Variational Bayes and beyond: Bayesian inference for big data

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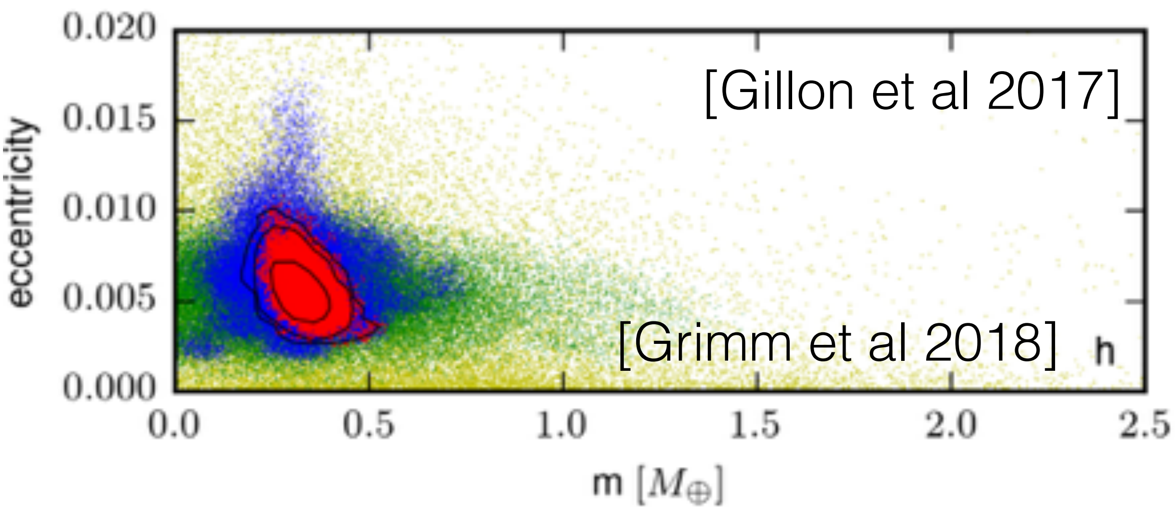
MIT

http://www.tamarabroderick.com/tutorial_2018_lugano.html

Bayesian inference

Bayesian inference



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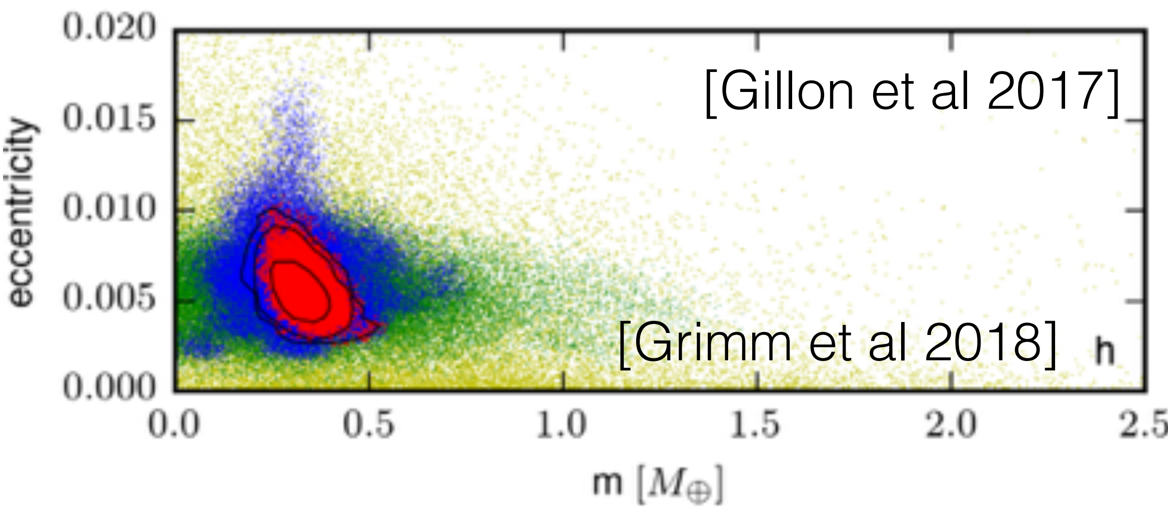

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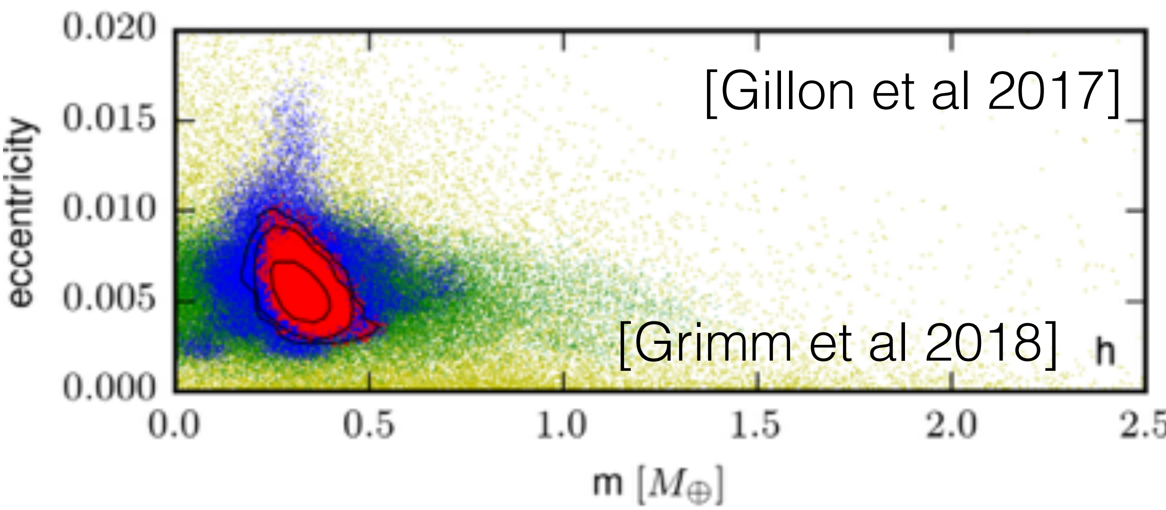

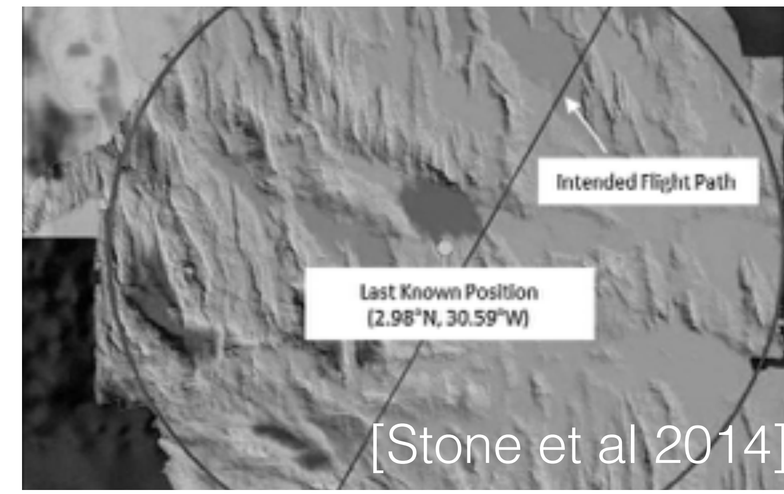

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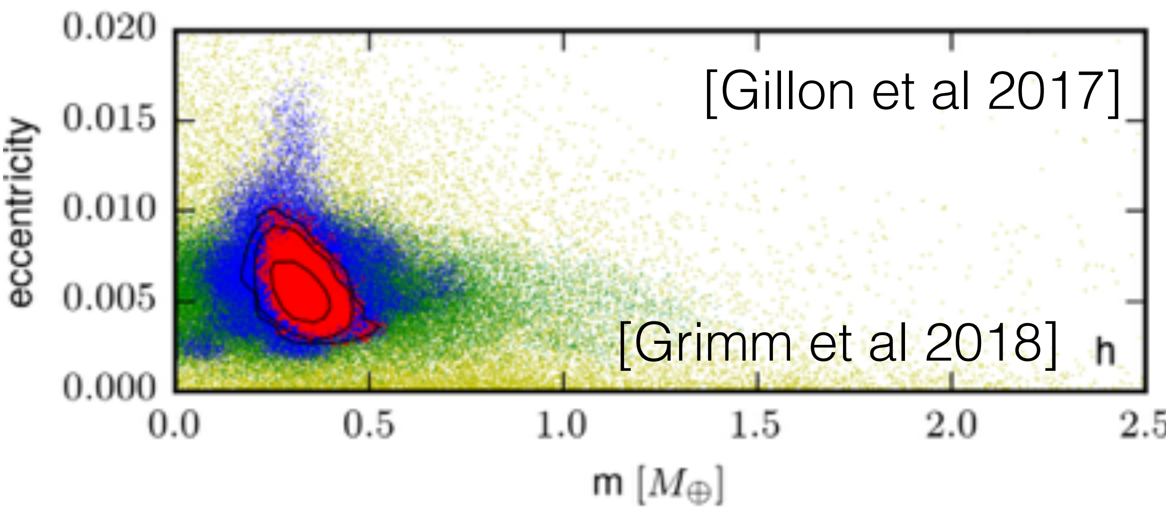

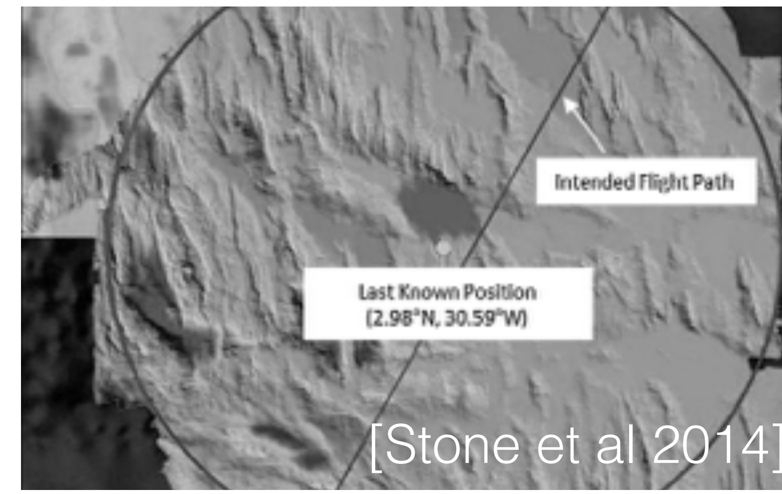

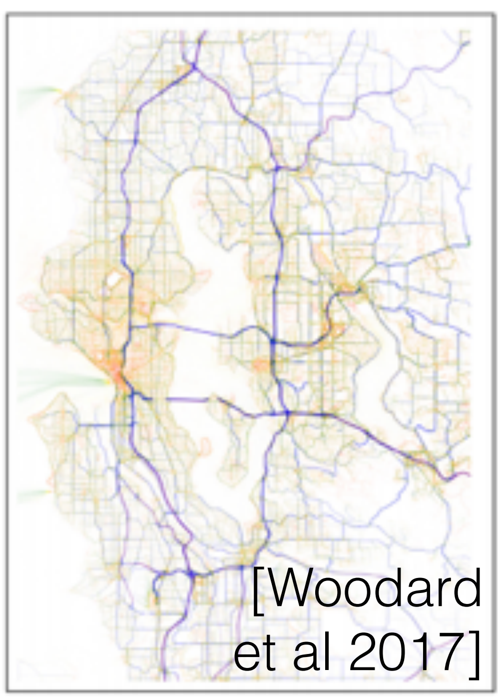

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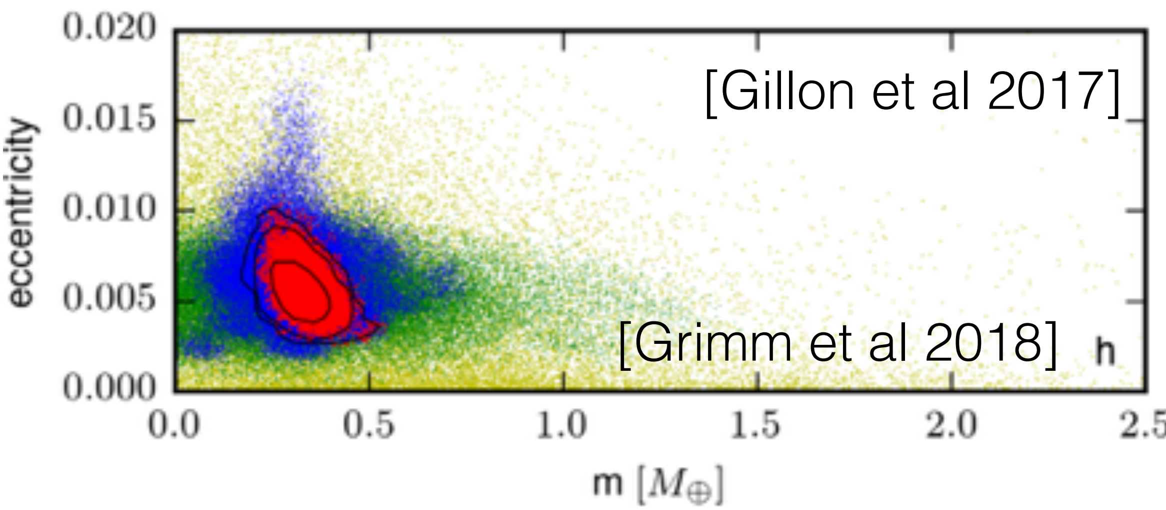

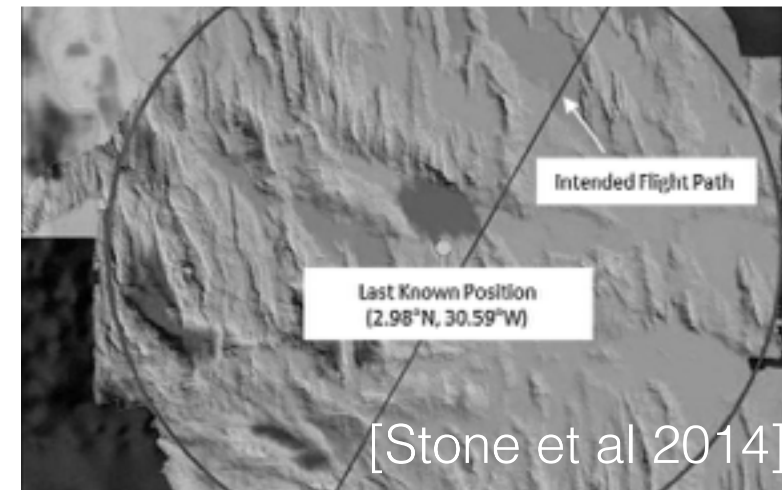

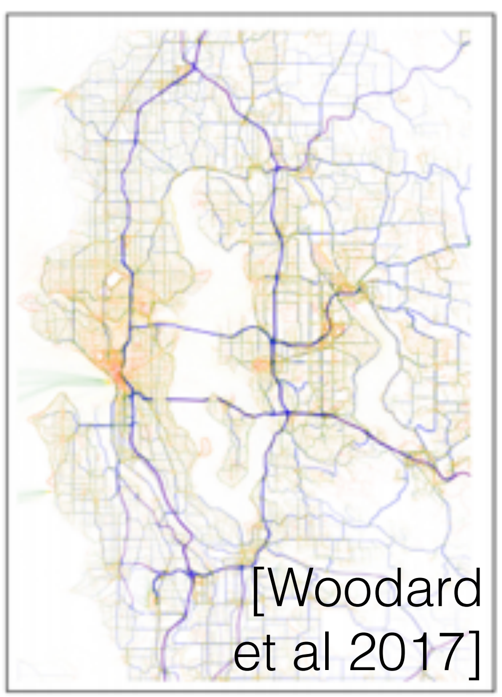

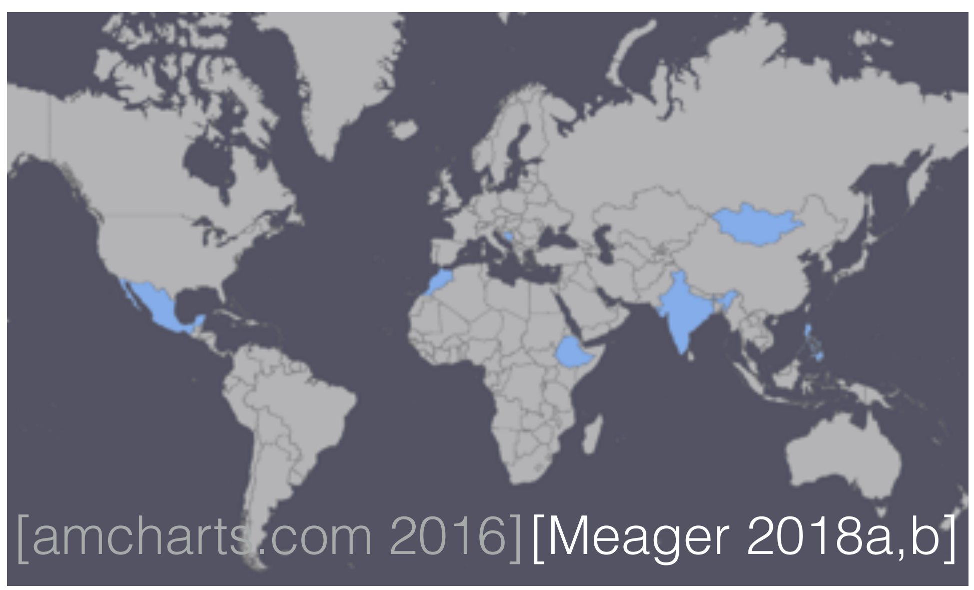

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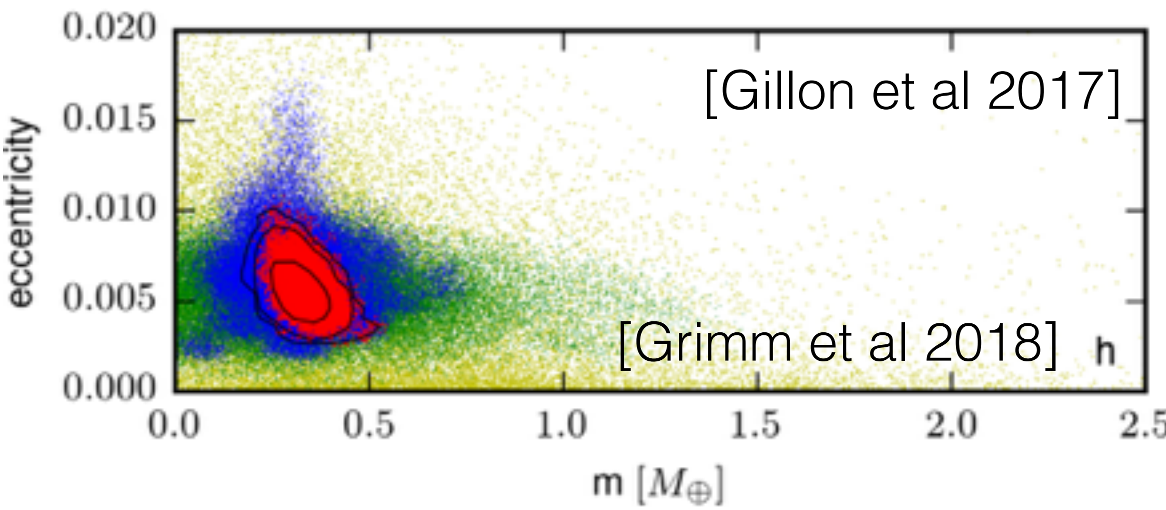

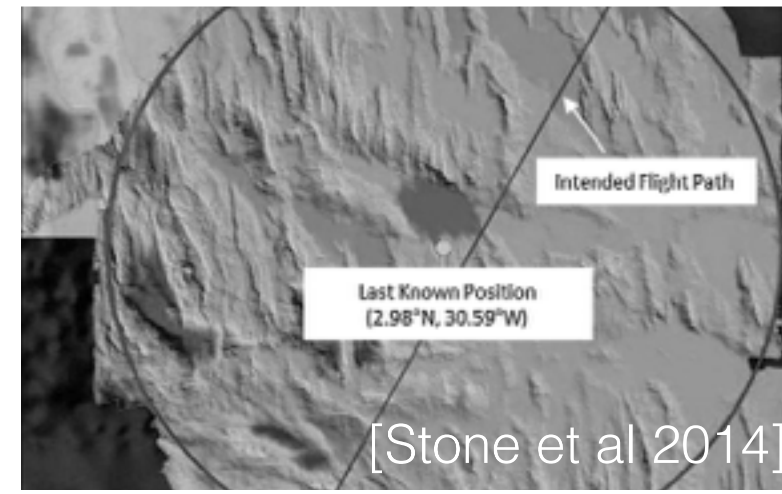

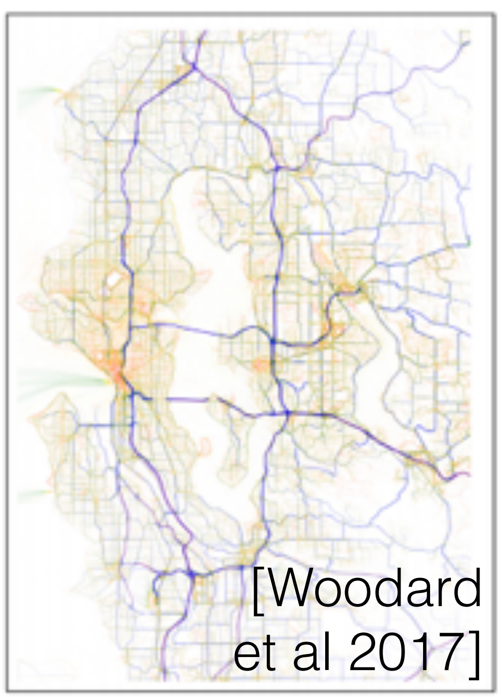

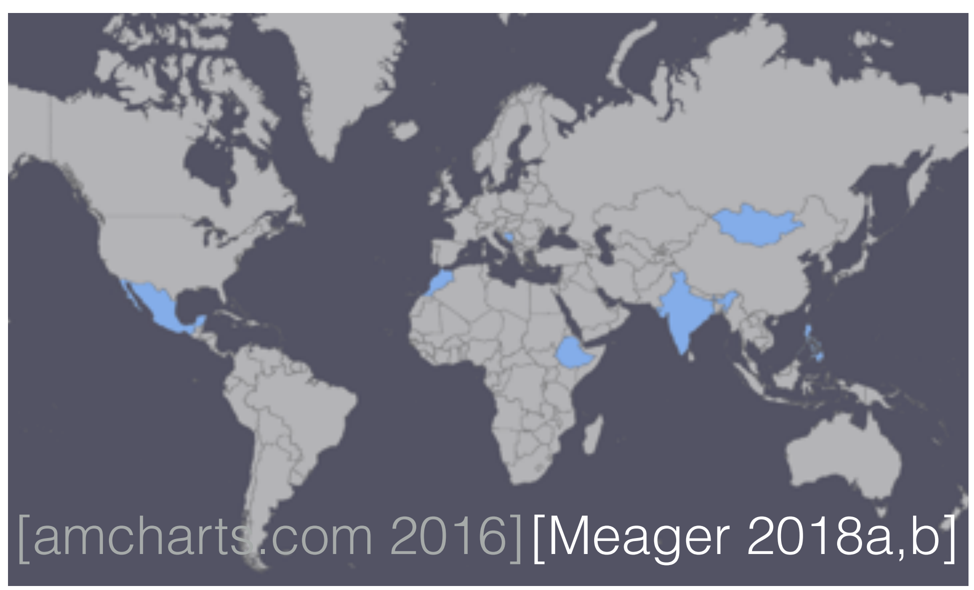

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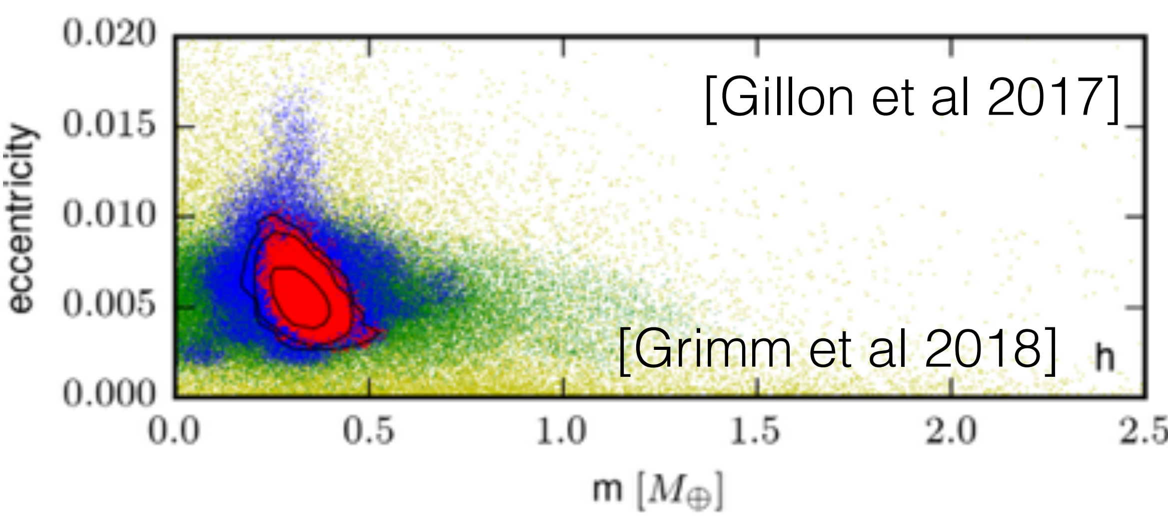
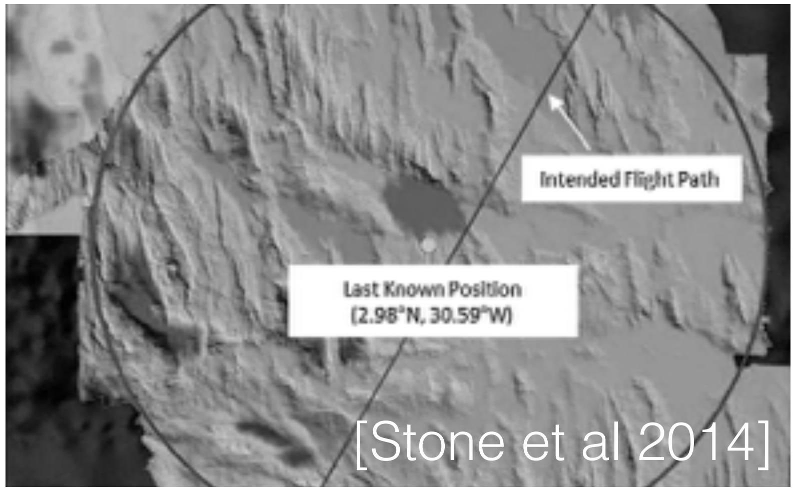
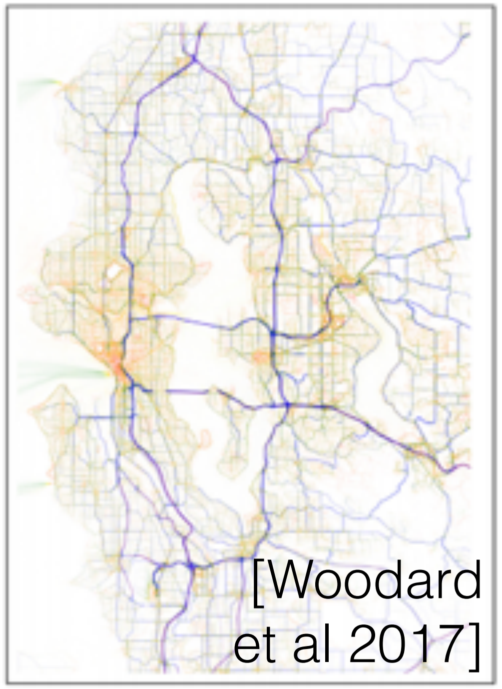
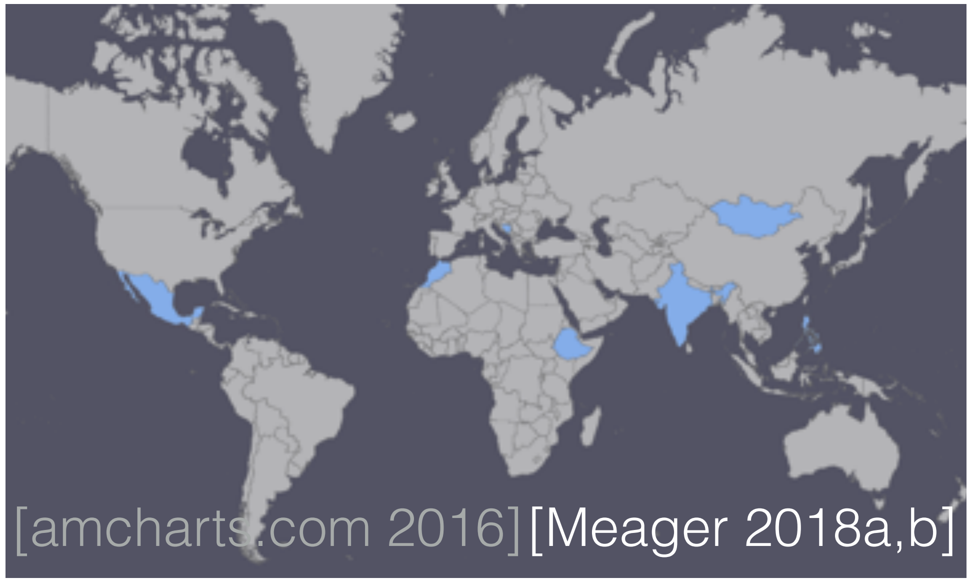


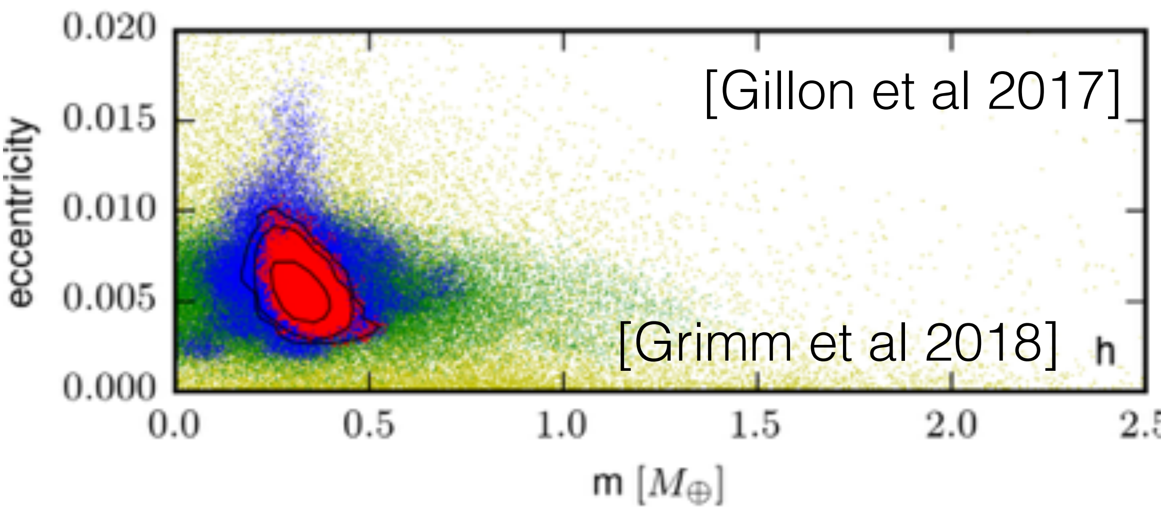


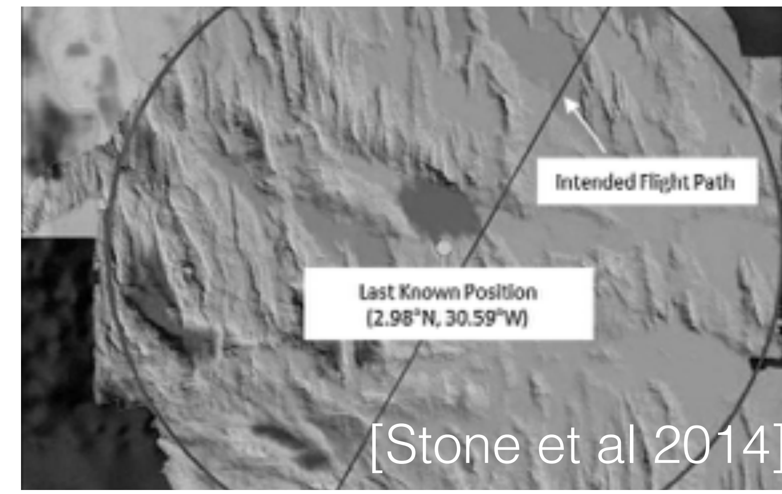


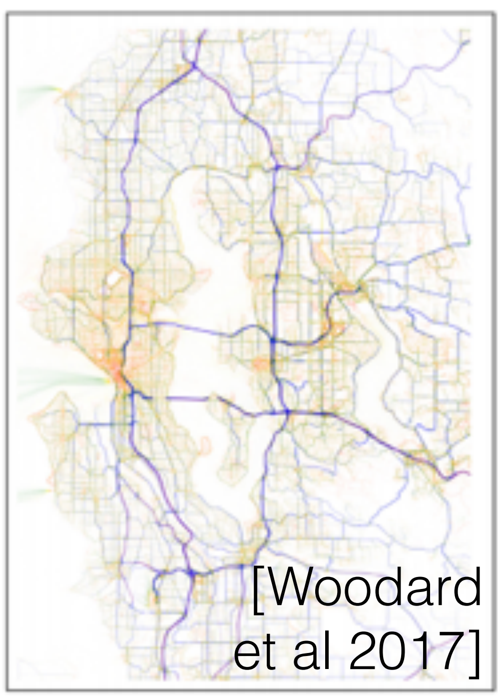


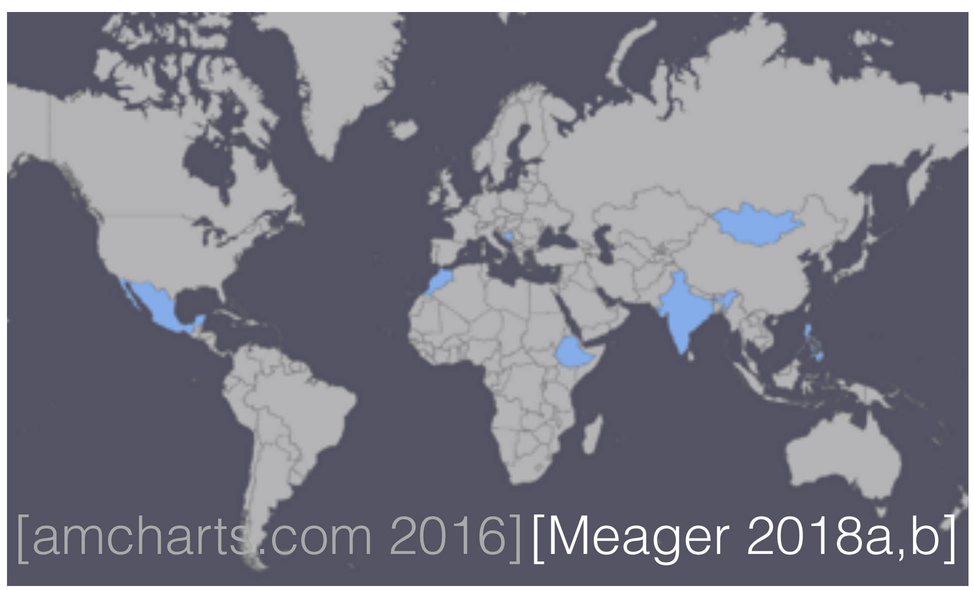
Bayesian inference

- Analysis goals: Point estimates, coherent uncertainties
    - Interpretable, complex, modular; expert information









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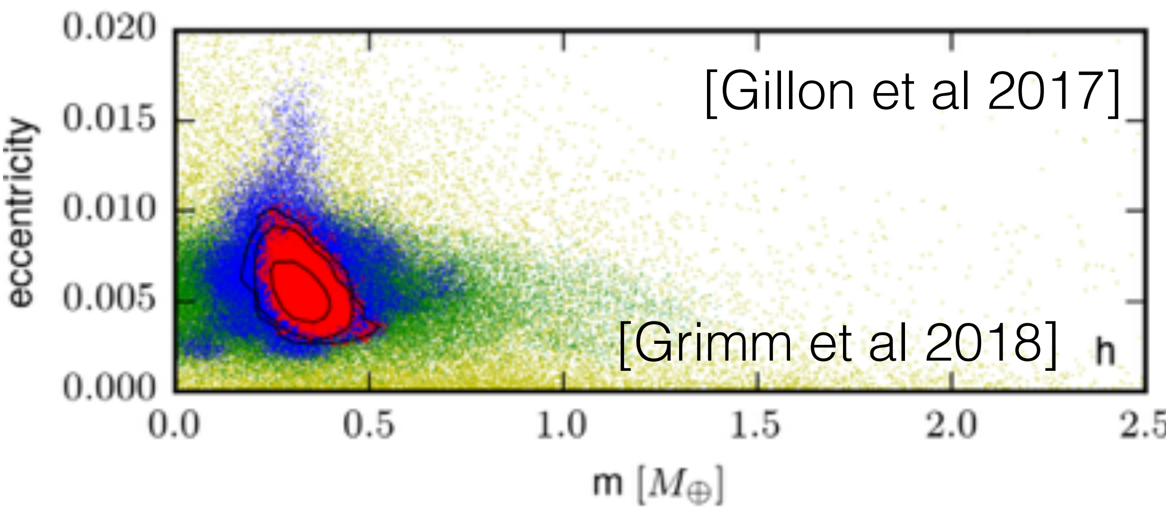

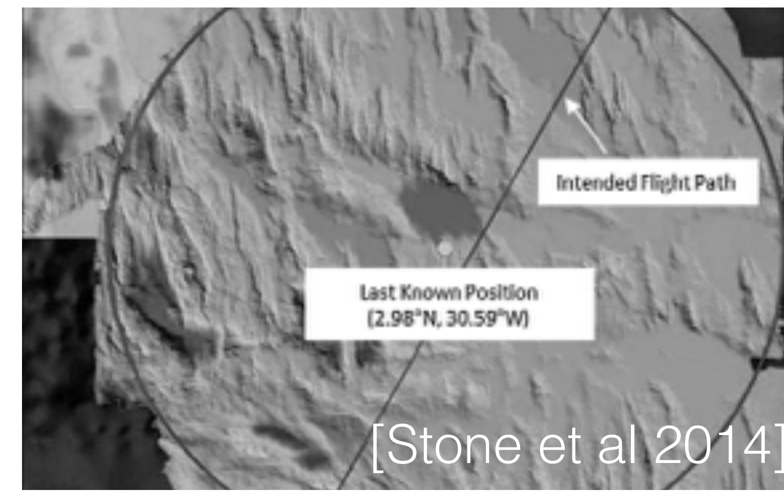

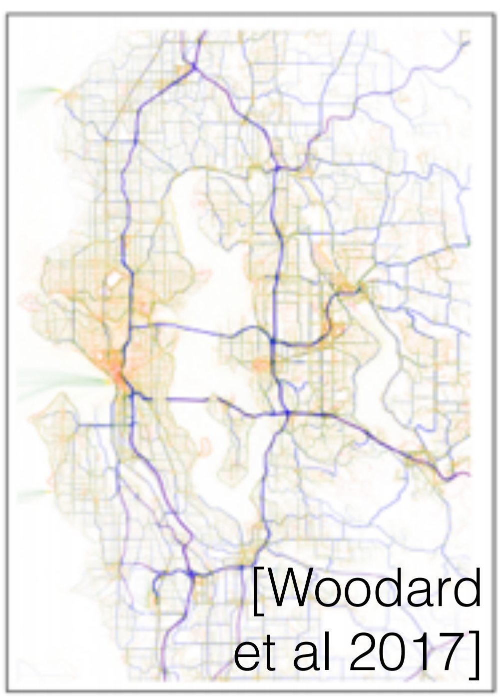

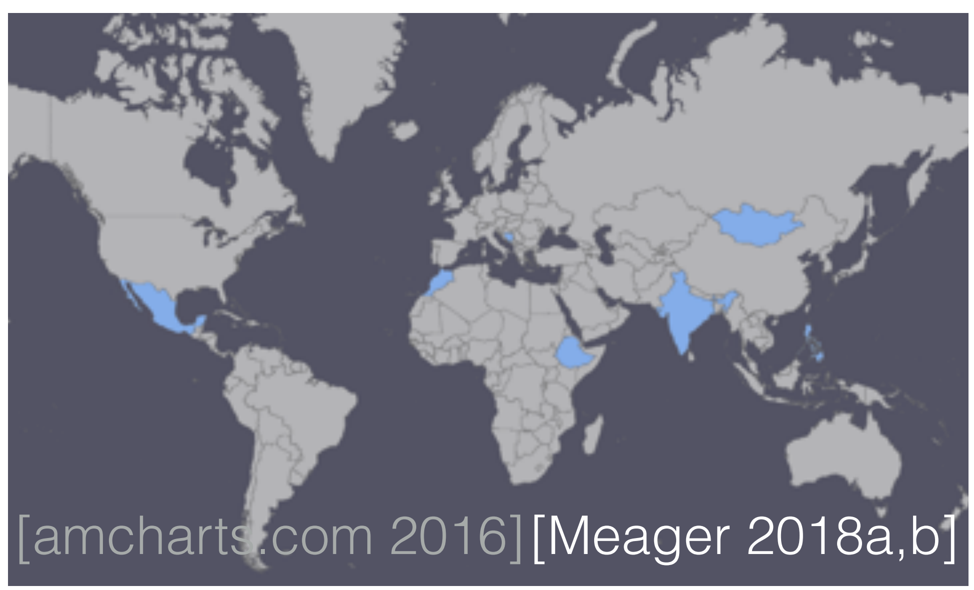

[mc-stan.org]

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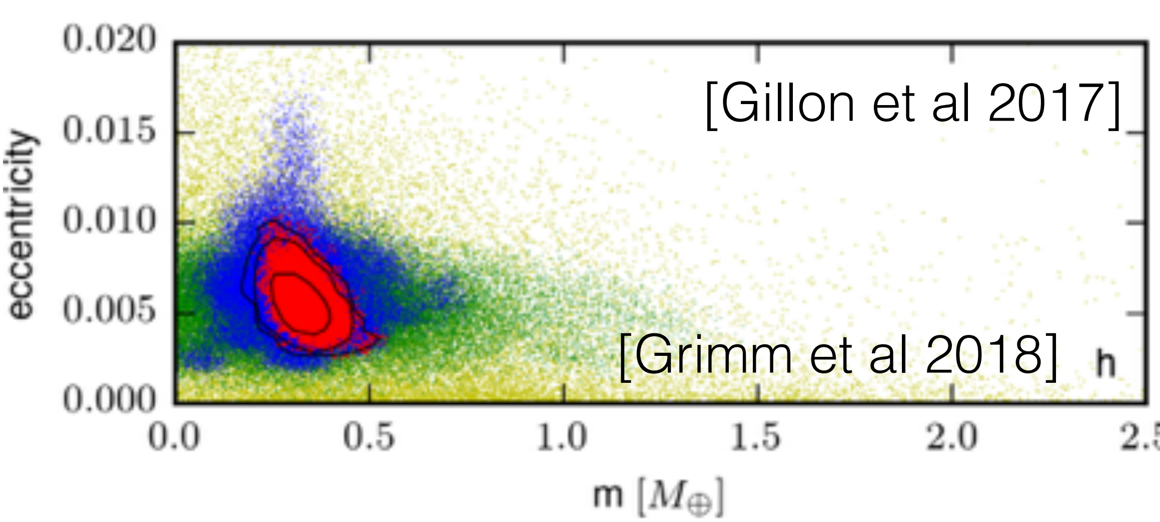

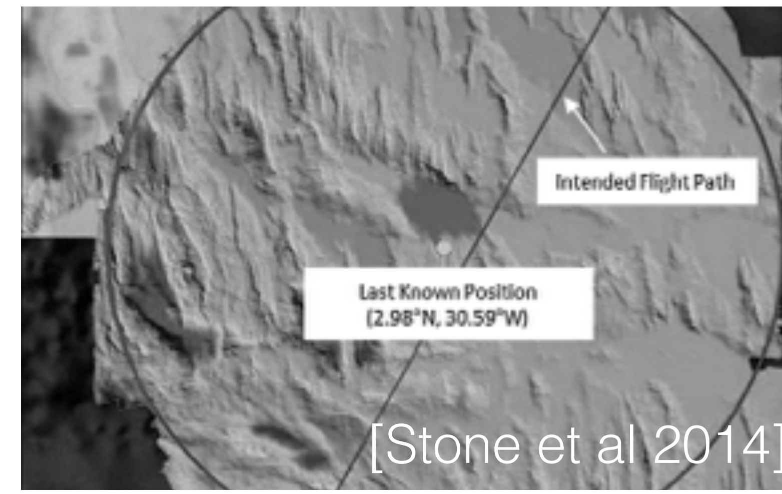

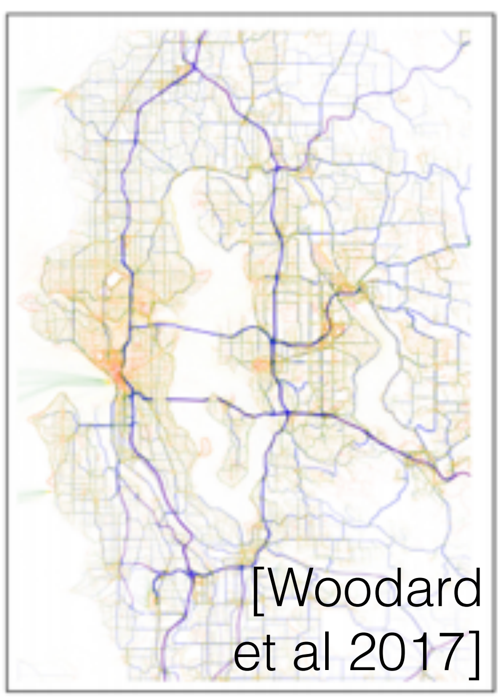

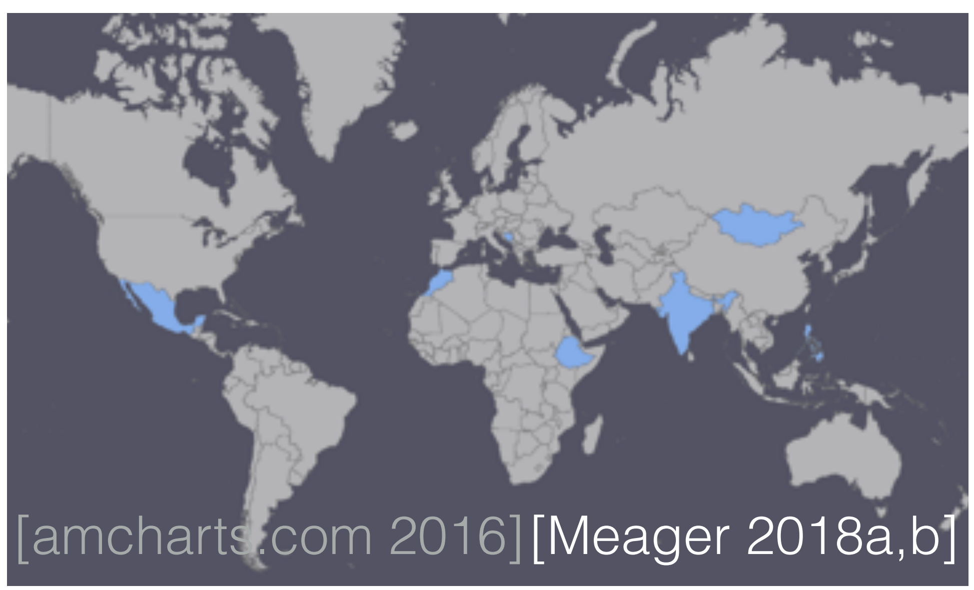

[mc-stan.org]

- Challenge: fast (compute, user), reliable inference

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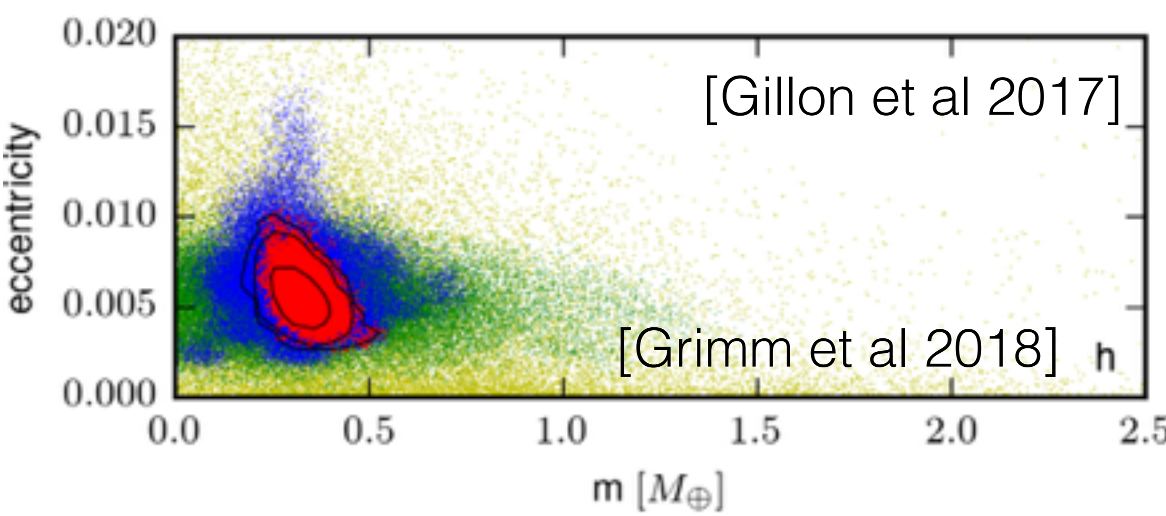
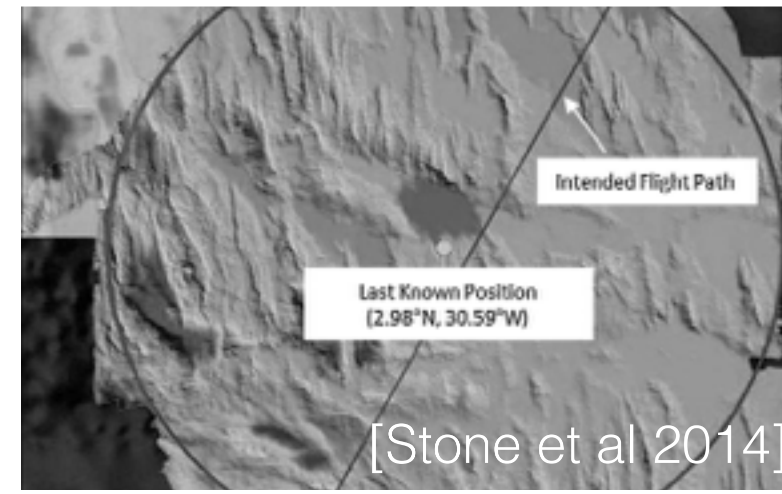


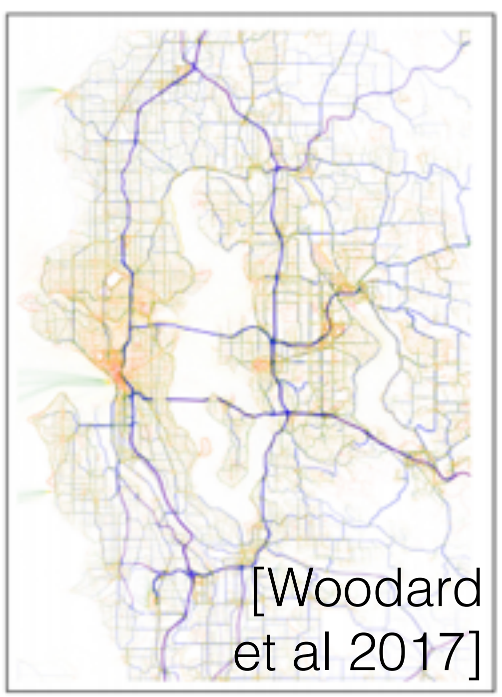
[Gillon et al 2017]

[Grimm et al 2018]

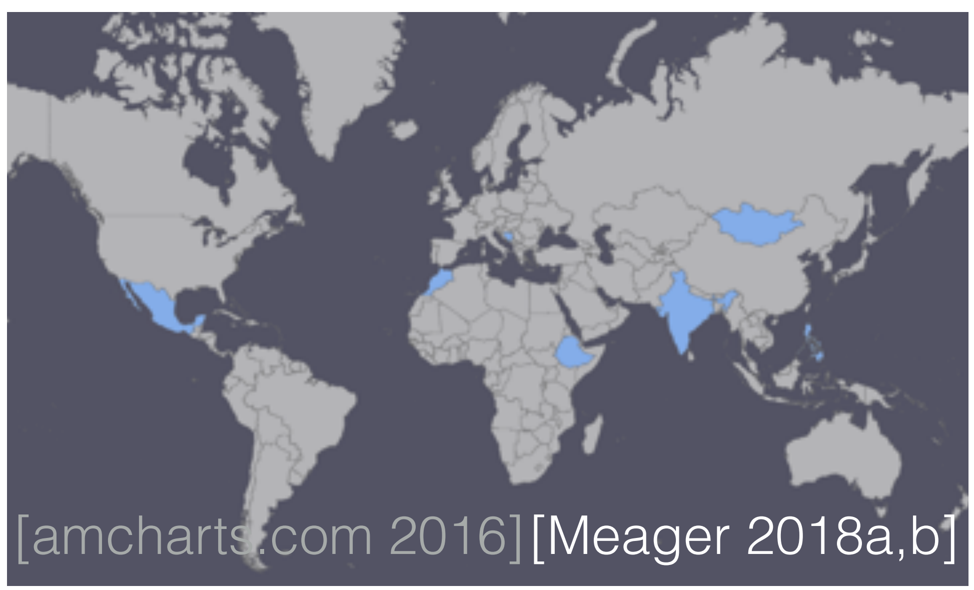
[ESO/ L. Calçada/ M. Kornmesser 2017] [Abbott et al 2016a,b]



[Stone et al 2014]



[Woodard et al 2017]



[amcharts.com 2016] [Meager 2018a,b]

[Kuikka et al 2014]
[Baltic Salmon Fund]

[Chati, Balakrishnan 2017]
[Julian Hertzog 2016]

[mc-stan.org]

- Challenge: fast (compute, user), reliable inference
- Uncertainty doesn't have to disappear in large data sets

Variational Bayes

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- Modern problems: often large data, large dimensions

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[Blei et al 2003]

The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.

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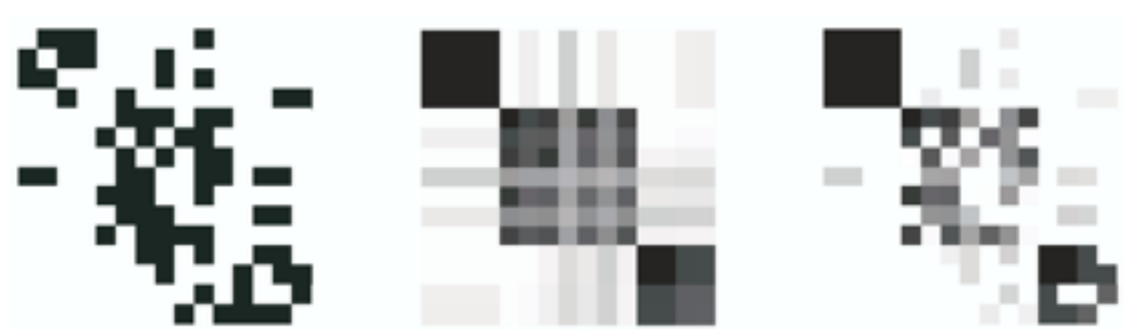

[Airoldi et al 2008]

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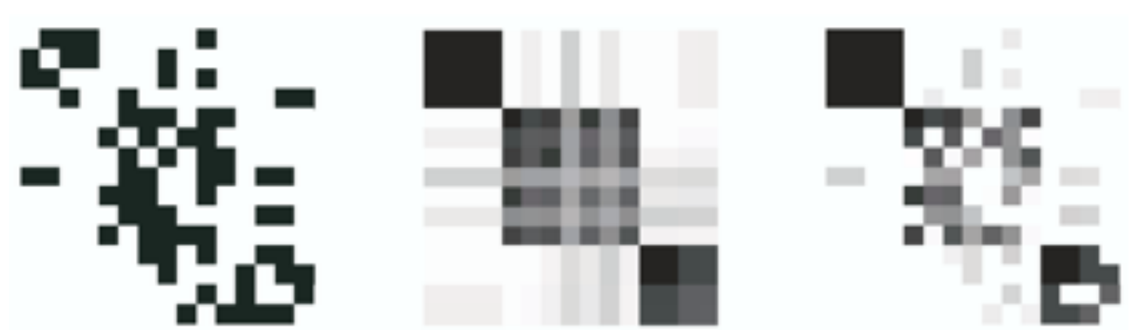

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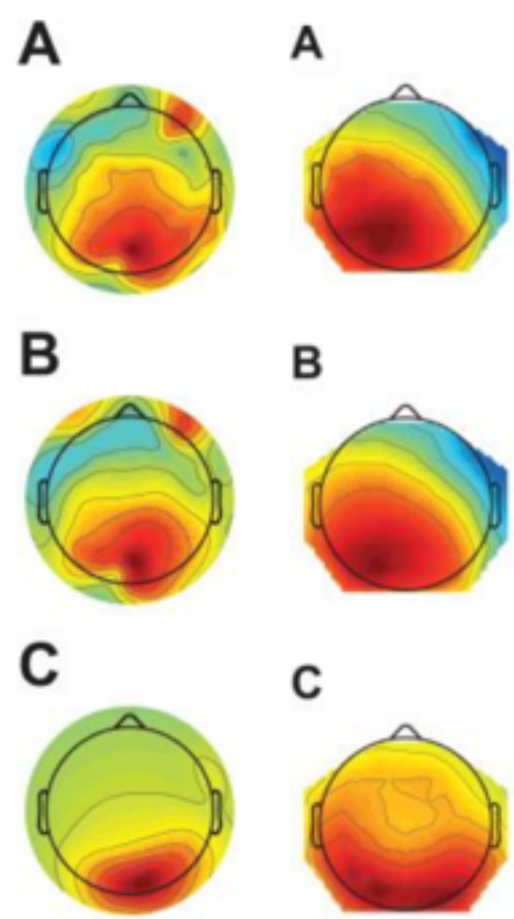

[Gershman et al 2014]

[Blei et al 2018]

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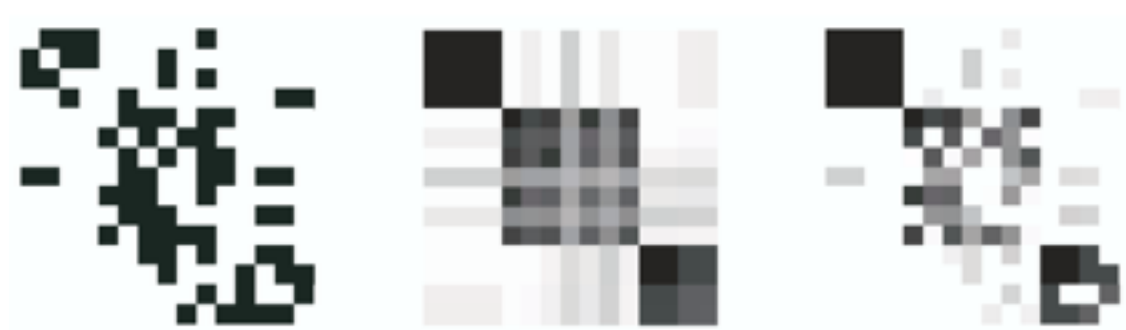

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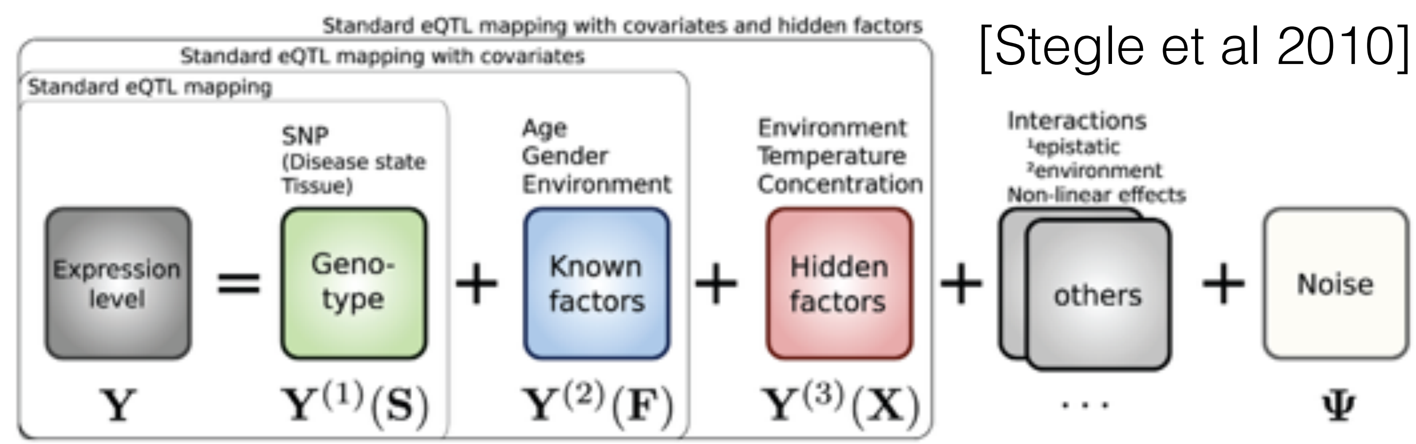
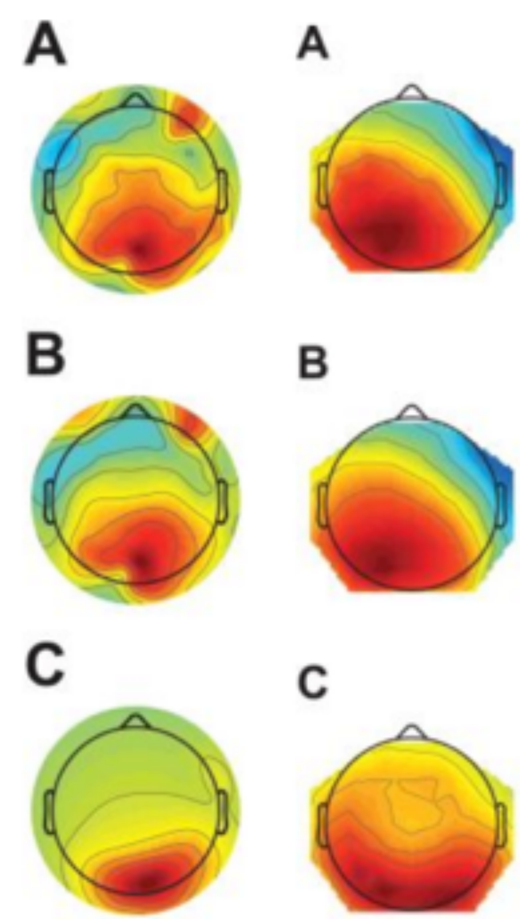


[Stegle et al 2010]



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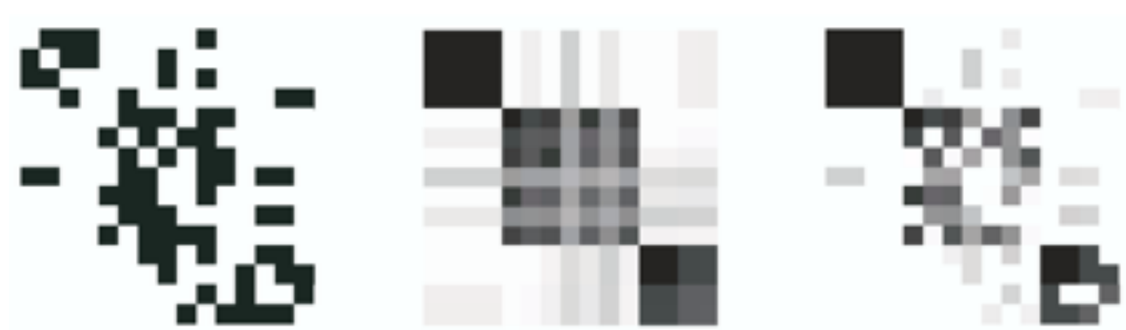

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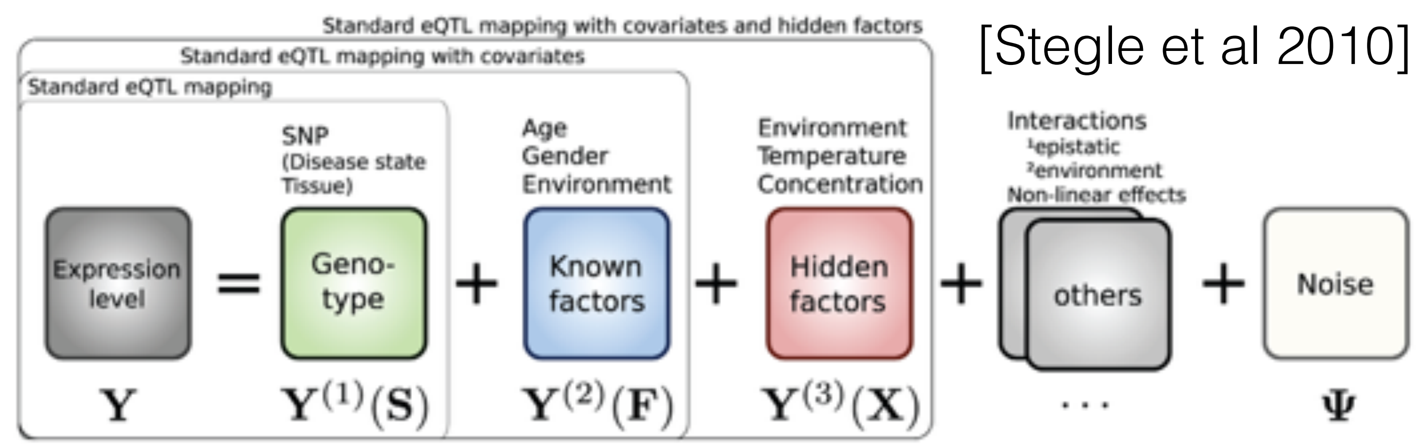

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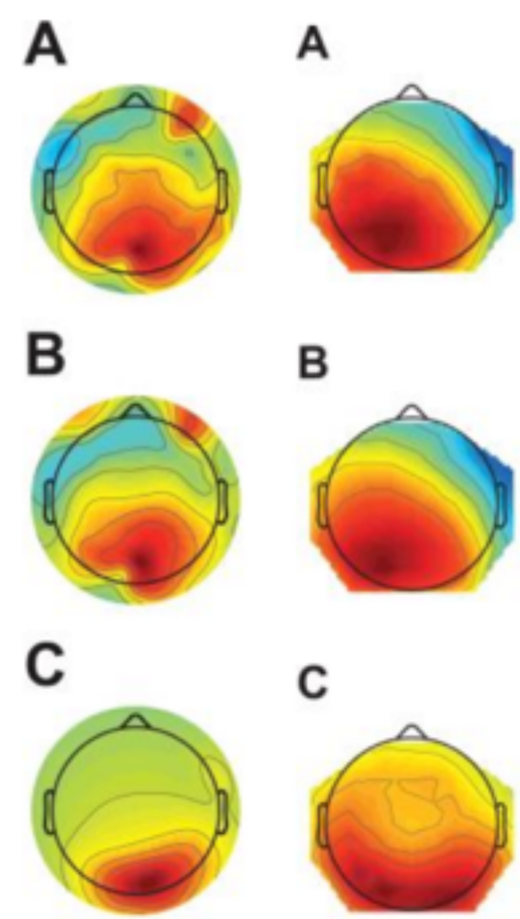

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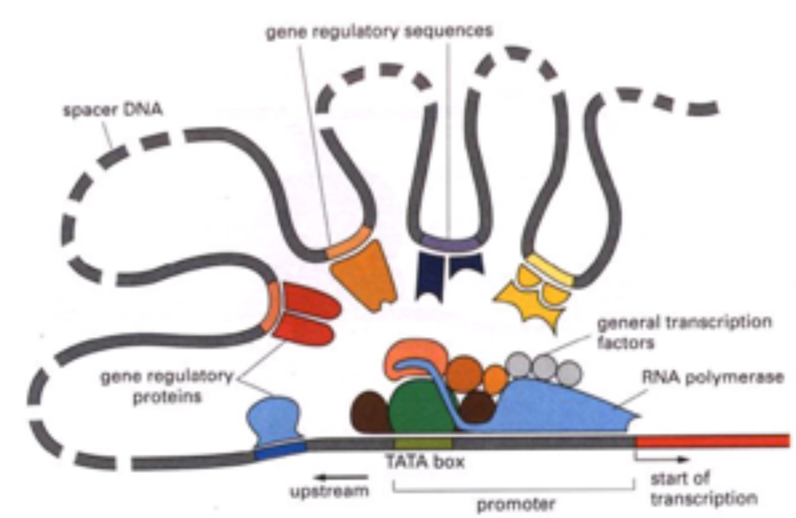

[Xing et al 2004]
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Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use MFVB?
- When can we trust MFVB?
- Where do we go from here?

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Bayesian inference

parameters

$p(\theta)$

prior

Bayesian inference

parameters

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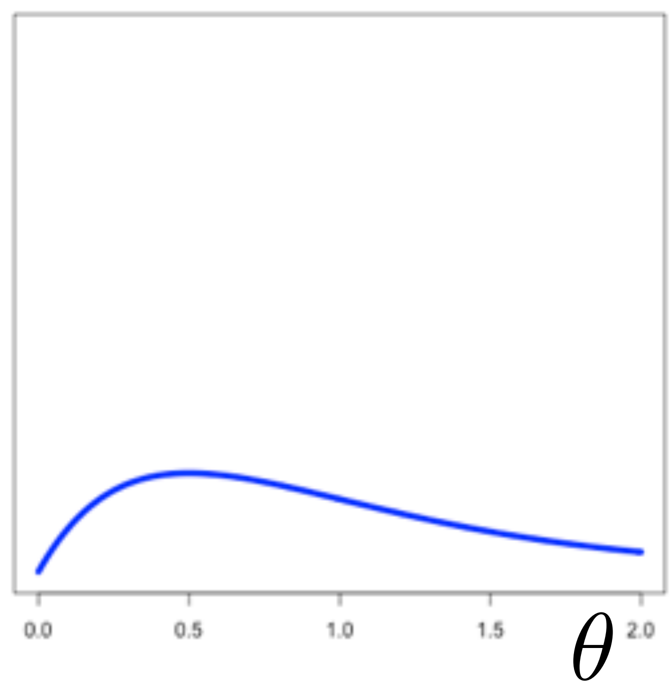
prior

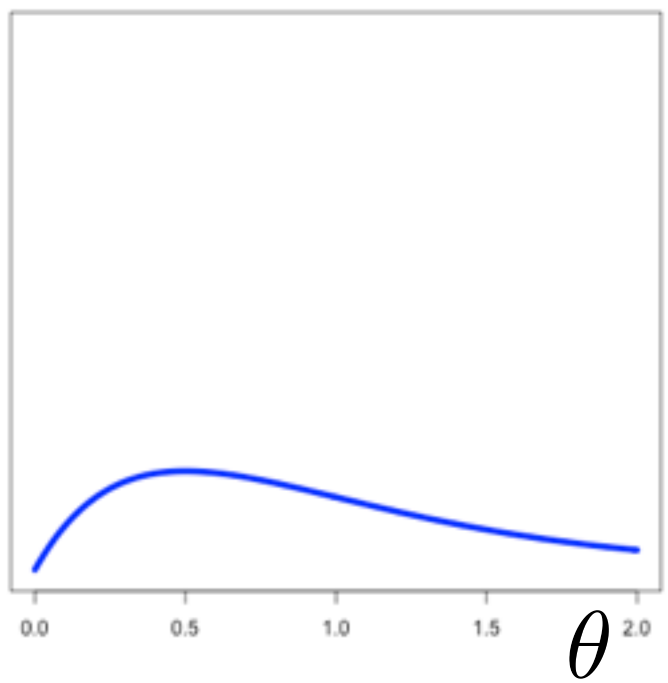


Bayesian inference

parameters

$$p(y_{1:N}|\theta)p(\theta)$$

likelihood prior

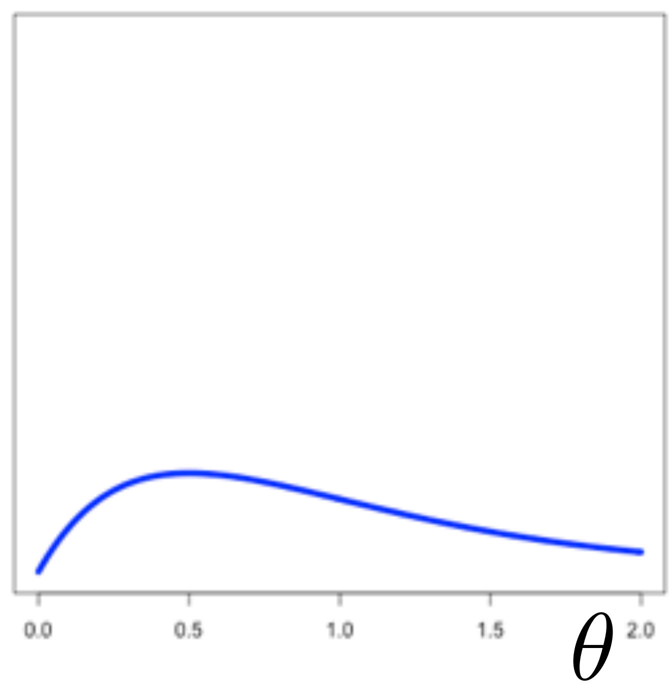


Bayesian inference

$$p(y_{1:N}|\theta)p(\theta)$$

data → $y_{1:N}$; parameters → θ

likelihood prior

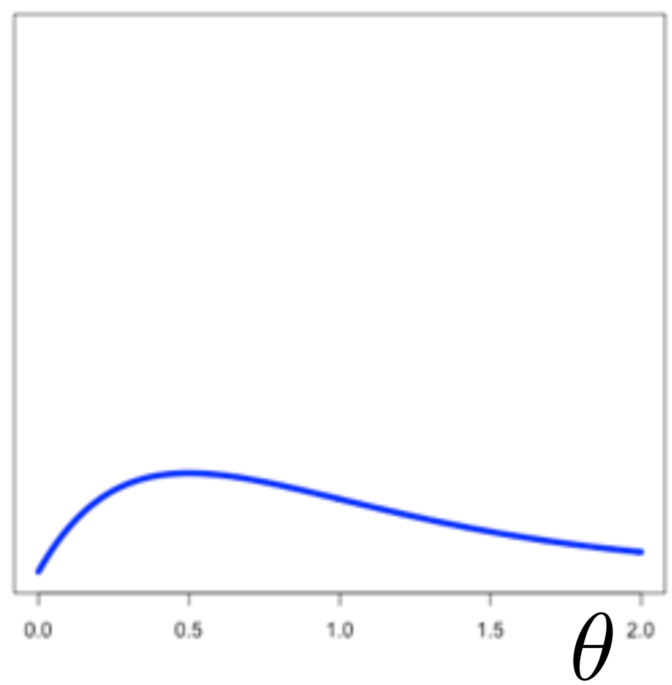


Bayesian inference

data → parameters

$$p(\theta|y_{1:N}) \propto_\theta p(y_{1:N}|\theta)p(\theta)$$

posterior likelihood prior



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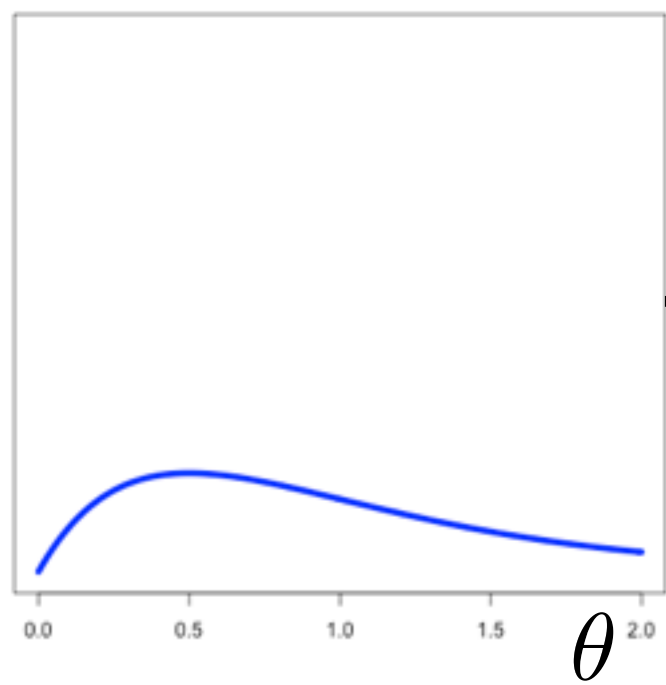

Bayes Theorem

Bayesian inference

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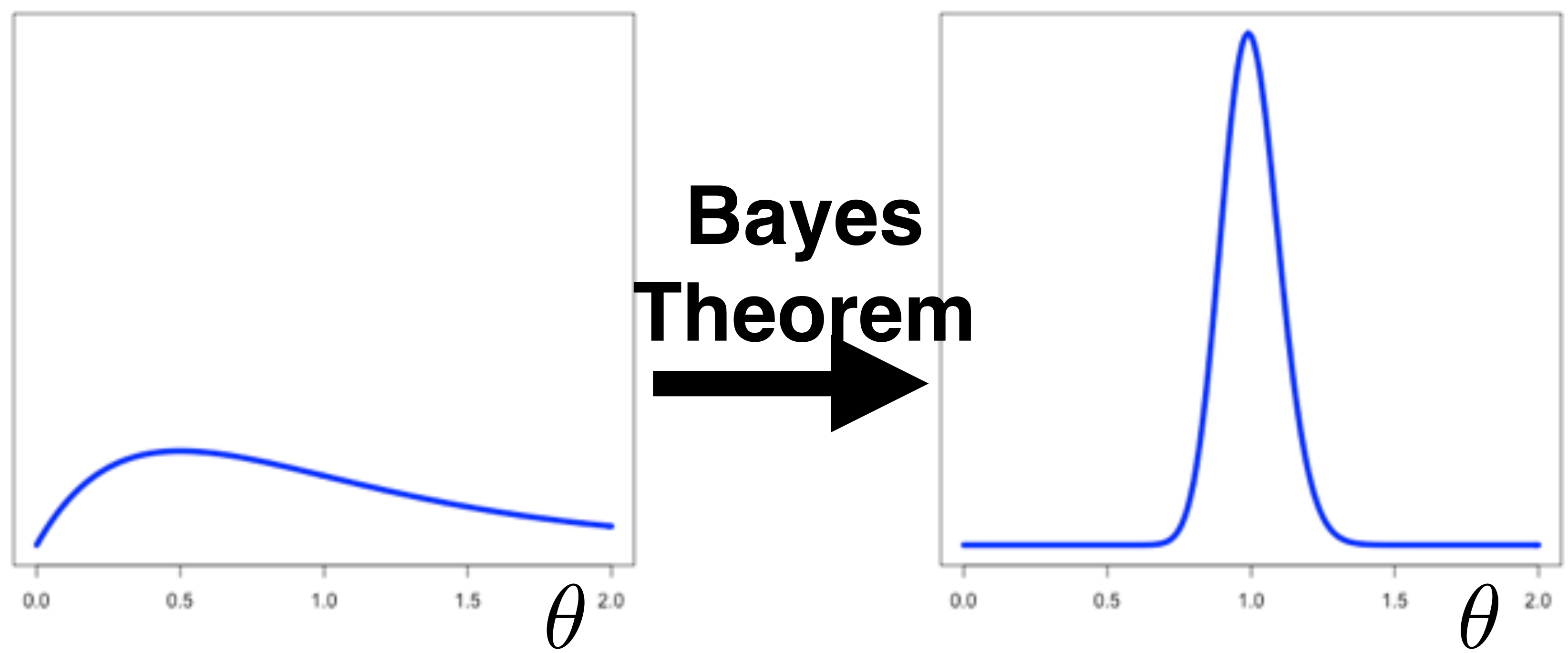

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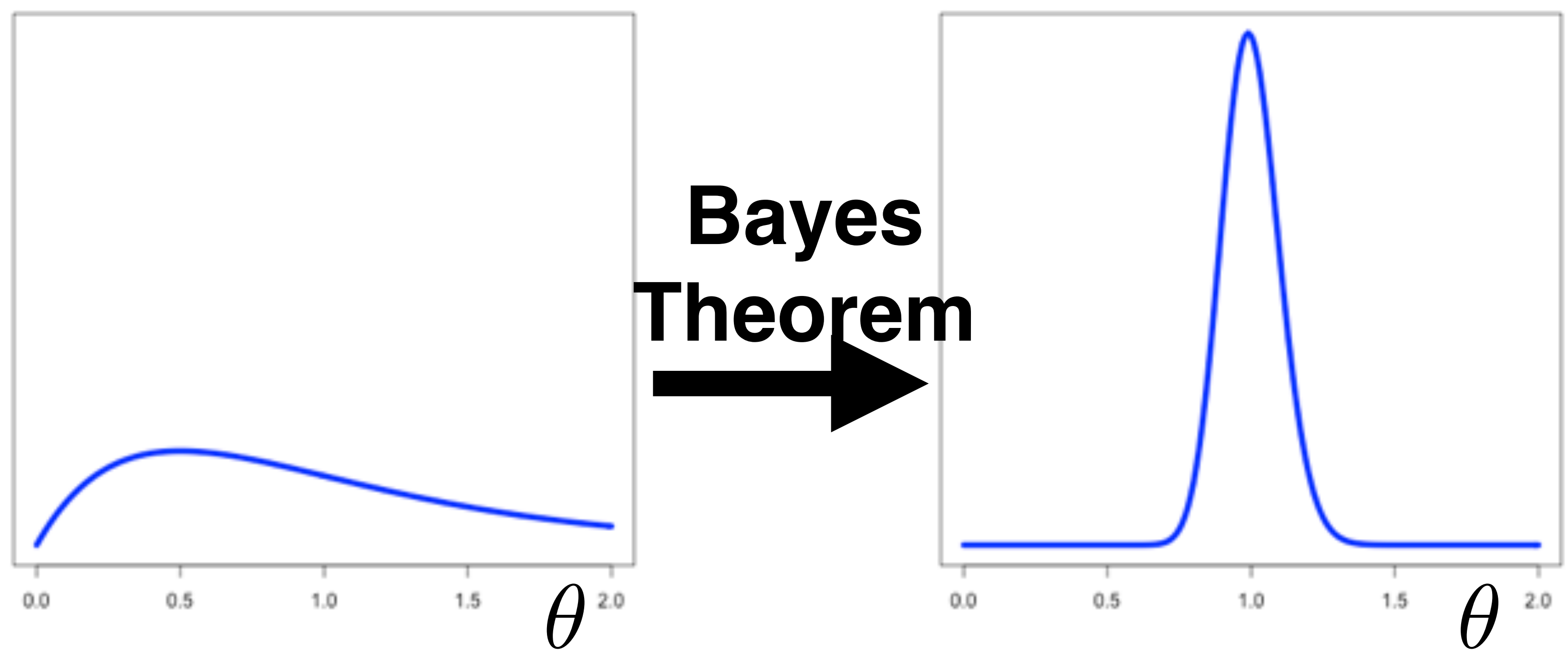

1. Build a model: choose prior & choose likelihood

Bayesian inference

data → $p(y_{1:N}|\theta)$; parameters → $p(\theta)$

$$p(\theta|y_{1:N}) \propto_\theta p(y_{1:N}|\theta)p(\theta)$$

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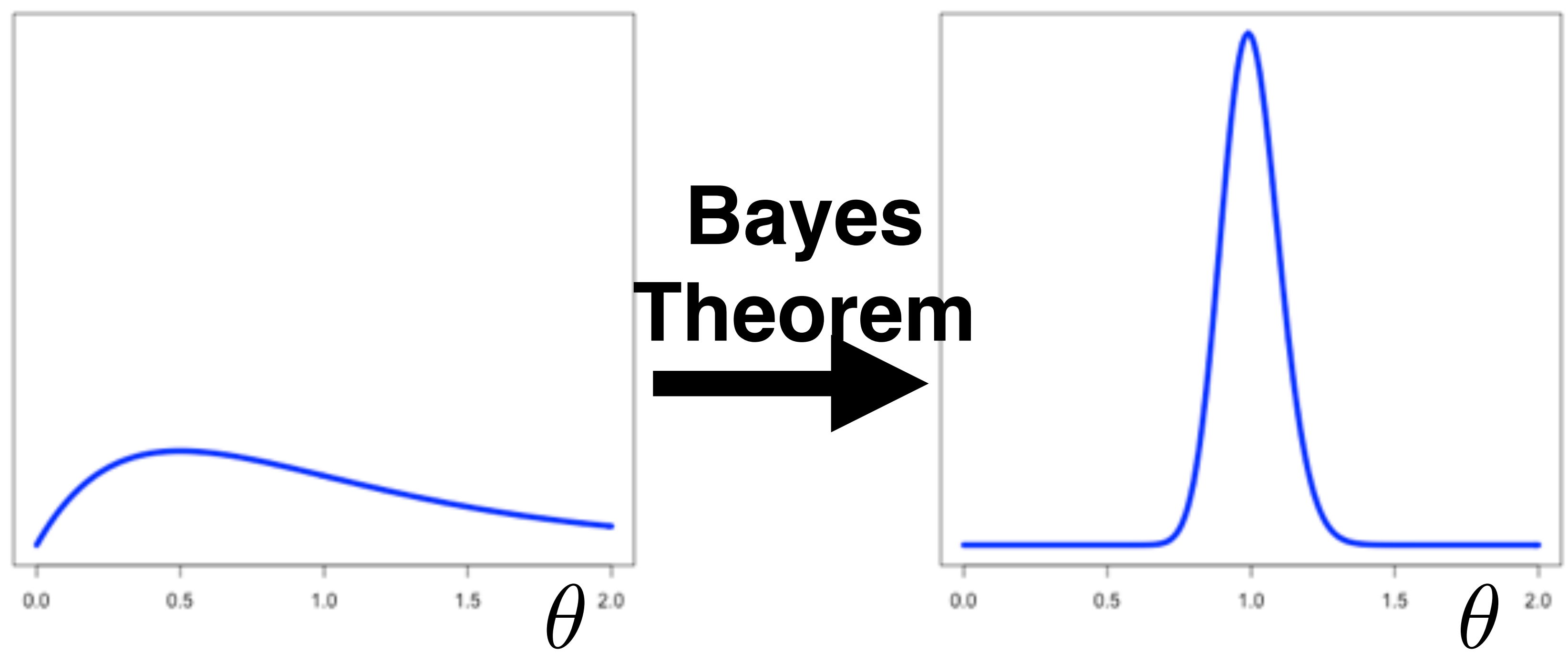

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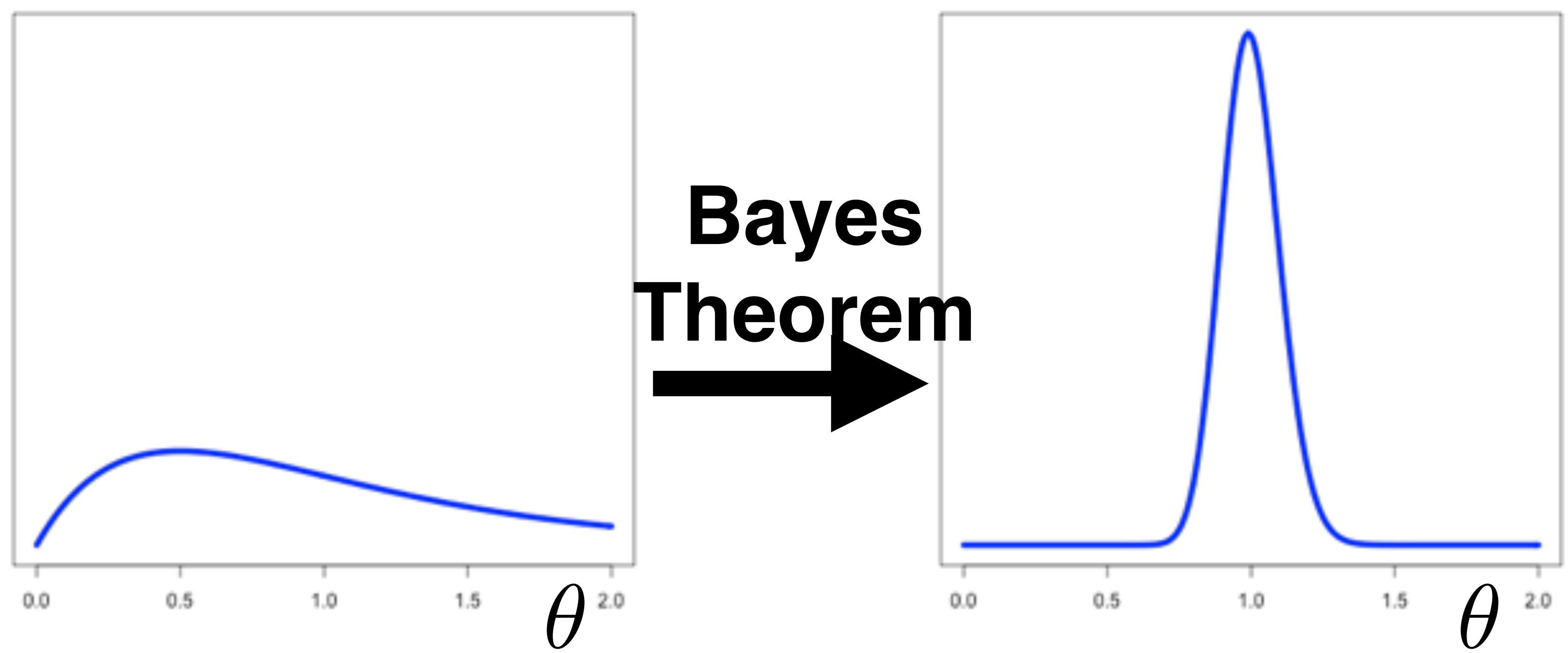

1. Build a model: choose prior & choose likelihood
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3. Report a summary, e.g. posterior means and (co)variances

Bayesian inference

data — parameters

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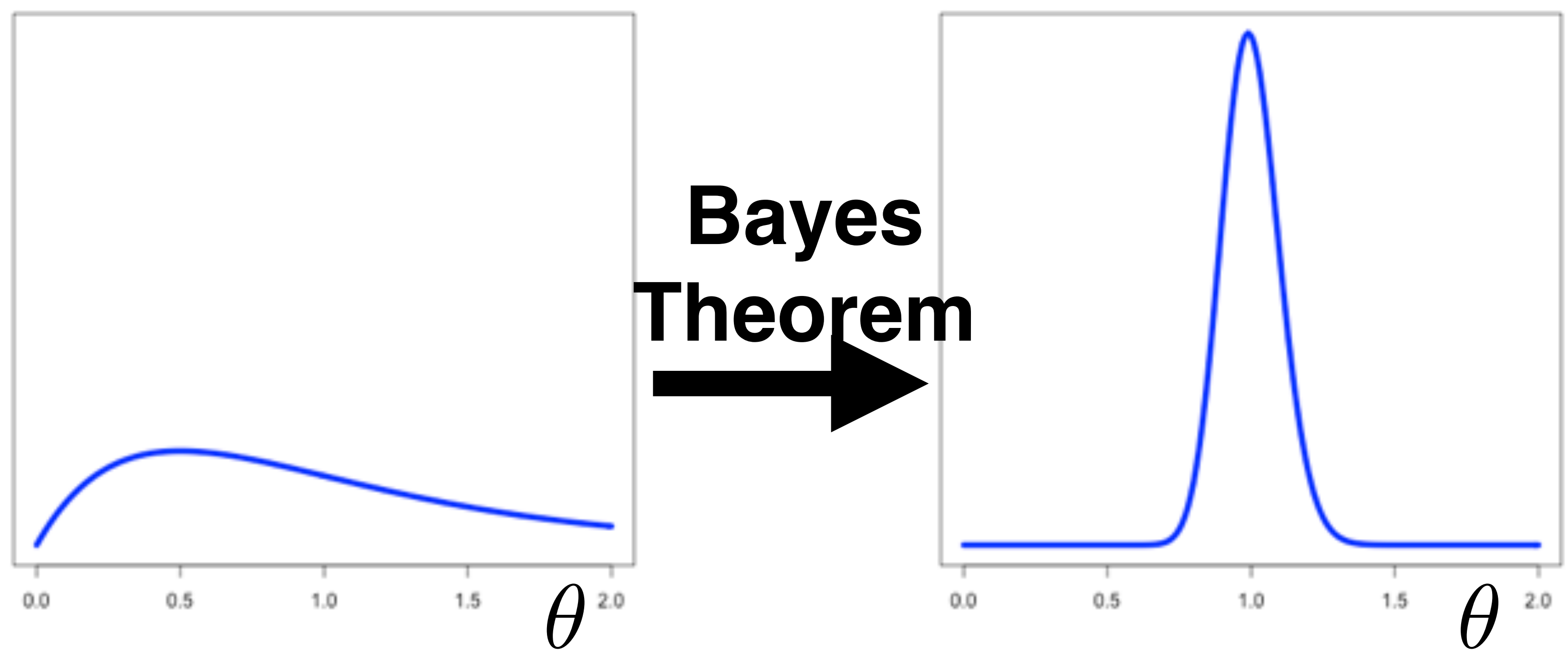
posterior — likelihood — prior



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- Why are steps 2 and 3 hard?

Bayesian inference

data parameters

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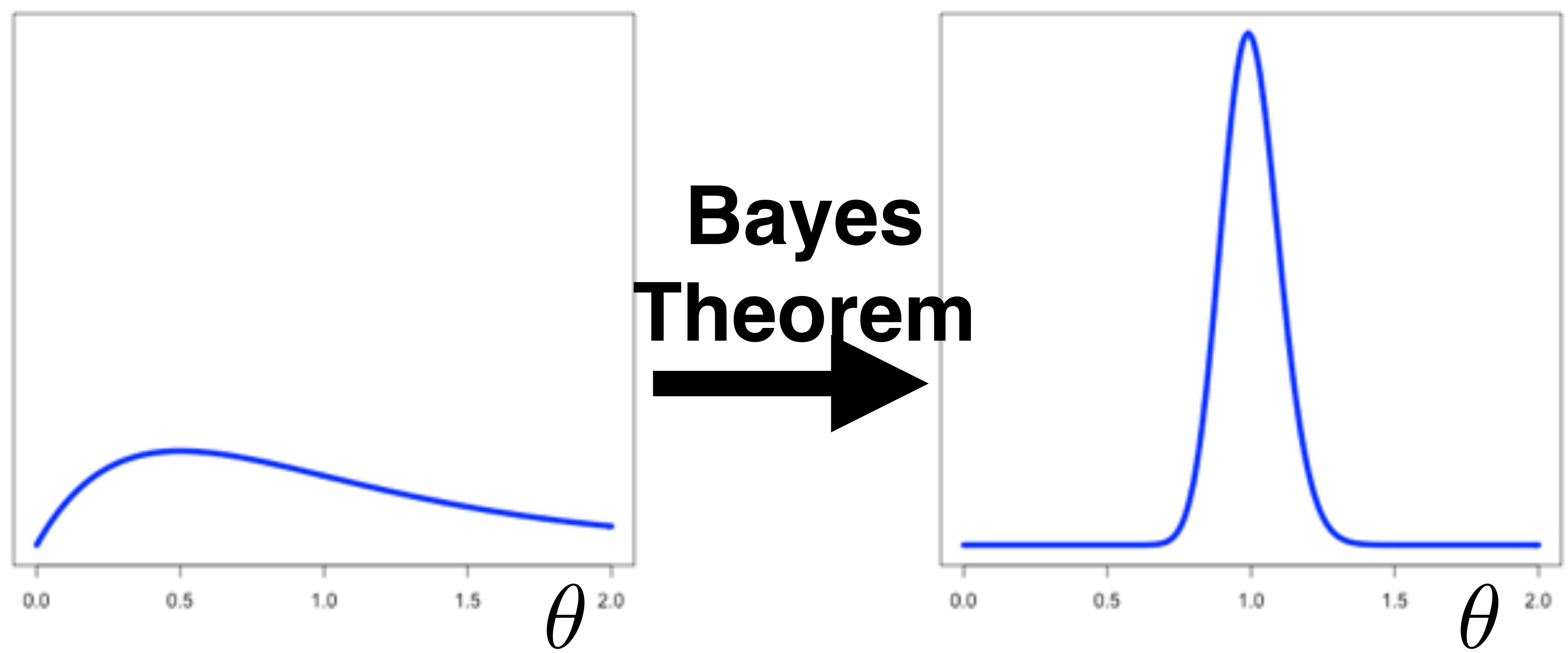
posterior likelihood prior



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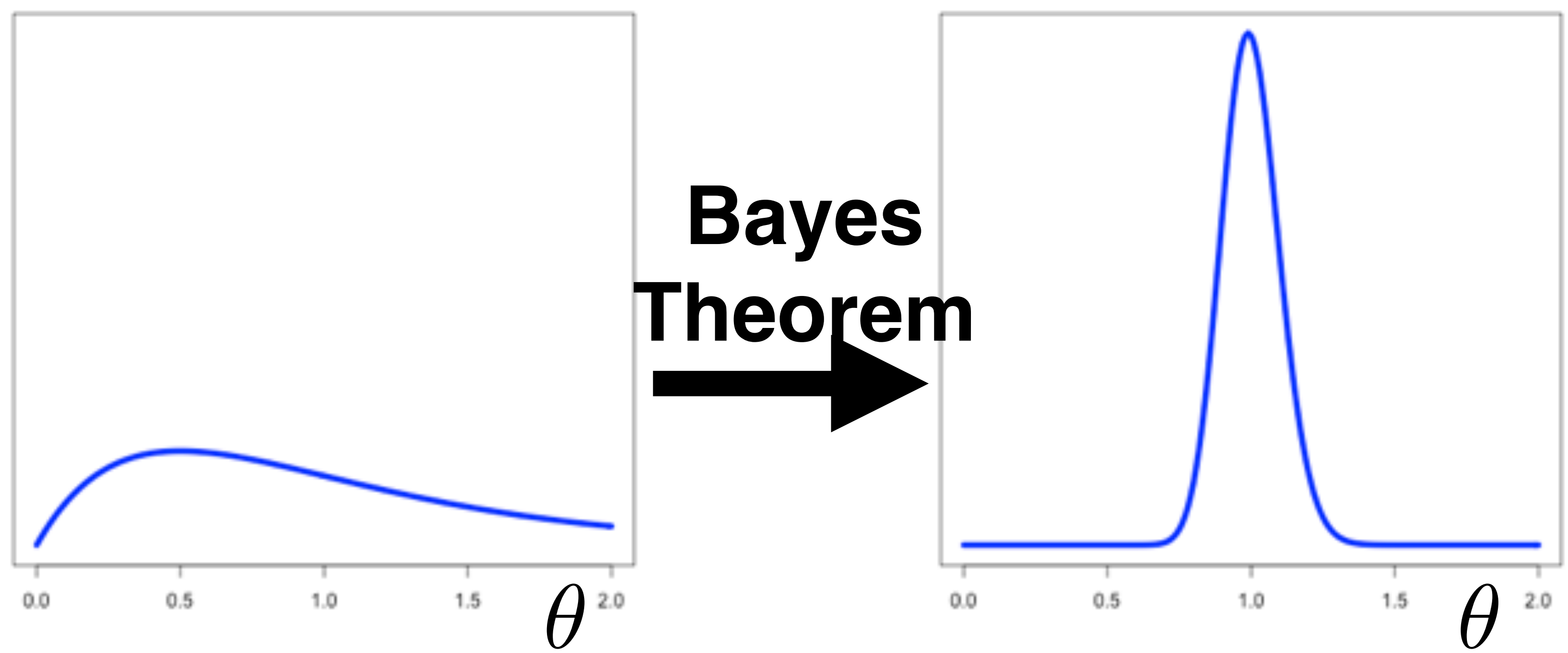

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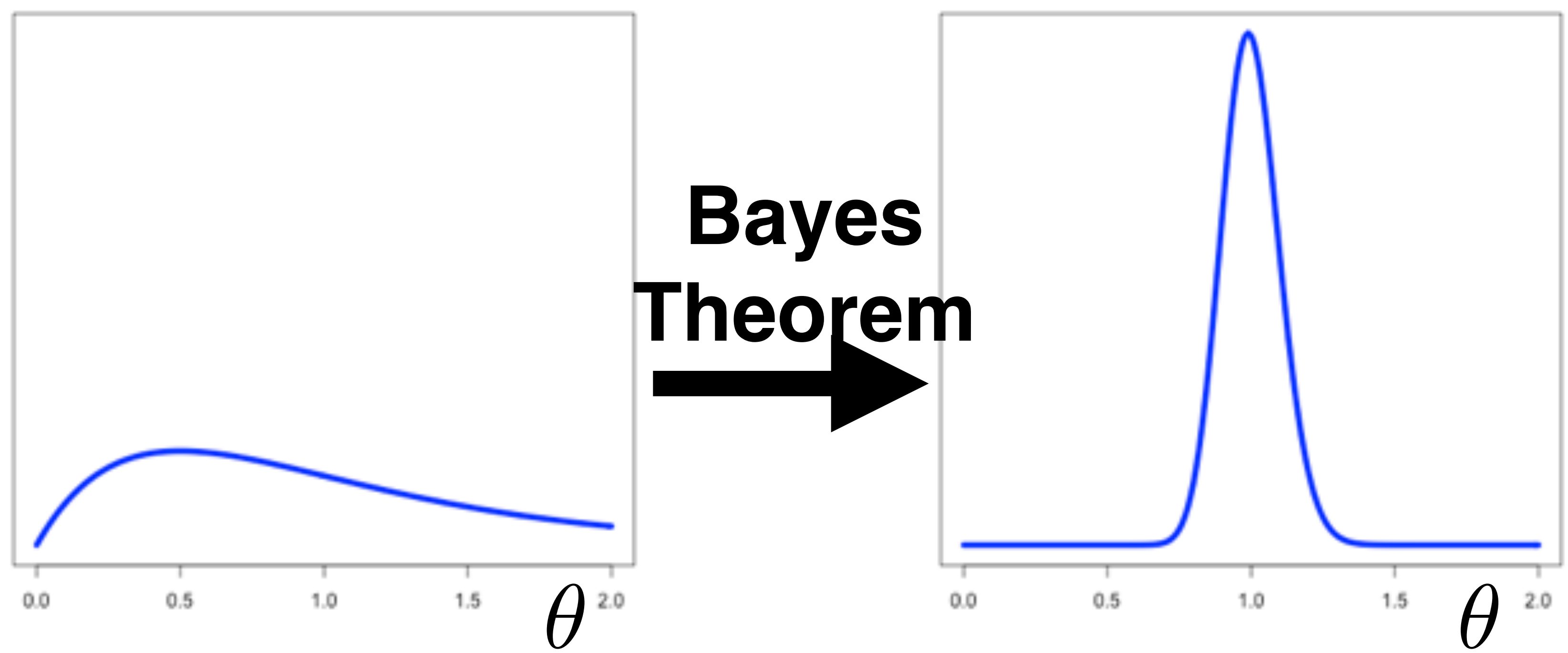
- Why are steps 2 and 3 hard?
    - Typically no closed form, high-dimensional integration

Bayesian inference

data parameters

$$p(\theta|y_{1:N}) = p(y_{1:N}|\theta)p(\theta)/p(y_{1:N})$$

posterior likelihood prior



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data → parameters

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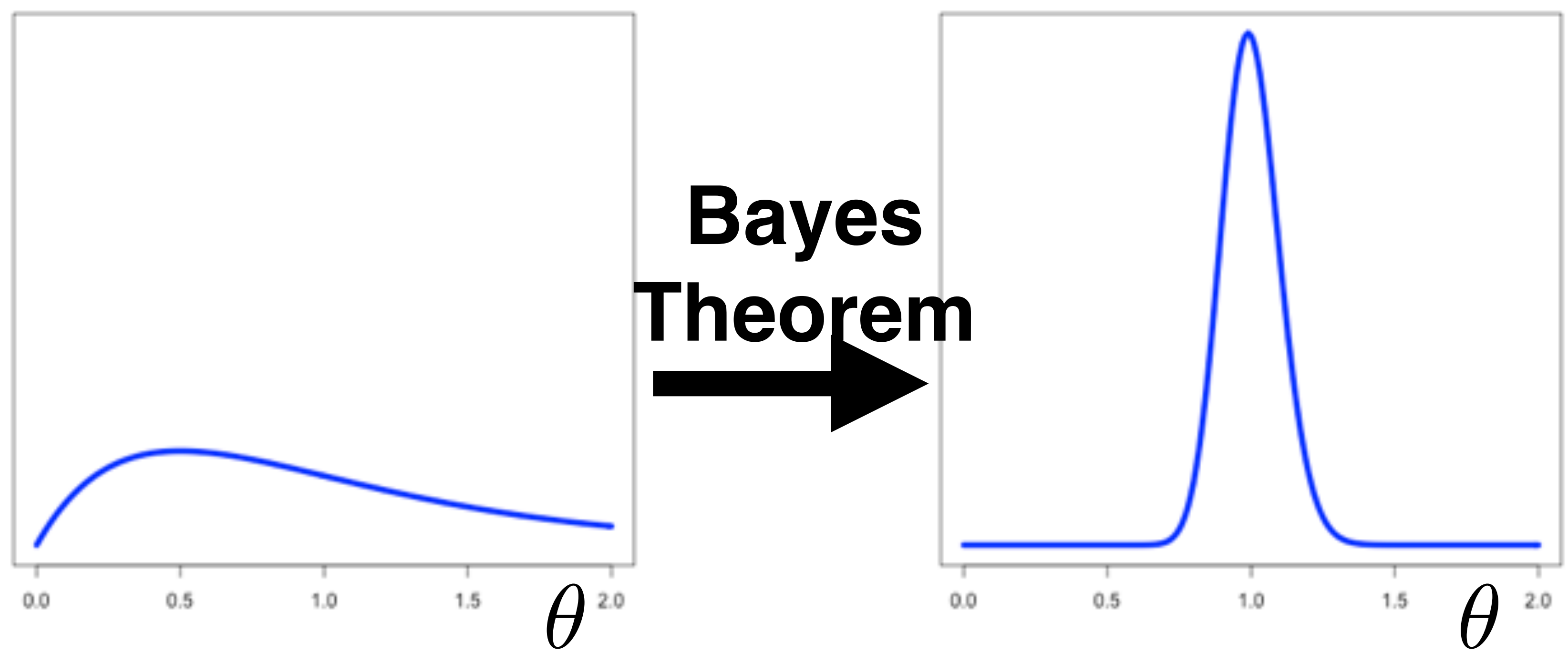
posterior likelihood prior evidence



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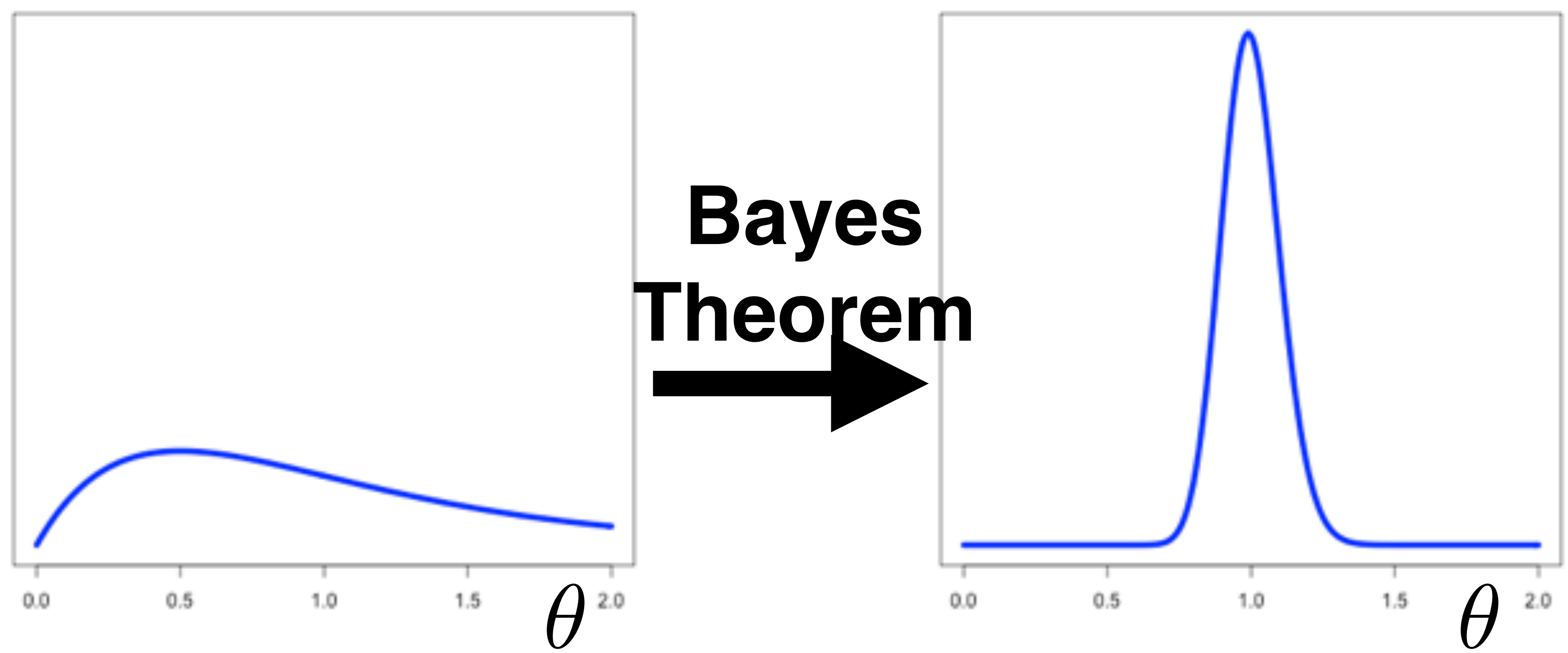
data → $y_{1:N}$, parameters → θ

$$p(\theta|y_{1:N}) = p(y_{1:N}|\theta)p(\theta) / \int p(y_{1:N},\theta)d\theta$$

posterior   likelihood prior   evidence



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- Gold standard: Markov Chain Monte Carlo (MCMC)

[Bardenet, Doucet, Holmes 2017]

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Instead: an optimization approach

- Approximate posterior with q^*

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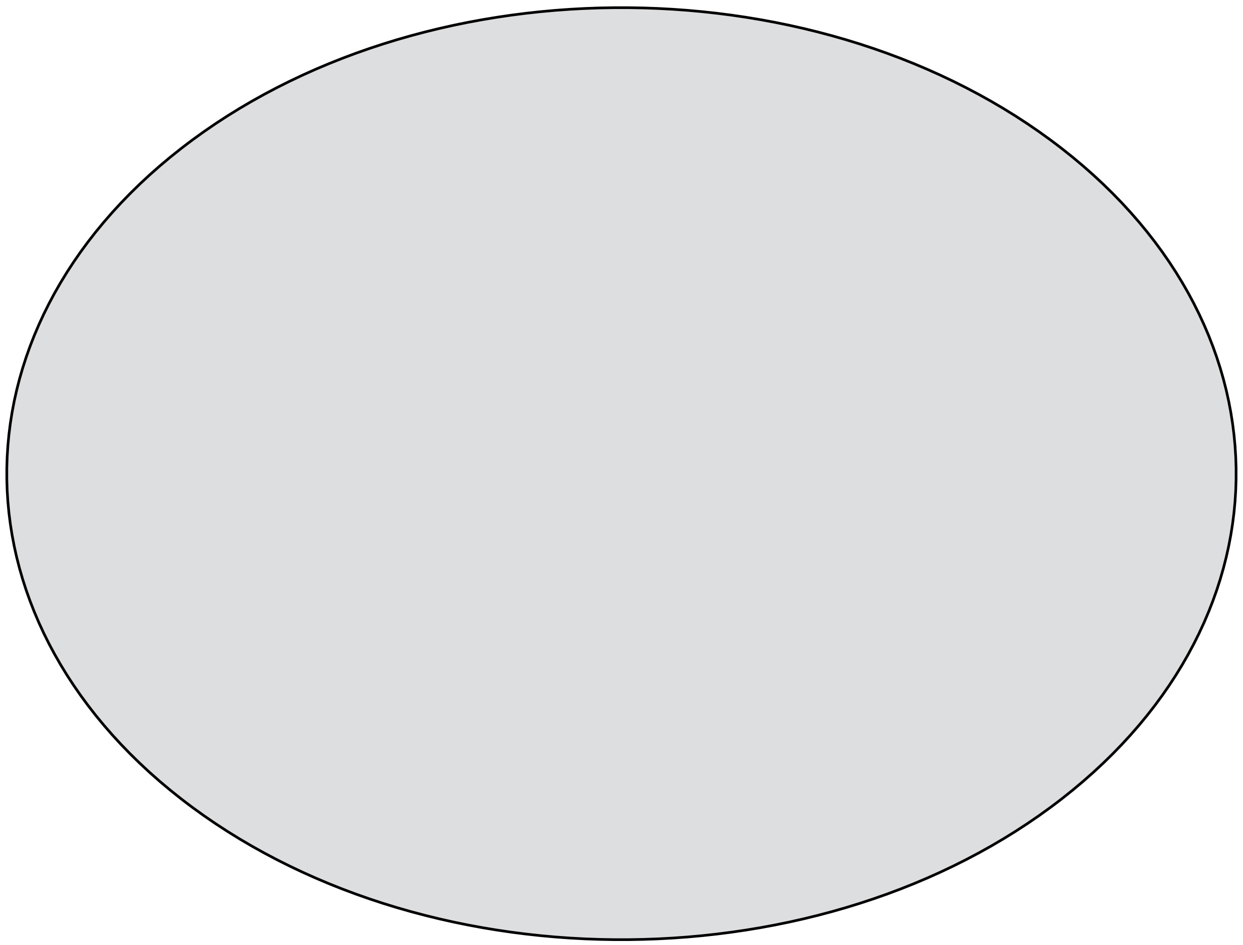

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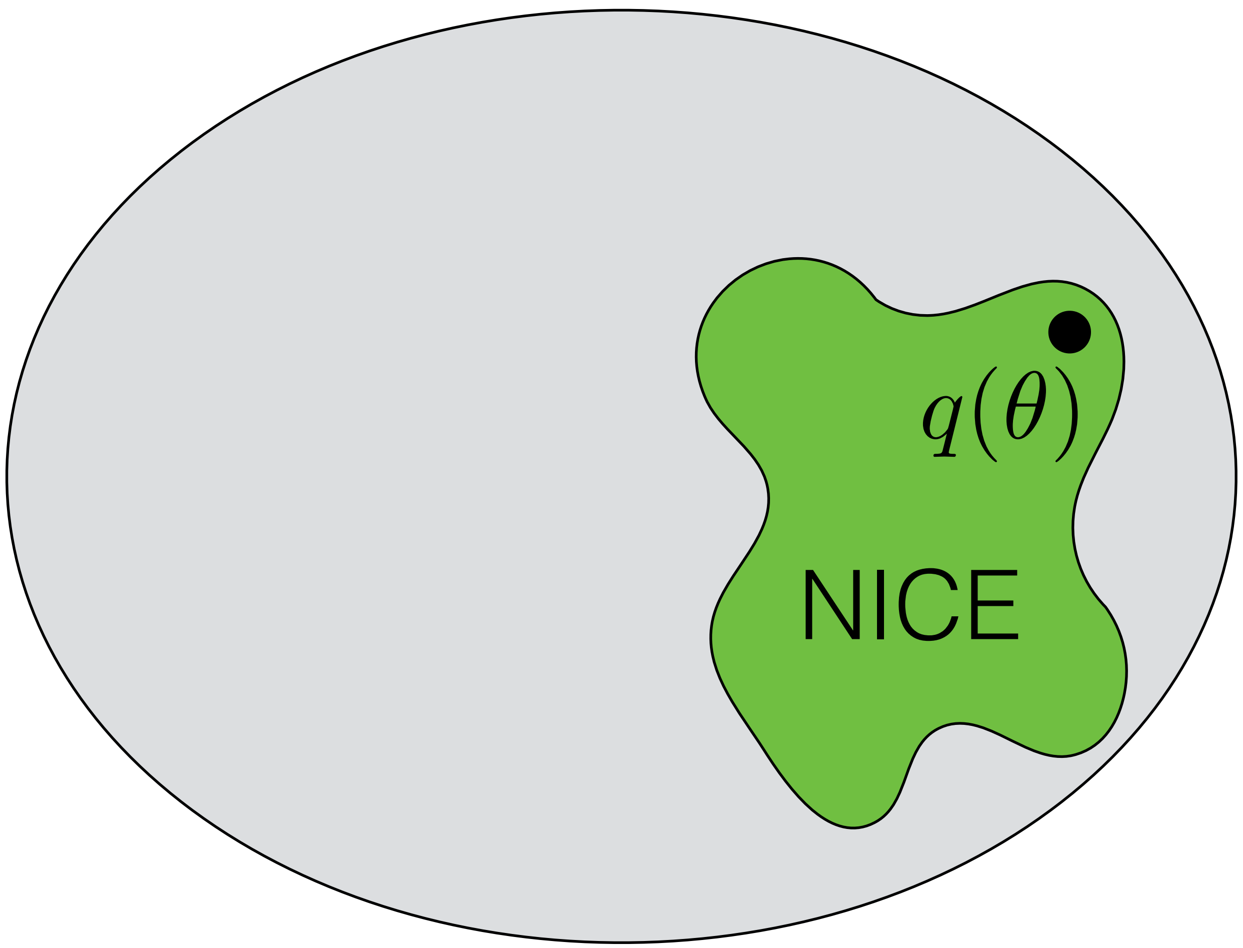

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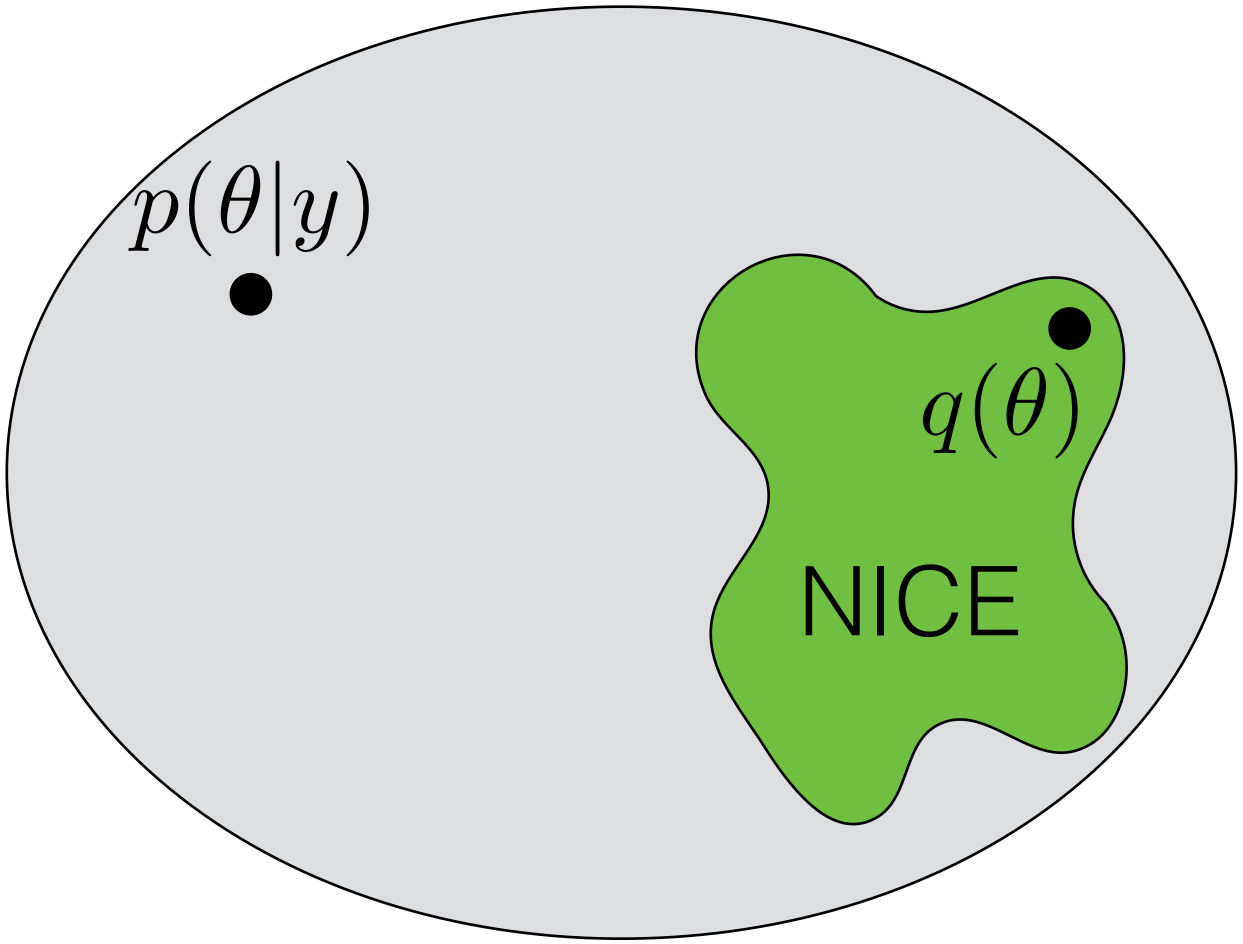

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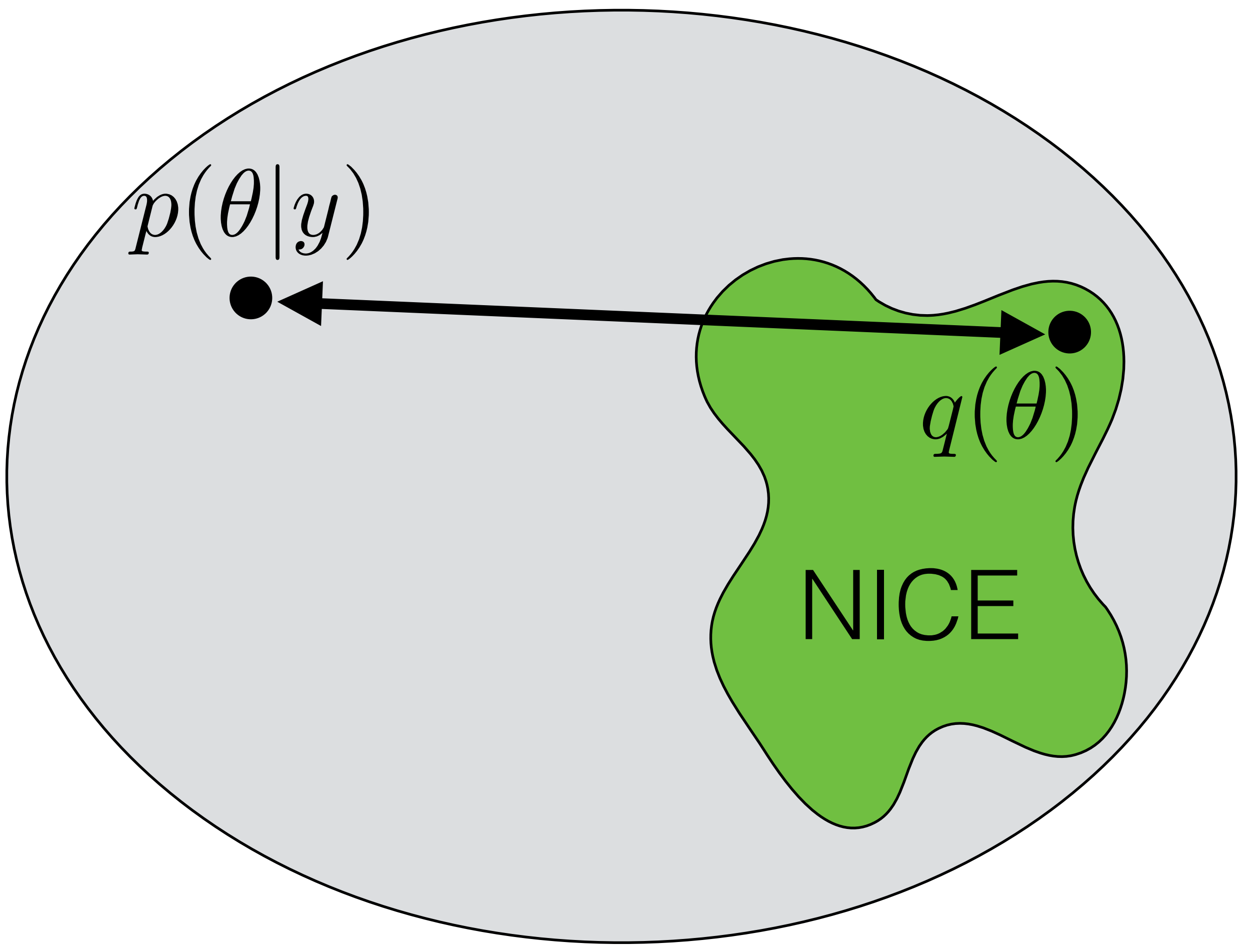

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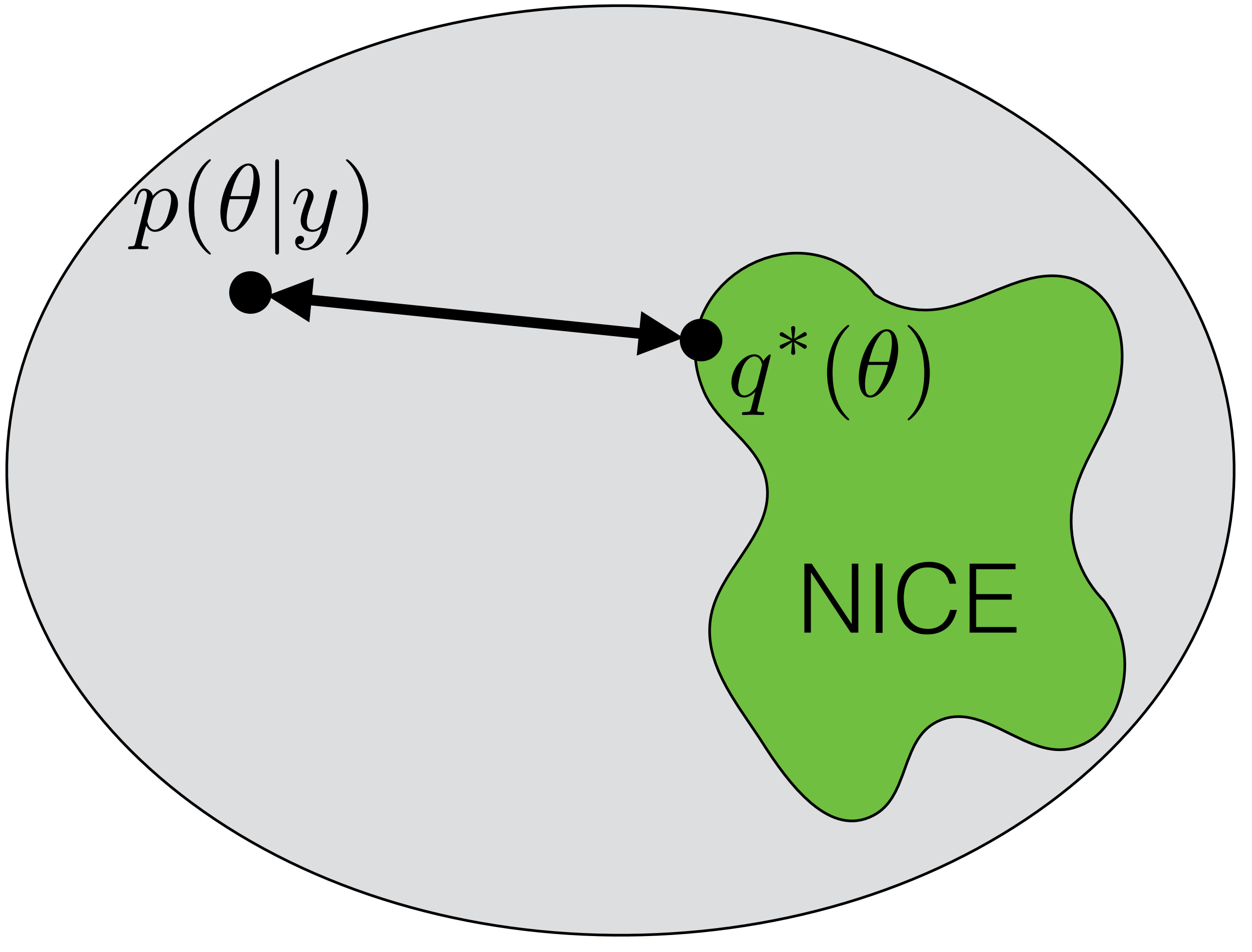

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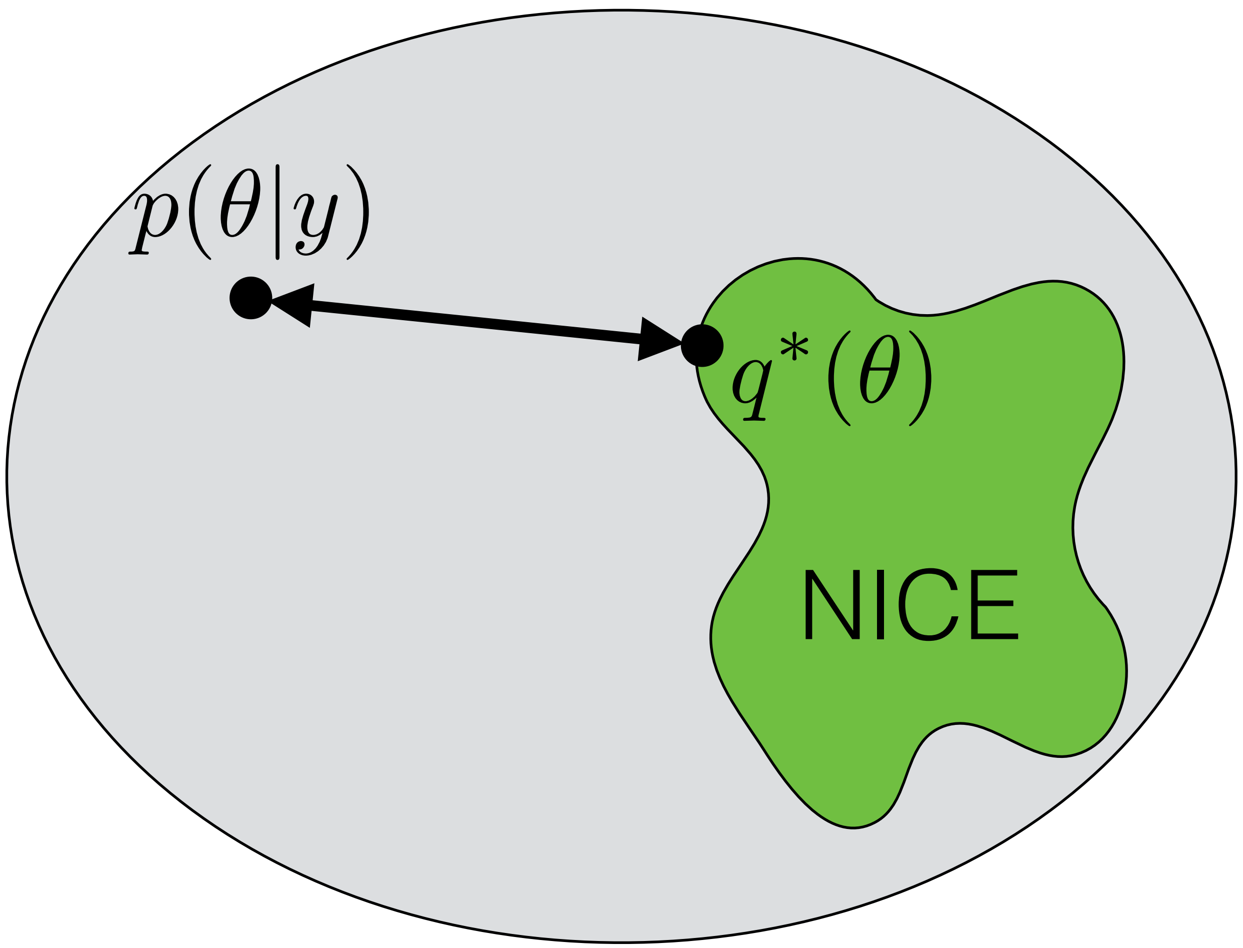

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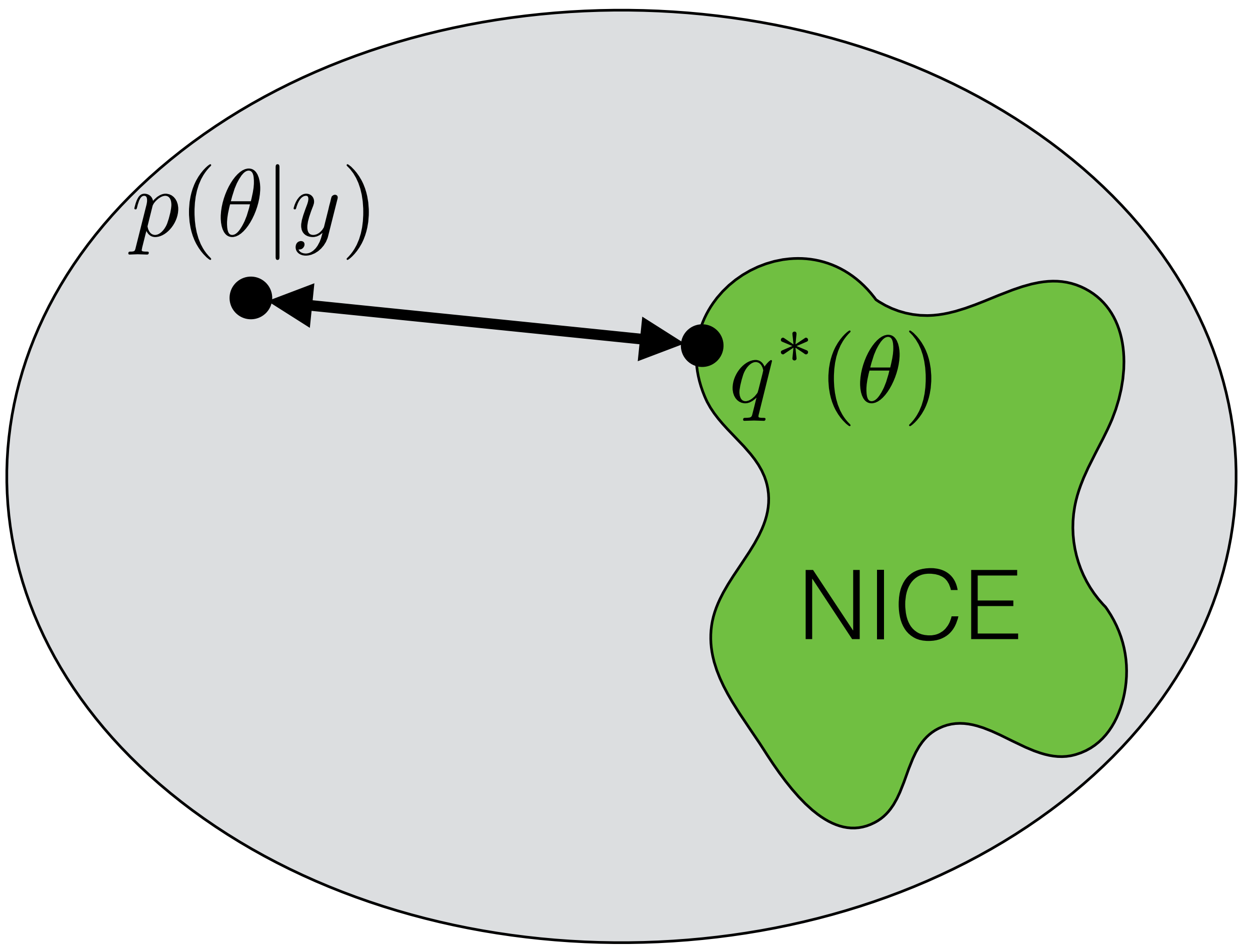

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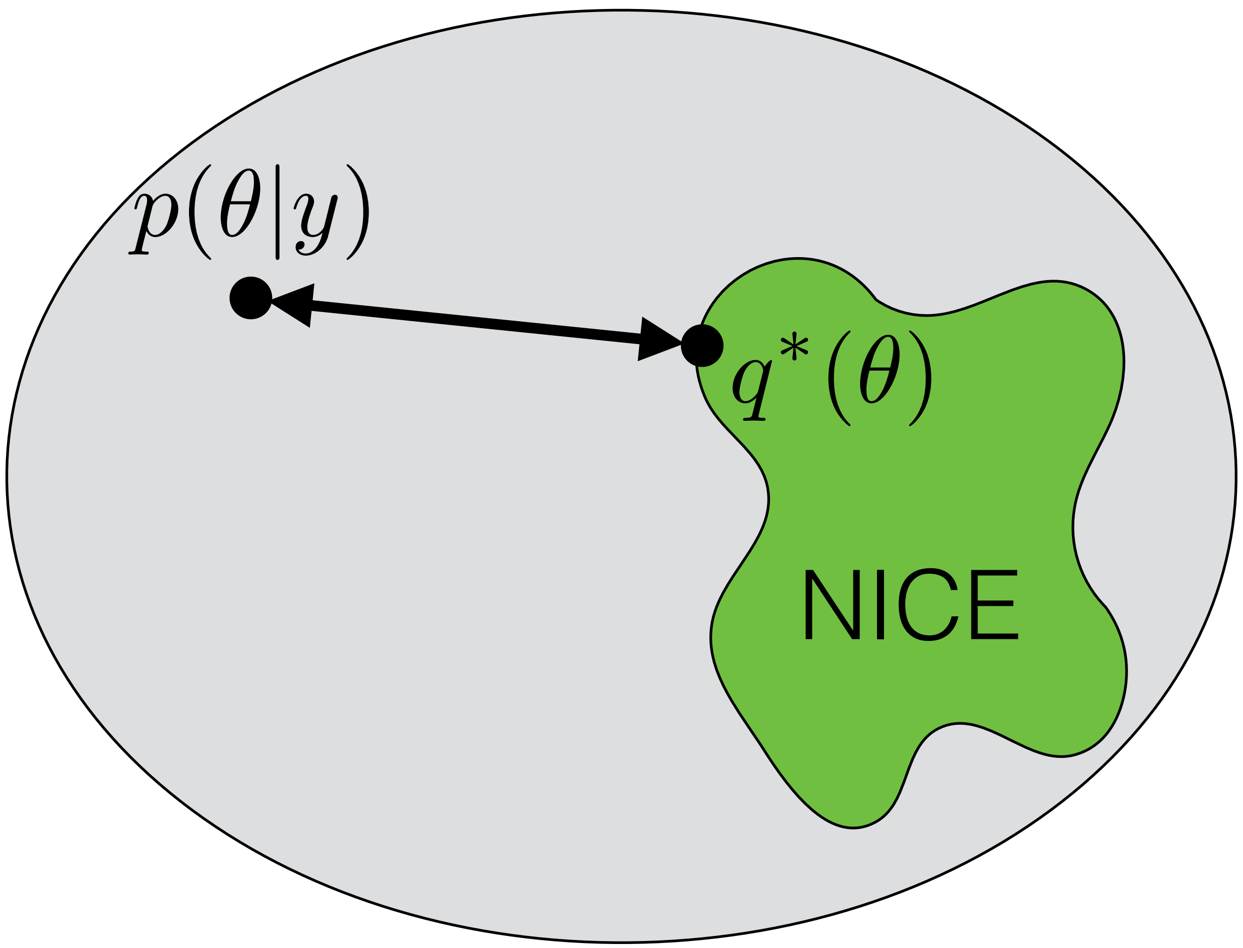

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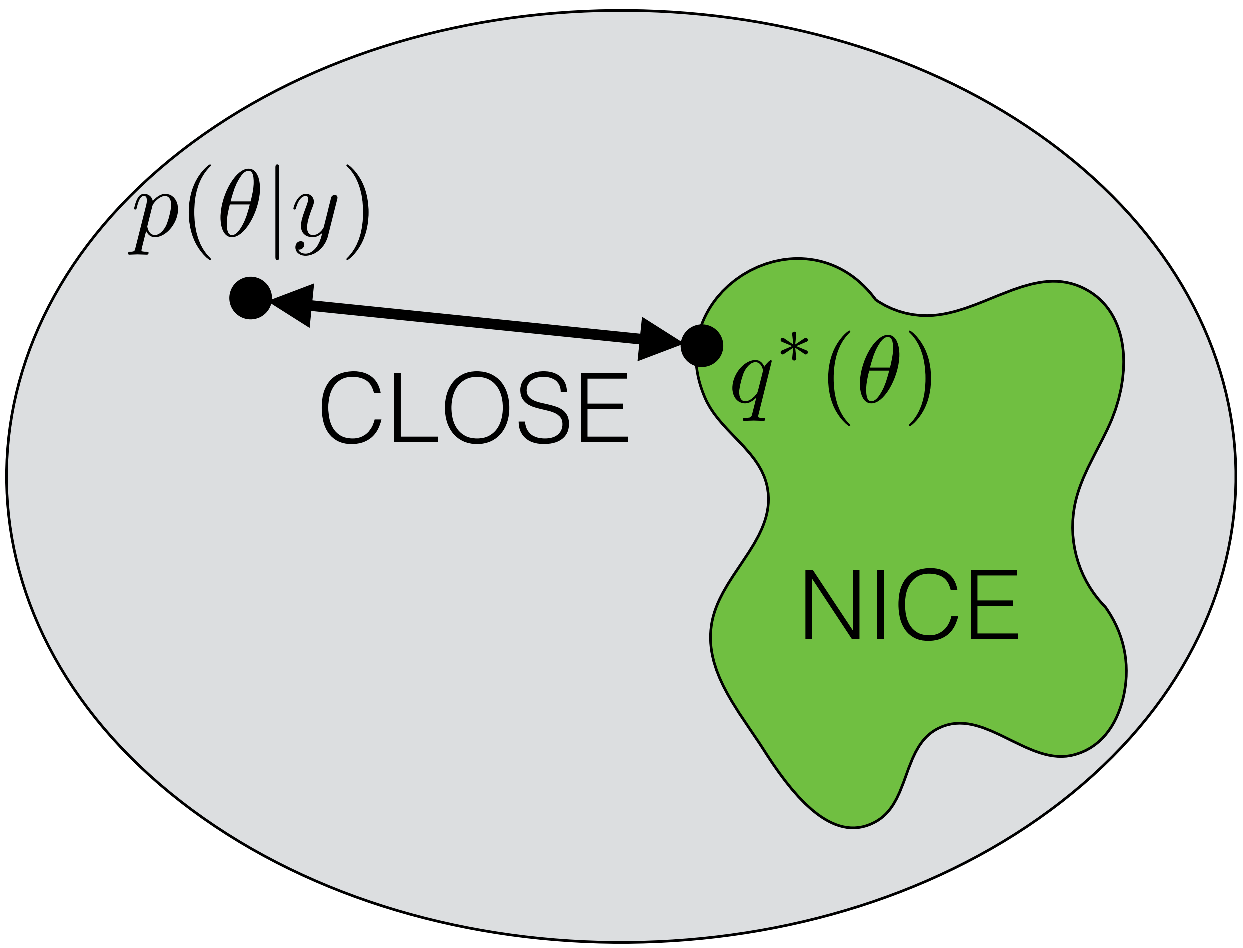

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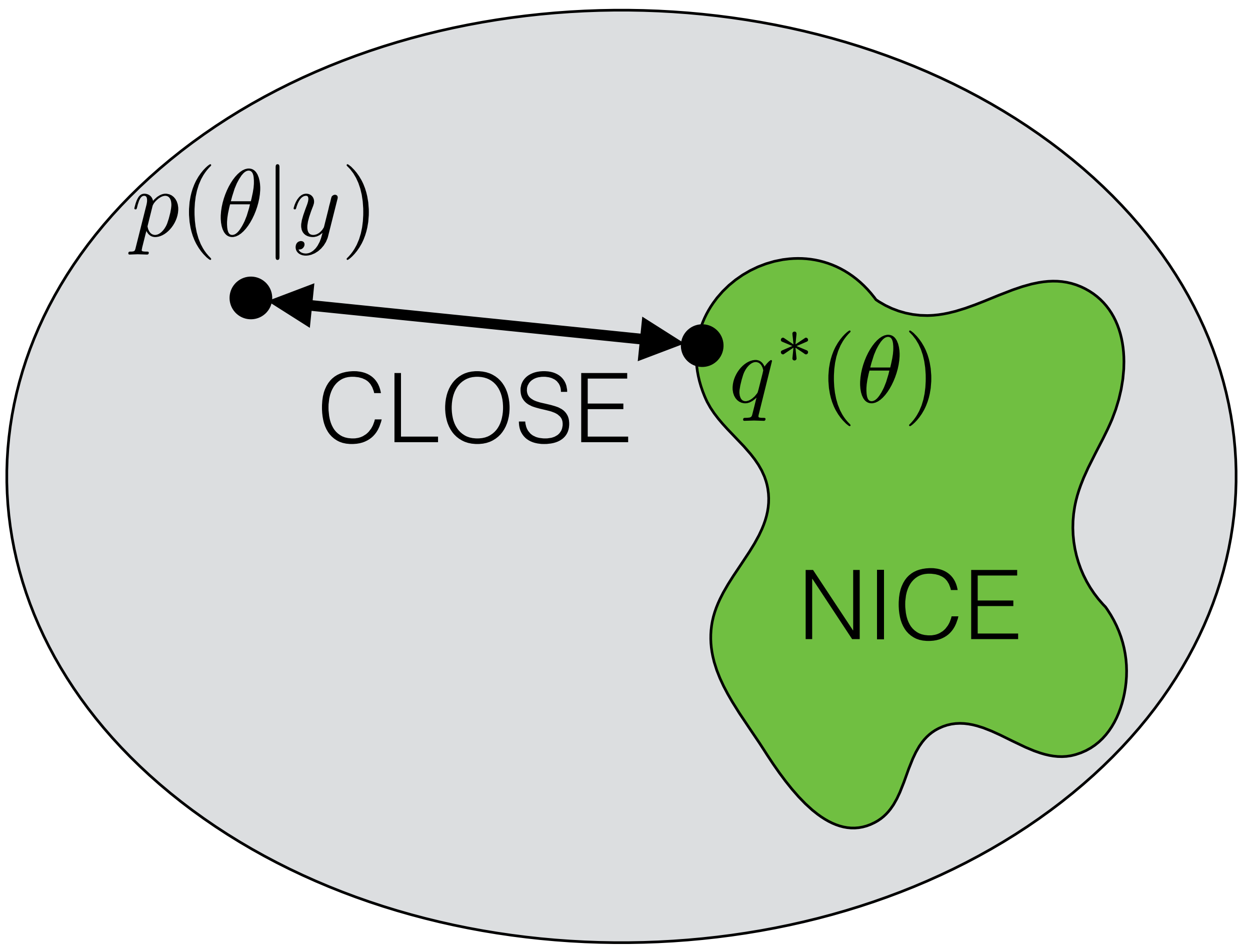

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[board]

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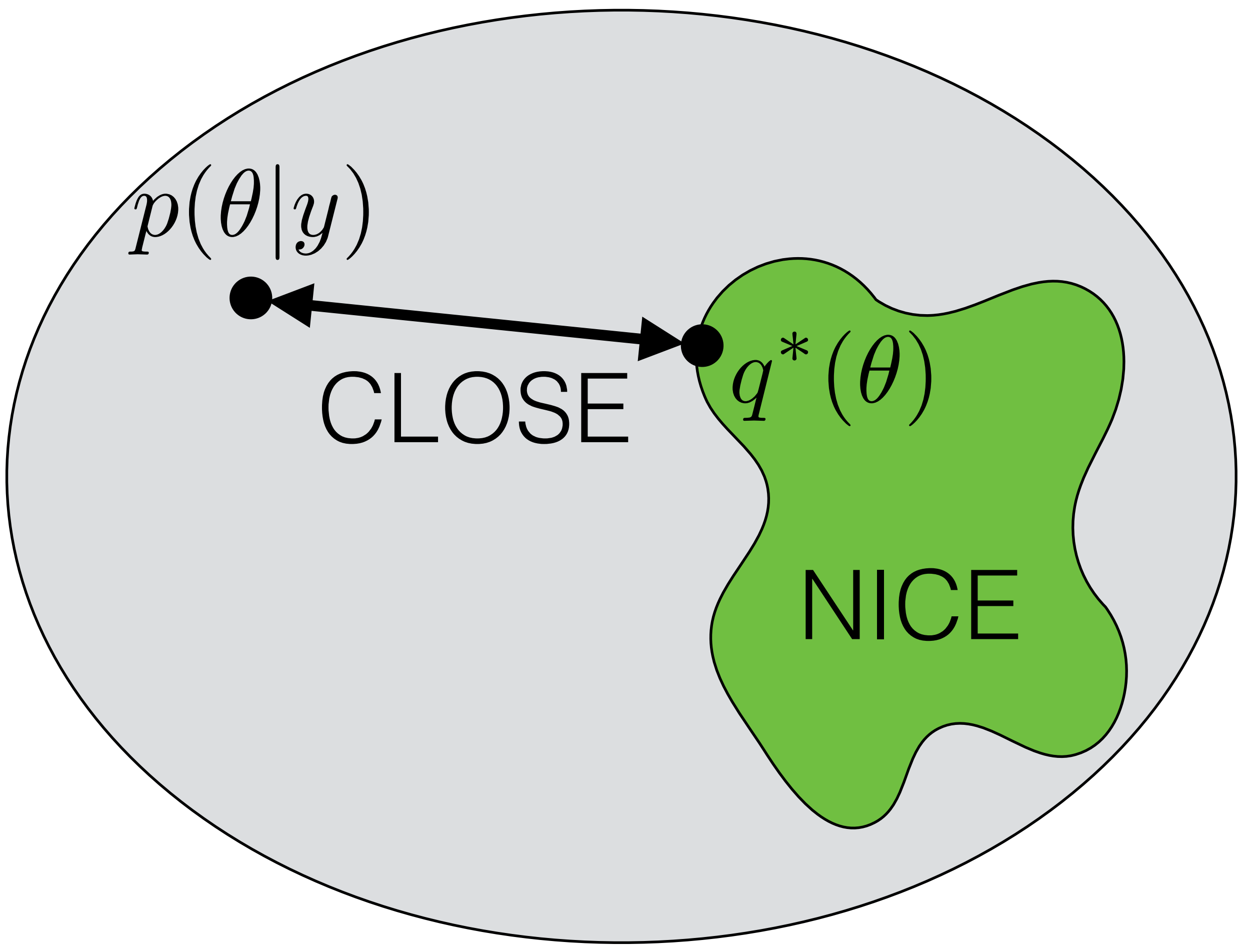

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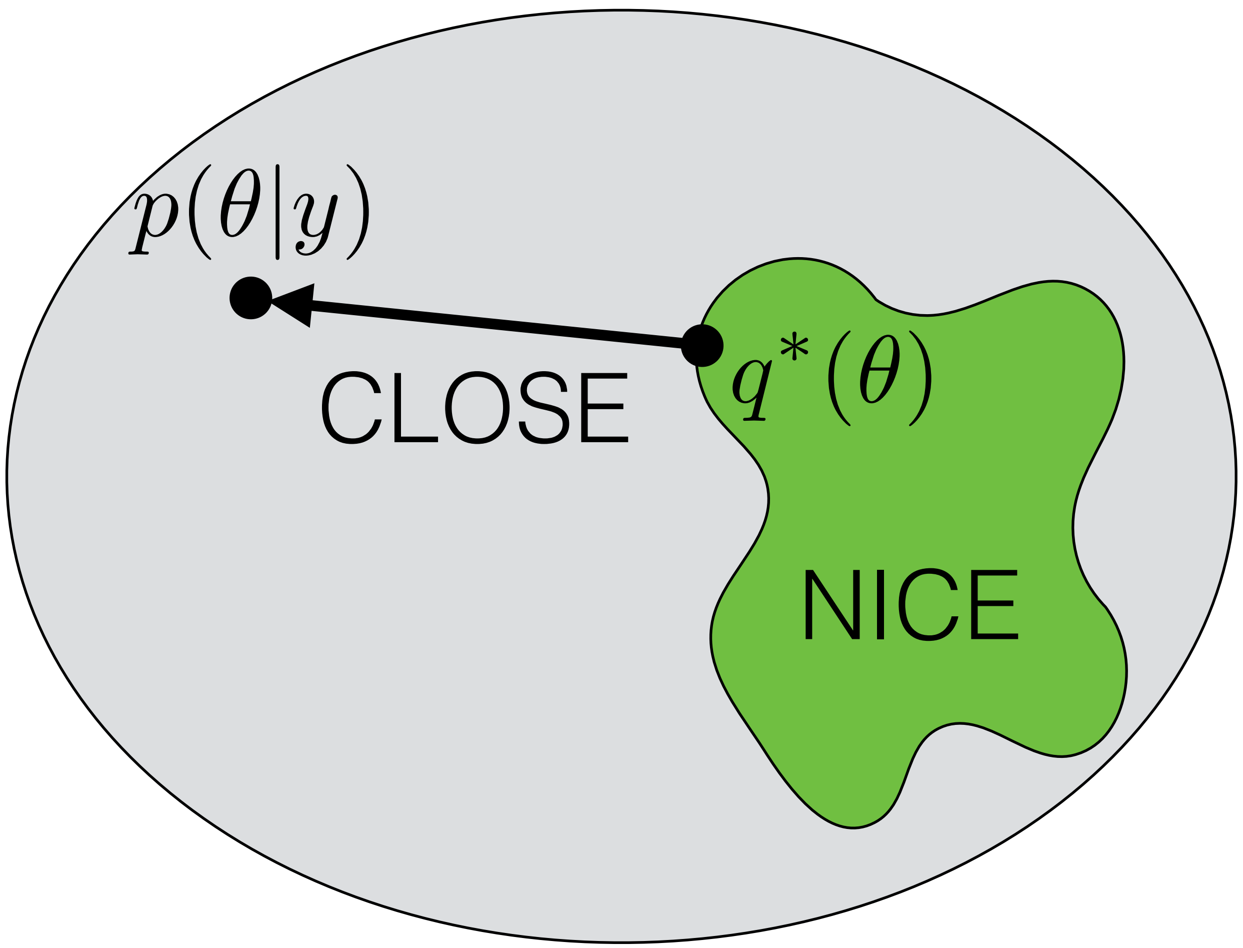

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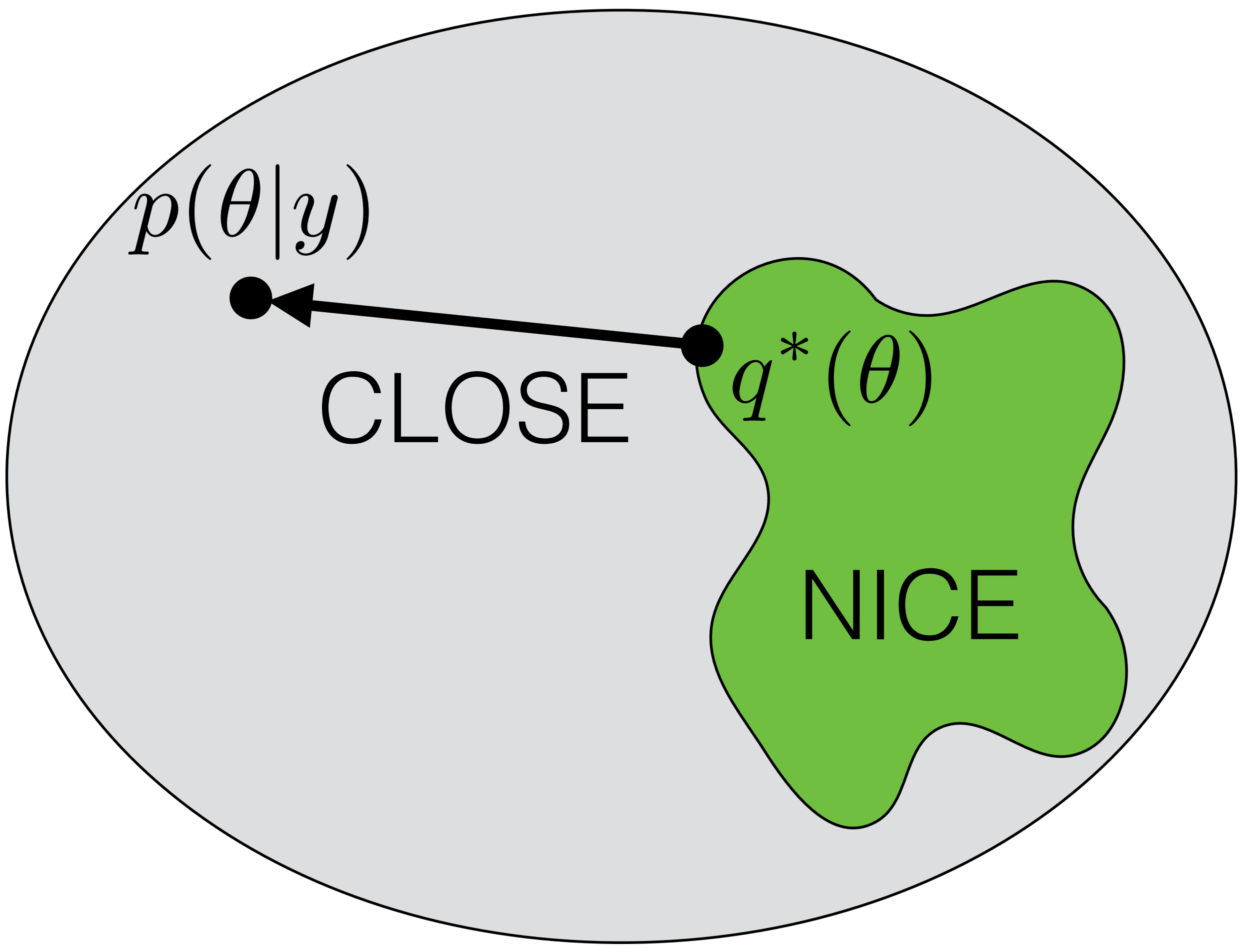

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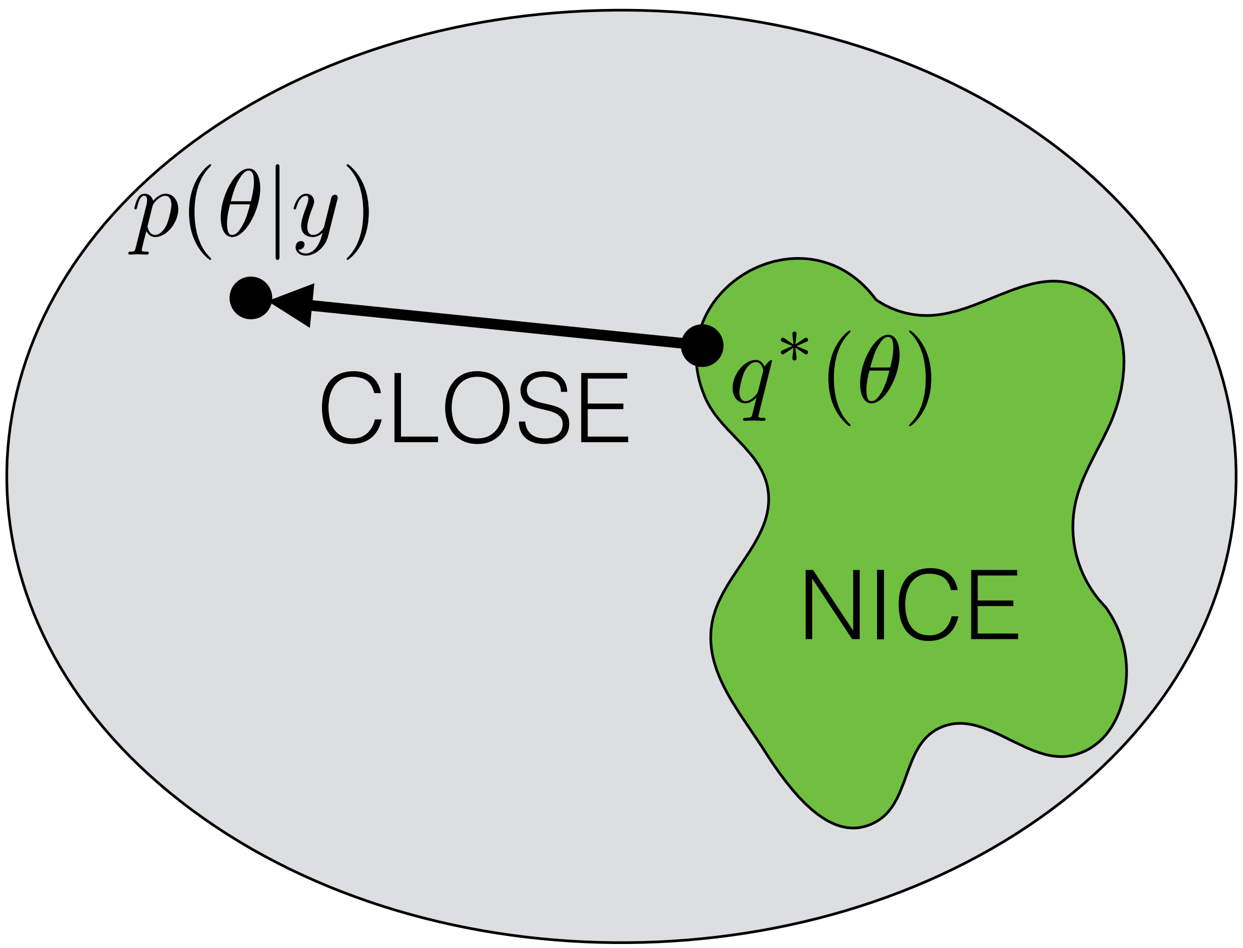

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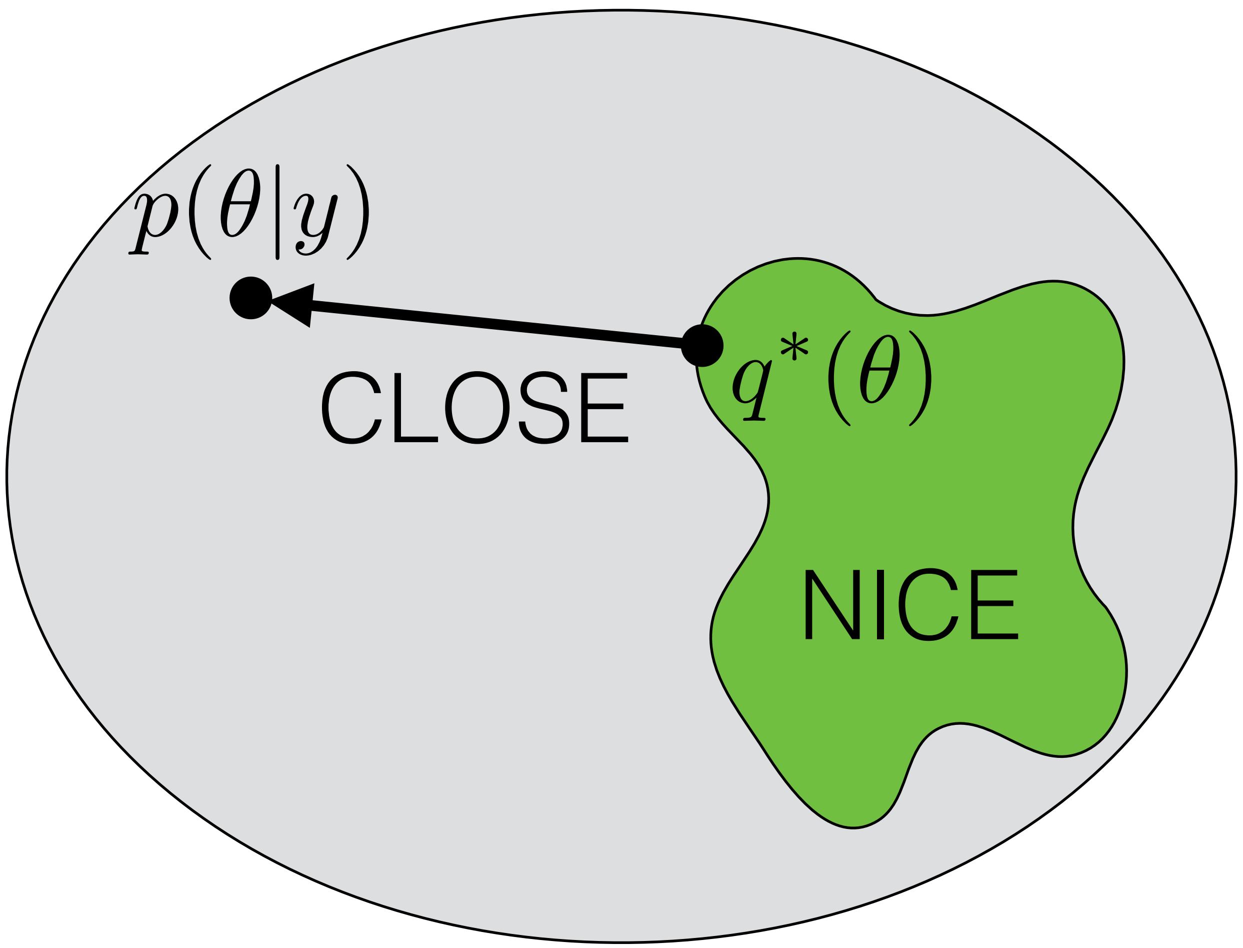

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- VB practical success: point estimates and prediction, fast

Approximate Bayesian Inference

[Bardenet, Doucet, Holmes 2017]

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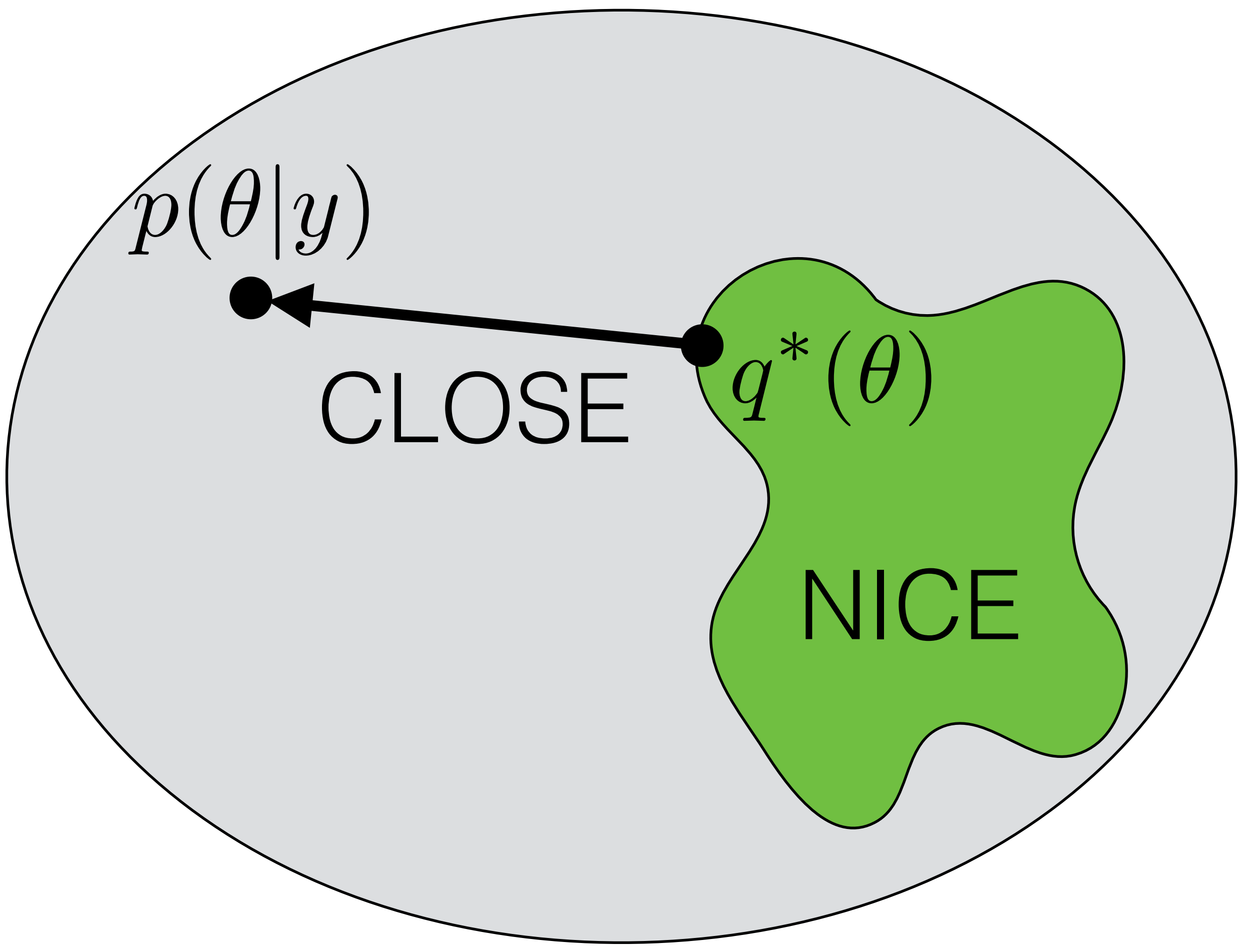

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$$KL(q(\cdot)||p(\cdot|y))$$

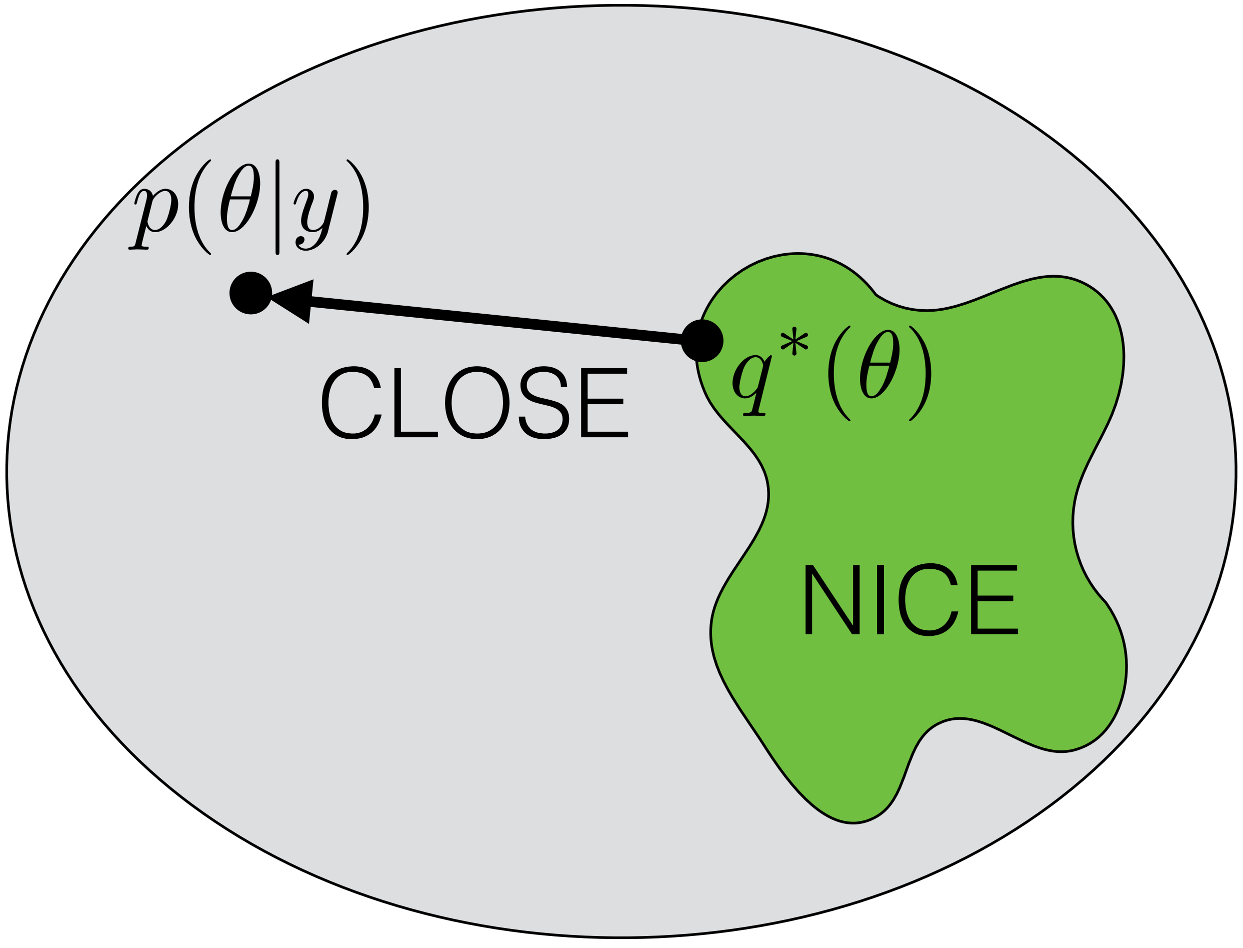
- VB practical success: point estimates and prediction, fast, streaming, distributed (3.6M Wikipedia, 350K Nature)

[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

Why KL?

- Variational Bayes

$$q^* = \operatorname{argmin}_{q \in Q} \mathrm{KL}\left(q(\cdot) || p(\cdot|y)\right)$$



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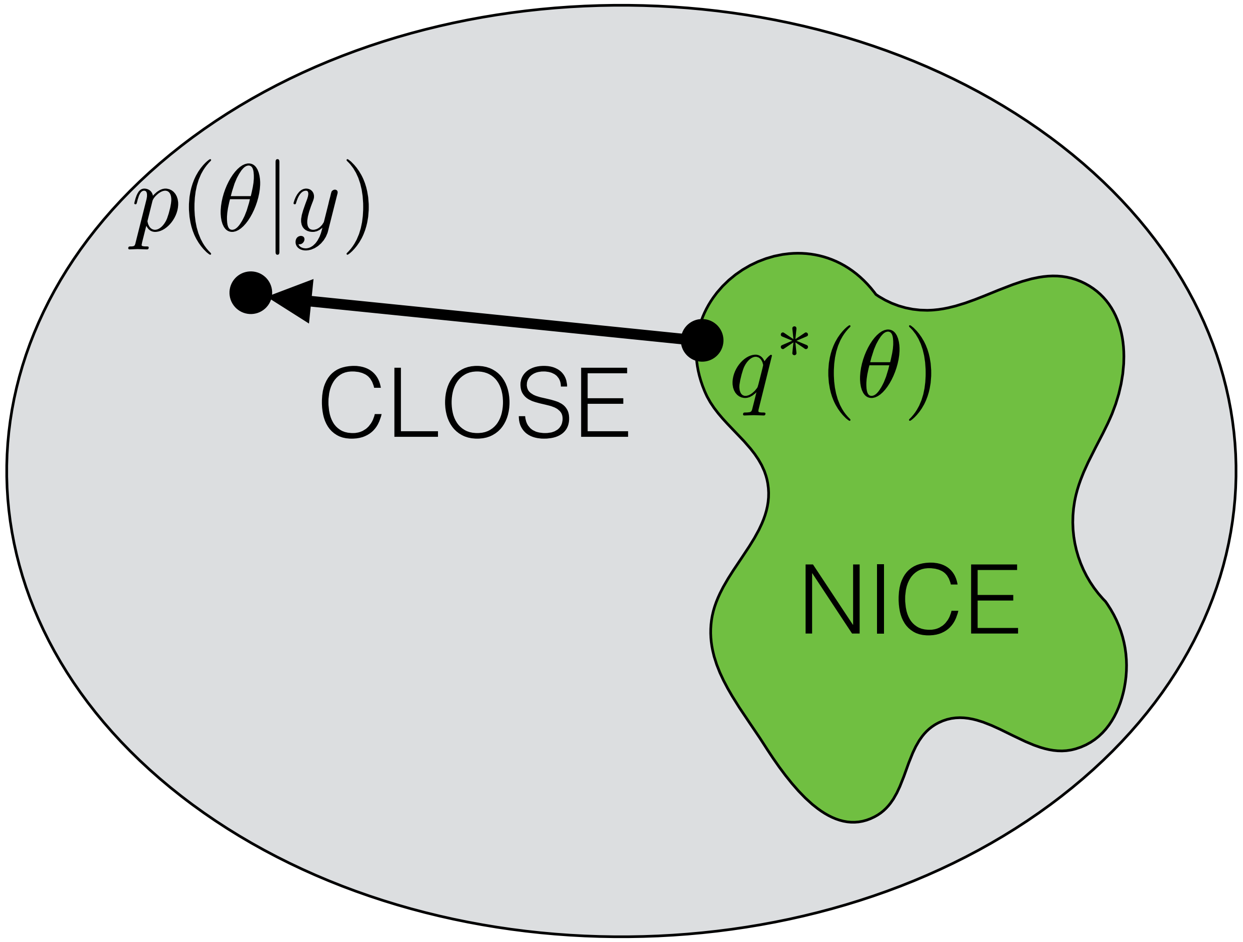

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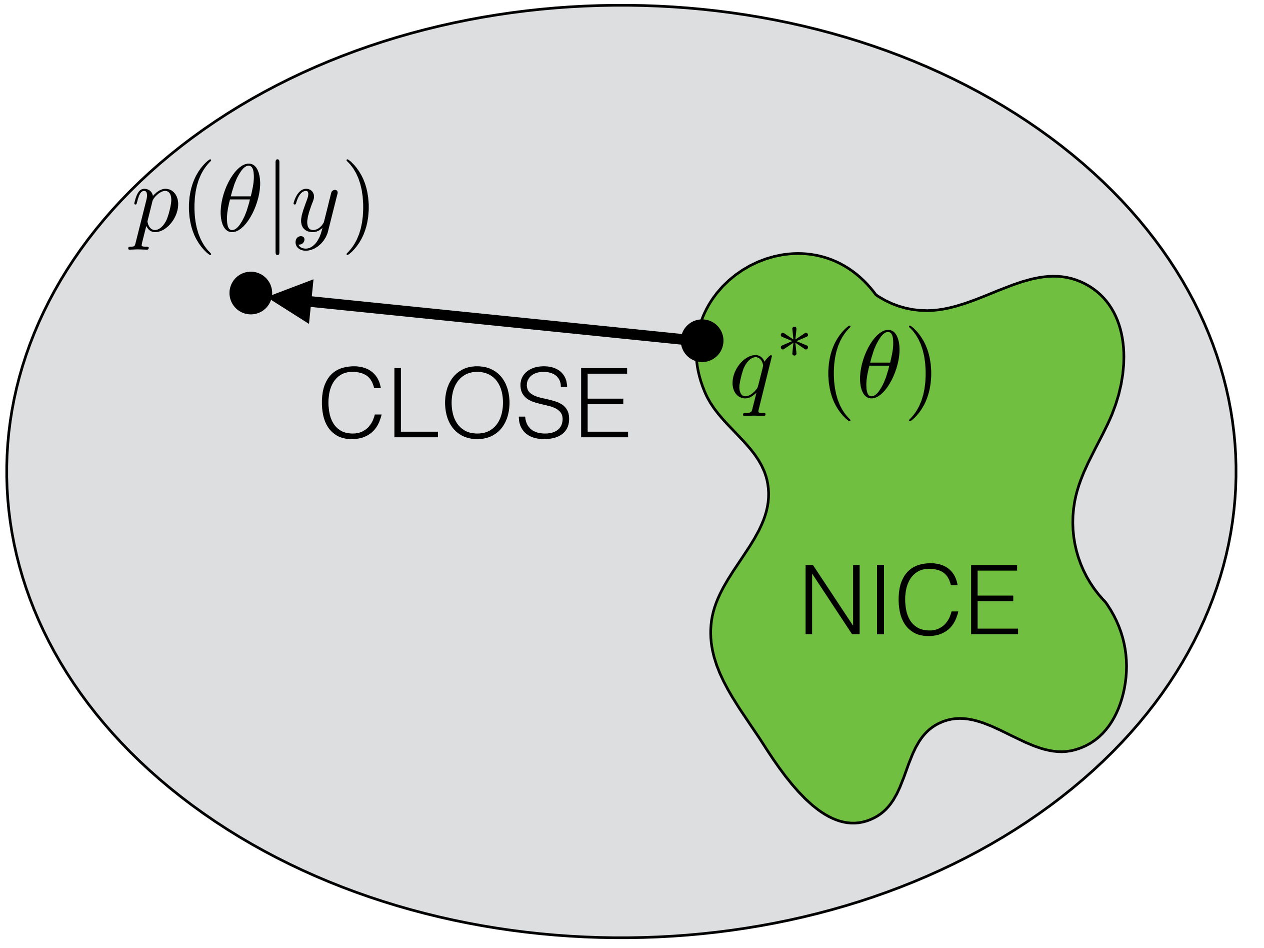

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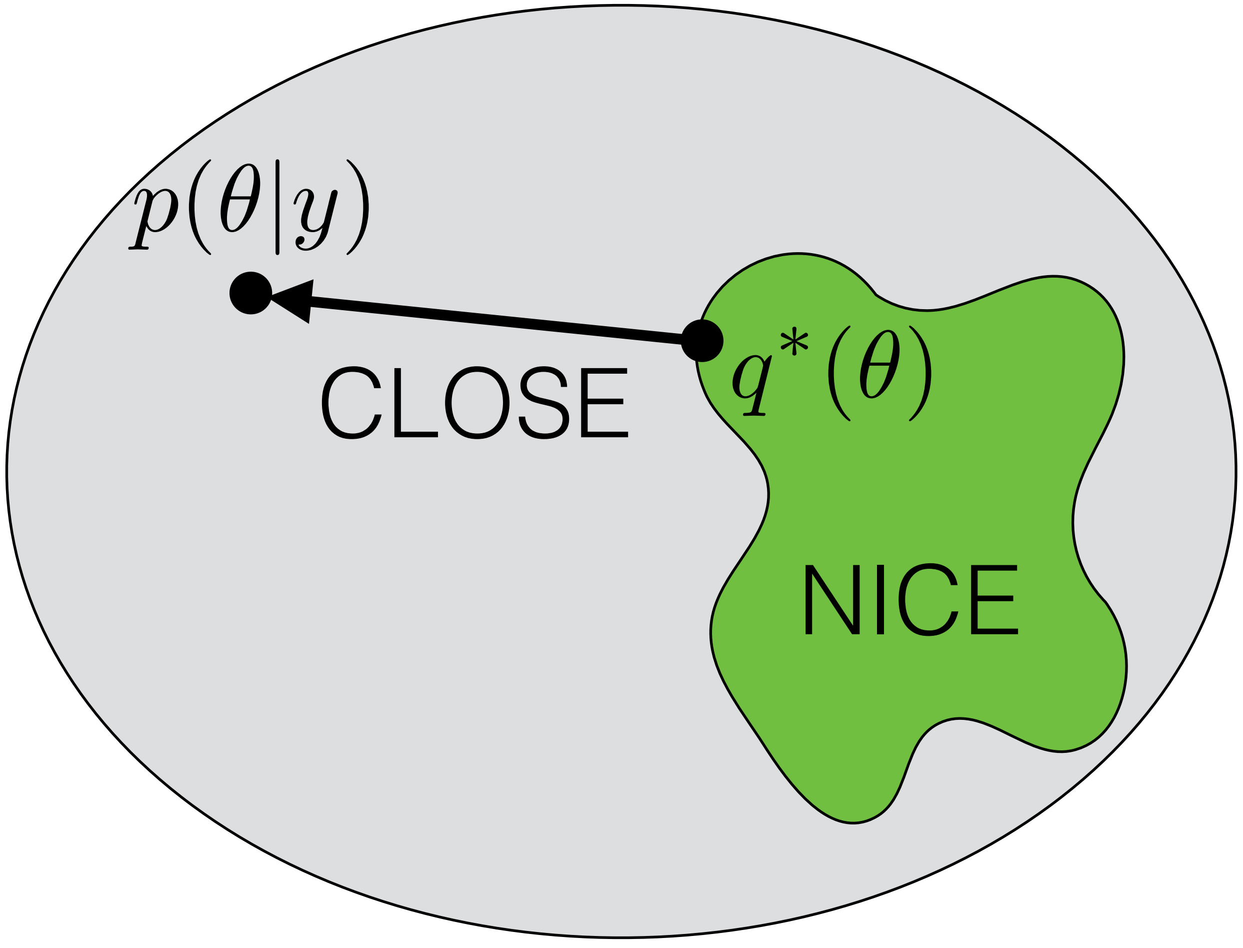

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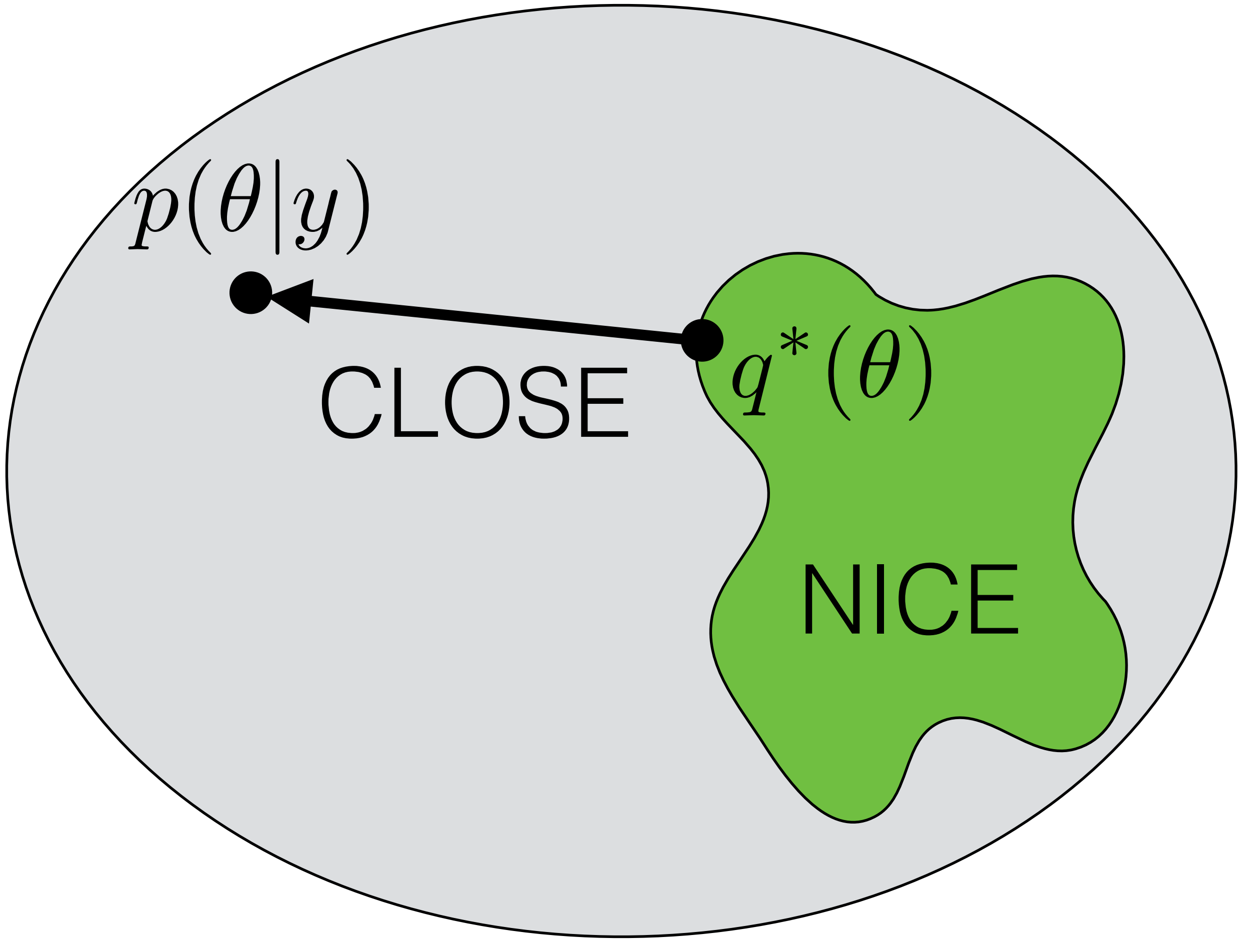

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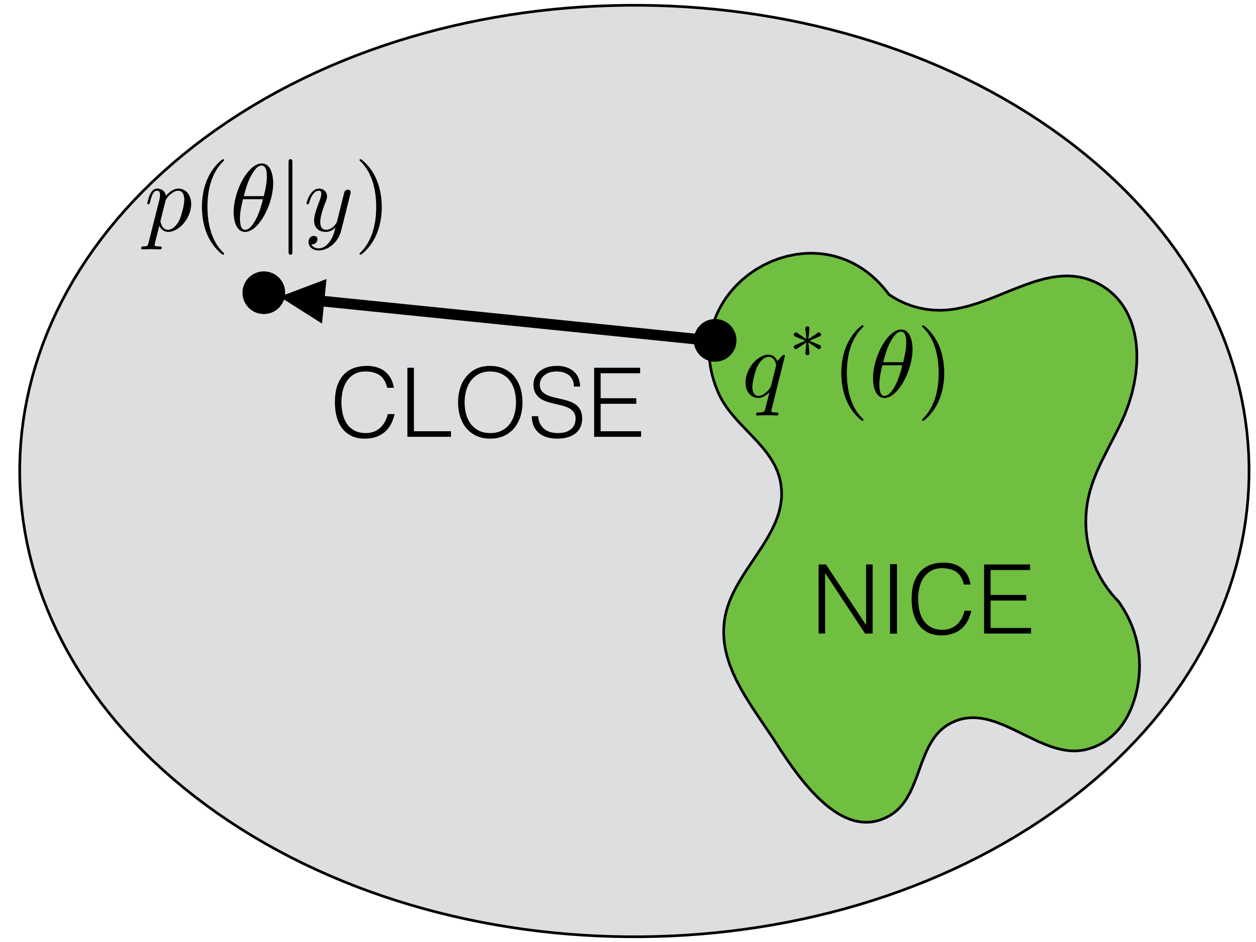

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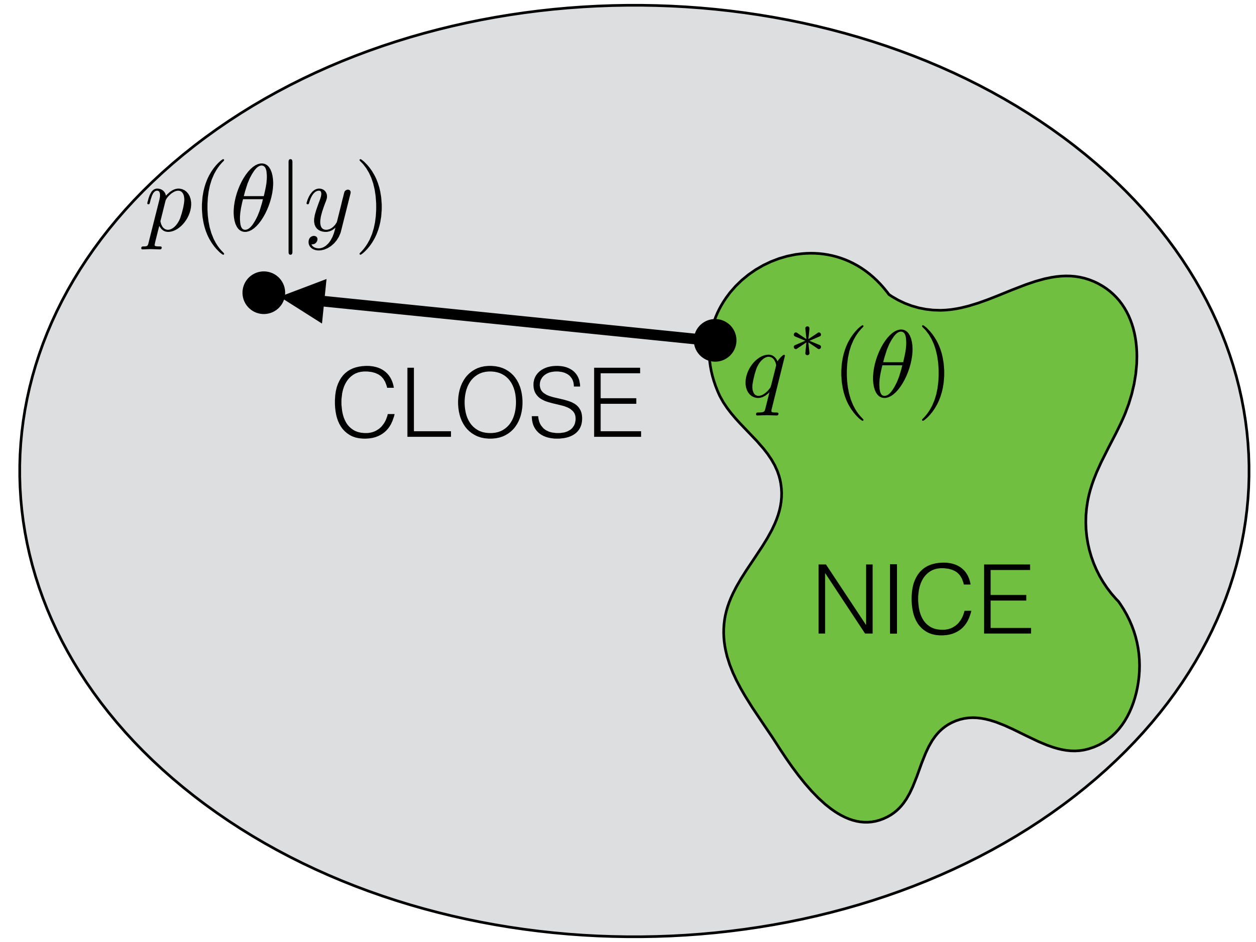

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"Evidence lower bound" (ELBO)

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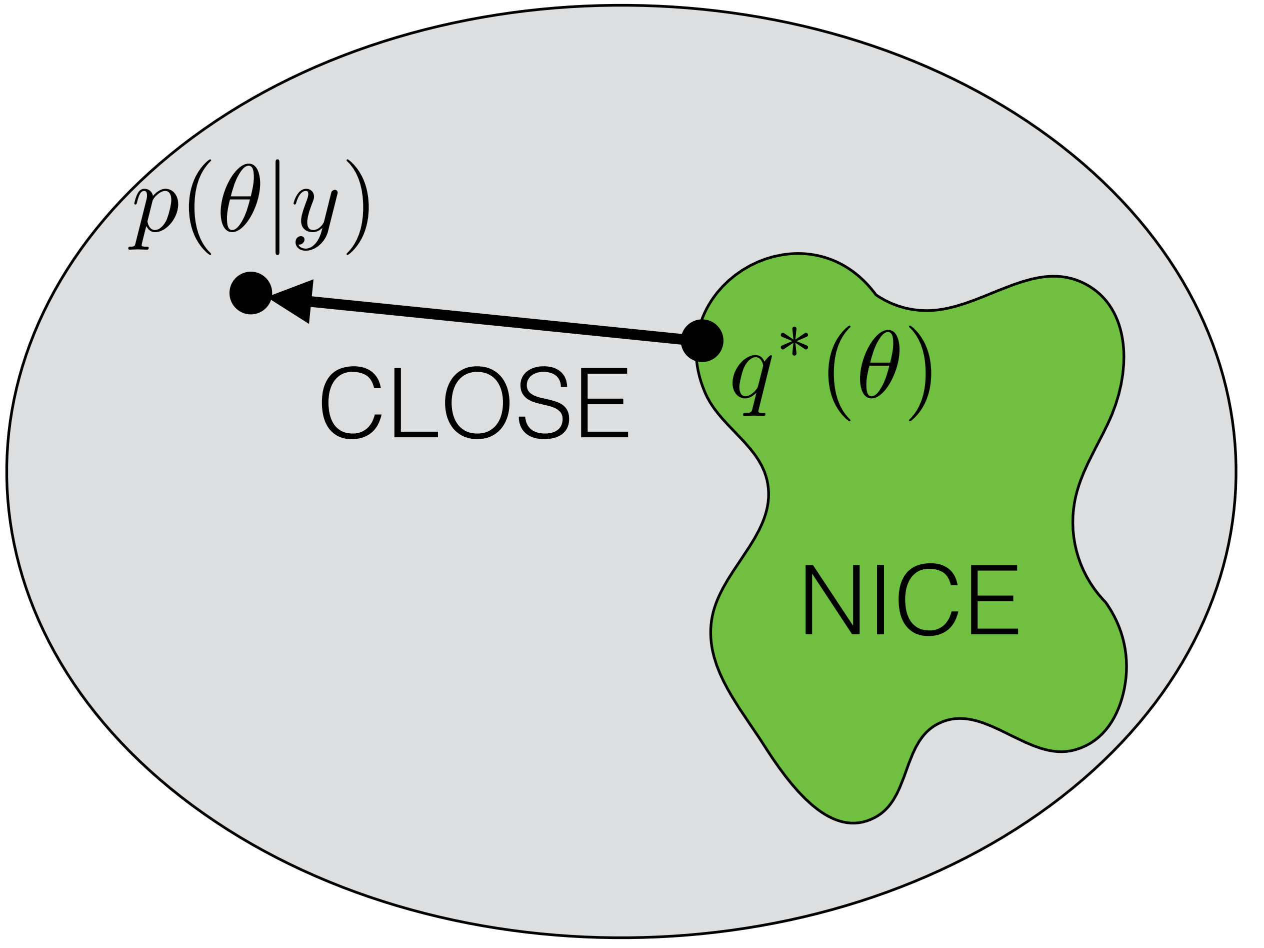

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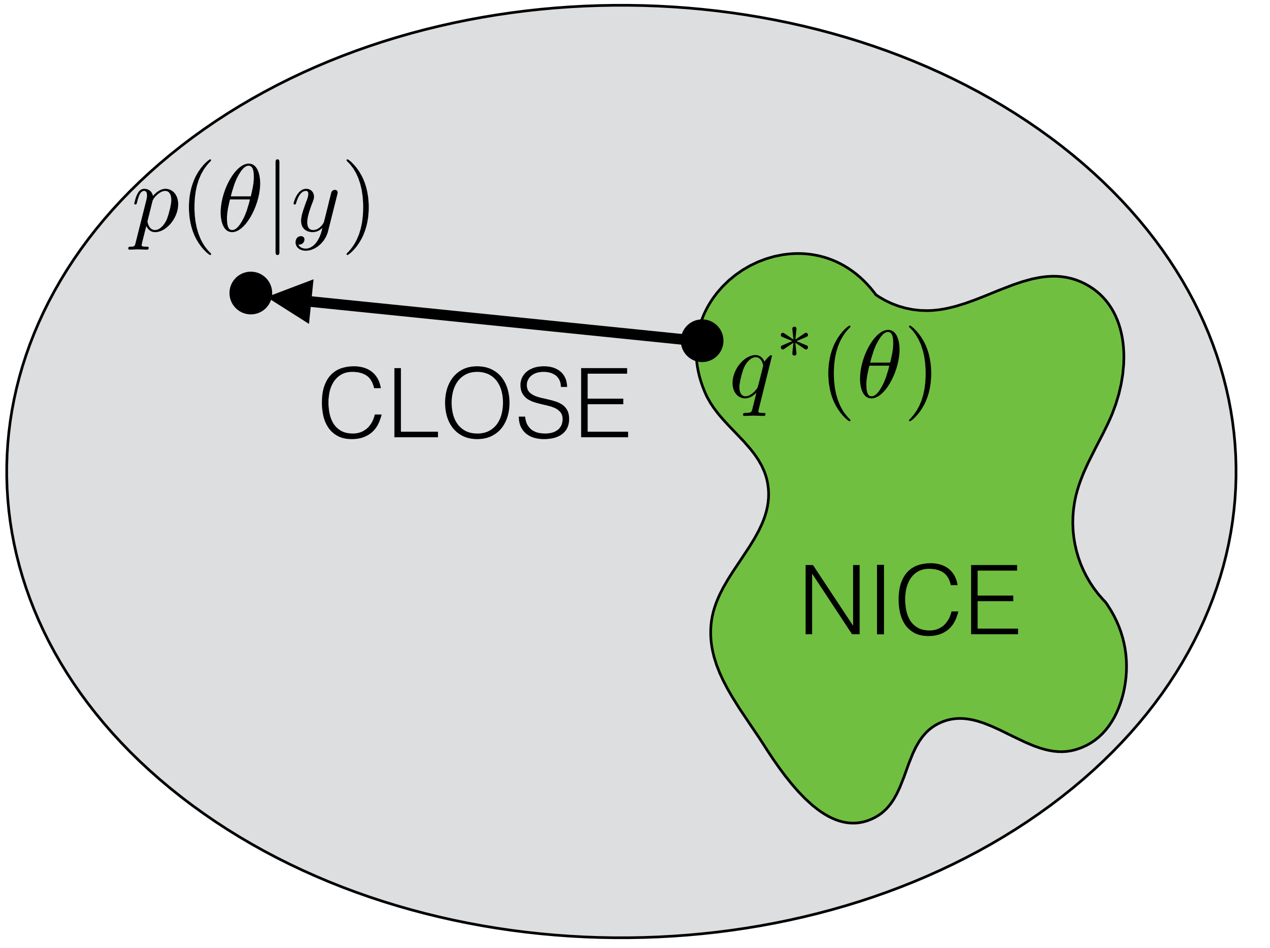

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"Evidence lower bound" (ELBO)

- $q^* = \text{argmax}_{q \in Q} \text{ELBO}(q)$

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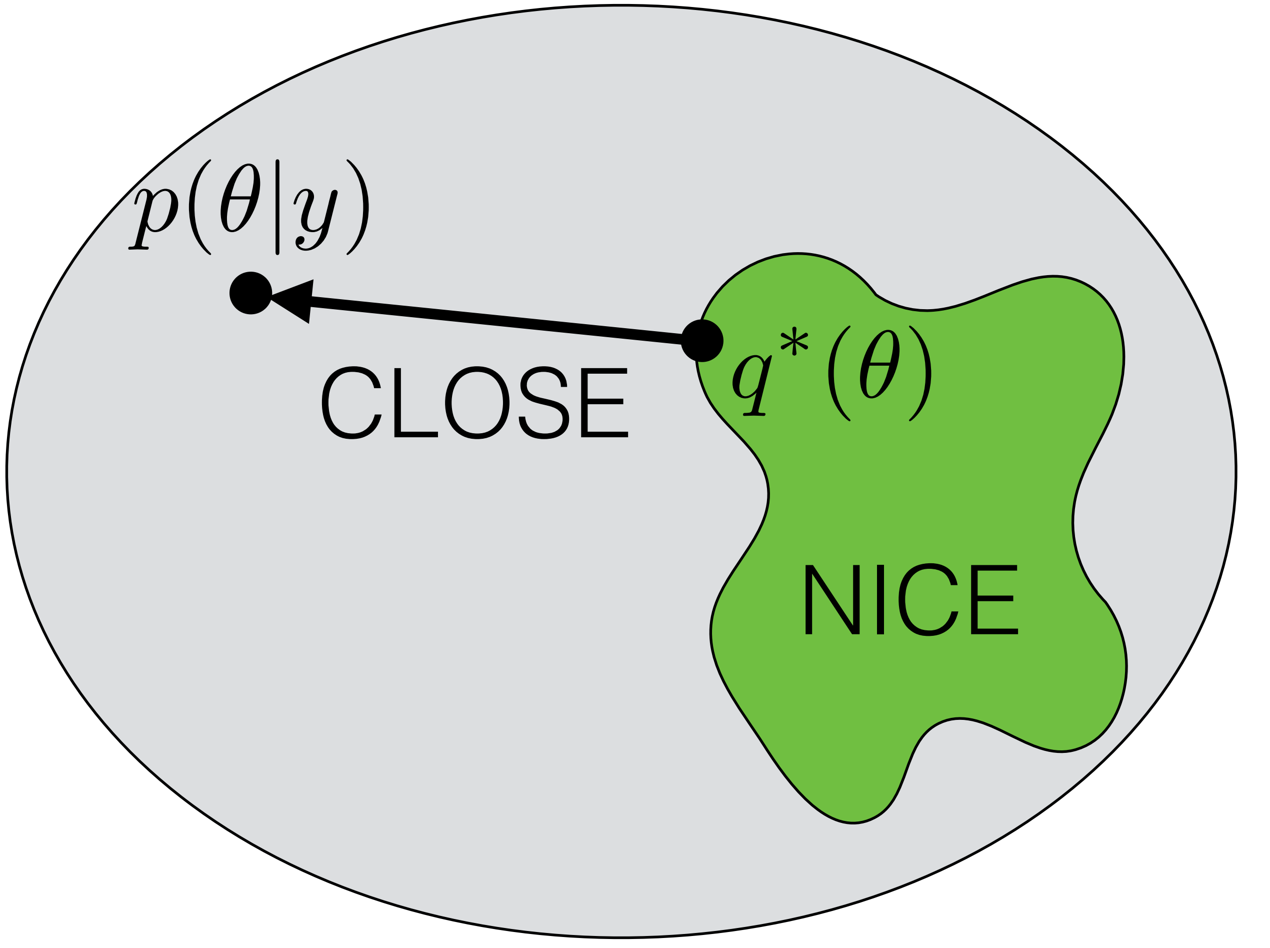

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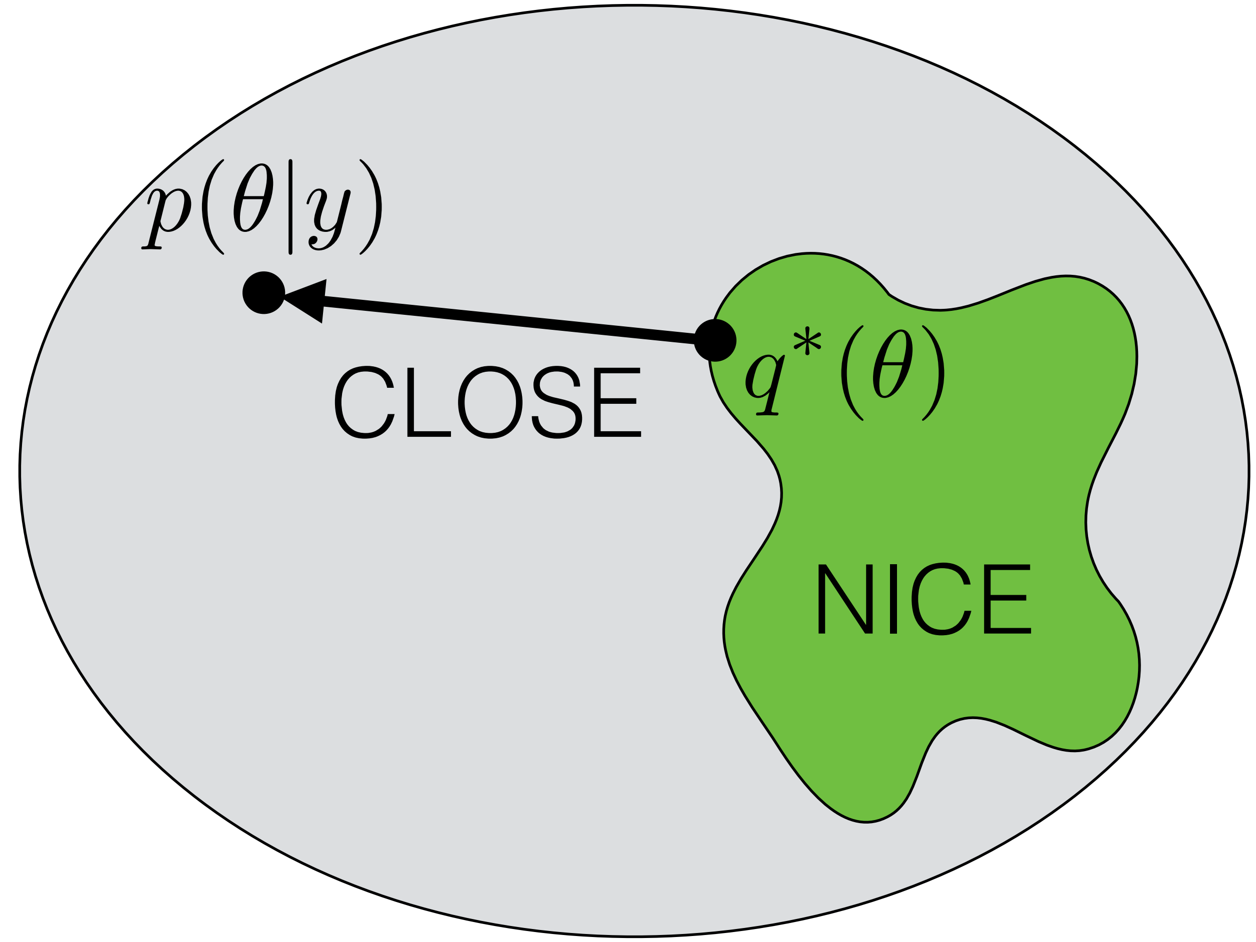

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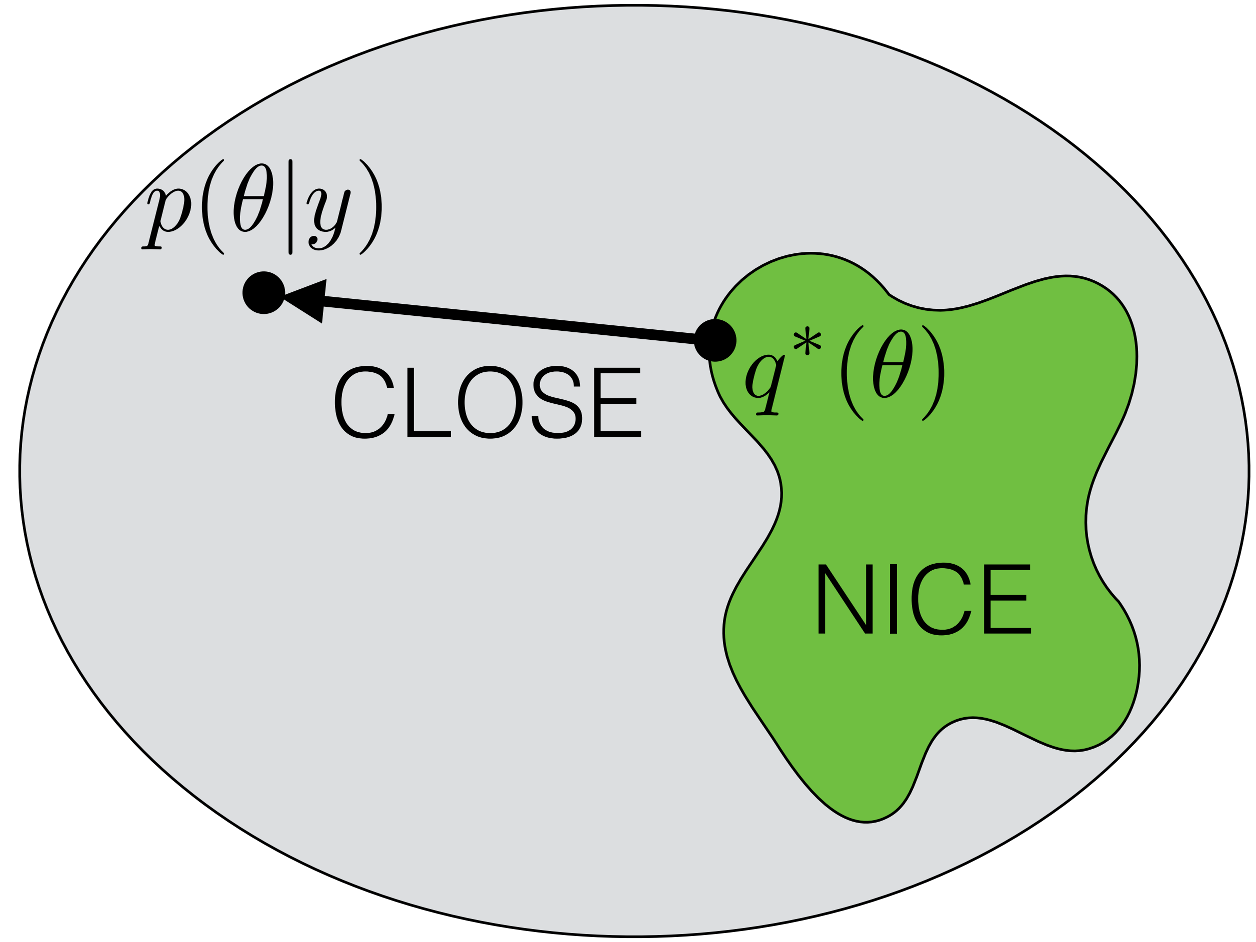

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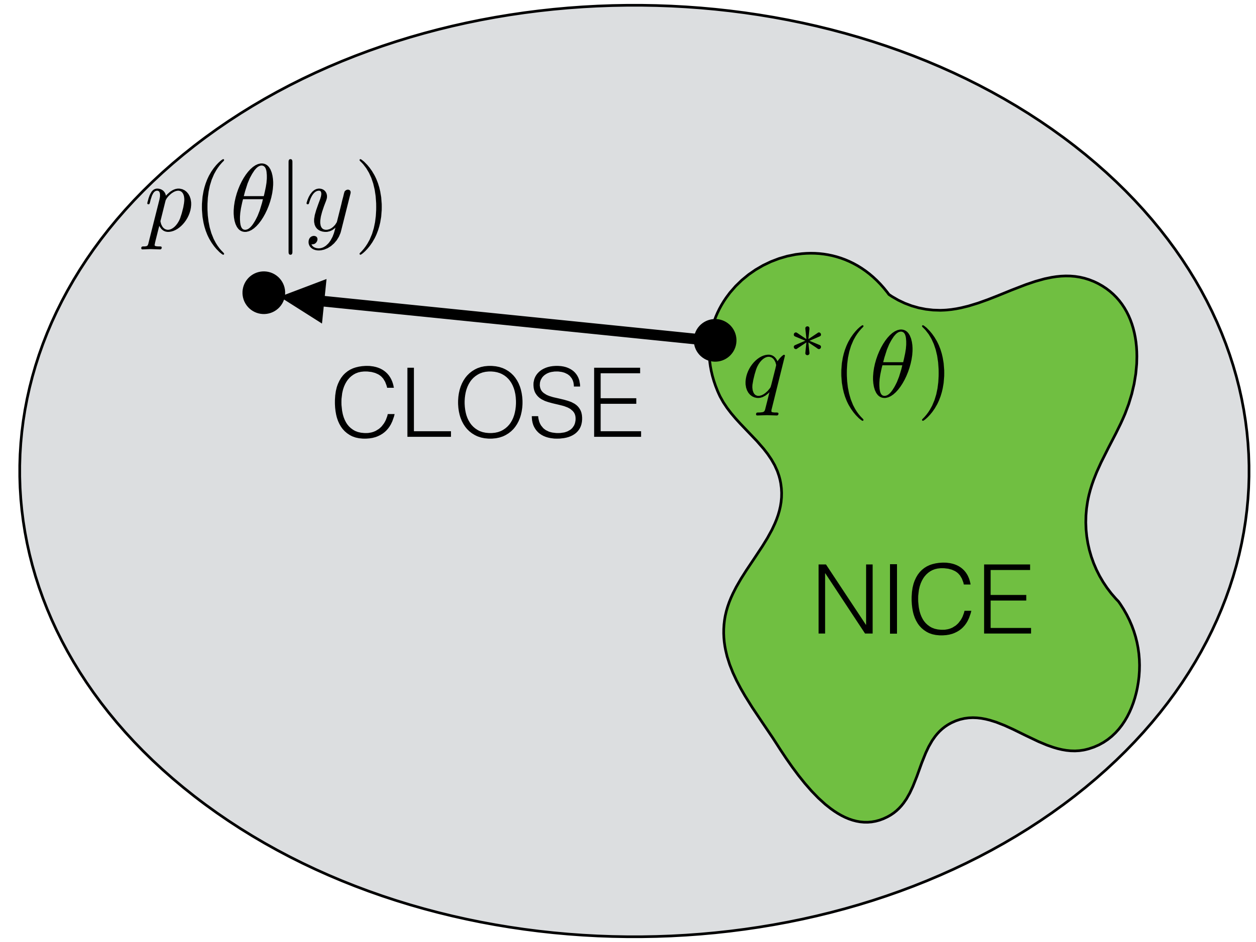

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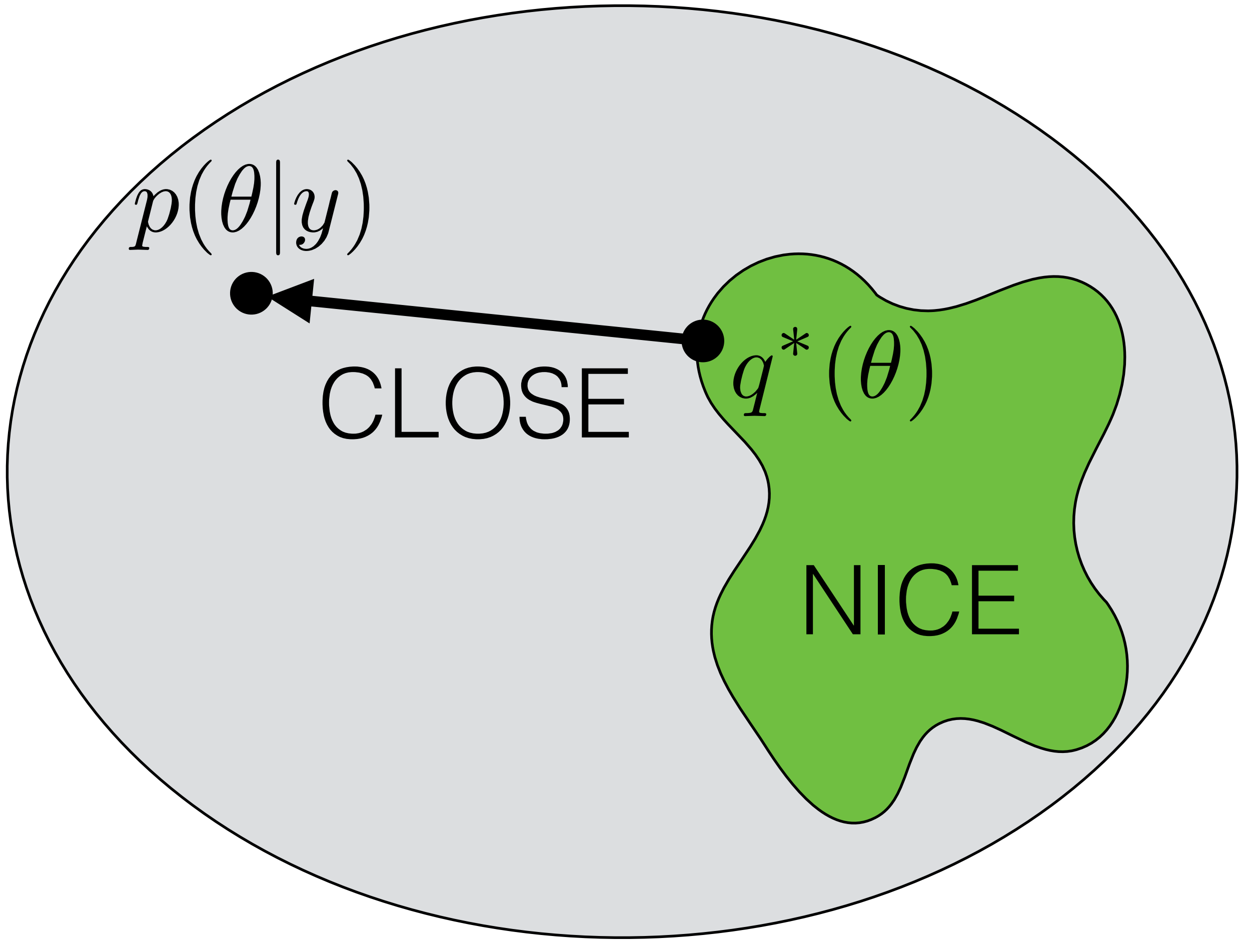

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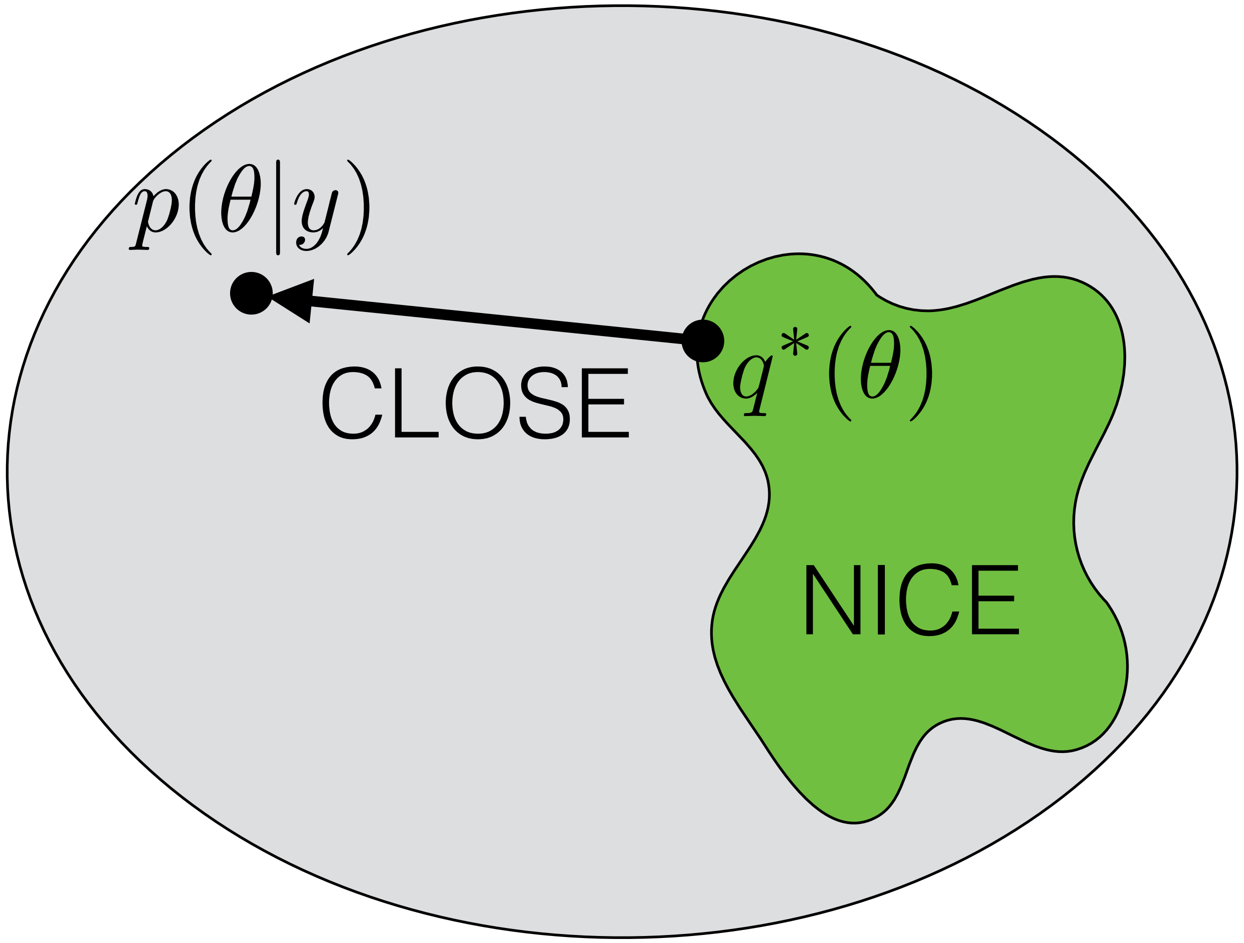

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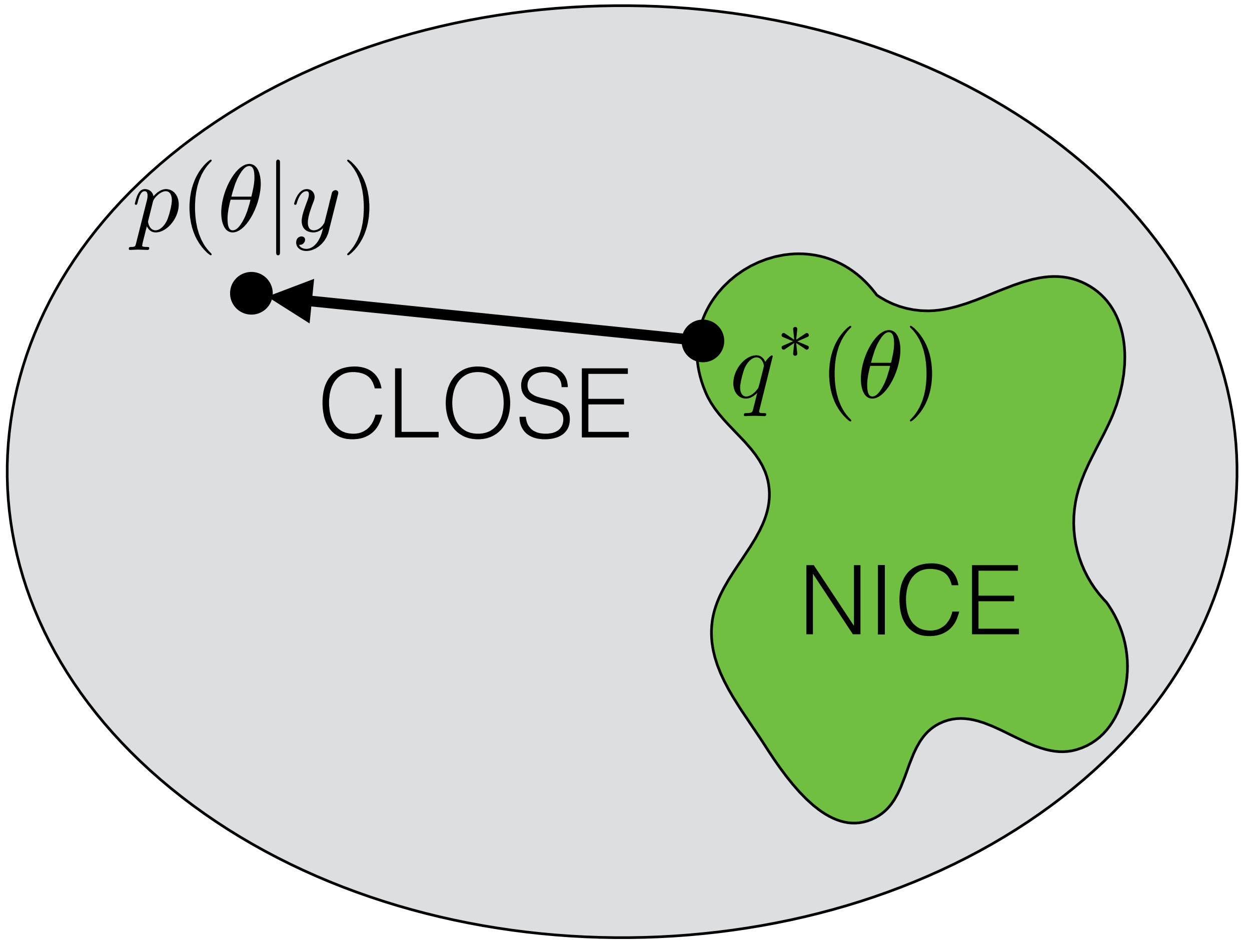

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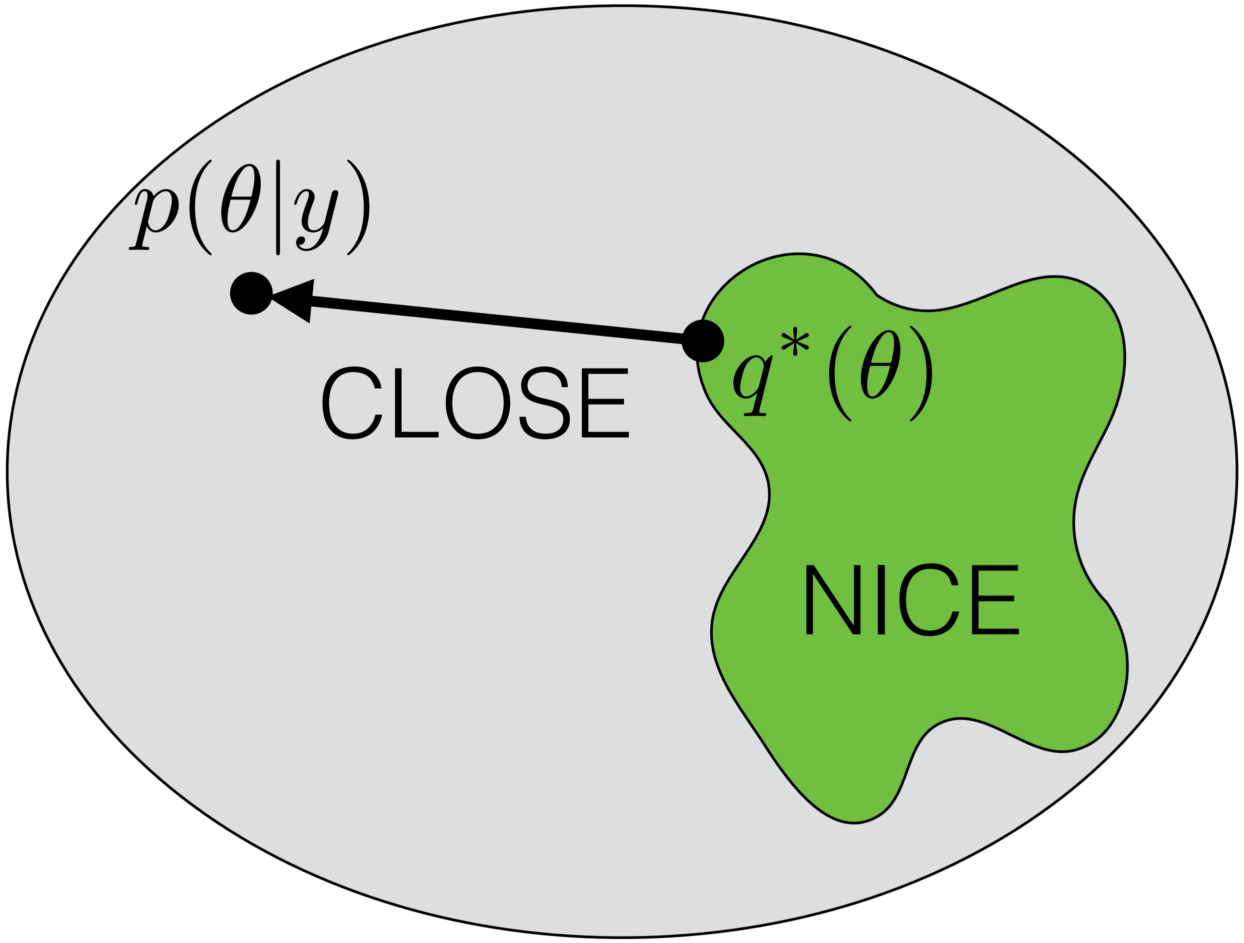

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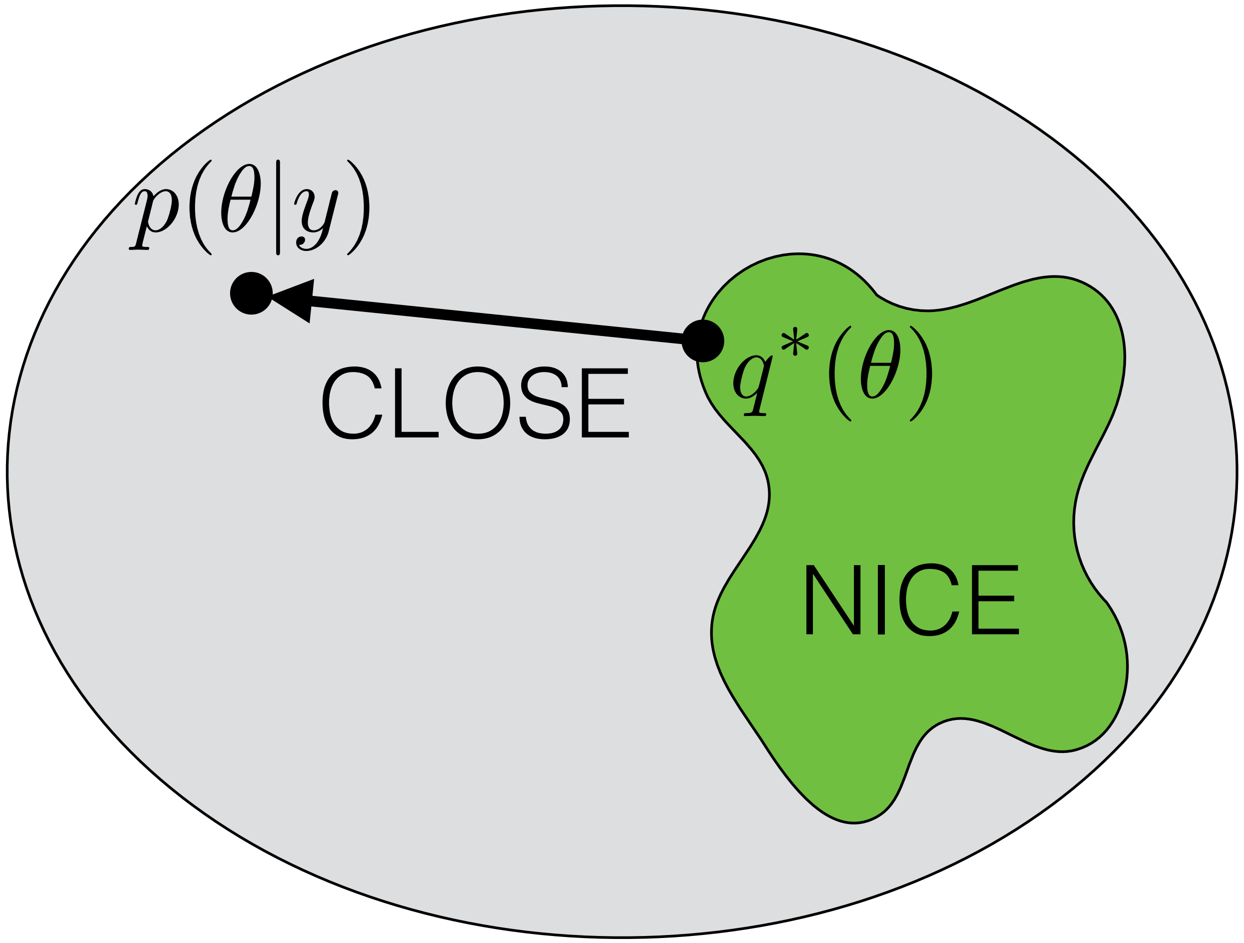

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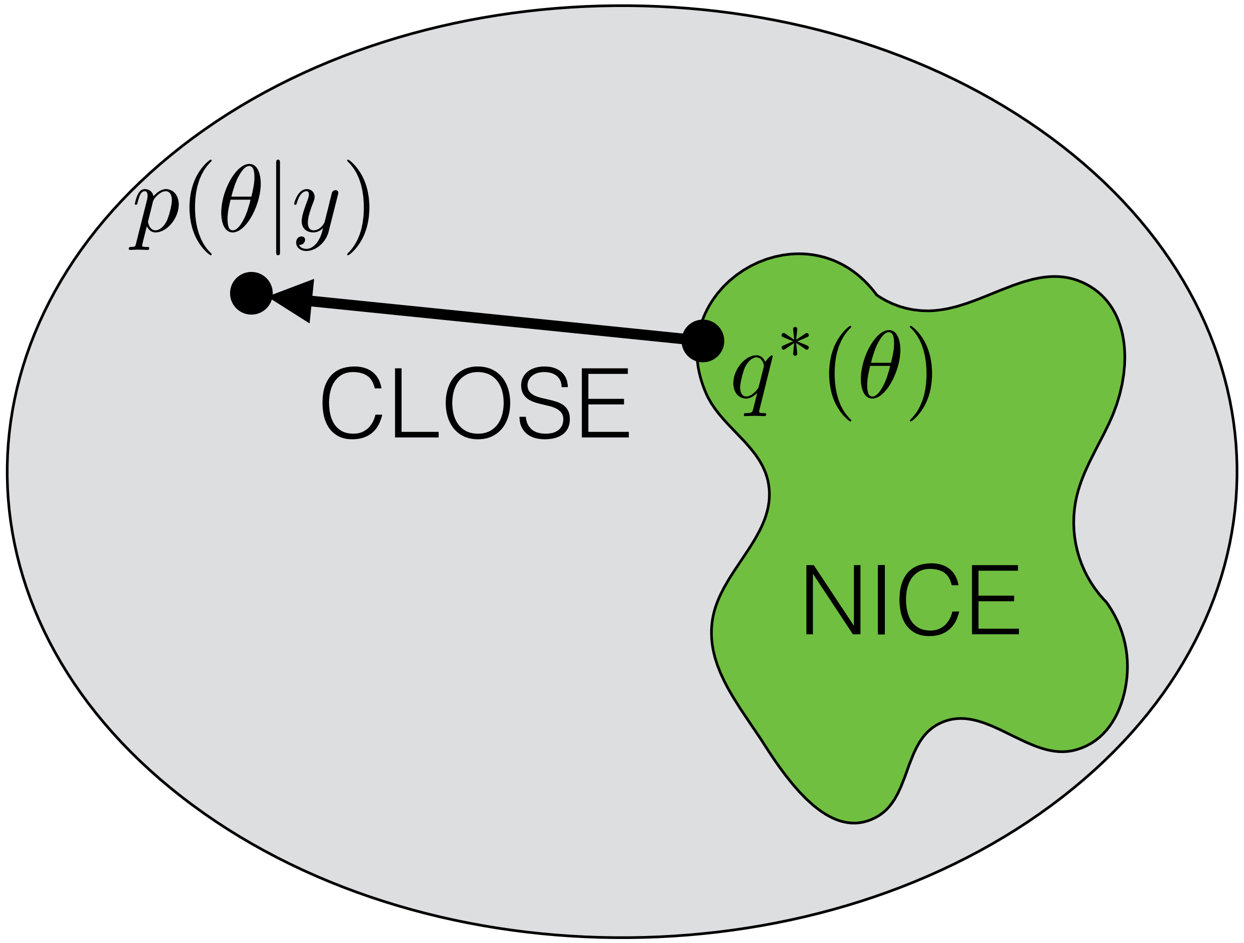

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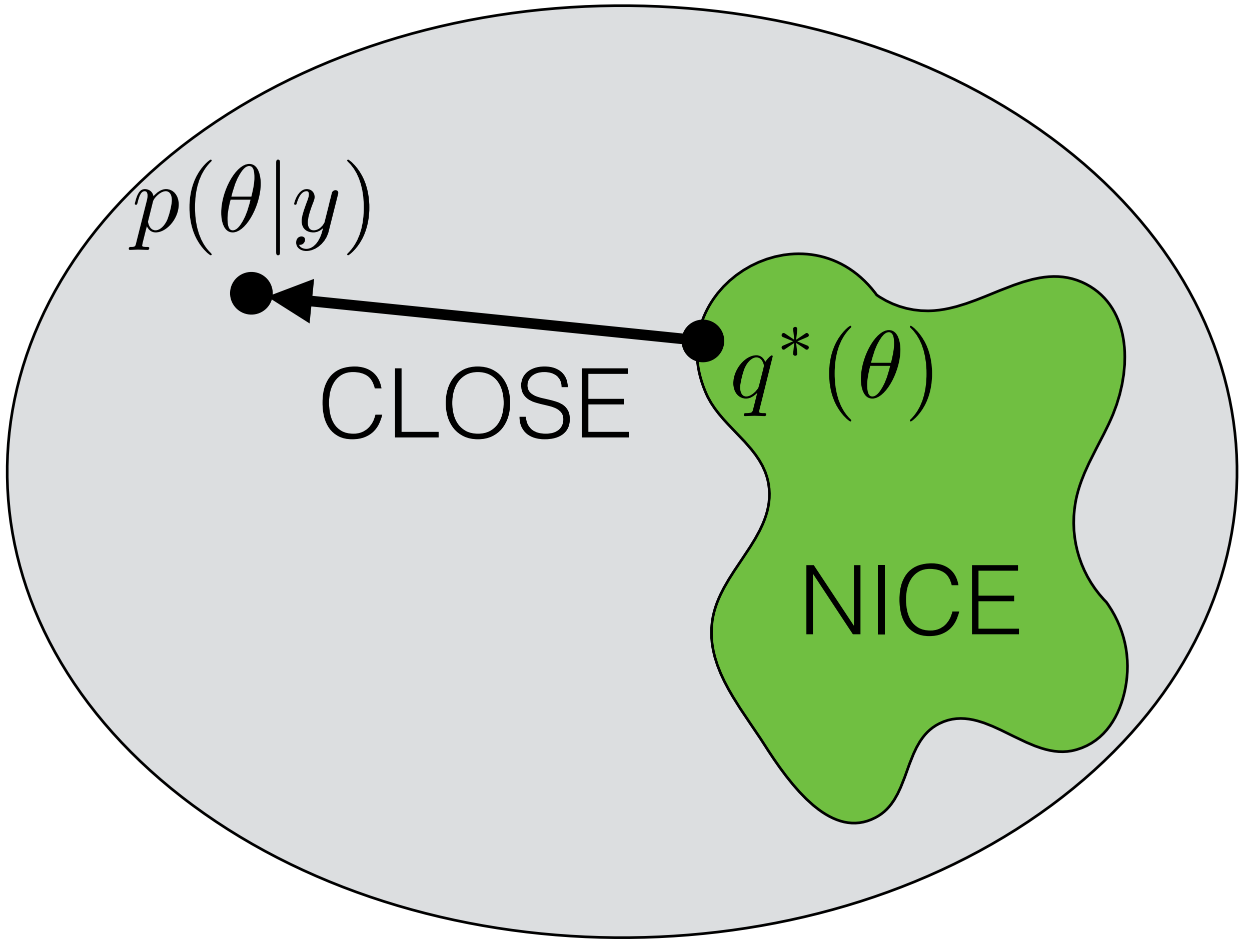

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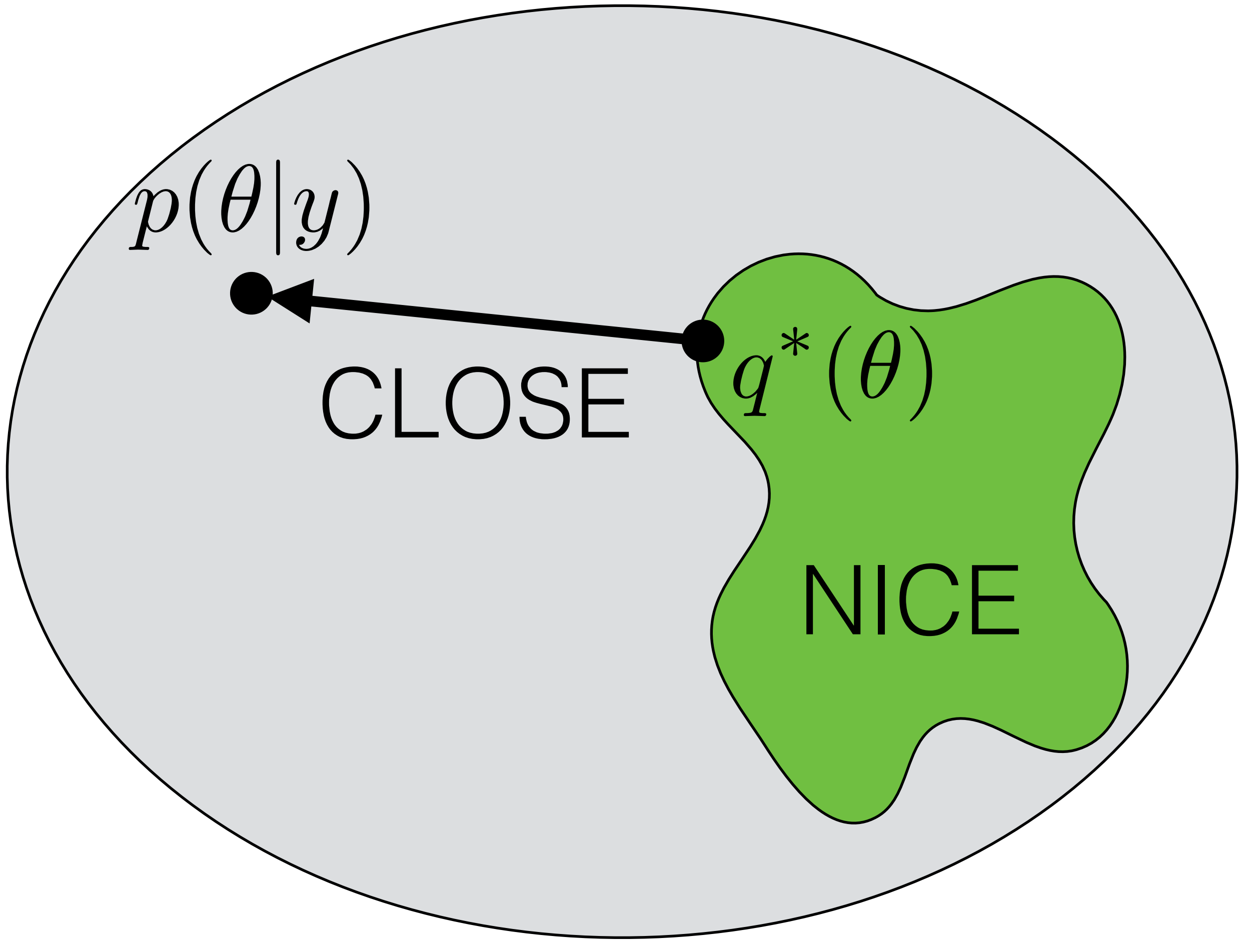

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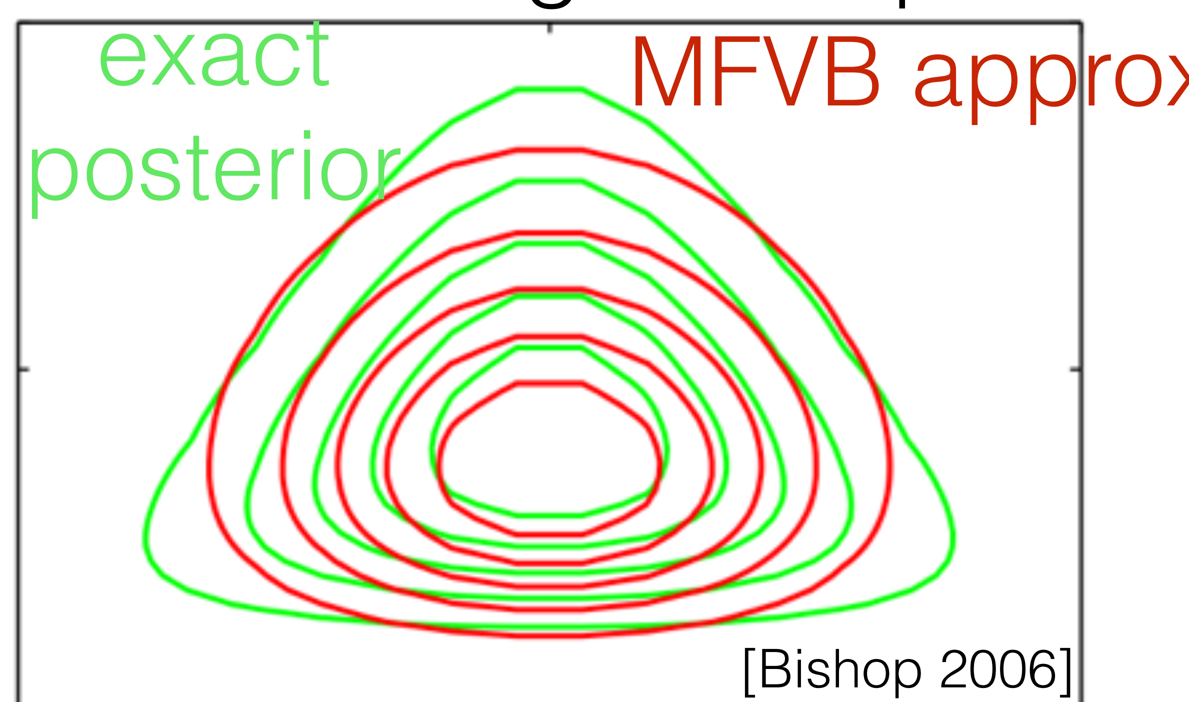

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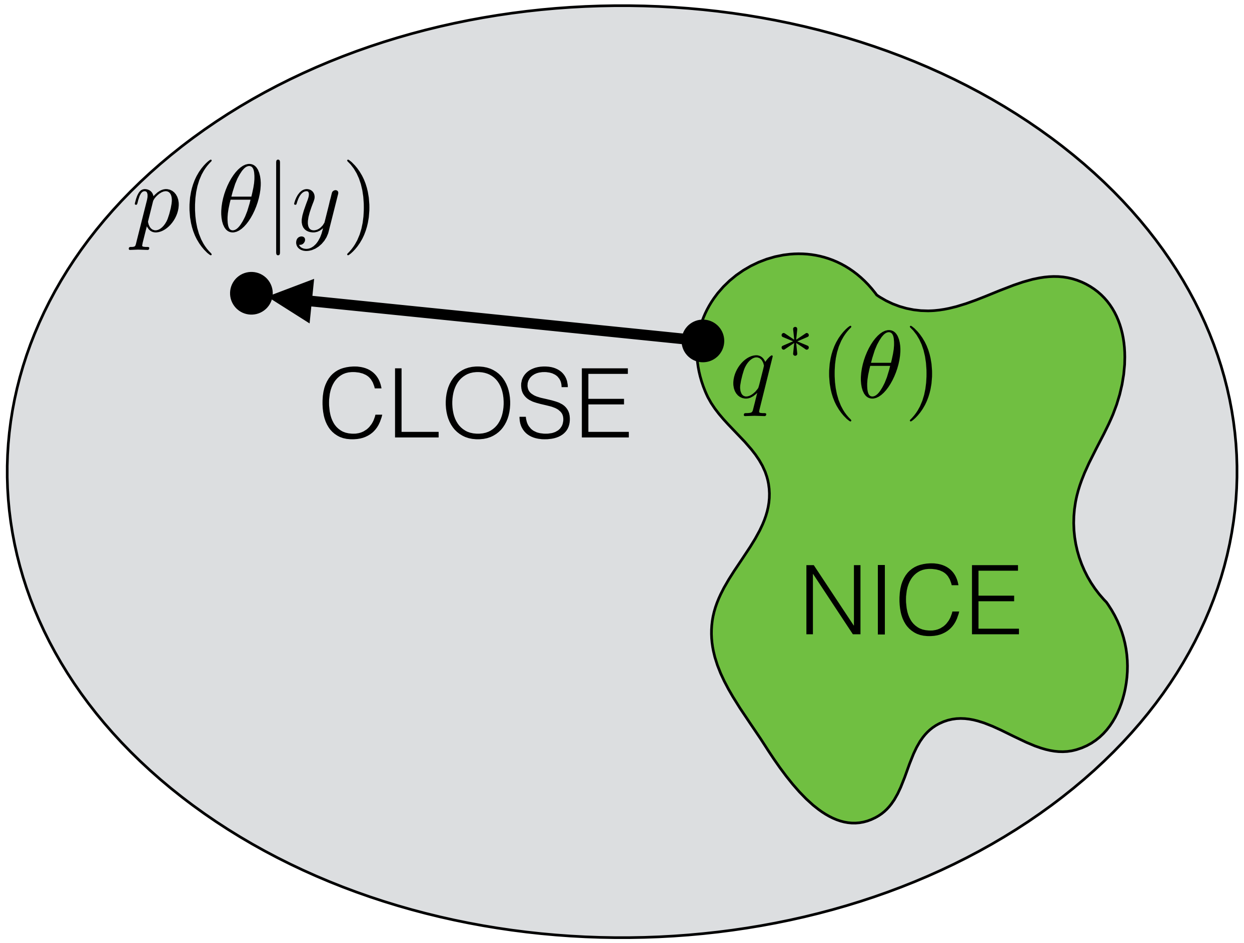

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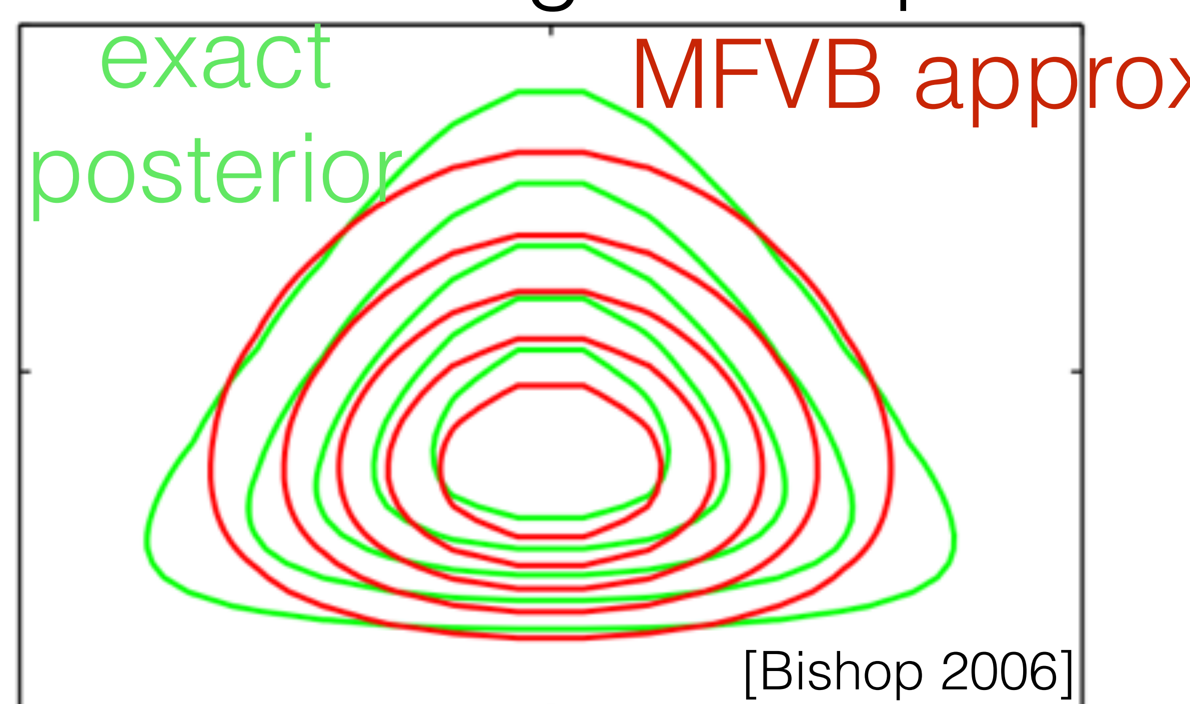

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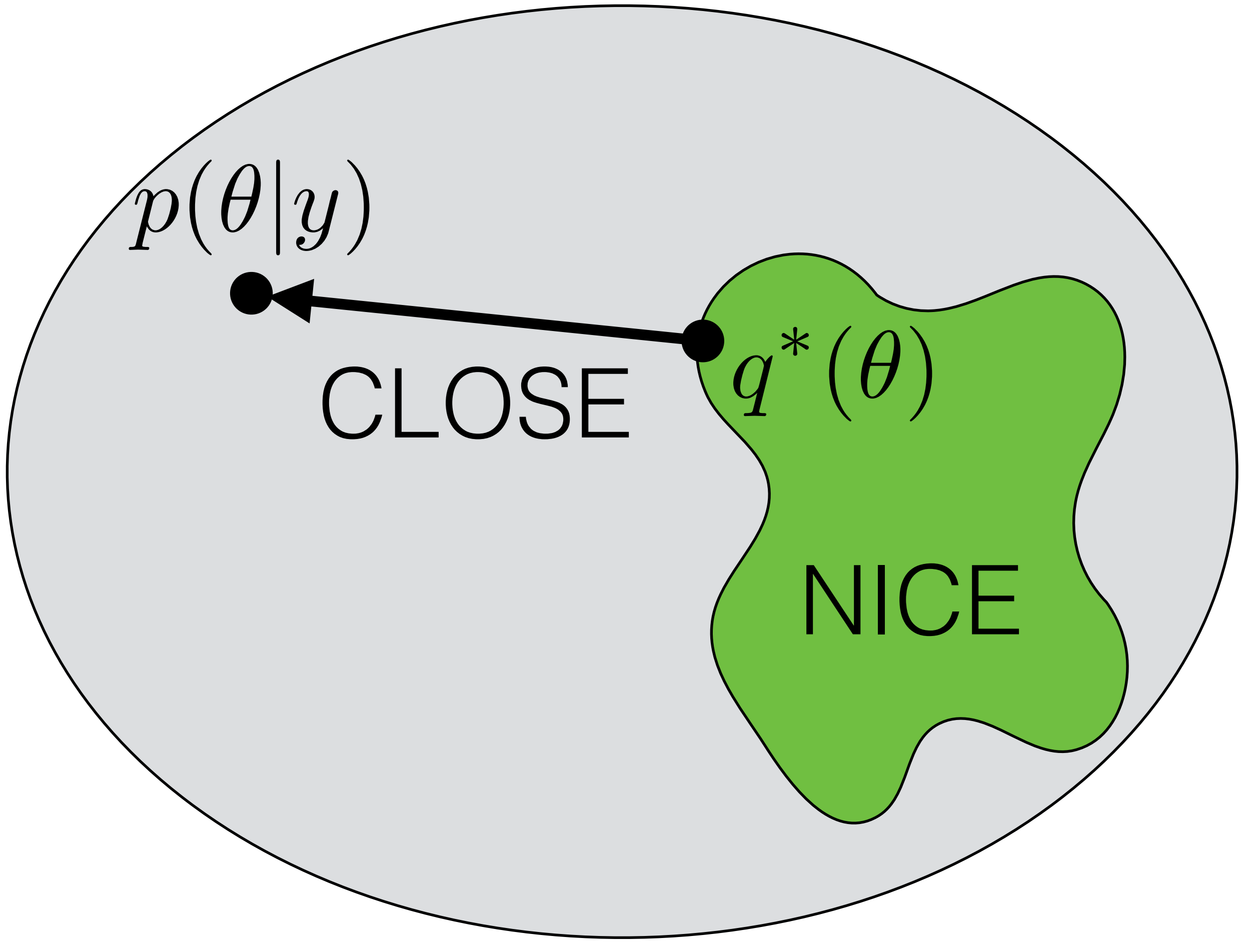

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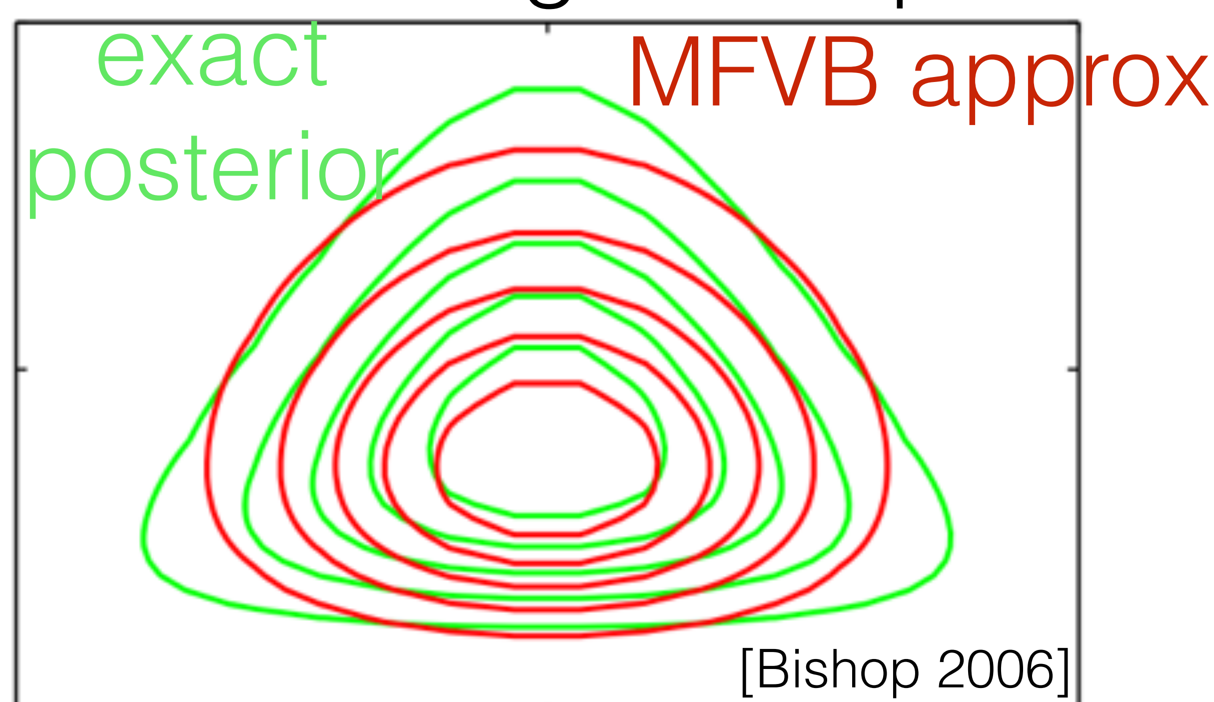

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- One option: Coordinate descent in $q_1, \ldots, q_J$

Approximate Bayesian inference

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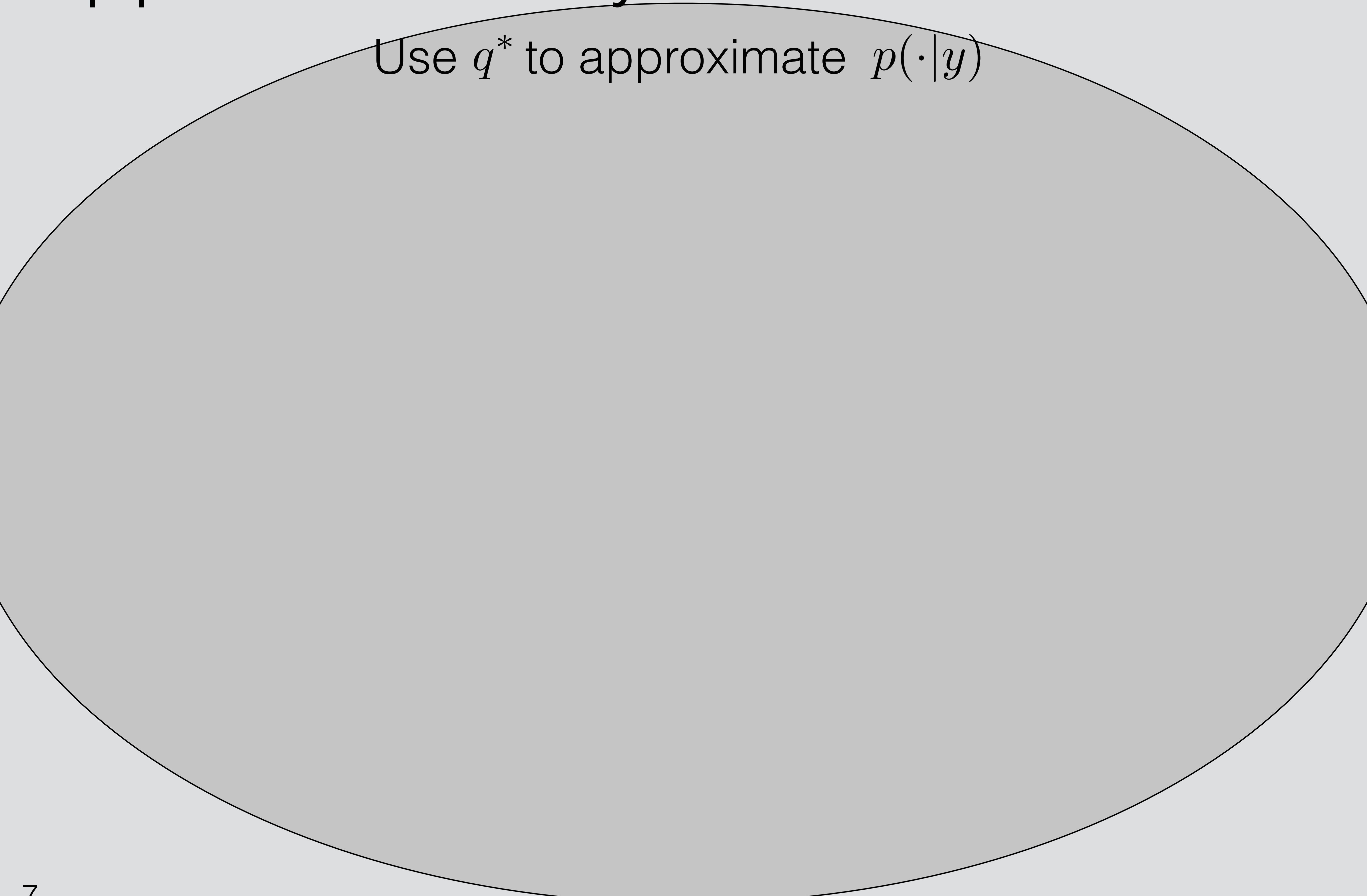
Use q^* to approximate $p(\cdot|y)$



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- Stochastic variational inference (SVI) [Hoffman et al 2013]

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- Automatic differentiation variational inference (ADVI) [Kucukelbir et al 2015, 2017]

Roadmap

- Bayes & Approximate Bayes review
- What is:
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- Why use MFVB?
- When can we trust MFVB?
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References

- See the end of Part II for reference list