Automated Scalable Bayesian Inference via Data Summarization

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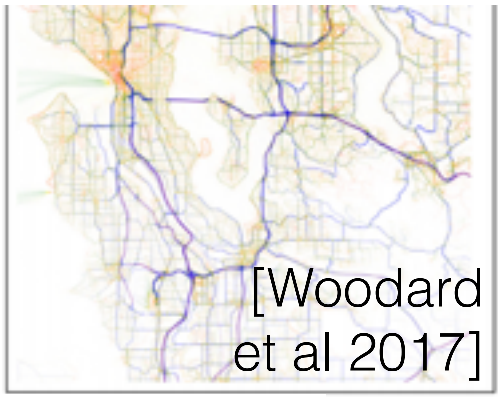
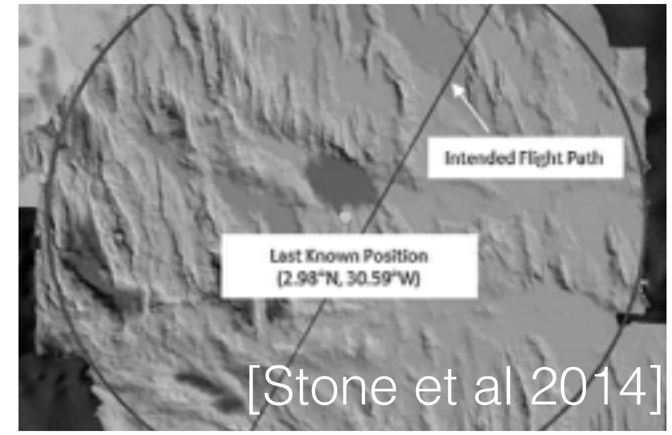
MIT

With: Trevor Campbell, Jonathan Huggins

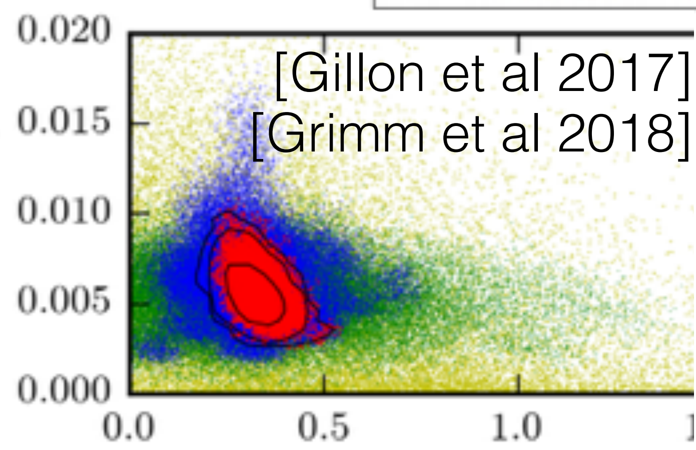
http://www.tamarabroderick.com/tutorials.html

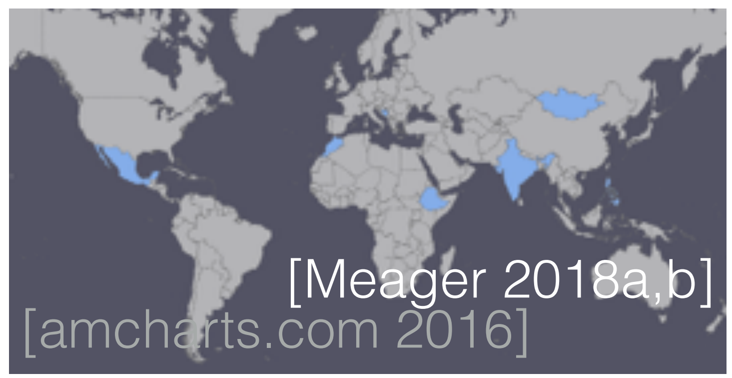
Bayesian inference





[mc-stan.org]





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  - Point estimates, coherent uncertainties
  - Interpretable, complex, modular; prior information

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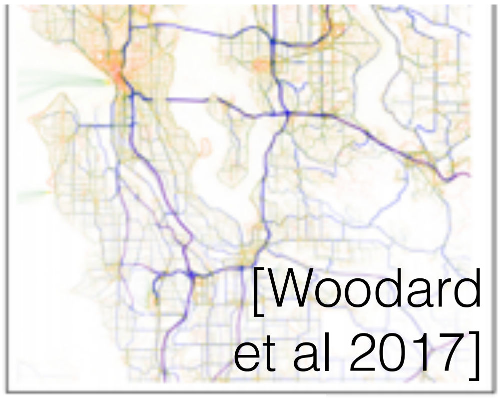

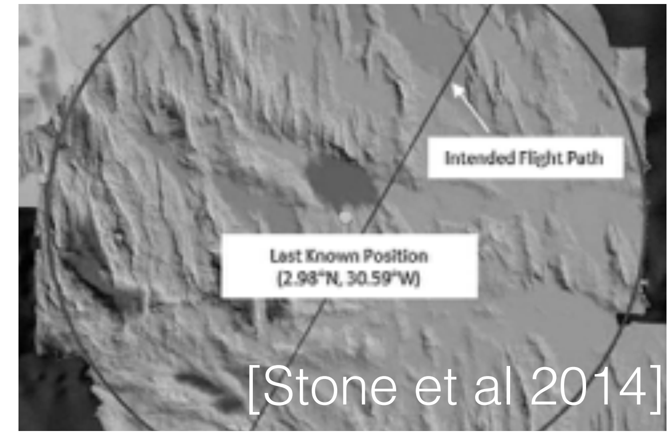

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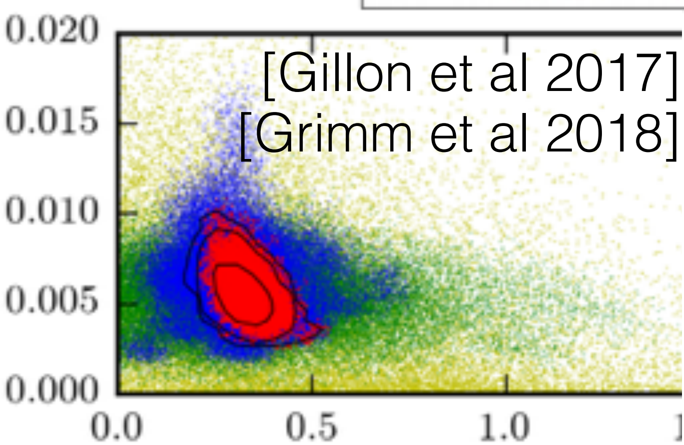

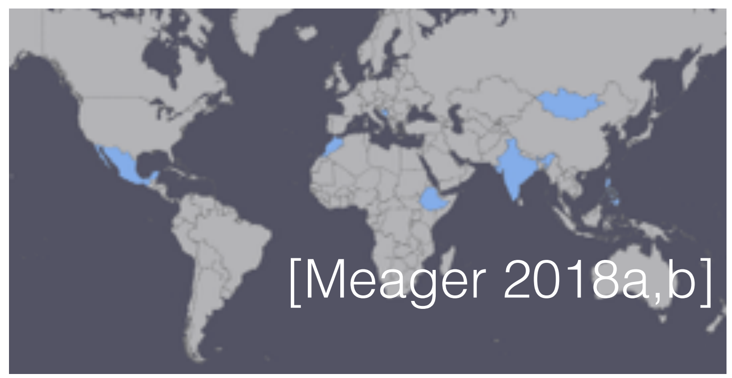

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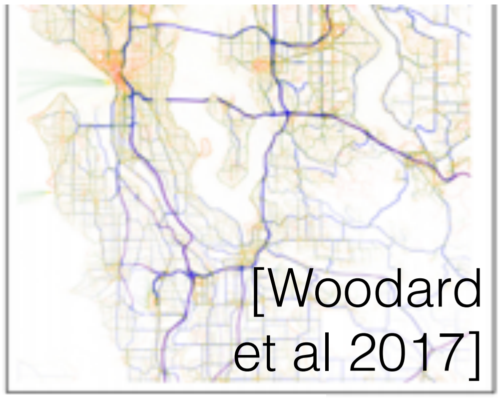

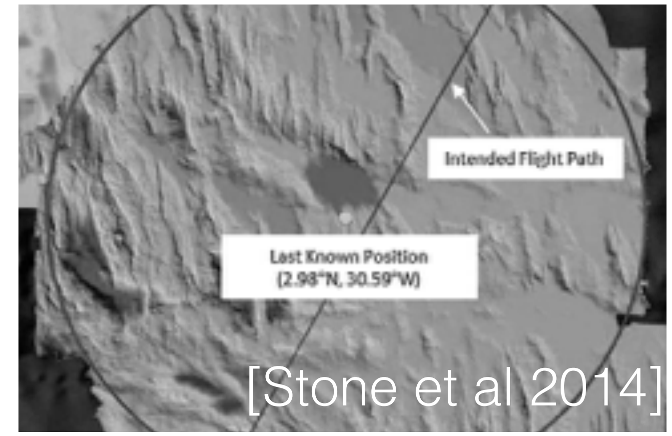

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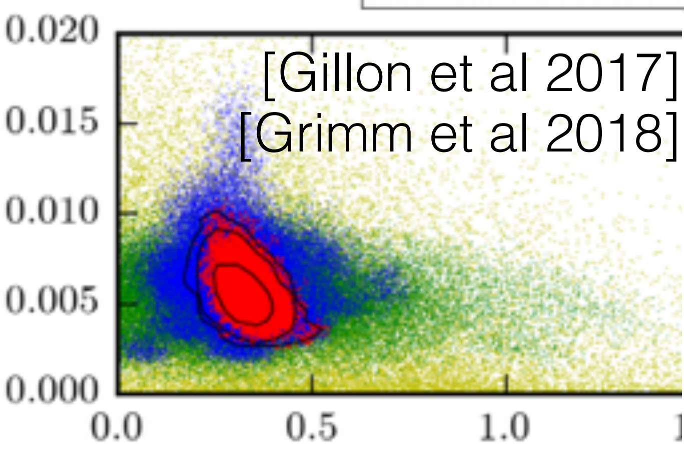

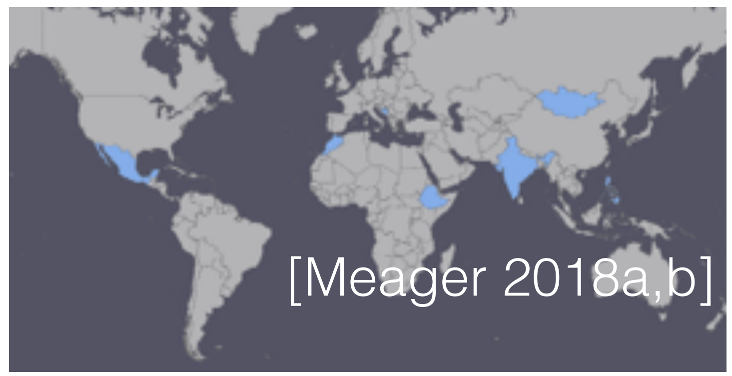

- Desiderata:
    - Point estimates, coherent uncertainties
    - Interpretable, complex, modular; prior information
- Challenge: existing methods can be slow, tedious, unreliable
- Our proposal: use *efficient data summaries* for **scalable**, **automated** algorithms with **error bounds for finite data**

Roadmap

- Approximate Bayes review
- The “core” of the data set
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$p(\theta)$

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$p(y|\theta)p(\theta)$

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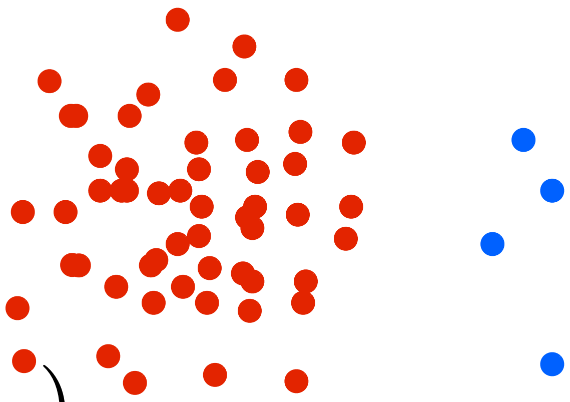

(x_n, y_n)

Bayesian inference

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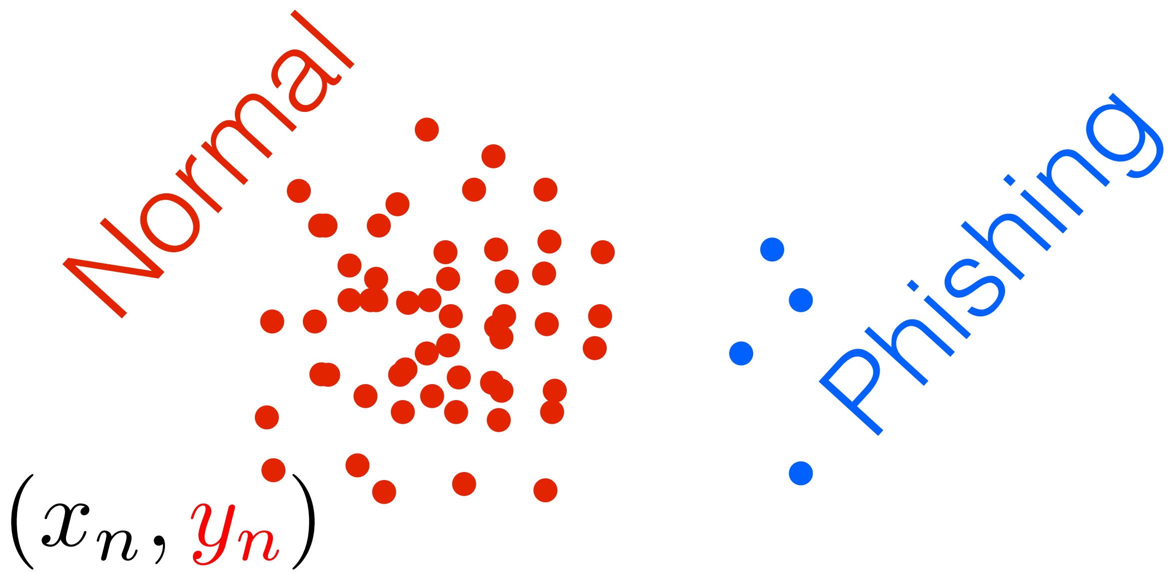

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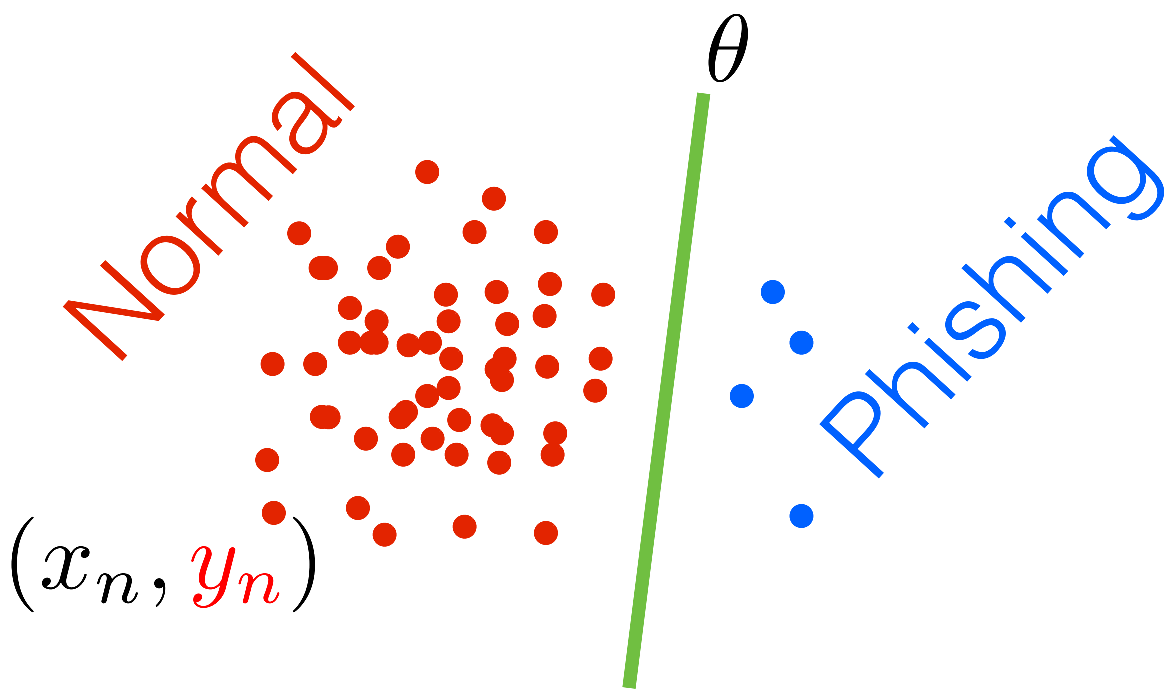

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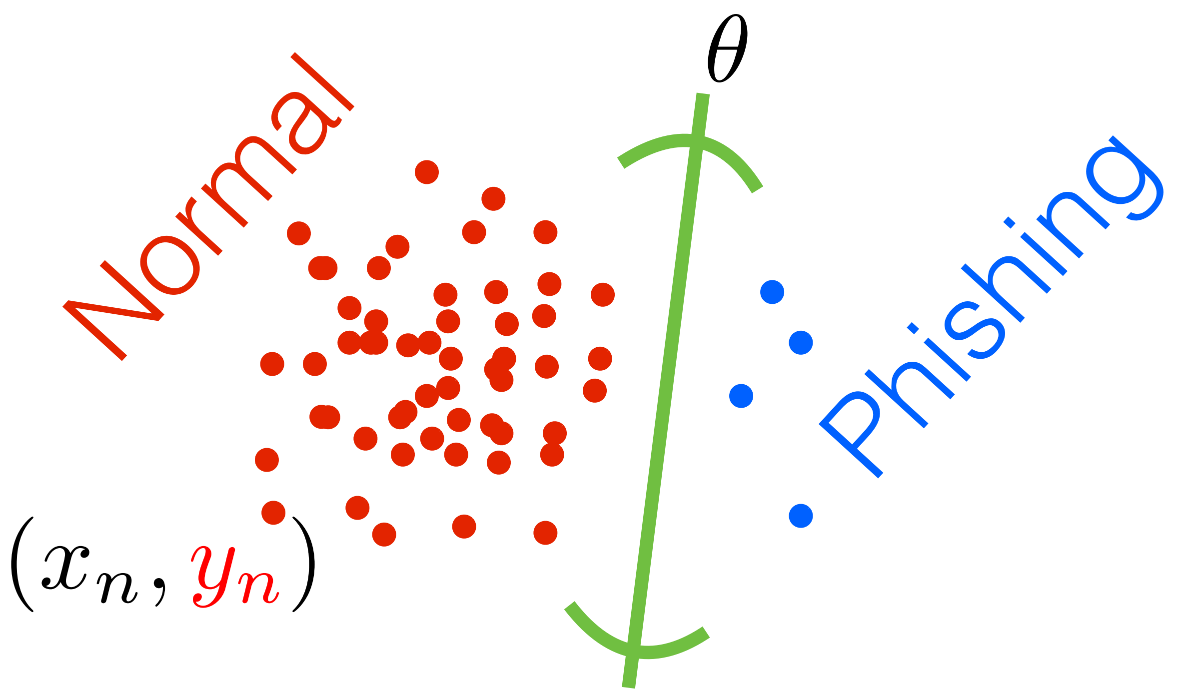

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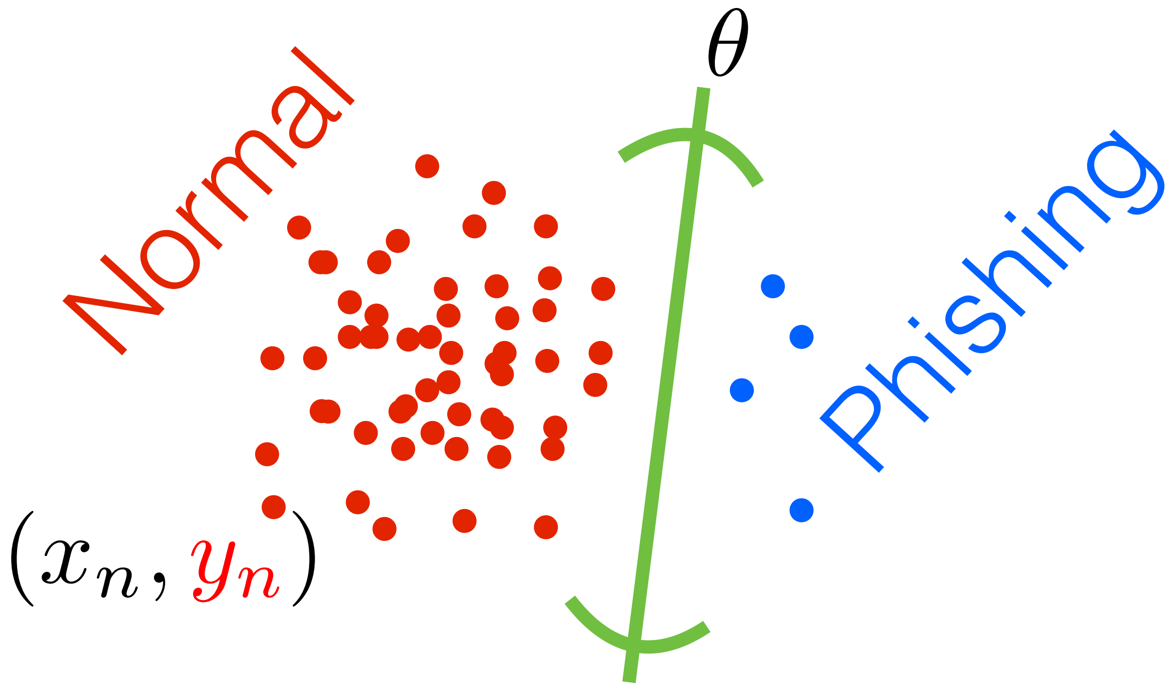

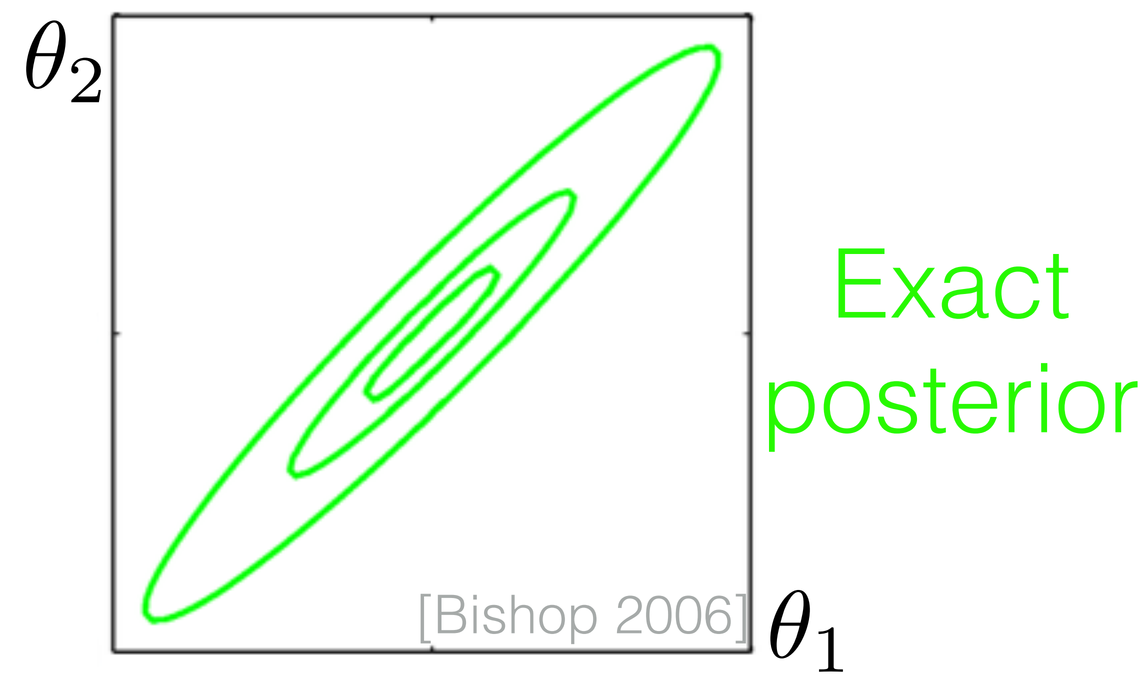

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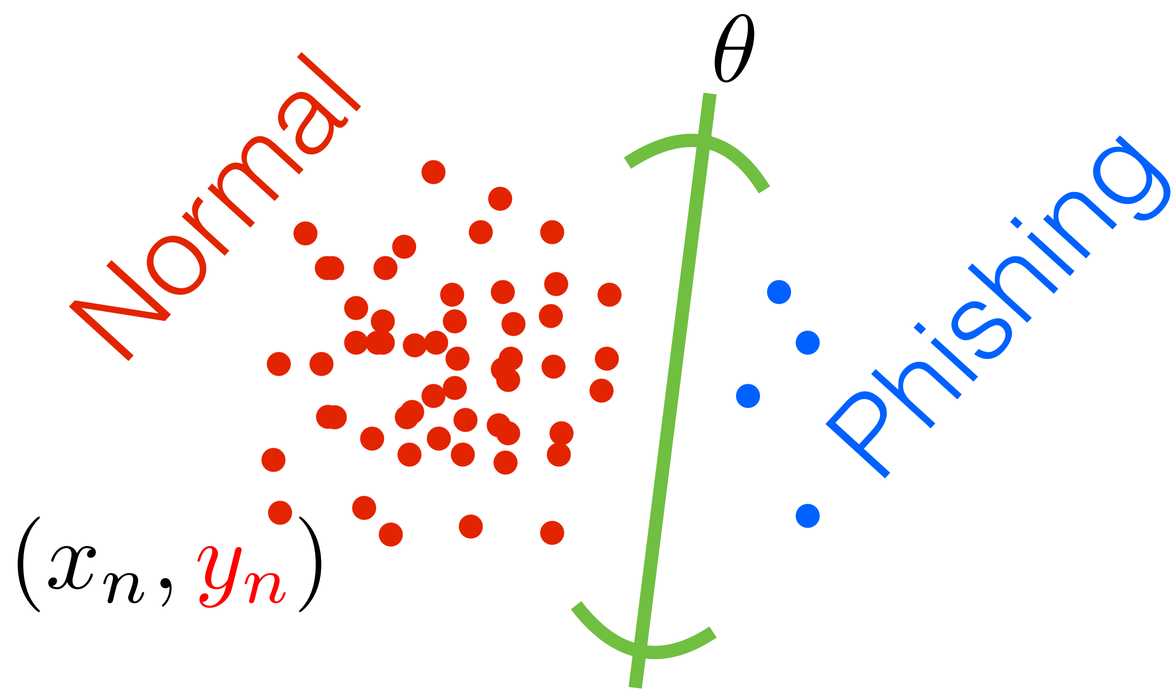

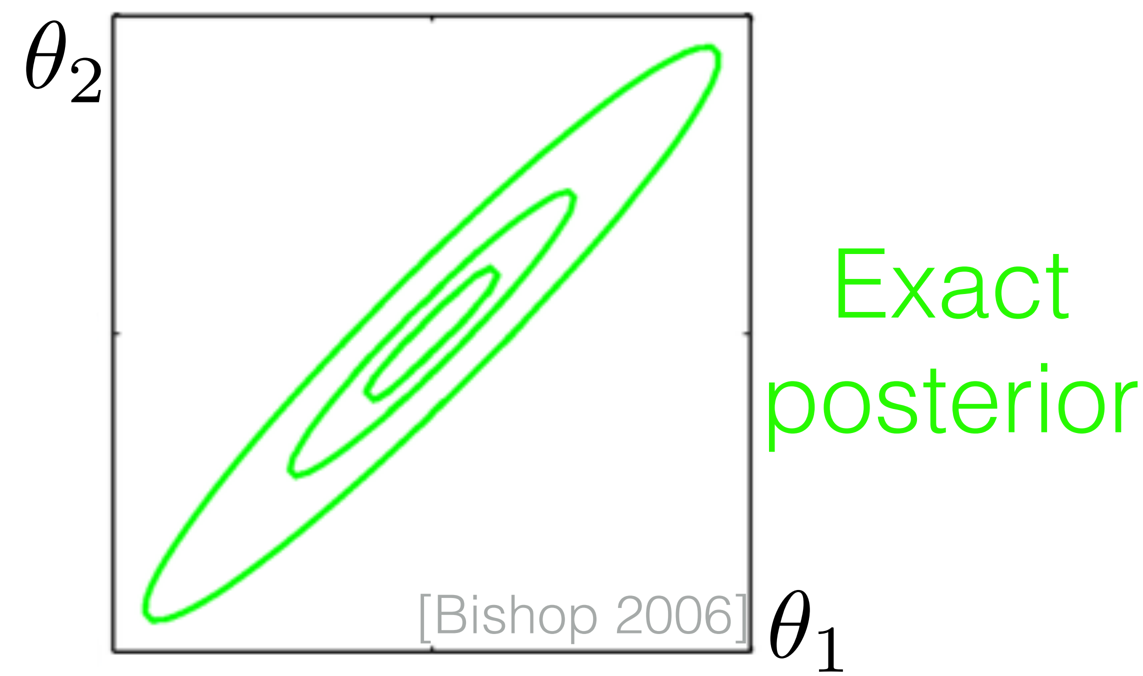

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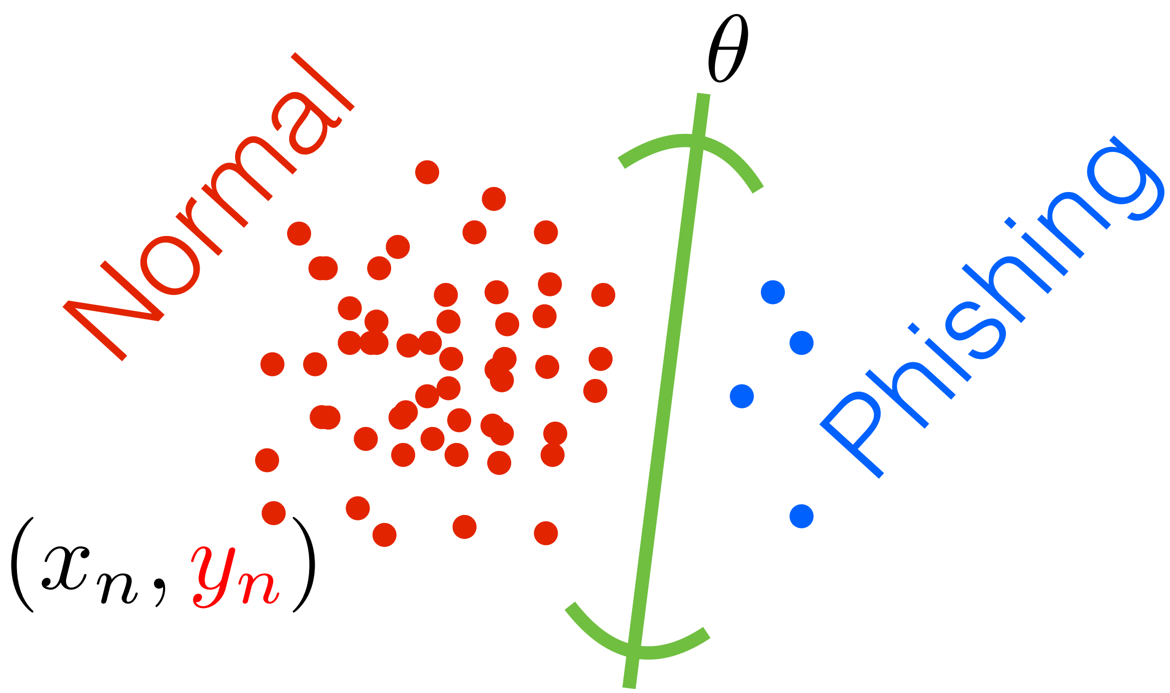

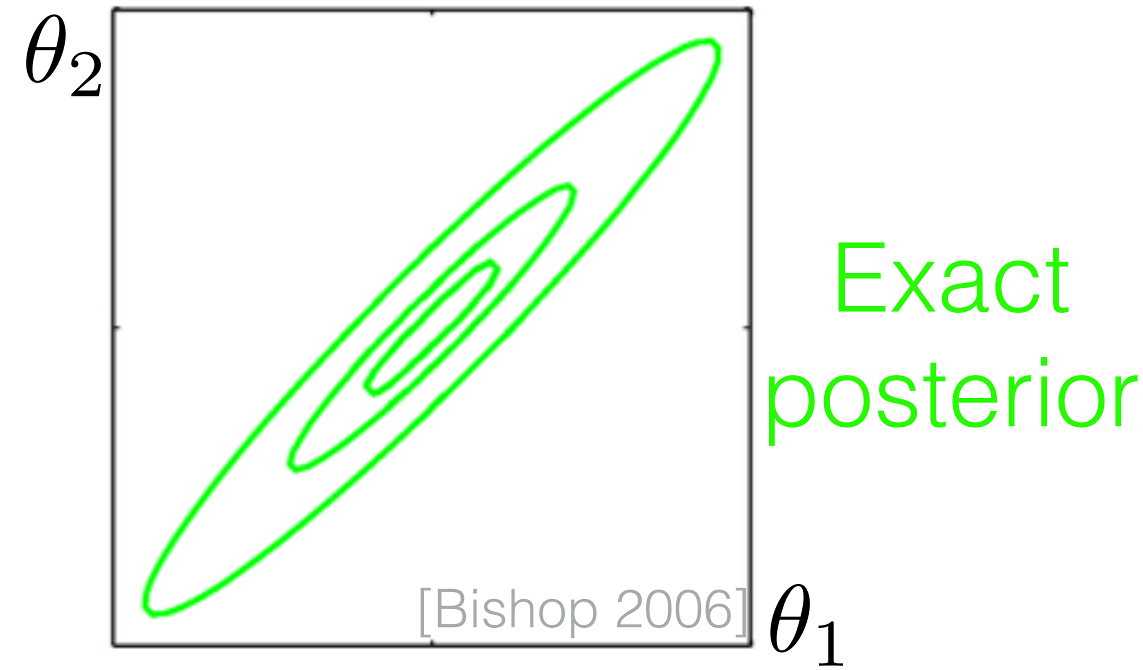

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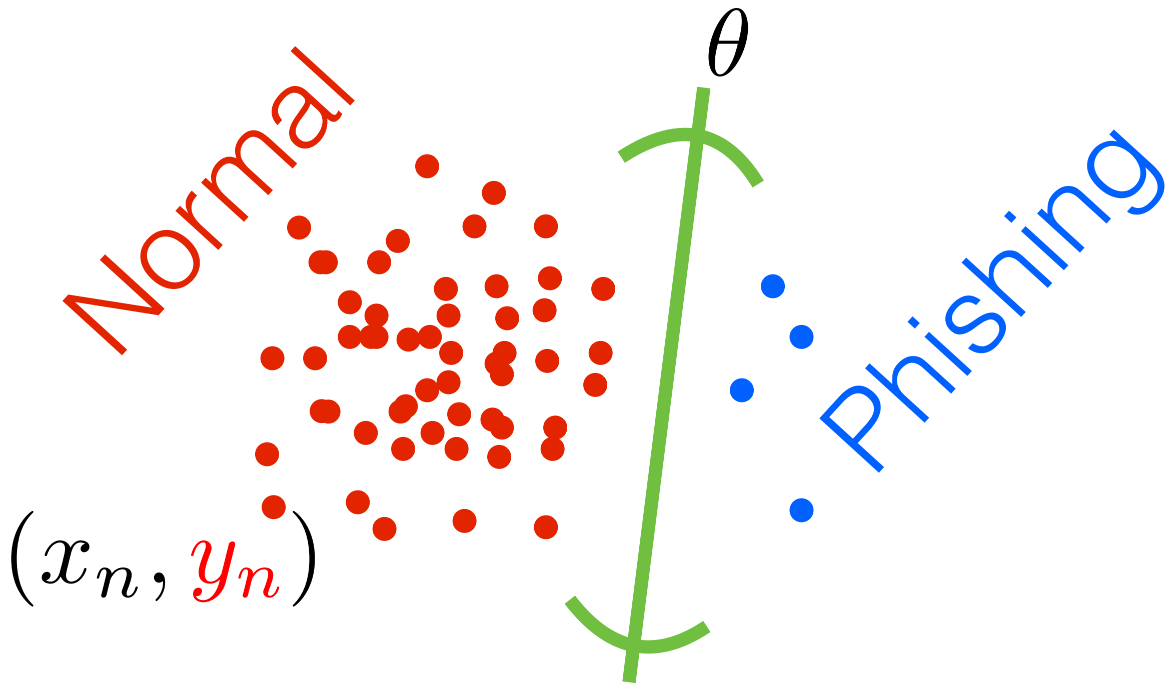

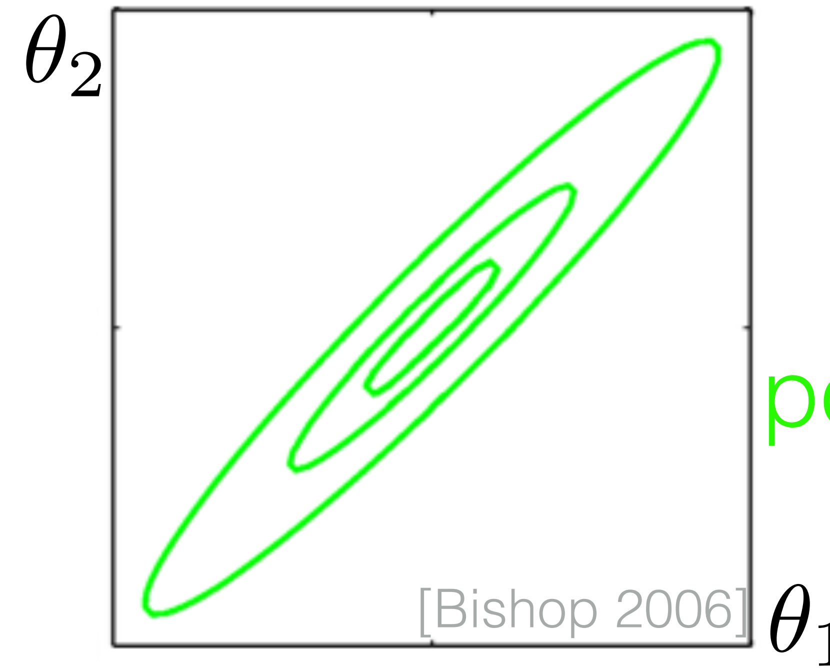

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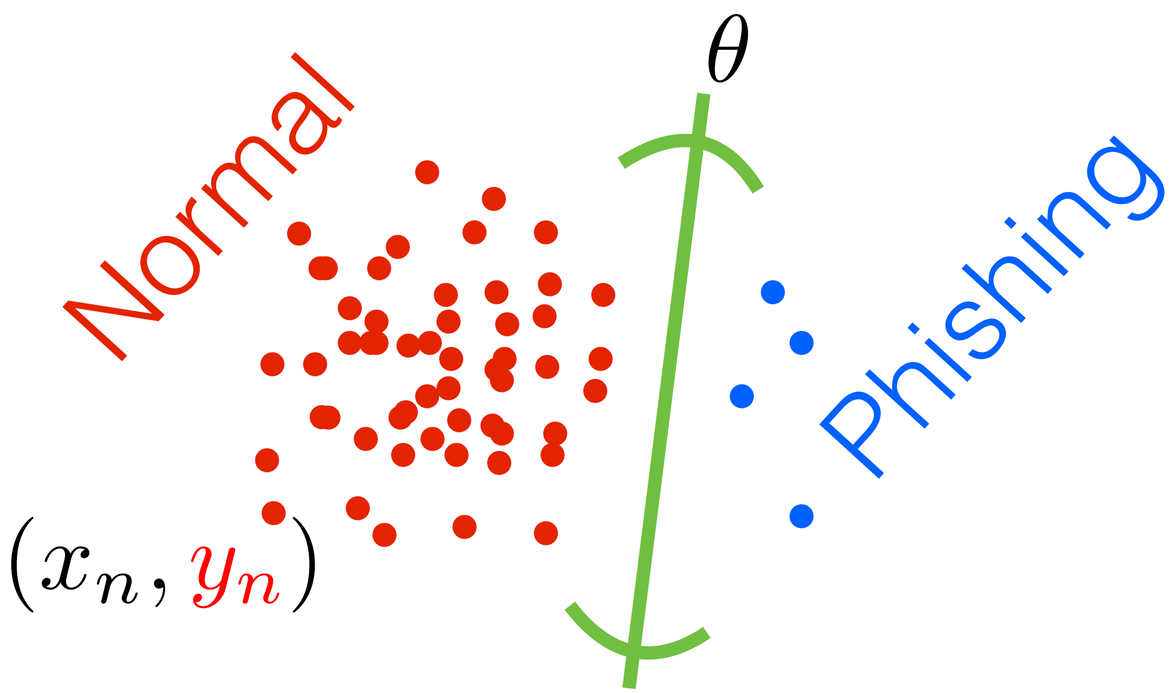

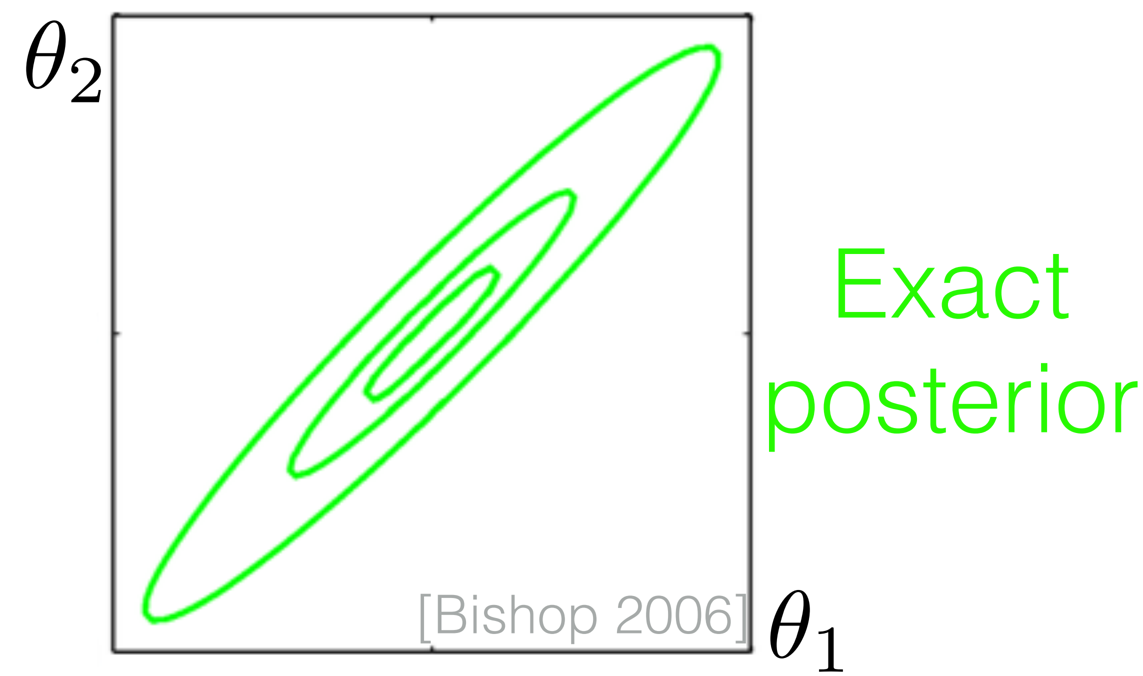

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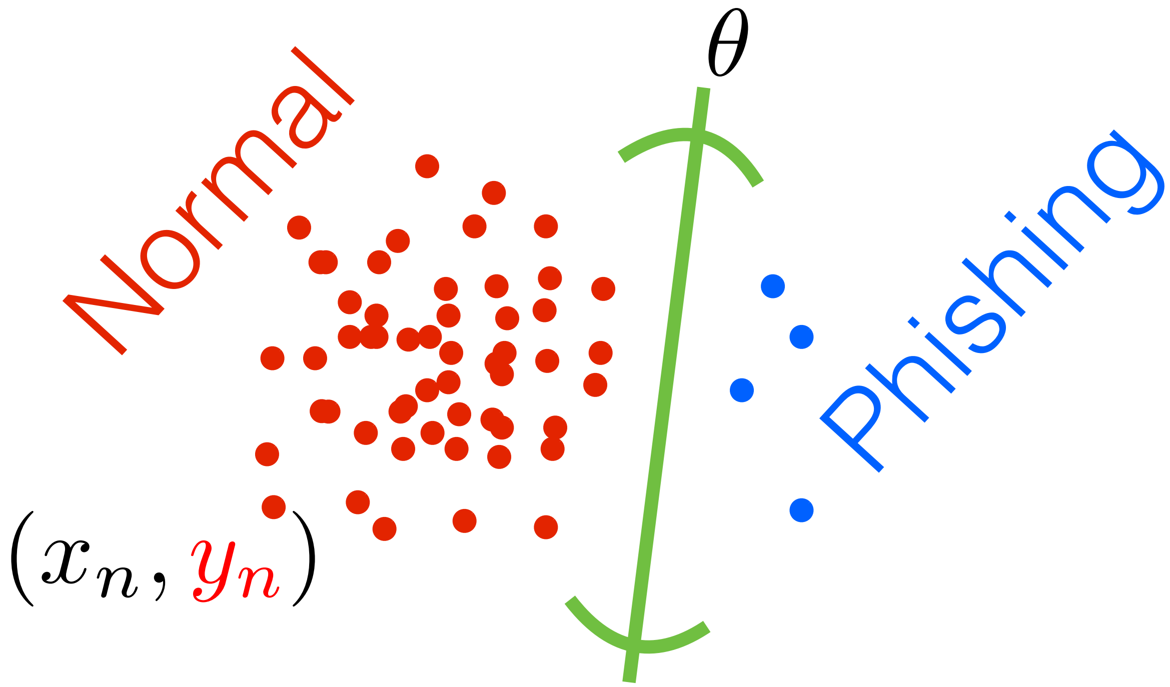

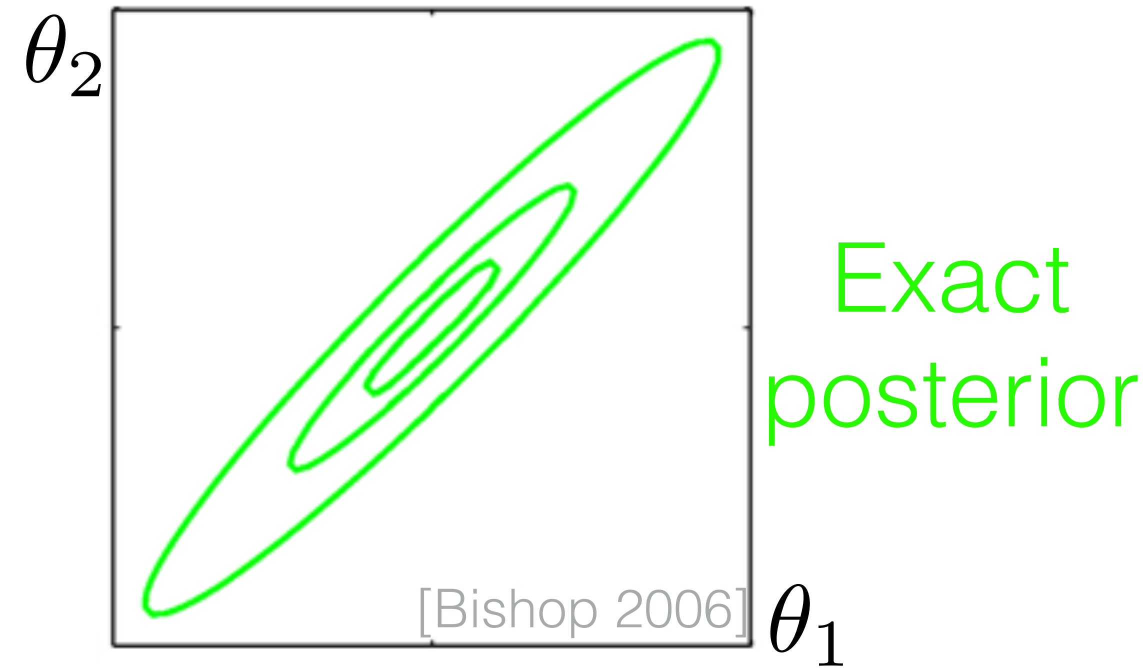

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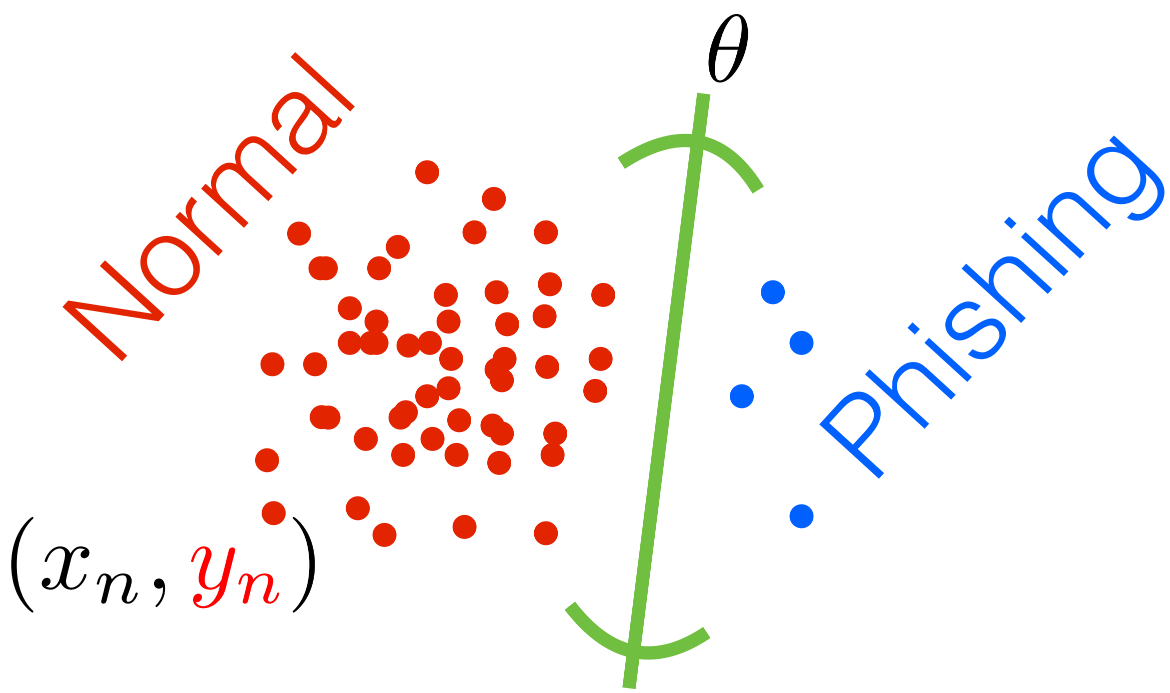

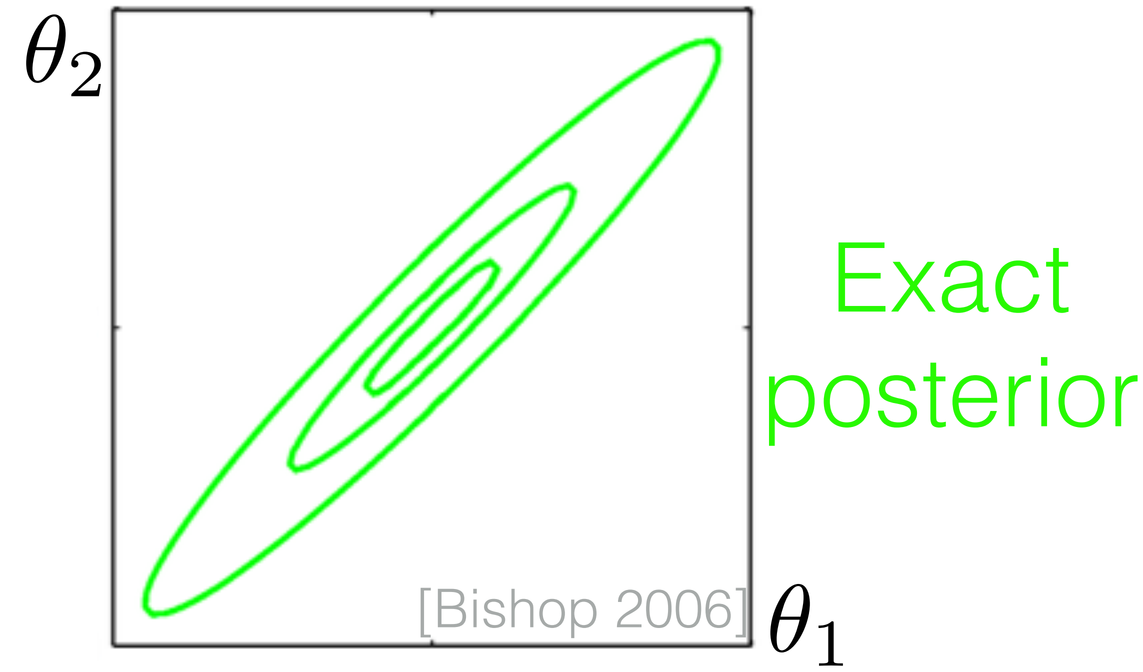

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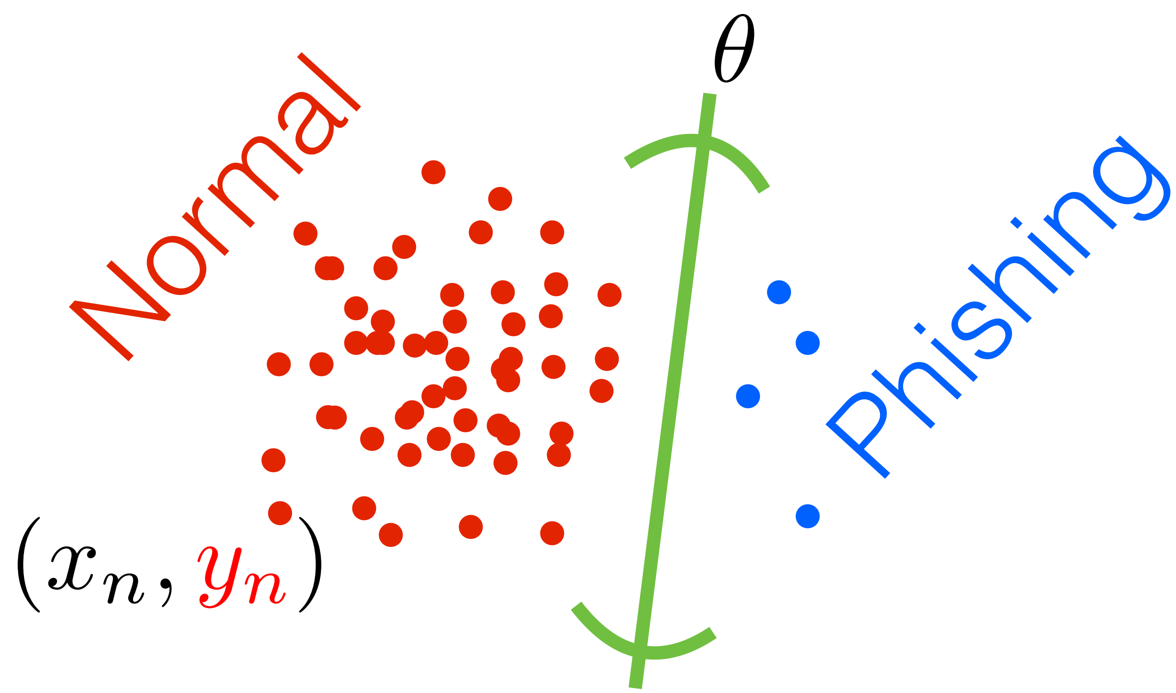

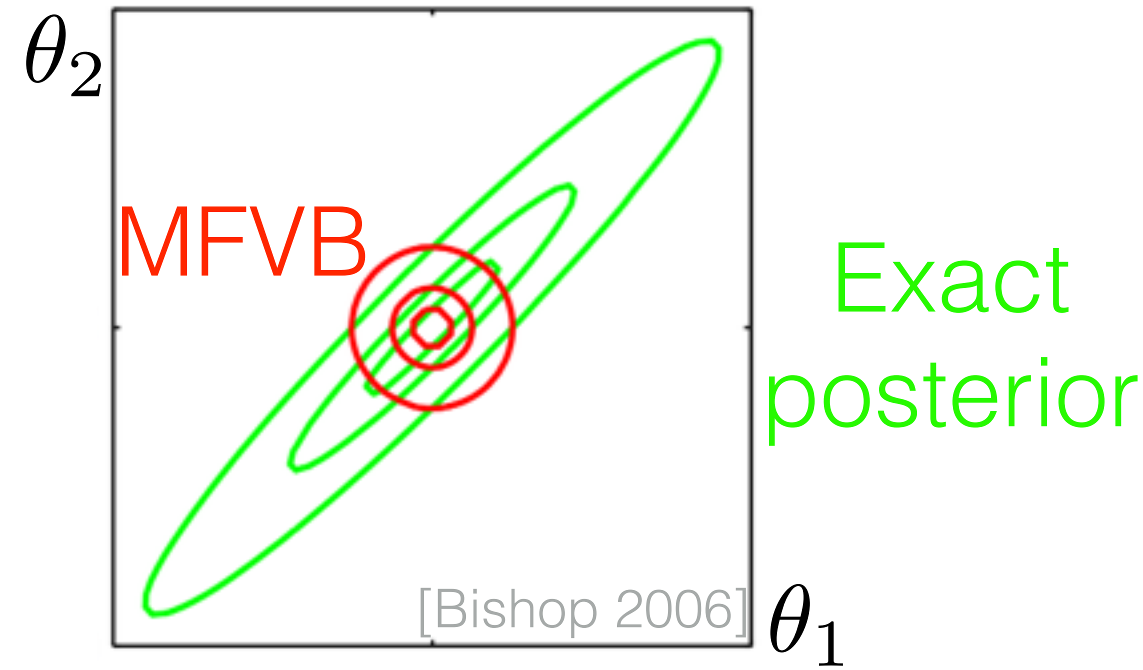

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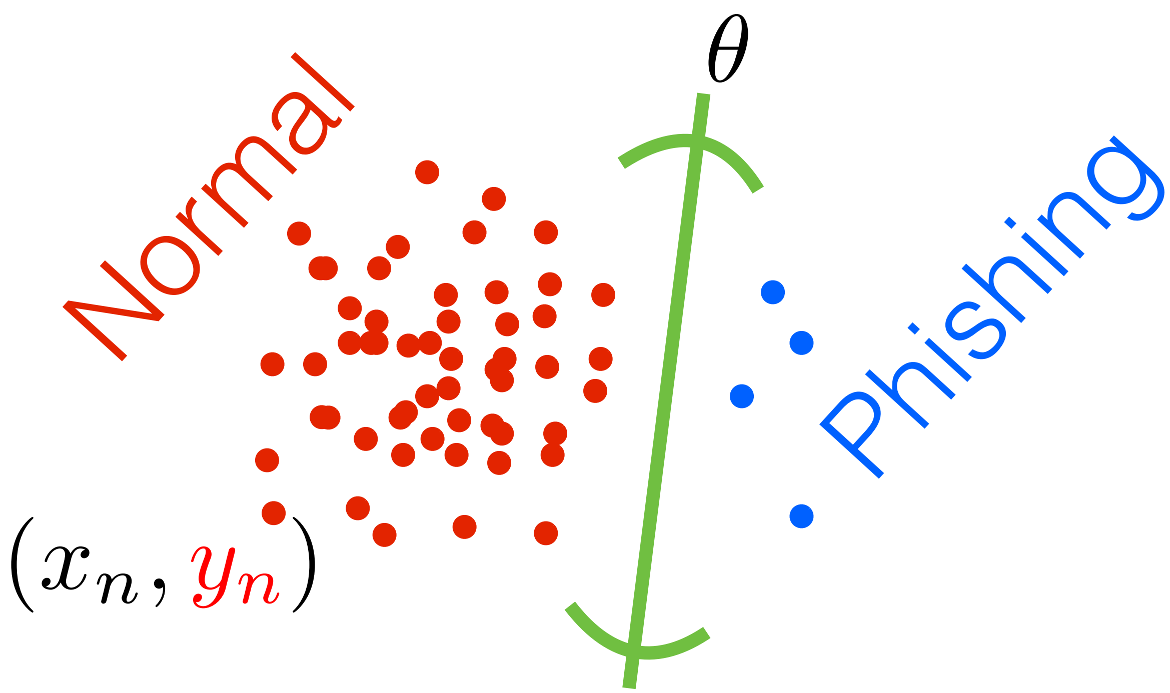

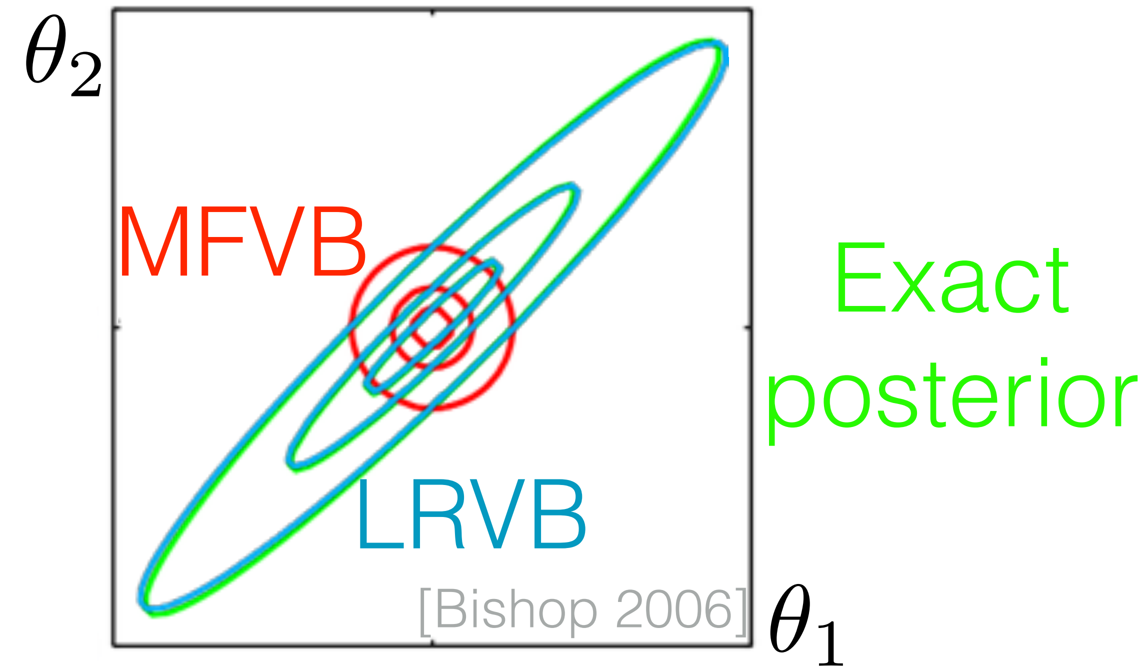

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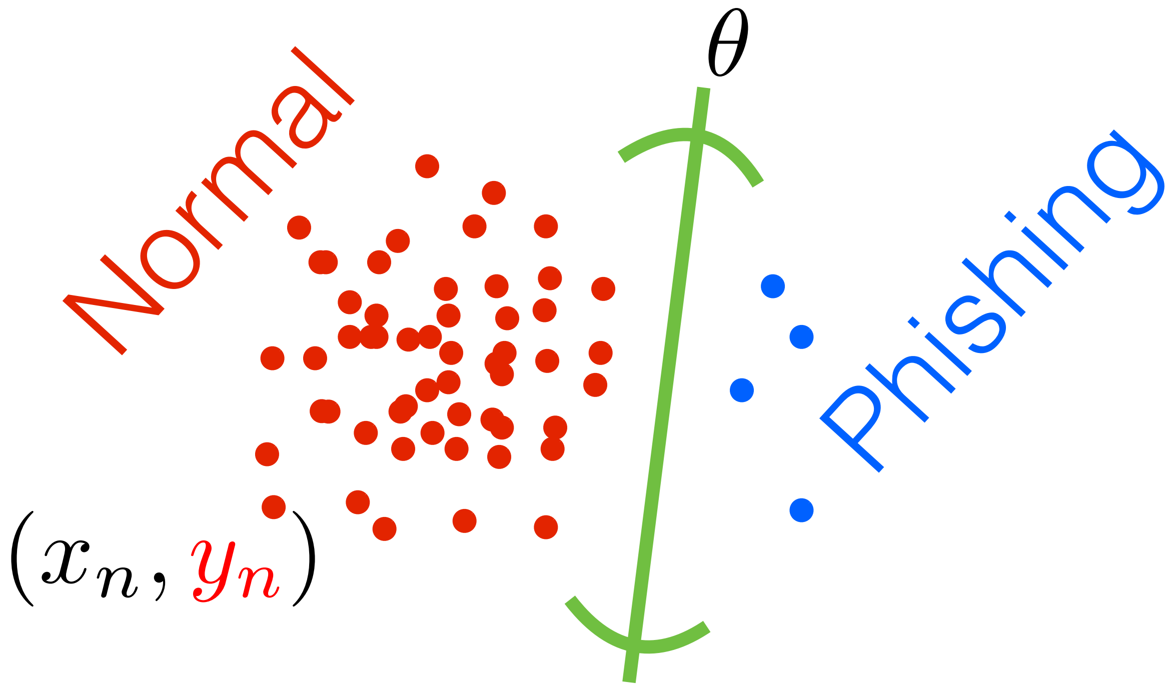

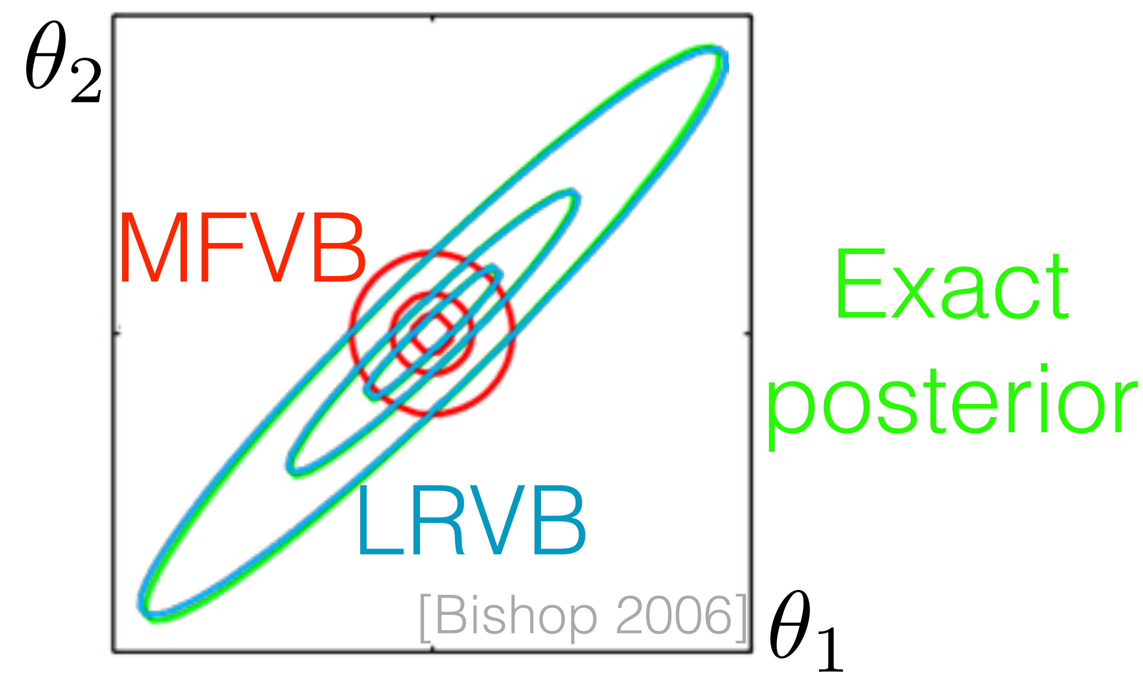

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- Automation: e.g. Stan, NUTS, ADVI

[http://mc-stan.org/ ; Hoffman, Gelman 2014; Kucukelbir, Tran, Ranganath, Gelman, Blei 2017]

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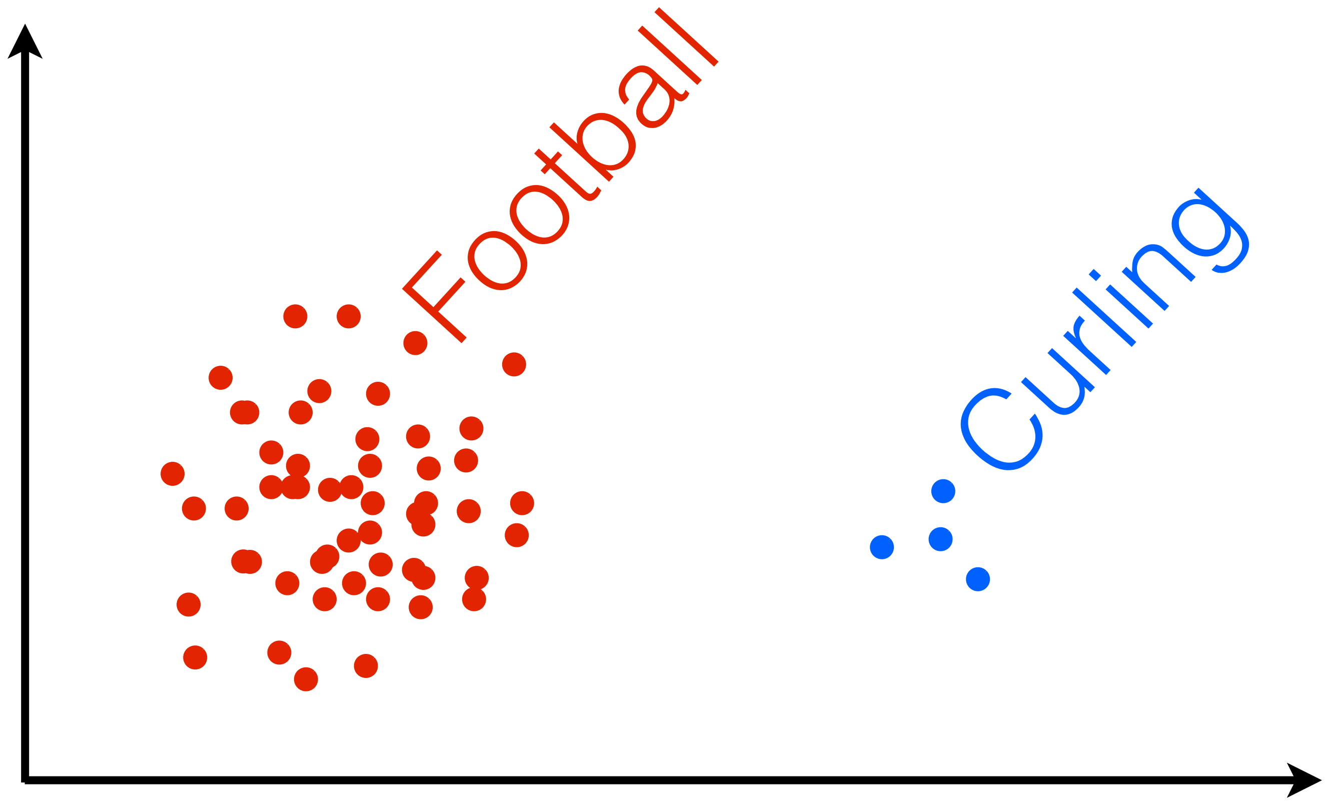

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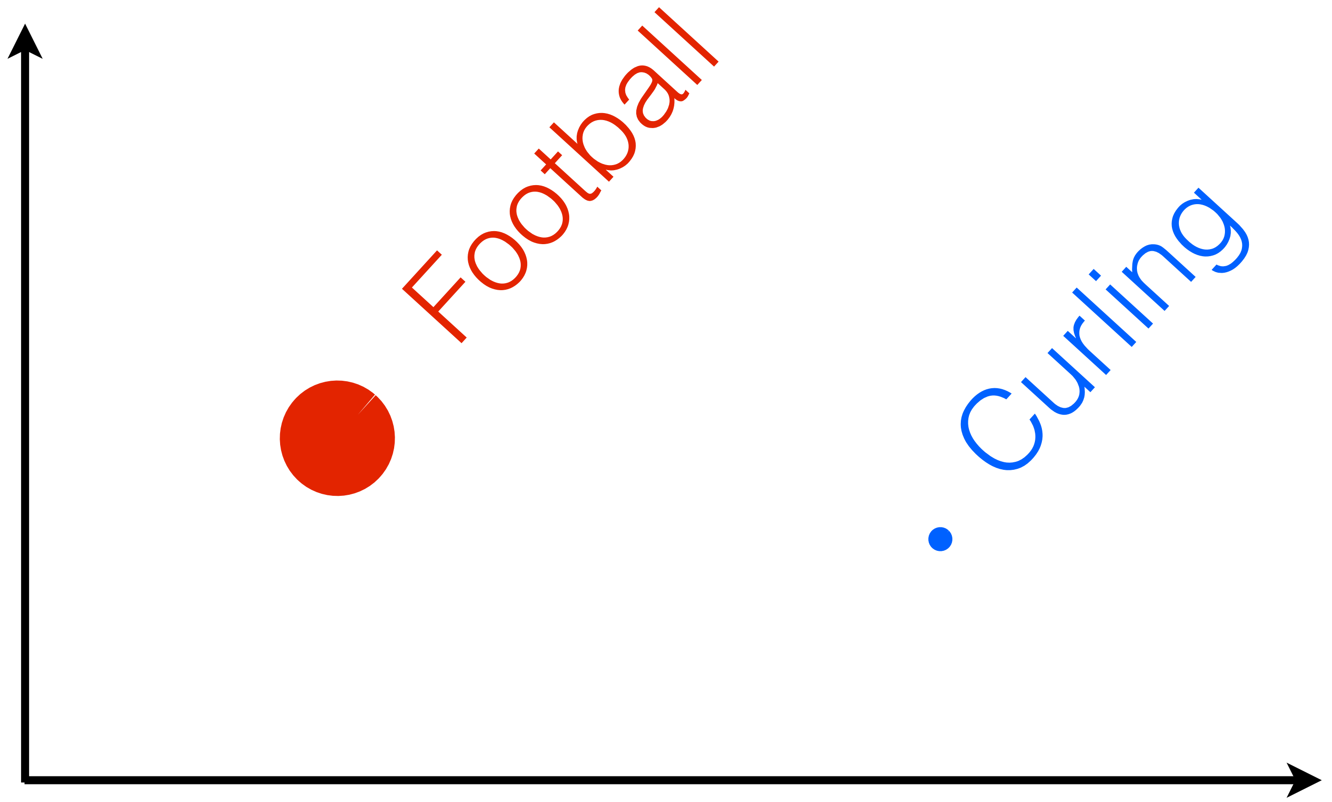

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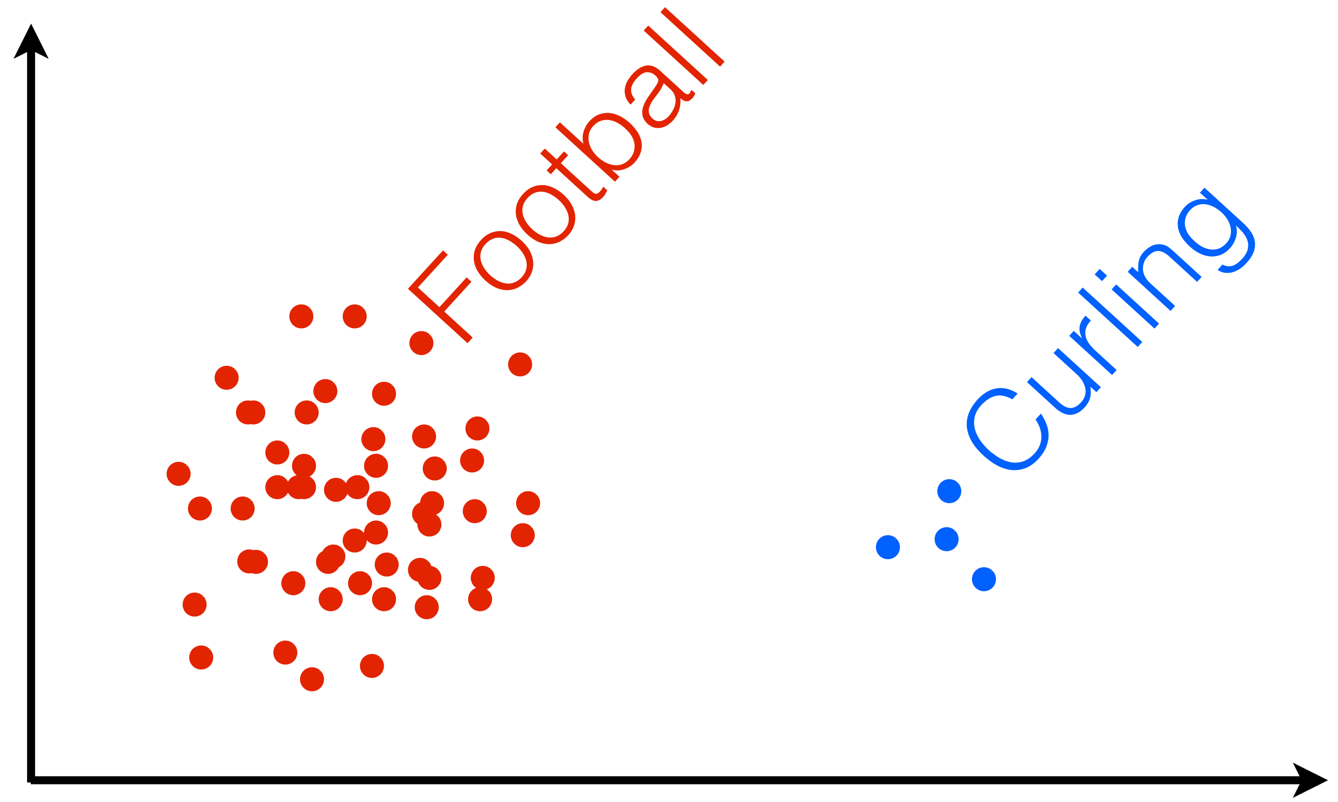

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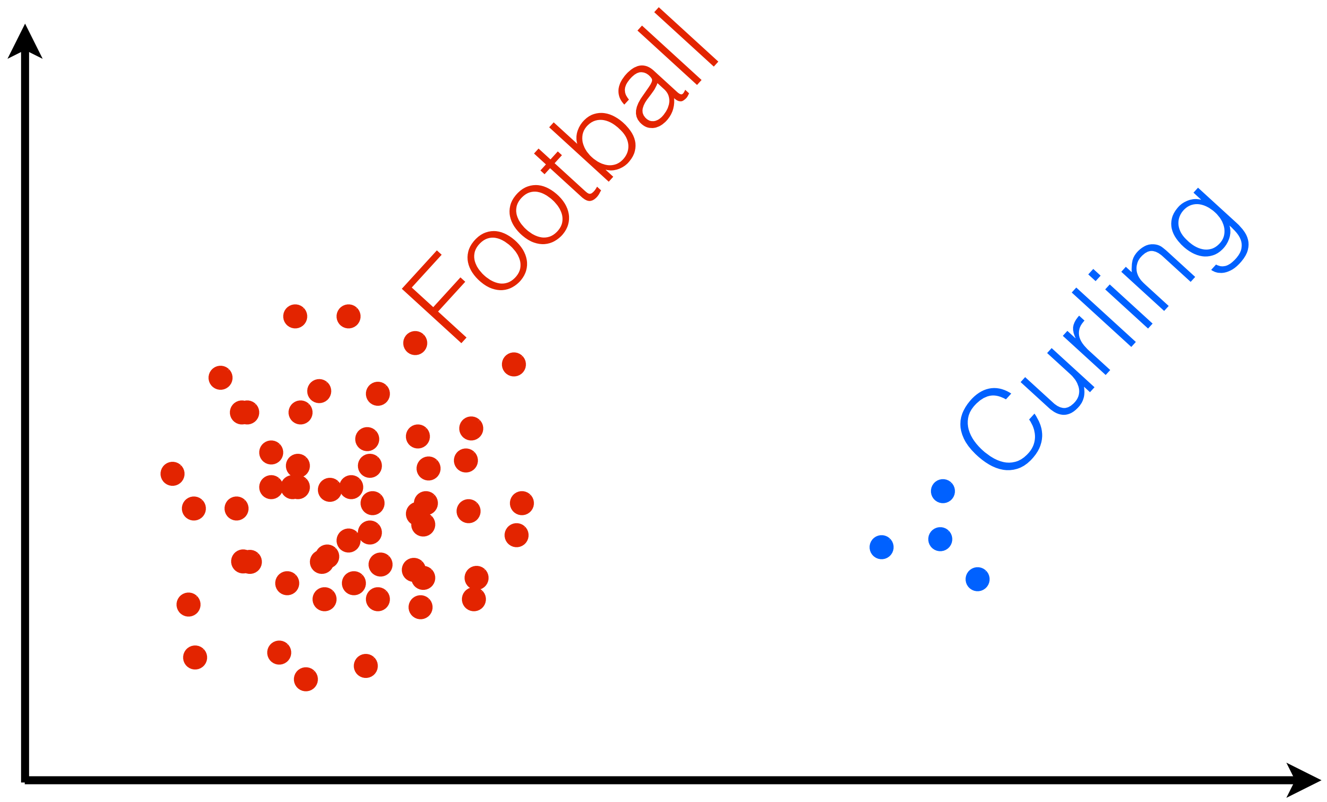

[Agarwal et al 2005; Feldman & Langberg 2011]

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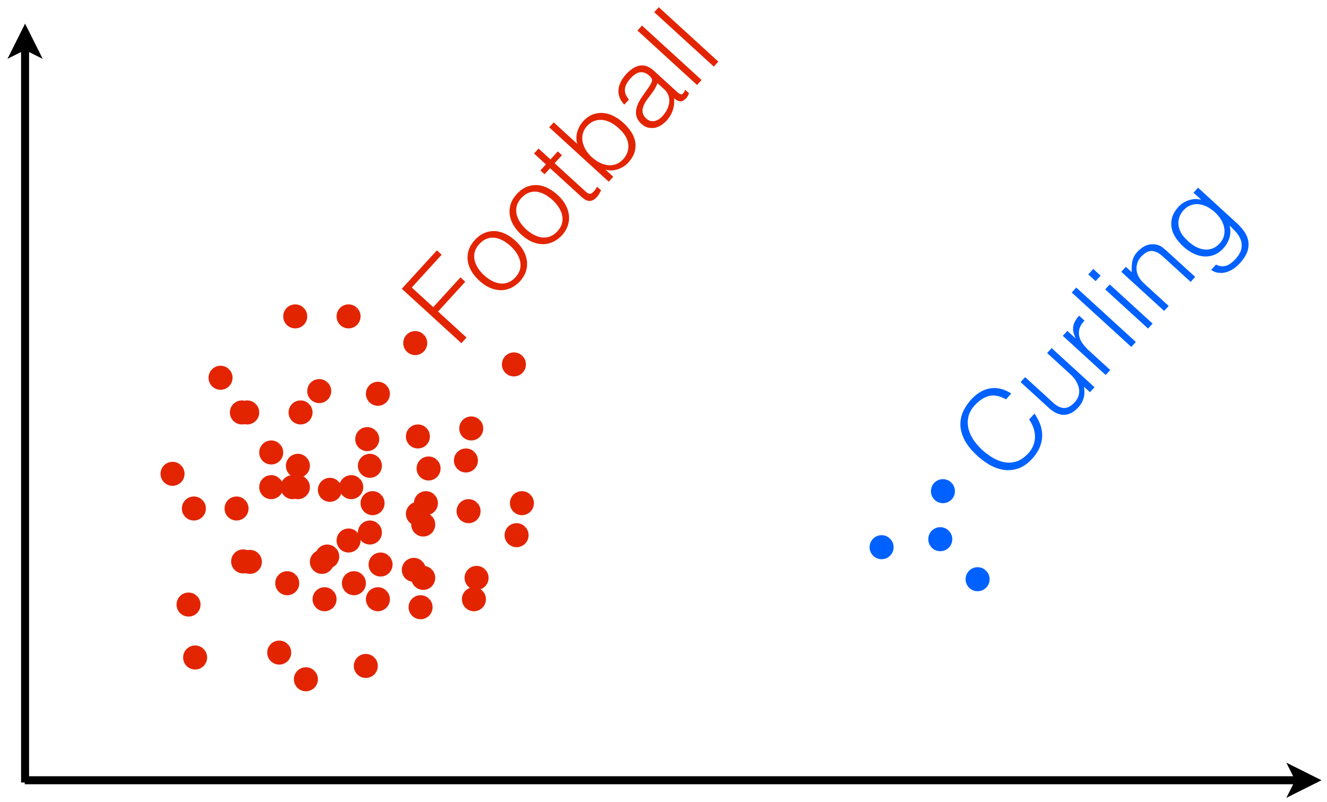

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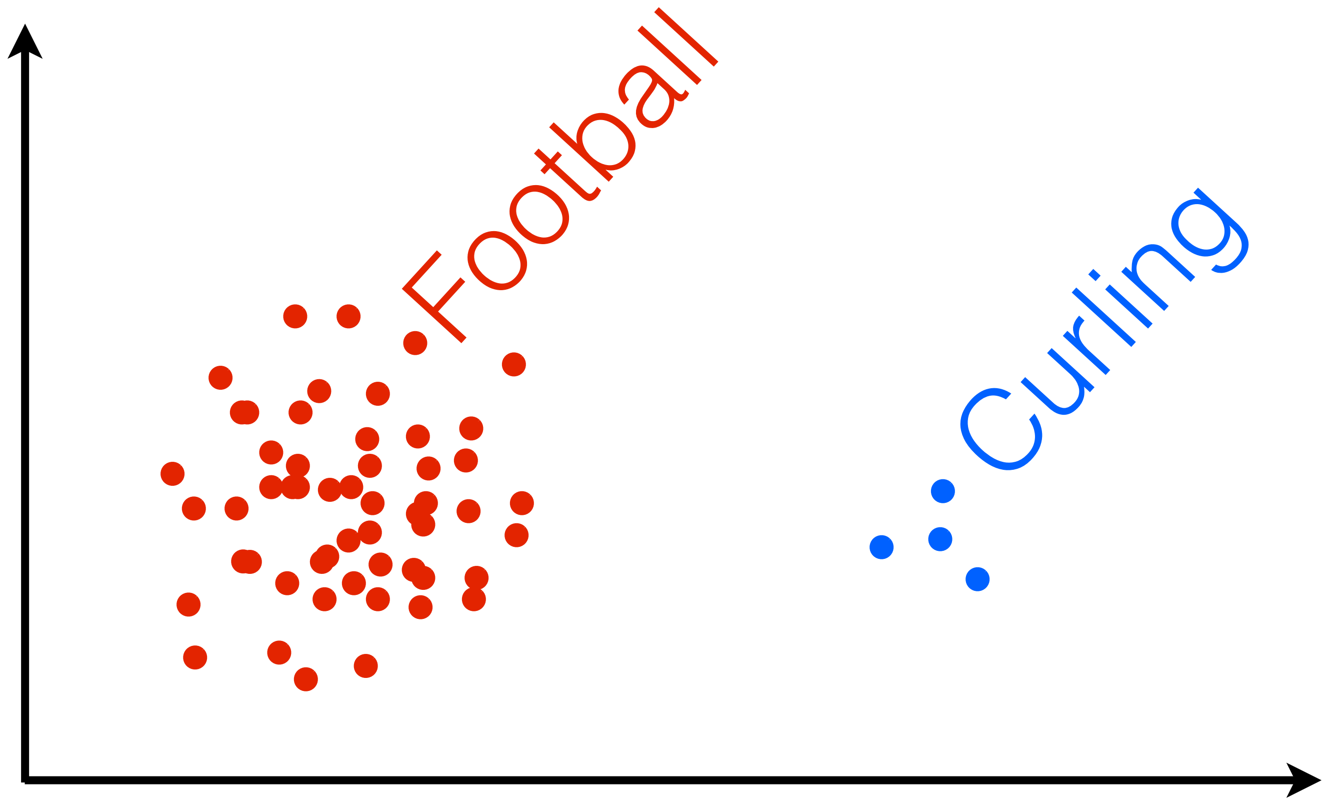

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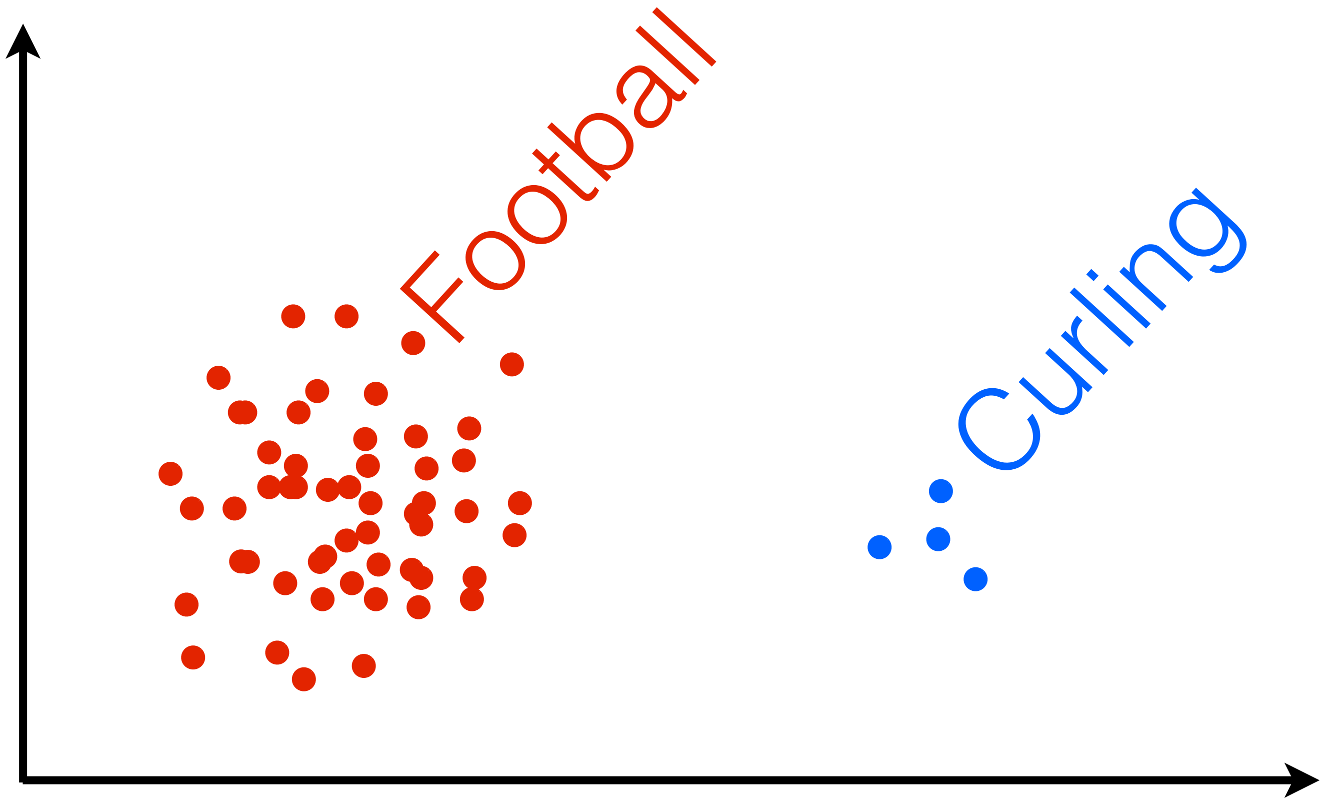

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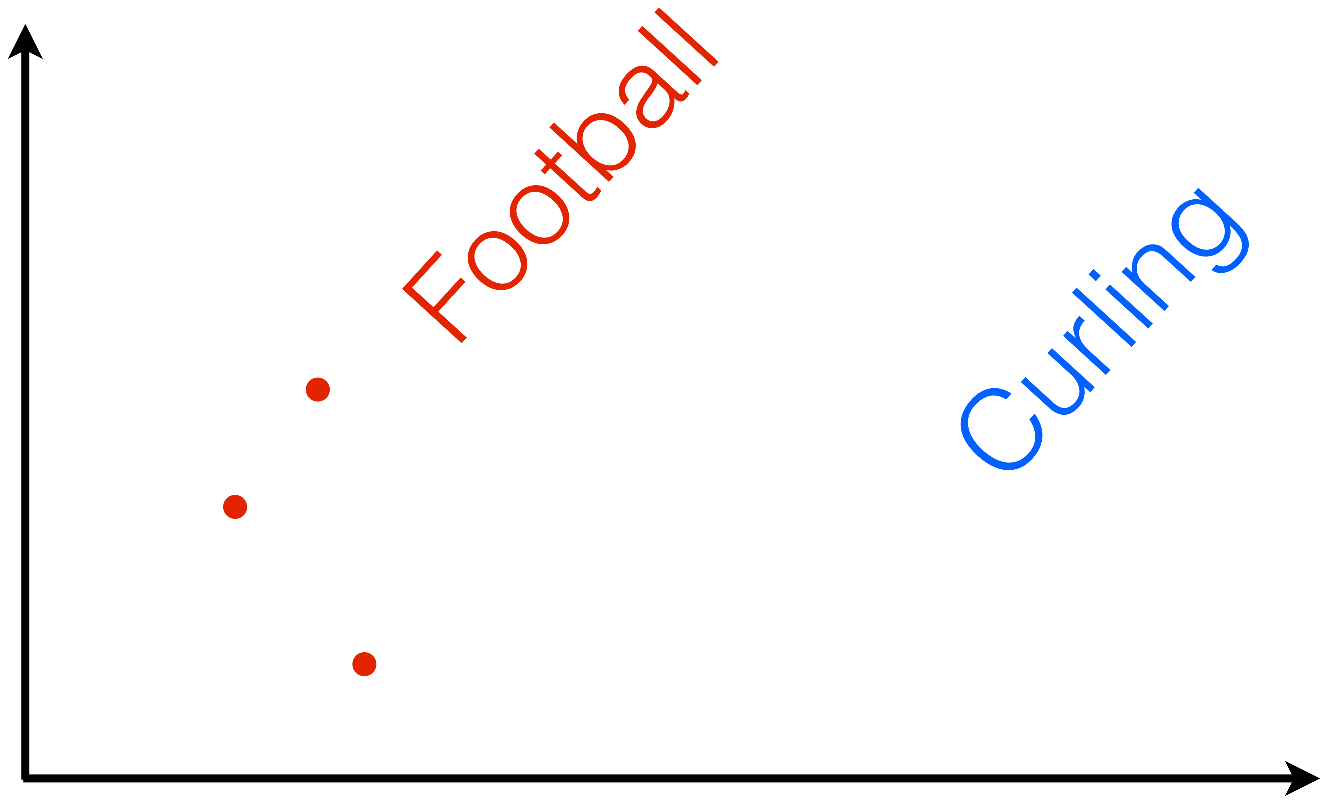

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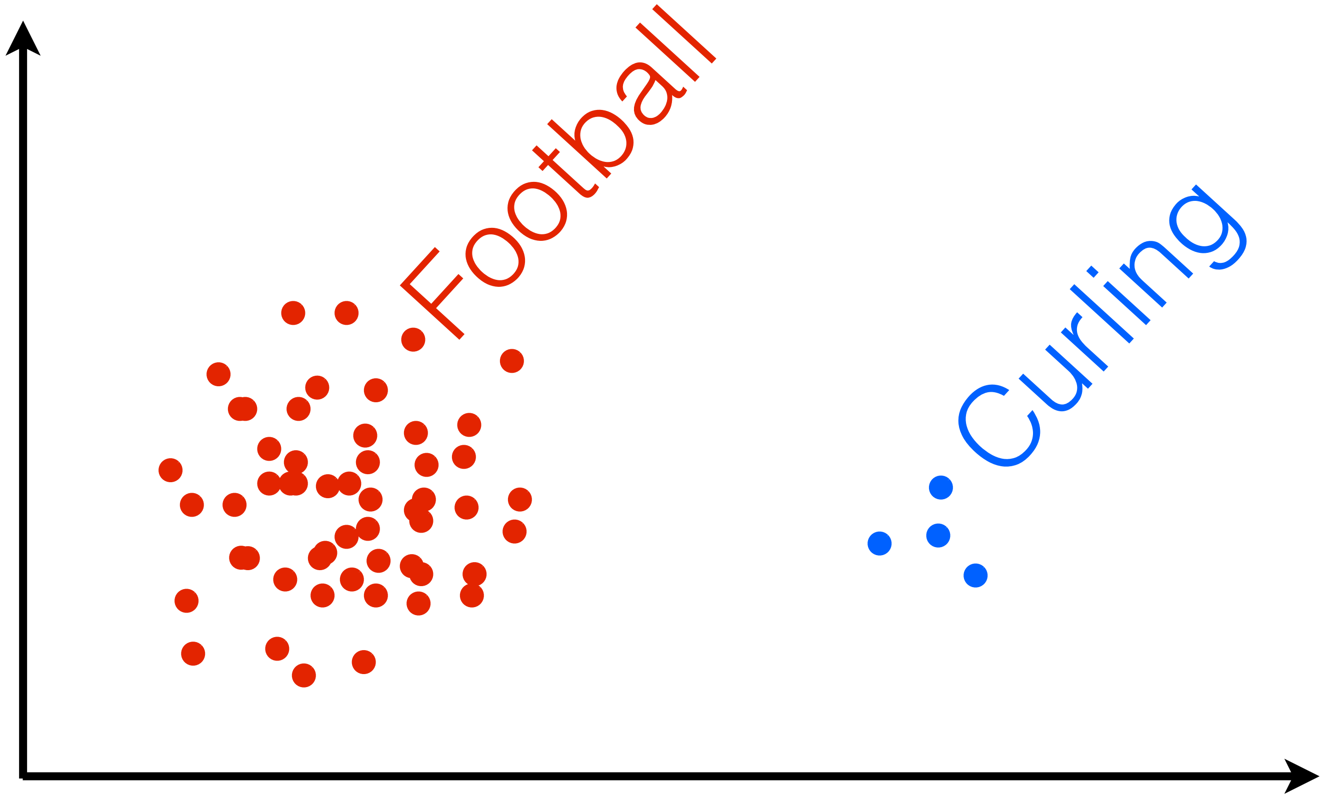

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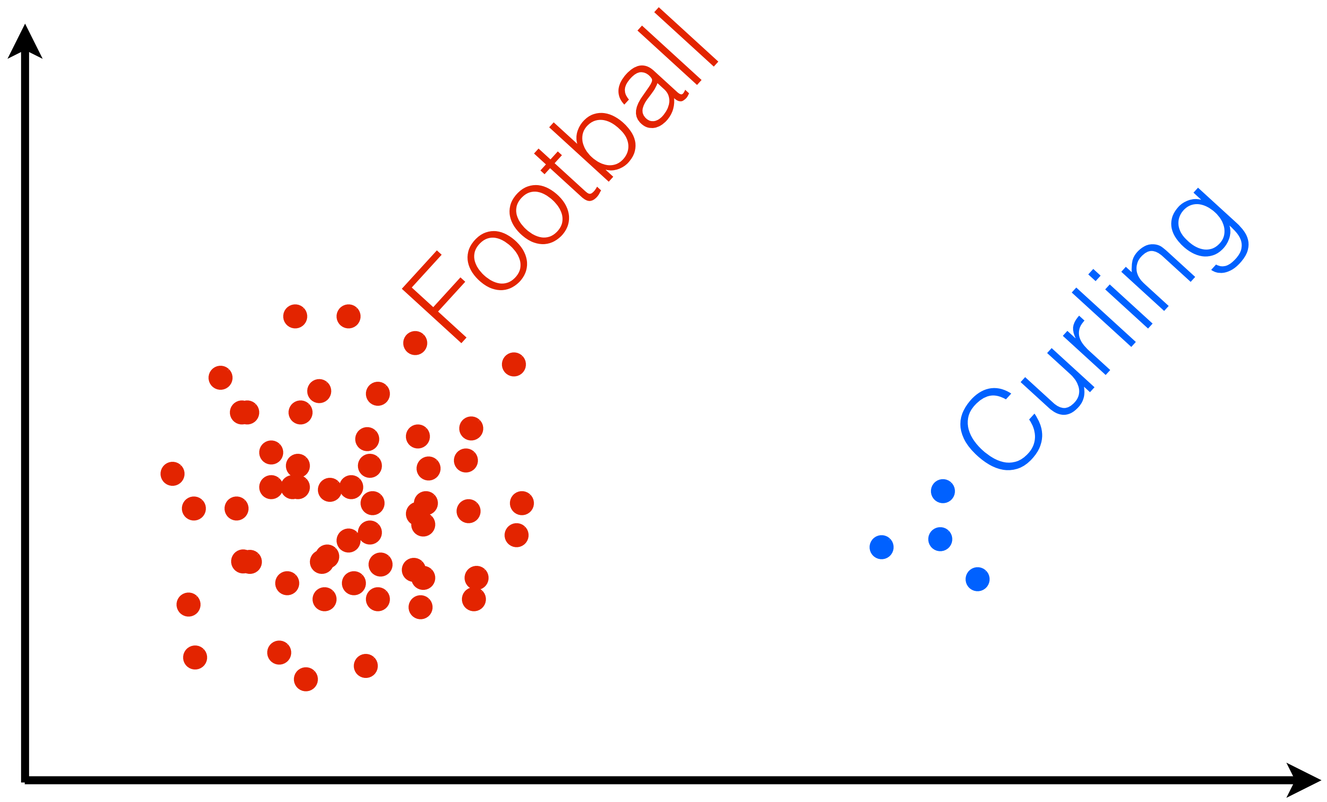

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- How to develop coresets for Bayes?

[Agarwal et al 2005; Feldman & Langberg 2011; DuMouchel et al 1999; Madigan et al 2002; Huggins, Campbell, Broderick 2016; Campbell, Broderick 2017; Campbell, Broderick 2018]

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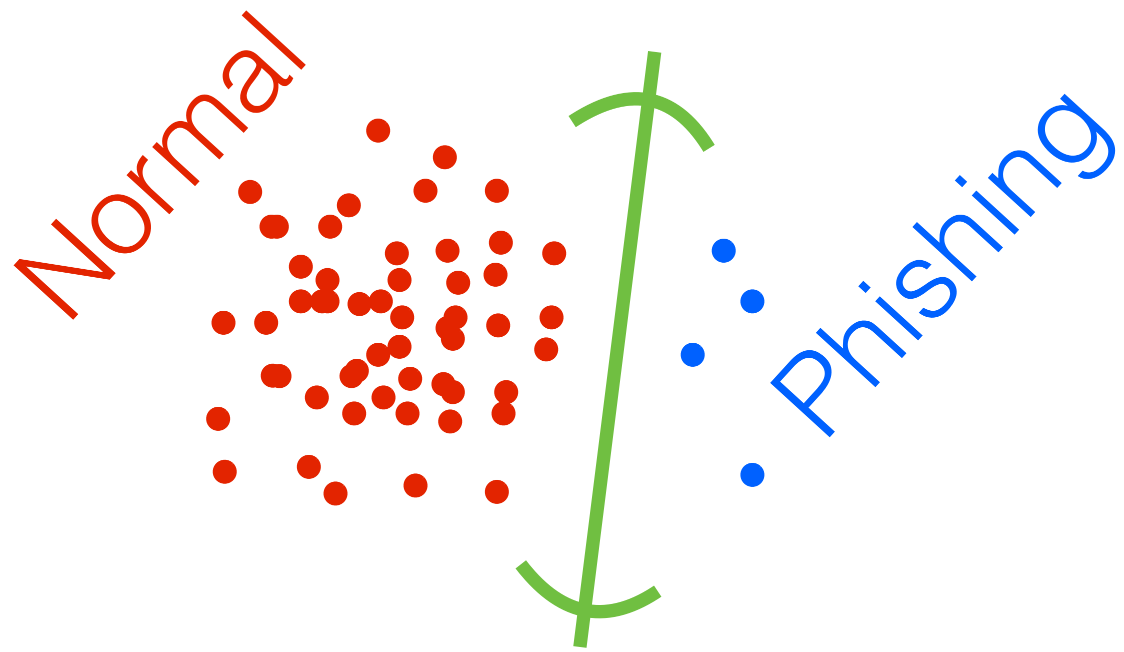

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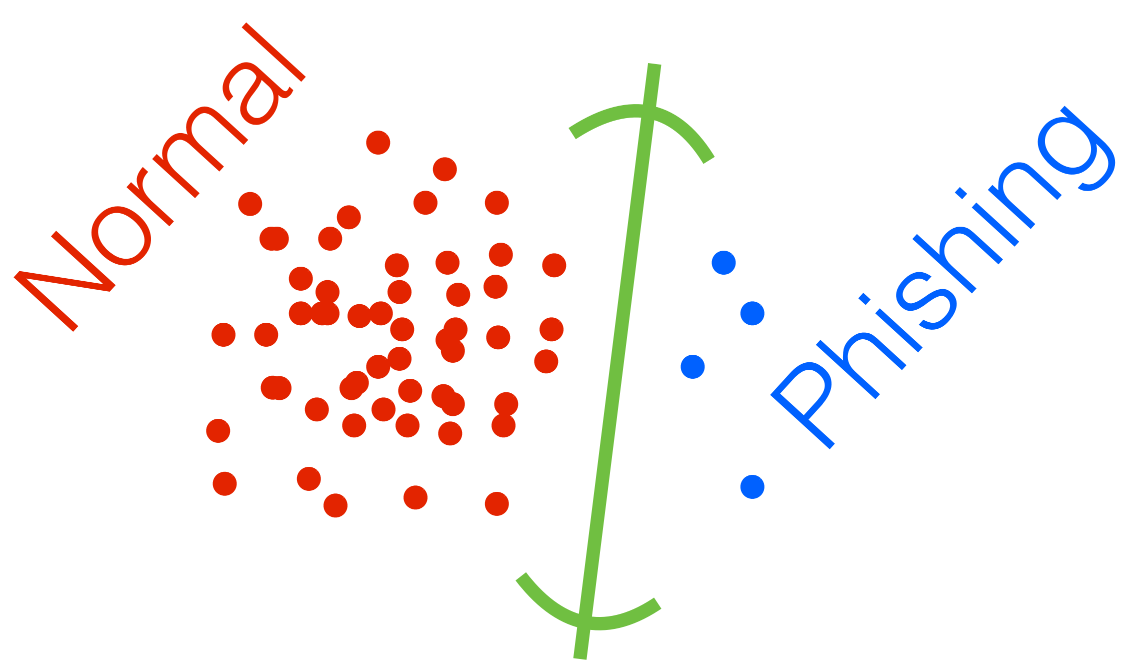

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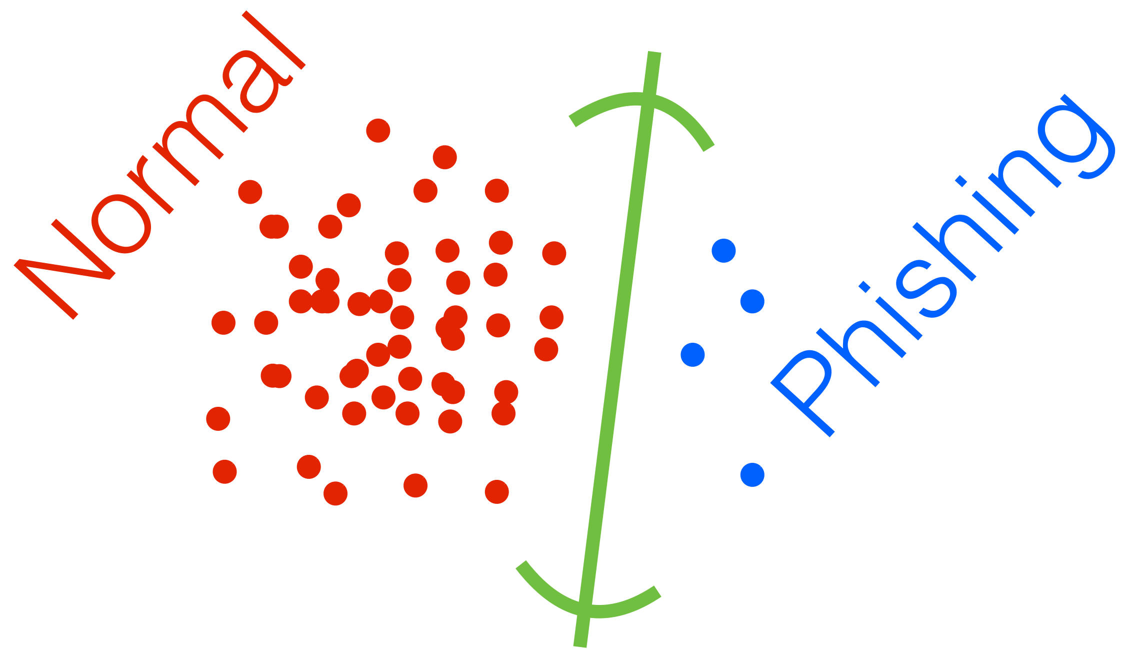

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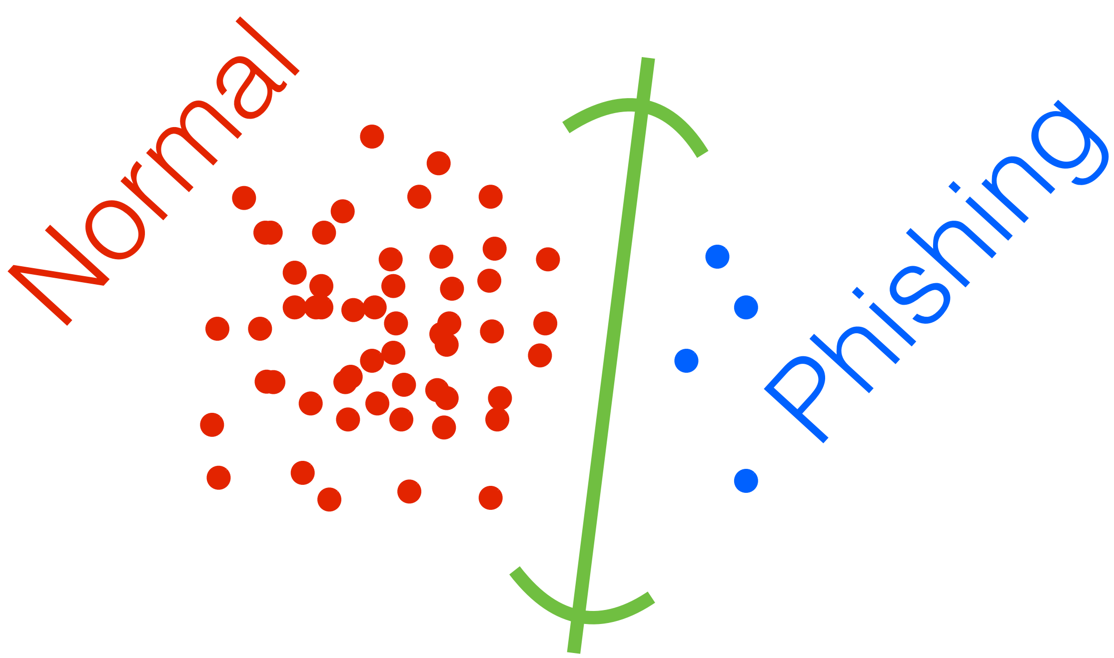
$$\|w\|_0 \ll N$$



Bayesian coresets

- Posterior $p(\theta|y) \propto_\theta p(y|\theta)p(\theta)$
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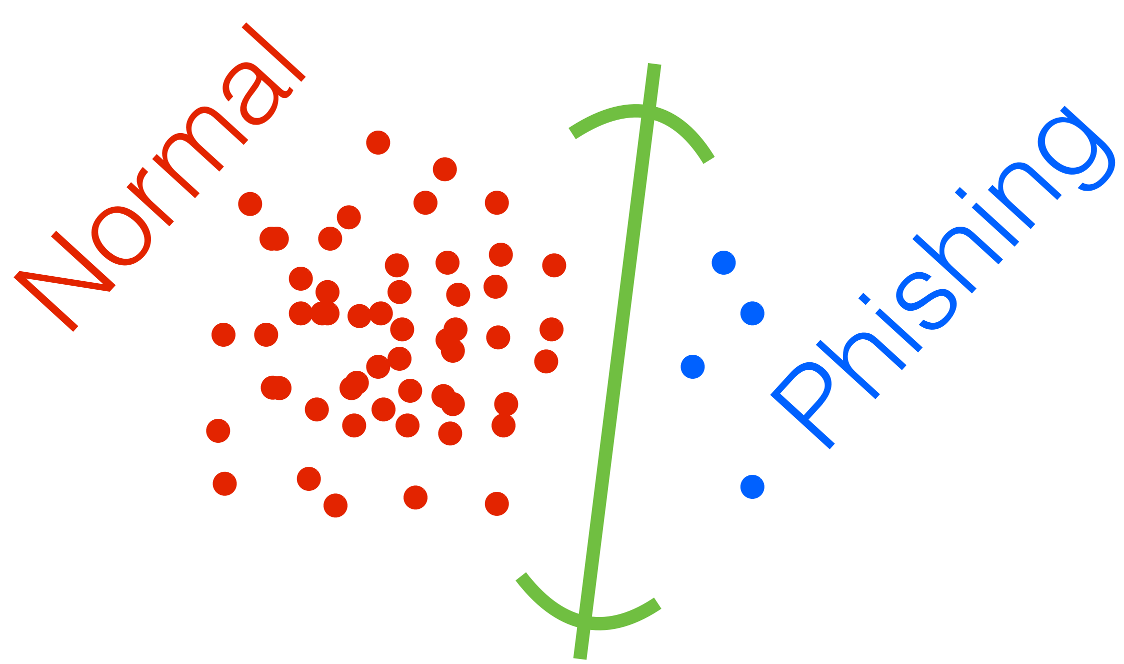

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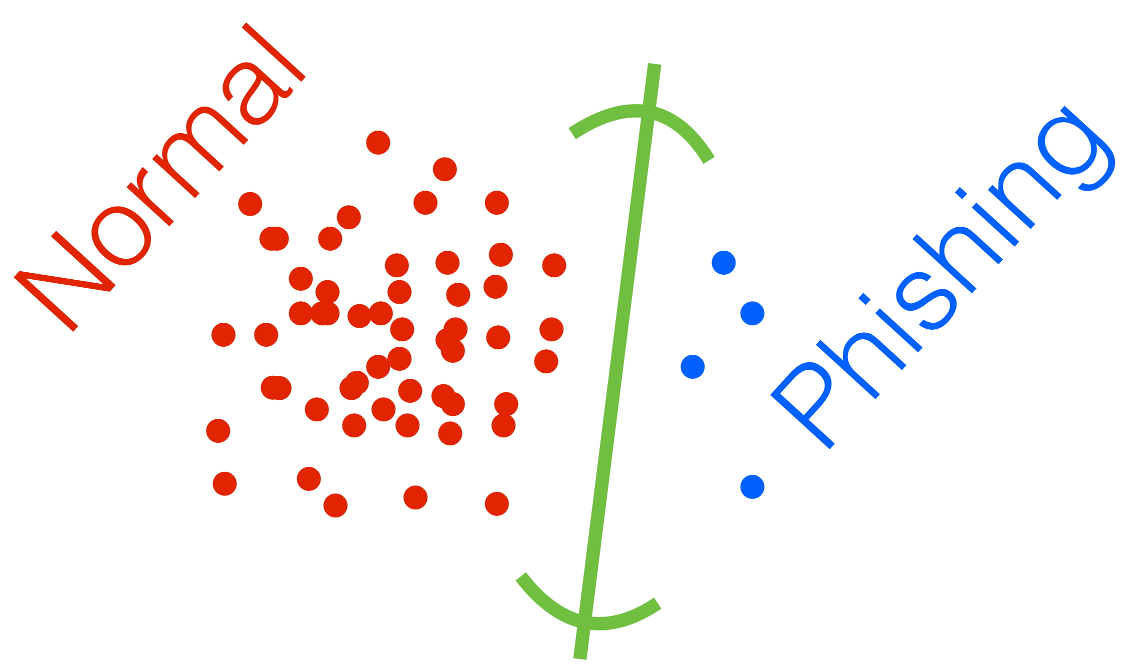

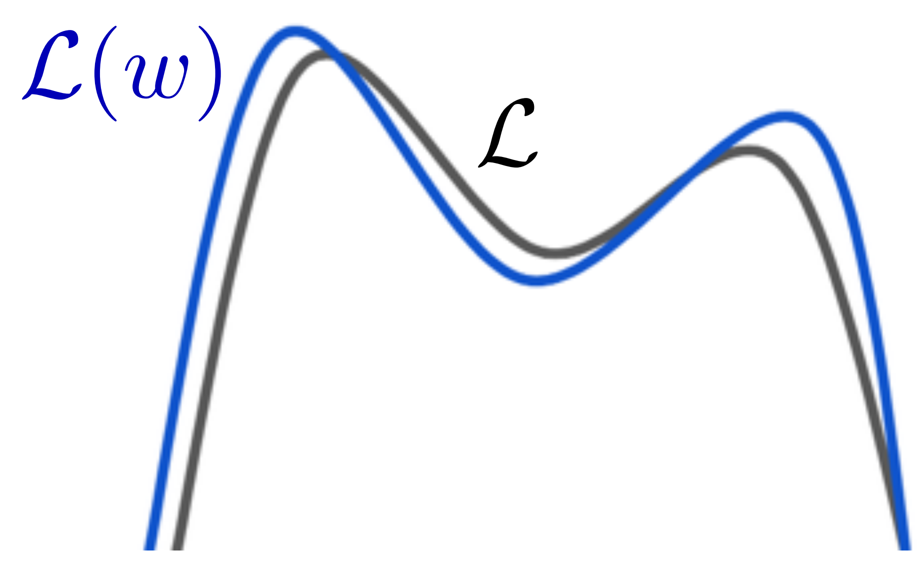

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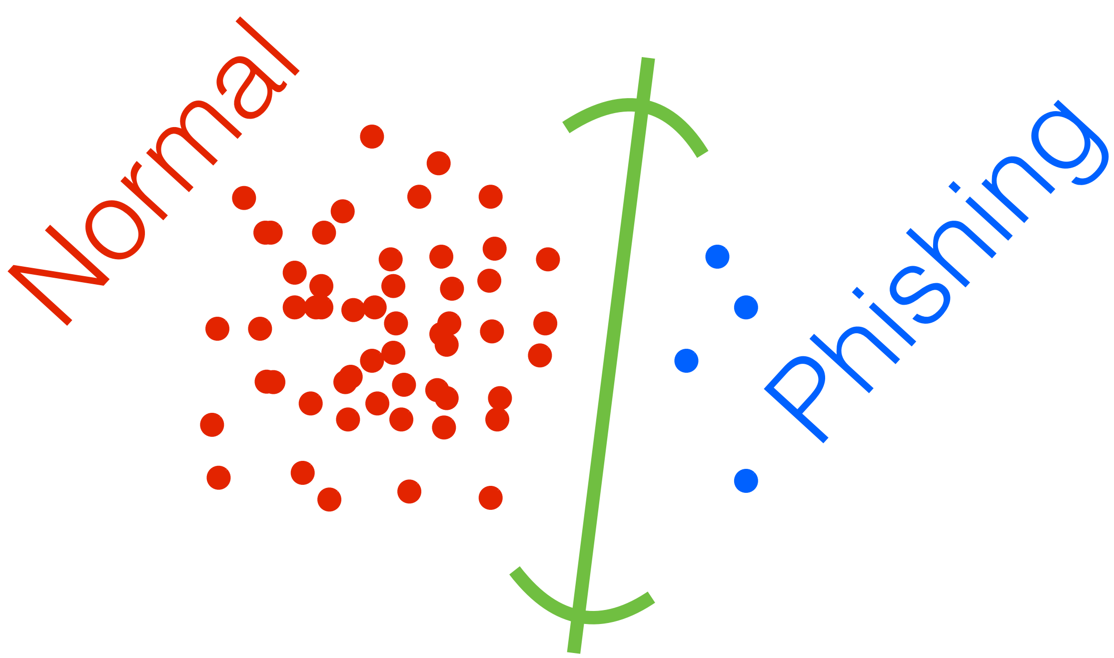

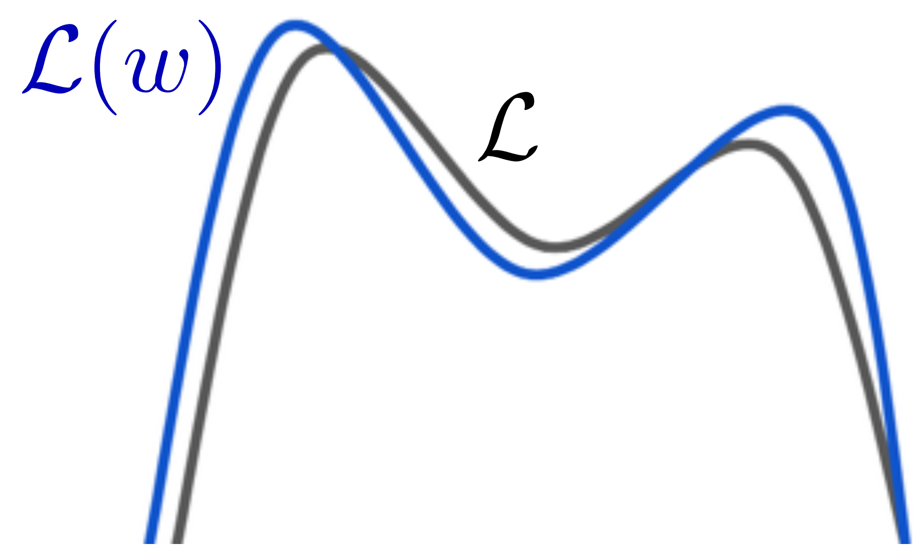

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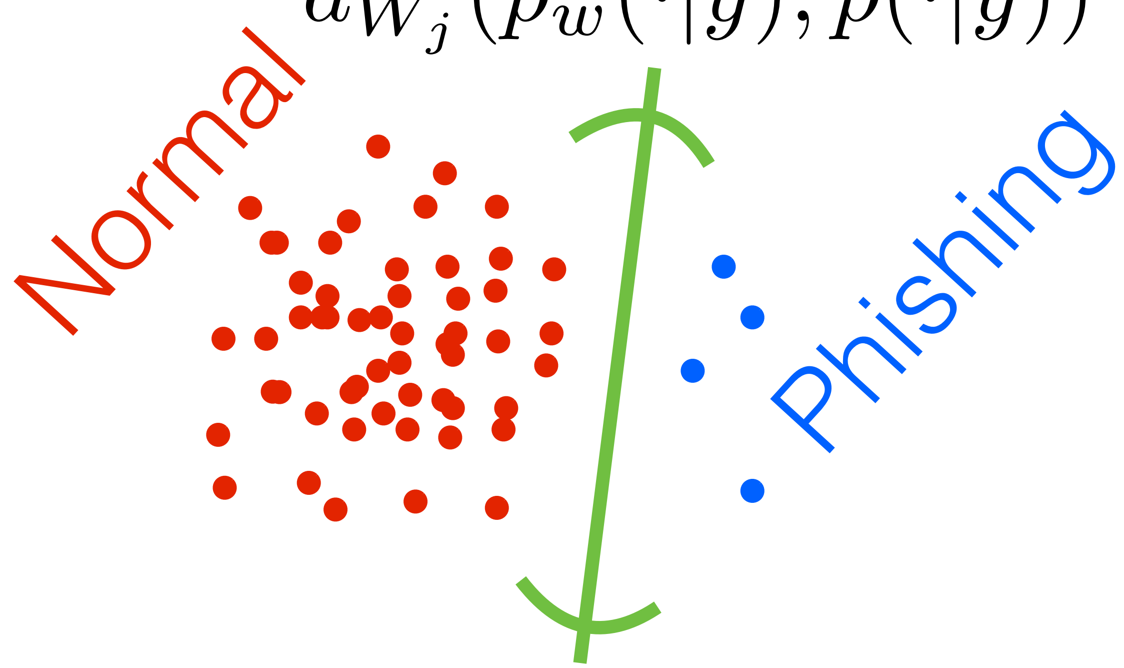
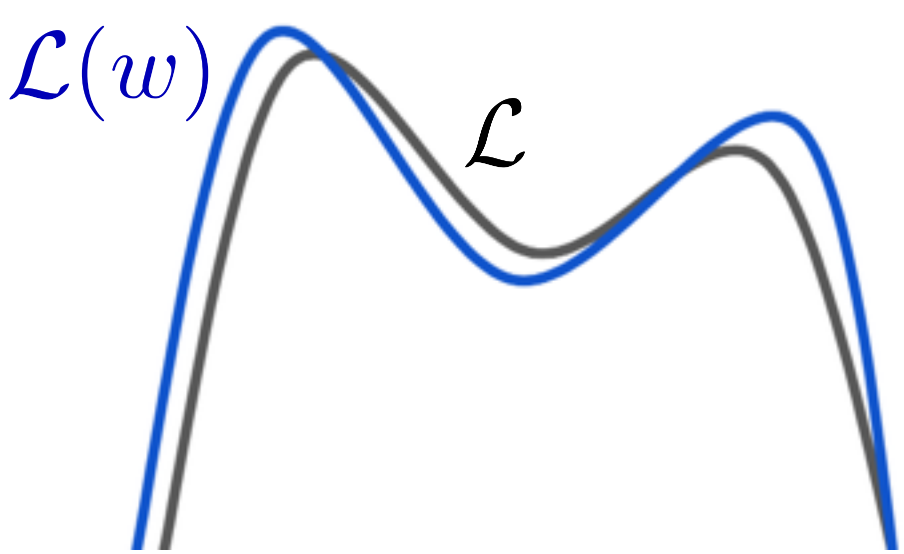
- ε-coreset: $\|\mathcal{L}(w) - \mathcal{L}\| \leq \epsilon$
    - Approximate posterior close in Wasserstein distance
      $d_{W_j}(p_w(\cdot|y), p(\cdot|y)) \leq C_j \|\mathcal{L}(w) - \mathcal{L}\|_{\mathrm{WFID}}, j \in \{1, 2\}$





[Huggins, Kasprzak, Campbell, Broderick, forthcoming]

Roadmap

- Approximate Bayes review
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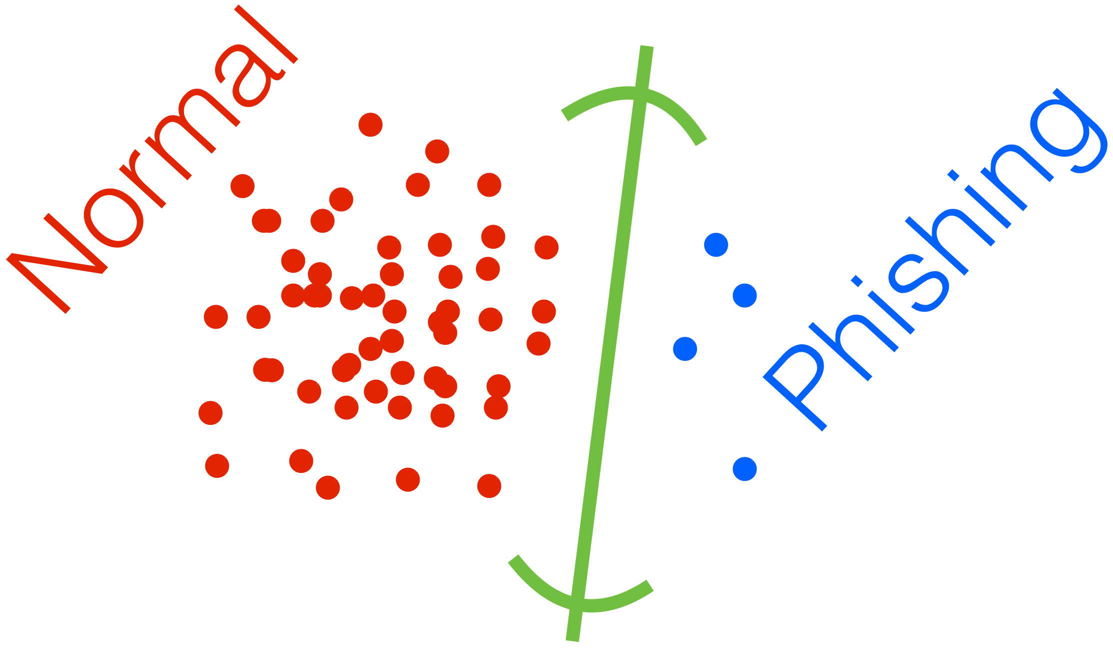

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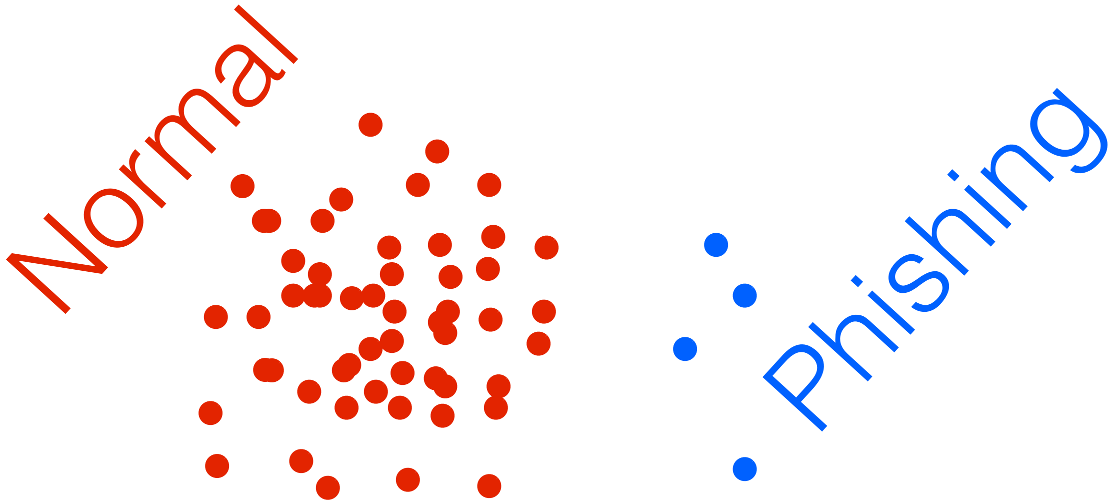

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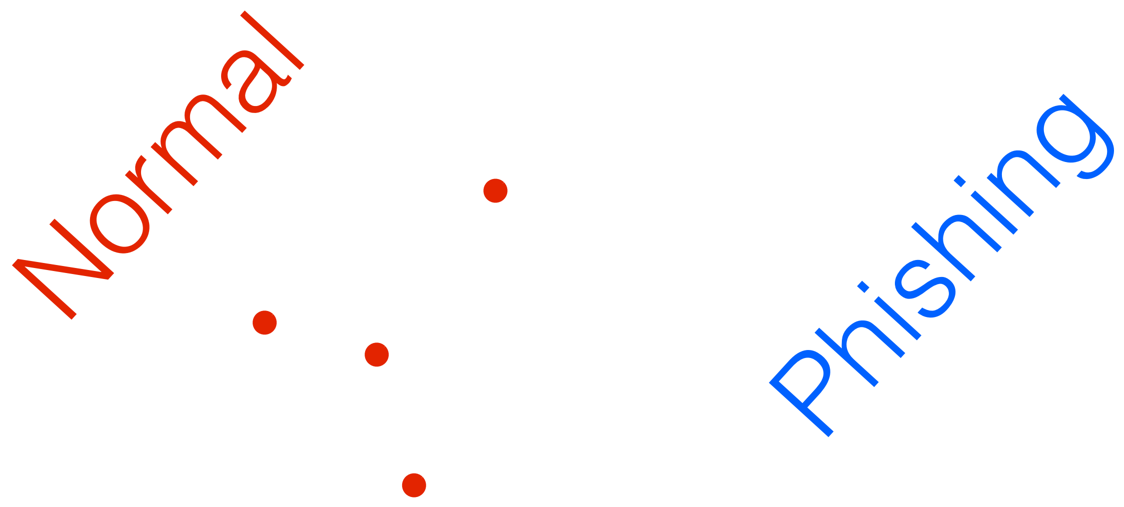

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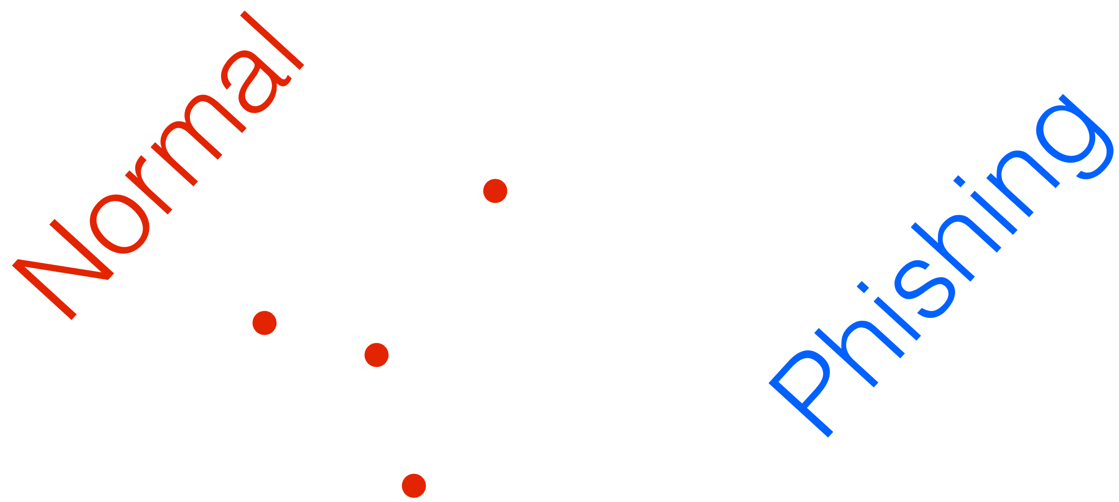

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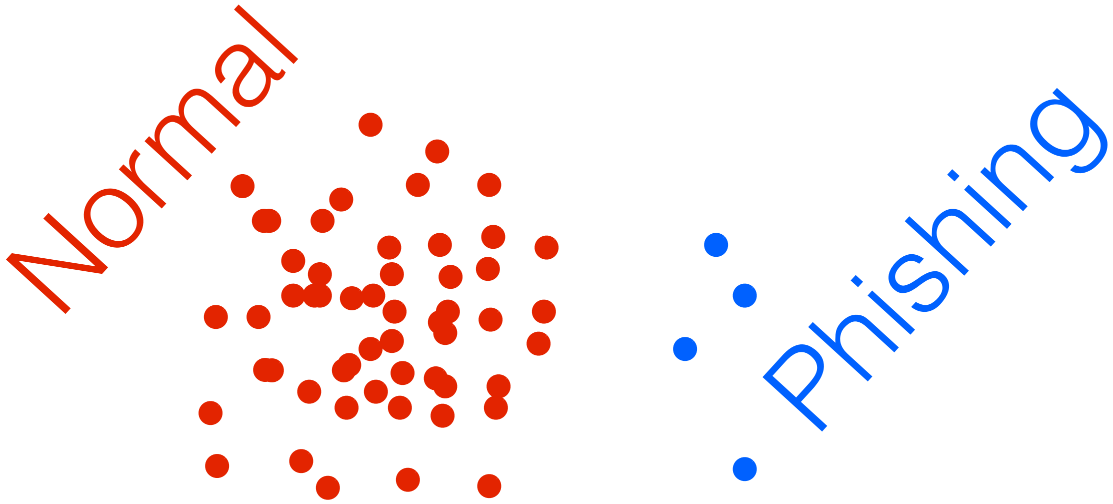

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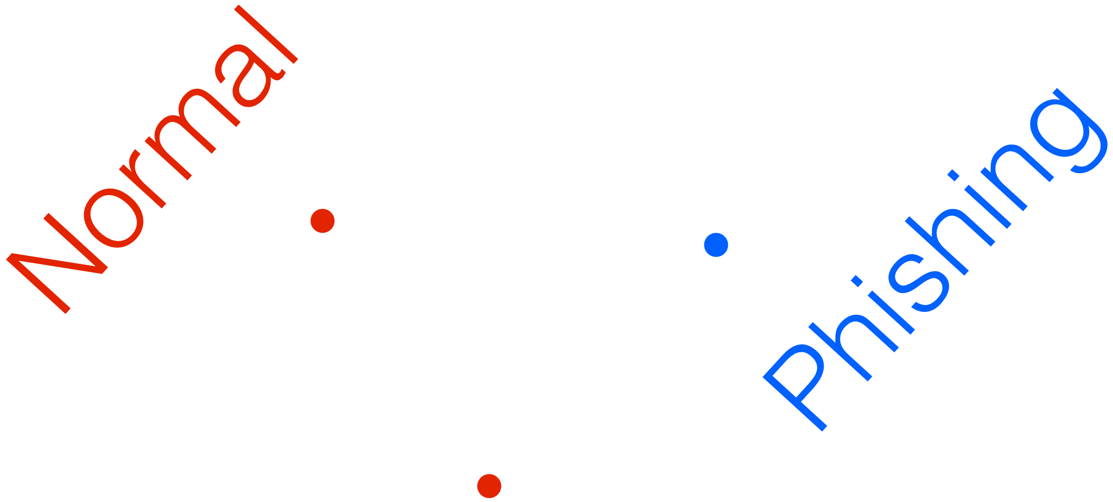

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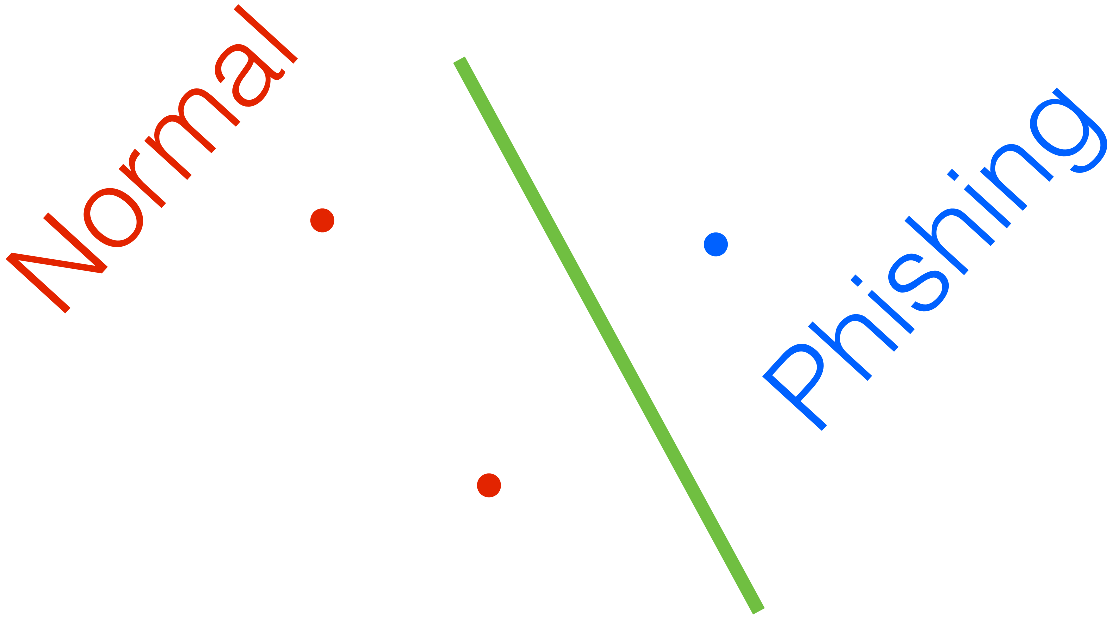

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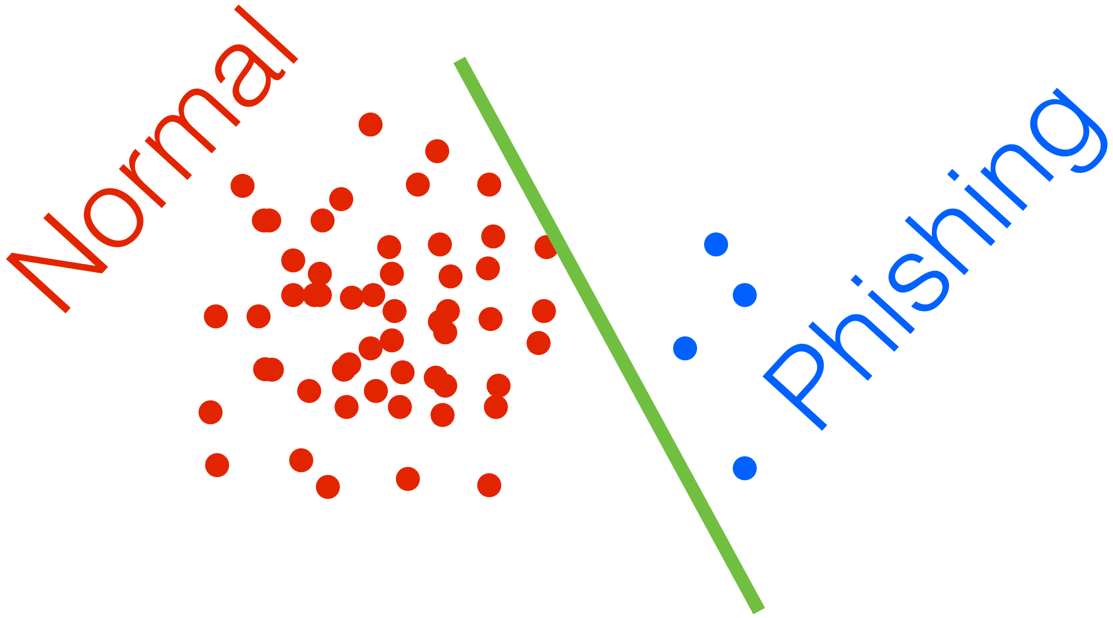

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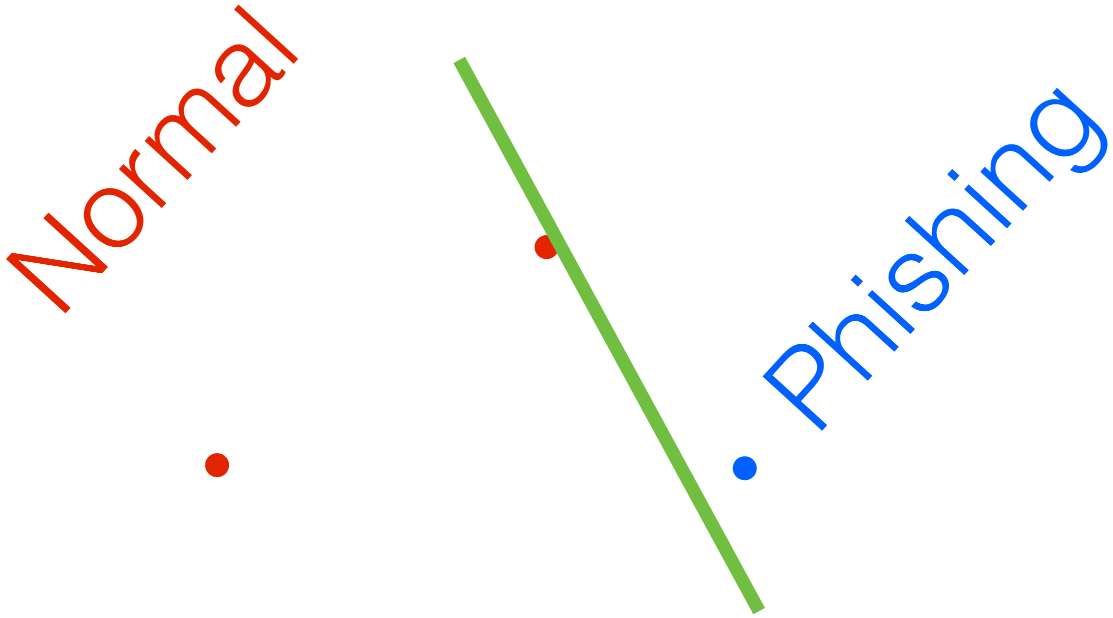

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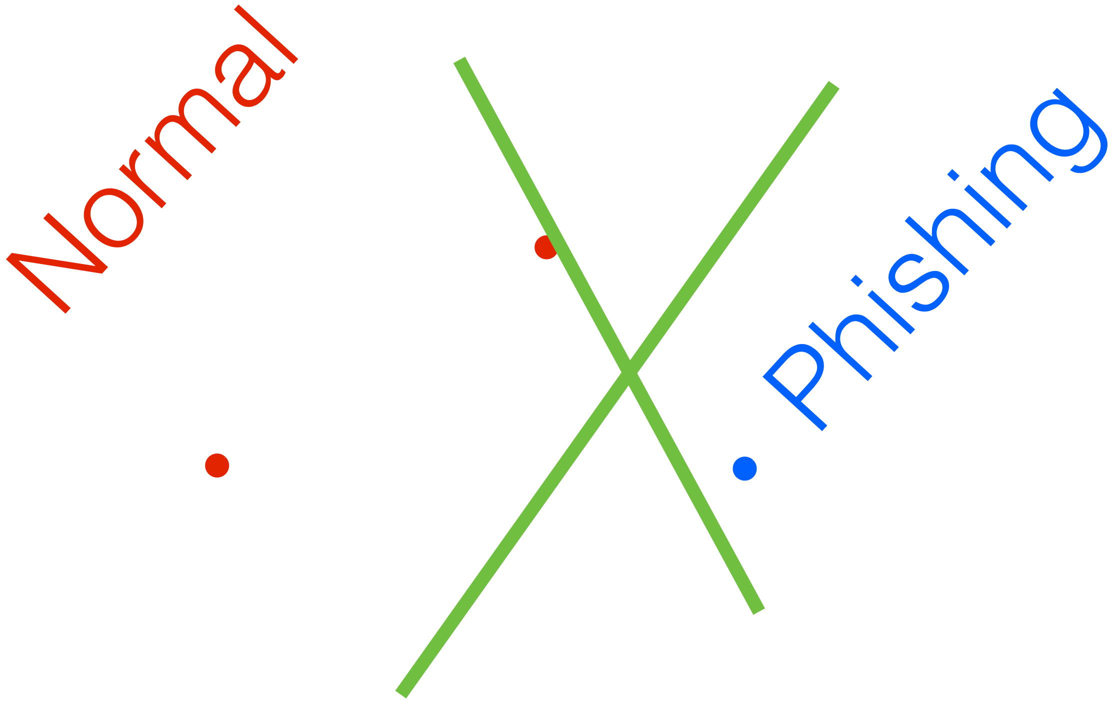

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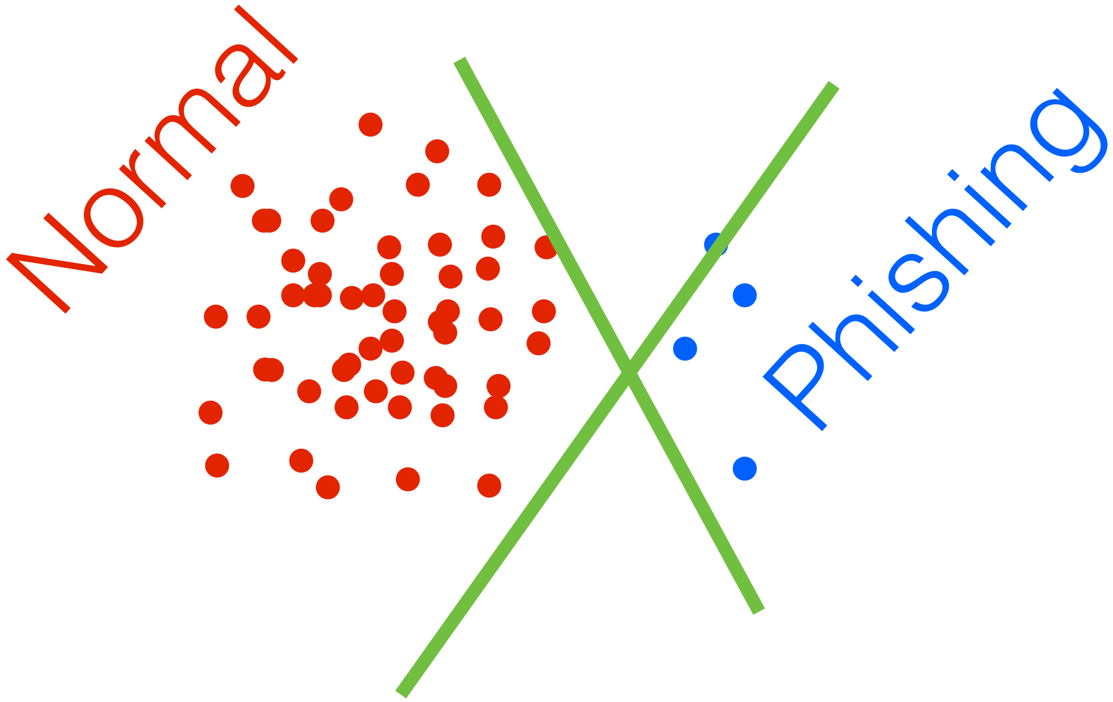

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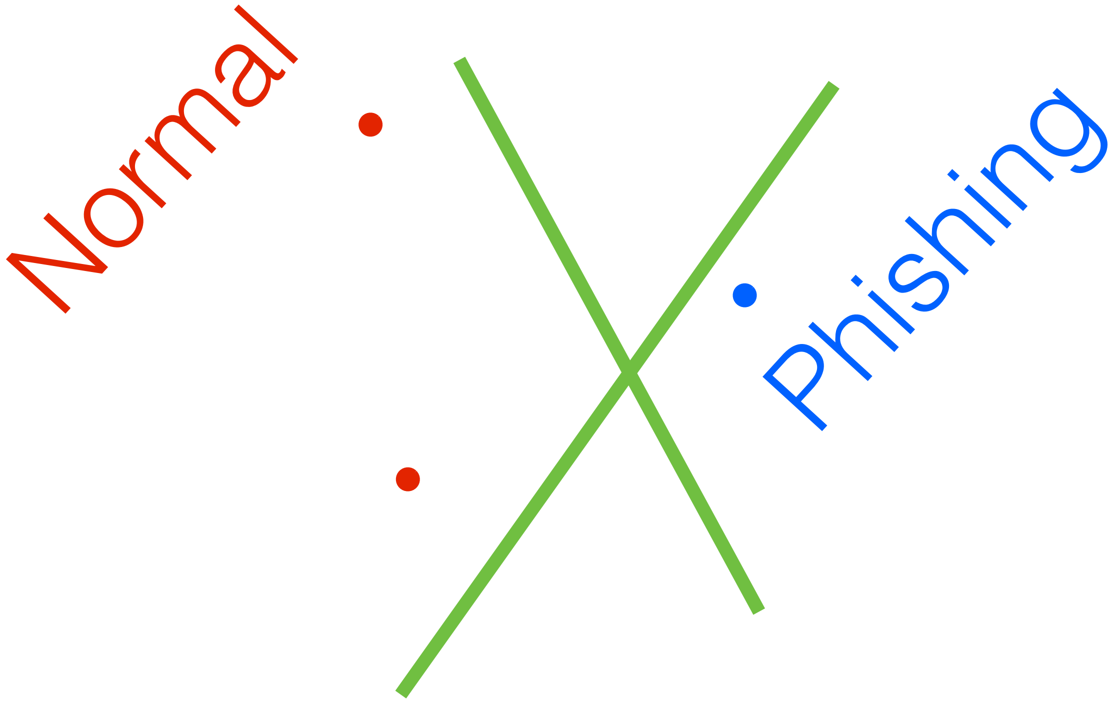

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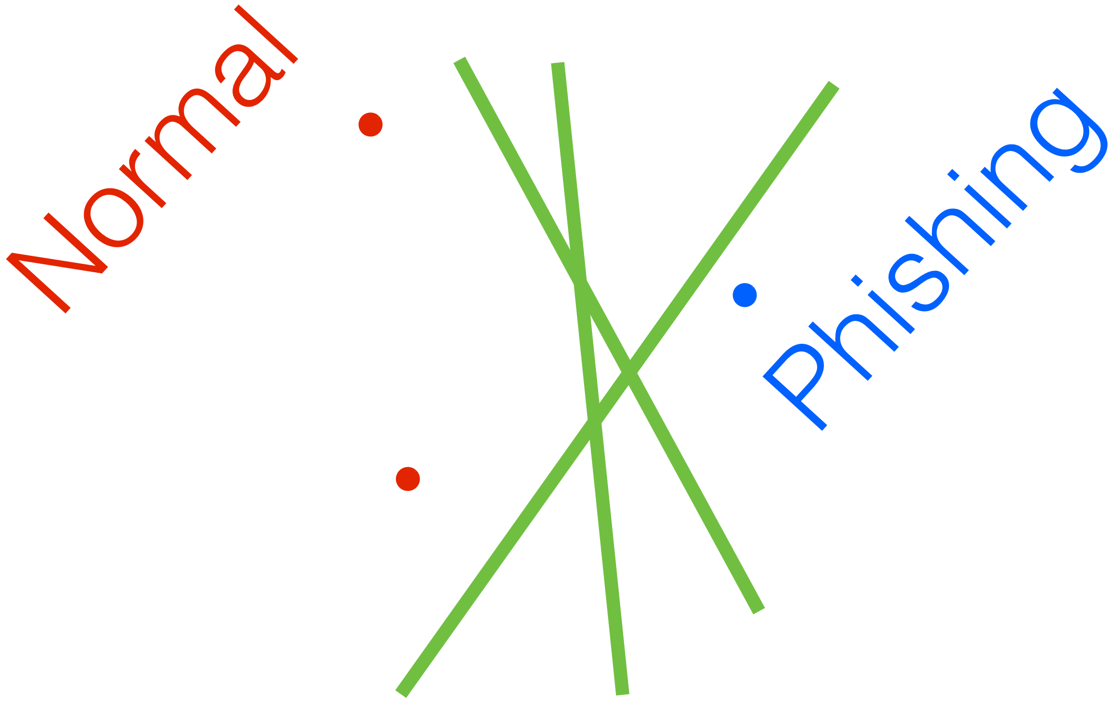

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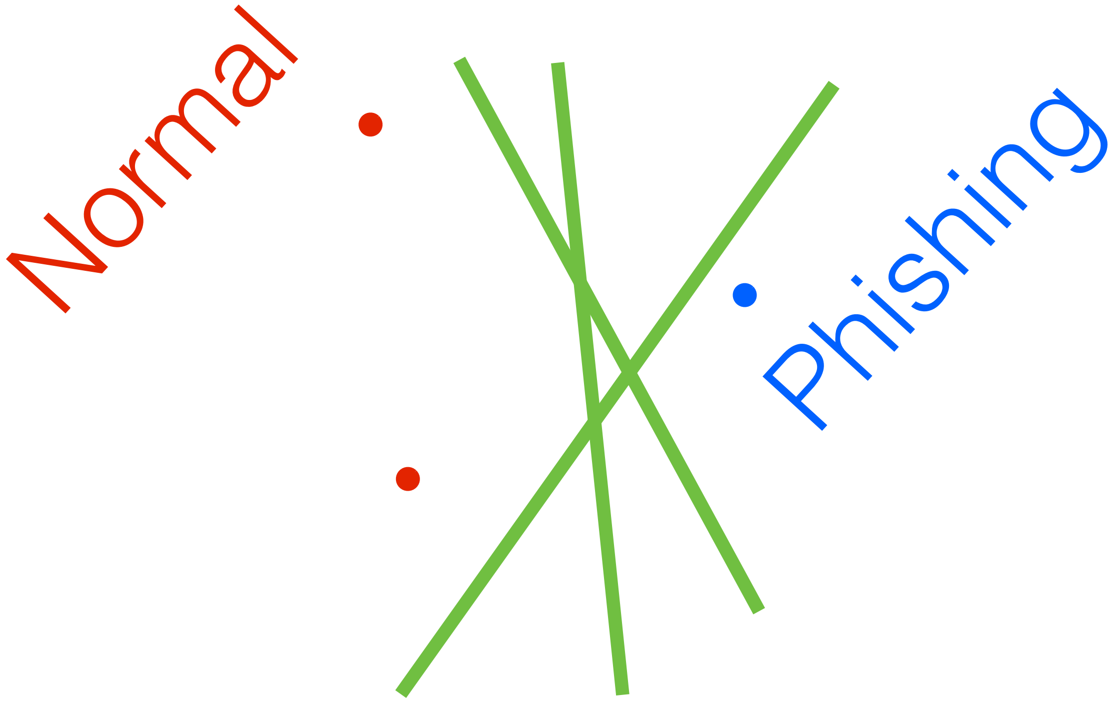
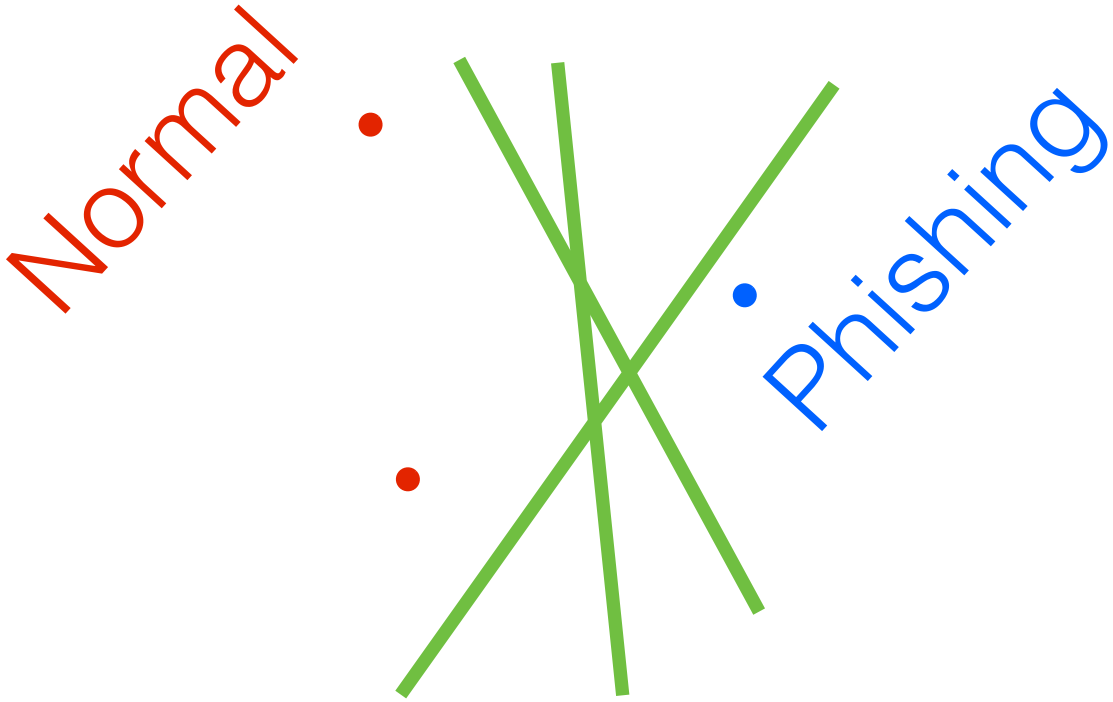


- Might miss important data
- Noisy estimates

Uniform subsampling revisited



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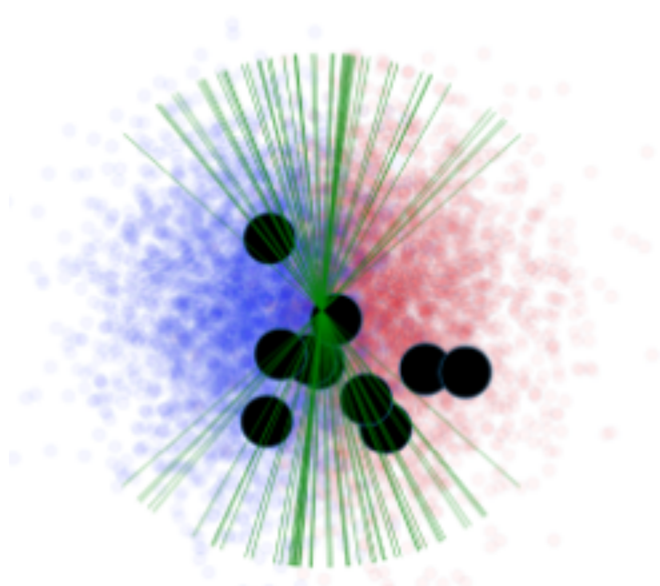
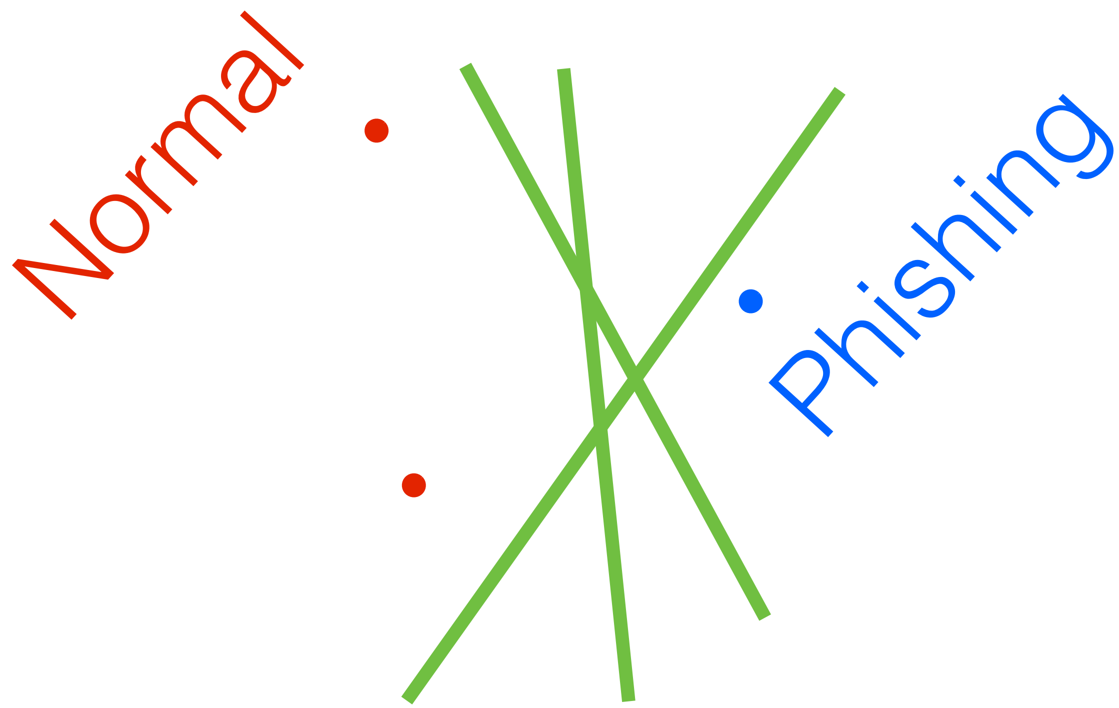


$M = 10$

Uniform subsampling revisited



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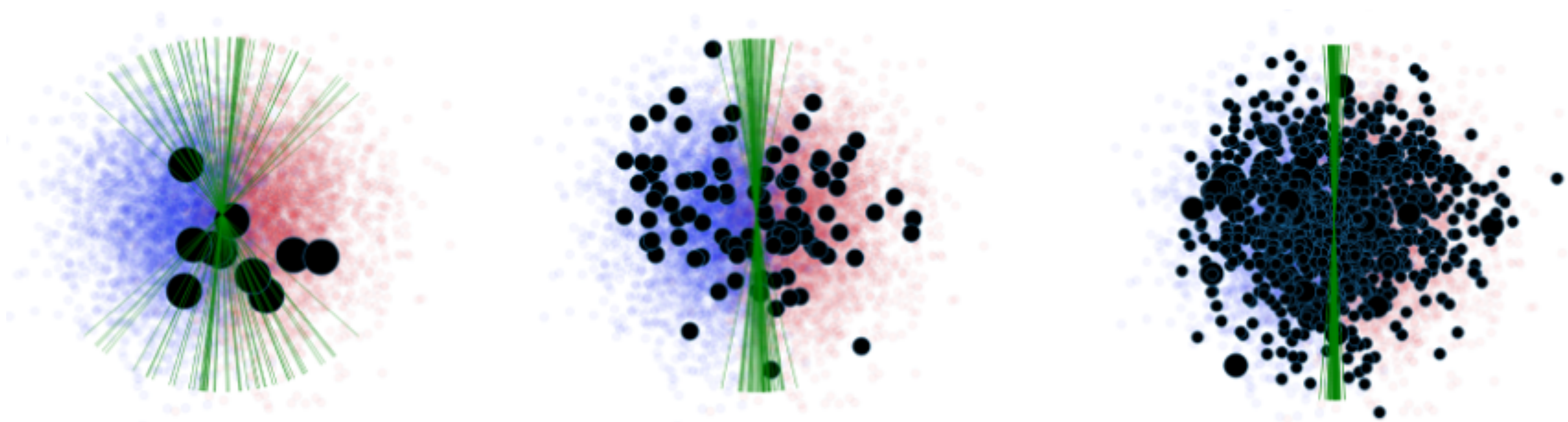

$M = 10$ $M = 100$ $M = 1000$

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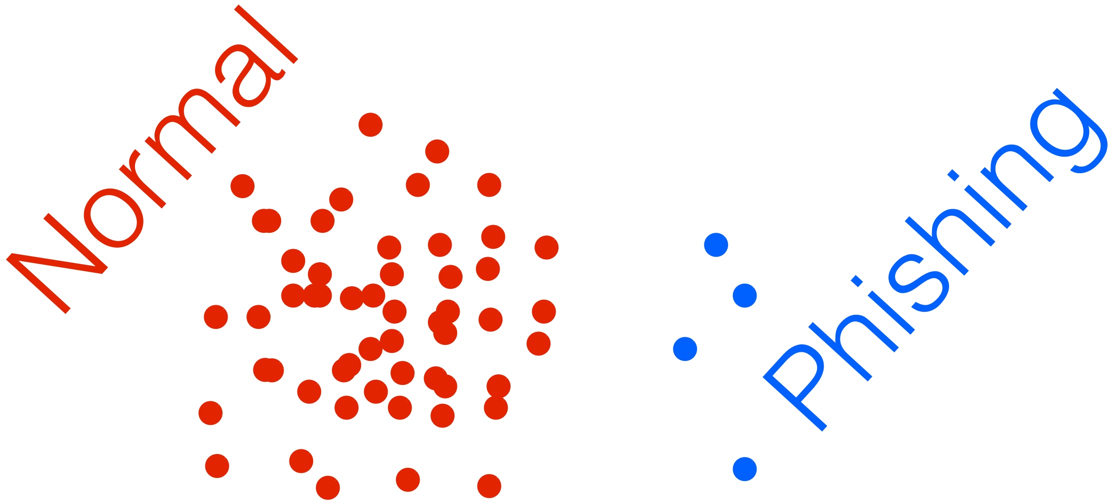

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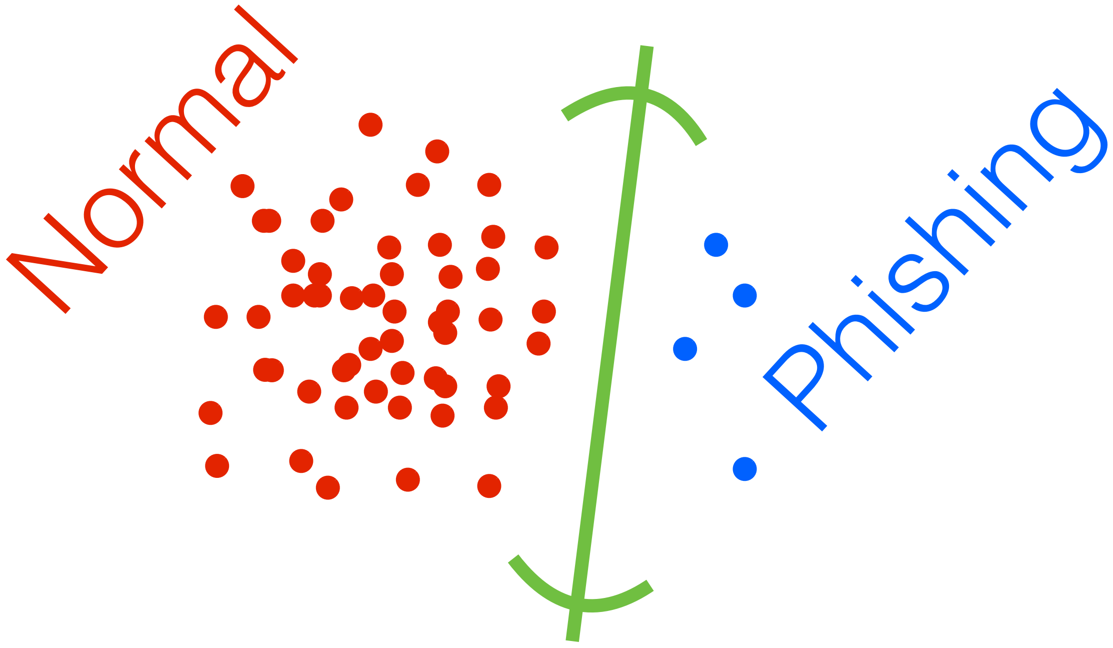

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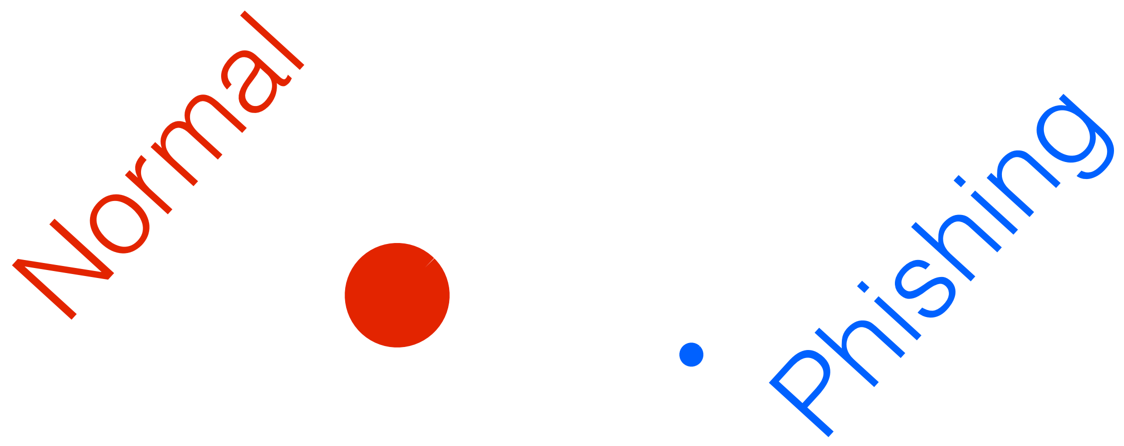

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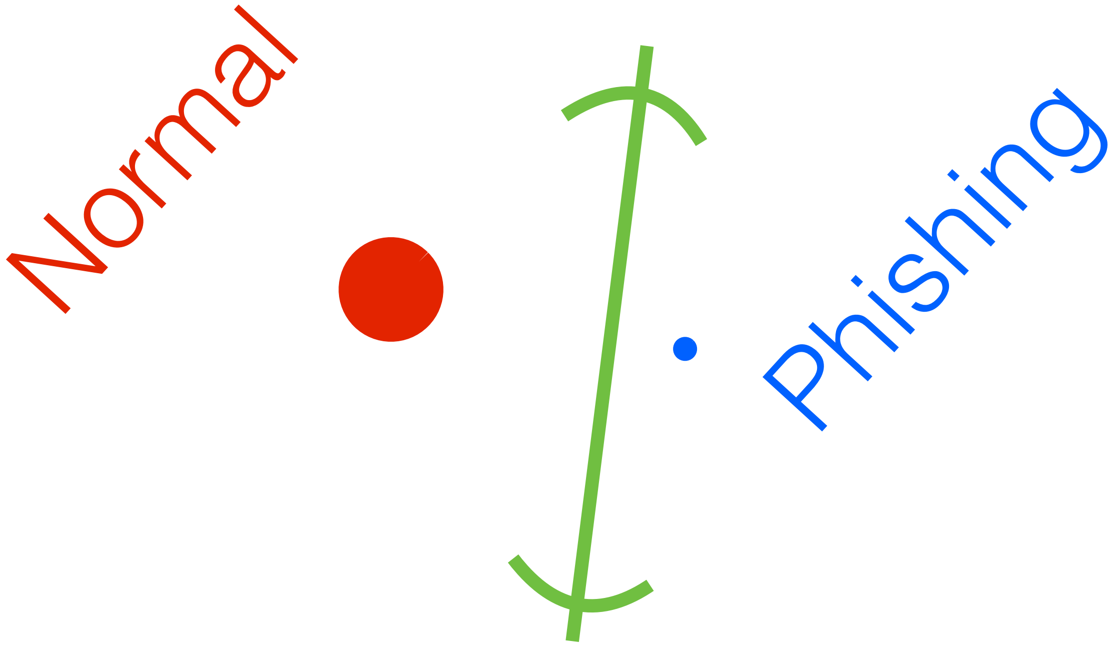

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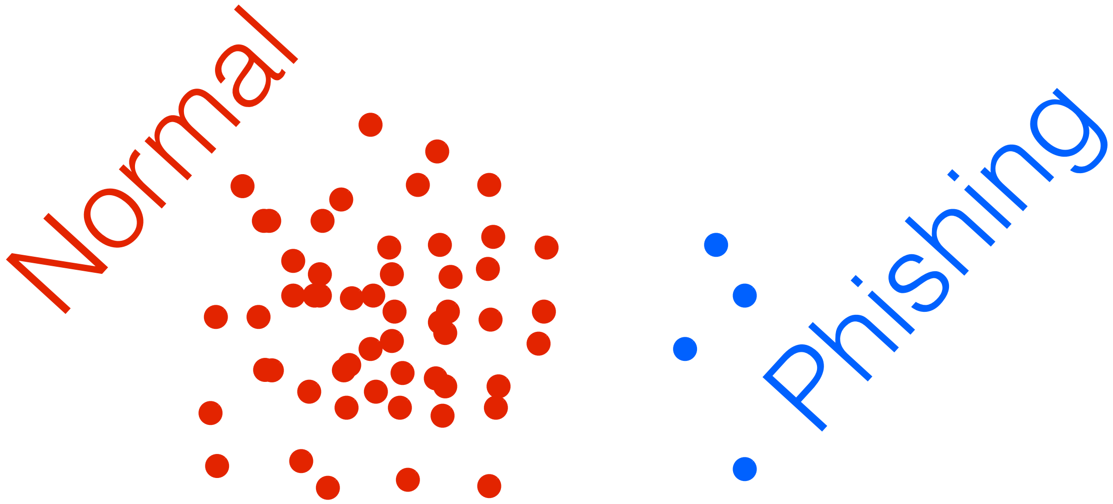

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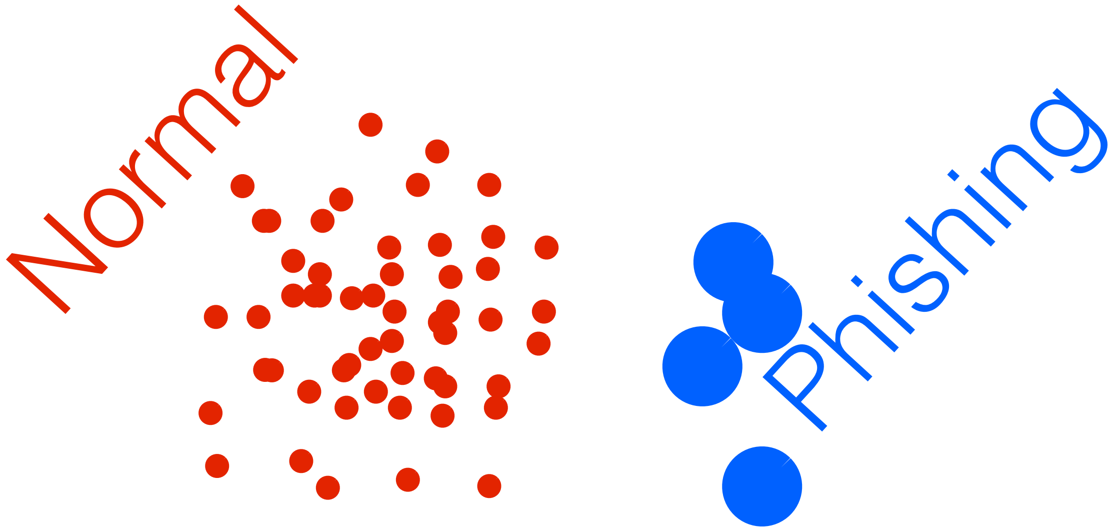

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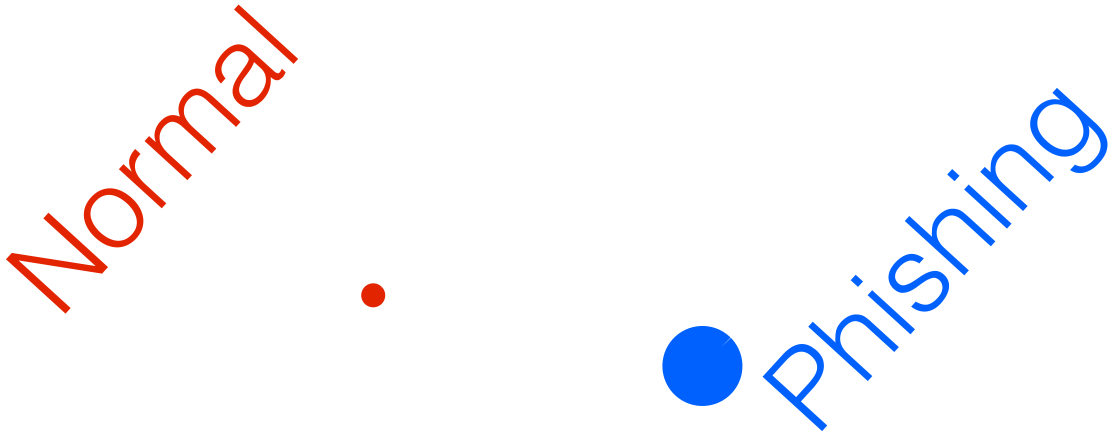

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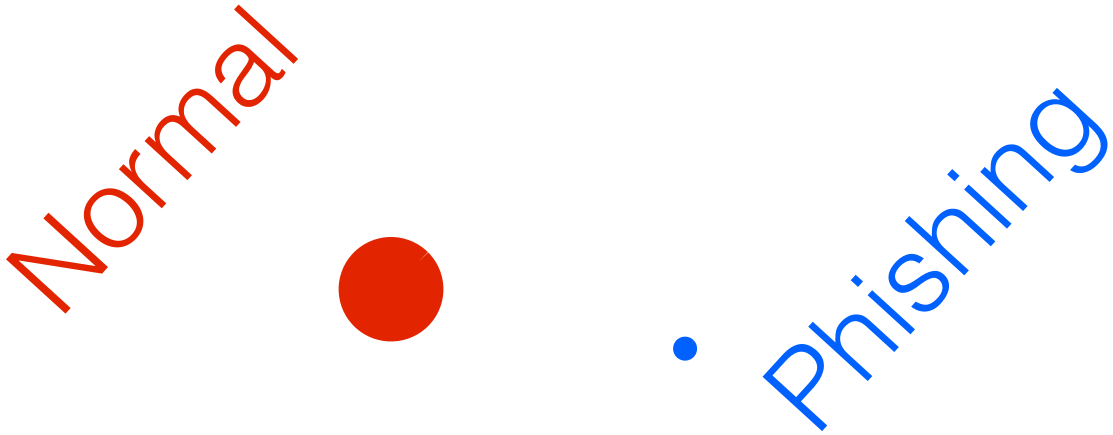

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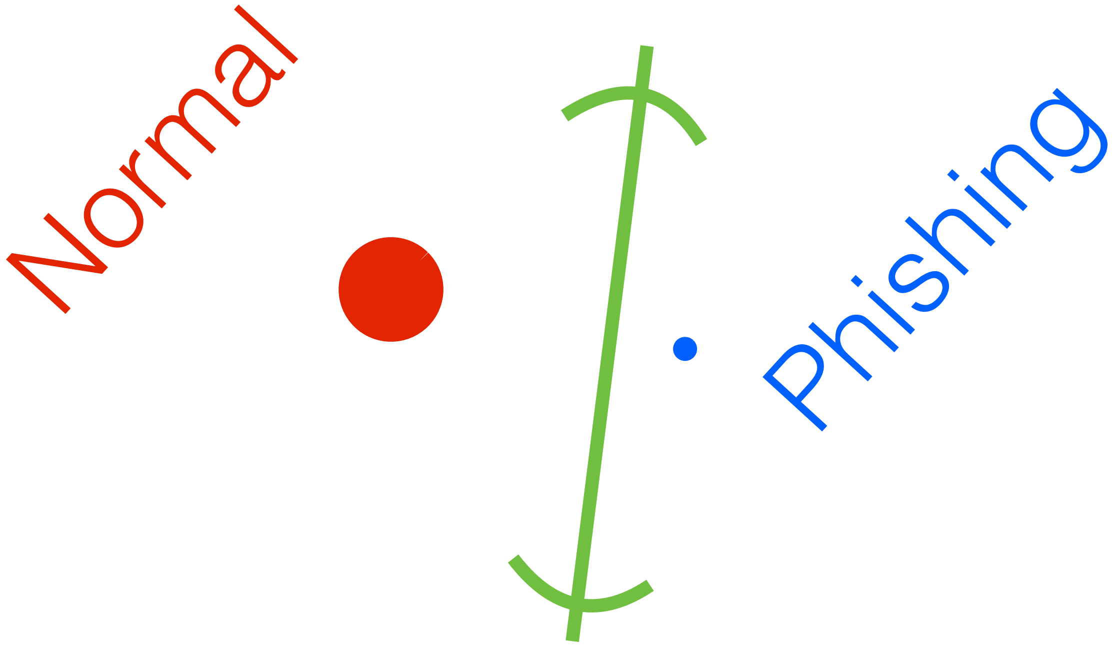

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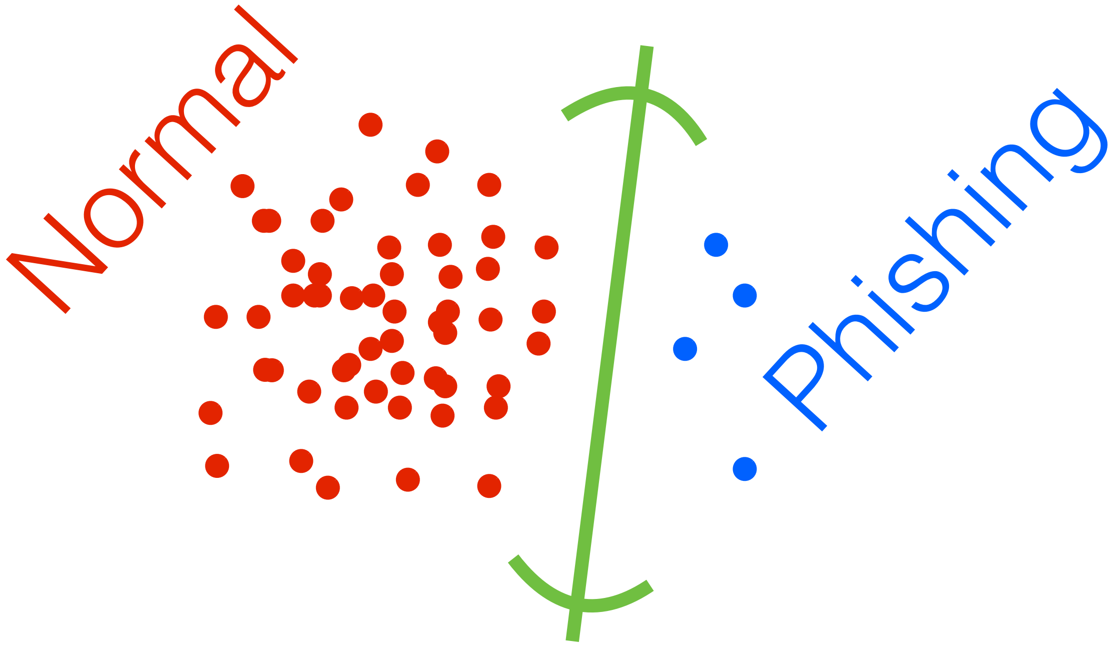

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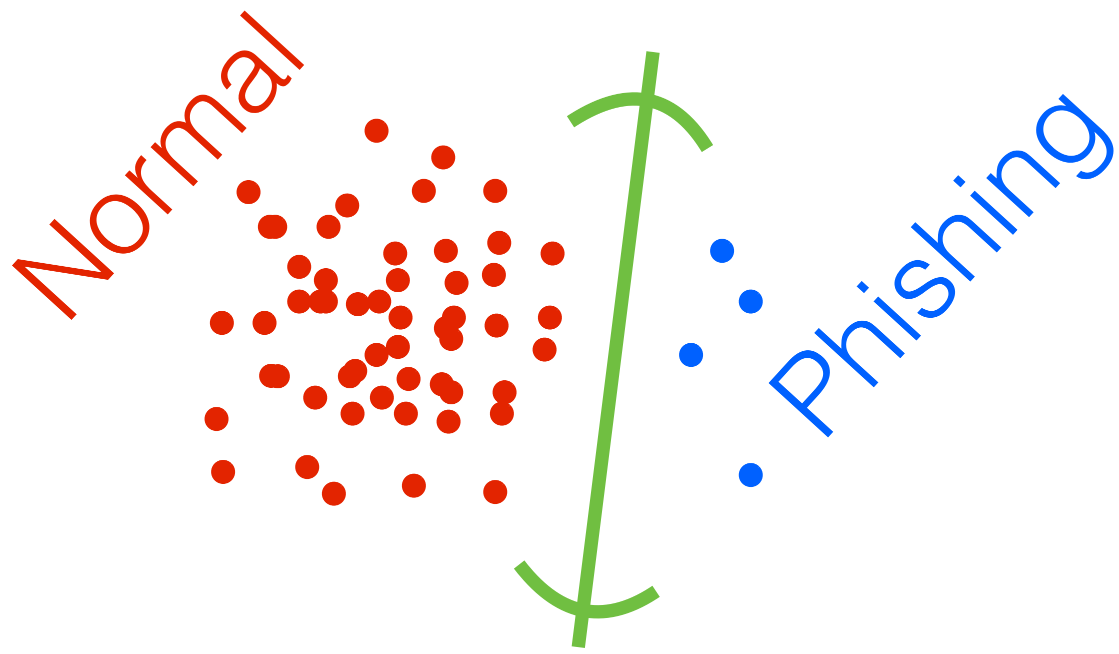
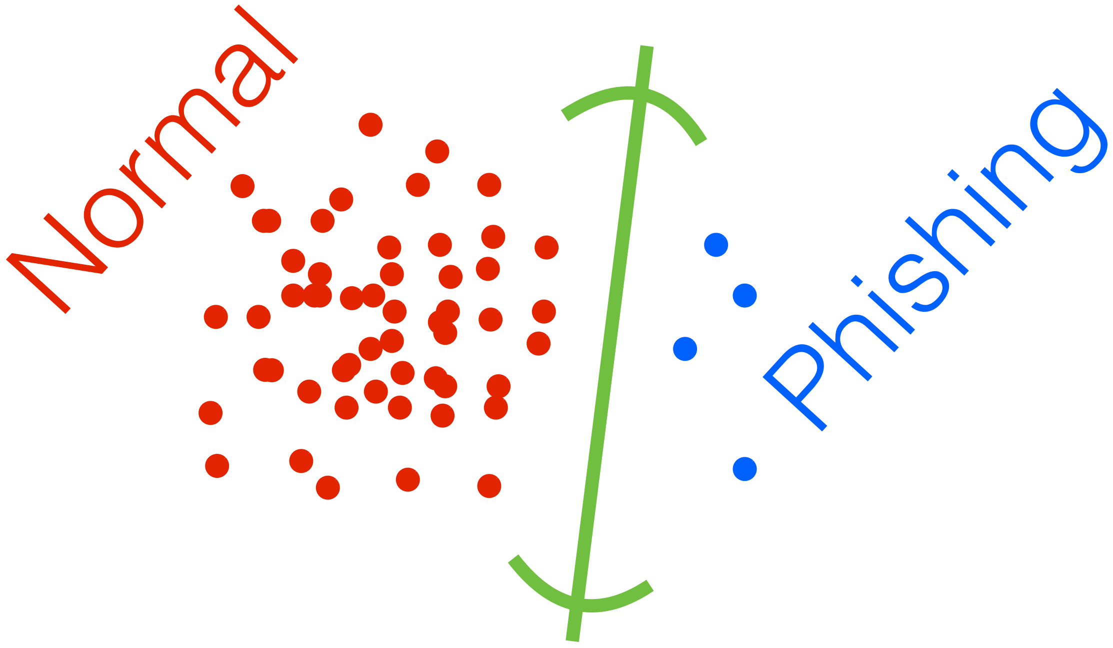


$$\sigma_n \propto \|\mathcal{L}_n\|$$

Importance sampling



$$\sigma := \sum_{n=1}^{N} \|\mathcal{L}_n\|$$

$$\sigma_n := \|\mathcal{L}_n\| / \sigma$$

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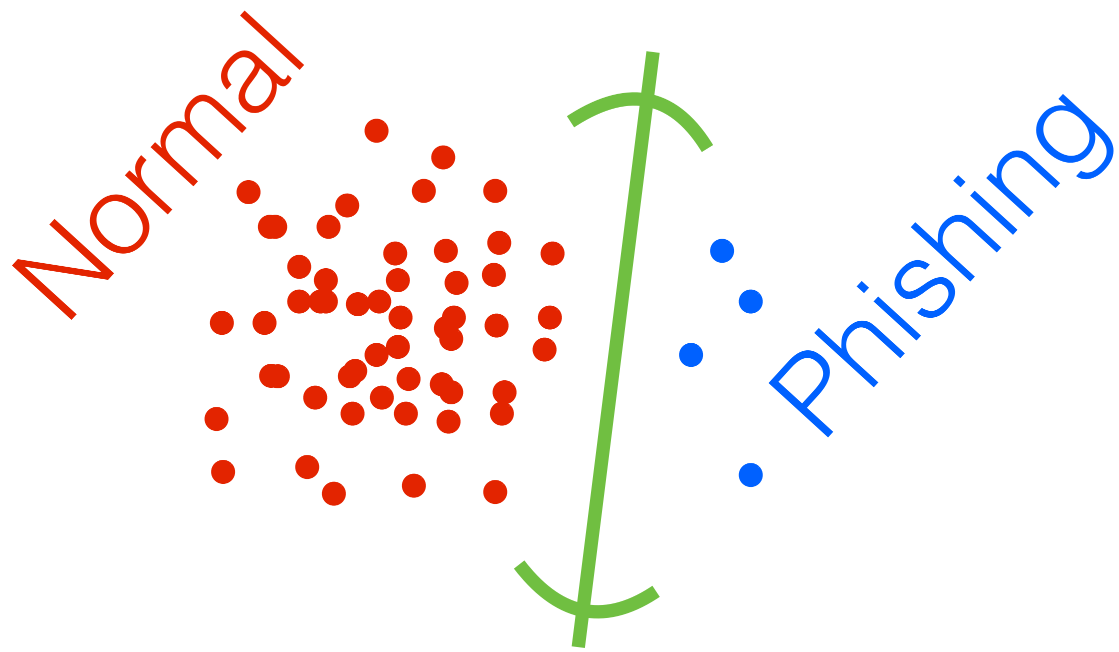

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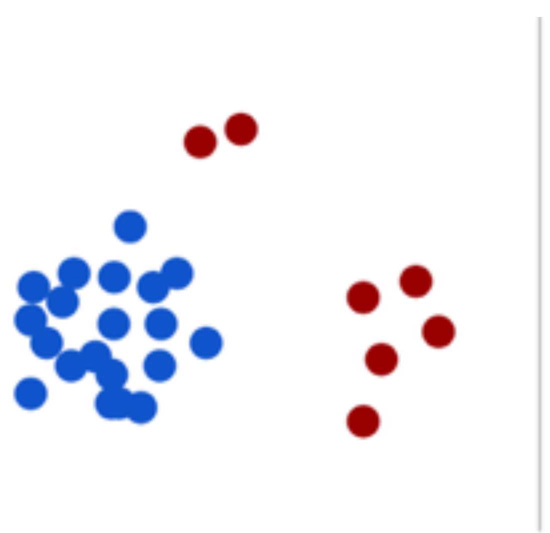

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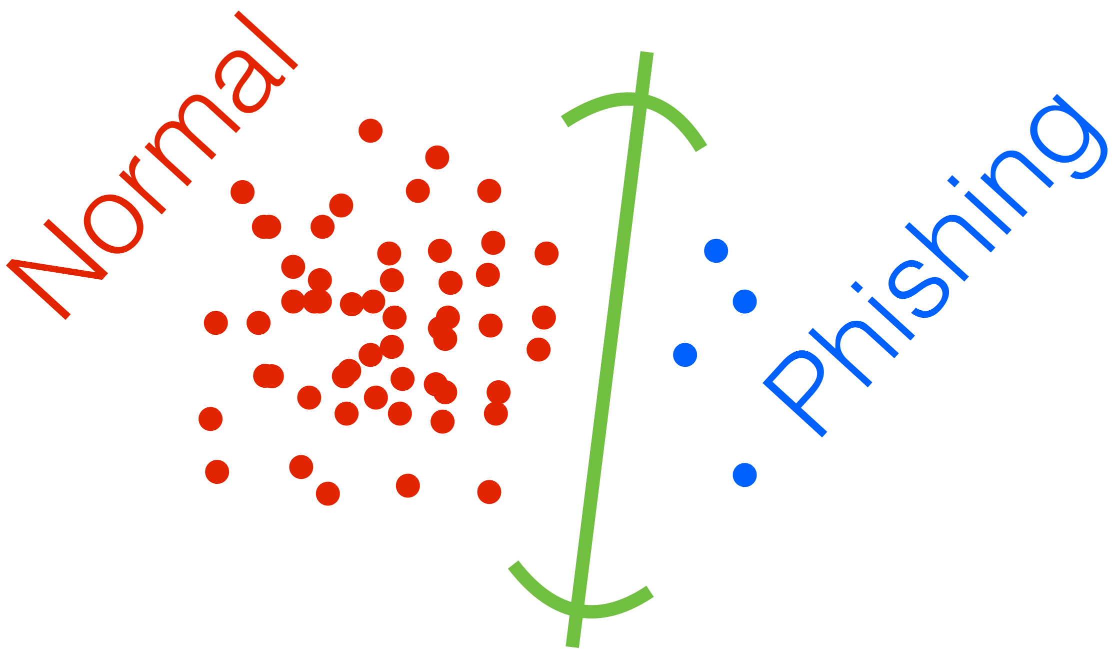

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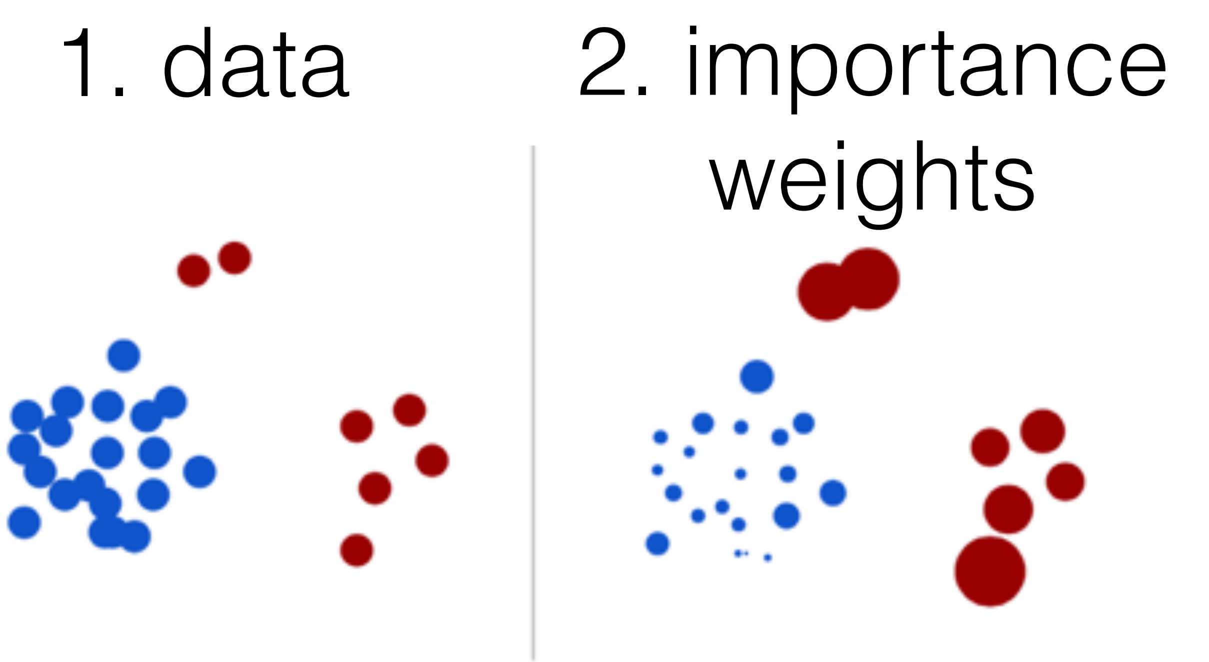

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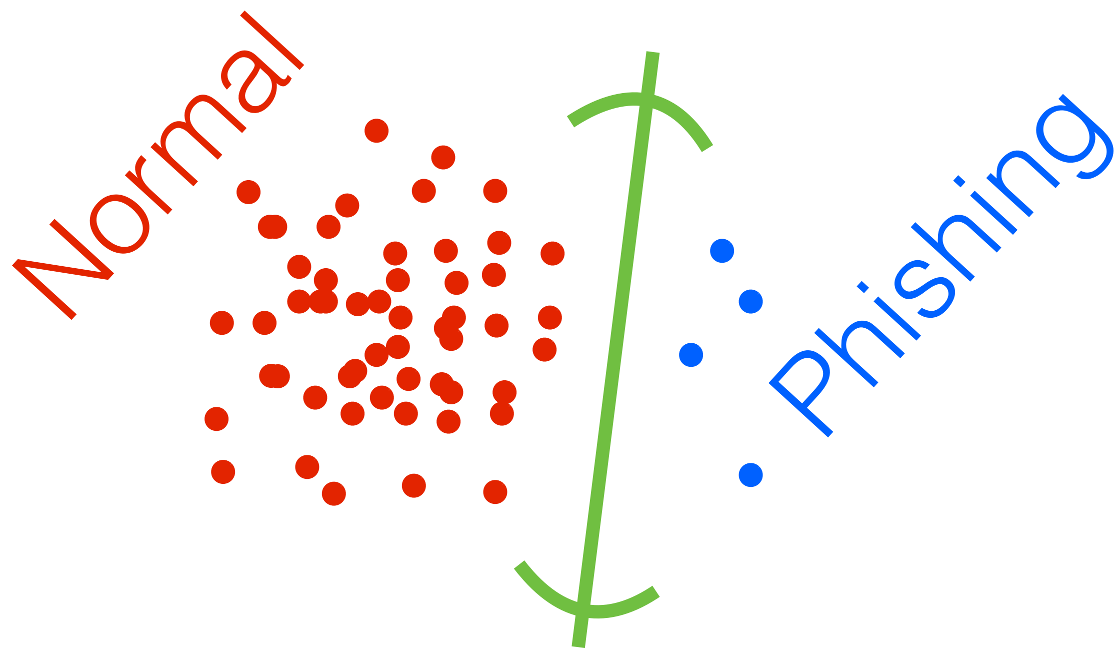

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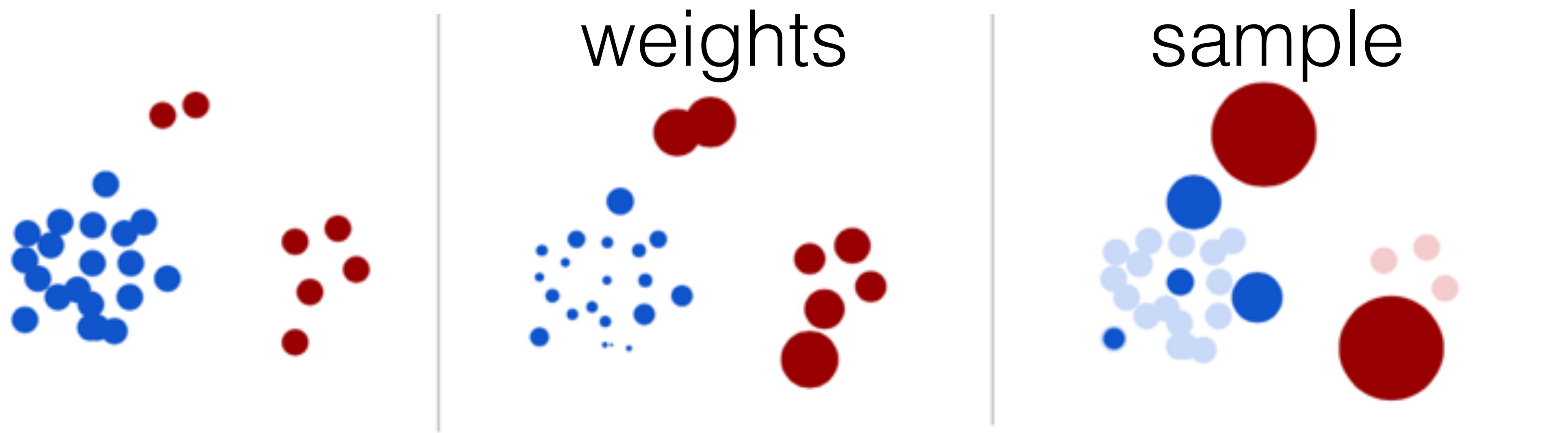
2. importance weights
3. importance sample



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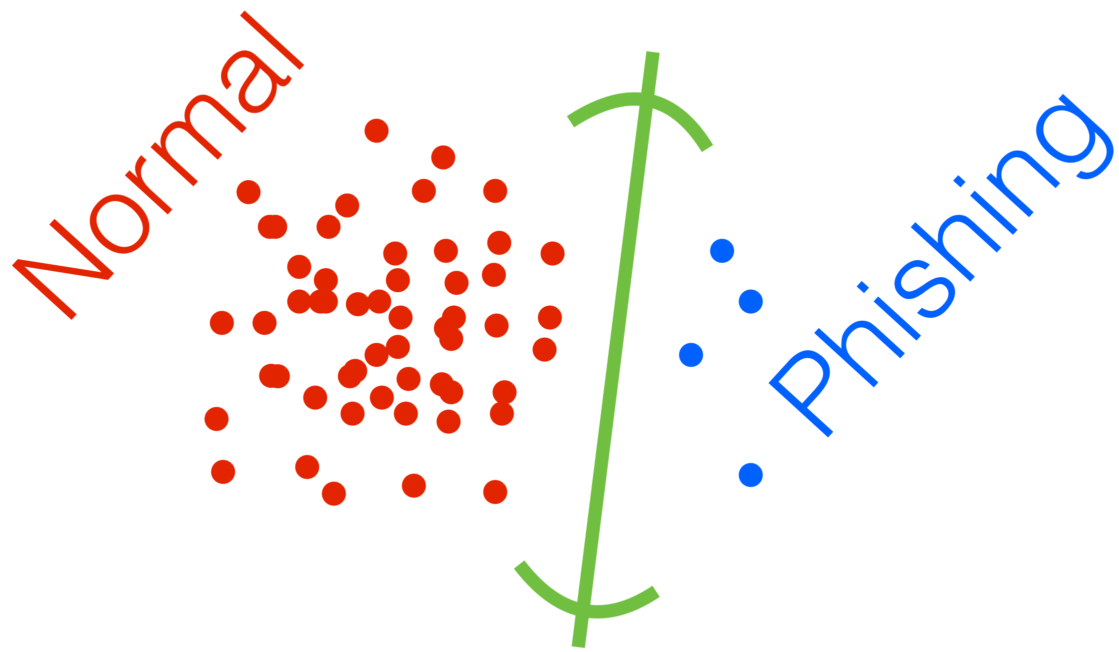

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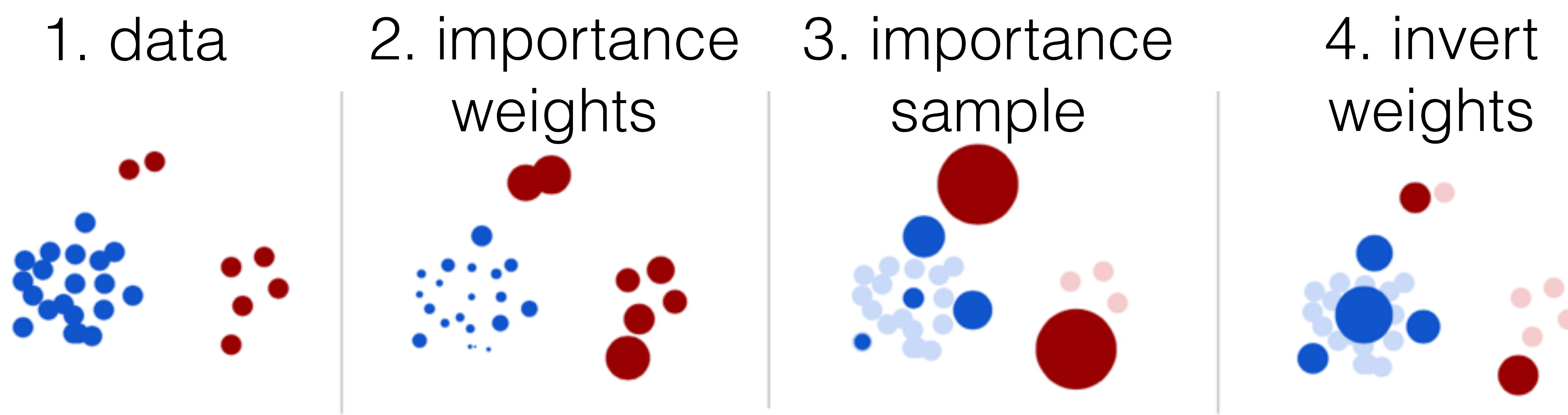

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Thm sketch (CB). δ ∈ (0,1). W.p. ≥1 - δ, after *M* iterations,

$$\|\mathcal{L}(w) - \mathcal{L}\| \leq \frac{\sigma\bar{\eta}}{\sqrt{M}}\left(1 + \sqrt{2\log\frac{1}{\delta}}\right)$$

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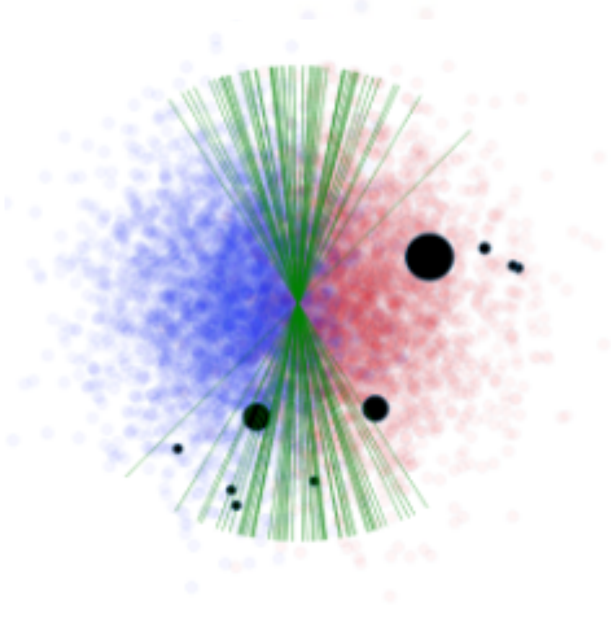

M = 10

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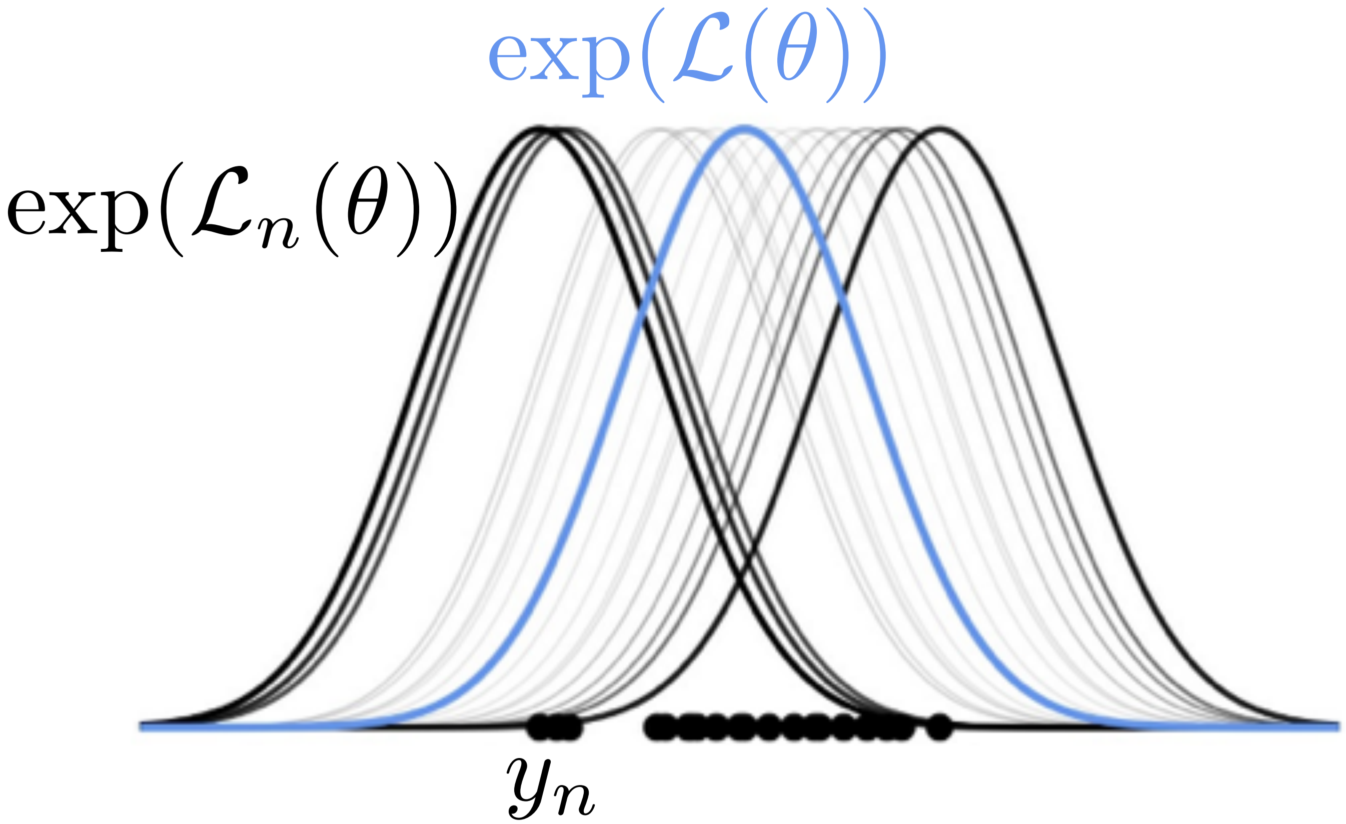

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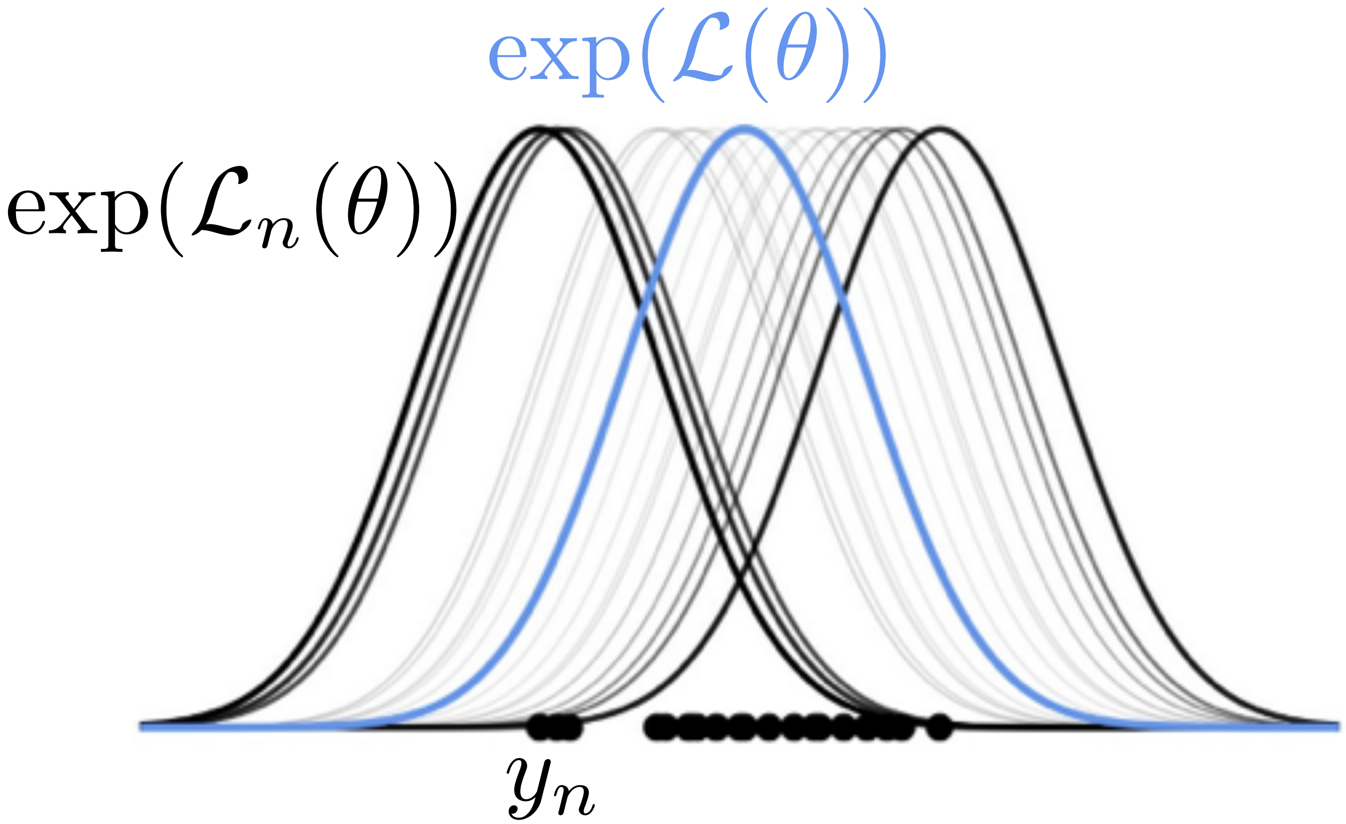

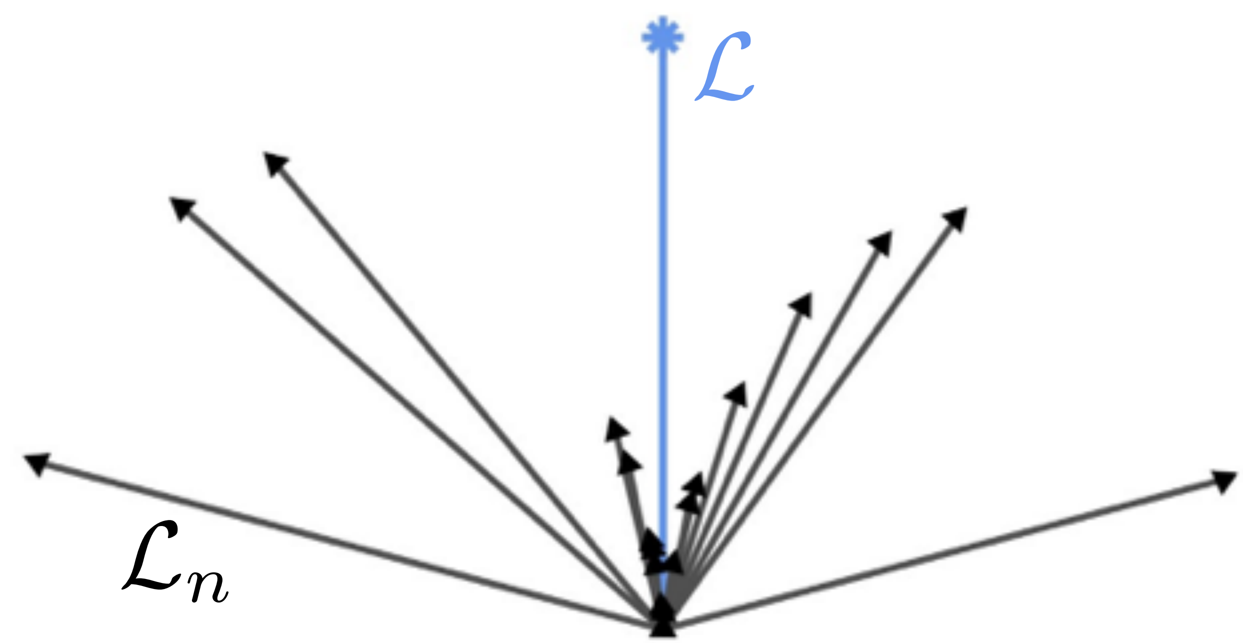

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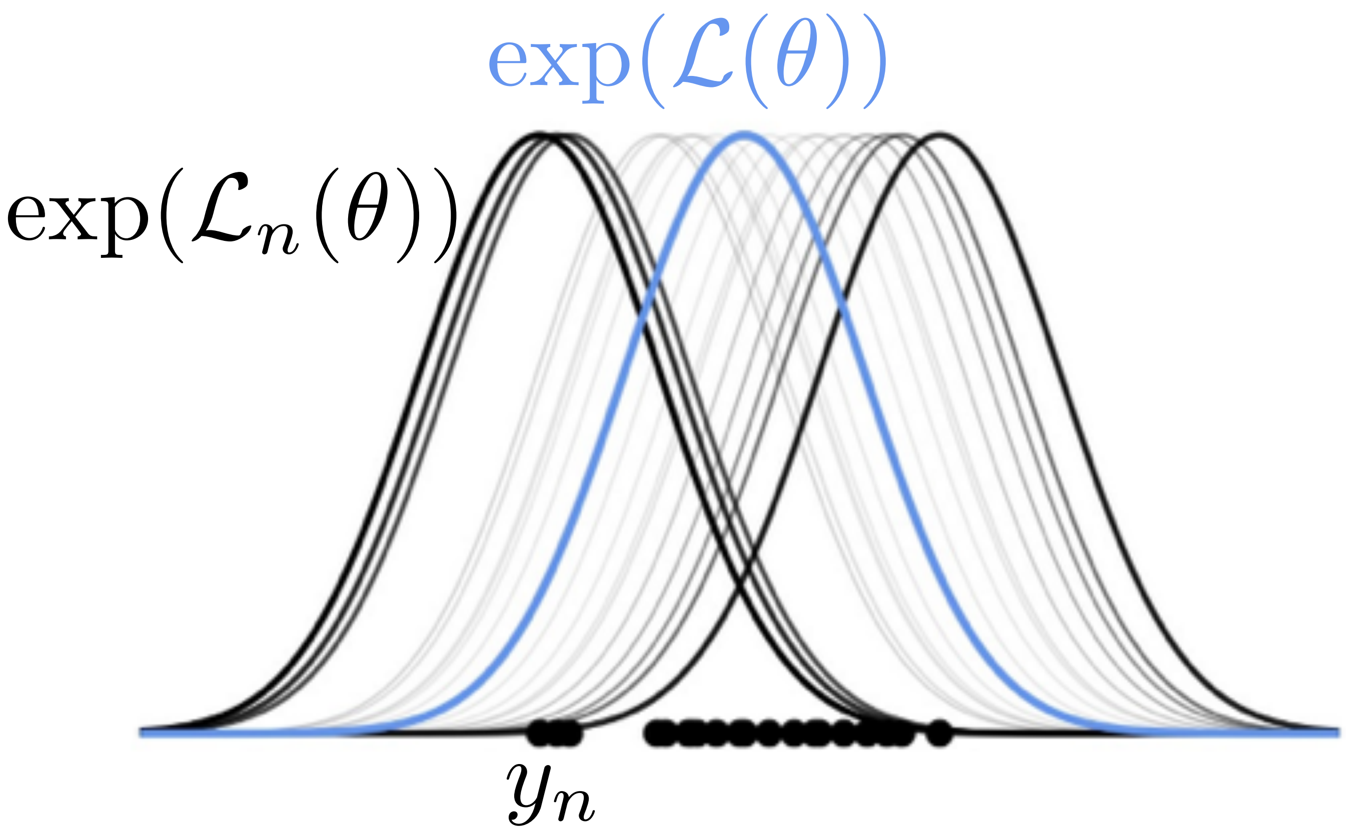

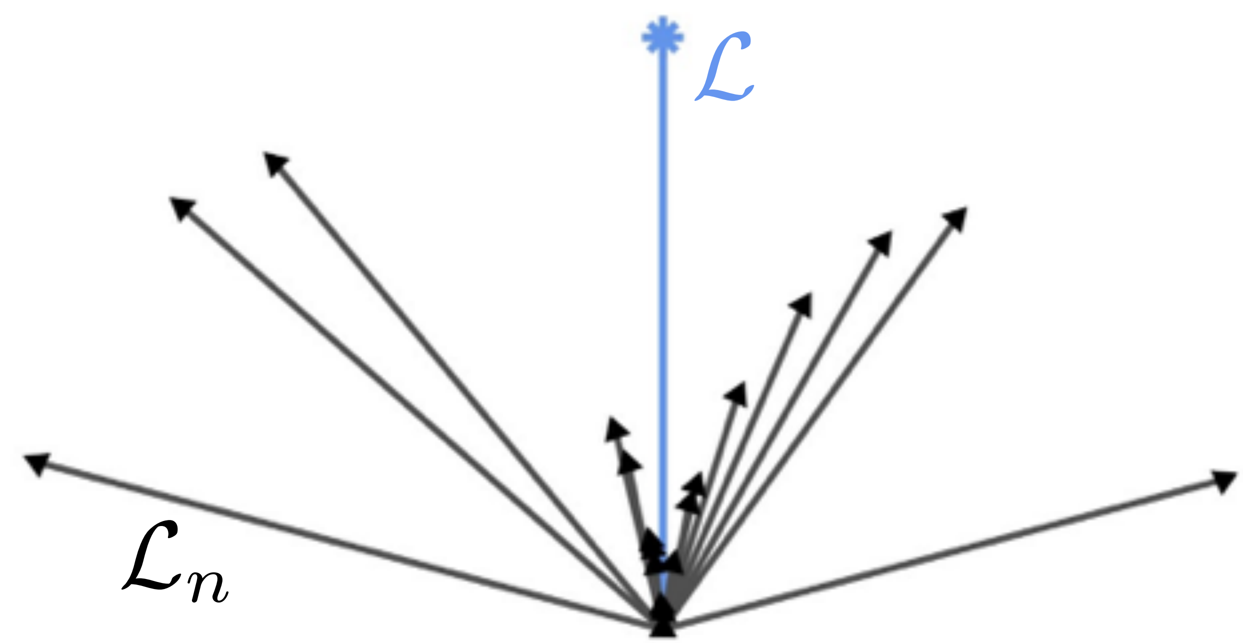

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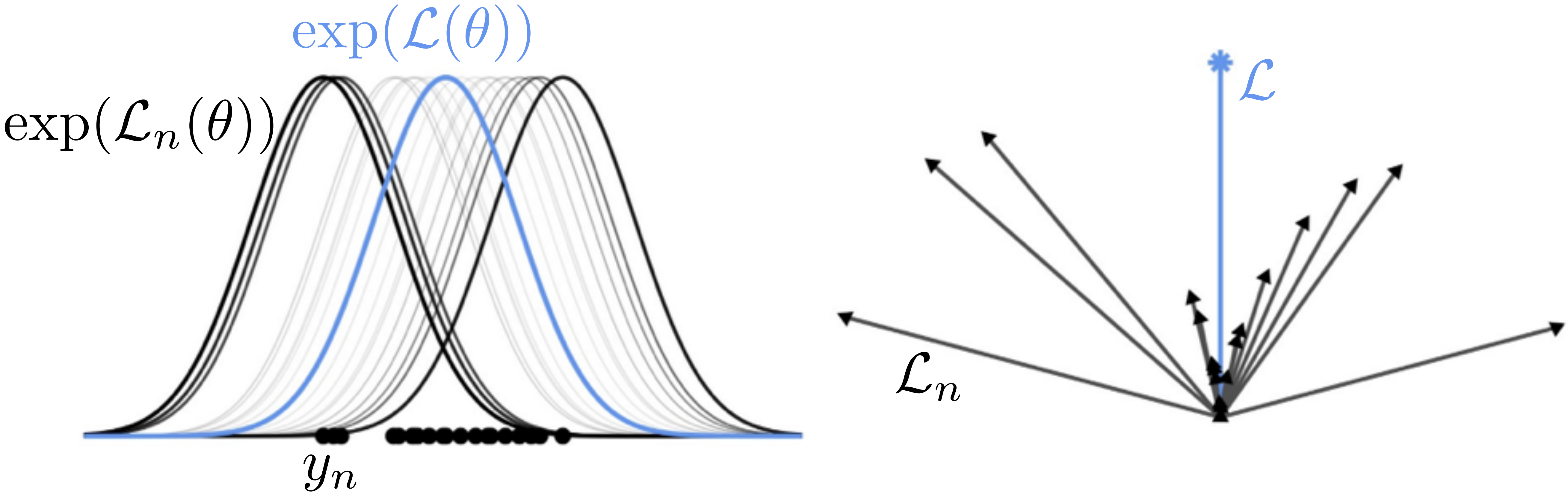

- need to consider (residual) error direction
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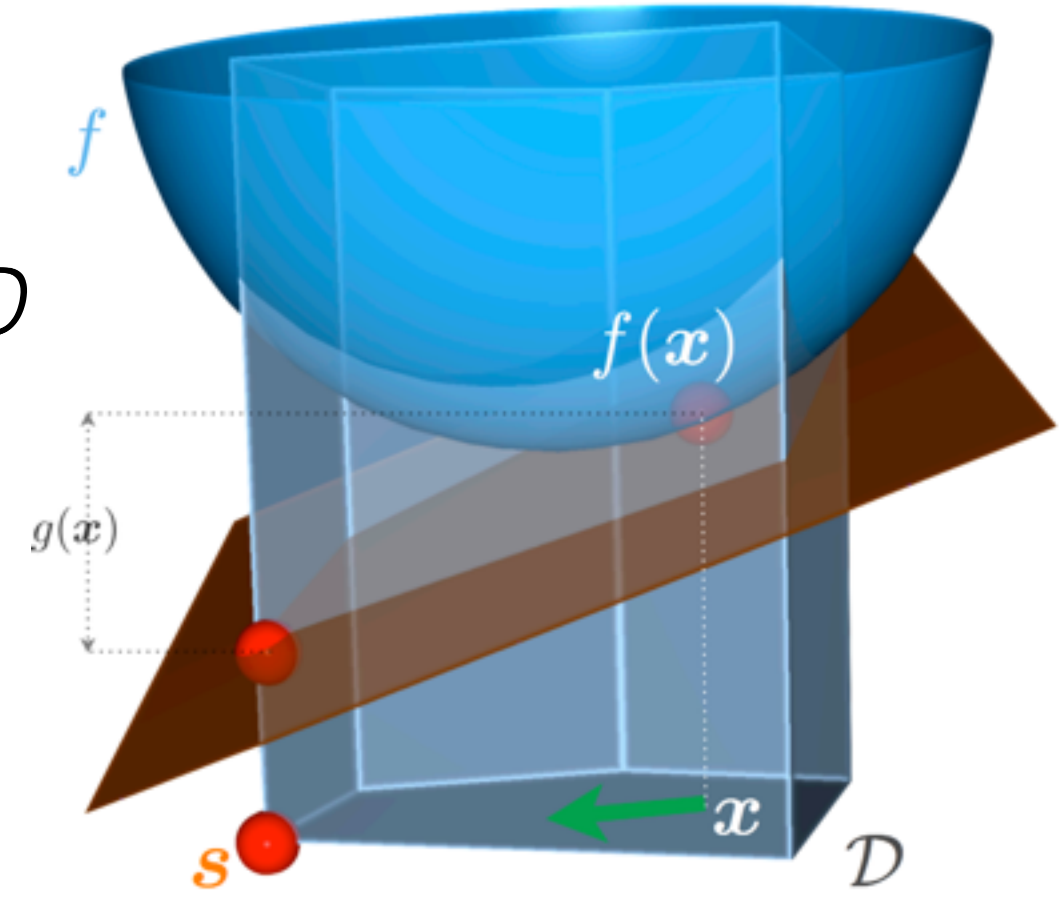
Frank-Wolfe

Convex optimization on a polytope *D*

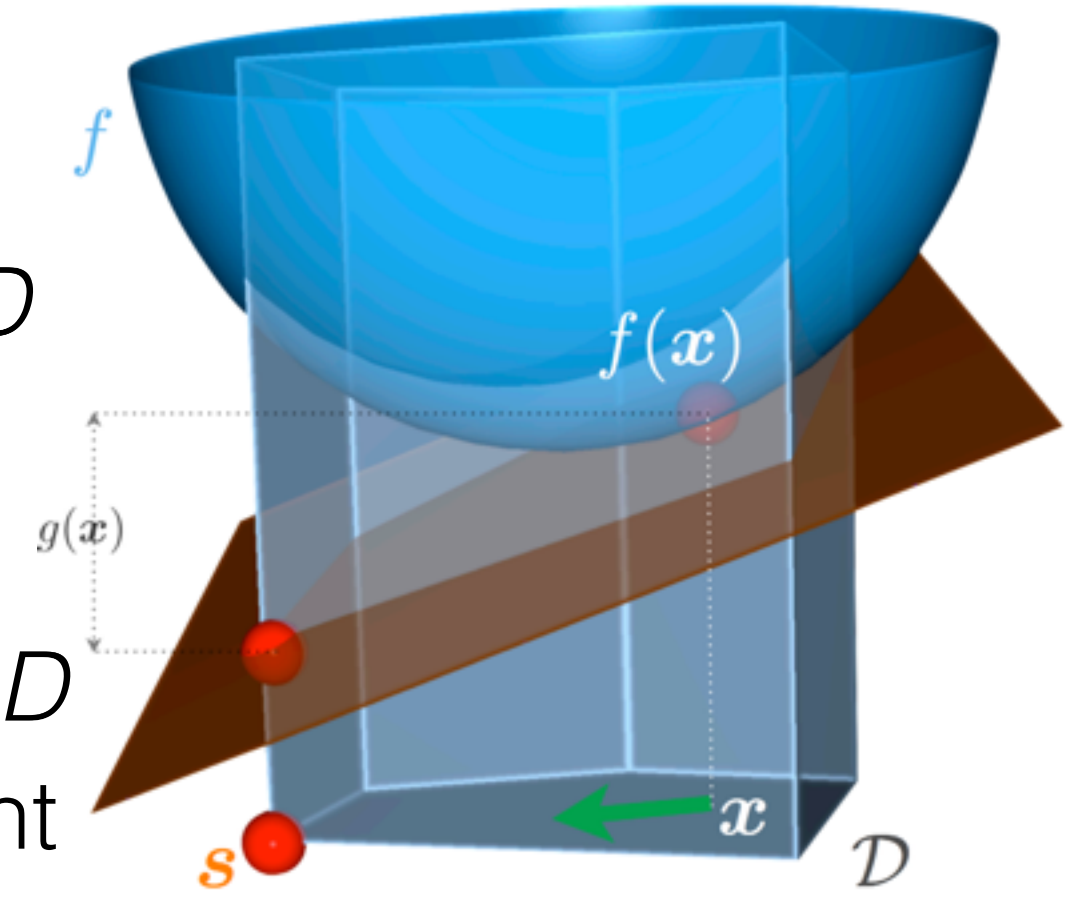


[Jaggi 2013]

Frank-Wolfe

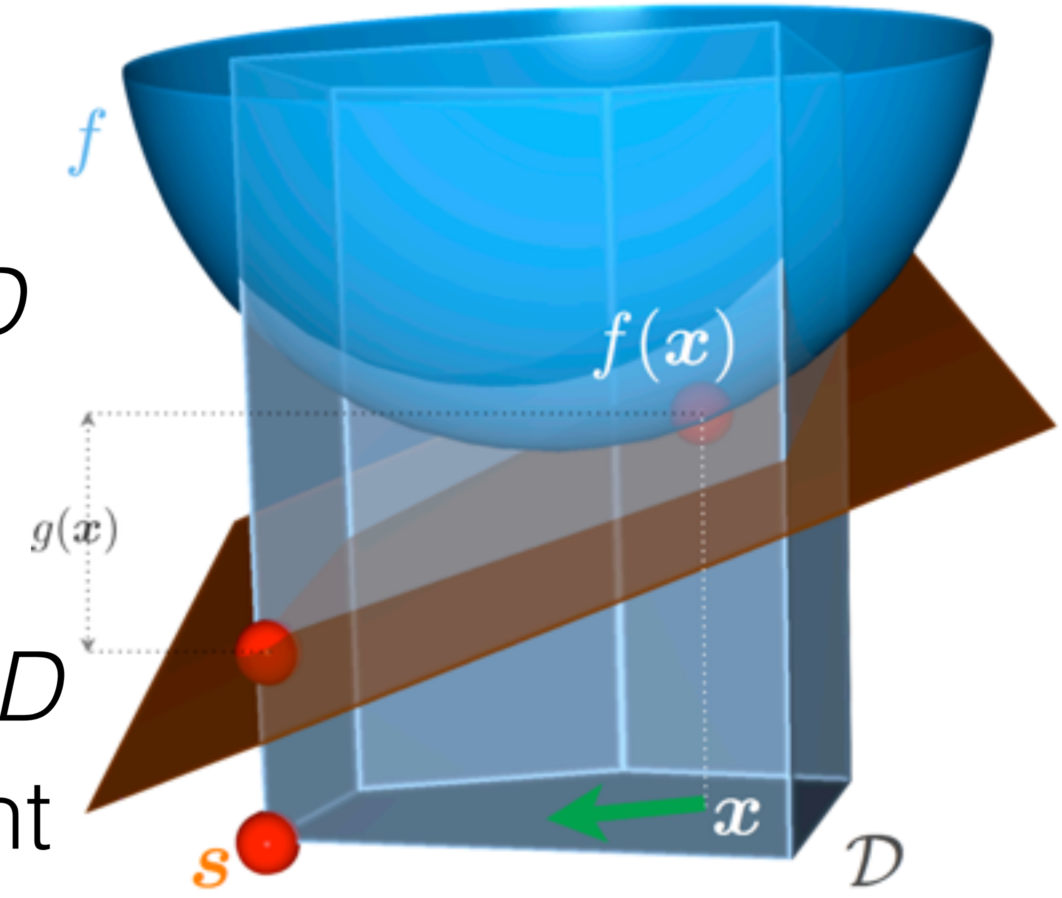
Convex optimization on a polytope *D*

- Repeat:
  1. Find gradient
  2. Find argmin point on plane in *D*
  3. Do line search between current point and argmin point



[Jaggi 2013]

Frank-Wolfe



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Convex optimization on a polytope *D*

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  1. Find gradient
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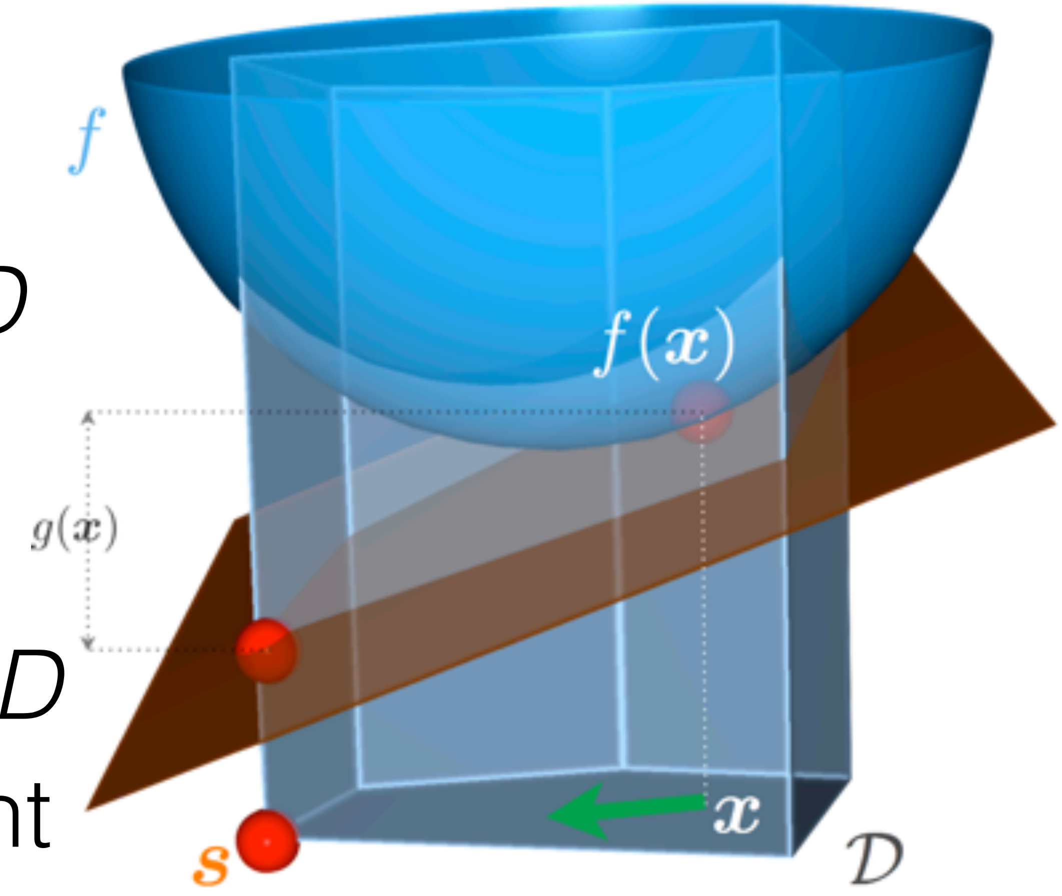

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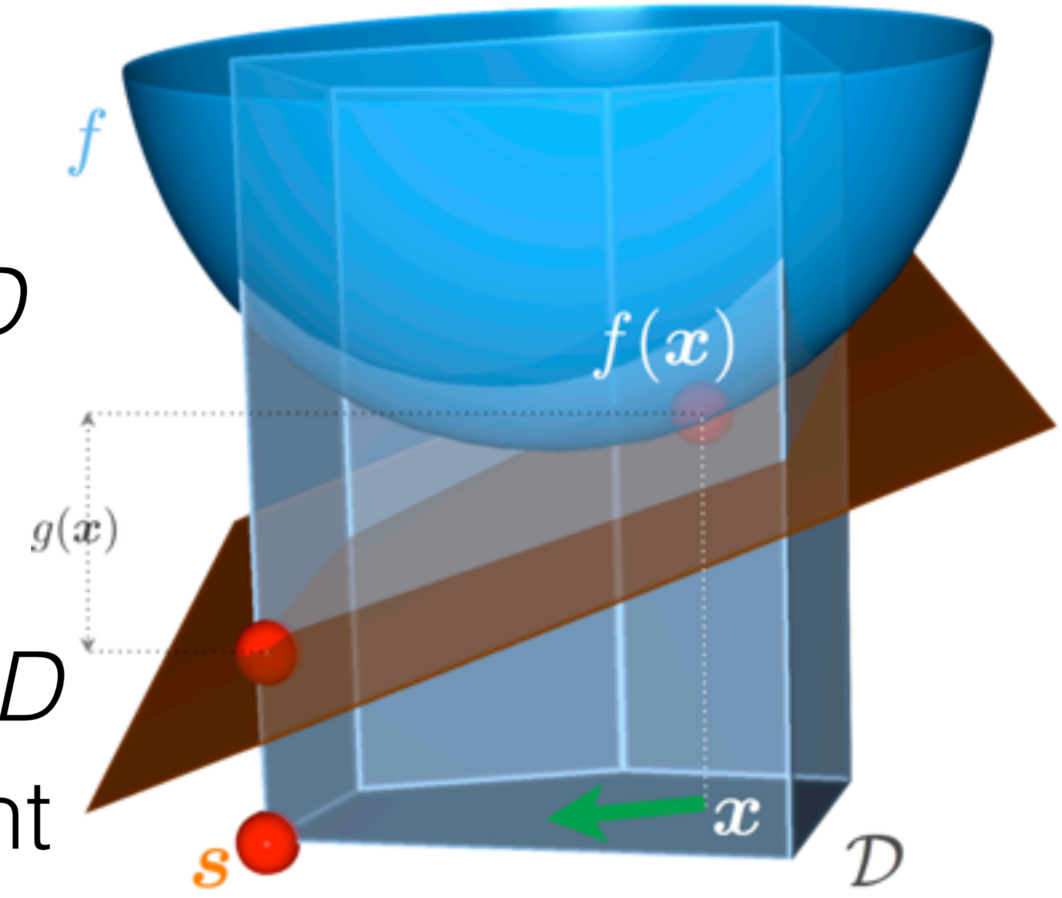

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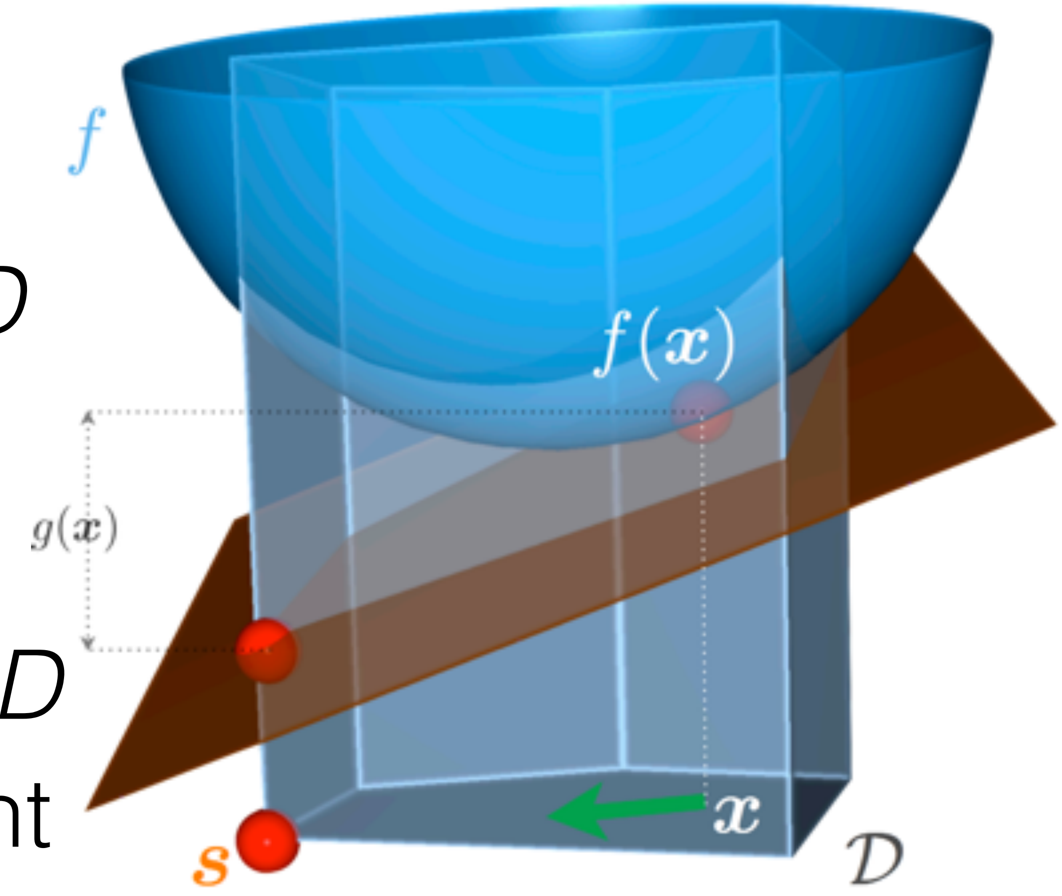

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$$\text{s.t. } w \geq 0, \|w\|_0 \leq M$$

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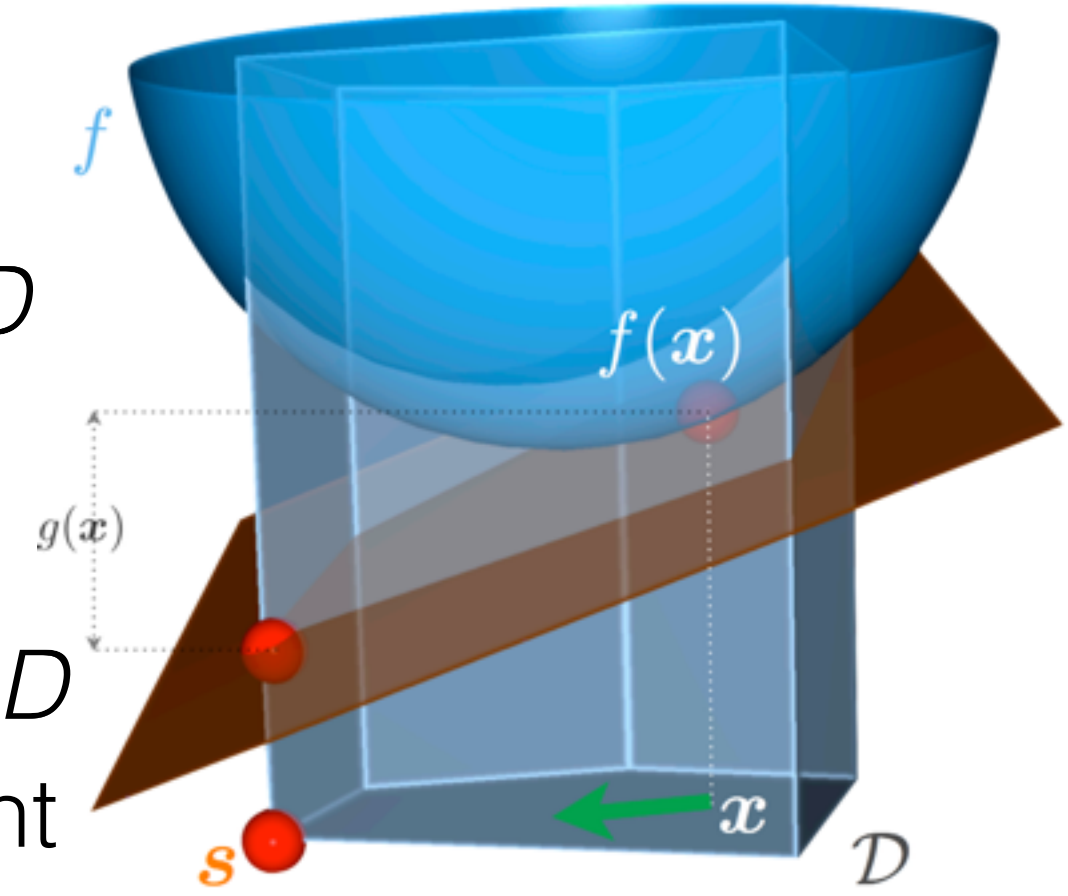

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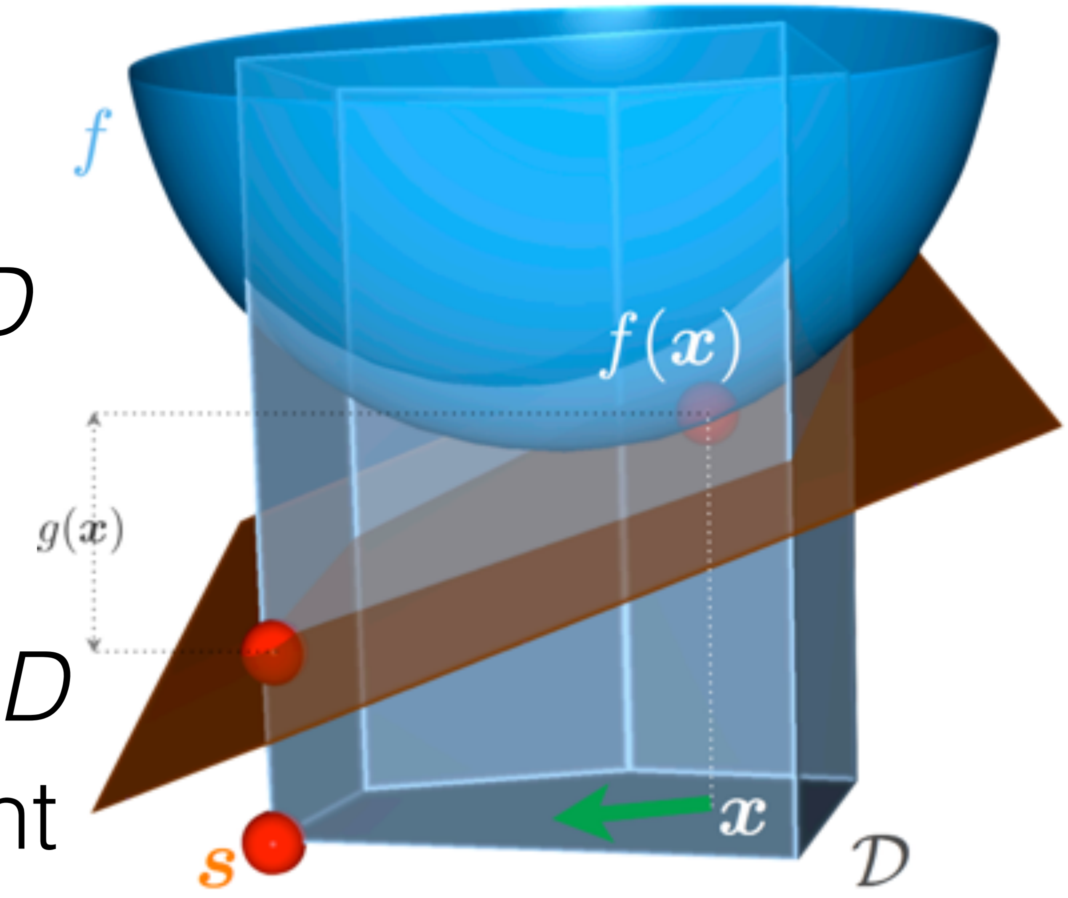

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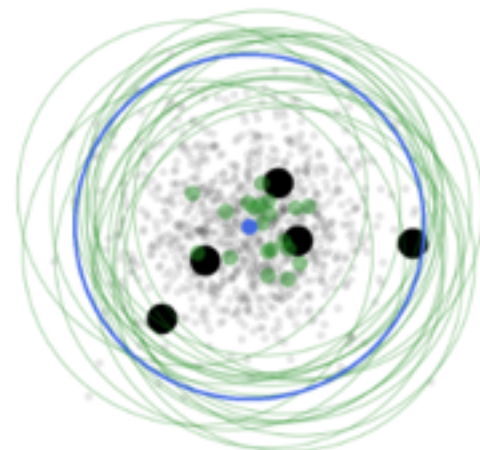
Thm sketch (CB). After *M* iterations,

$$\|\mathcal{L}(w) - \mathcal{L}\| \leq \frac{\sigma \bar{\eta}}{\sqrt{\alpha^{2M} + M}}$$

Gaussian model (simulated)

- 10K pts; norms, inference: closed-form

Uniform subsampling



$M = 5$

Gaussian model (simulated)

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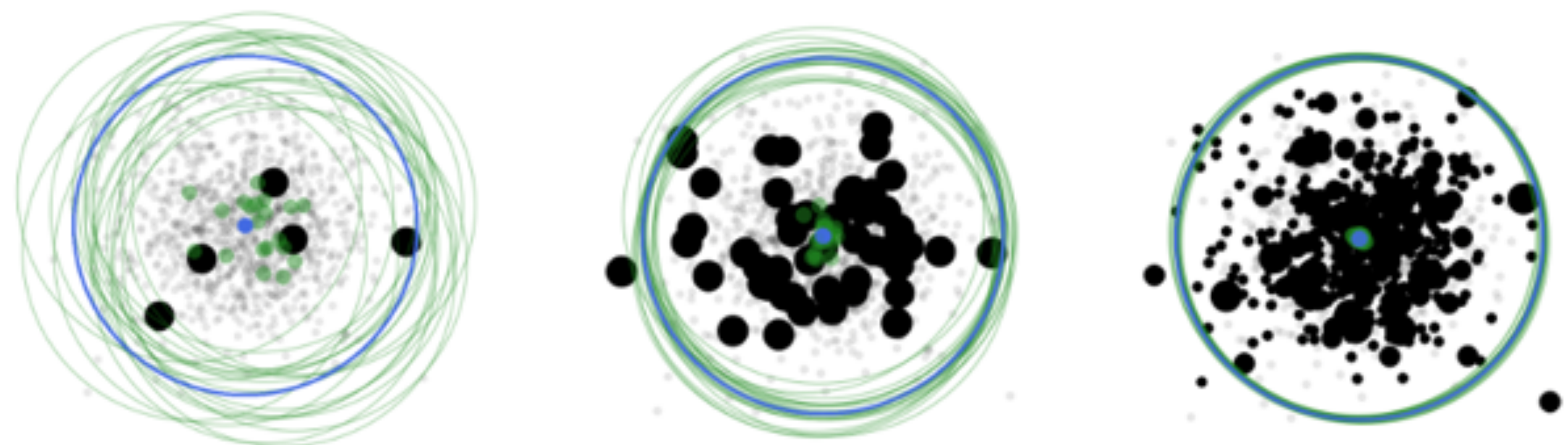

$M = 5$ $M = 50$ $M = 500$

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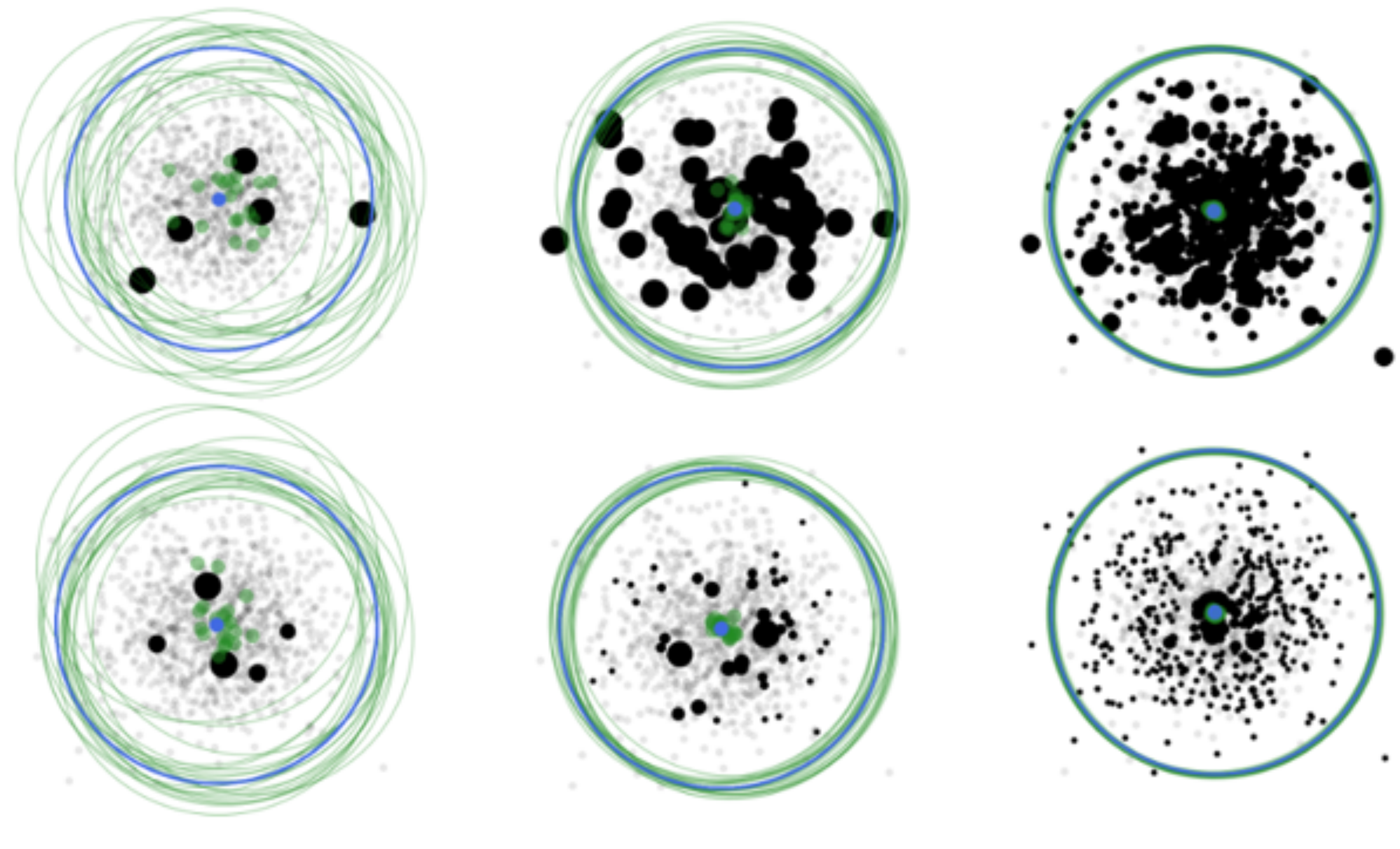
Uniform subsampling

Importance sampling



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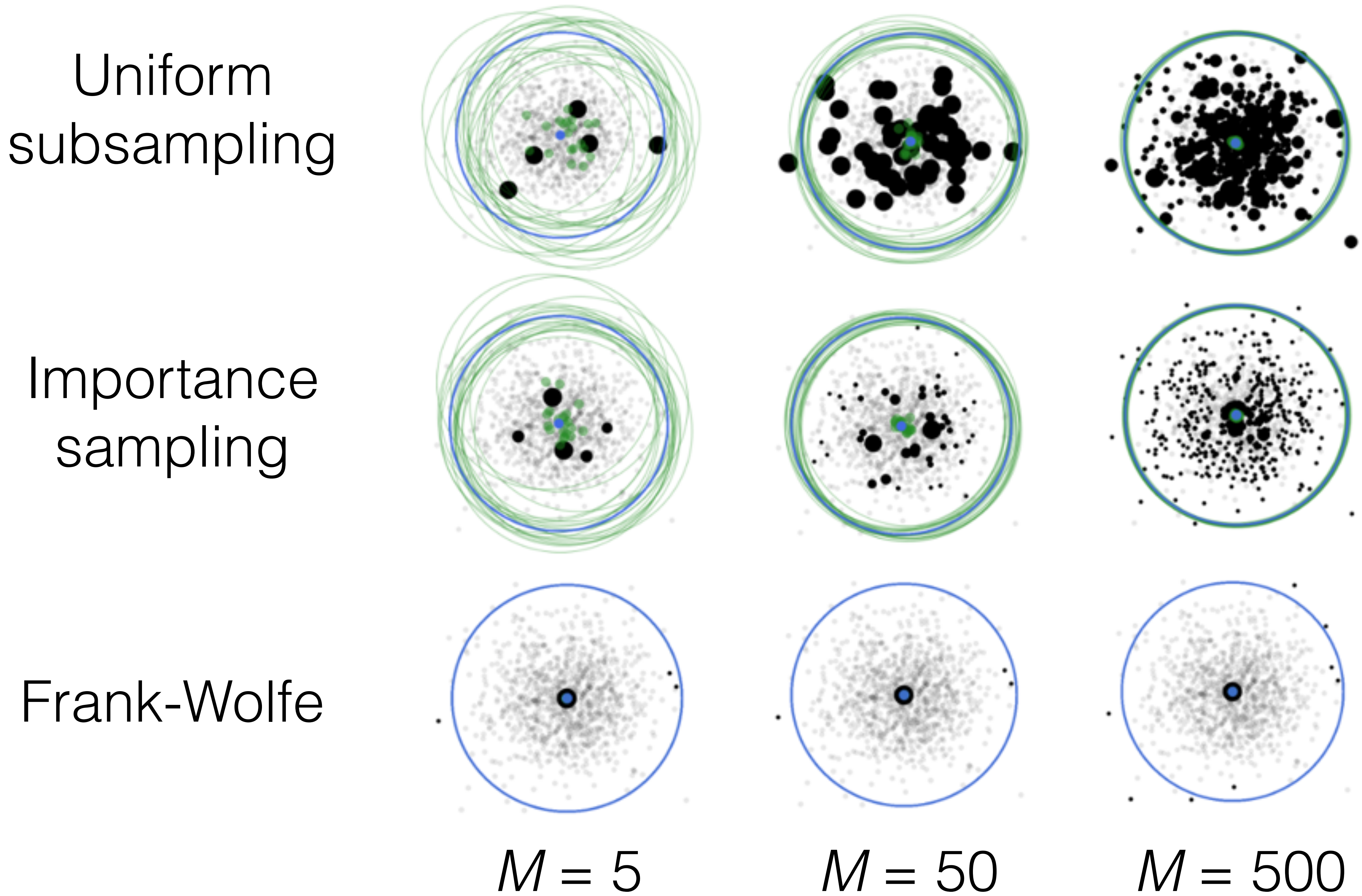

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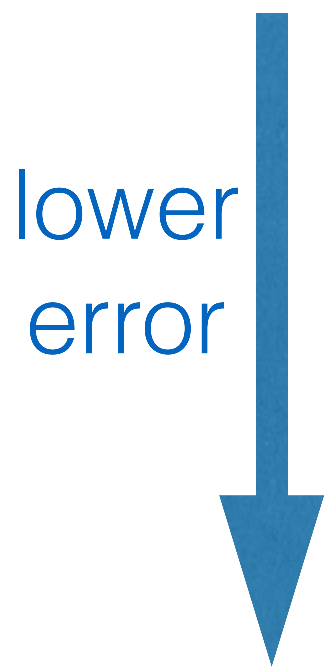
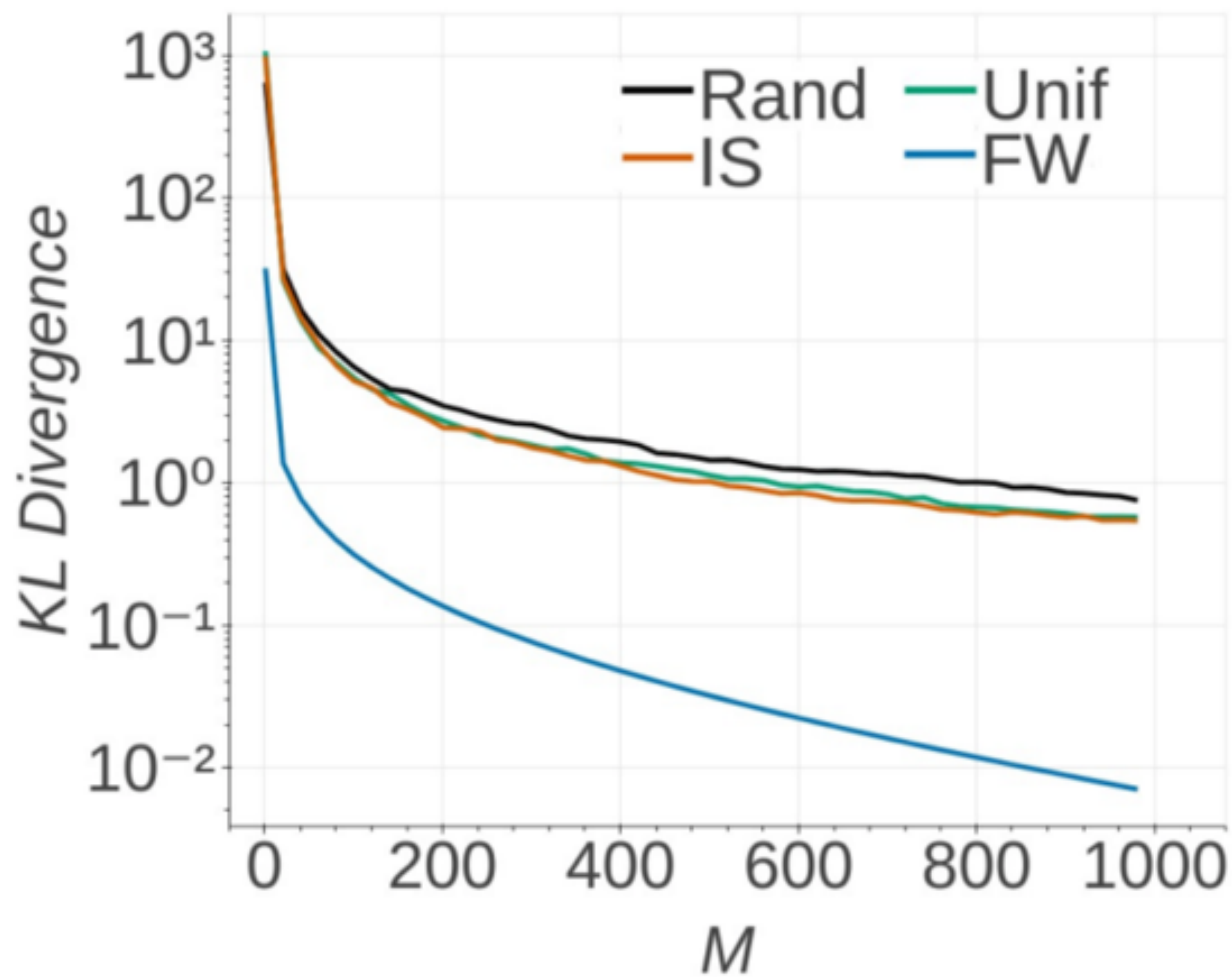
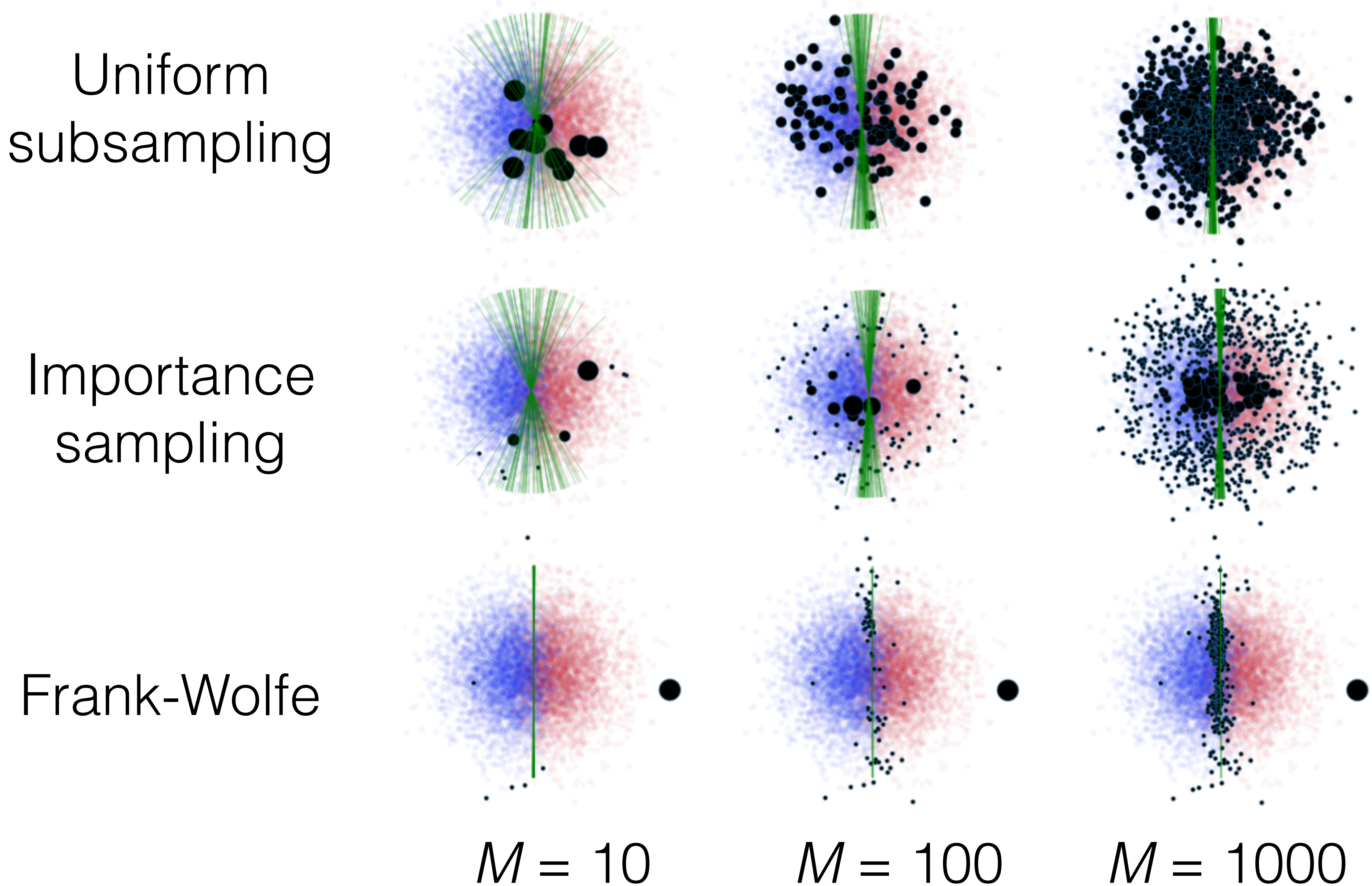




Logistic regression (simulated)

- 10K pts; general inference



Poisson regression (simulated)

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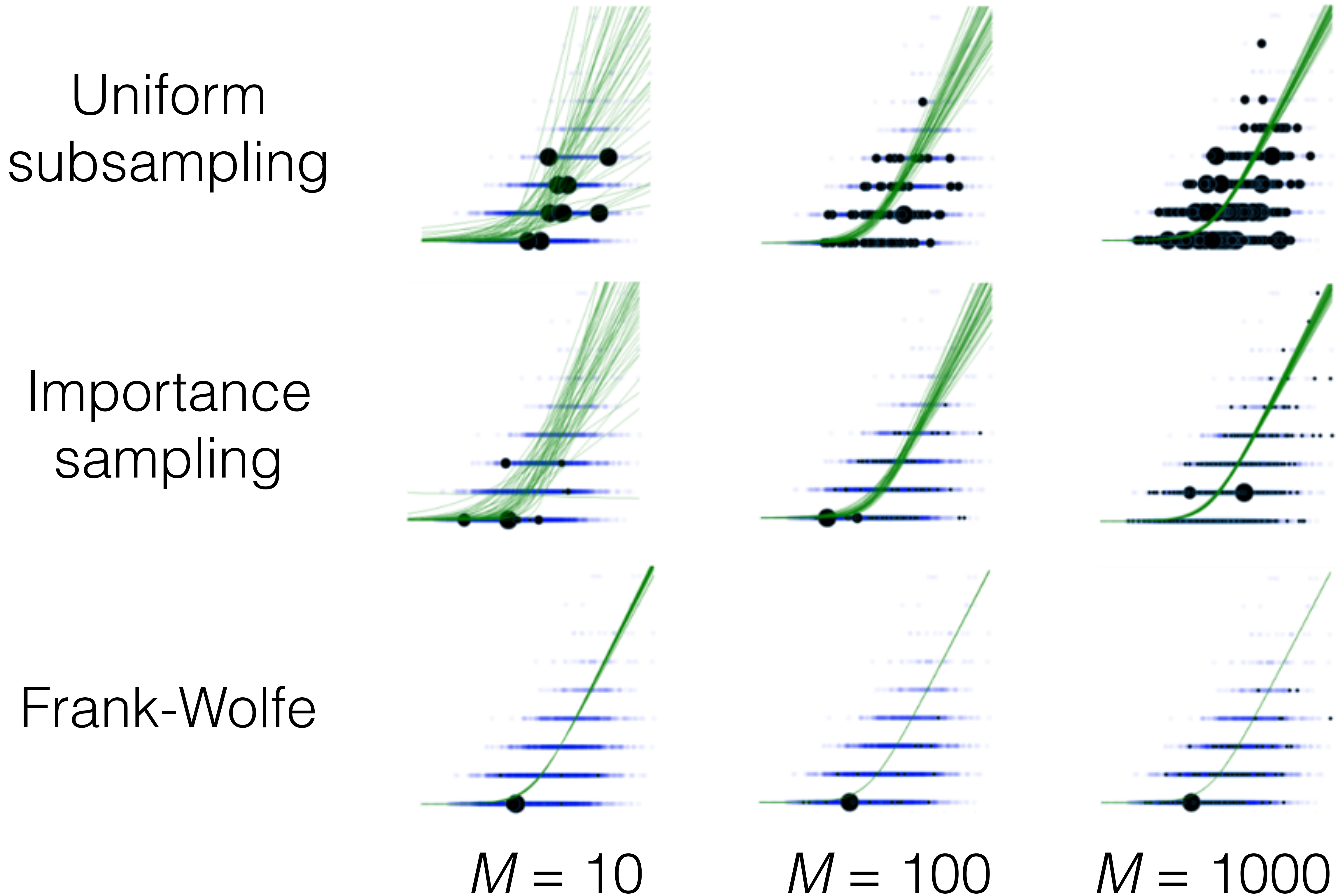
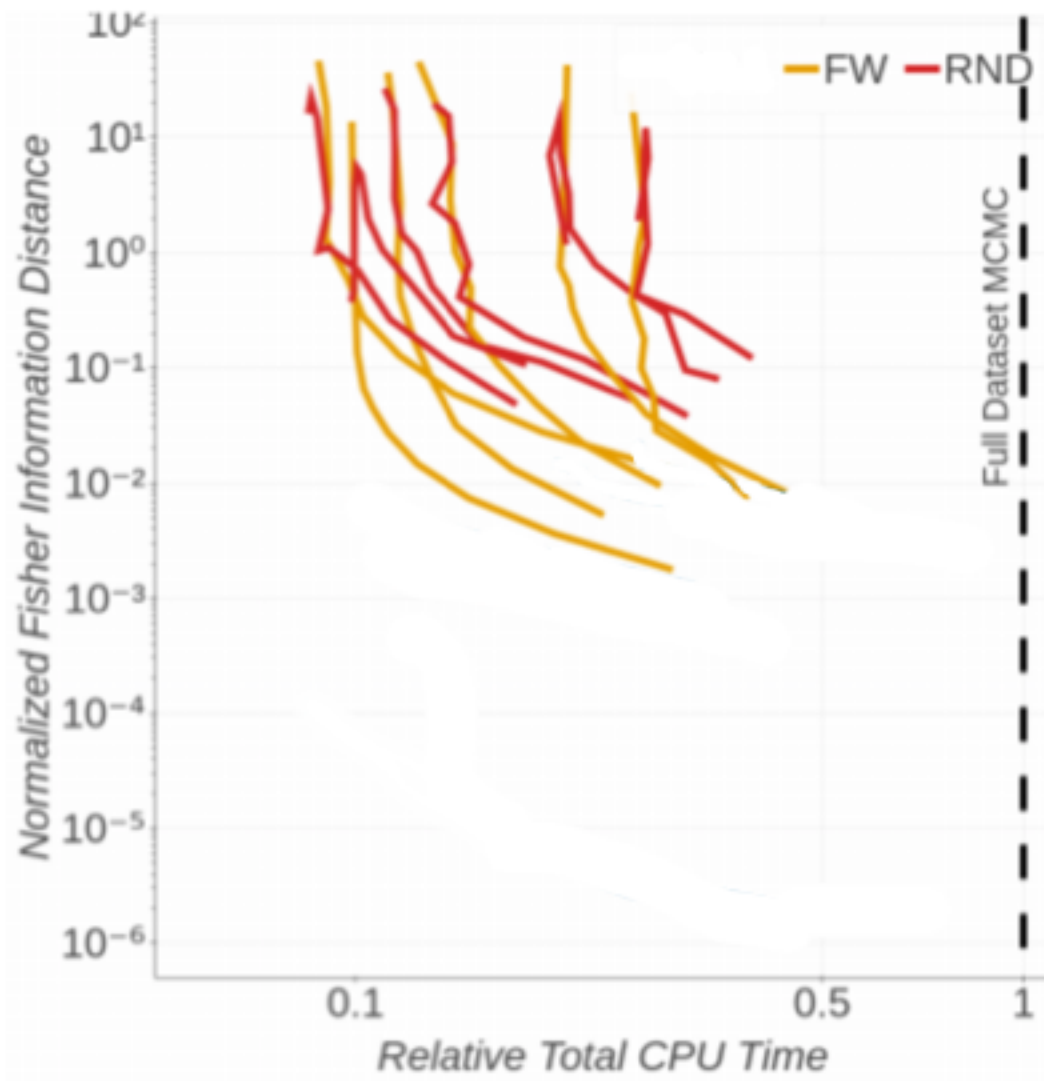


Real data experiments

lower error



less total time

Uniform subsampling

Frank Wolfe coresets

Data sets include:

- Phishing
- Chemical reactivity
- Bicycle trips
- Airport delays

[Campbell, Broderick 2017]

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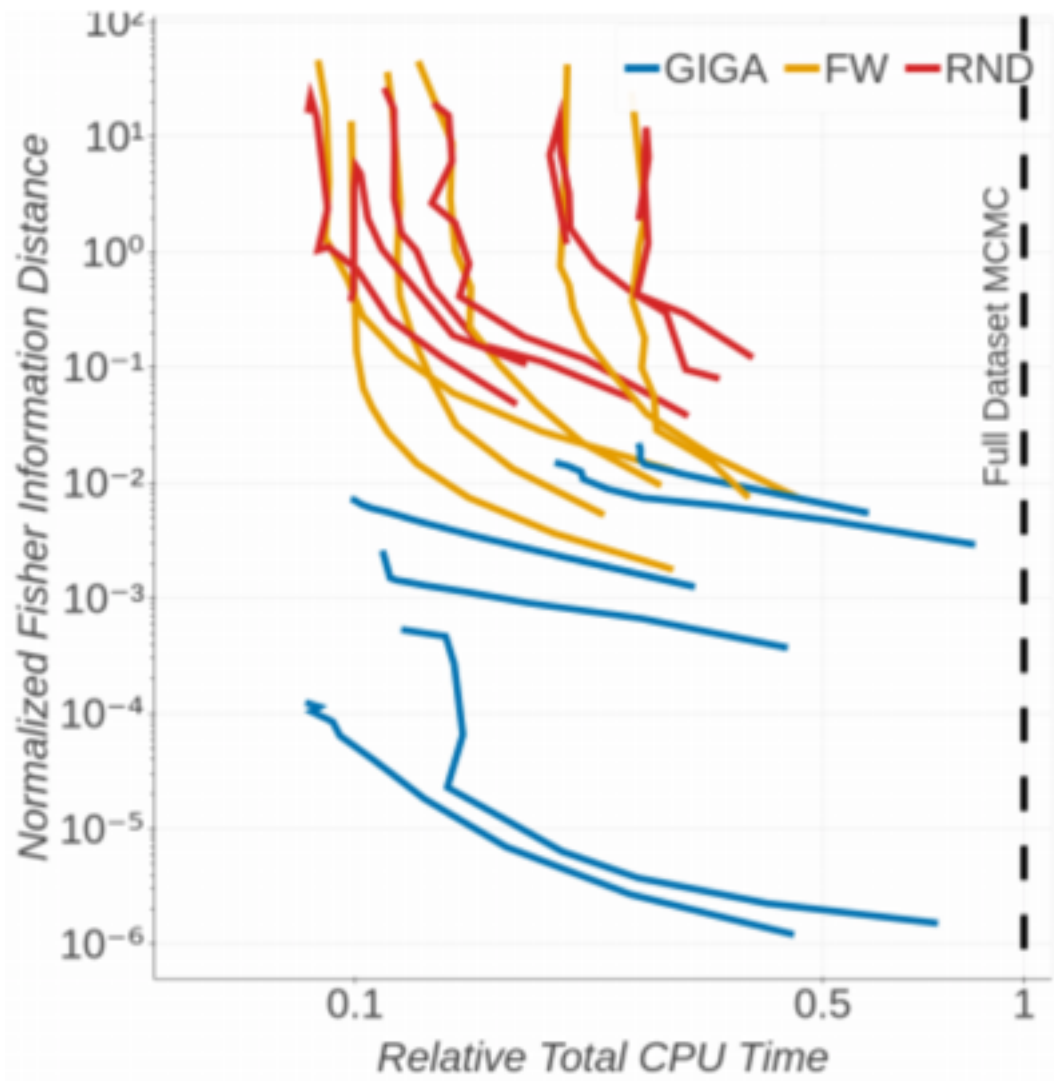

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GIGA coresets

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[Campbell, Broderick 2017, 2018]

Roadmap

- Approximate Bayes review
- The “core” of the data set
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- Our proposal: (polynomial) *approximate* sufficient statistics

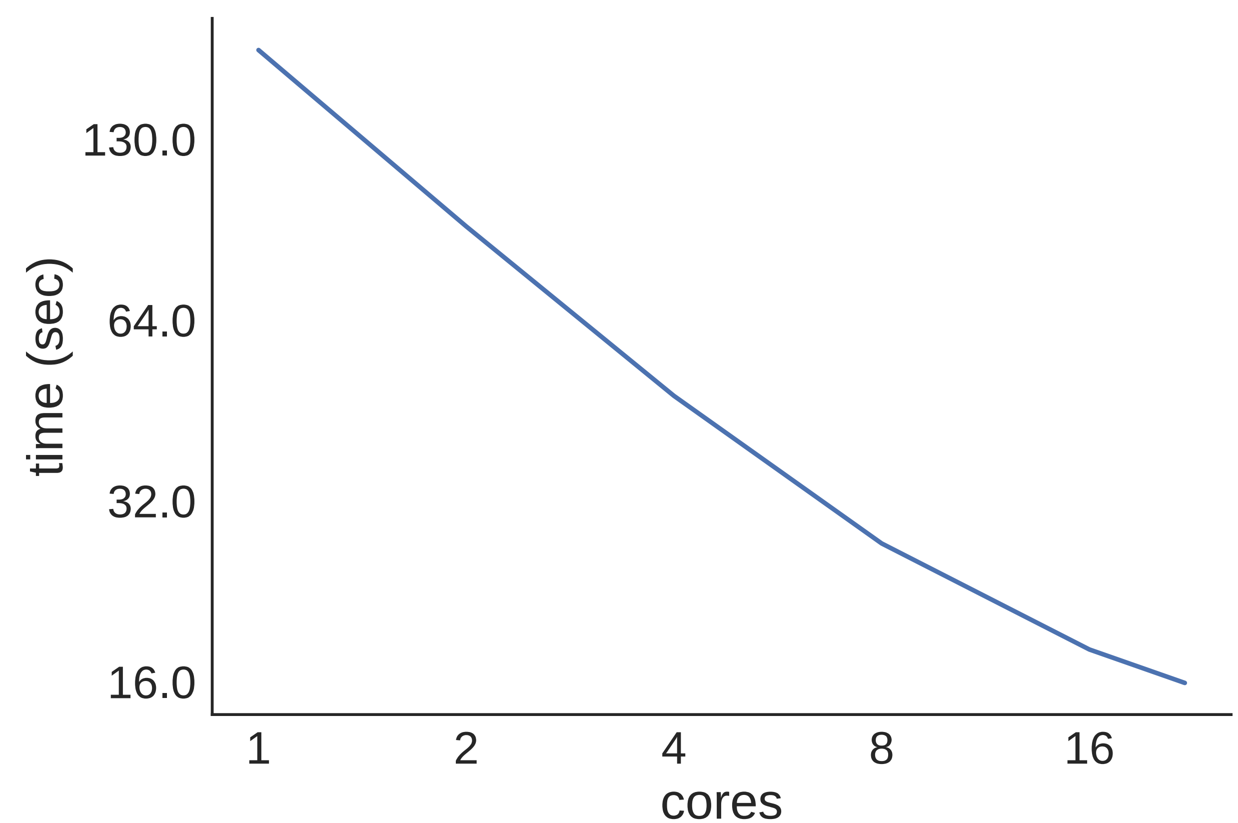
Data summarization

Criteo Labs > Algorithms > Criteo Releases its New Dataset

Criteo Releases its New Dataset

By: CriteoLabs / 31 Mar 2015

- 6M data points, 1000 features
- Streaming, distributed; minimal communication
- 22 cores, 16 sec
- Finite-data guarantees on Wasserstein distance to exact posterior



[Huggins, Adams, Broderick 2017]

Conclusions

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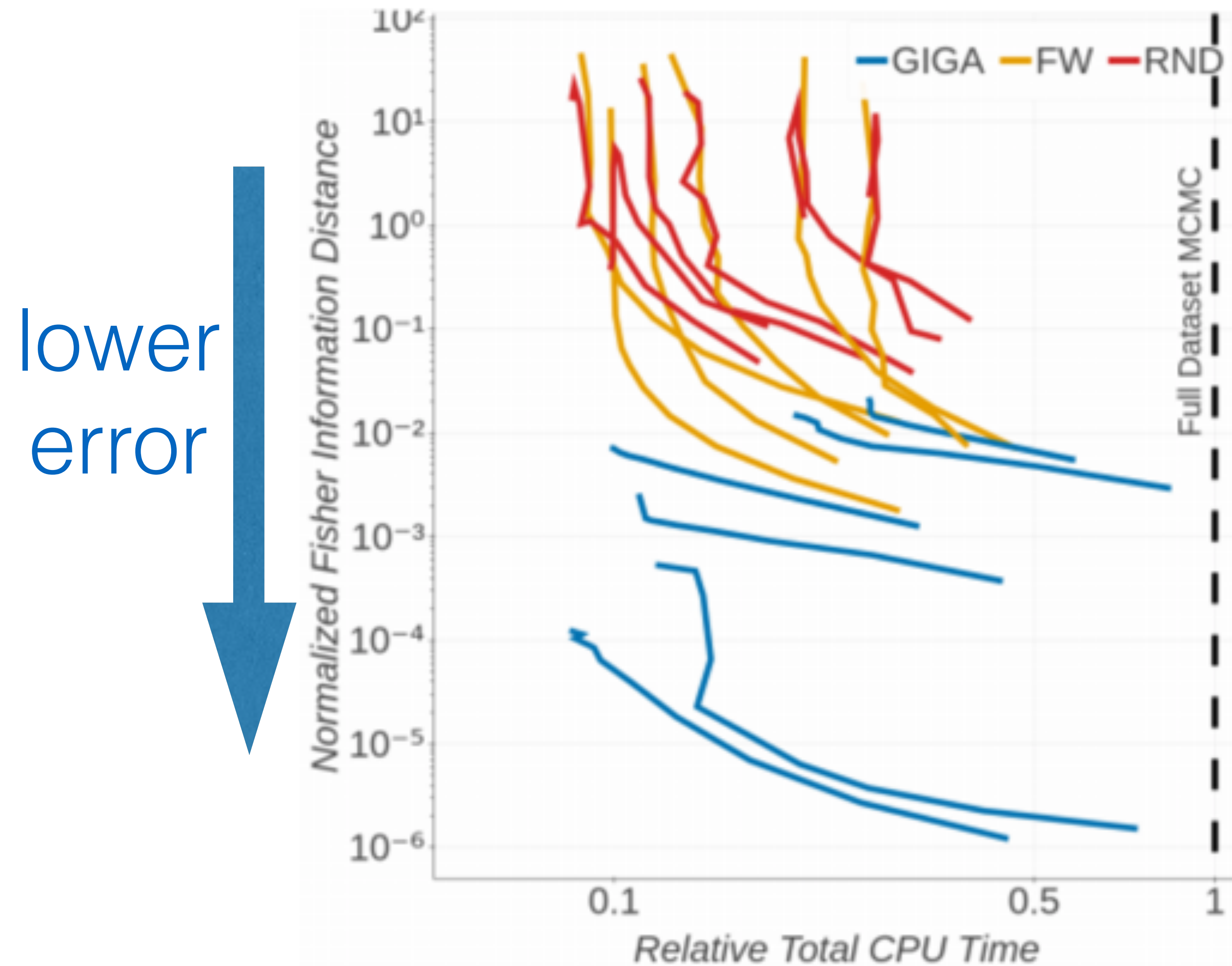

[Campbell, Broderick 2018]

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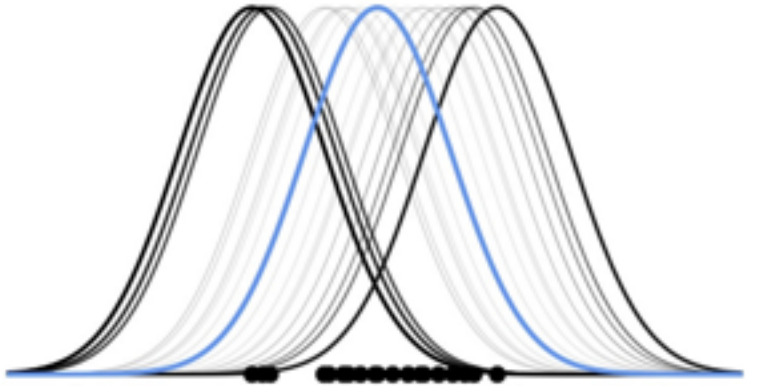
J. Herzog. 3 June 2016, 17:17:30. Obtained from: https://commons.wikimedia.org/wiki/File:Airbus_A350-941_F-WWCF_MSN002_ILA_Berlin_2016_17.jpg (Creative Commons Attribution 4.0 International License)

Practicalities



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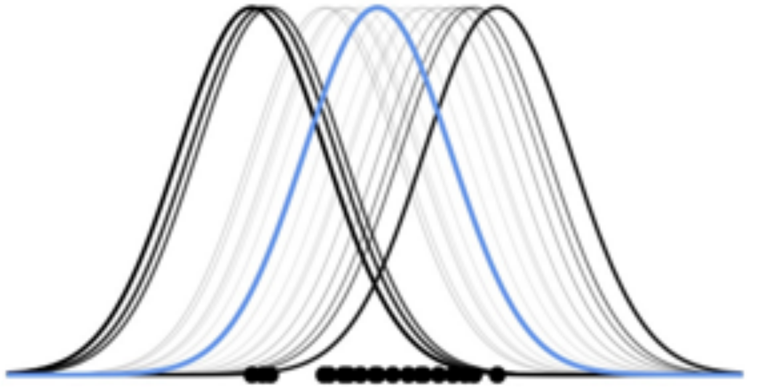

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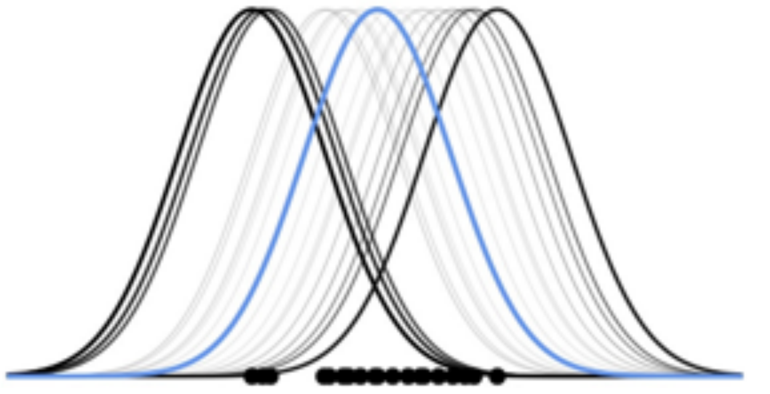

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  - E.g. (weighted) Fisher information distance

$$\|\mathcal{L}(w) - \mathcal{L}\|^2_{\hat{\pi},F} := \mathbb{E}_{\hat{\pi}}\left[\|\nabla\mathcal{L}(\theta) - \nabla\mathcal{L}(w,\theta)\|_2^2\right]$$

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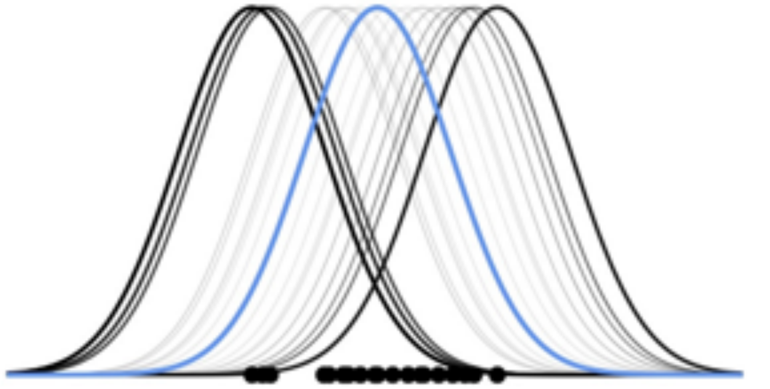

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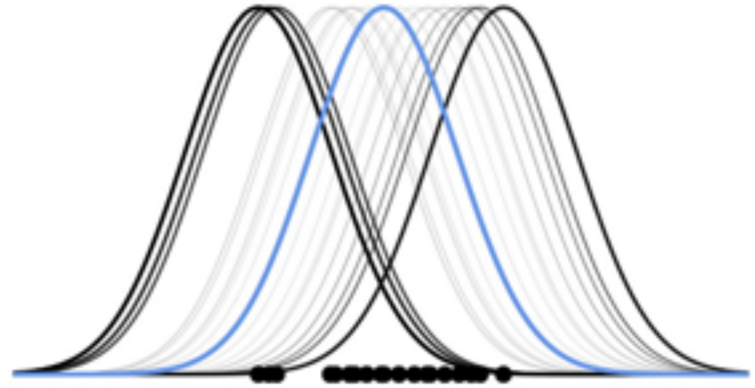

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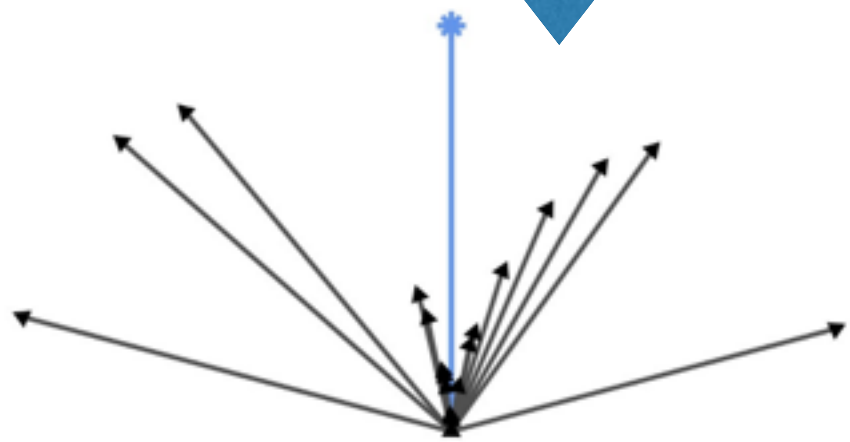

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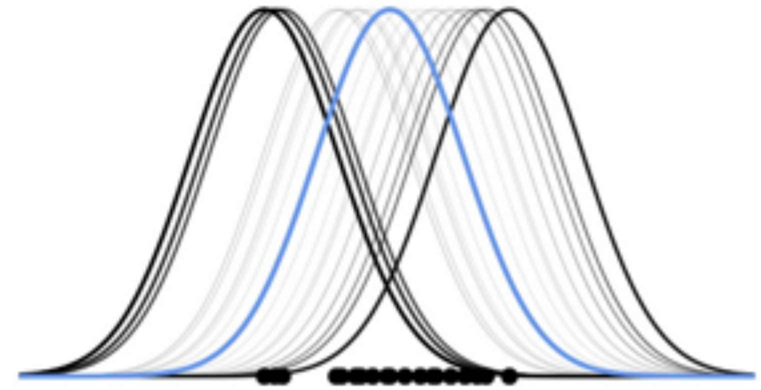

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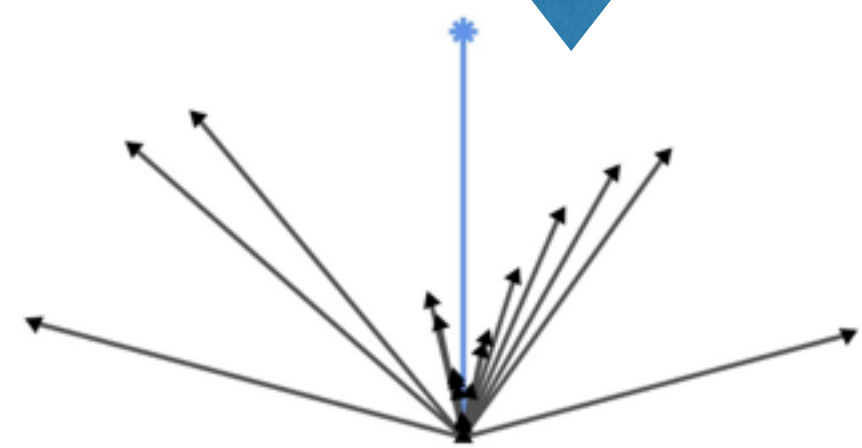

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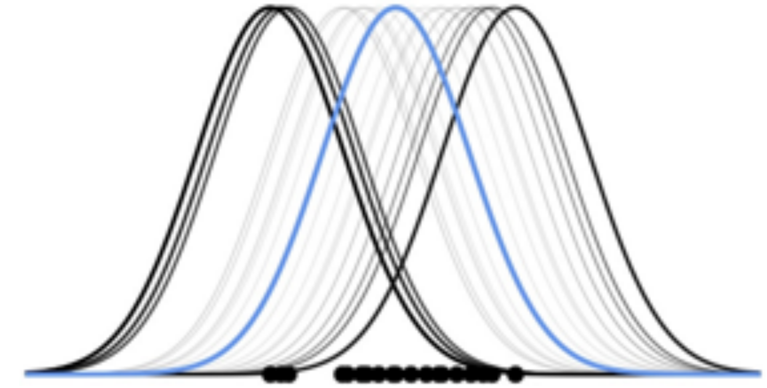

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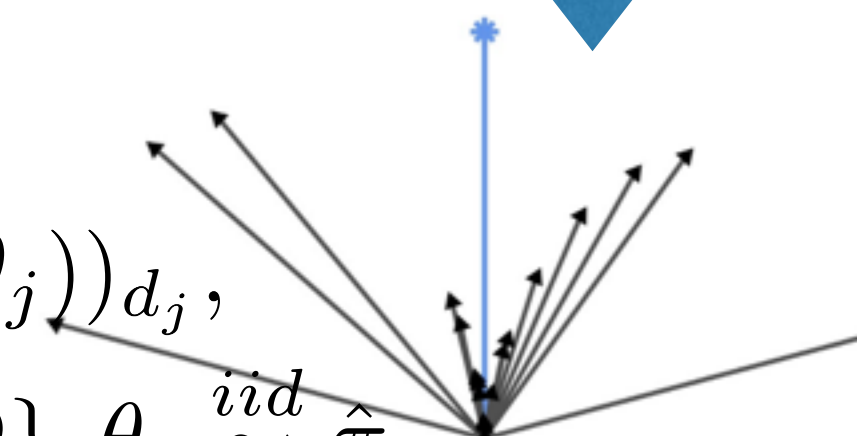
$$\langle\mathcal{L}_n, \mathcal{L}_m\rangle_{\hat{\pi},F} \approx \frac{D}{J}\sum_{j=1}^{J}(\nabla\mathcal{L}_n(\theta_j))_{d_j}(\nabla\mathcal{L}_m(\theta_j))_{d_j},$$

$$d_j \overset{iid}{\sim} \text{Unif}\{1,\ldots,D\}, \theta_j \overset{iid}{\sim} \hat{\pi}$$



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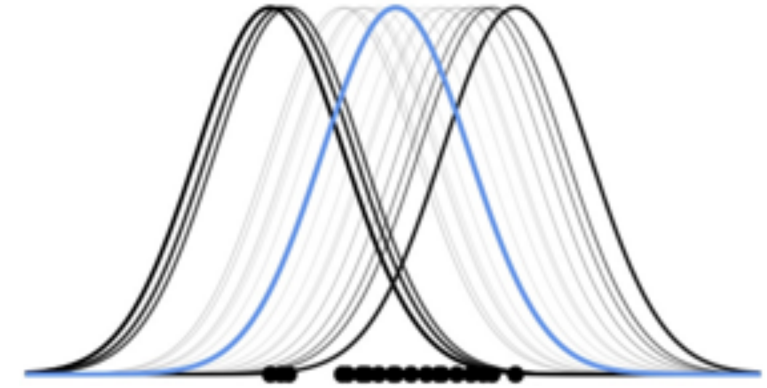

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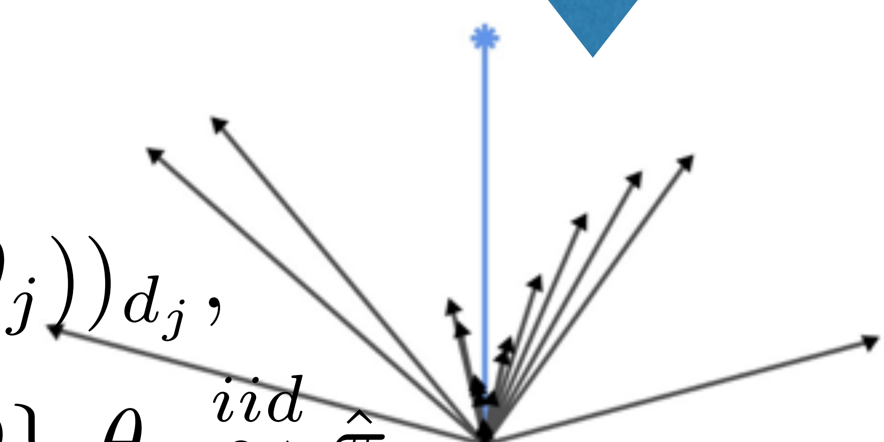
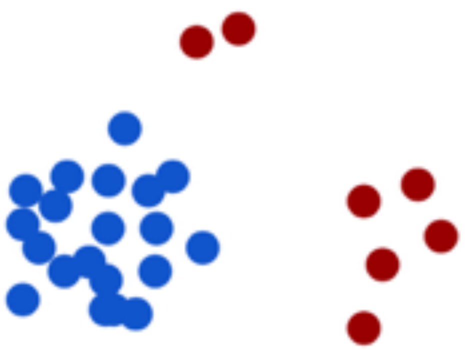
$$\langle\mathcal{L}_n, \mathcal{L}_m\rangle_{\hat{\pi},F} \approx \frac{D}{J}\sum_{j=1}^{J}(\nabla\mathcal{L}_n(\theta_j))_{d_j}(\nabla\mathcal{L}_m(\theta_j))_{d_j},$$

$$d_j \overset{iid}{\sim} \text{Unif}\{1,\ldots,D\}, \theta_j \overset{iid}{\sim} \hat{\pi}$$



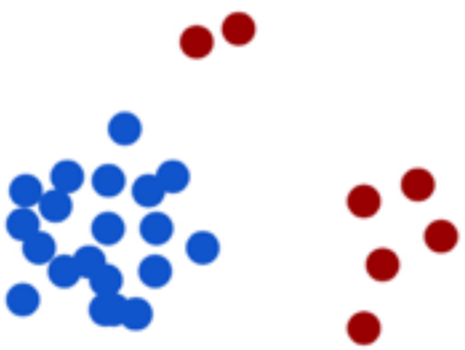
Thm sketch (CB). With high probability and large enough J, a good coreset after random feat. proj. is a good coreset for $(\mathcal{L}_n)_{n=1}^N$

Full pipeline



N
dataset size

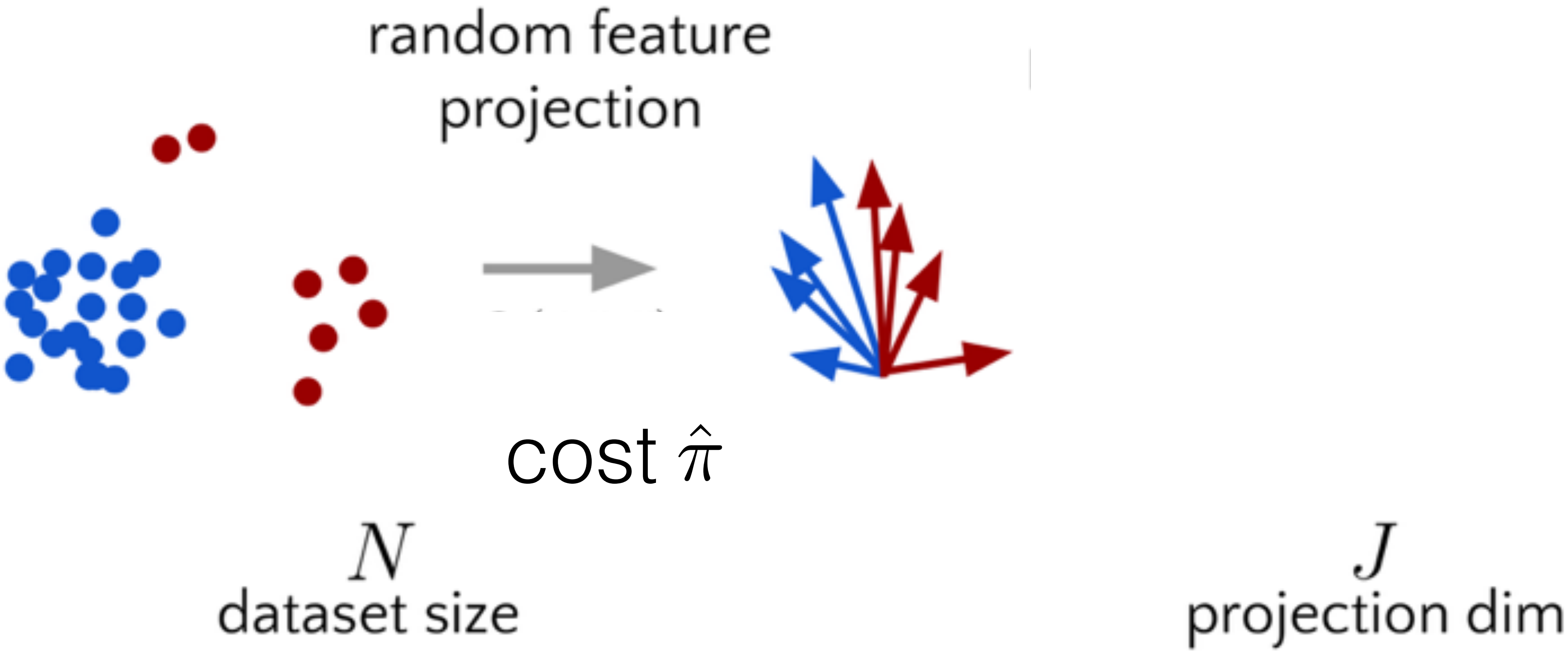
Full pipeline



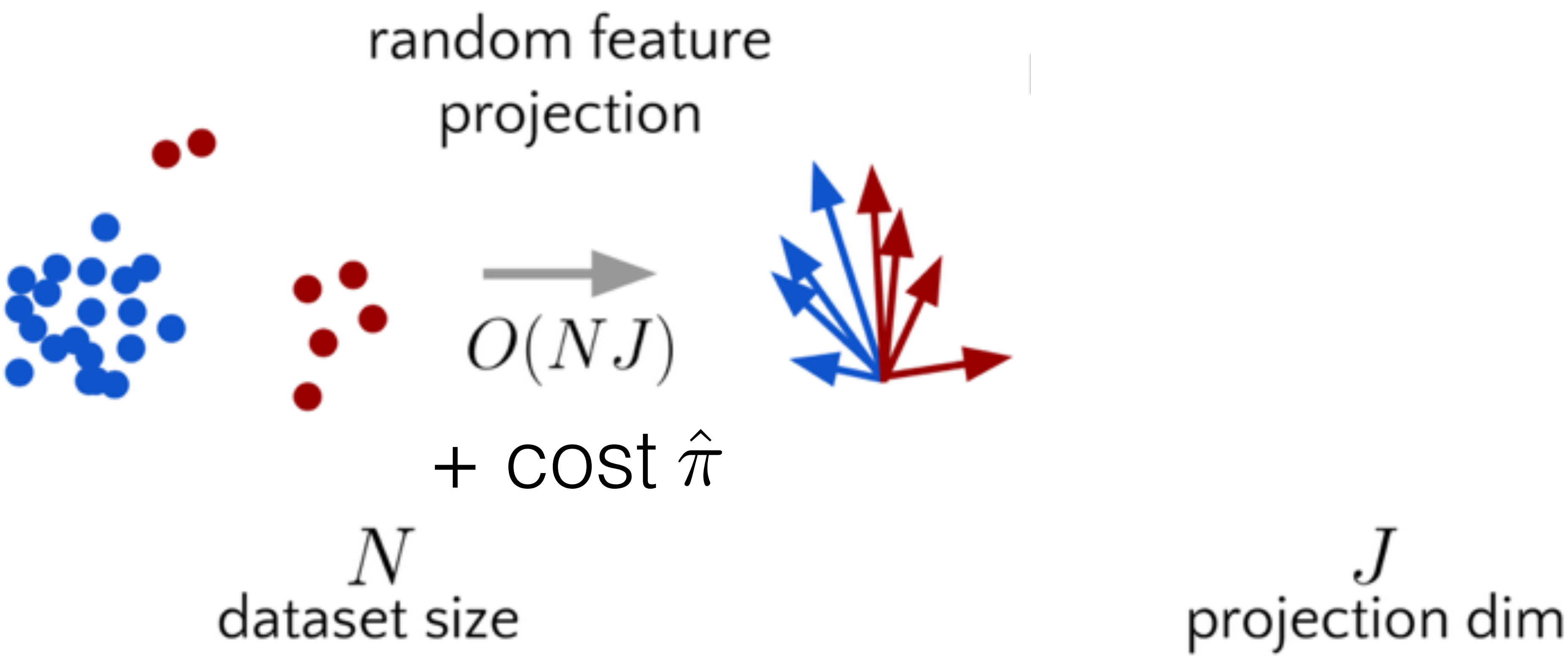
cost $\hat{\pi}$

N
dataset size

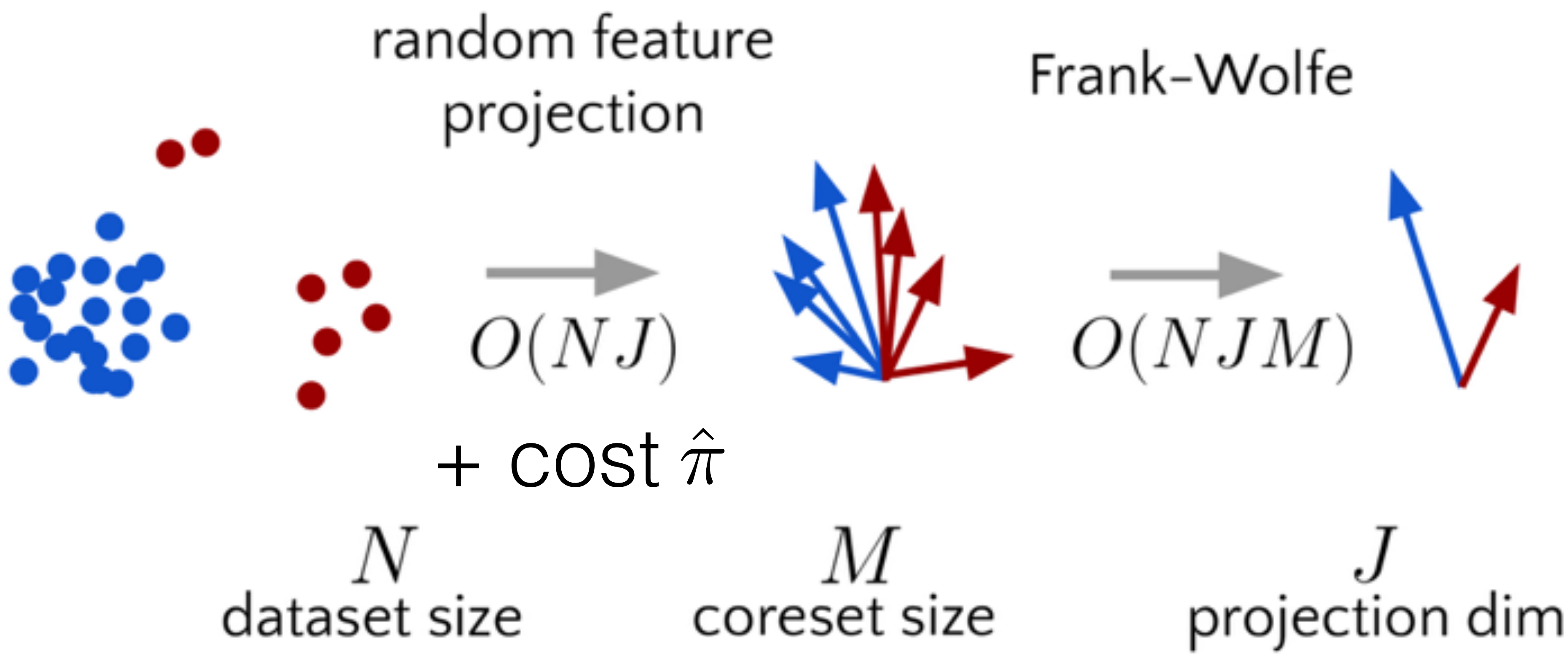
Full pipeline



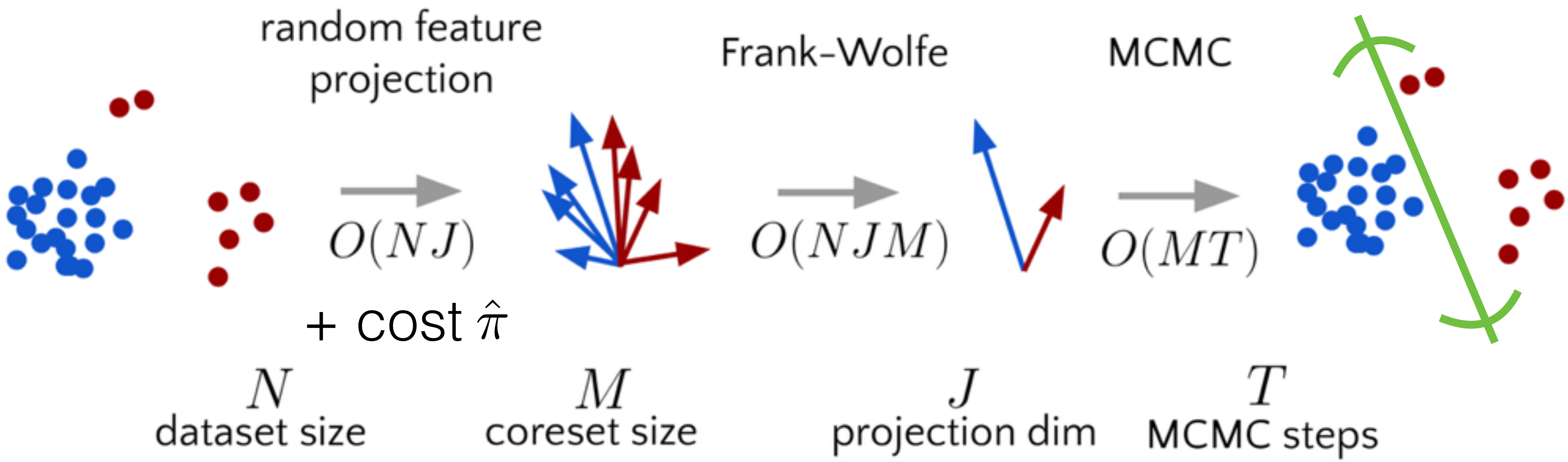
Full pipeline



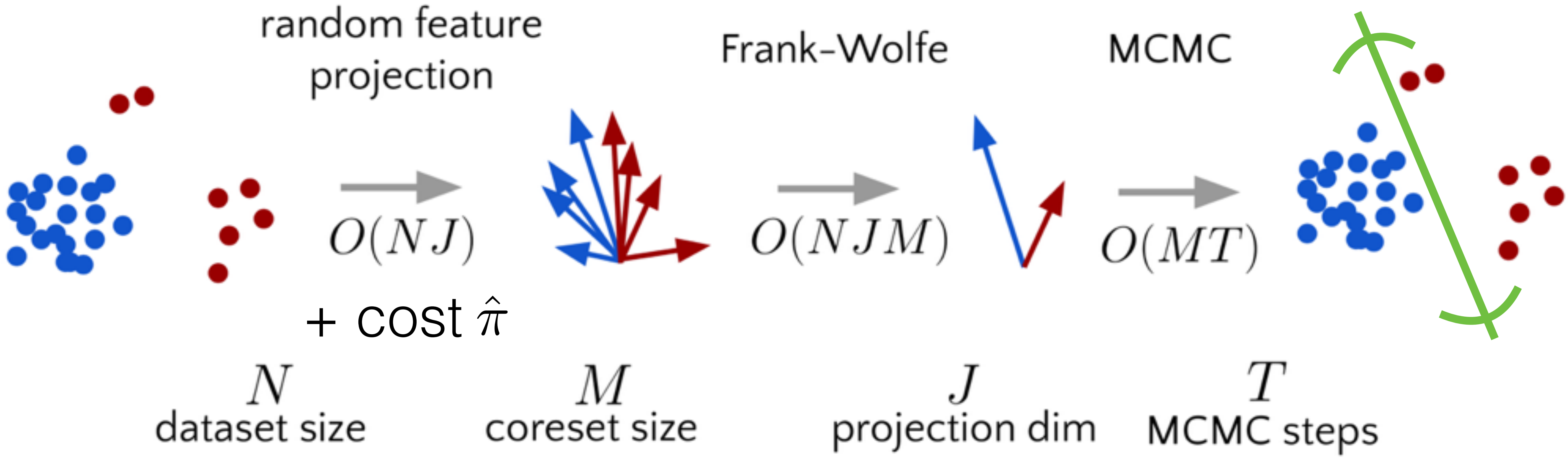
Full pipeline



Full pipeline

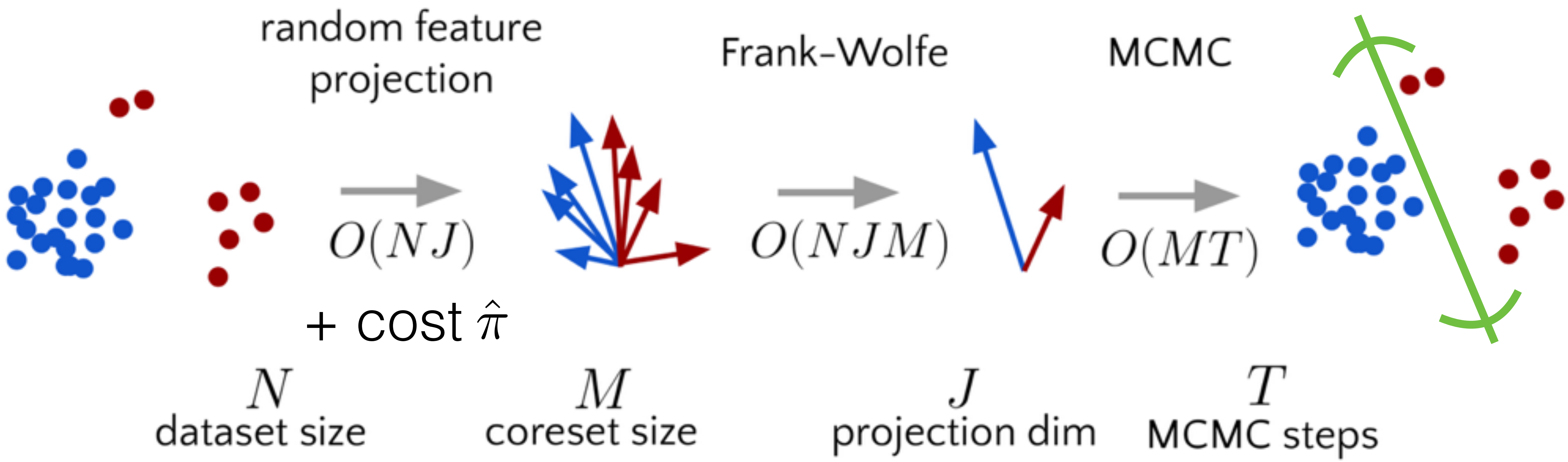


Full pipeline



- vs. $O(NT)$

Full pipeline



- vs. $O(NT)$
- Can make streaming, distributed