Nonparametric Bayesian Methods

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Nonparametric Bayesian Methods: Part I

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Nonparametric Bayes

Nonparametric Bayes

- Bayesian

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"Wikipedia phenomenon"

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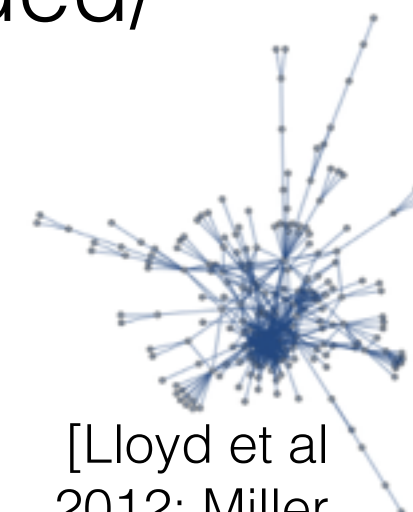

[Lloyd et al 2012; Miller et al 2010]

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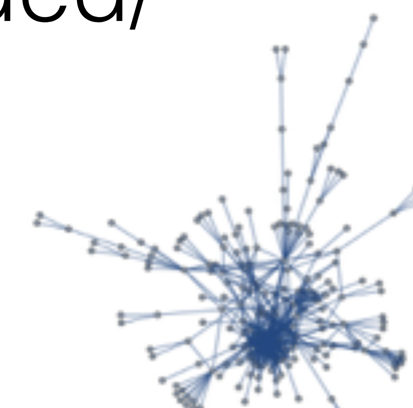

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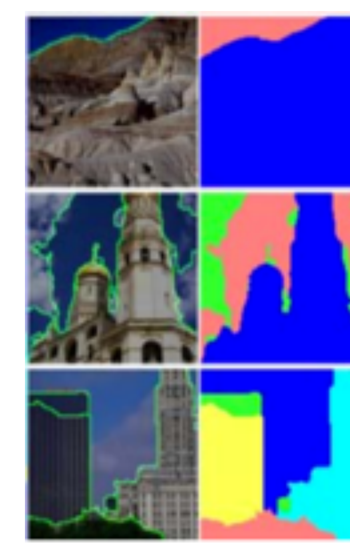

[Sudderth, Jordan 2009]

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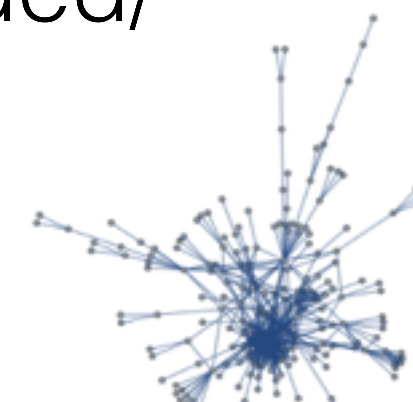
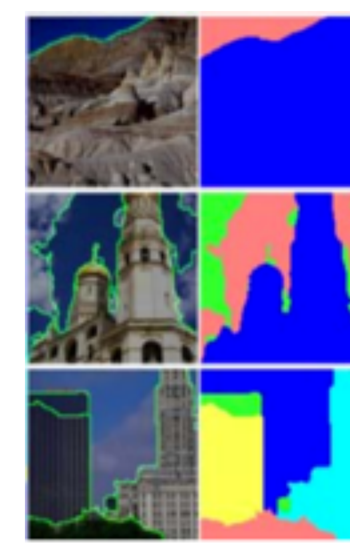

[Lloyd et al 2012; Miller et al 2010]

[Ewens 1972; Hartl, Clark 2003]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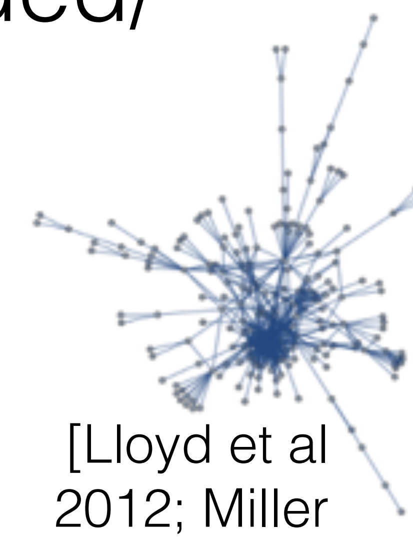

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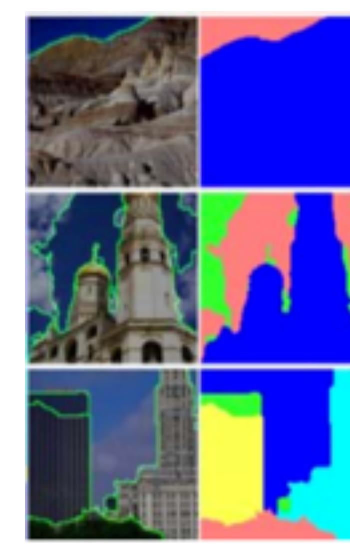
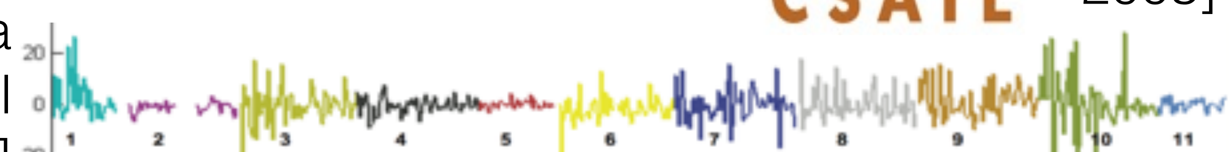


[Sudderth, Jordan 2009]

[Saria et al 2010]



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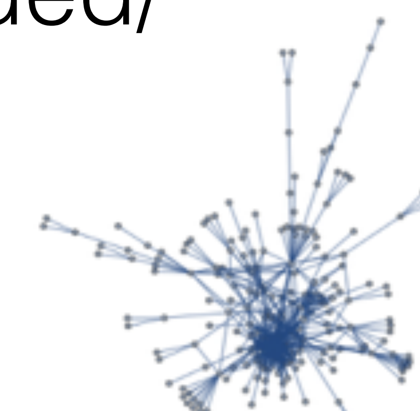
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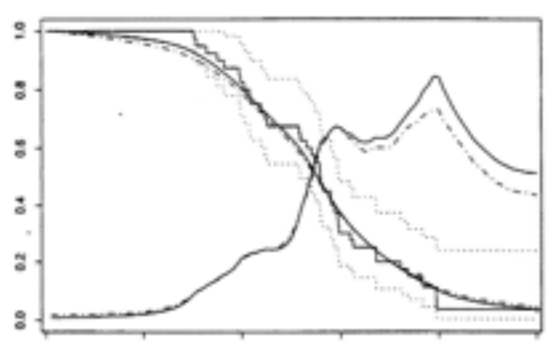
[Arjas, Gasbarra 1994]

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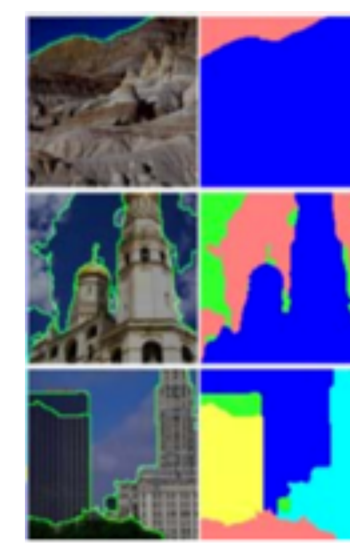
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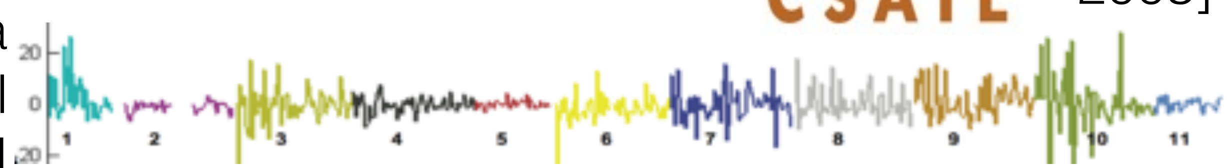
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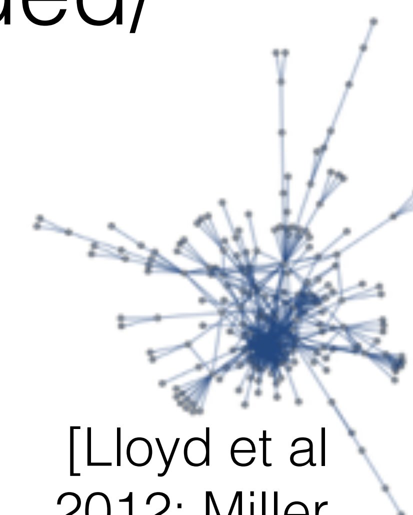

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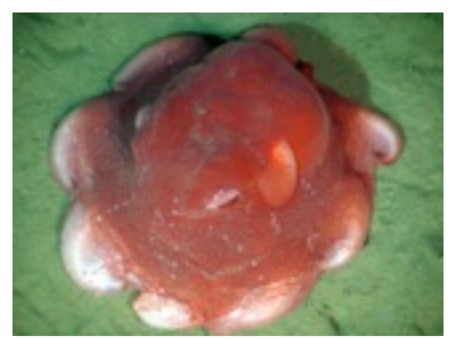

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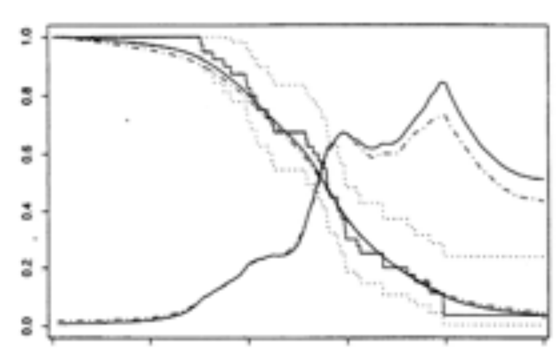

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CSAIL

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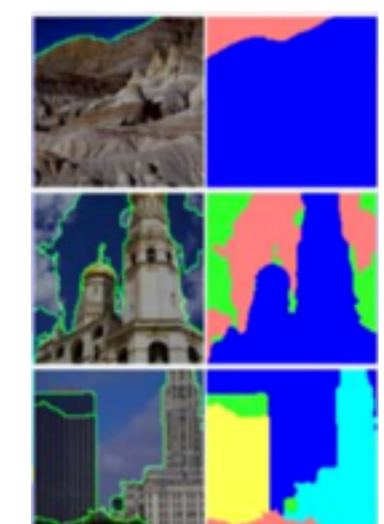

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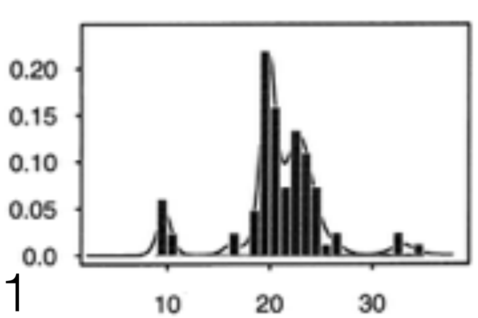

[Escobar, West 1995; Ghosal et al 1999]

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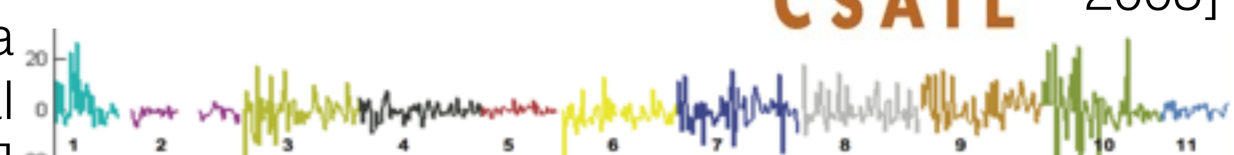
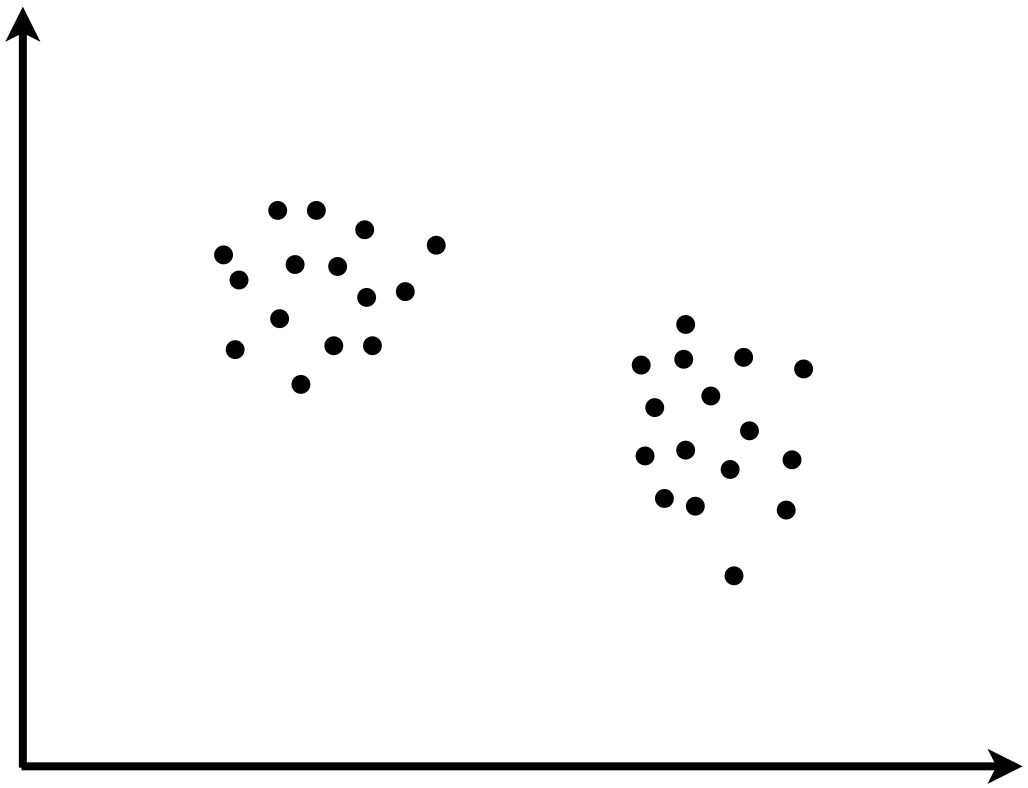


Roadmap

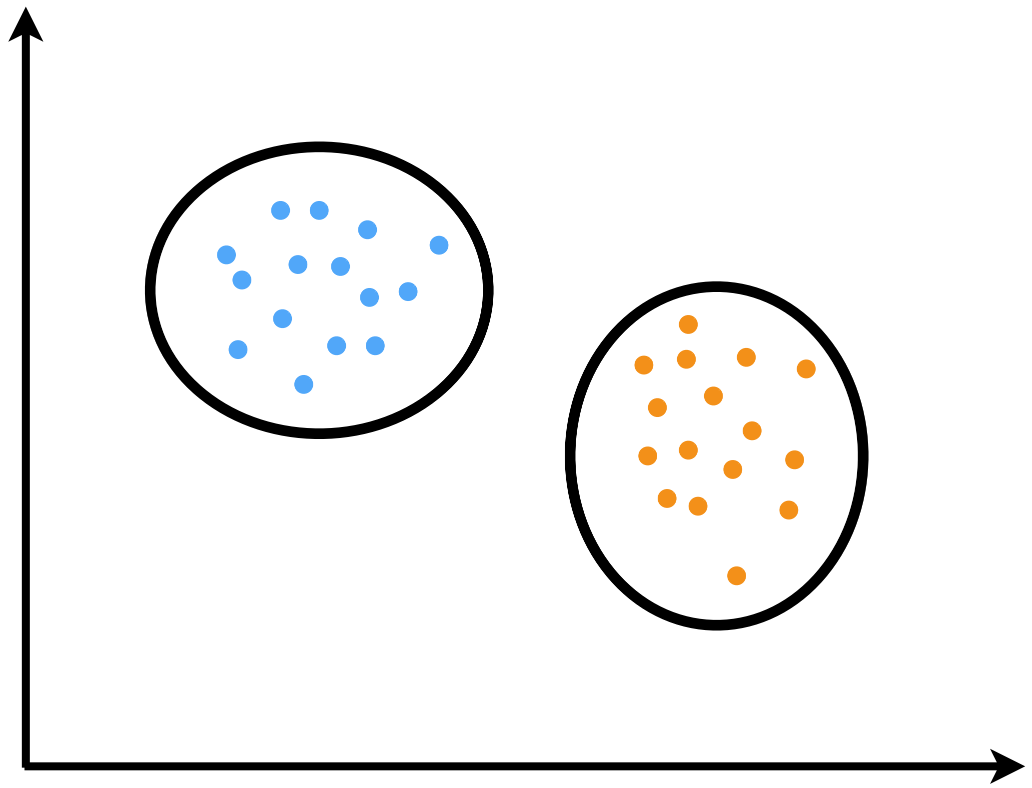
- Example problem: clustering
- Example NPBayes model: Dirichlet process

- Big questions
  - Why NPBayes?
  - What does a growing/infinite number of parameters really mean (in NPBayes)?
  - Why is NPBayes challenging but practical?

Clustering



Clustering



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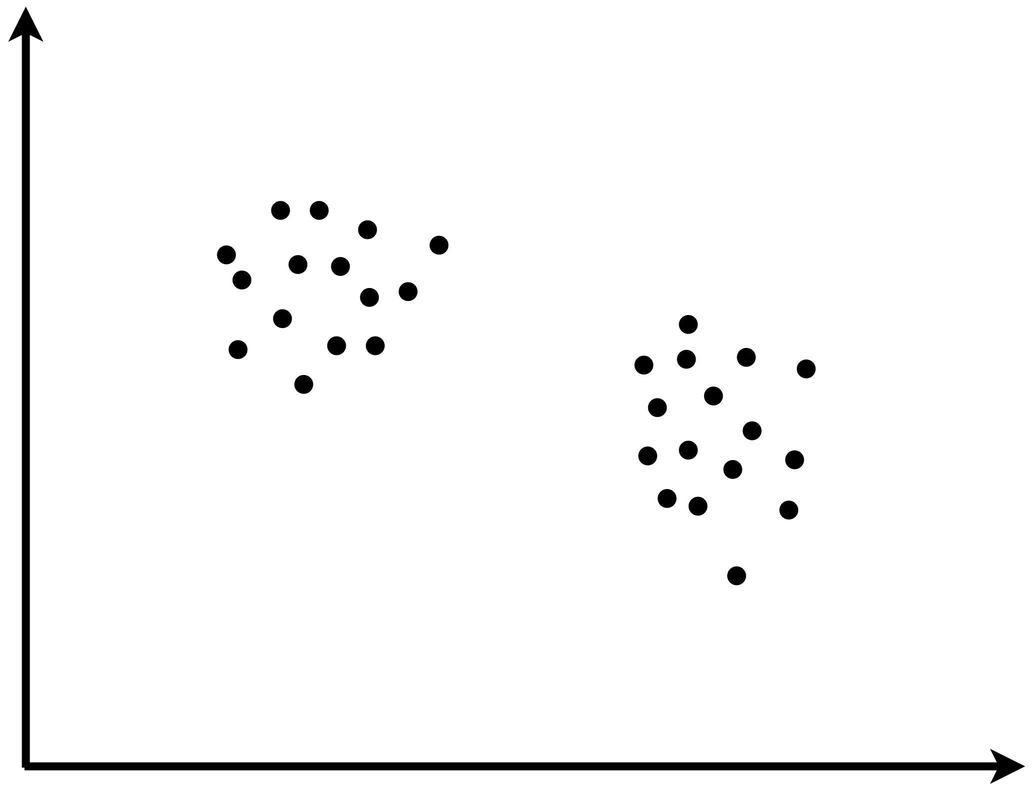

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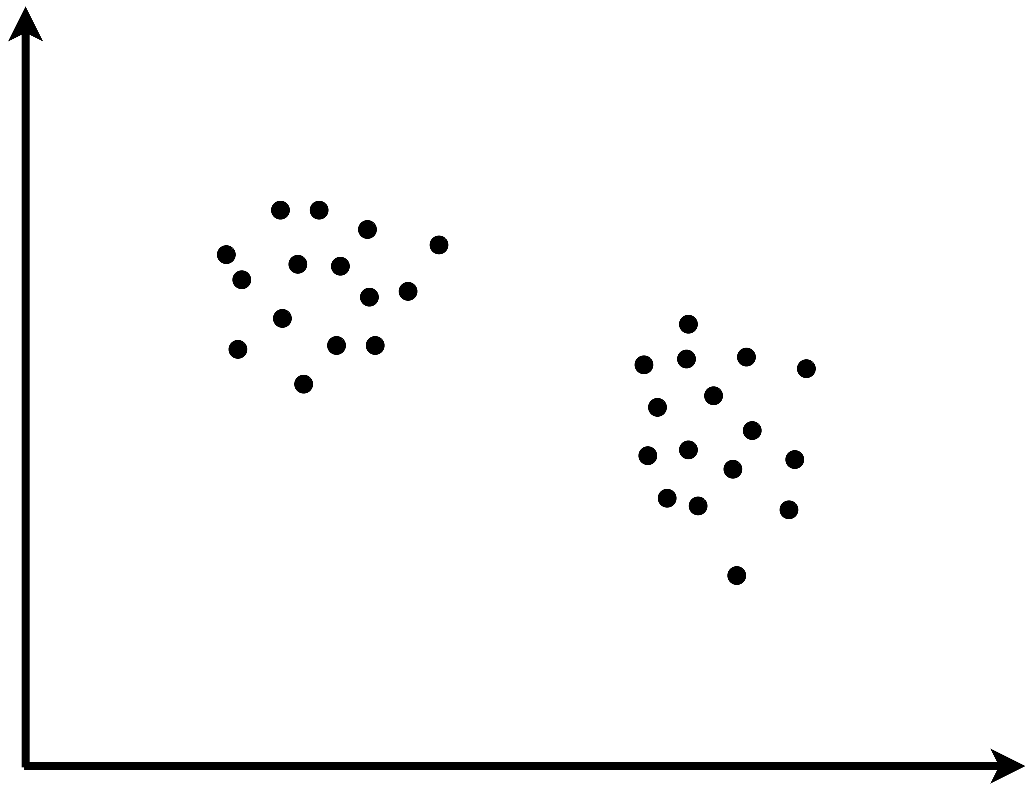

Generative model

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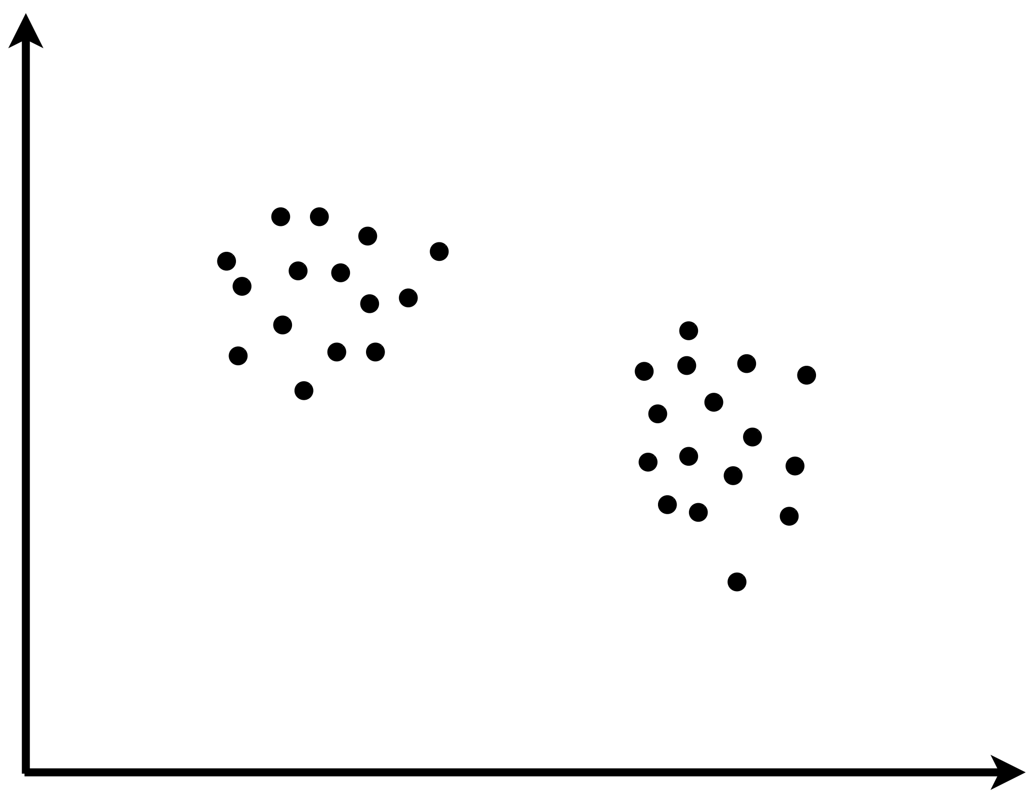

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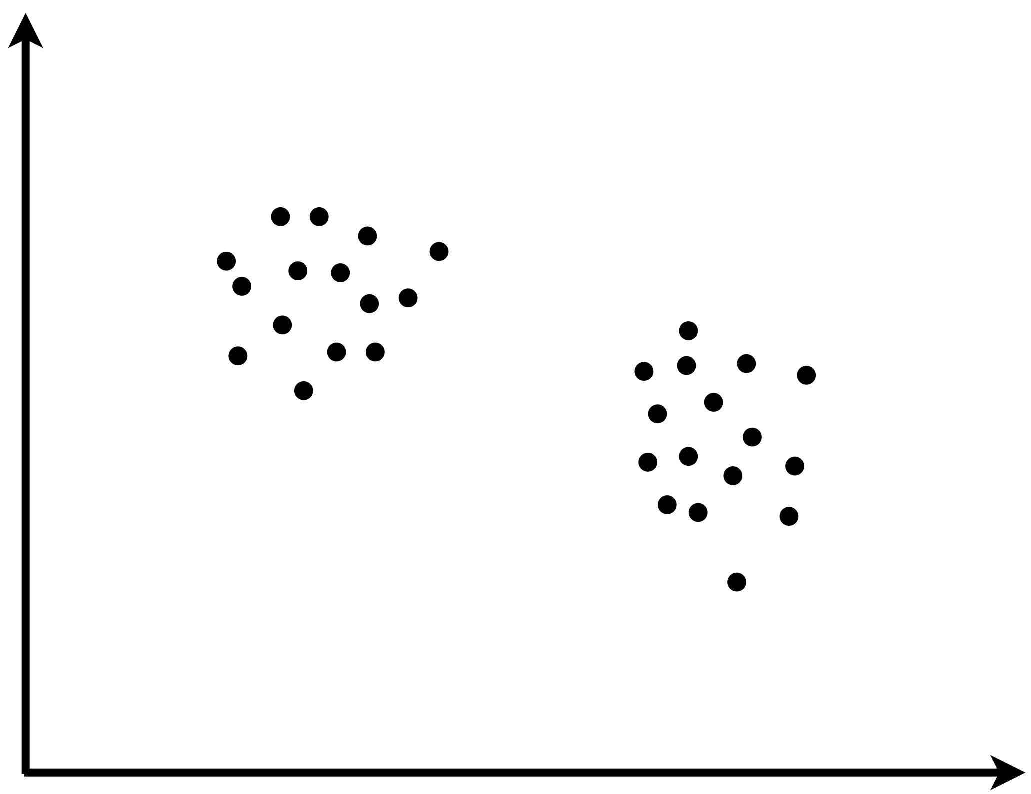

- Finite Gaussian mixture model (K=2 clusters)

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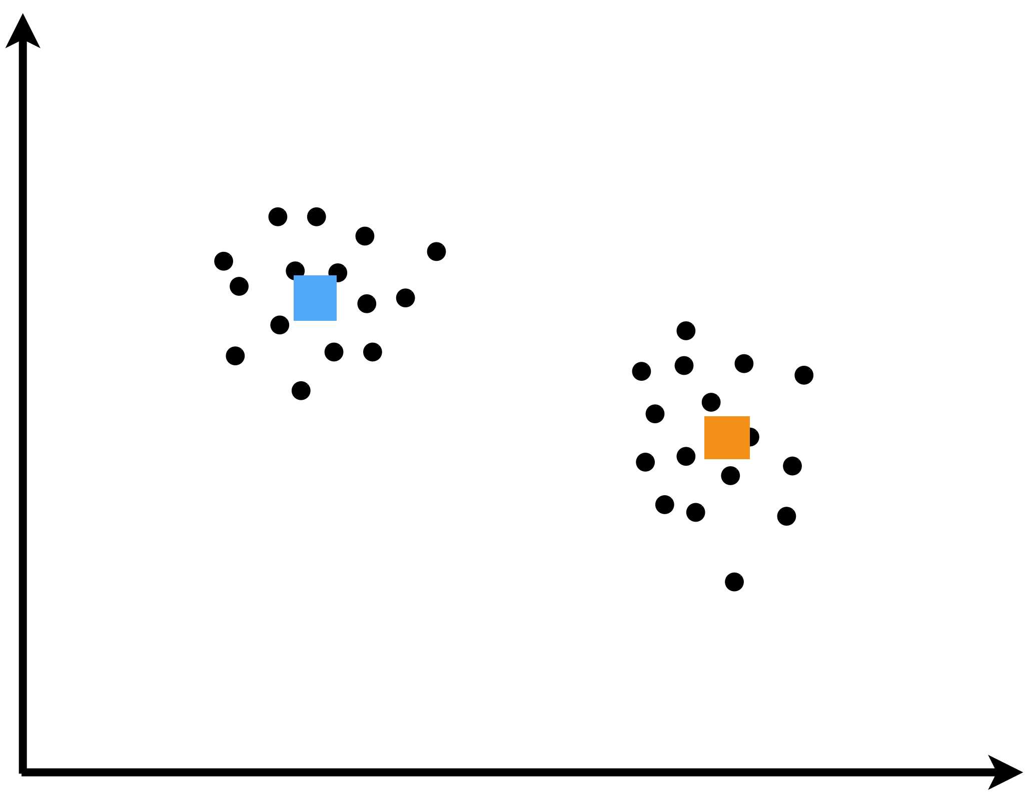

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μ_k

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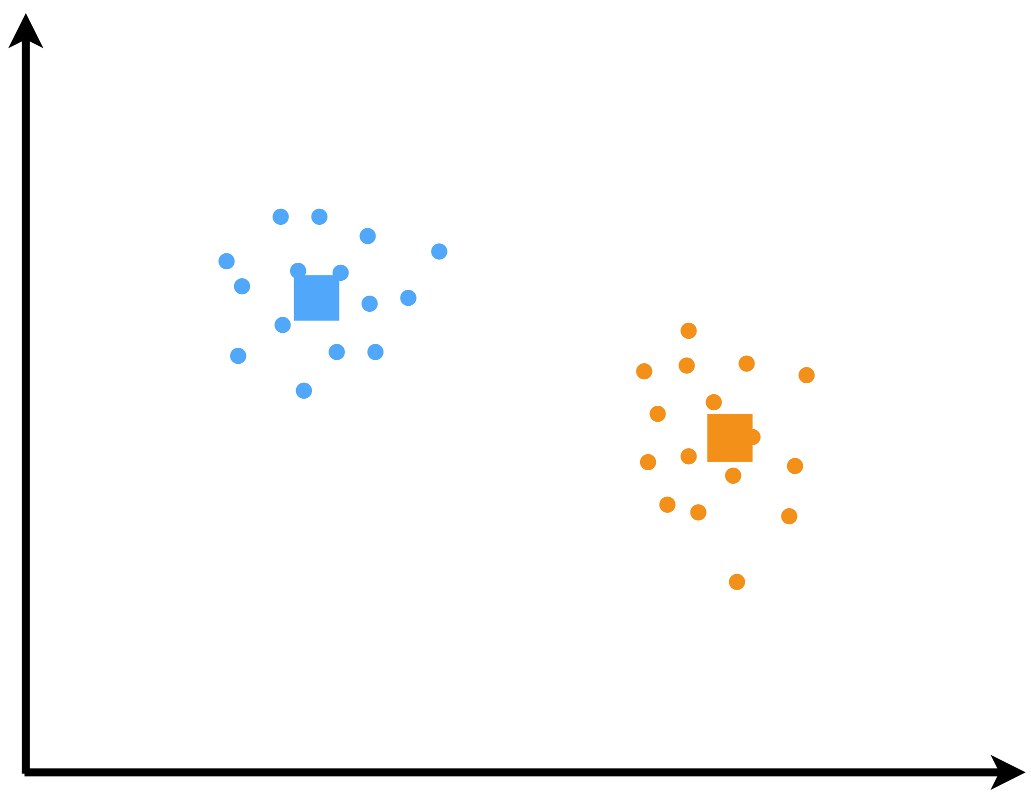

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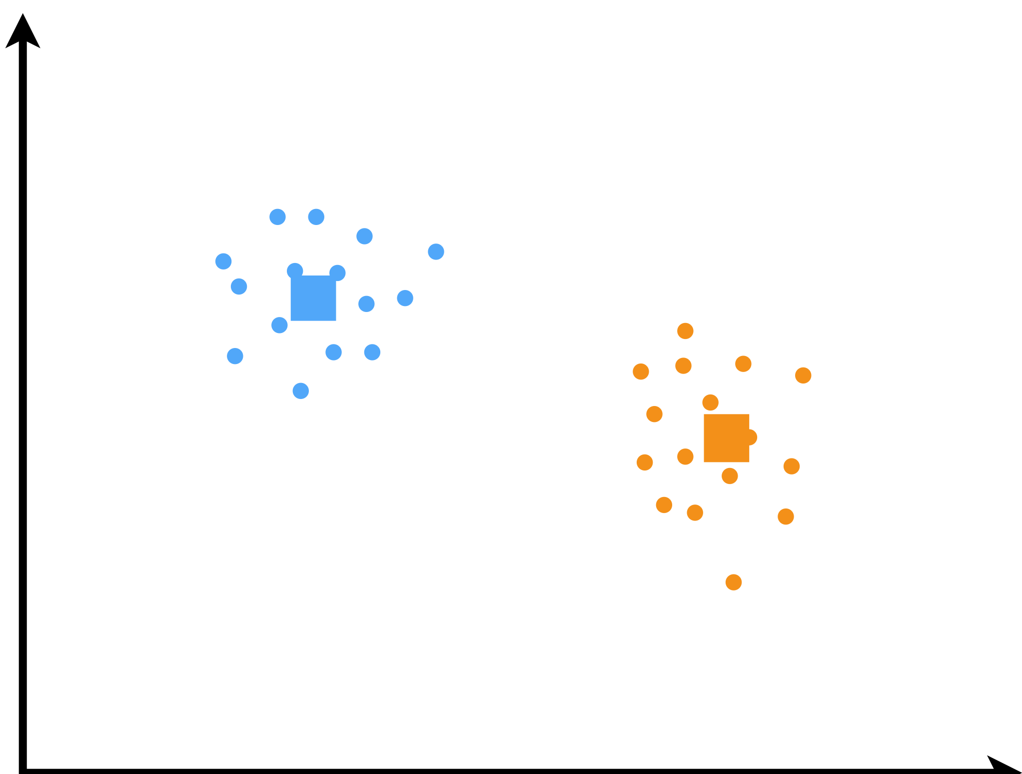

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$x_n \overset{indep}{\sim} \mathcal{N}(\mu_{z_n}, \Sigma)$

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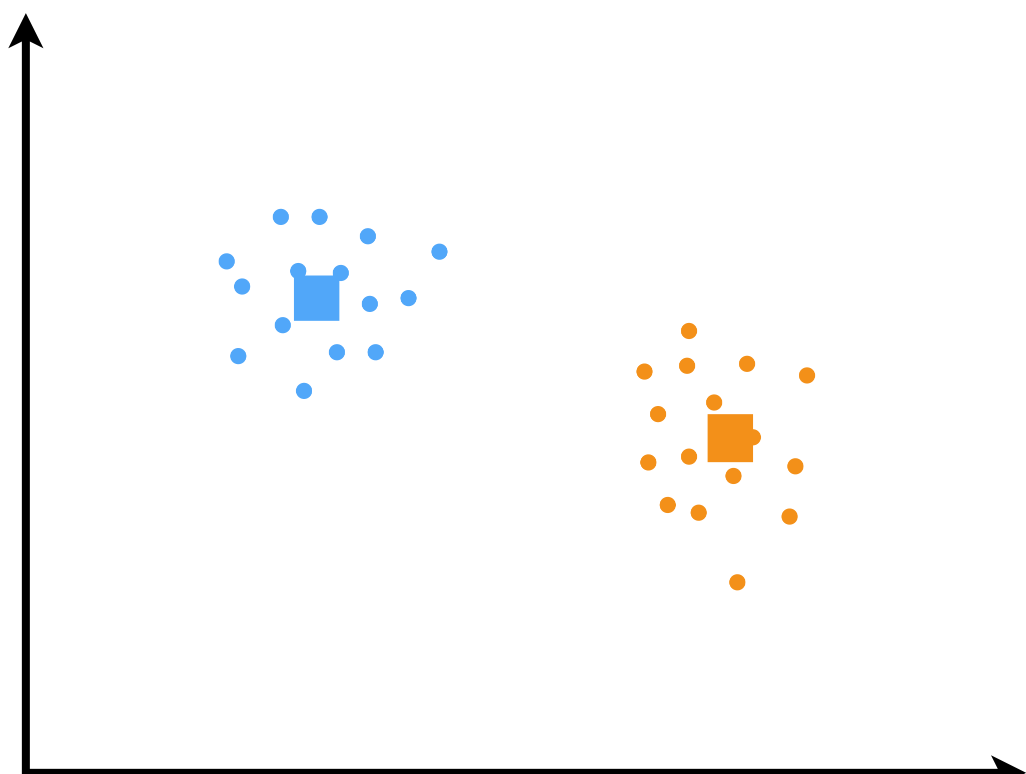

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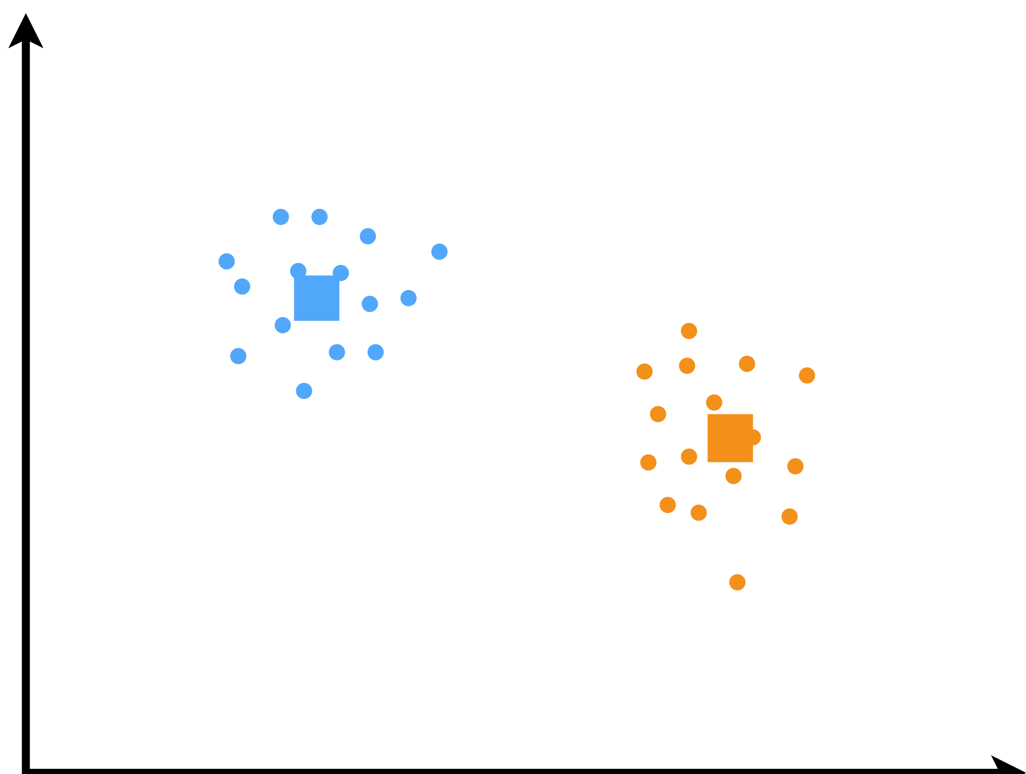

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$$\mu_k \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
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$$\rho_2 = 1 - \rho_1$$
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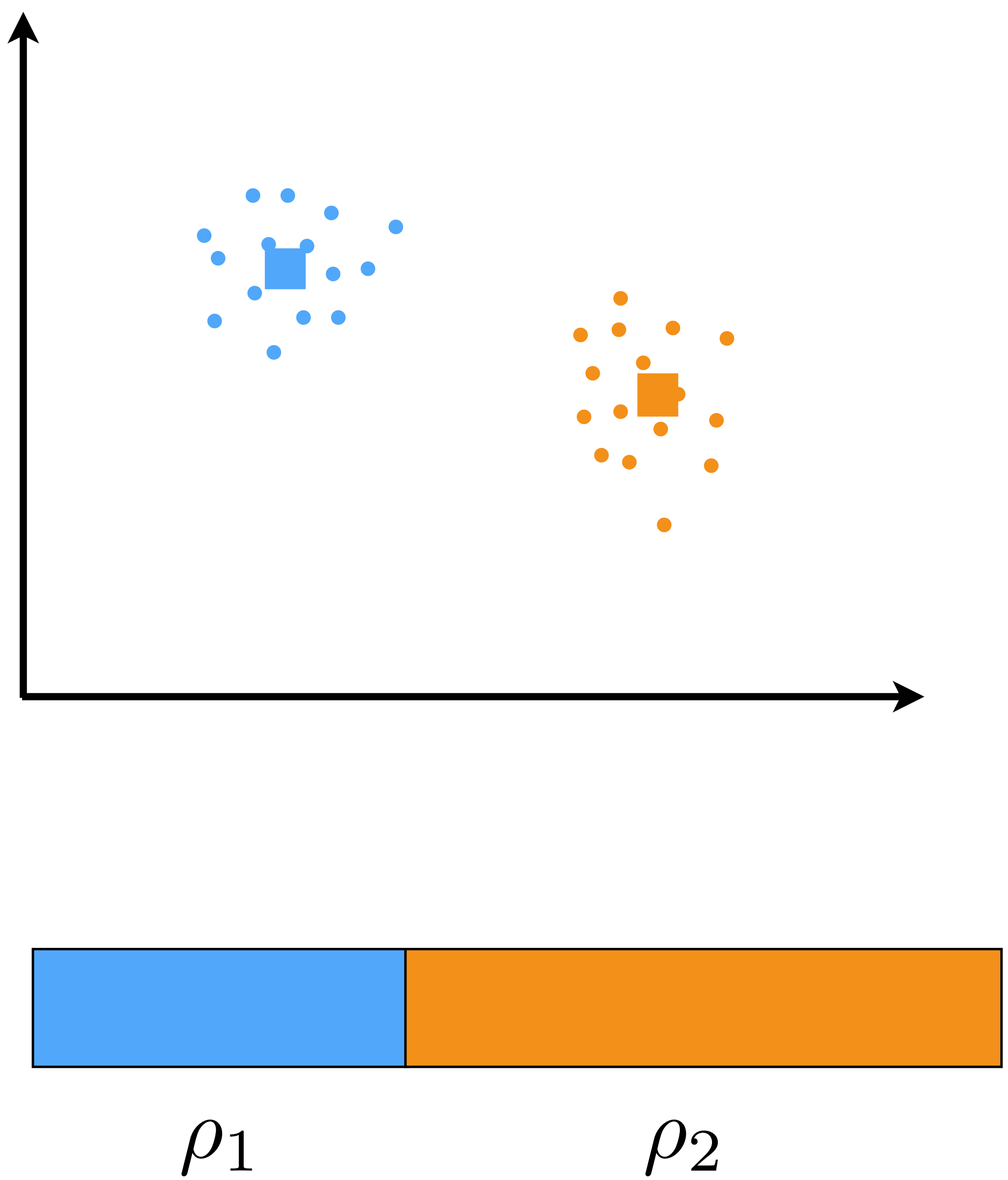

- Finite Gaussian mixture model (*K*=2 clusters)

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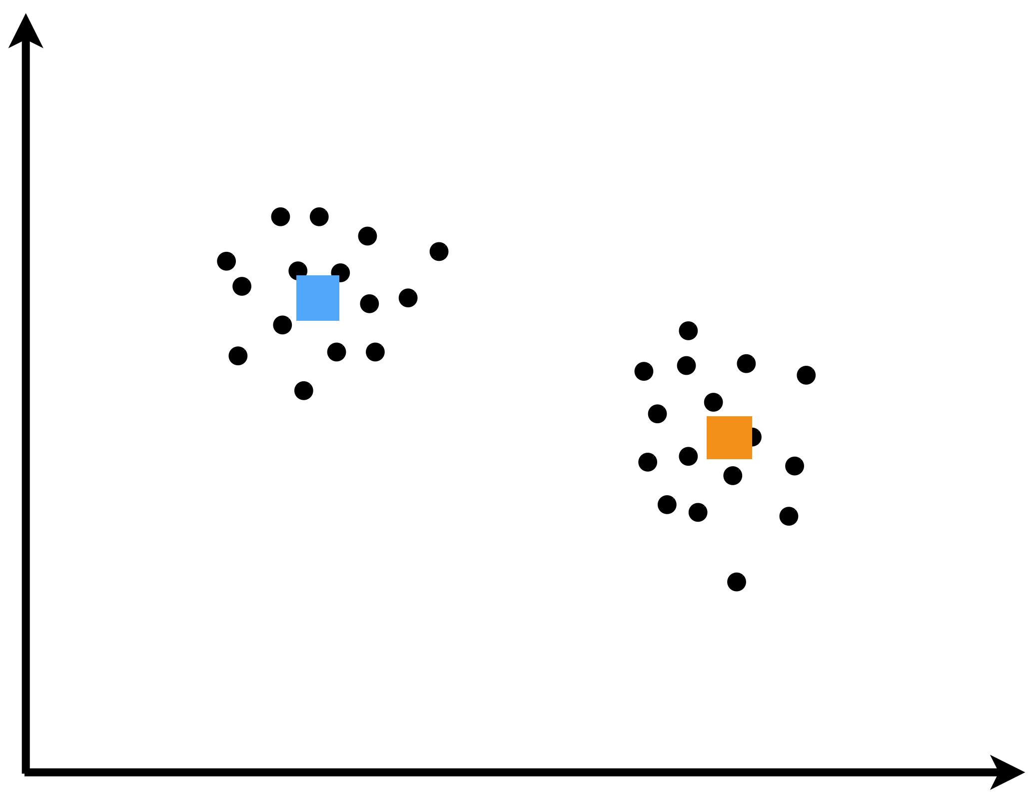

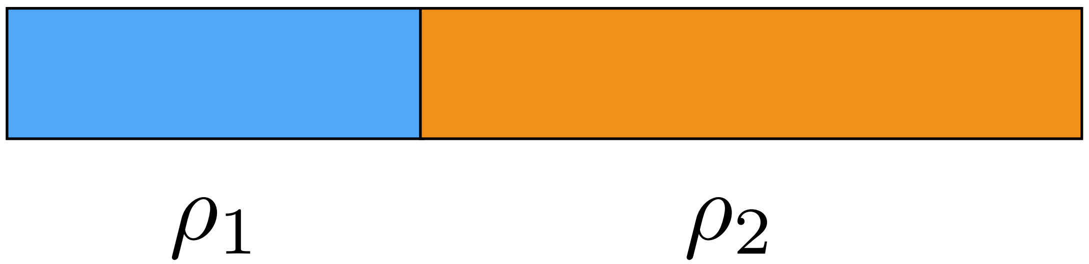

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Beta distribution review

$$\mathrm{Beta}(\rho_1|a_1, a_2) = \frac{\Gamma(a_1 + a_2)}{\Gamma(a_1)\Gamma(a_2)}\rho_1^{a_1-1}(1-\rho_1)^{a_2-1}$$

$$\rho_1 \in (0, 1)$$

$$a_1, a_2 > 0$$

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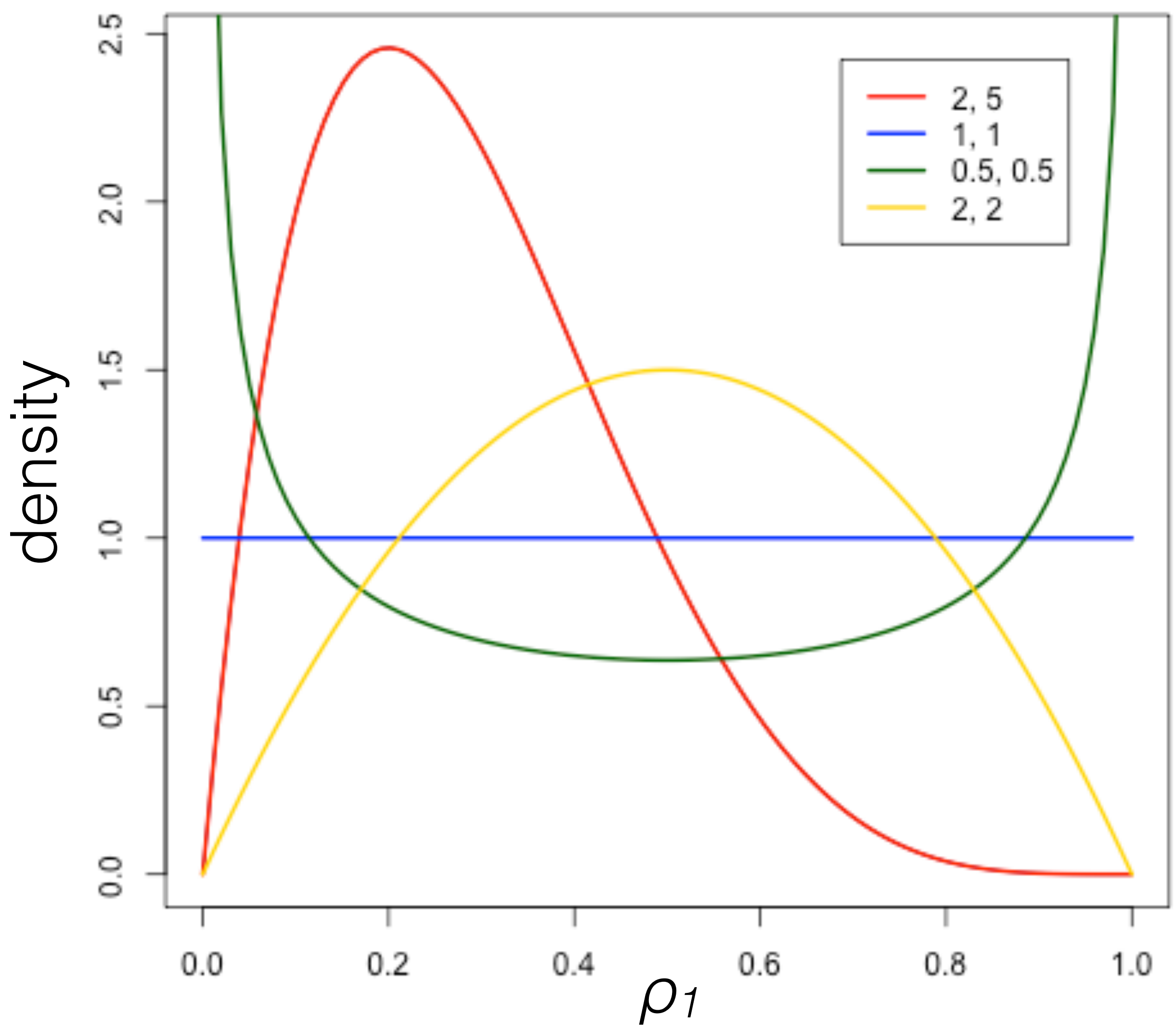

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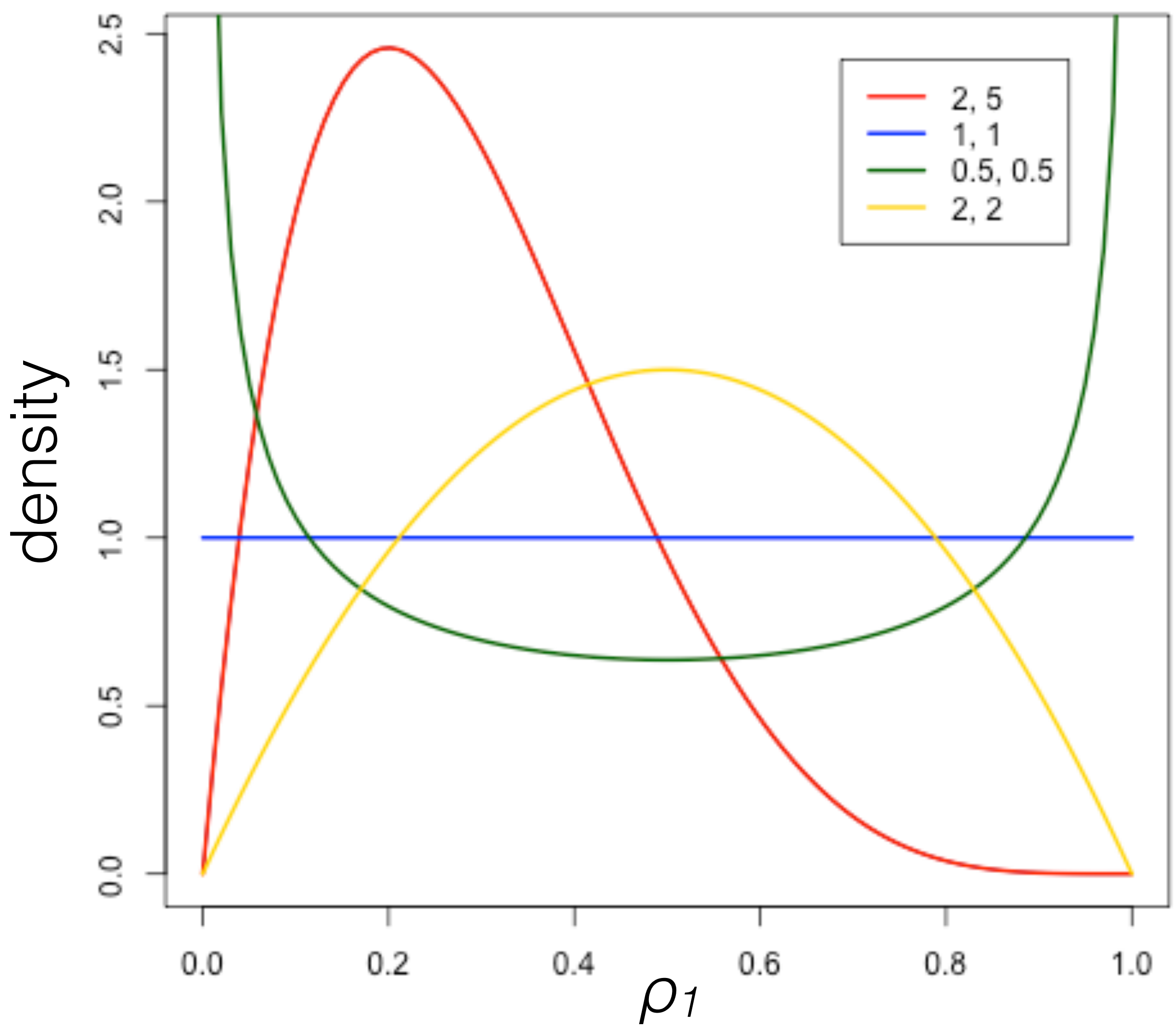

- What happens?

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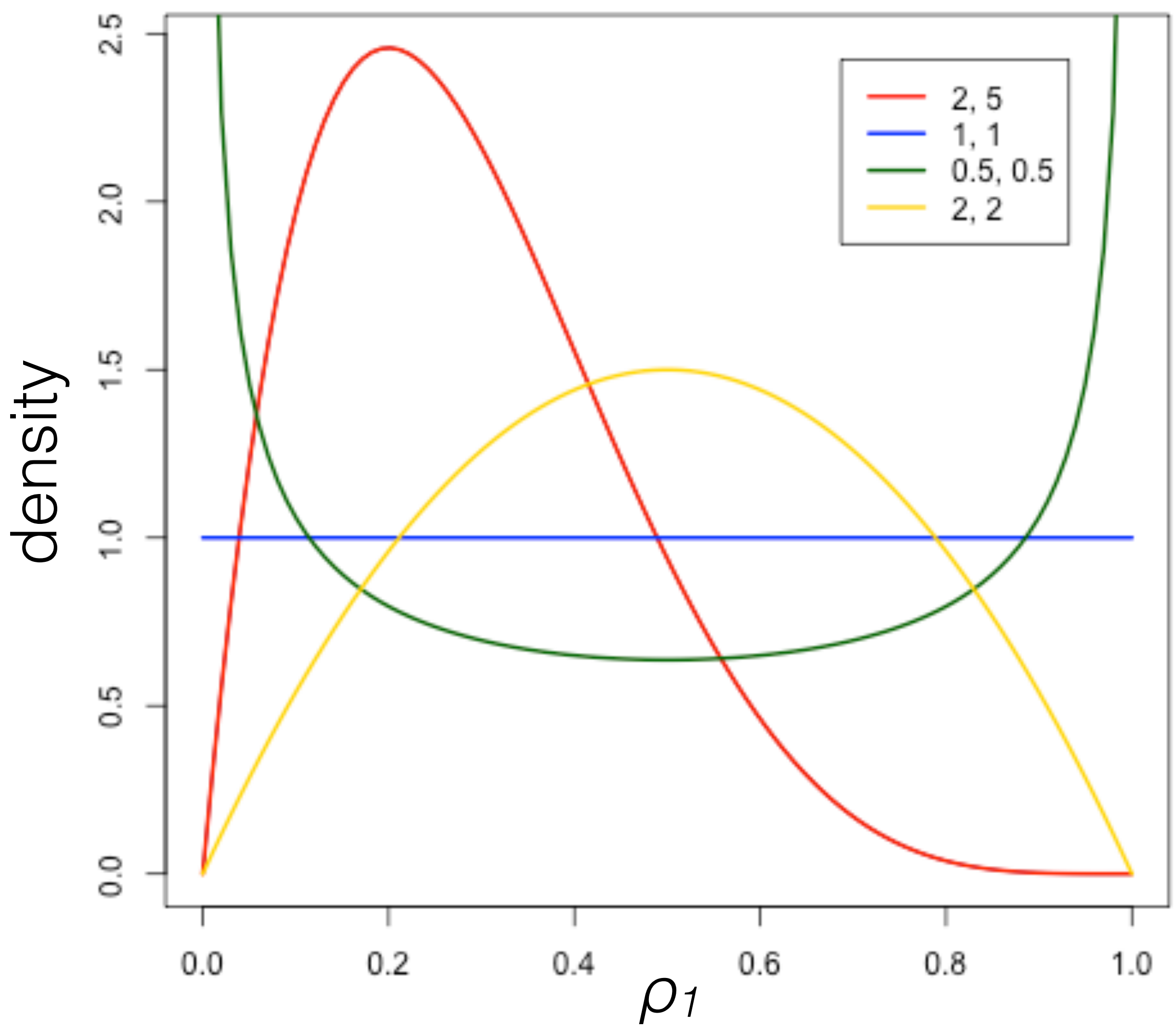

- What happens?

$$a = a_1 = a_2 \to 0$$
$$a = a_1 = a_2 \to \infty$$
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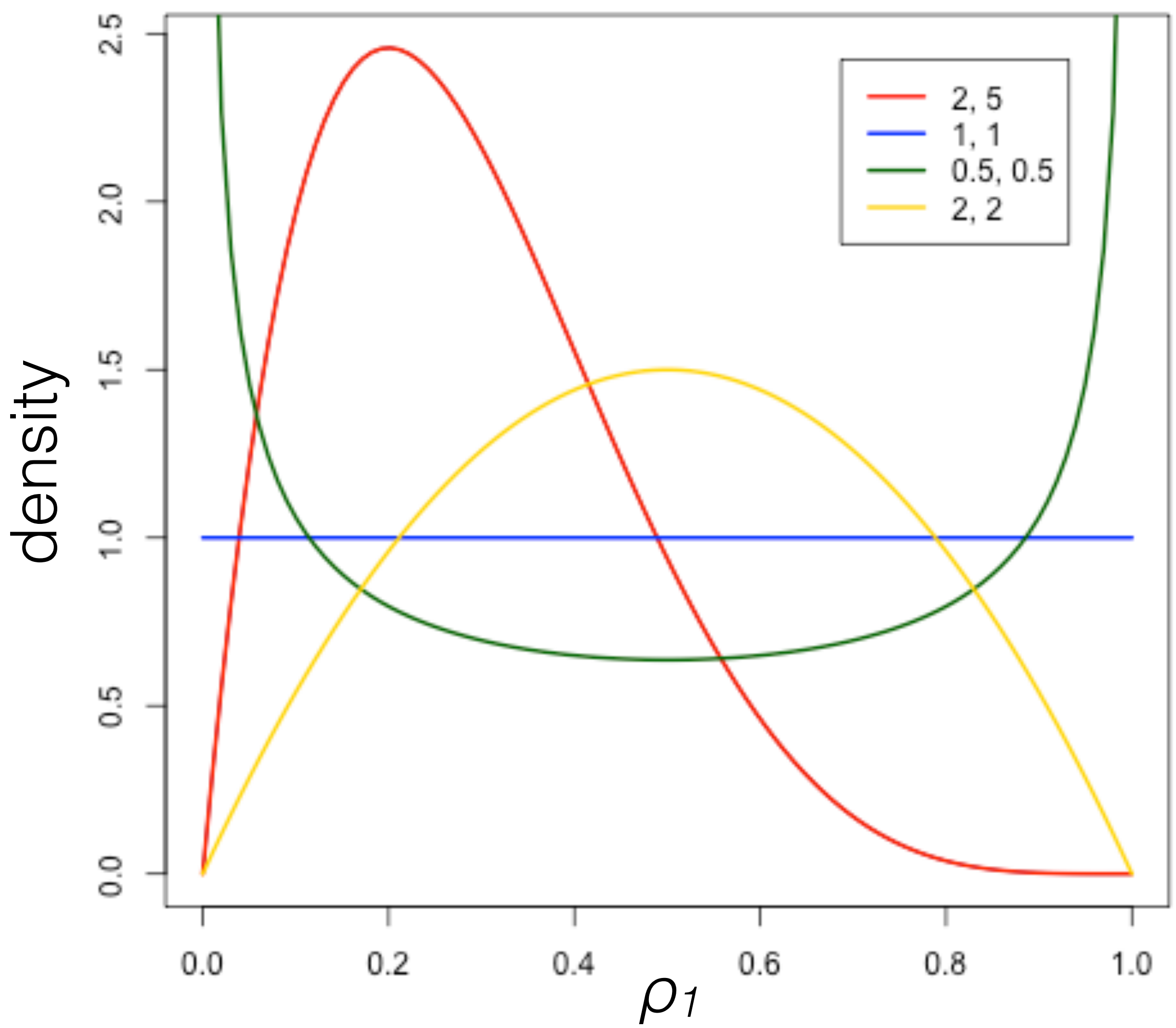

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[demo]

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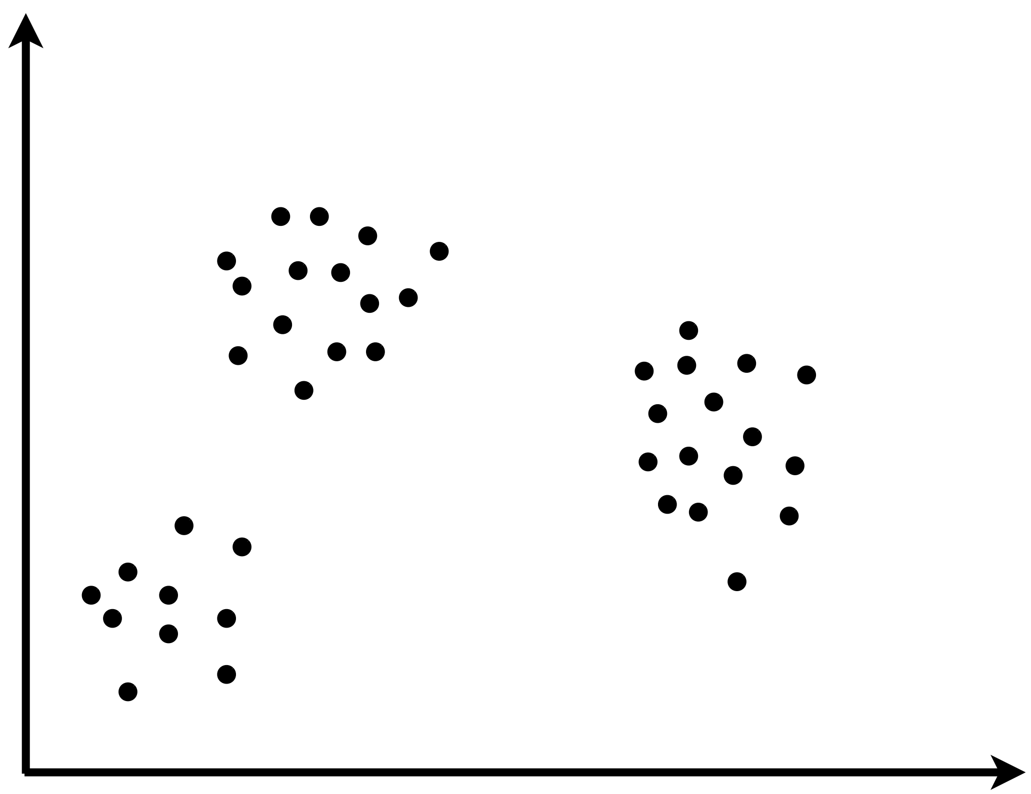

- Finite Gaussian mixture model (K clusters)

Generative model

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- Finite Gaussian mixture model (*K* clusters)

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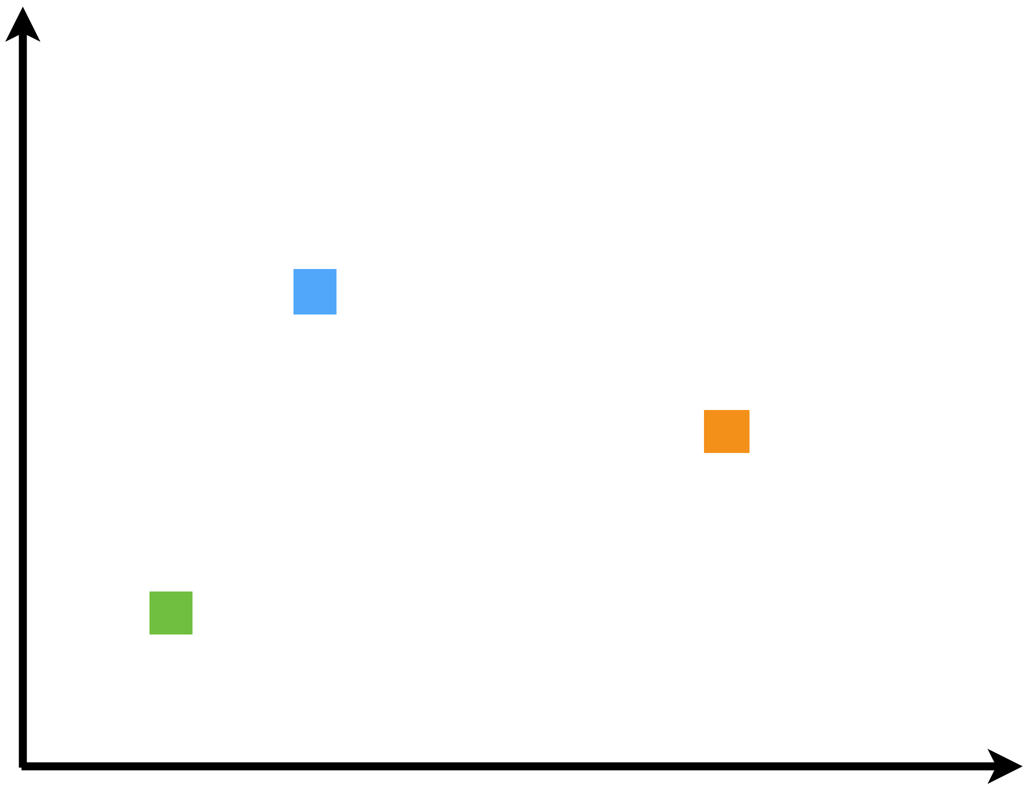

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$$\mu_k \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$

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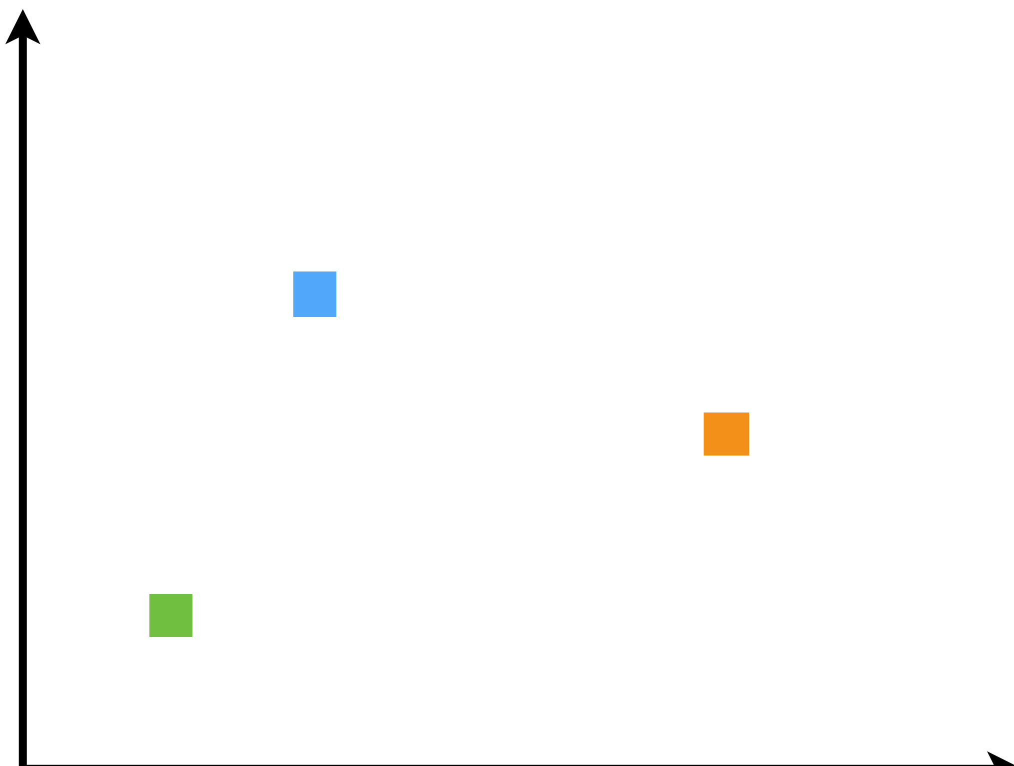

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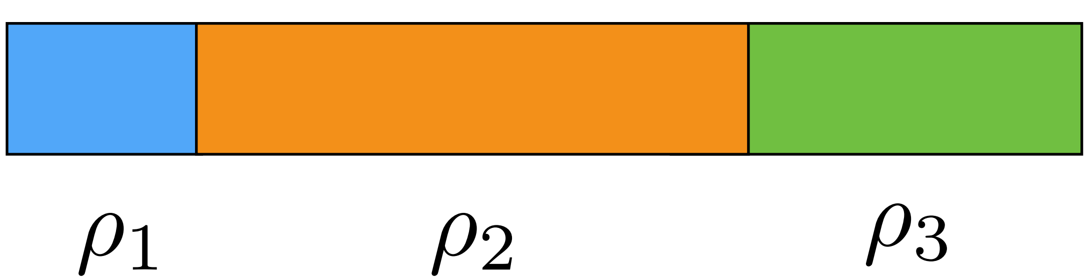

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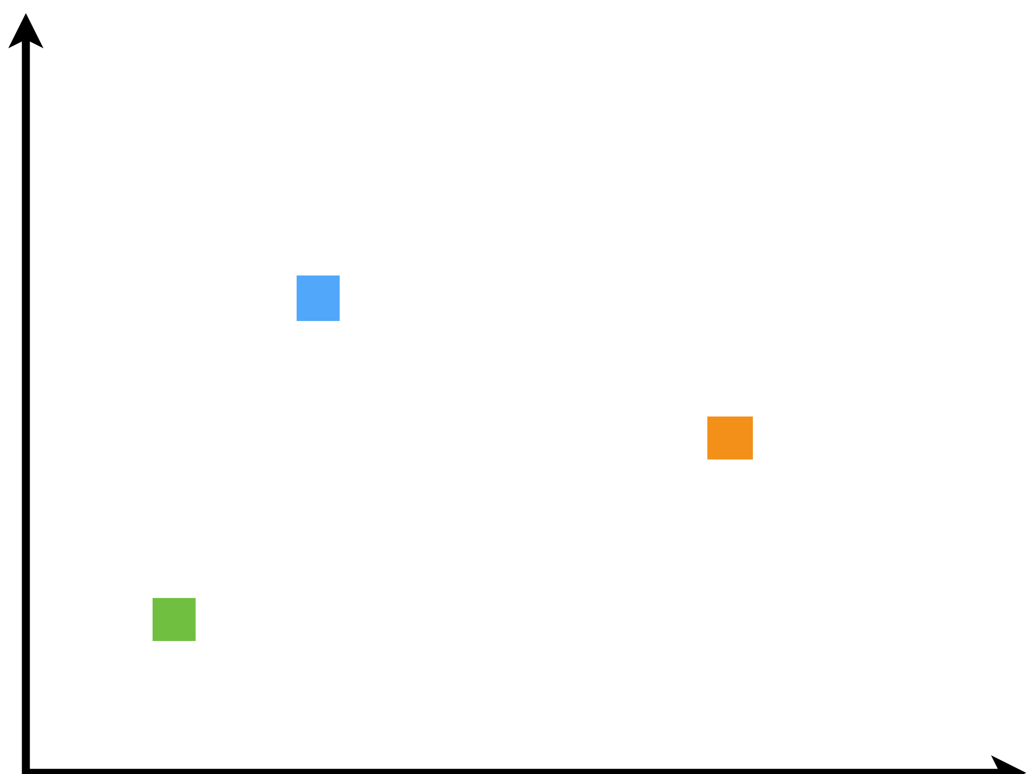

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ρ_1 ρ_2 ρ_3

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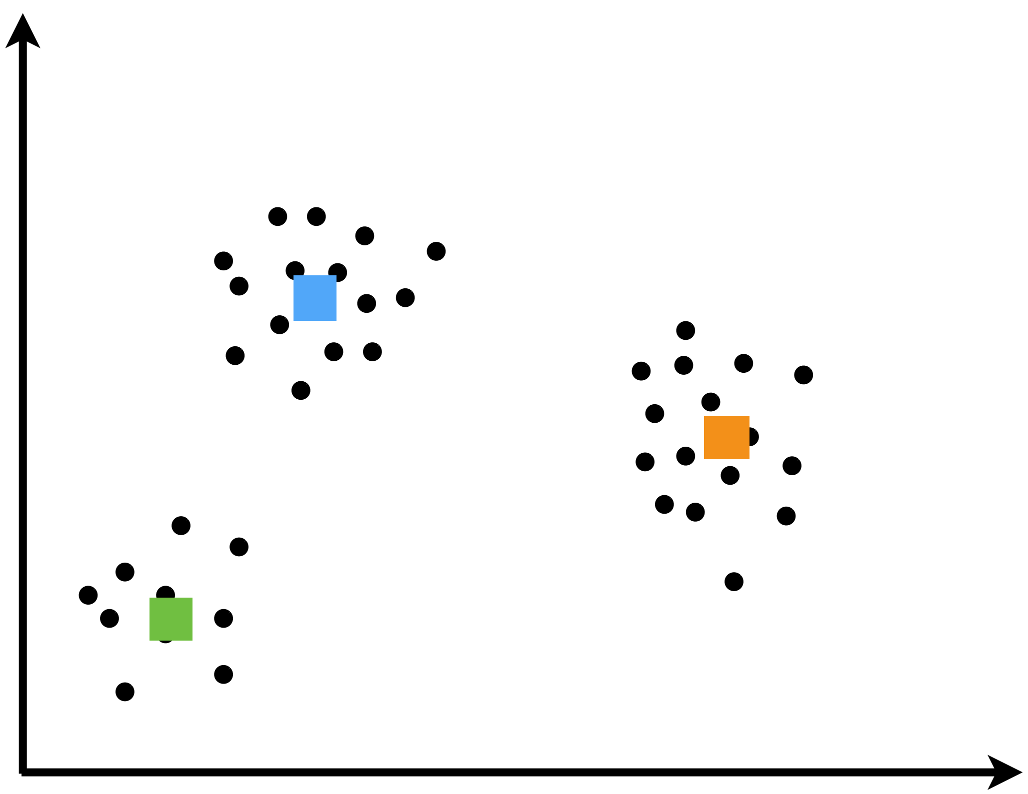

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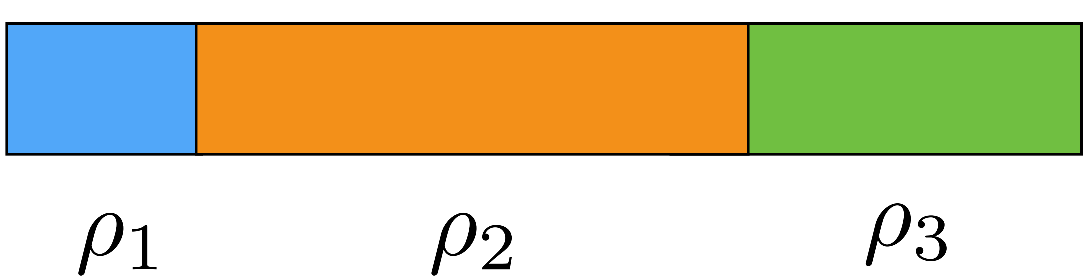
$$z_n \overset{iid}{\sim} \text{Categorical}(\rho_{1:K})$$

$$x_n \overset{indep}{\sim} \mathcal{N}(\mu_{z_n}, \Sigma)$$



Dirichlet distribution review

$$\mathrm{Dirichlet}(\rho_{1:K}|a_{1:K}) = \frac{\Gamma(\sum_{k=1}^{K} a_k)}{\prod_{k=1}^{K} \Gamma(a_k)} \prod_{k=1}^{K} \rho_k^{a_k - 1} \qquad a_k > 0$$

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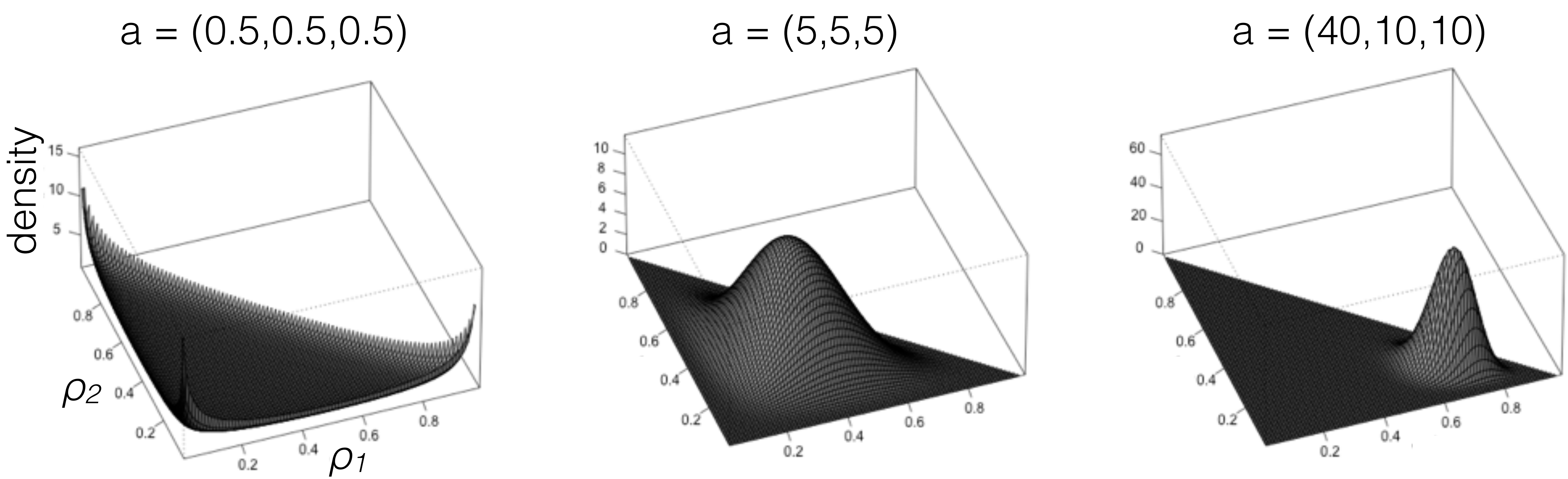

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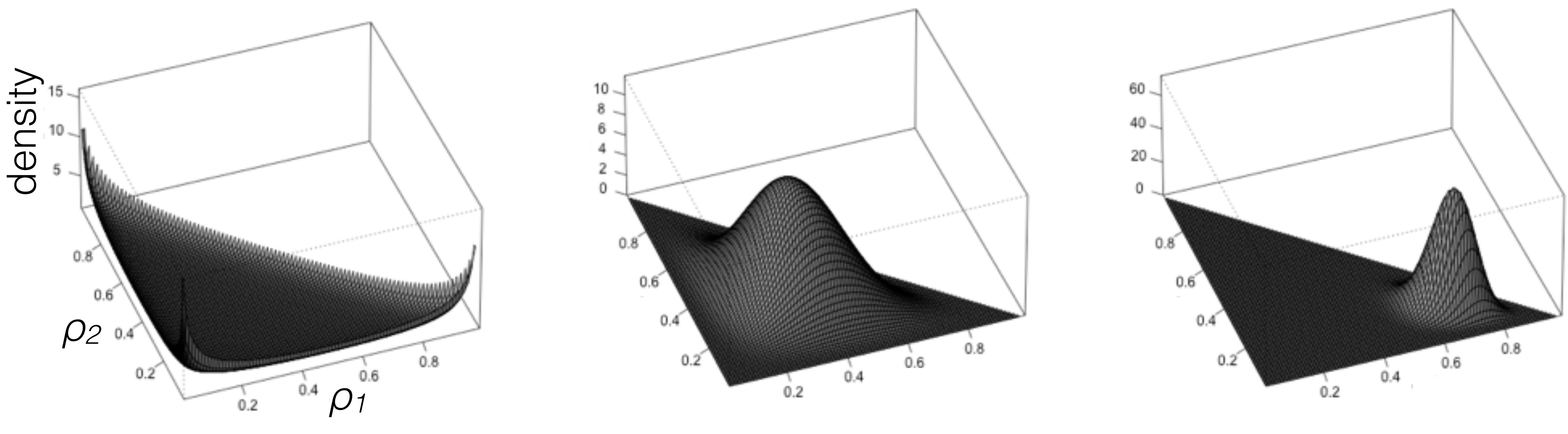
a = (0.5,0.5,0.5) a = (5,5,5) a = (40,10,10)



- What happens? $a = a_k = 1$ $a = a_k \to 0$ $a = a_k \to \infty$

[demo]

So far $K << N$. What if not?

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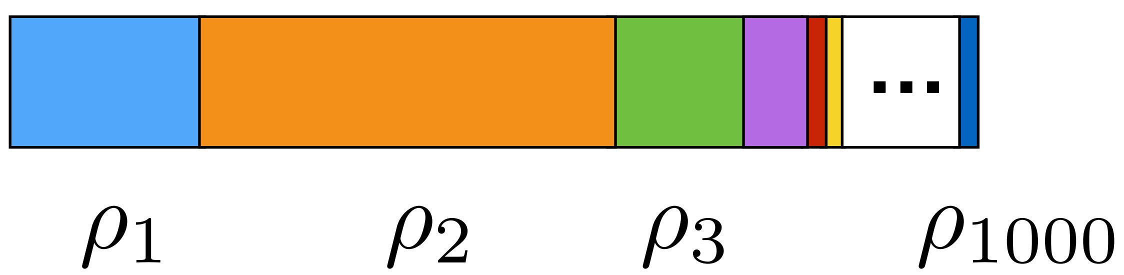
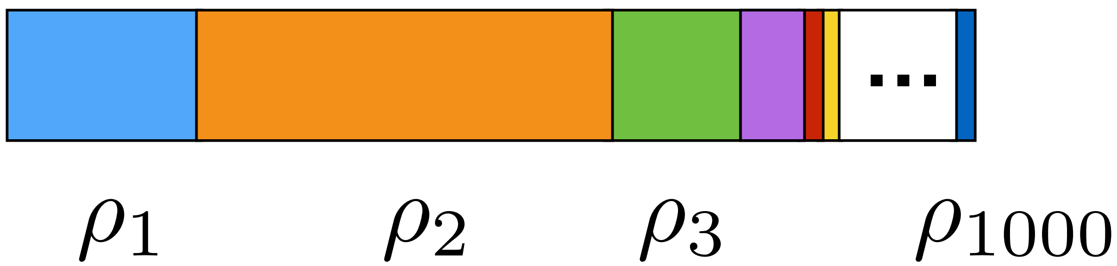


So far $K << N$. What if not?

- e.g. species sampling, topic modeling, groups on a social network, etc.



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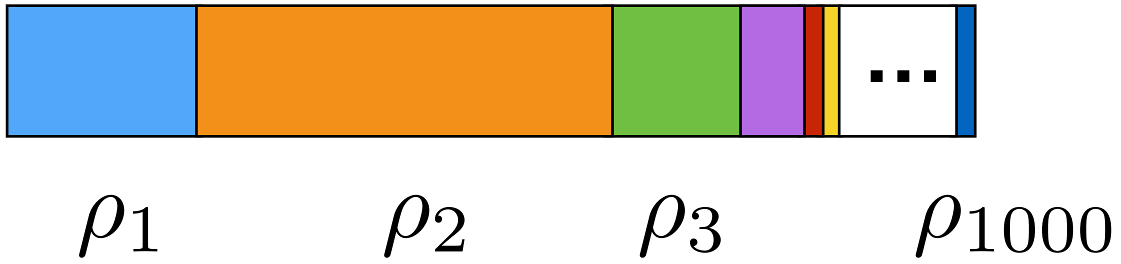

- Components: number of latent groups

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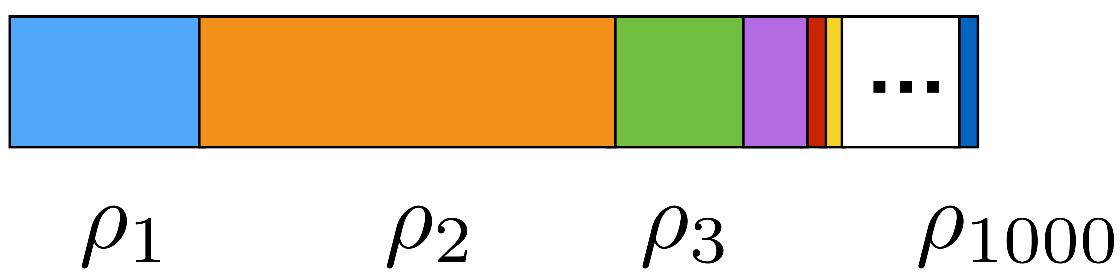

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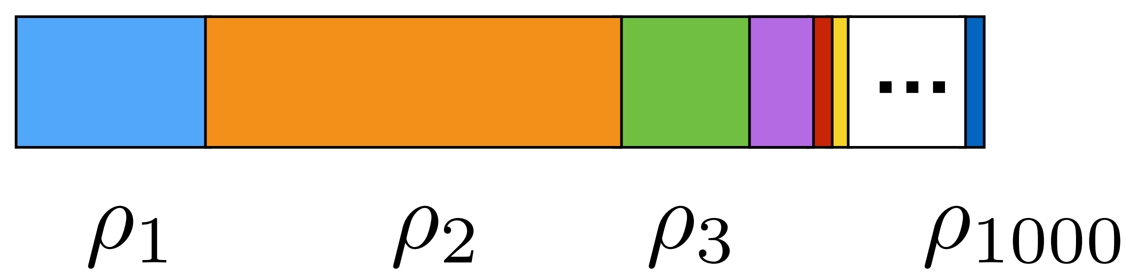

- Components: number of latent groups
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- [demo 1, demo 2]

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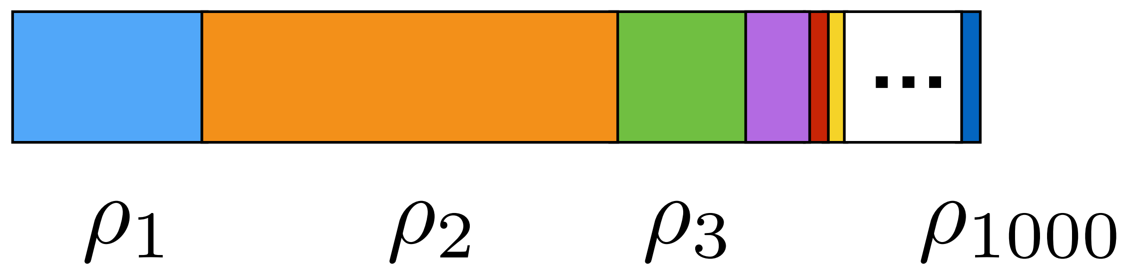

- Components: number of latent groups
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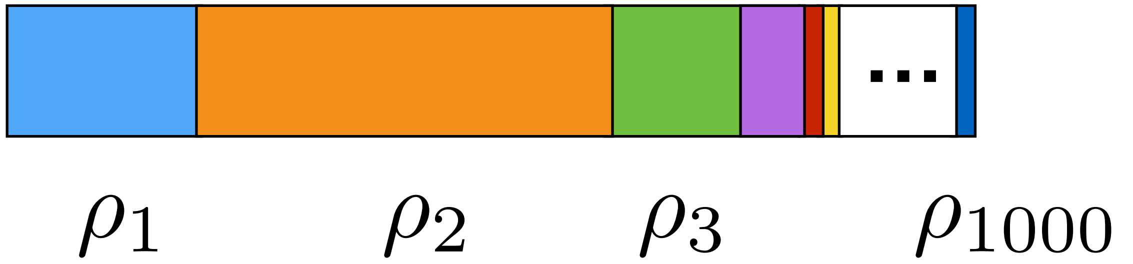

- Components: number of latent groups
- Clusters: number of components represented in the data
- [demo 1, demo 2]
- Number of clusters is random
- Number of clusters grows with N

- Here, difficult to choose finite K in advance (contrast with small K): don't know K, difficult to infer, streaming data

Choosing $K = \infty$

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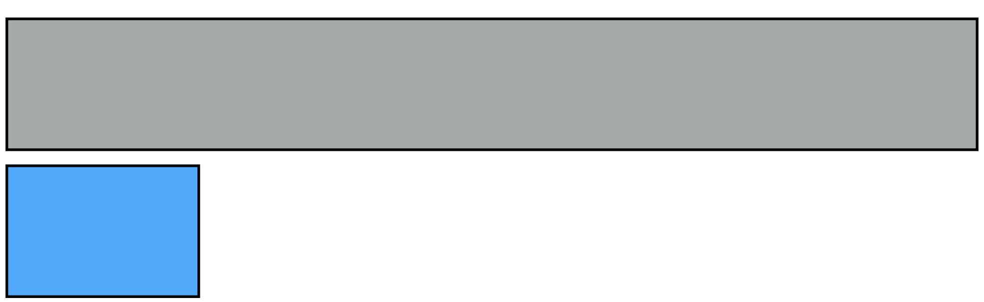

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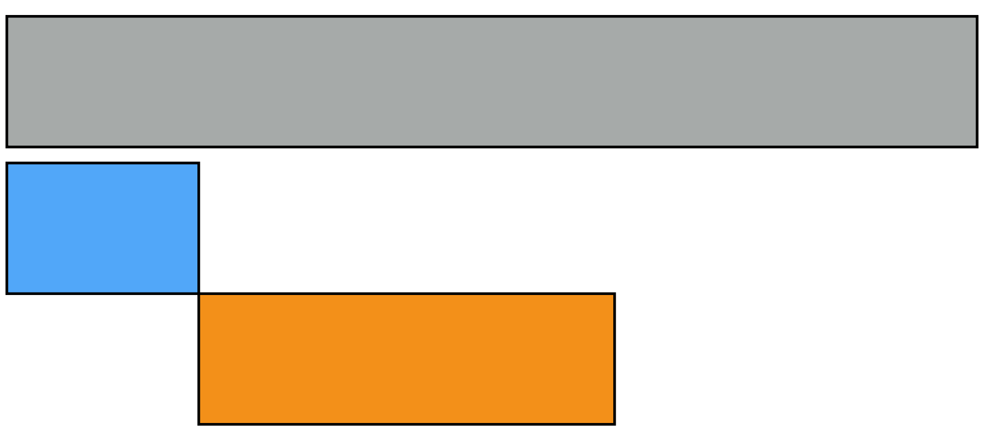

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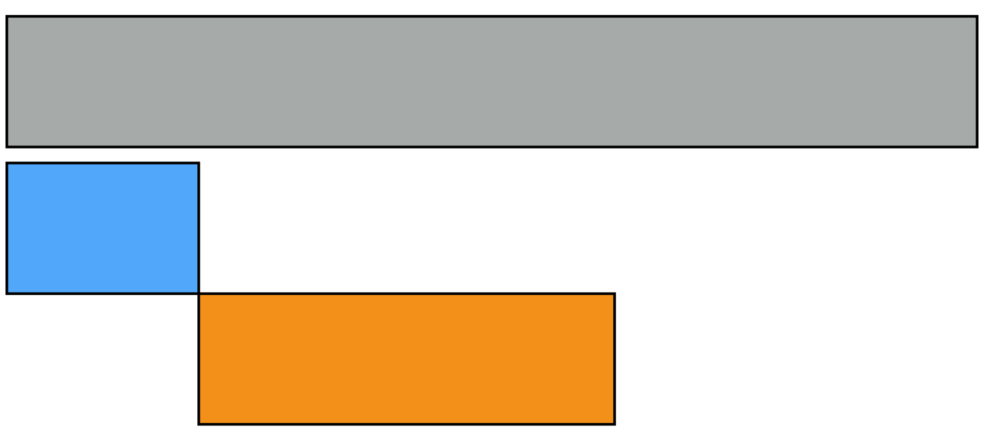

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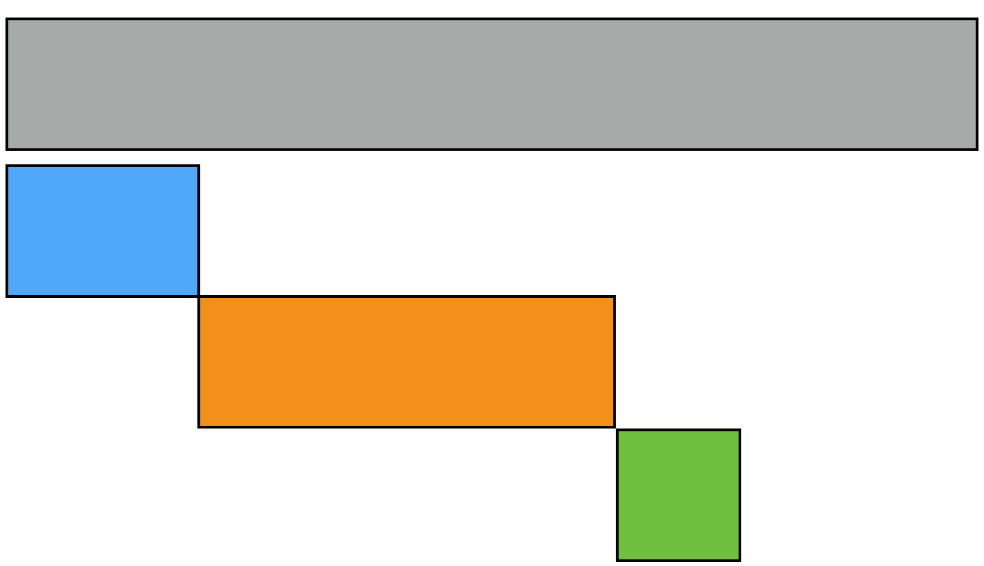

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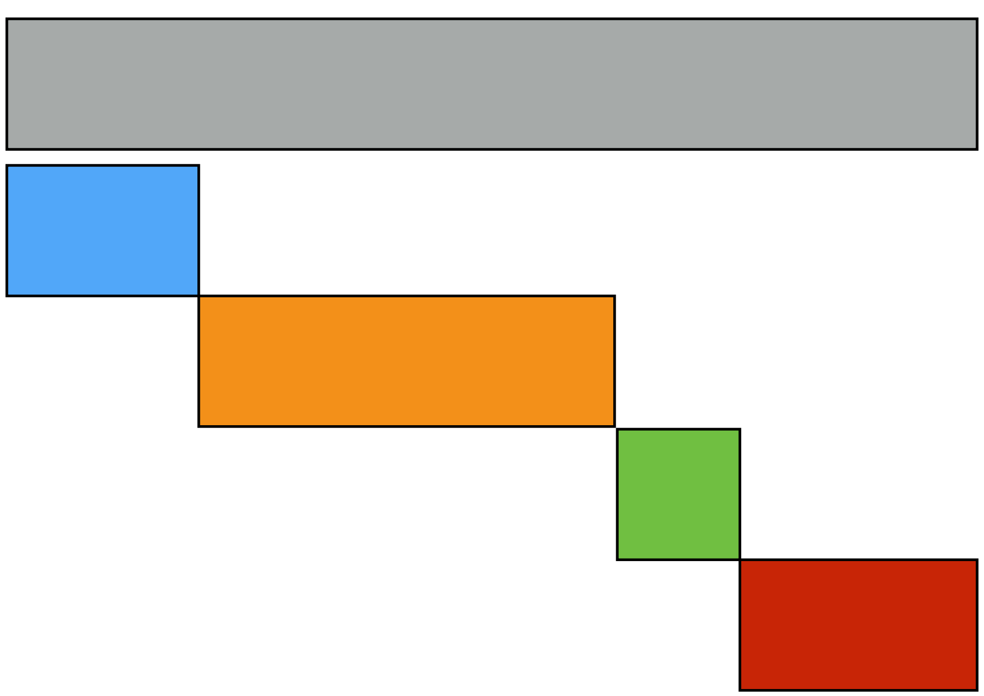

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$$\rho_4 = 1 - \sum_{k=1}^{3} \rho_k$$

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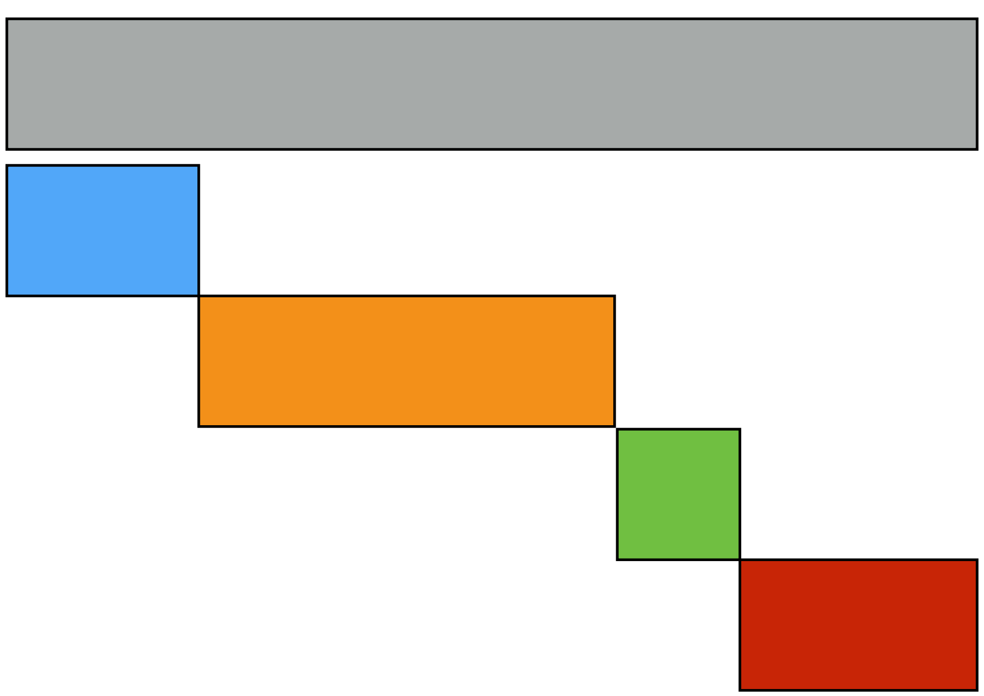

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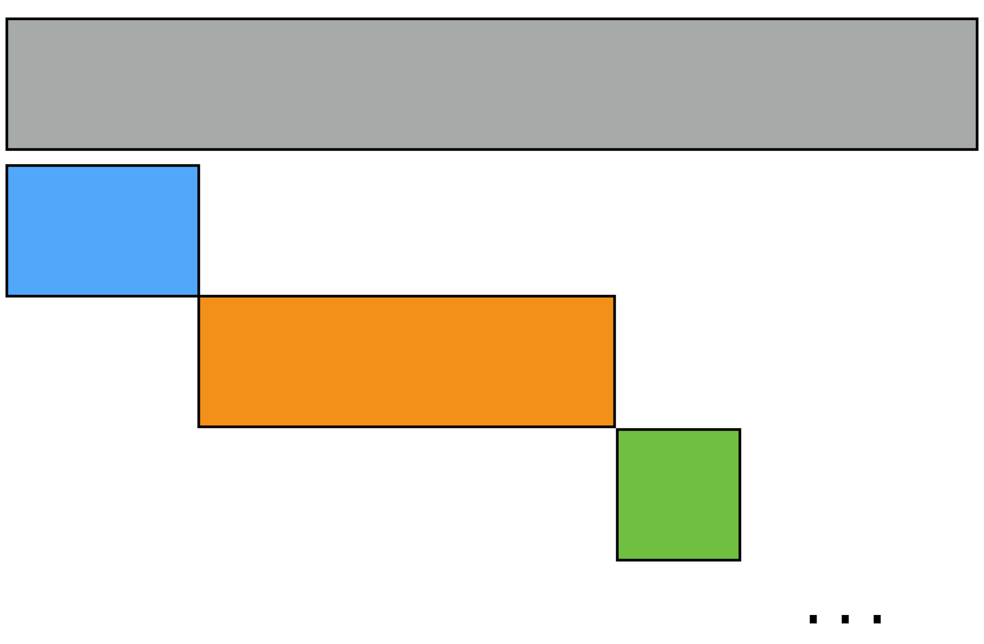

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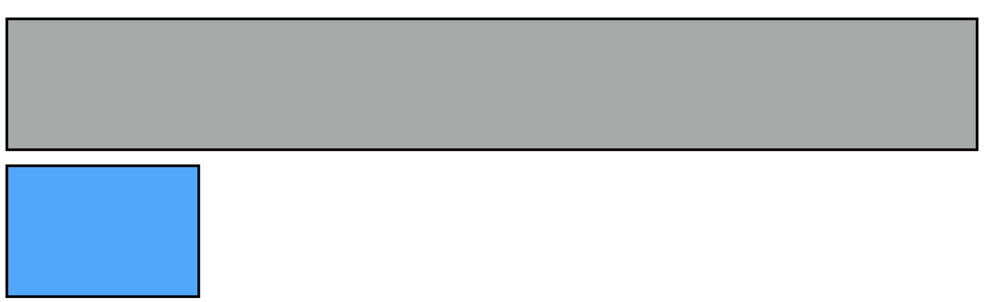

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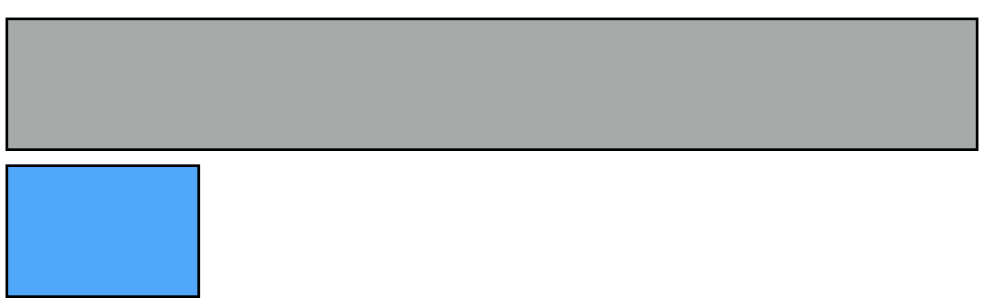

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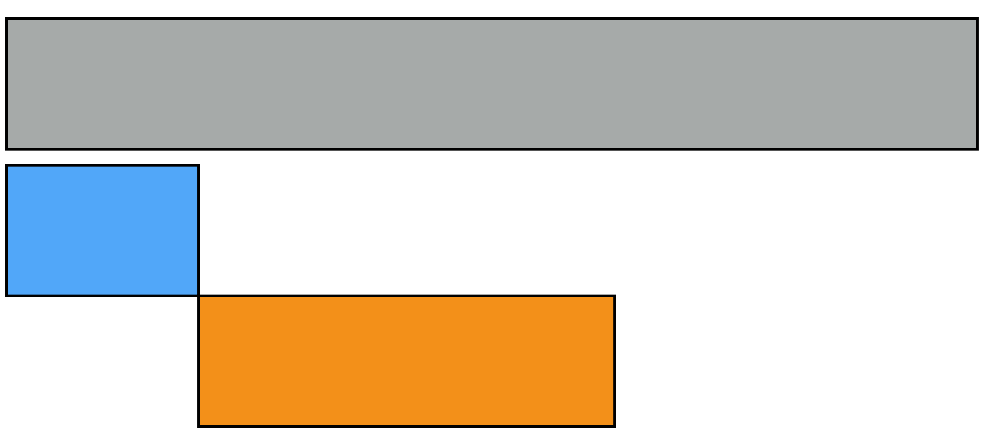

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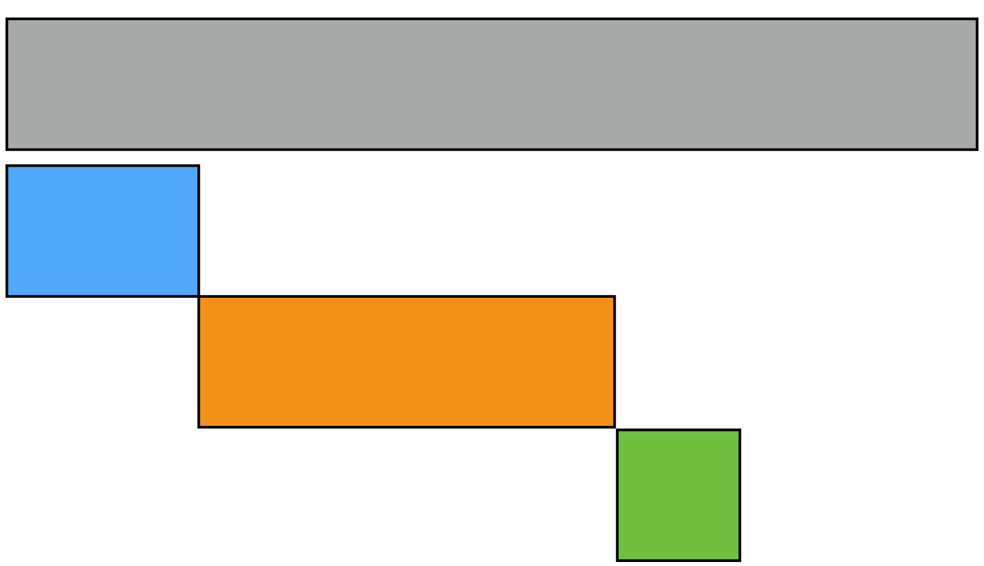

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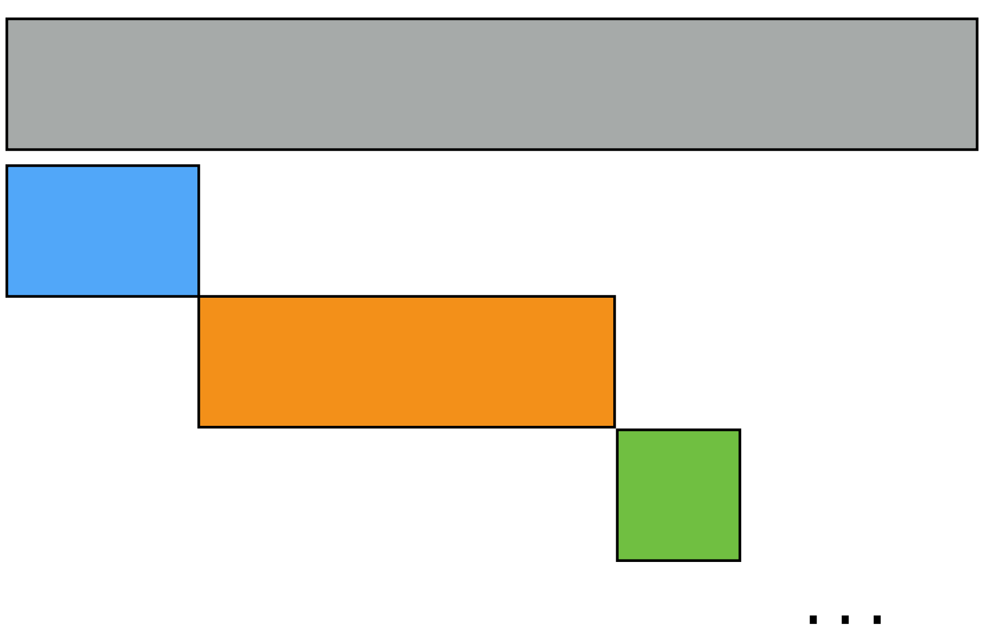

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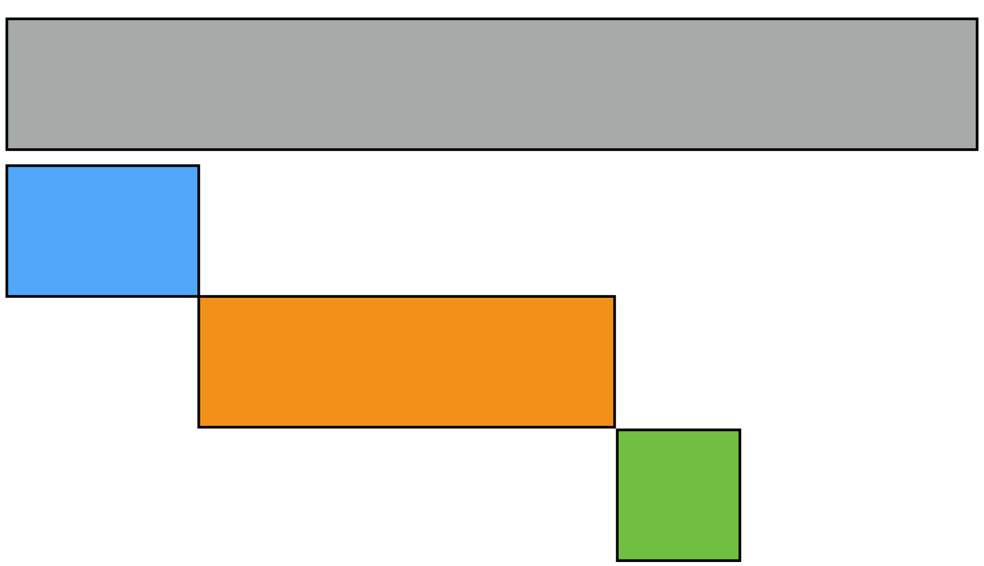

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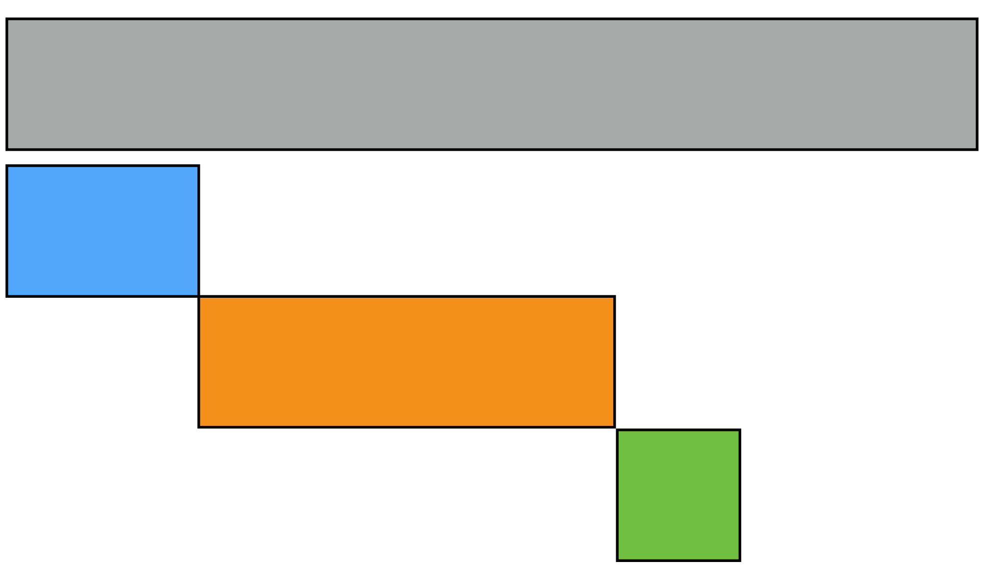

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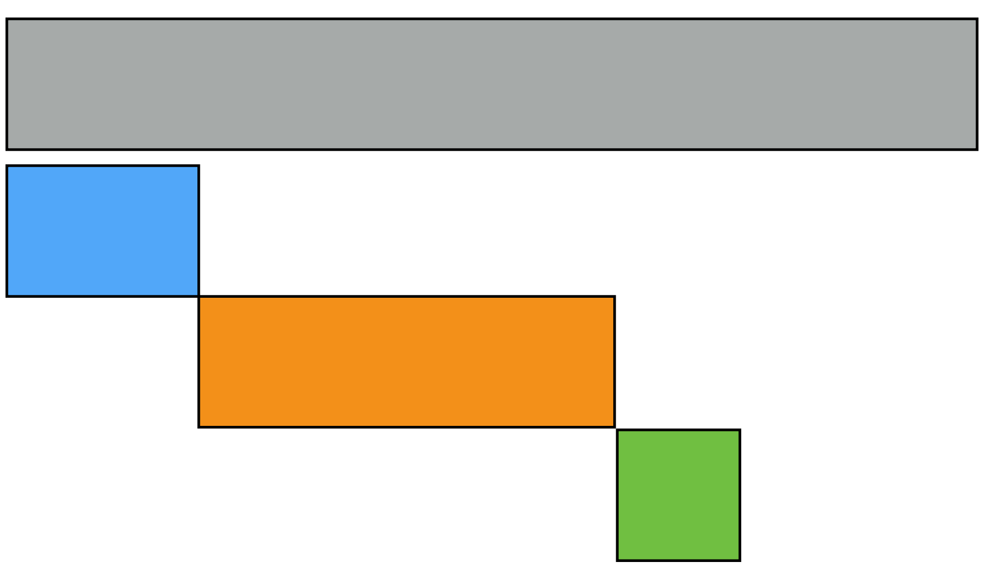

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[Ishwaran, James 2001]

Choosing $K = \infty$

- Here, difficult to choose finite K in advance (contrast with small K): don't know K, difficult to infer, streaming data
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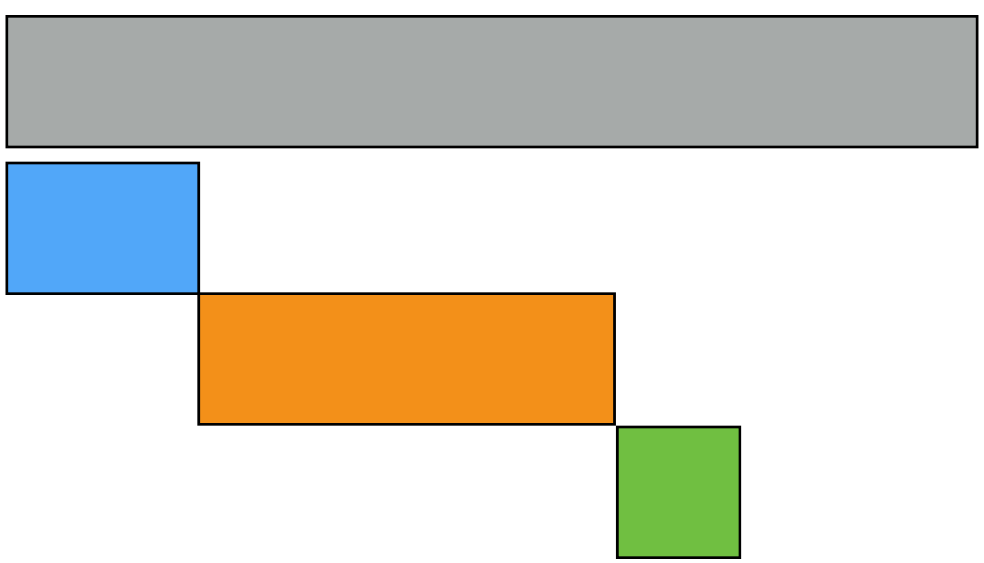
  - **Dirichlet process stick-breaking**: $a_k = 1, b_k = \alpha > 0$



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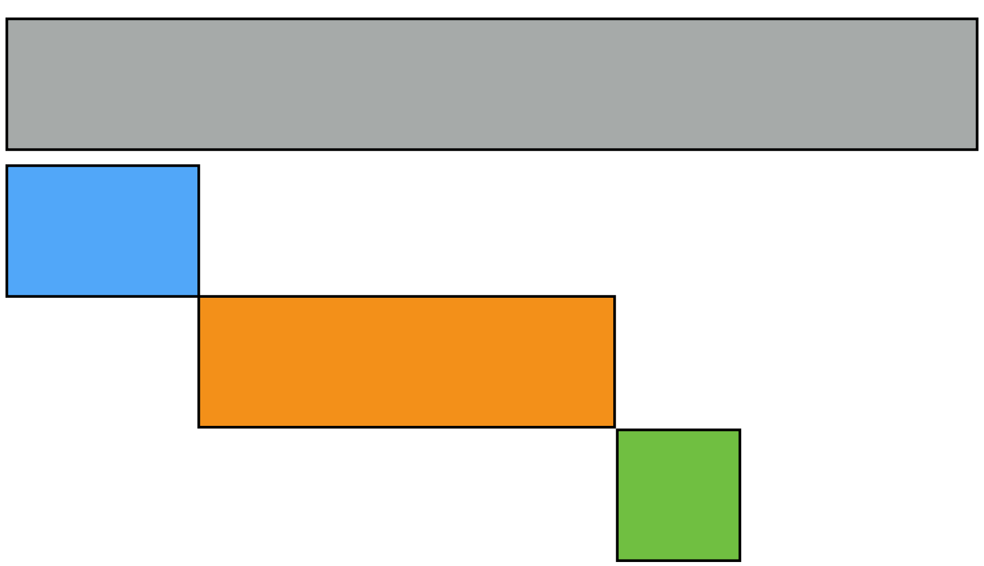

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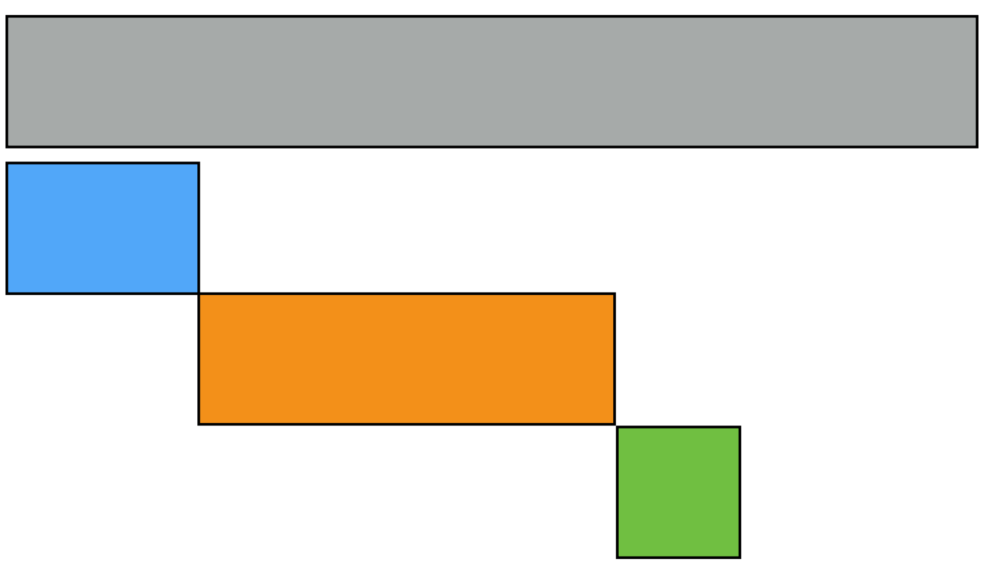
[demo]



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Dirichlet process mixture model

Dirichlet process mixture model

- Gaussian mixture model

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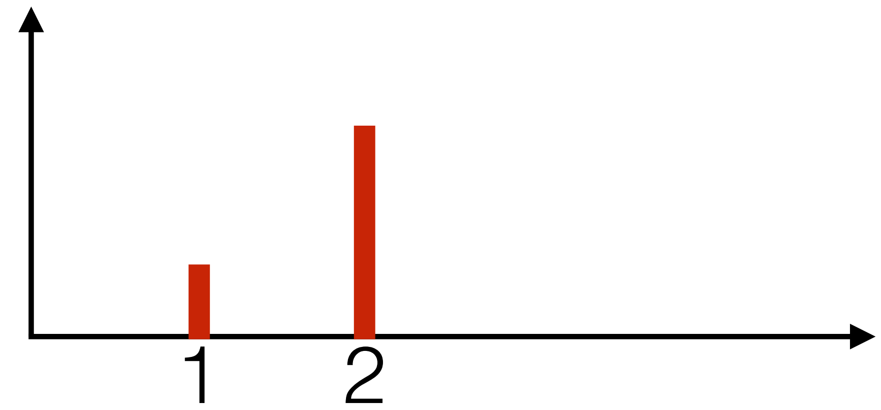

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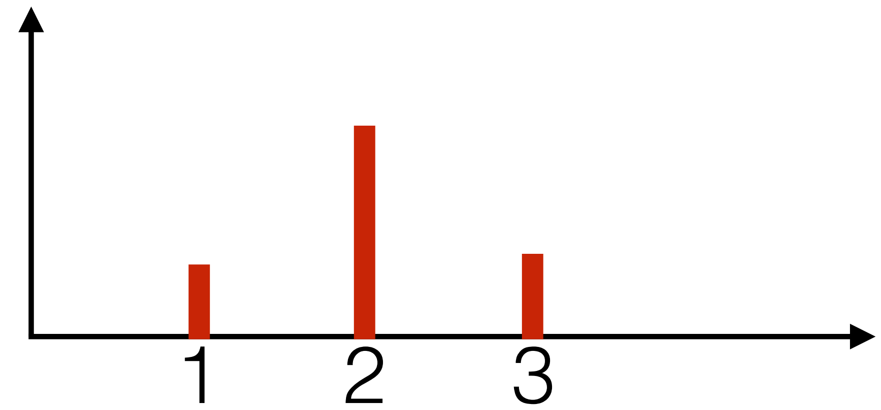
$$\rho = (\rho_1, \ldots, \rho_K) \sim \mathrm{Dir}(a_{1:K})$$



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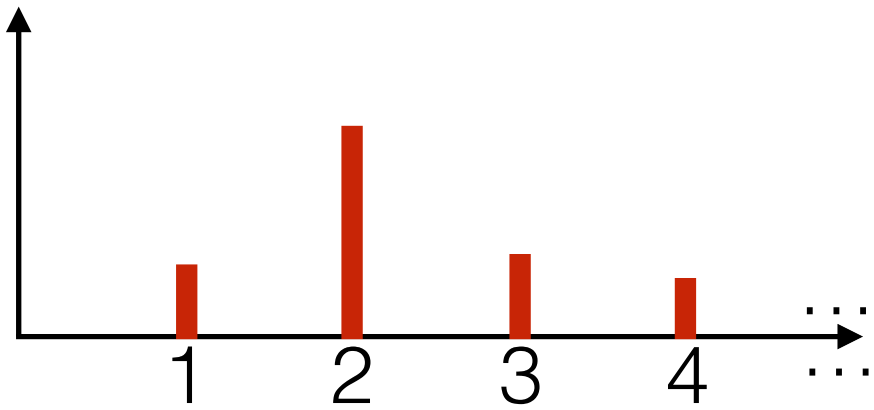

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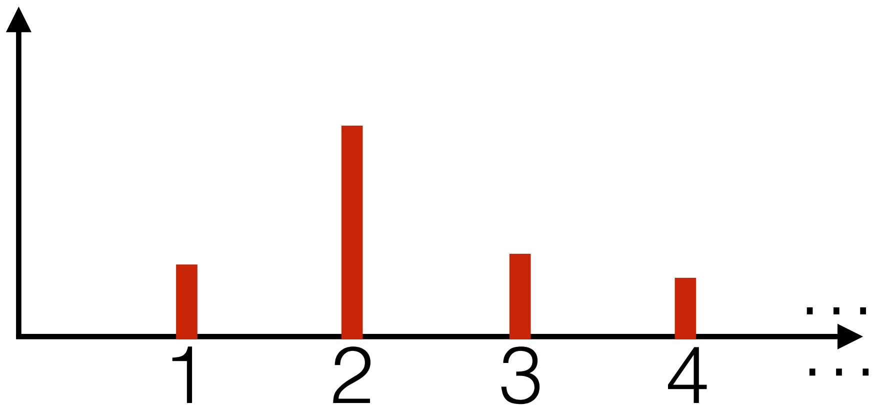

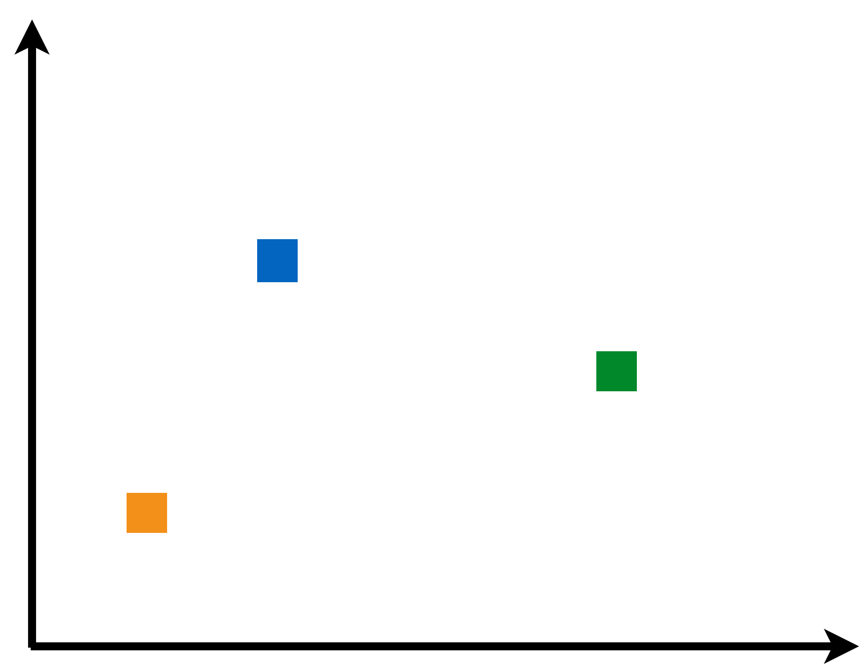

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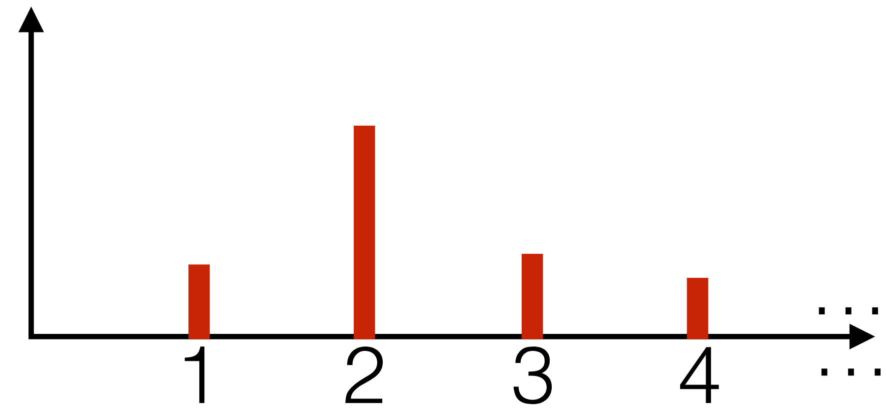

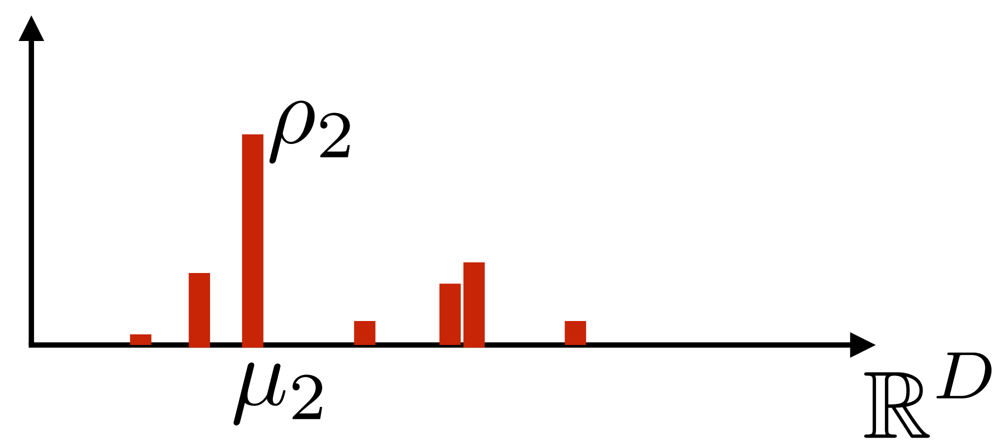

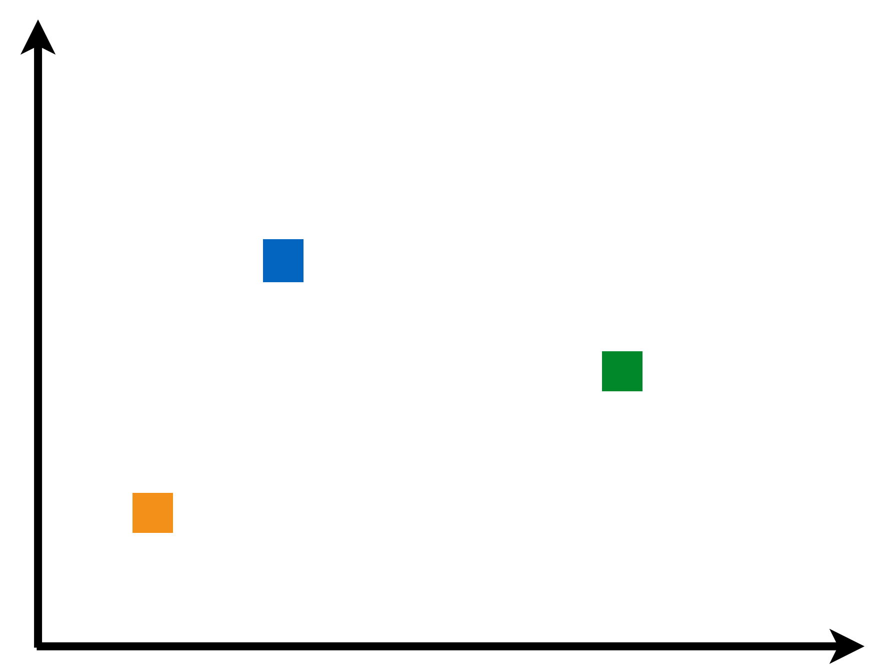

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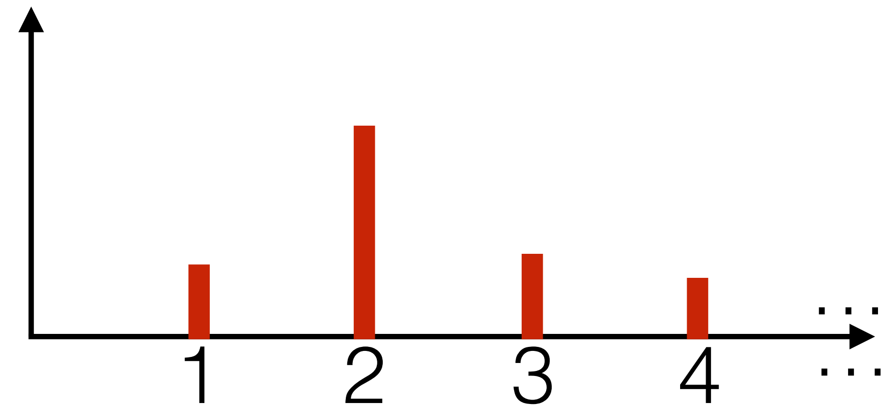

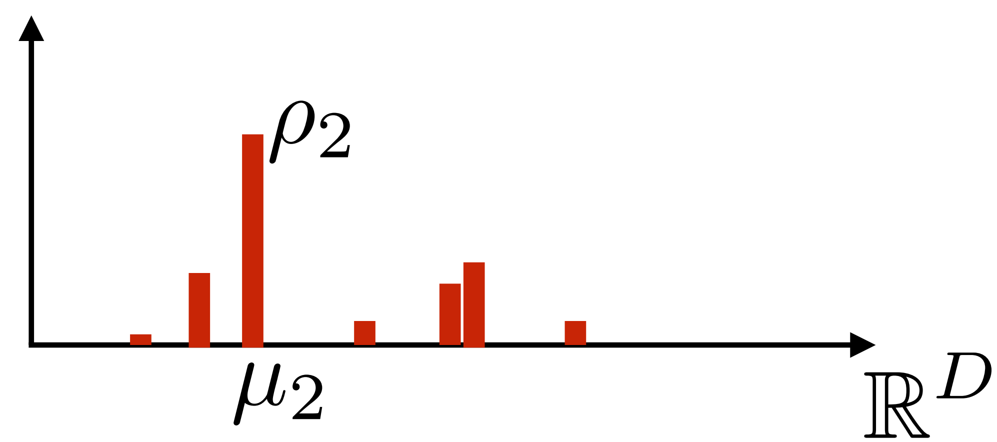

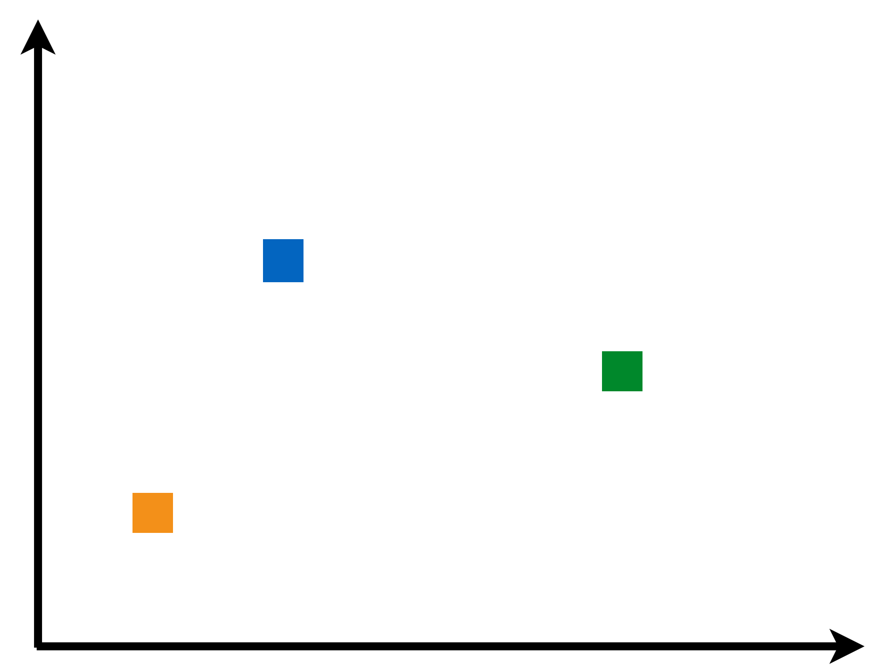

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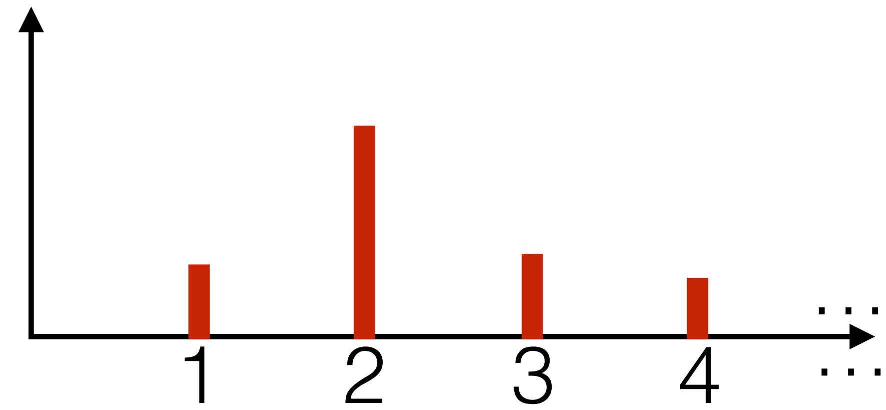

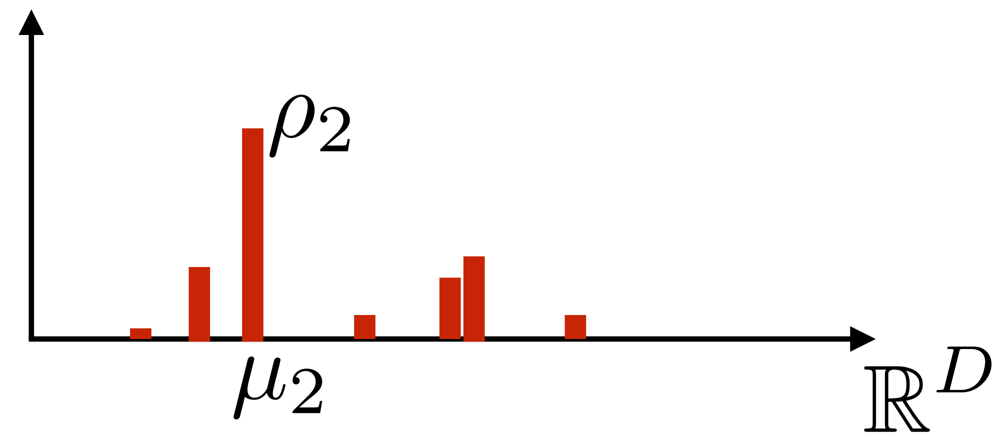

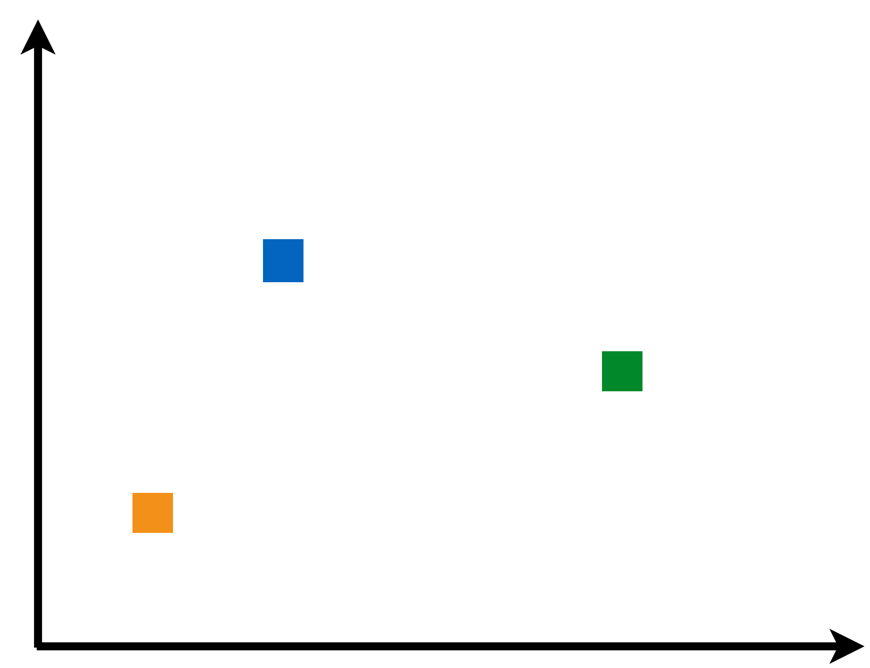

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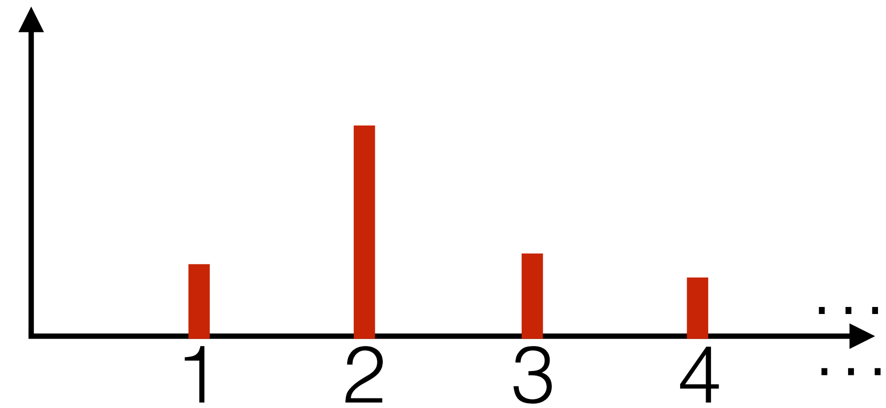
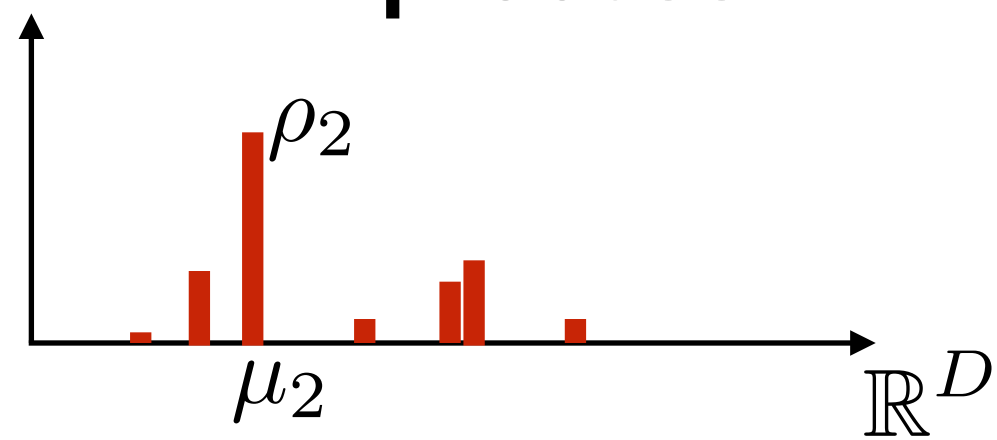
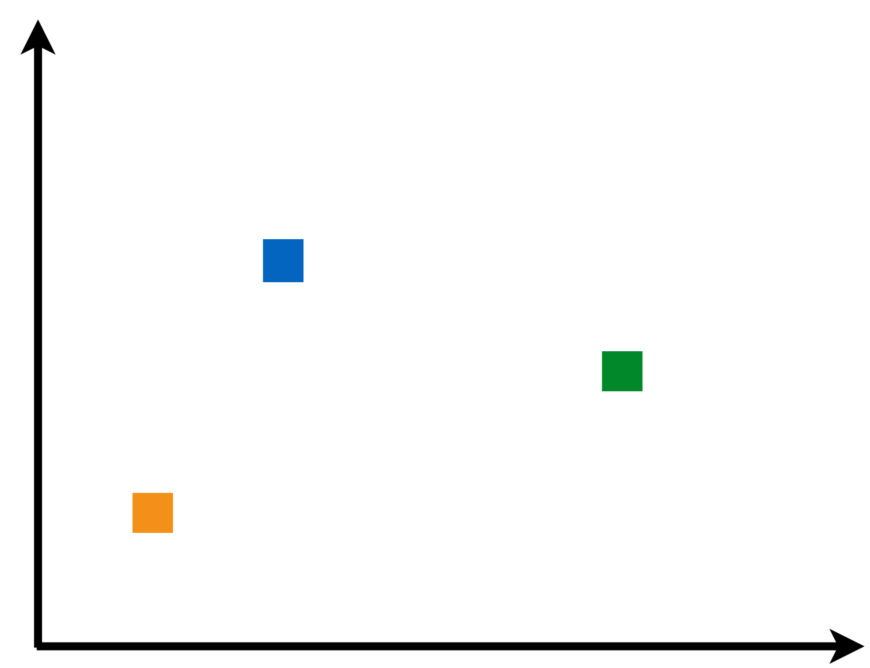


Dirichlet process





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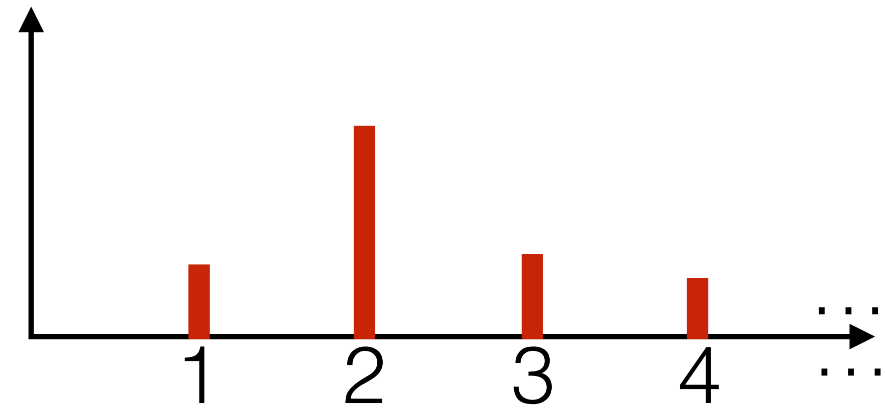

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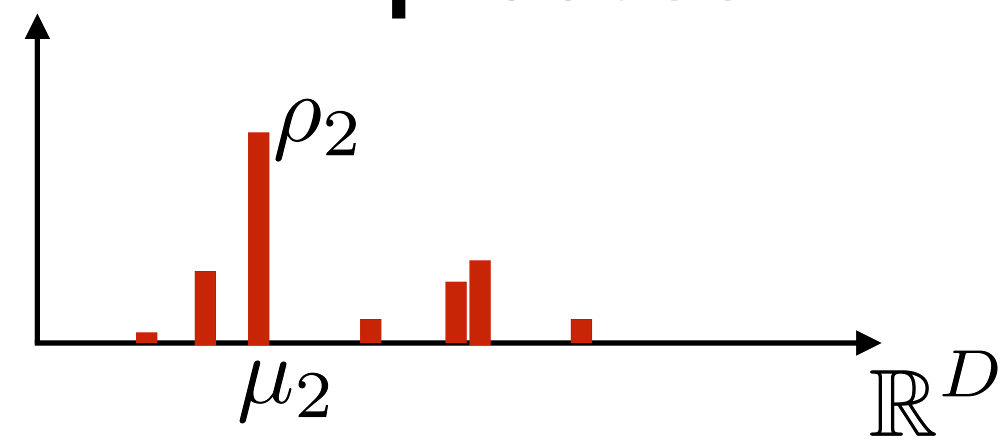

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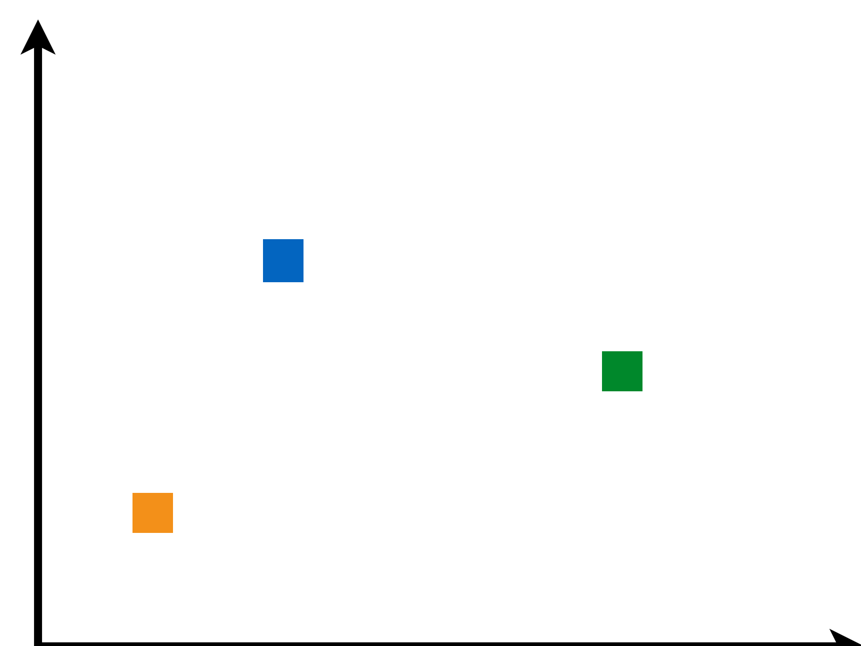

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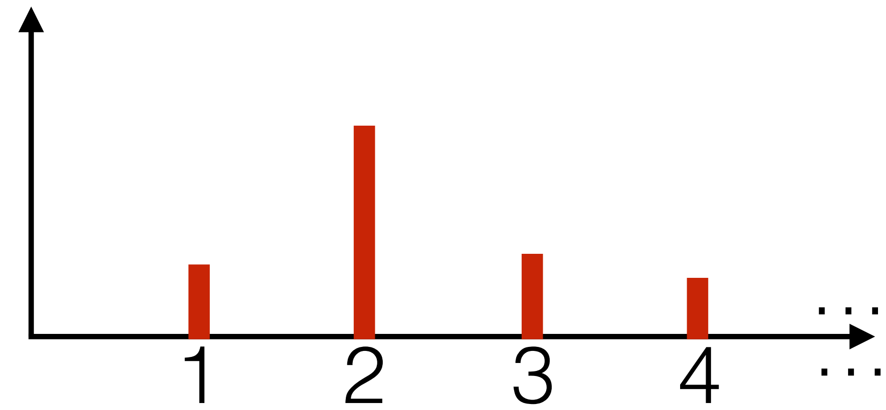

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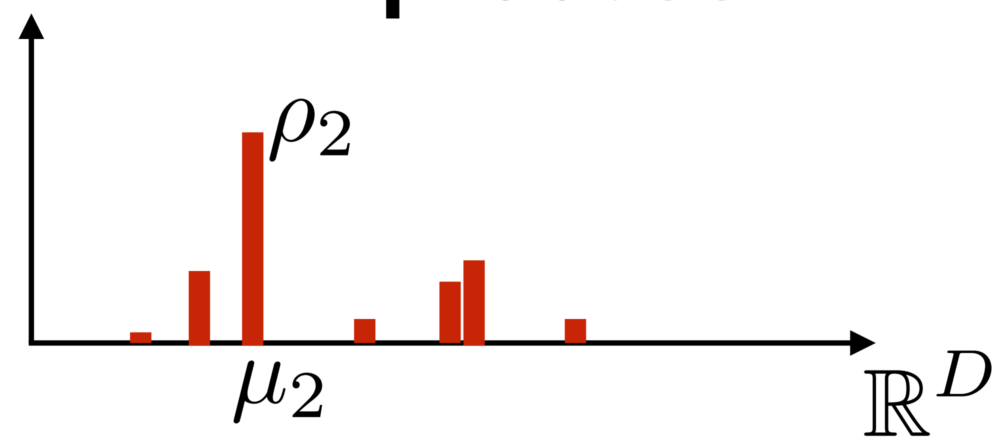

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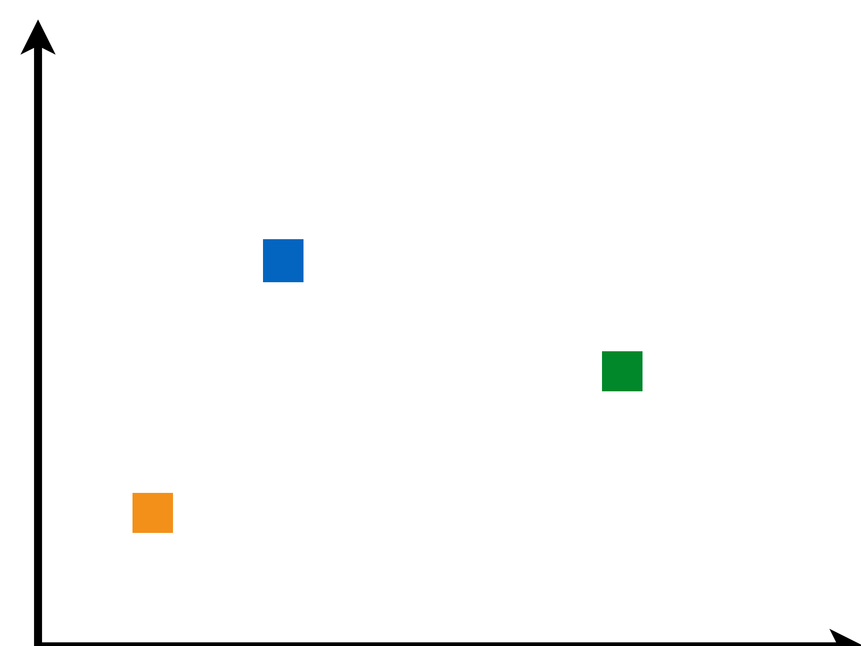

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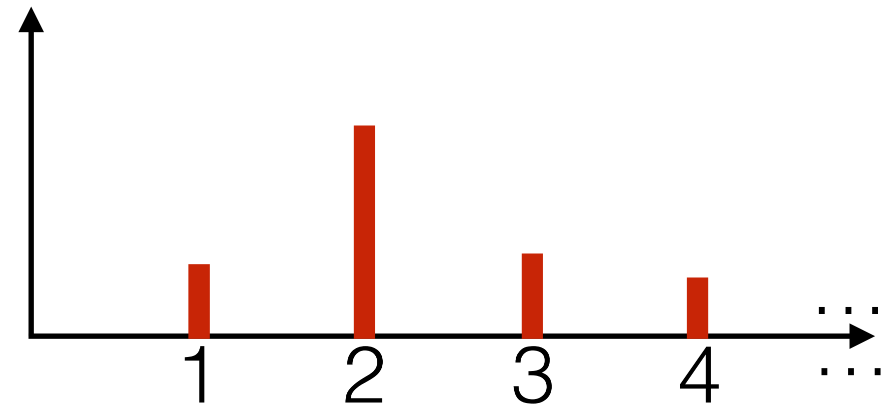

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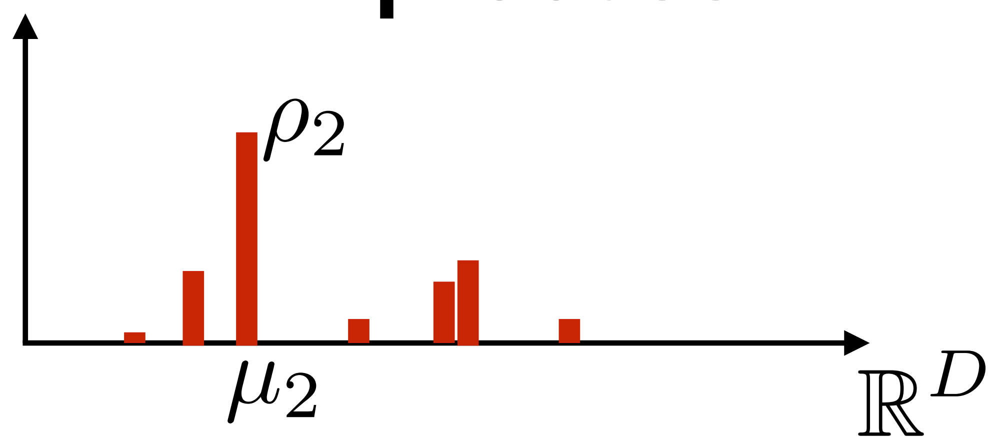

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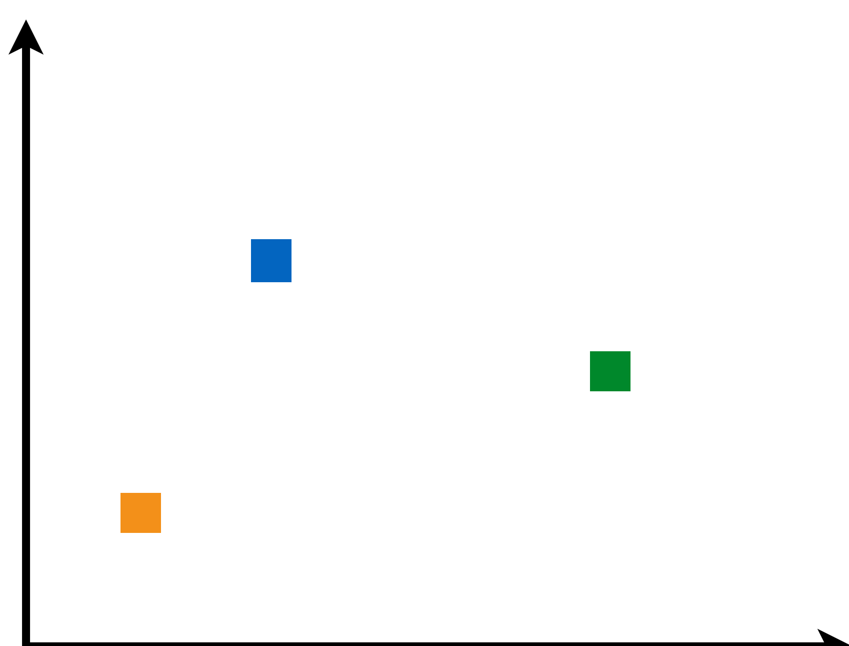

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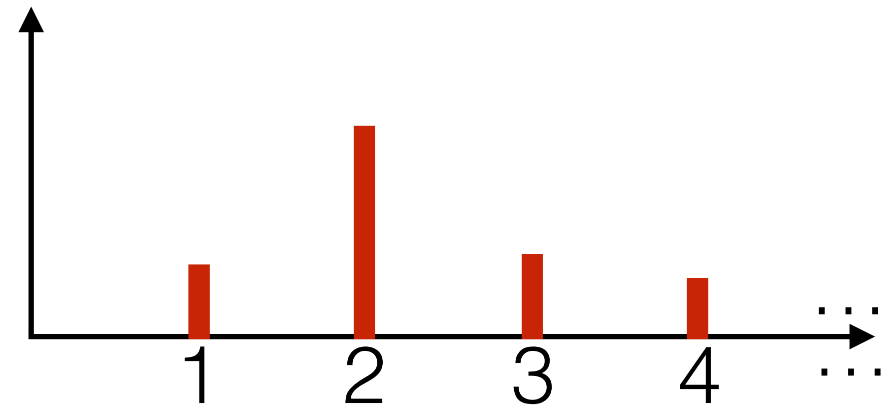

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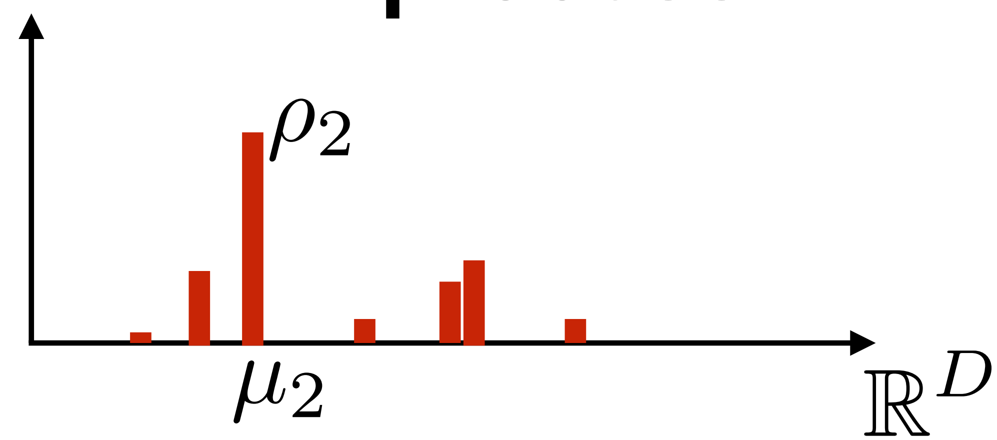

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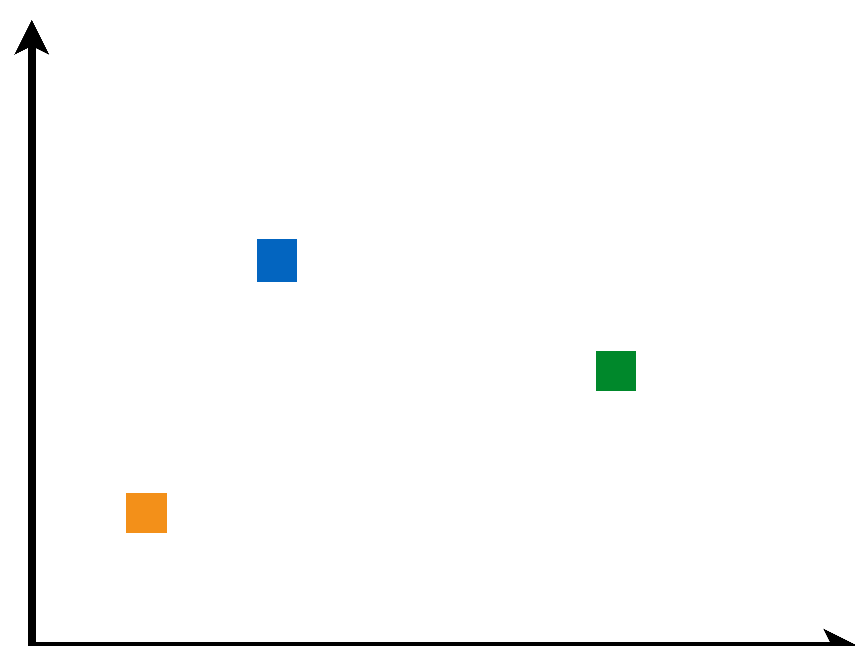

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Dirichlet process mixture model

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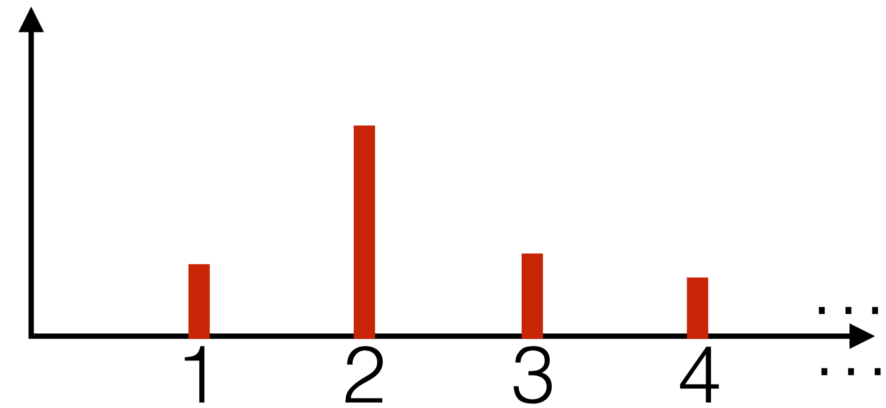

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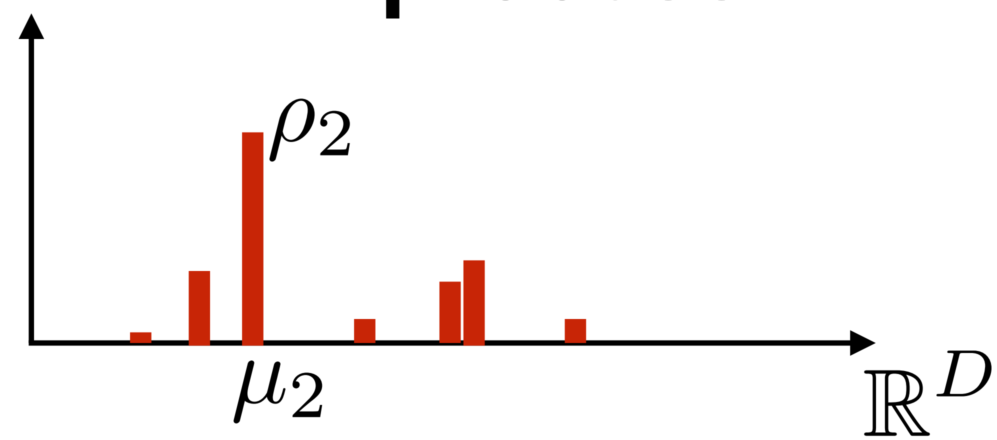

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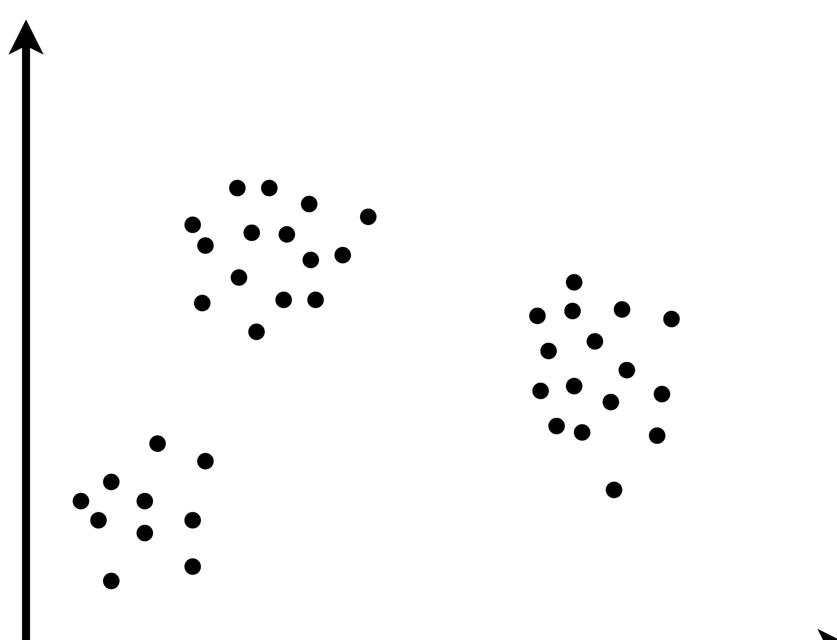

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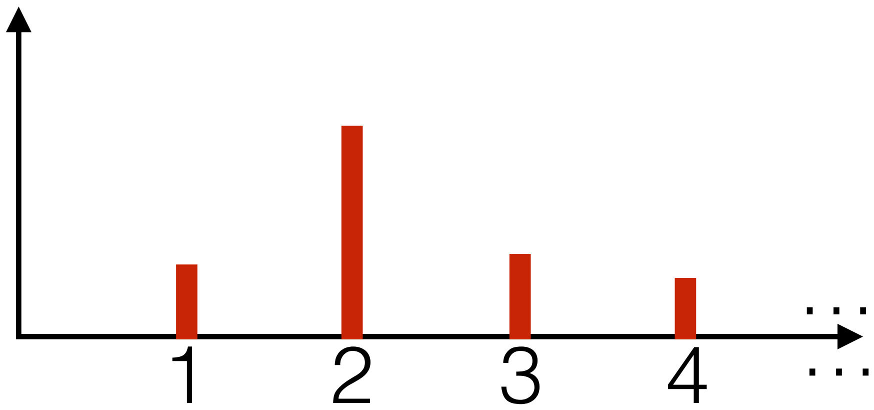

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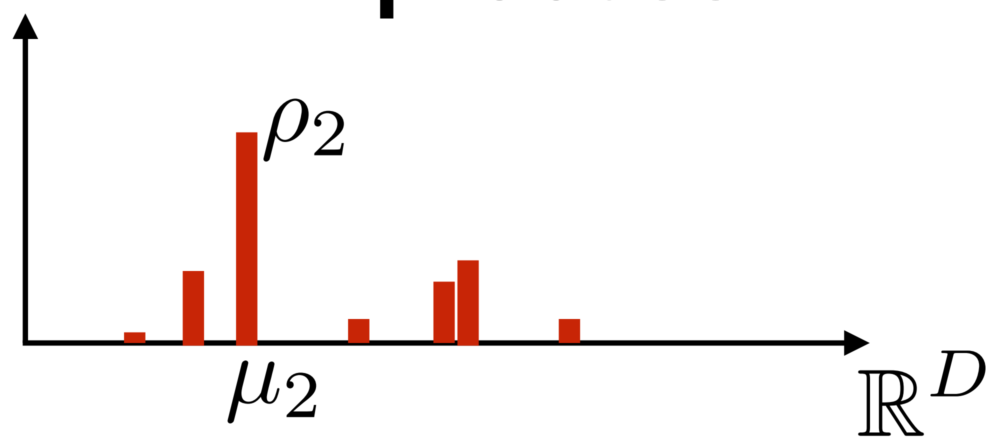

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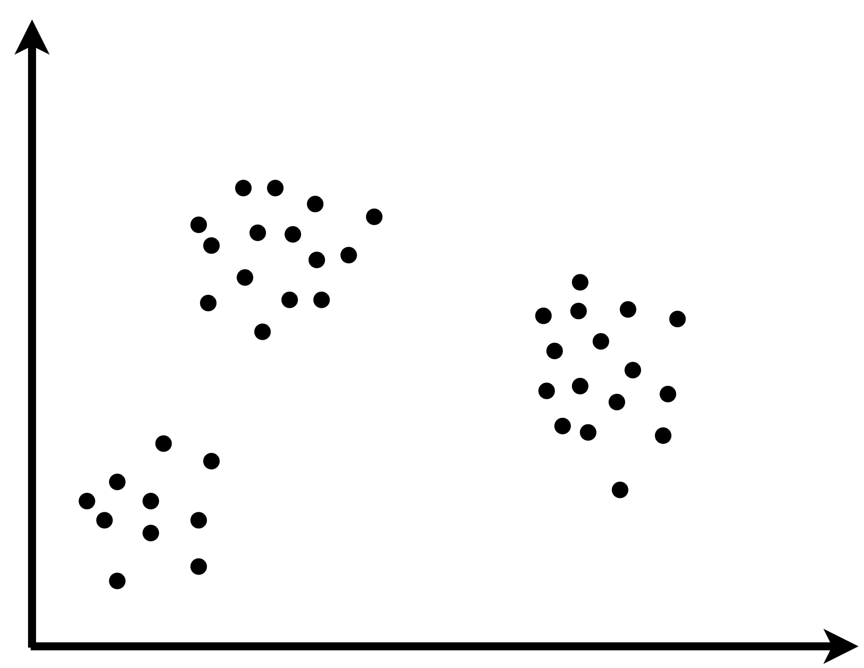

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[demo]

Roadmap

- Example problem: clustering
- Example NPBayes model: Dirichlet process

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  - Why is NPBayes challenging but practical?

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[Broderick, Wilson, Jordan, *to appear*]

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[Broderick, Wilson, Jordan, *to appear*; Campbell*, Huggins*, How, Broderick, *in review*]

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