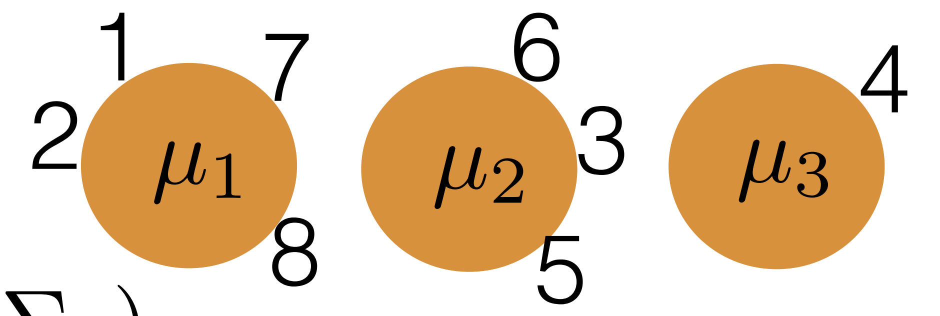
- Want: posterior

CRP mixture model: inference

- Data $x_{1:N}$



- Generative model



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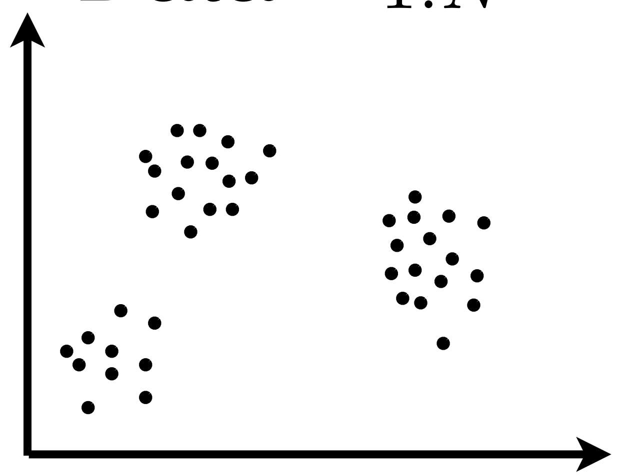
- Want: posterior $p(\Pi_N | x_{1:N})$

CRP mixture model: inference

- Data $x_{1:N}$



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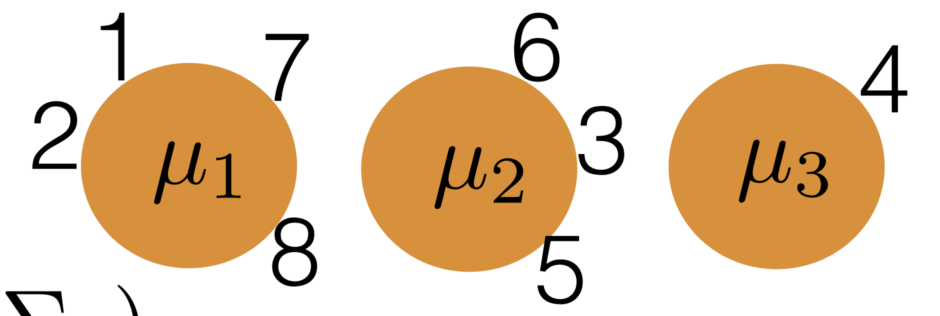
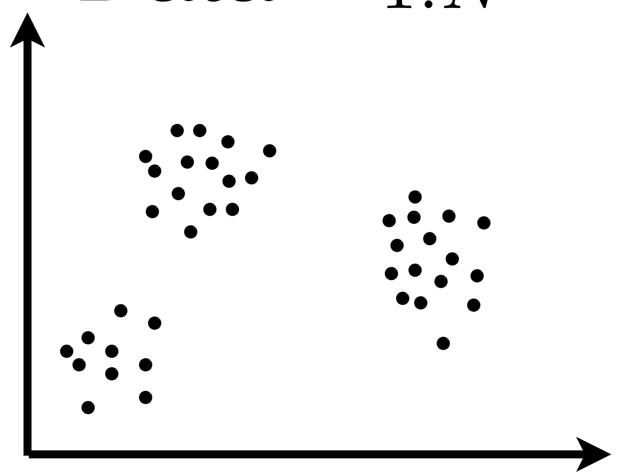


- Want: posterior $p(\Pi_N | x_{1:N})$
- Gibbs sampler:

CRP mixture model: inference

- Data $x_{1:N}$



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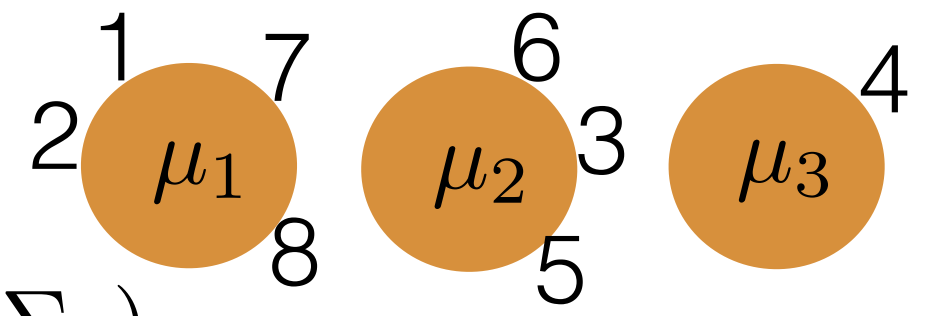


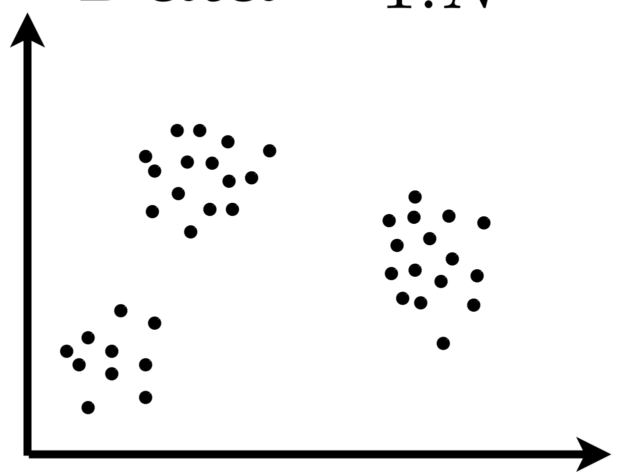
- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x)$$

CRP mixture model: inference

- Data $x_{1:N}$



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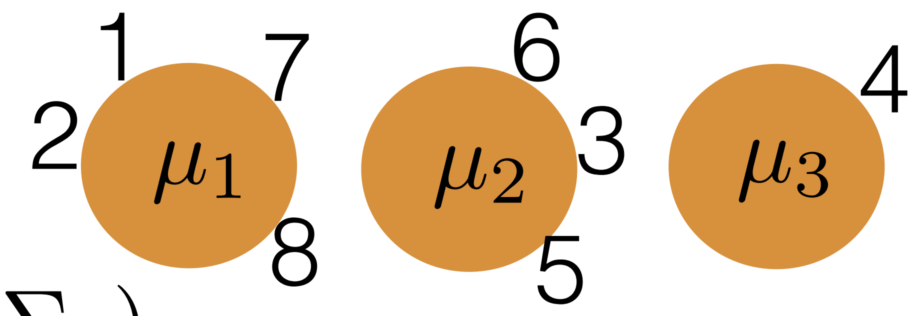

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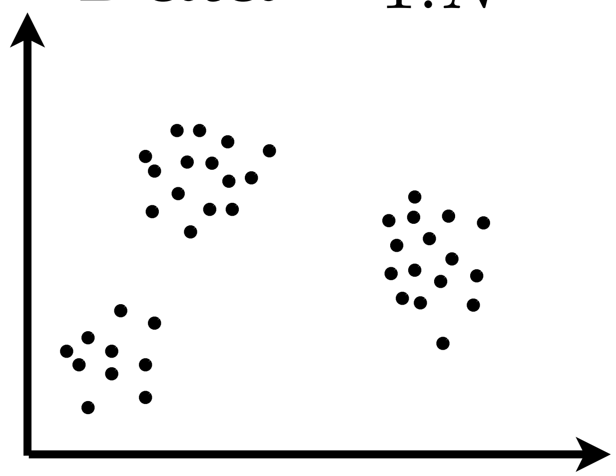
- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) = \Bigg\{$$

CRP mixture model: inference

- Data $x_{1:N}$



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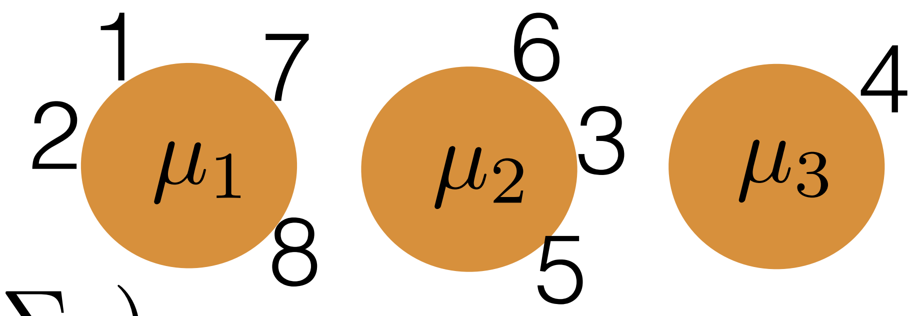

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- Gibbs sampler:

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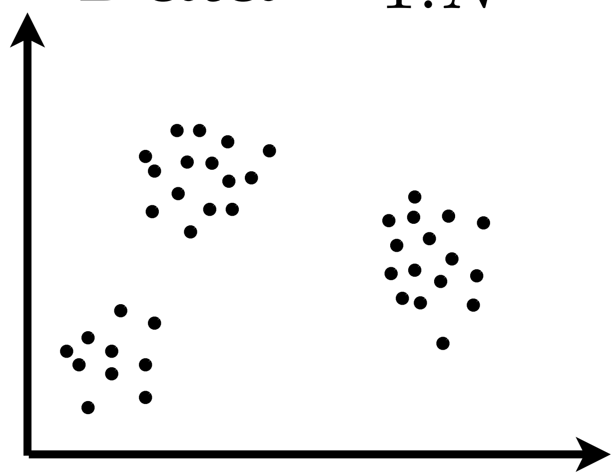
CRP mixture model: inference

- Data $x_{1:N}$



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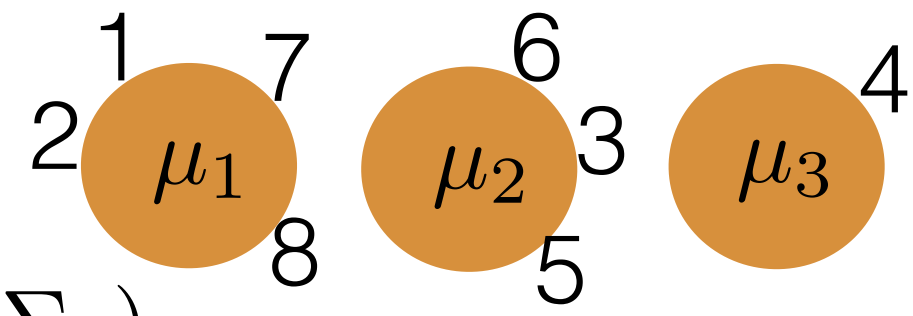

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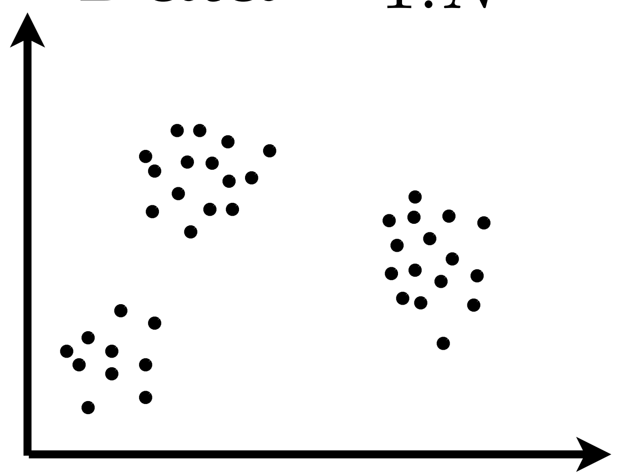
CRP mixture model: inference

- Data $x_{1:N}$



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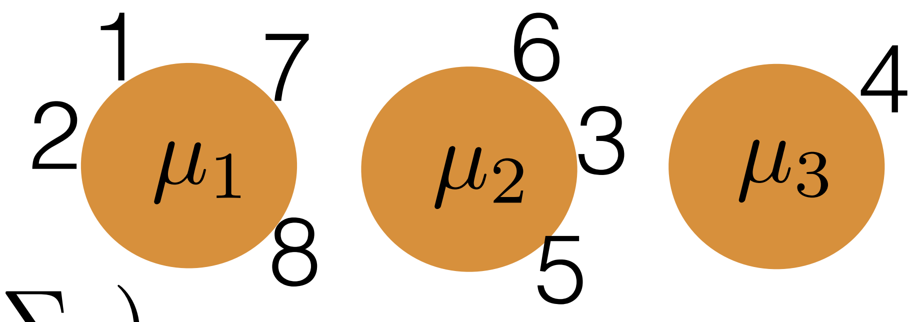

- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

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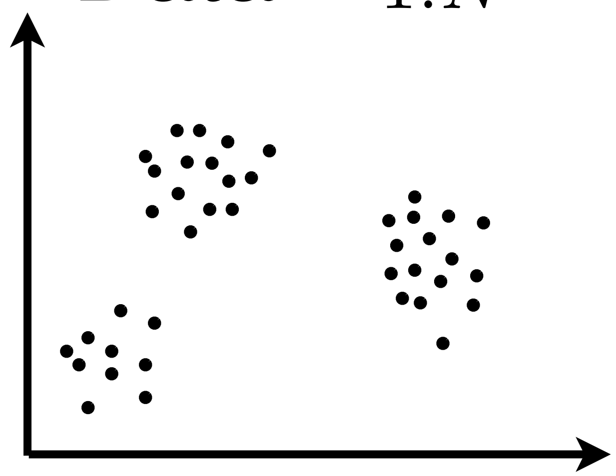
CRP mixture model: inference

- Data $x_{1:N}$



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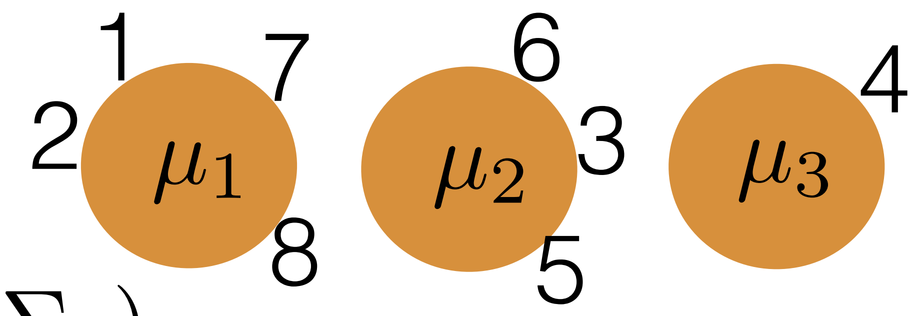

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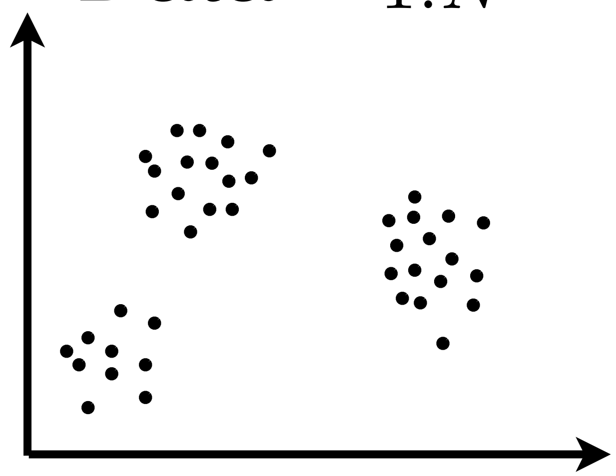
CRP mixture model: inference

- Data $x_{1:N}$



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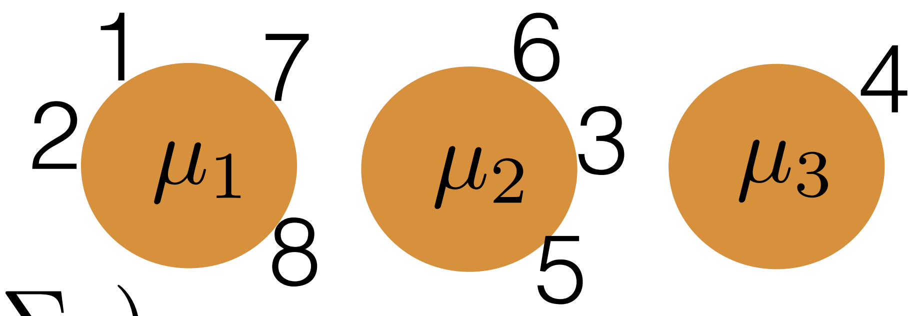

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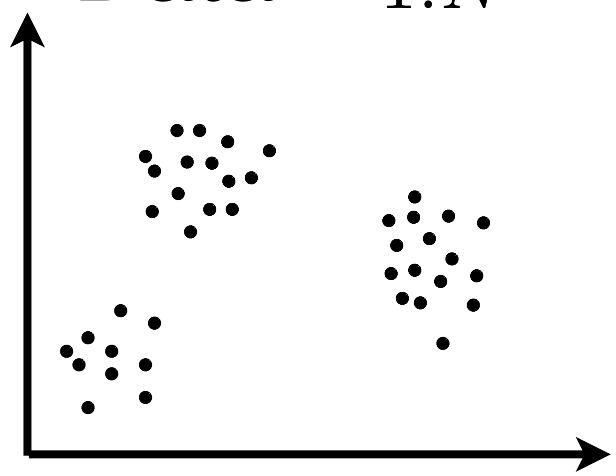

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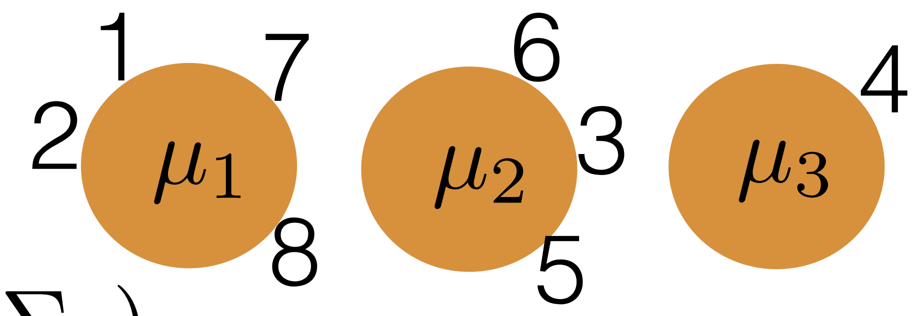

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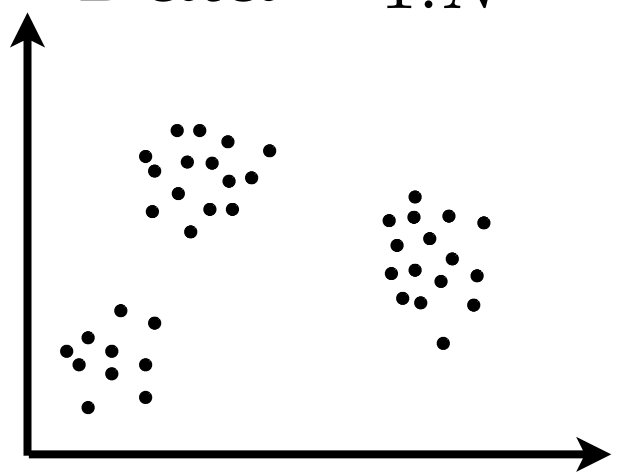

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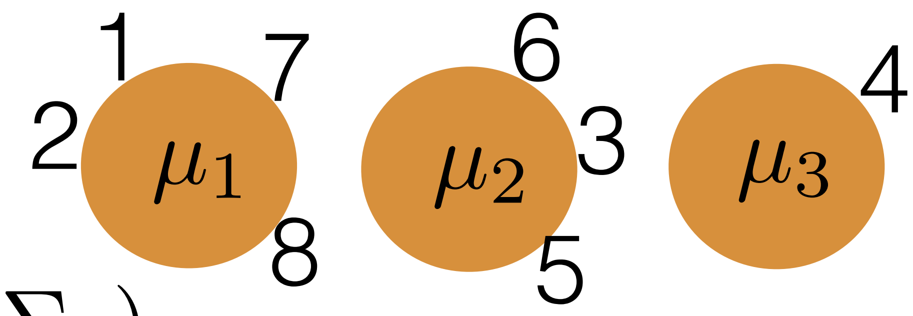

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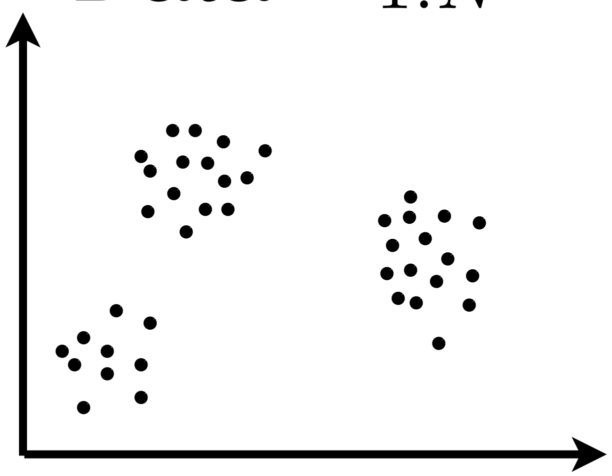
CRP mixture model: inference

- Data $x_{1:N}$



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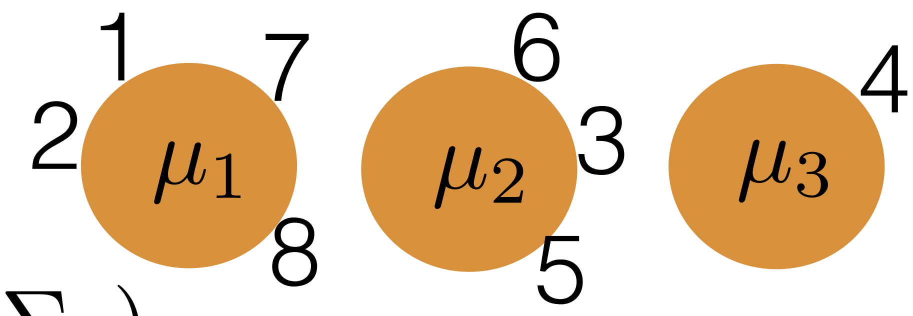

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[MacEachern 1994; Neal 1992; Neal 2000]

CRP mixture model: inference

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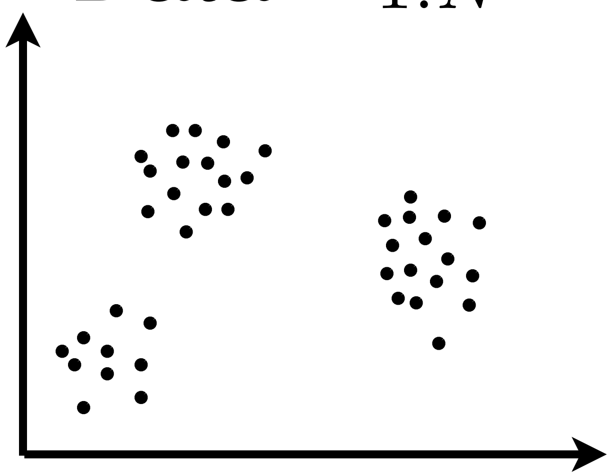

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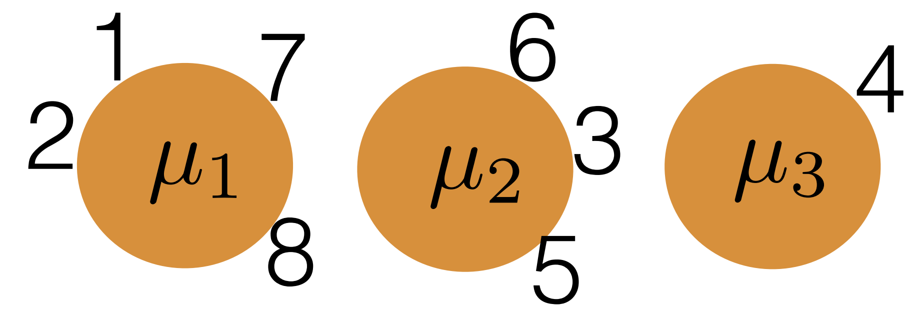

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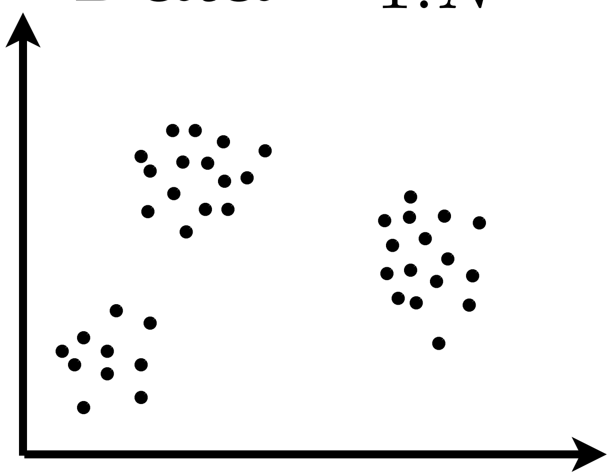

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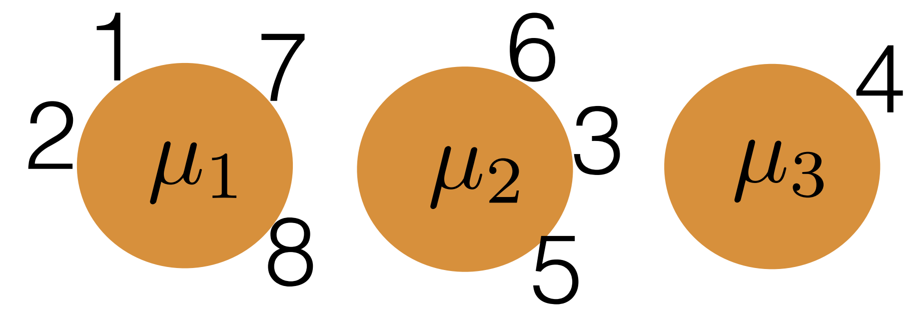

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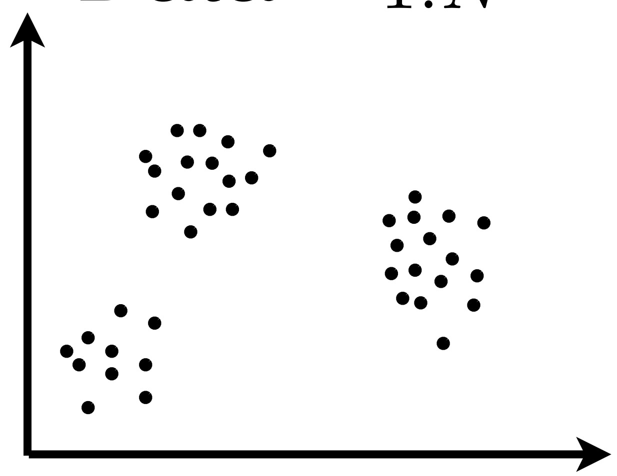

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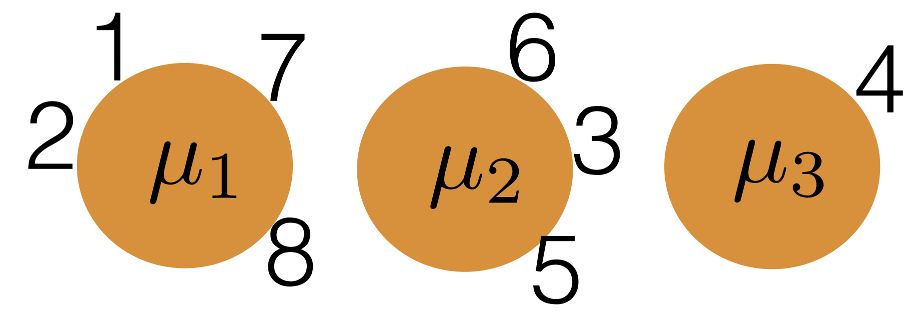

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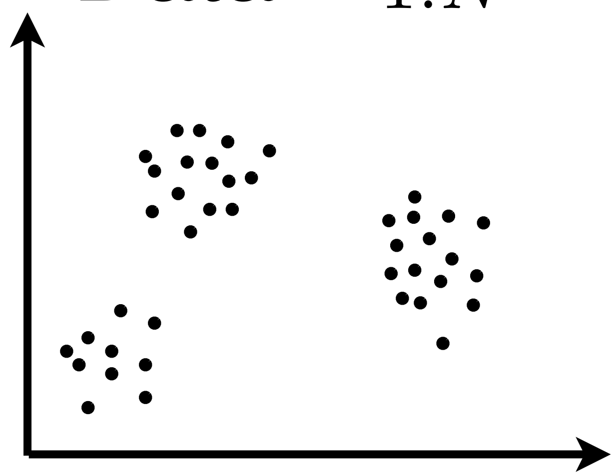

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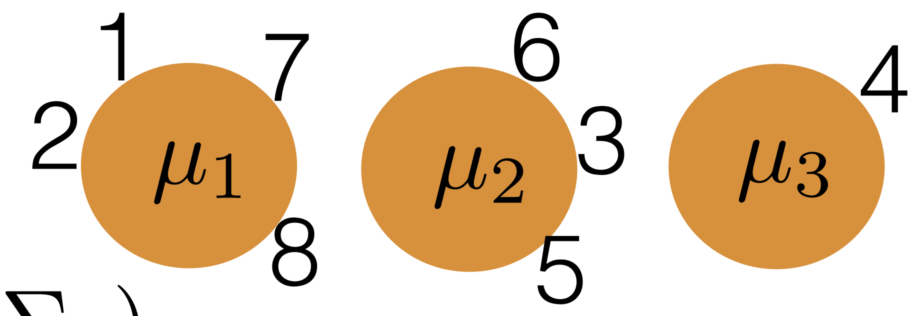

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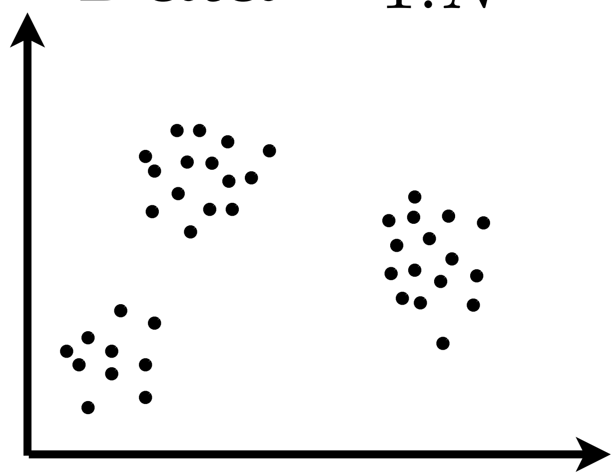

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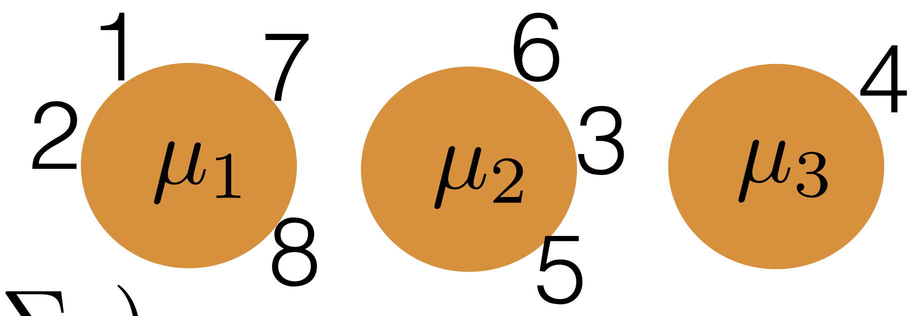

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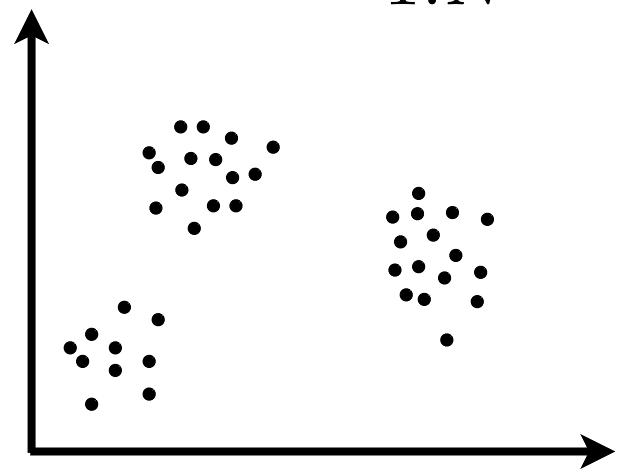
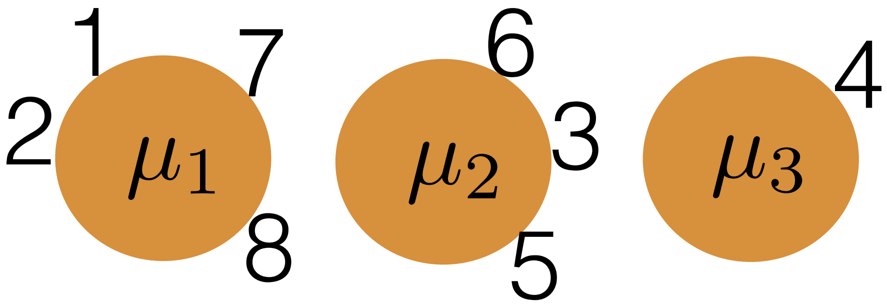
[demo]

[MacEachern 1994; Neal 1992; Neal 2000]

CRP mixture model exercises

- Data $x_{1:N}$

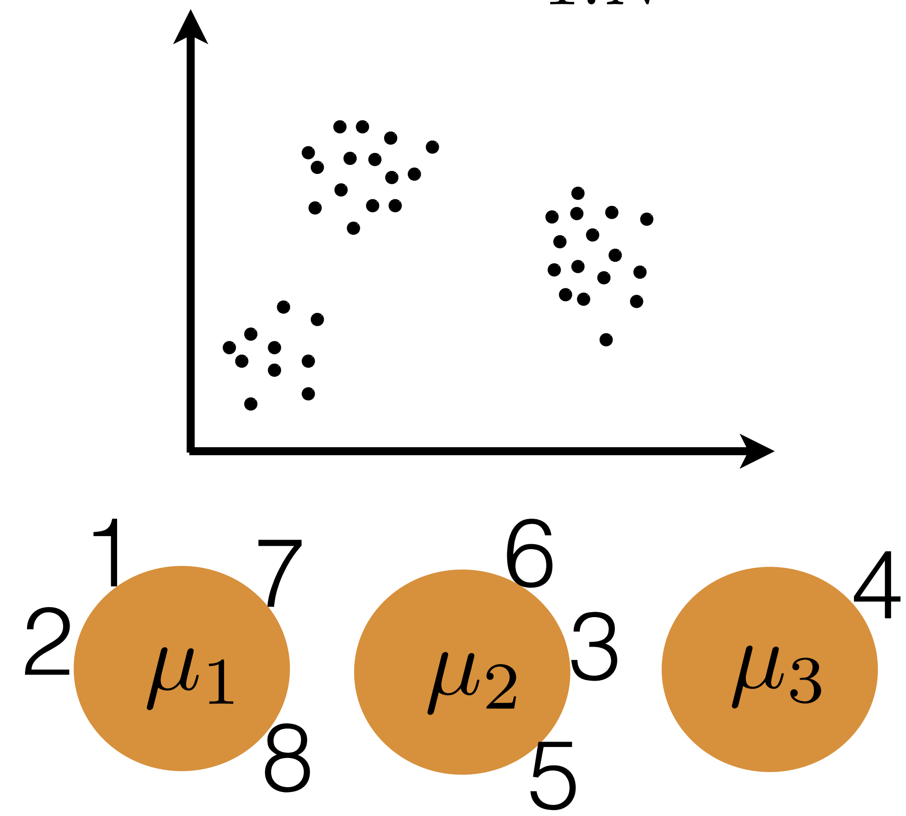




CRP mixture model exercises

- Code a CRP mixture model simulator

- Data $x_{1:N}$



CRP mixture model exercises

- Code a CRP mixture model simulator
- Derive the CRP mixture model Gibbs sampler in the slides; derive $p(x_{C \cup \{n\}} | x_C)$ explicitly for a Gaussian mixture

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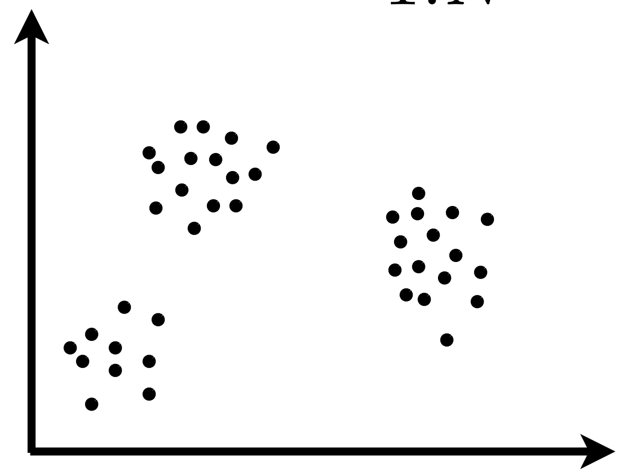

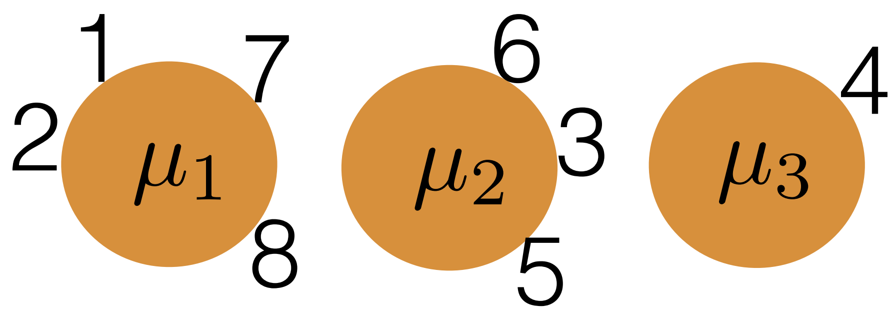

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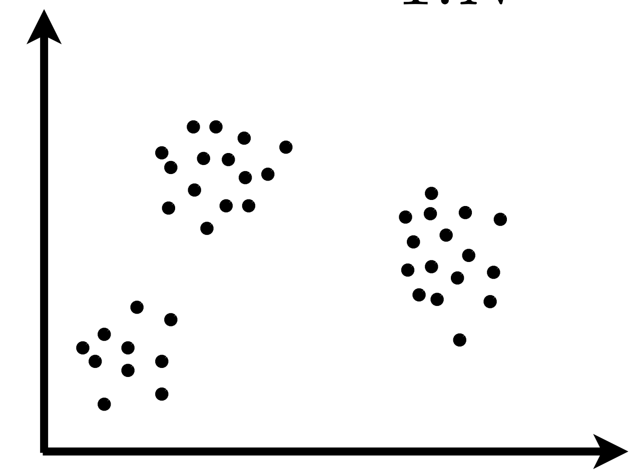

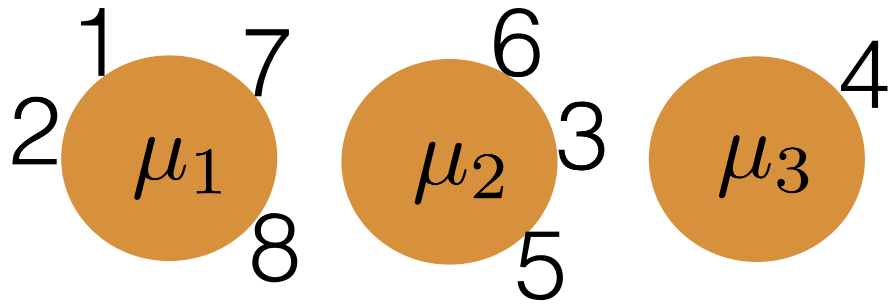

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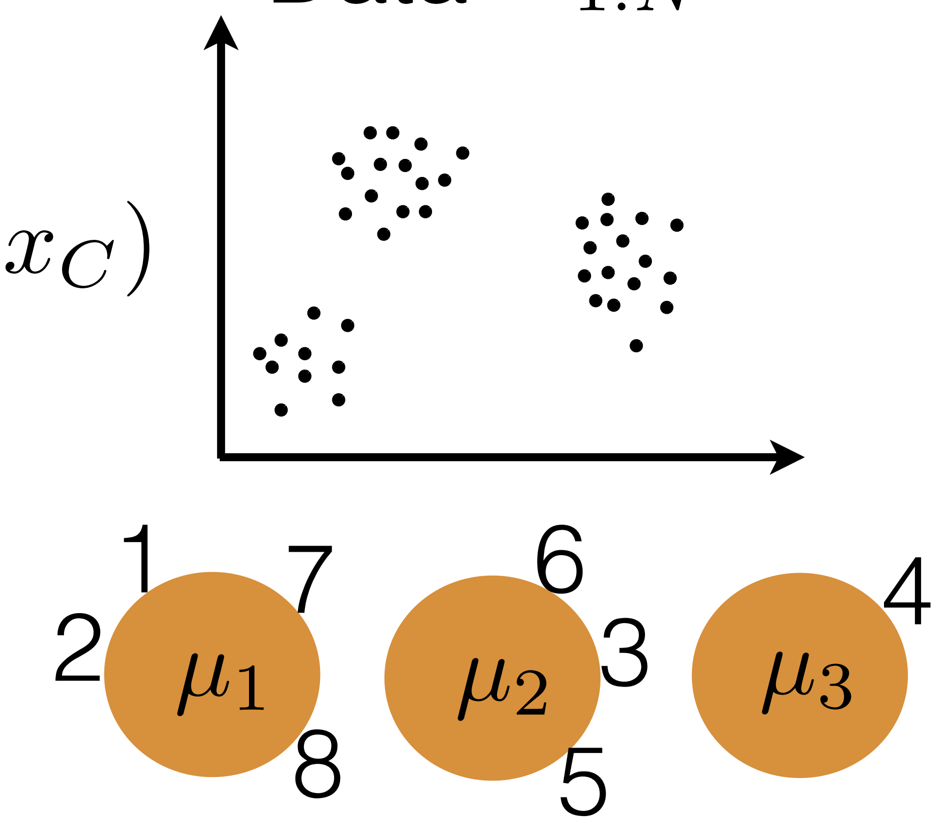

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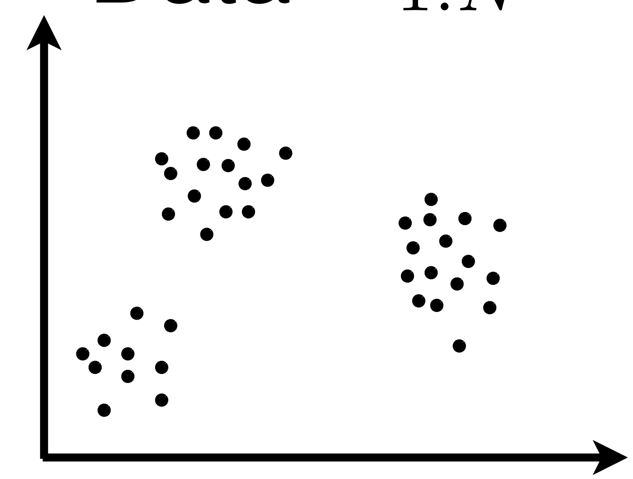

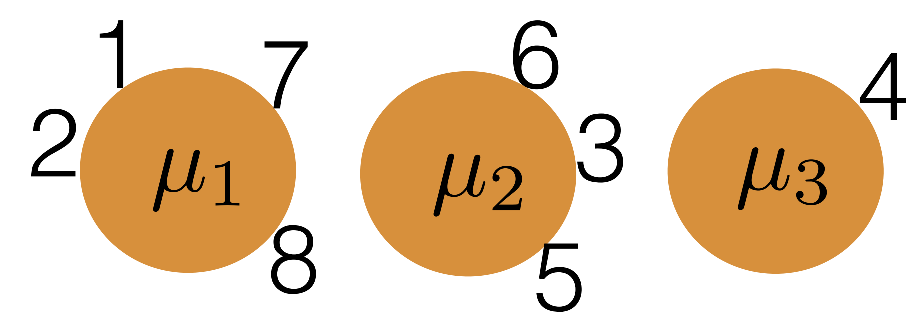

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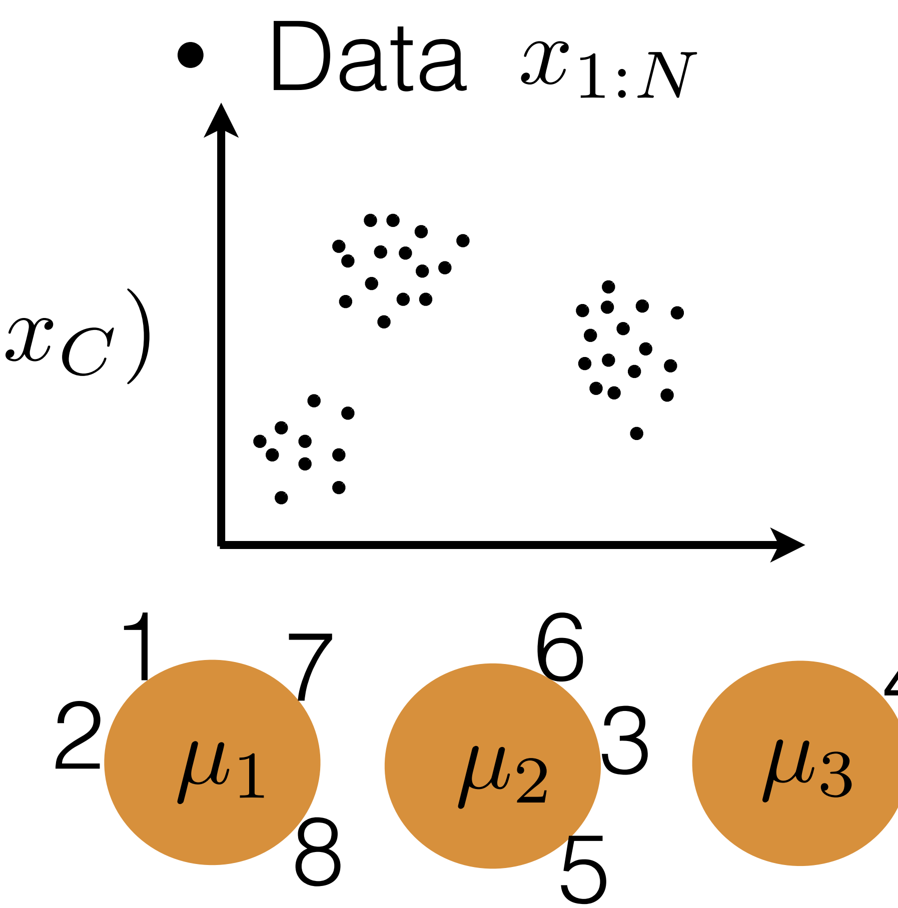

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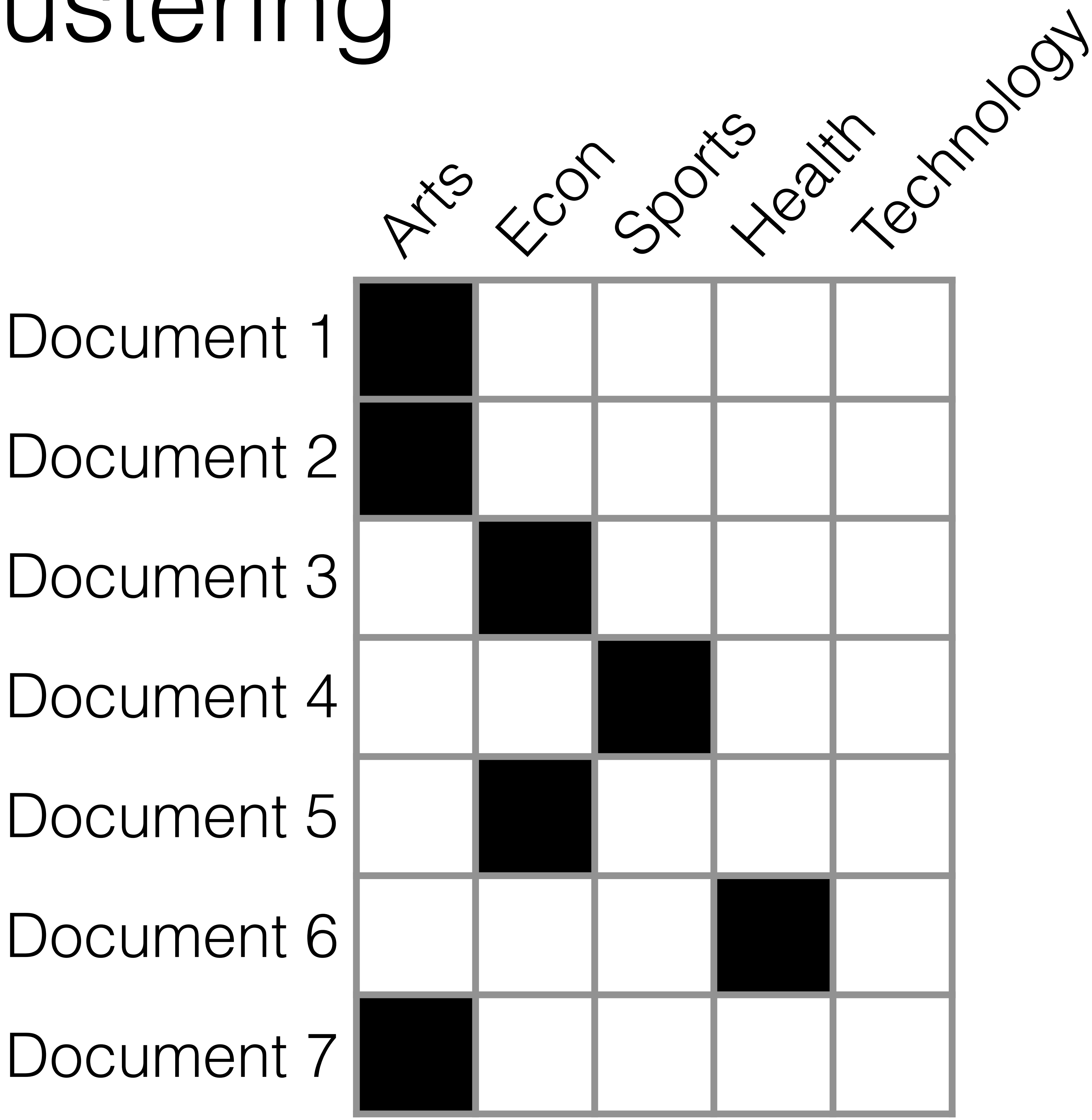
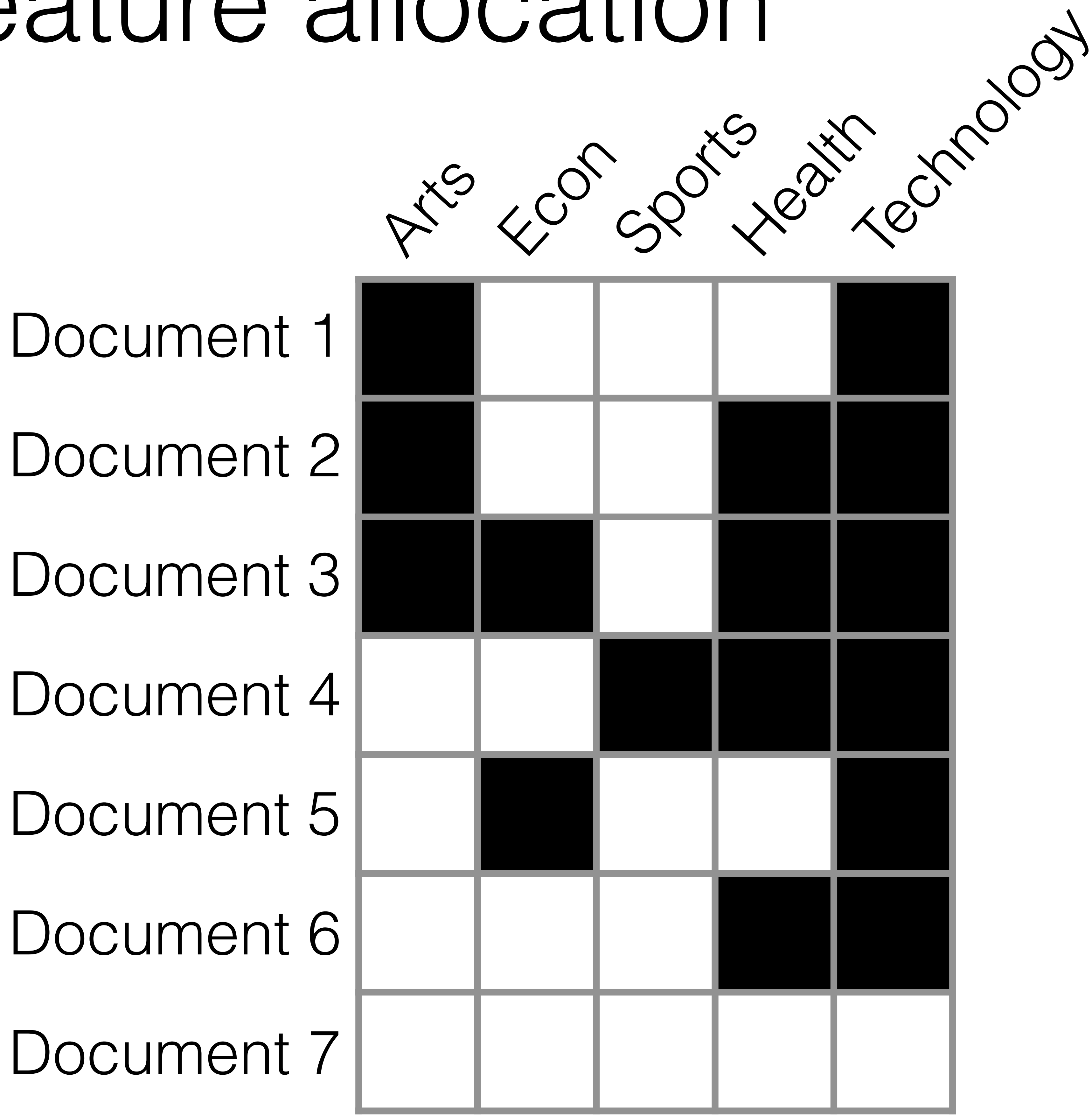
- Read Broderick, Jordan, Pitman 2013 "Cluster and feature modeling […]" for more background/extensions

[Tiia Monto, https://commons.wikimedia.org/wiki/File:Trail_and_mountain.jpg]

Clustering


Arts
Econ
Sports
Health
Technology
Document 1
Document 2
Document 3
Document 4
Document 5
Document 6
Document 7

Feature allocation



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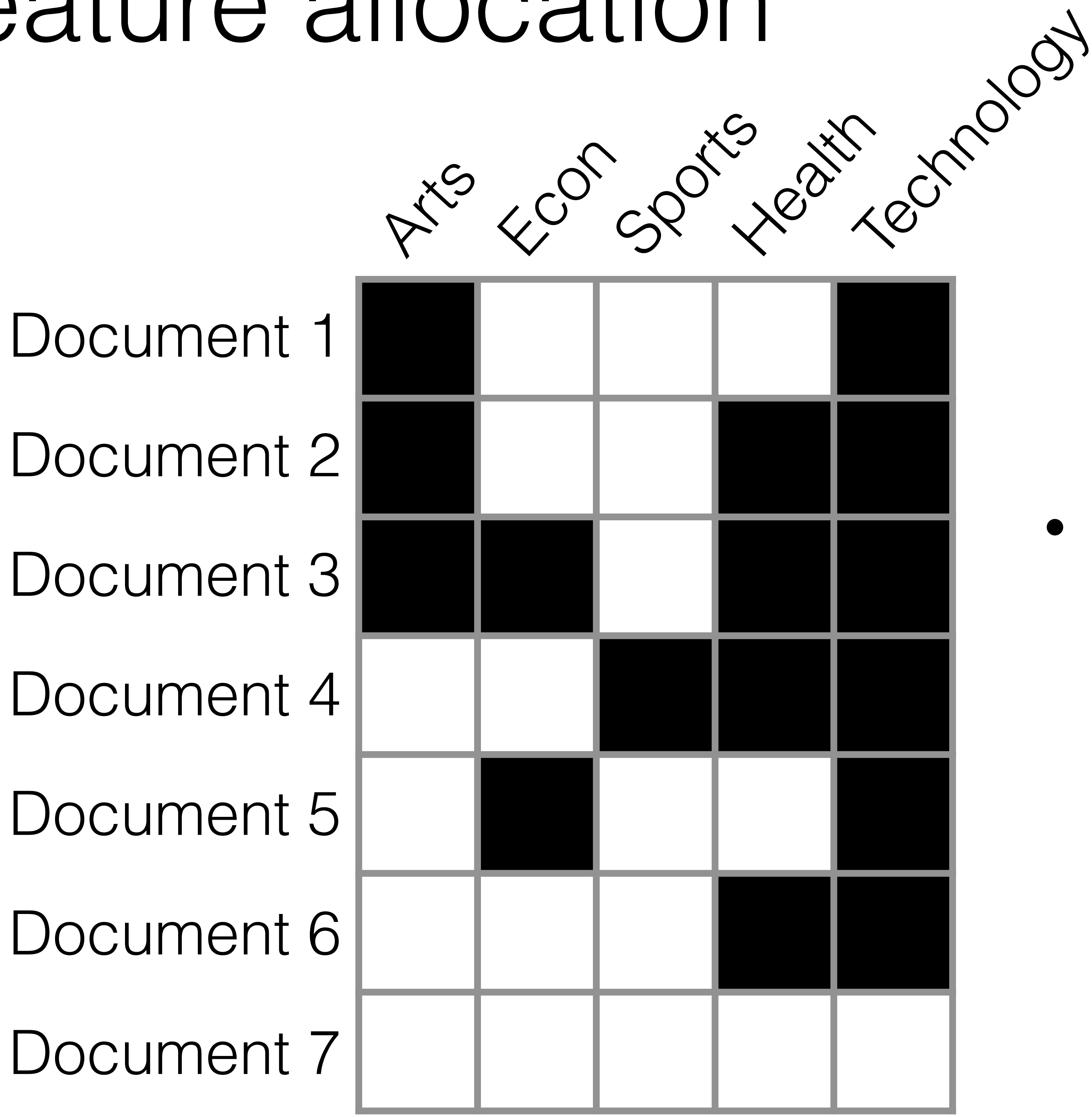
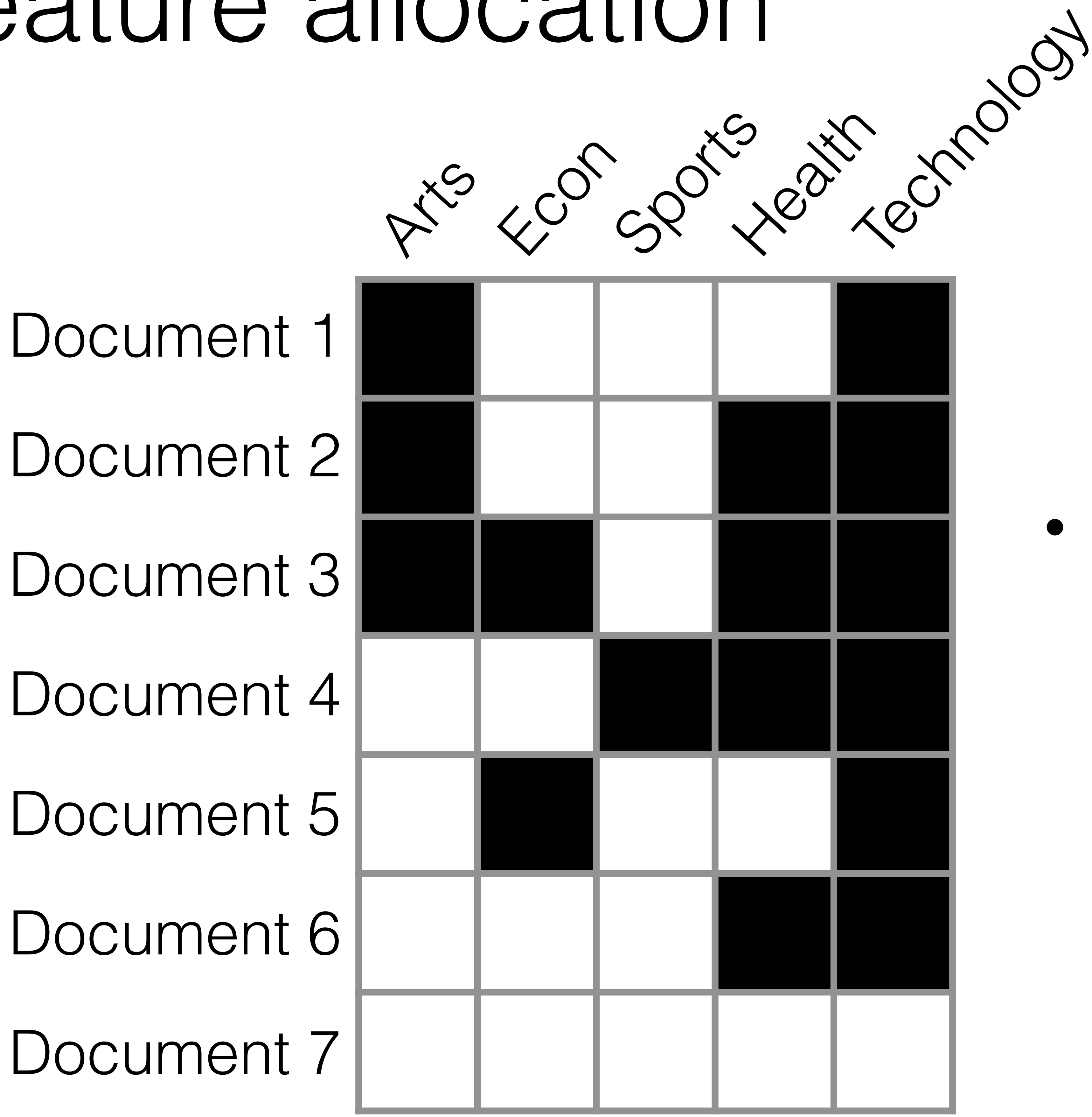


- Indian buffet process

Feature allocation



- Indian buffet process

[Griffiths, Ghahramani 2005

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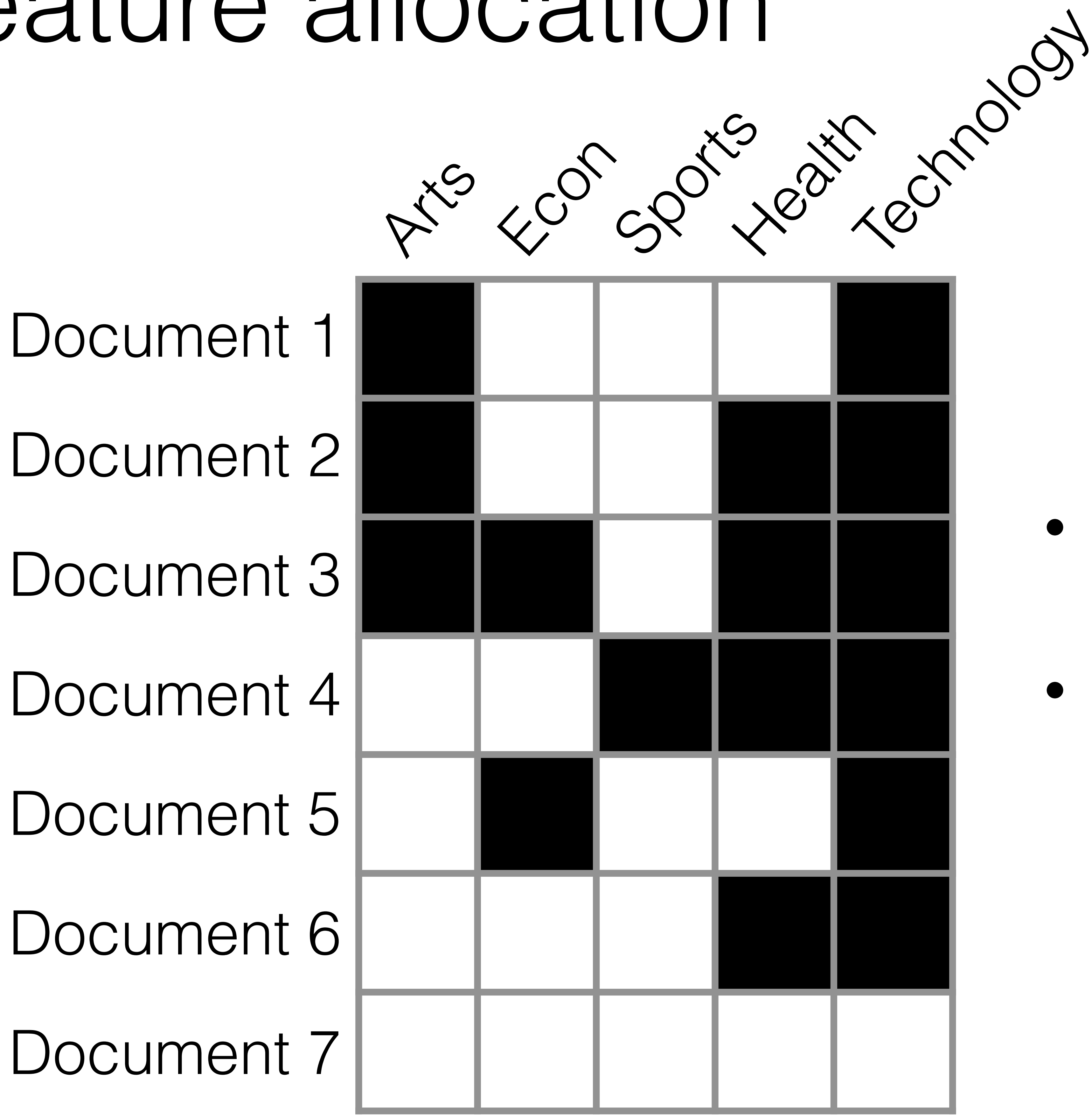

- Indian buffet process
- Beta process

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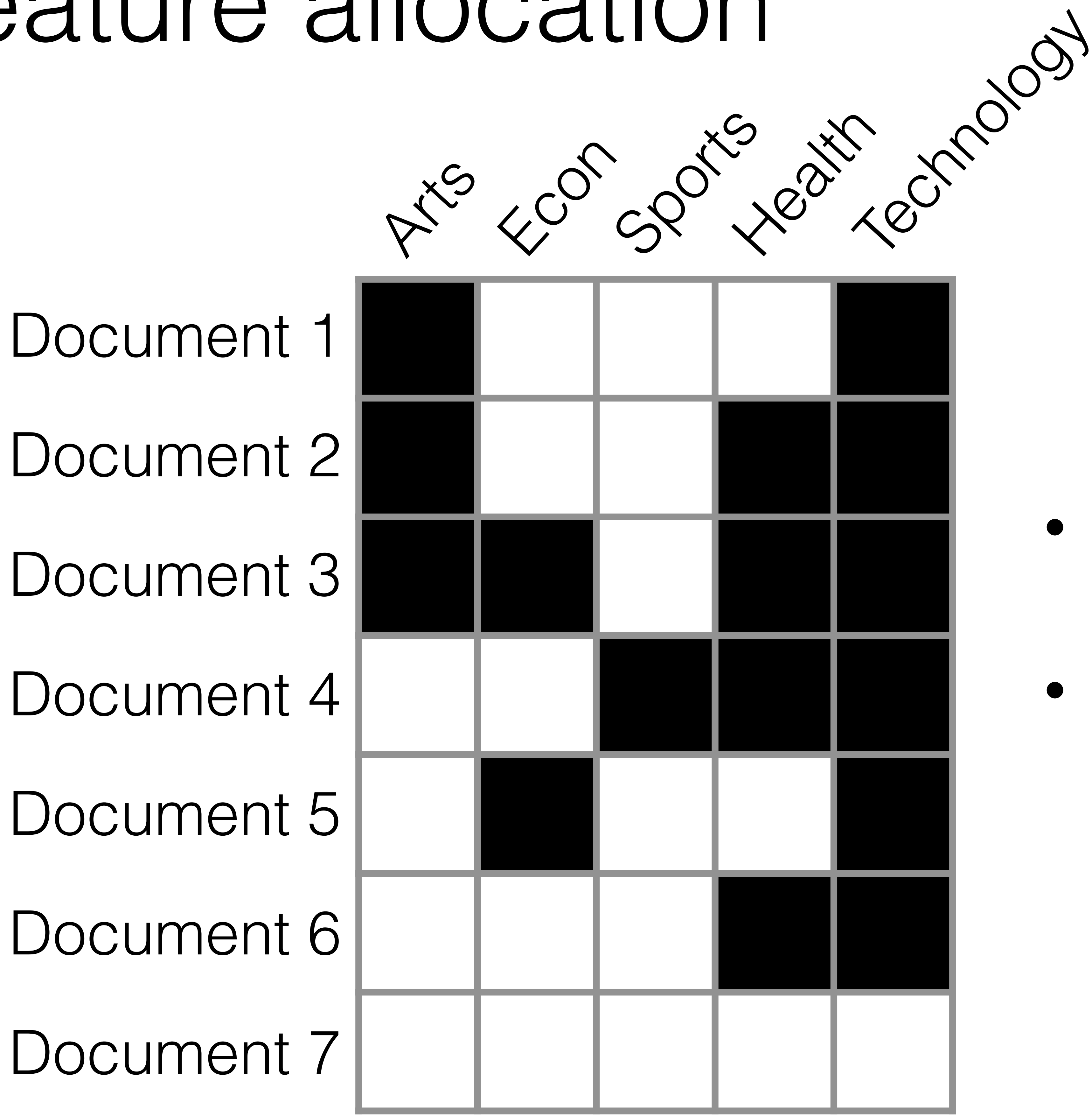

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[Griffiths, Ghahramani 2005, Hjort 1990

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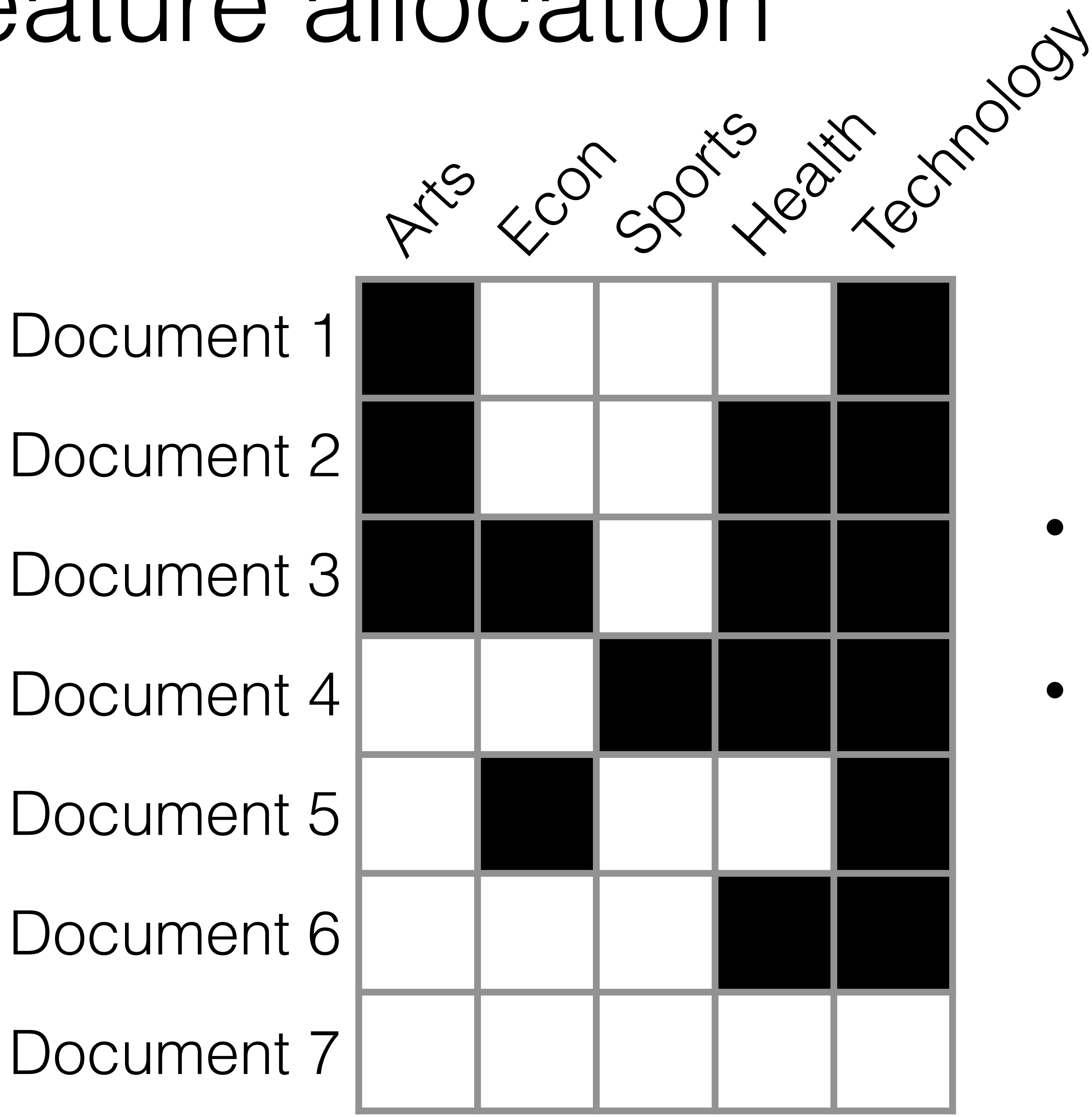

- Indian buffet process
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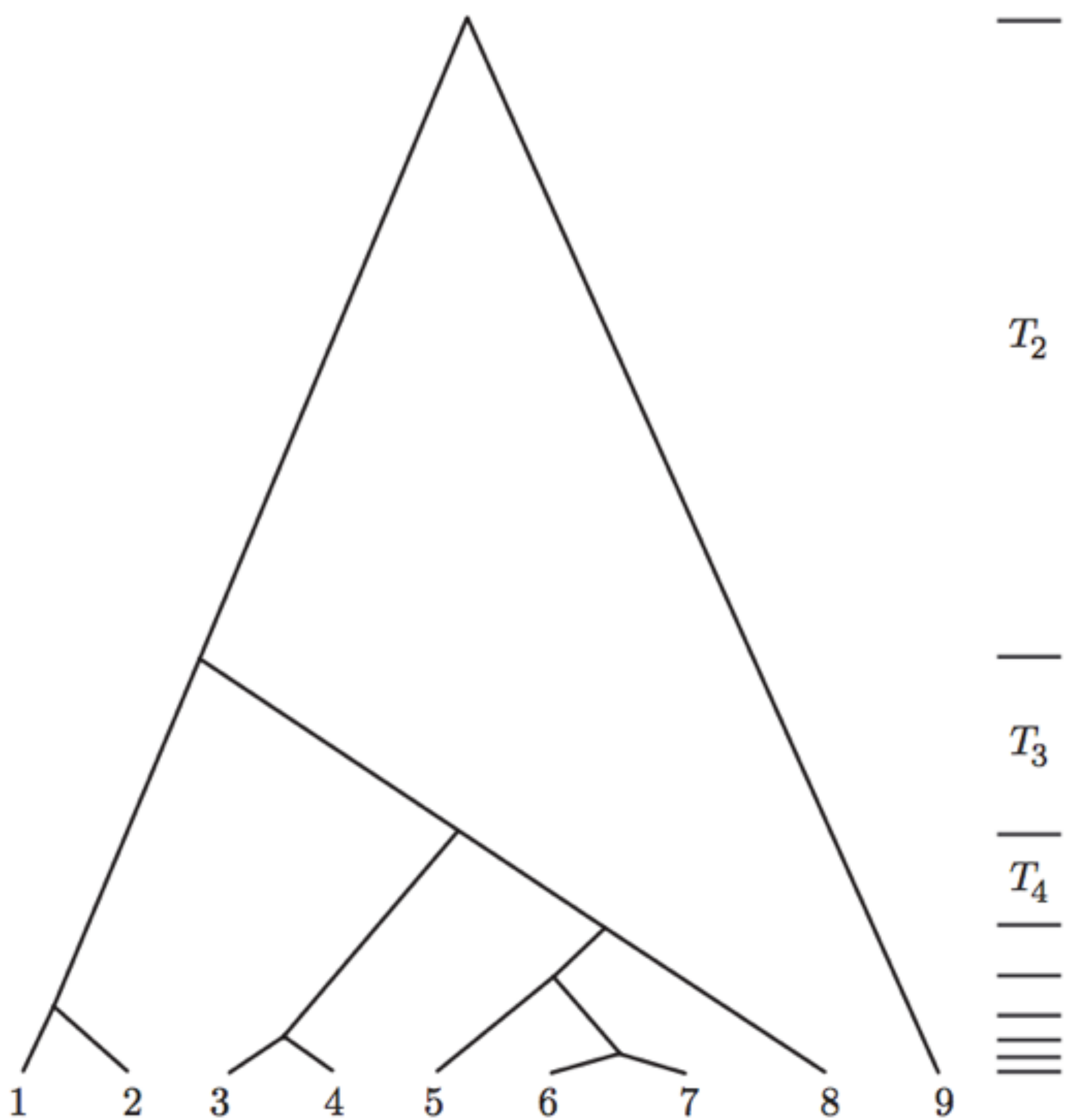
[Griffiths, Ghahramani 2005, Hjort 1990, Kim 1999, Thibaux, Jordan 2007, Broderick, Jordan, Pitman 2013]

Genealogy, trees, beyond trees



[Wakeley 2008]

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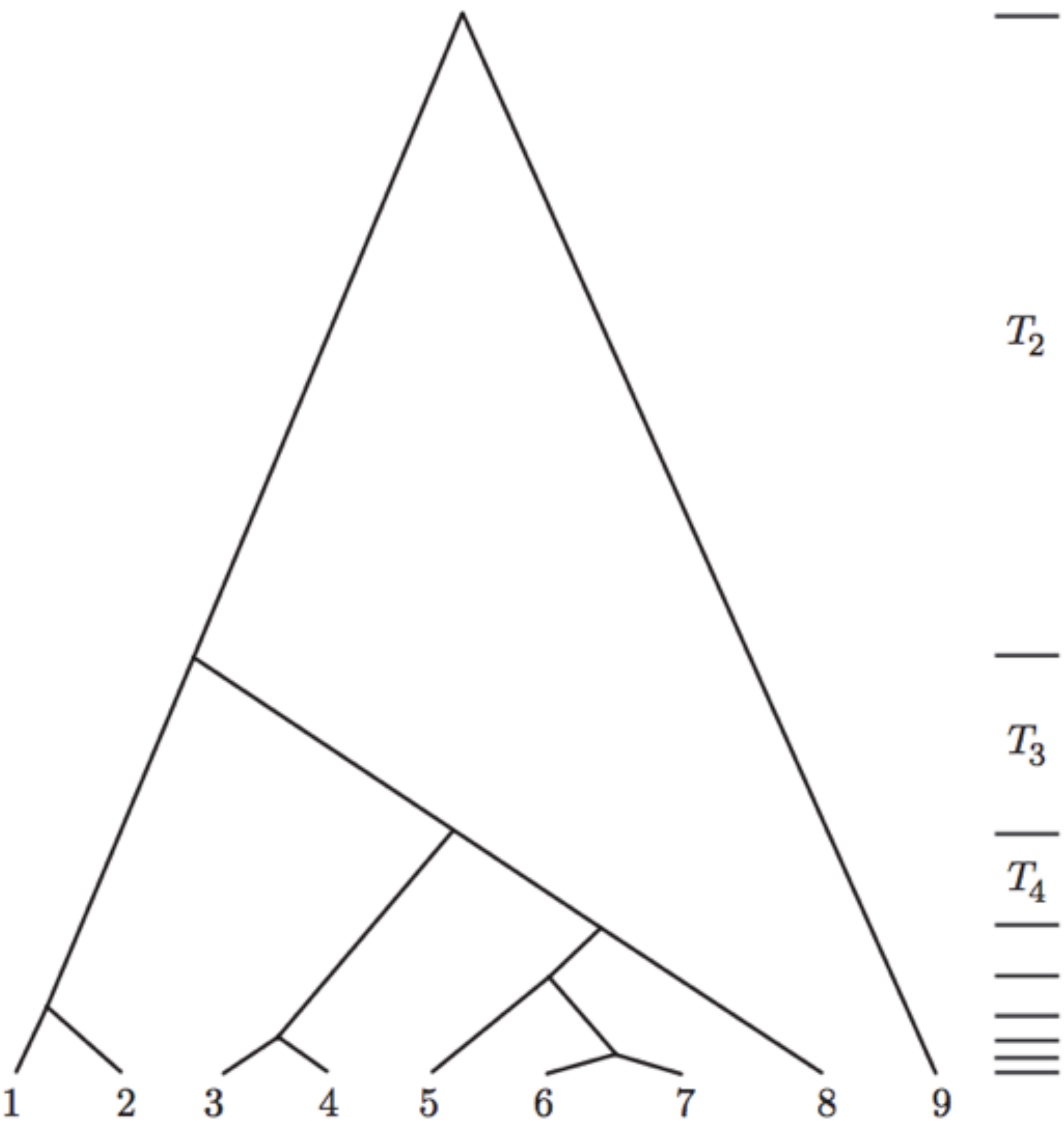

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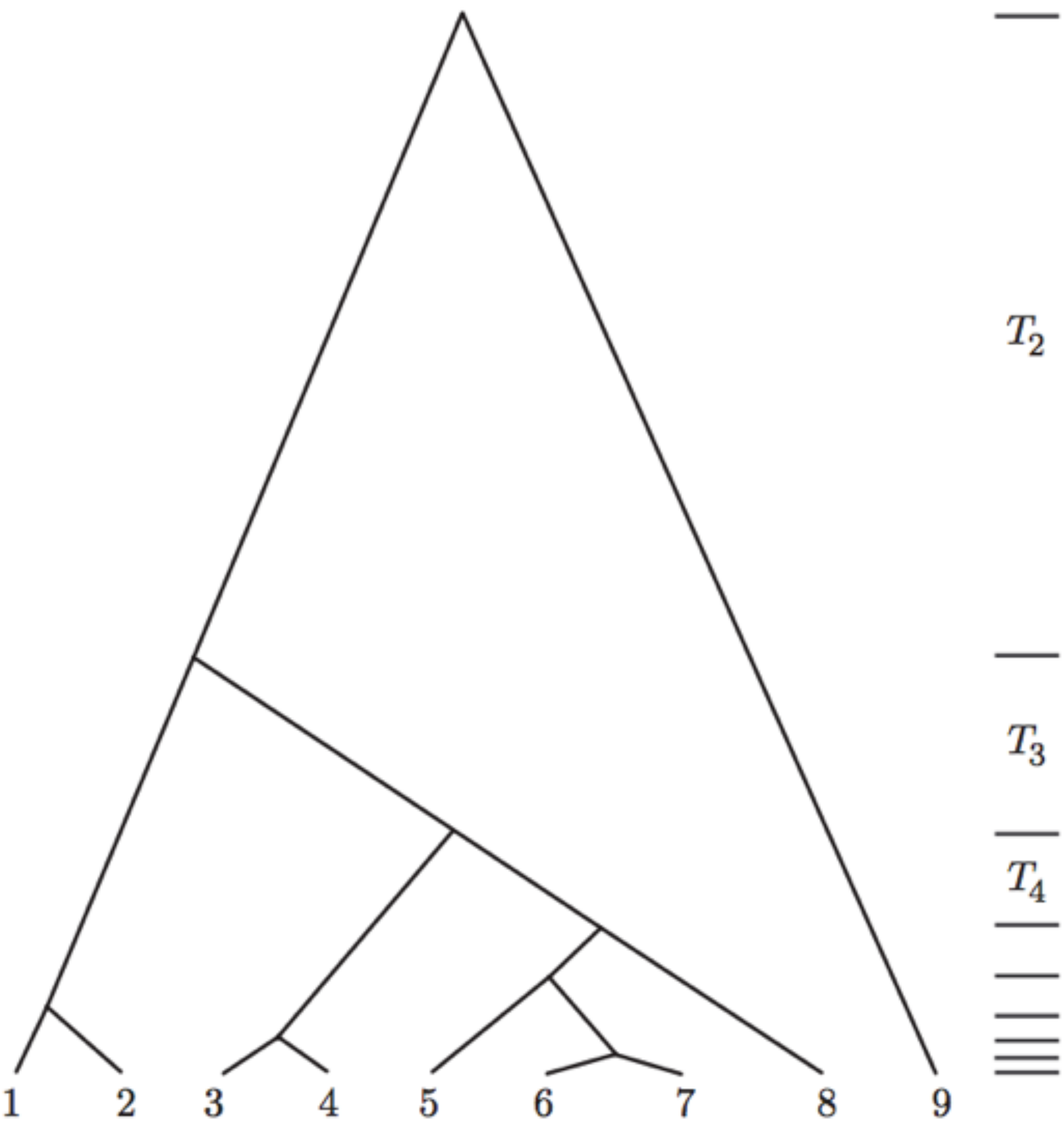

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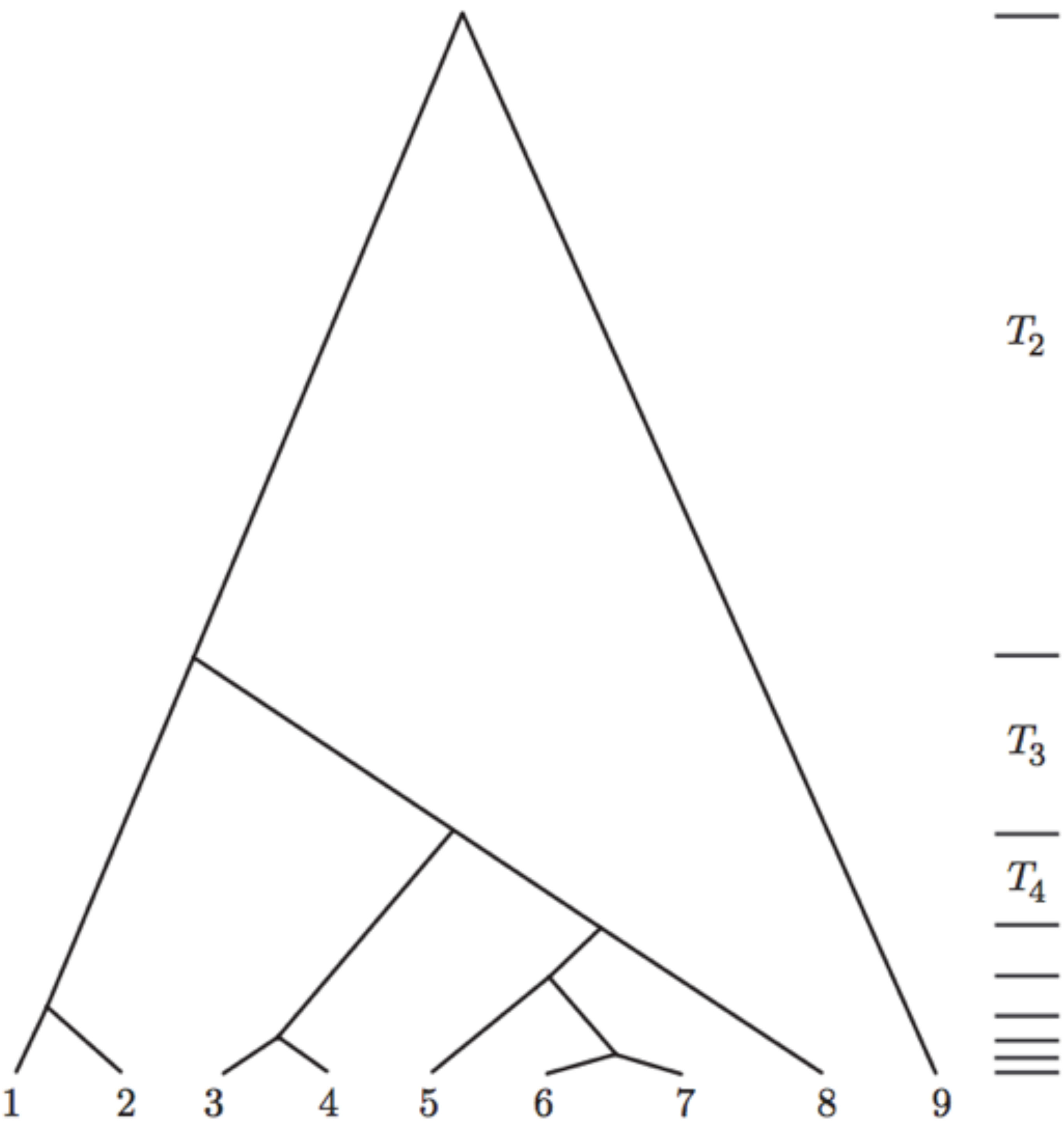
[Kingman 1982

Genealogy, trees, beyond trees



[Wakeley 2008]

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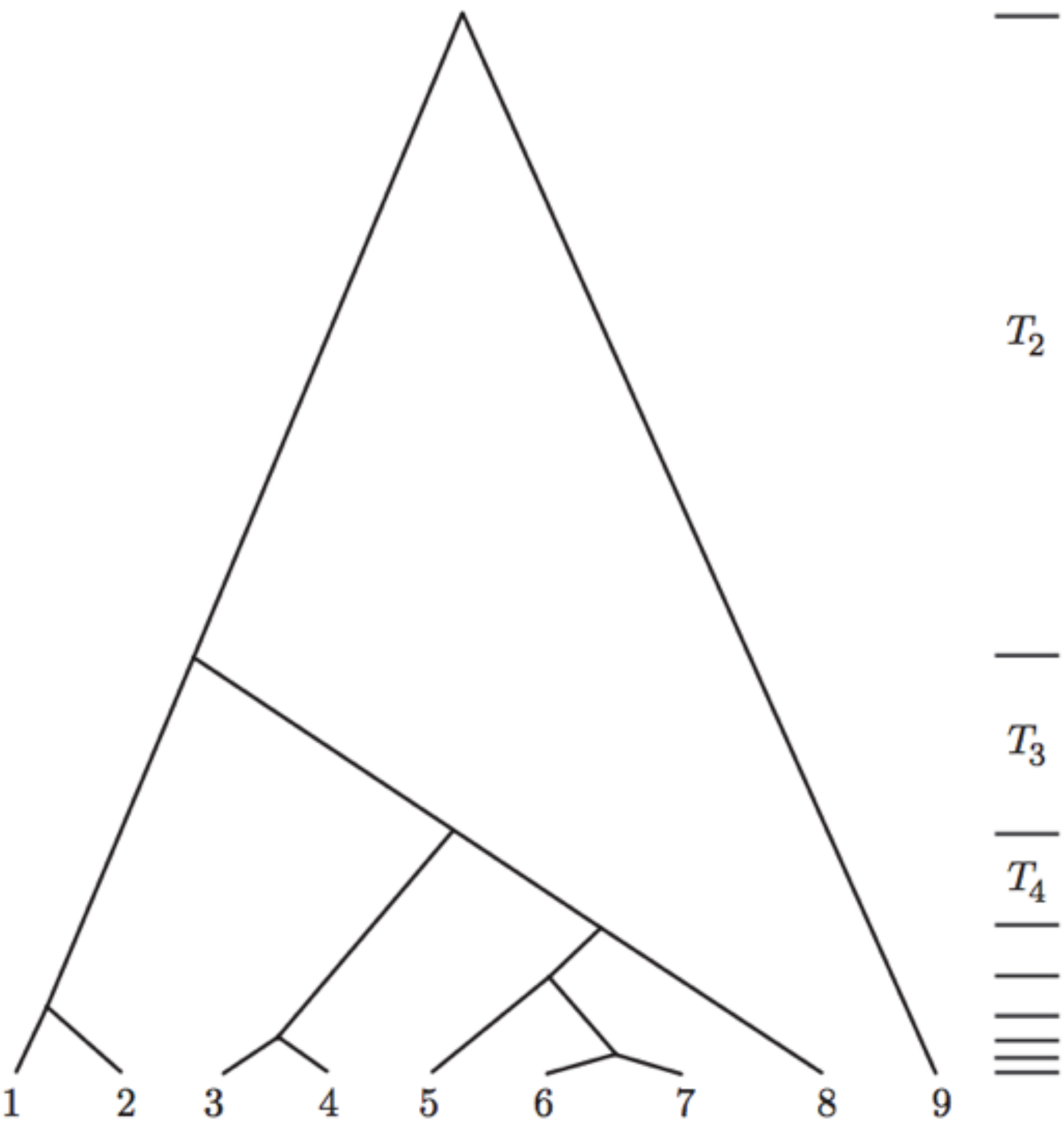

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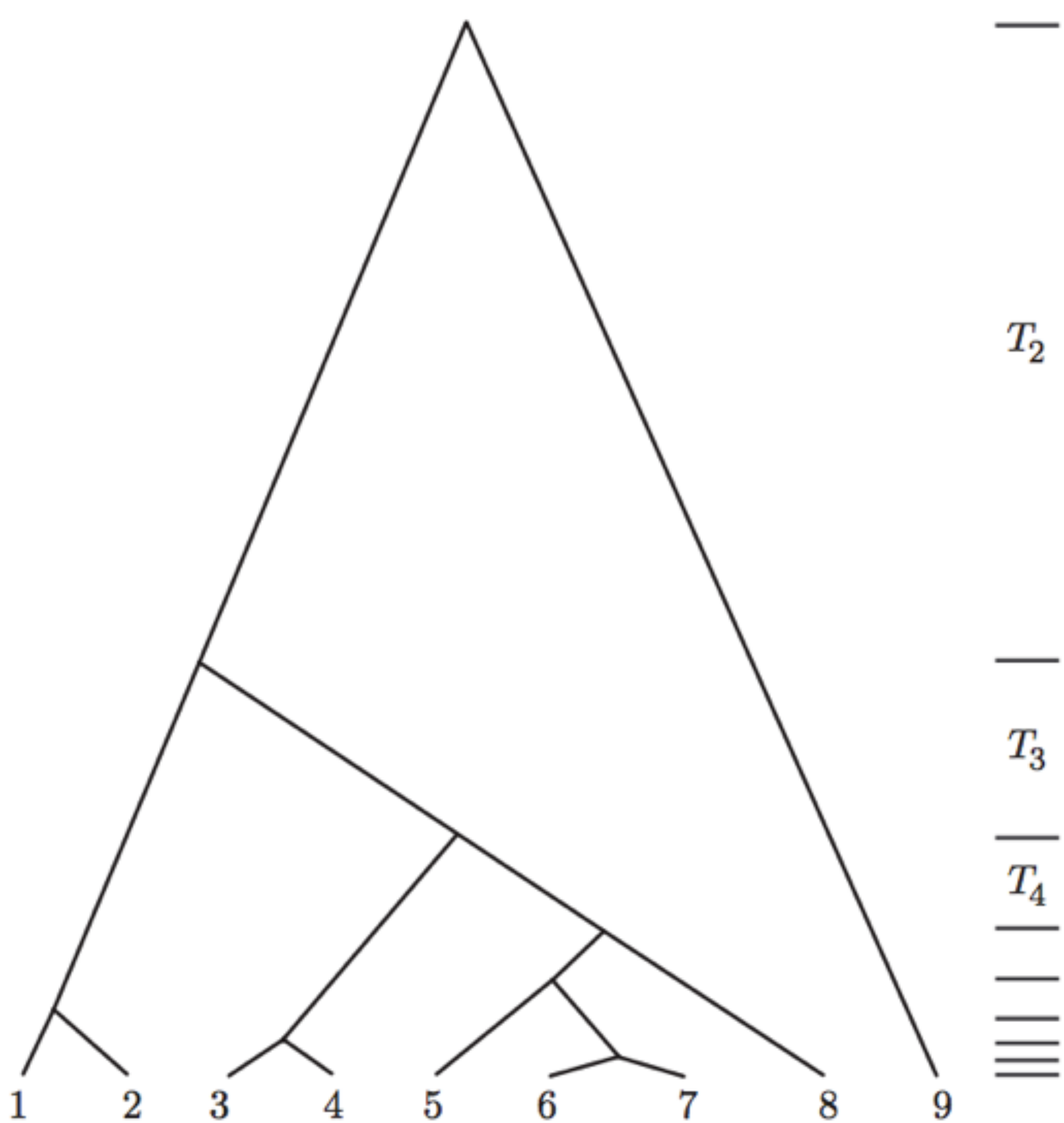

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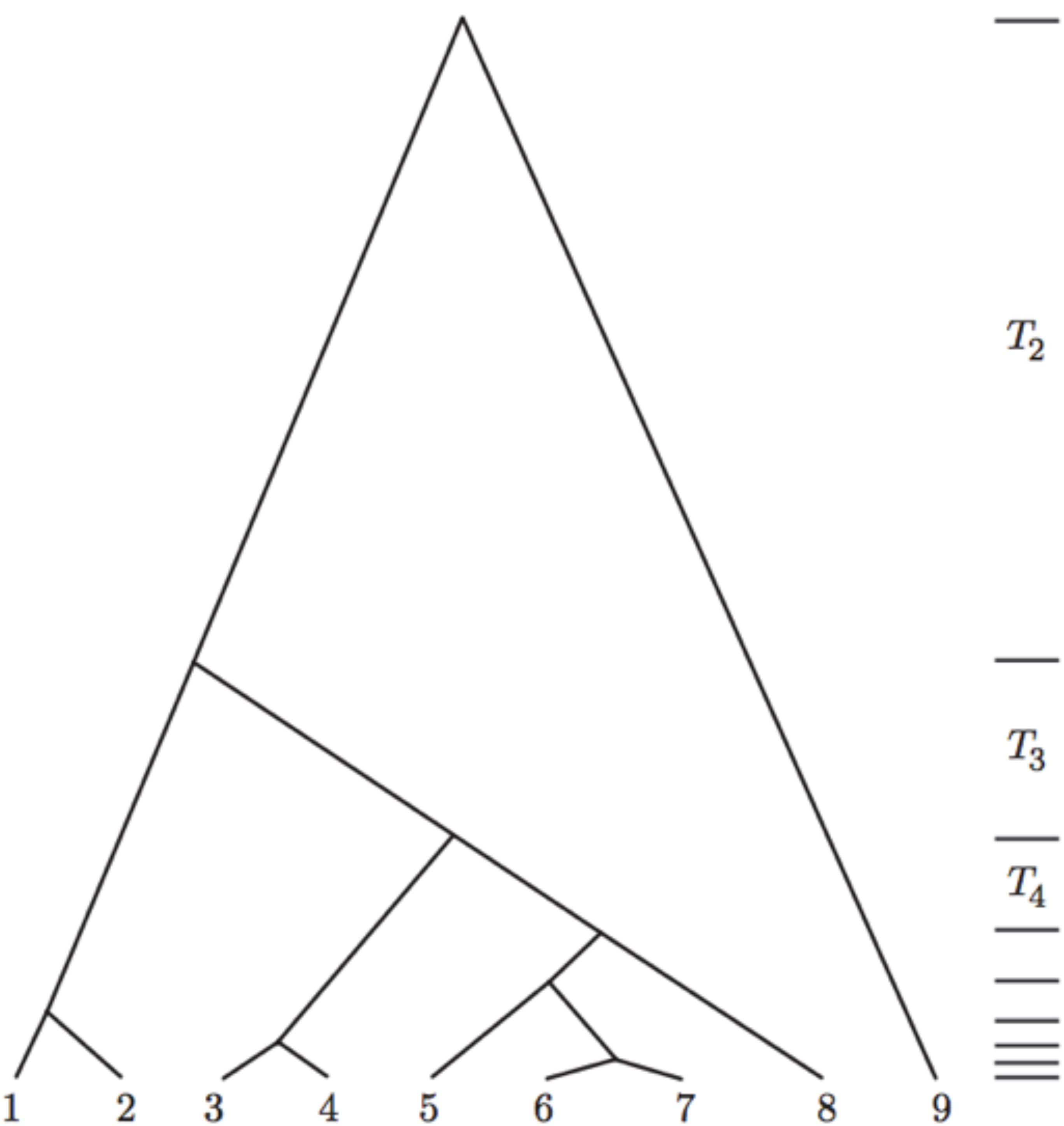

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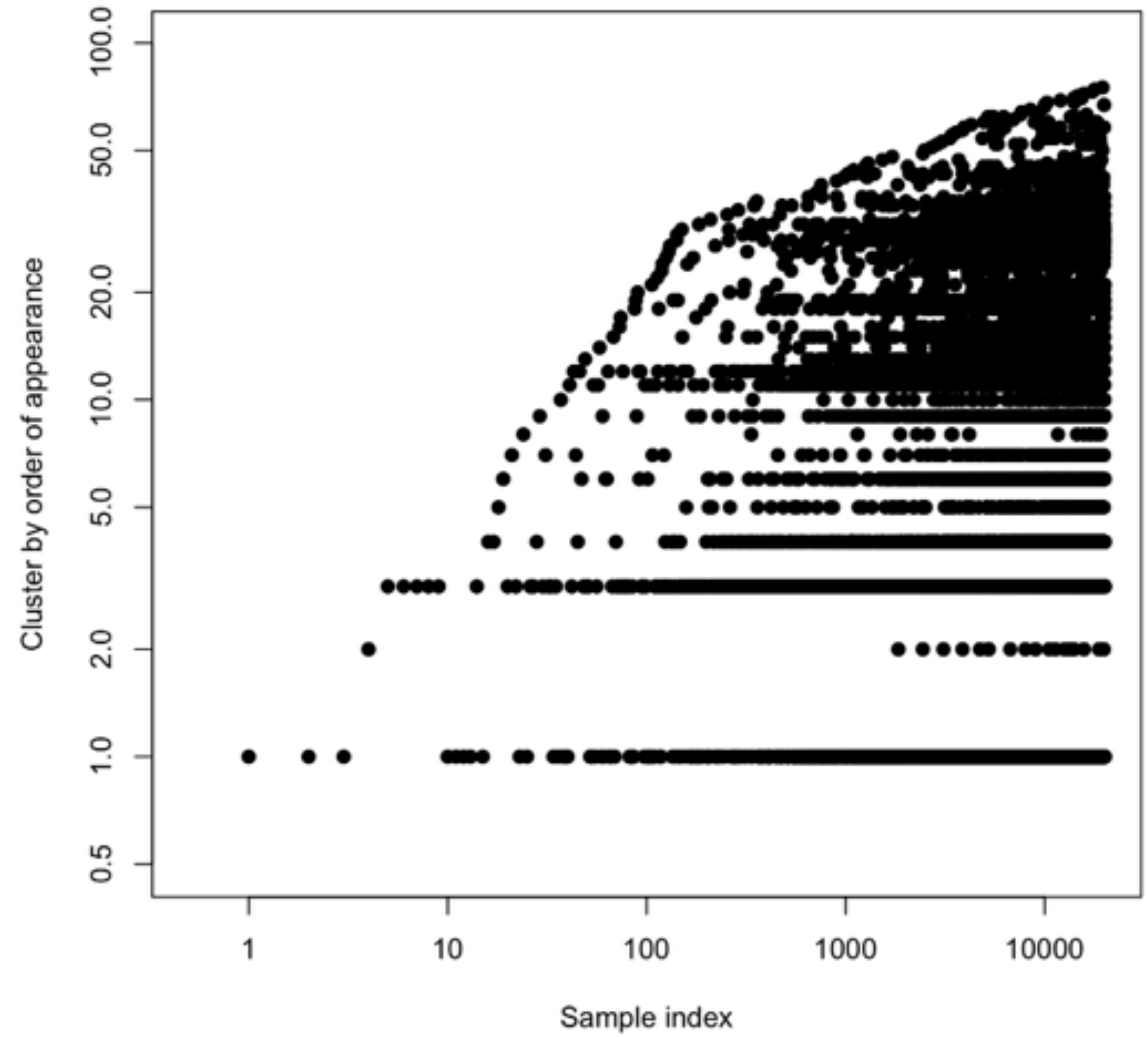
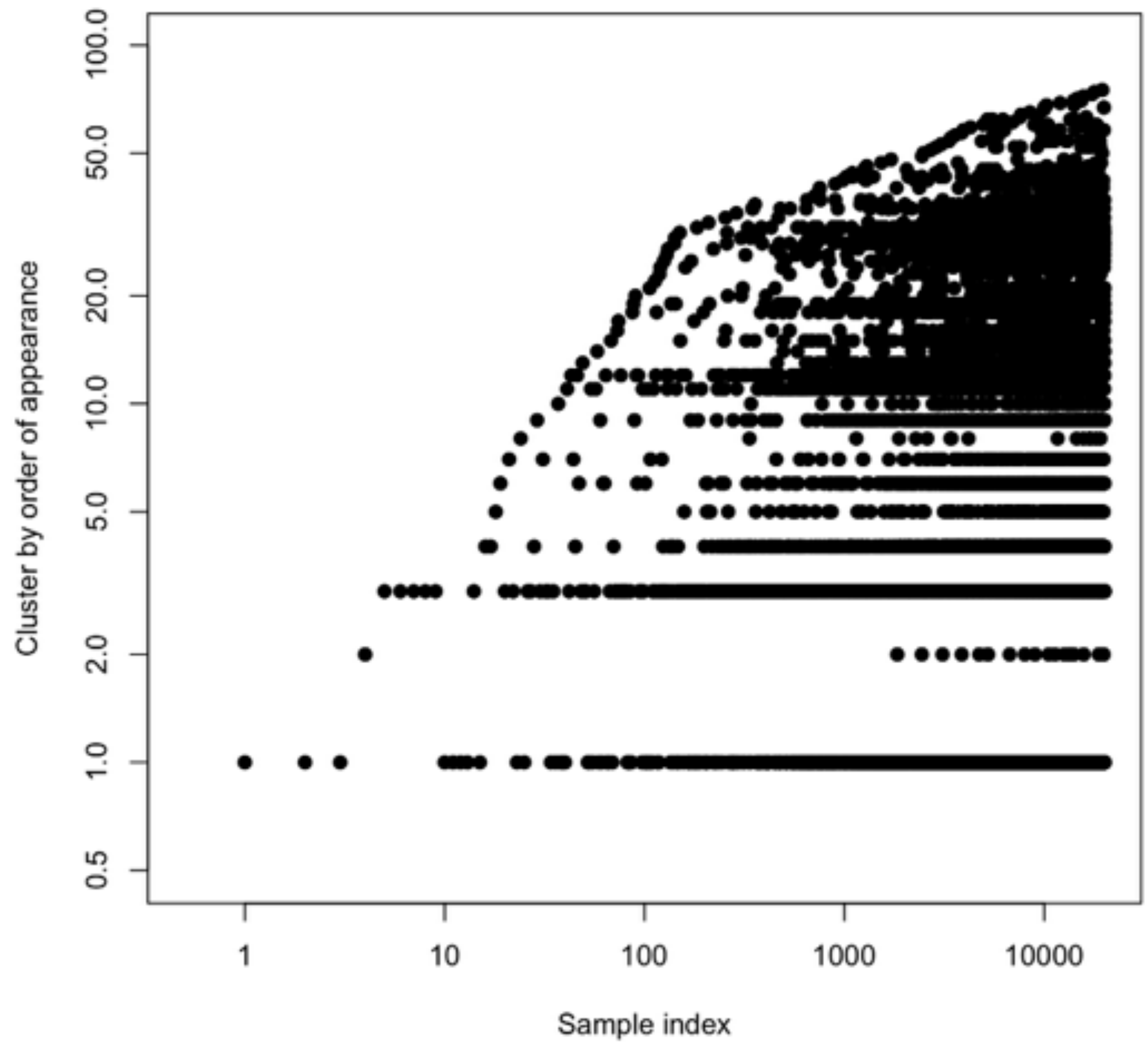
Power laws



Cluster by order of appearance
0.5
1.0
2.0
5.0
10.0
20.0
50.0
100.0
1
10
100
1000
10000
Sample index

Power laws

- K_N := # clusters occupied by N data points



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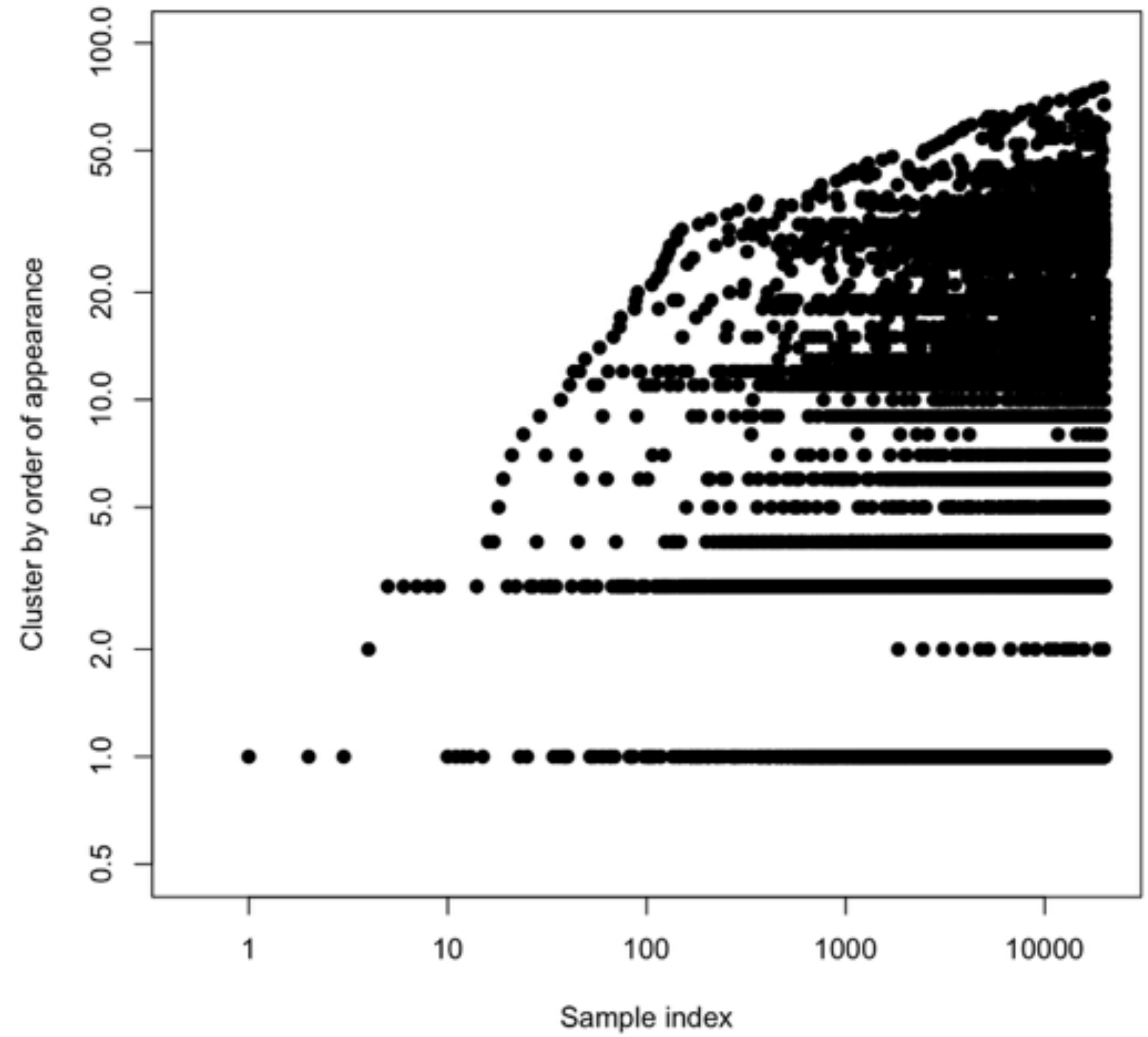

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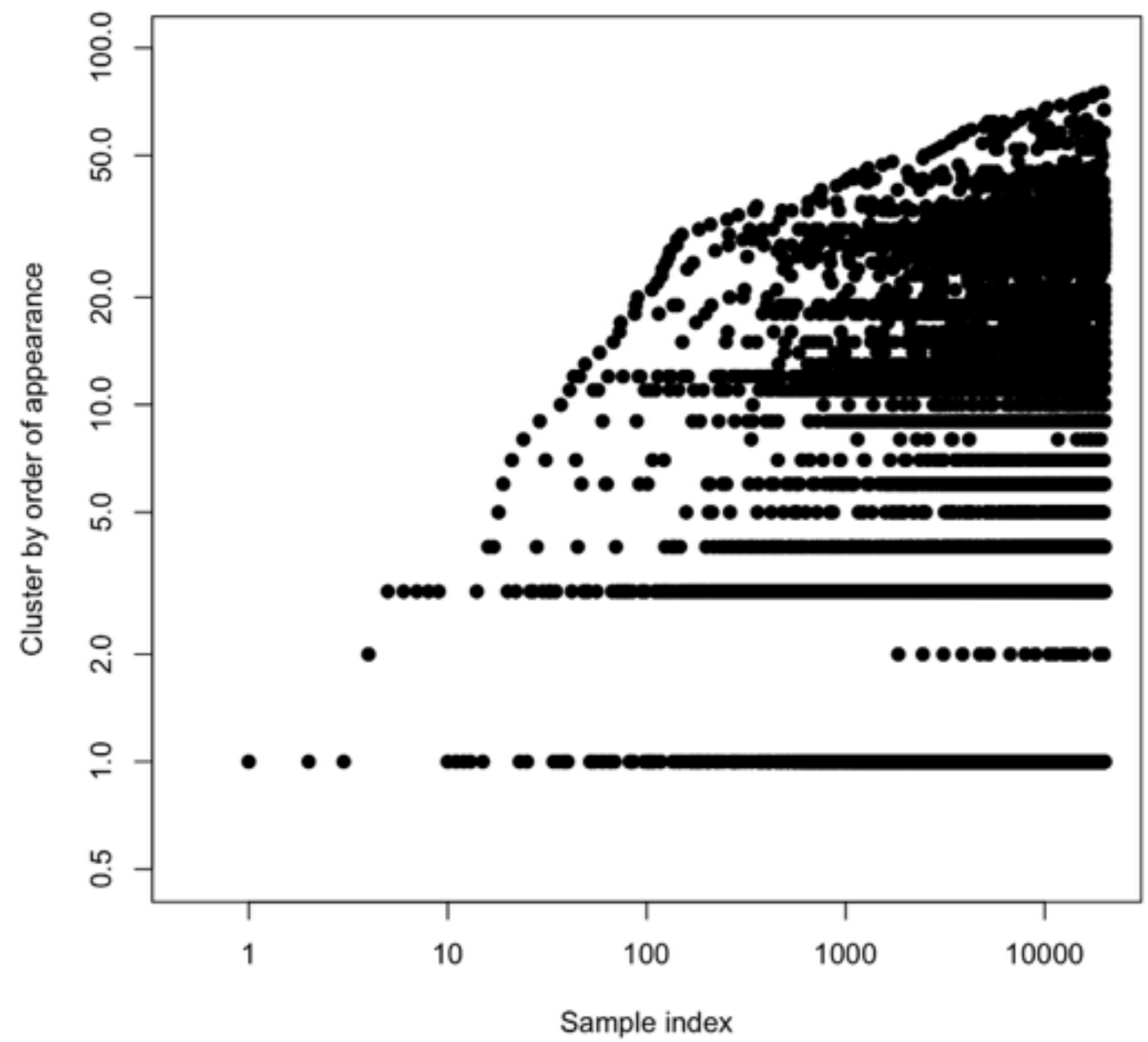

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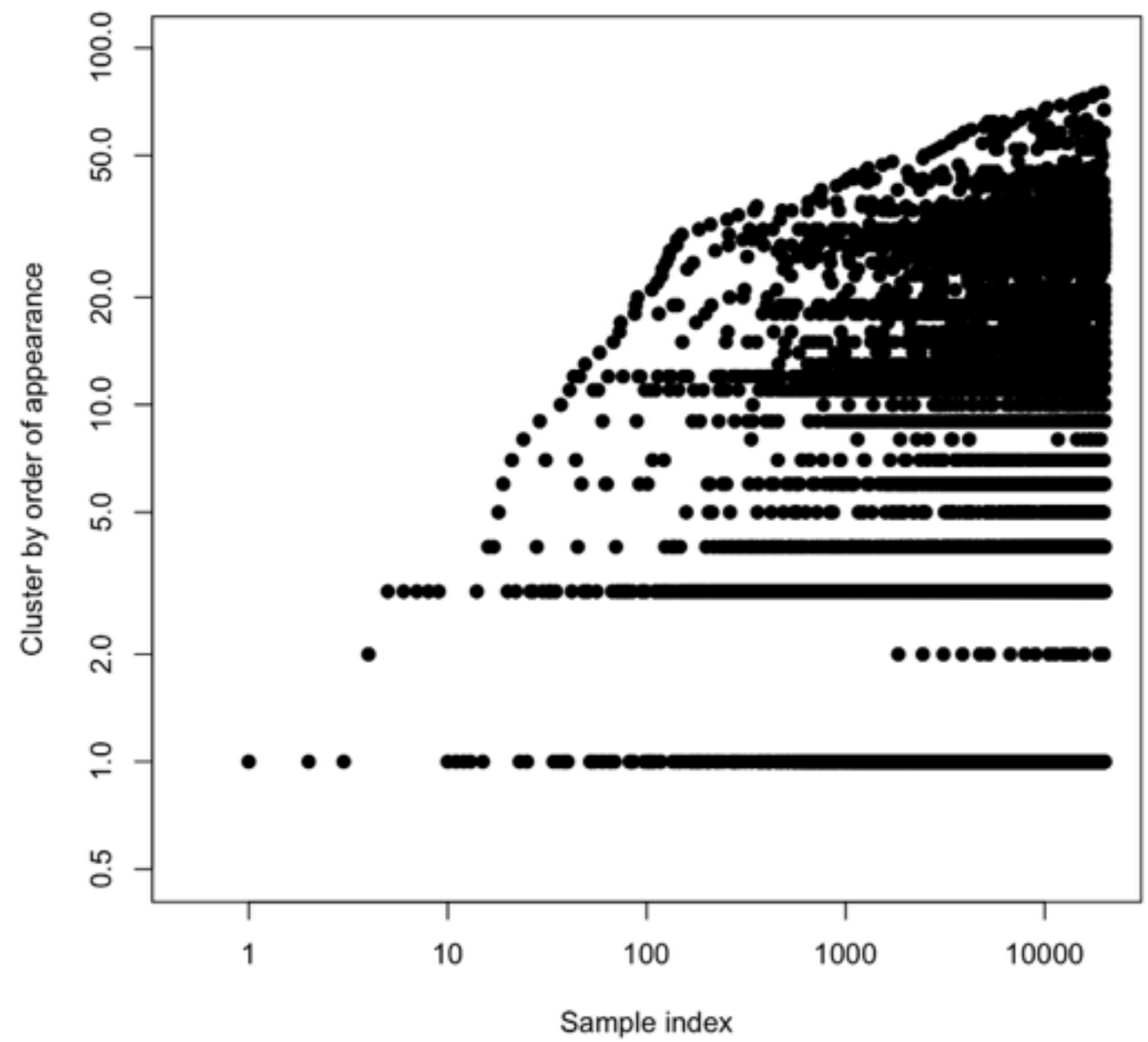

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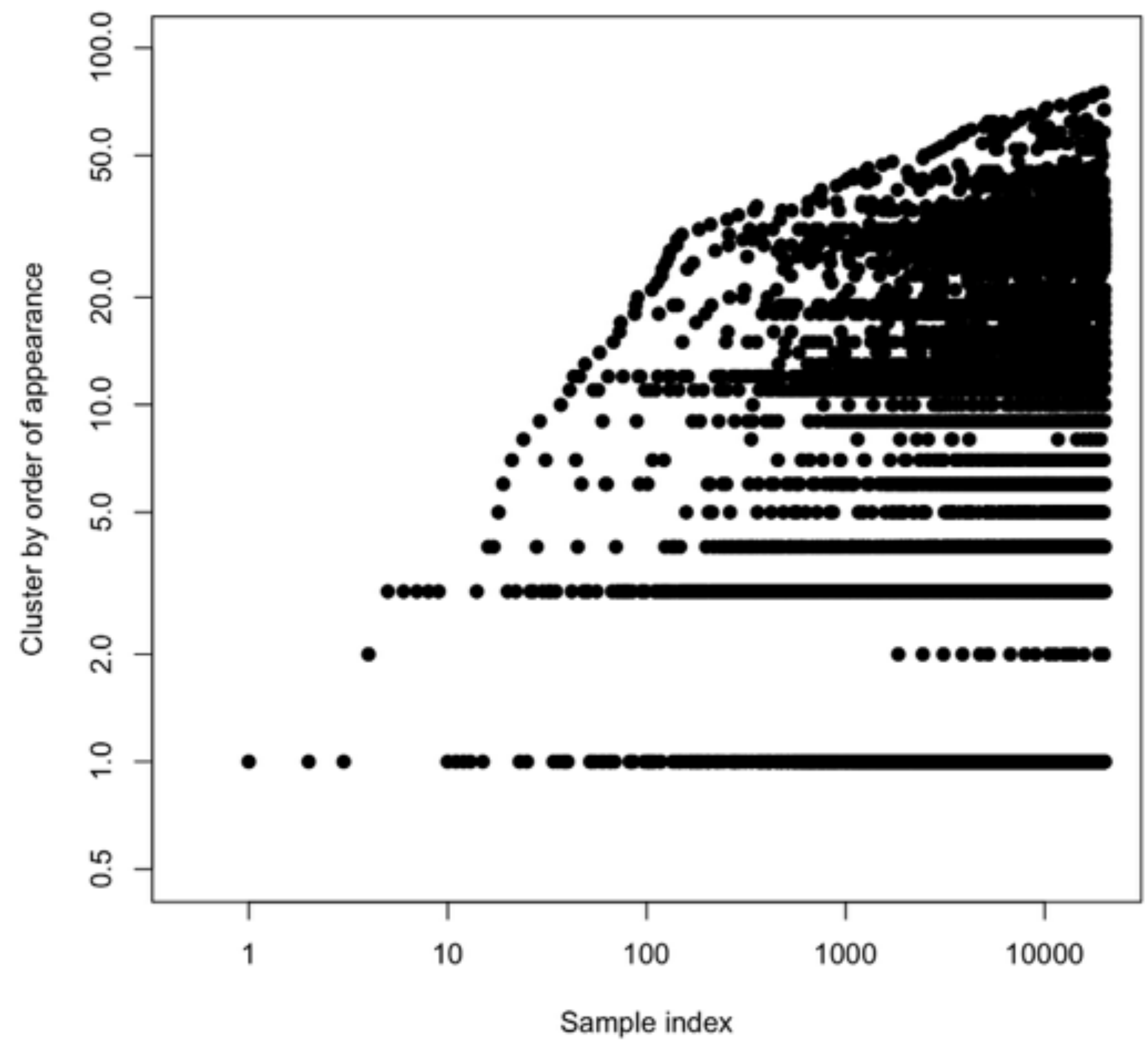

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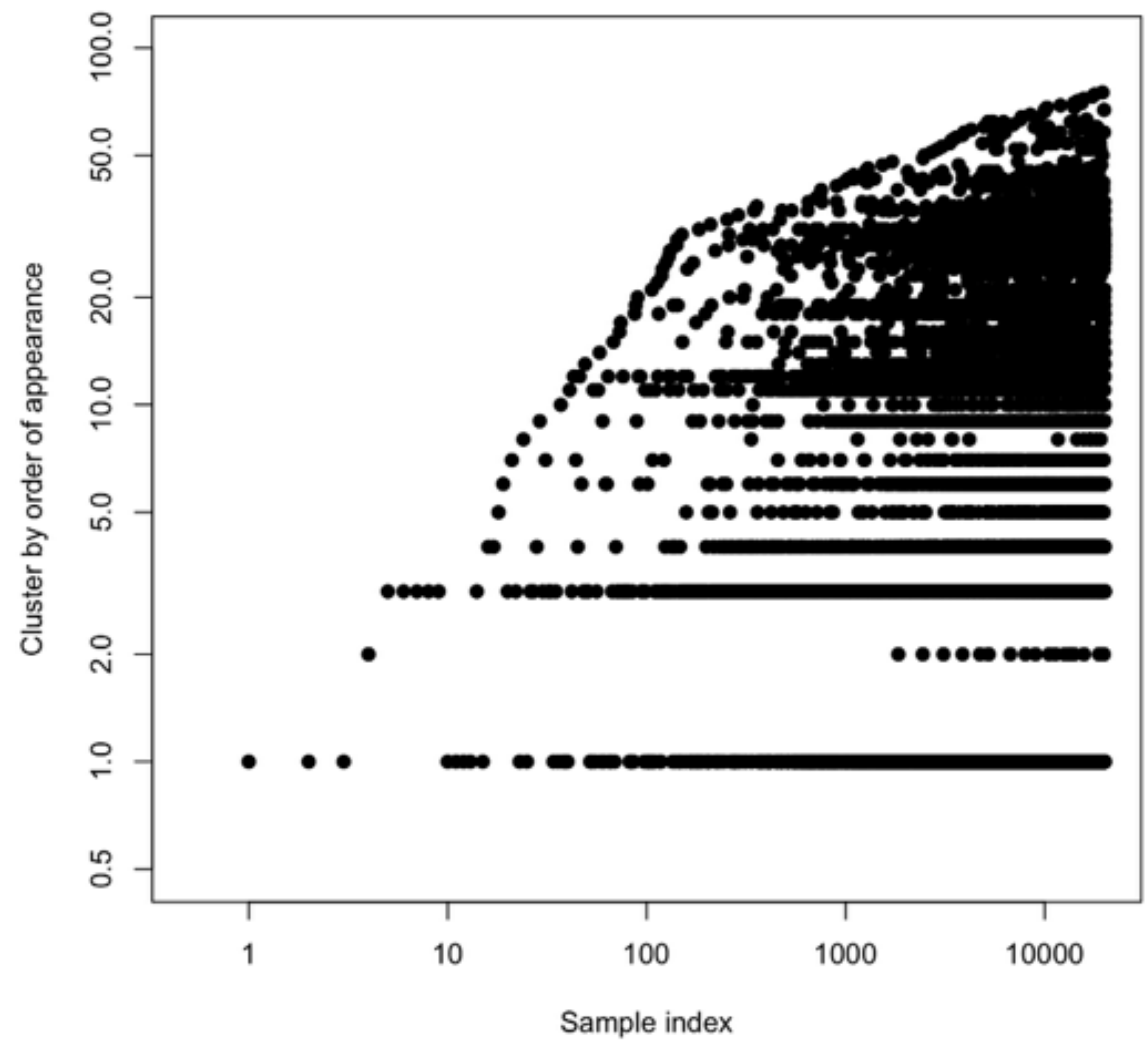

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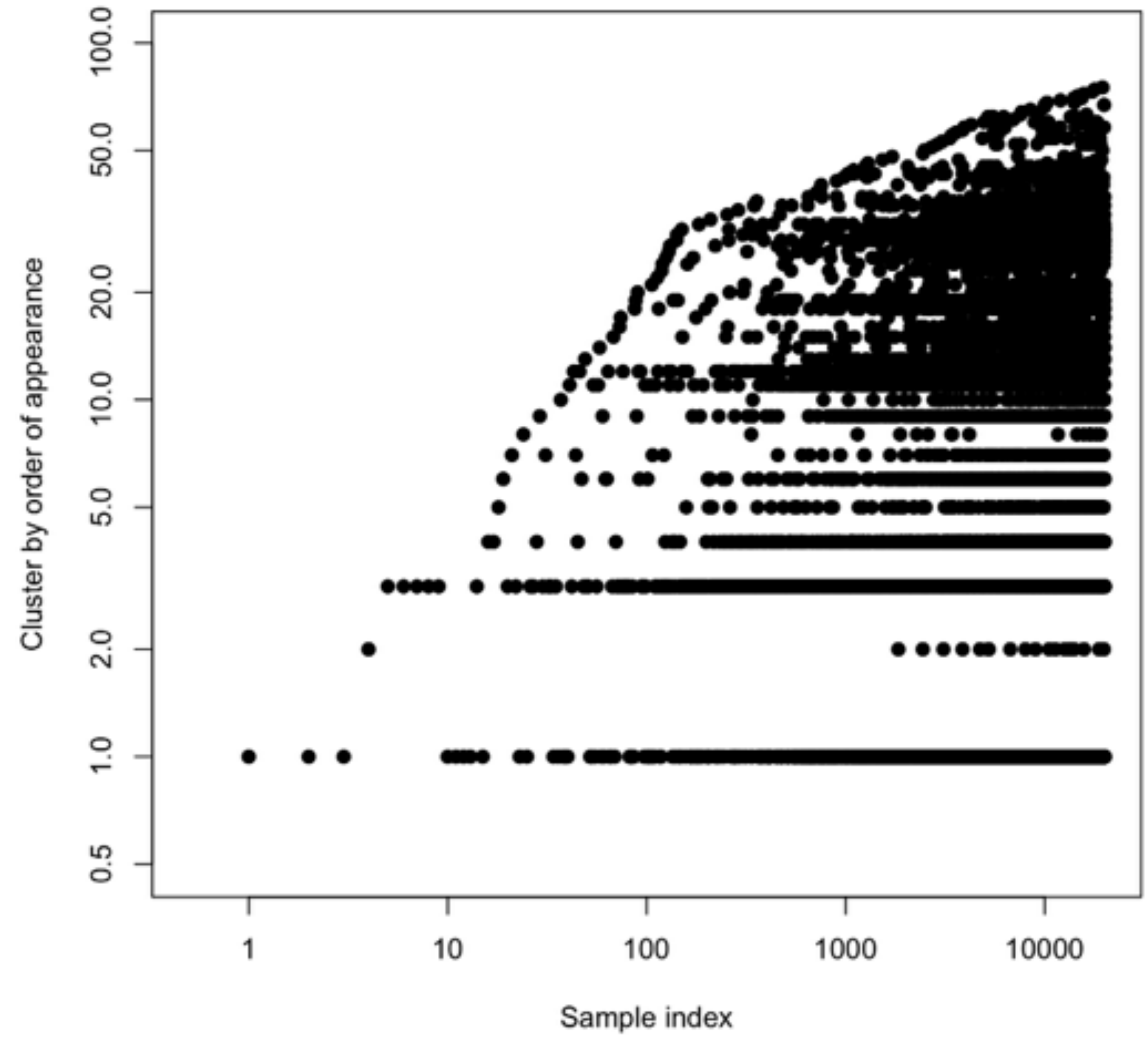

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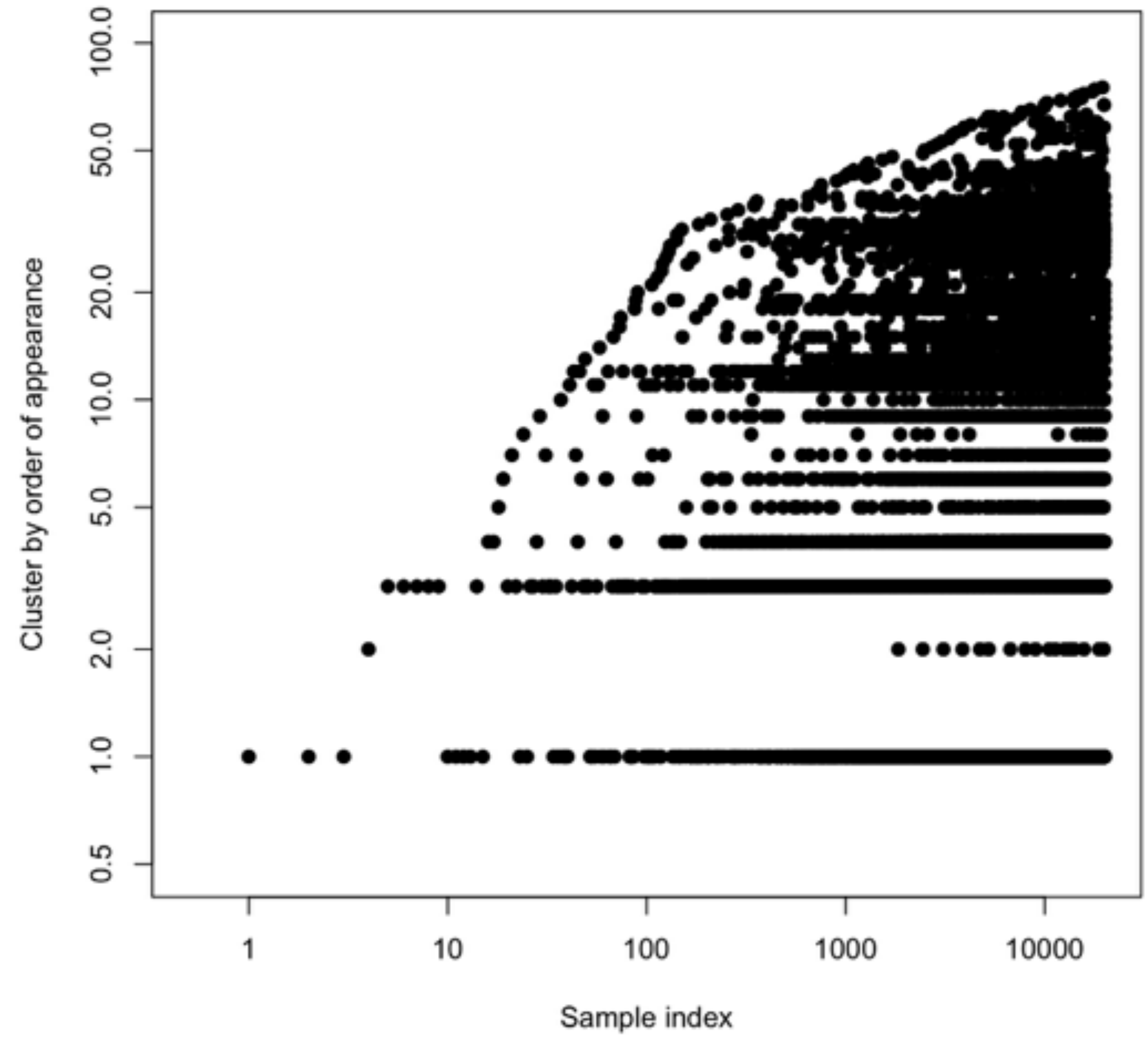

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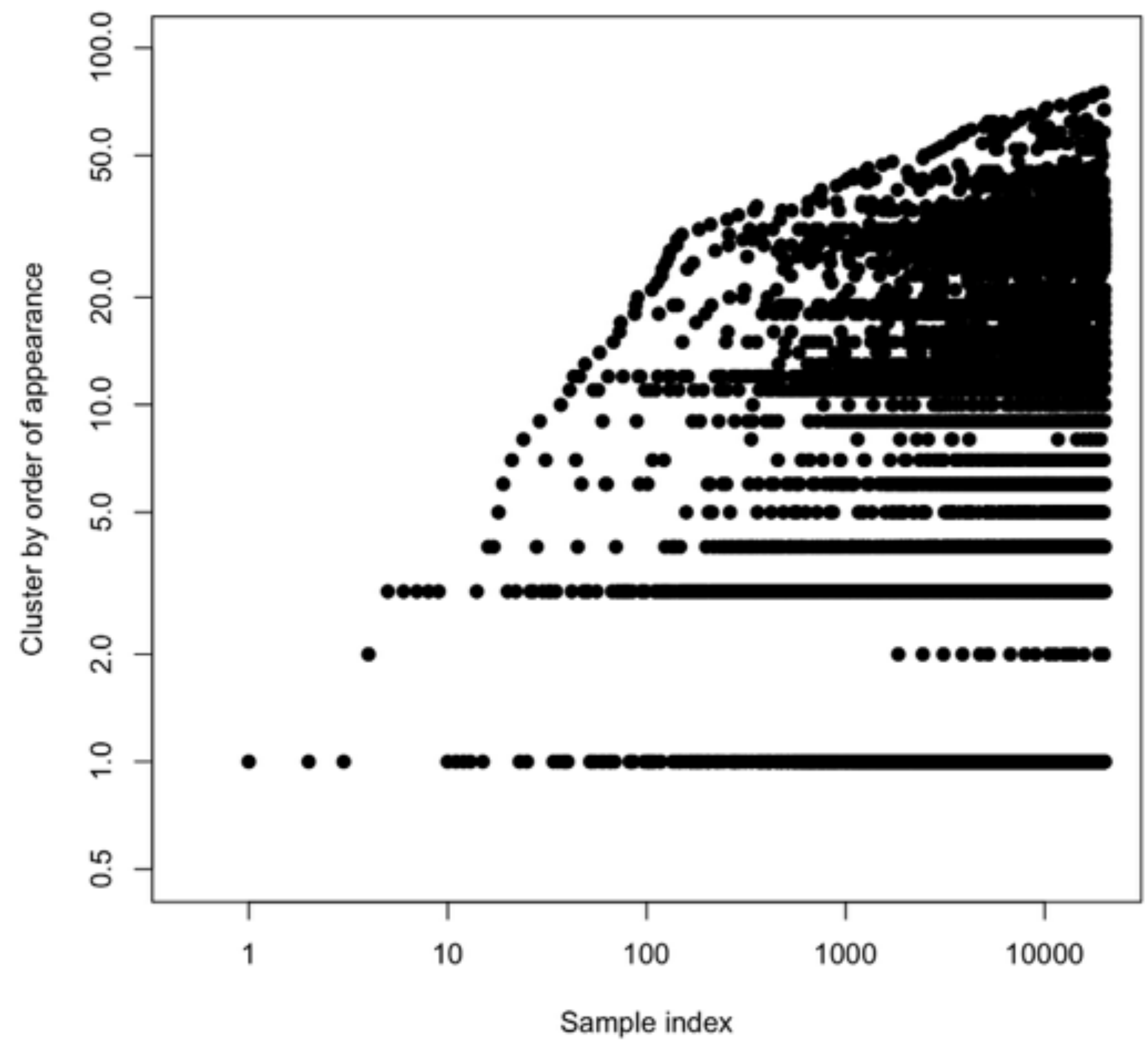

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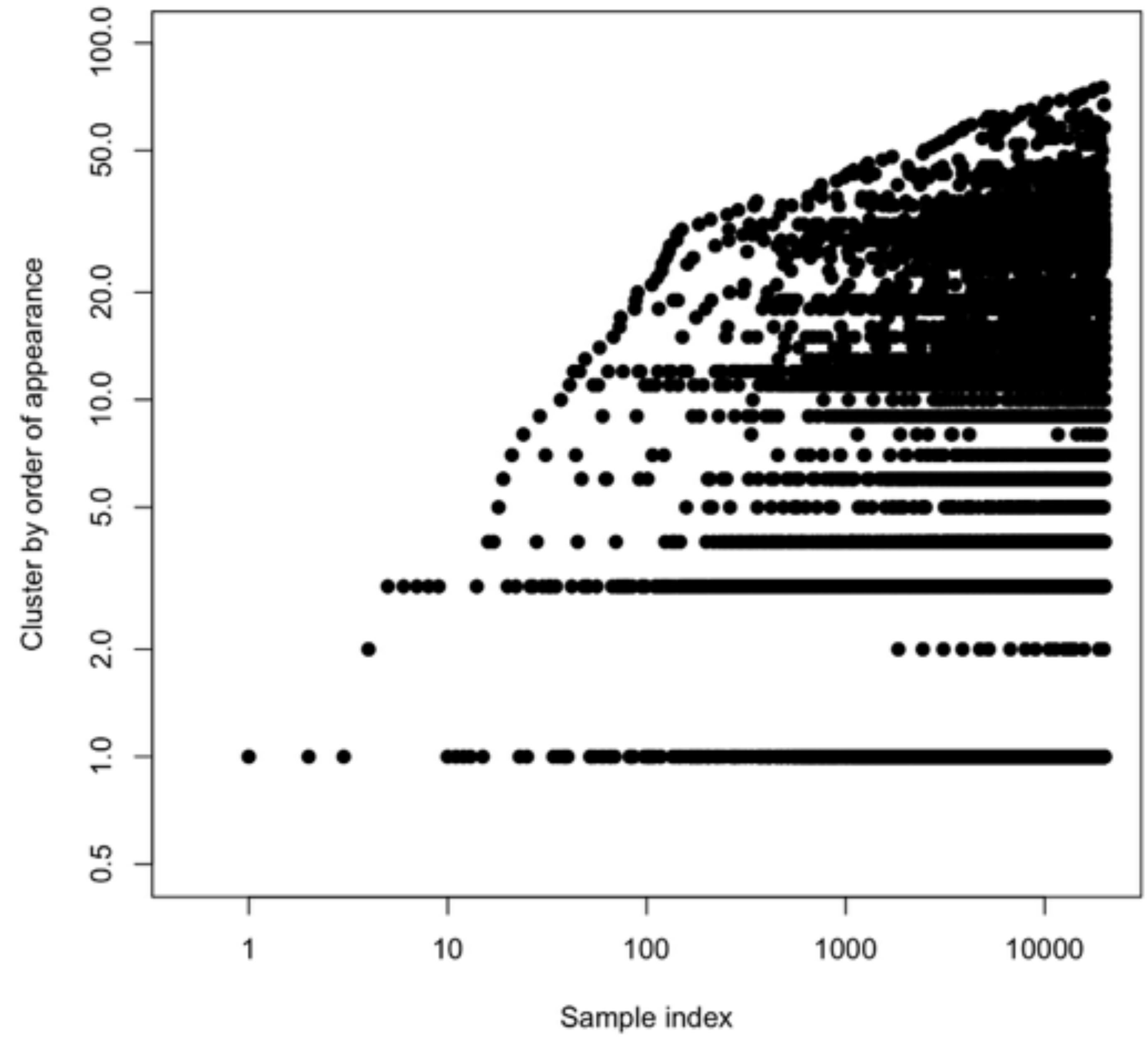

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Hierarchies

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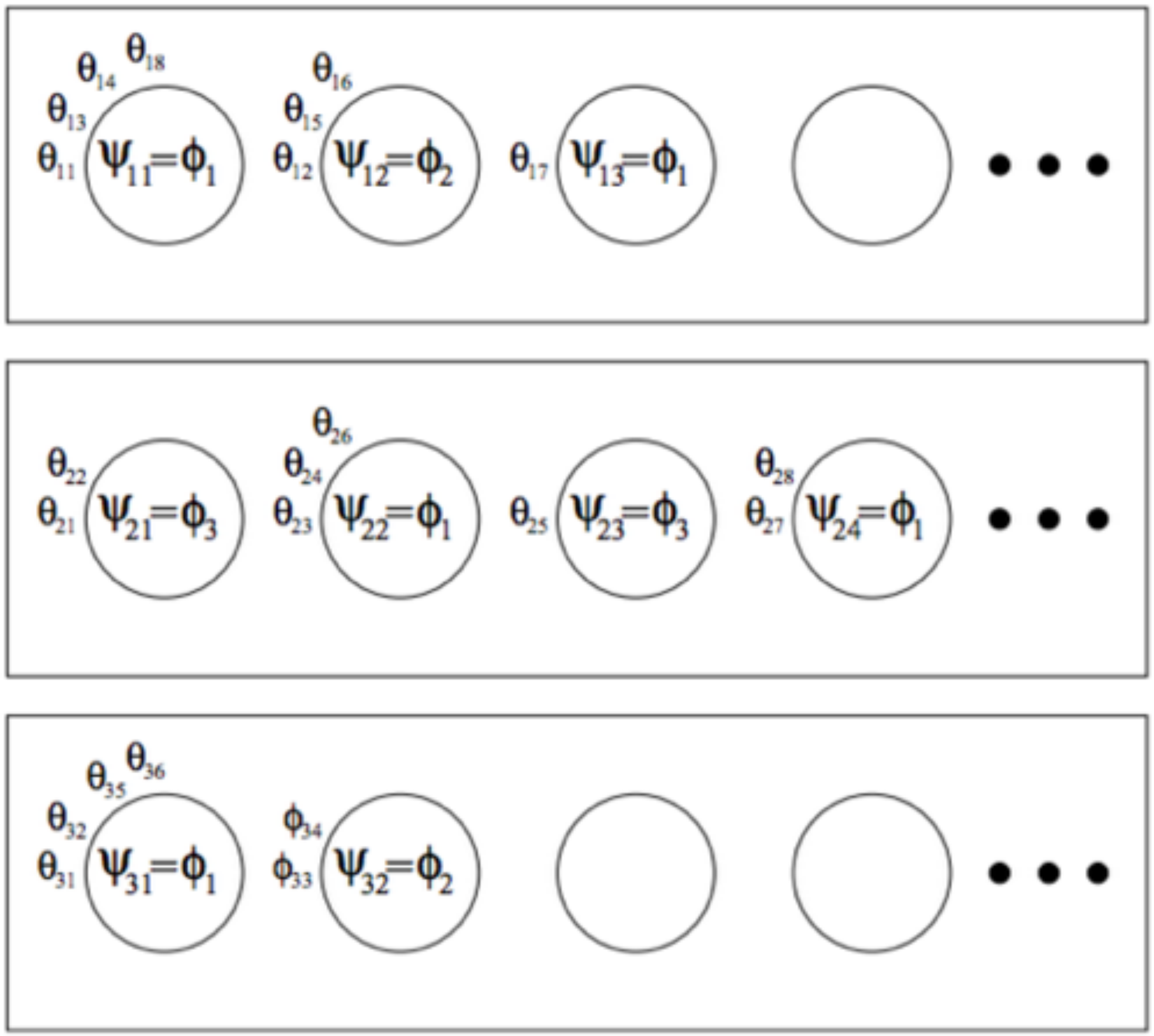

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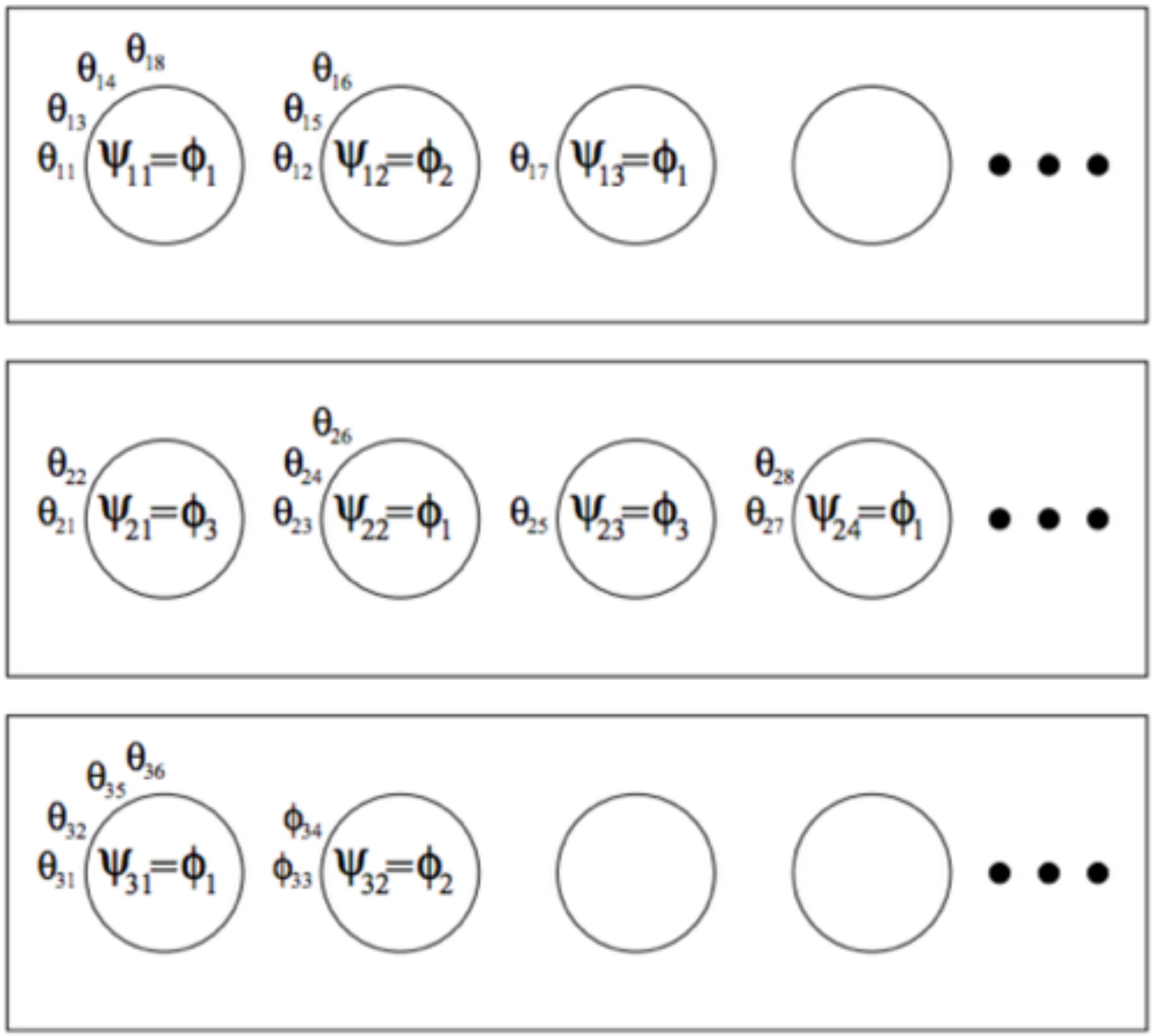

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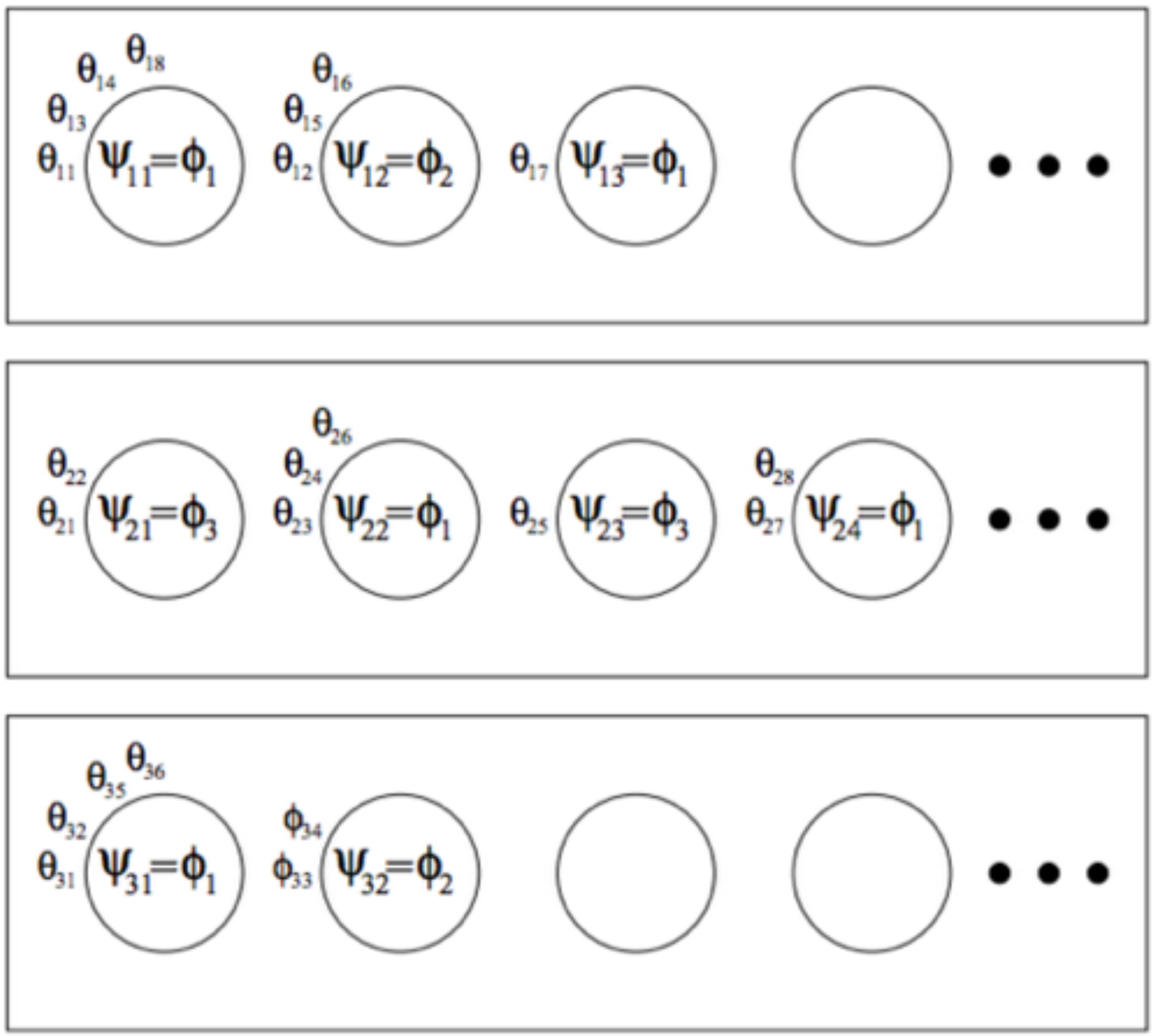

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De Finetti mixing measures

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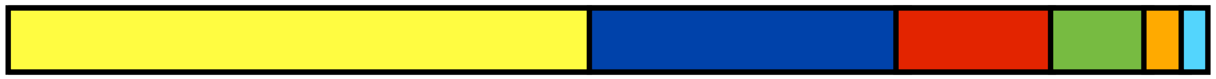

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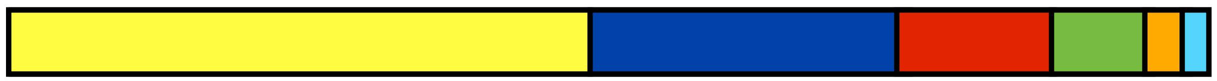

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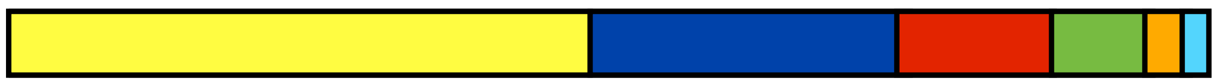

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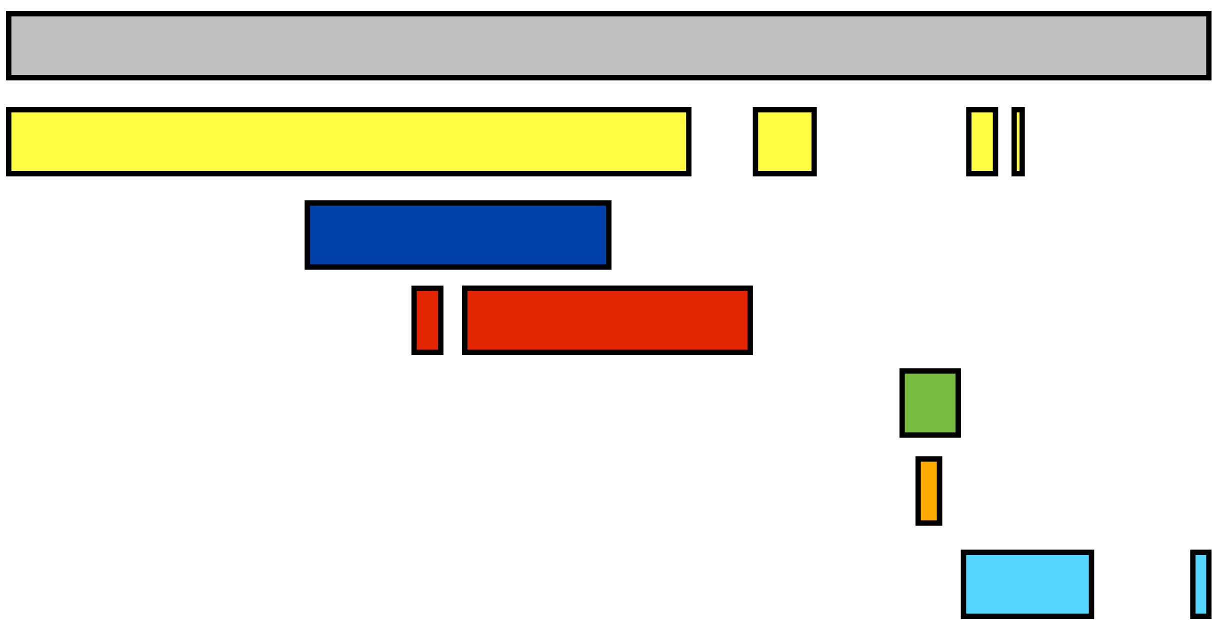

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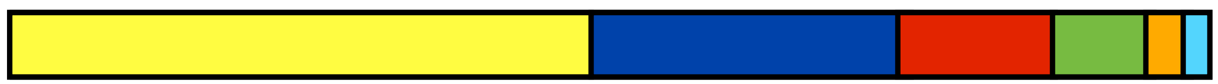

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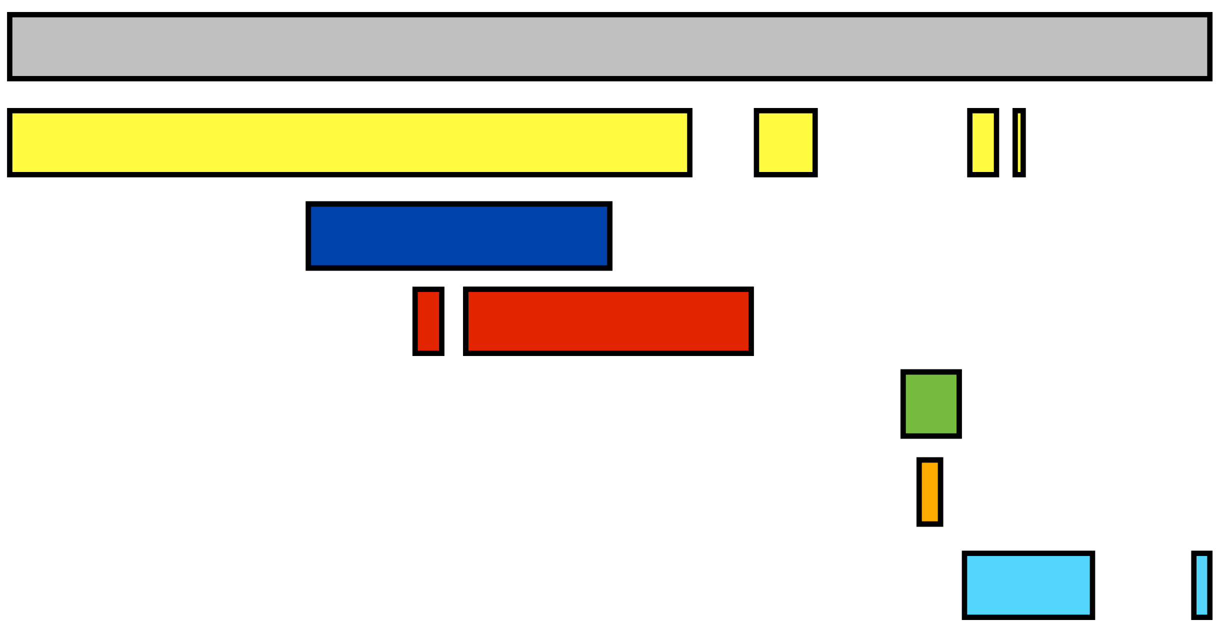

[Kingman 1978, Broderick, Pitman, Jordan 2013]

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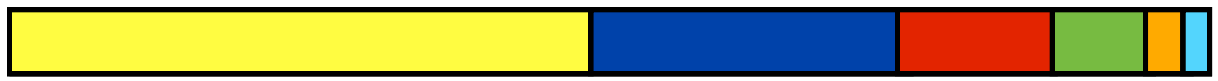

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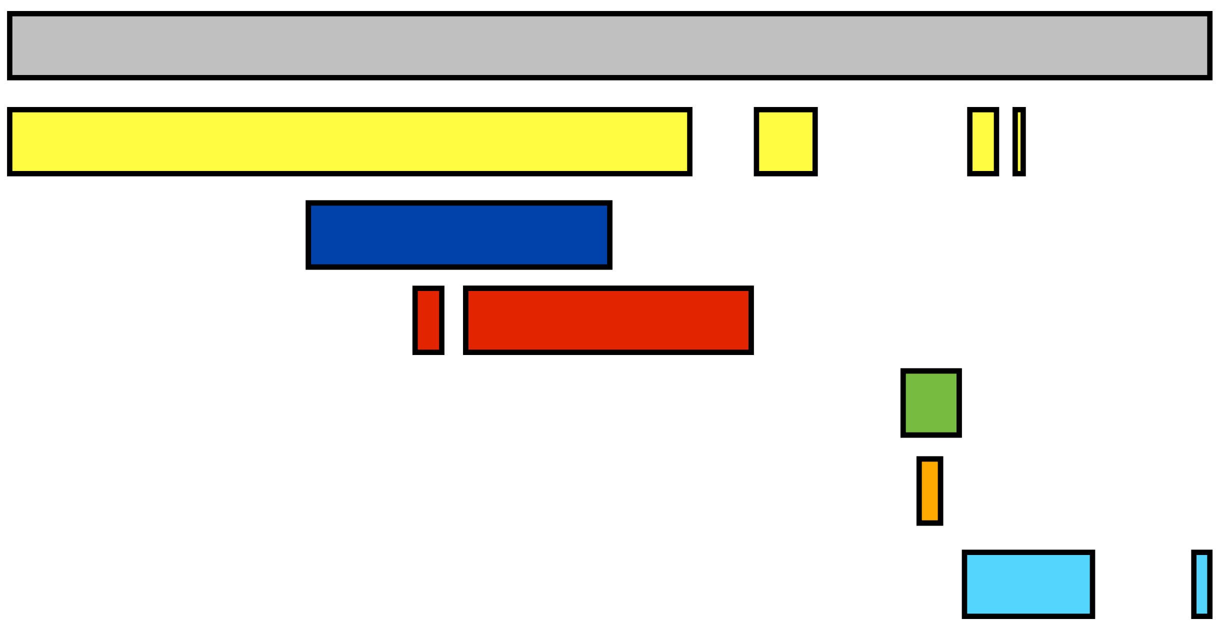
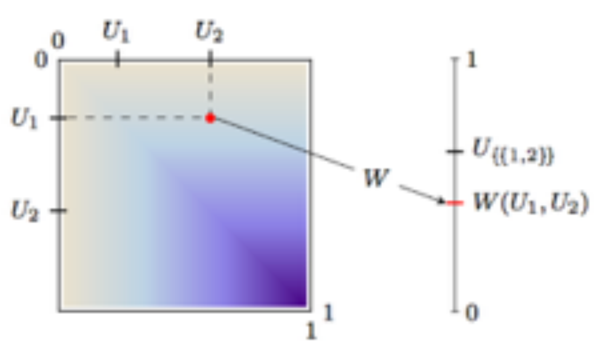
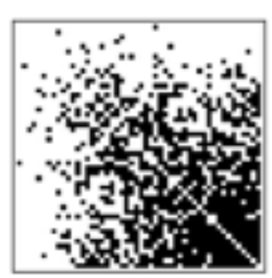
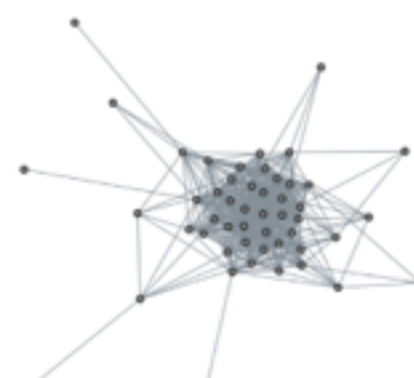


- Graphs/networks: Aldous-Hoover theorem







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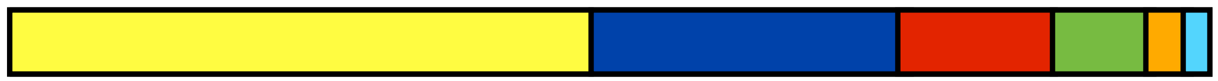

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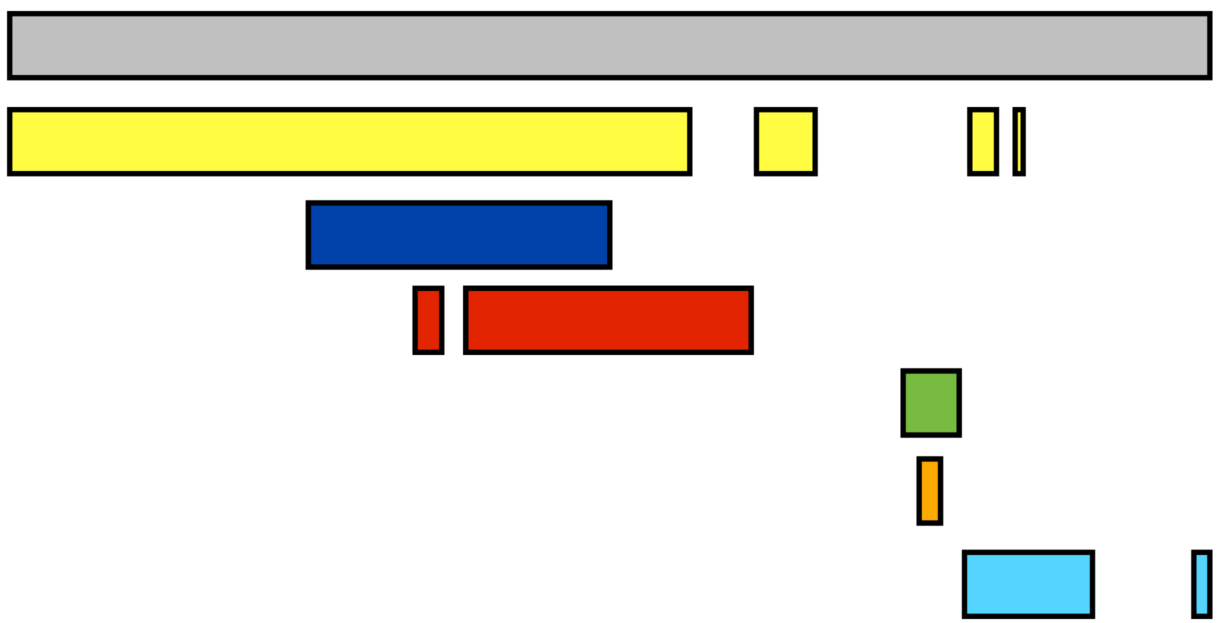

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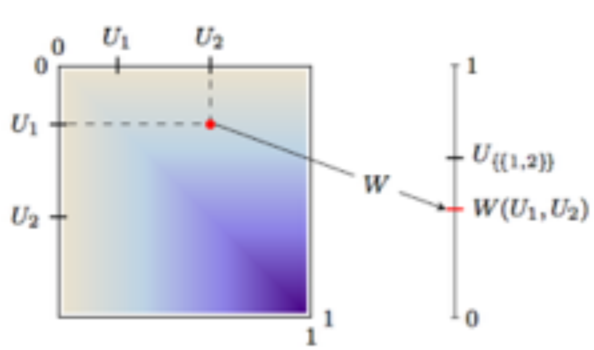

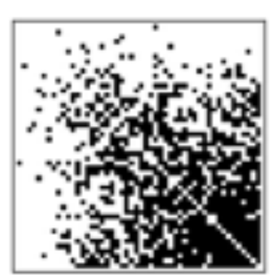

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[Kingman 1978, Broderick, Pitman, Jordan 2013, Aldous 1983, Hoover 1979, Orbanz, Roy 2015]

Conjugacy & Poisson point processes

Orbanz 2009, Orbanz 2010

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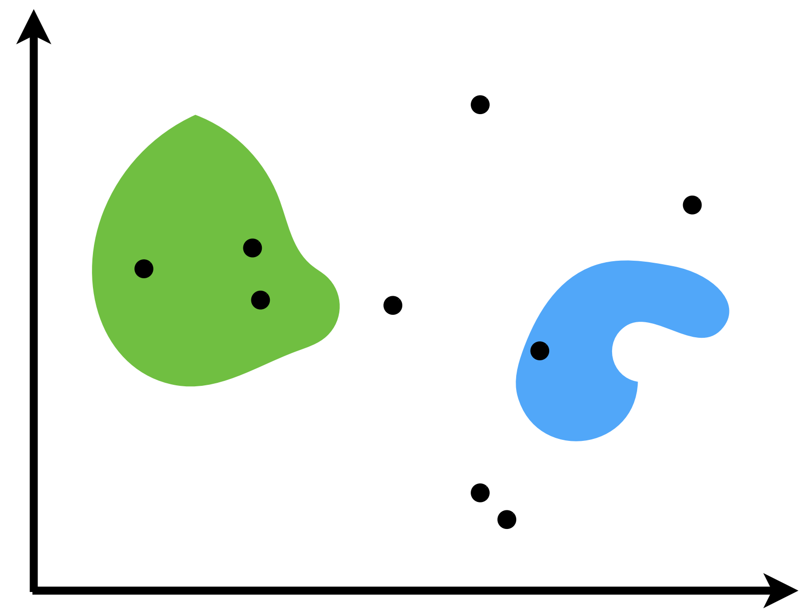

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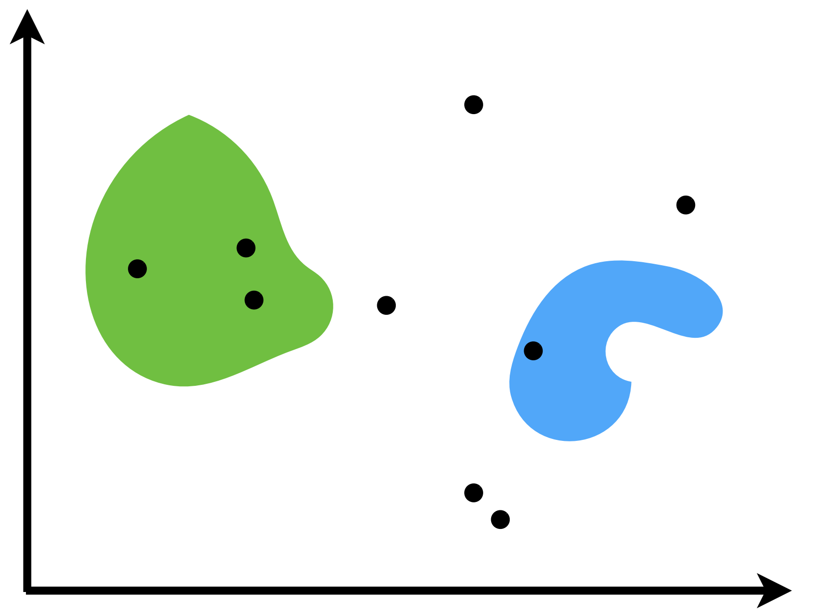

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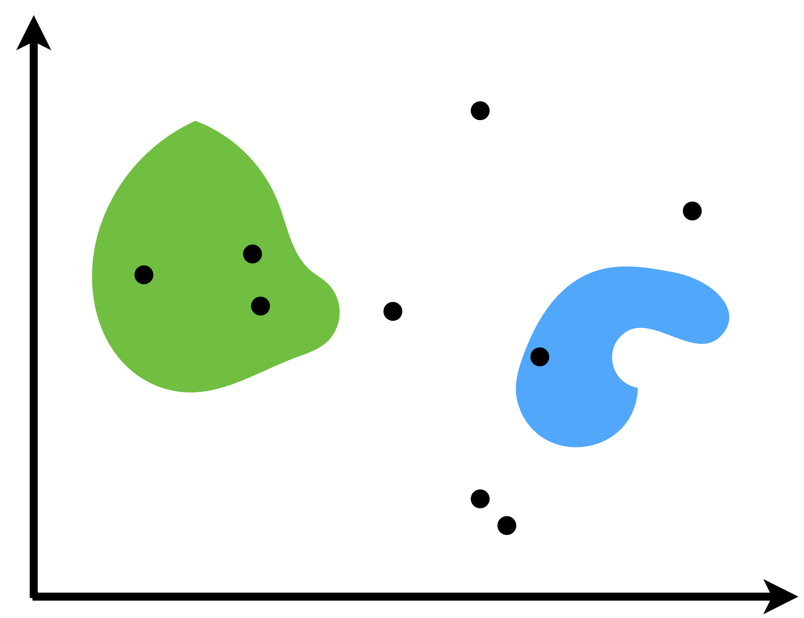

- Posteriors, conjugacy, and exponential families for completely random measures

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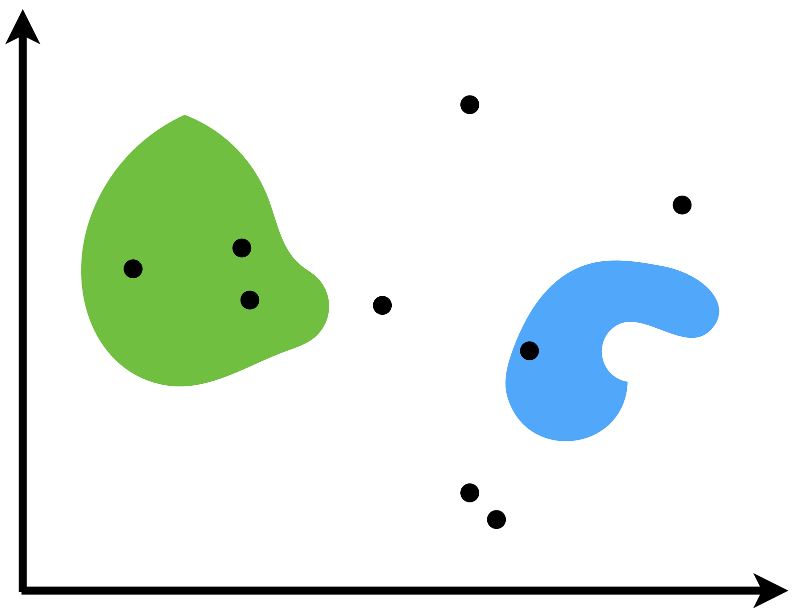

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[Kingman 1992, Orbanz 2009, Orbanz 2010, Broderick et al 2014, James 2014]

Nonparametric Bayes

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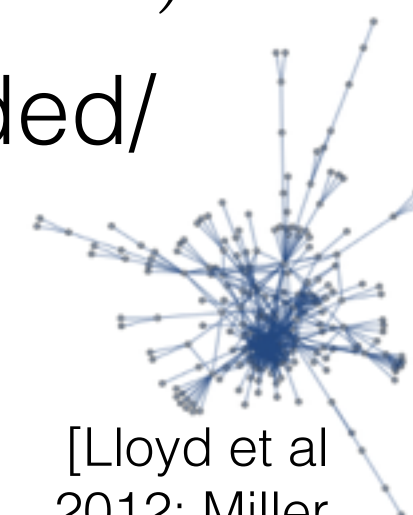
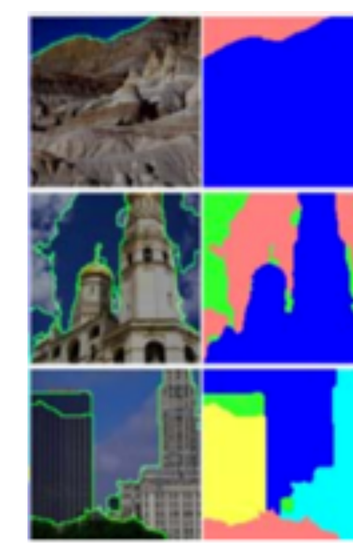


[Lloyd et al 2012; Miller et al, 2010]

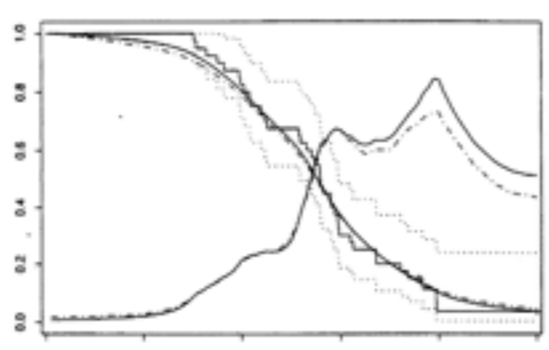
[wikipedia.org]

[Ed Bowlby, NOAA]

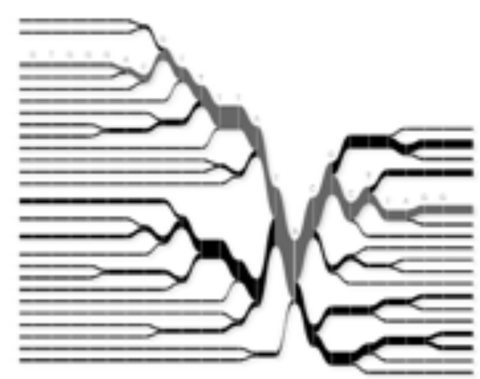
[Fox, et al 2014]



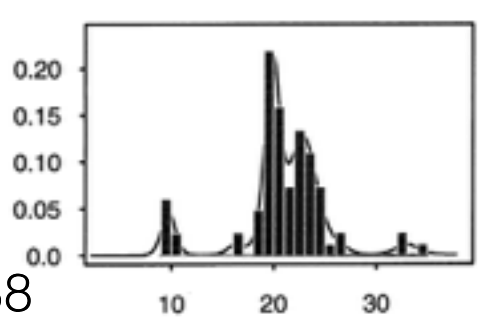
[Sudderth, Jordan 2009]



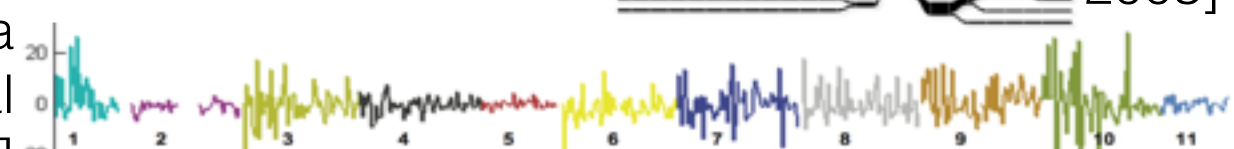
[Arjas, Gasbarra 1994]



[Ewens, 1972; Hartl, Clark 2003]



[Escobar, West 1995; Ghosal, et al 1999]



[Saria et al 2010]

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