Part II: Variational Bayes and beyond

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Approximate Bayesian inference

Use q^* to approximate $p(\cdot|y)$

Optimization

$$q^* = \mathrm{argmin}_{q \in Q} f(q(\cdot), p(\cdot|y))$$

Variational Bayes

$$q^* = \mathrm{argmin}_{q \in Q} KL(q(\cdot)||p(\cdot|y))$$

Mean-field variational Bayes

$$q^* = \mathrm{argmin}_{q \in Q_{\mathrm{MFVB}}} KL(q(\cdot)||p(\cdot|y))$$

- Coordinate descent
- Stochastic variational inference (SVI) [Hoffman et al 2013]
- Automatic differentiation variational inference (ADVI) [Kucukelbir et al 2015, 2017]

Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use MFVB?
- When can we trust MFVB?
- Where do we go from here?

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Midge wing length

- Catalogued midge wing lengths (mm) $y = (y_1, \ldots, y_N)$

[CSIRO 2004]

[Hoff 2009; Grogan, Wirth 1981; MacKay 2003; Bishop 2006]

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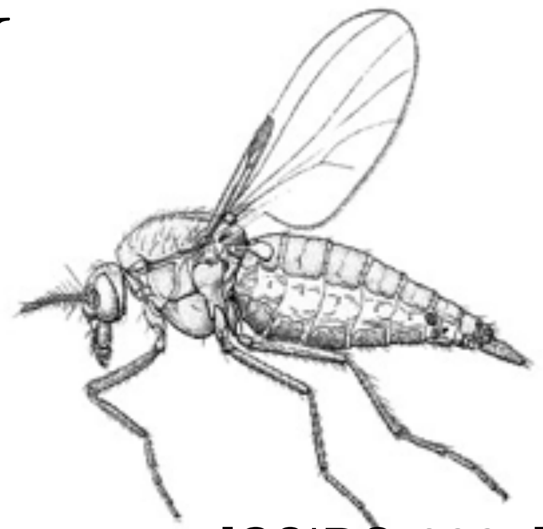

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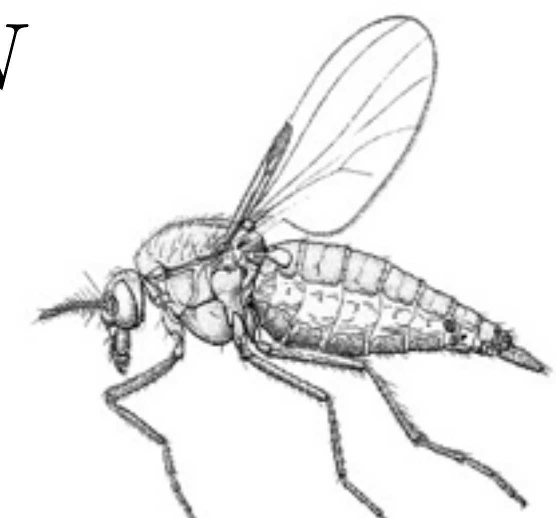

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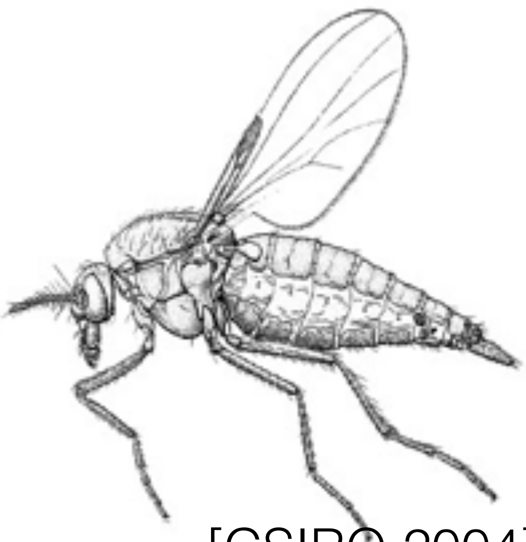

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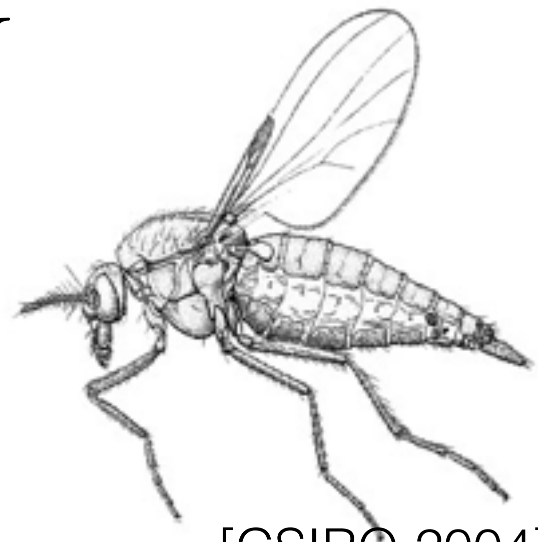

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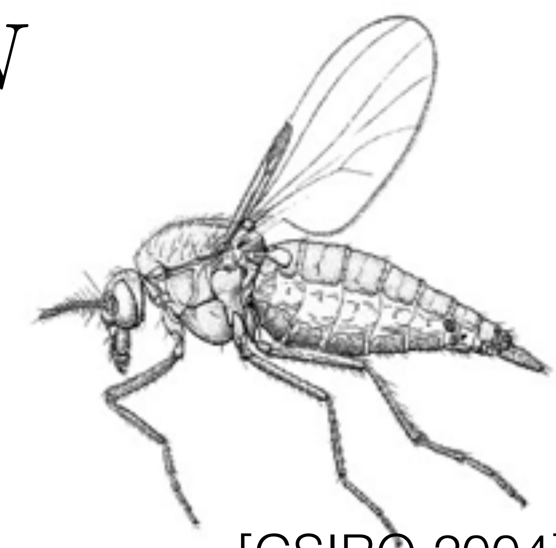

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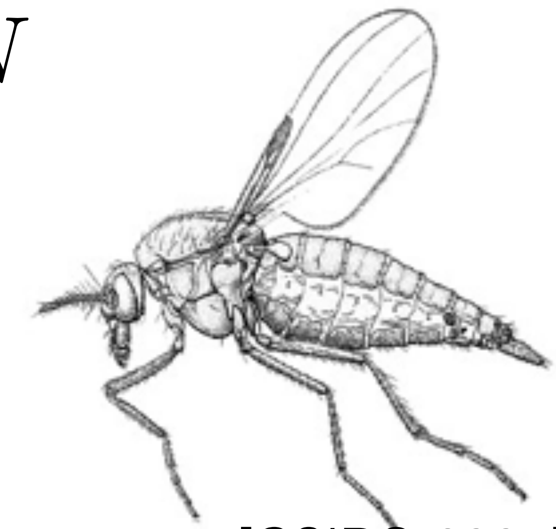

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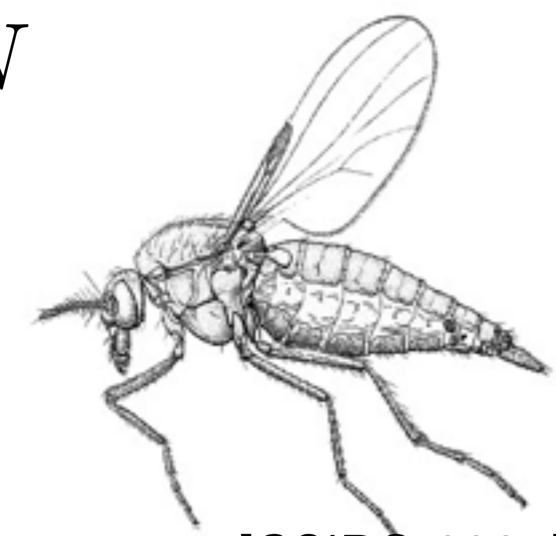

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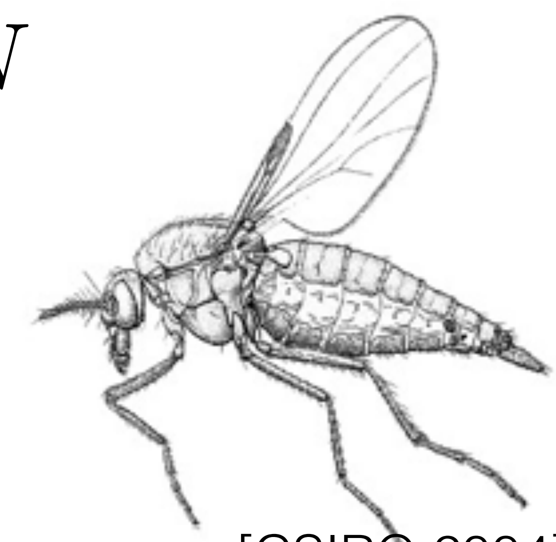

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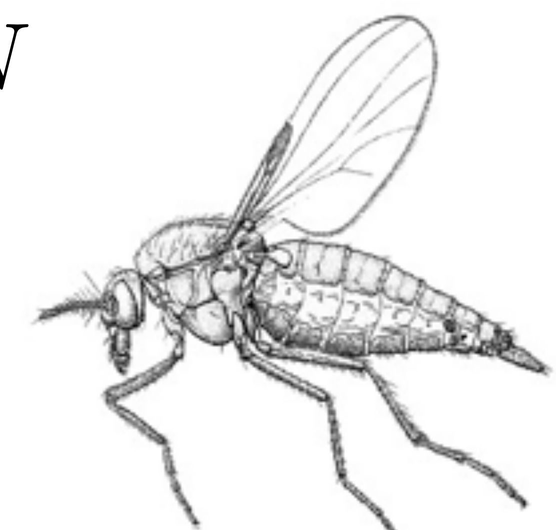

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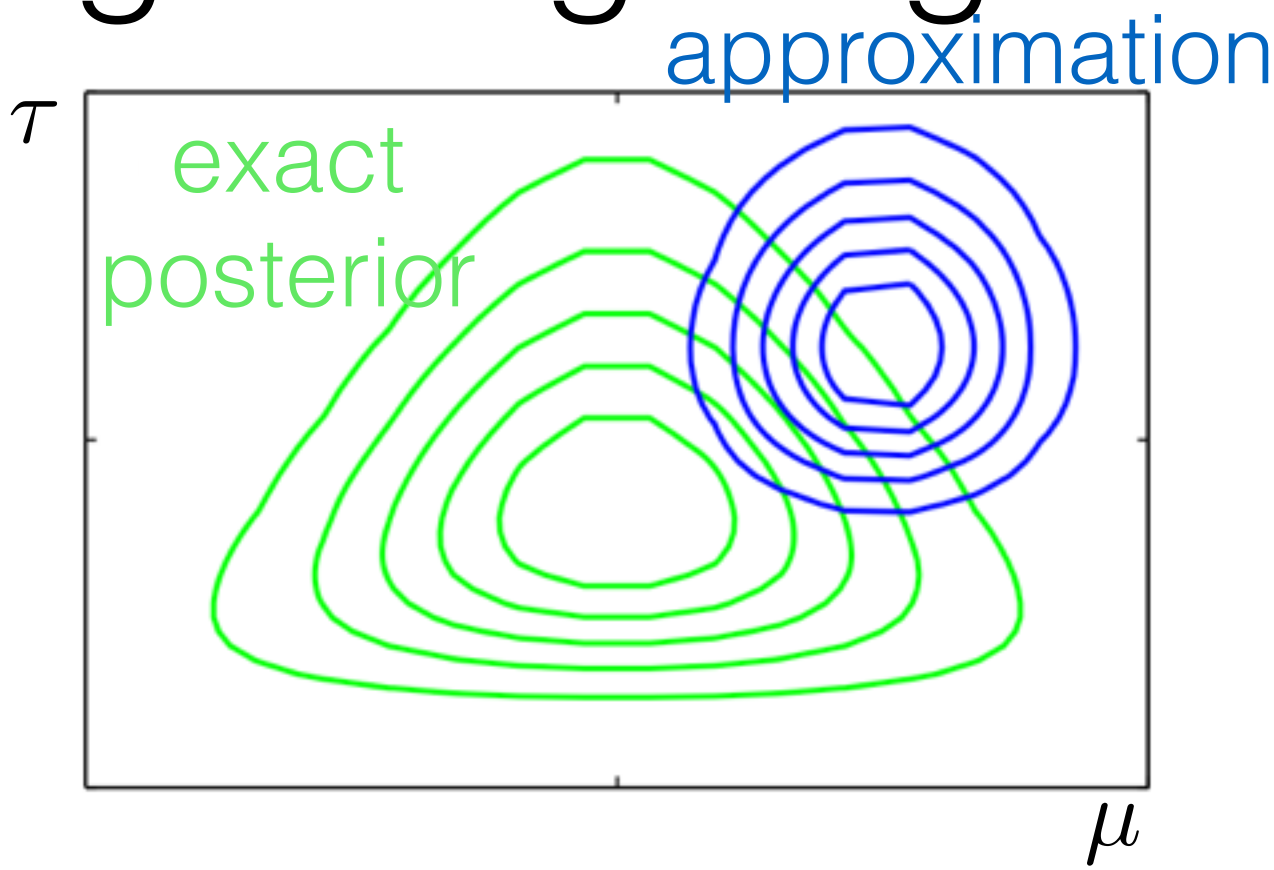

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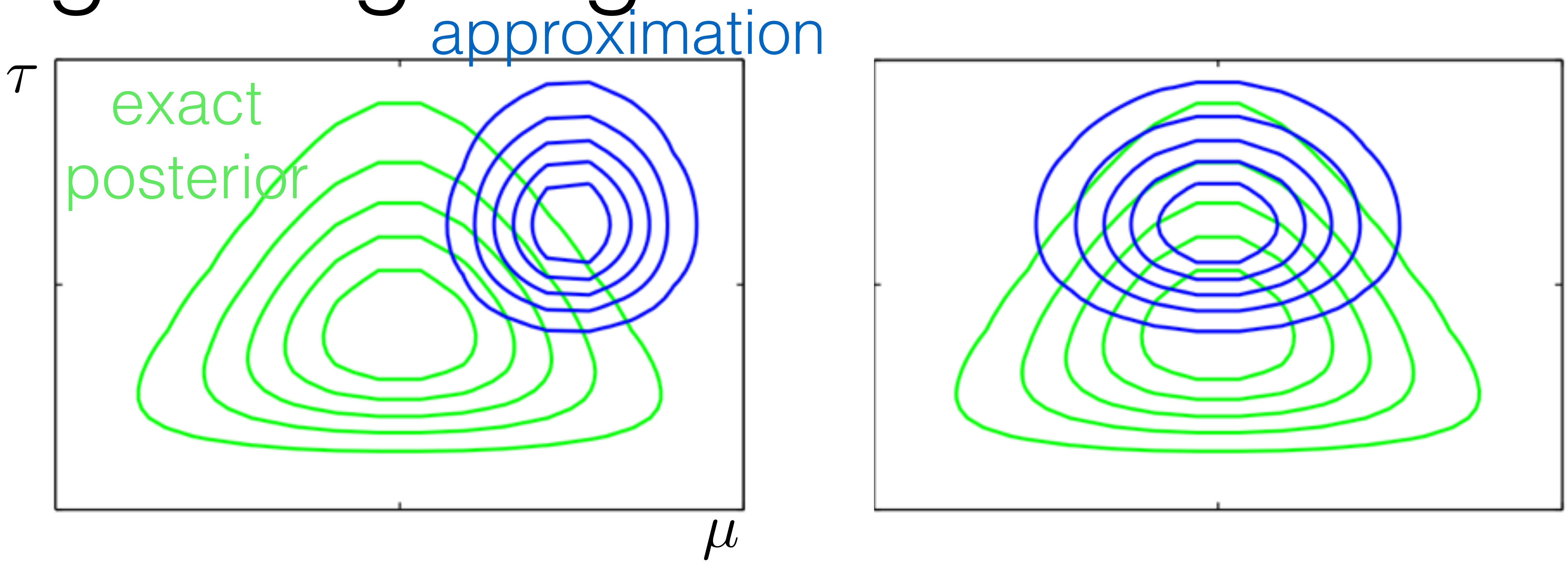

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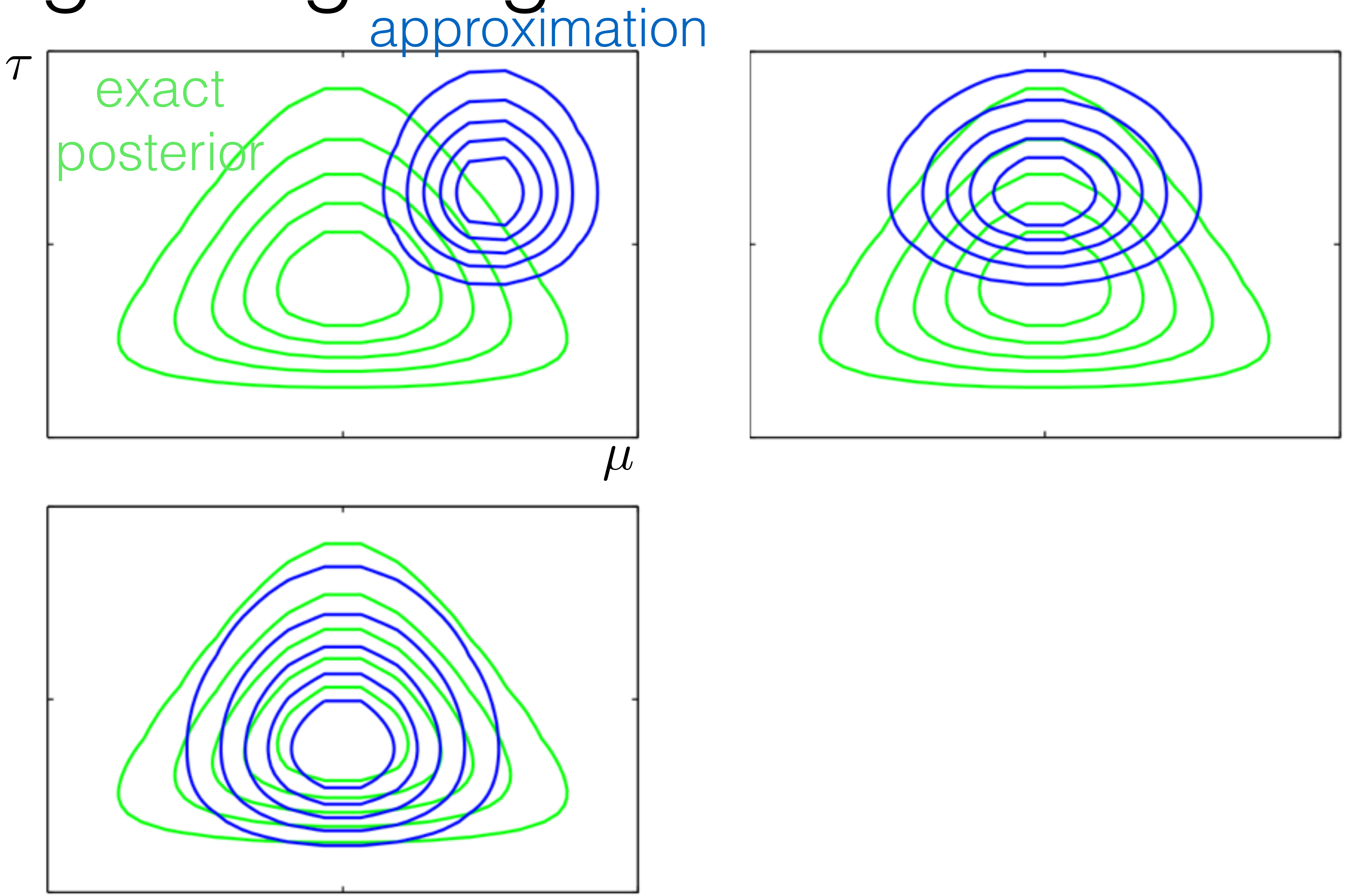

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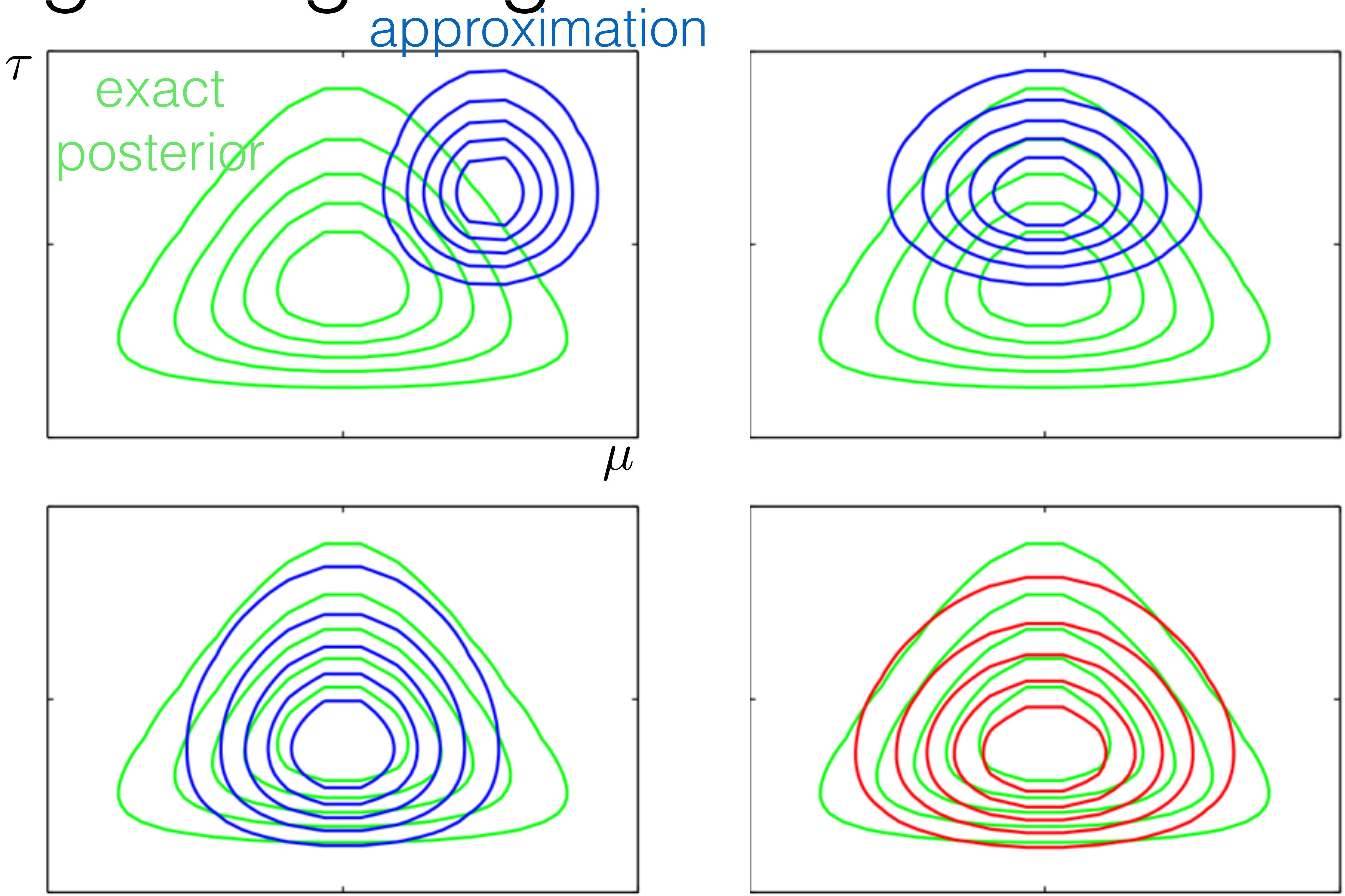

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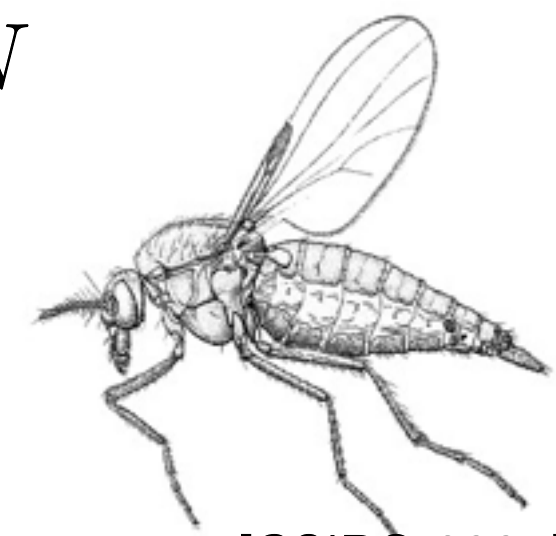

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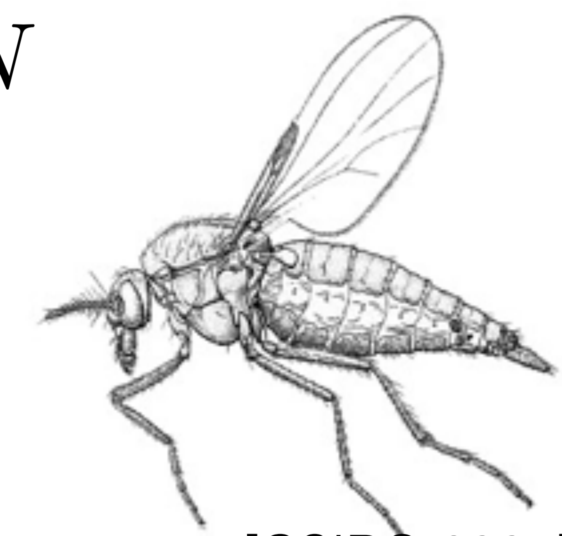

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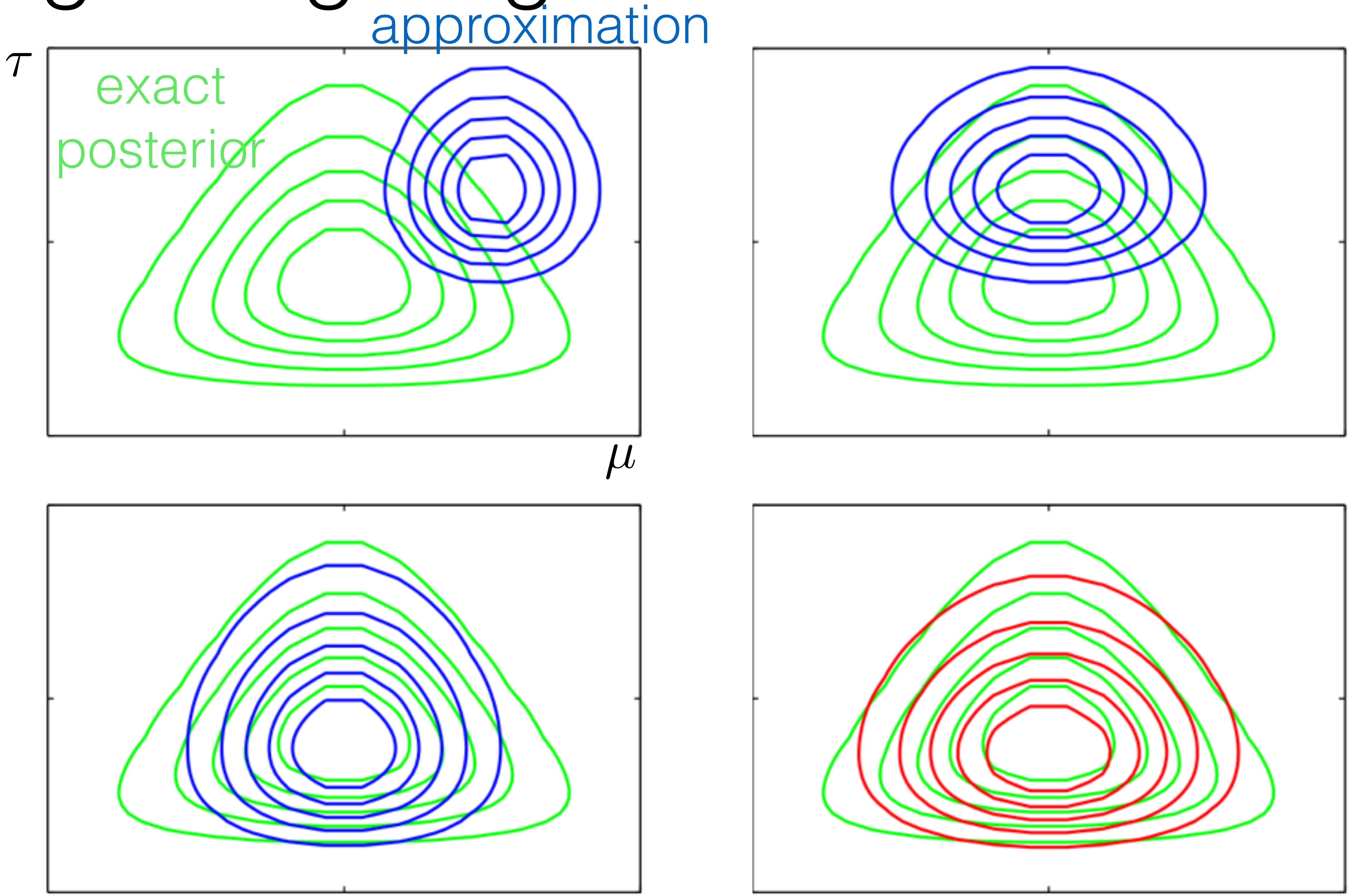

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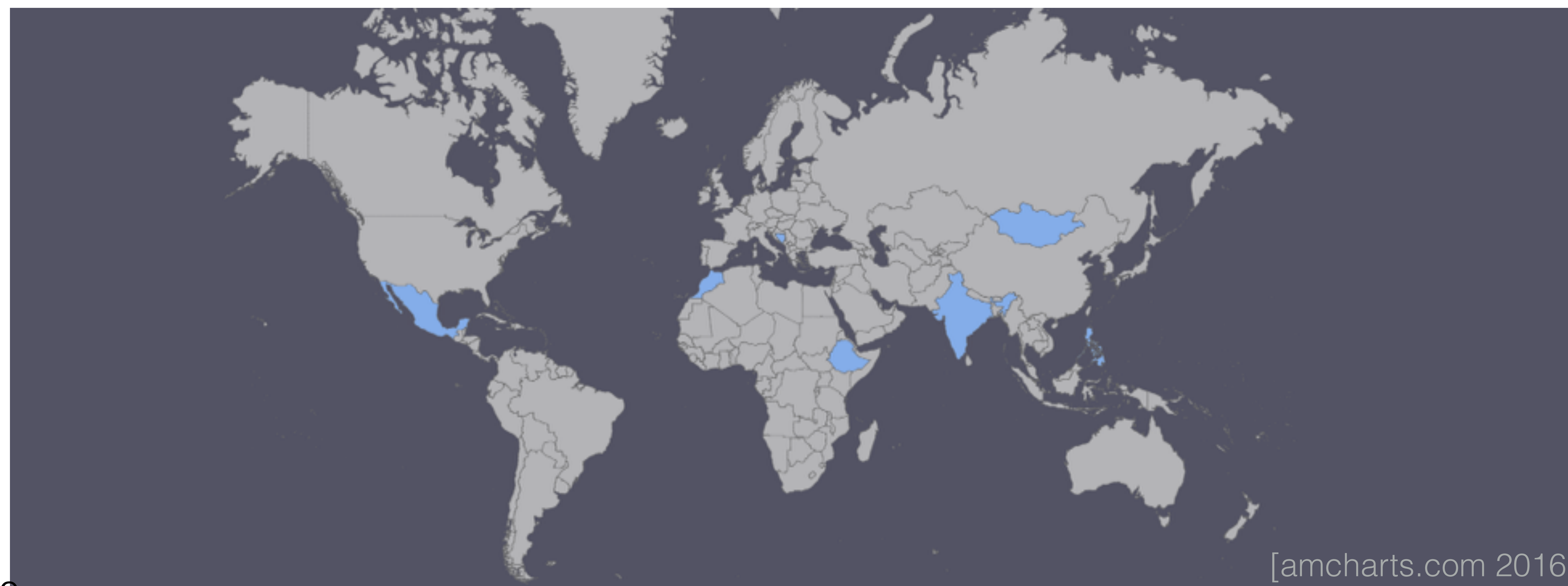
Microcredit Experiment



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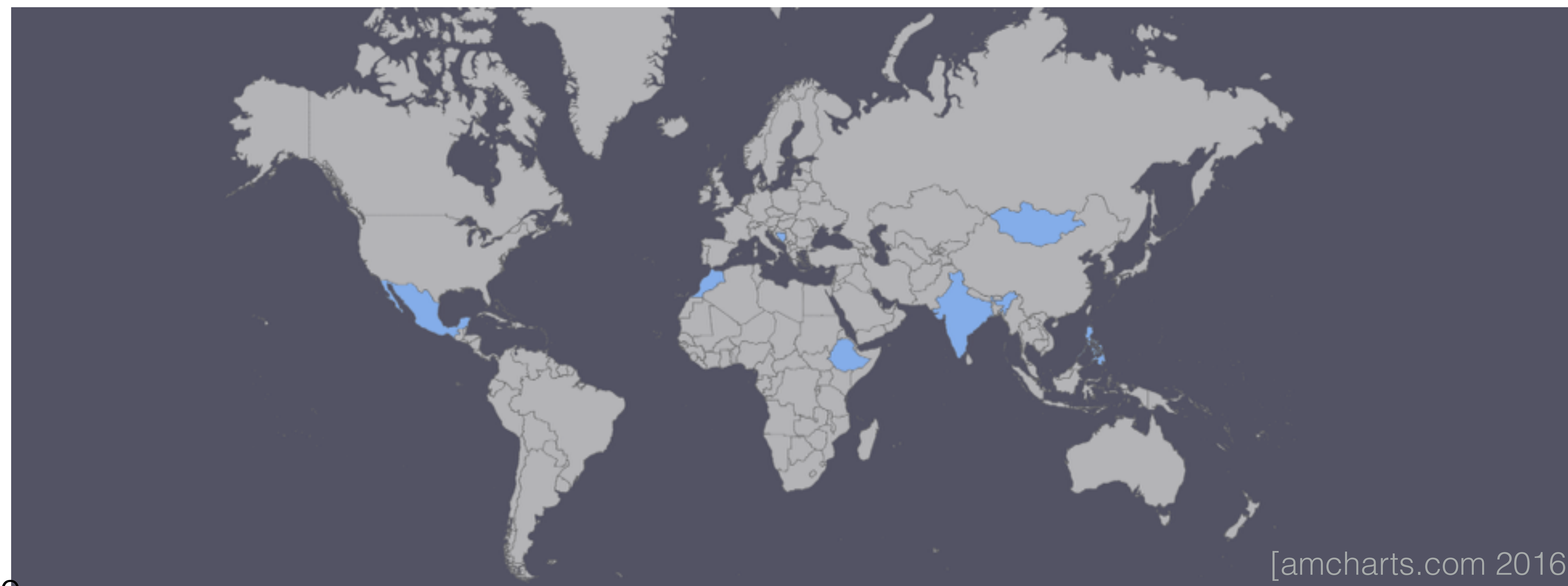

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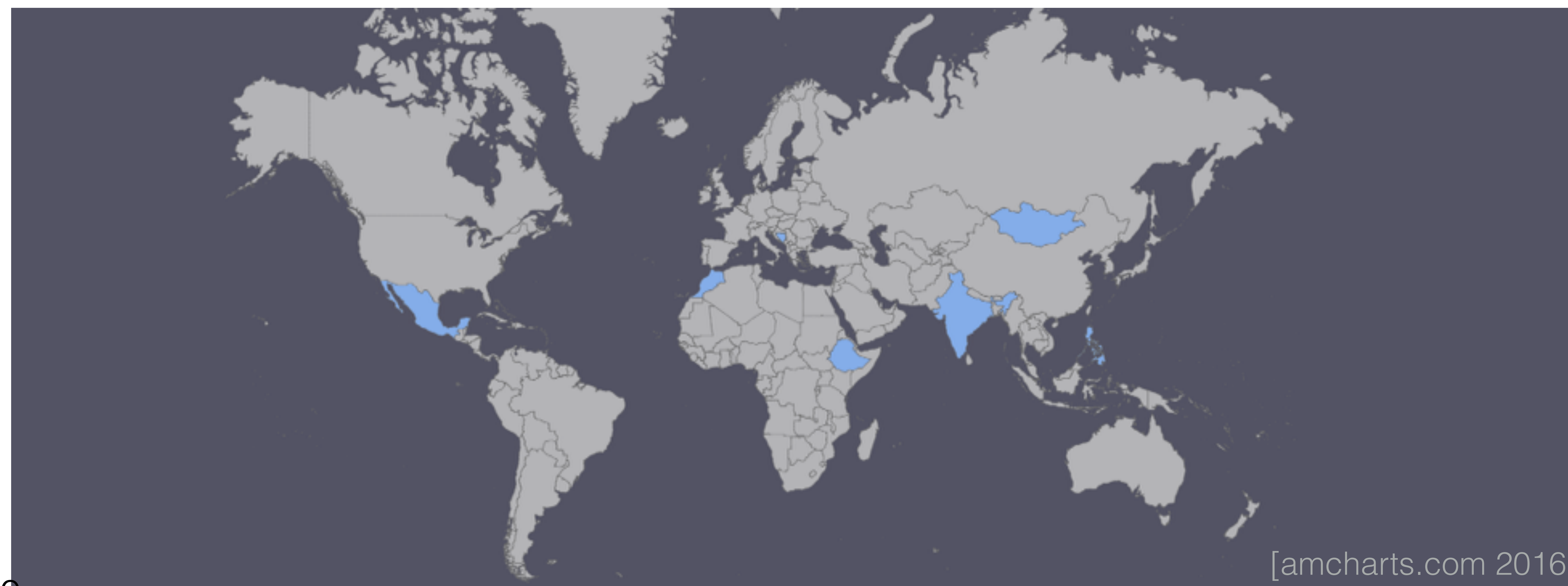

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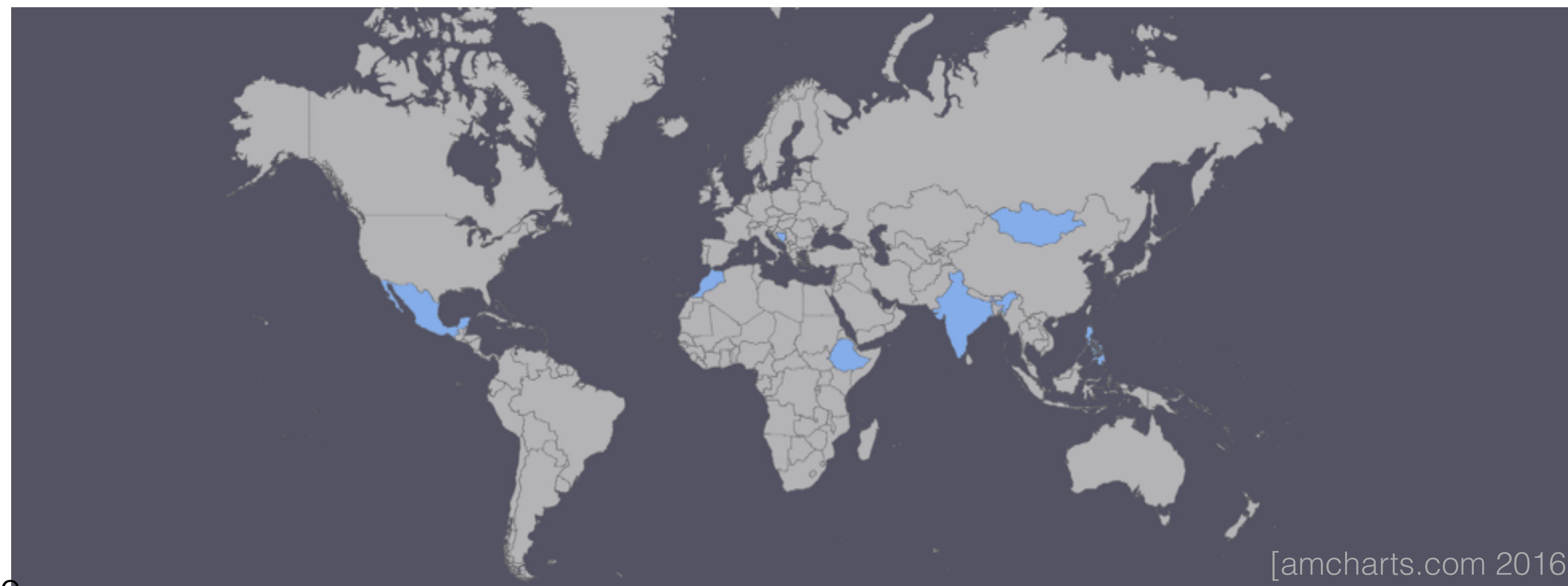

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$$y_{kn}$$

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- N_k businesses in kth site (~900 to ~17K)
- Profit of nth business at kth site:

1 if microcredit

profit

$$y_{kn} \overset{indep}{\sim} \mathcal{N}(\mu_k + T_{kn}\tau_k, \boxed{\sigma_k^2})$$

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$$\left(\begin{array}{c} \mu_k \\ \tau_k \end{array} \right) \stackrel{iid}{\sim} \mathcal{N}\left(\left(\begin{array}{c} \mu \\ \tau \end{array} \right), C \right)$$

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- Priors and hyperpriors:

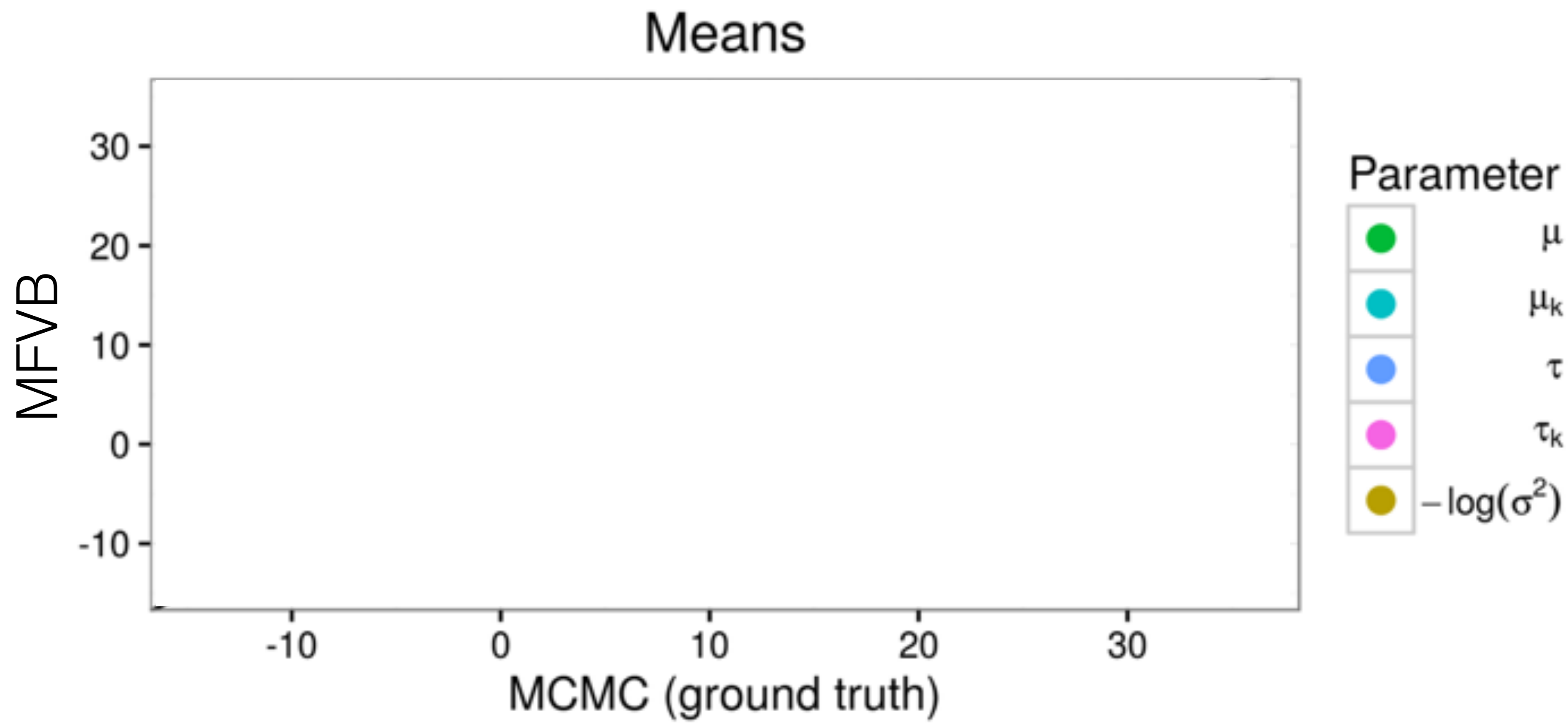
$$\begin{pmatrix} \mu_k \\ \tau_k \end{pmatrix} \stackrel{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu \\ \tau \end{pmatrix}, C\right) \qquad \begin{pmatrix} \mu \\ \tau \end{pmatrix} \stackrel{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu_0 \\ \tau_0 \end{pmatrix}, \Lambda^{-1}\right)$$

$$\sigma_k^{-2} \stackrel{iid}{\sim} \Gamma(a, b) \qquad C \sim \mathrm{Sep\&LKJ}(\eta, c, d)$$

Microcredit

MFVB: How will we know if it’s working?

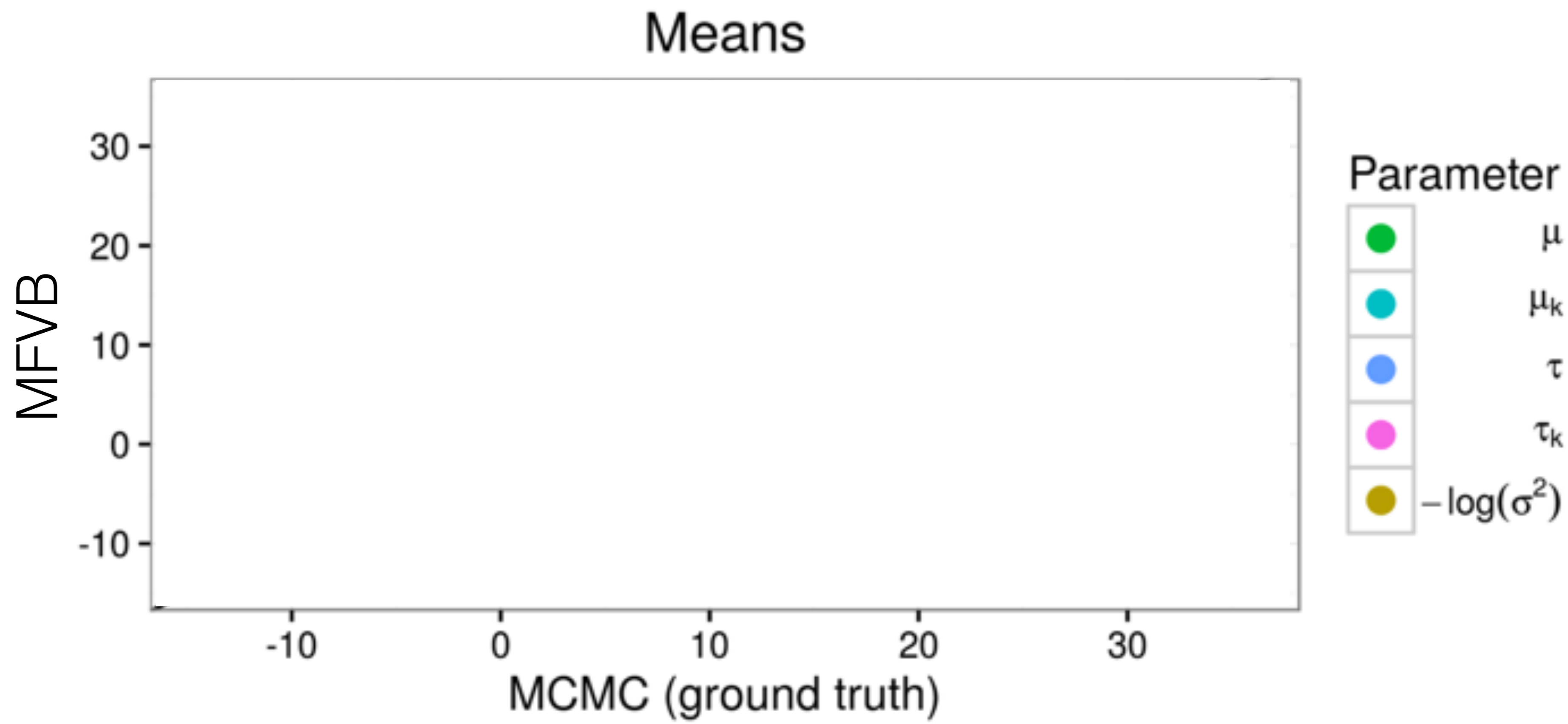
Microcredit



[Giordano, Broderick, Meager, Huggins, Jordan 2016]

Microcredit

- *One set* of 2500 MCMC draws: **45 minutes**



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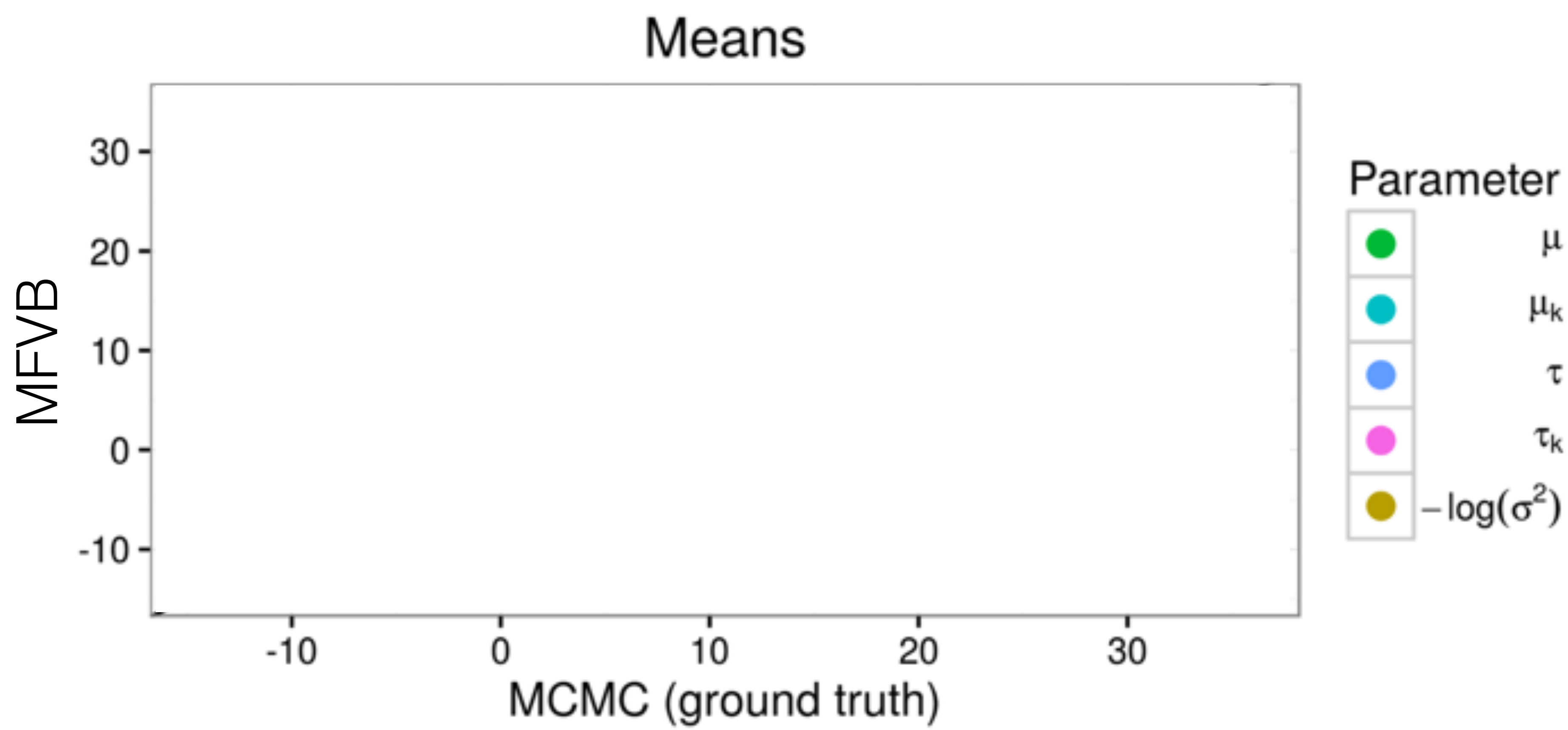
- MFVB optimization: **<1 min**



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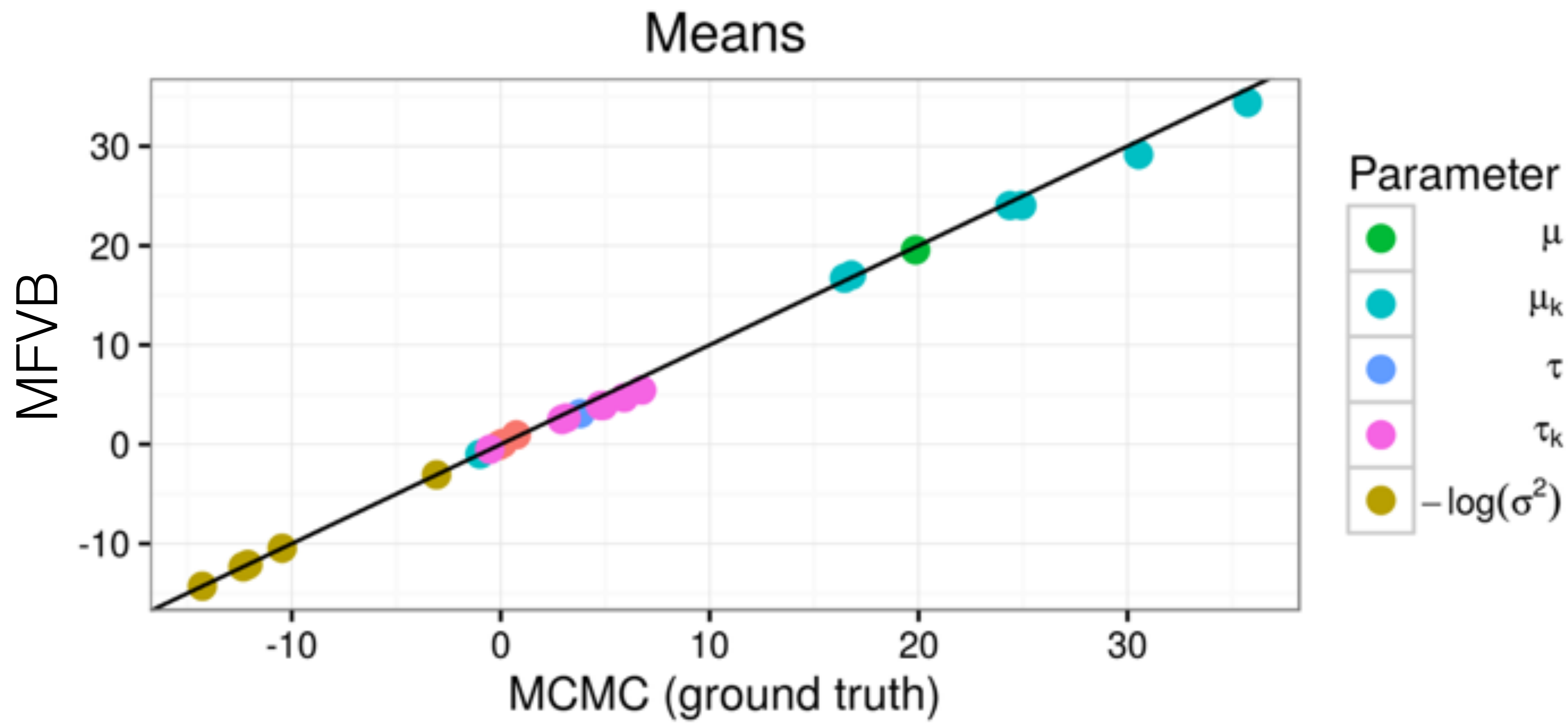

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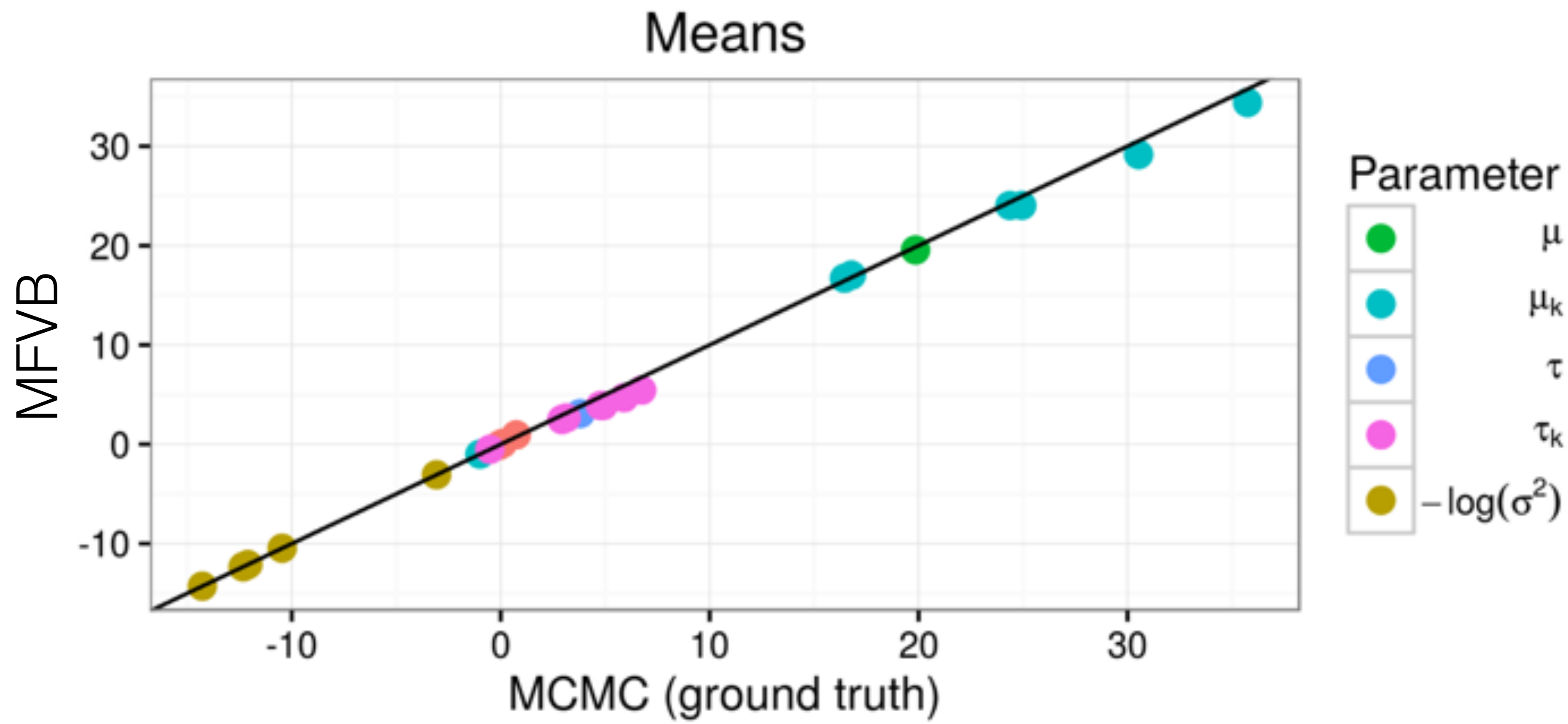

Criteo Online Ads Experiment

- Click-through conversion prediction
- Q: Will a customer (e.g.) buy a product after clicking?

[Giordano, Broderick, Meager, Huggins, Jordan 2016]

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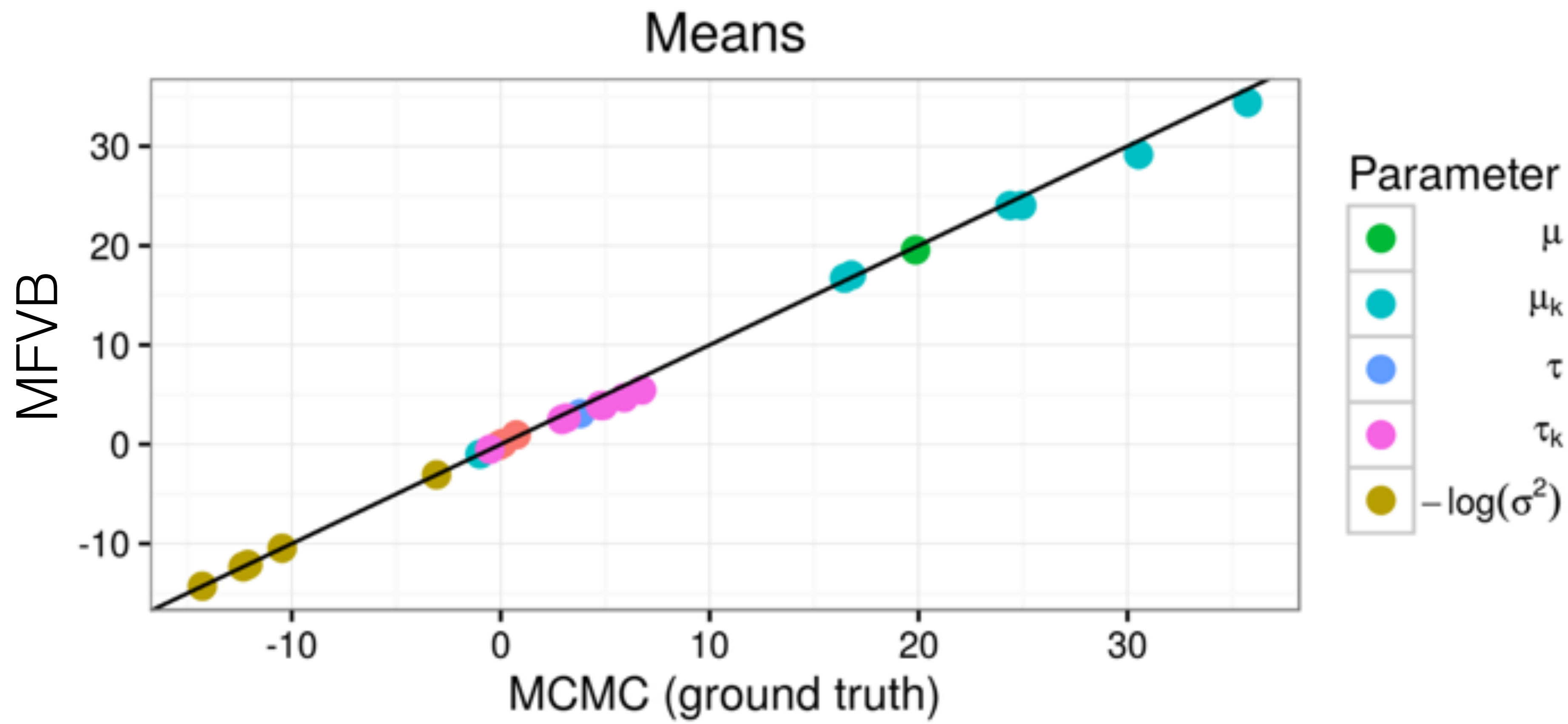

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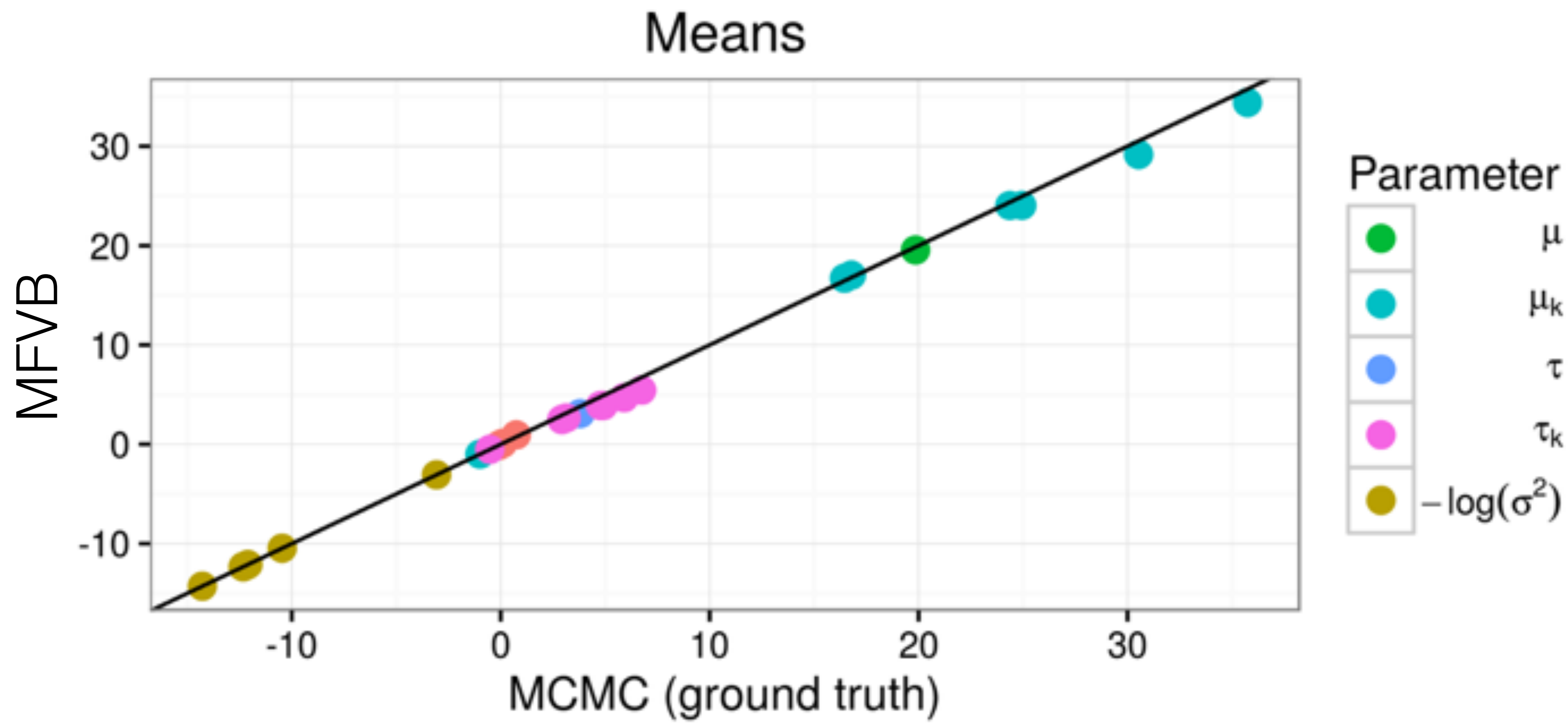

Criteo Online Ads Experiment

- Click-through conversion prediction
- Q: Will a customer (e.g.) buy a product after clicking?
- Q: How predictive of conversion are different features?
- Logistic GLMM [board]

[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2017]

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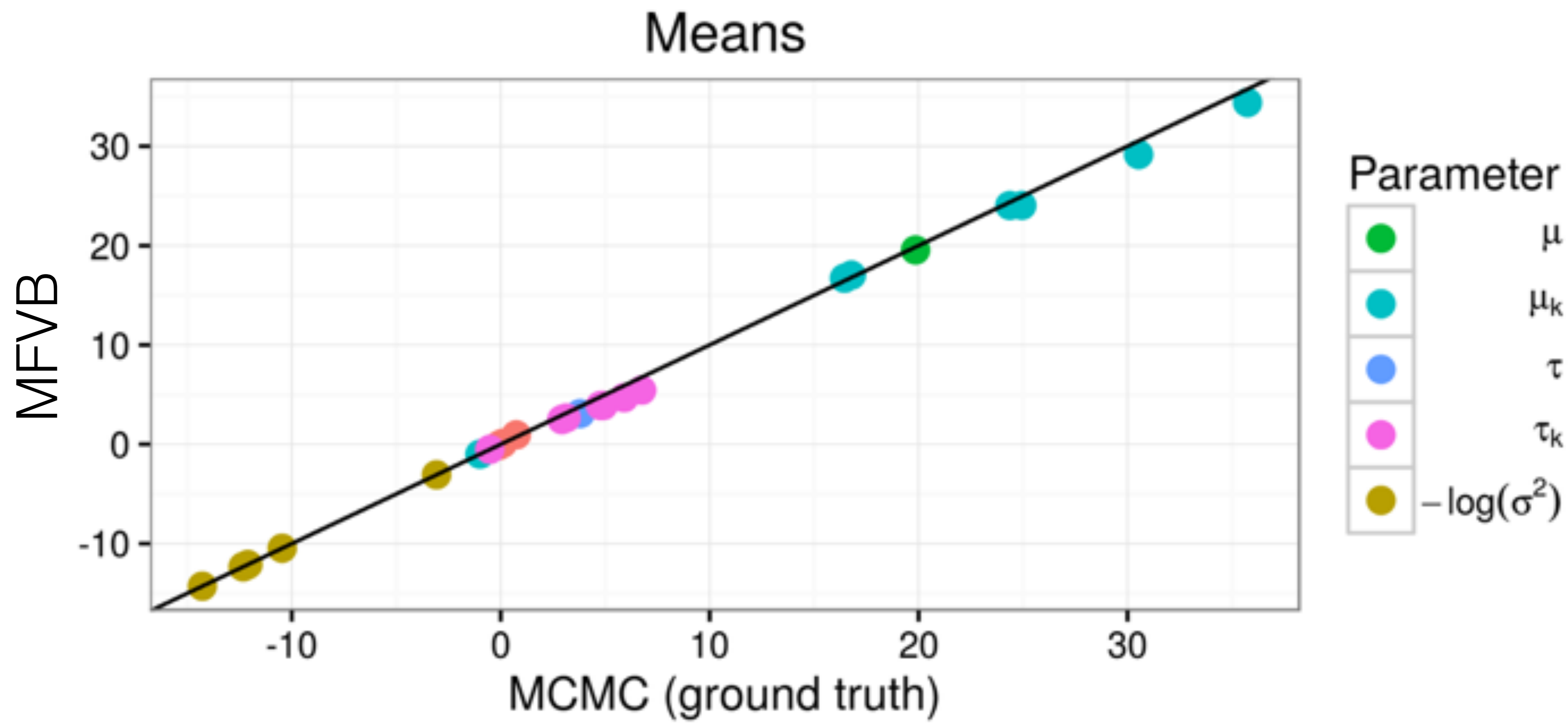


Criteo Online Ads Experiment

- Click-through conversion prediction
- Q: Will a customer (e.g.) buy a product after clicking?
- Q: How predictive of conversion are different features?
- Logistic GLMM; N = 61,895 subset to compare to MCMC

[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2017]

Criteo Online Ads Experiment

[Giordano, Broderick, Jordan 2017]

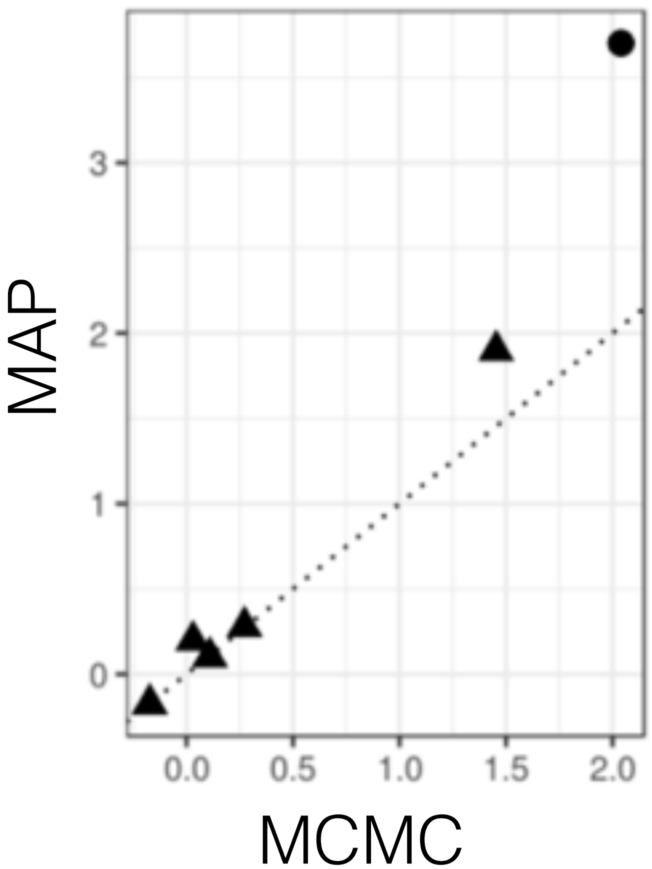
Criteo Online Ads Experiment

- MAP: **12 s**

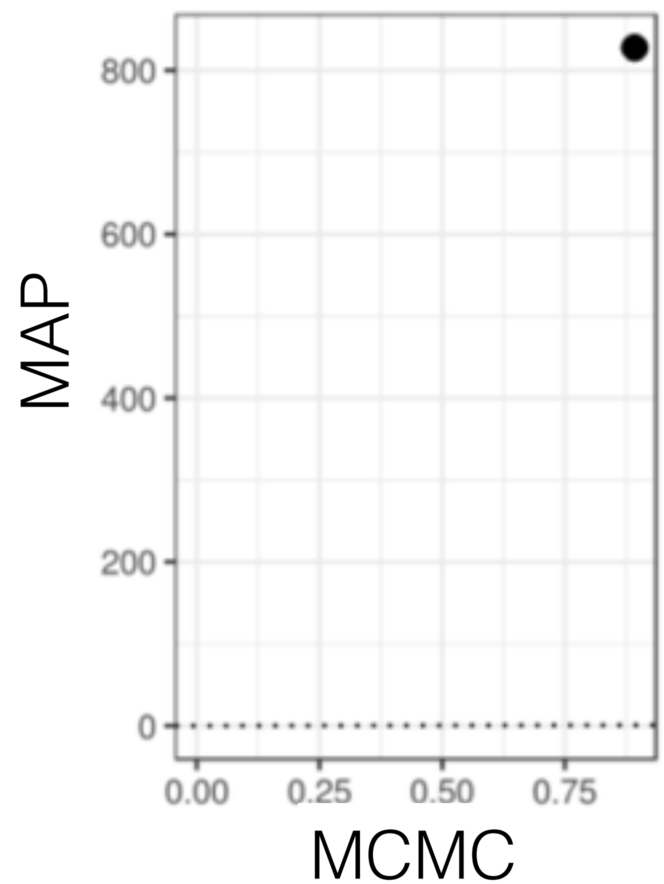
[Giordano, Broderick, Jordan 2017]

Criteo Online Ads Experiment

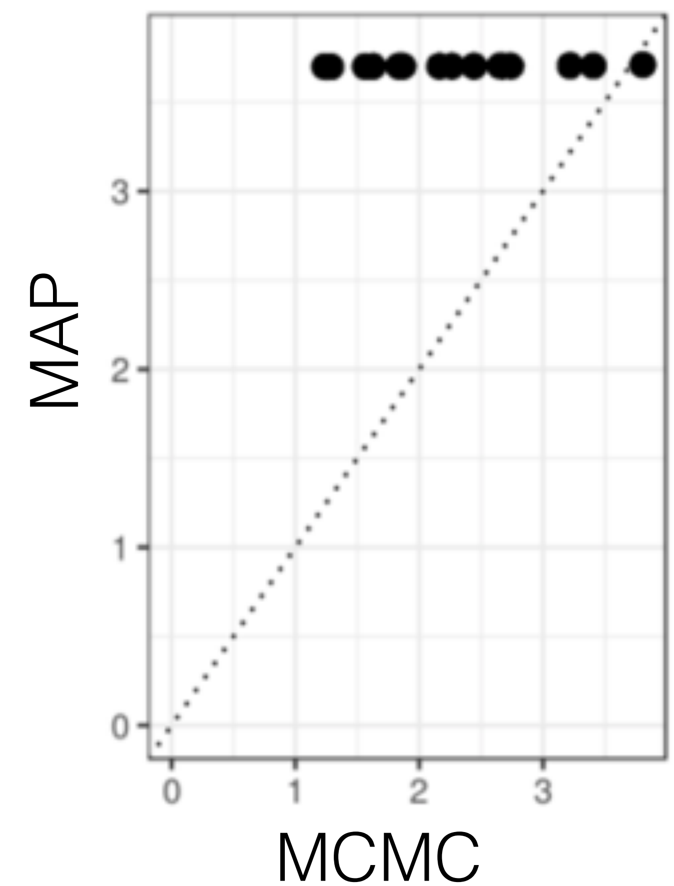
Global parameters (-τ)



Global parameter τ



Local parameters



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[Giordano, Broderick, Jordan 2017]

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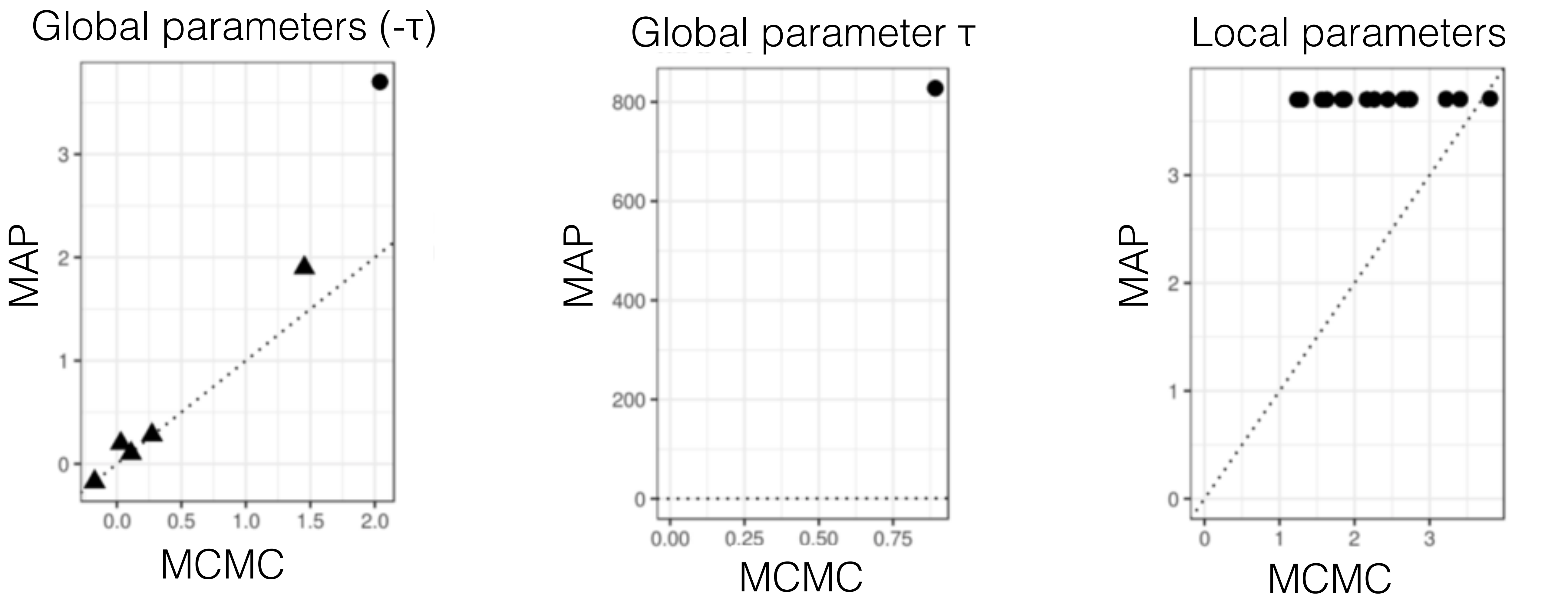
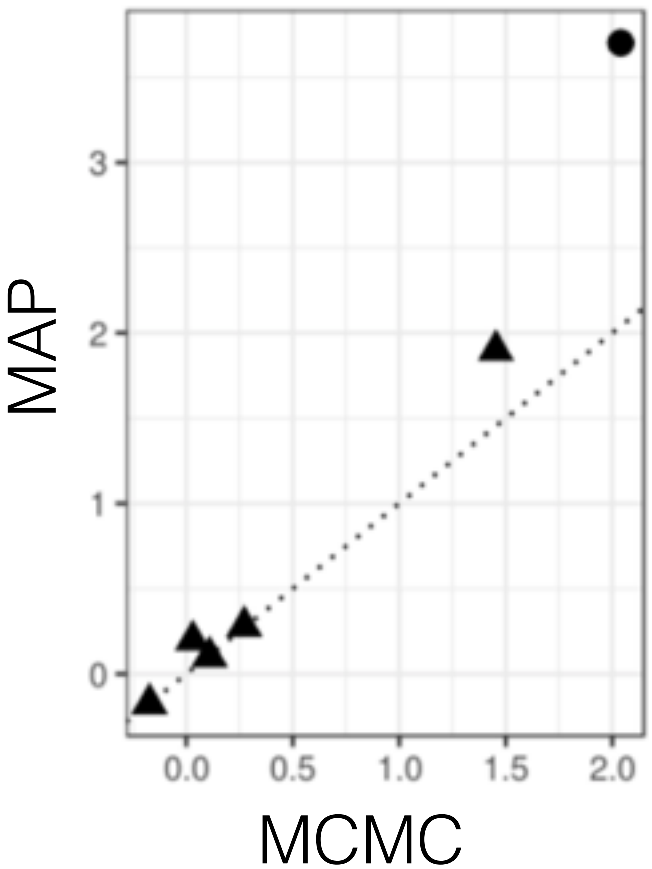


- MAP: **12 s**
- MFVB: **57 s**

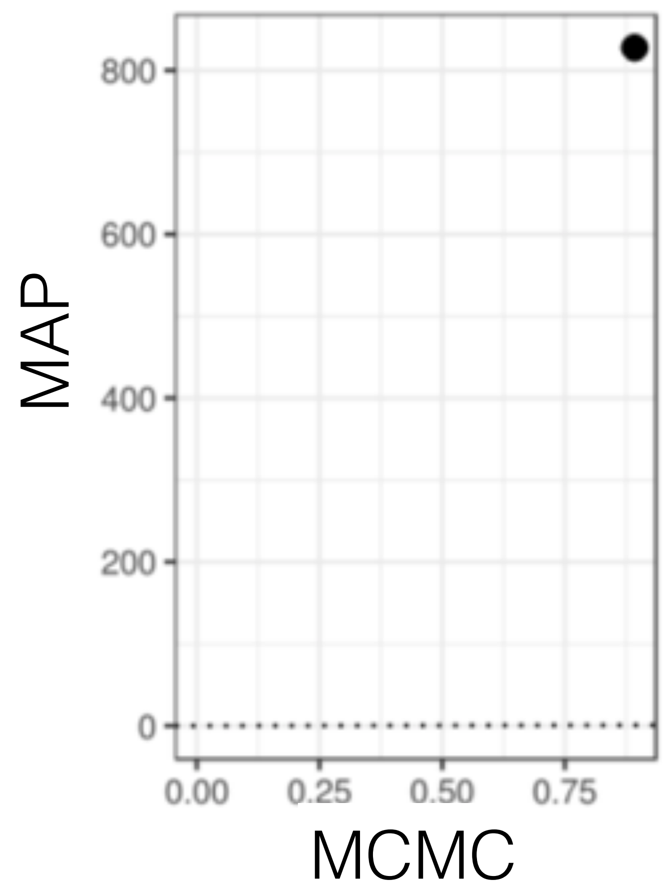
[Giordano, Broderick, Jordan 2017]

Criteo Online Ads Experiment

Global parameters (-τ)



Global parameter τ



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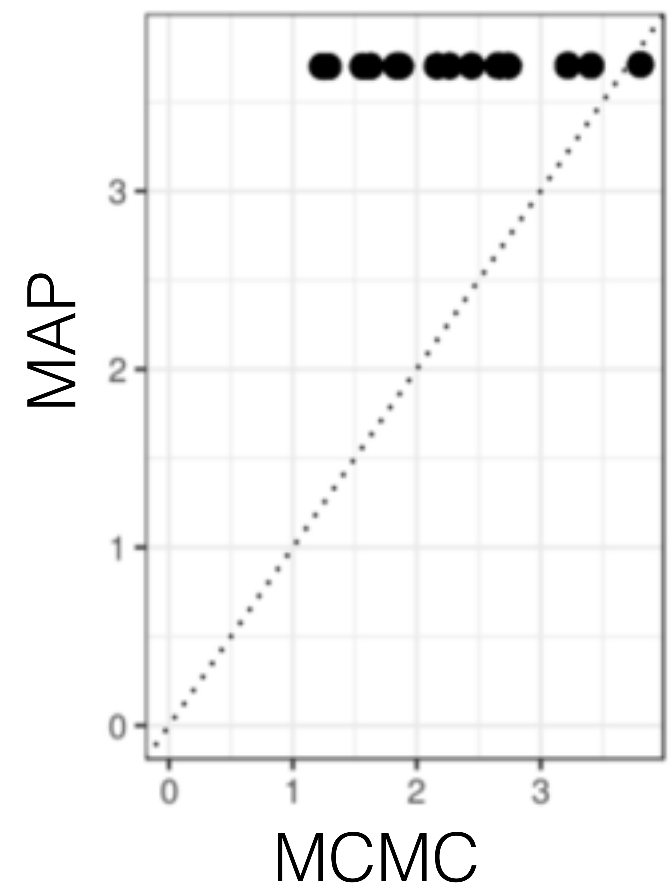

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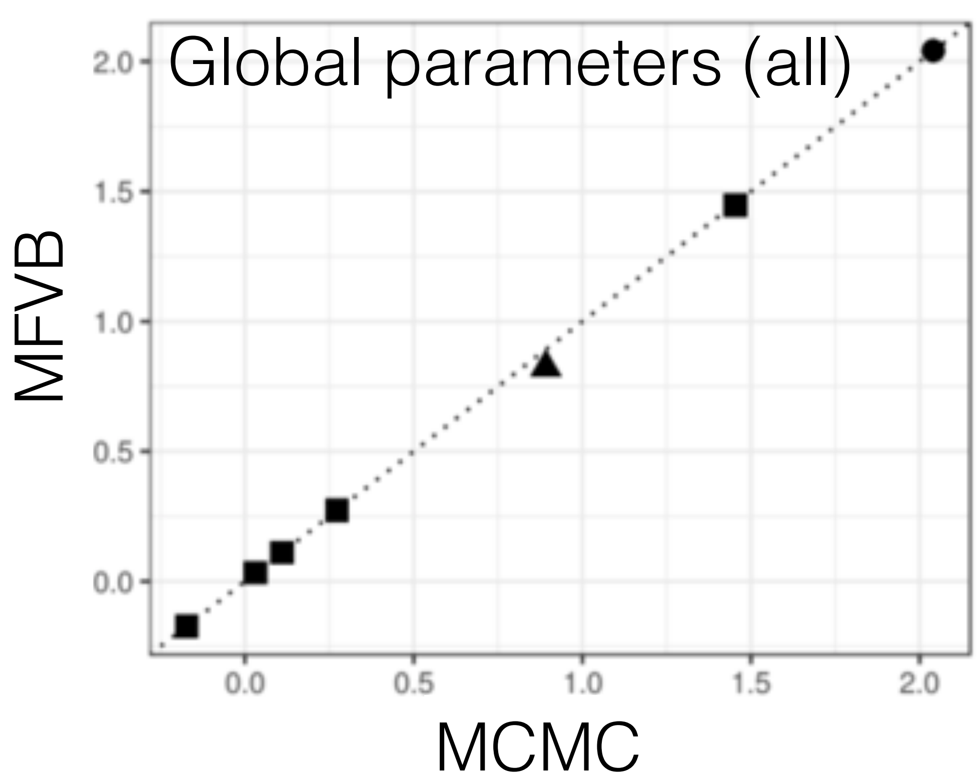

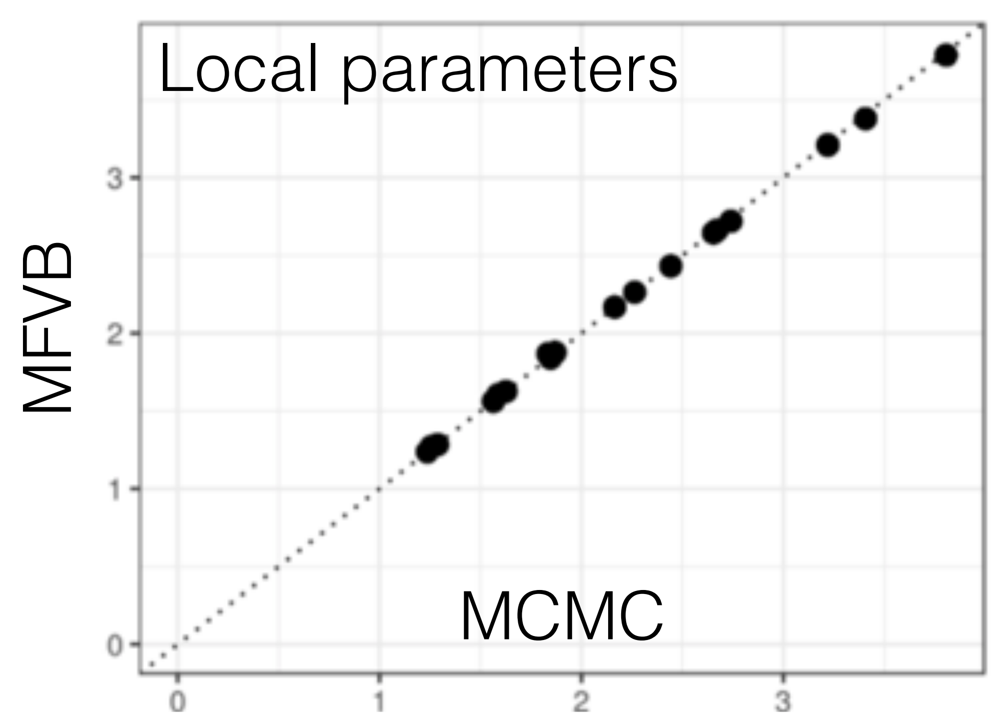

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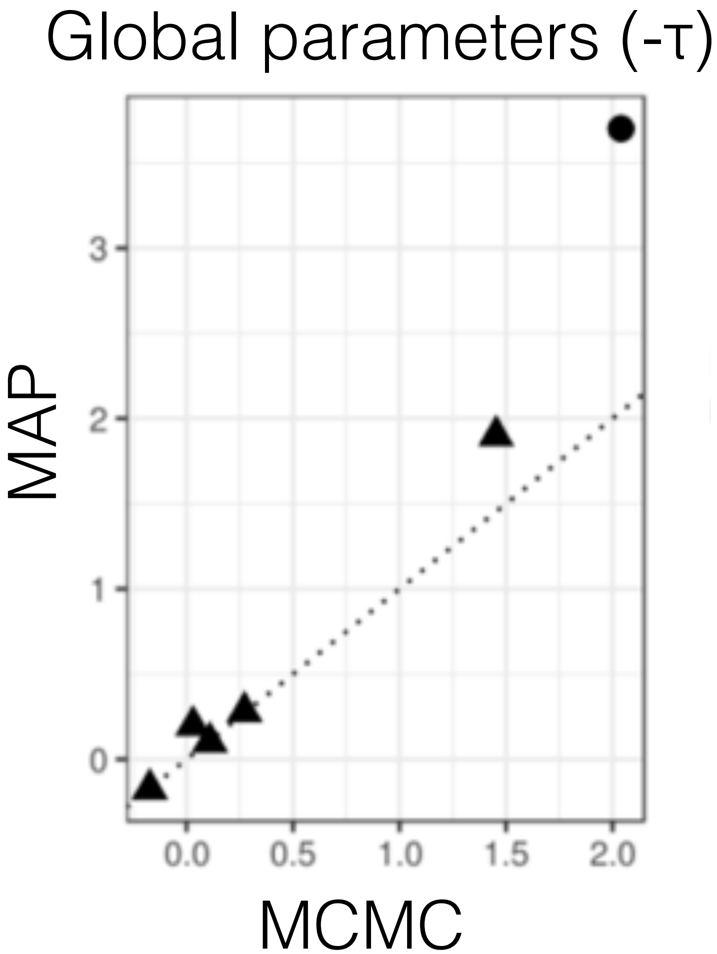
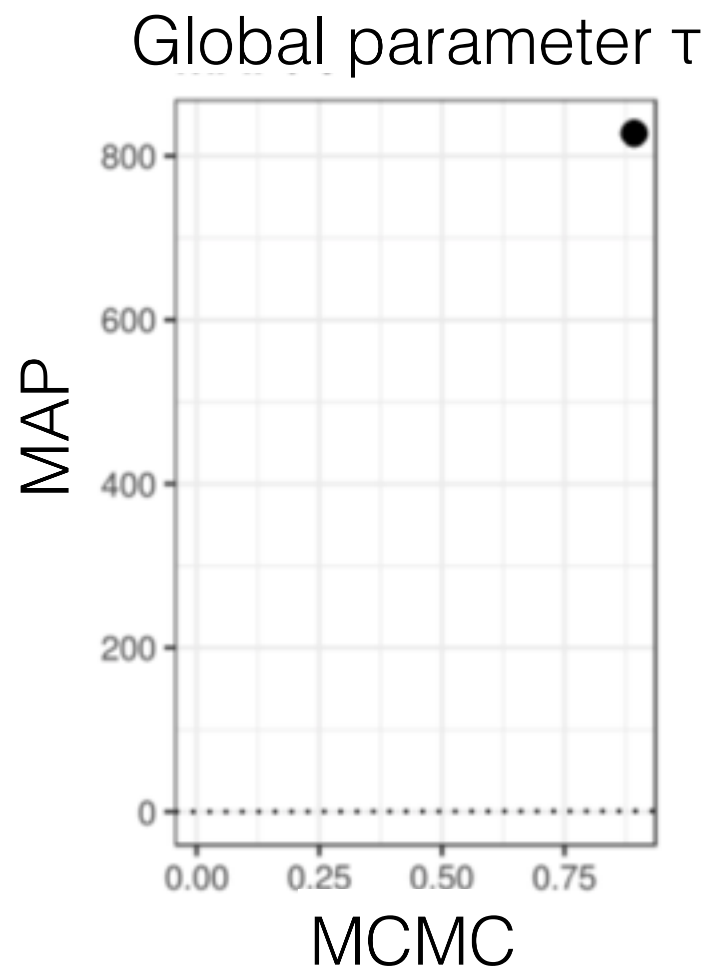
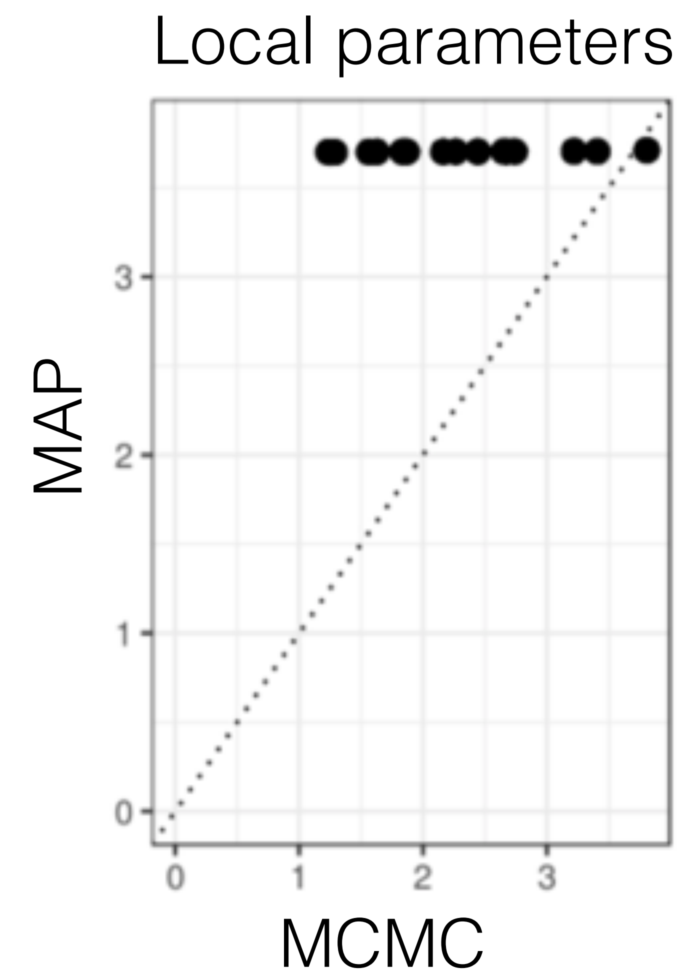
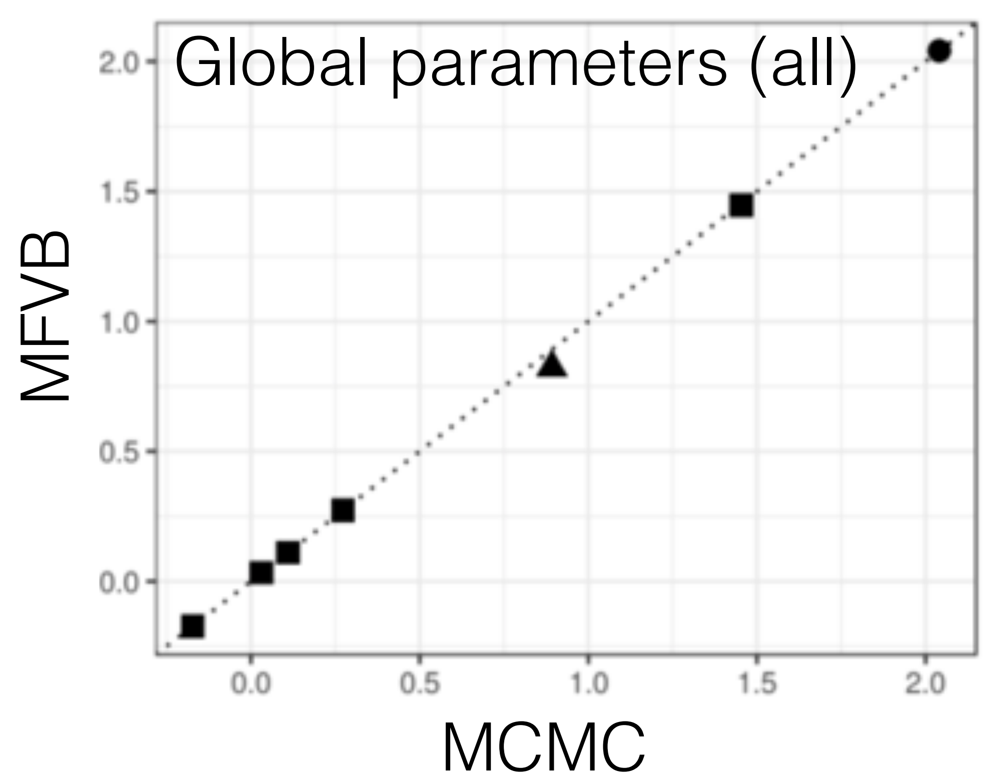
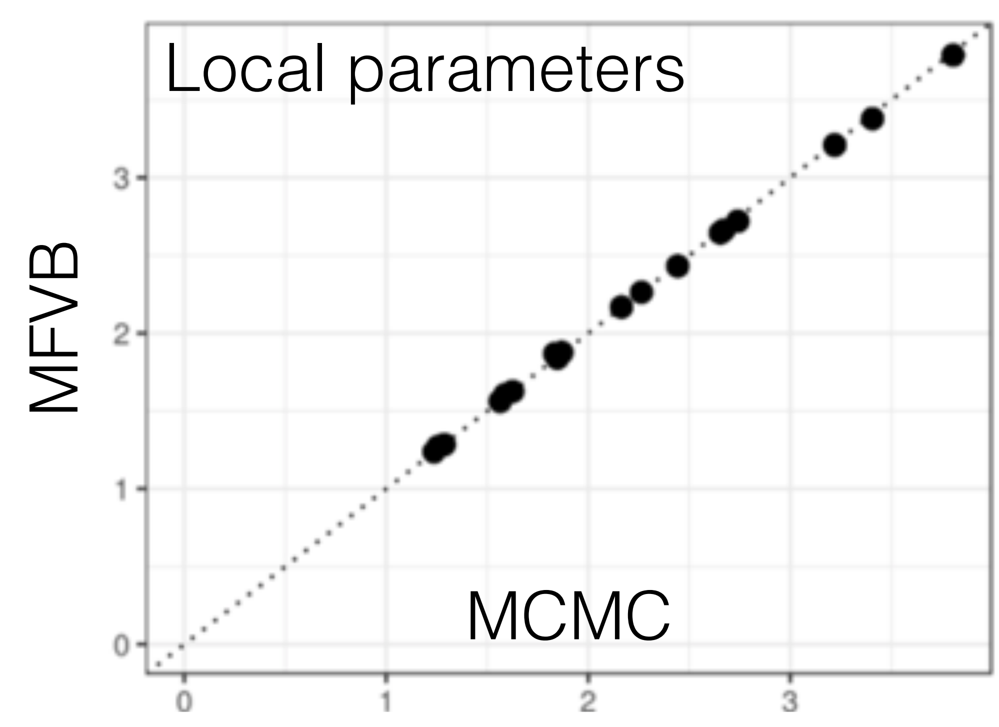






- MAP: **12 s**
- MFVB: **57 s**
- MCMC (5K samples): 21,066 s (**5.85 h**)





[Giordano, Broderick, Jordan 2017]

Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use MFVB?
- When can we trust MFVB?
- Where do we go from here?

Latent Dirichlet Allocation (LDA)

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- Topics

Latent Dirichlet Allocation (LDA)

- Topics: two perspectives
  - Each document can belong to multiple groups
  - Cluster words in documents

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"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHER
ACTOR	NEW	SAYS	BENNETT
FIRST	STATE	FAMILY	MANIGAT
YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
THEATER	PROGRAMS	PERCENT	PRESIDENT
ACTRESS	GOVERNMENT	CARE	ELEMENTARY

The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.

[Blei, Ng, Jordan 2003]

Latent Dirichlet Allocation (LDA)

- Topics: two perspectives
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"Arts"	"Budgets"	"Children"	"Education"
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[Blei, Ng, Jordan 2003; Pritchard, Stephens, Donnelly 2000]

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Approximate Bayesian inference

Use q^* to approximate $p(\cdot|y)$

Optimization

$$q^* = \mathrm{argmin}_{q \in Q} f(q(\cdot), p(\cdot|y))$$

Variational Bayes

$$q^* = \mathrm{argmin}_{q \in Q} KL(q(\cdot)||p(\cdot|y))$$

Mean-field variational Bayes

$$q^* = \mathrm{argmin}_{q \in Q_{\mathrm{MFVB}}} KL(q(\cdot)||p(\cdot|y))$$

- Coordinate descent
- Stochastic variational inference (SVI) [Hoffman et al 2013]
- Automatic differentiation variational inference (ADVI) [Kucukelbir et al 2015, 2017]

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