Part III: Variational Bayes and beyond

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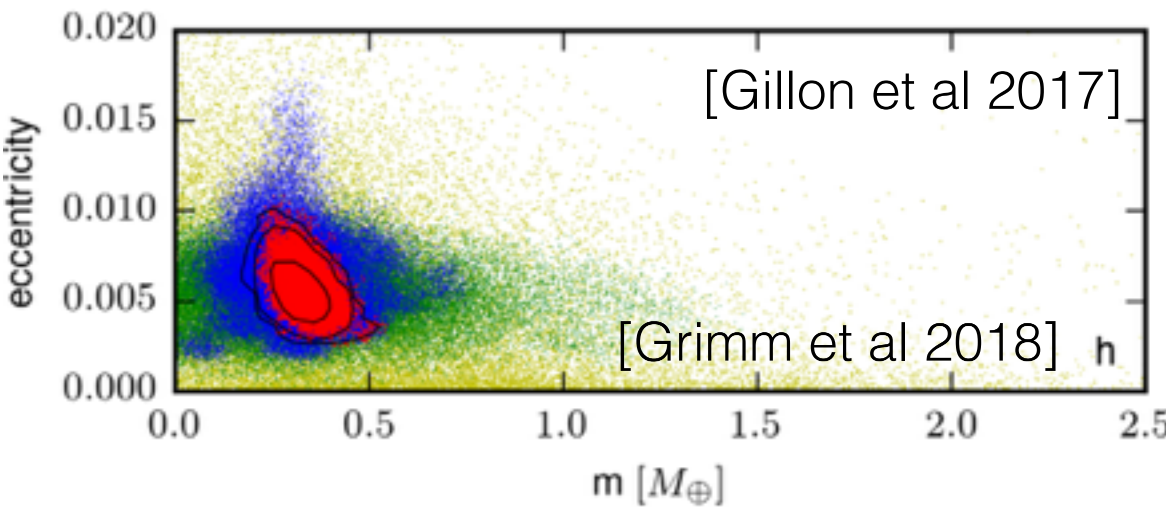
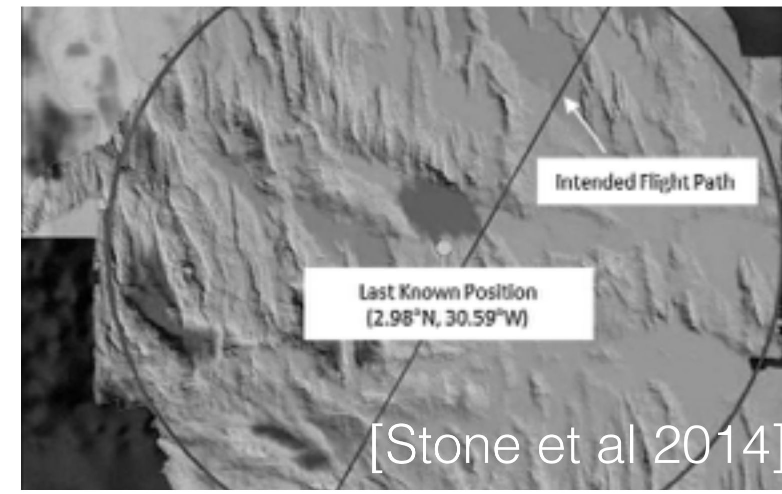
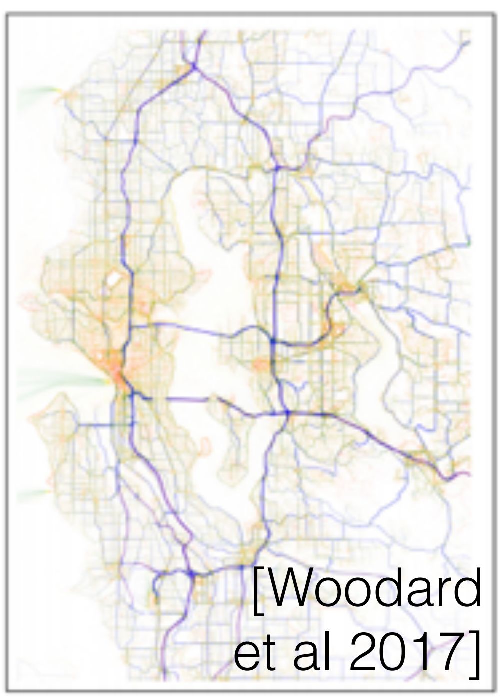
http://www.tamarabroderick.com/tutorial_2018_lugano.html

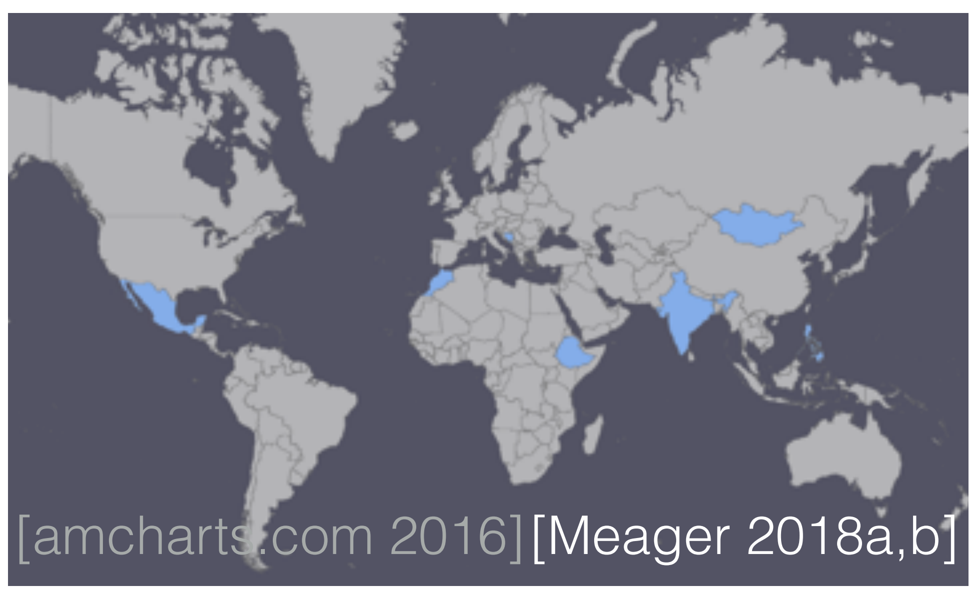
Bayesian inference

- Analysis goals: Point estimates, coherent uncertainties









[mc-stan.org]

- Challenge: fast (compute, user), reliable inference

Approximate Bayesian inference

Use q^* to approximate $p(\cdot|y)$

Optimization

$$q^* = \mathrm{argmin}_{q \in Q} f(q(\cdot), p(\cdot|y))$$

Variational Bayes

$$q^* = \mathrm{argmin}_{q \in Q} KL(q(\cdot)||p(\cdot|y))$$

Mean-field variational Bayes

$$q^* = \mathrm{argmin}_{q \in Q_{\mathrm{MFVB}}} KL(q(\cdot)||p(\cdot|y))$$

- Coordinate descent
- Stochastic variational inference (SVI) [Hoffman et al 2013]
- Automatic differentiation variational inference (ADVI) [Kucukelbir et al 2015, 2017]

What about uncertainty?

$$KL(q||p(\cdot|y)) = \int_\theta q(\theta) \log \frac{q(\theta)}{p(\theta|y)} d\theta$$

$$q(\theta) = \prod_{j=1}^{J} q_j(\theta_j)$$

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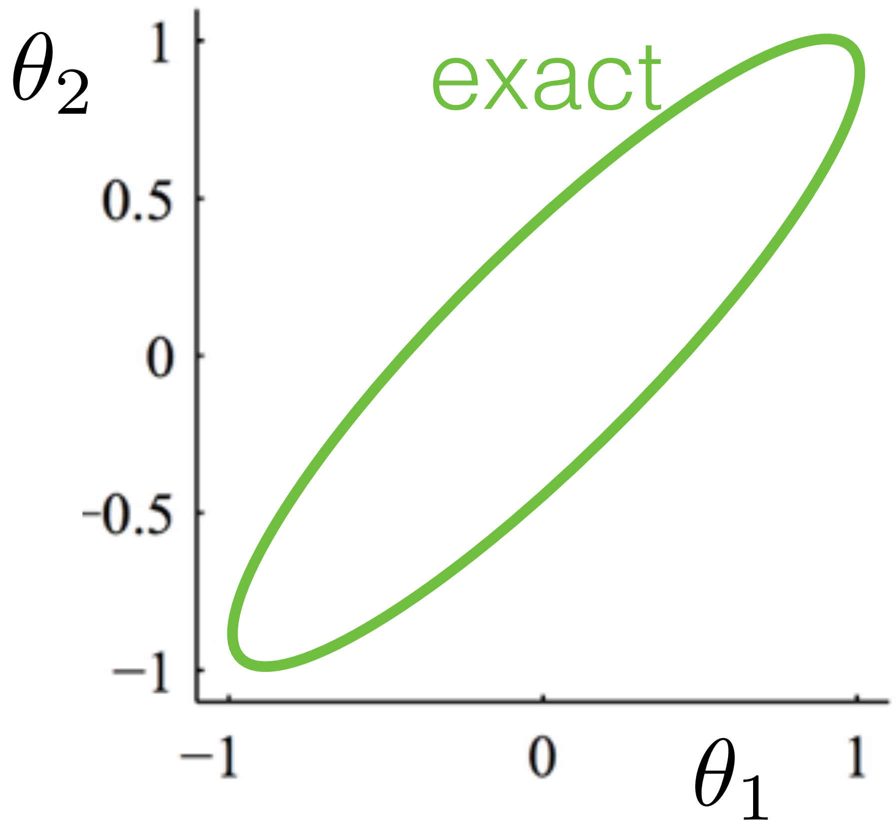

[Turner & Sahani 2011; MacKay 2003; Bishop 2006; Wang, Titterington 2004]

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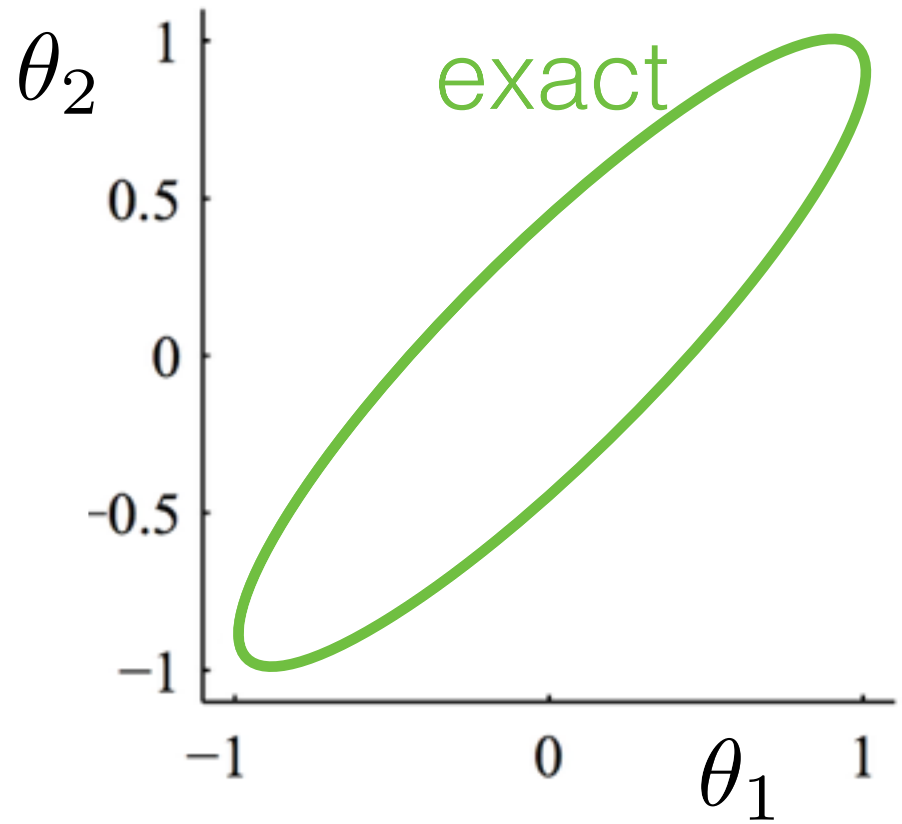

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- Conjugate linear regression

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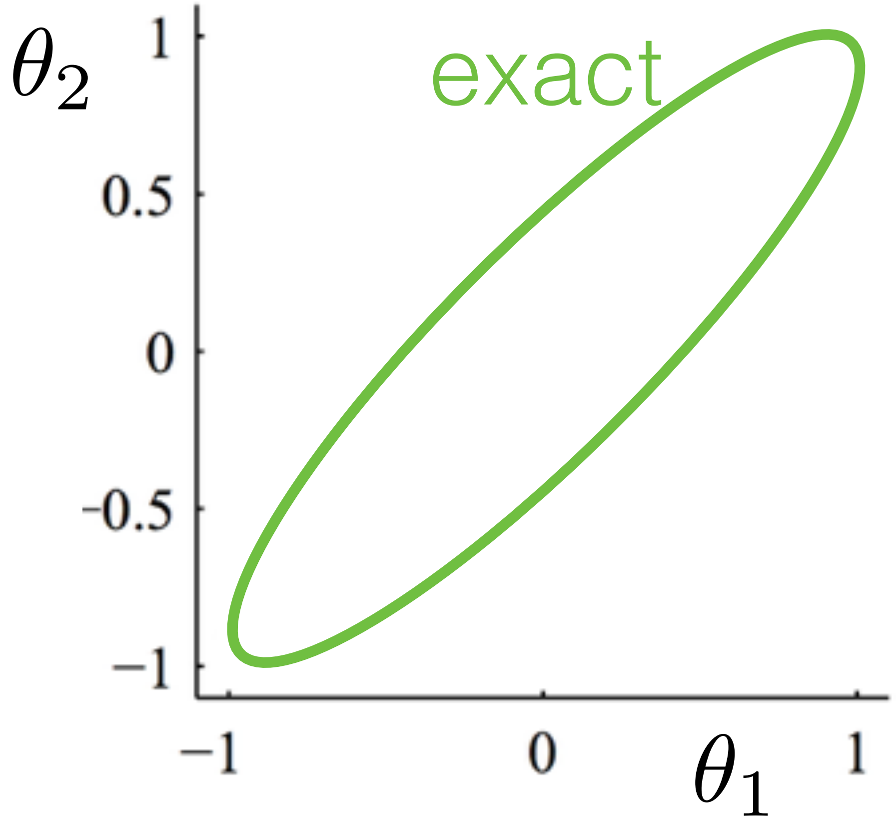

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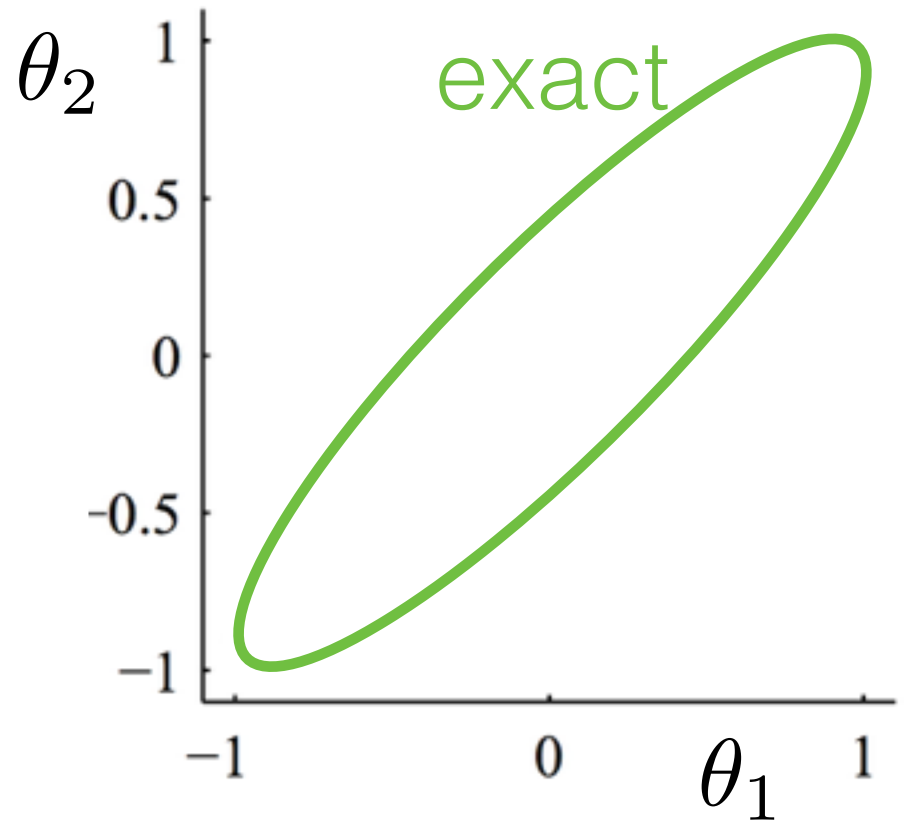

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[board]

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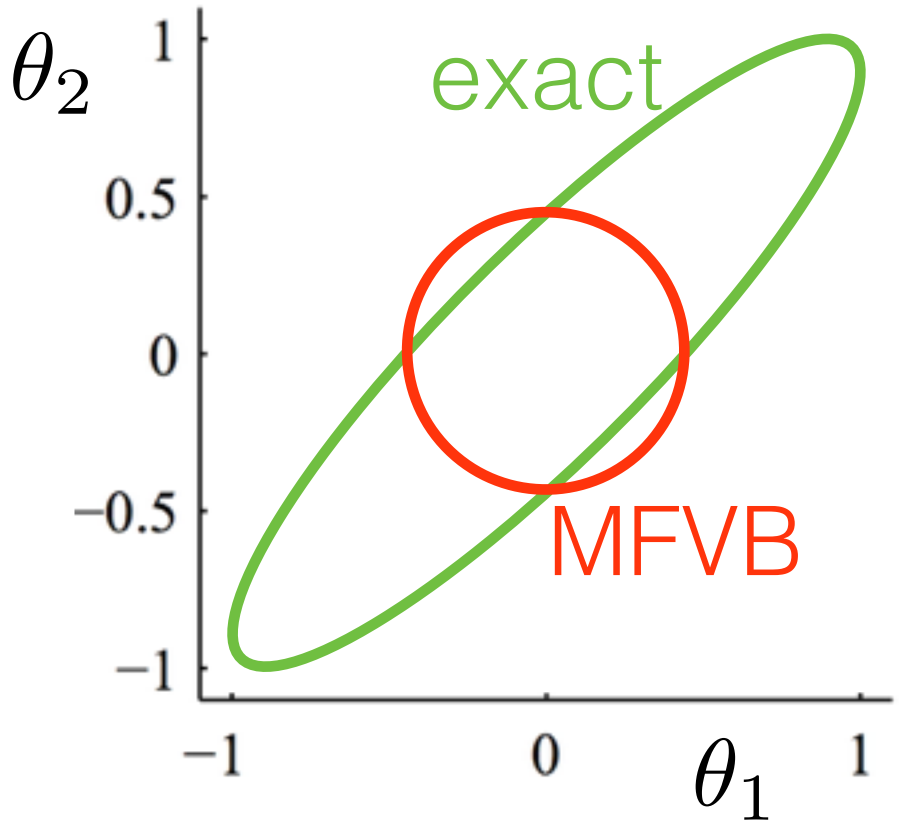

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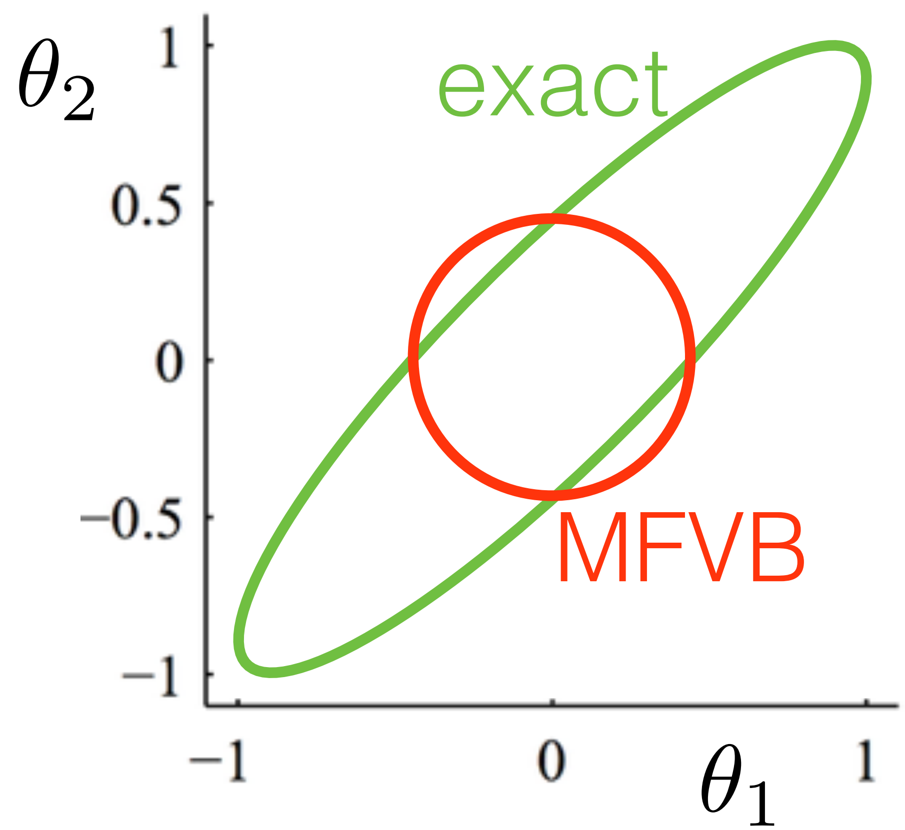

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- Underestimates variance (sometimes severely)

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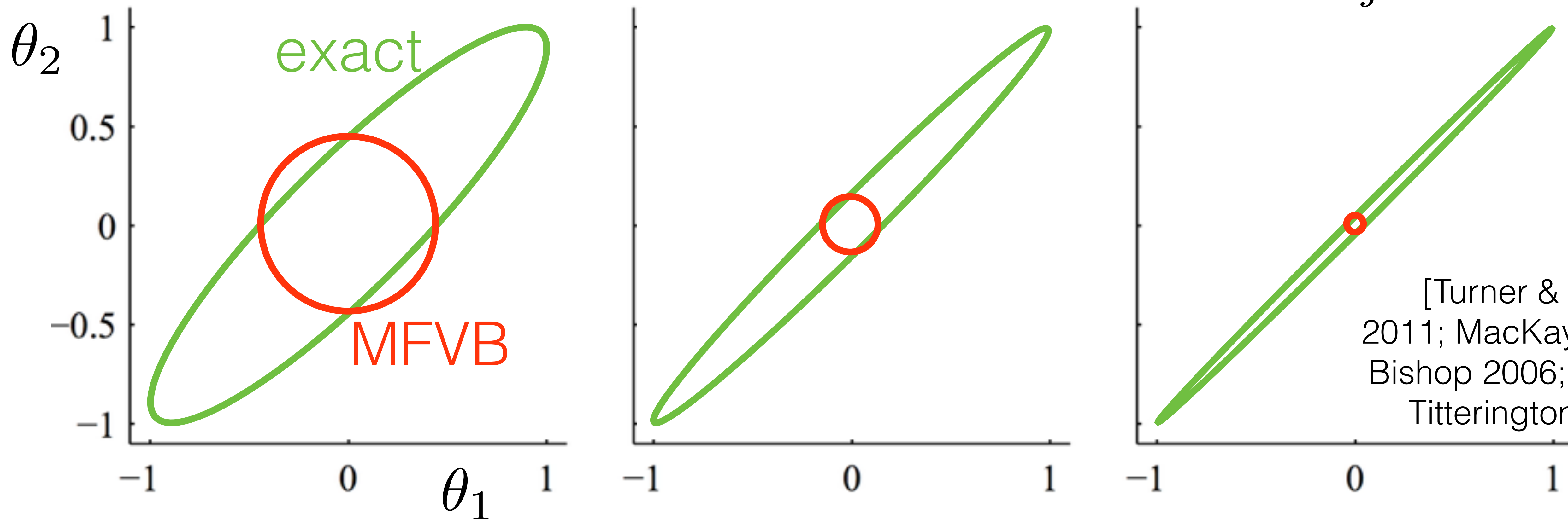

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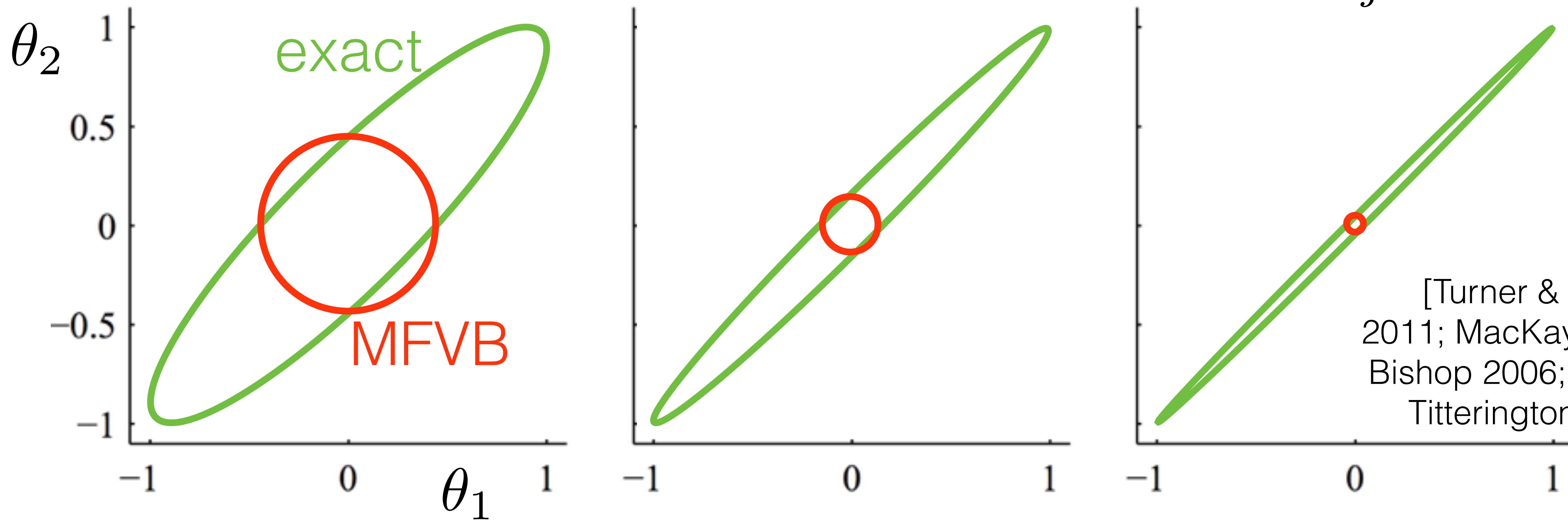

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- Underestimates variance (sometimes severely)
- No covariance estimates

- Conjugate linear regression
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What about uncertainty?

- Microcredit

[Giordano, Broderick, Meager, Huggins, Jordan 2016]

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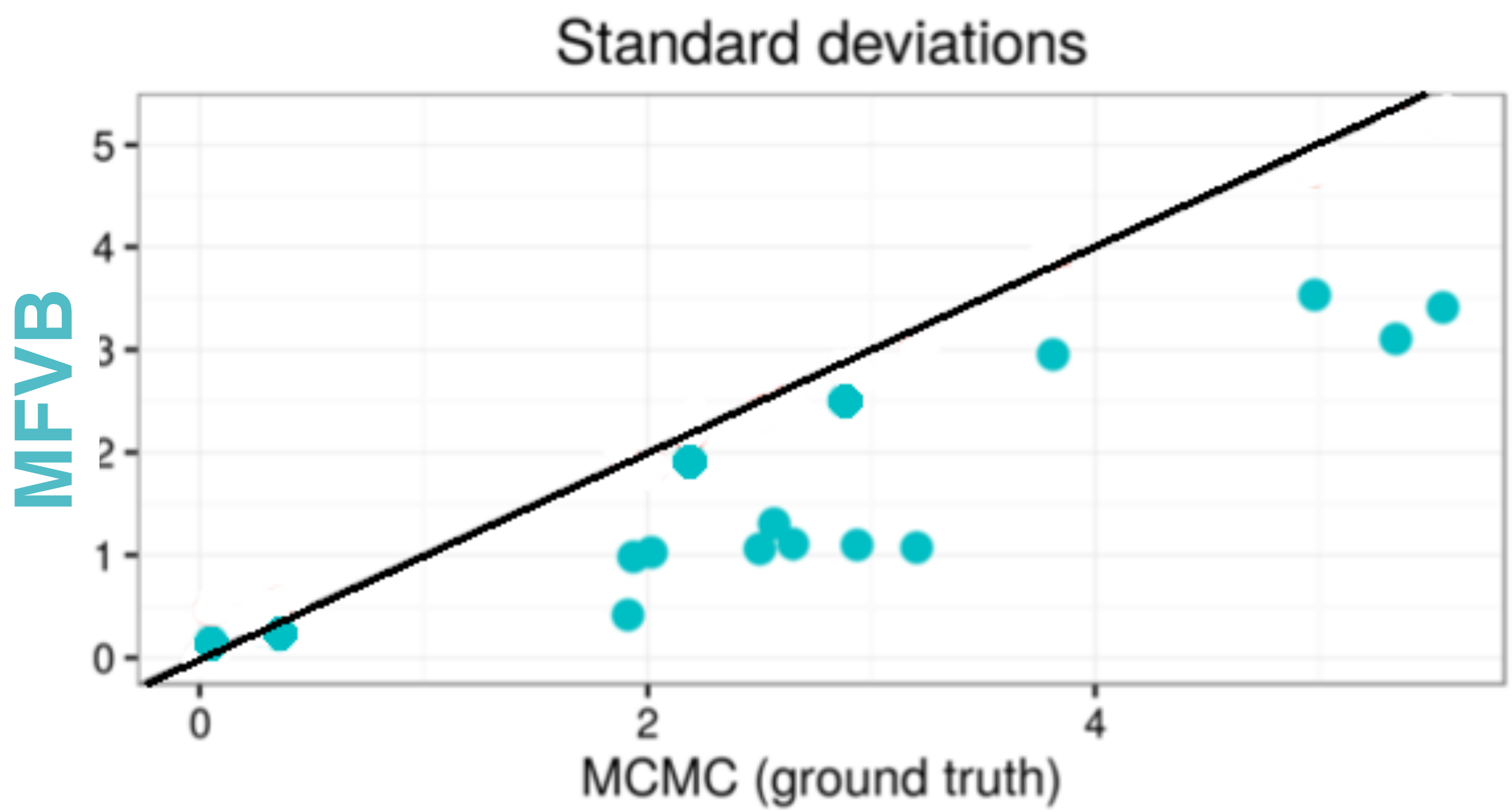
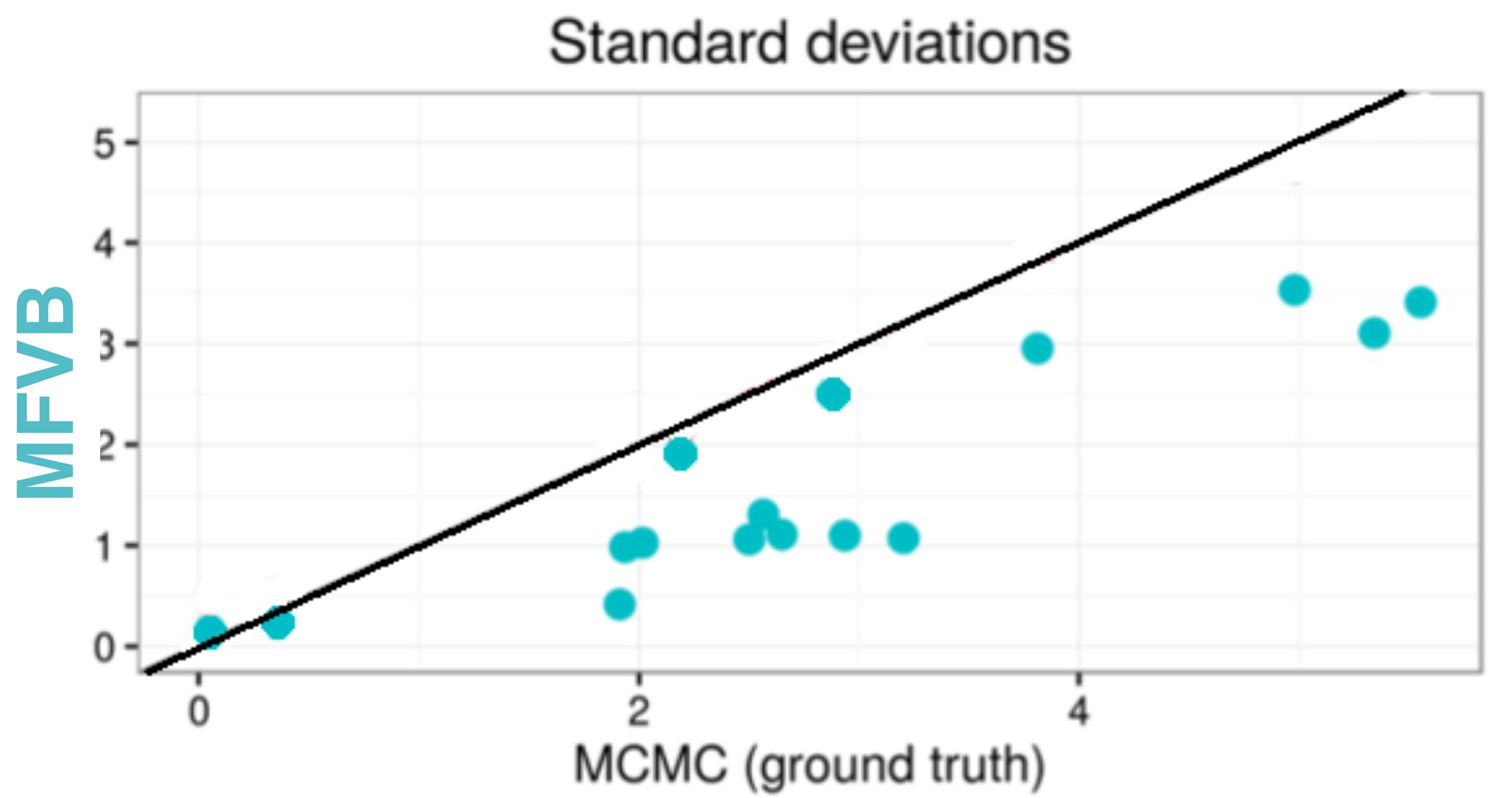


[Giordano, Broderick, Meager, Huggins, Jordan 2016]

What about uncertainty?

- Microcredit effect
- τ mean: 3.08 USD PPP



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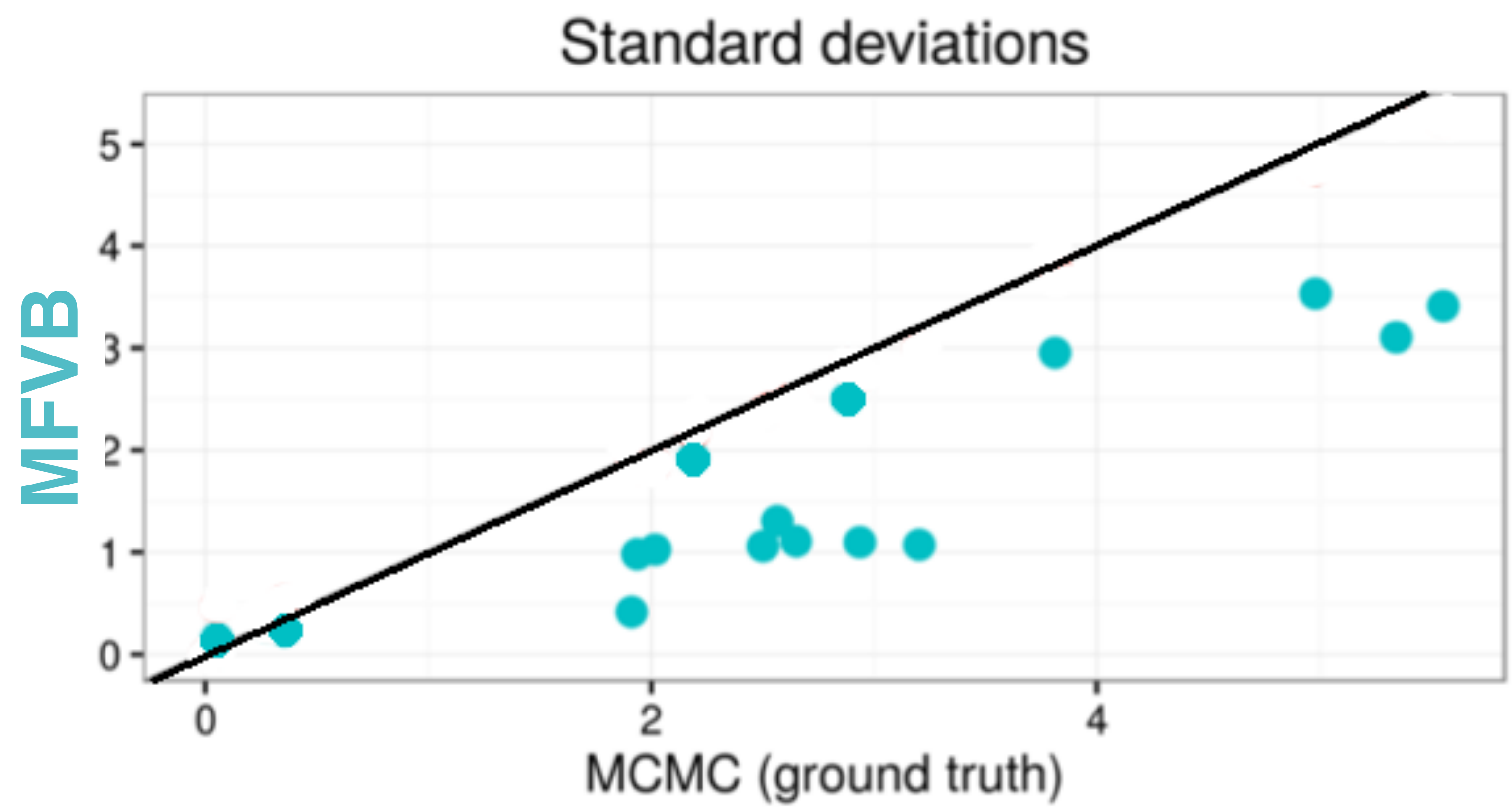

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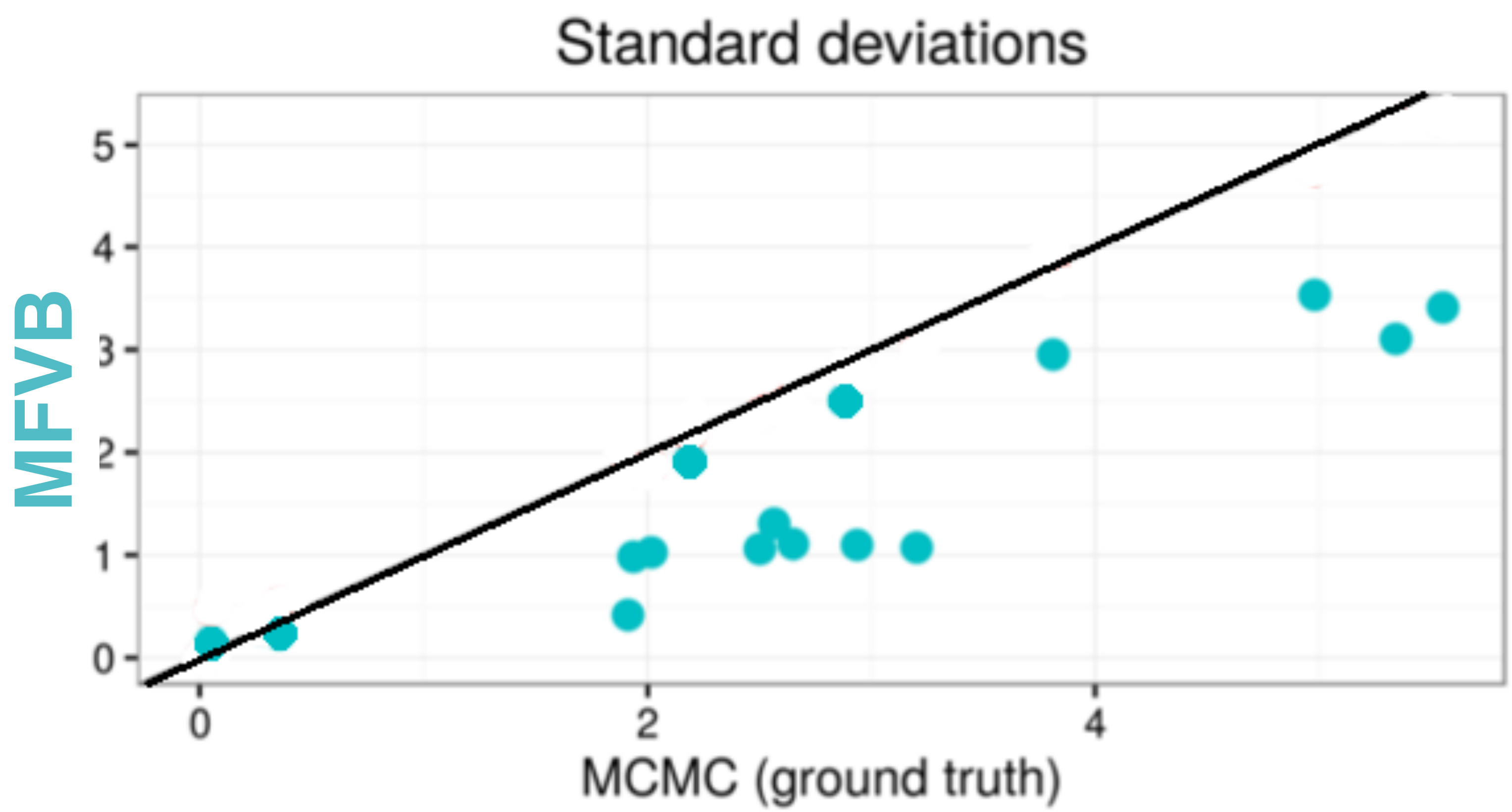

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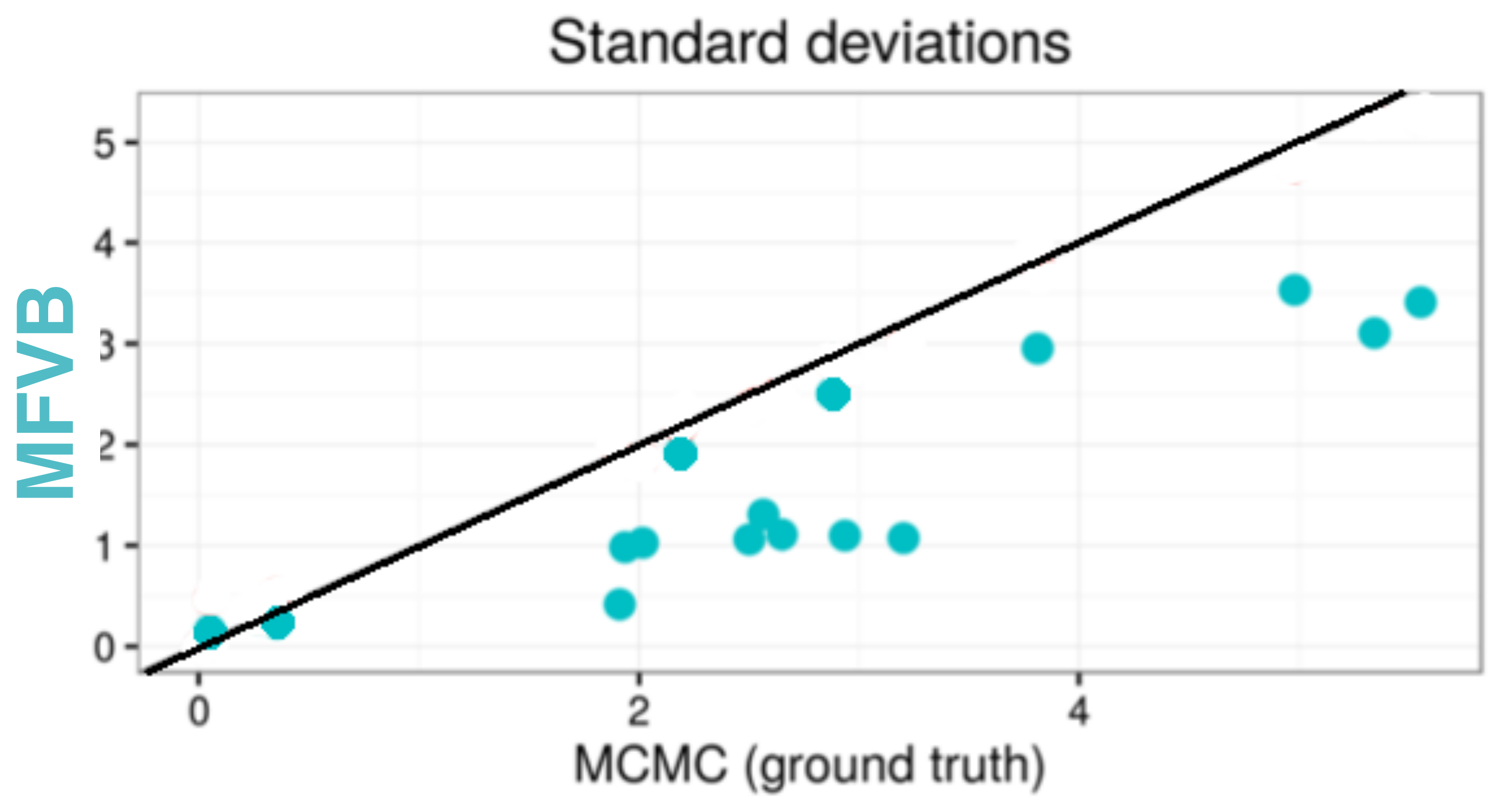
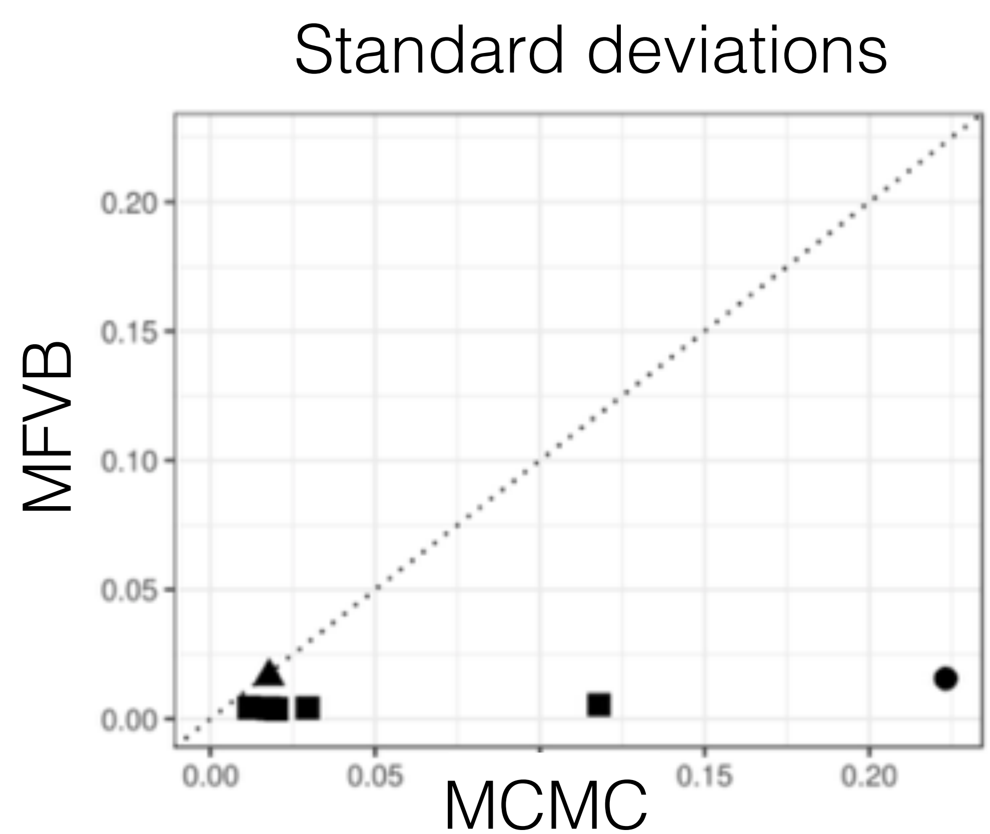


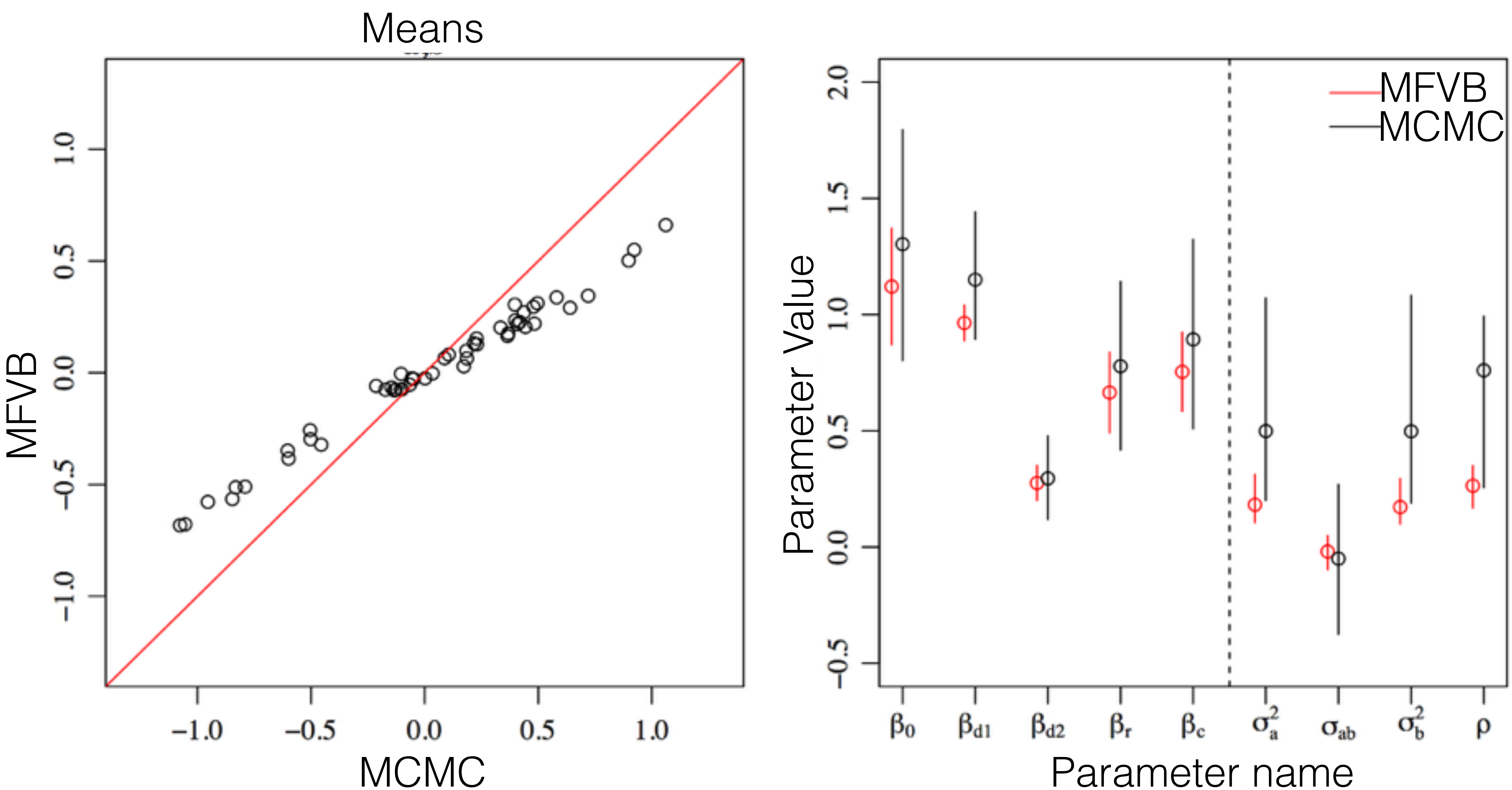
- Criteo online ads experiment



[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2017]

What about means?

- Model for relational data with covariates
- When 1000+ nodes, MCMC > 1 day [Fosdick 2013, Ch 4]



[Fosdick 2013, Ch 4, Fig 4.3]

Posterior means: revisited

- Want to predict college GPA y_n

[Giordano, Broderick, Jordan 2015]

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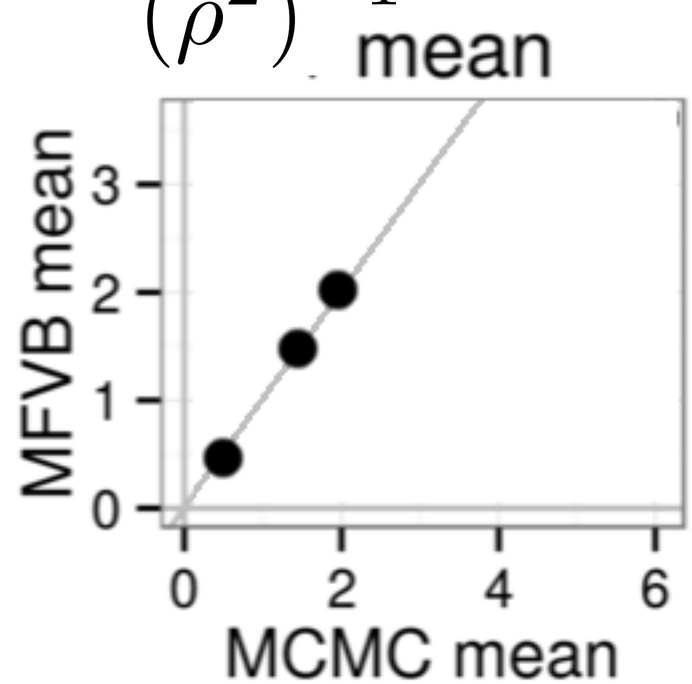
- Data simulated from model (3 data sets, 300 data points):

$(\rho^2)^{-1}$



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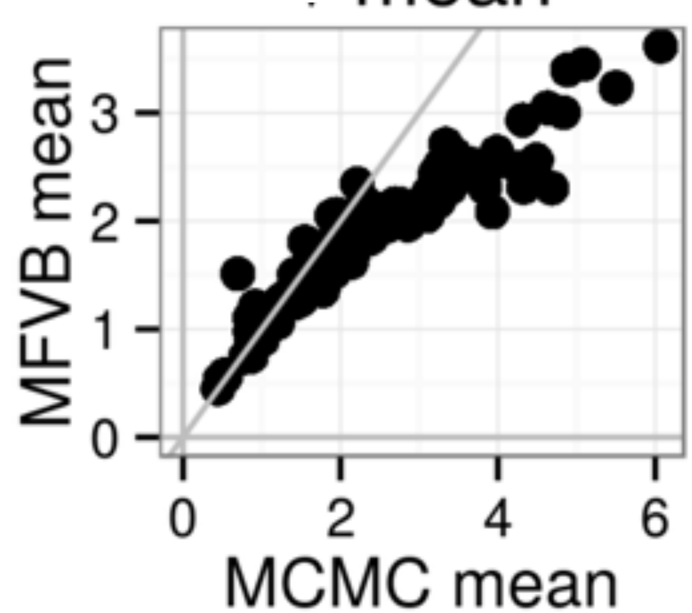
- Data simulated from model (100 data sets, 300 data points):

$(\rho^2)^{-1}$ mean



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Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use MFVB?
- When can we trust MFVB?
- Where do we go from here?

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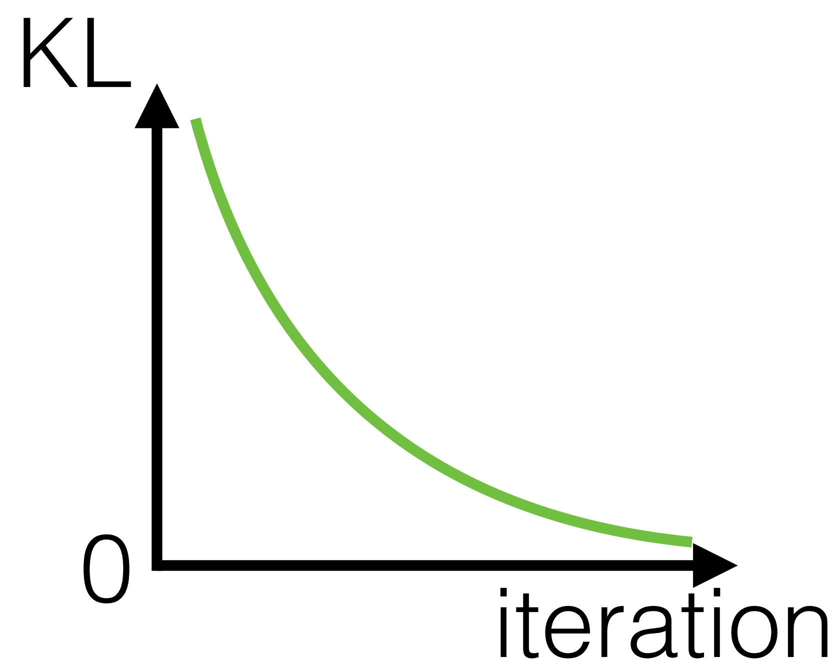

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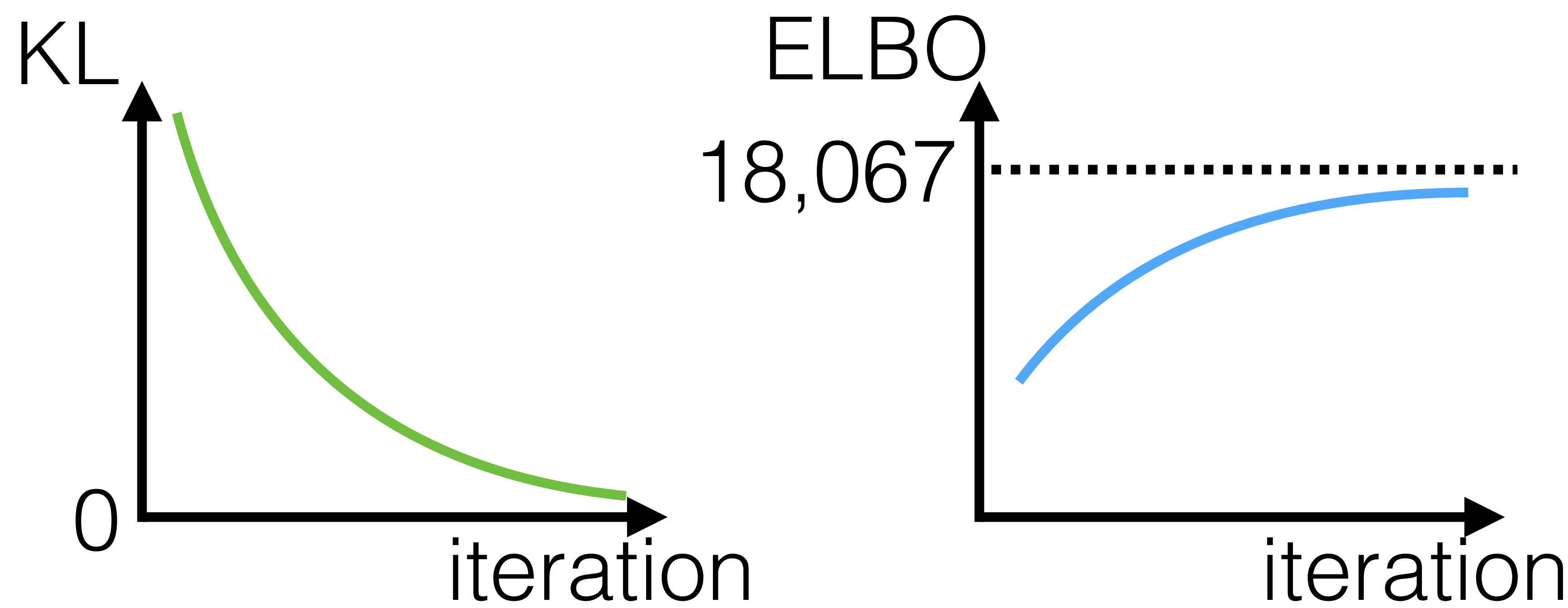

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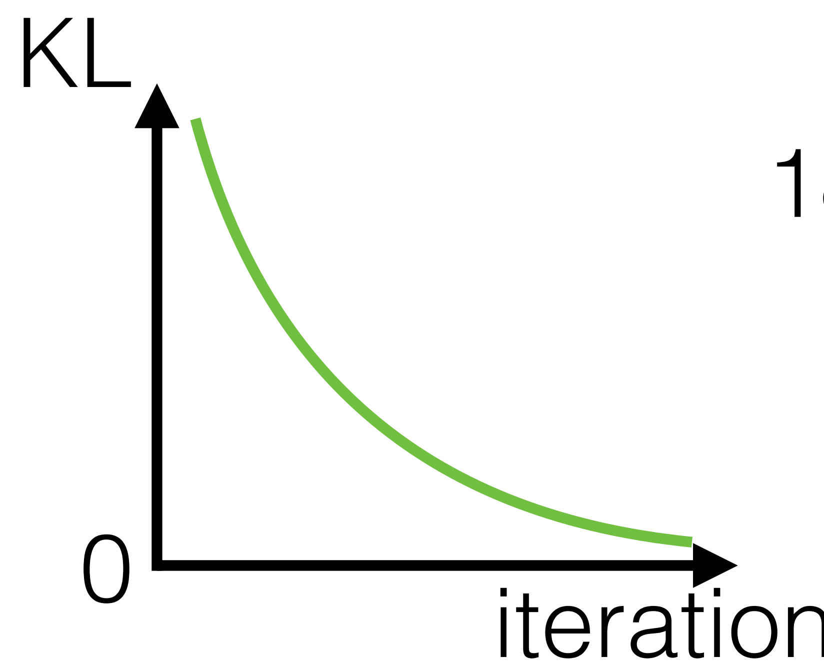

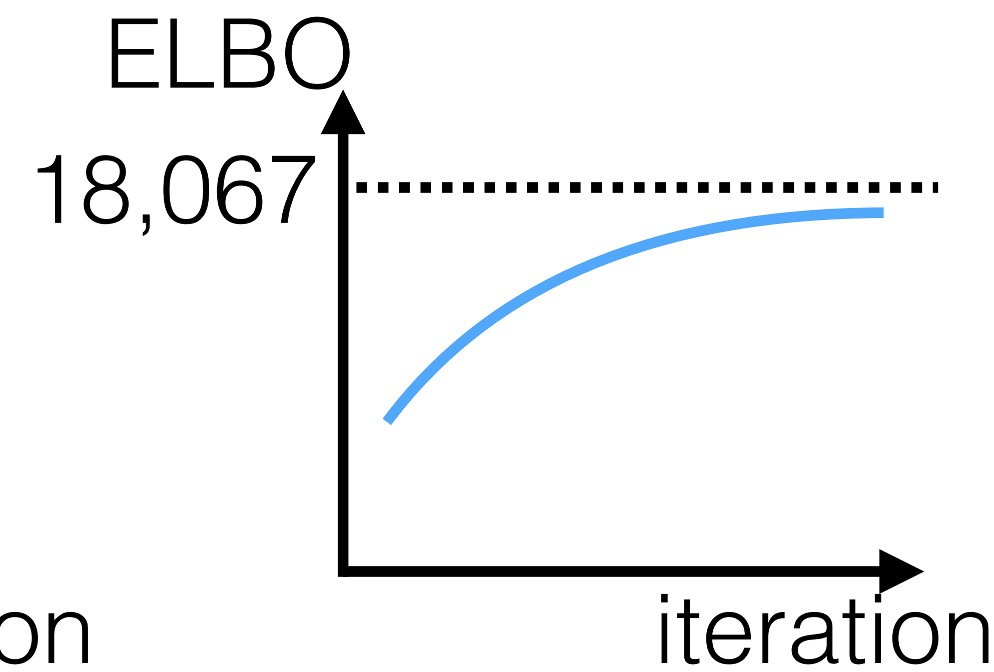

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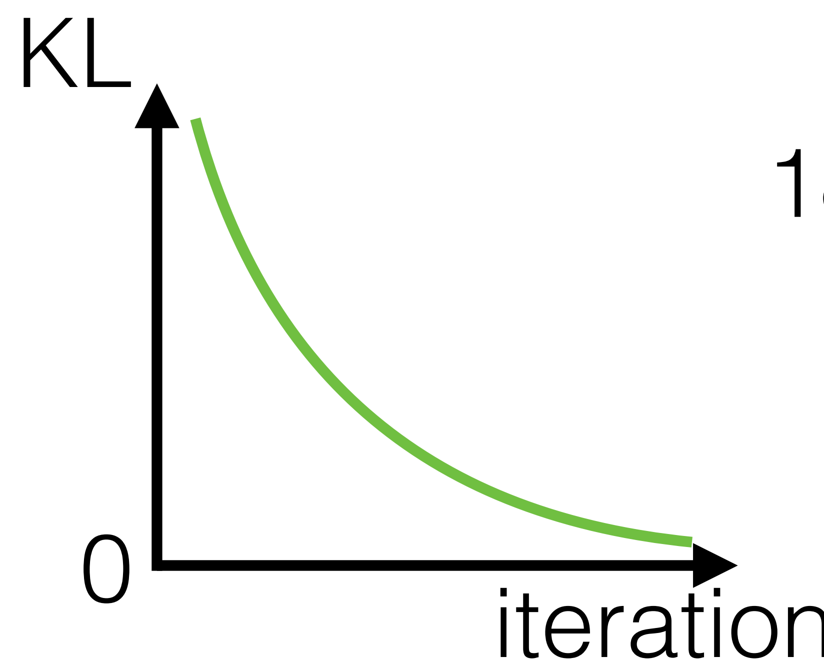

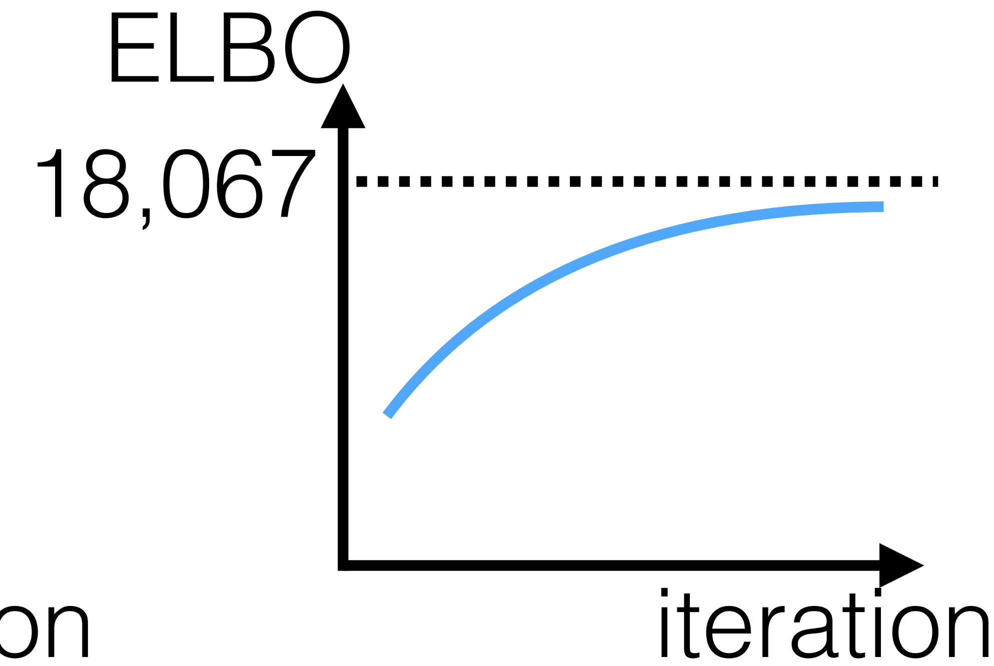

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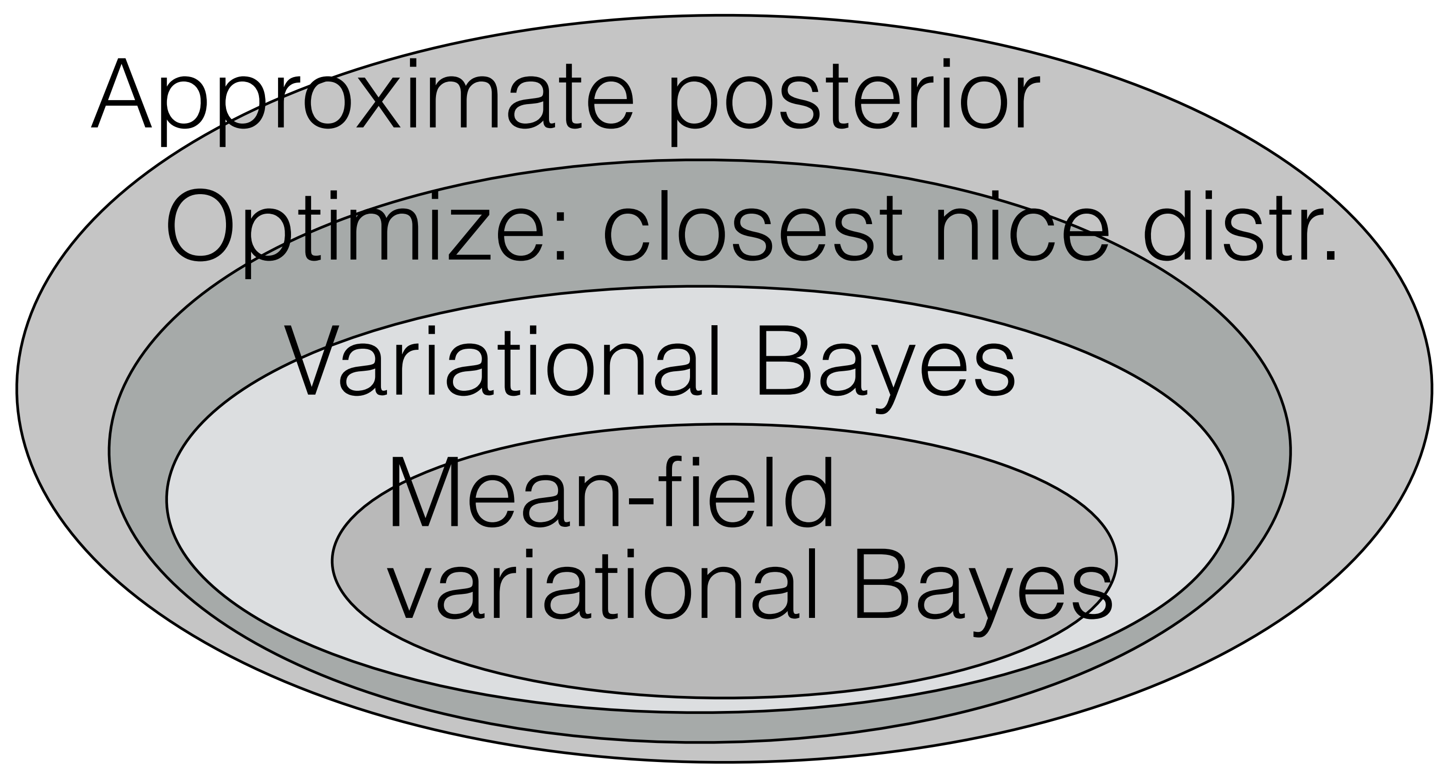

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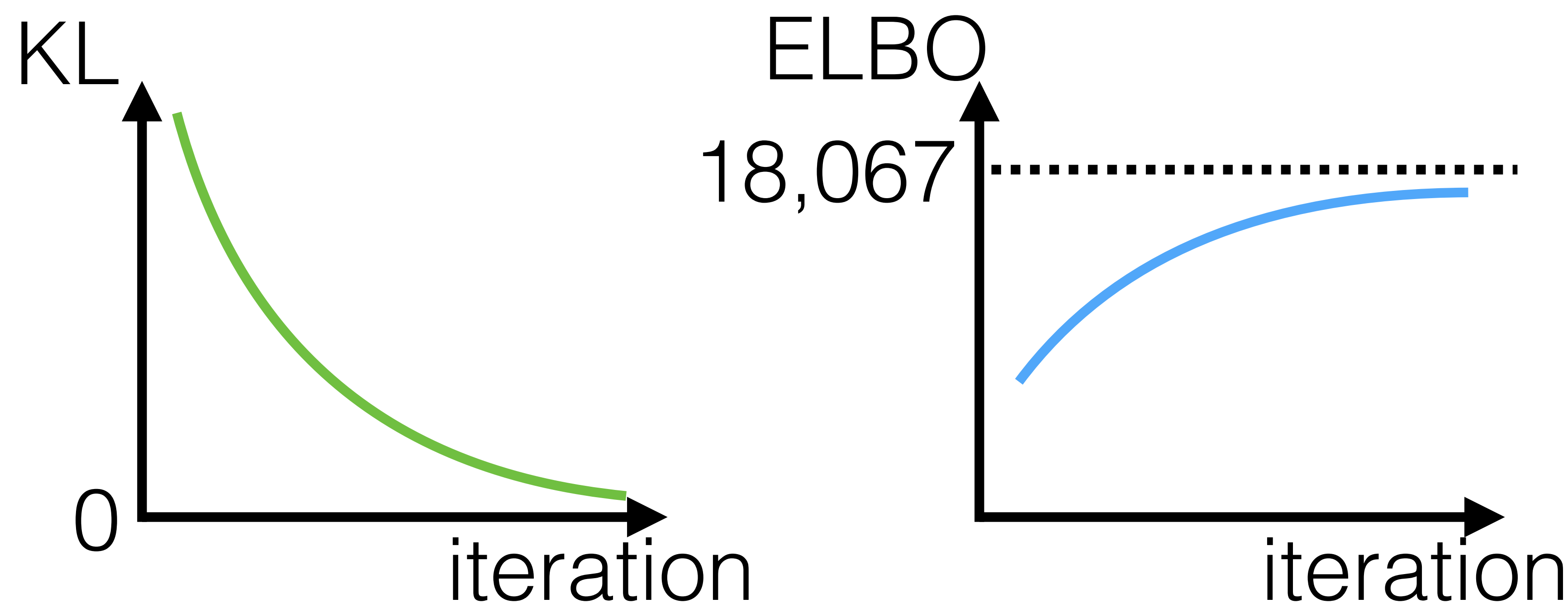

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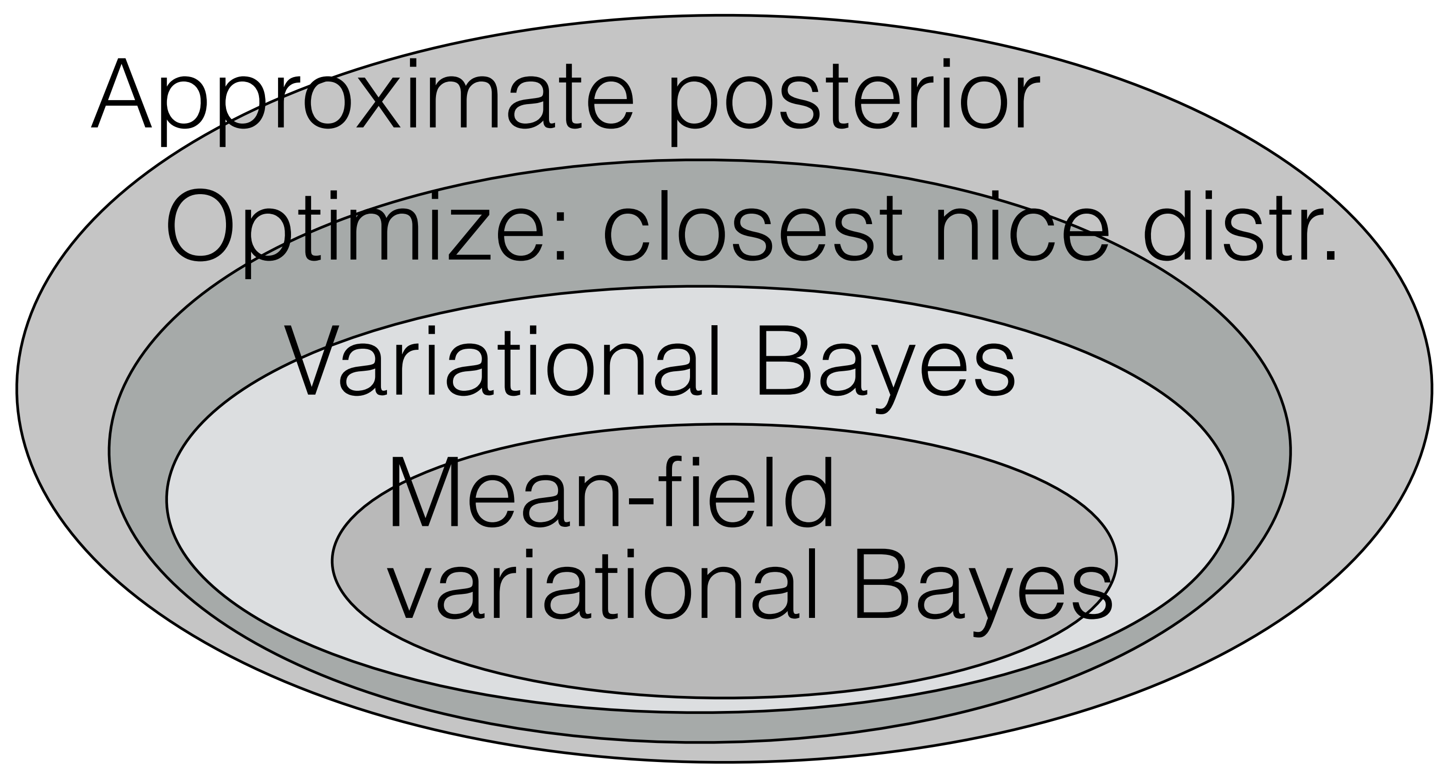

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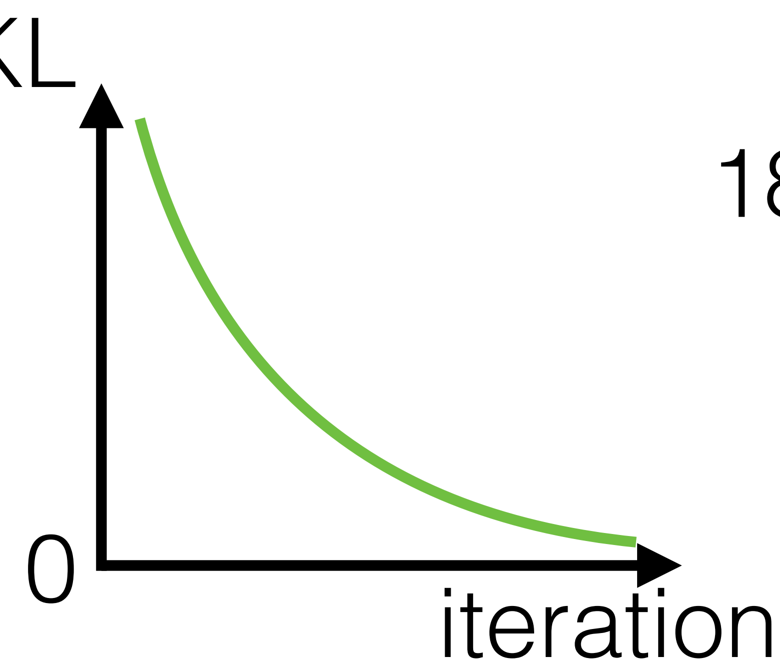

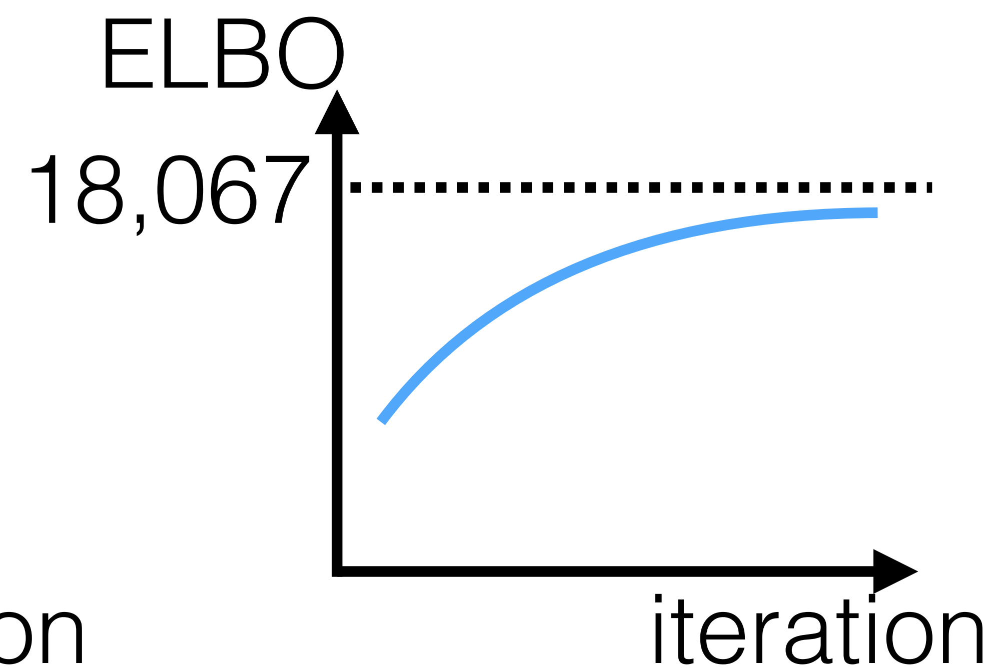

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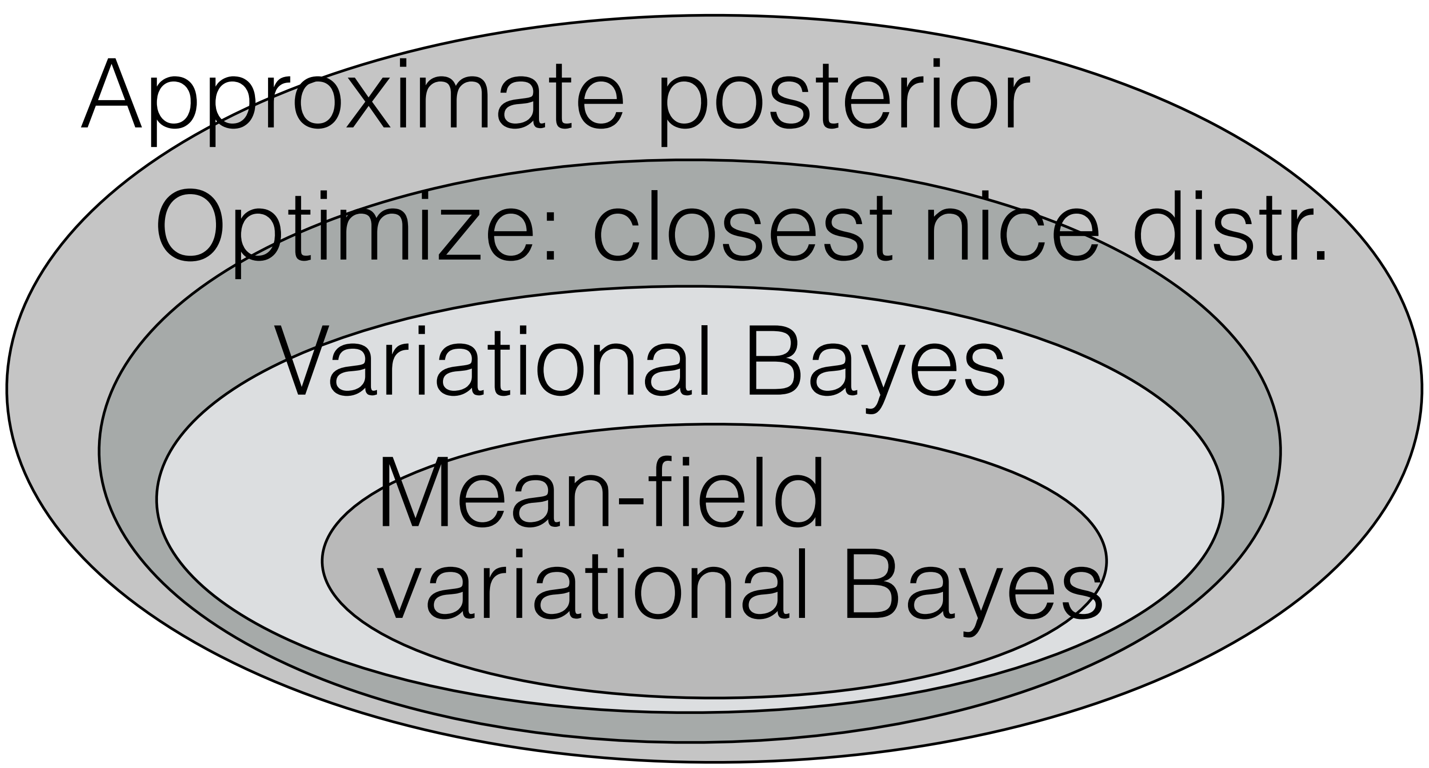

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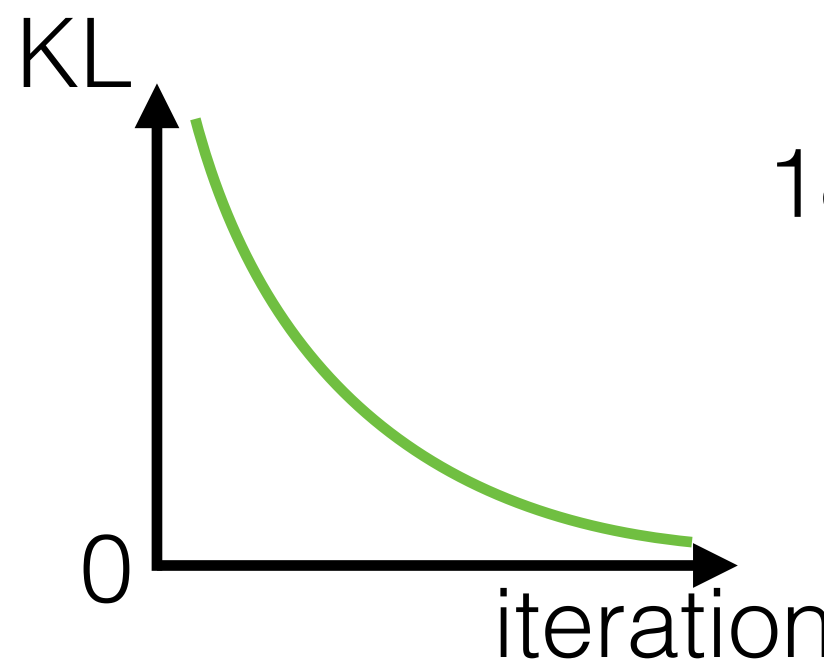

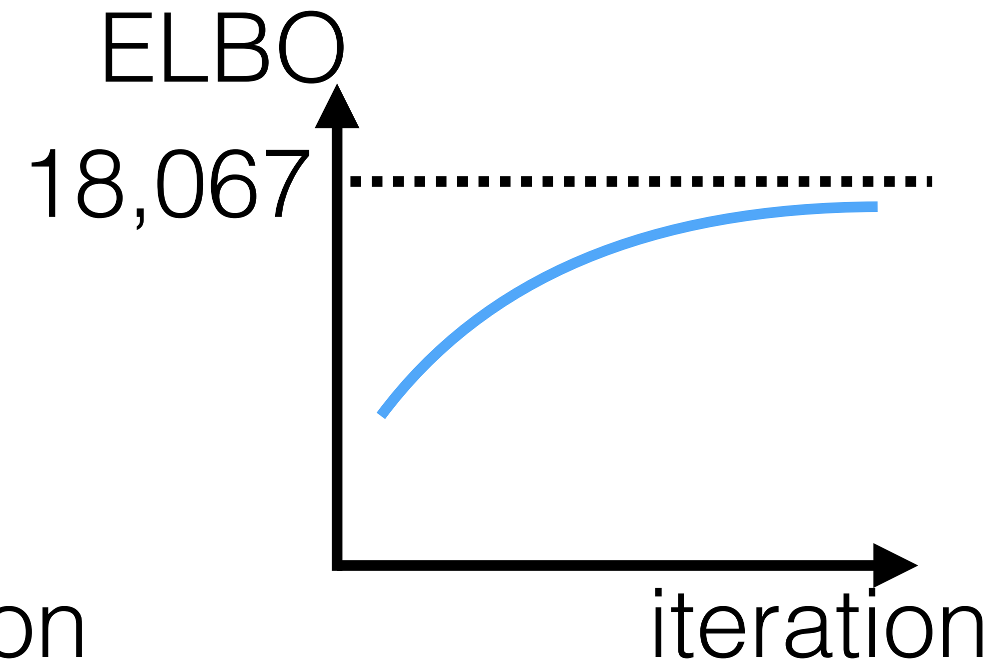

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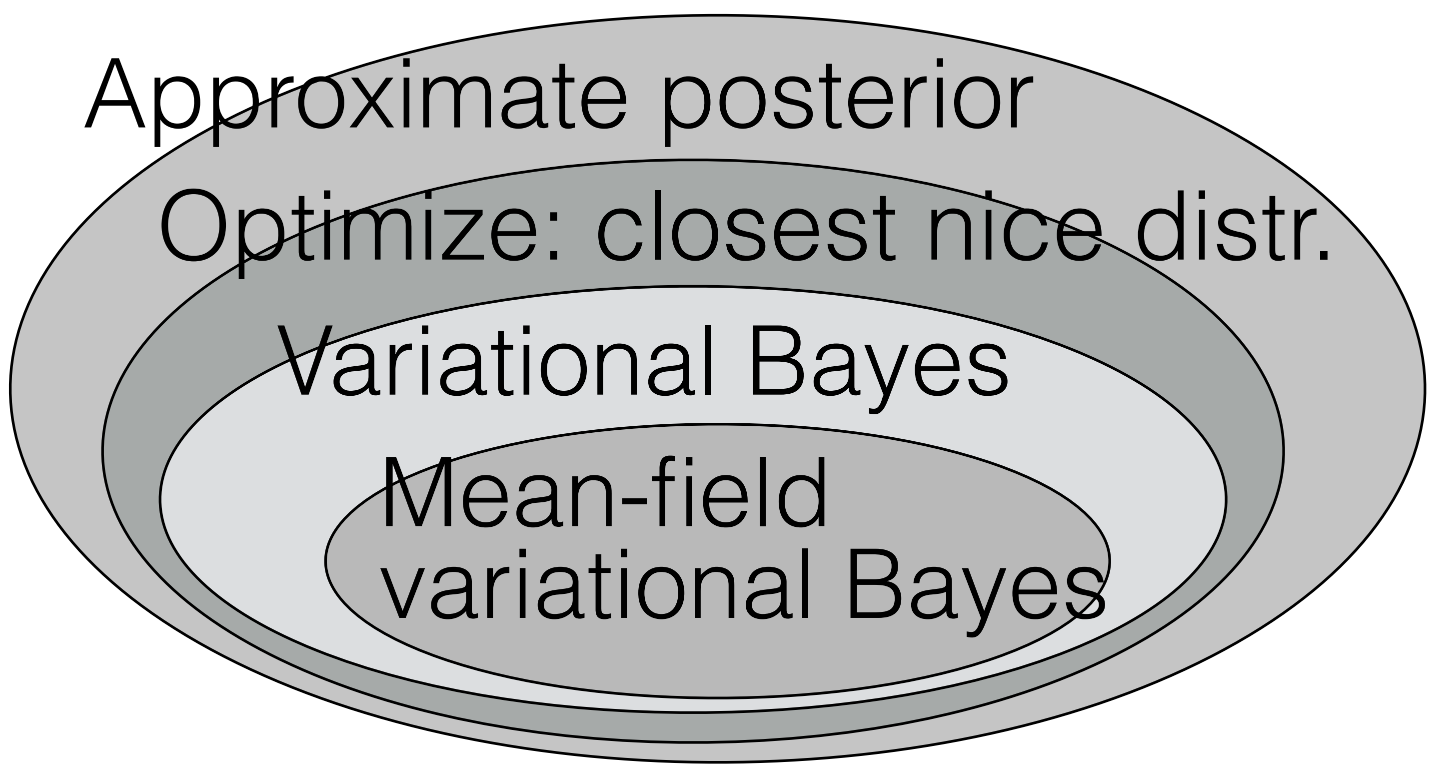

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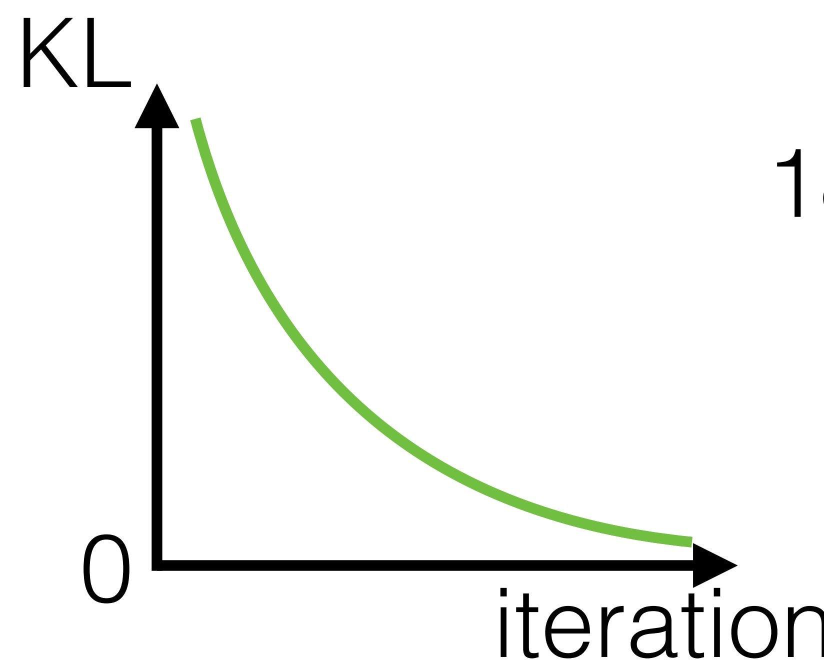

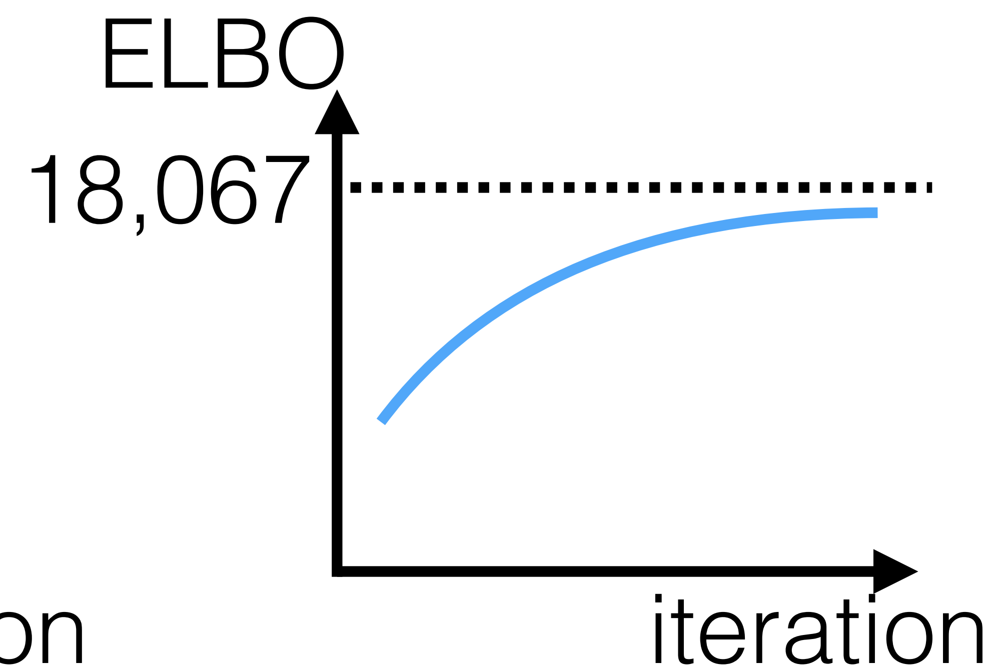

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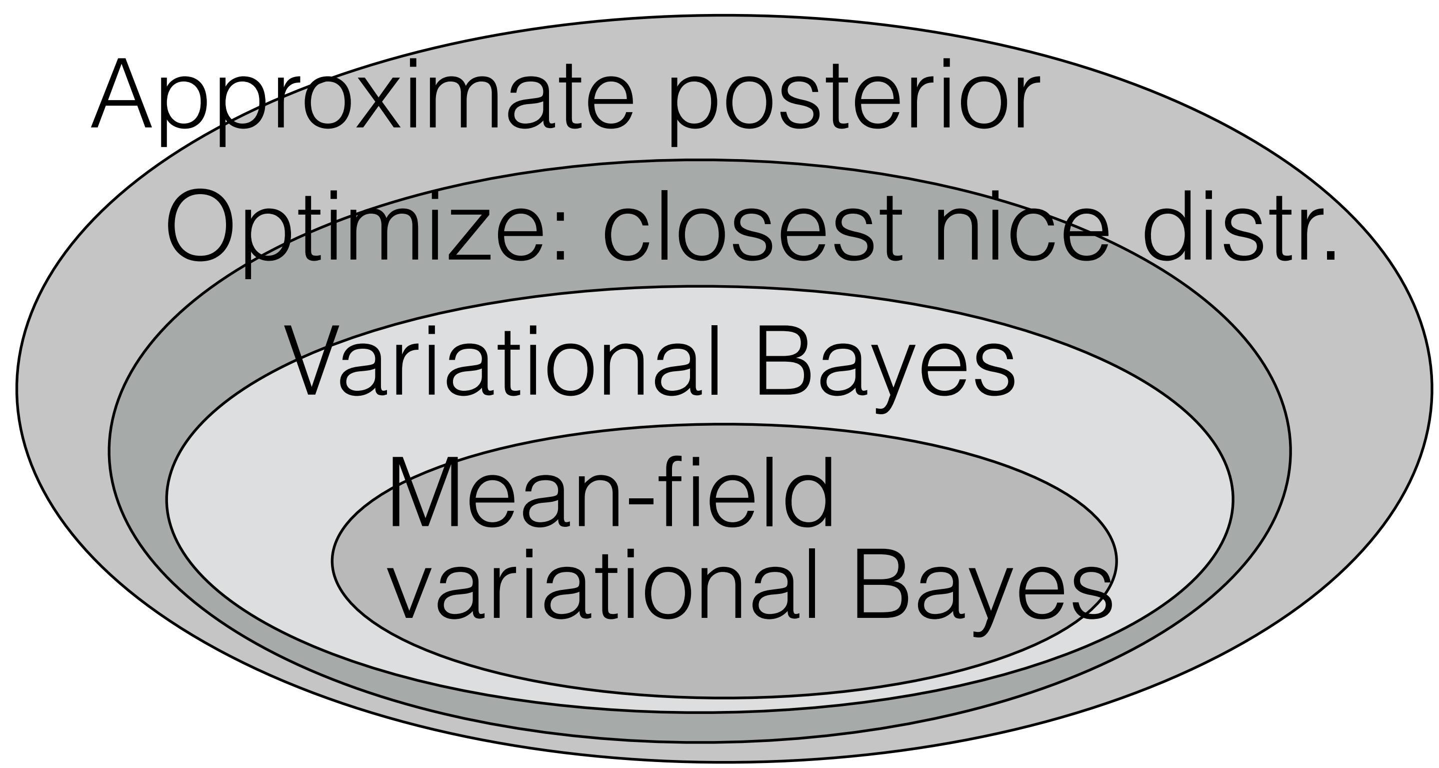

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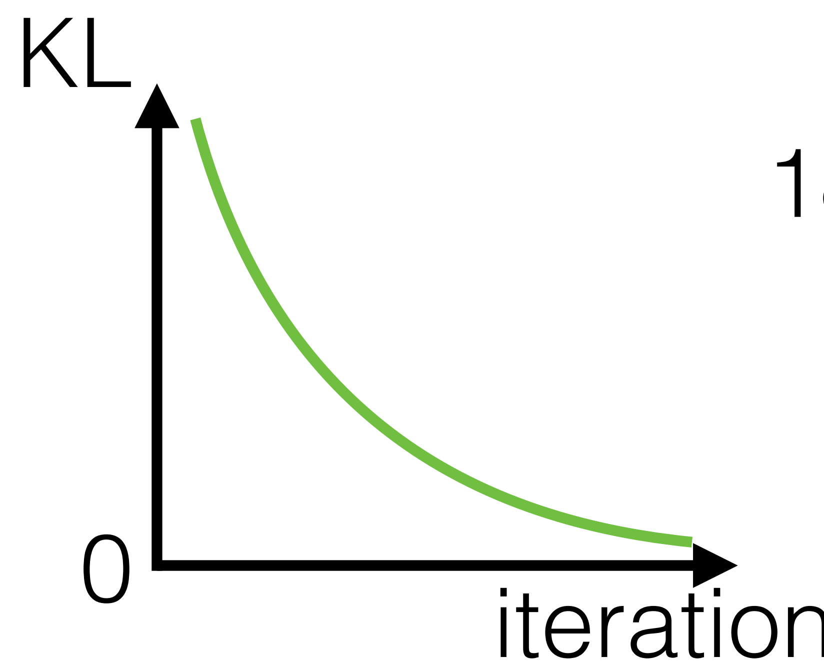

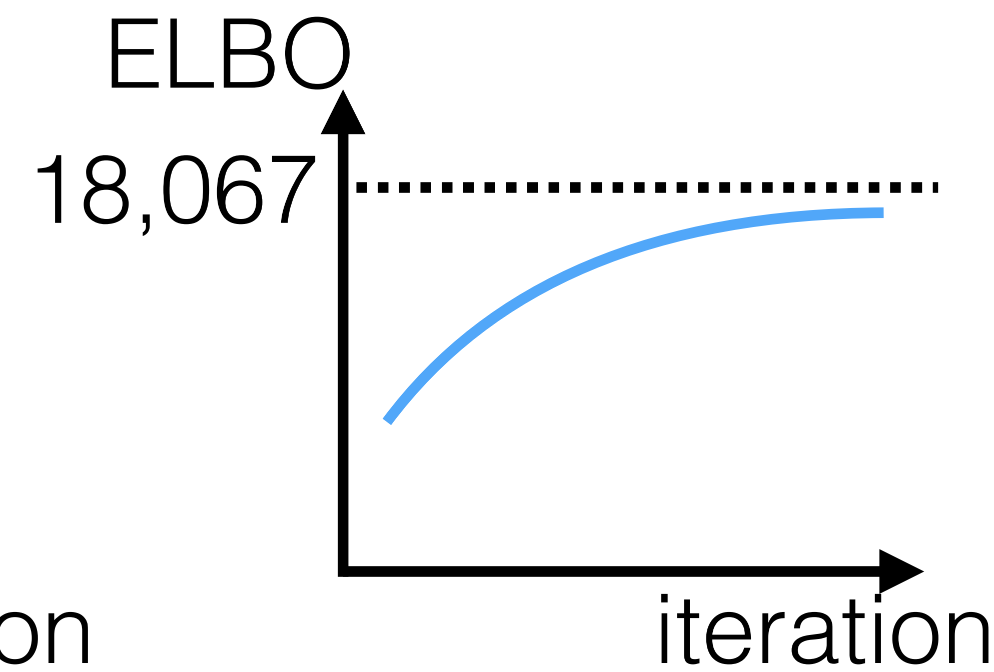

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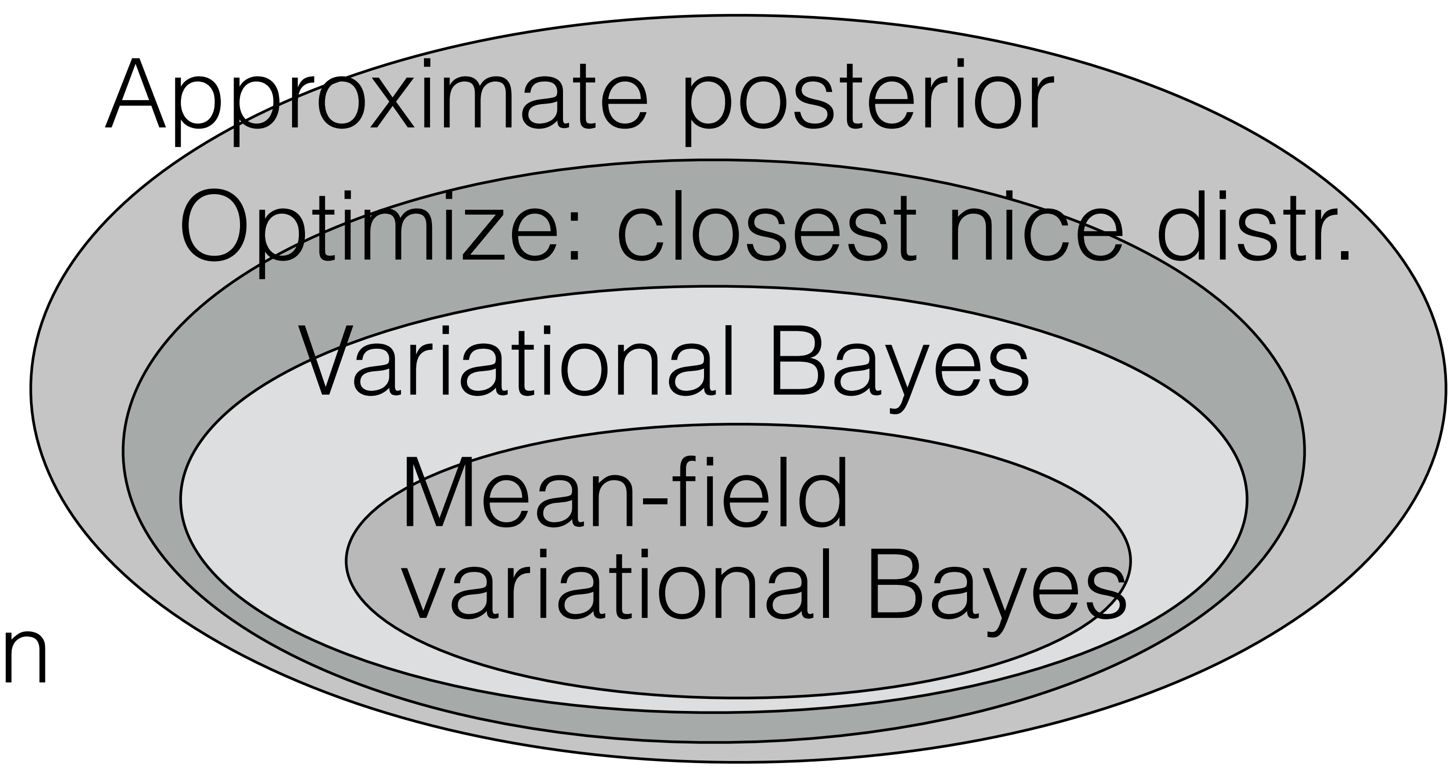
- Theoretical guarantees on finite-data quality [next]



What to read next

- Textbooks and Reviews
  - Bishop. *Pattern Recognition and Machine Learning*, Ch 10. 2006.
  - Blei, Kucukelbir, McAuliffe. Variational inference: A review for statisticians, *JASA* 2016.
  - MacKay. *Information Theory, Inference, and Learning Algorithms*, Ch 33. 2003.
  - Murphy. *Machine Learning: A Probabilistic Perspective*, Ch 21. 2012.
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- See the end of Part II for reference list up to this point

Covariances, Robustness, and Variational Bayes

Tamara Broderick
ITT Career Development
Assistant Professor,
MIT

With: Ryan Giordano, Rachael Meager,
Jonathan H. Huggins, Michael I. Jordan

- Bayesian inference

- Bayesian inference $p(\theta)$

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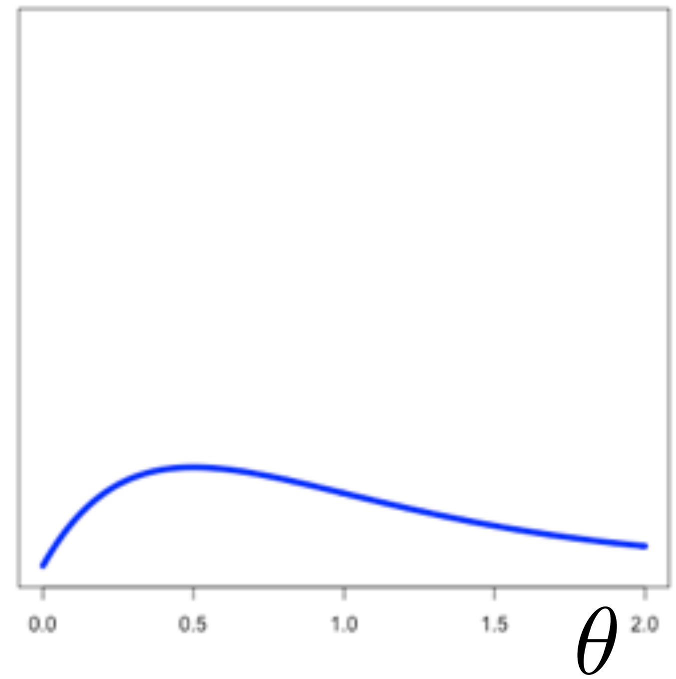

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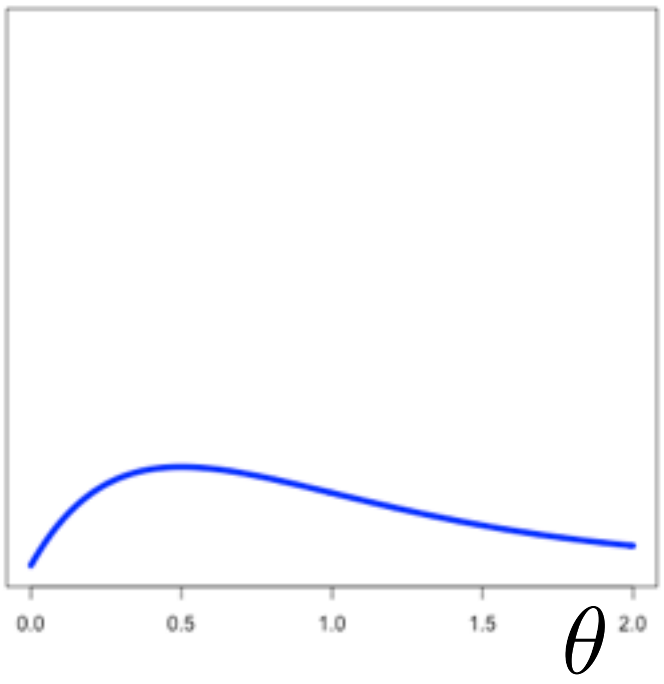

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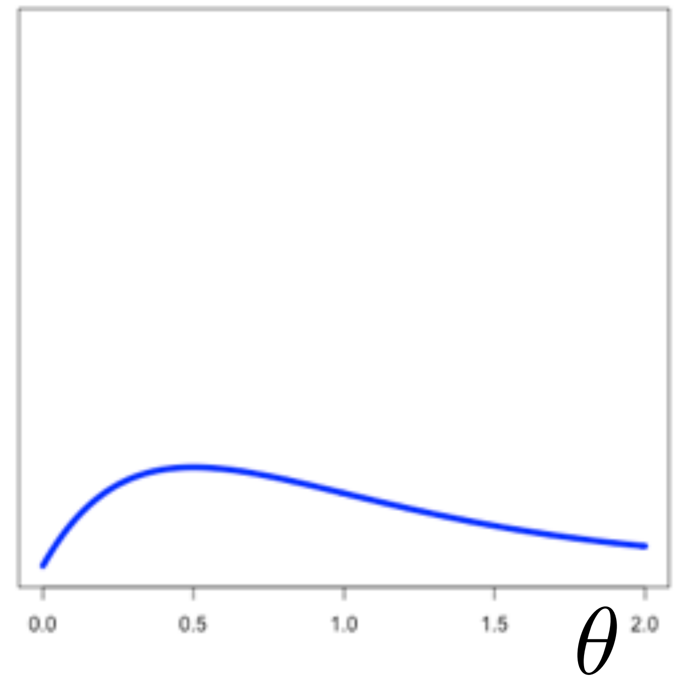

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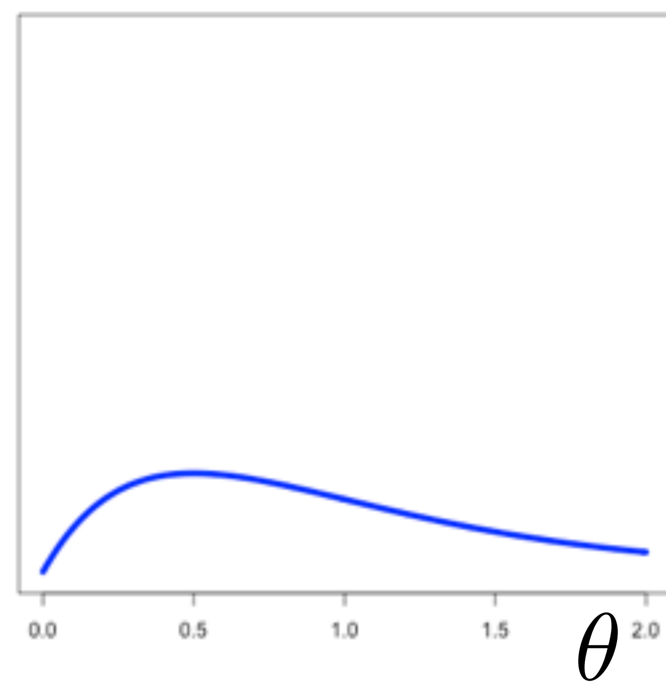

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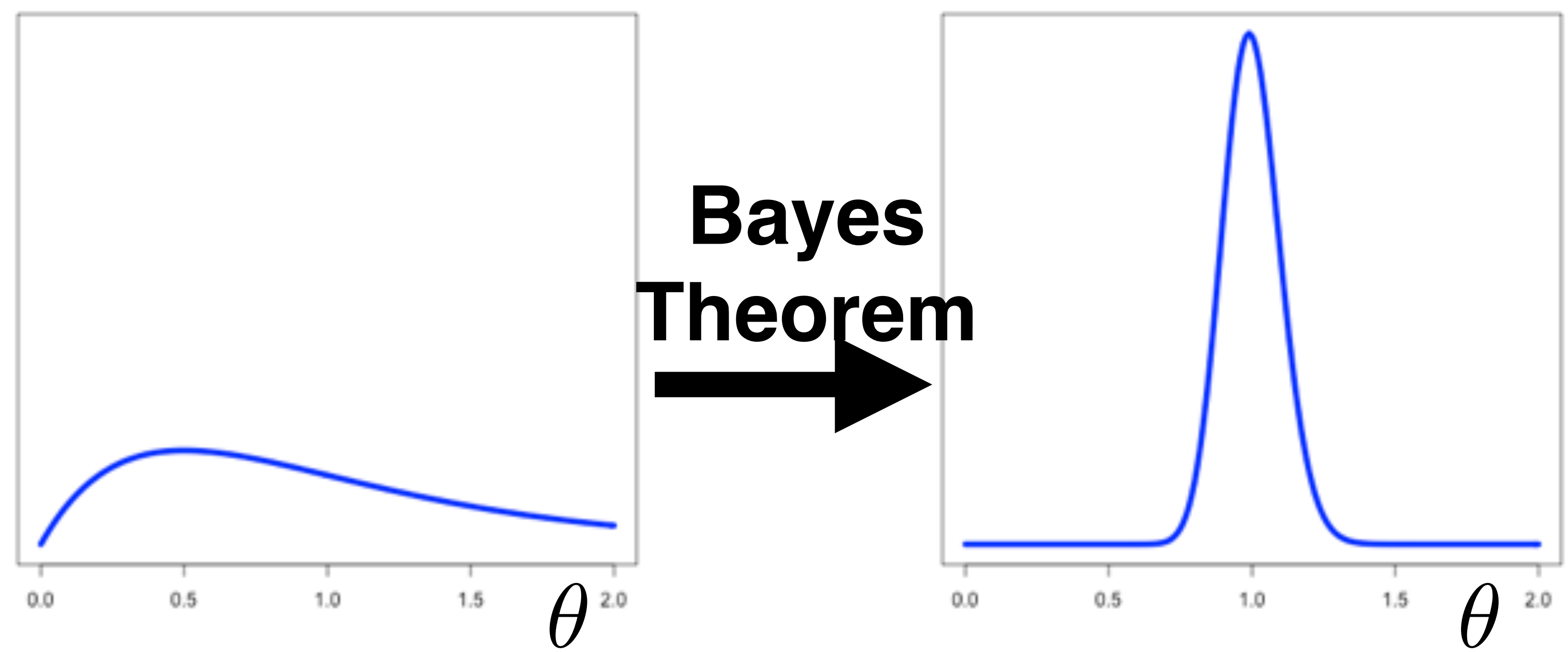

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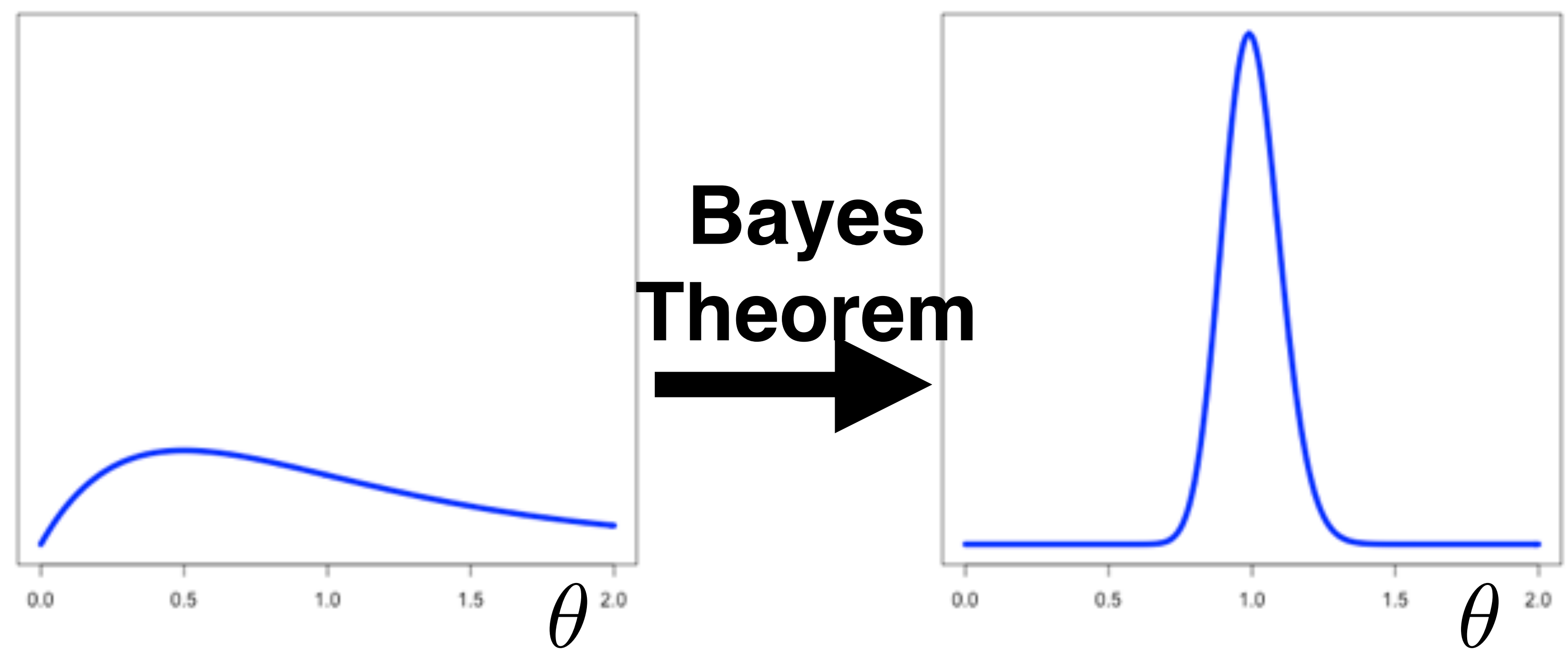

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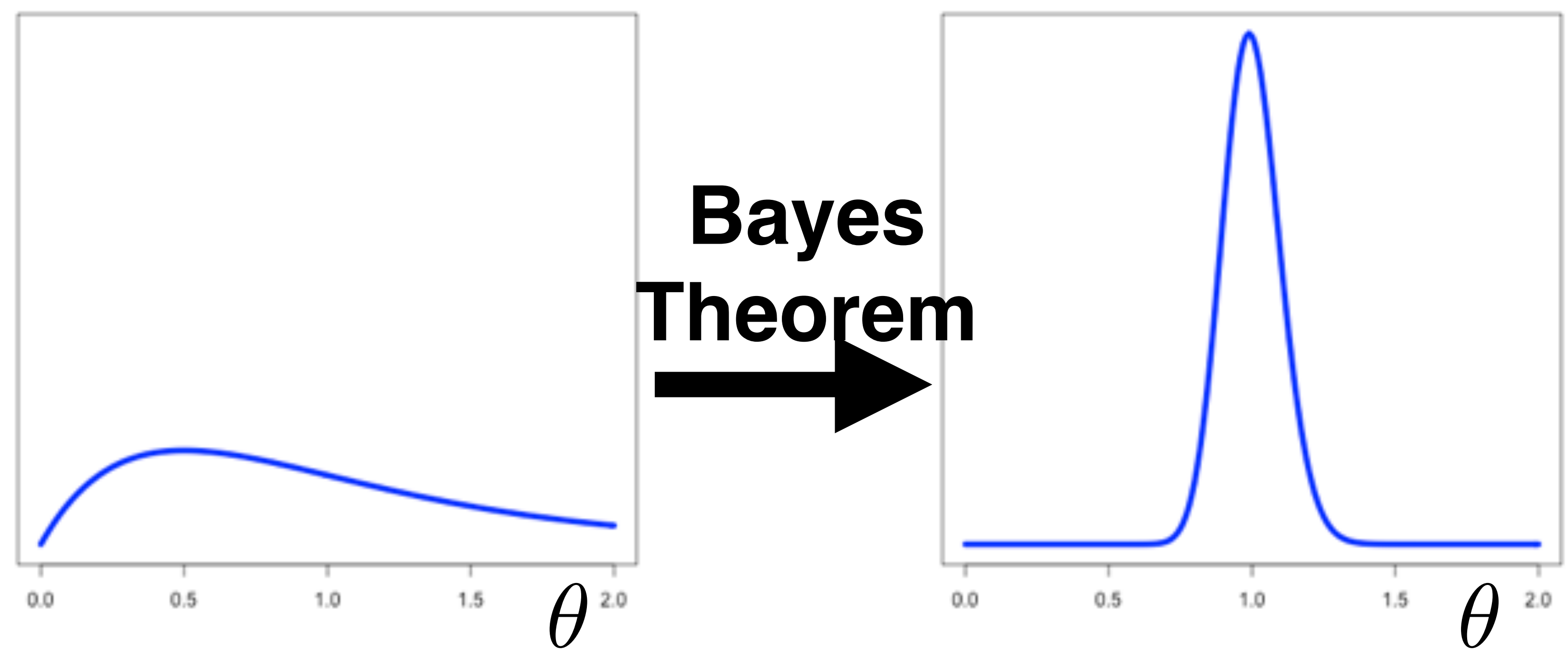

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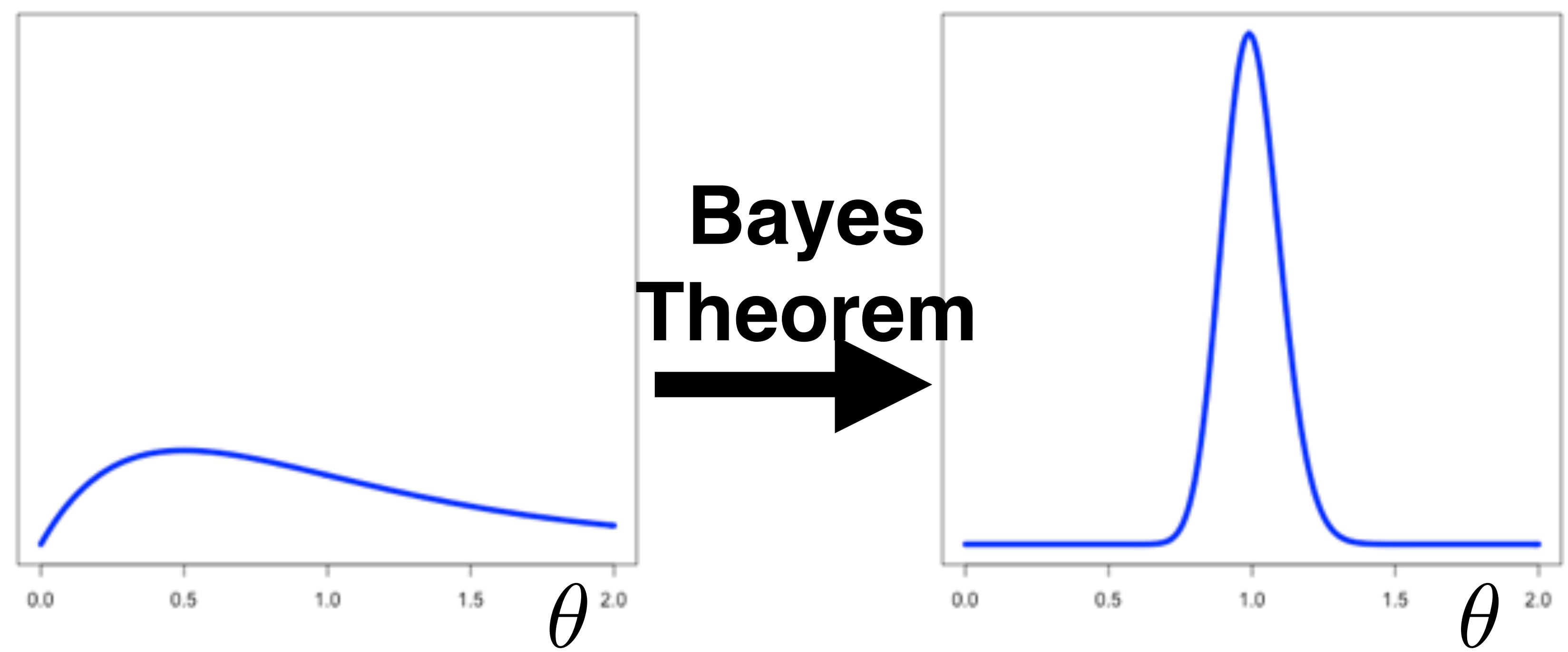

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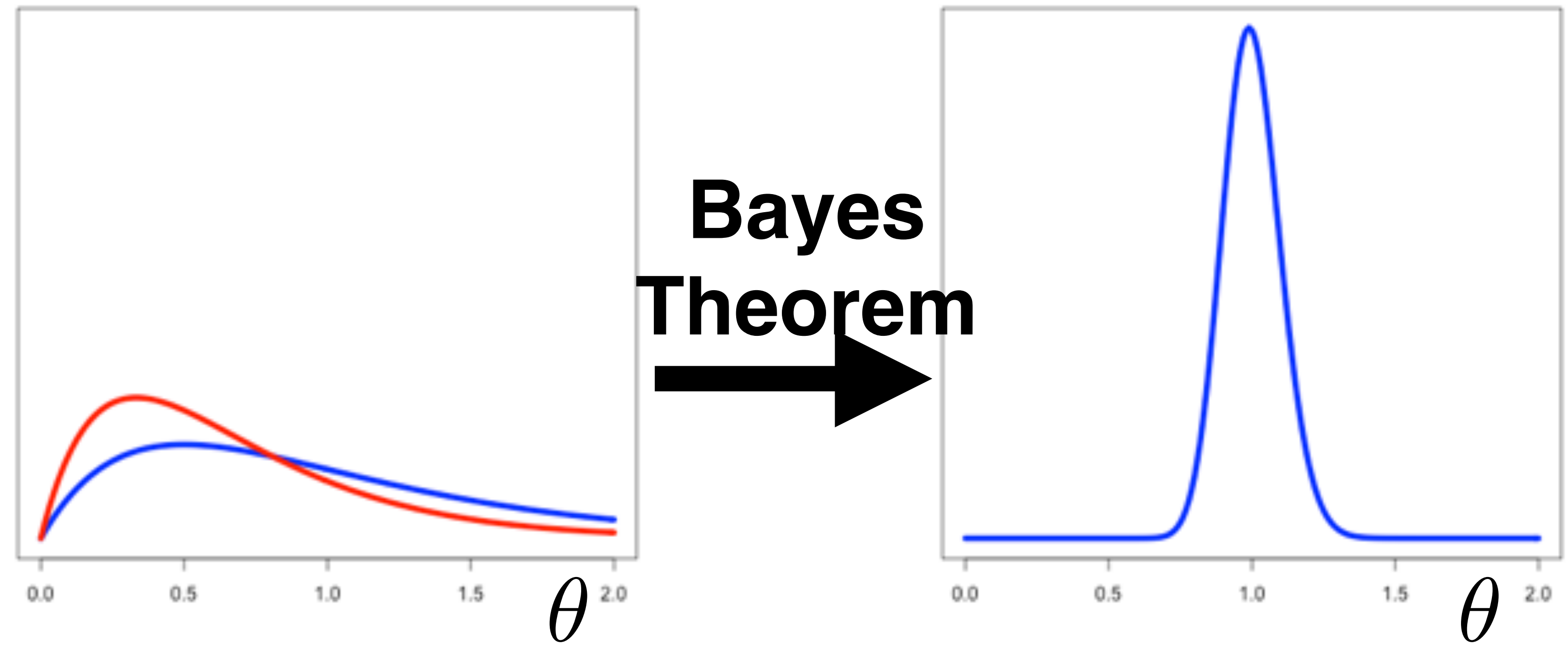

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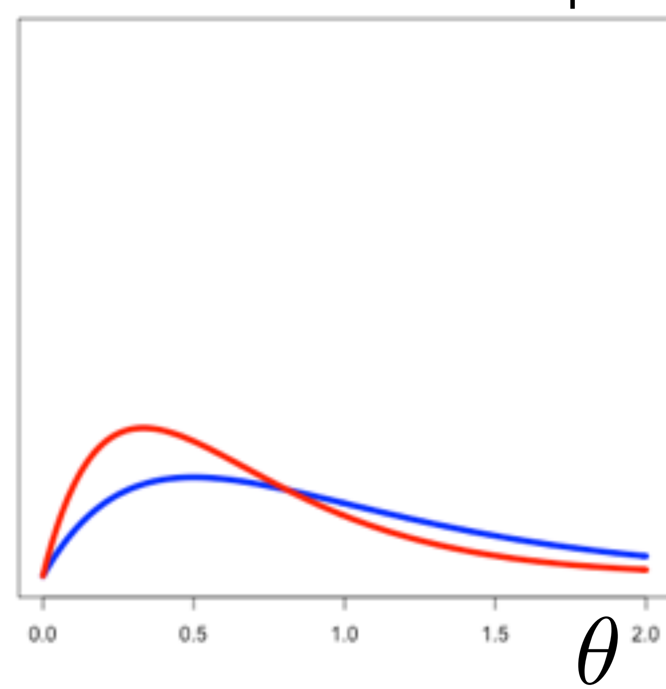

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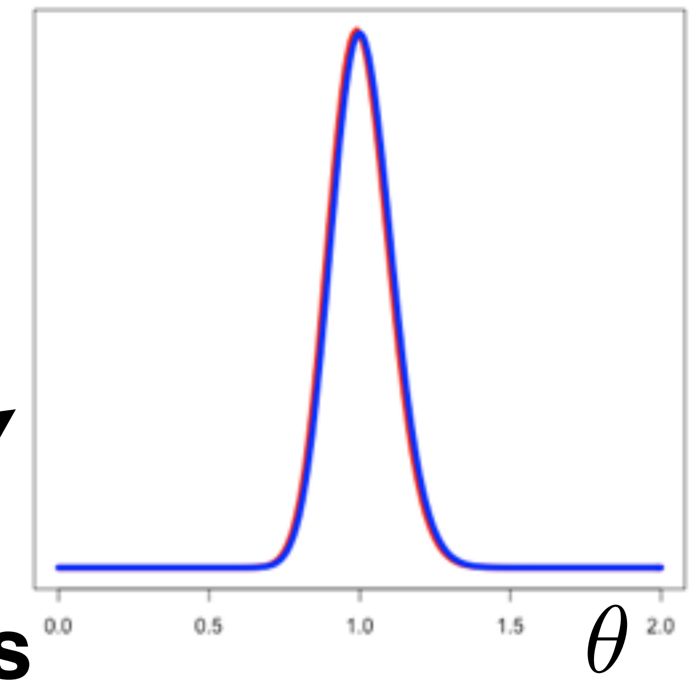

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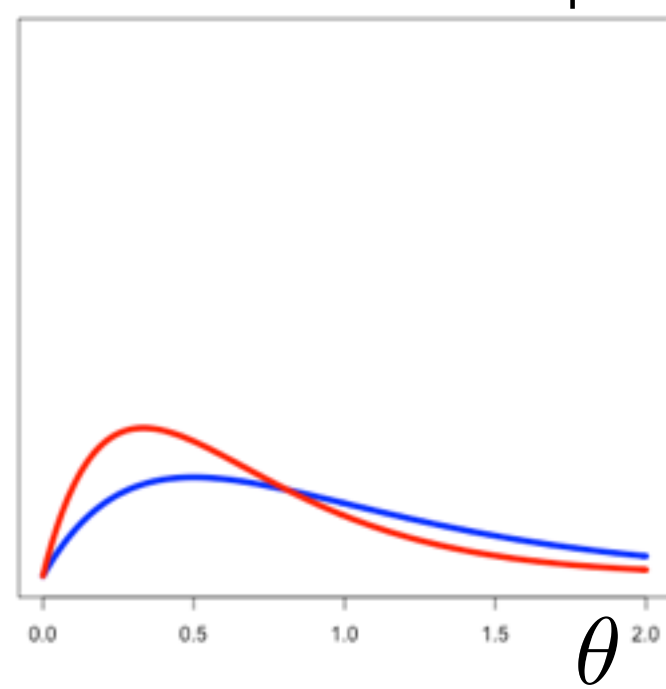

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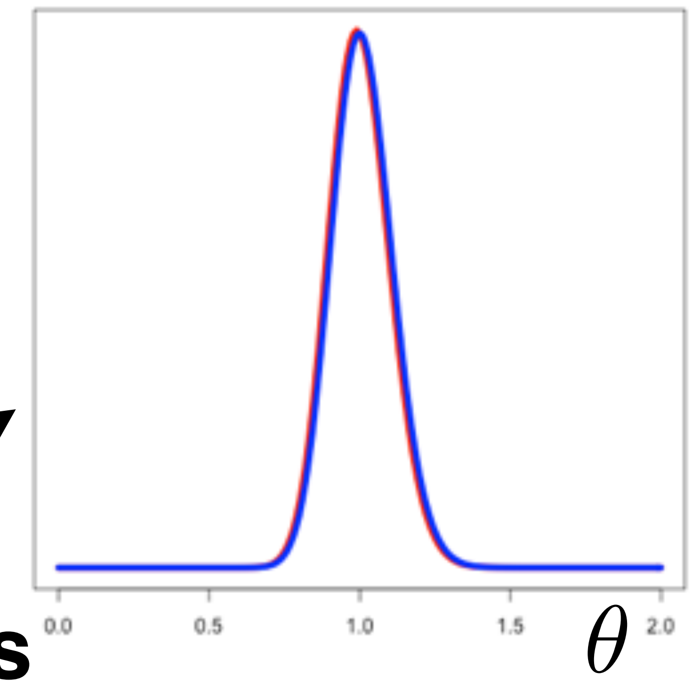

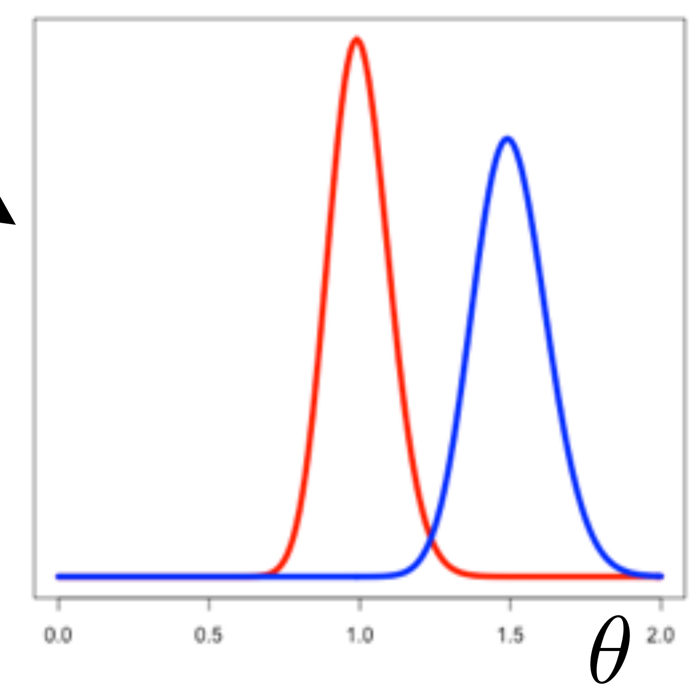

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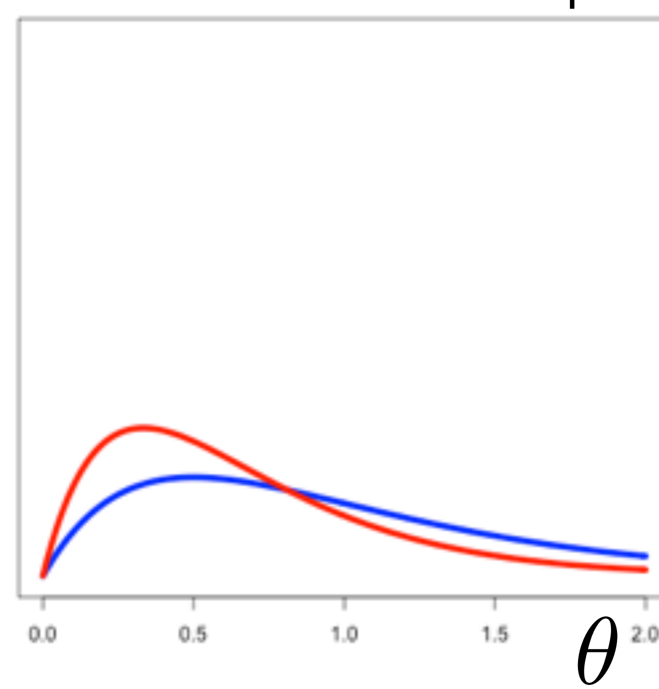

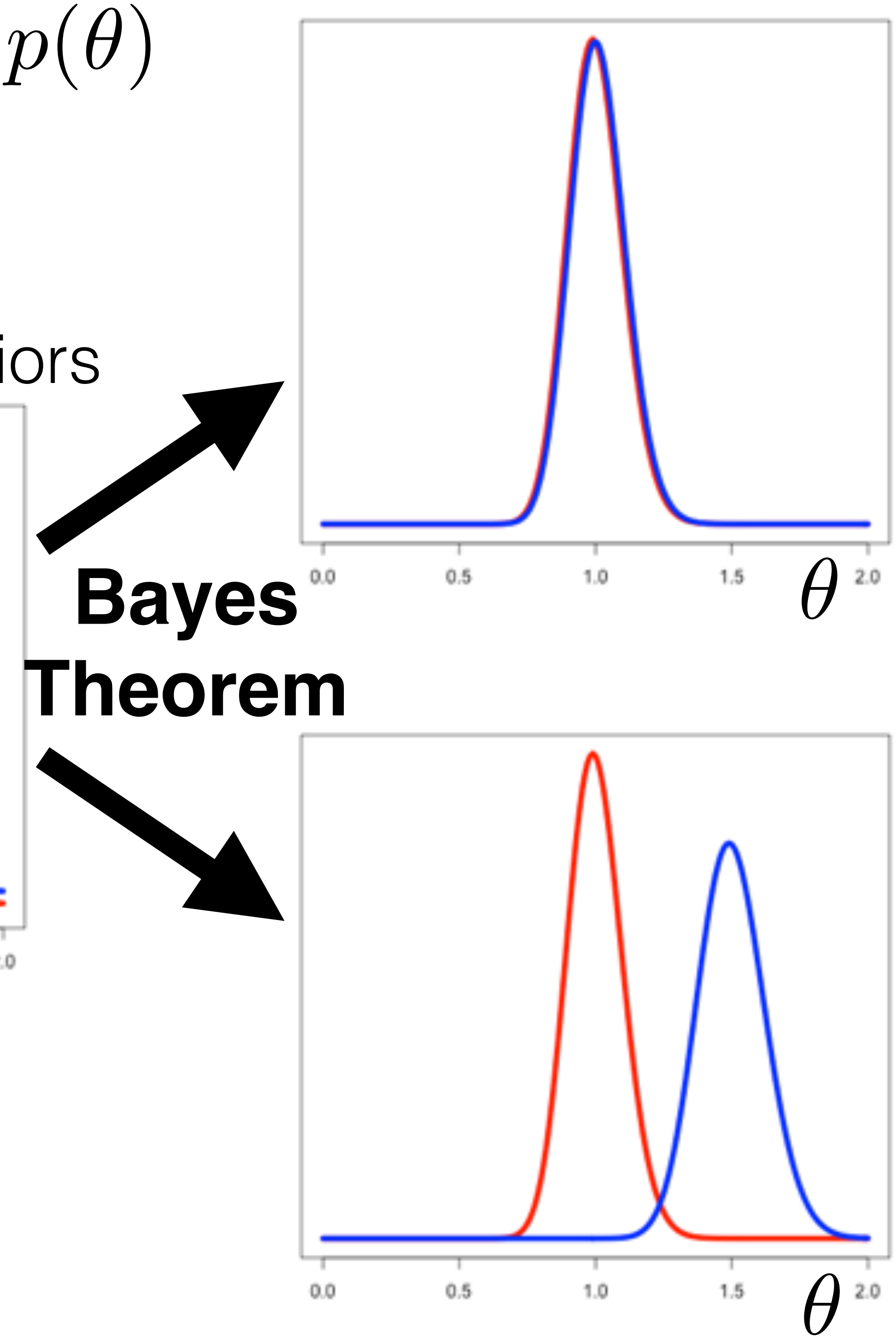

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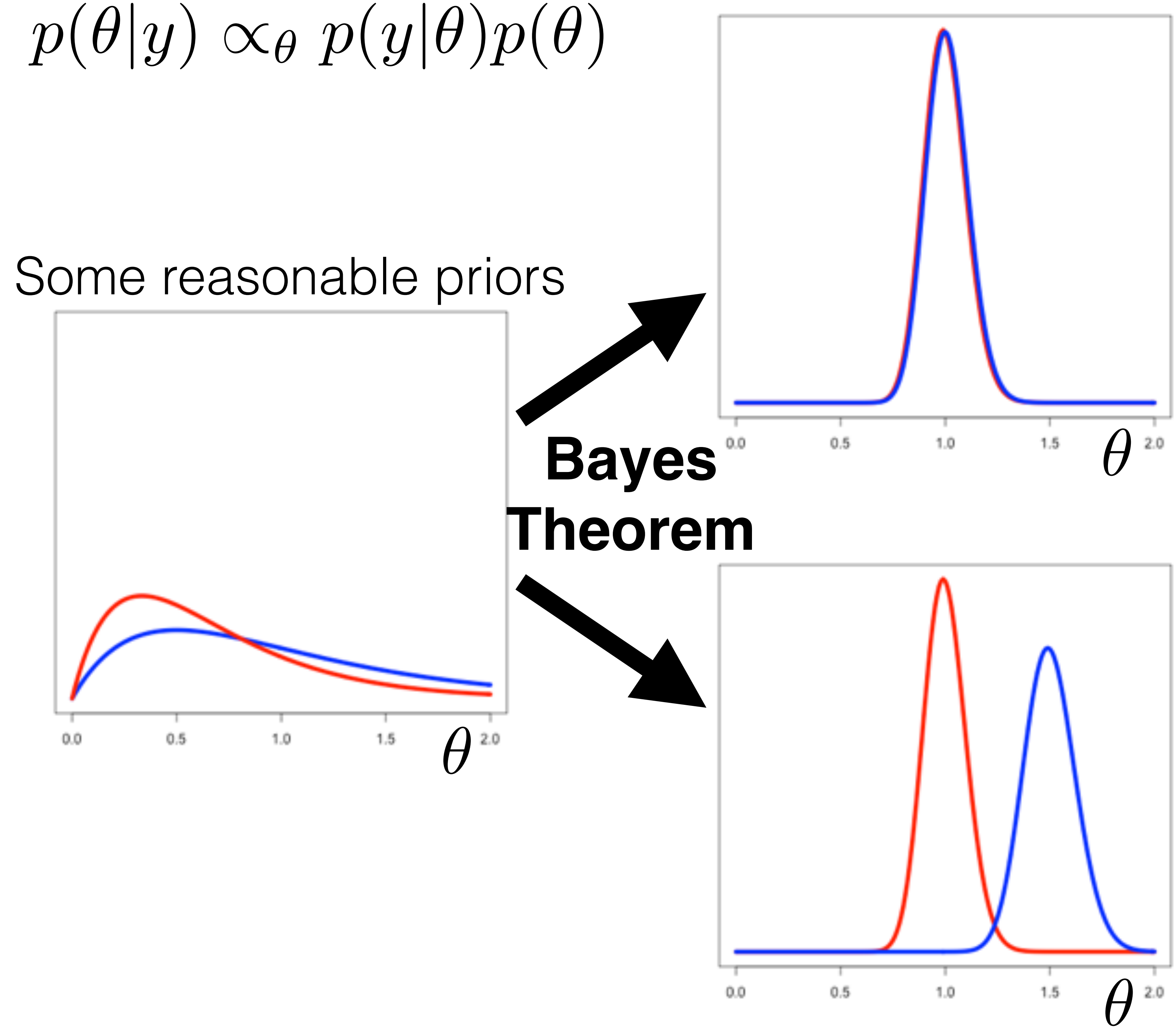

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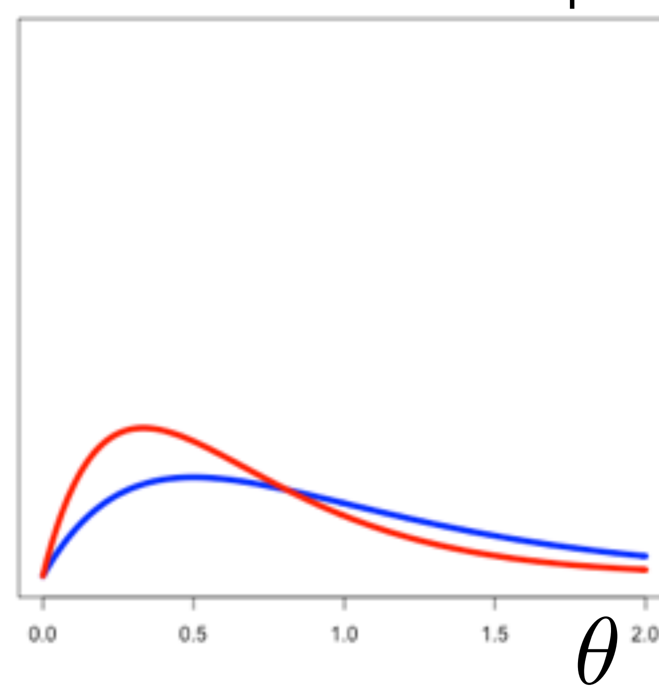

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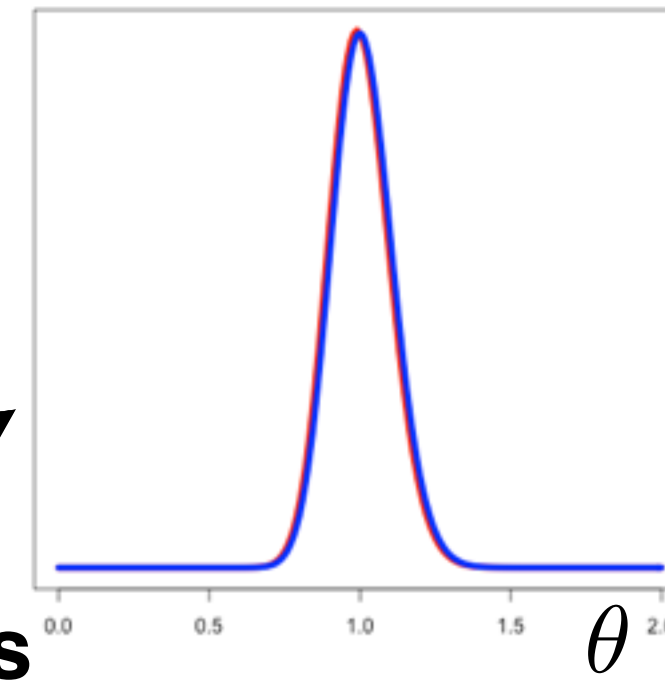

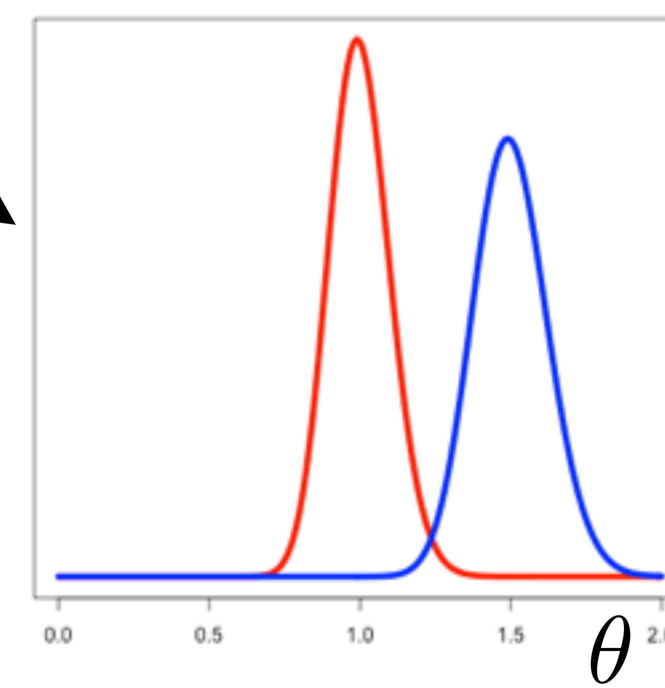

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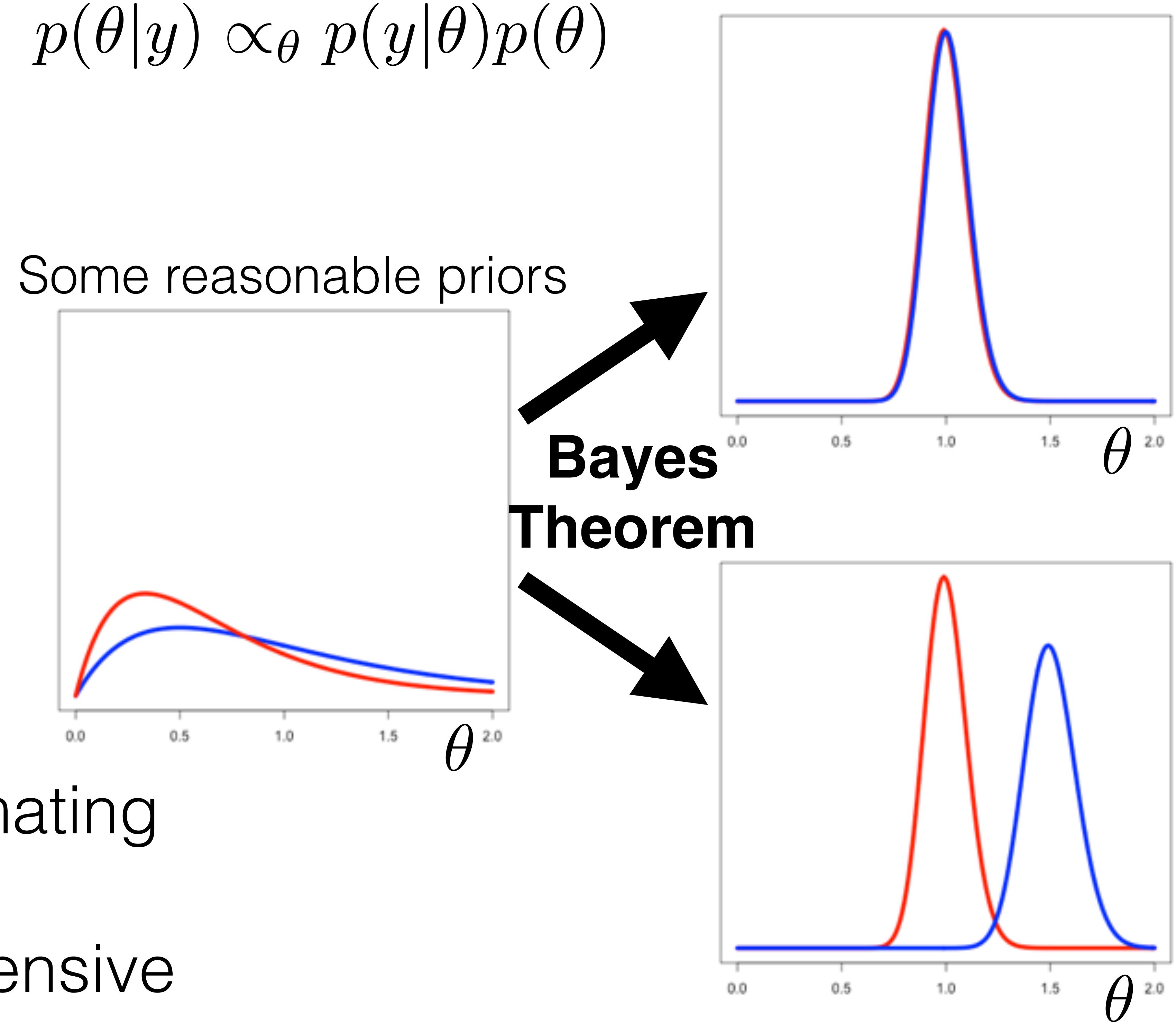

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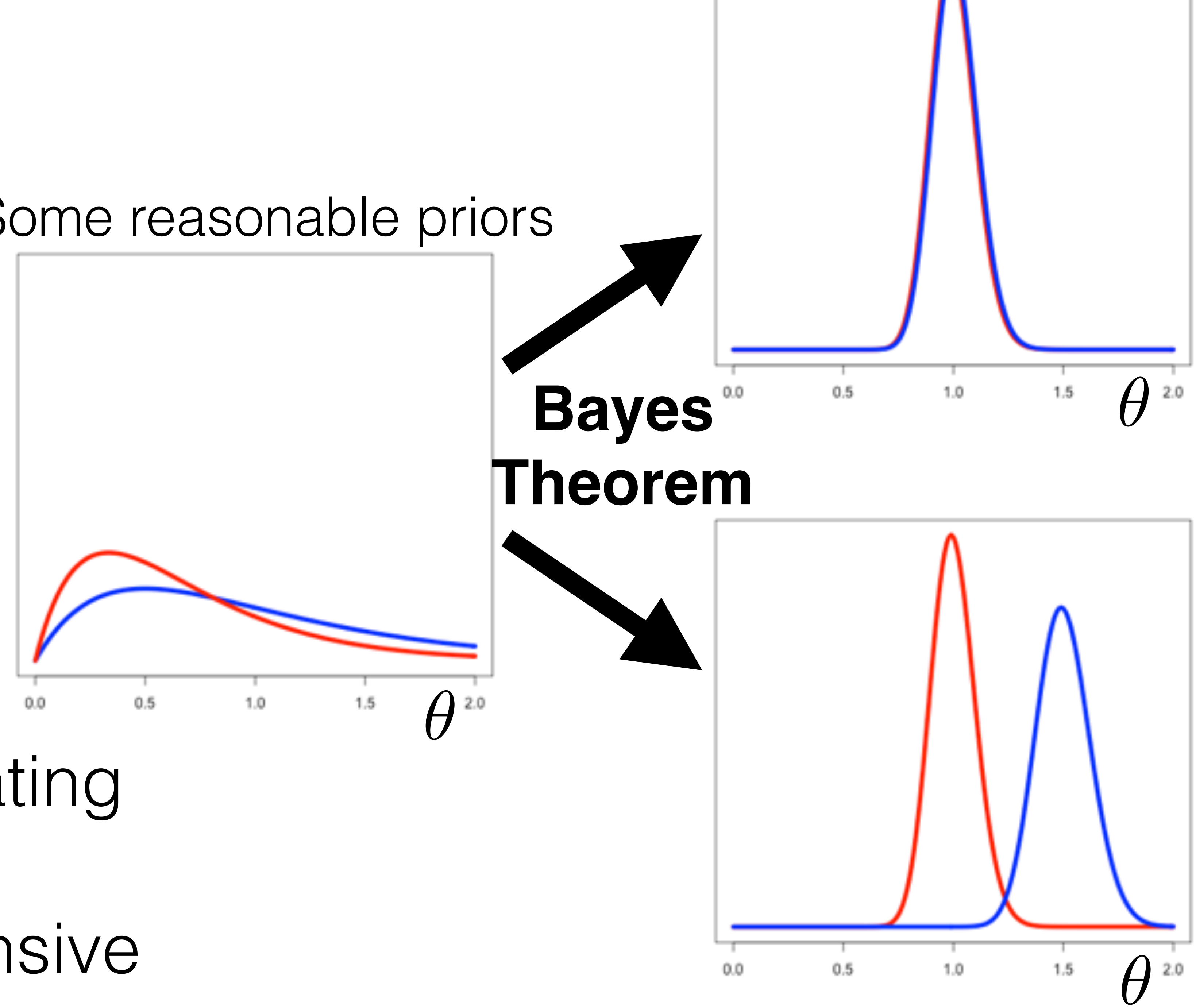

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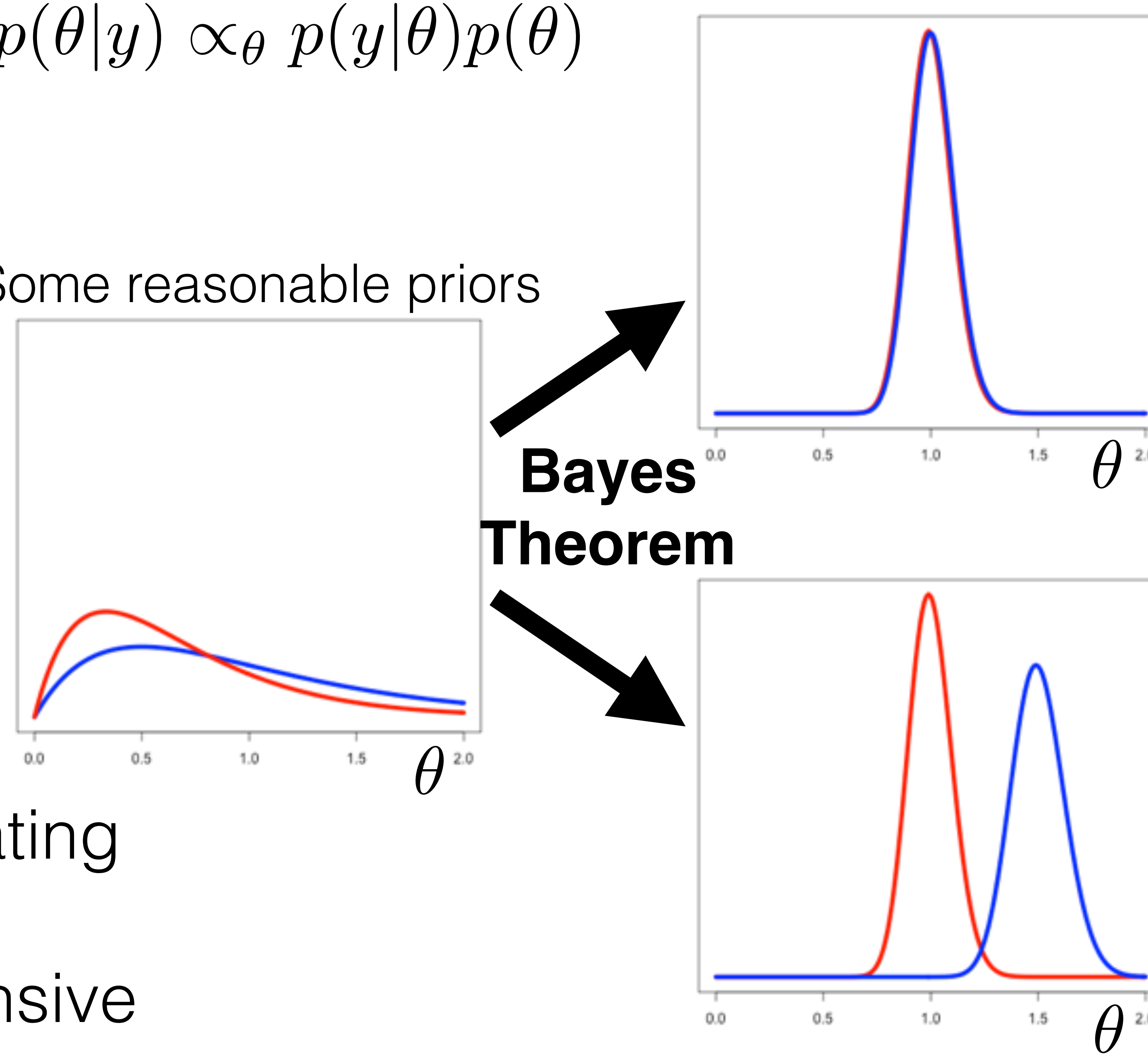

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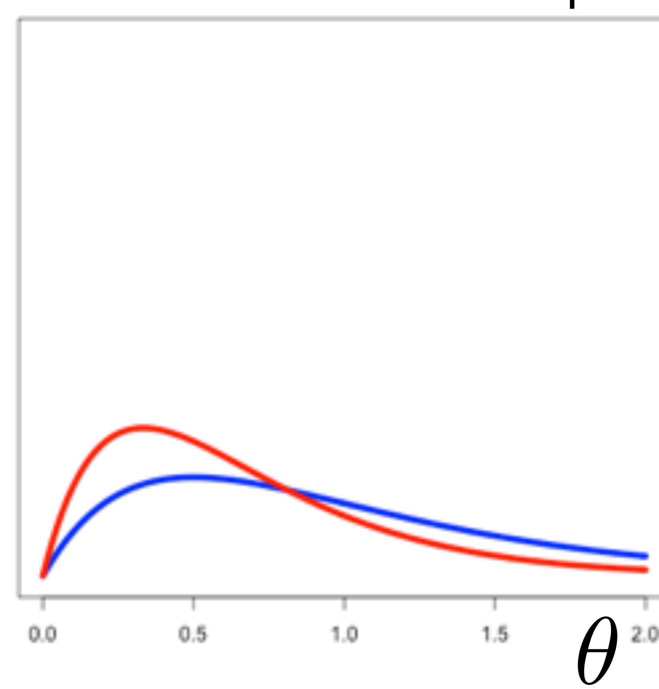

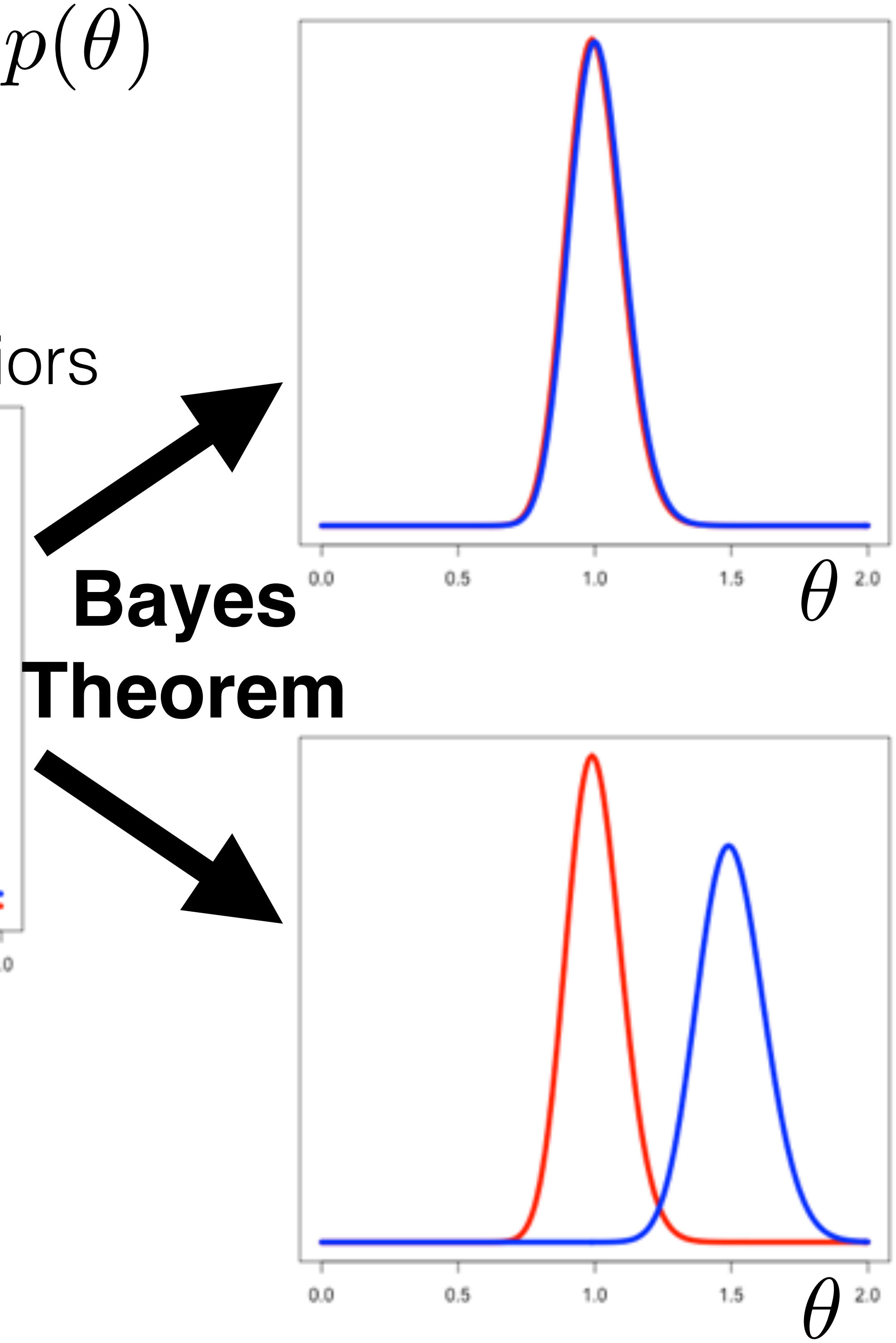

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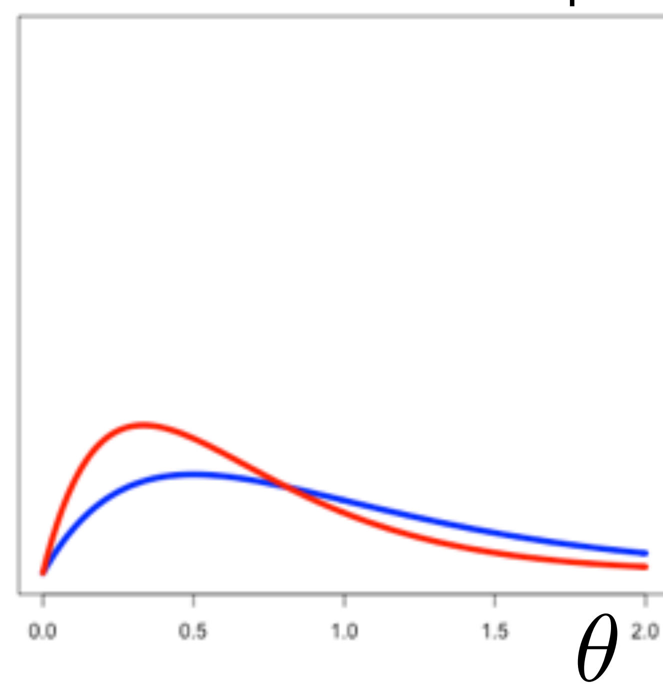

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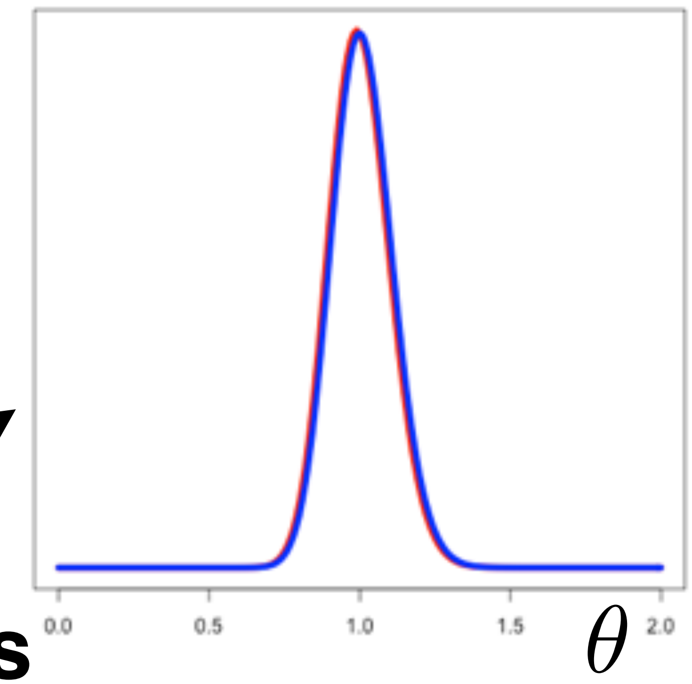

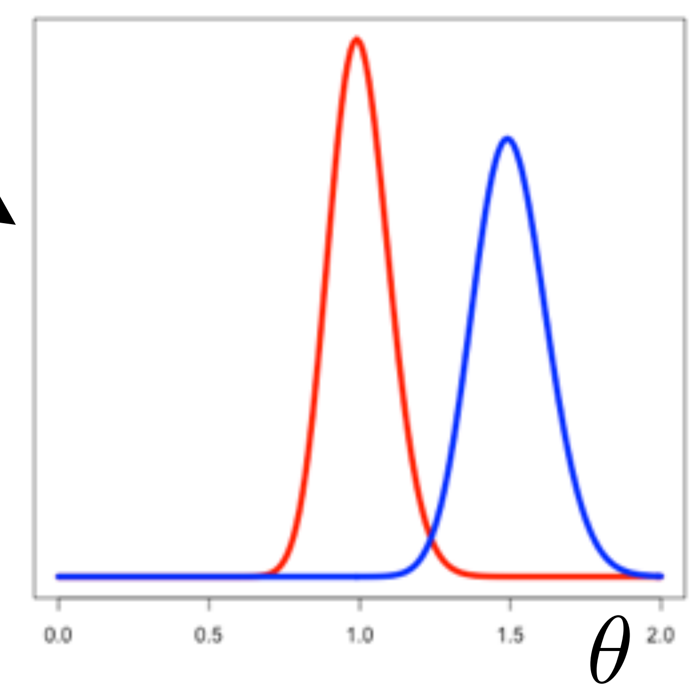

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[see also Opper, Winther 2003]

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- Accurate uncertainties from VB
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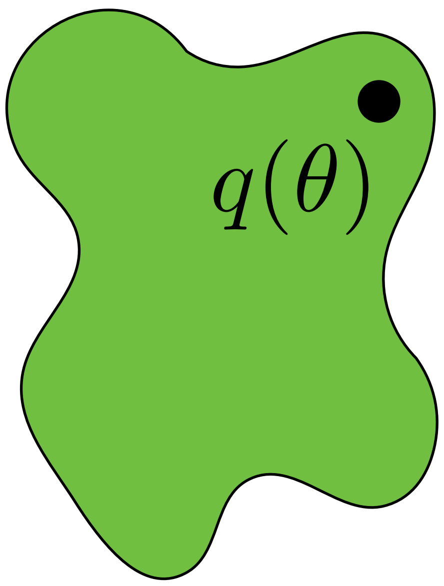

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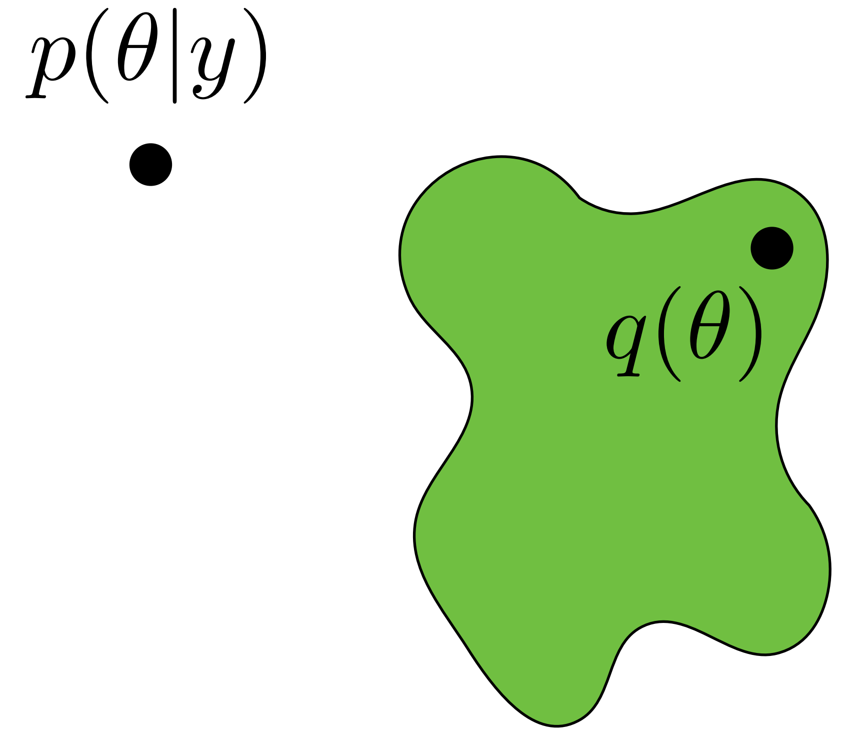

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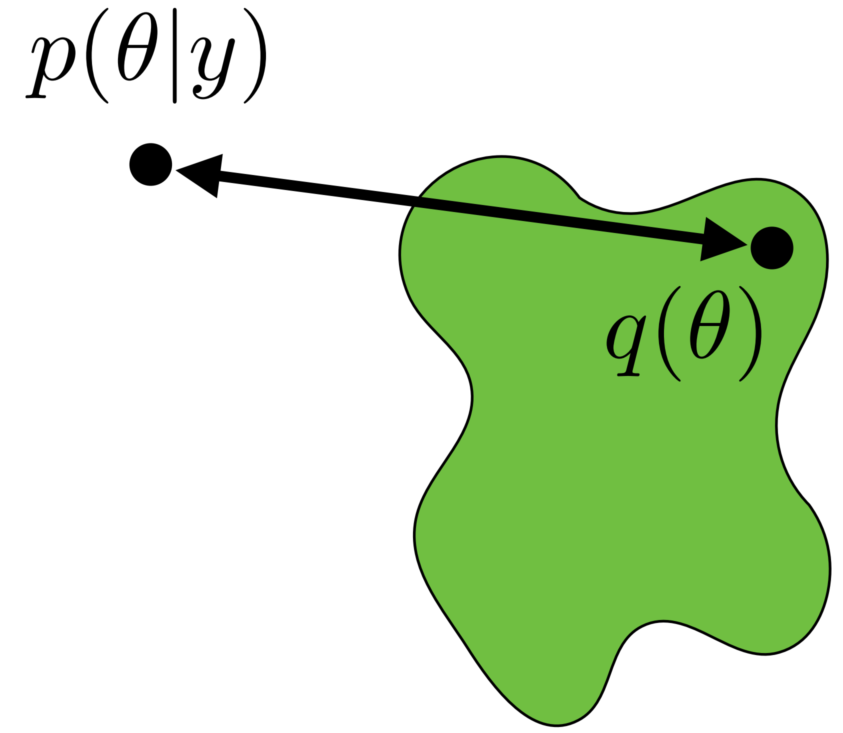

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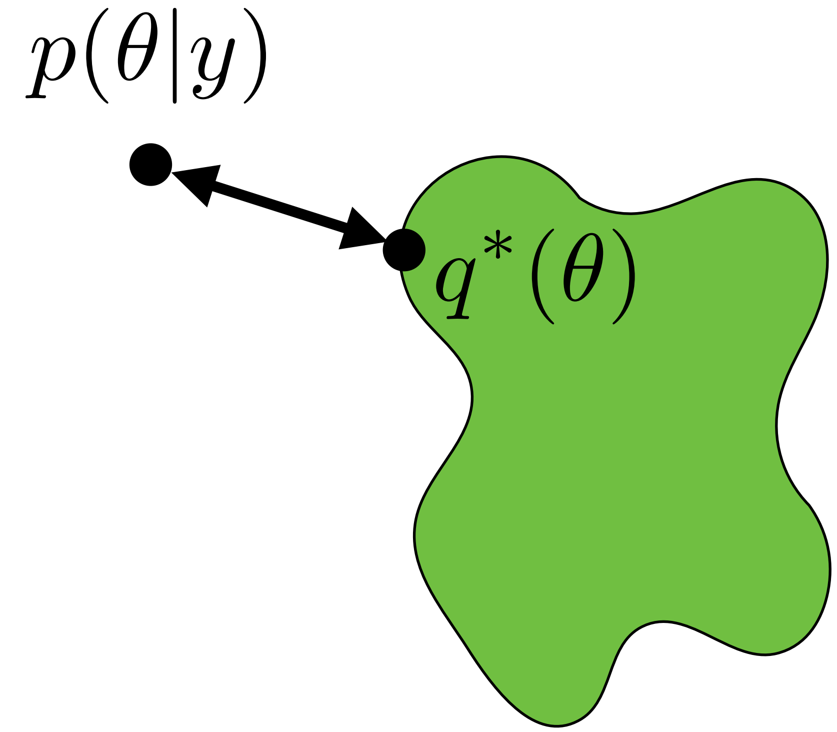

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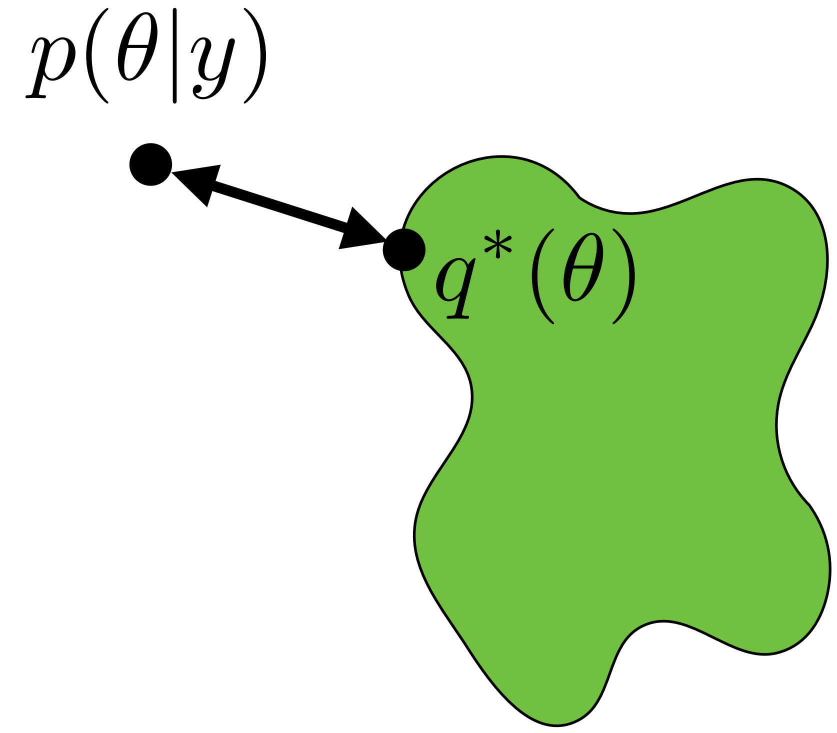

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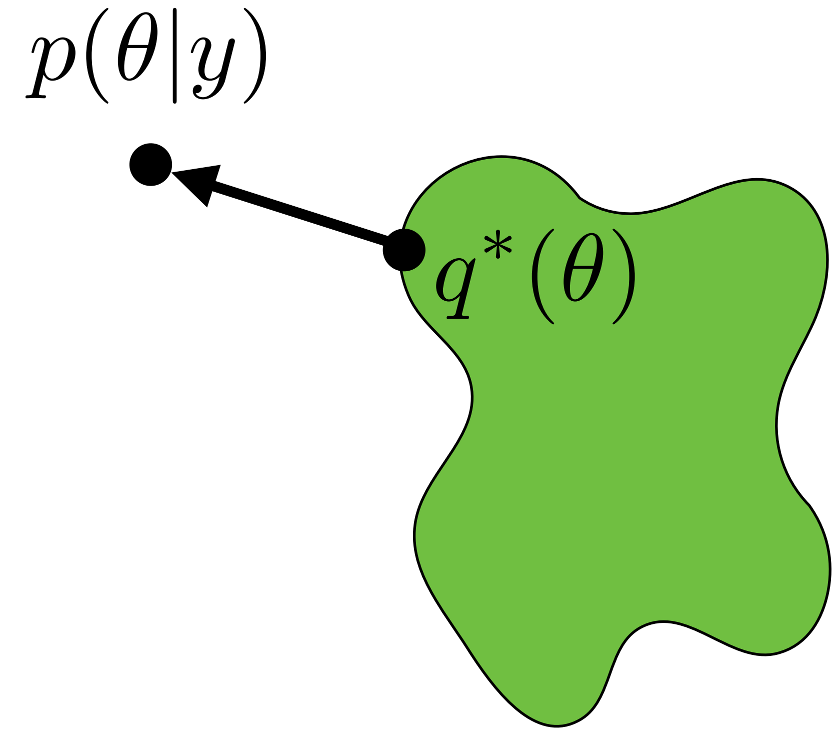

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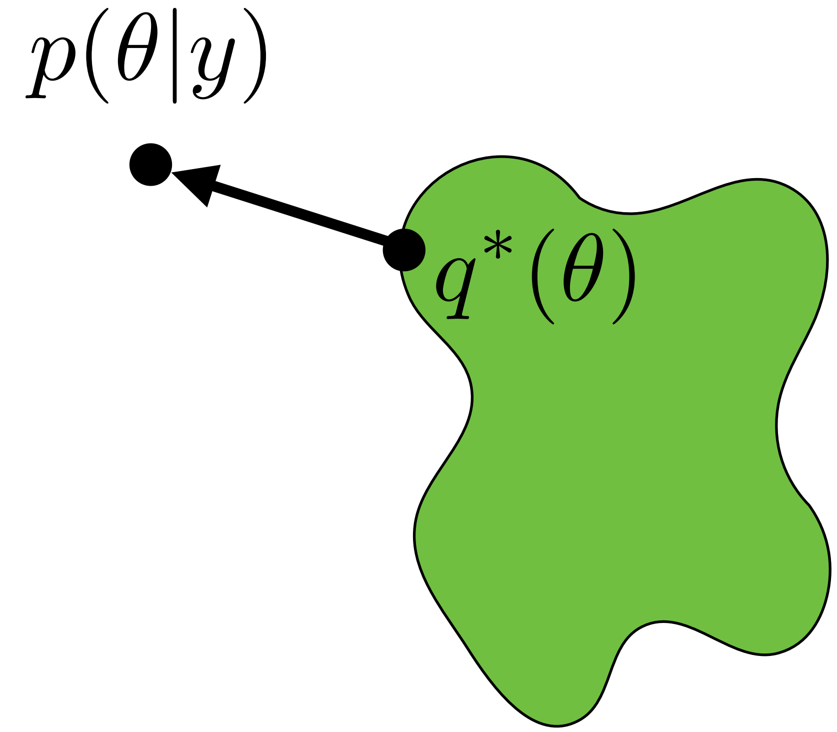

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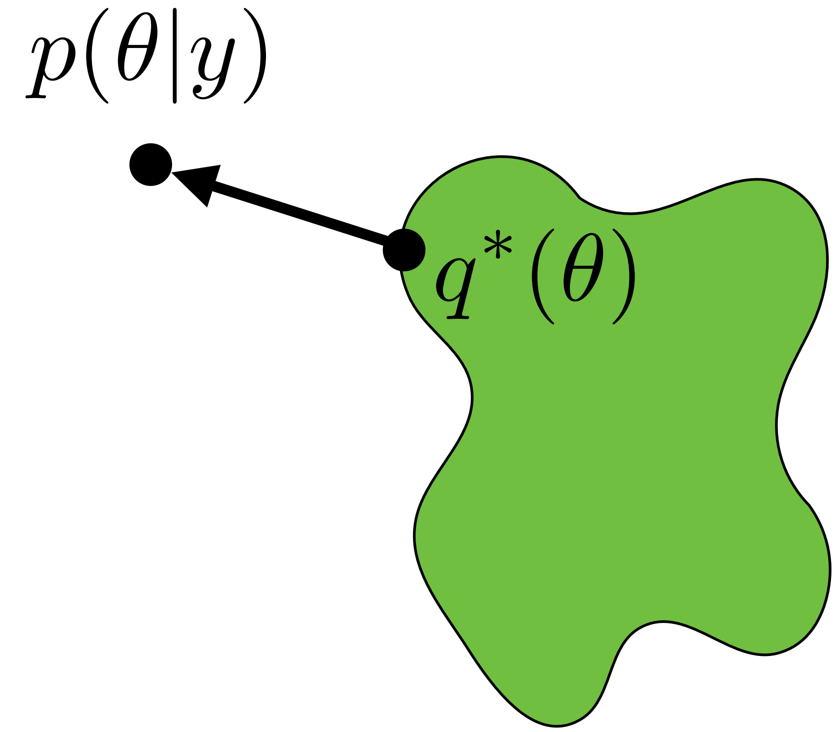

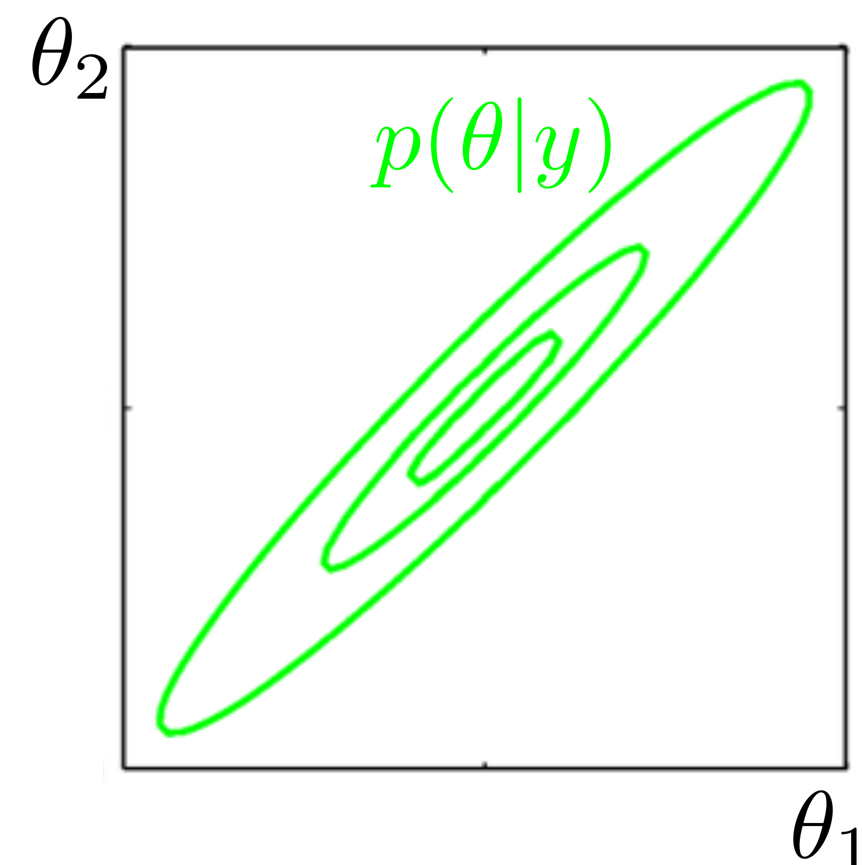

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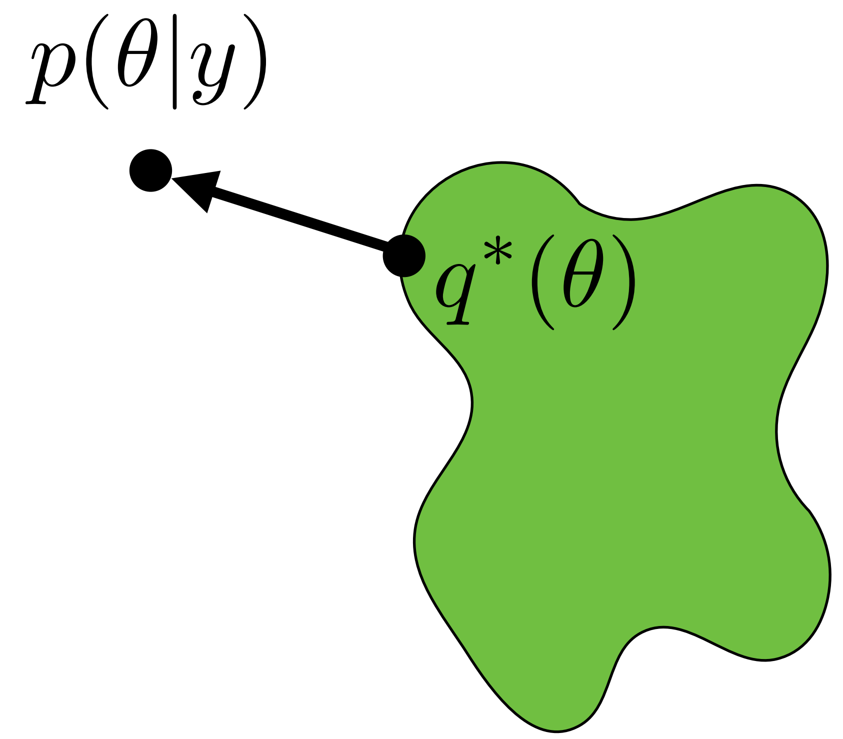

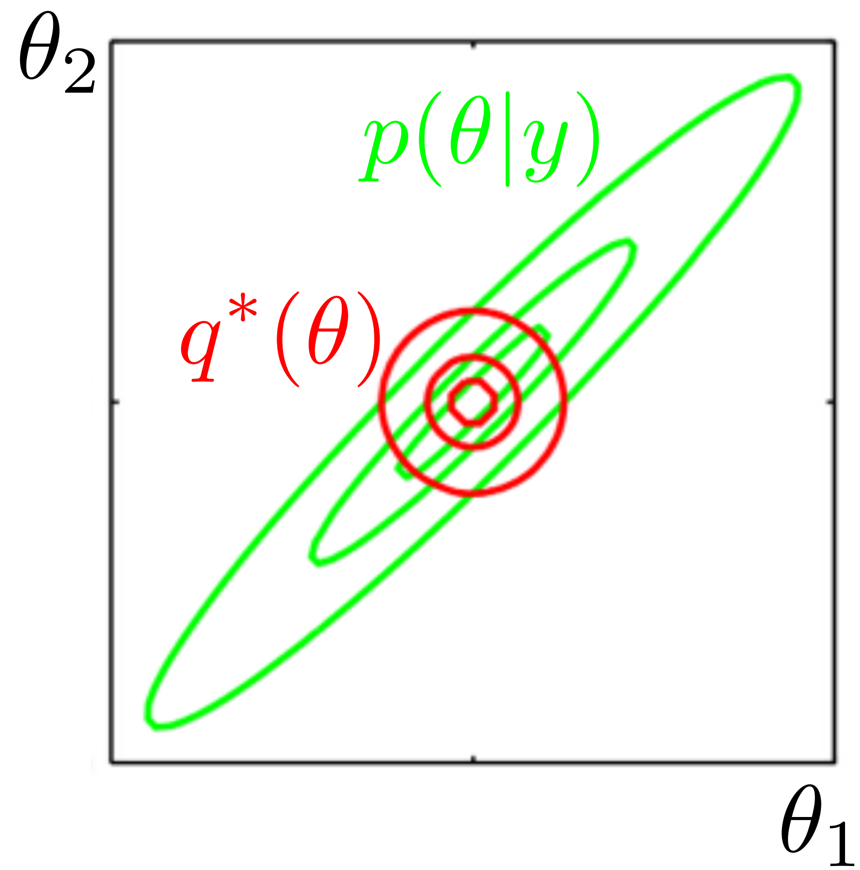

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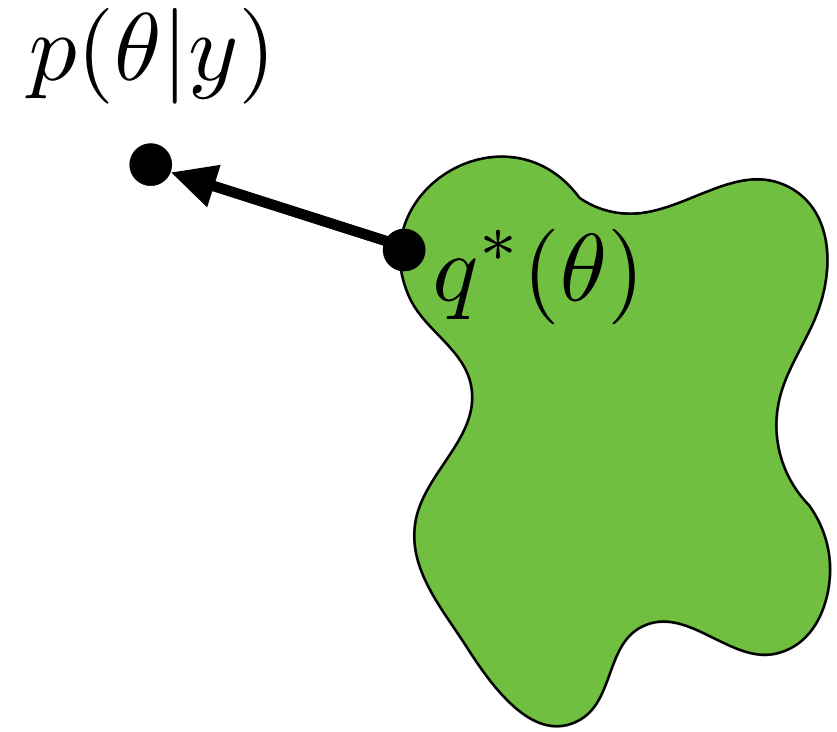

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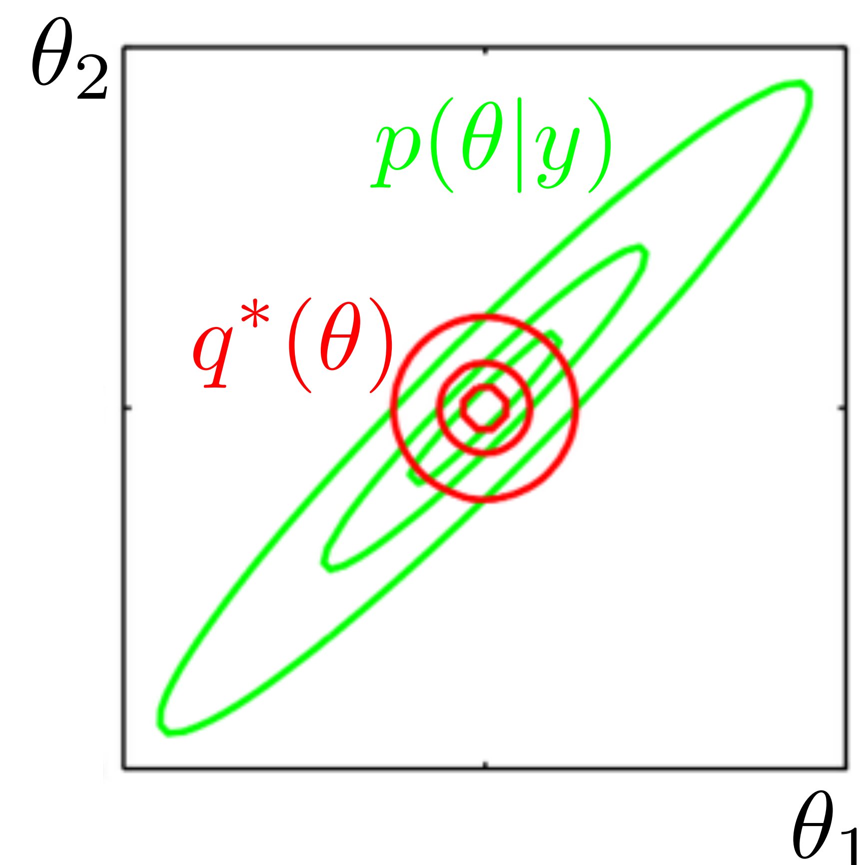

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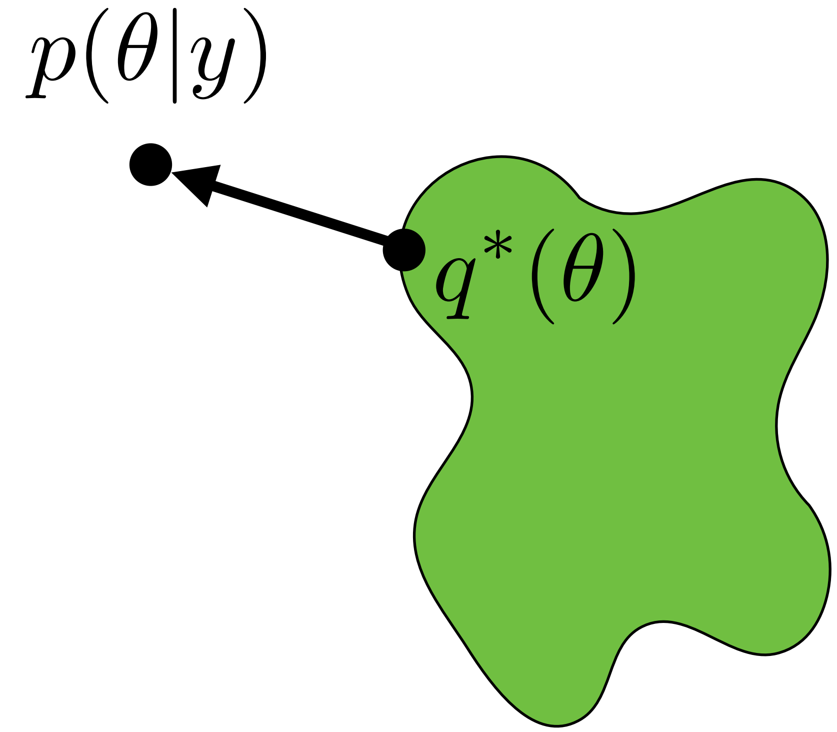

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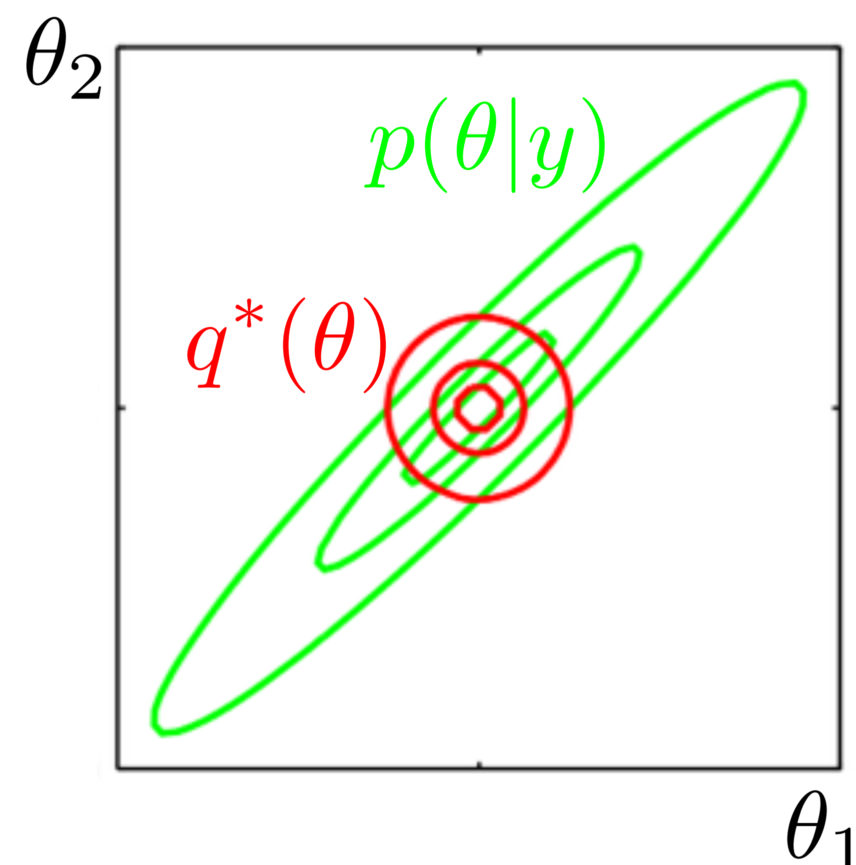

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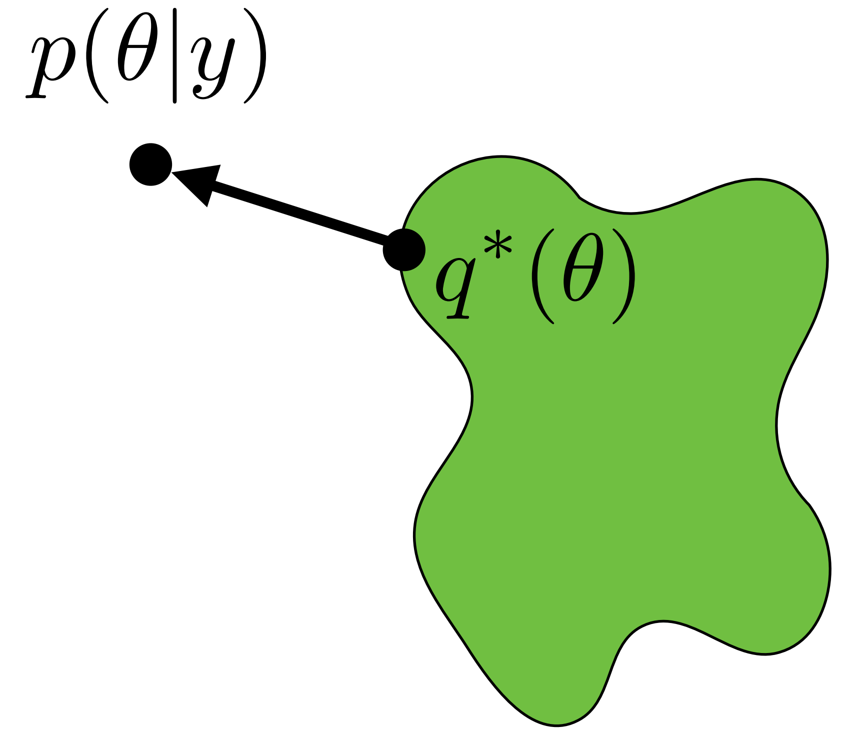

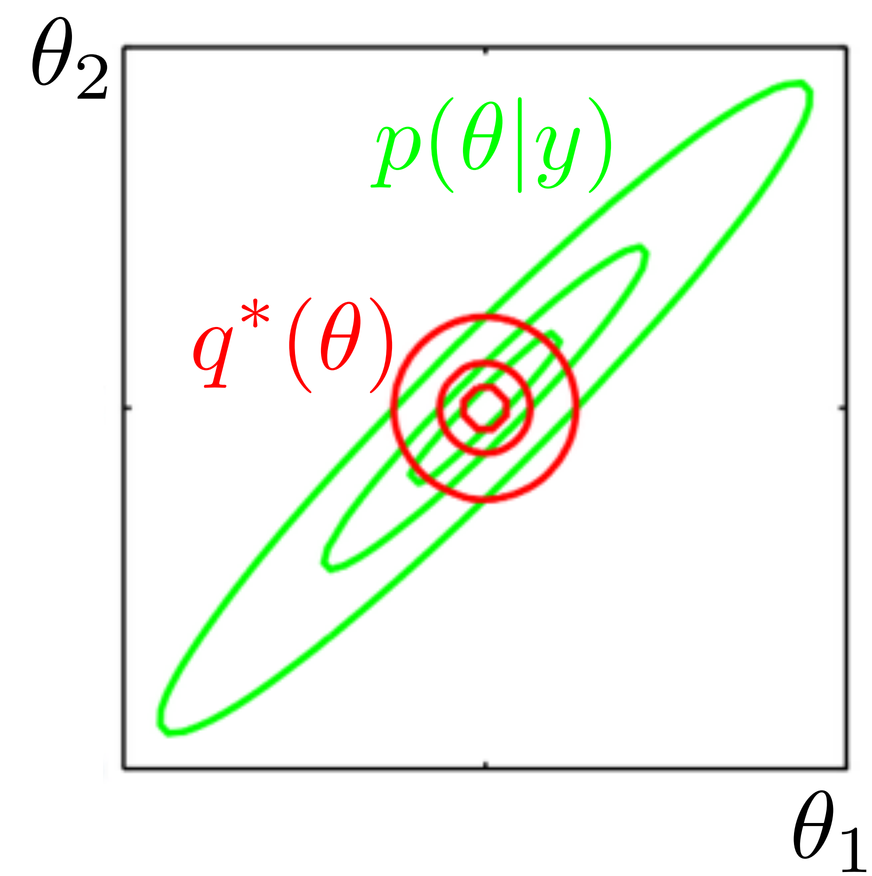

[MacKay 2003; Bishop 2006; Wang, Titterington 2004; Turner, Sahani 2011]

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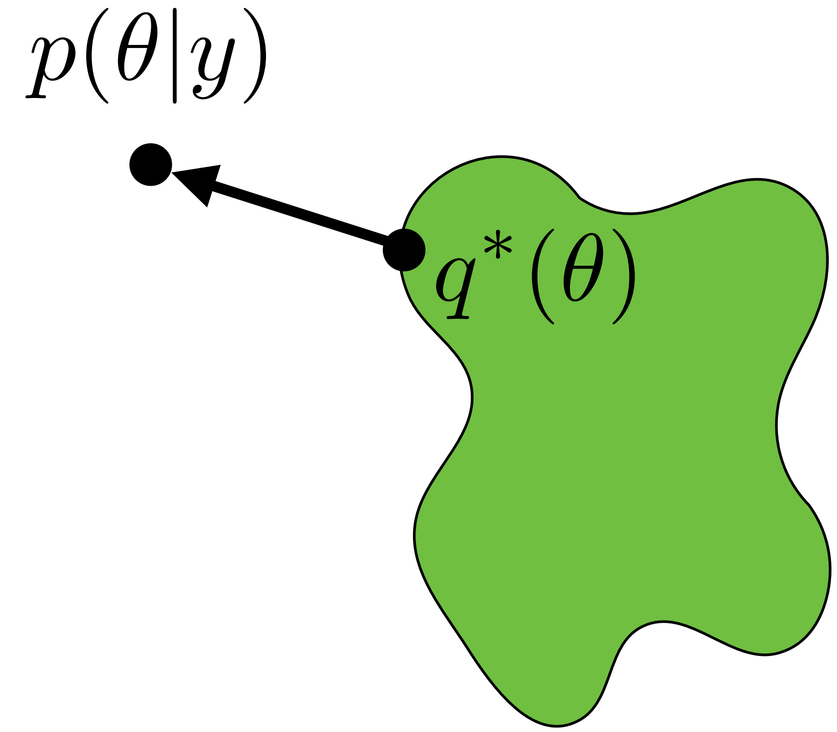

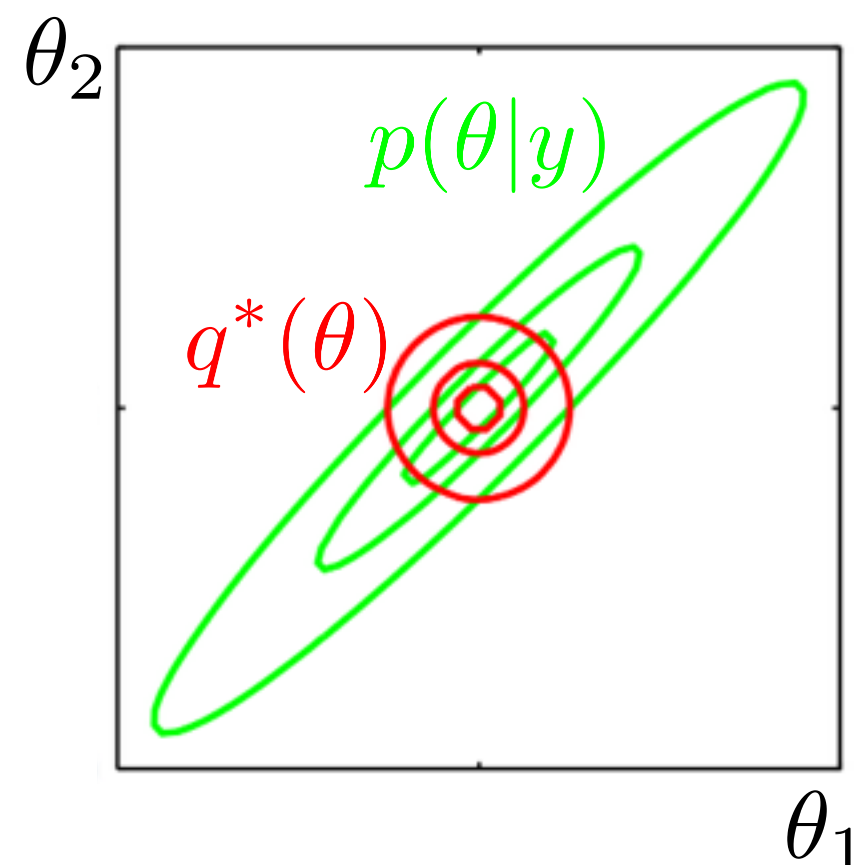

[MacKay 2003; Bishop 2006; Wang, Titterington 2004; Turner, Sahani 2011]
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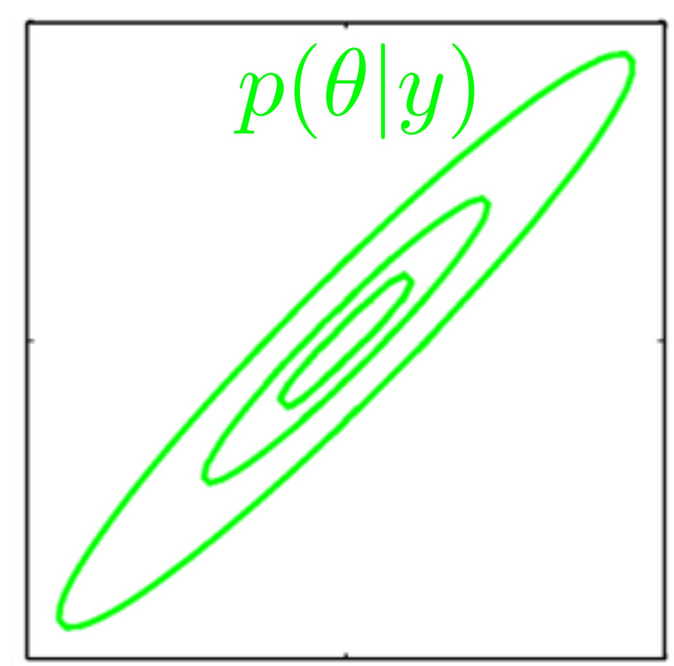

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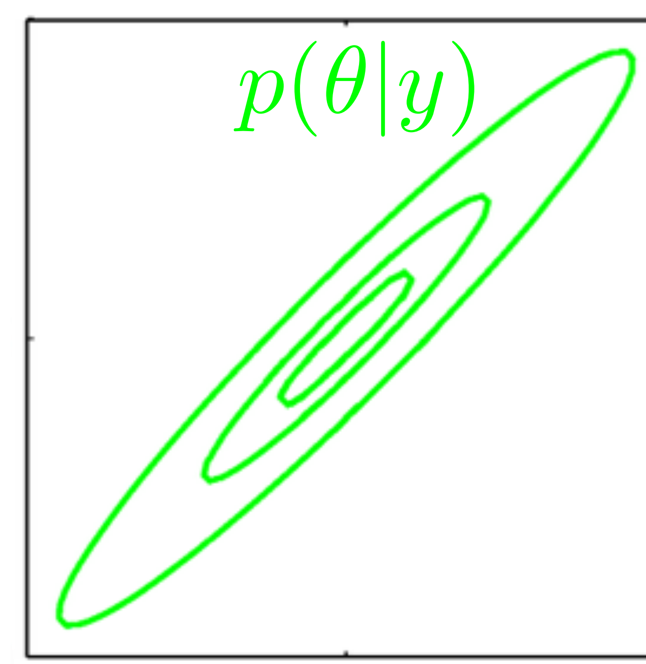

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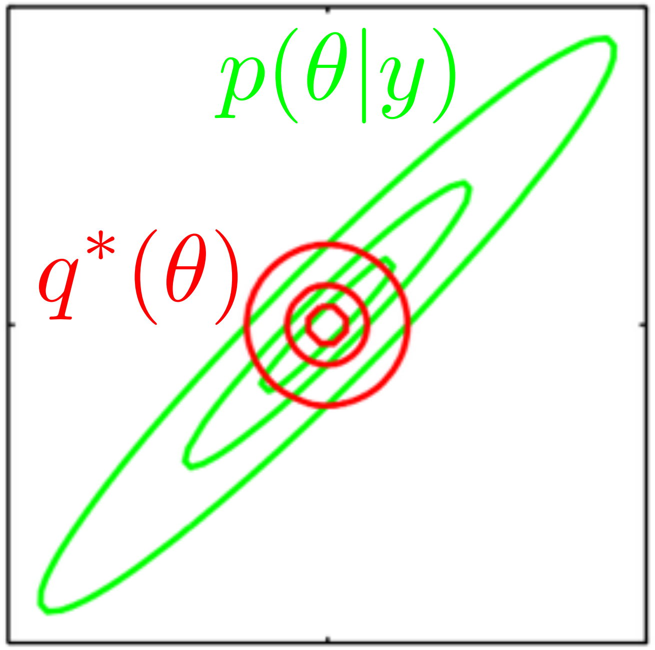

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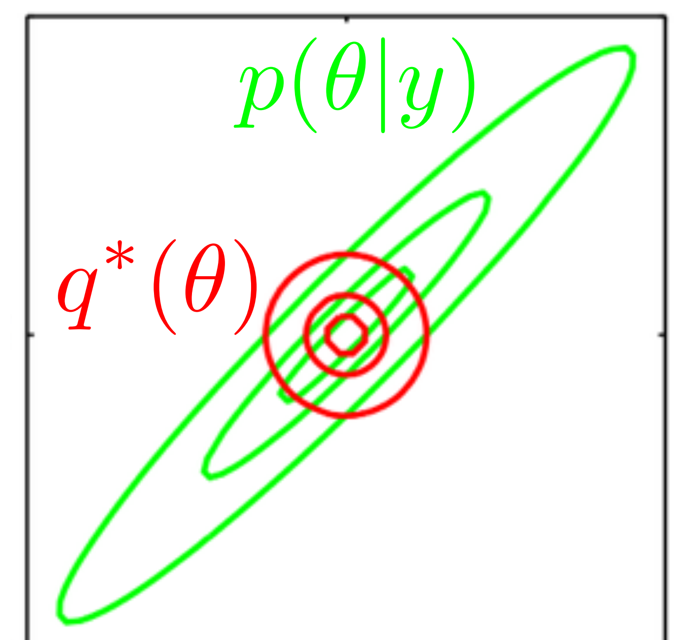

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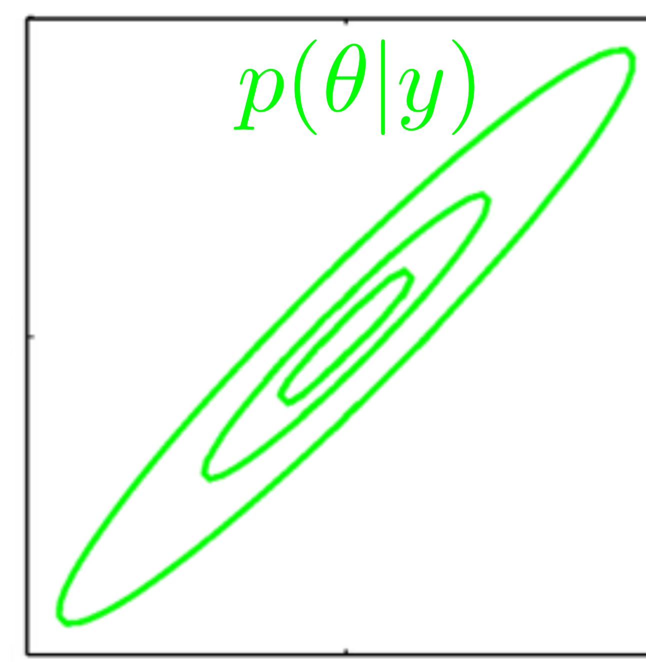

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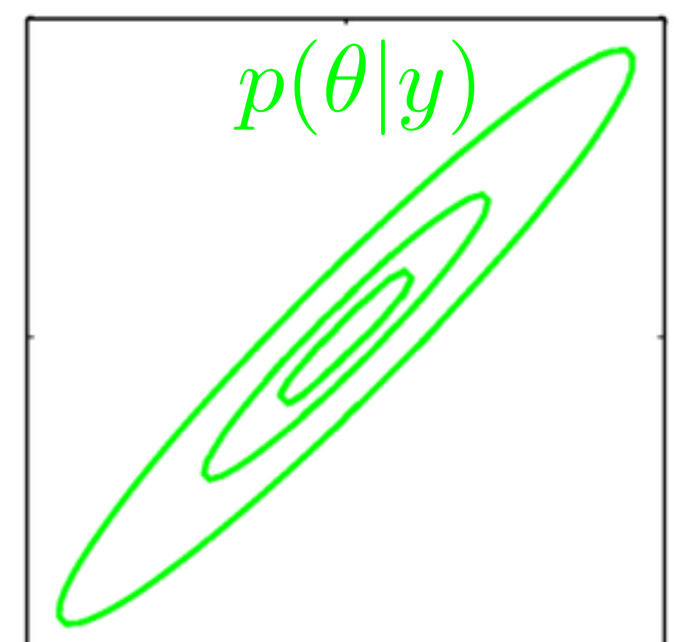

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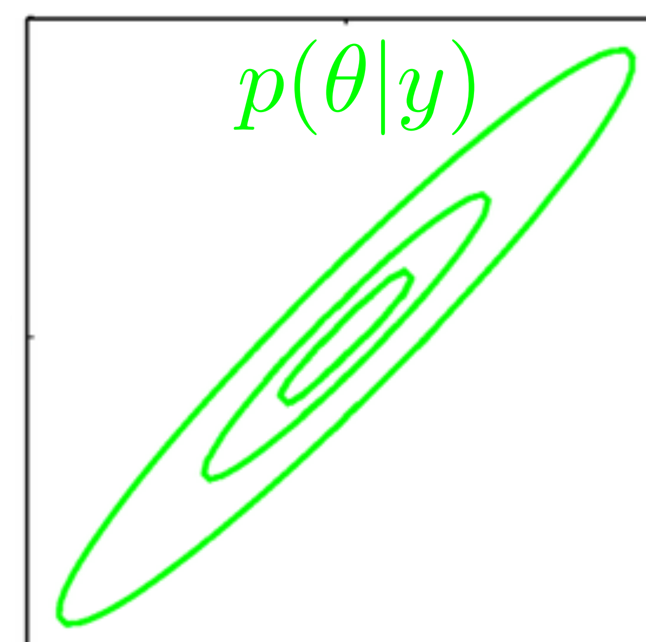

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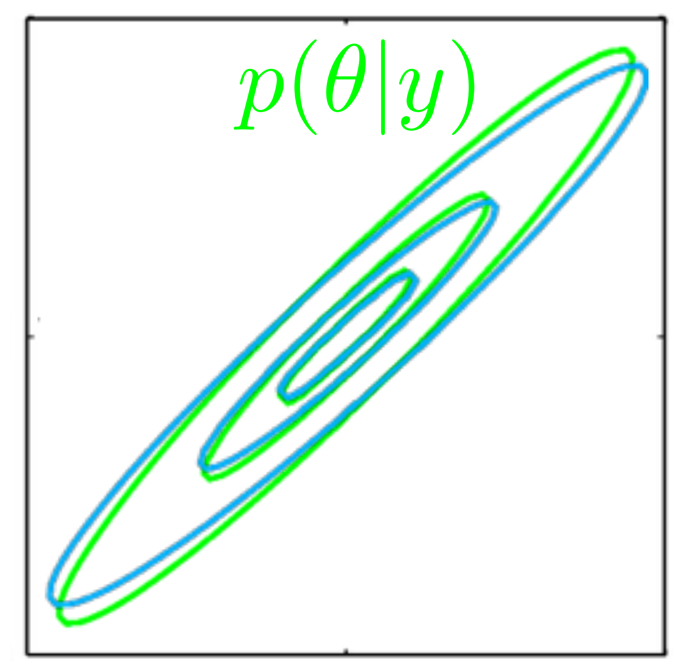

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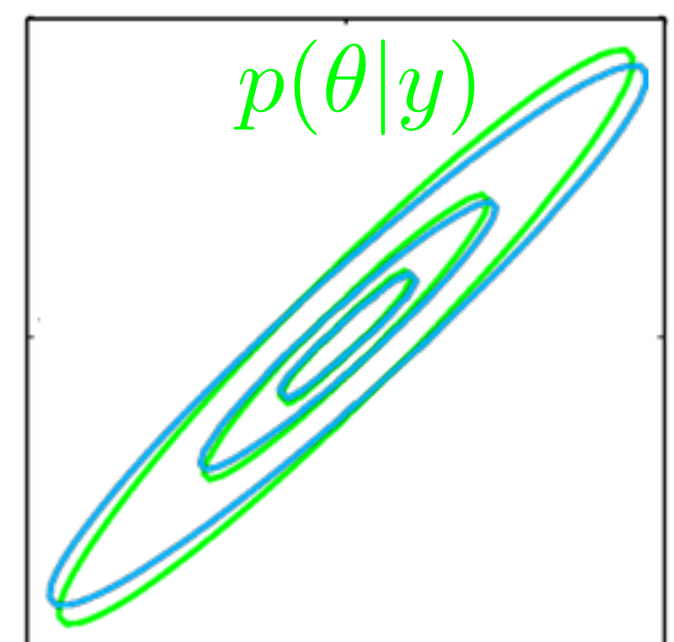

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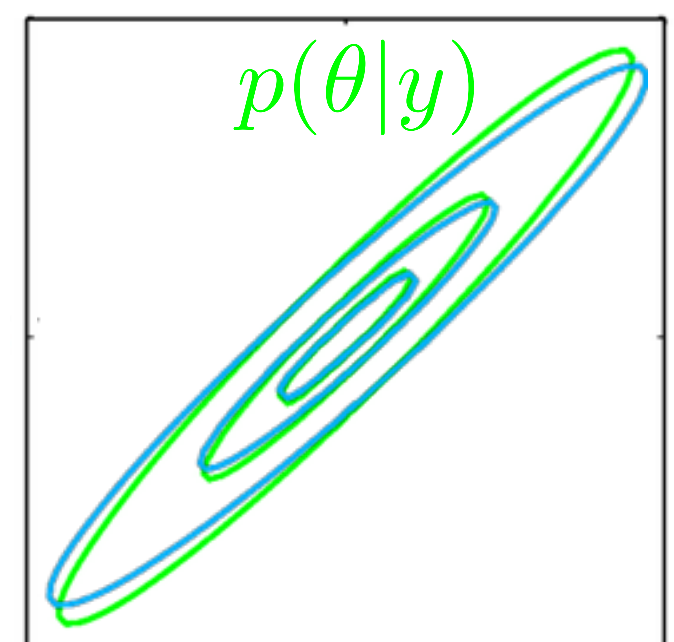

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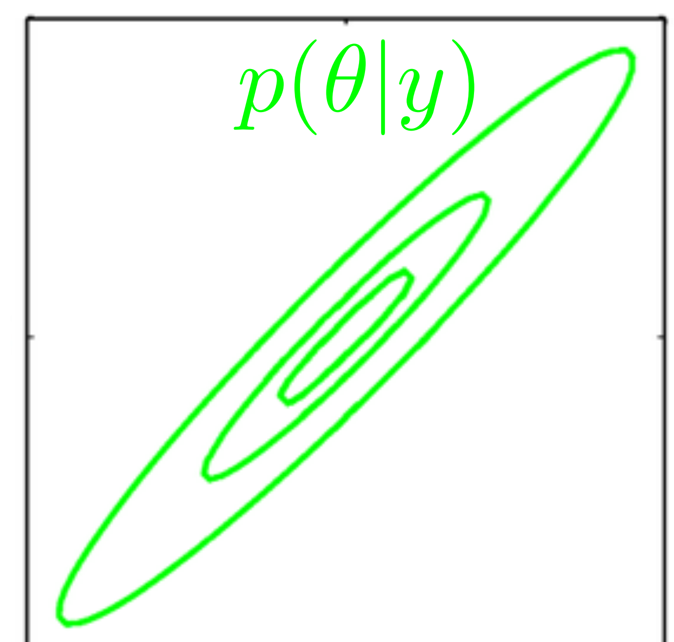

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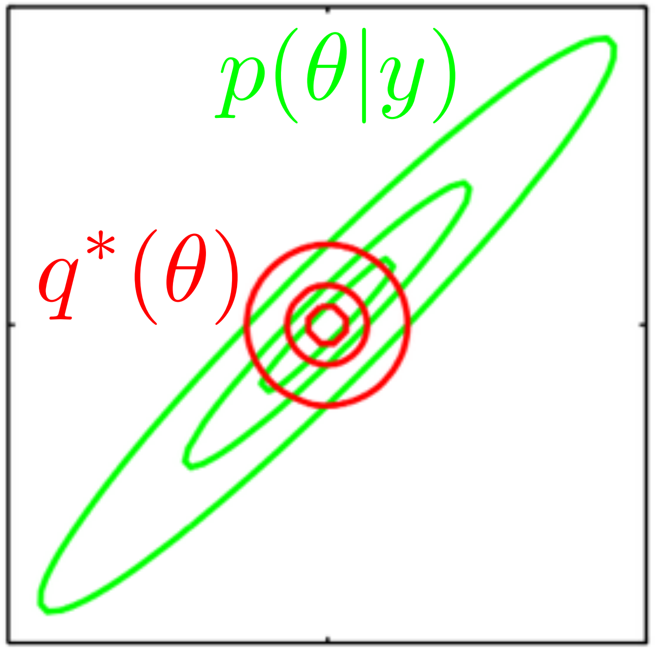

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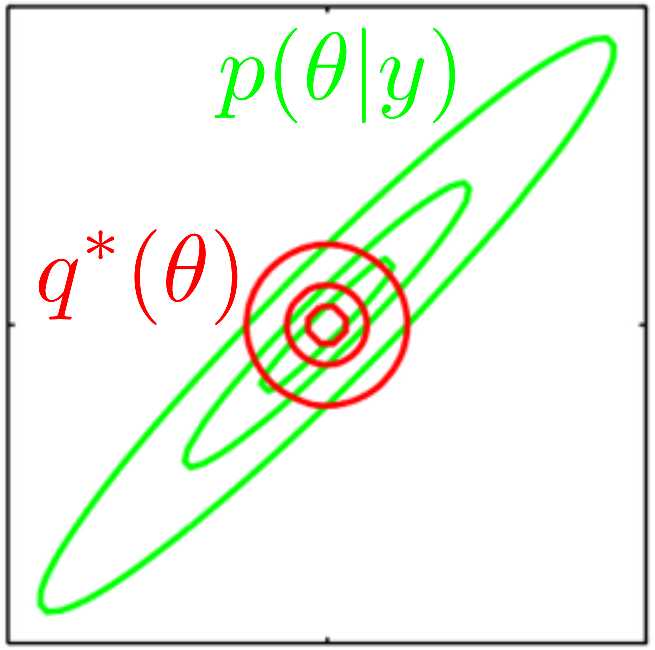

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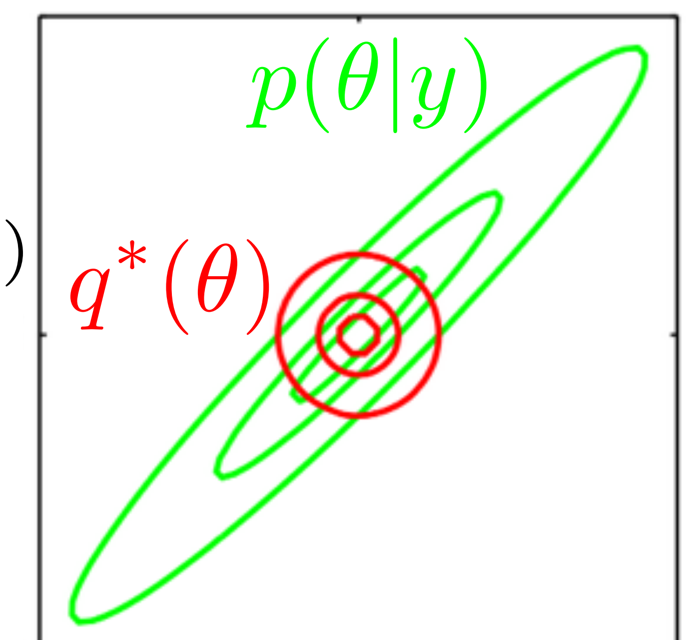

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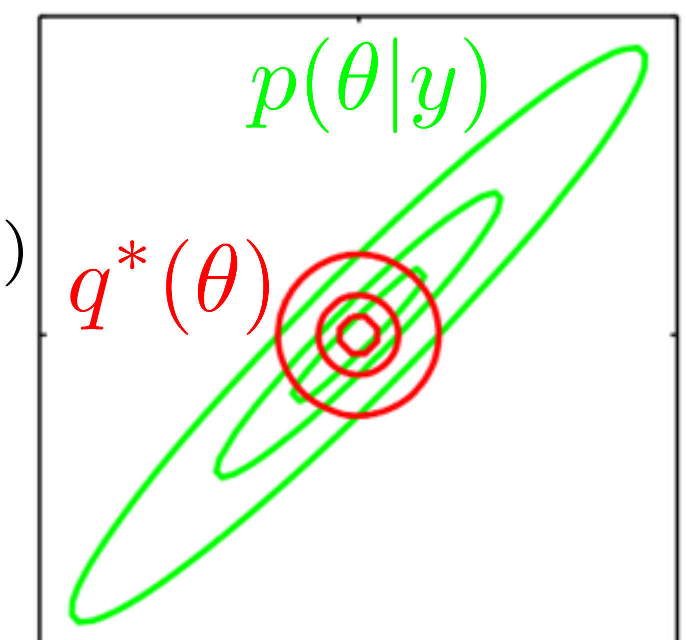

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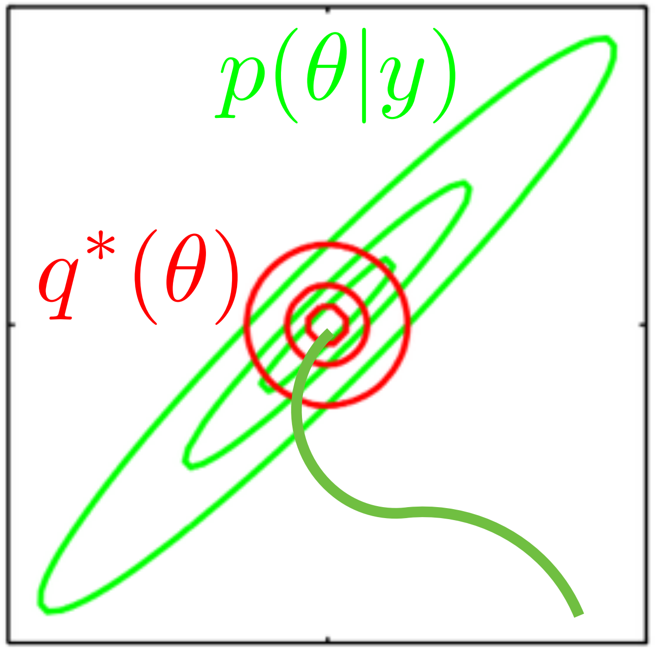

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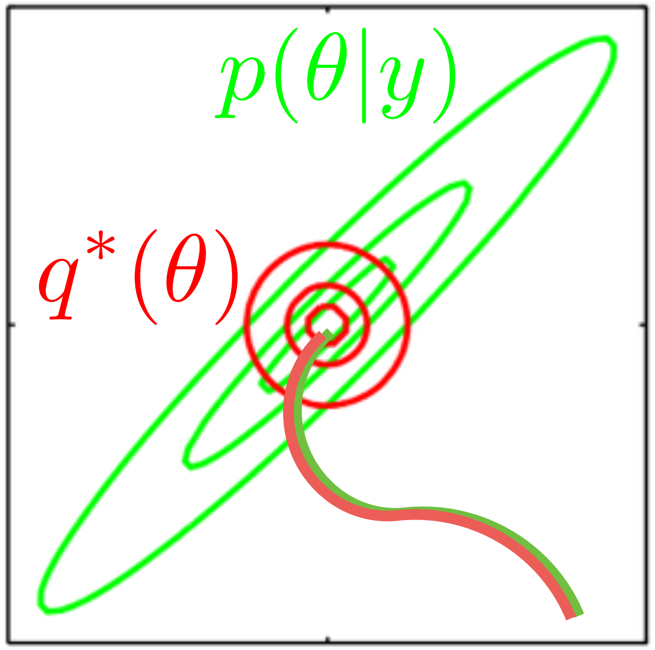

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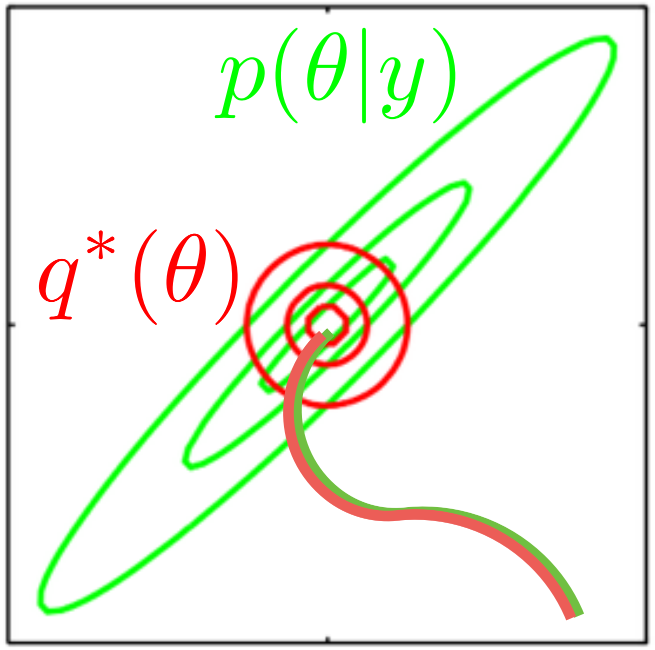

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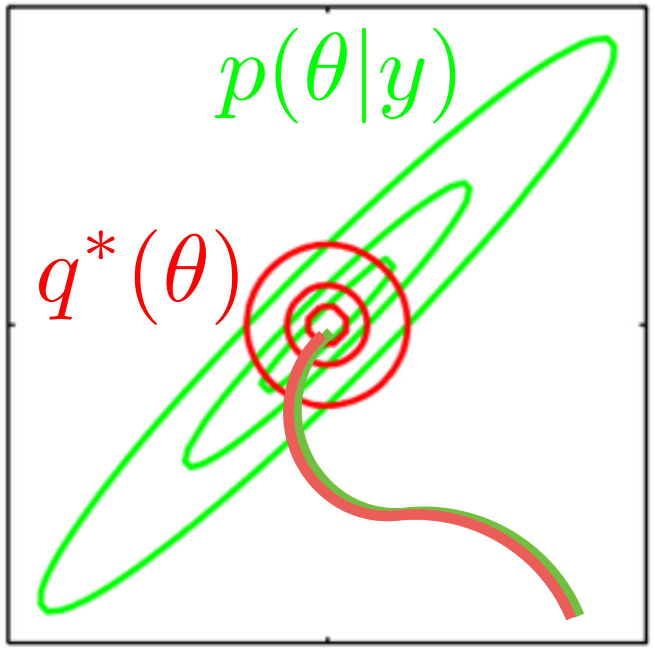

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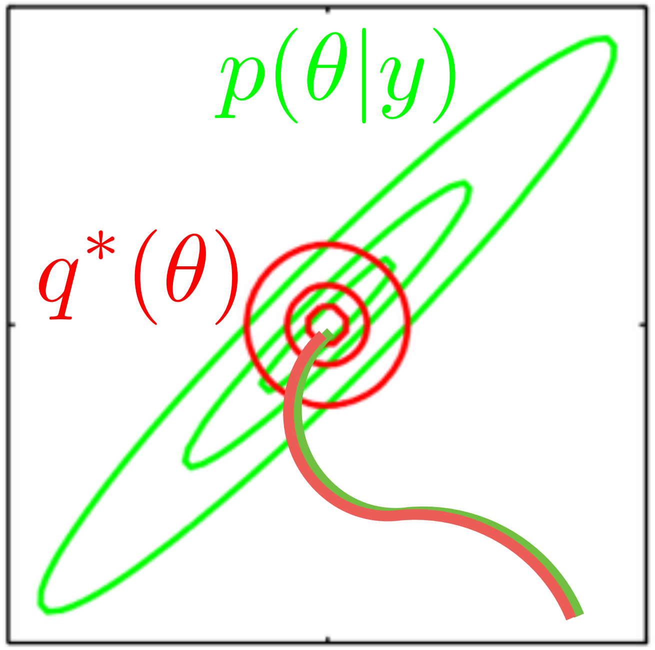

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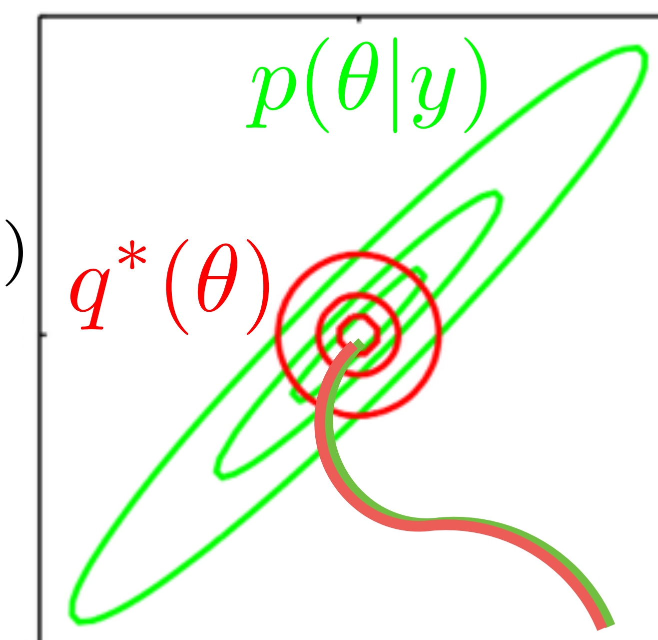
[board]



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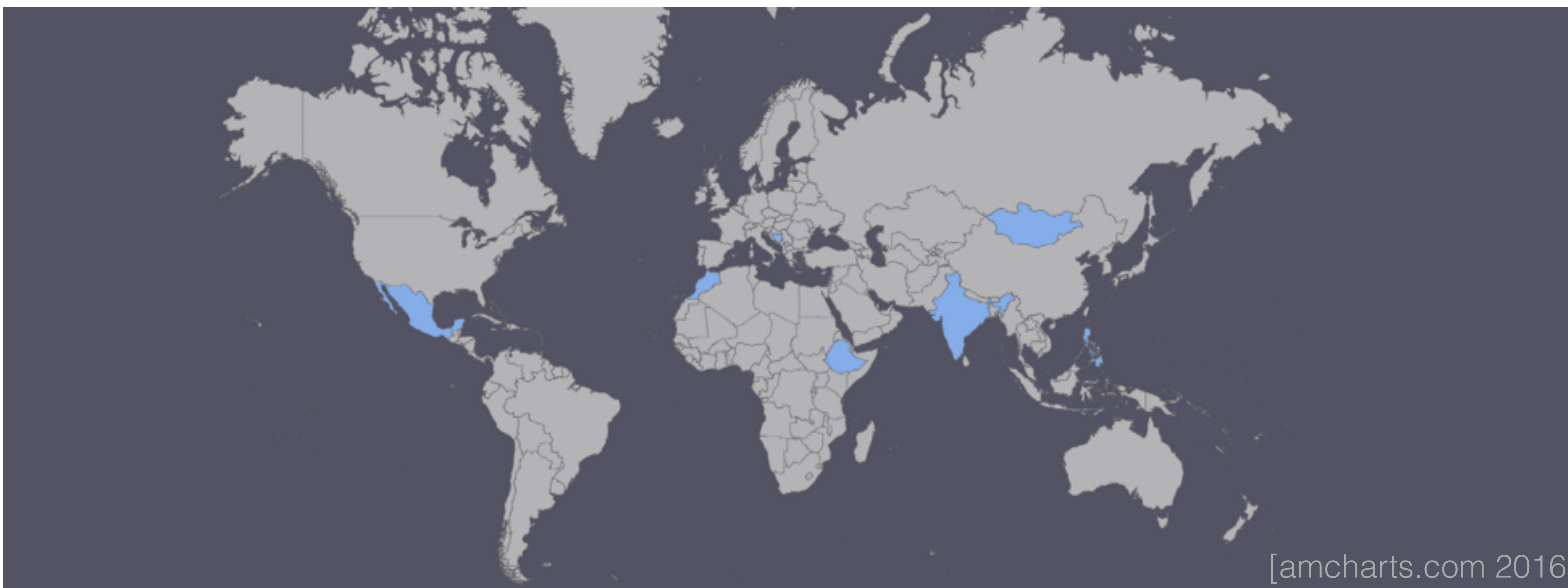
Microcredit Experiment



Microcredit Experiment

- Simplified from Meager (2018a)
- *K* microcredit trials (Mexico, Mongolia, Bosnia, India, Morocco, Philippines, Ethiopia)
- N_k businesses in *k*th site (~900 to ~17K)
- Profit of *n*th business at *k*th site:

1 if microcredit

profit

$$y_{kn} \overset{indep}{\sim} \mathcal{N}(\mu_k + T_{kn}\tau_k, \sigma_k^2)$$

- Priors and hyperpriors:

$$\begin{pmatrix} \mu_k \\ \tau_k \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu \\ \tau \end{pmatrix}, C\right) \qquad \begin{pmatrix} \mu \\ \tau \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu_0 \\ \tau_0 \end{pmatrix}, \Lambda^{-1}\right)$$

$$\sigma_k^{-2} \overset{iid}{\sim} \Gamma(a, b) \qquad C \sim \mathrm{Sep\&LKJ}(\eta, c, d)$$

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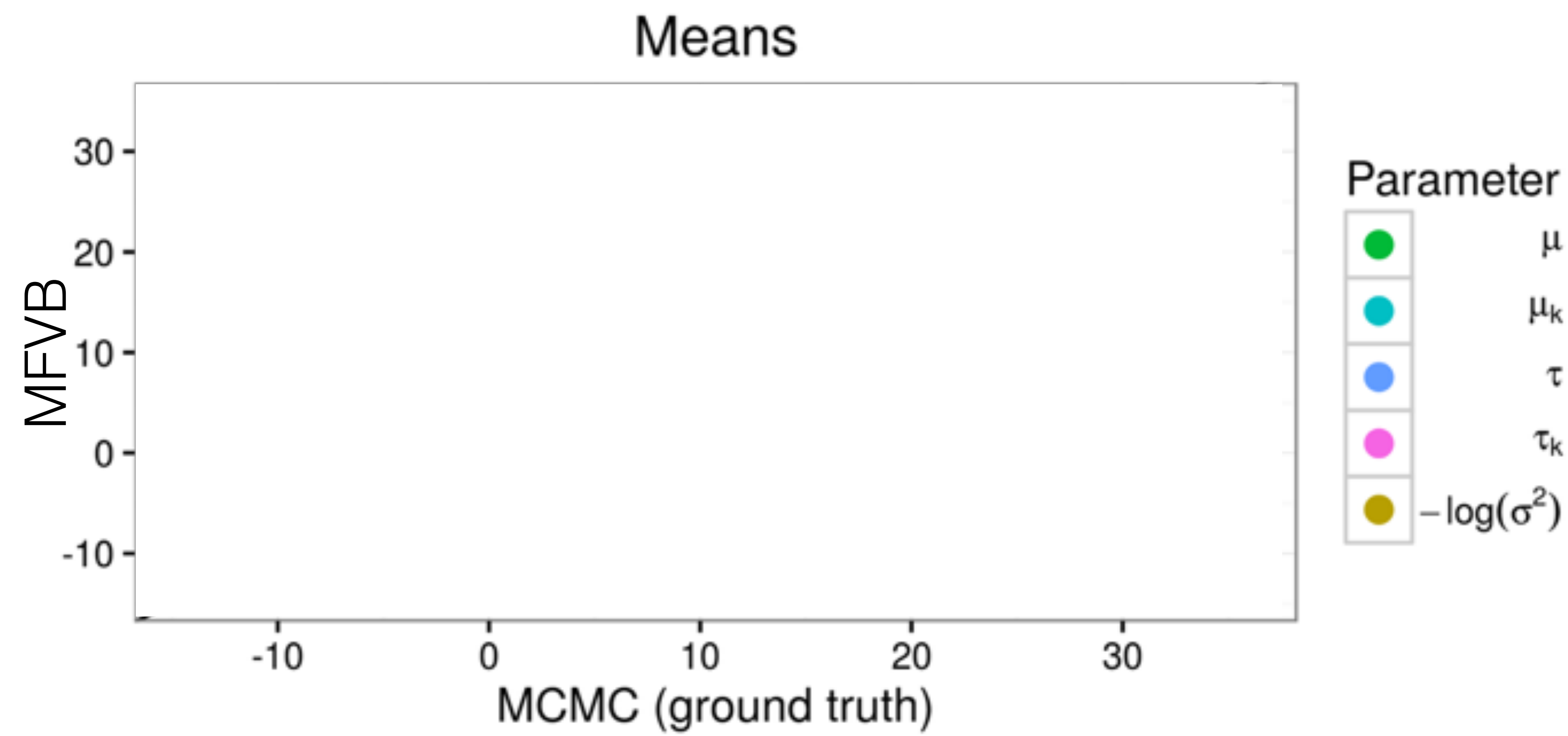

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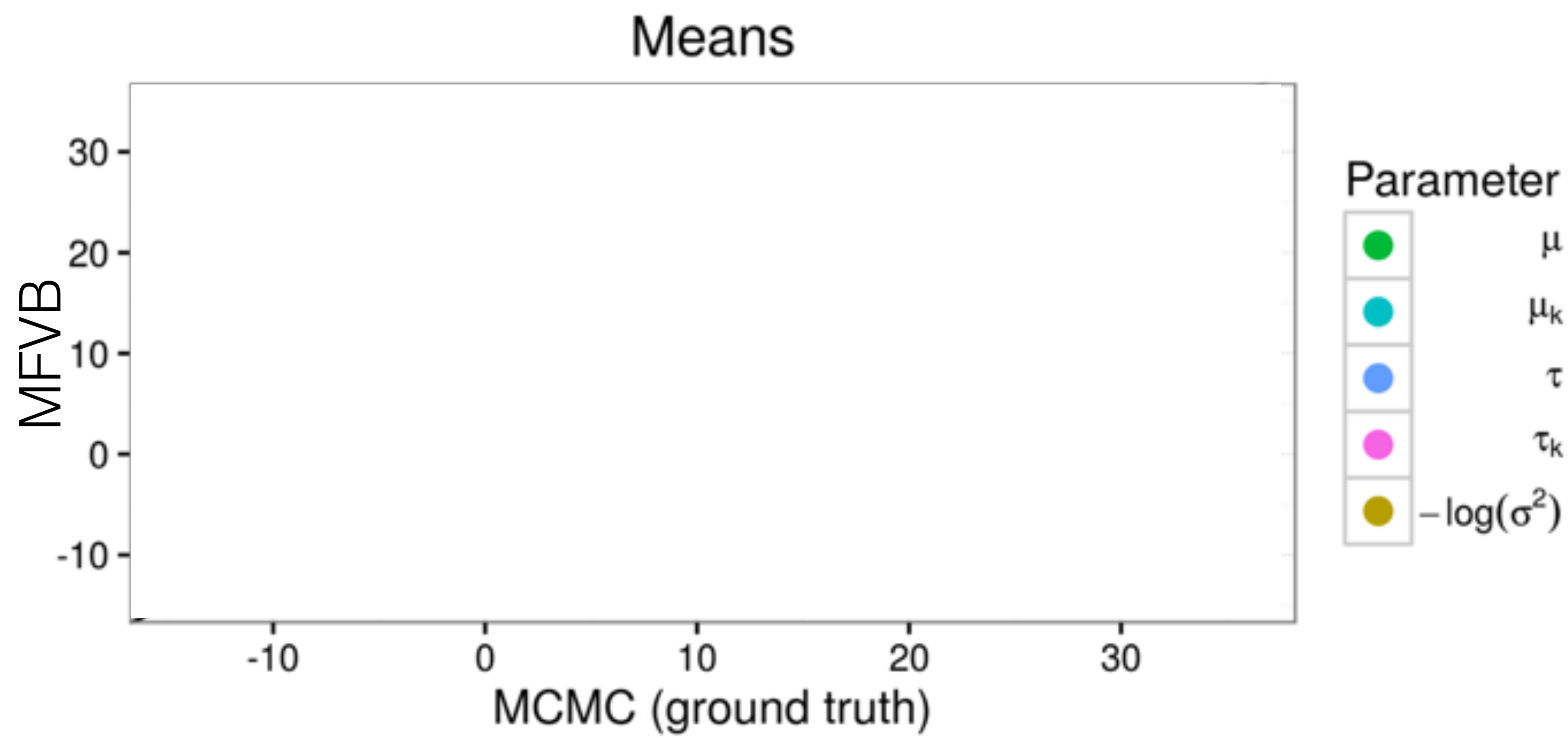
- *One set* of 2500 MCMC draws: **45 minutes**



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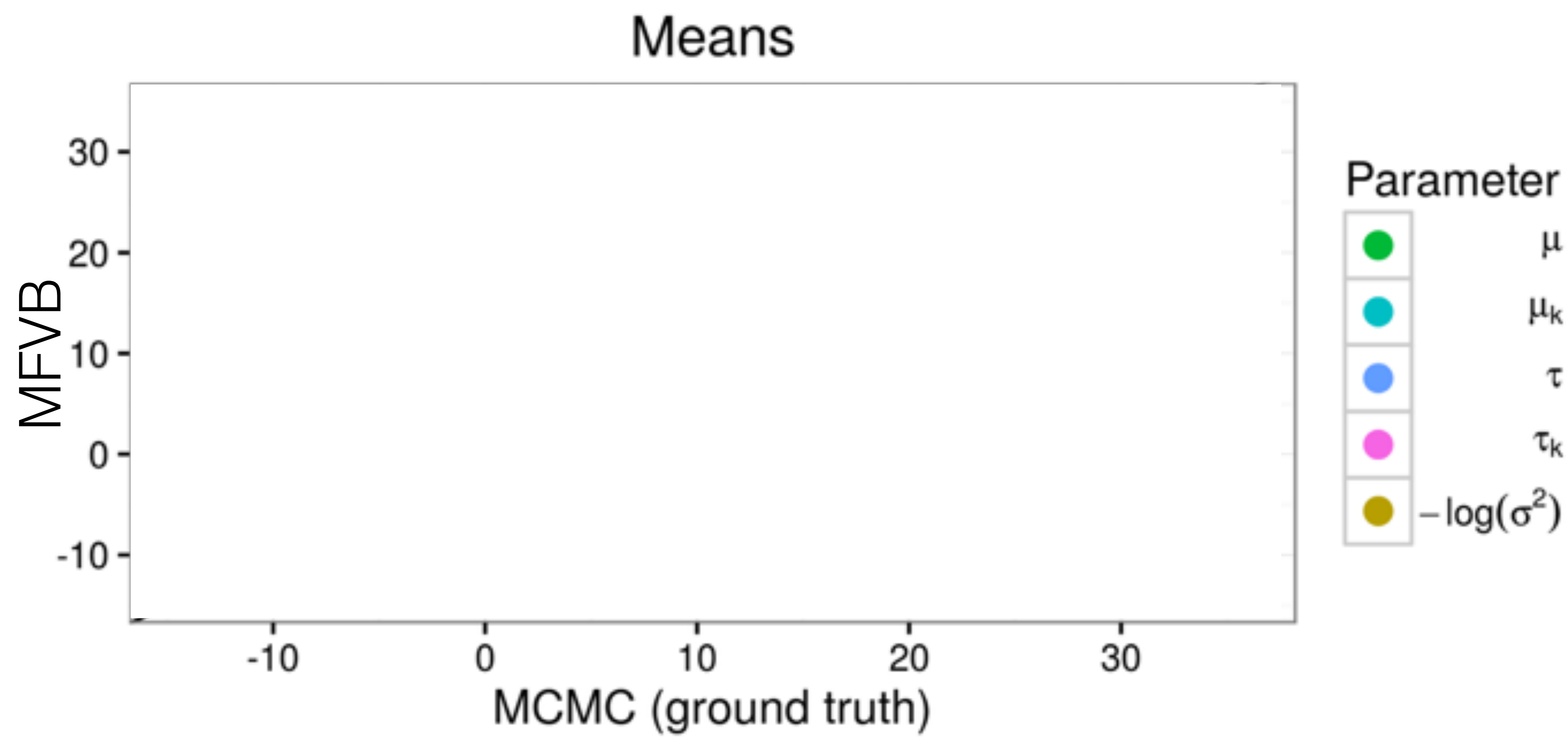

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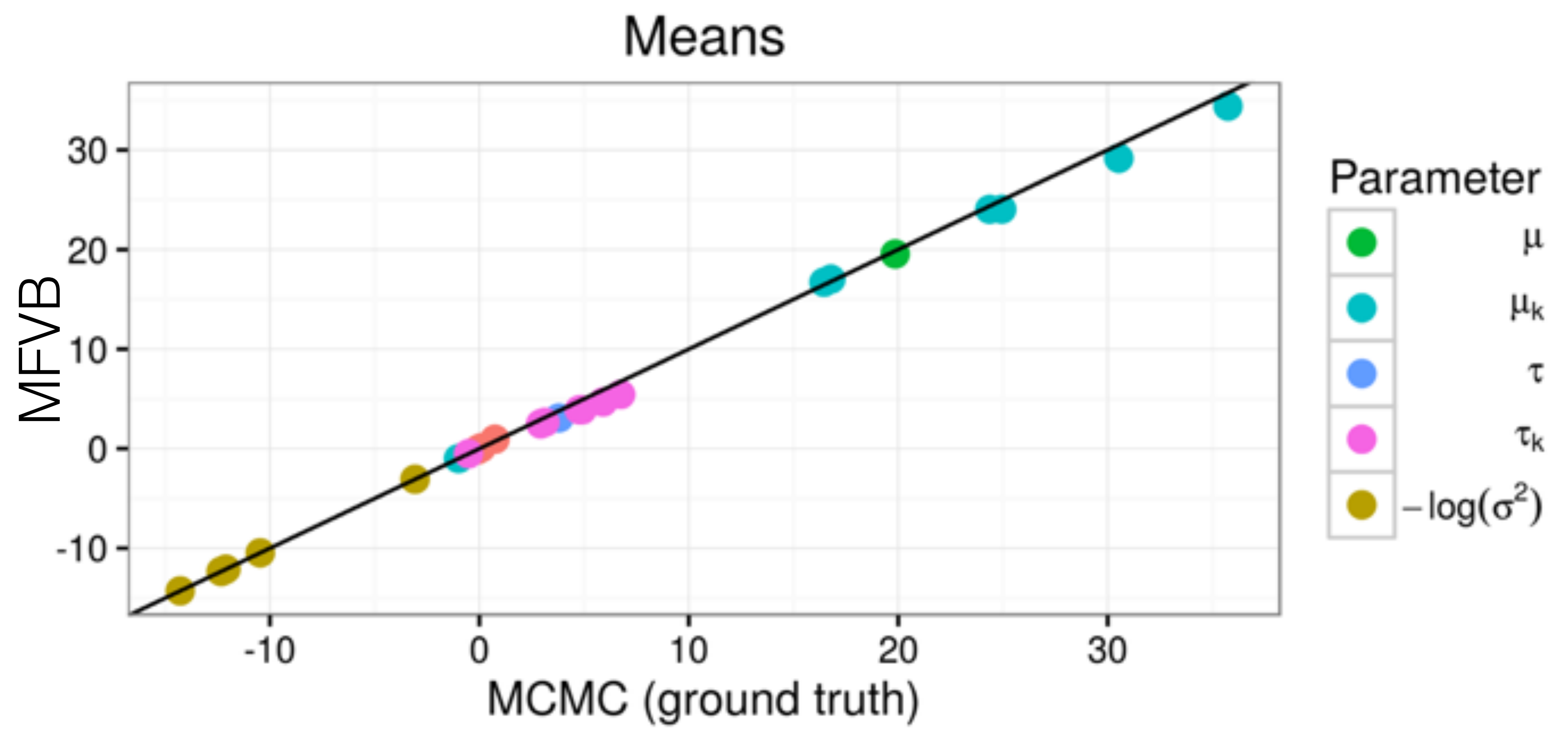

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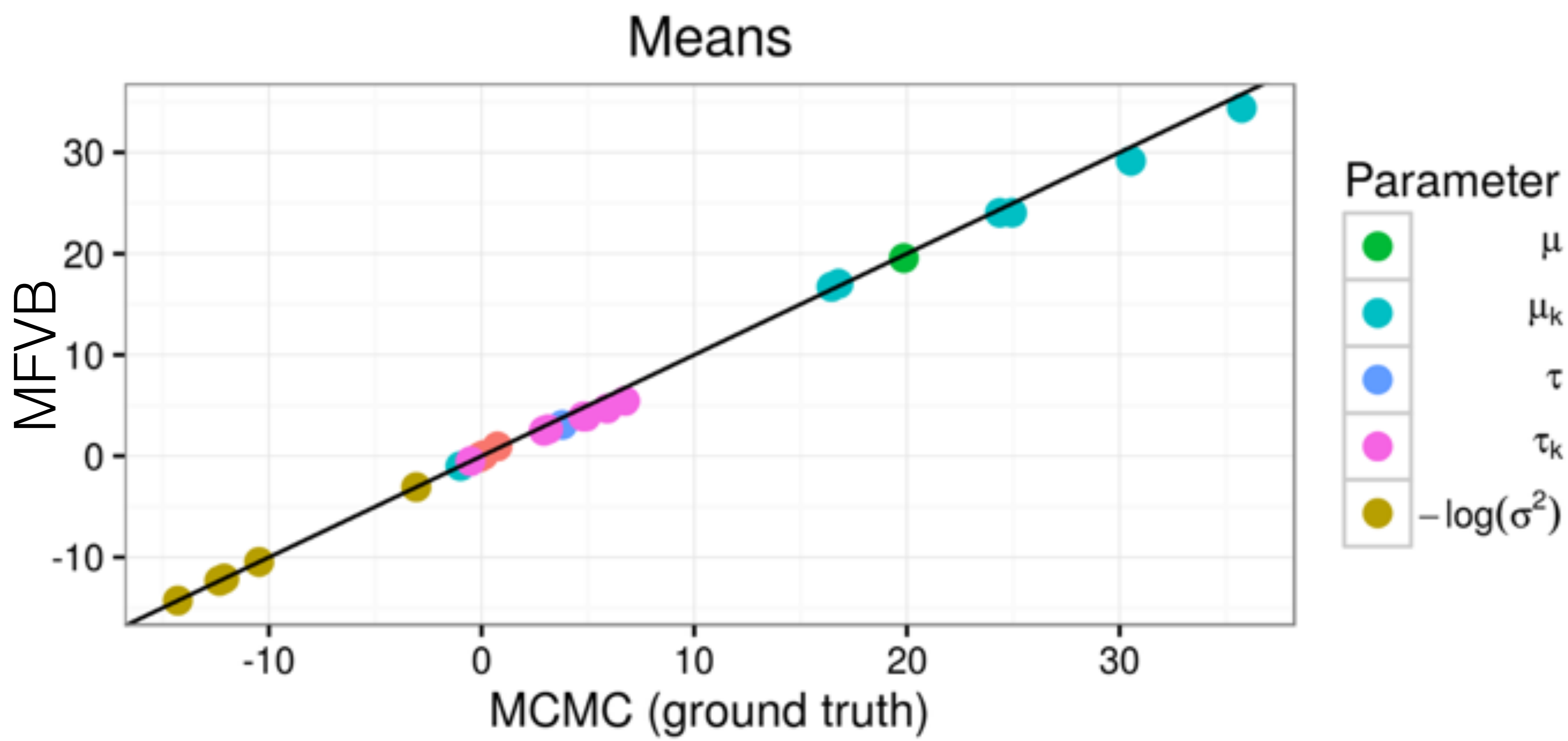

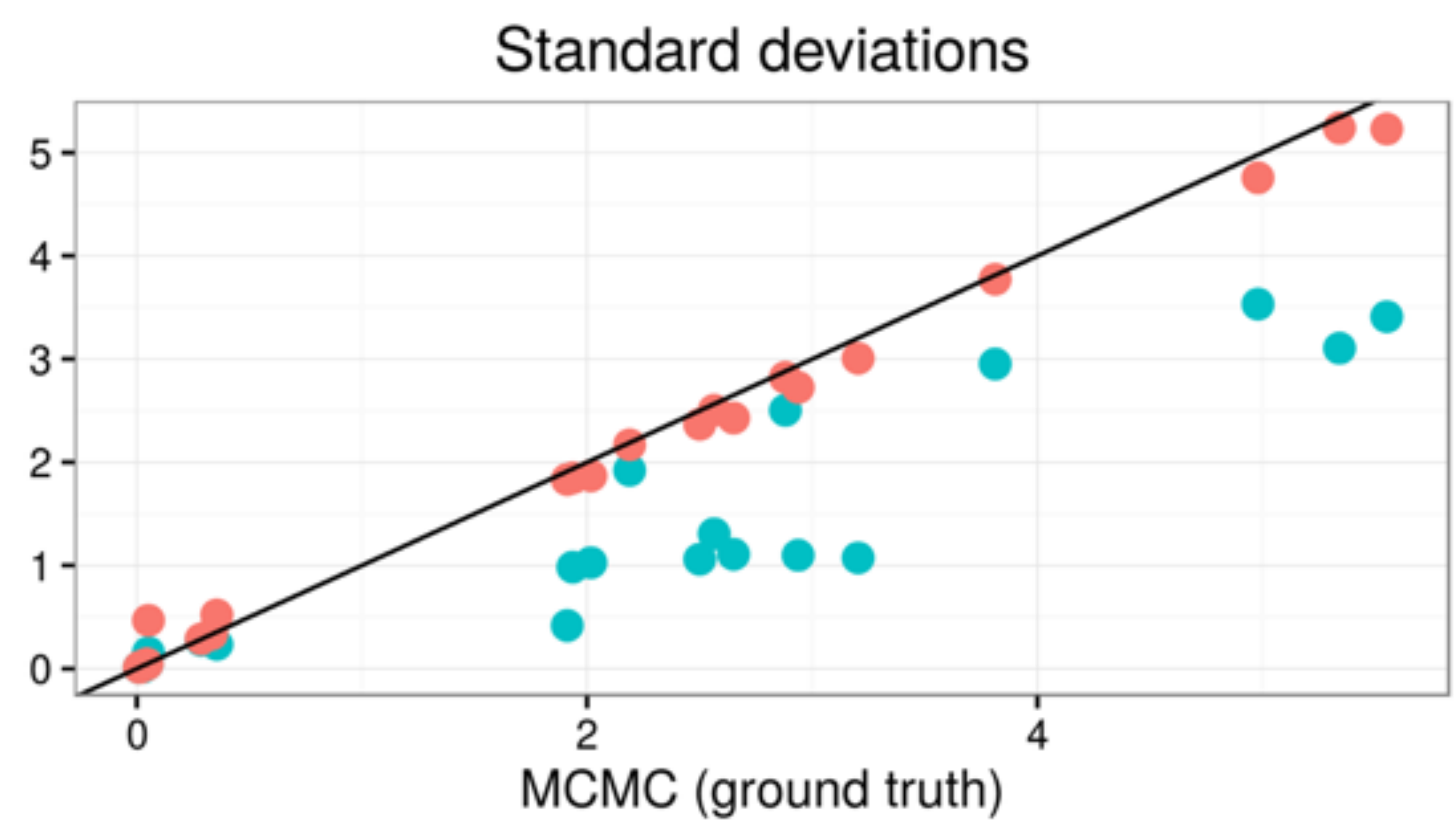

LRVB, **MFVB**

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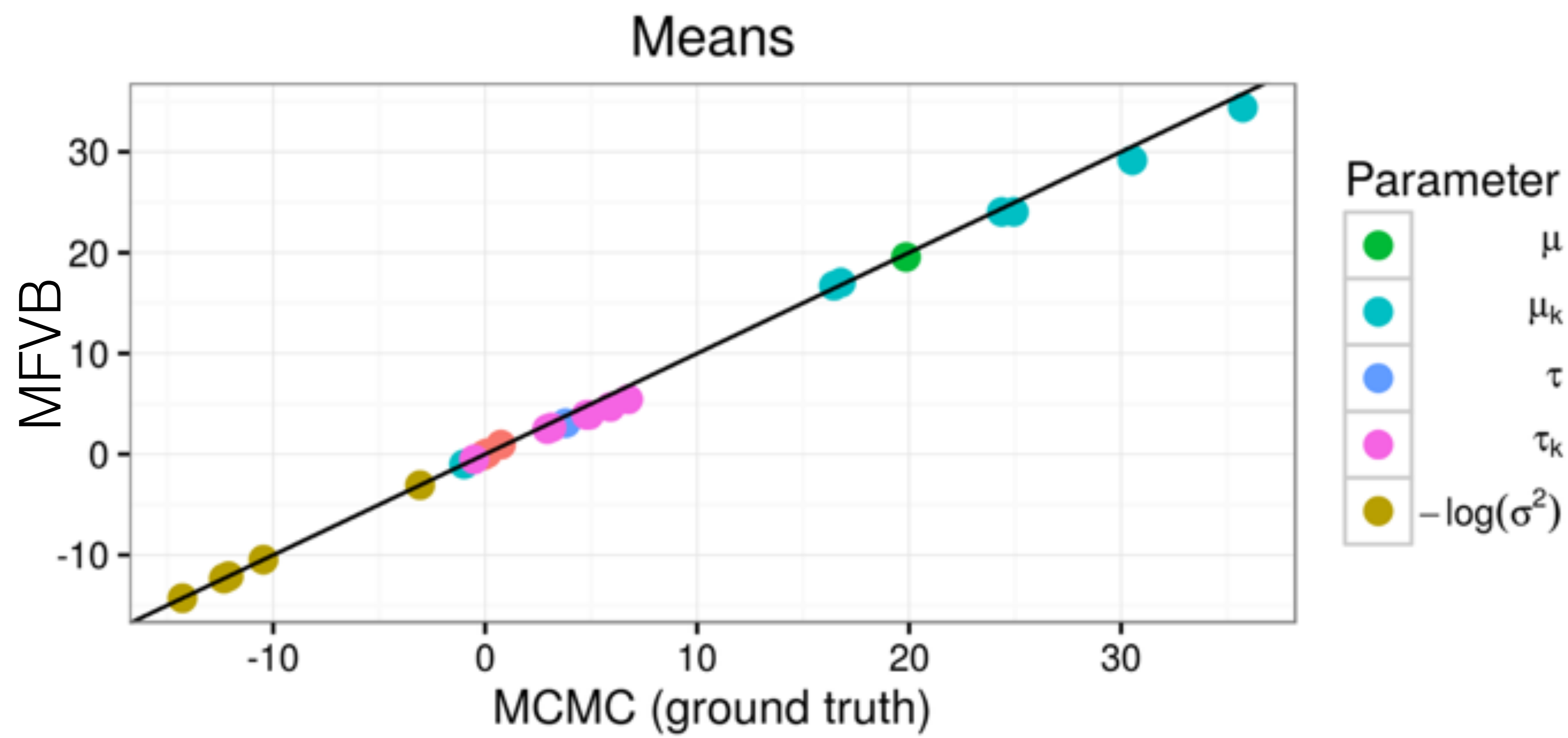

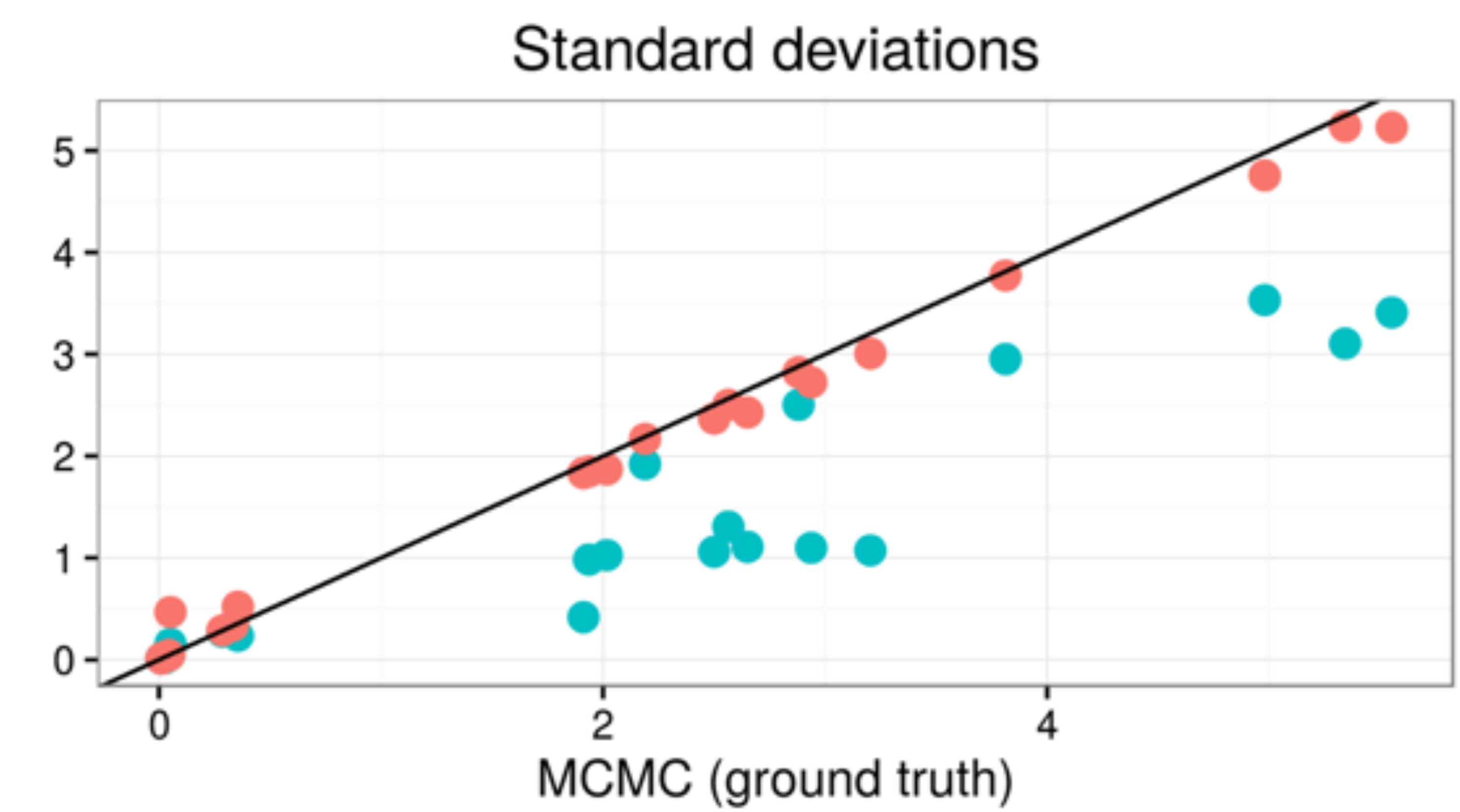

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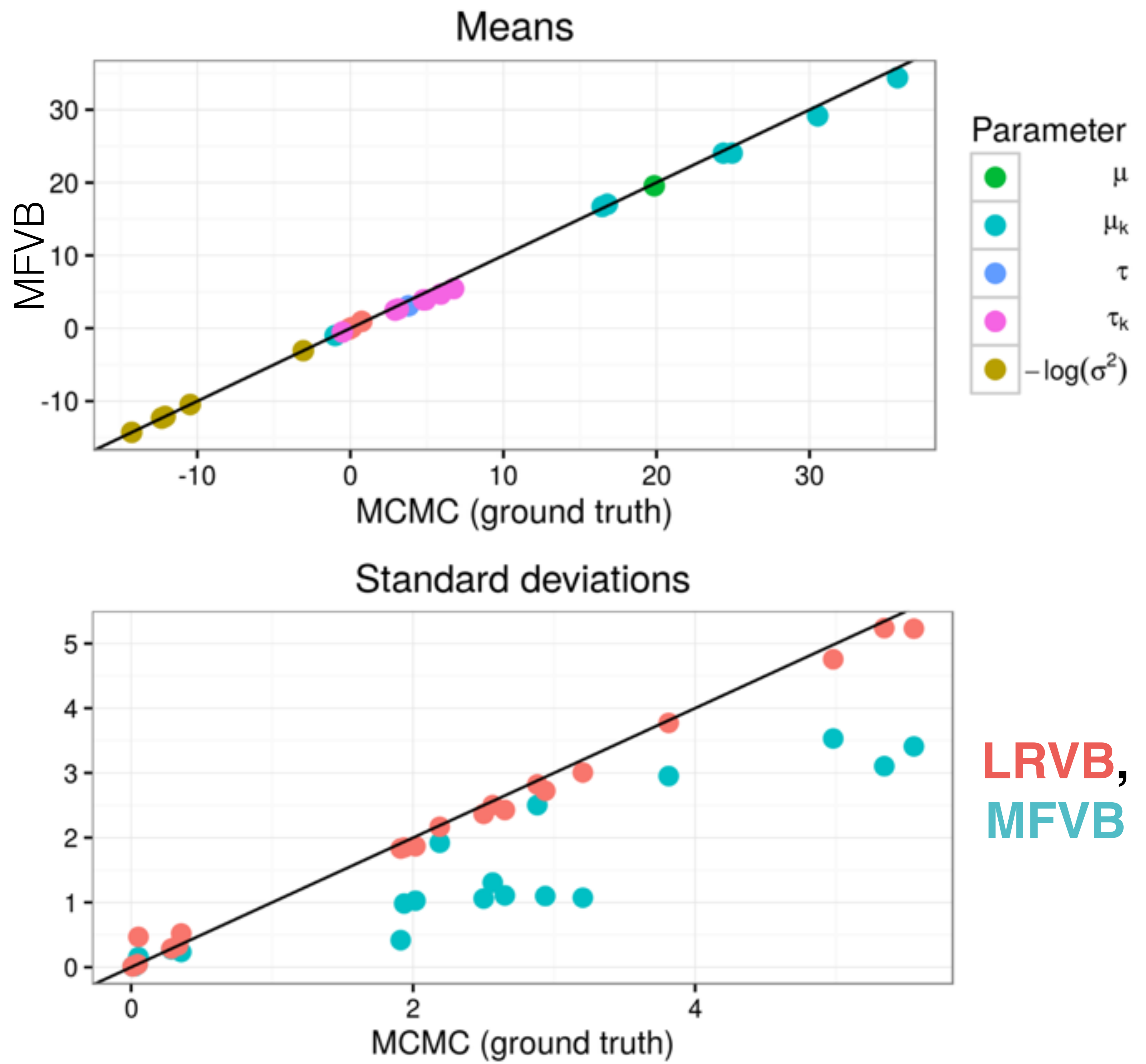

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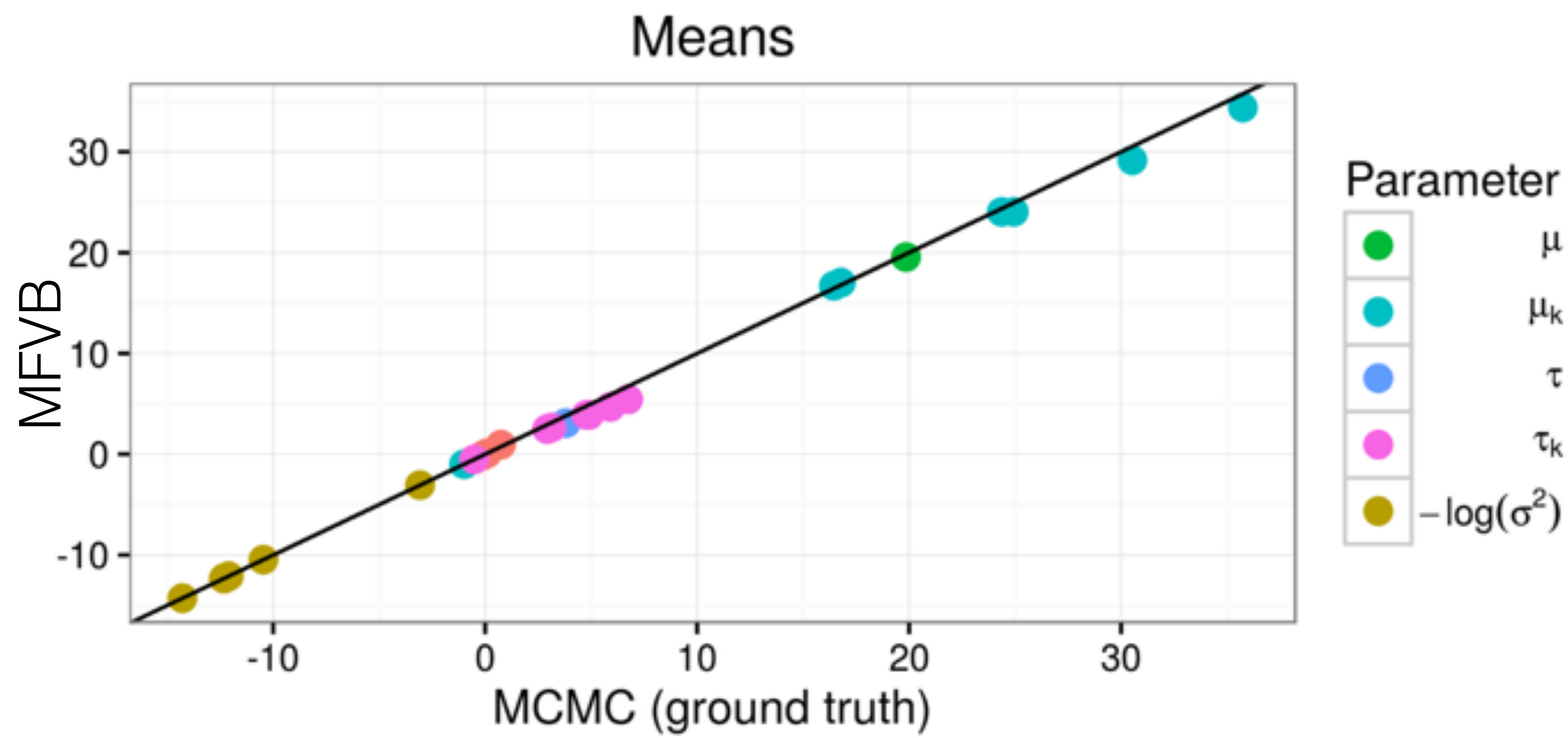

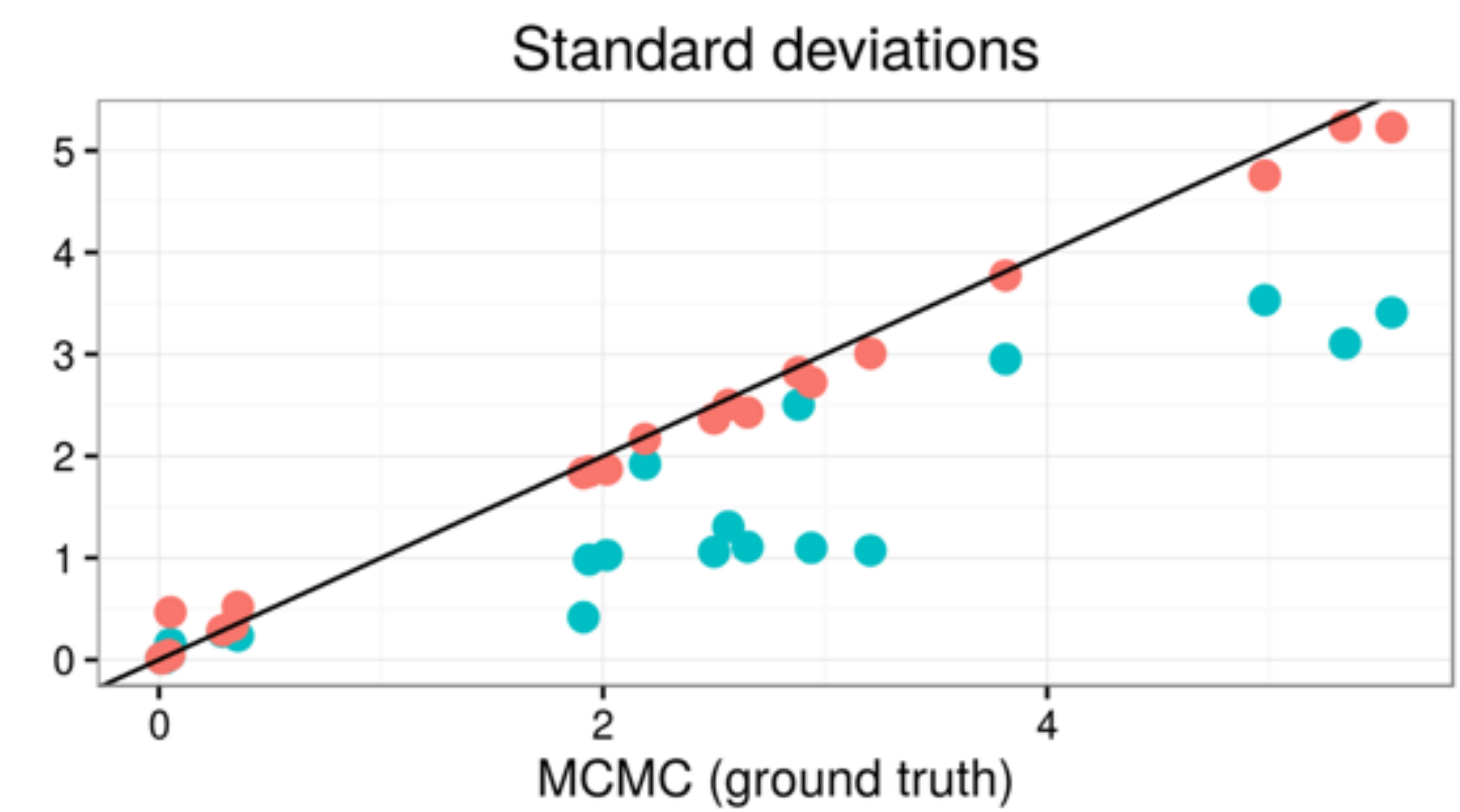

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Roadmap

- Challenges of VB
- Accurate uncertainties from VB
- Accurate robustness quantification from VB

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Robustness quantification

- Bayes Theorem

$$p(\theta|y)$$

$$\propto_\theta p(y|\theta)p(\theta)$$

Robustness quantification

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$$p(\theta|y,\alpha)$$

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Robustness quantification

- Bayes Theorem

$$
\begin{aligned}
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&\propto_\theta p(y|\theta)p(\theta|\alpha)
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- Sensitivity

Robustness quantification

- Bayes Theorem

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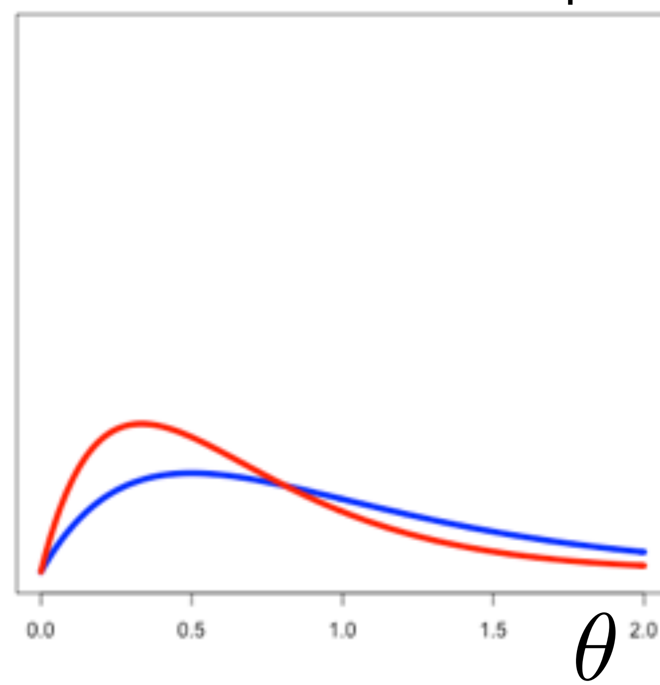
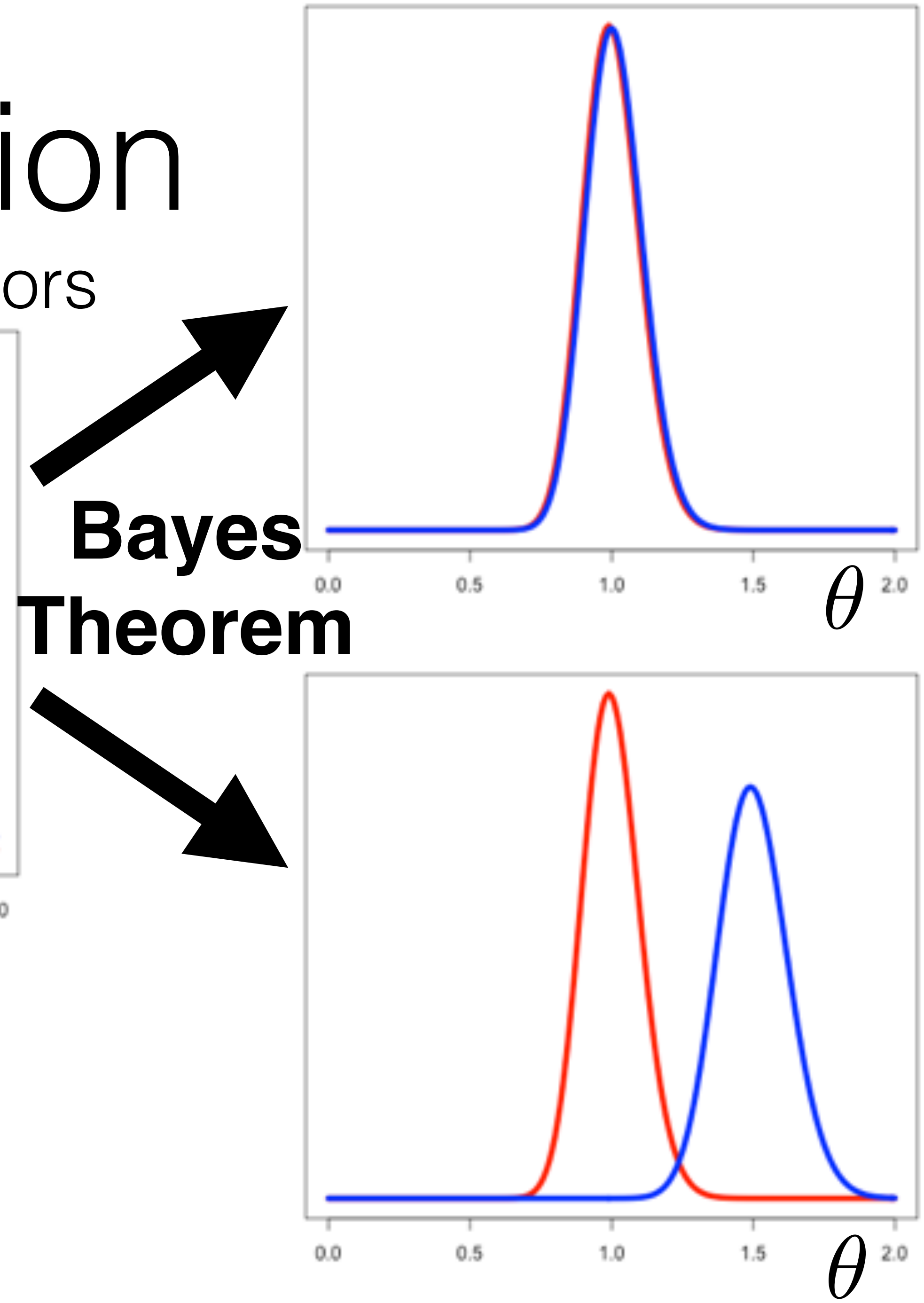
$$\propto_\theta p(y|\theta)p(\theta|\alpha)$$

- Sensitivity

Some reasonable priors





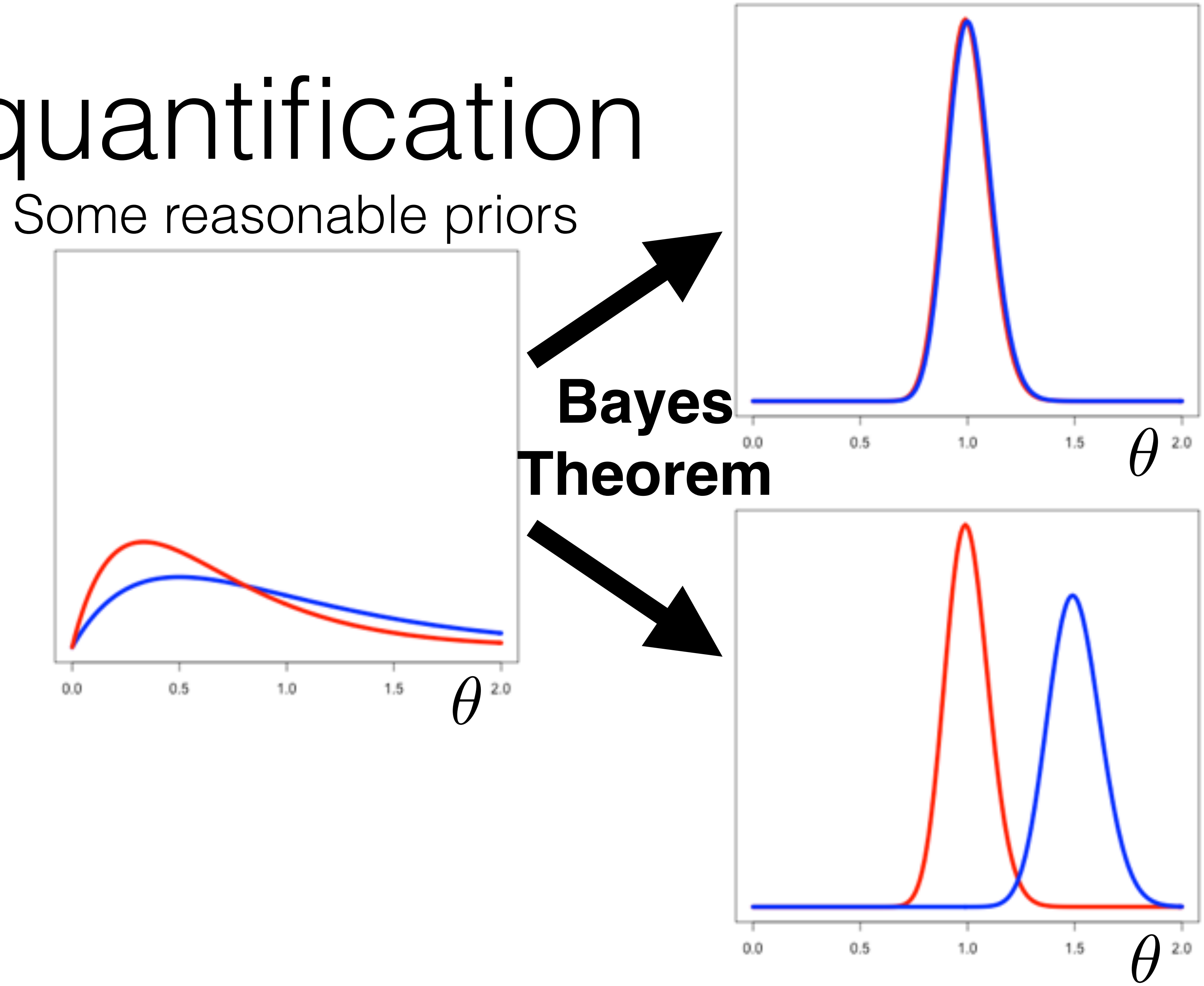
Robustness quantification

- Bayes Theorem

$$p_\alpha(\theta) := p(\theta|y,\alpha)$$
$$\propto_\theta p(y|\theta)p(\theta|\alpha)$$

- Sensitivity

$$\mathbb{E}_{p_\alpha}[g(\theta)]$$



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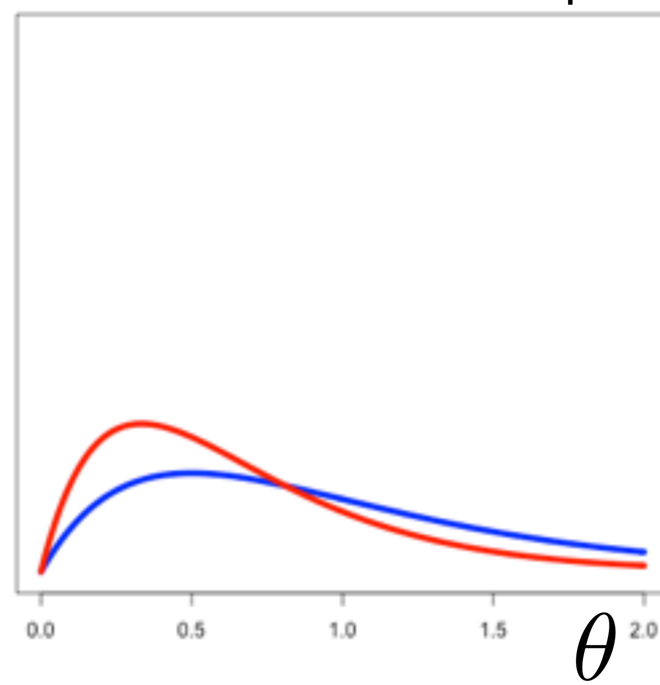

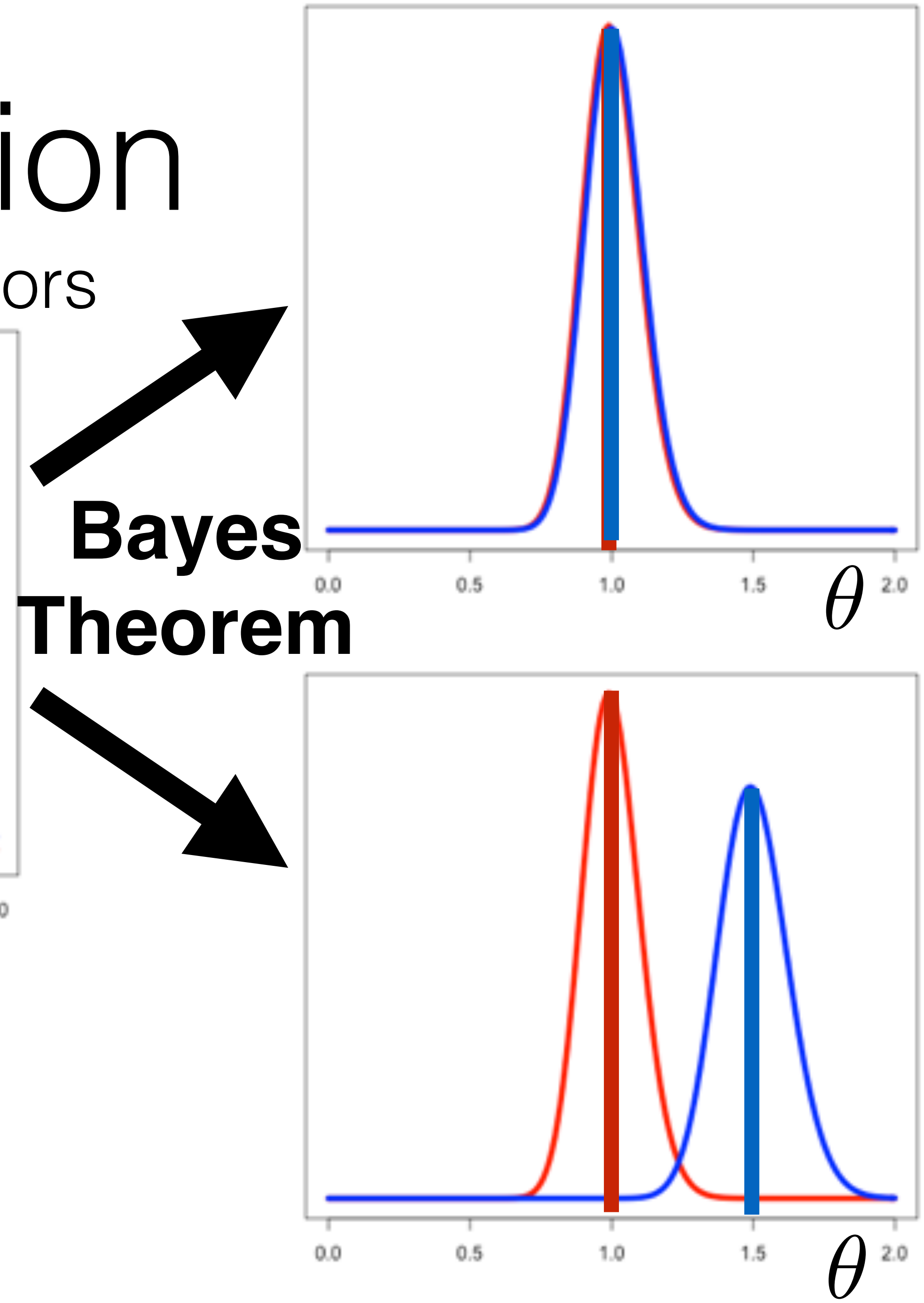

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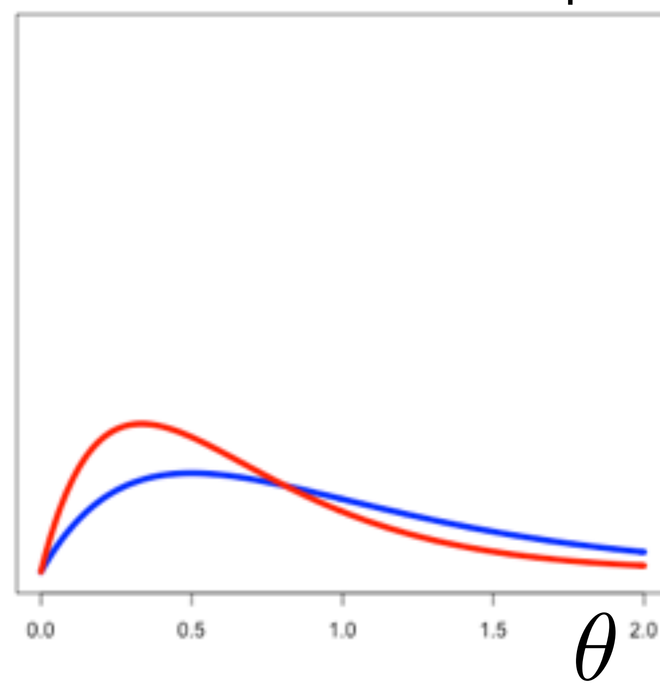

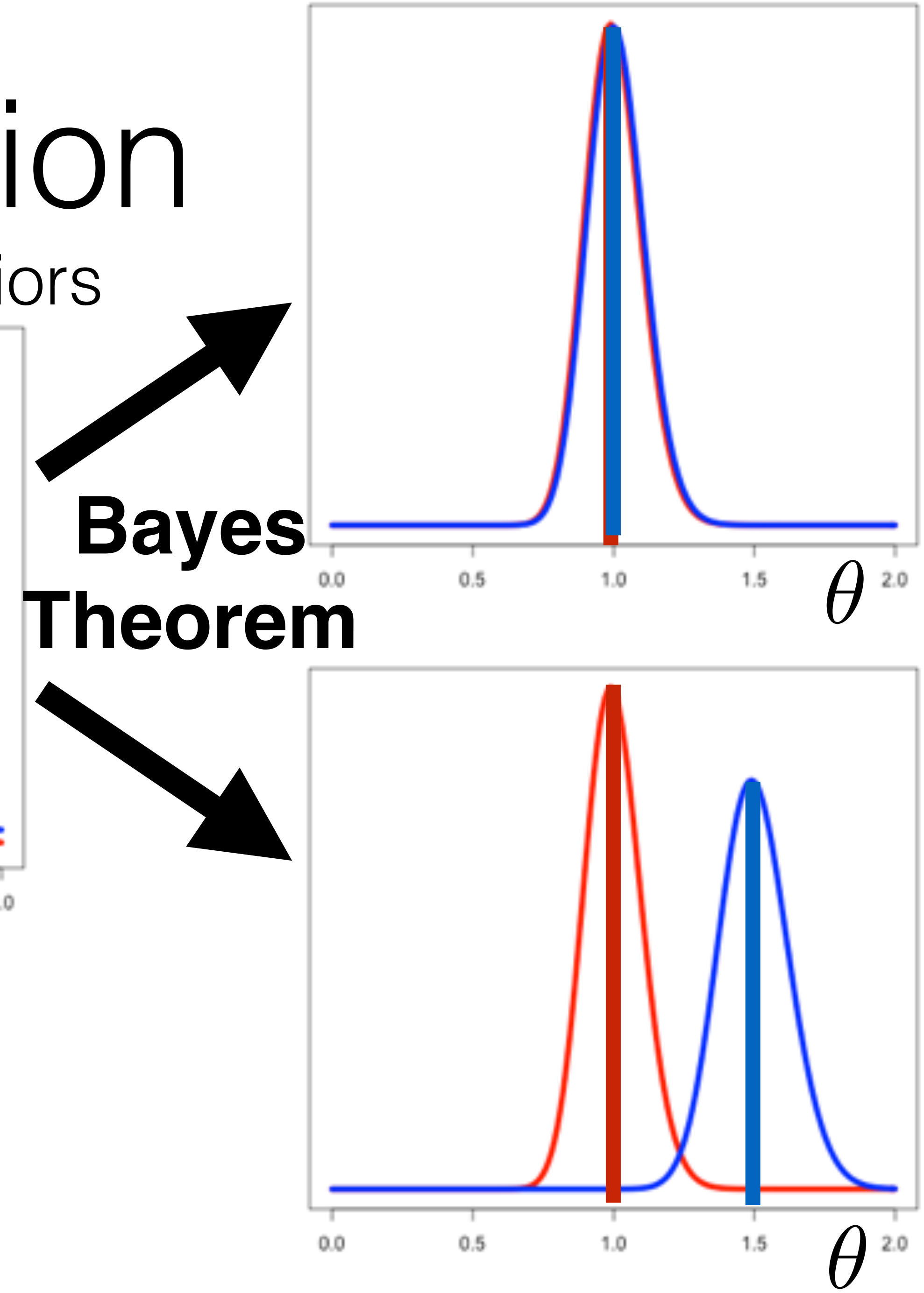

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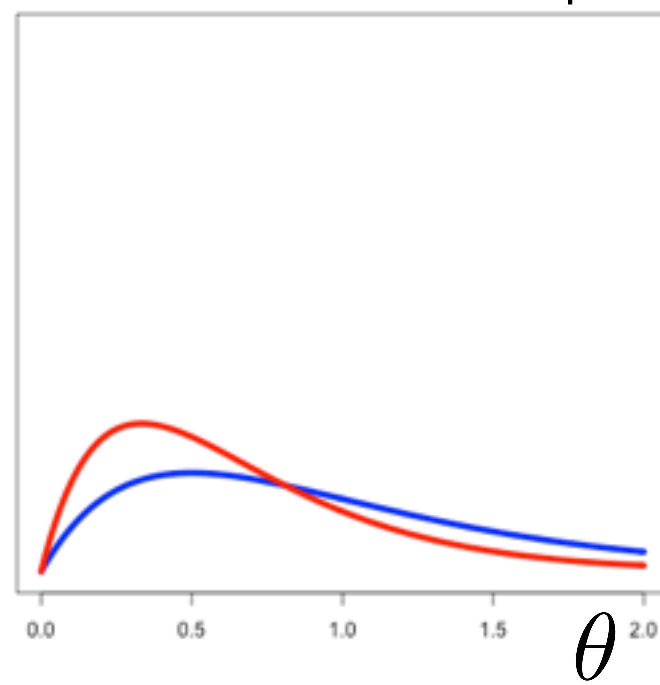

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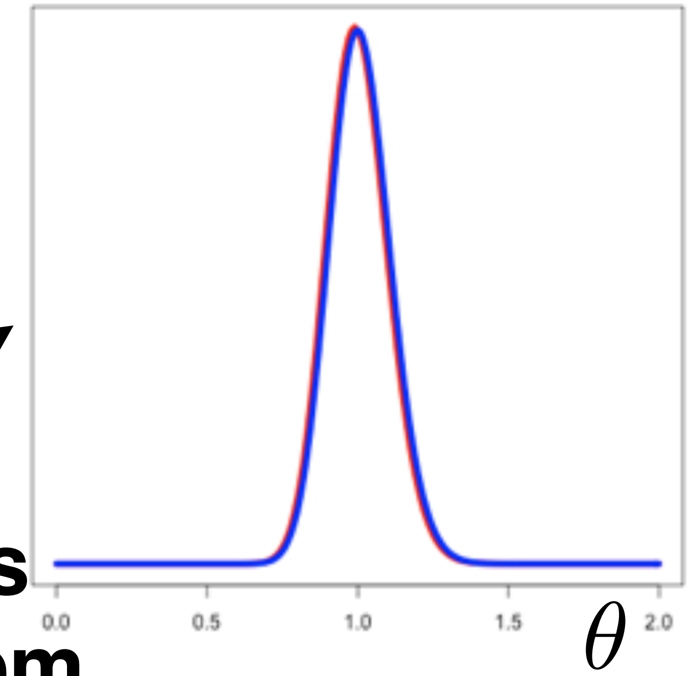

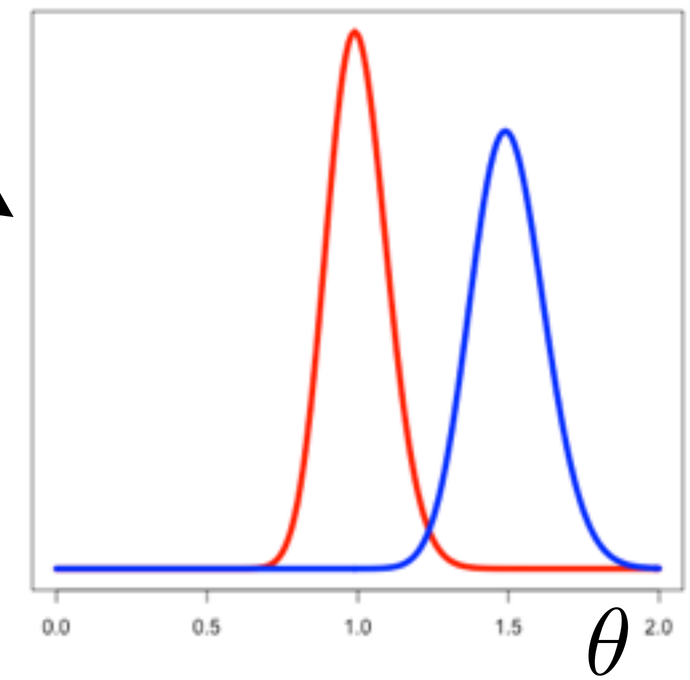

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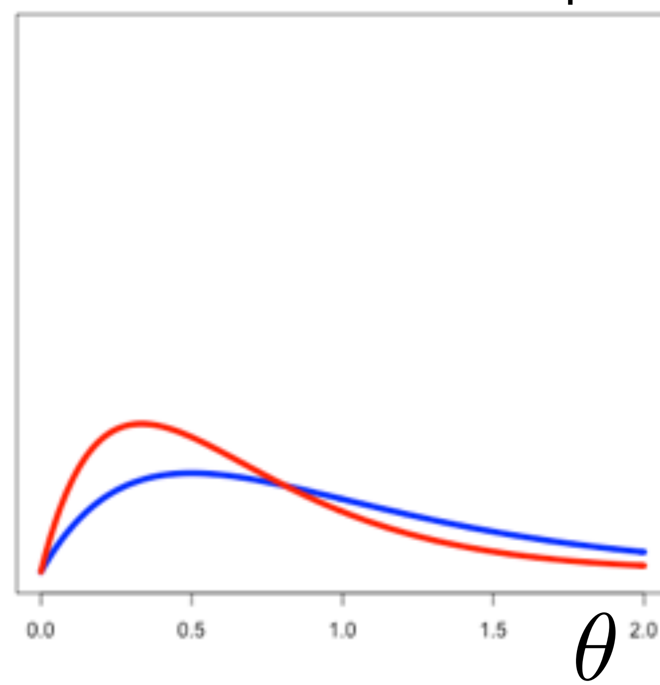

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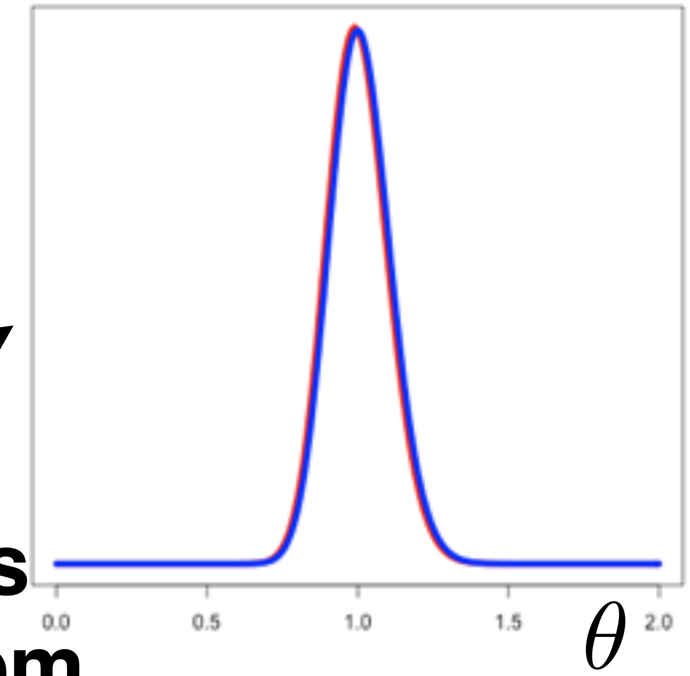

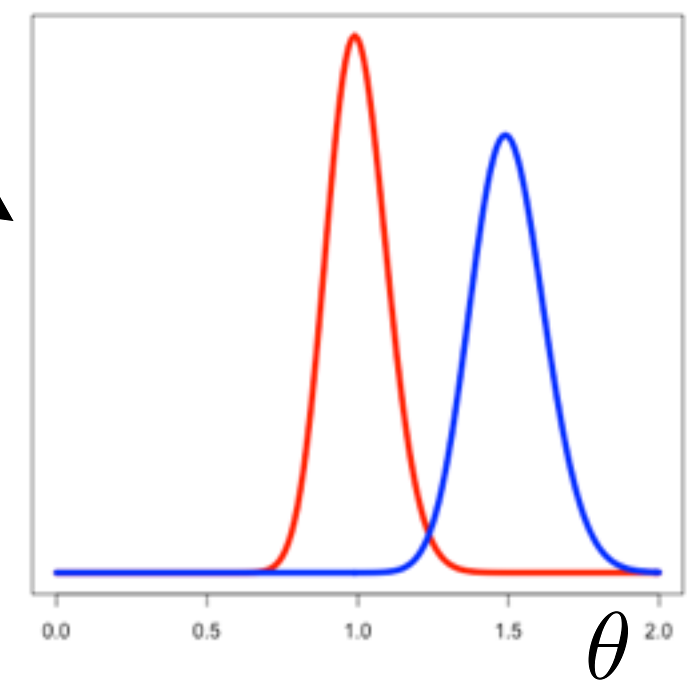

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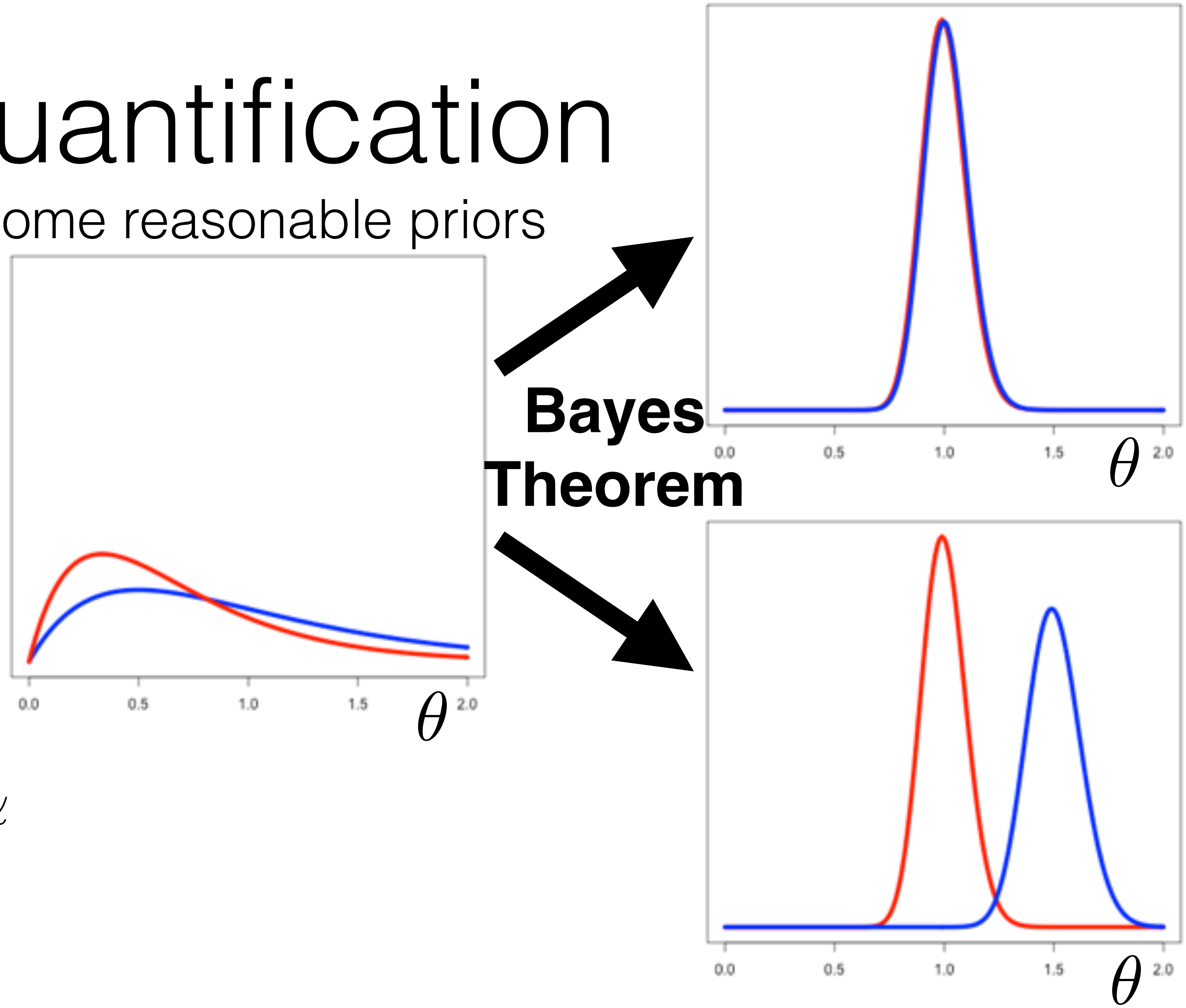

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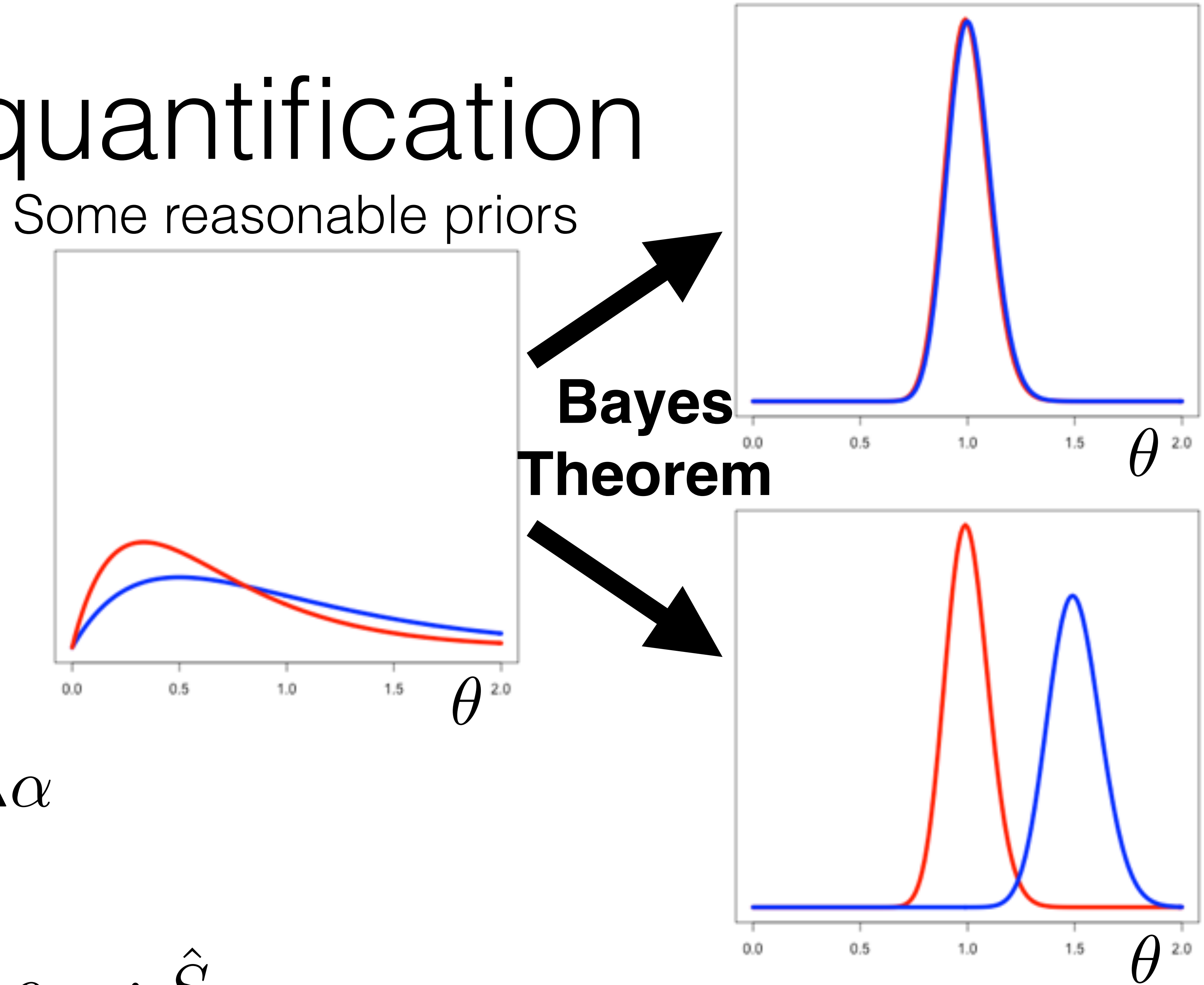

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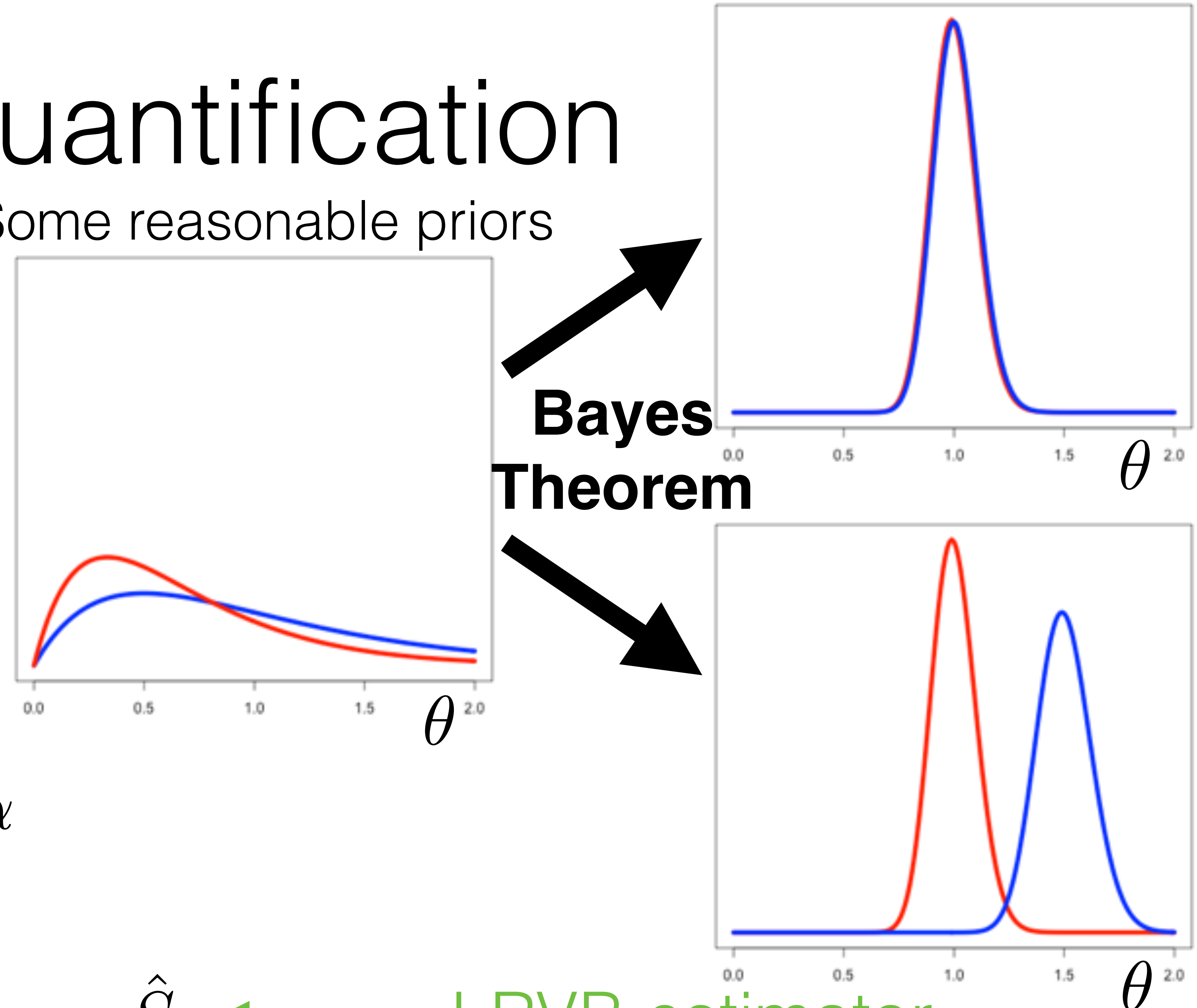
LRVB estimator



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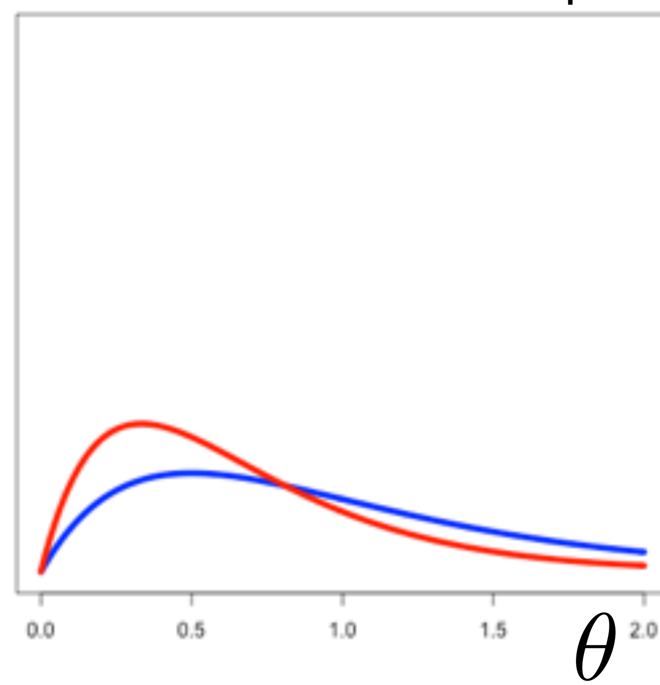

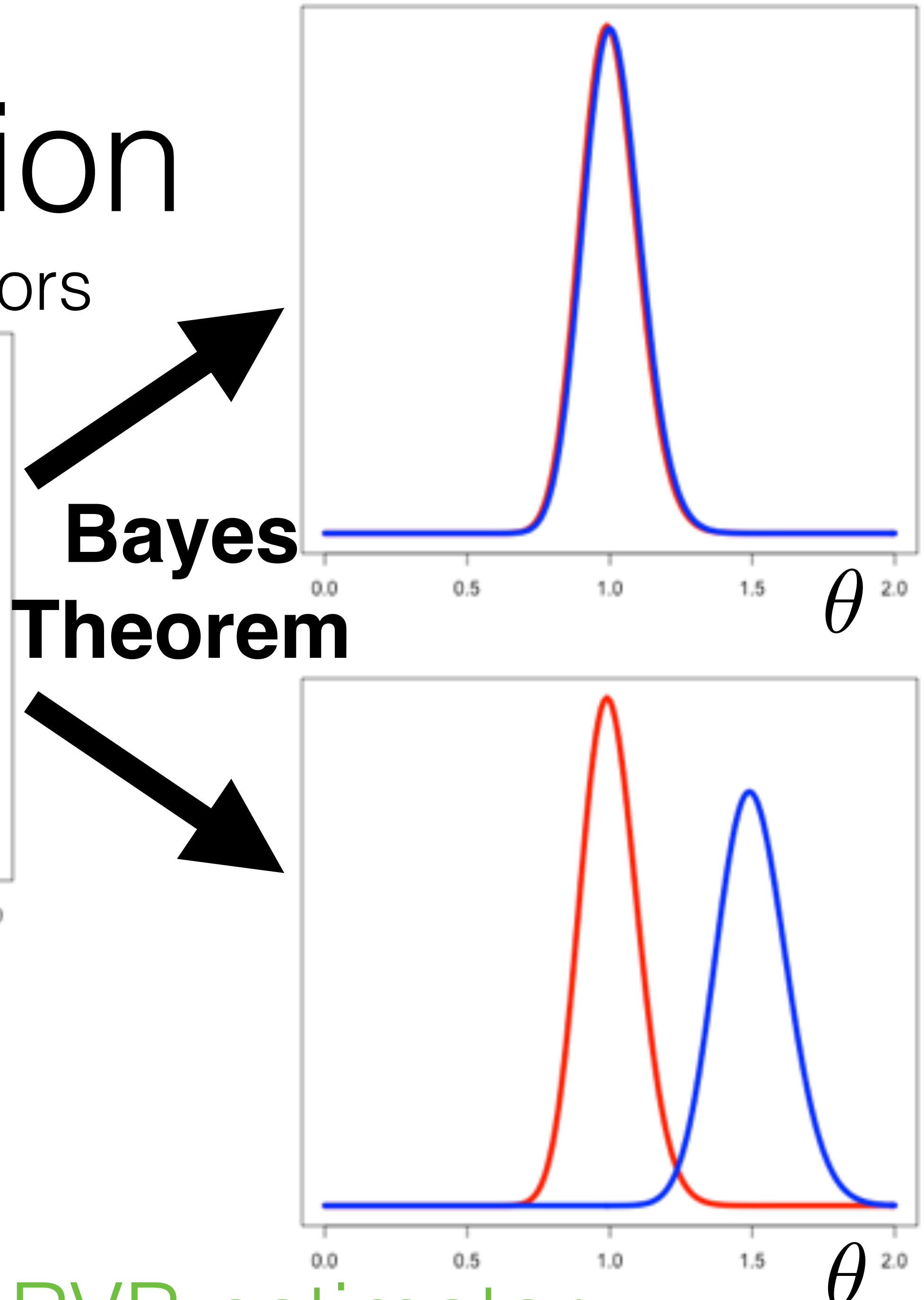

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- Recall: our general LRVB formula applies for:

$$\log p_t(\theta) = \log p(\theta|y) + f(\theta,t) - \mathrm{Const}(t)$$

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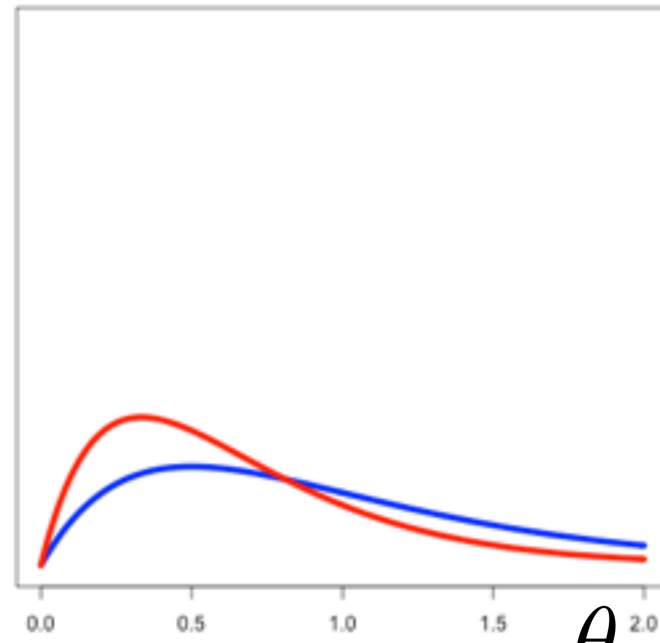

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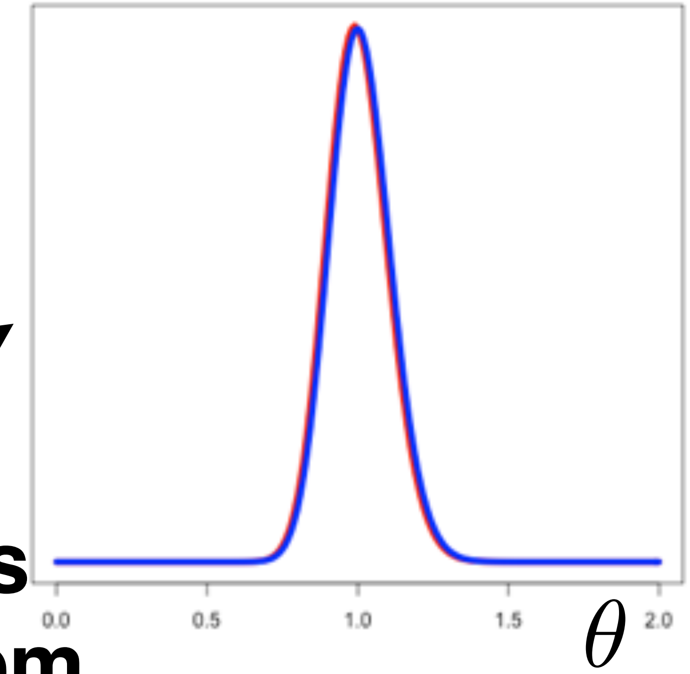

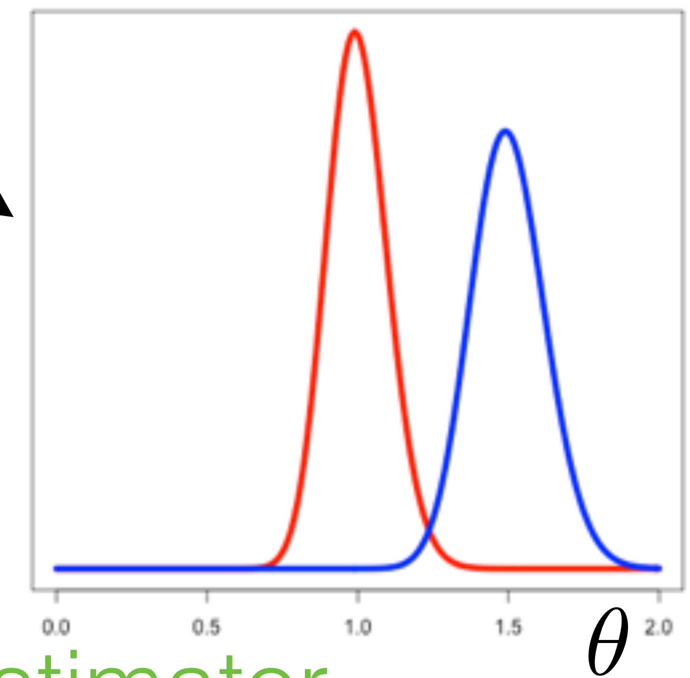

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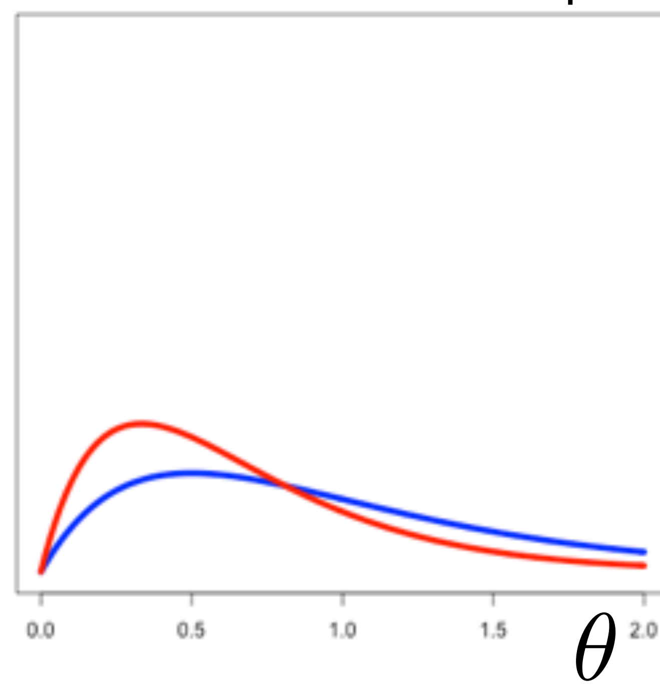

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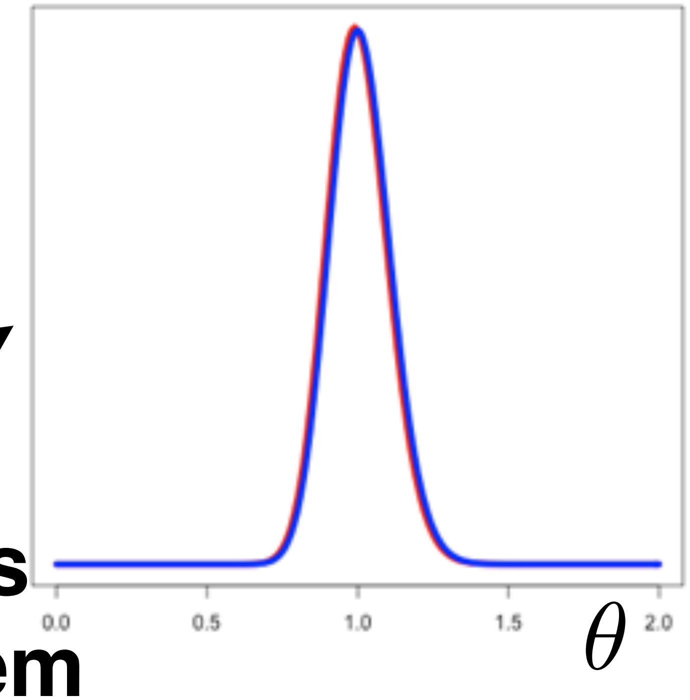

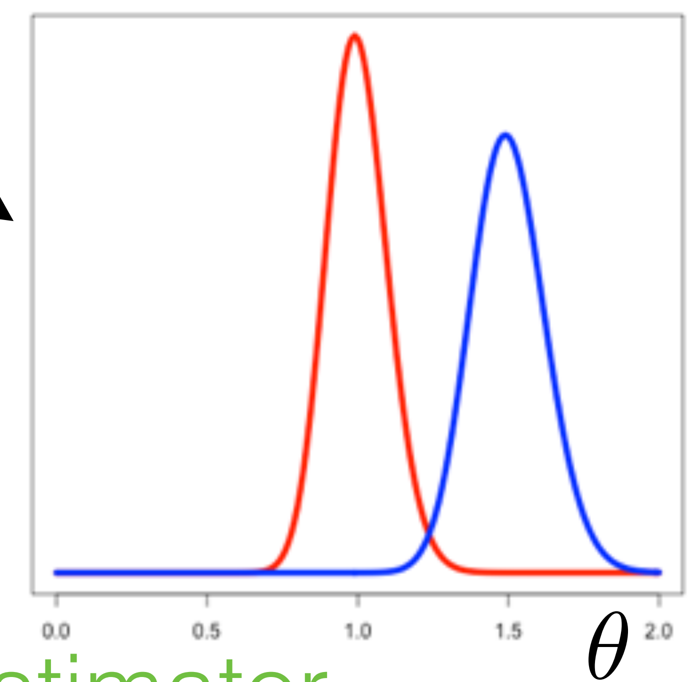

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- Here: $f(\theta,\alpha) = \log p(\theta|\alpha) - \log p(\theta|\alpha_0)$

Microcredit Experiment

- Simplified from Meager (2015)
- K microcredit trials (Mexico, Mongolia, Bosnia, India, Morocco, Philippines, Ethiopia)
- N_k businesses in kth site (~900 to ~17K)
- Profit of nth business at kth site:

$$\underbrace{y_{kn}}_{\text{profit}} \overset{indep}{\sim} \mathcal{N}(\mu_k + \underbrace{T_{kn}}_{\text{1 if microcredit}} \tau_k, \sigma_k^2)$$

- Priors and hyperpriors:

$$\begin{pmatrix} \mu_k \\ \tau_k \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu \\ \tau \end{pmatrix}, C \right) \qquad \begin{pmatrix} \mu \\ \tau \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu_0 \\ \tau_0 \end{pmatrix}, \Lambda^{-1} \right)$$

$$\sigma_k^{-2} \overset{iid}{\sim} \Gamma(a, b) \qquad C \sim \mathrm{Sep\&LKJ}(\eta, c, d)$$

Microcredit Experiment

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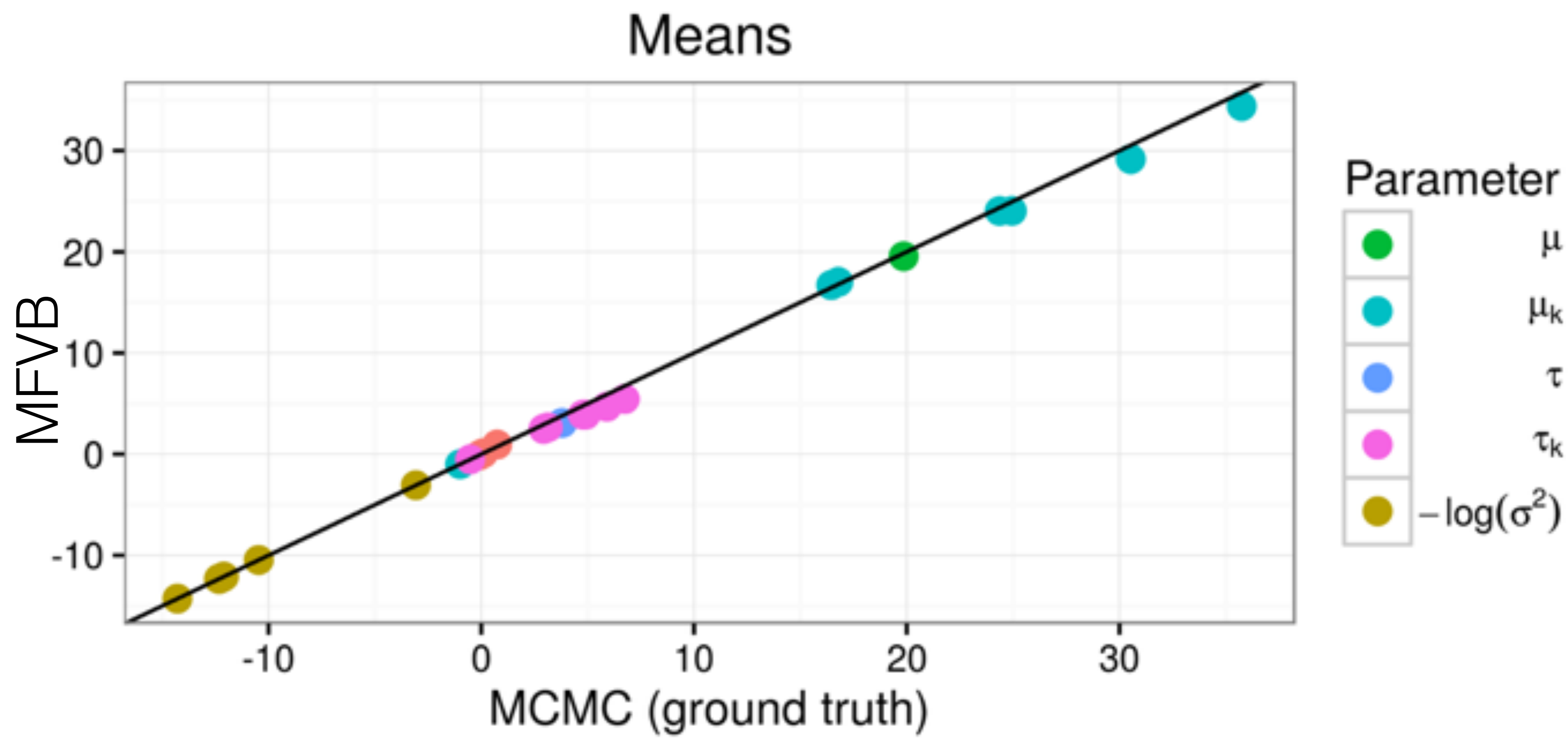
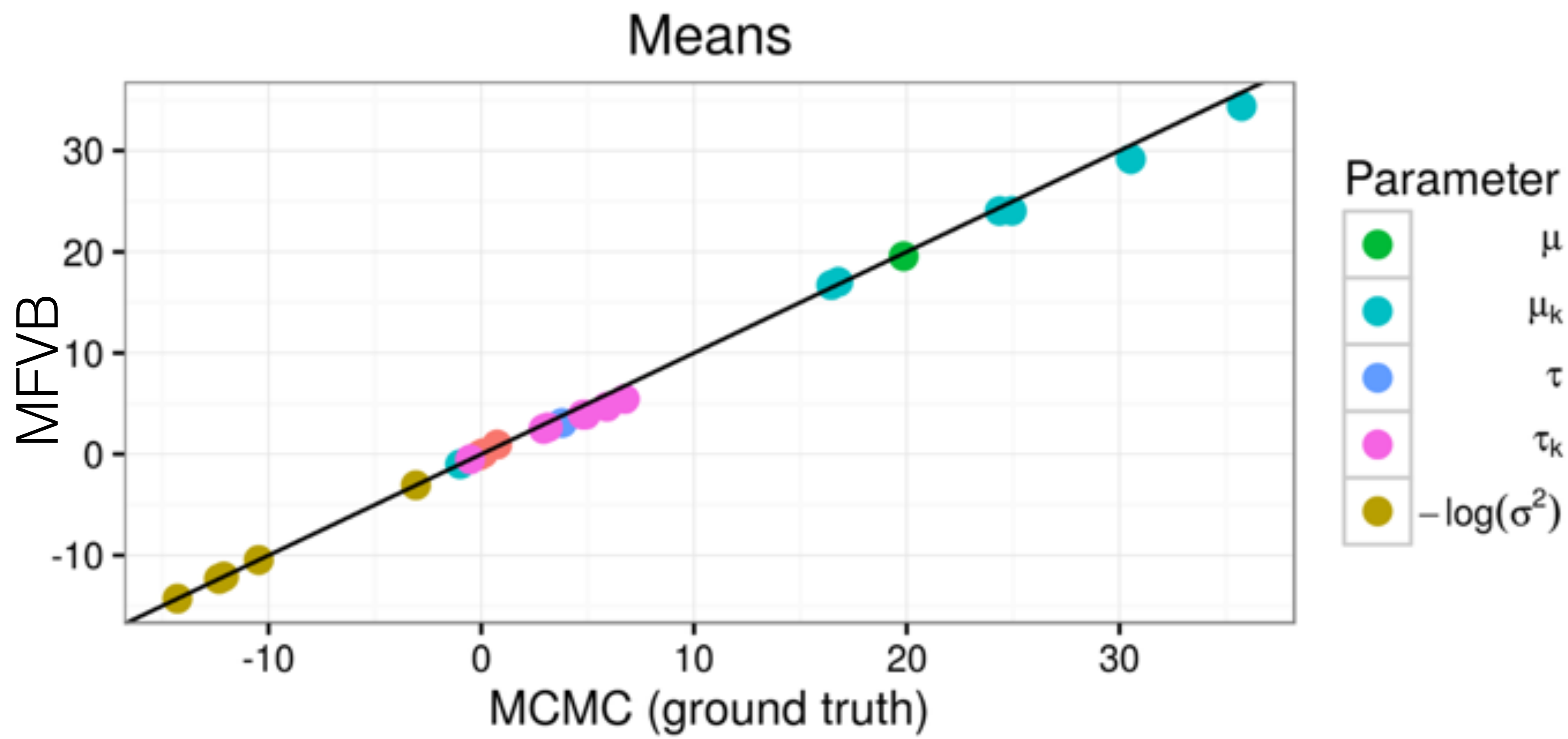


Microcredit Experiment

- Perturb Λ_{11}: 0.03 ➔ 0.04



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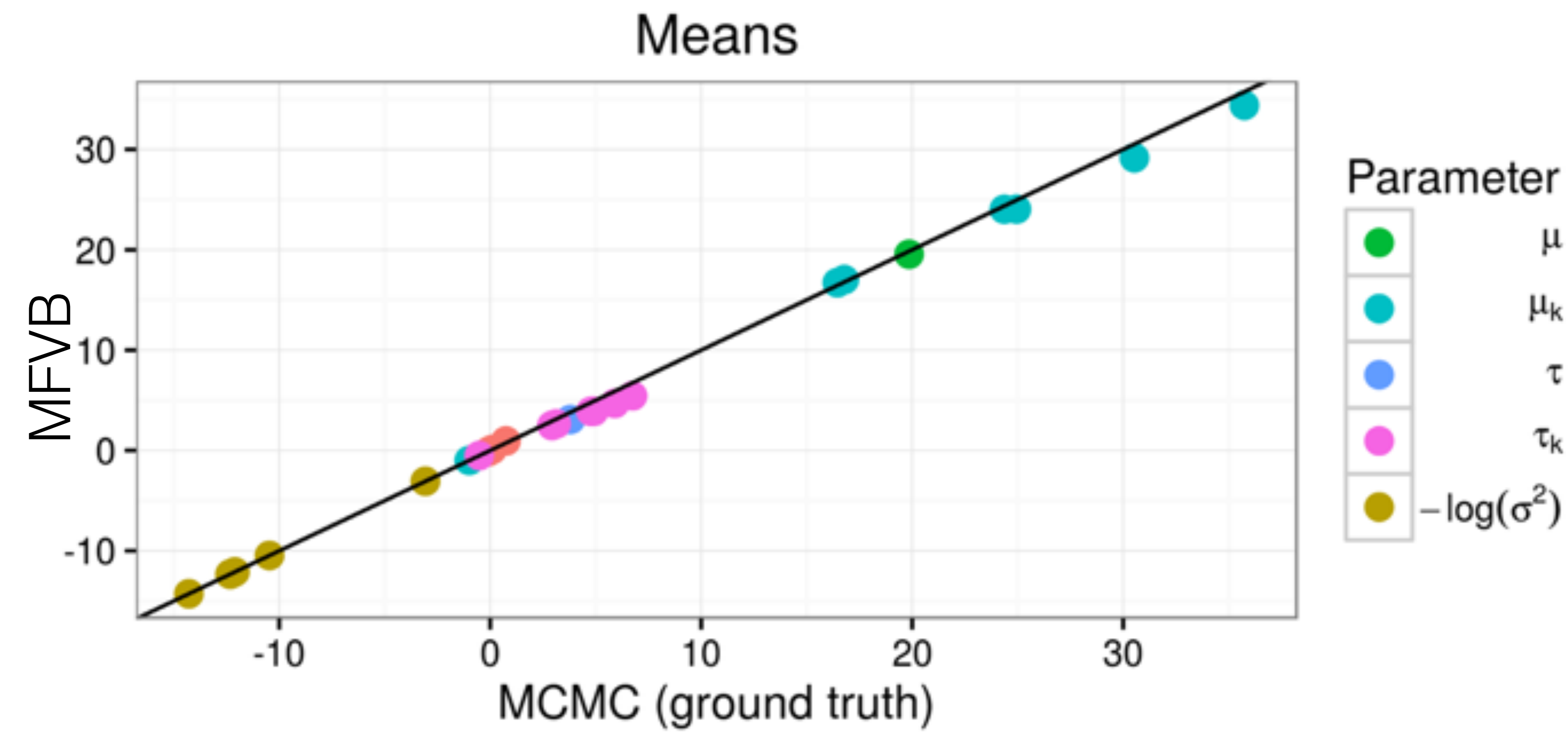

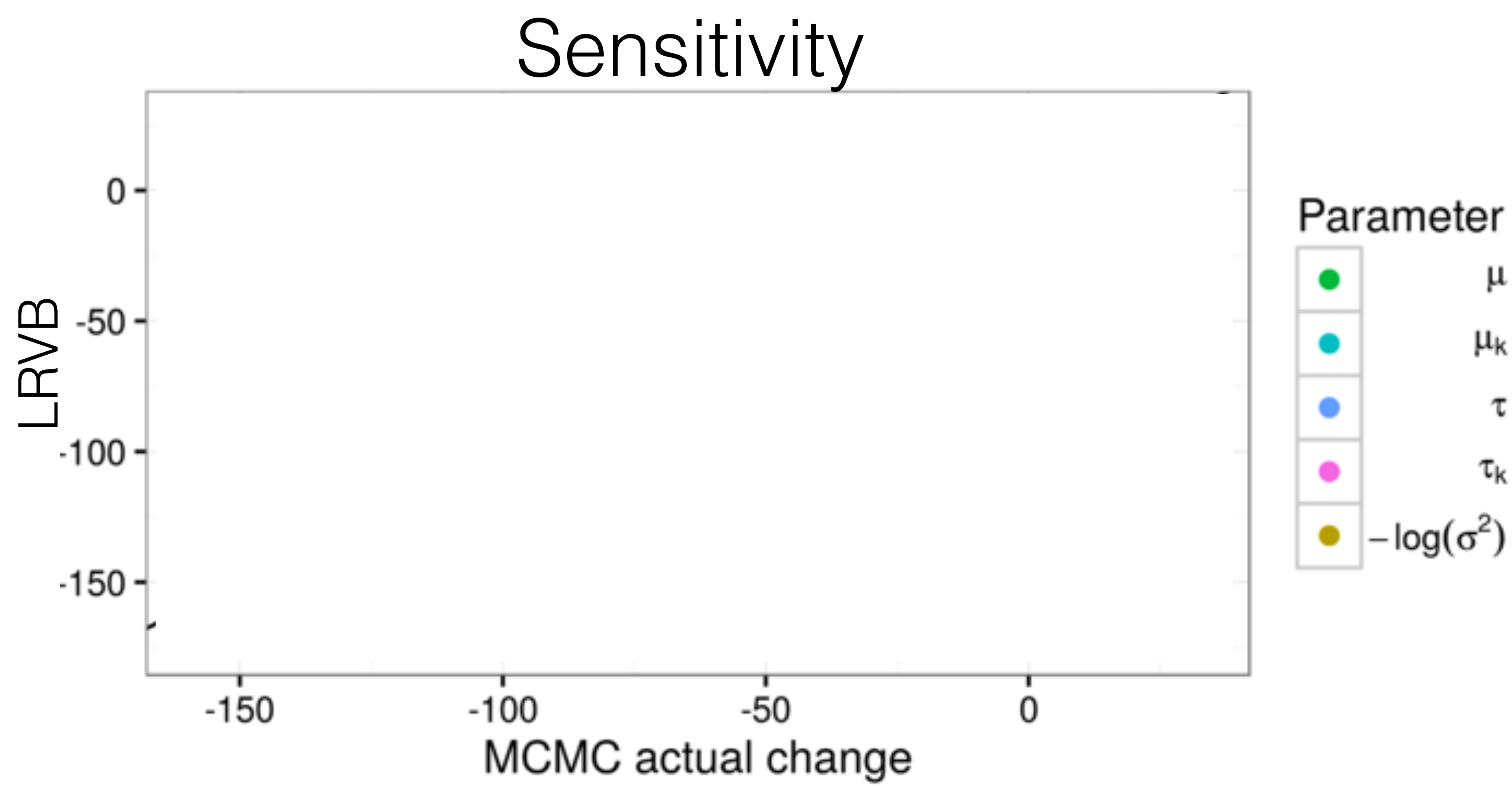

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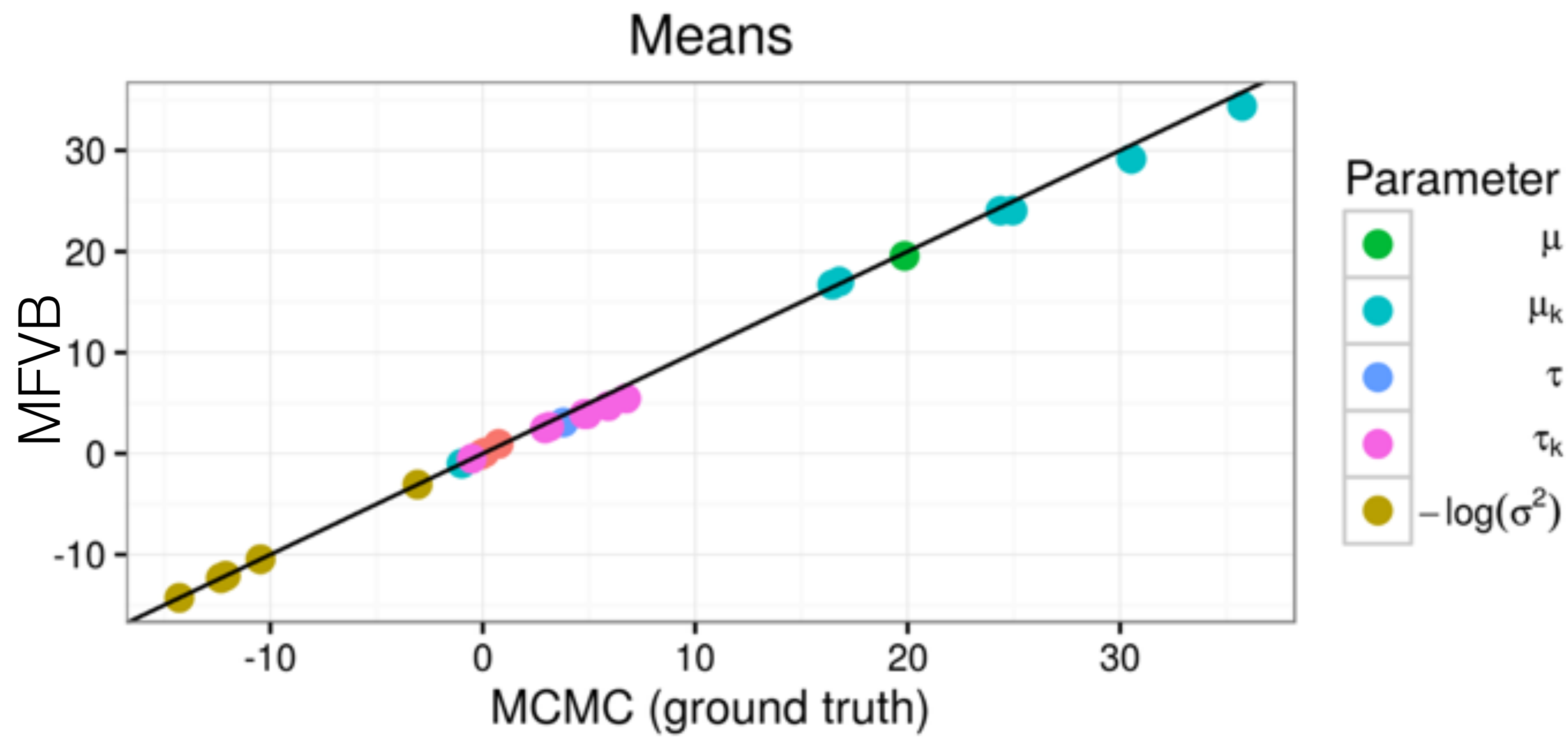

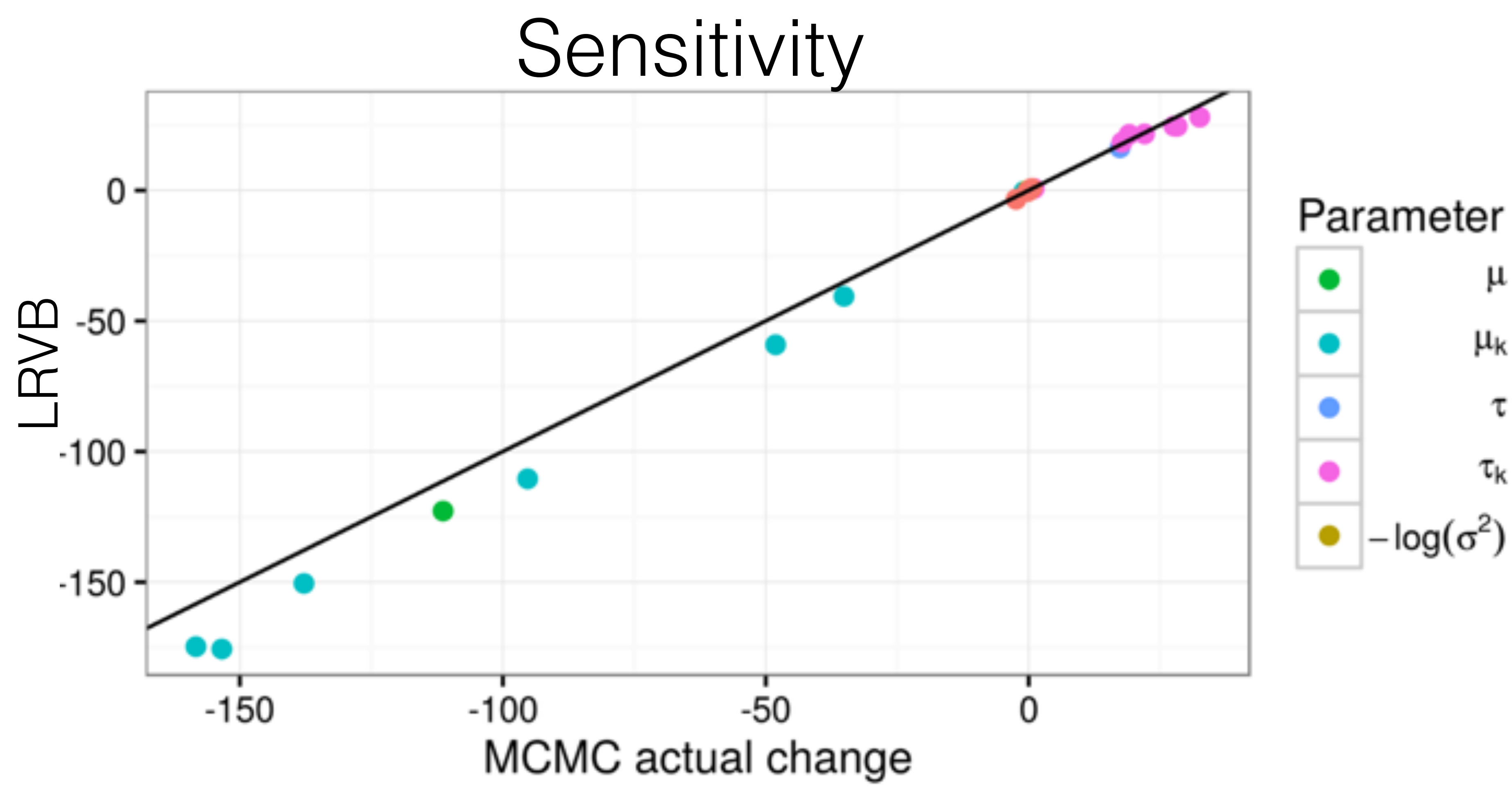

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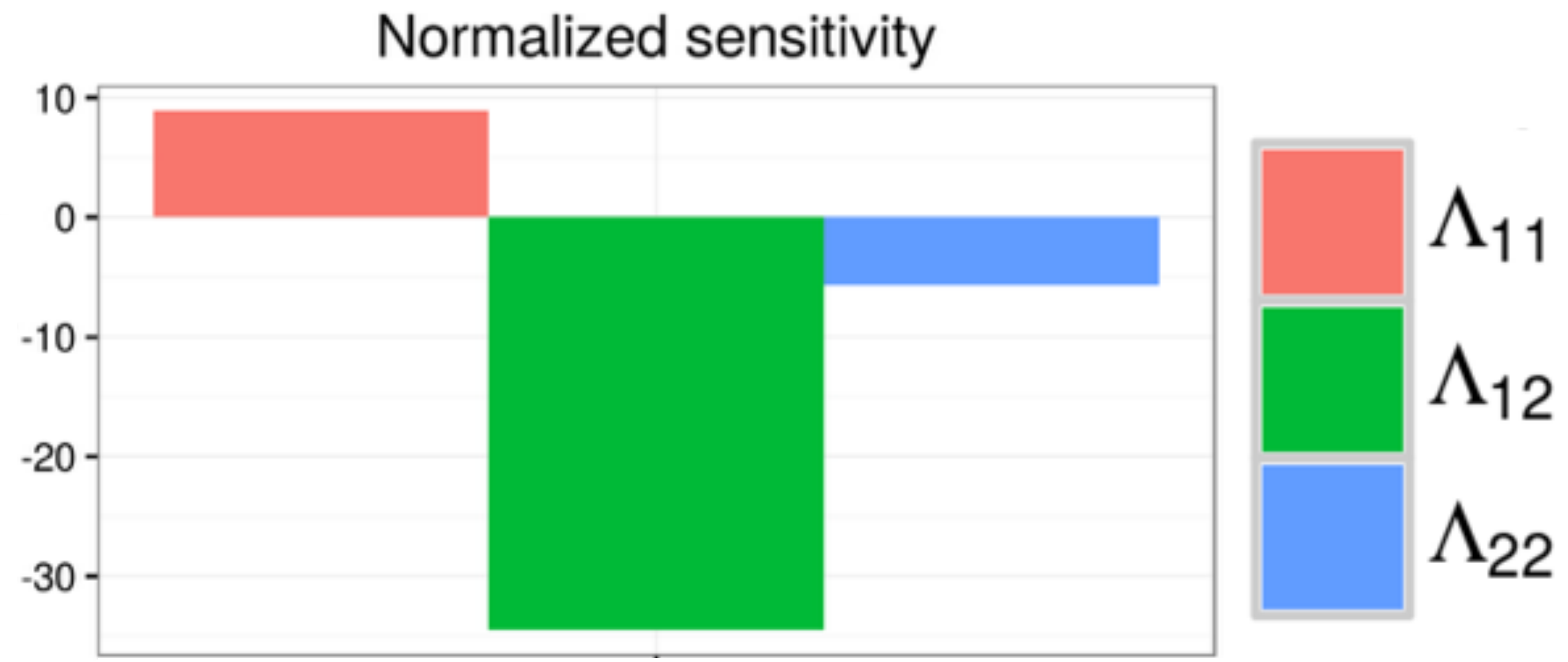

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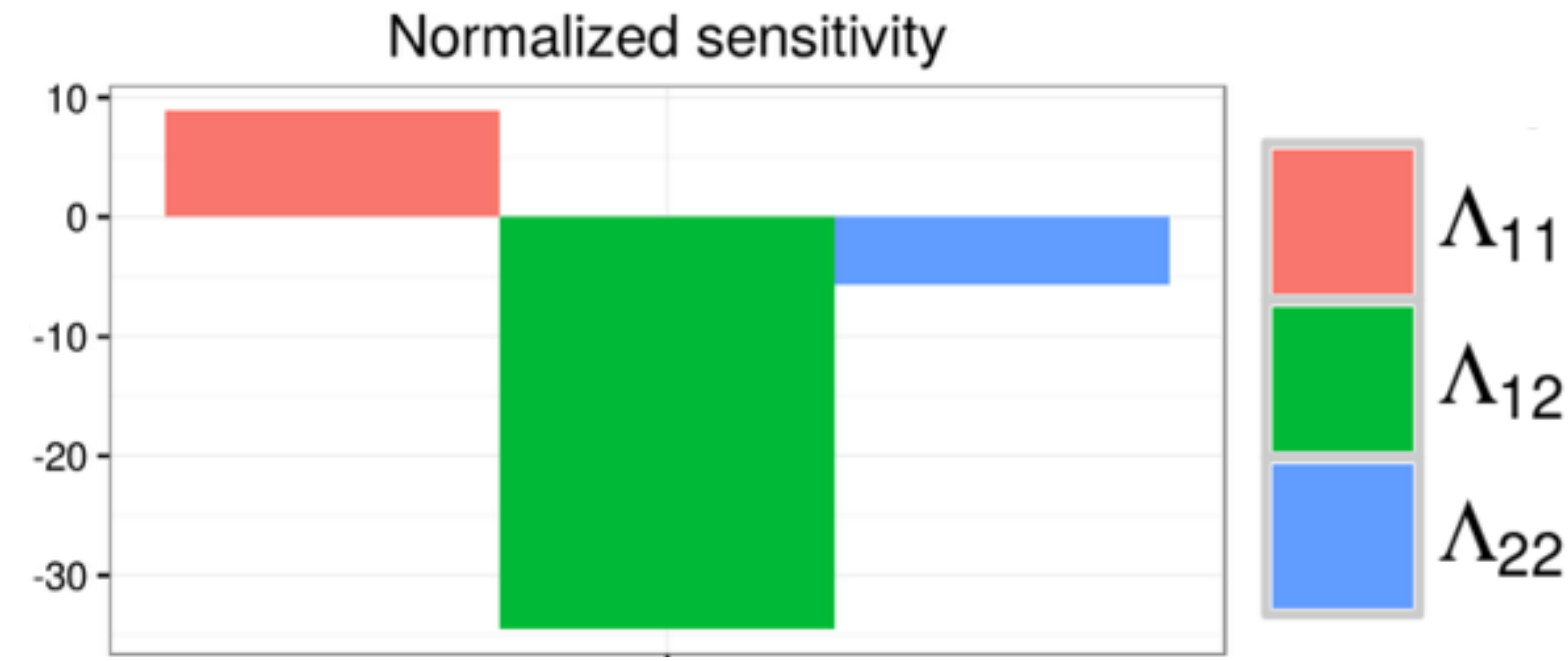

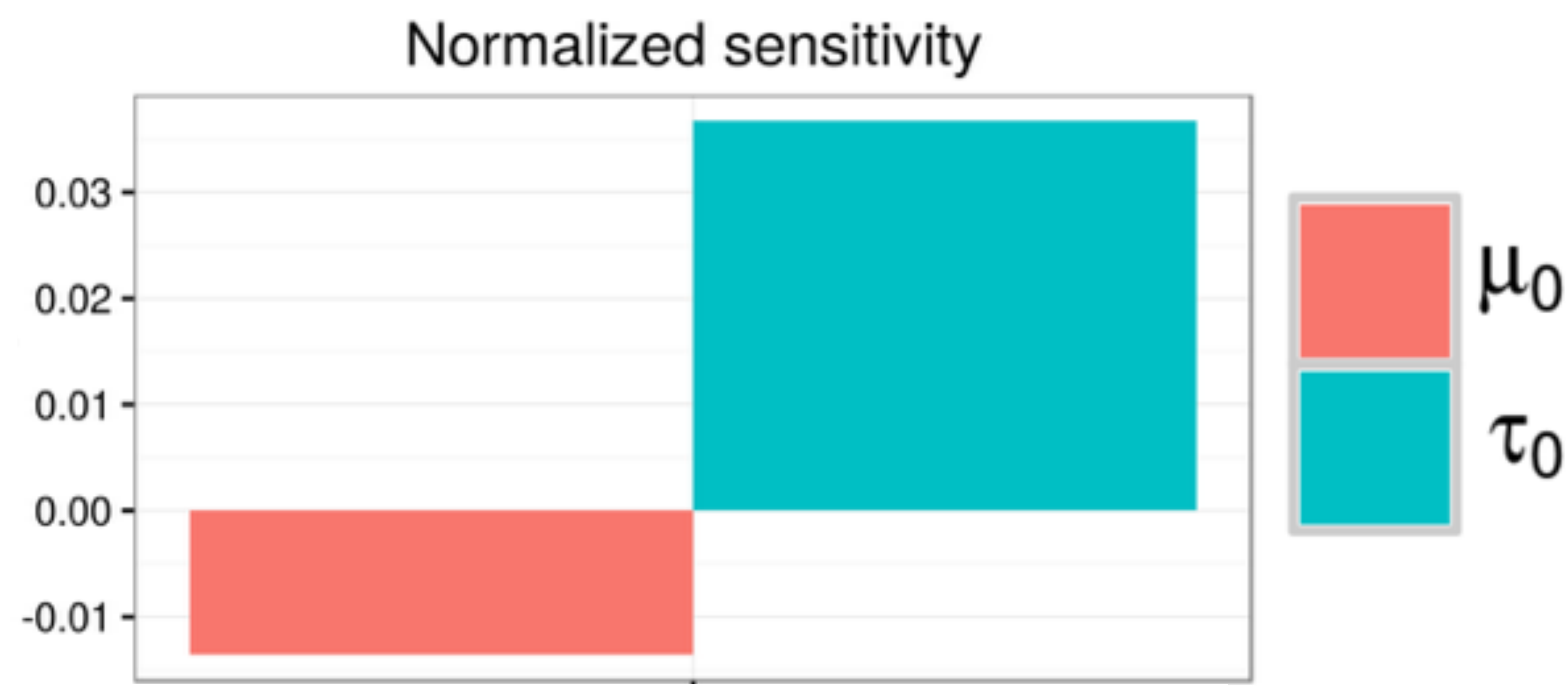

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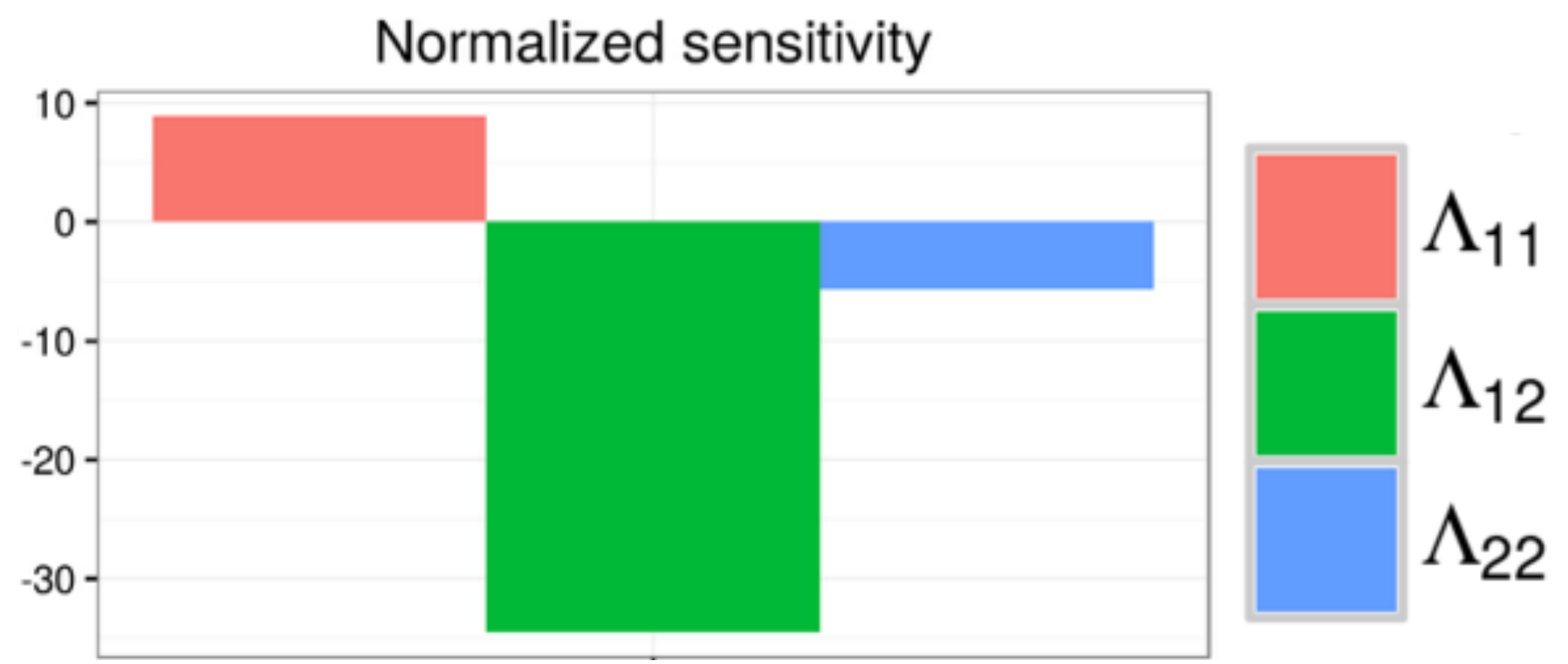

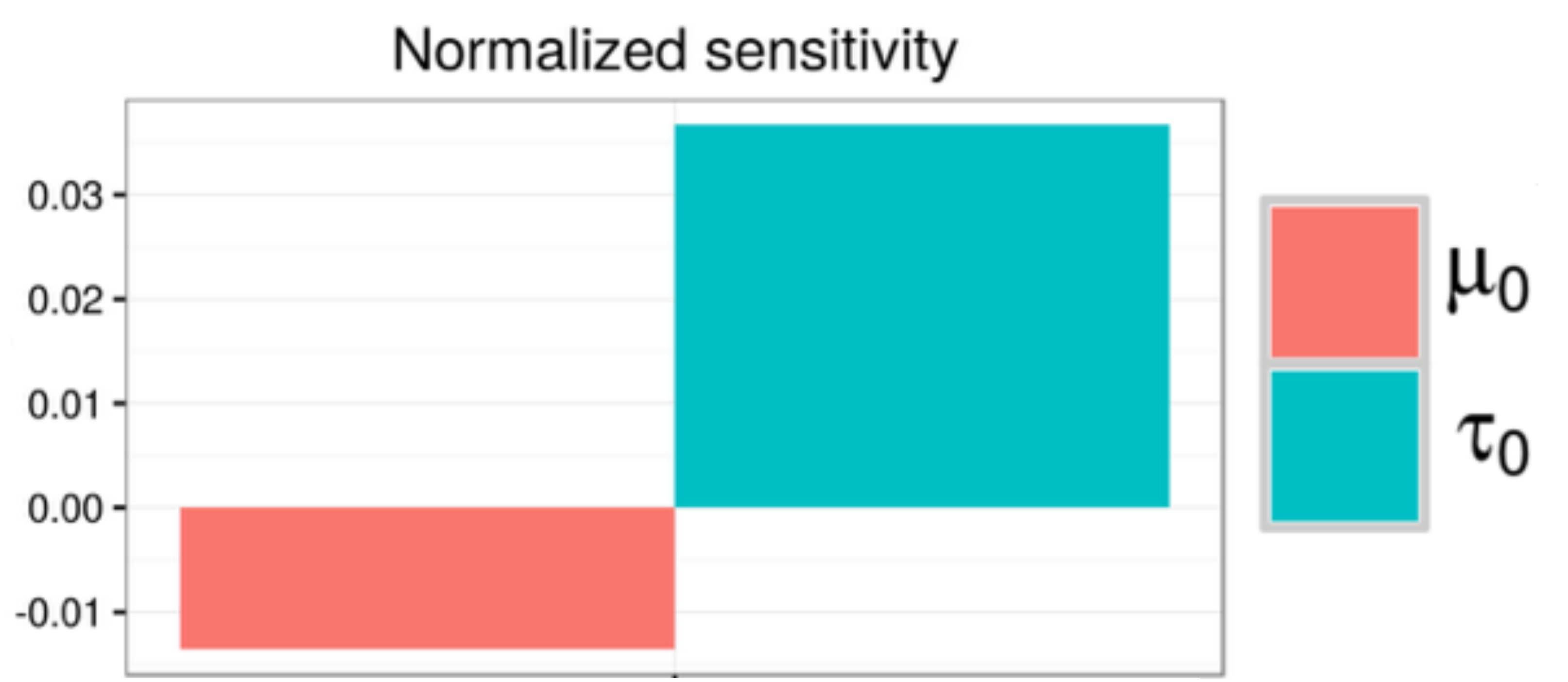

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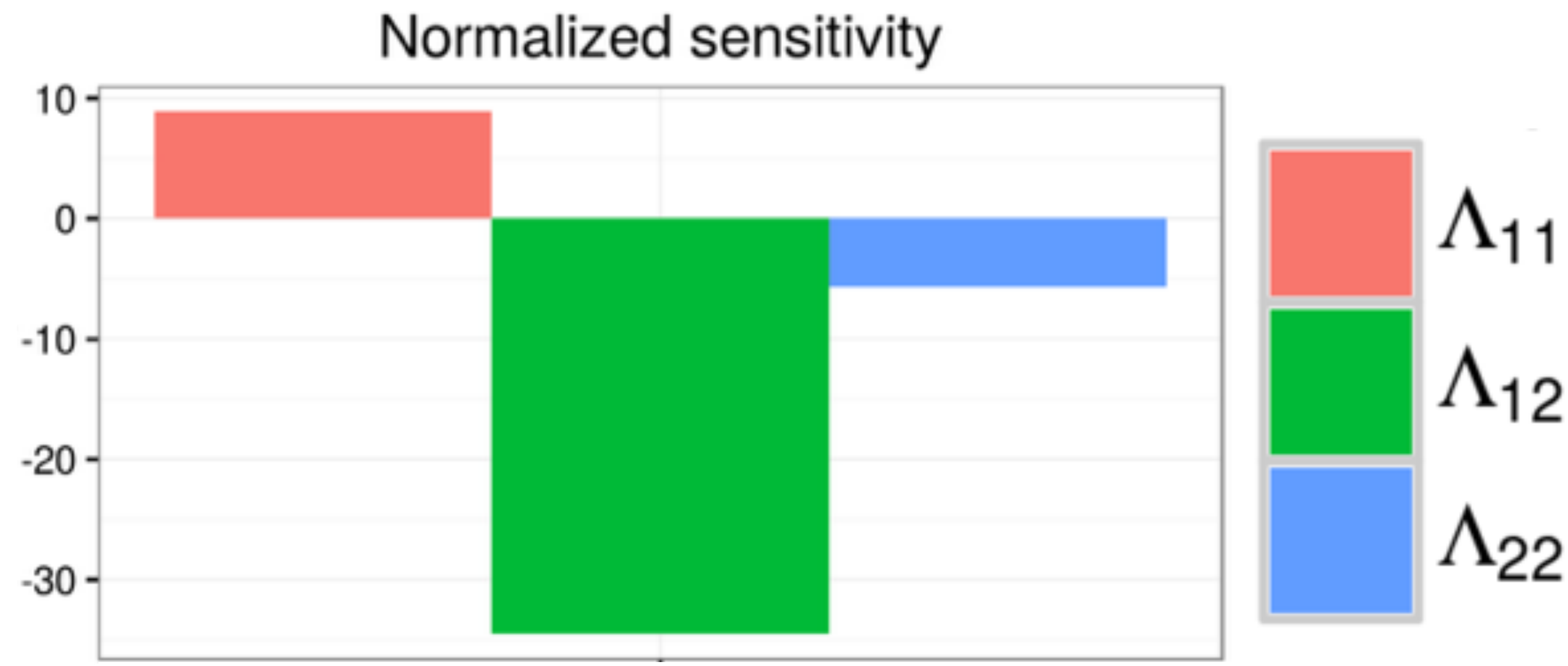

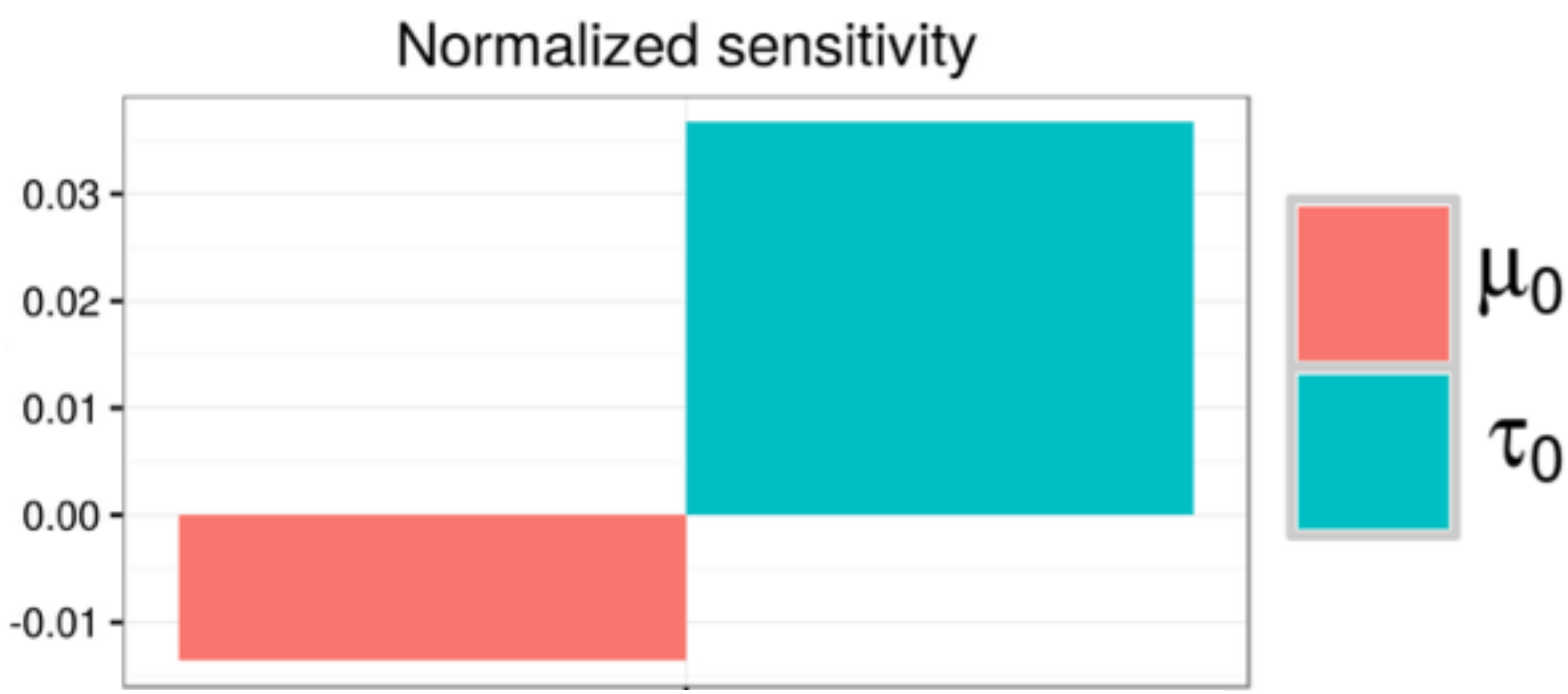

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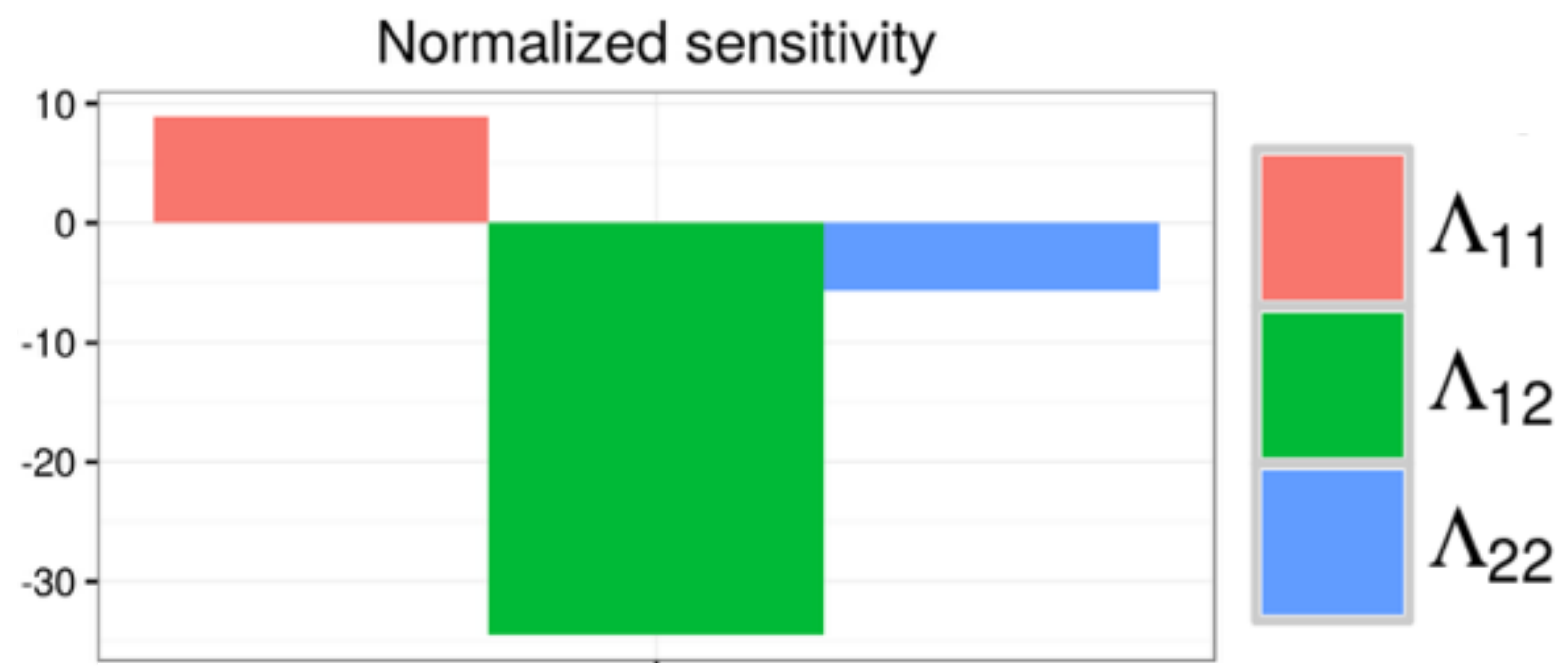
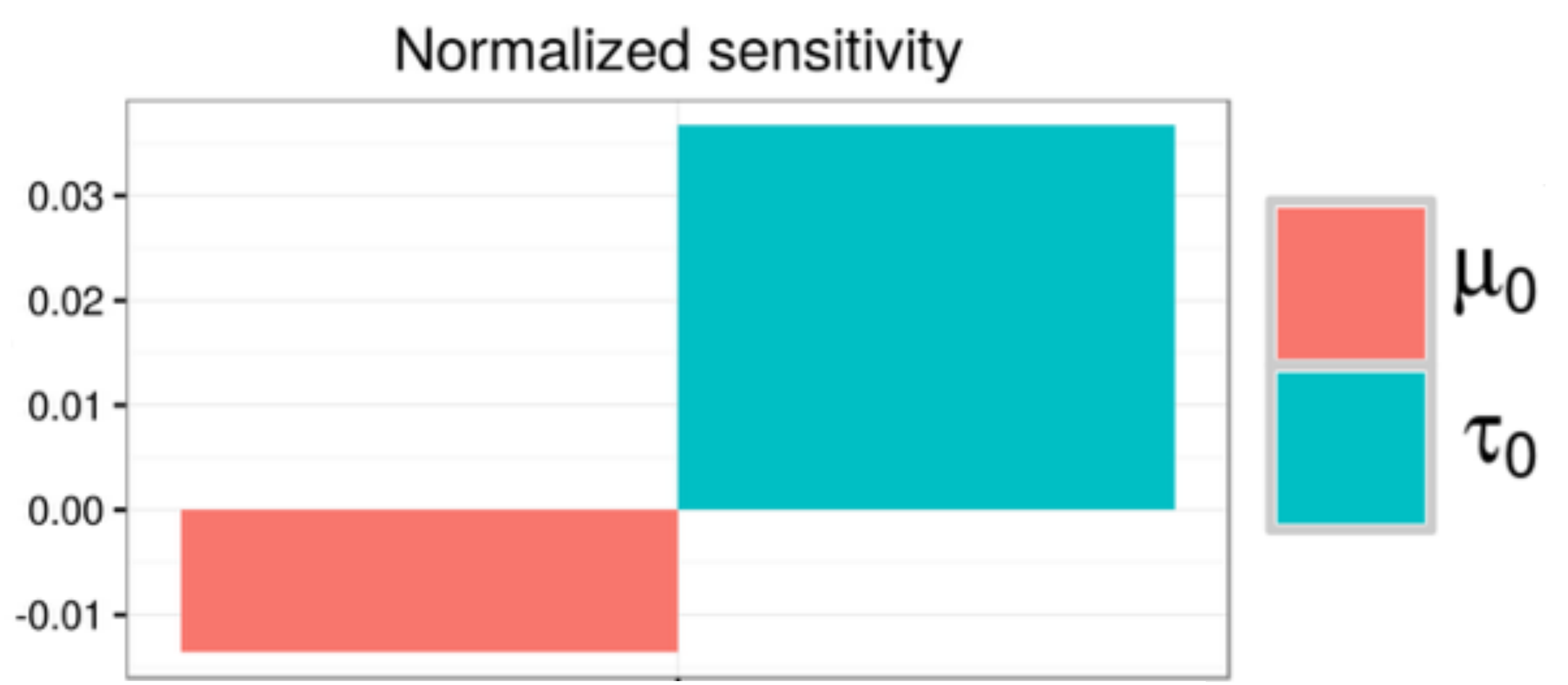
$\Rightarrow$ Mean > 2 std dev





Criteo Online Ads Experiment

- Click-through conversion prediction
- Q: Will a customer (e.g.) buy a product after clicking?
- Q: How predictive of conversion are different features?
- Logistic GLMM; *N* = 61,895 subset to compare to MCMC
- Model:

$$y_{kn} \sim \mathrm{Bernoulli}(p_{kn}) \qquad p_{kn} = \frac{\exp(\rho_{kn})}{1+\exp(\rho_{kn})}$$

$$\rho_{kn} = x_{kn}^T \beta + u_k$$

  - Priors and hyperpriors:

$$u_k \sim \mathcal{N}(\mu, \sigma^2) \qquad \beta \sim \mathcal{N}(\beta_0, \mathrm{diag}(\gamma))$$

$$\mu \sim \mathcal{N}(\mu_0, \sigma_0^2)$$

$$(\sigma^2)^{-1} \sim \mathrm{Gamma}(a, b)$$

Criteo Online Ads Experiment

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- VB: 57 sec

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- VB: 57 sec;
  VB + LRVB:
  553 sec
  (**9.2 min**)

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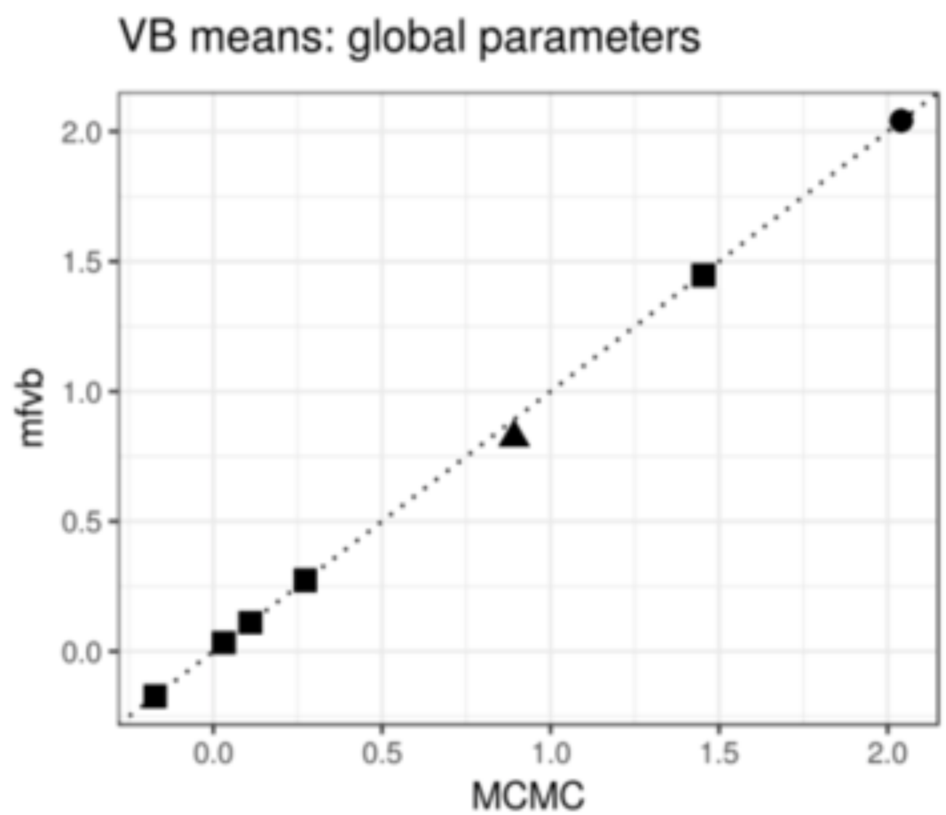

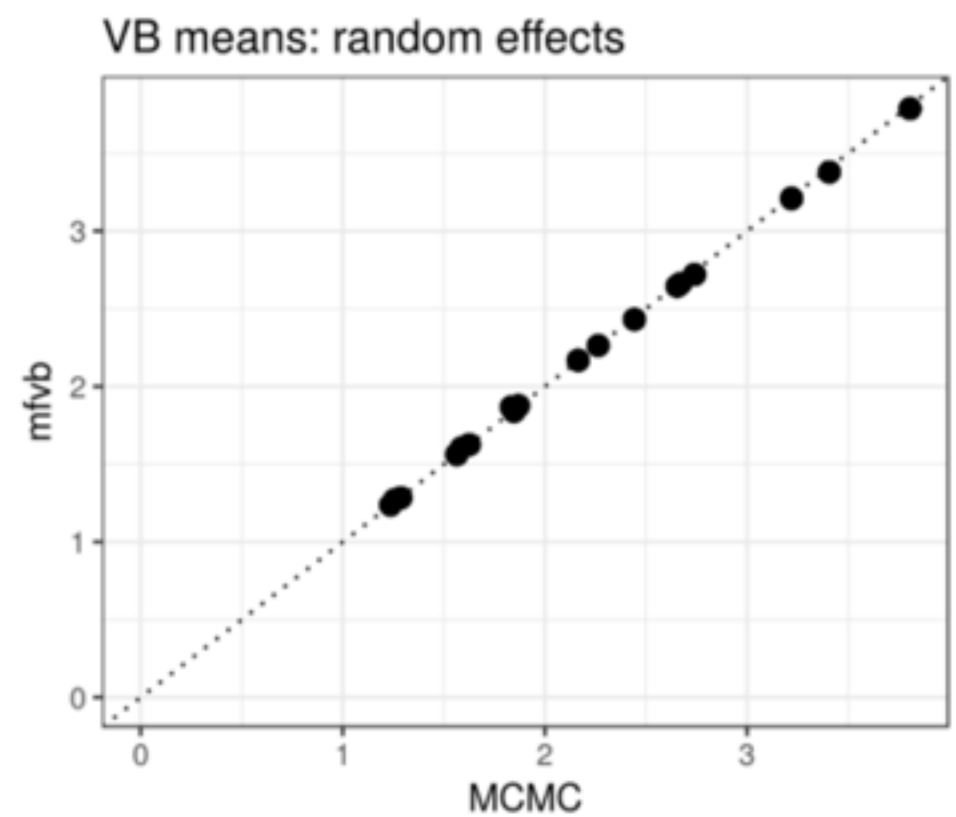

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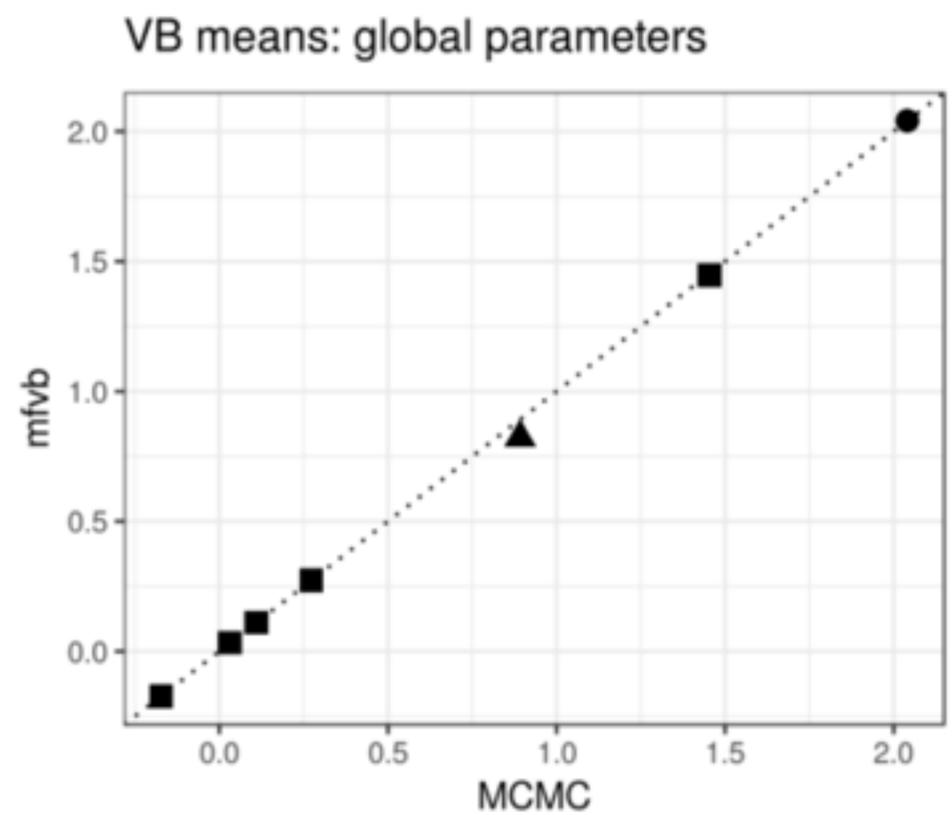

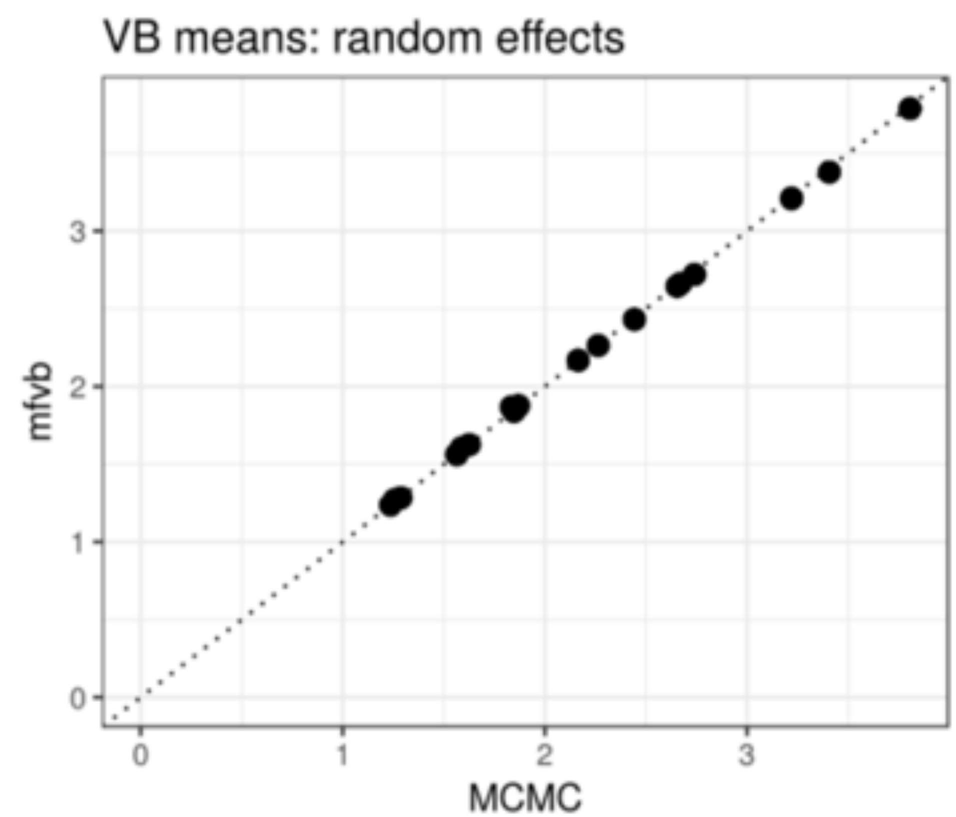

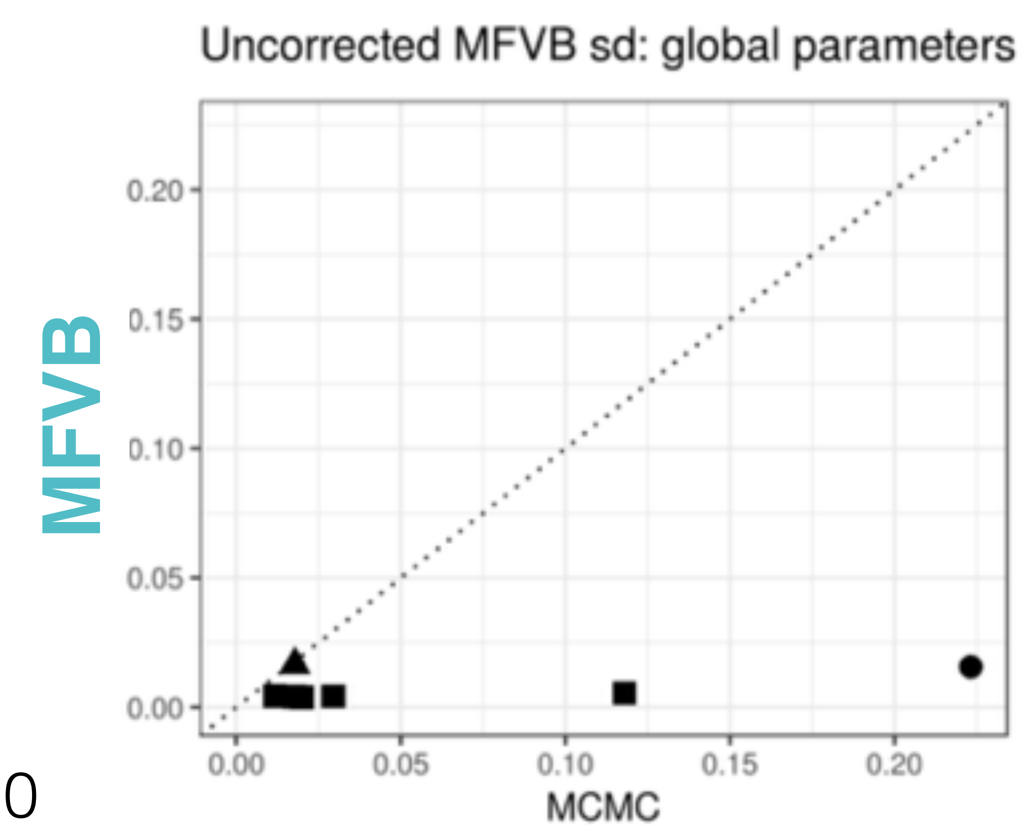

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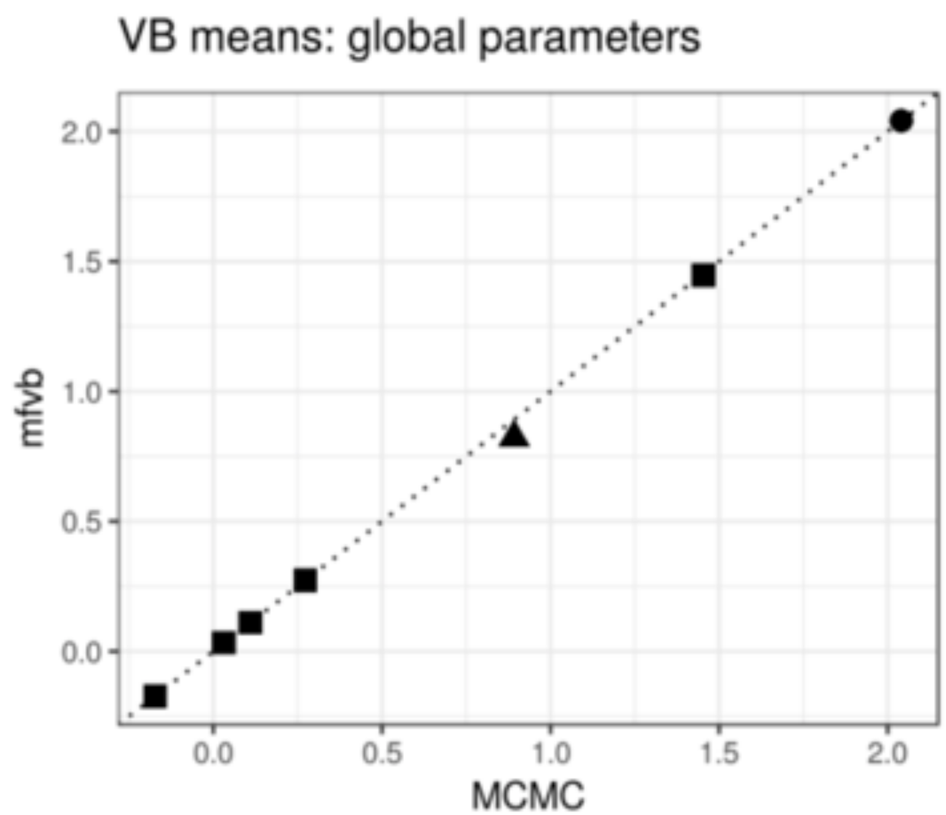

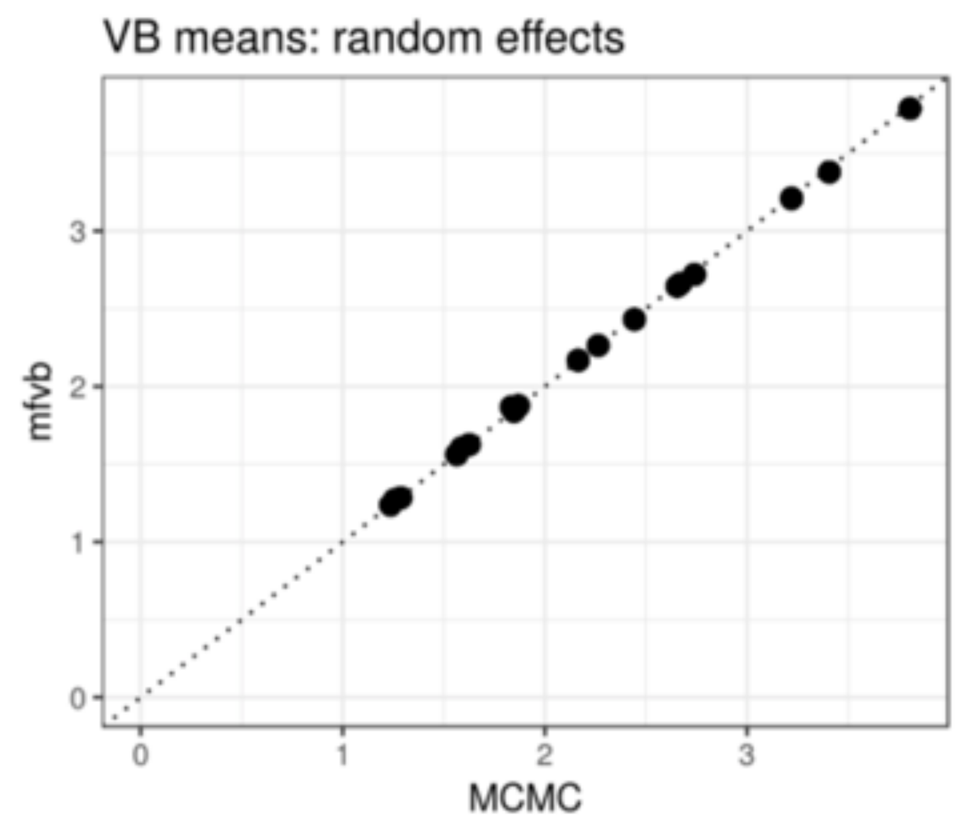

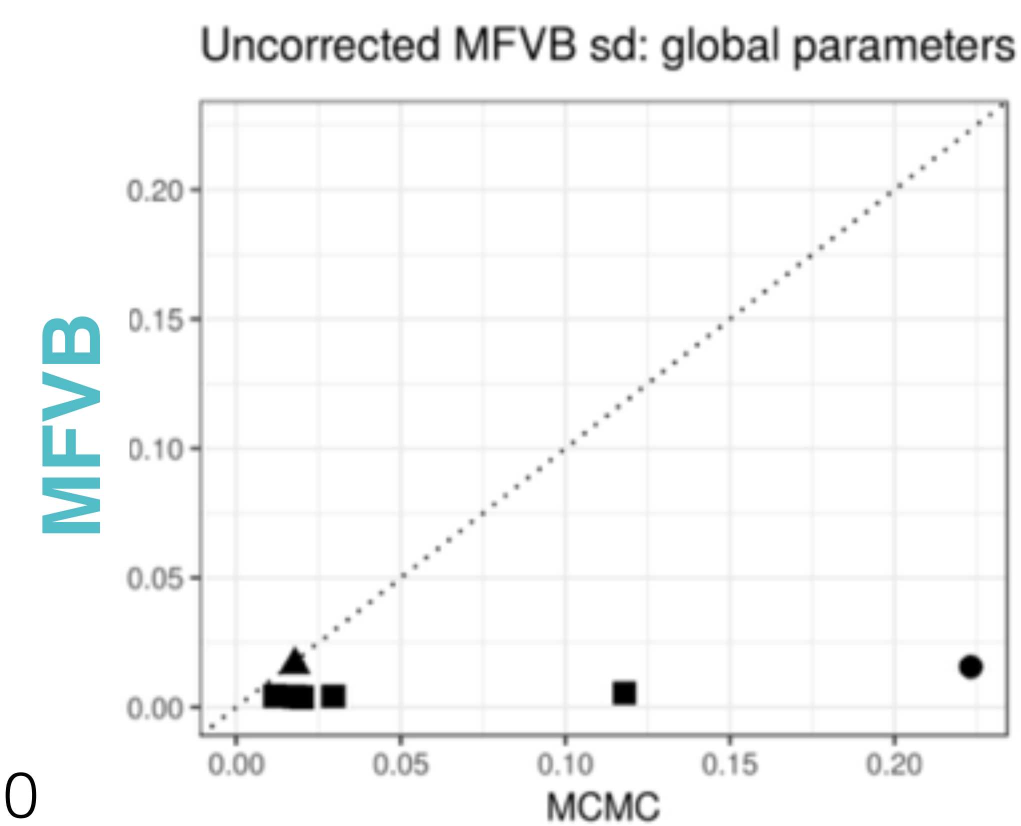

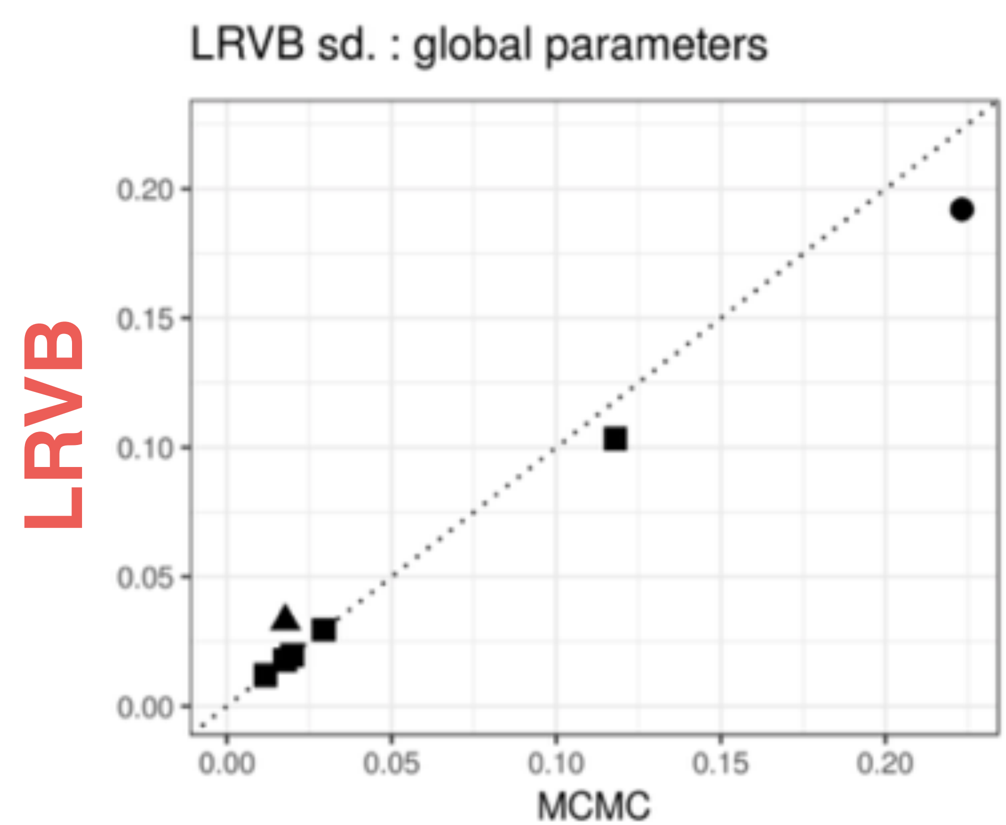

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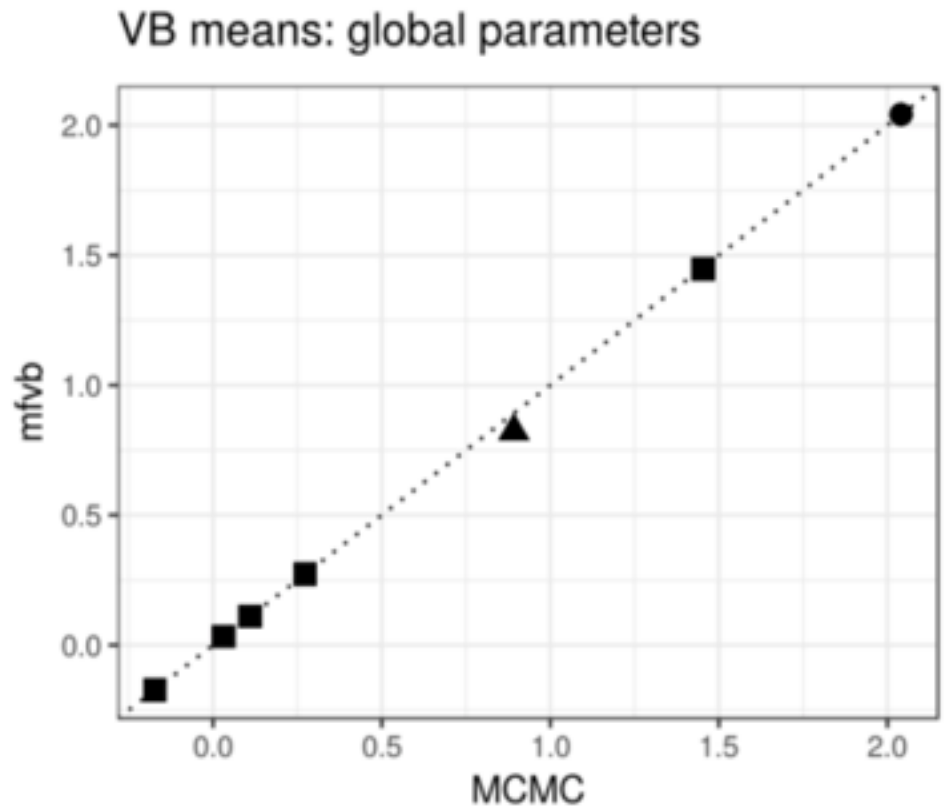

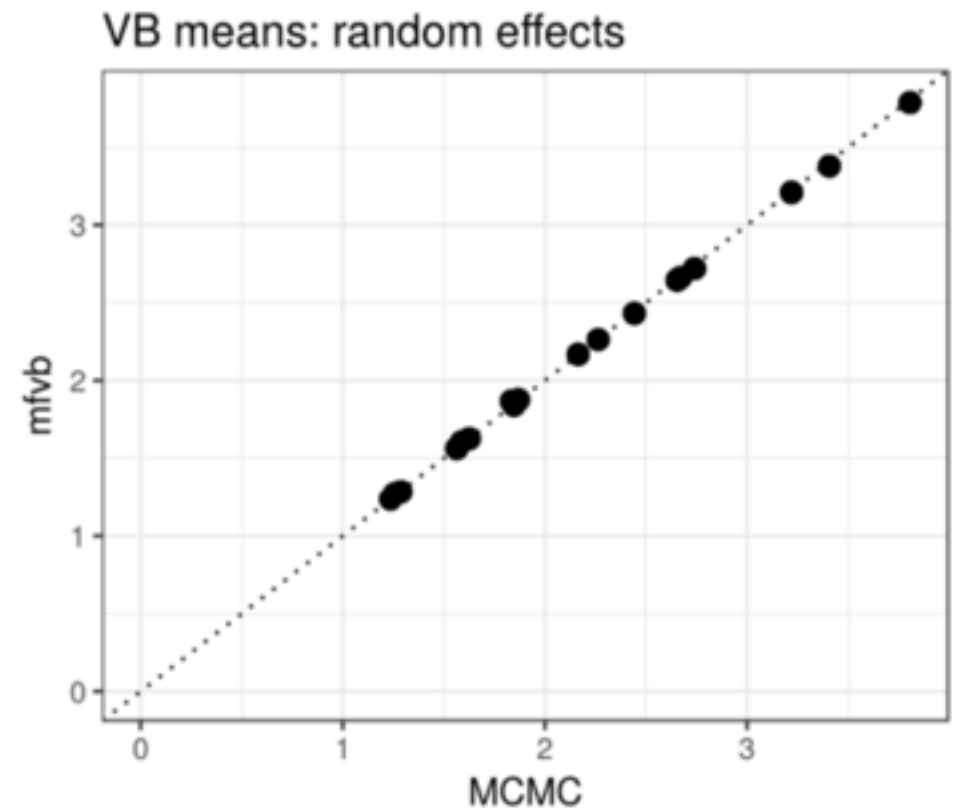

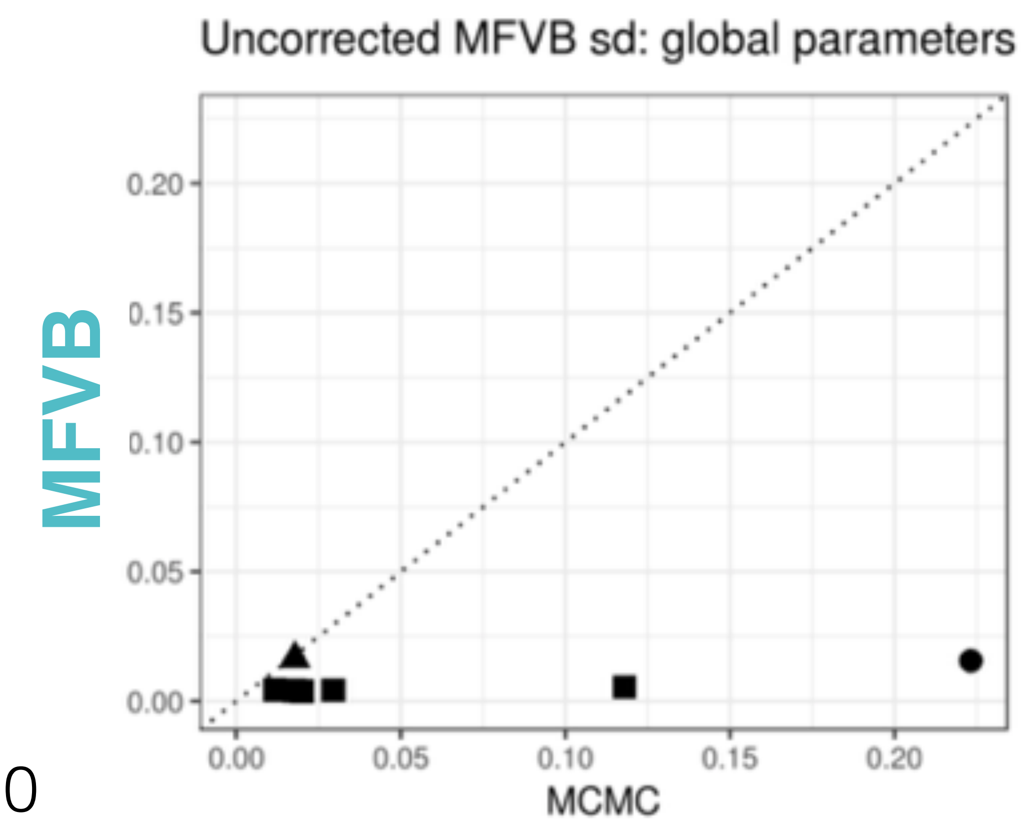

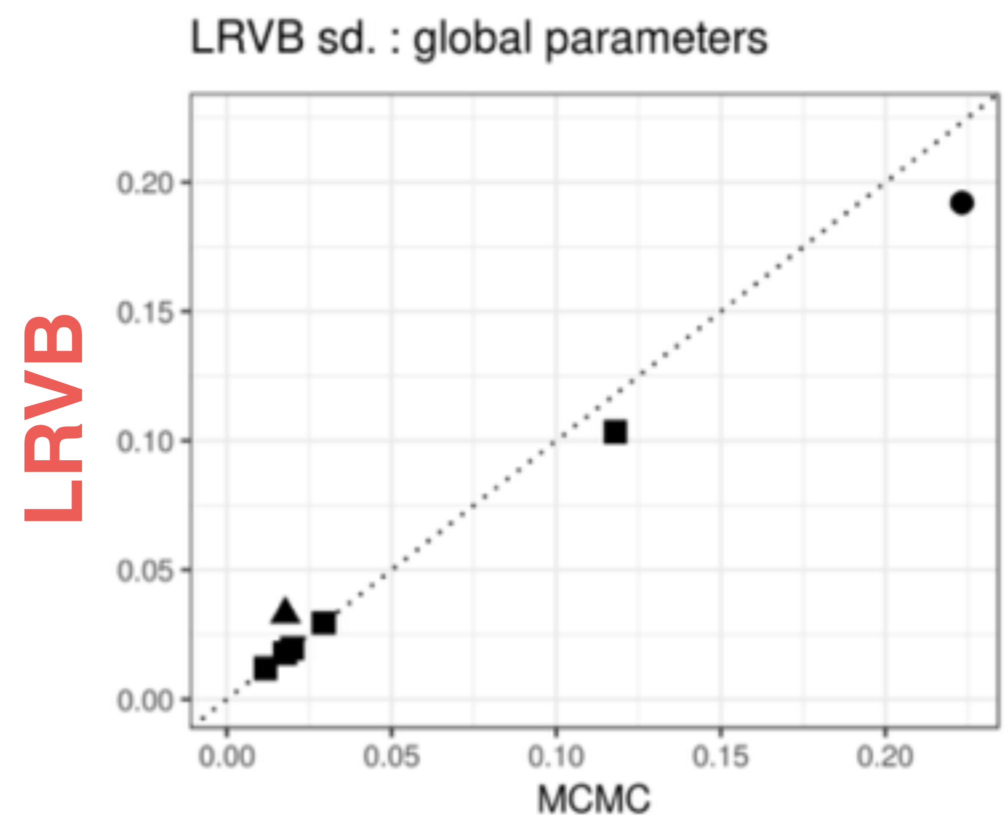

Also good random effects sd and covariances

Criteo Online Ads Experiment

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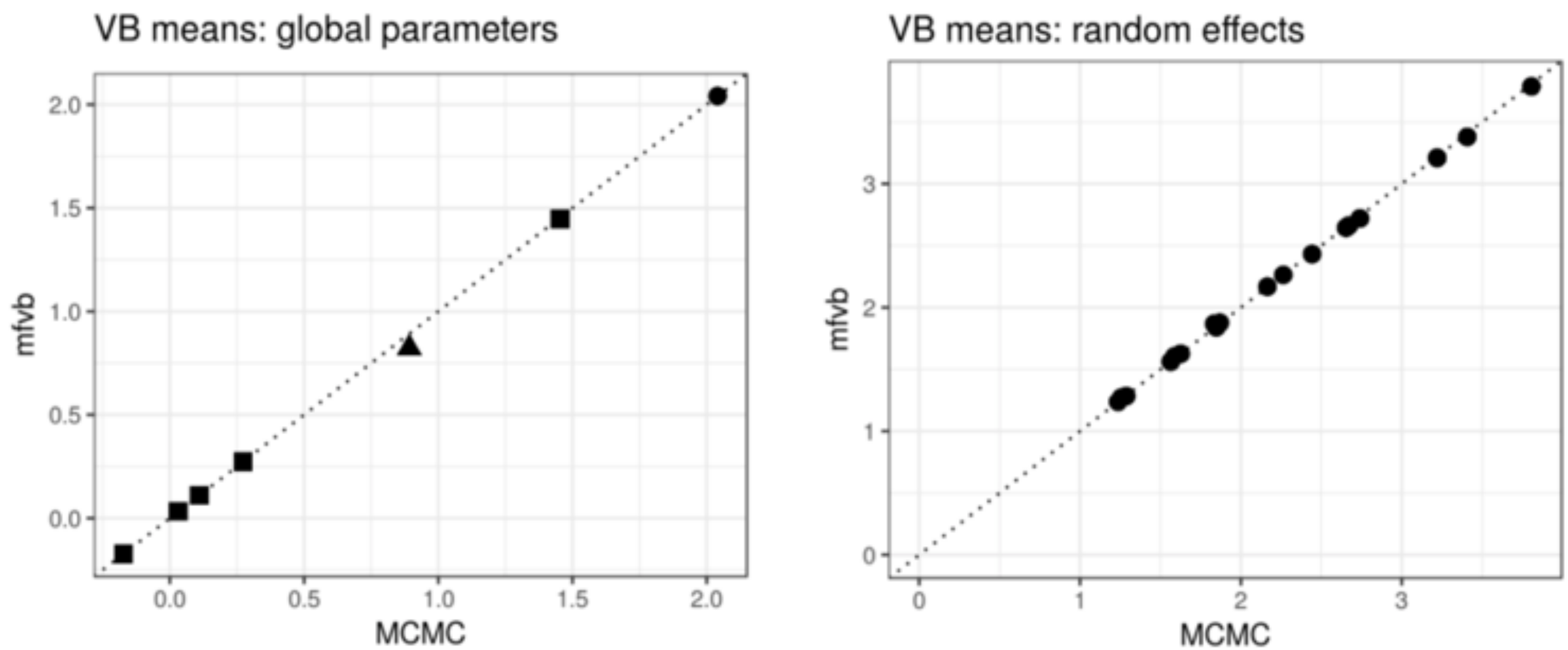
- MCMC (5k samples): 21,066 sec (**5.85 h**)



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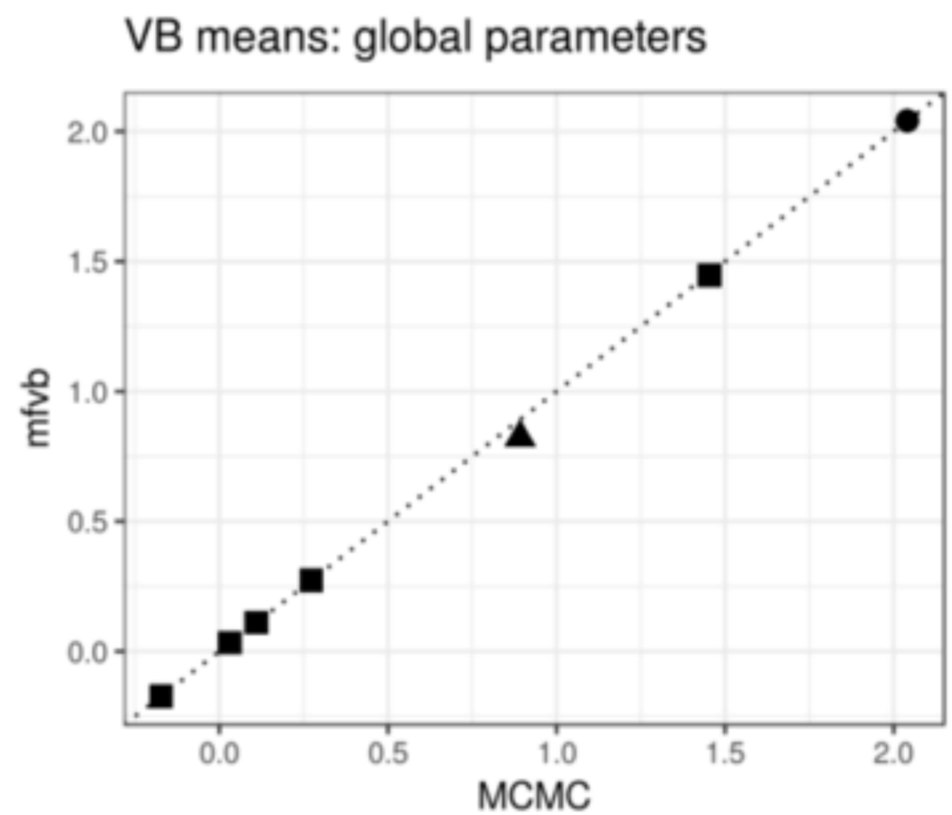

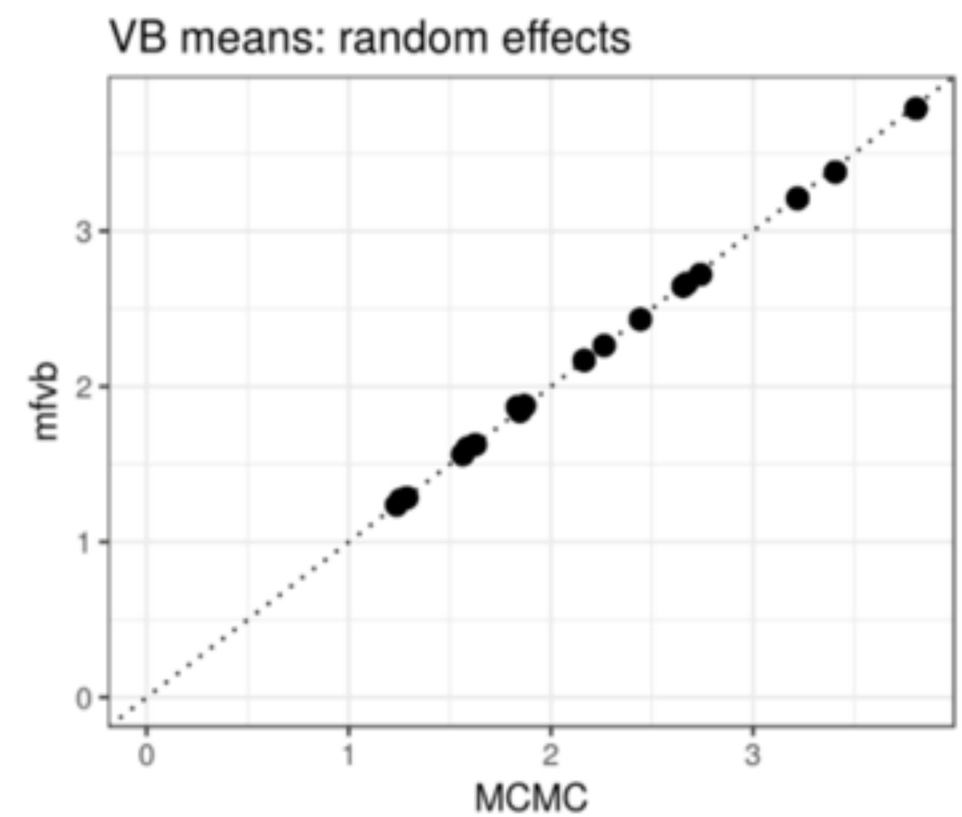

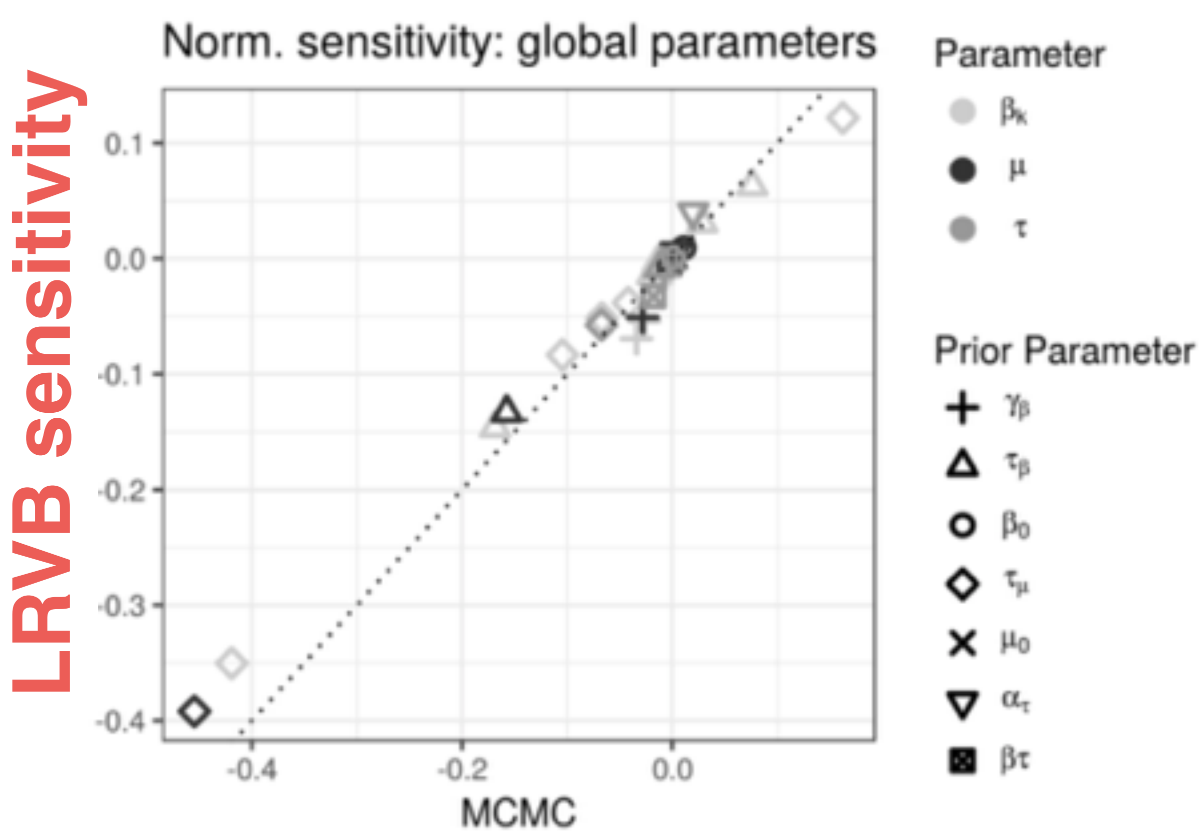

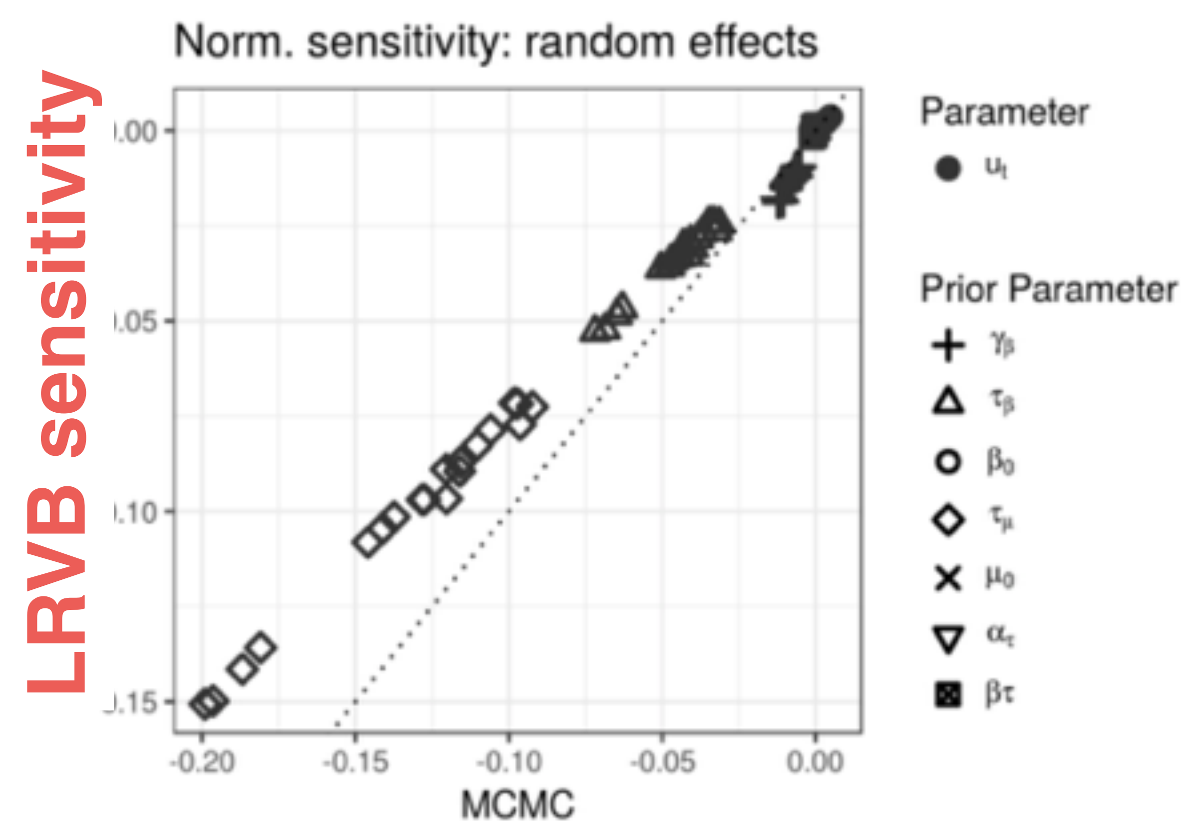

Computational complexity

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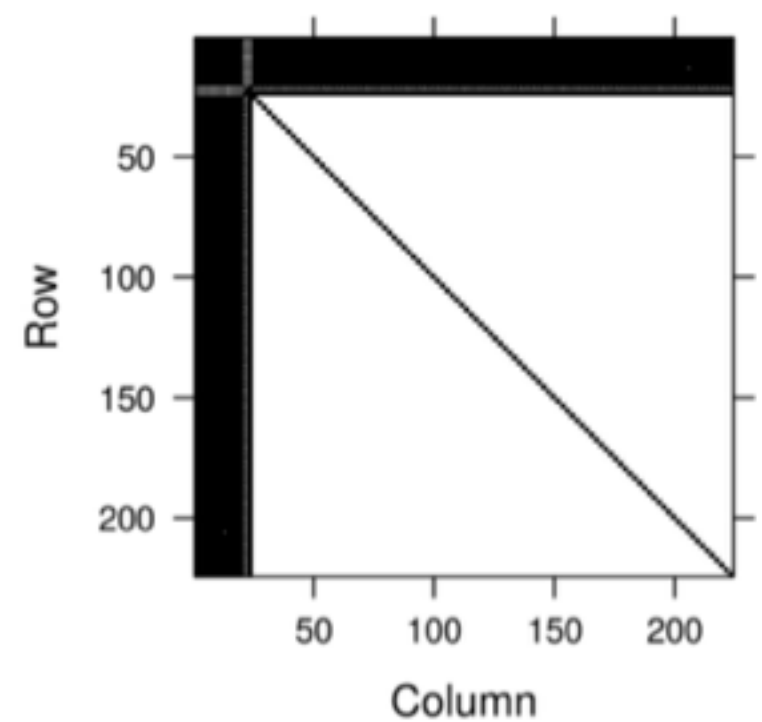
- Top left submatrix for Criteo analysis



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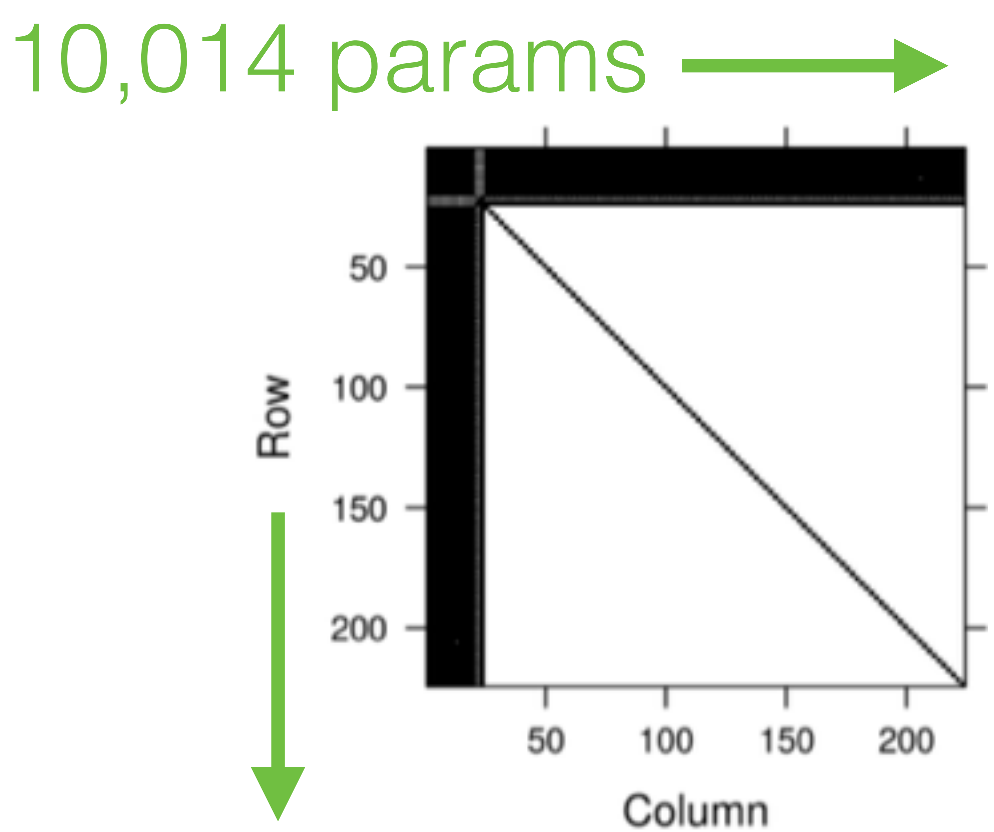

Posterior means: revisited

- Want to predict college GPA y_n

[Giordano, Broderick, Jordan 2015]

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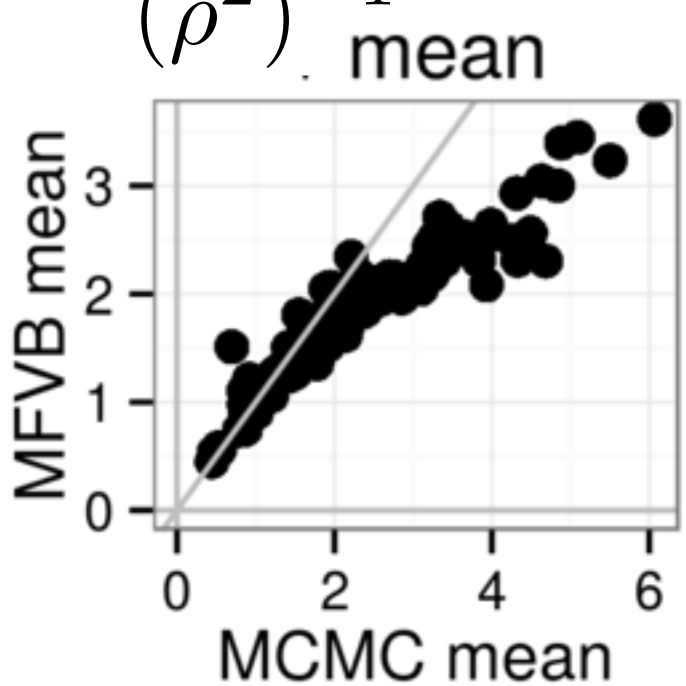
$(\rho^2)^{-1}$



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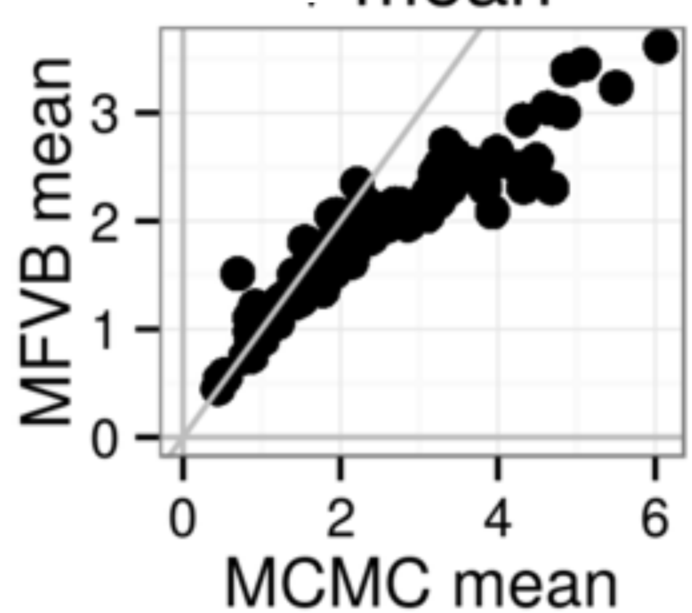
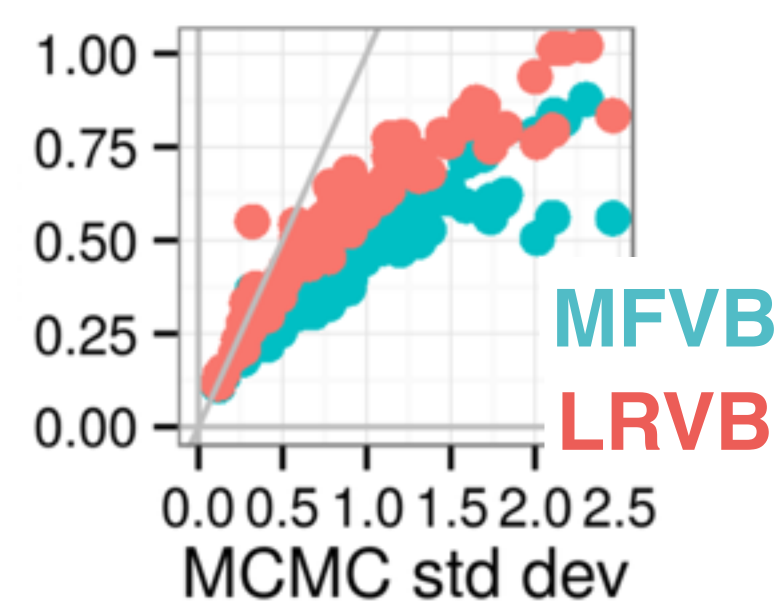


$(\rho^2)^{-1}$ std dev



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Conclusions

- *Linear response variational Bayes (LRVB)*: supplements VB for:
    - Fast **covariance** estimates
    - Fast **robustness** quantification
        - Priors, likelihood, data

[Giordano, Broderick, Jordan 2015, 2017]

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Conclusions

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  - Fast **robustness** quantification
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- Data summarization for scalability (Part IV)

[Giordano, Broderick, Jordan 2015, 2017]

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