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CSAIL

Variational Bayes and beyond: Foundations of scalable Bayesian inference

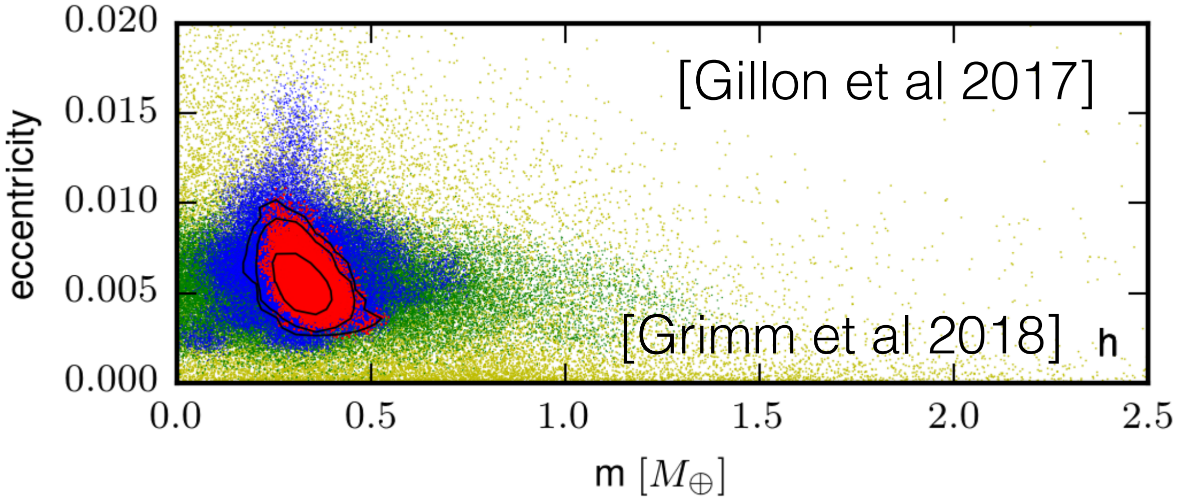
Tamara Broderick

Associate Professor
MIT

http://www.tamarabroderick.com/tutorials.html

Bayesian inference

Bayesian inference



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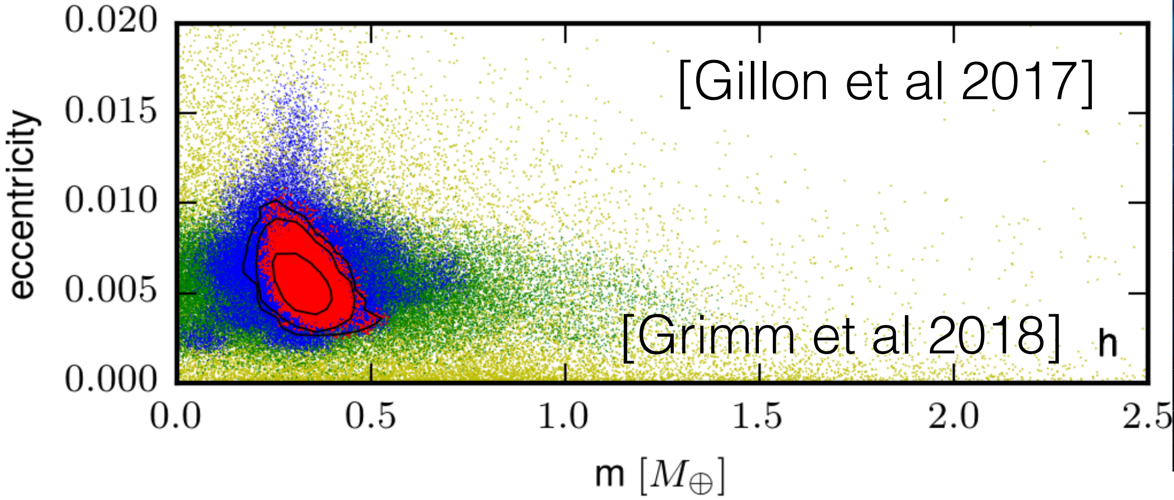

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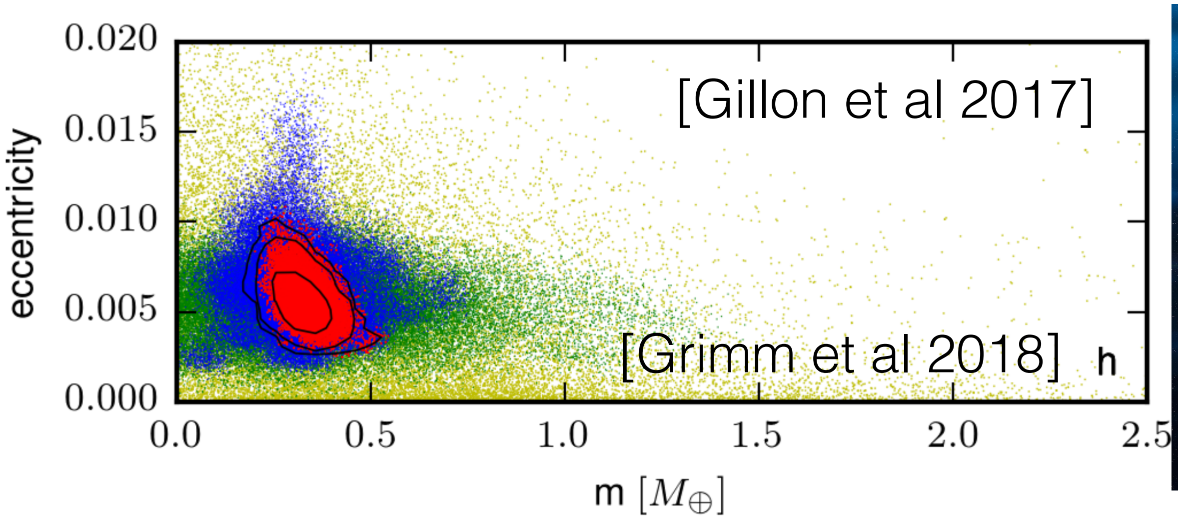

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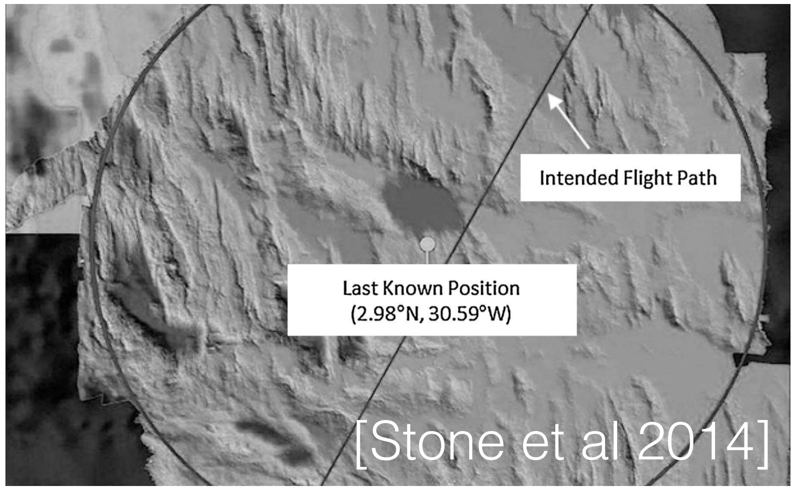

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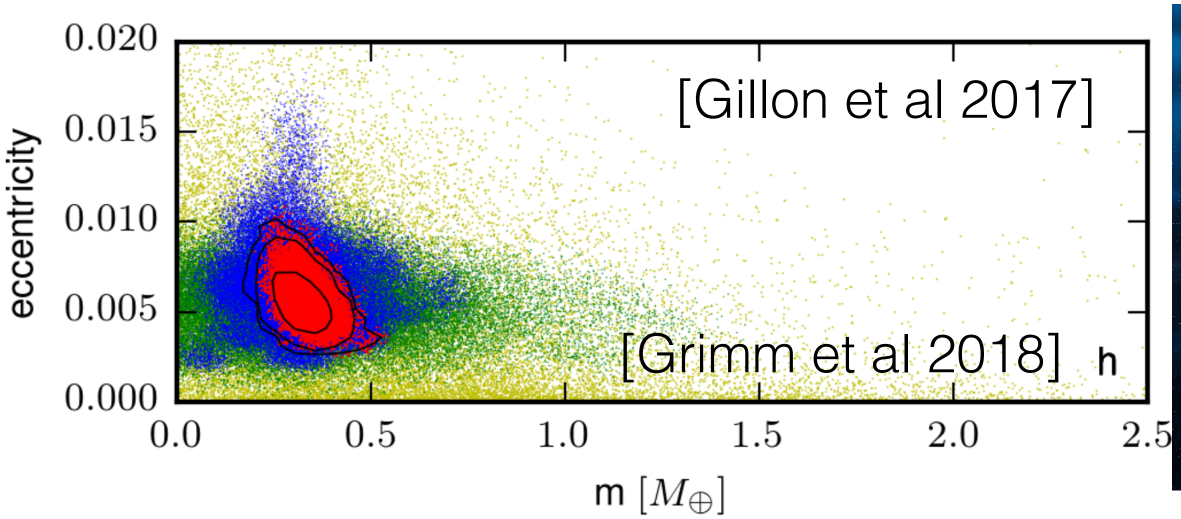

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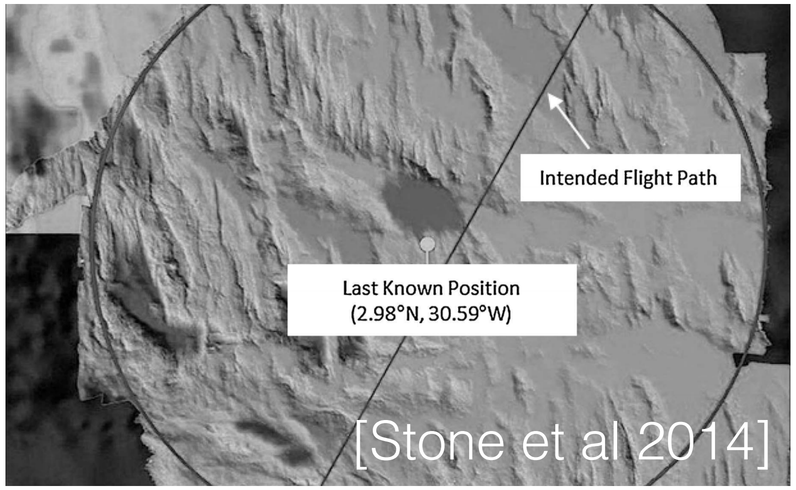

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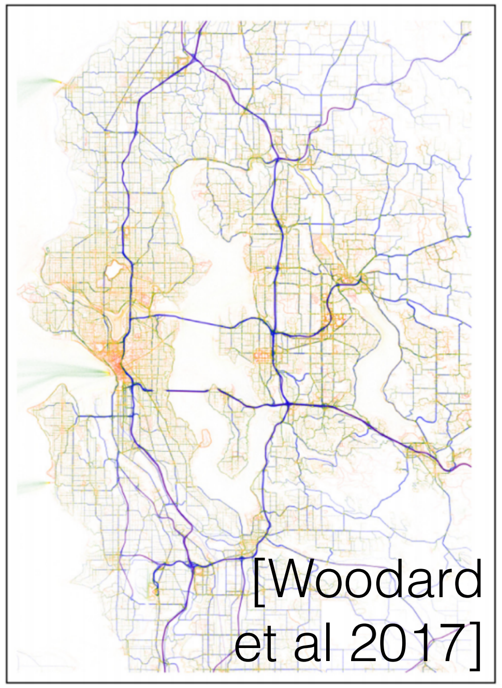

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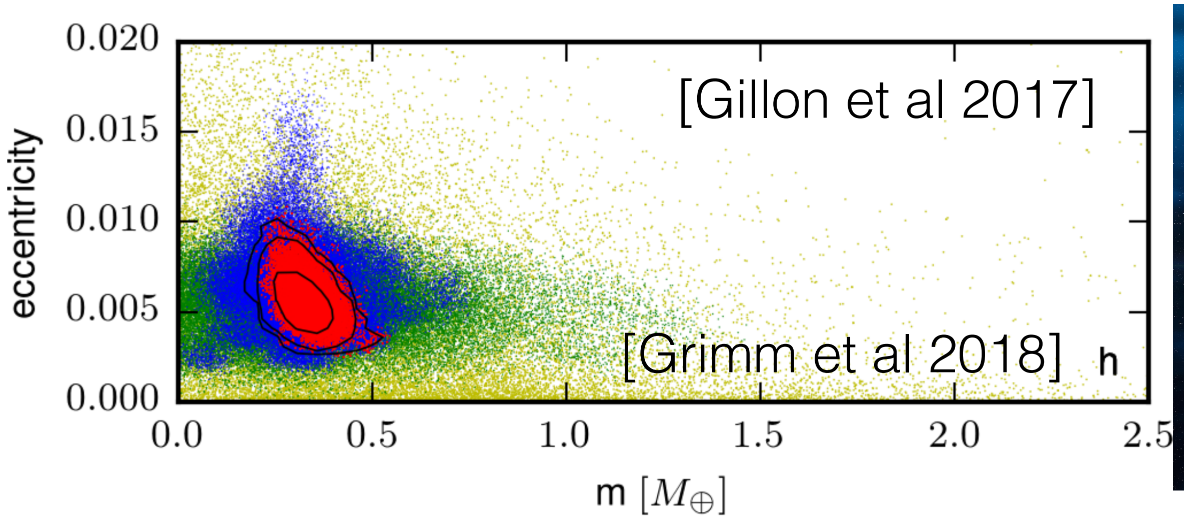

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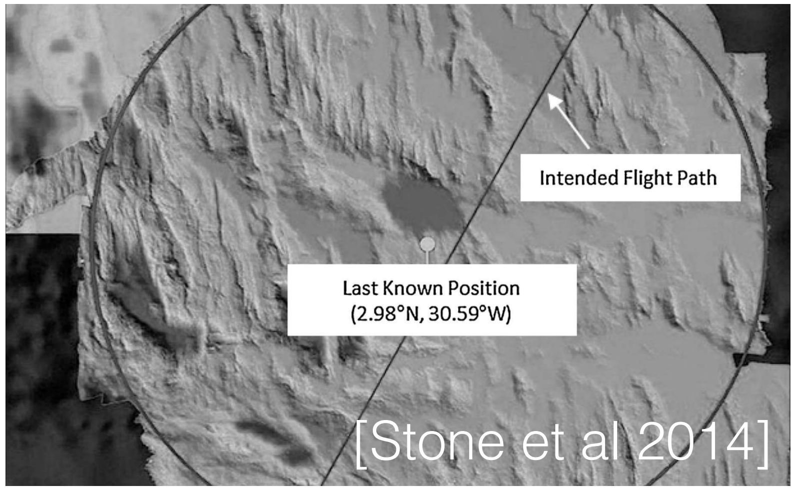

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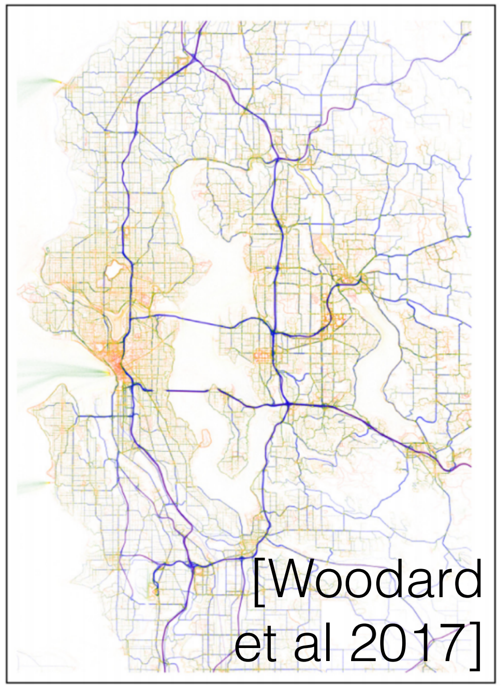

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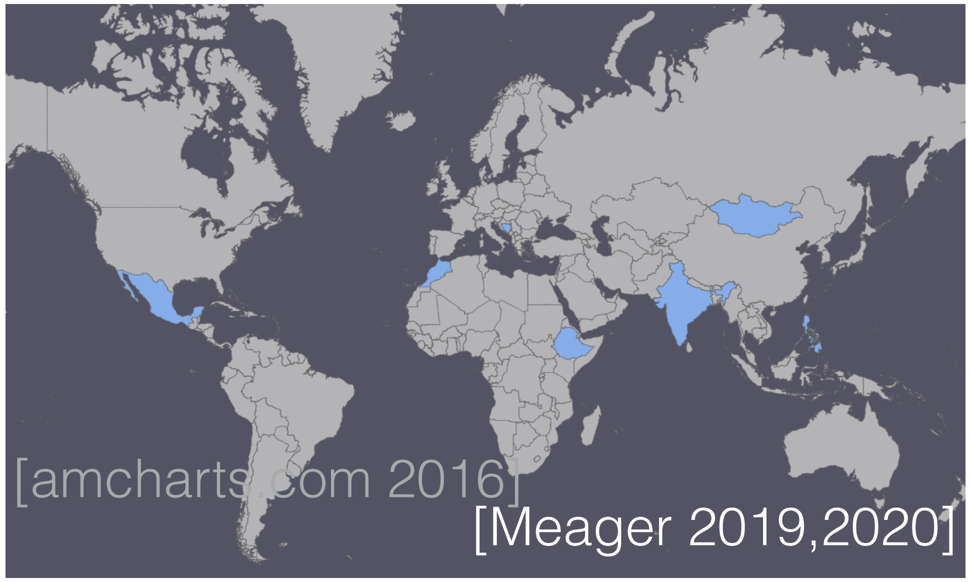

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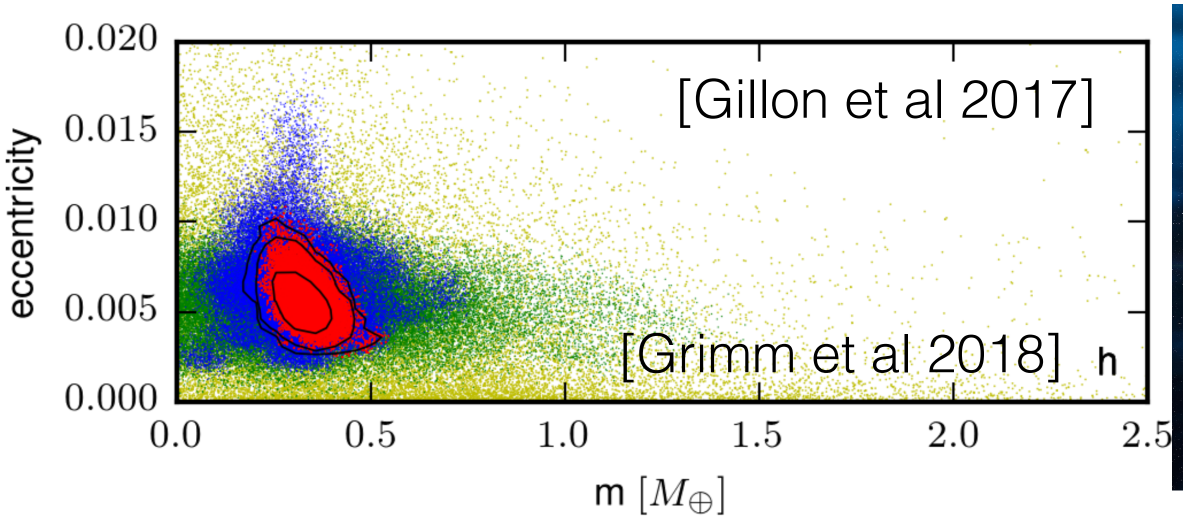

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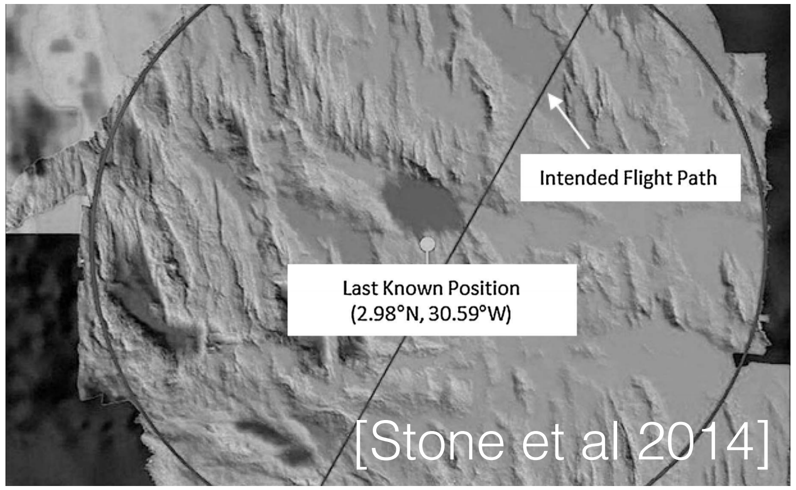

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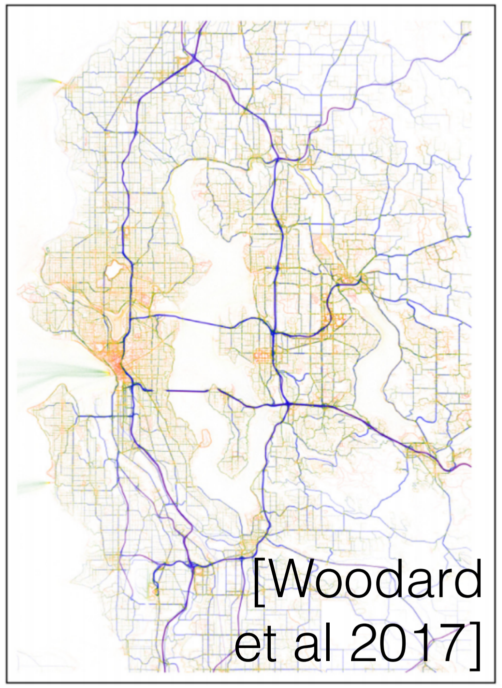

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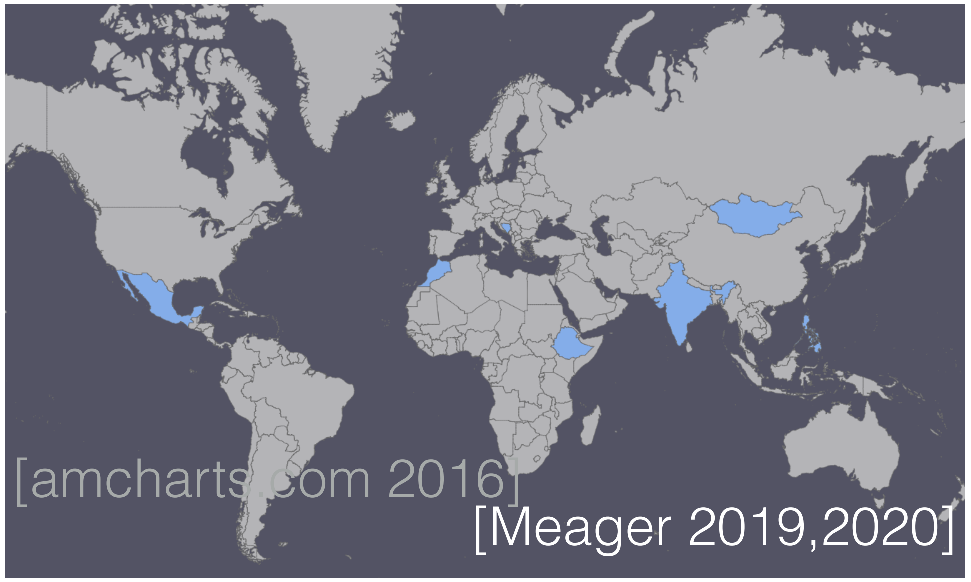

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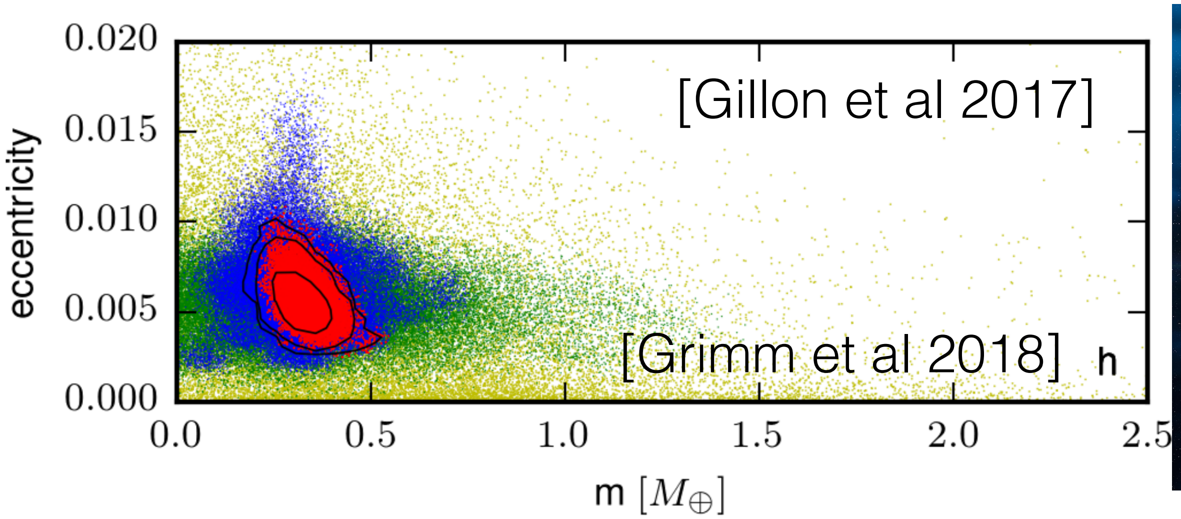

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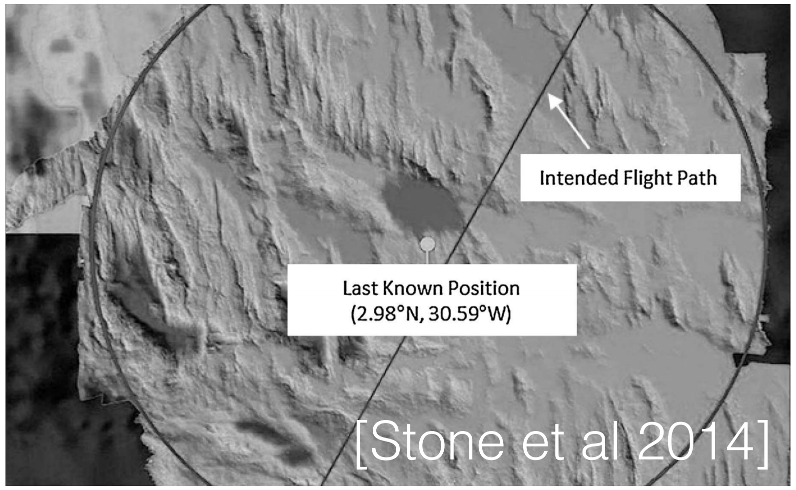

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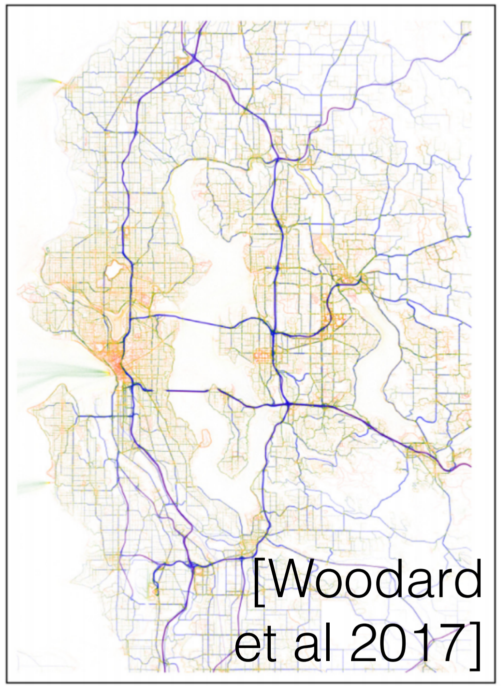

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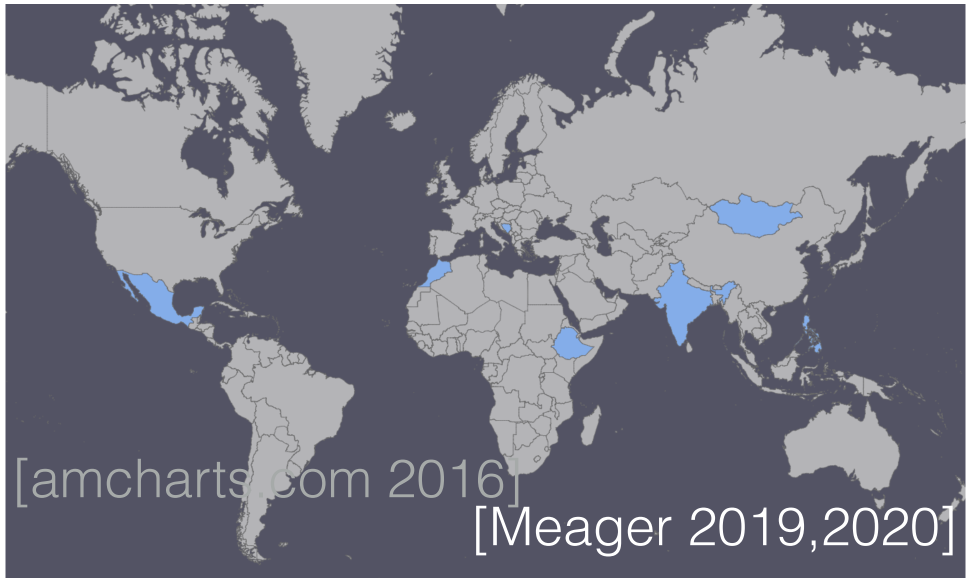

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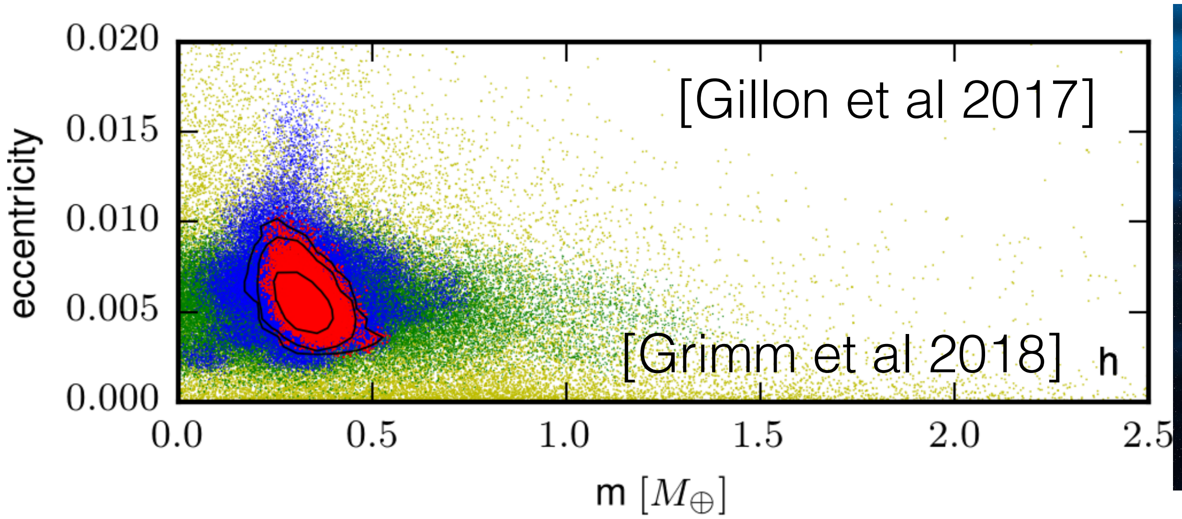

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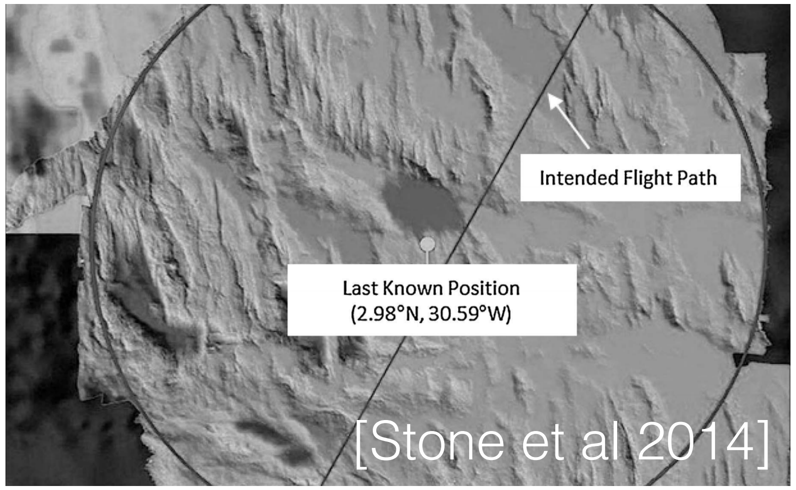

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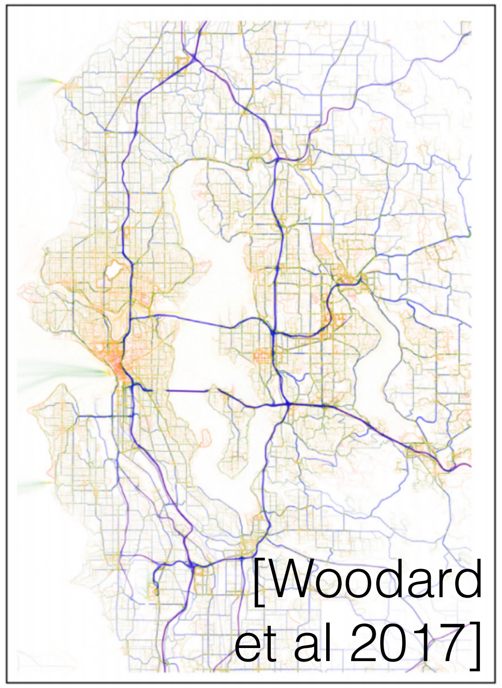

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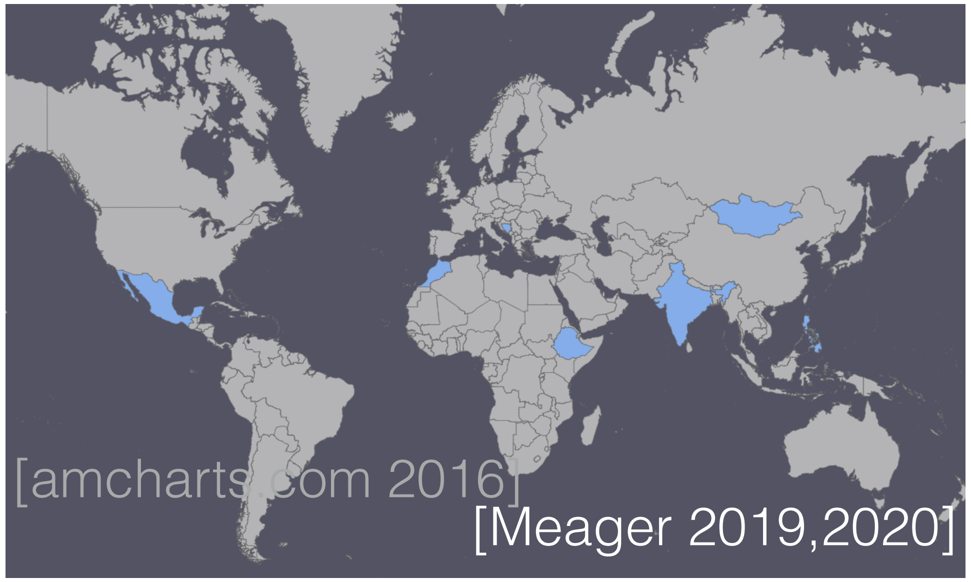

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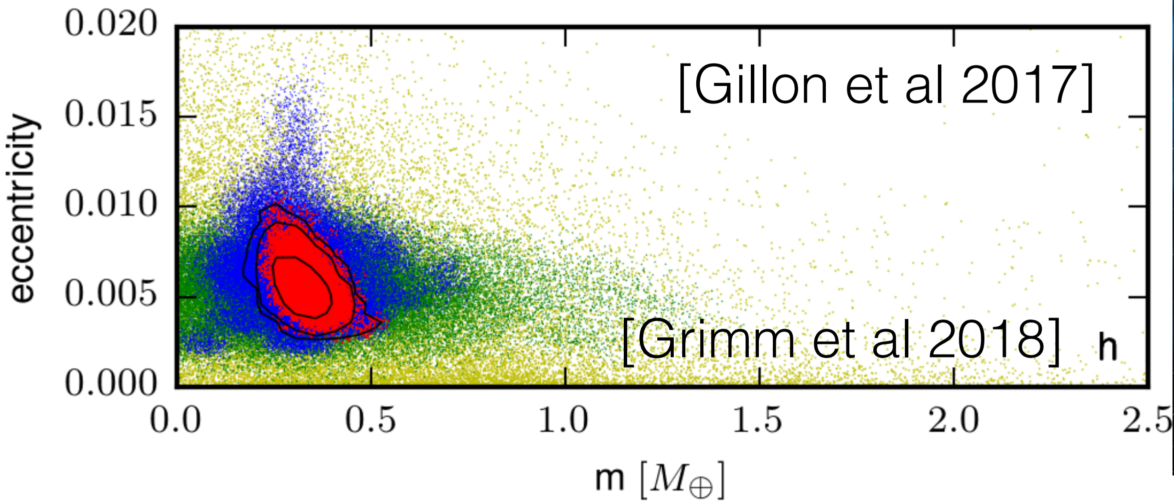

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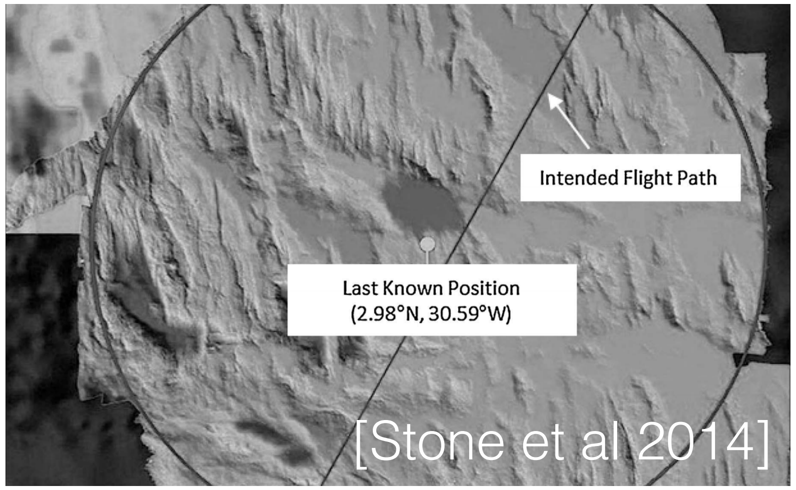

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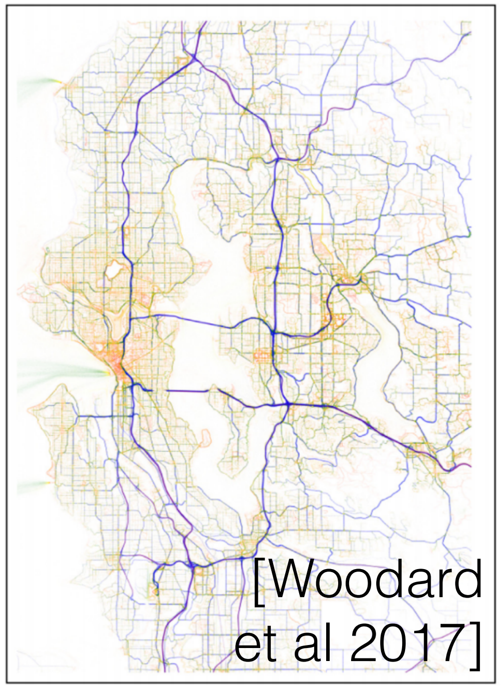

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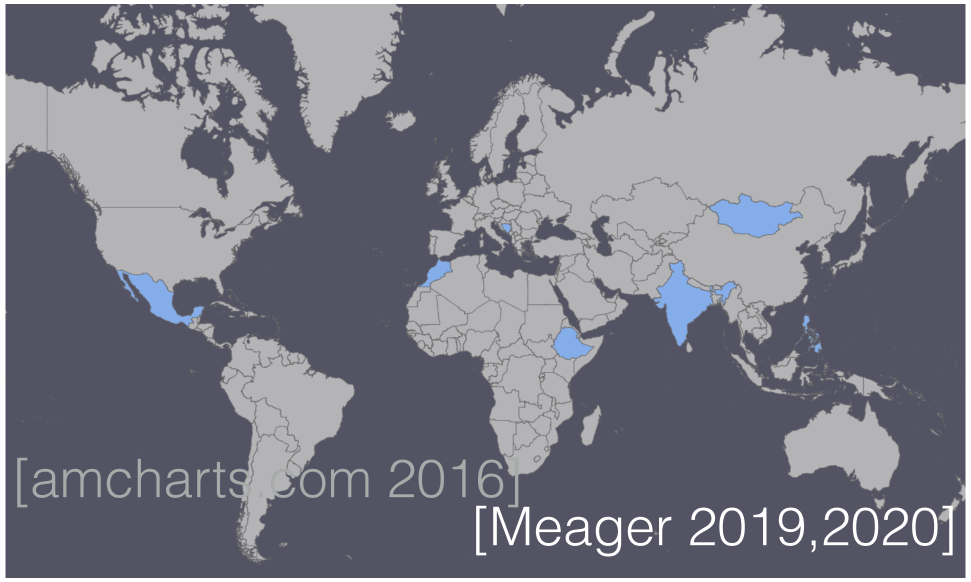

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- Goals: good point estimates, uncertainty estimates
    - More: interpretable, modular, expert info

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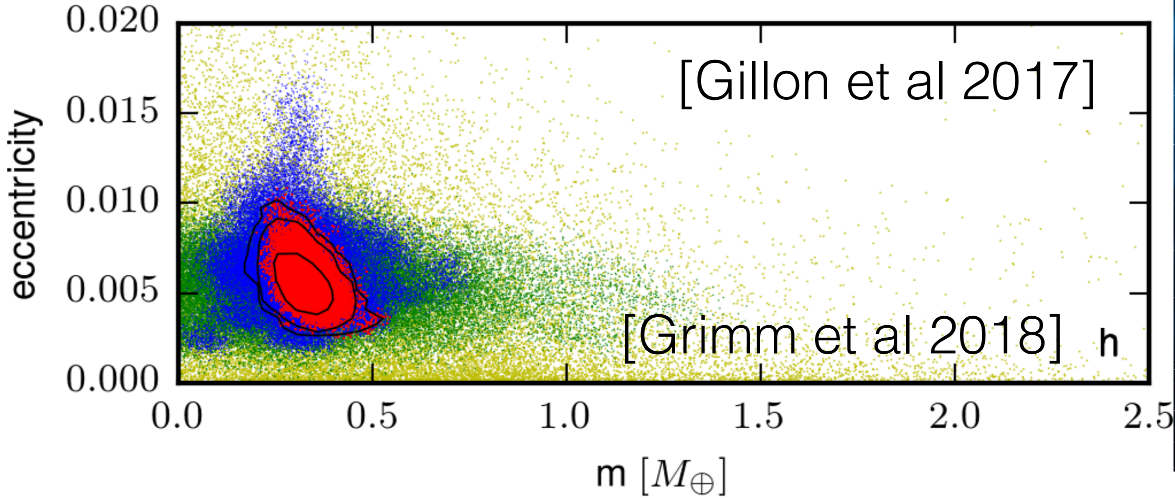

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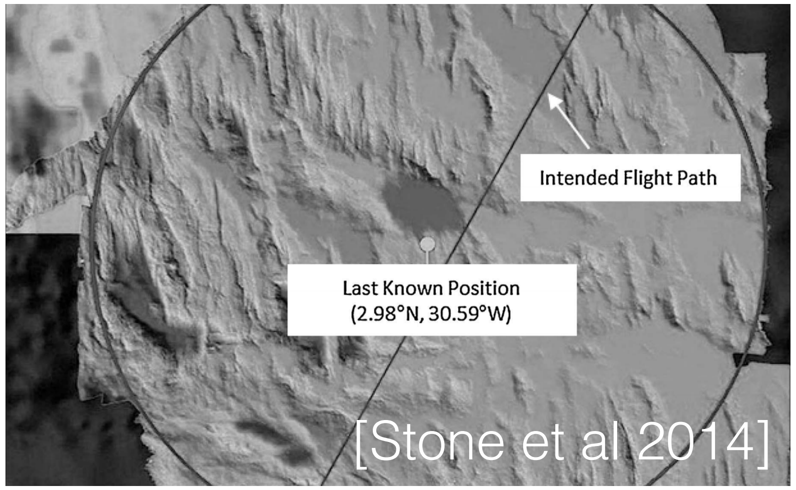

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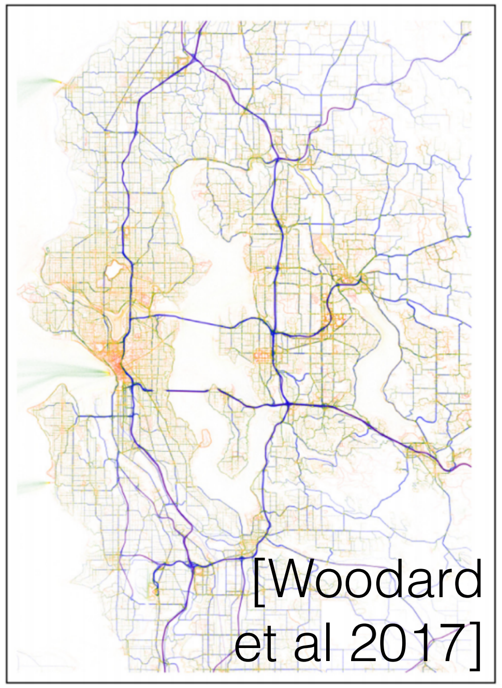

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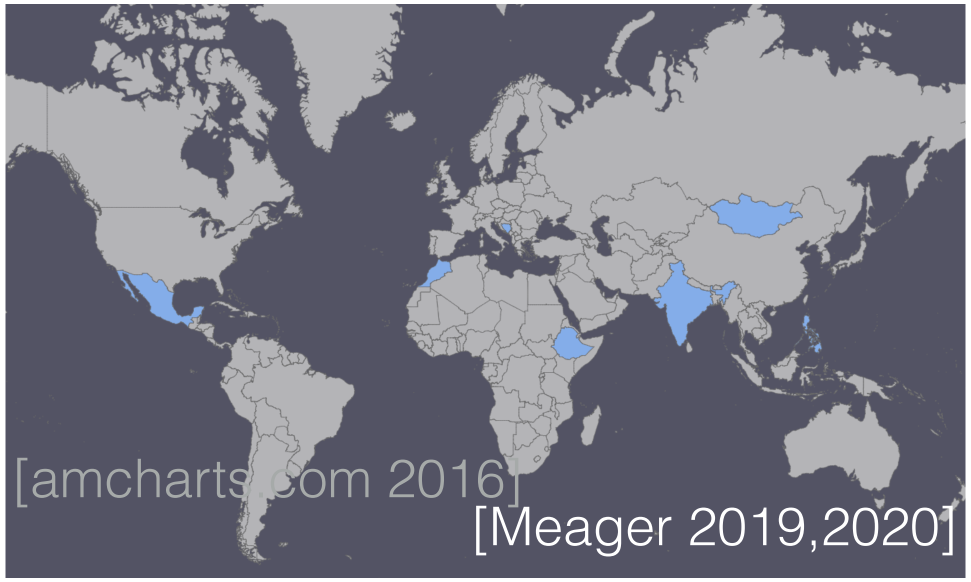

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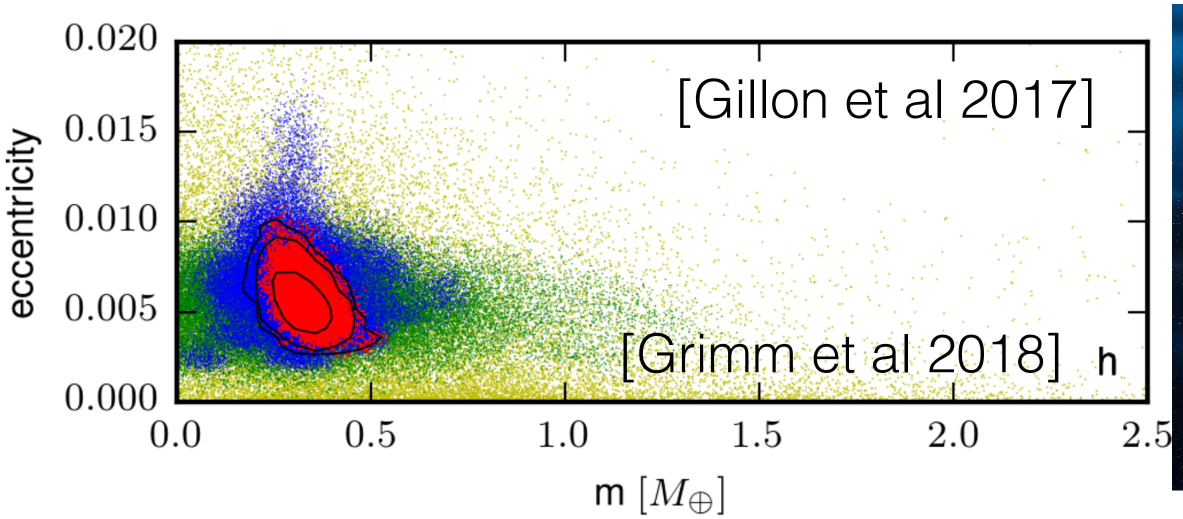

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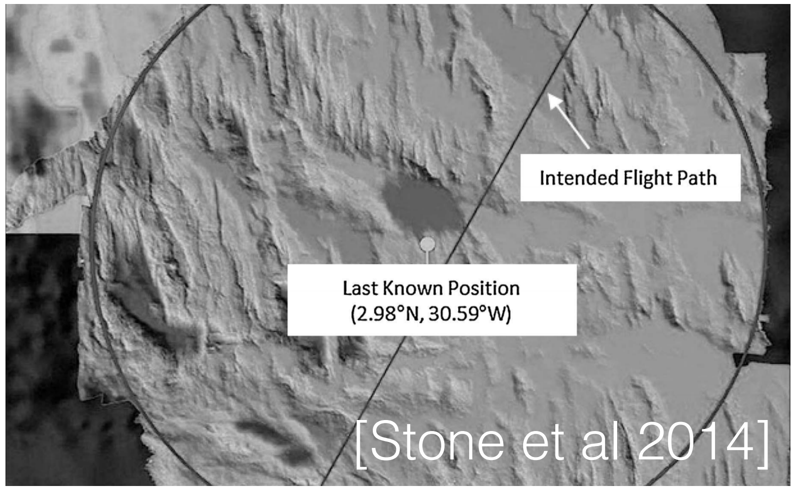

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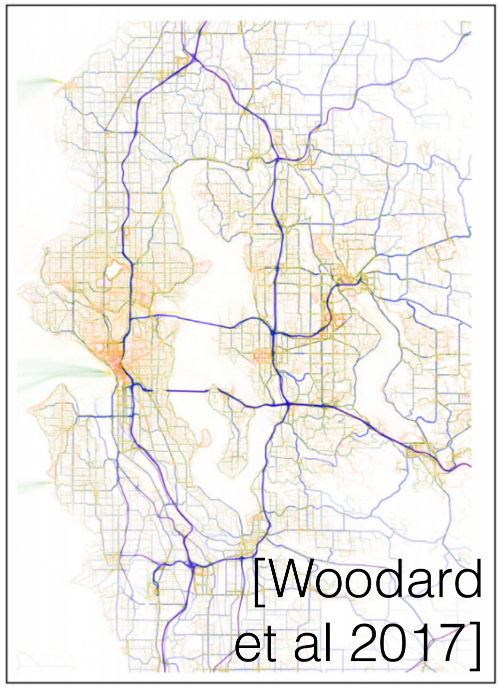

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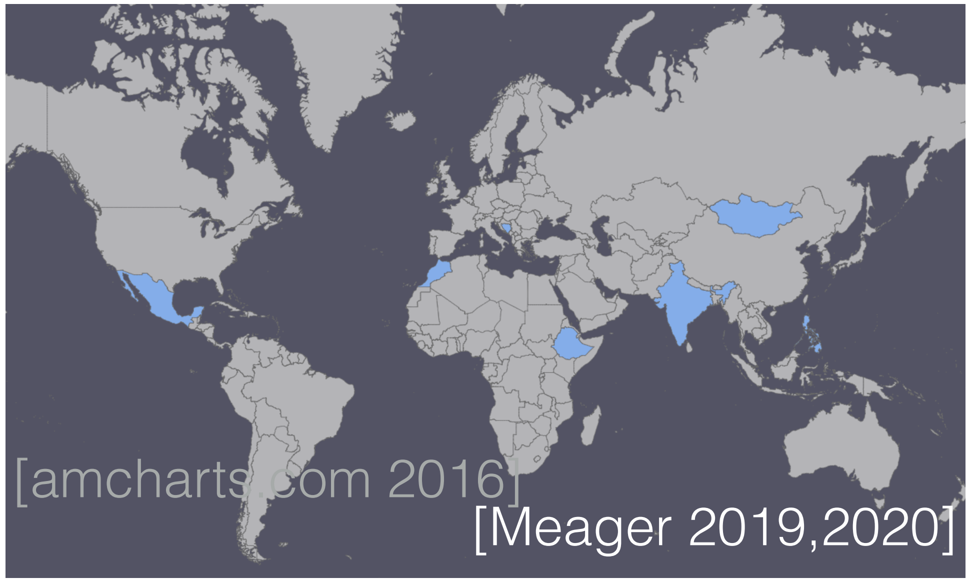

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- Goals: good point estimates, uncertainty estimates
  - More: interpretable, modular, expert info
- Challenge: speed (compute, user), reliable inference

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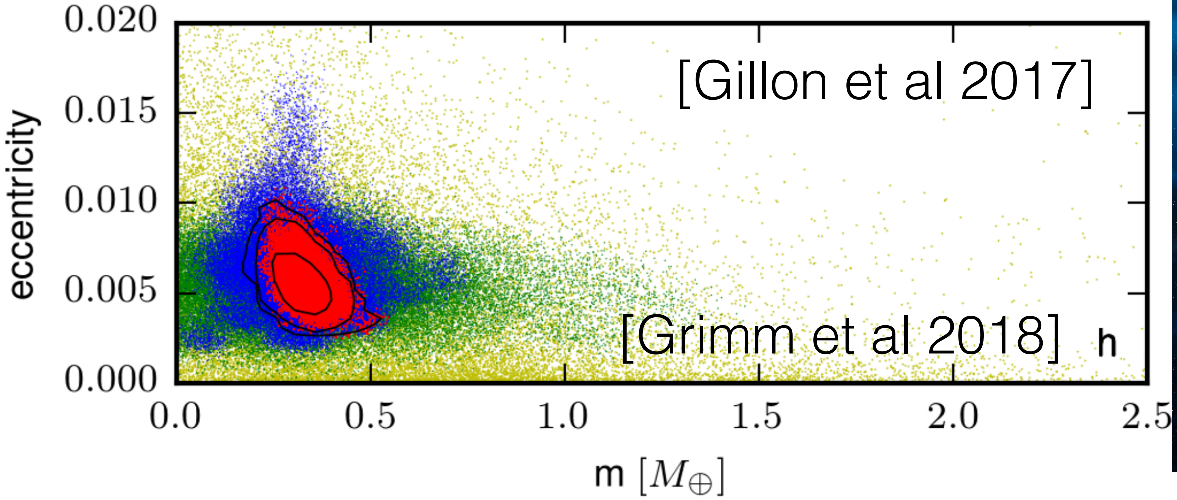

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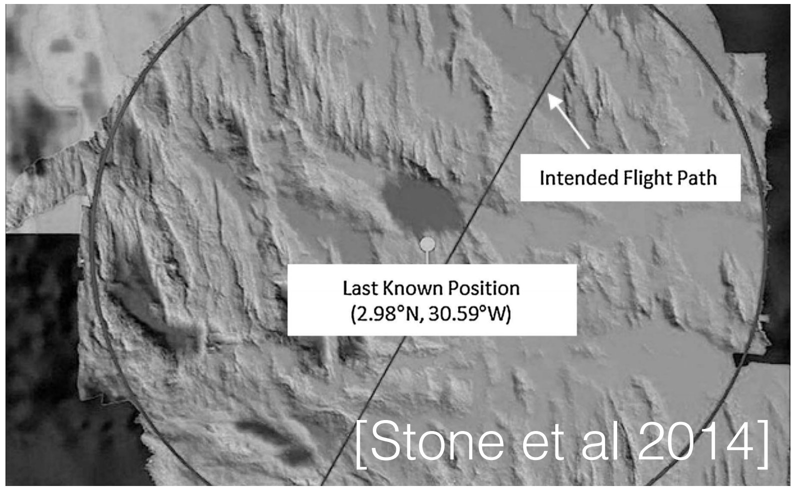

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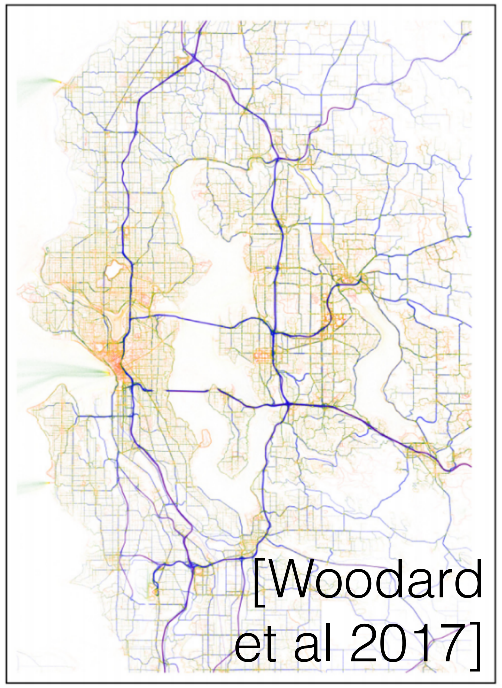

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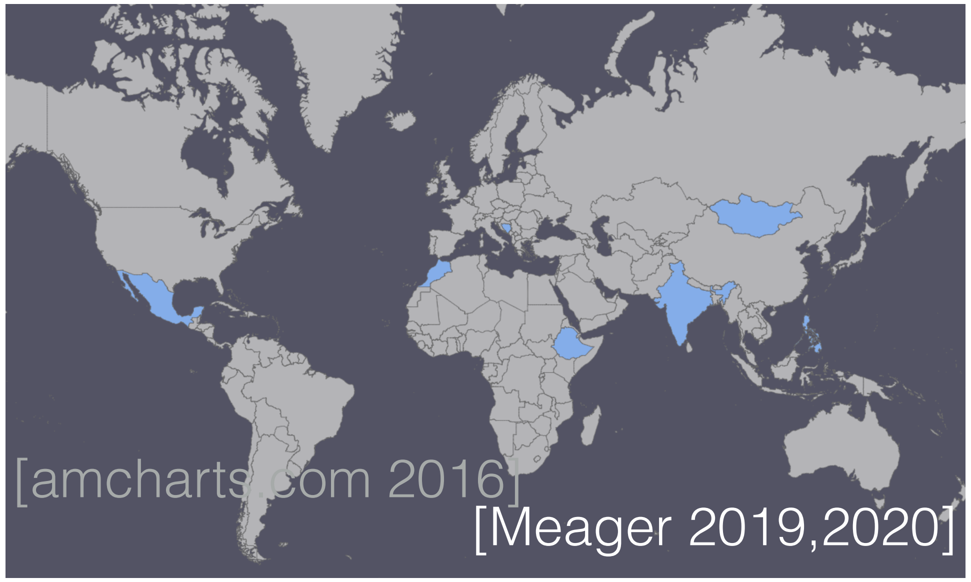

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- Challenge: speed (compute, user), reliable inference
- Uncertainty doesn't have to disappear in large data sets

Variational Bayes

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FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
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[Blei et al 2003]

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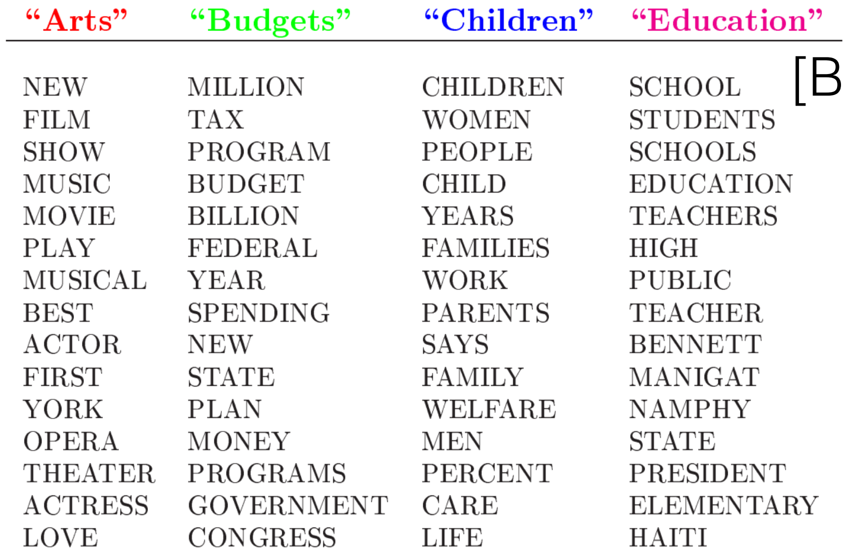

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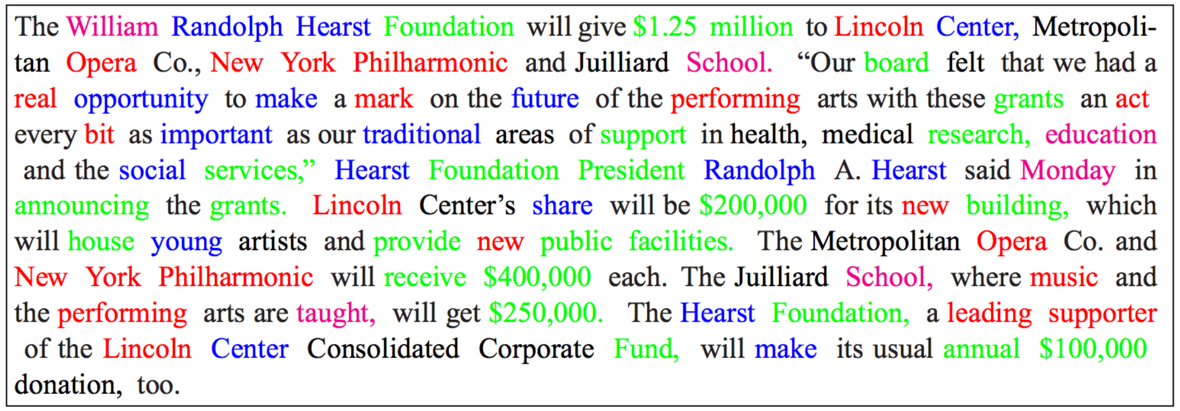

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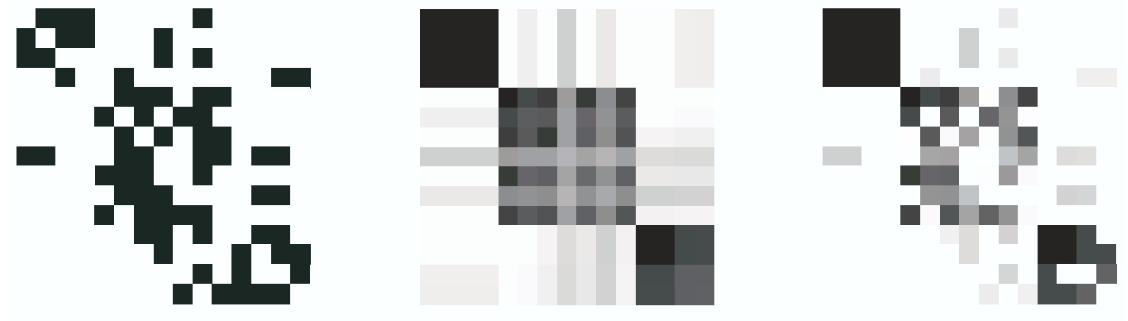

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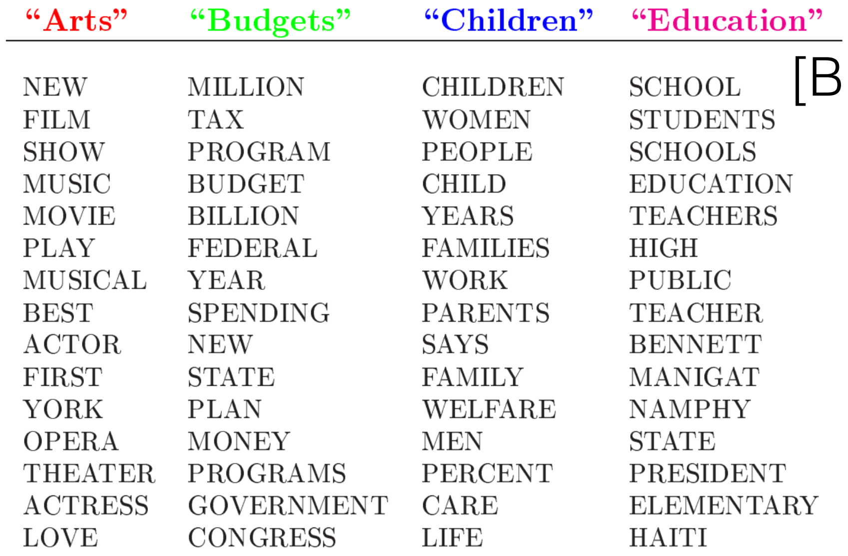

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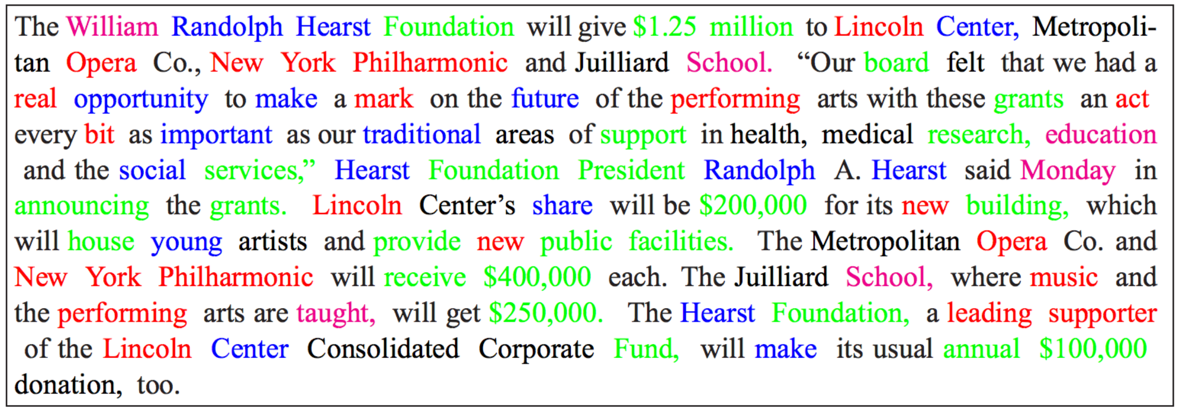

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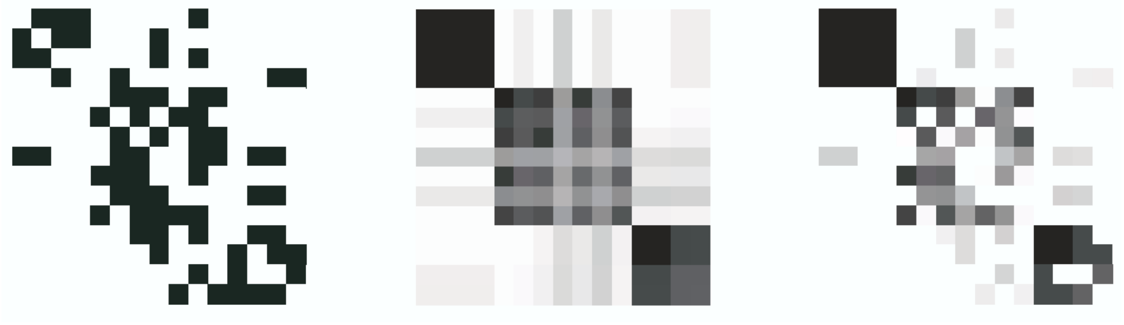

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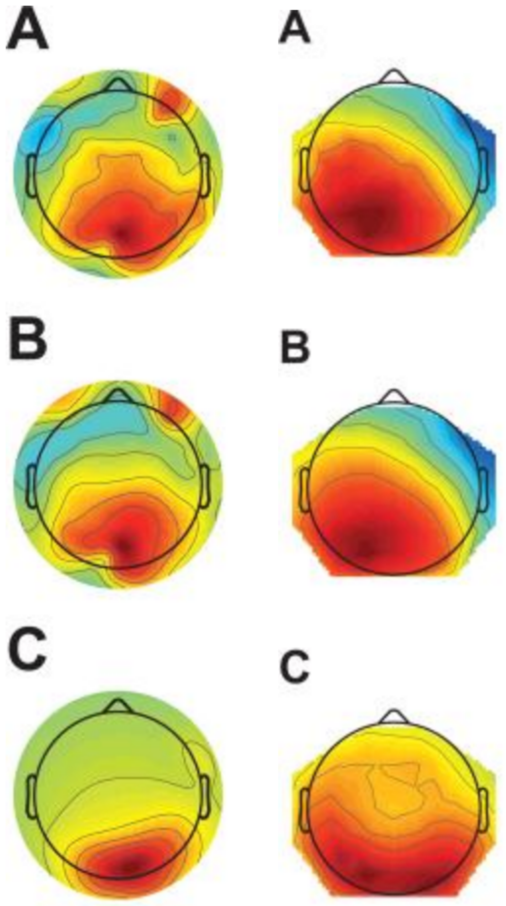

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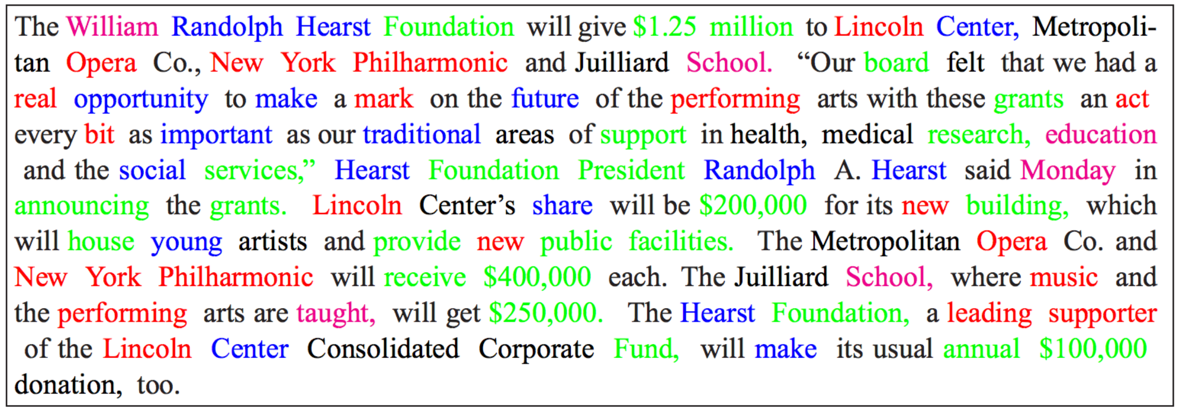
[Blei et al 2003]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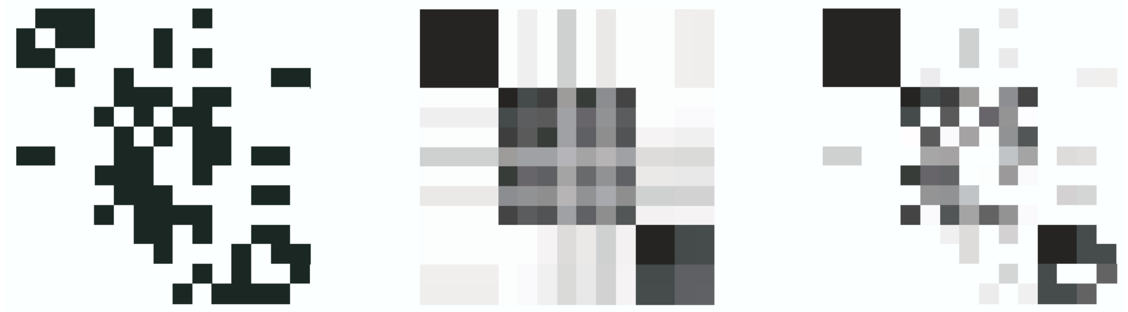
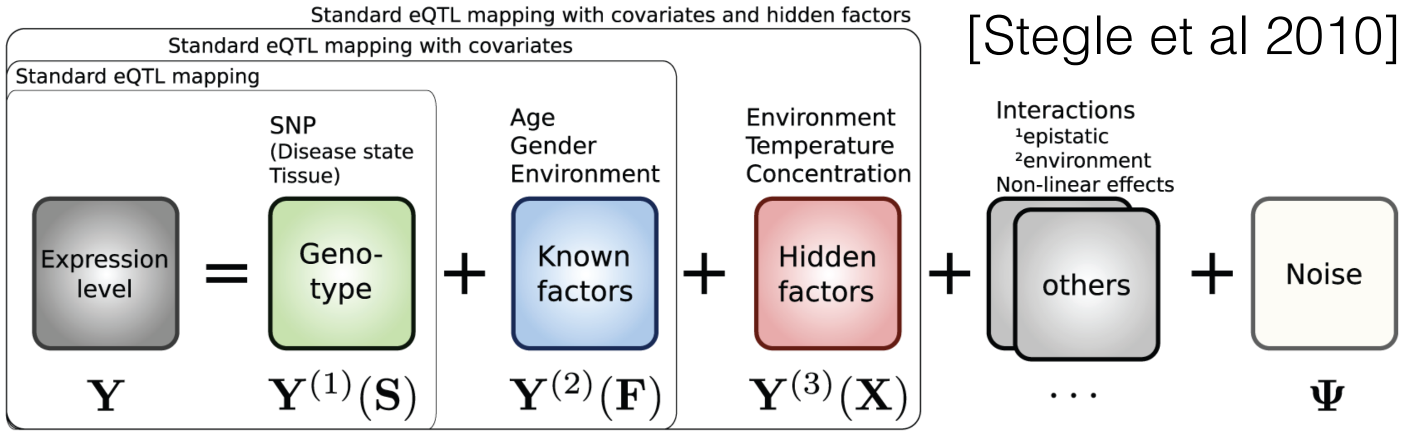
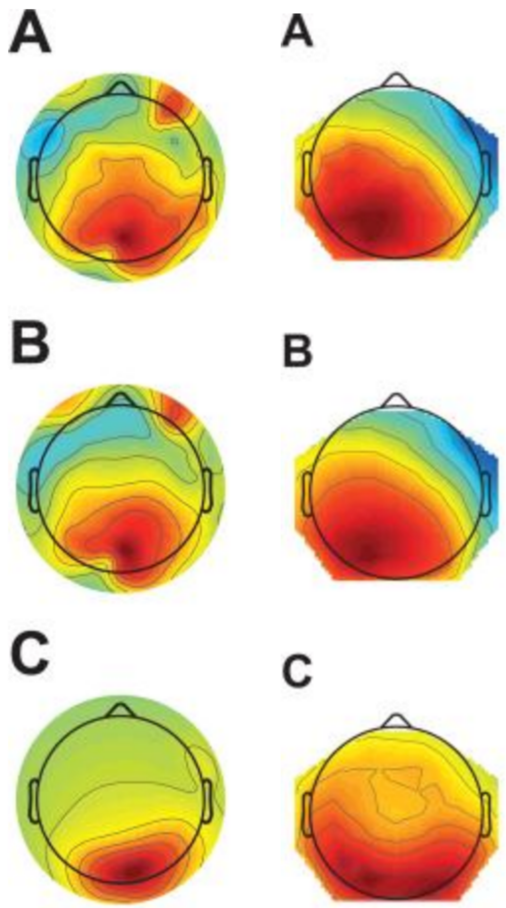


[Airoldi et al 2008]



[Stegle et al 2010]

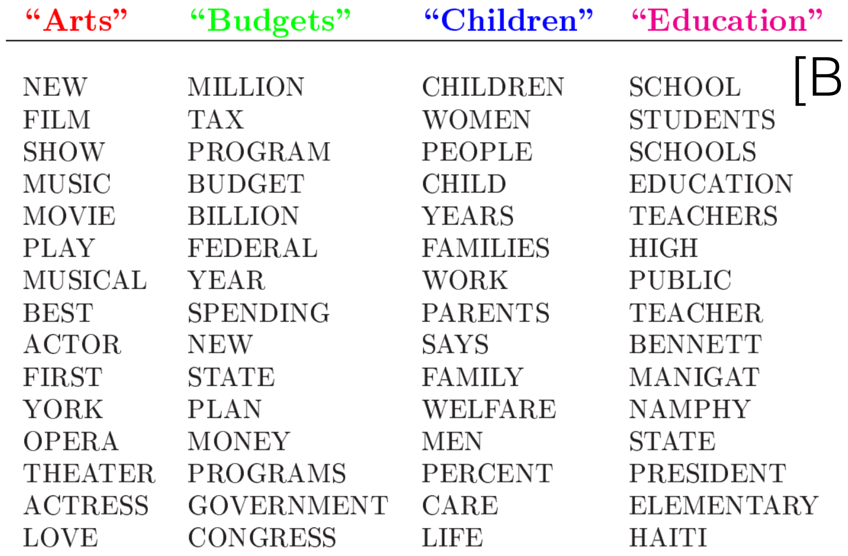


[Gershman et al 2014]

[Blei et al 2018]

Variational Bayes

- Modern problems: often large data, large dimensions
- Variational Bayes can be very fast



"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHER
ACTOR	NEW	SAYS	BENNETT
FIRST	STATE	FAMILY	MANIGAT
YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
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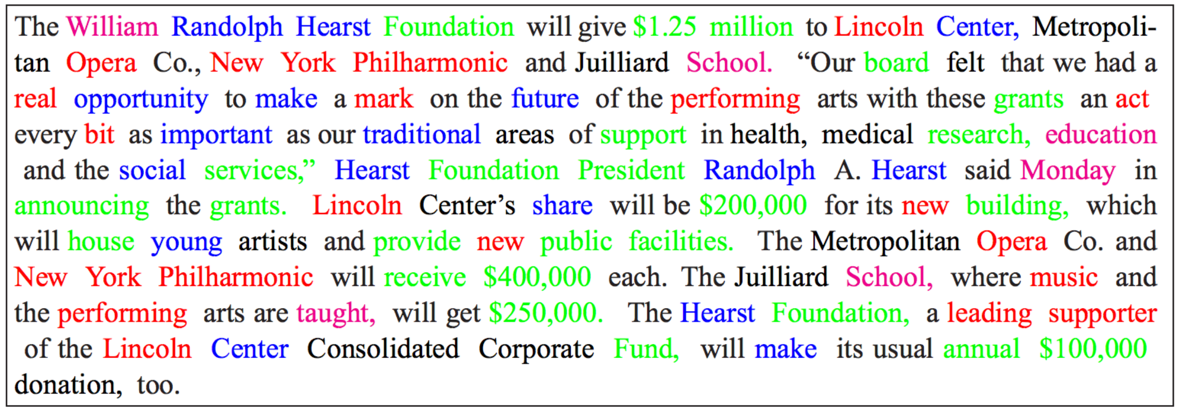

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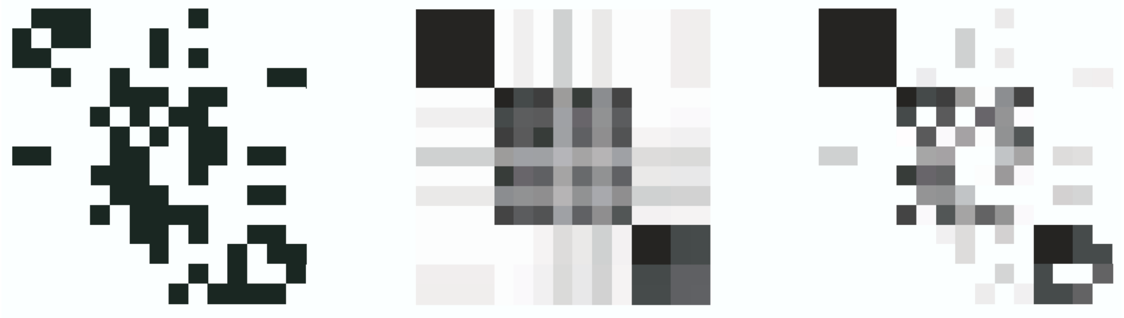

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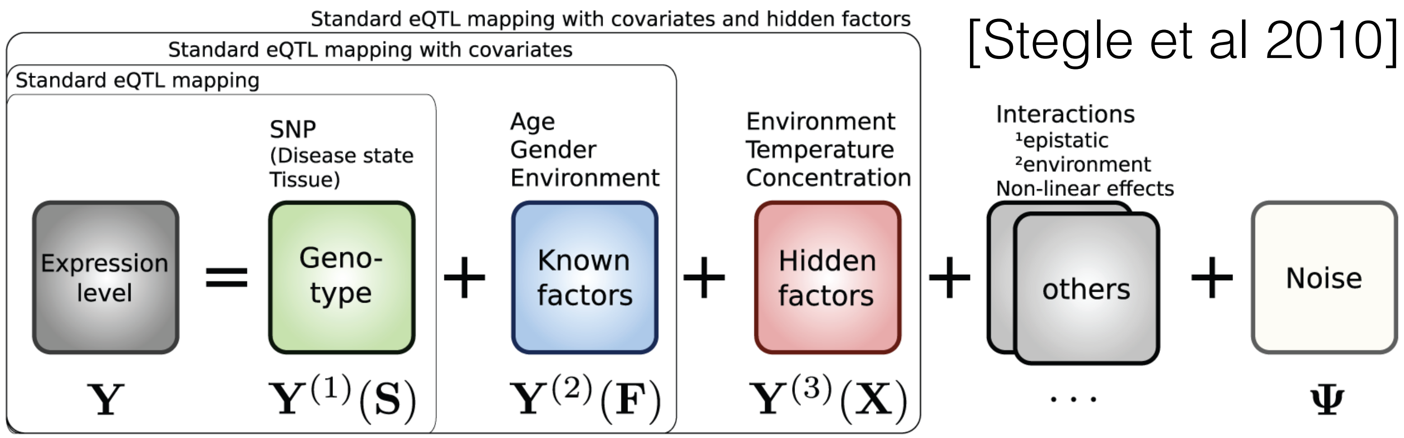

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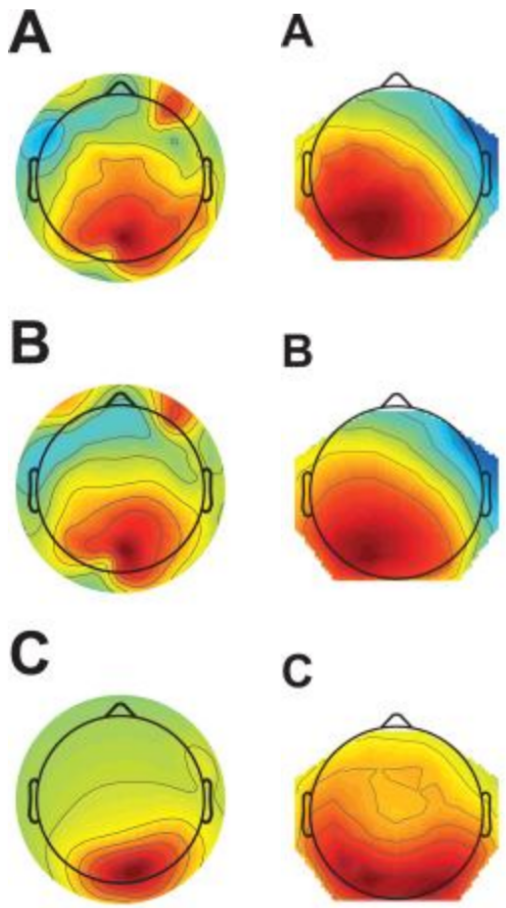

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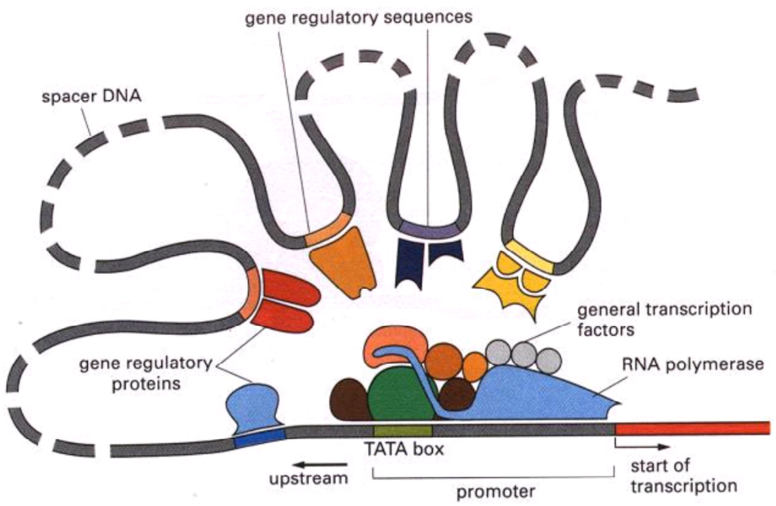

[Xing et al 2004]

[Xing 2003]

[Blei et al 2018]

Roadmap

- Bayes & Approximate Bayes review

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- What is:
    - Variational Bayes (VB)
    - Mean-field variational Bayes (MFVB)

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Bayesian inference

Bayesian inference

parameters

$$\theta$$

Bayesian inference

parameters

$p(\theta)$

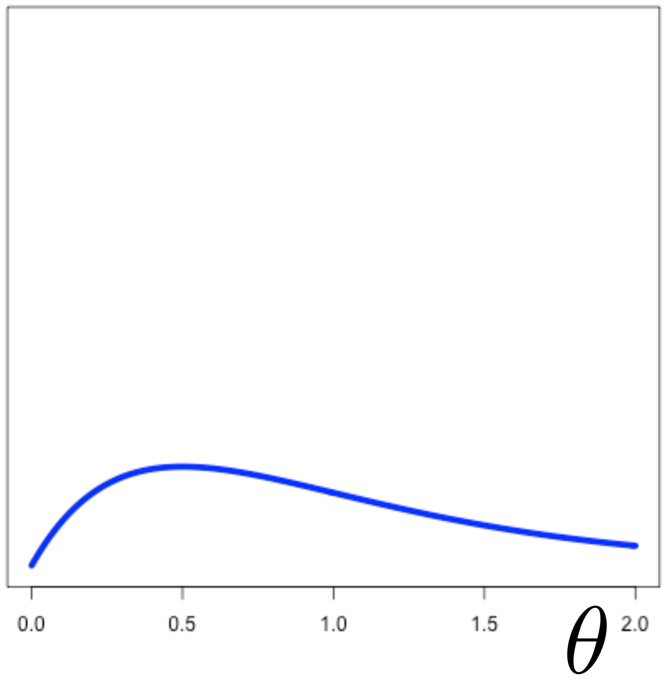
prior

Bayesian inference

parameters

$p(\theta)$

prior

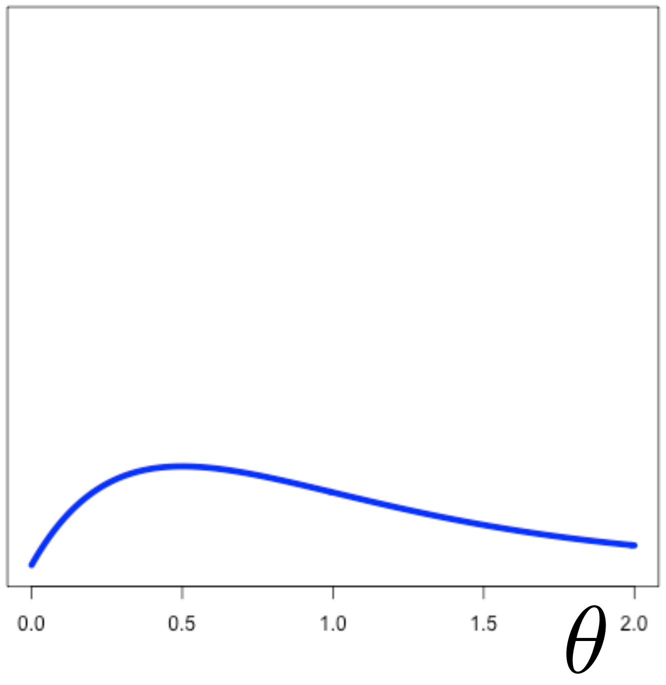


Bayesian inference

parameters

$$p(y_{1:N}|\theta)p(\theta)$$

likelihood prior



Bayesian inference

data parameters

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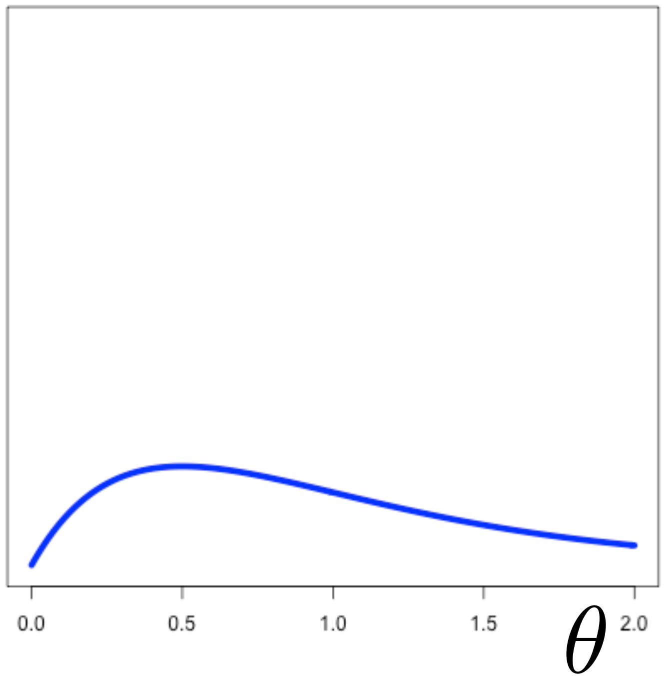
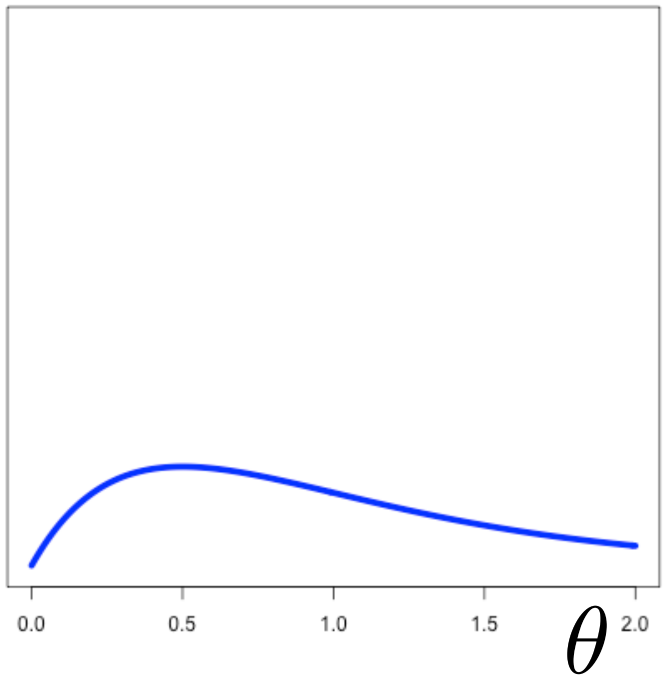


Bayesian inference

data parameters

$$p(\theta|y_{1:N}) \propto_\theta p(y_{1:N}|\theta)p(\theta)$$

posterior likelihood prior



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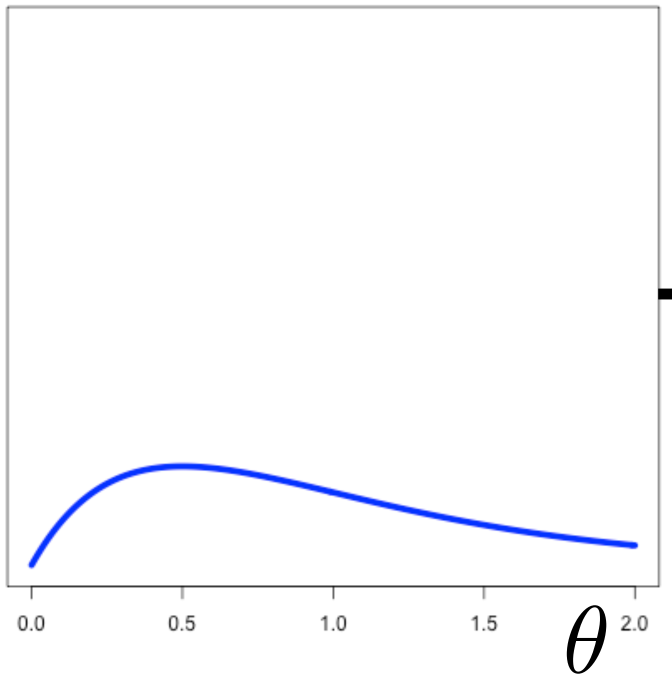

Bayes Theorem

Bayesian inference

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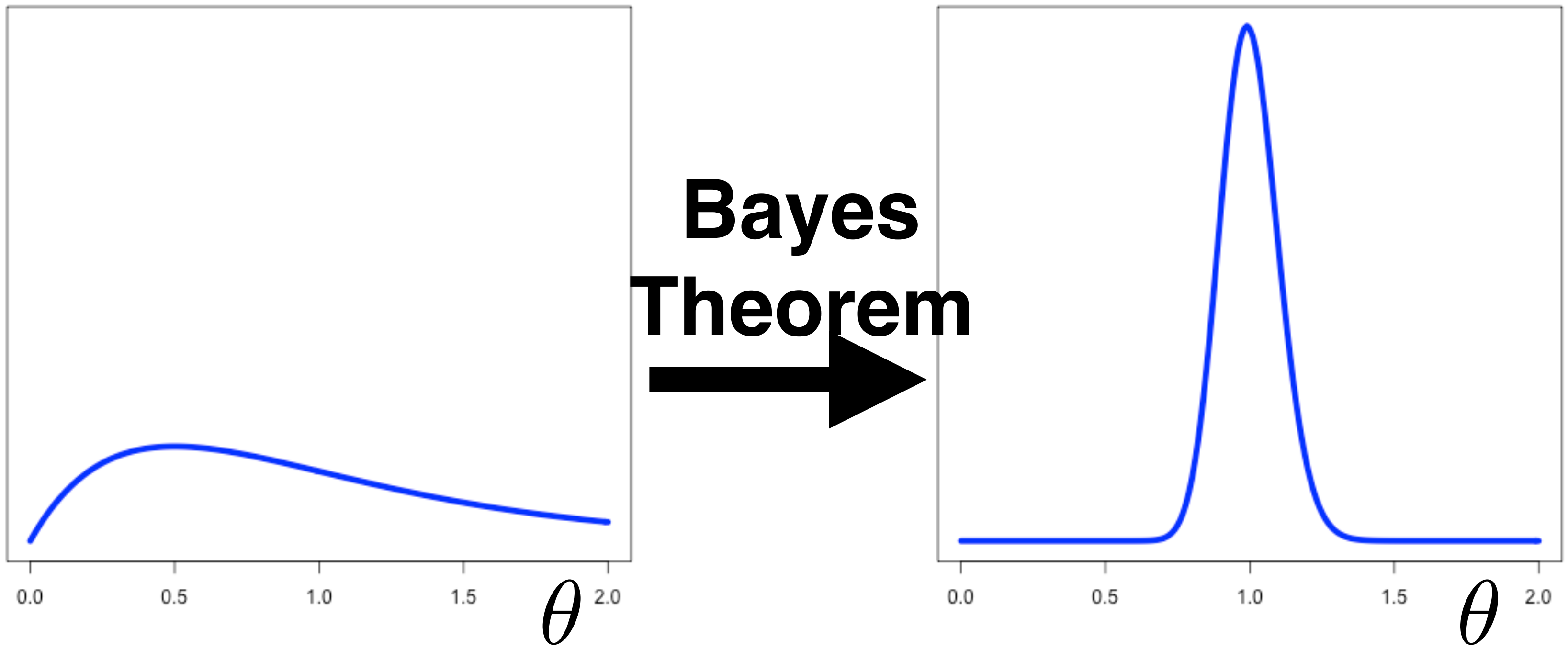

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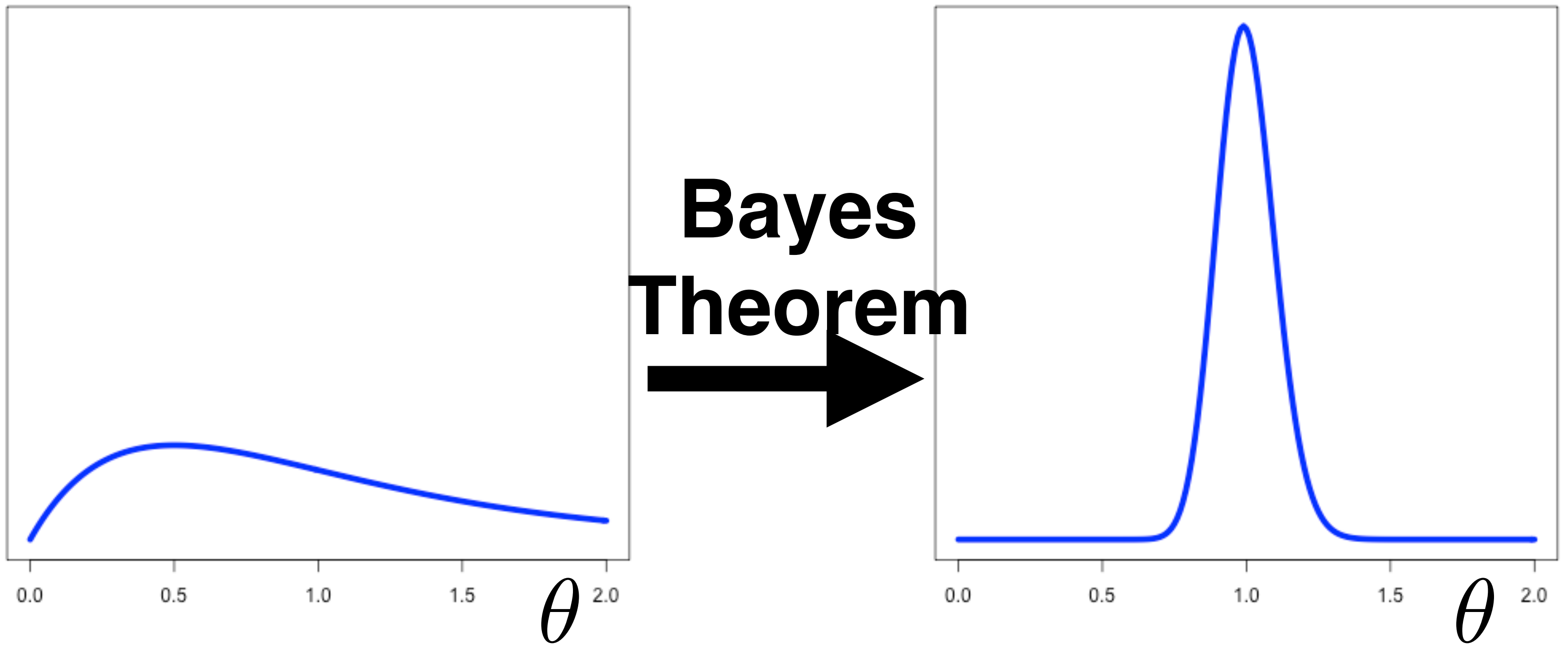

Bayes Theorem

1. Build a model: choose prior & choose likelihood

Bayesian inference

data parameters

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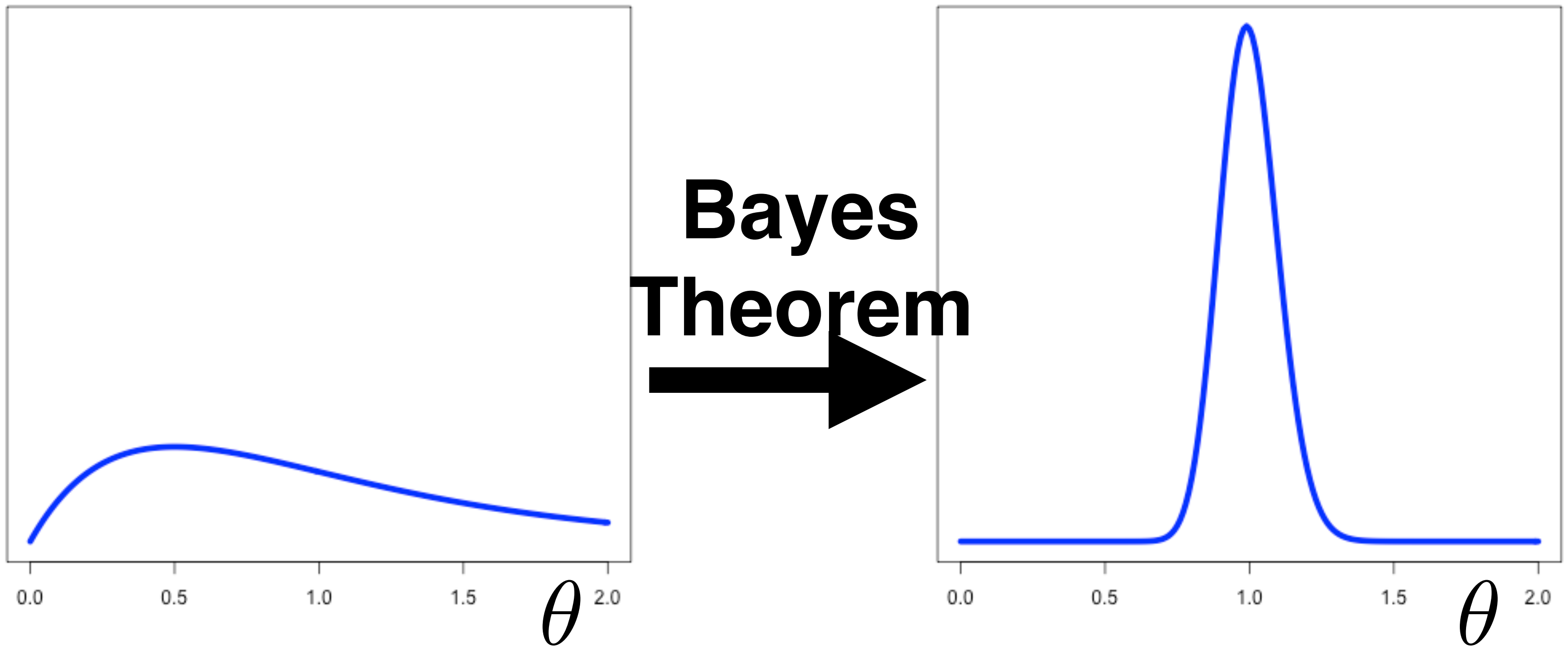
posterior likelihood prior



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2. Compute the posterior

Bayesian inference

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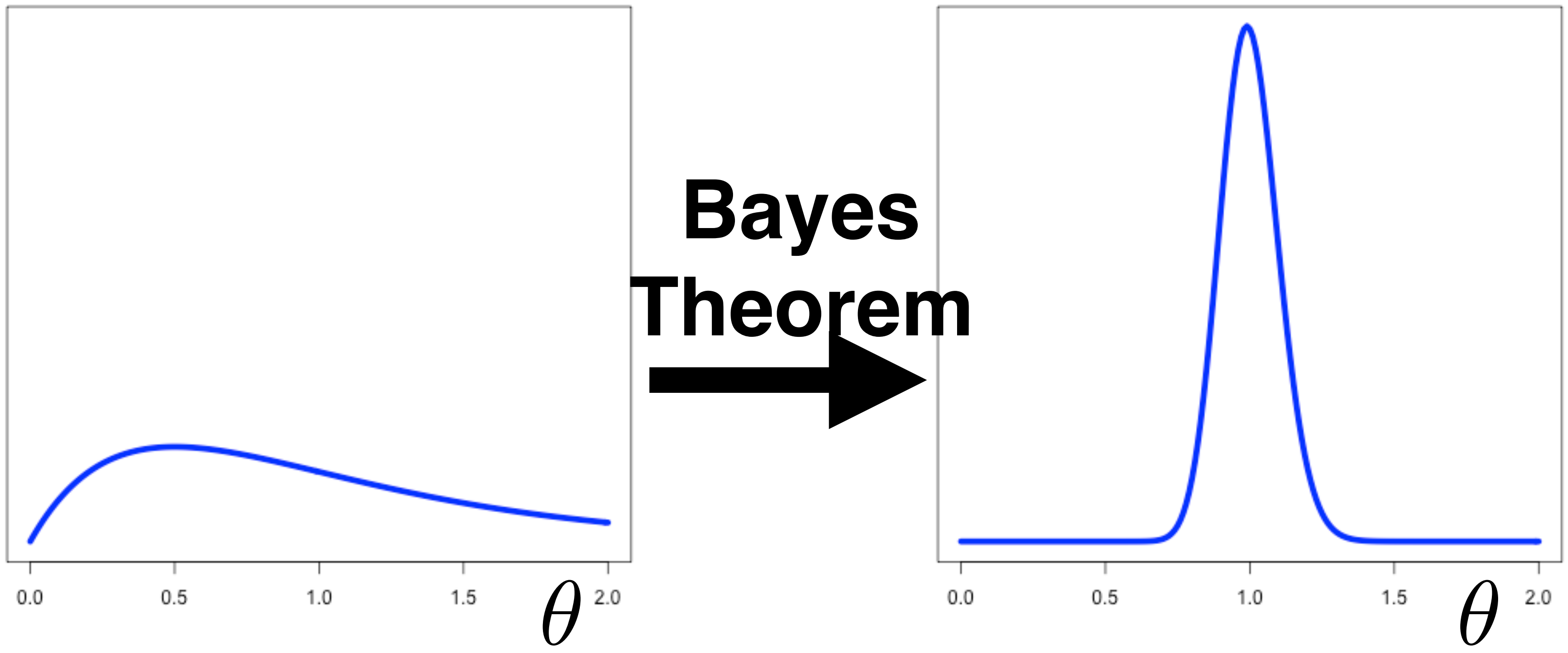

Bayes Theorem

1. Build a model: choose prior & choose likelihood
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3. Report a summary, e.g. posterior means and (co)variances

Bayesian inference

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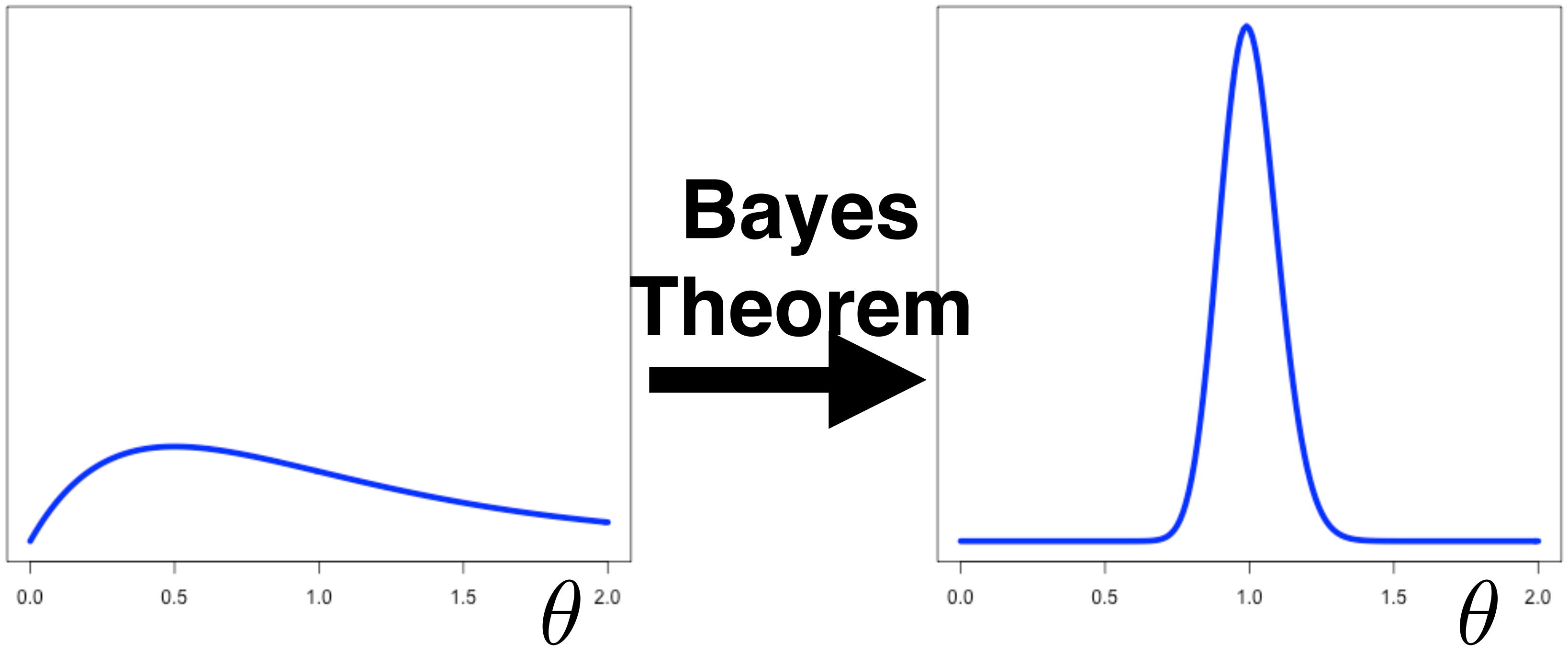

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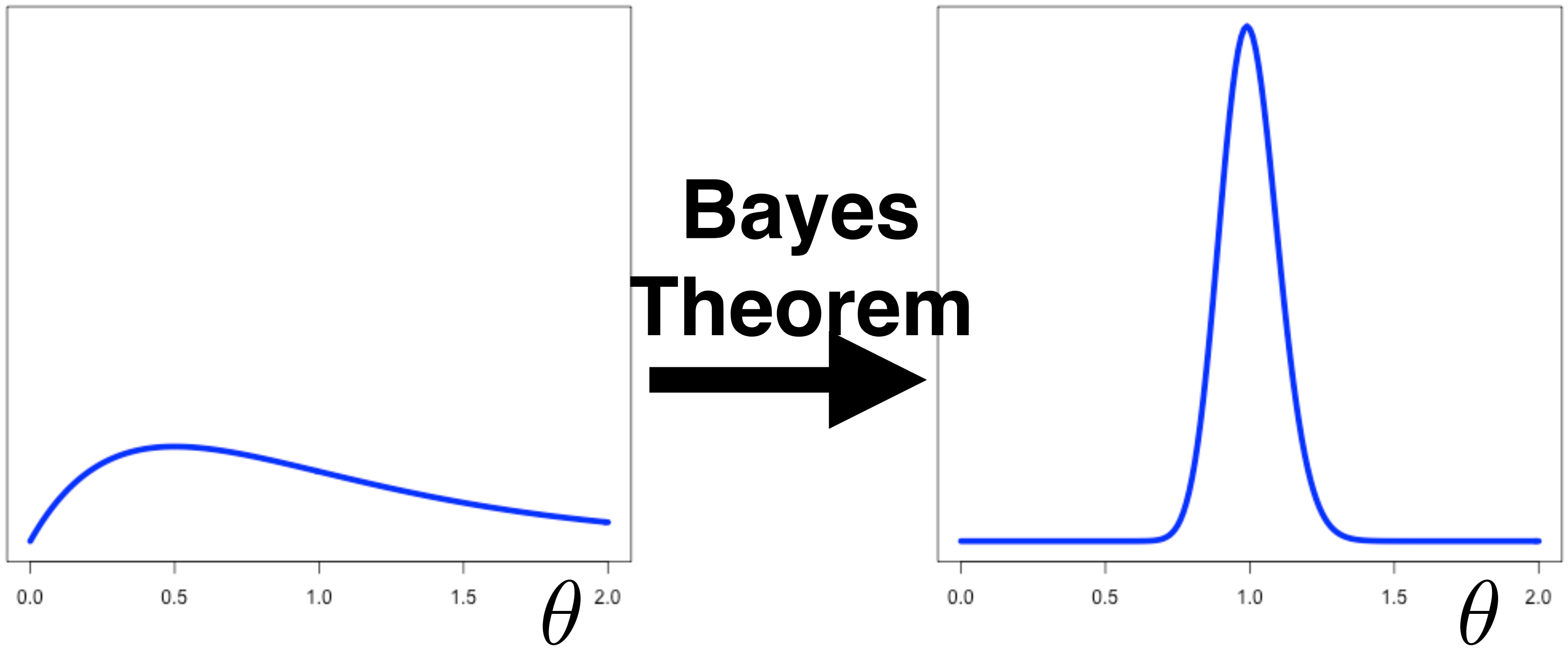

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  - Typically no closed form

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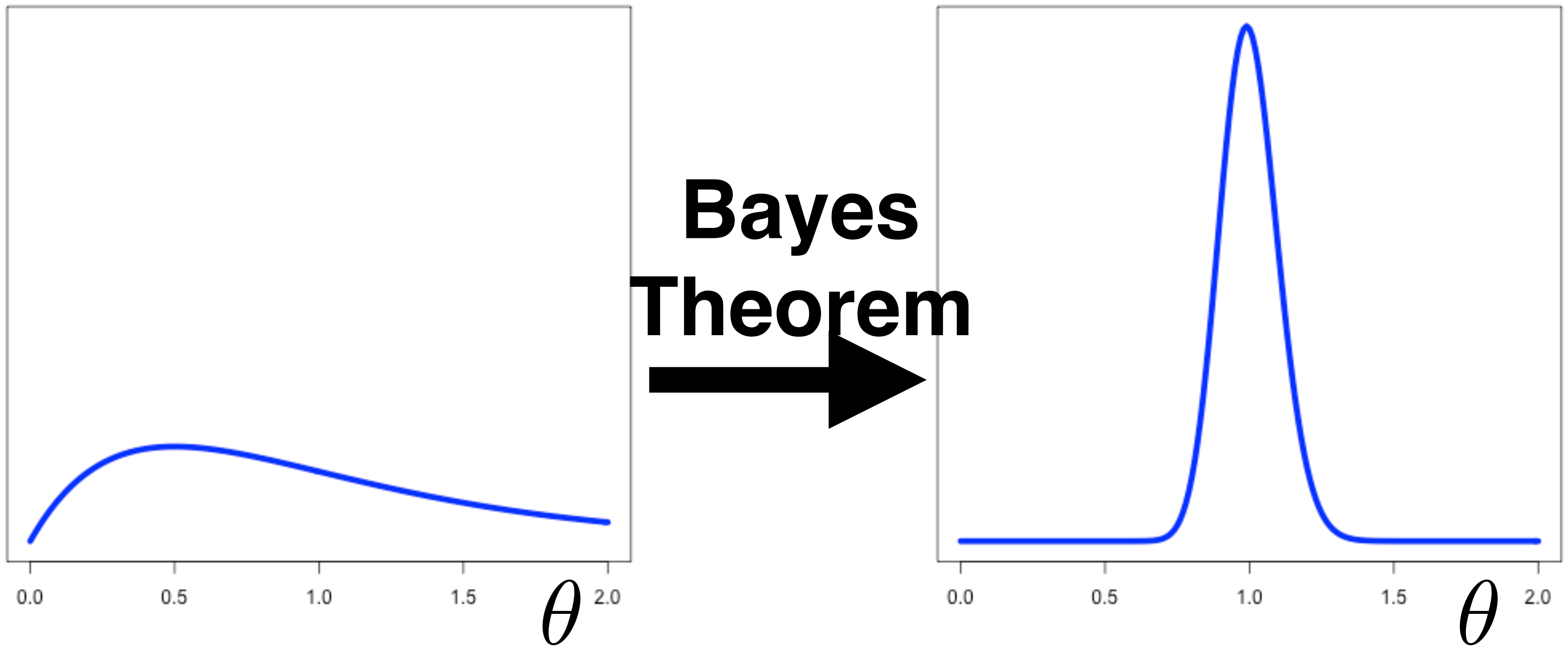

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Bayesian inference

data parameters

$$p(\theta|y_{1:N}) = p(y_{1:N}|\theta)p(\theta)/p(y_{1:N})$$

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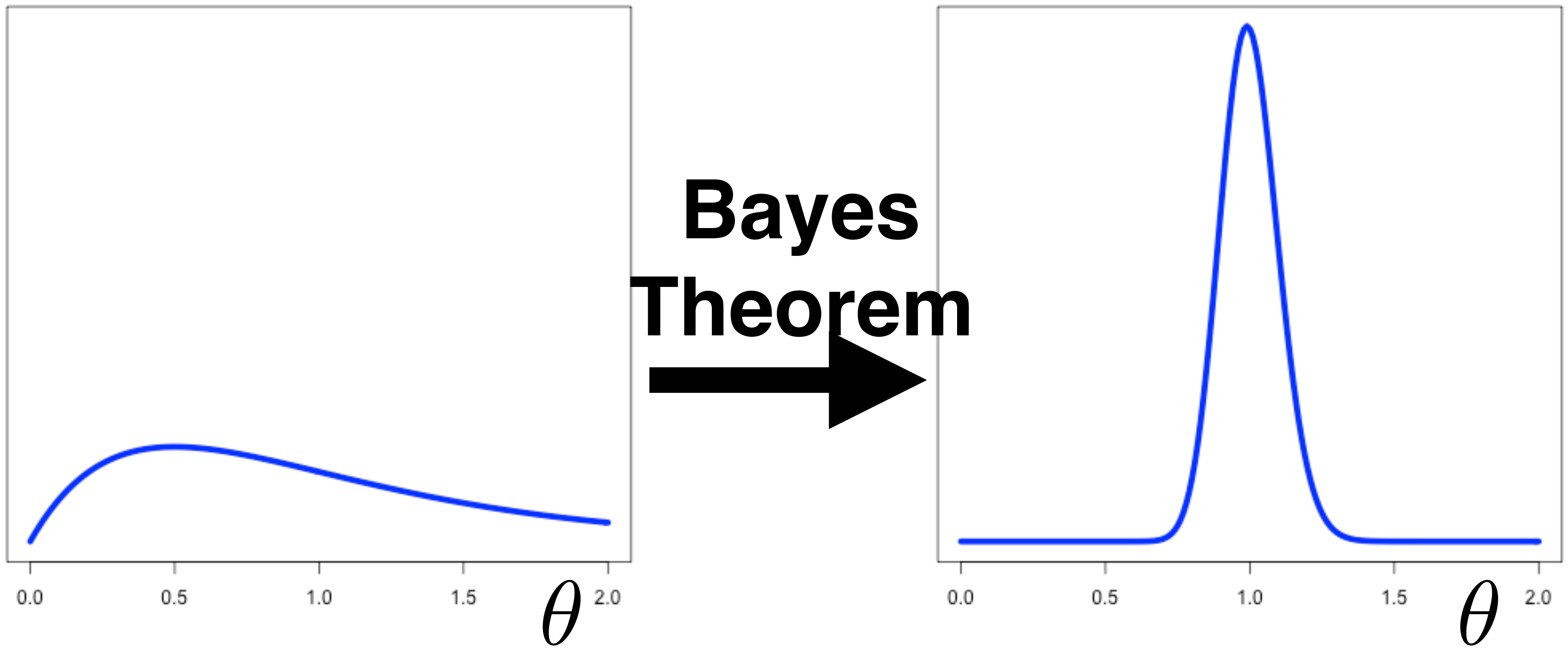

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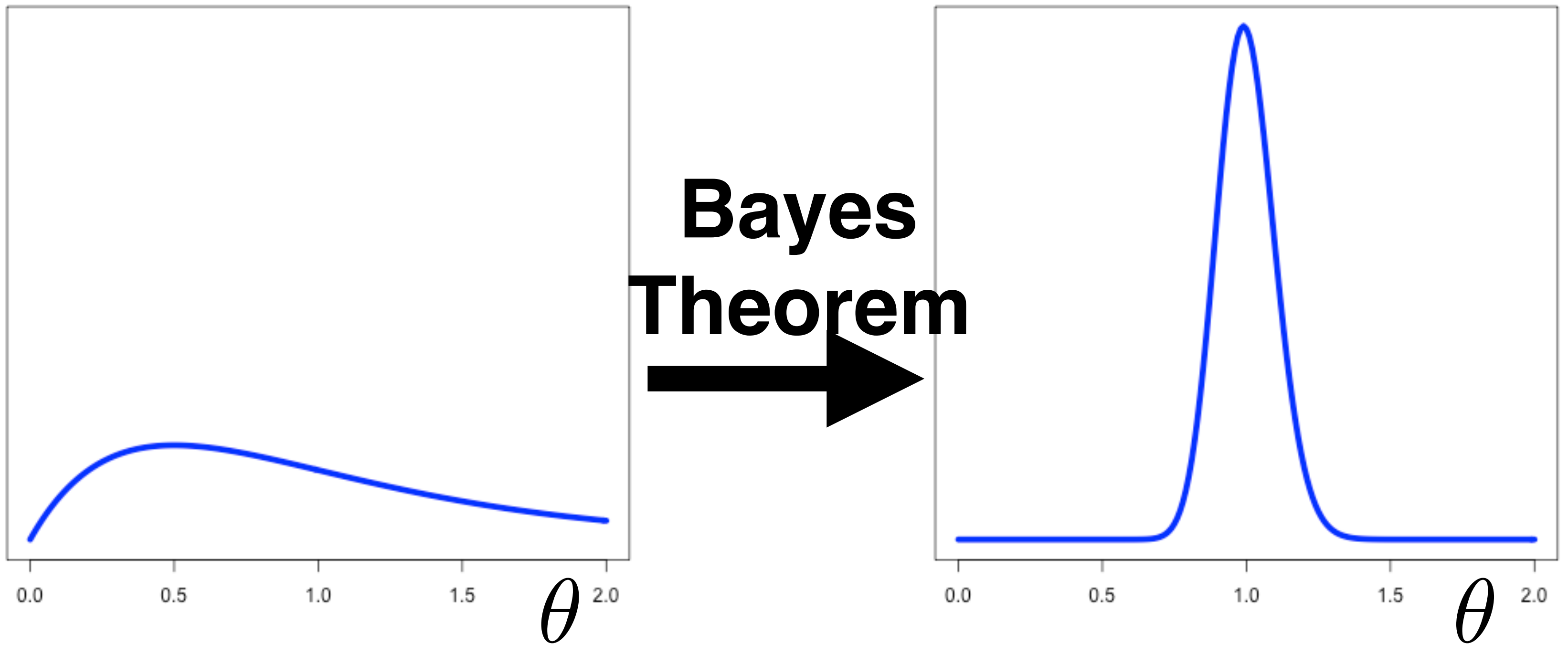

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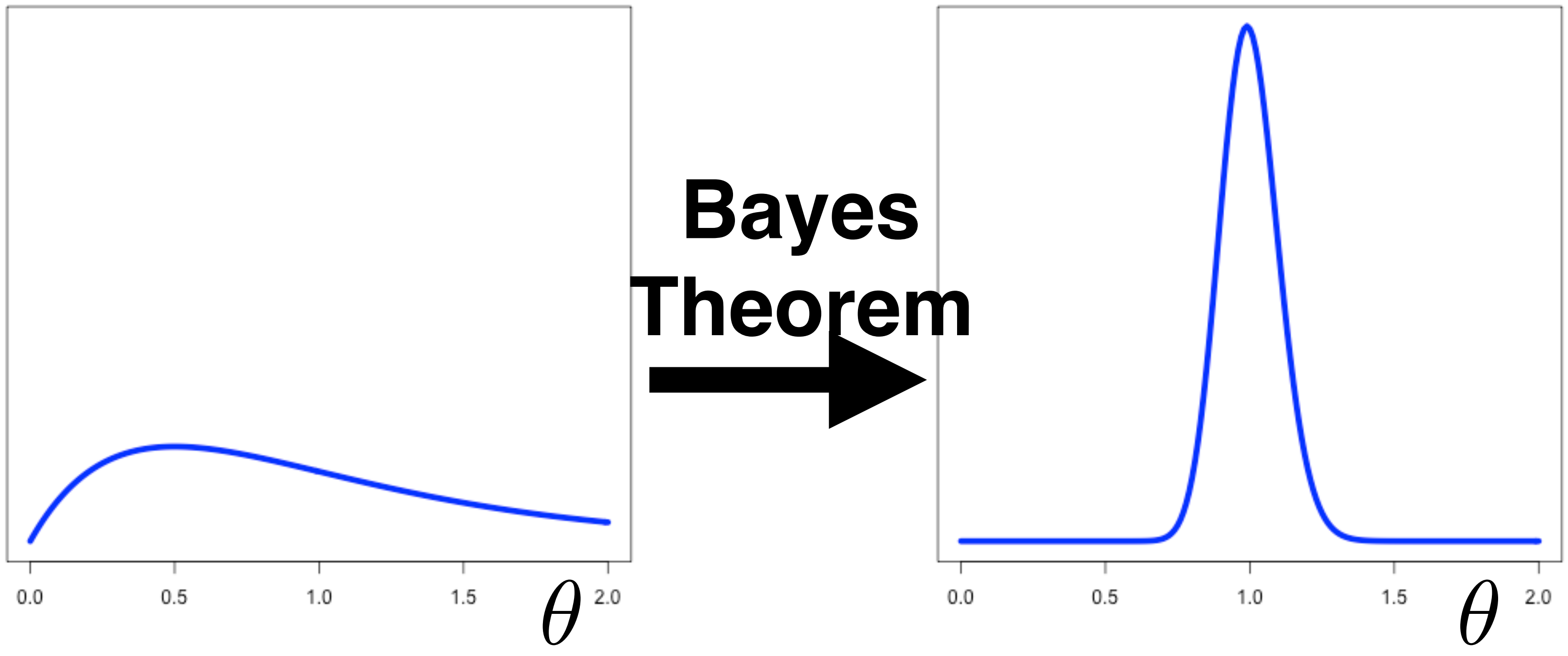
$$p(\theta|y_{1:N}) = p(y_{1:N}|\theta)p(\theta)/\int p(y_{1:N},\theta)d\theta$$

posterior likelihood prior evidence



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Approximate Bayesian Inference

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- Gold standard: Markov Chain Monte Carlo (MCMC)

[Bardenet,
Doucet,
Holmes
2017]

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  - Eventually accurate but can be slow

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Instead: an optimization approach

- Approximate posterior with *q**

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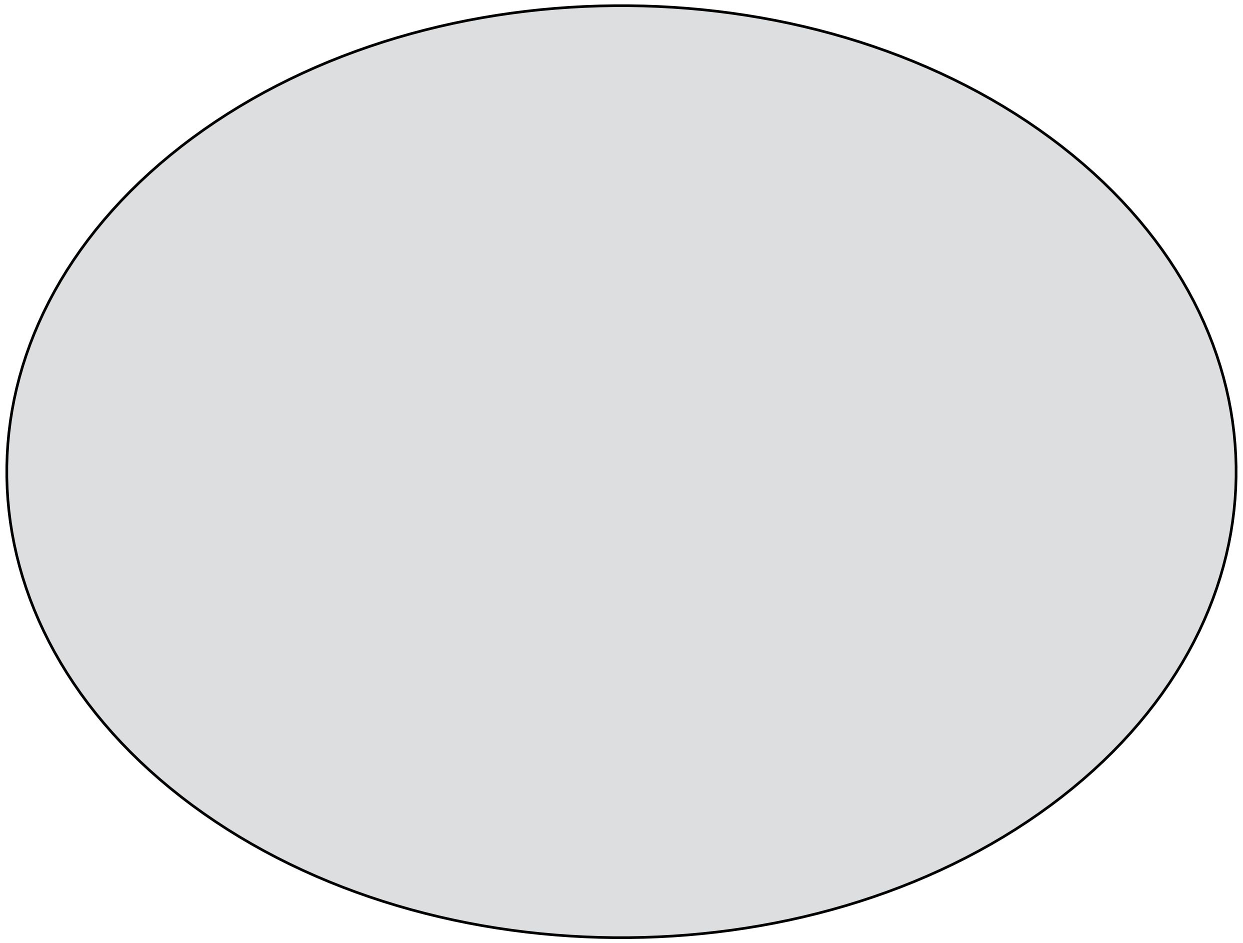

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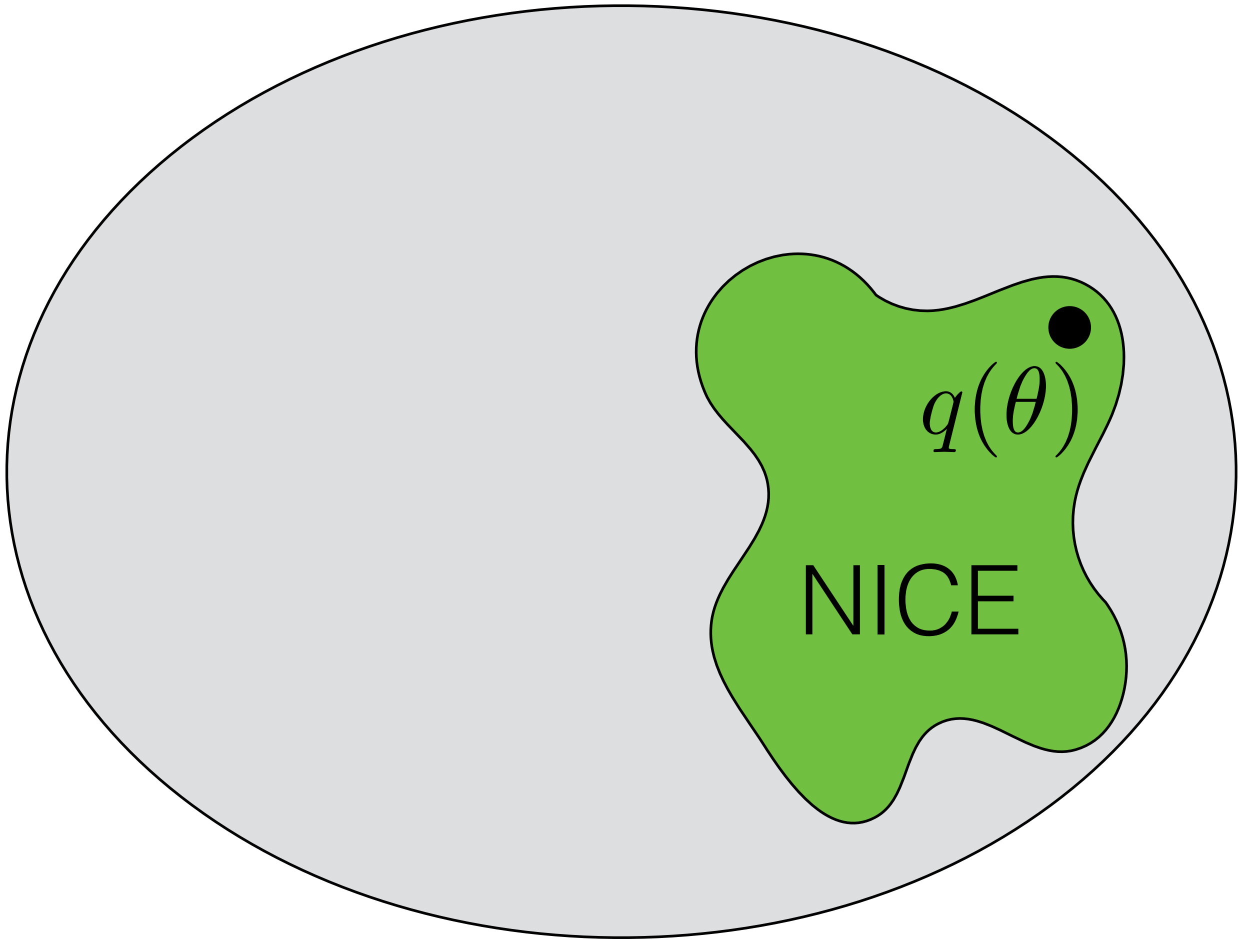

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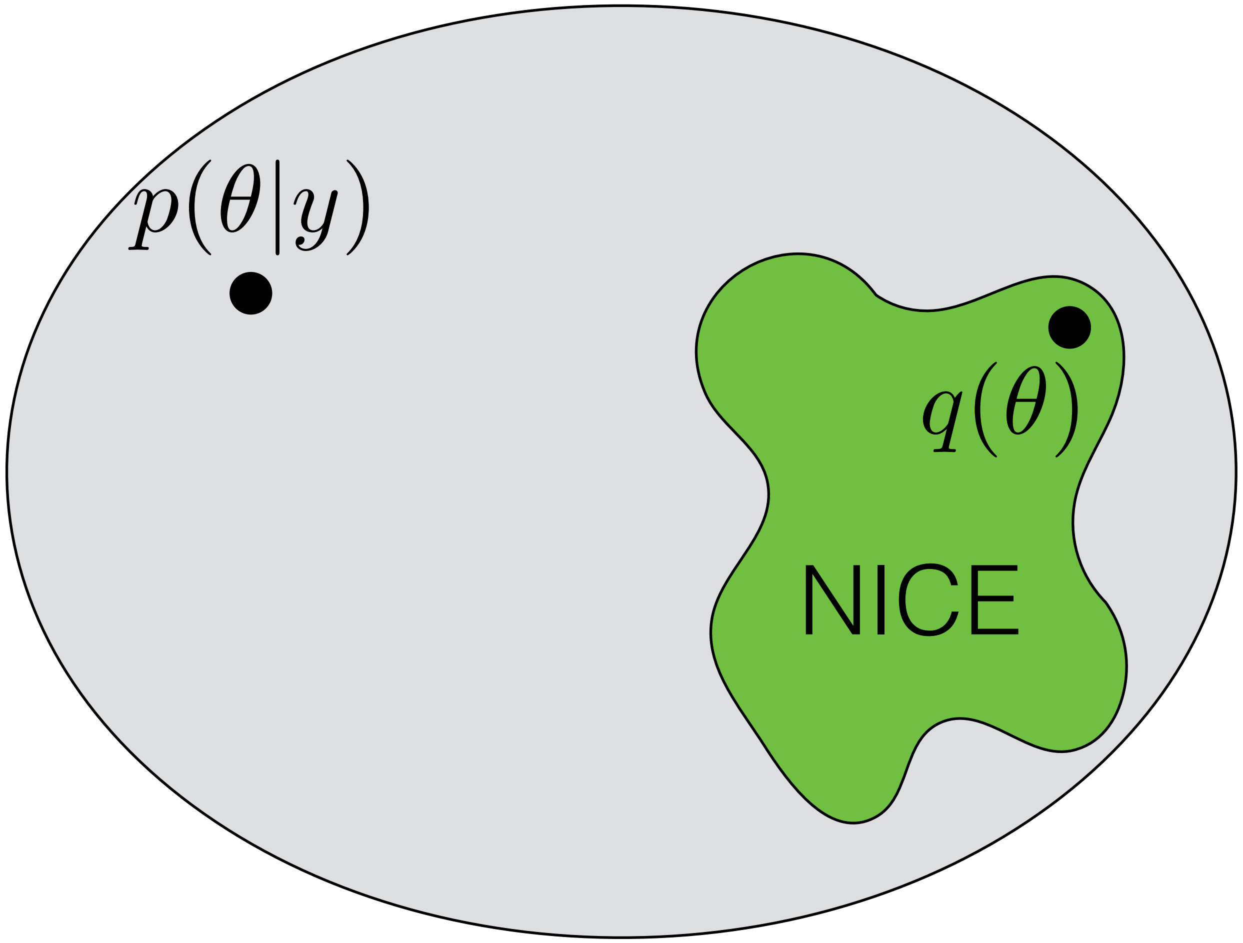

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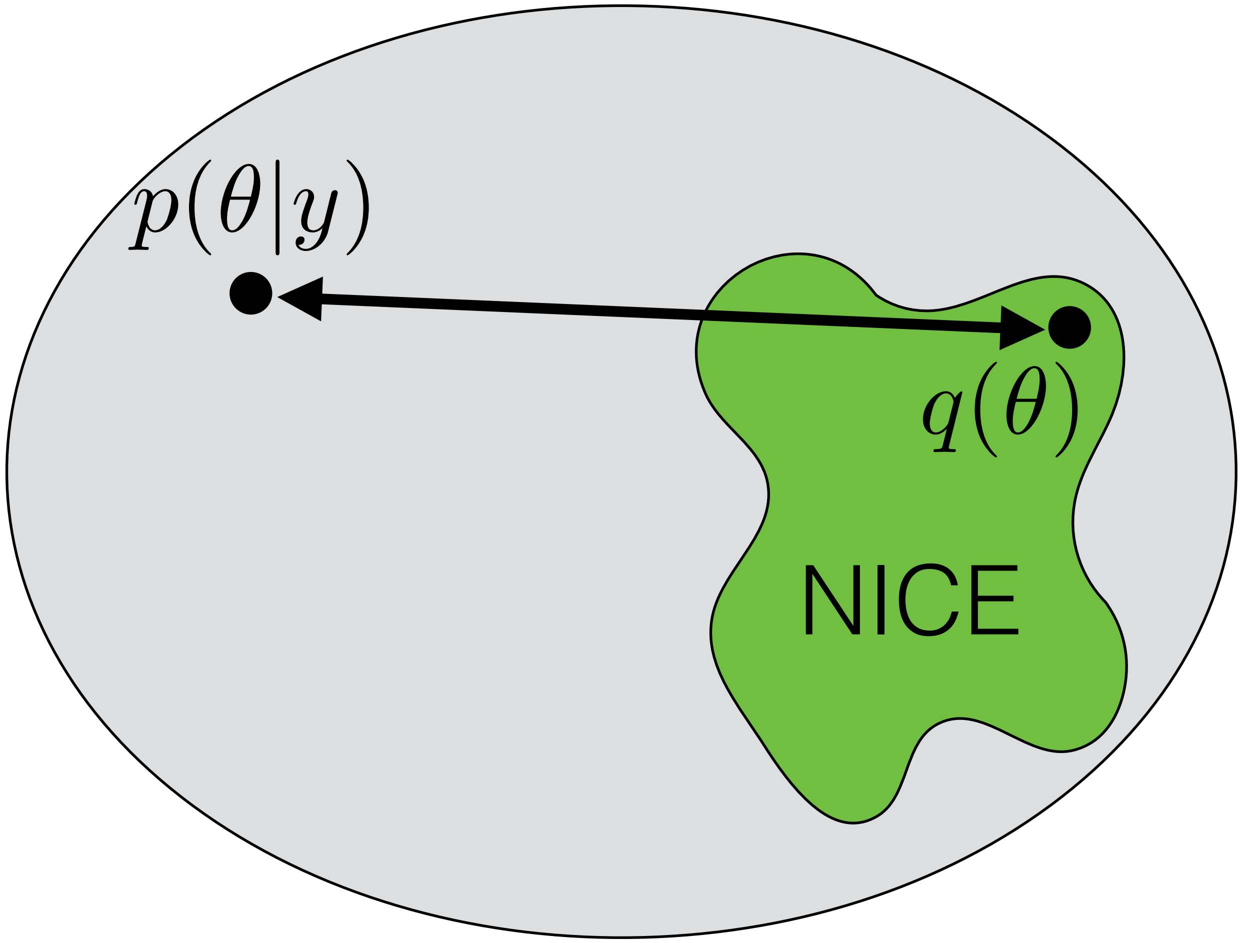

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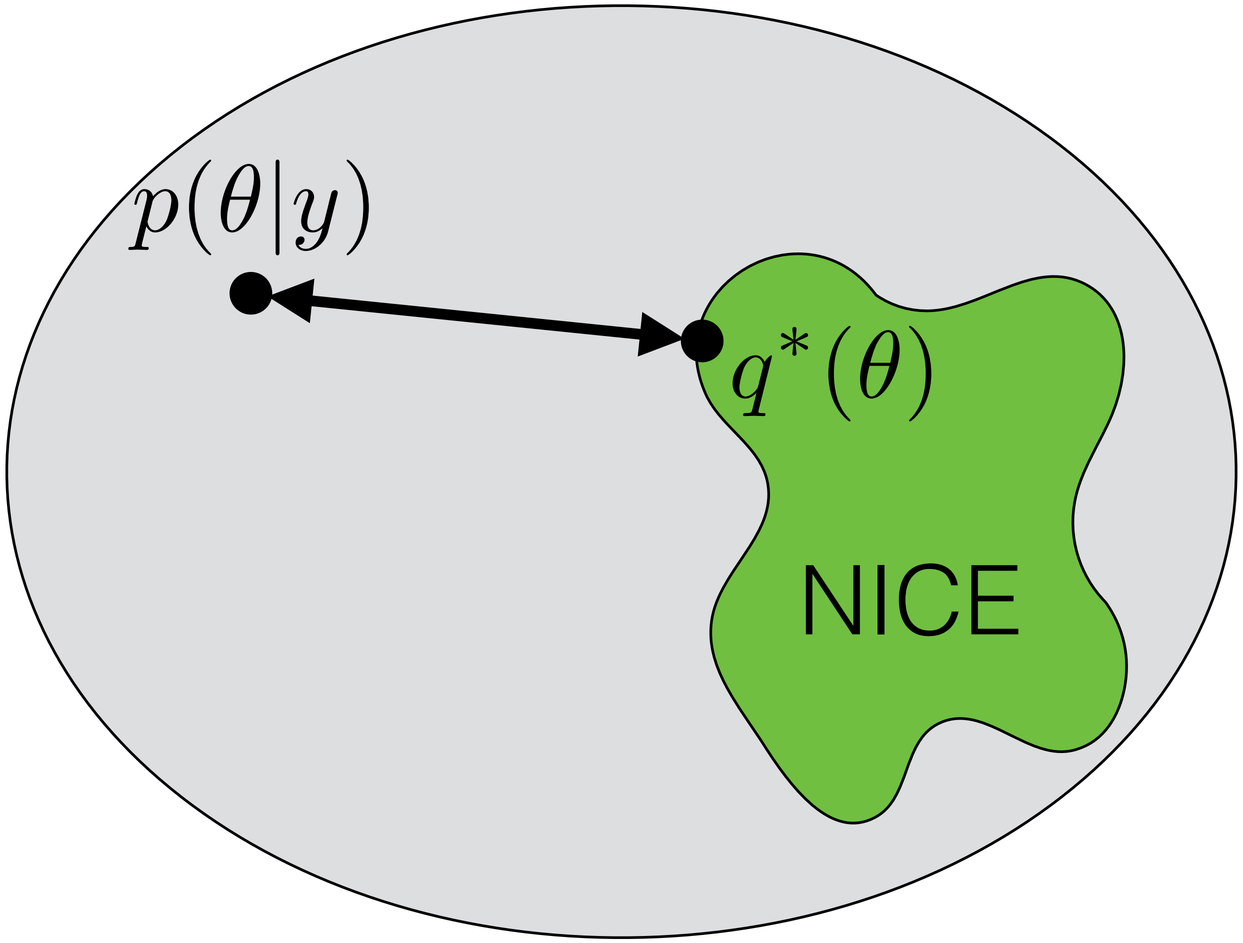

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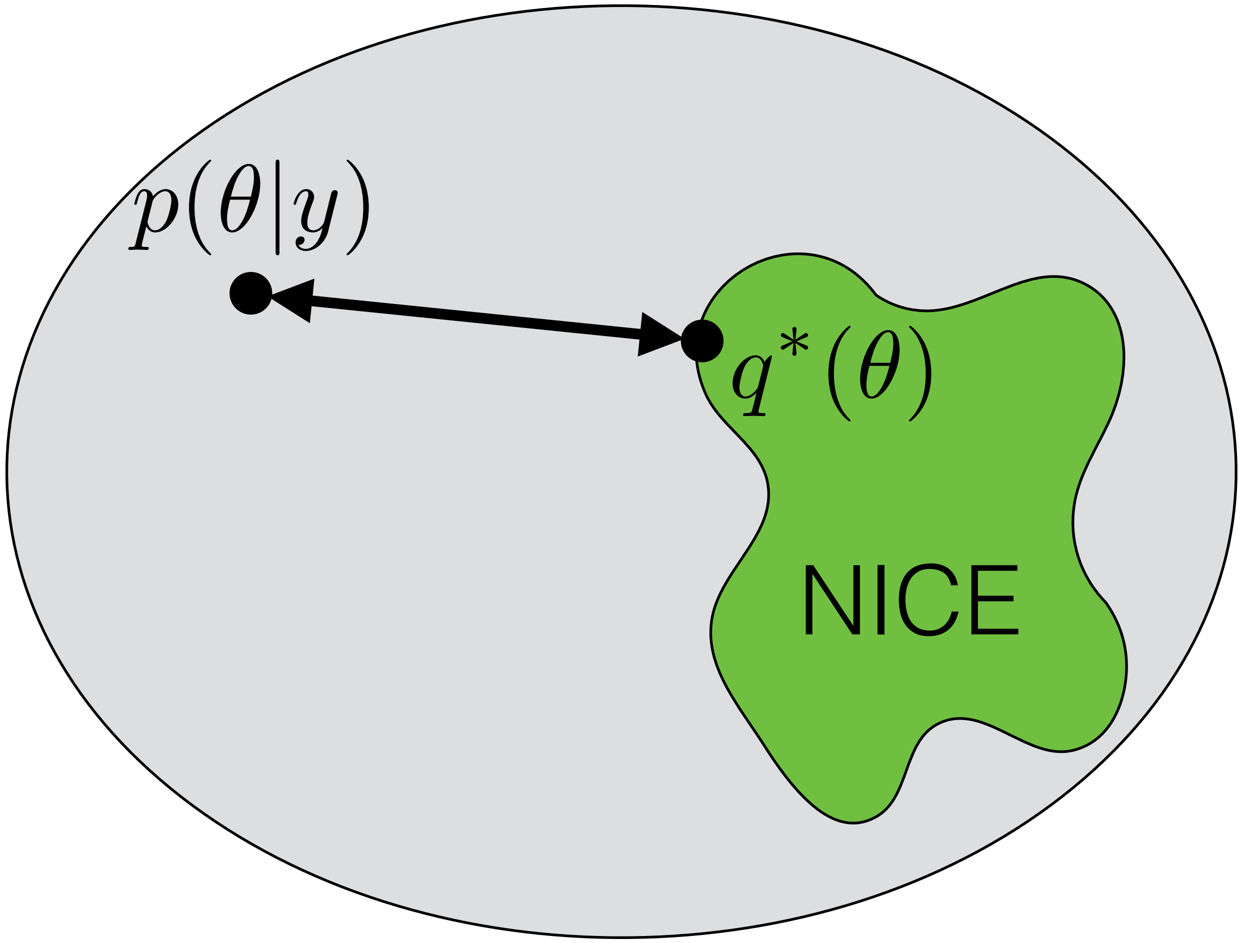

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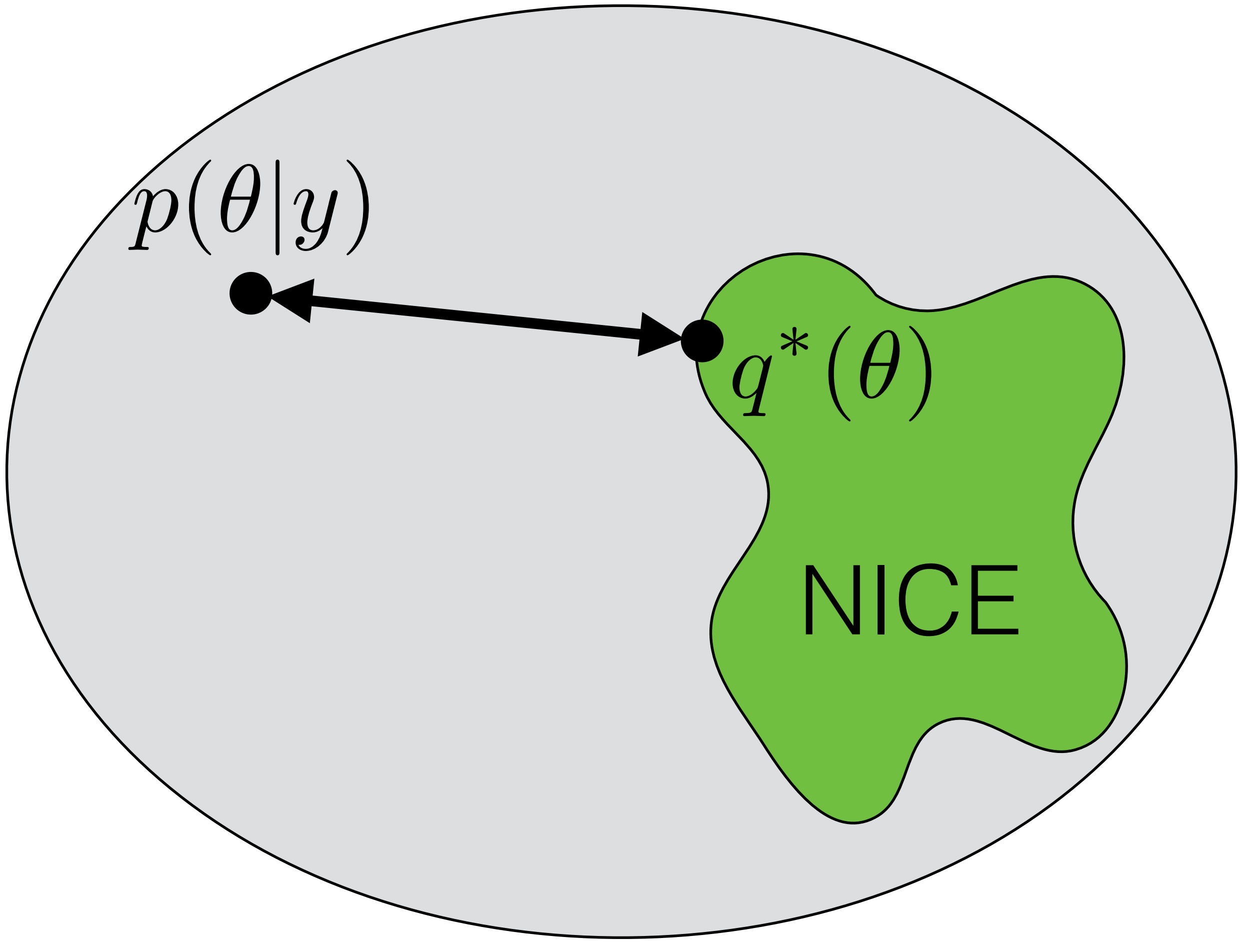

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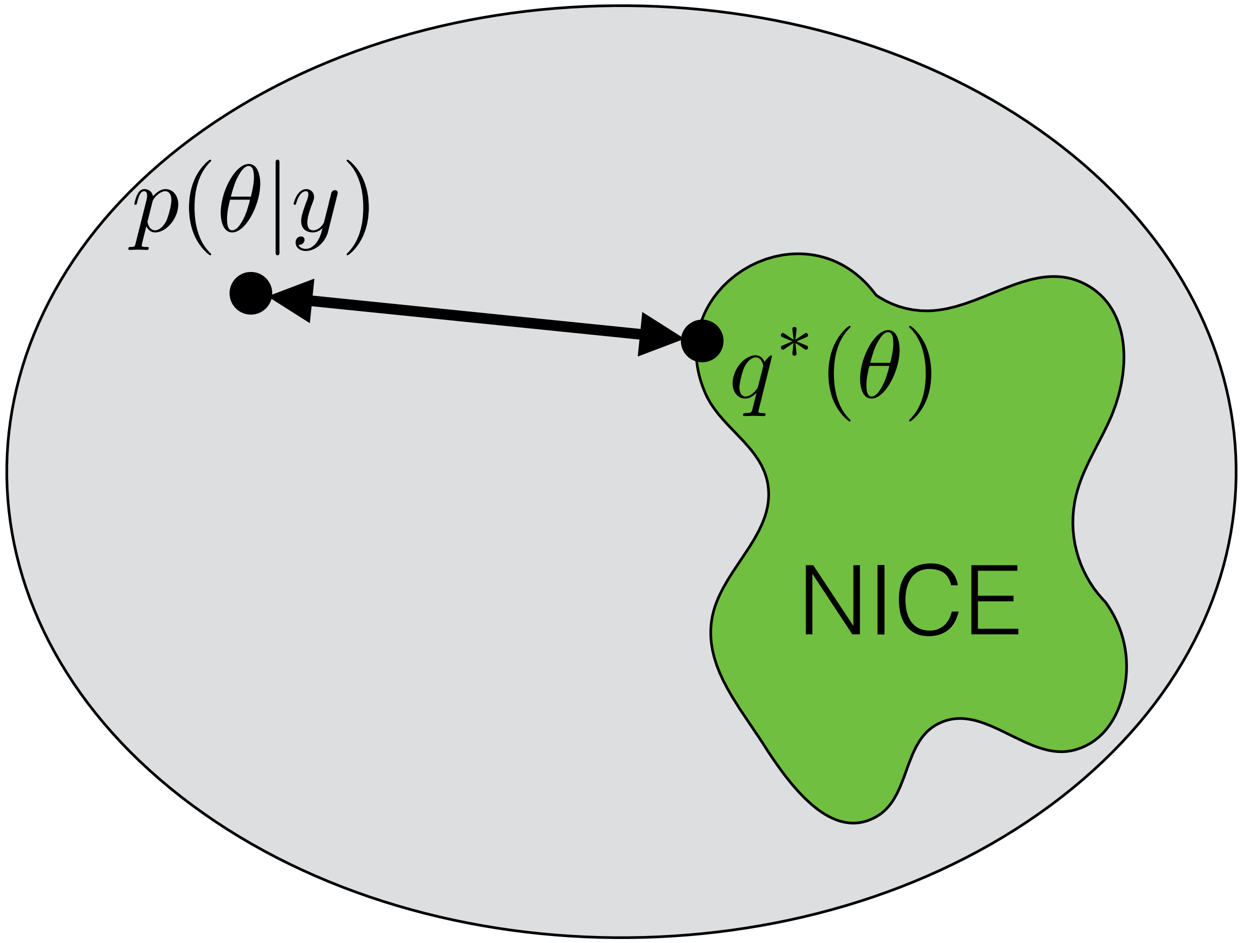

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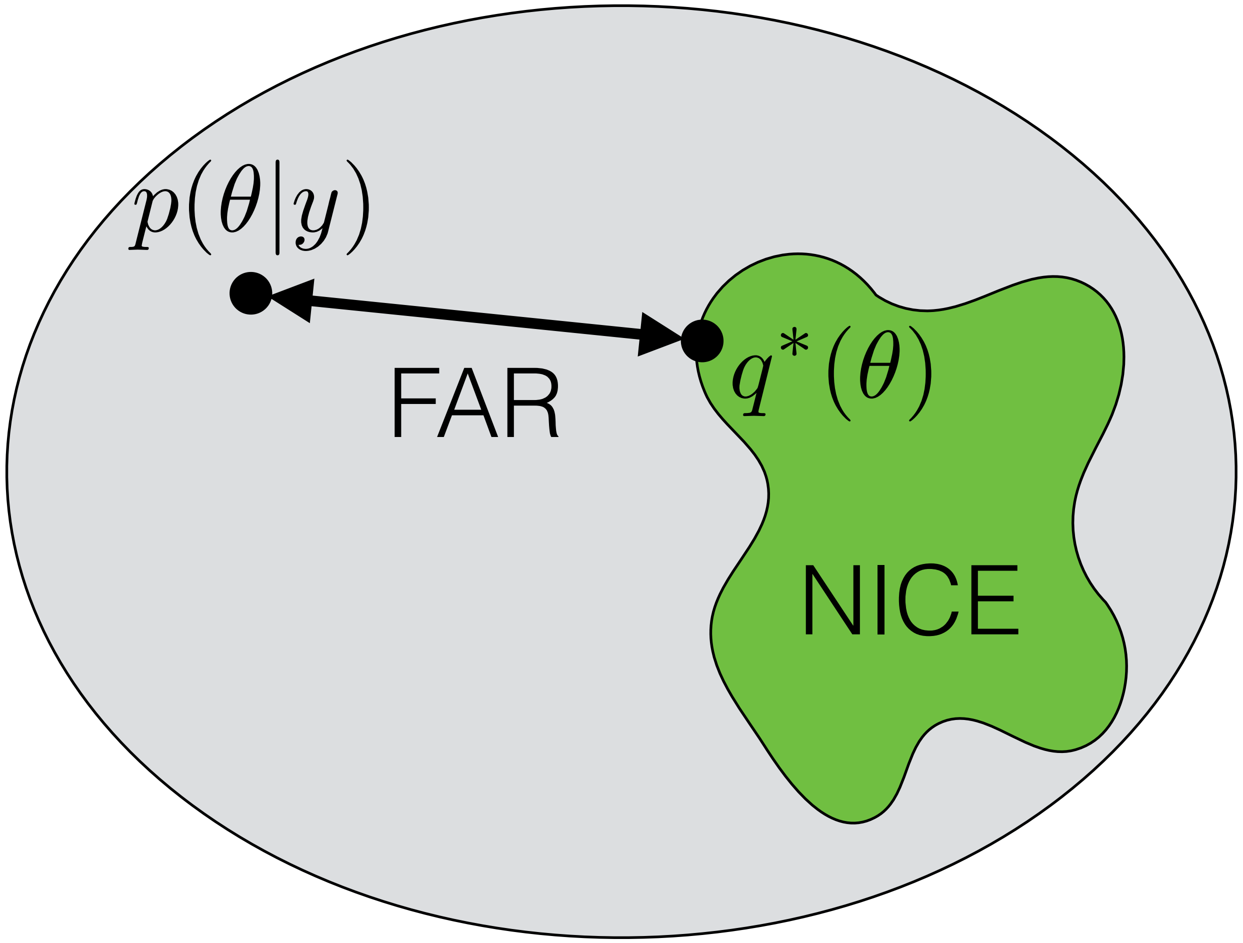

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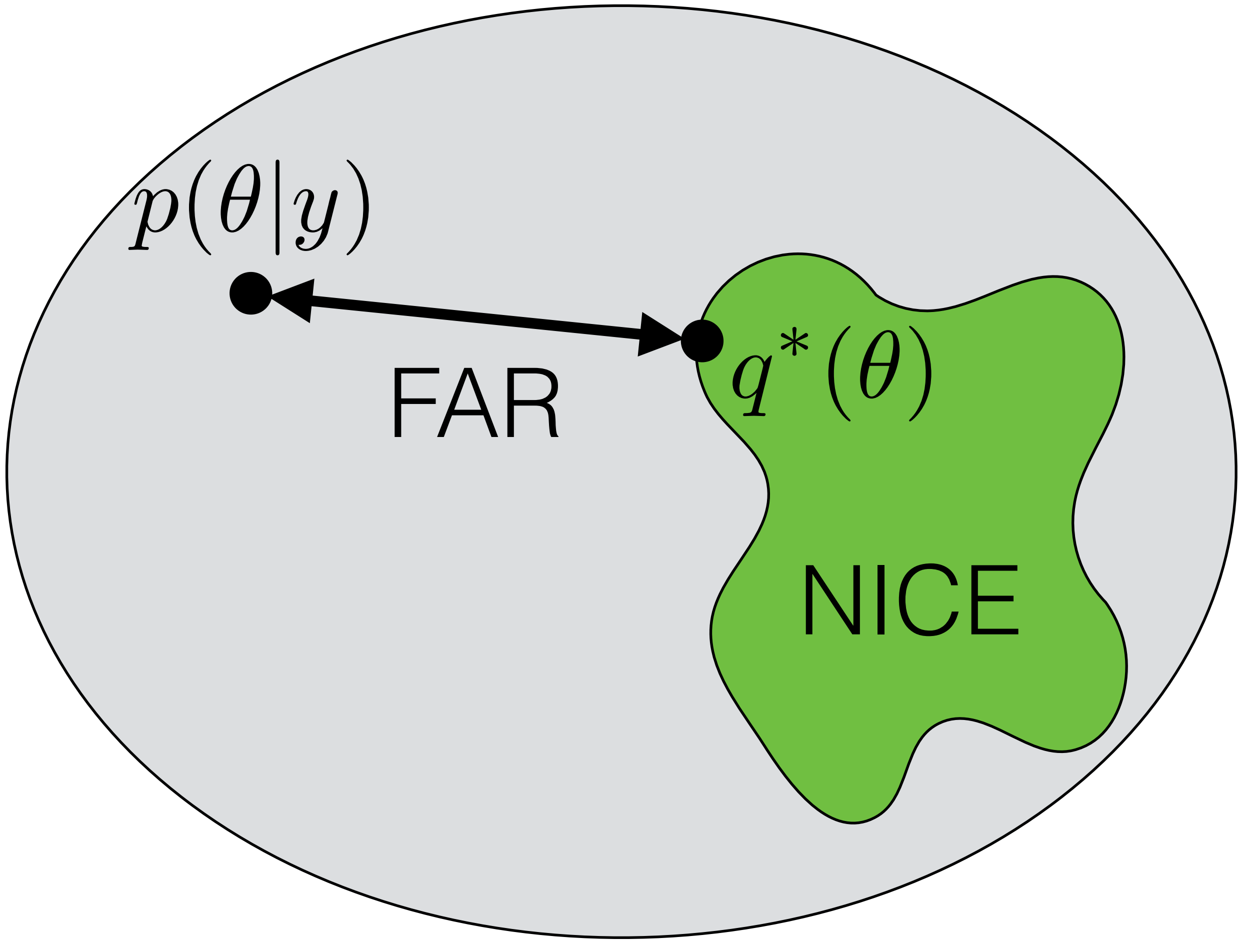

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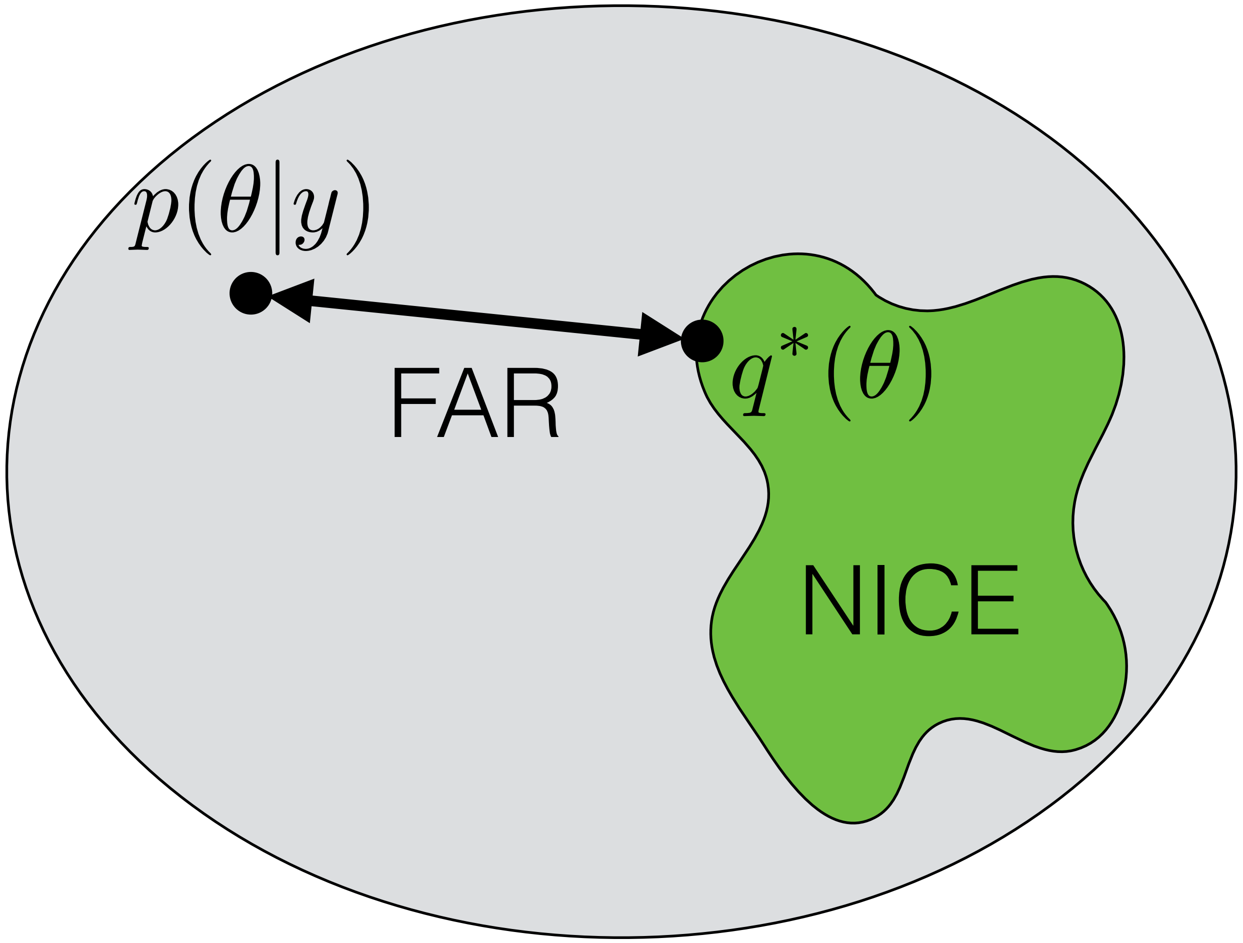

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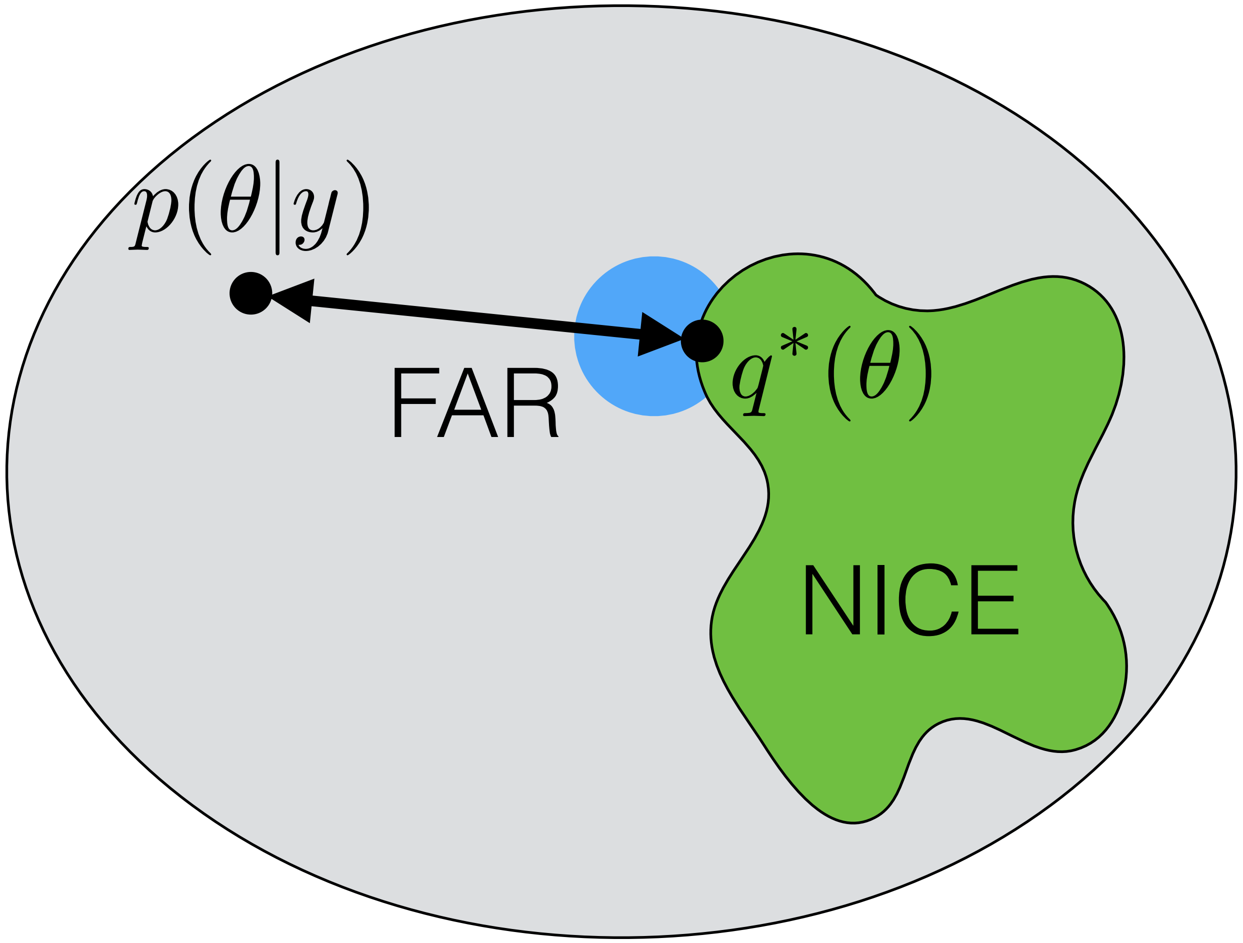

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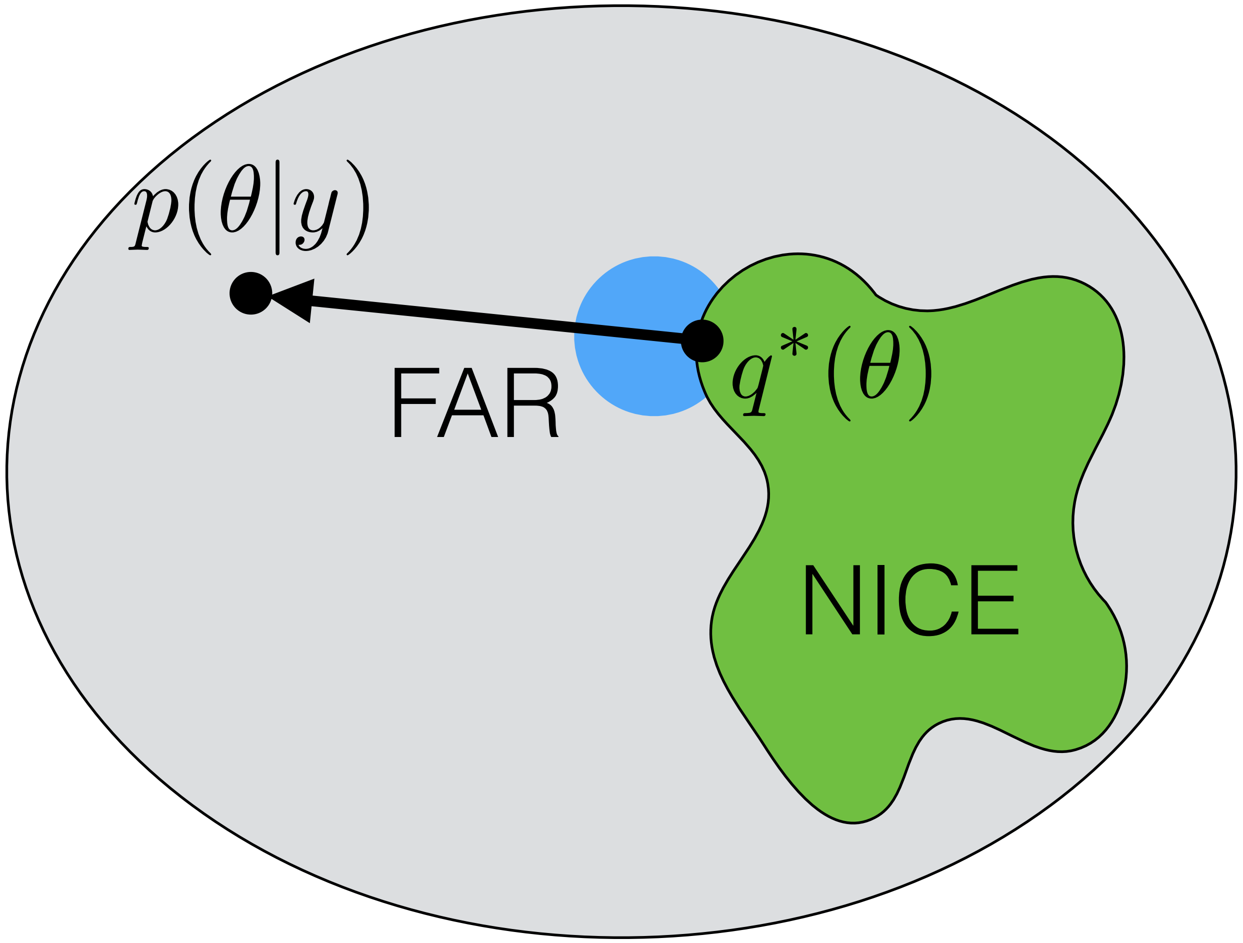

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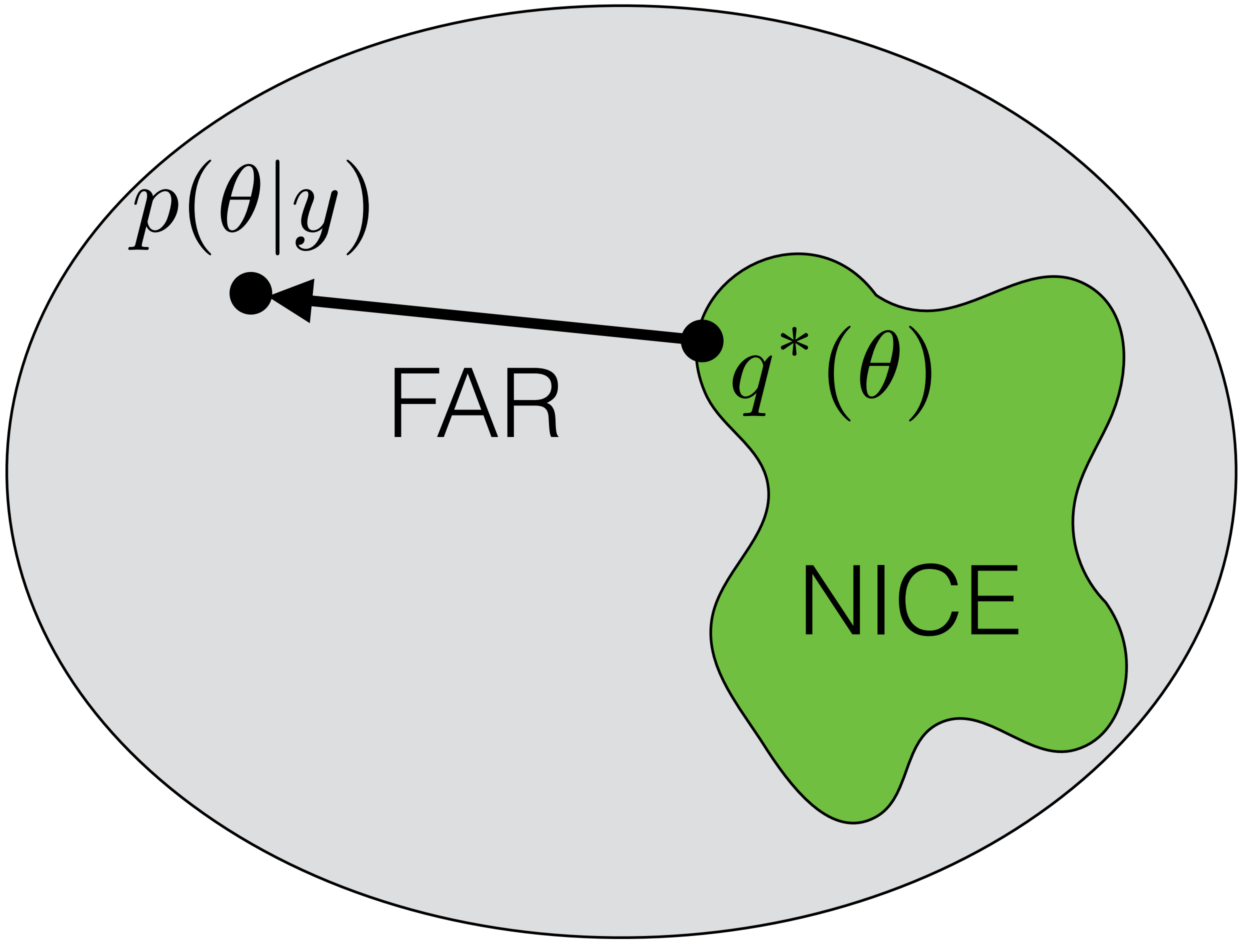

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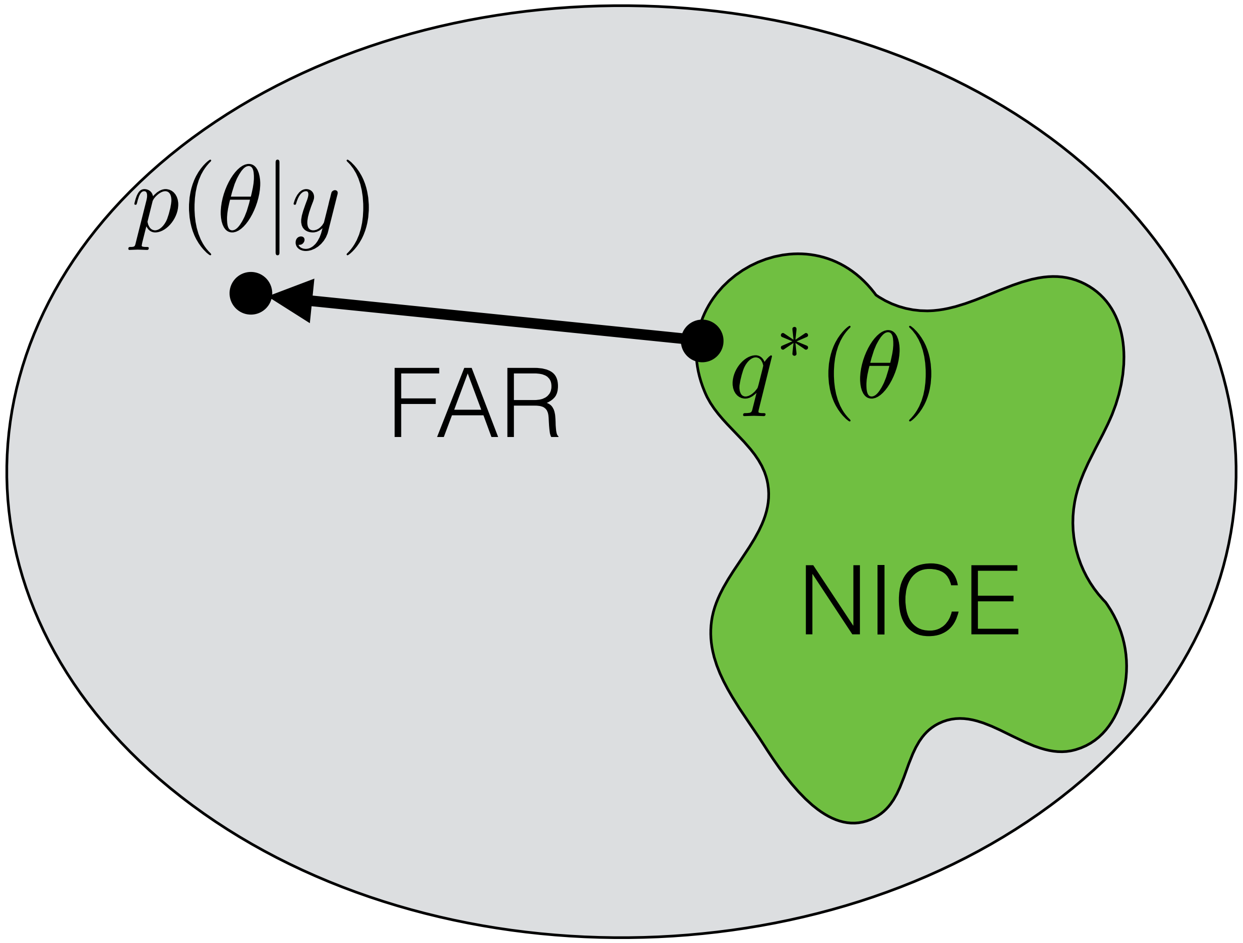

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$$KL(q(\cdot)||p(\cdot|y))$$

- VB practical success

Approximate Bayesian Inference

[Bardenet, Doucet, Holmes 2017]

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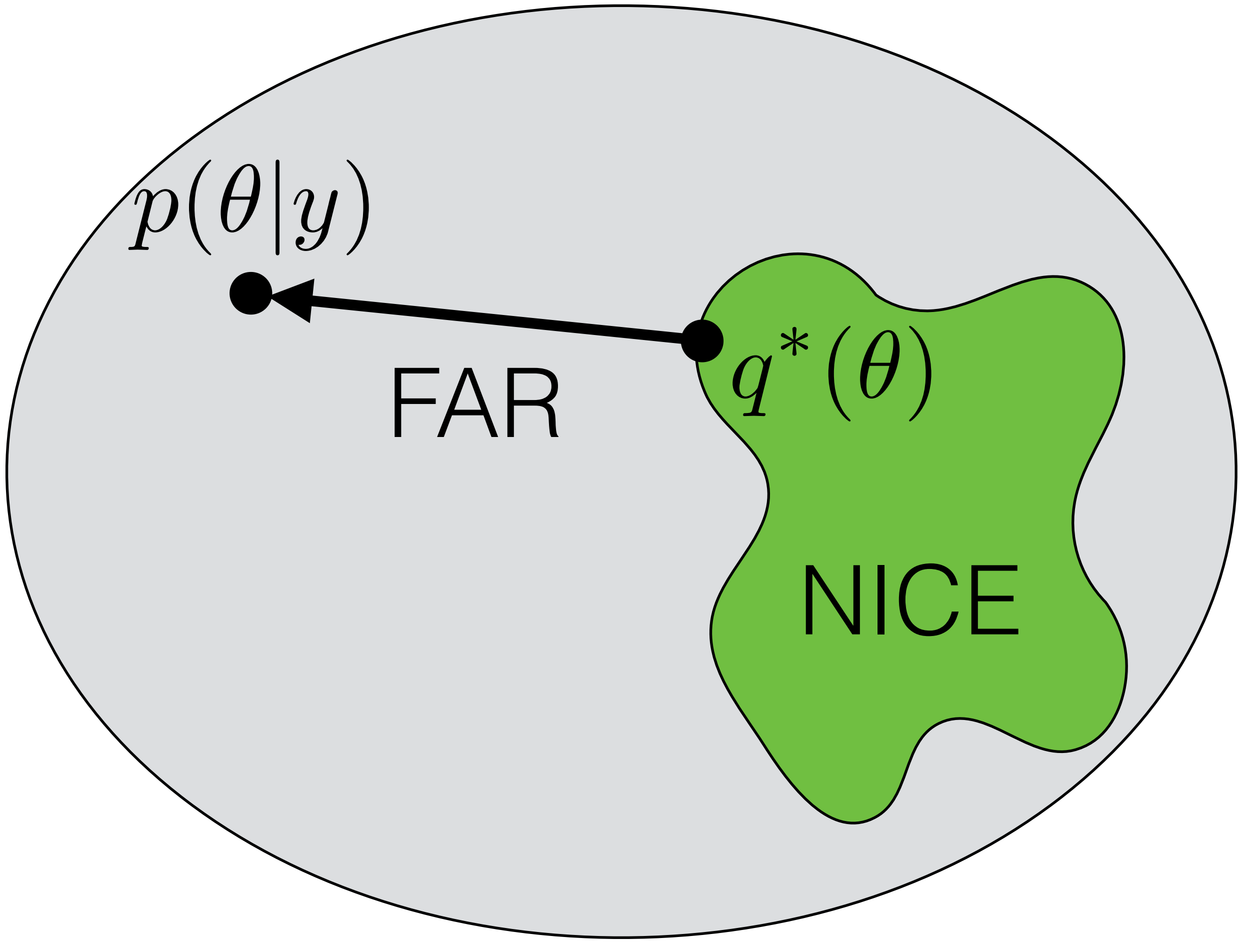

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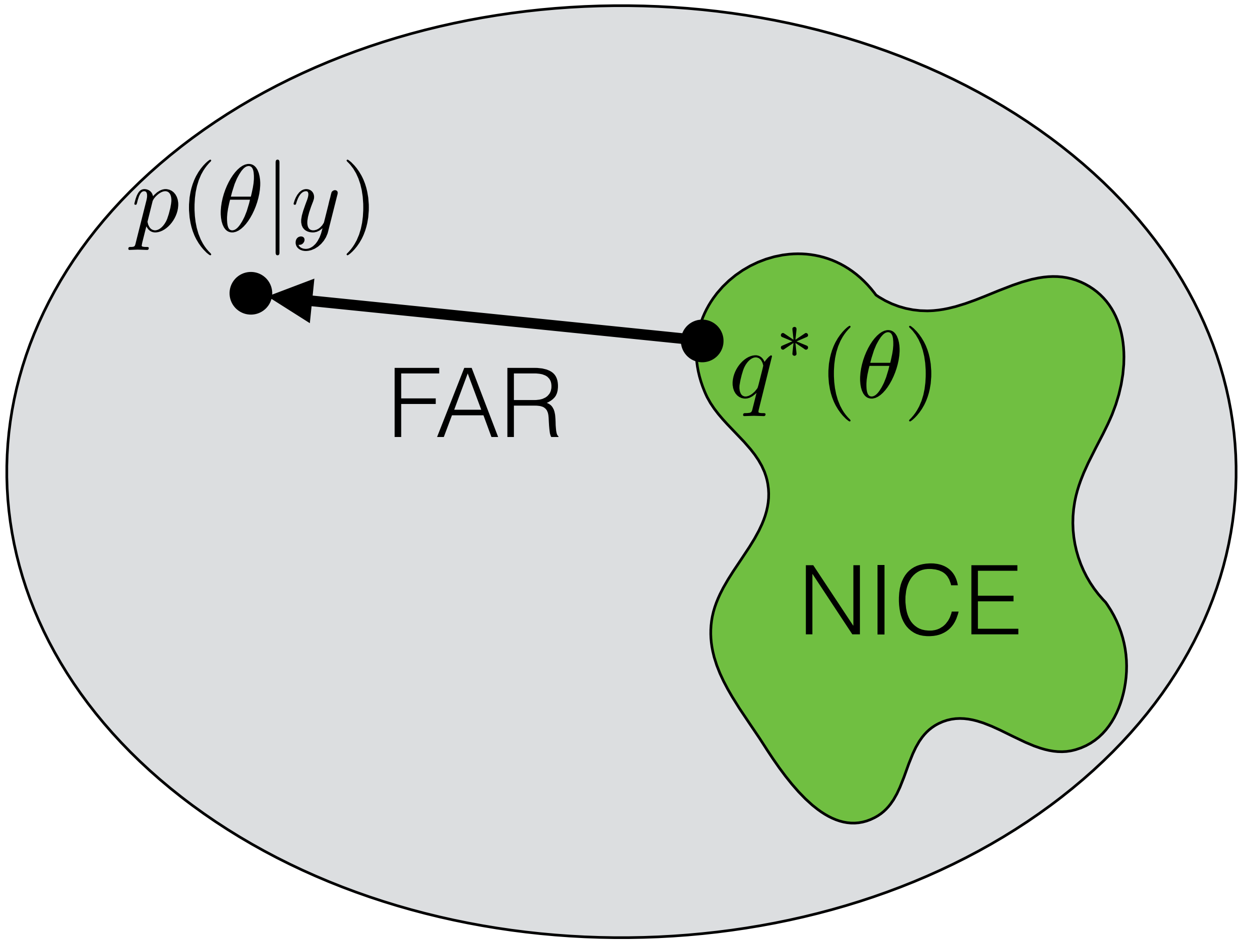

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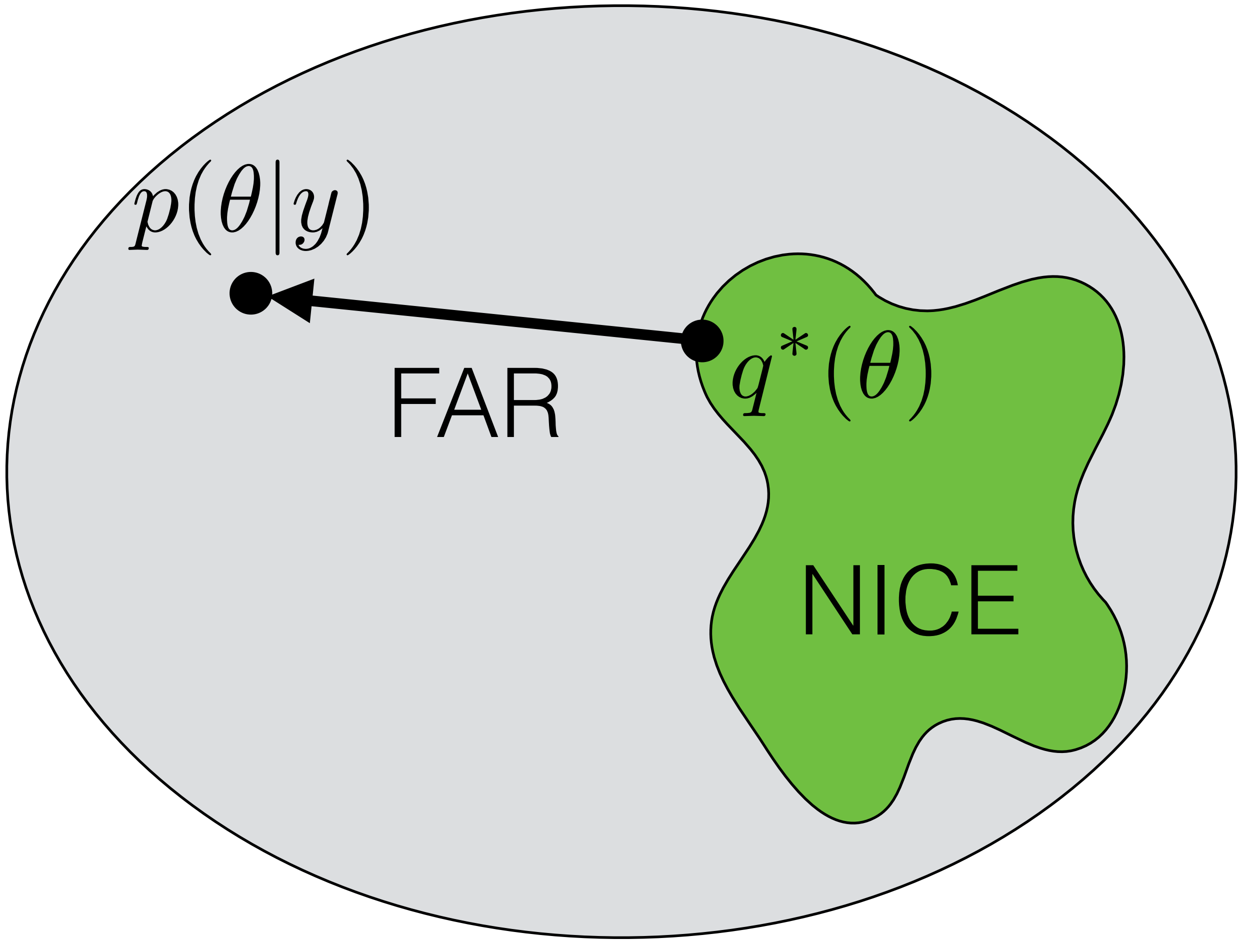

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[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

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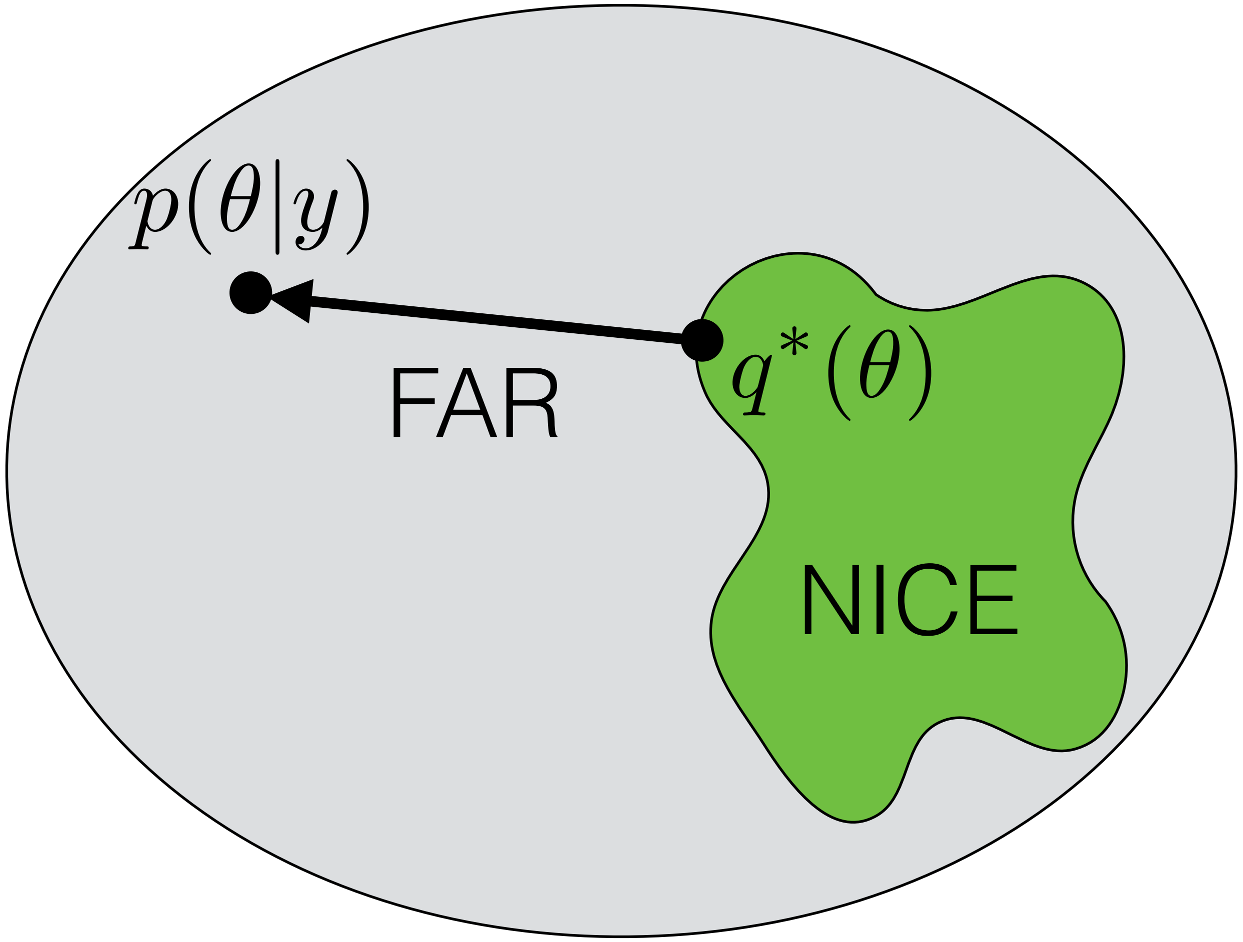

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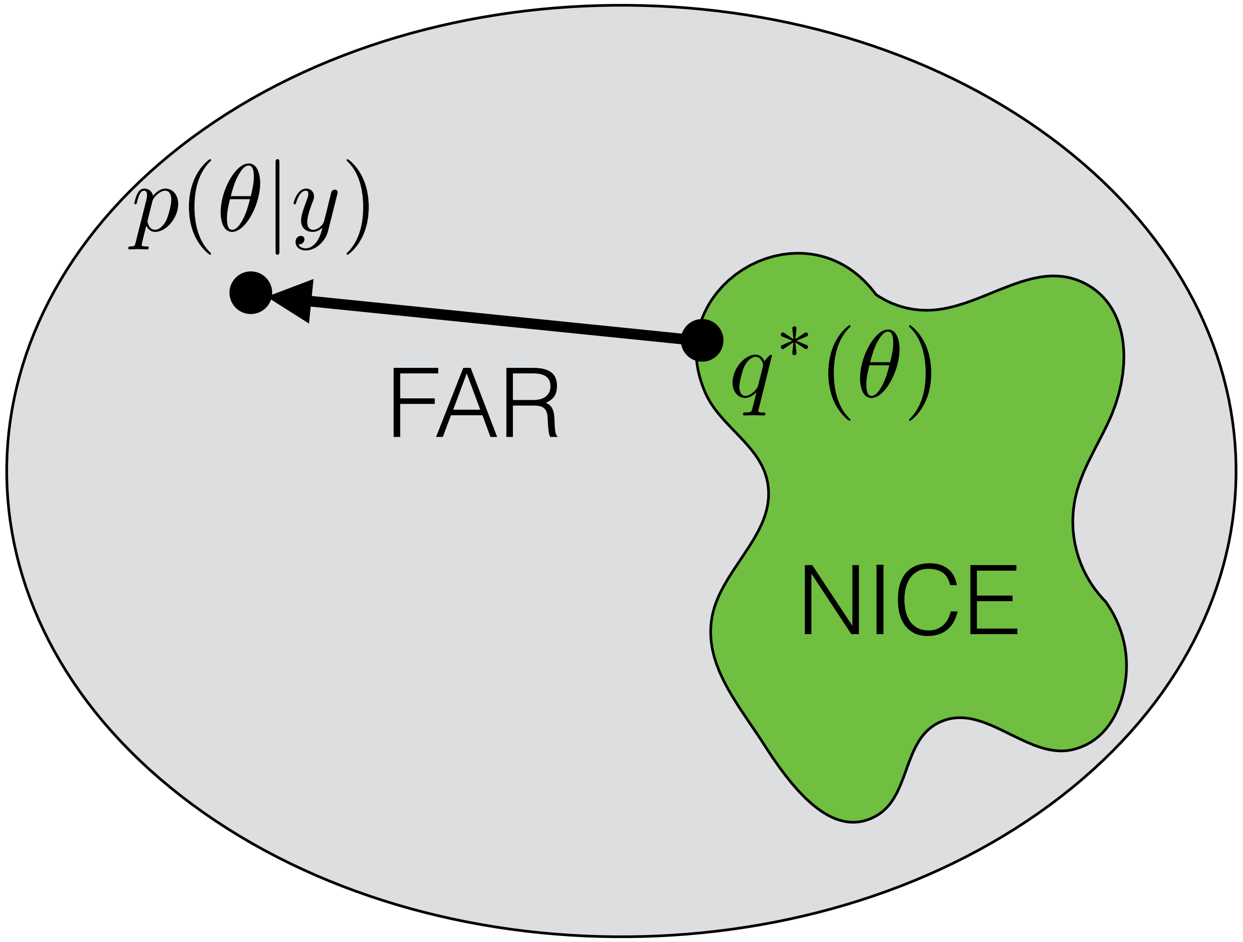

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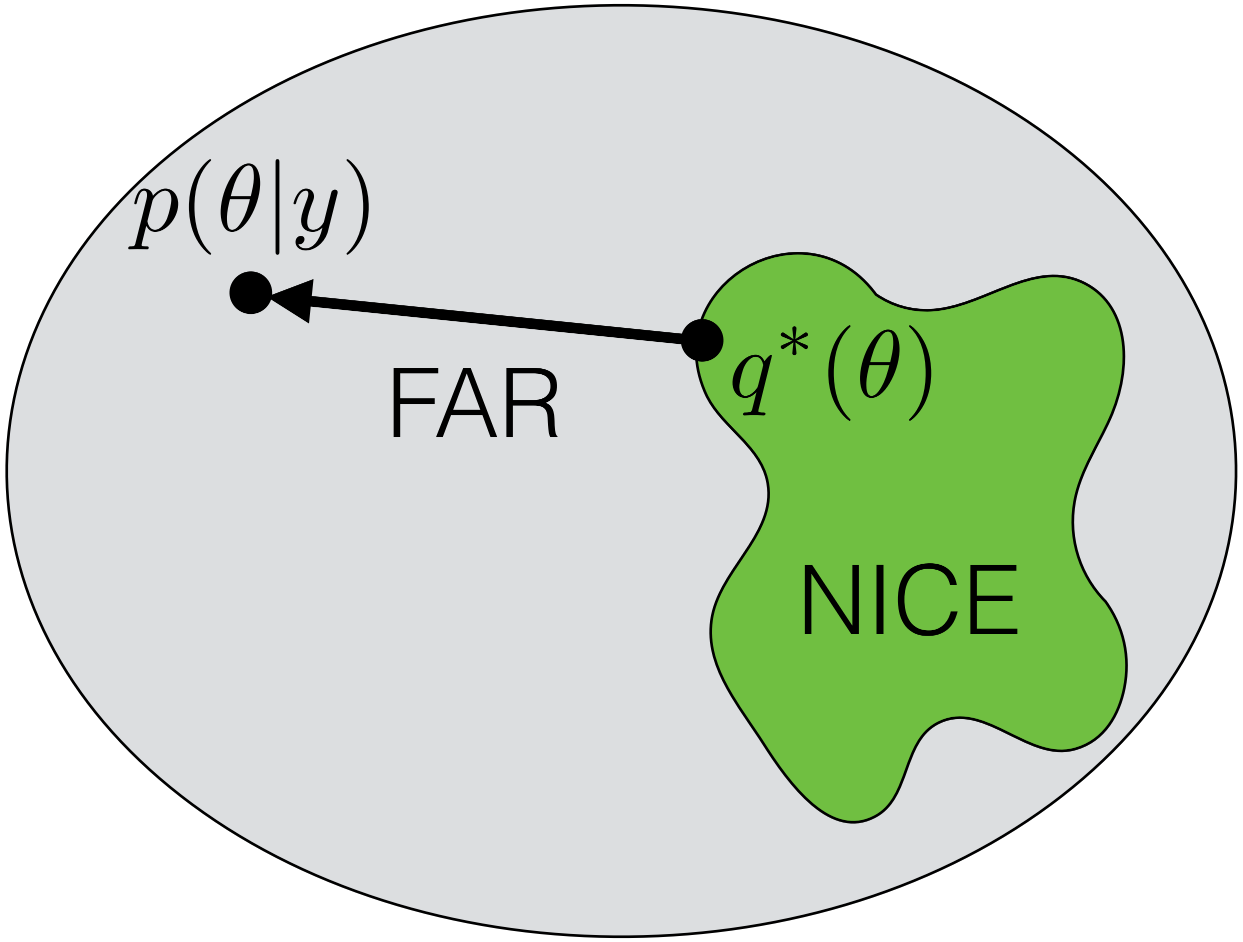

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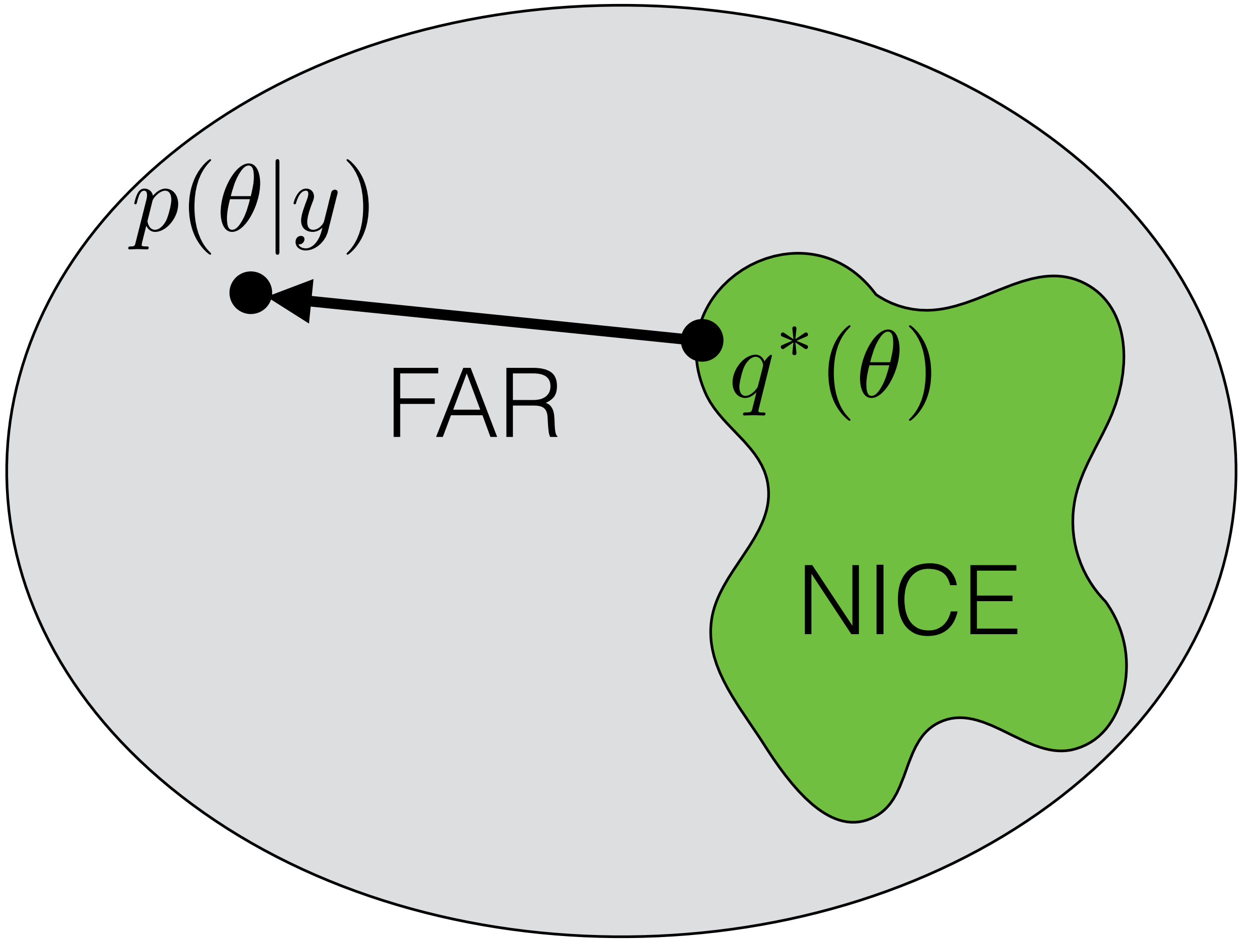

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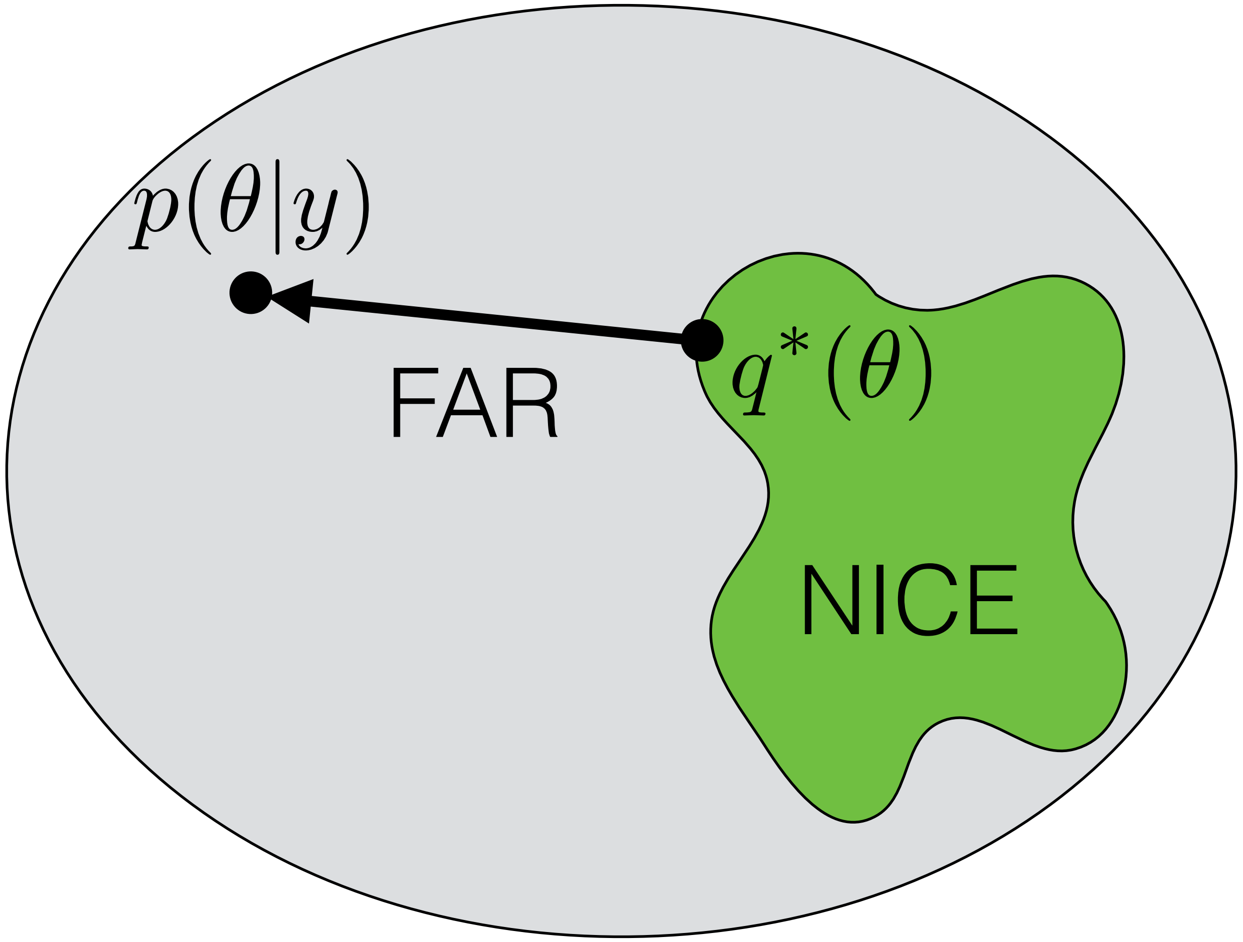

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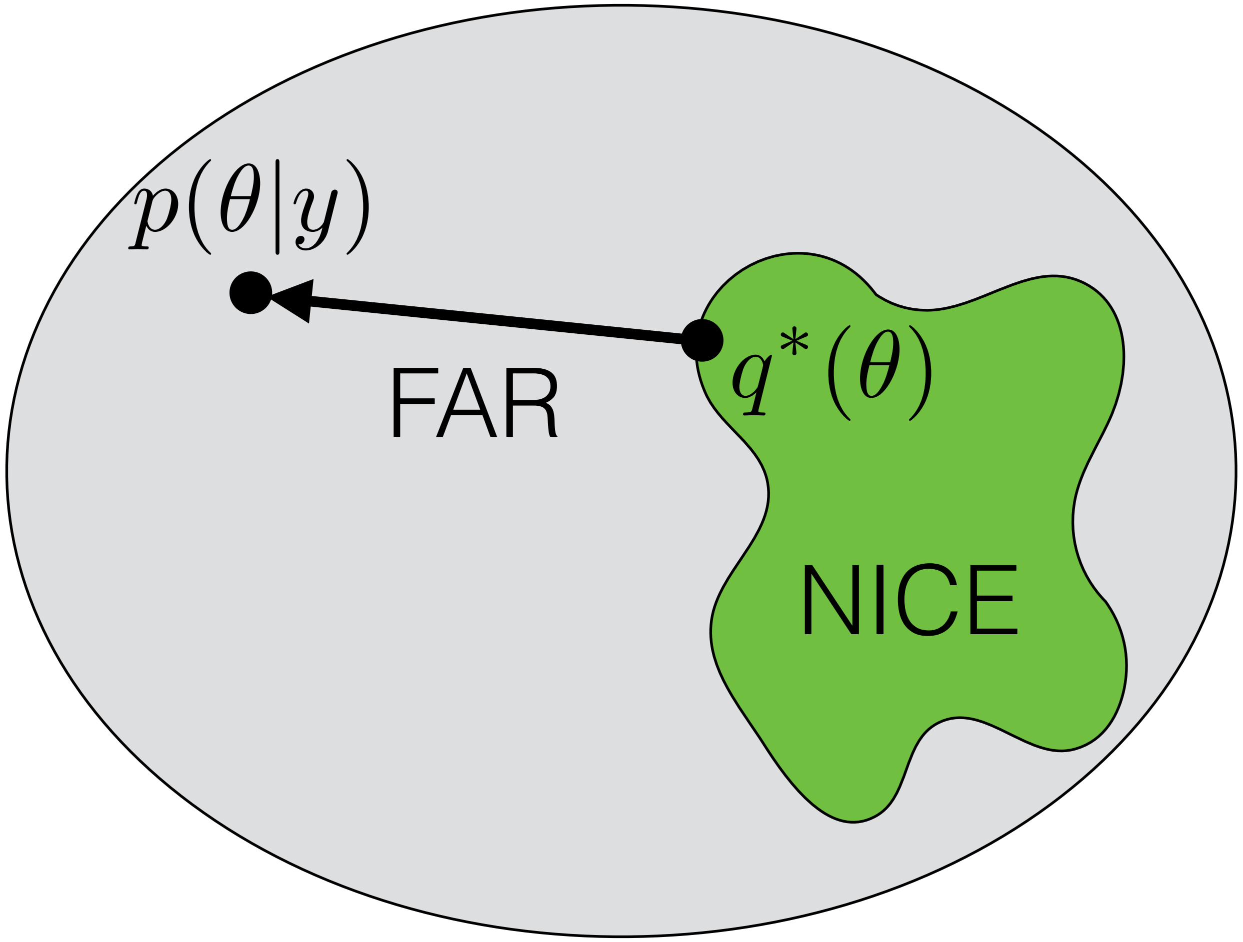

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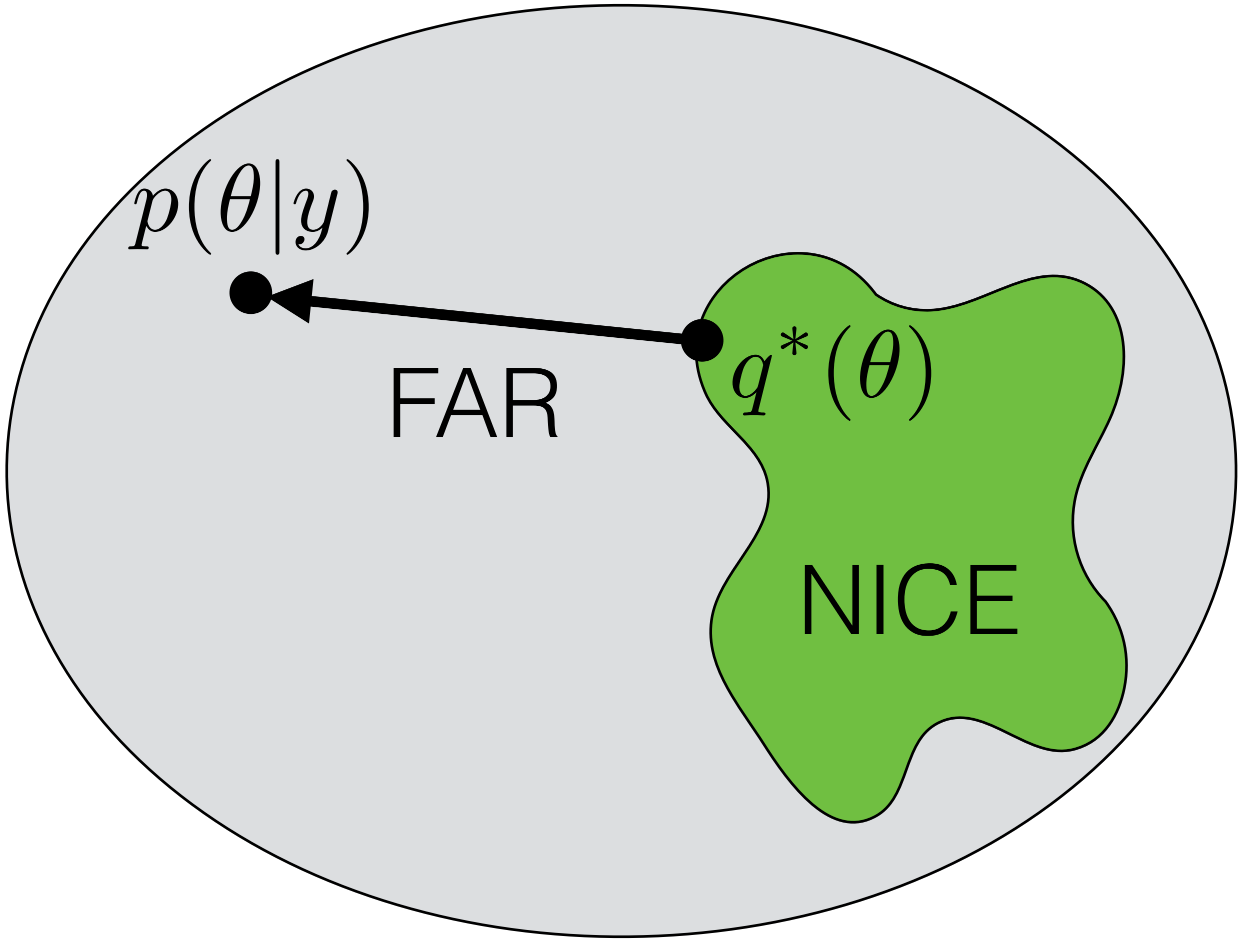

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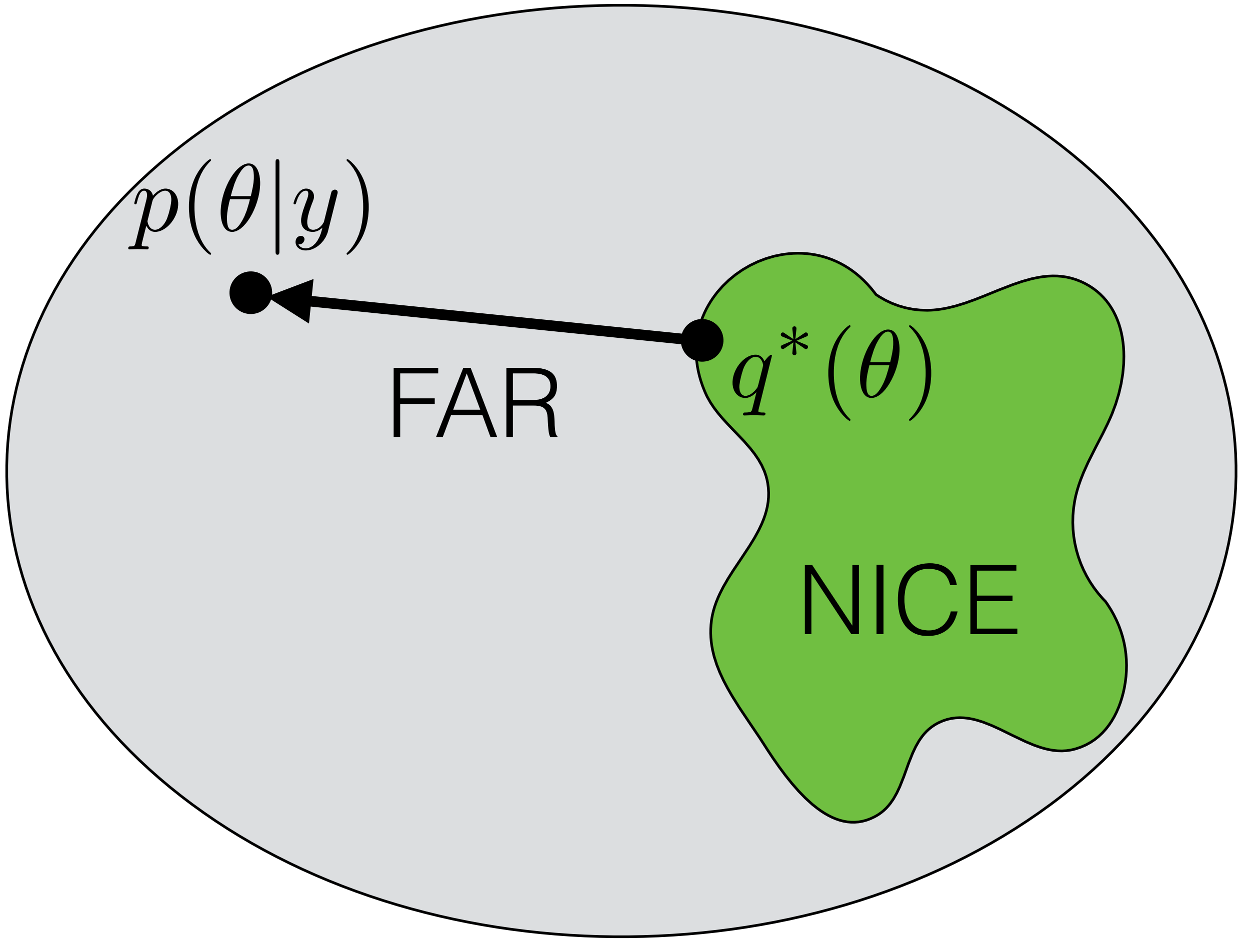

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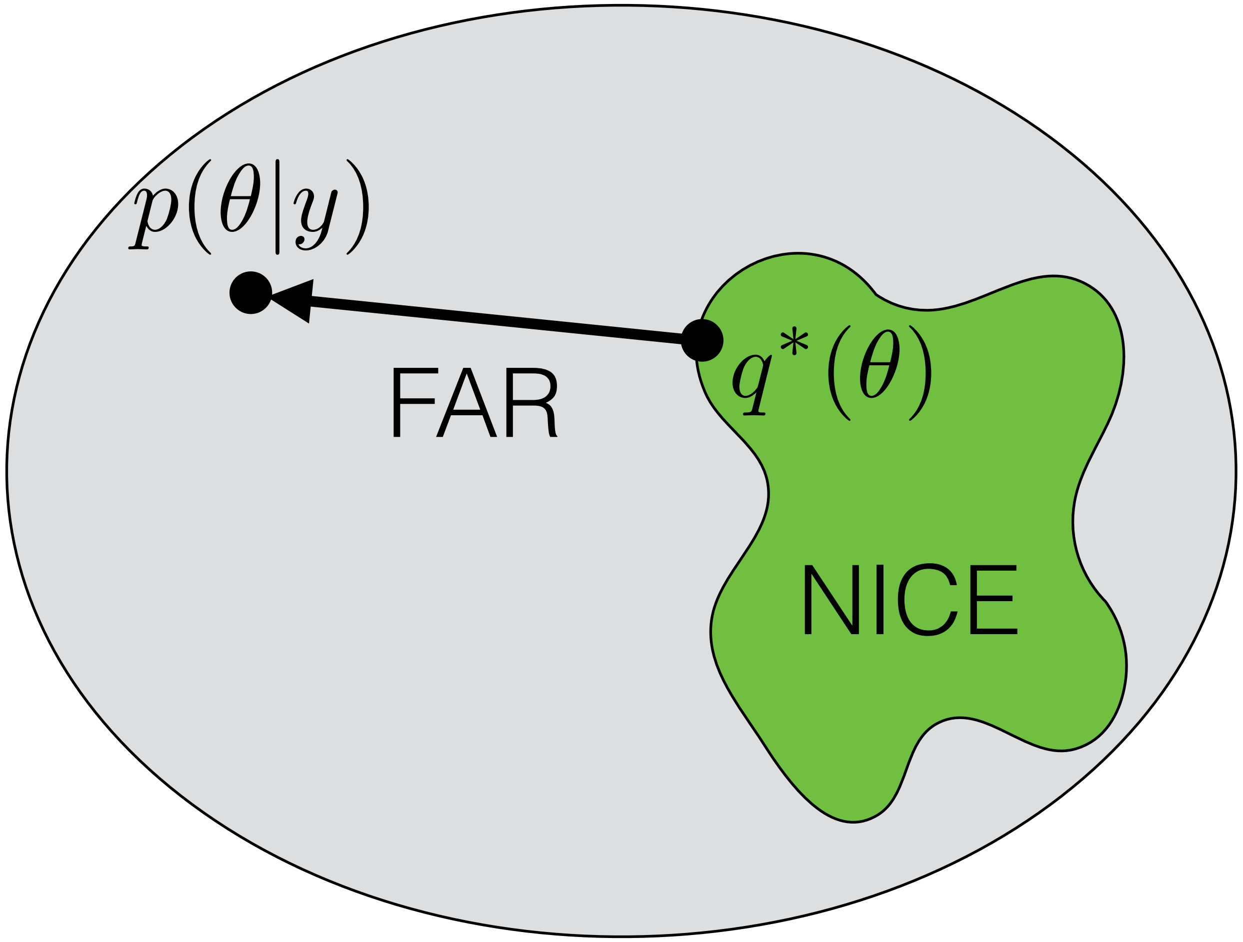

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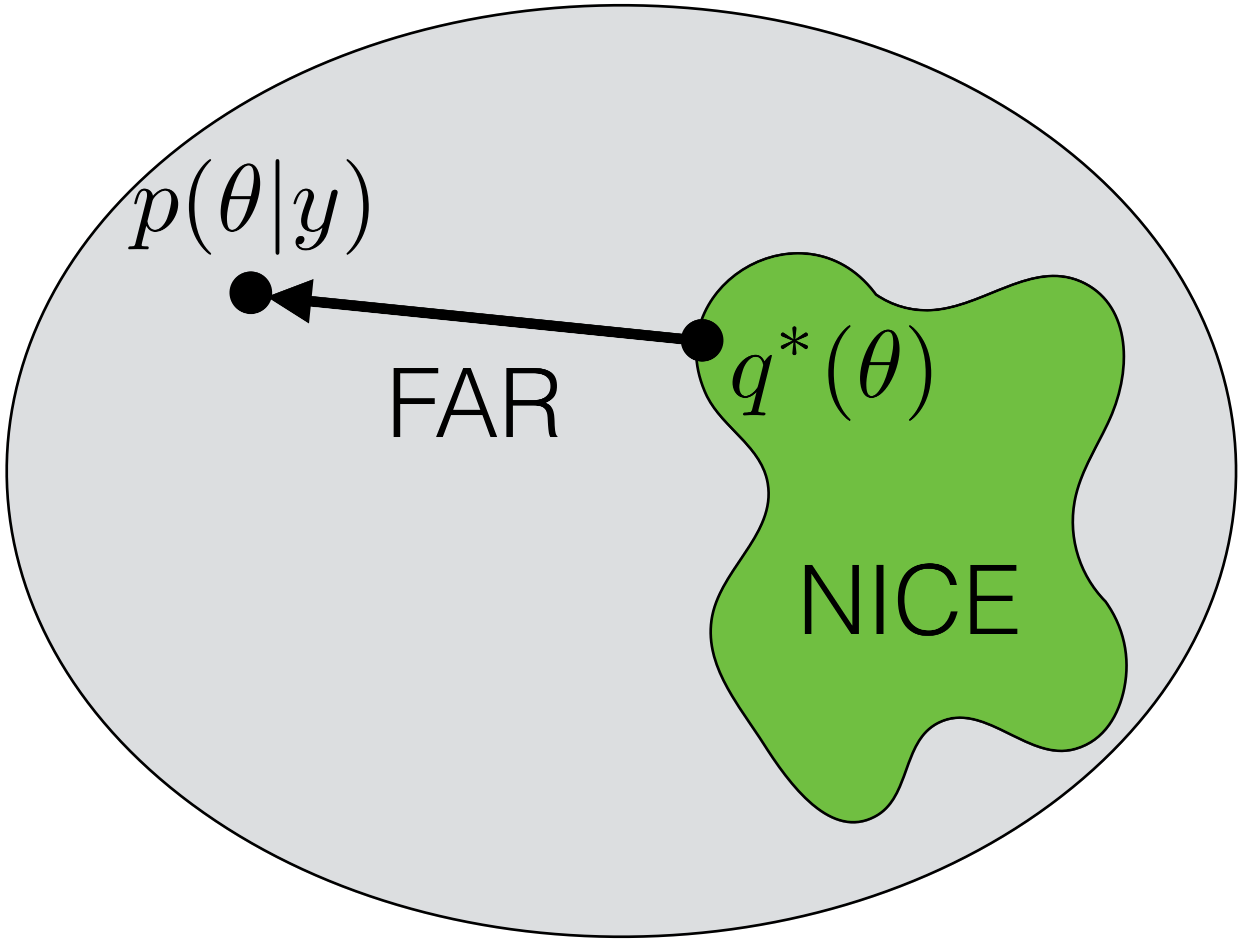

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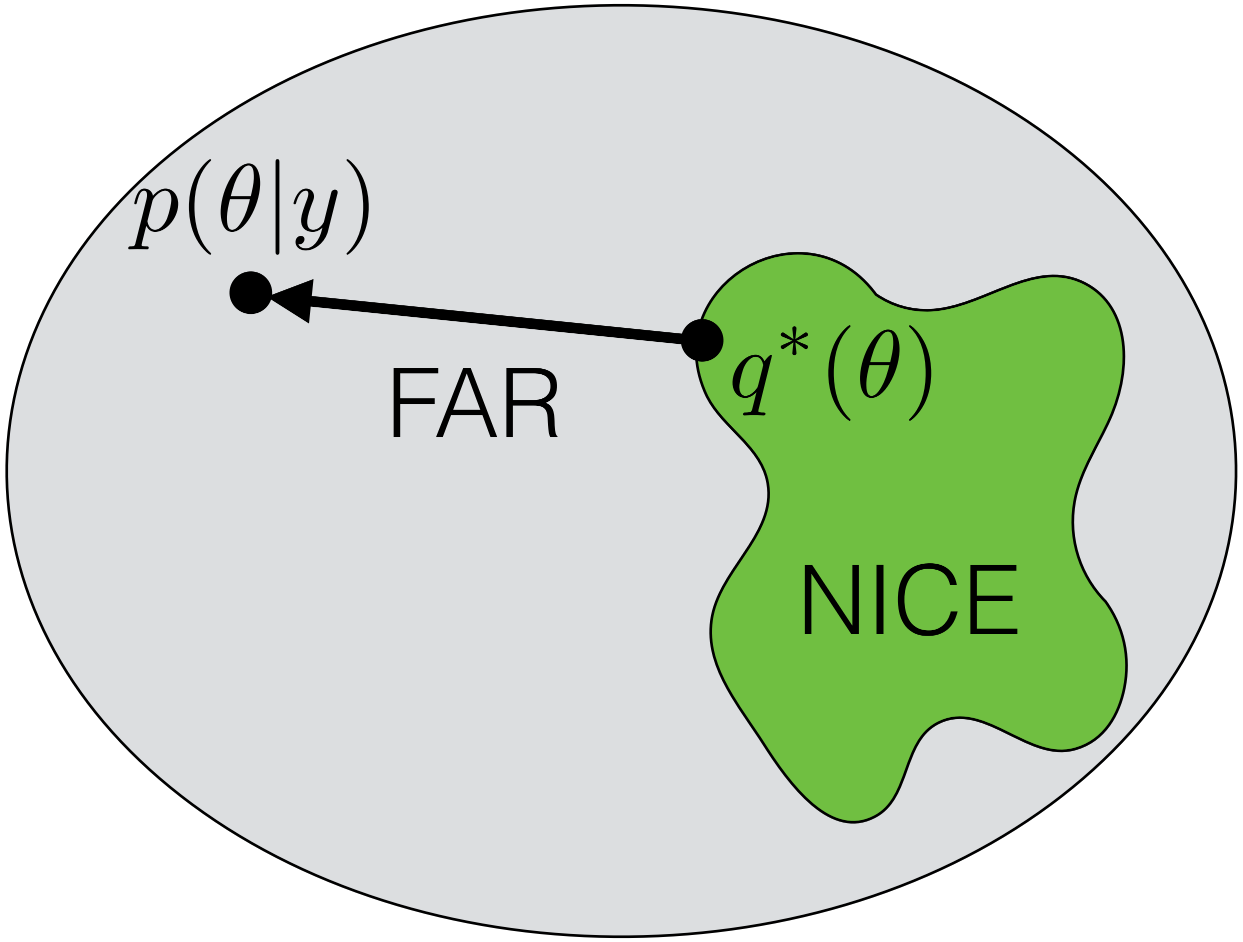

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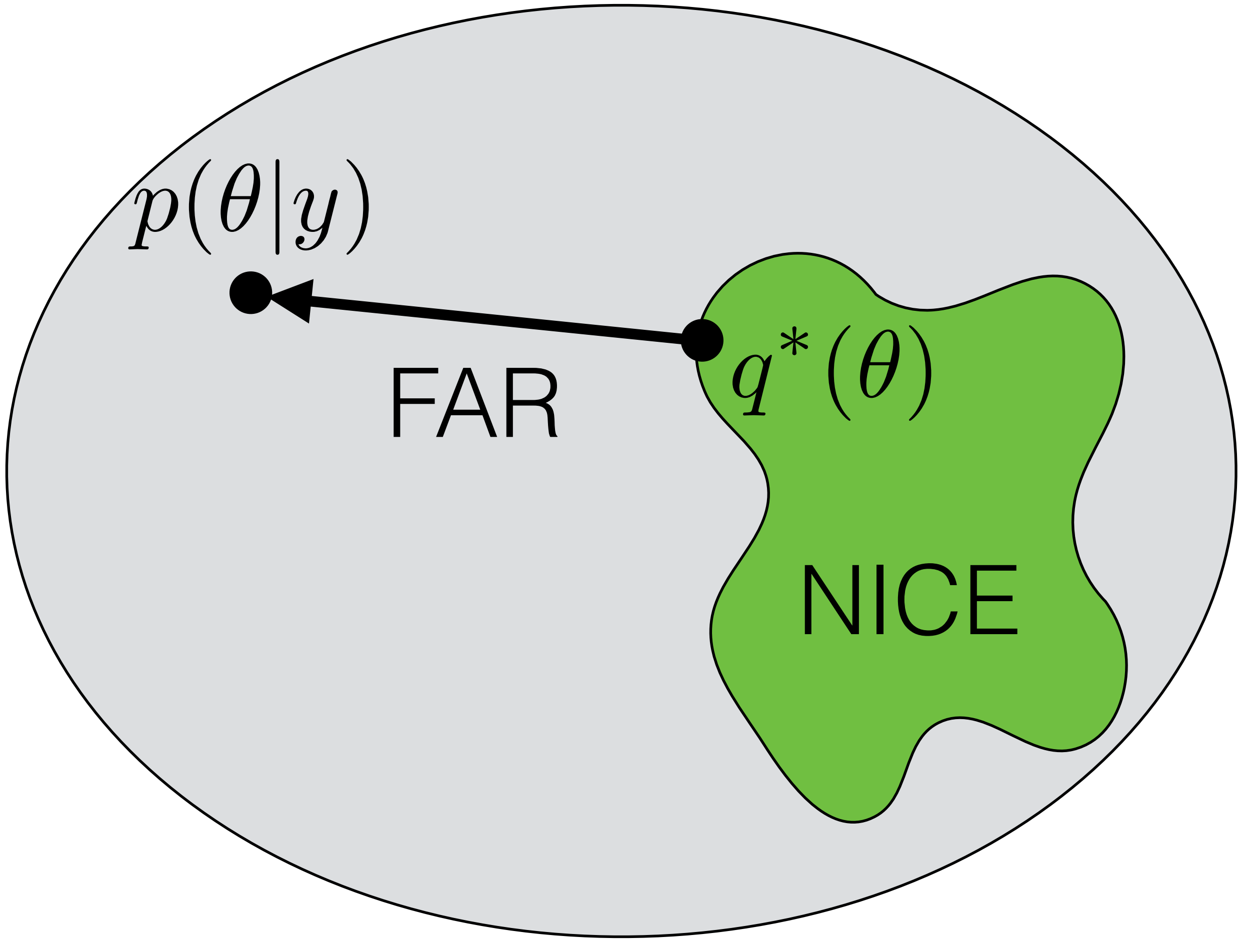

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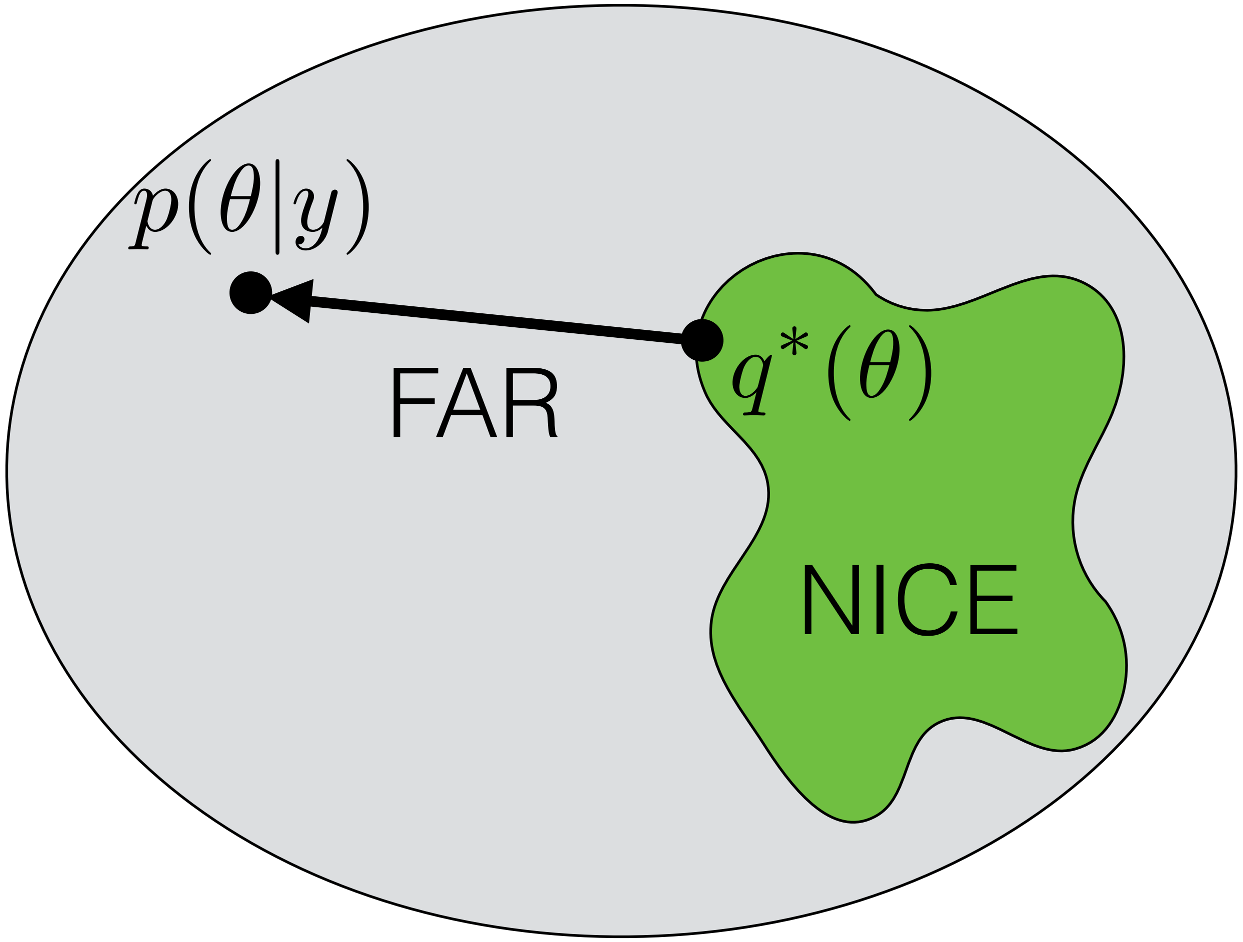

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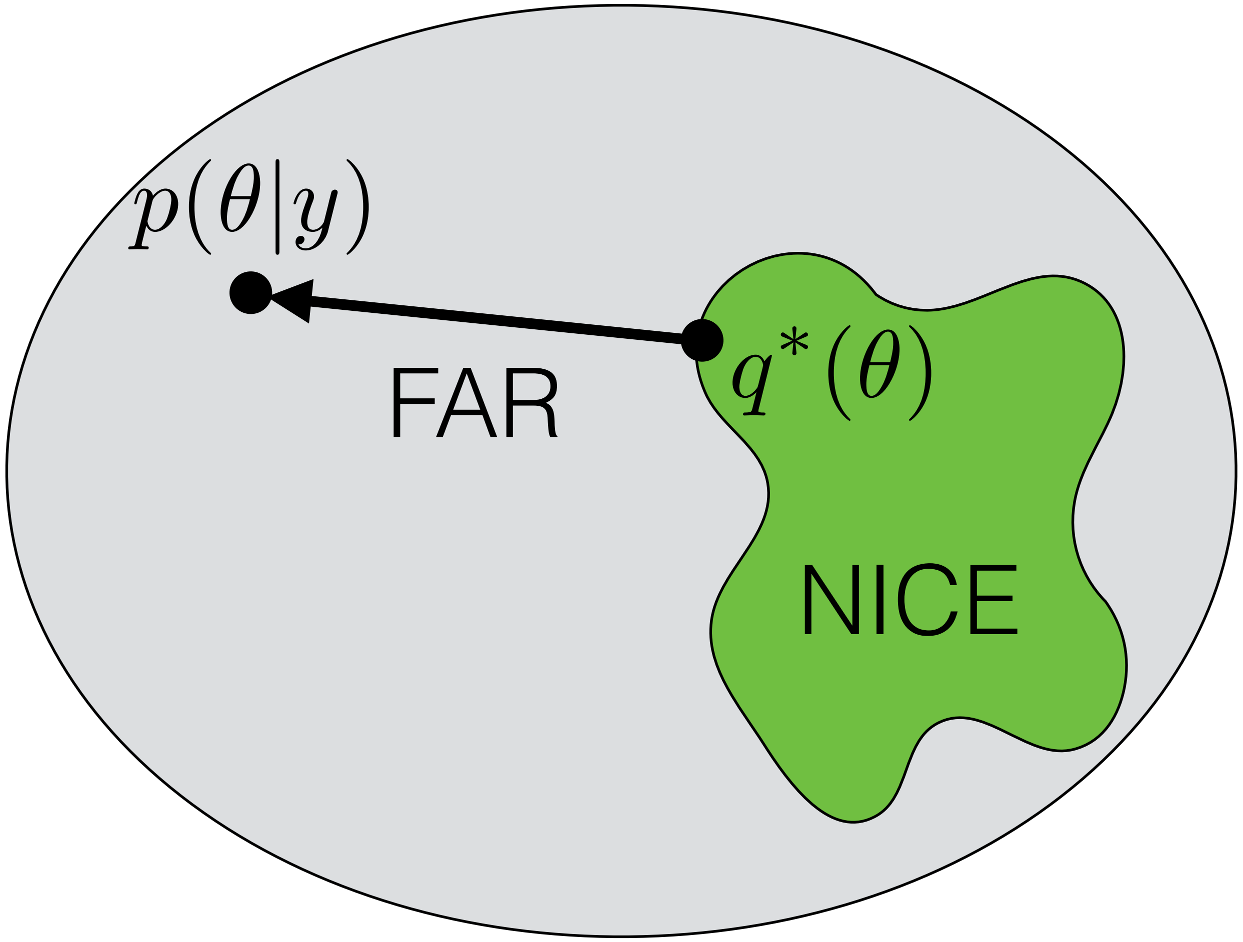

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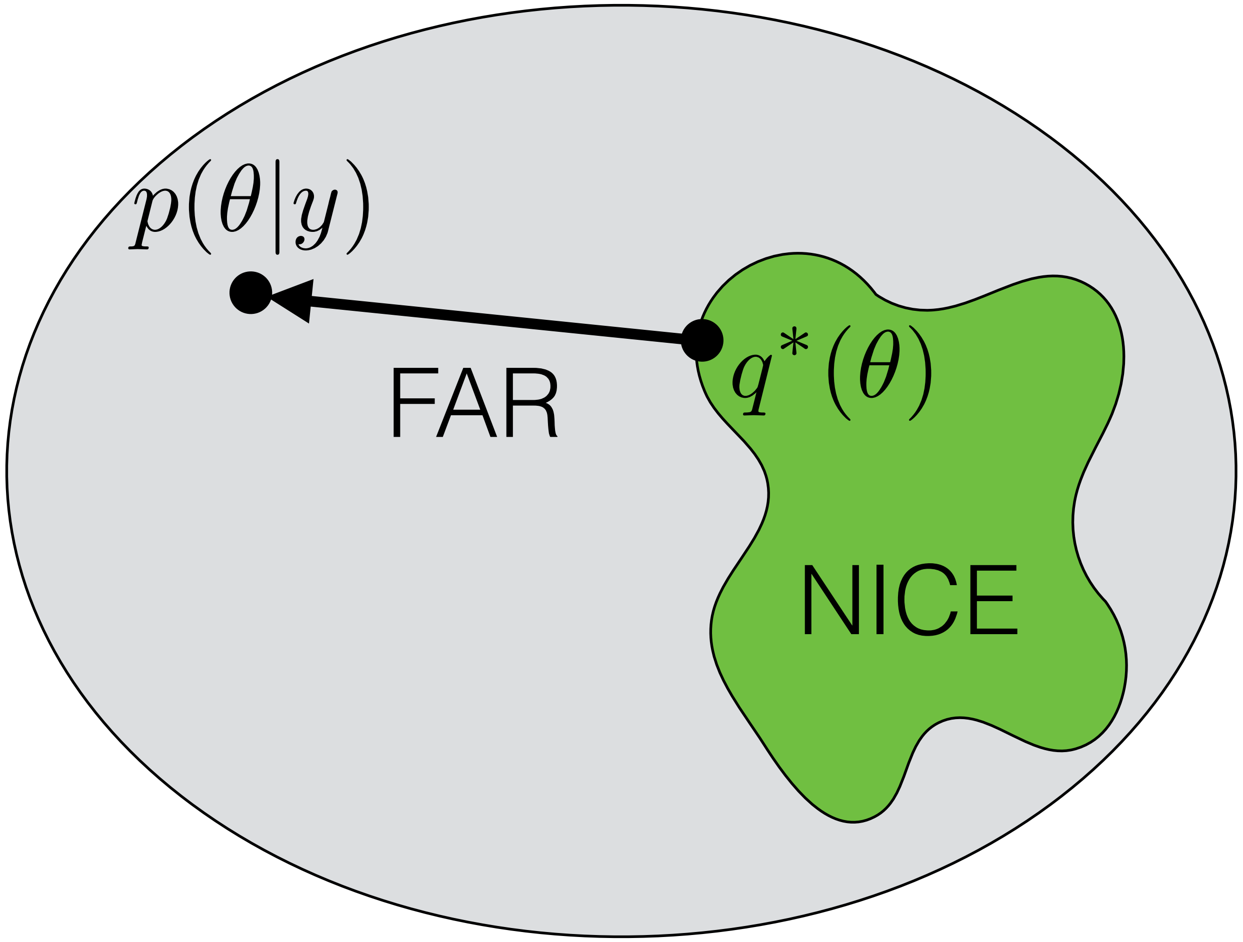

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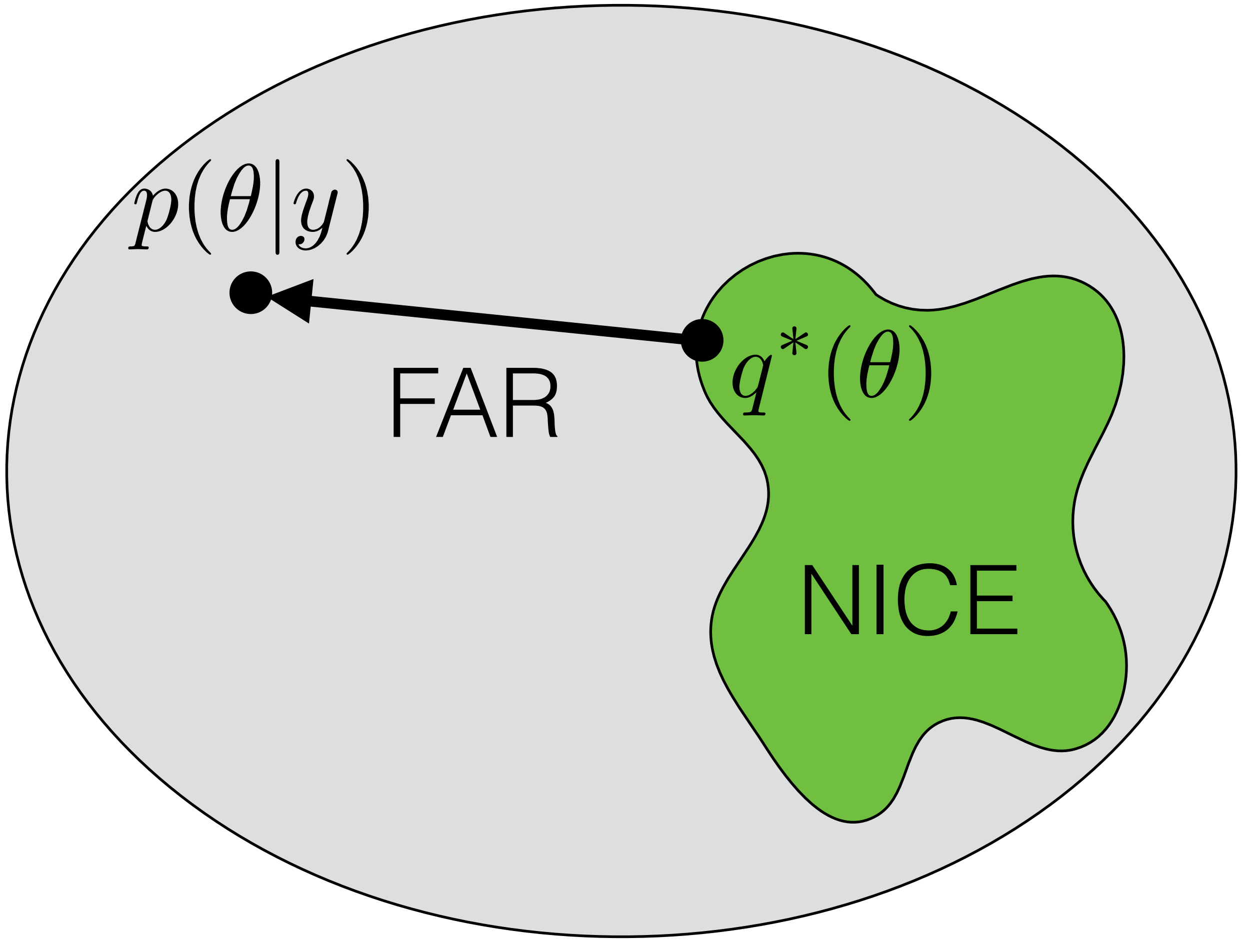

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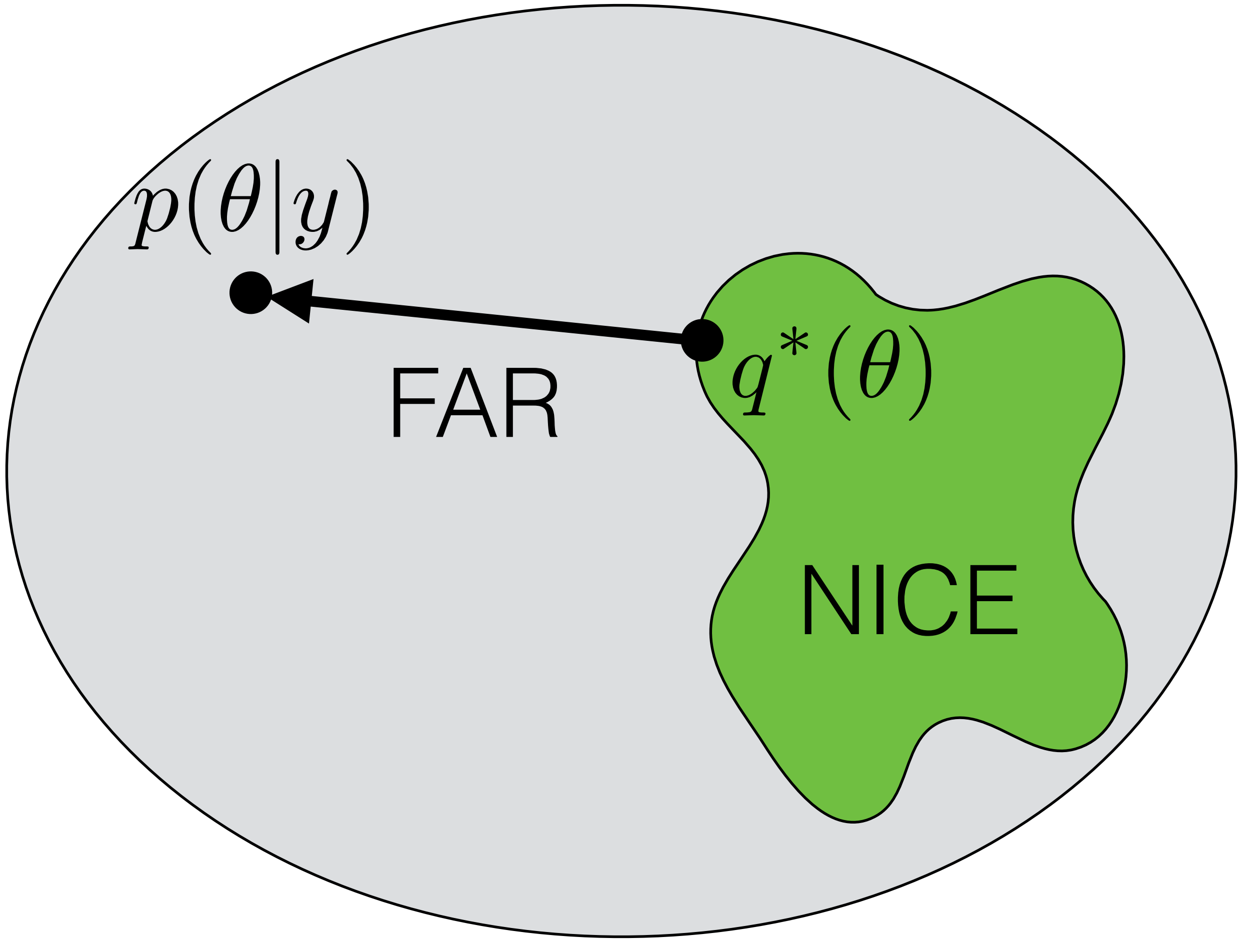

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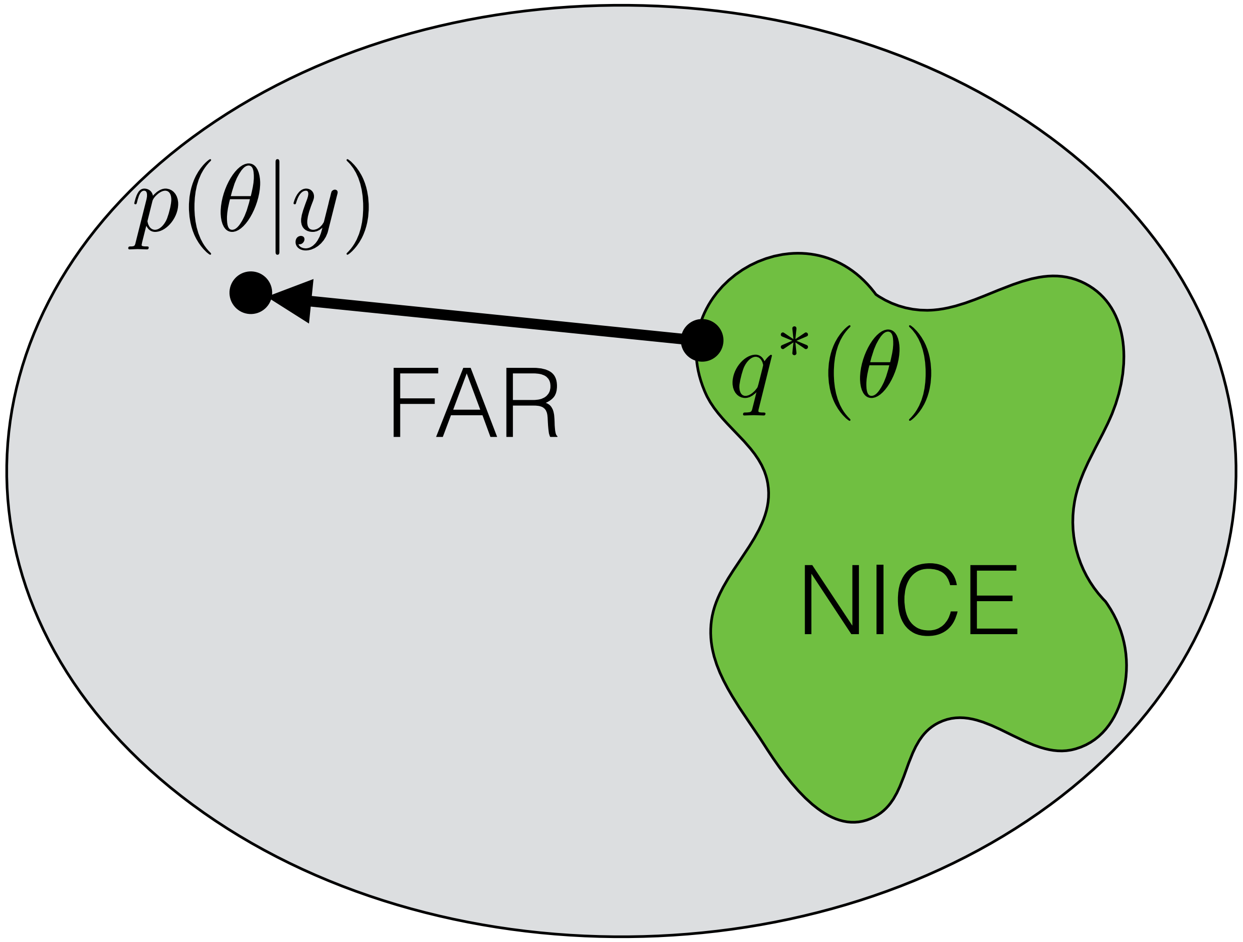

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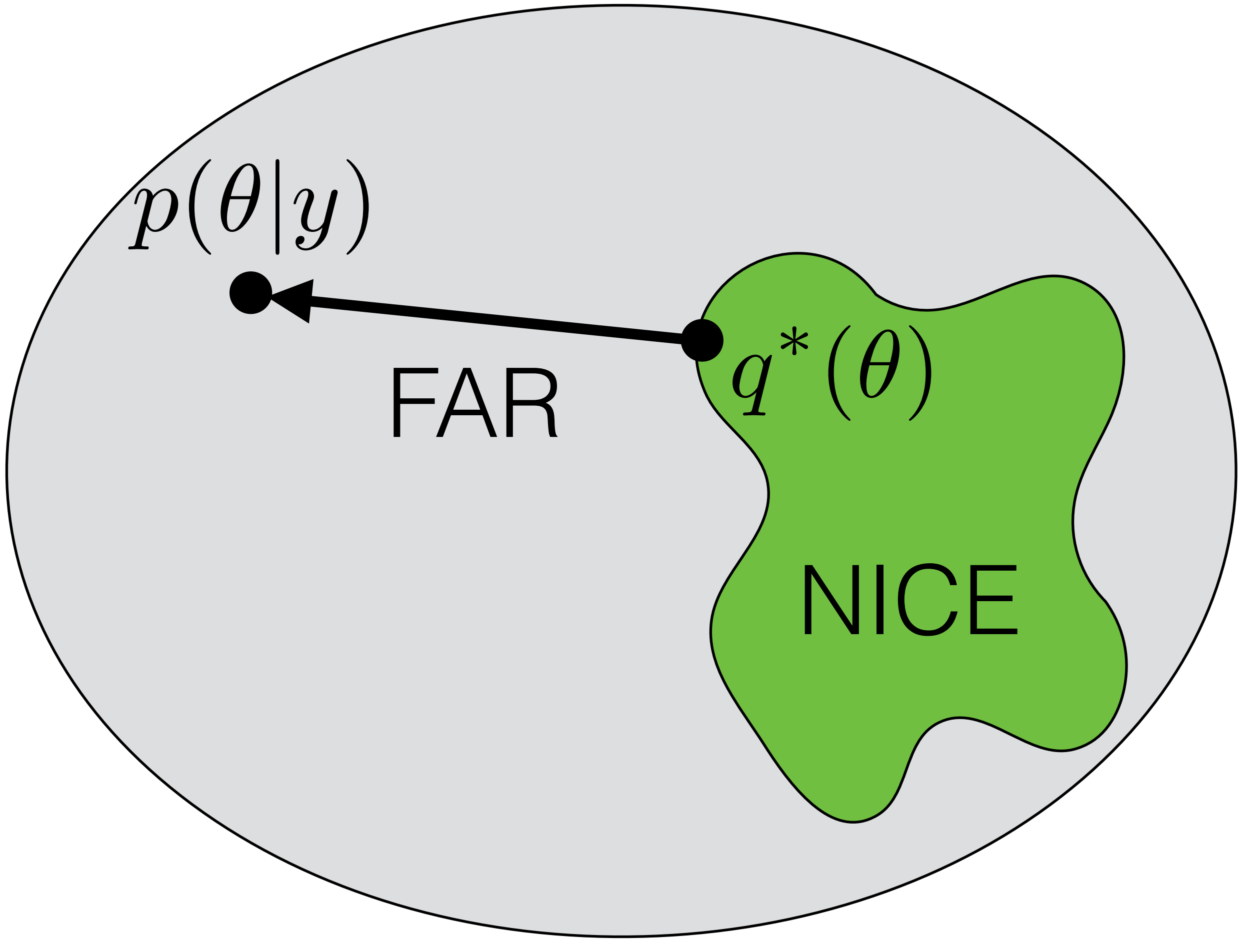

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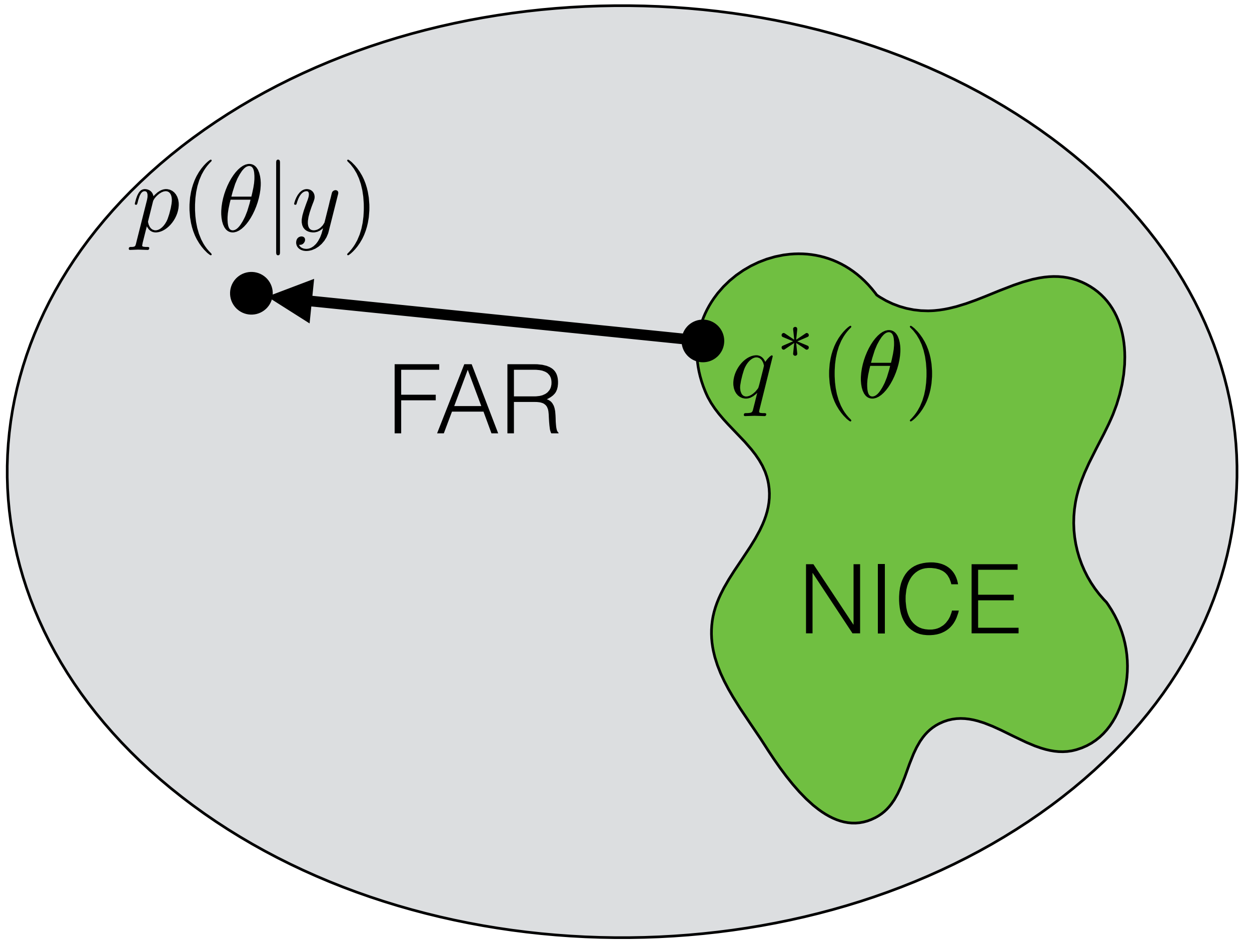

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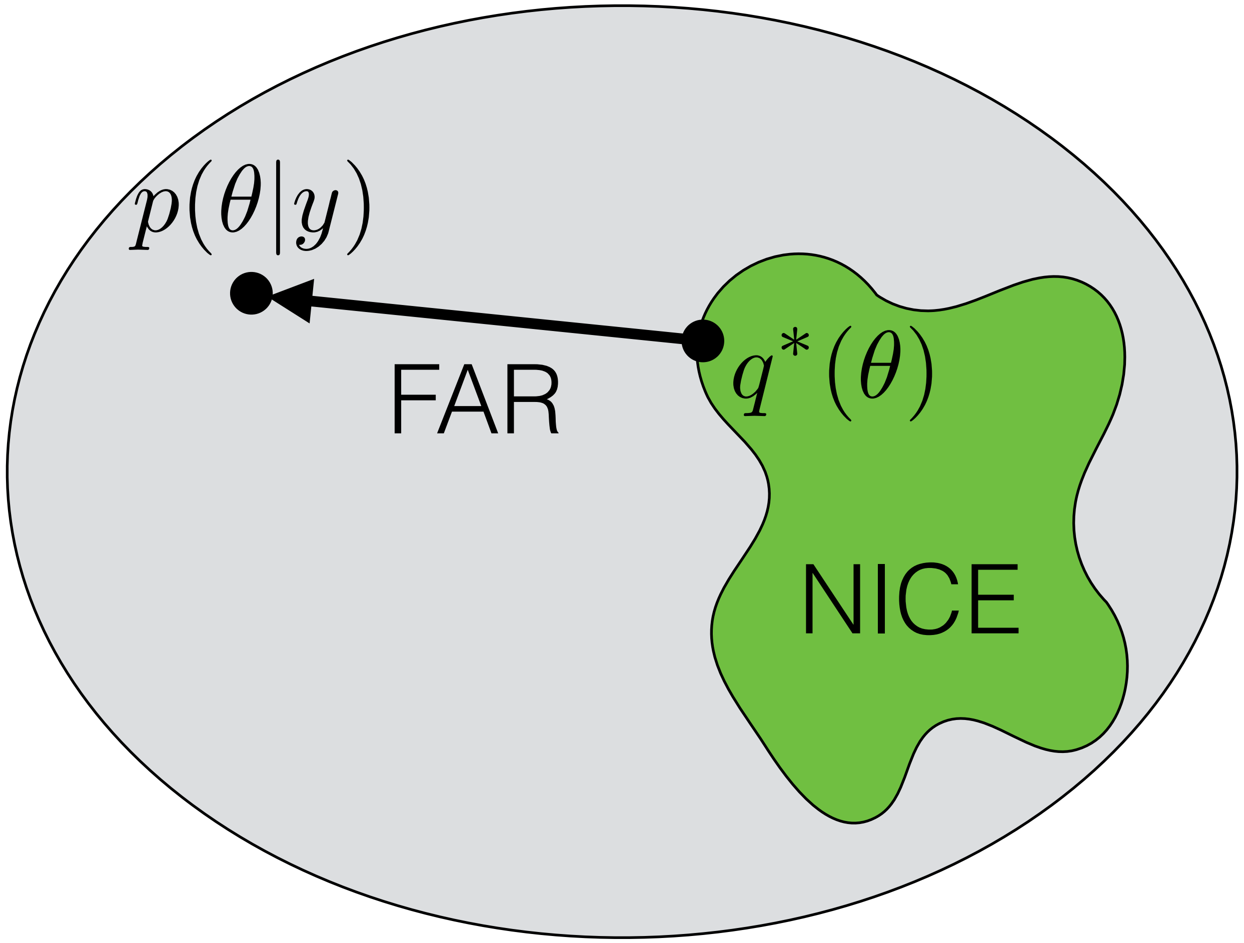
Choose “NICE” distributions



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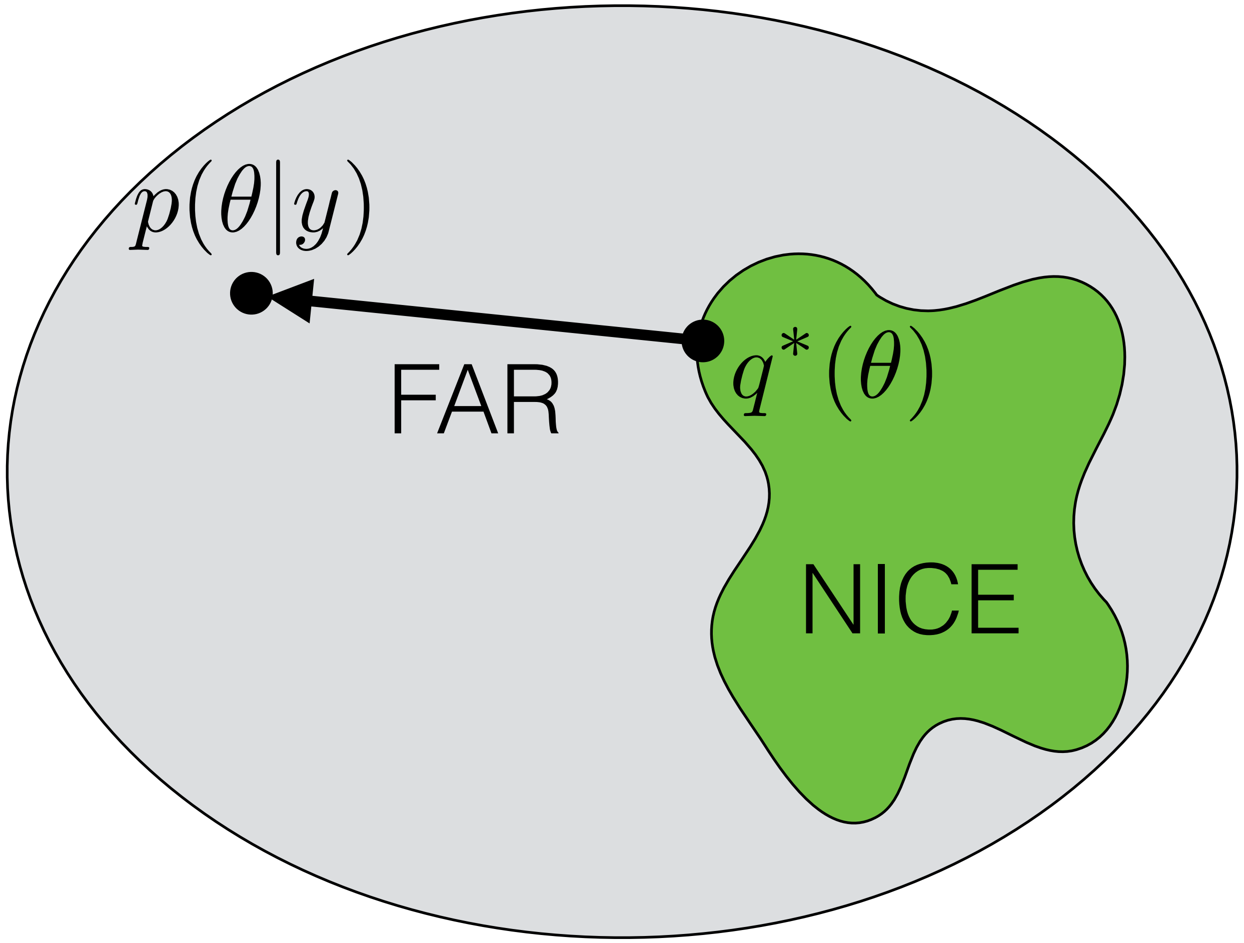

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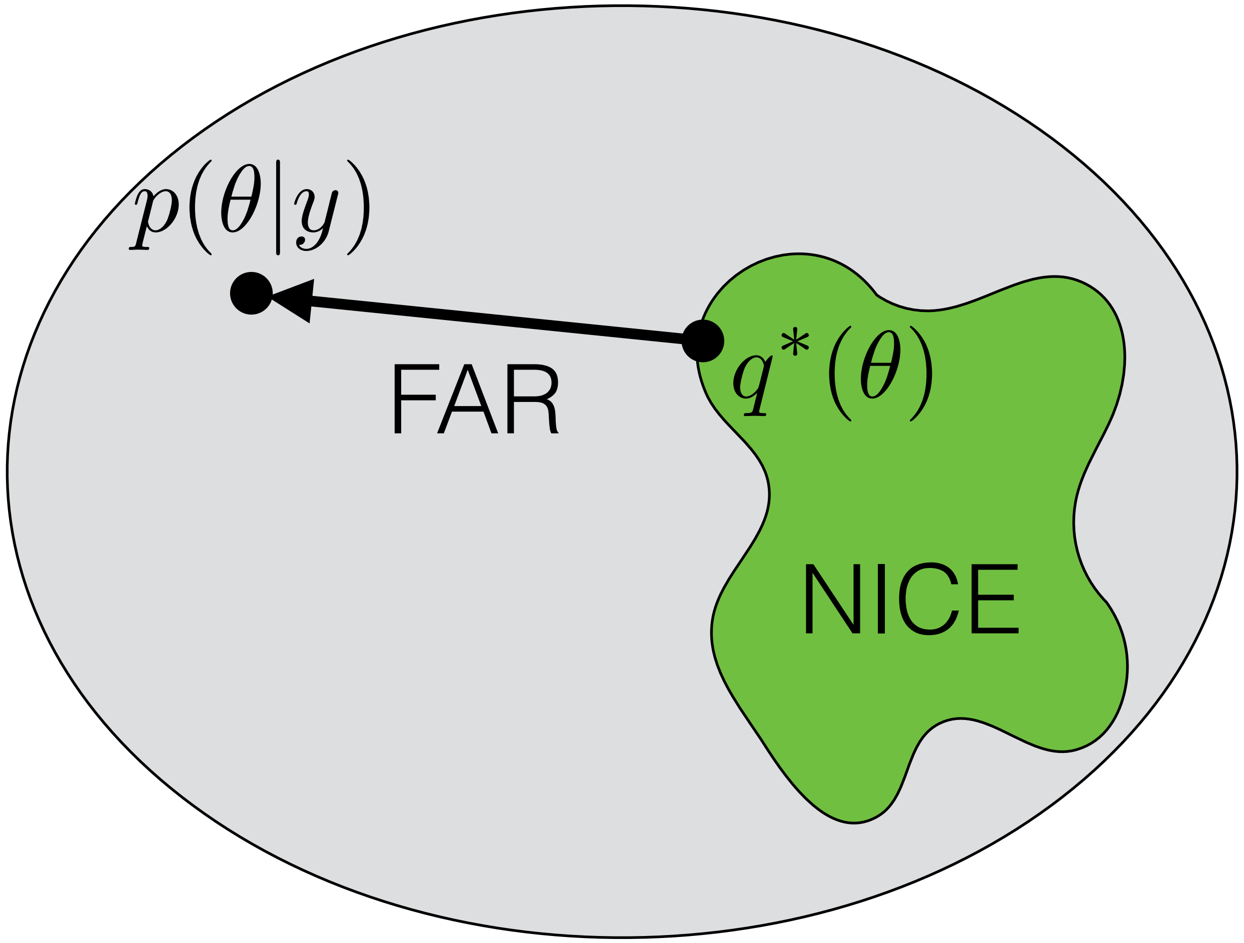

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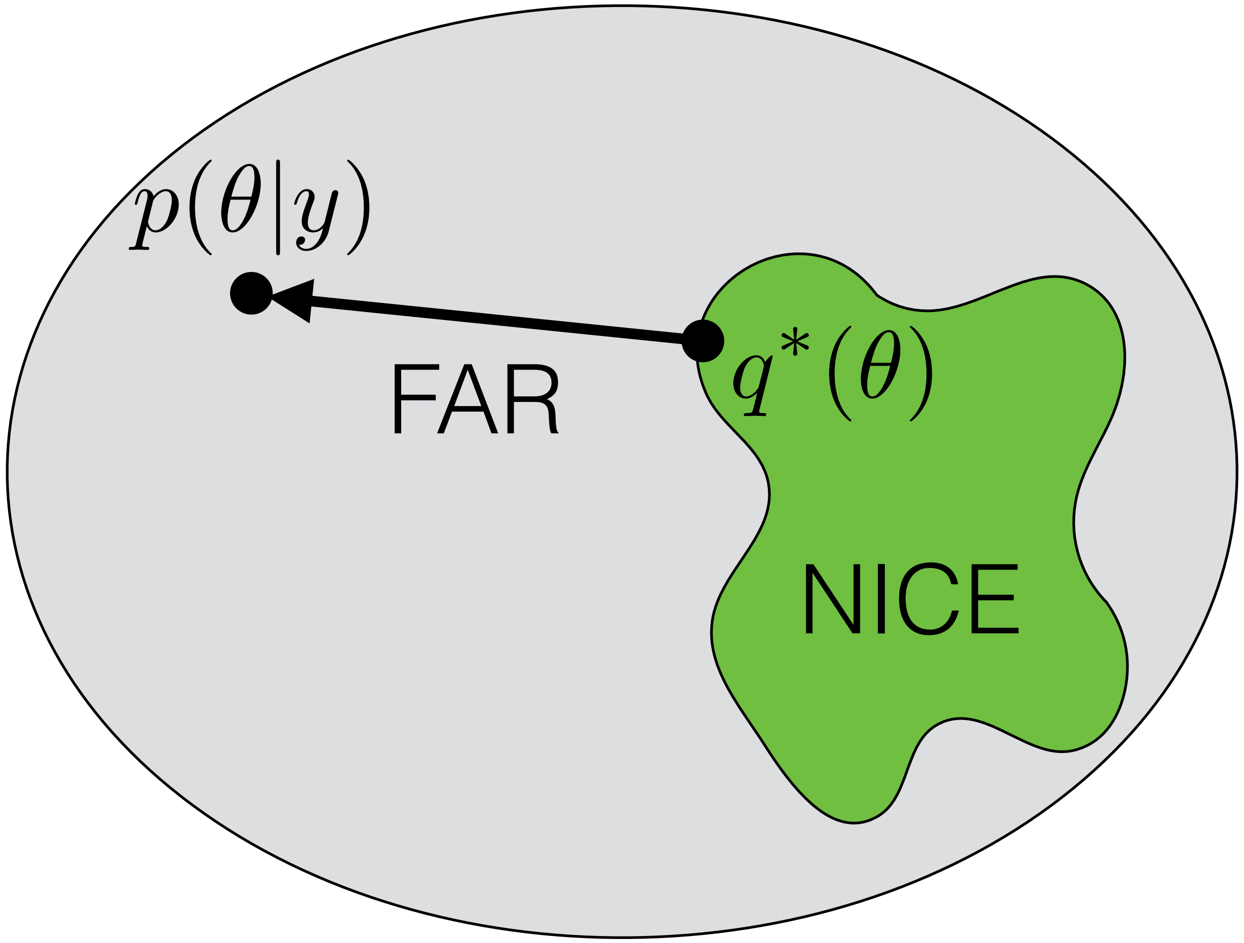

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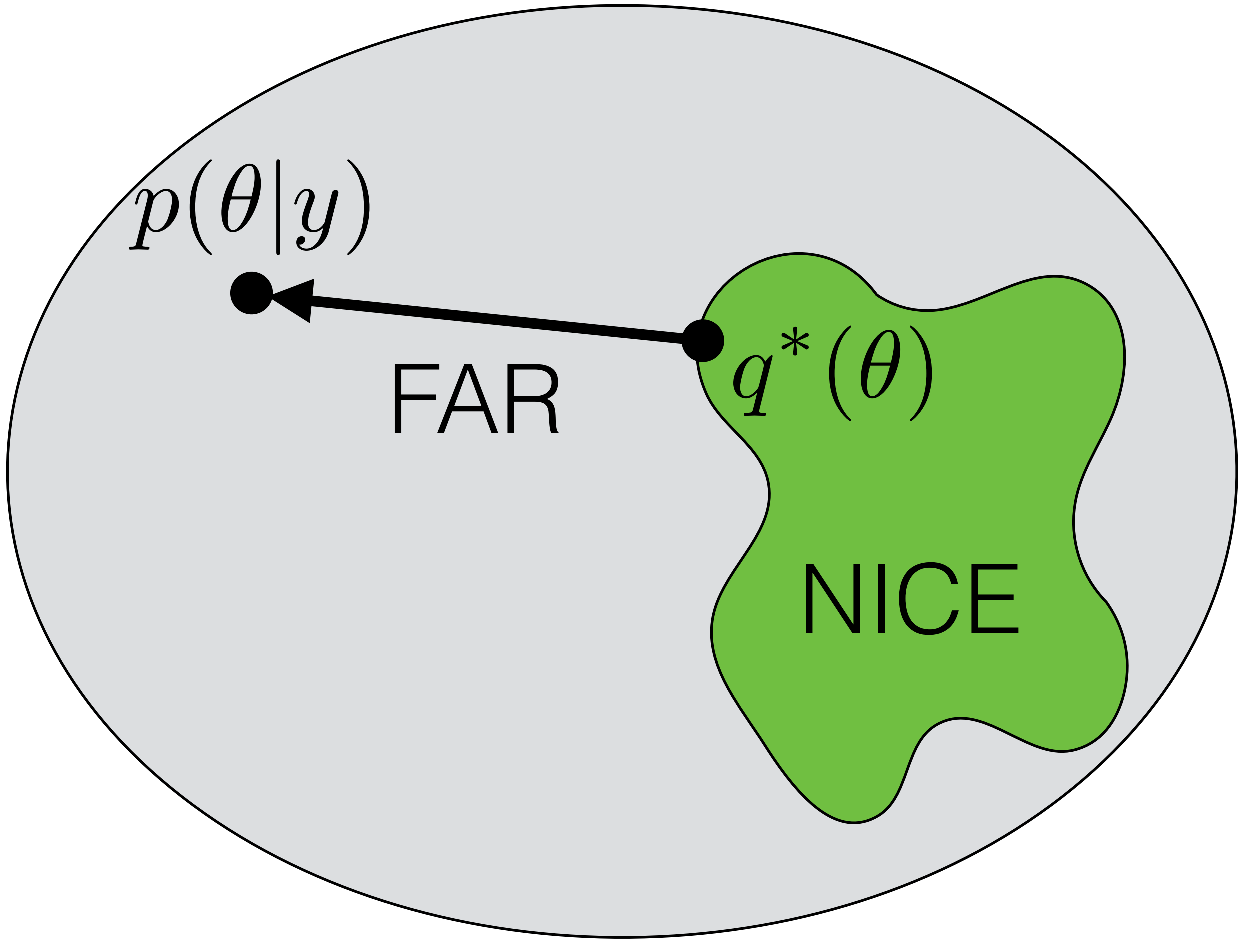

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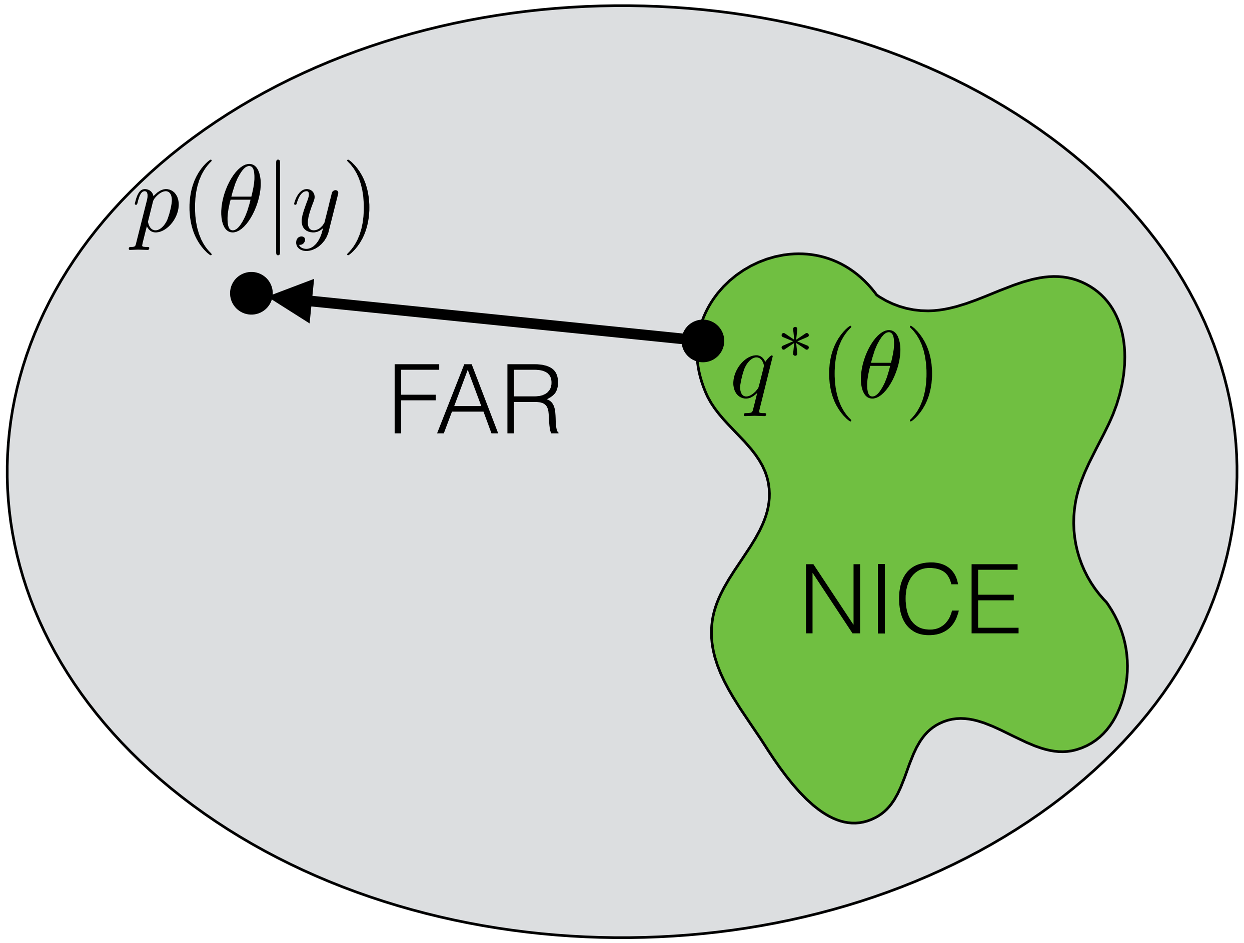

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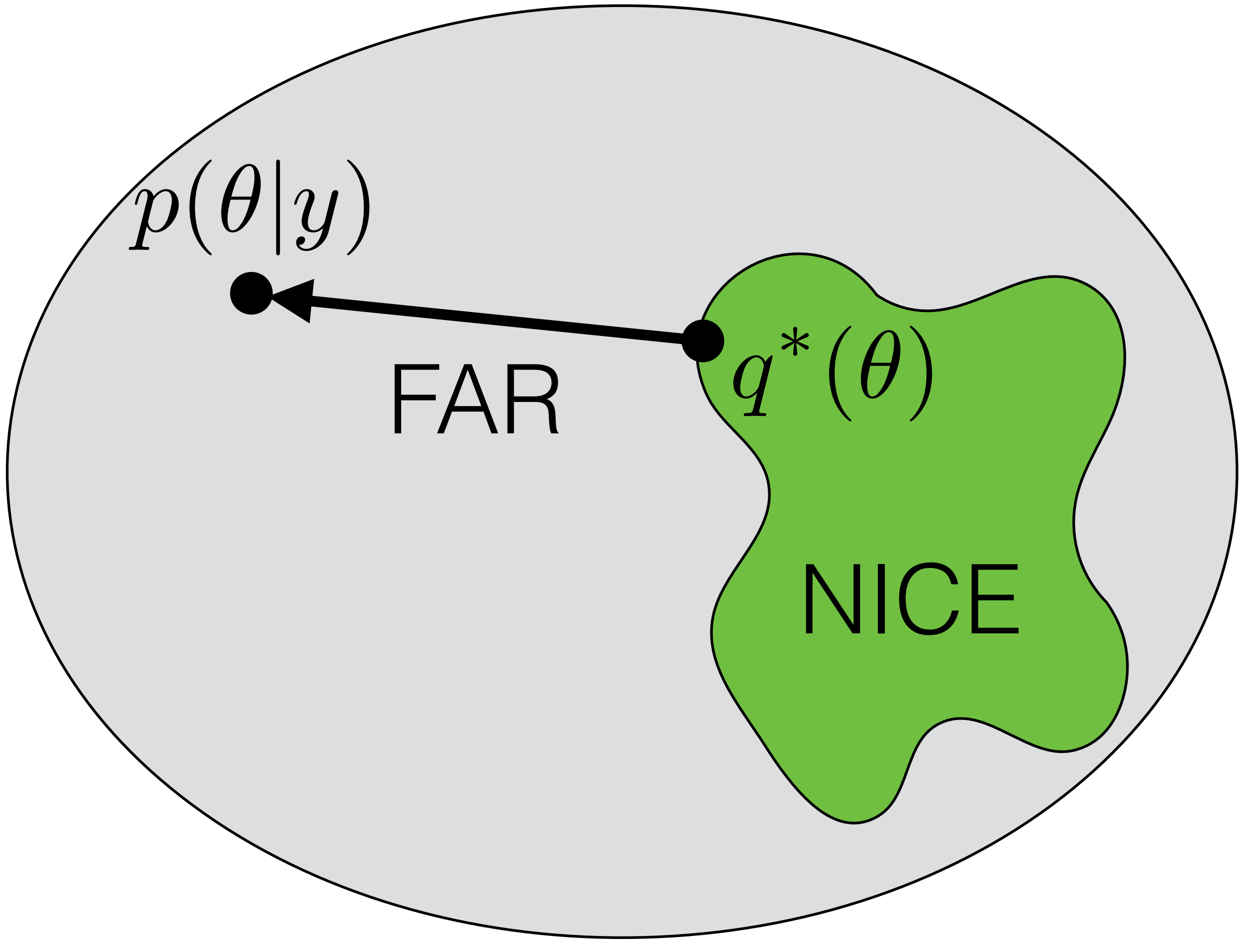

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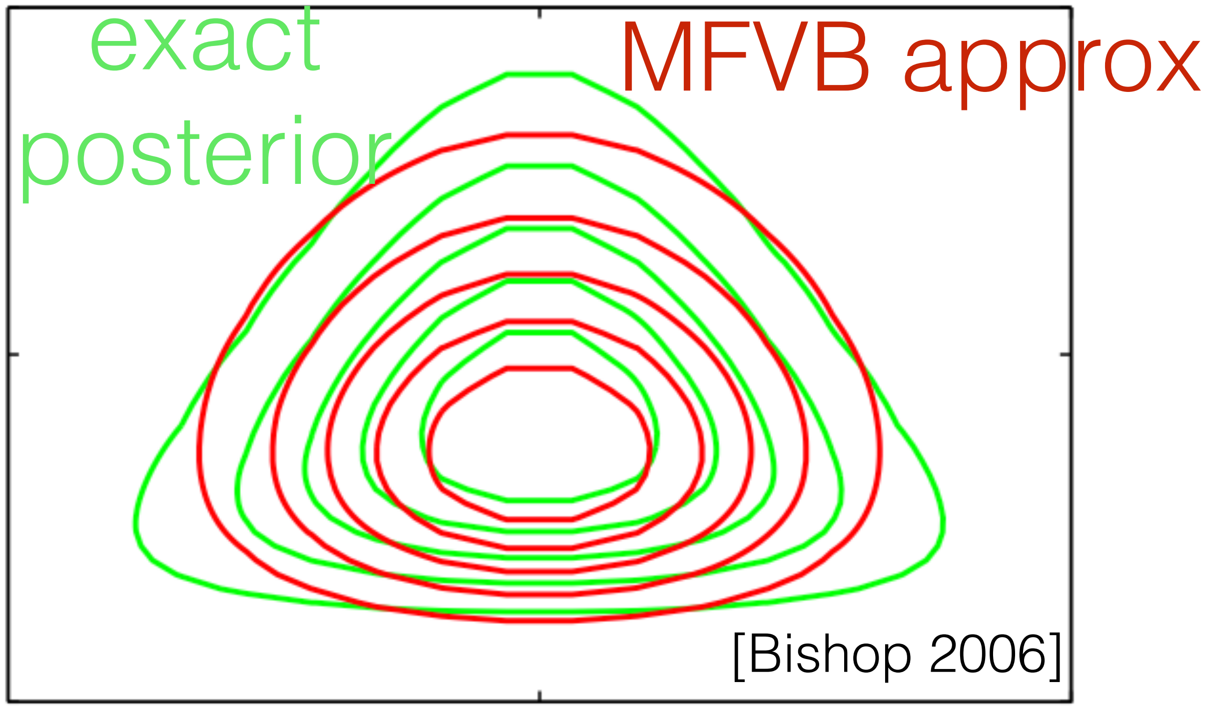

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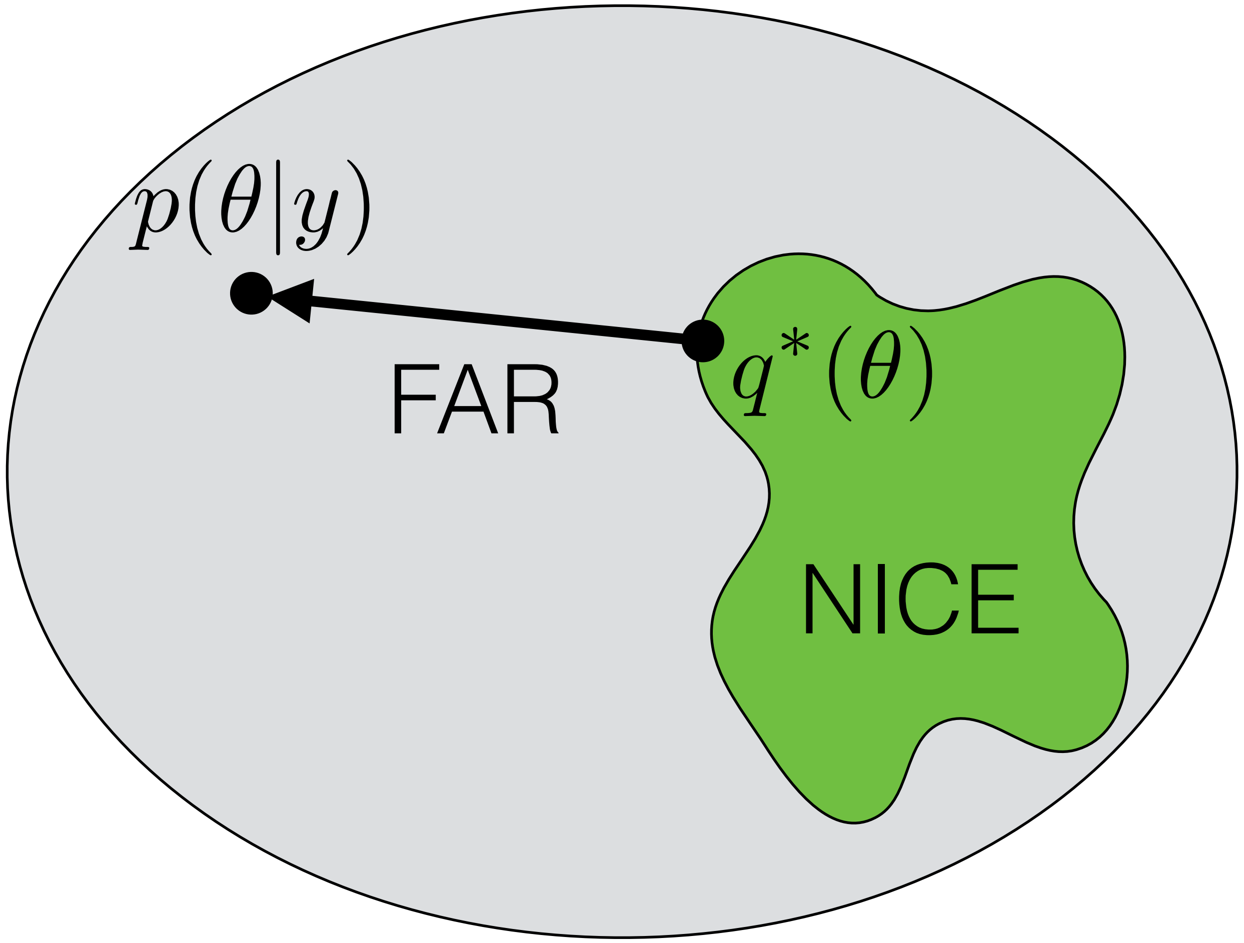

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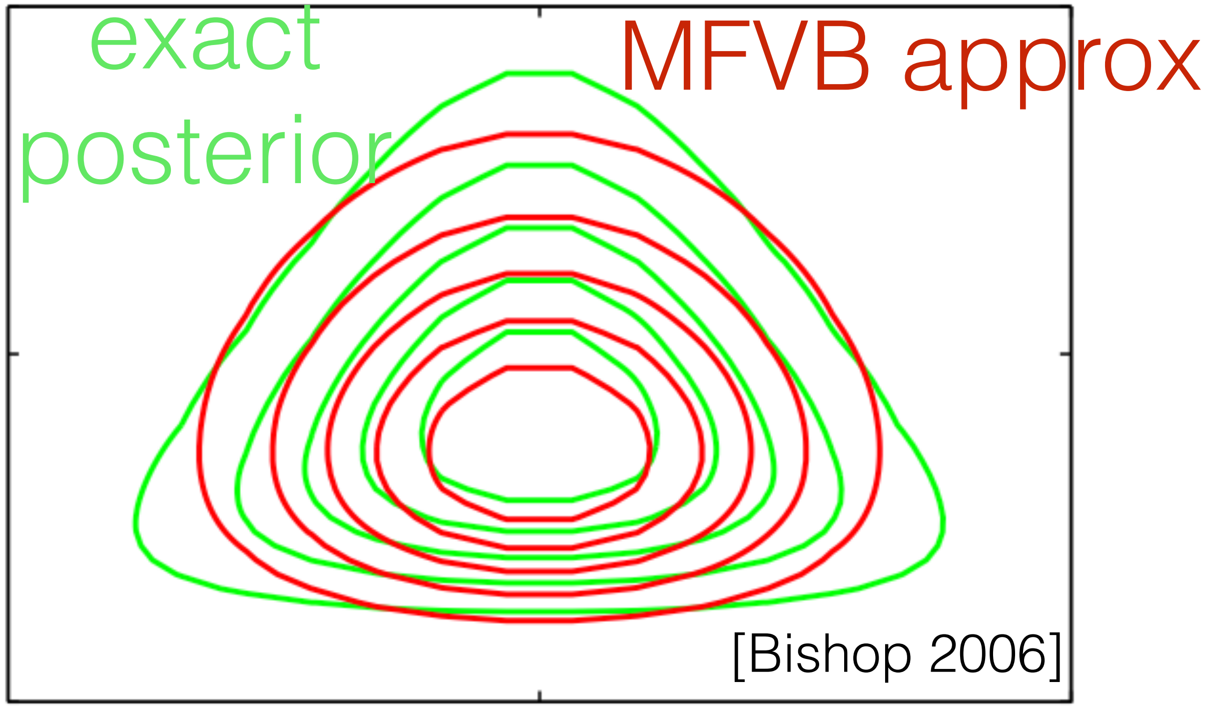

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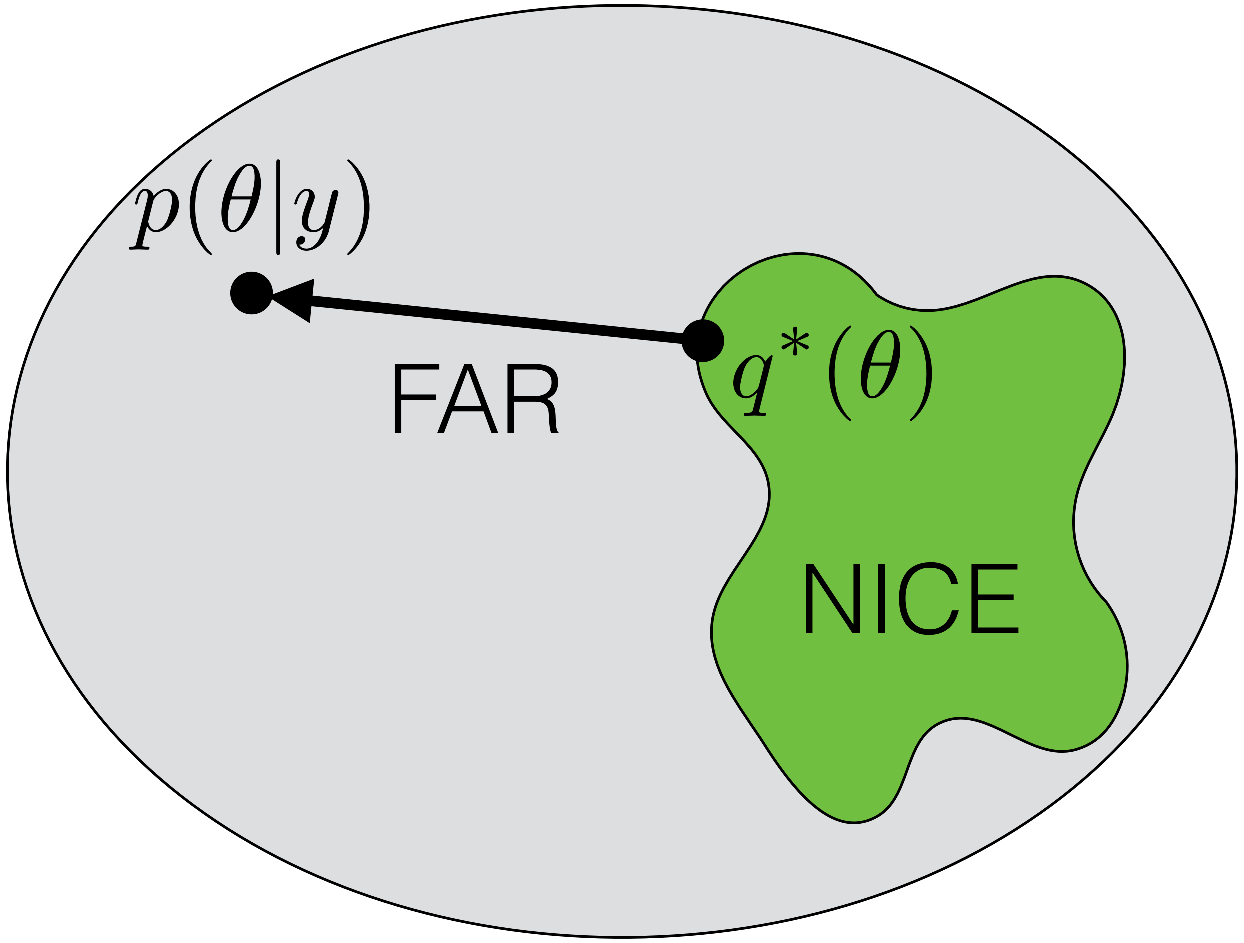

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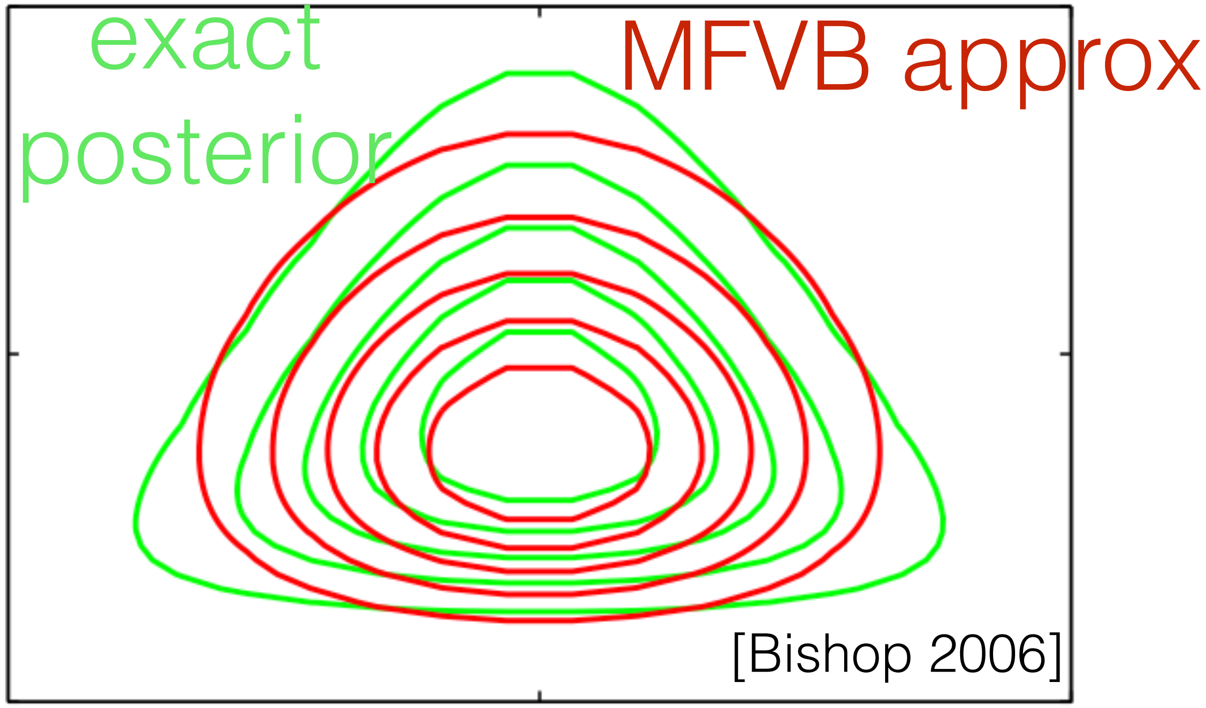
- One option: Coordinate descent in $q_1, \ldots, q_J$



[Bishop 2006]

Approximate Bayesian inference

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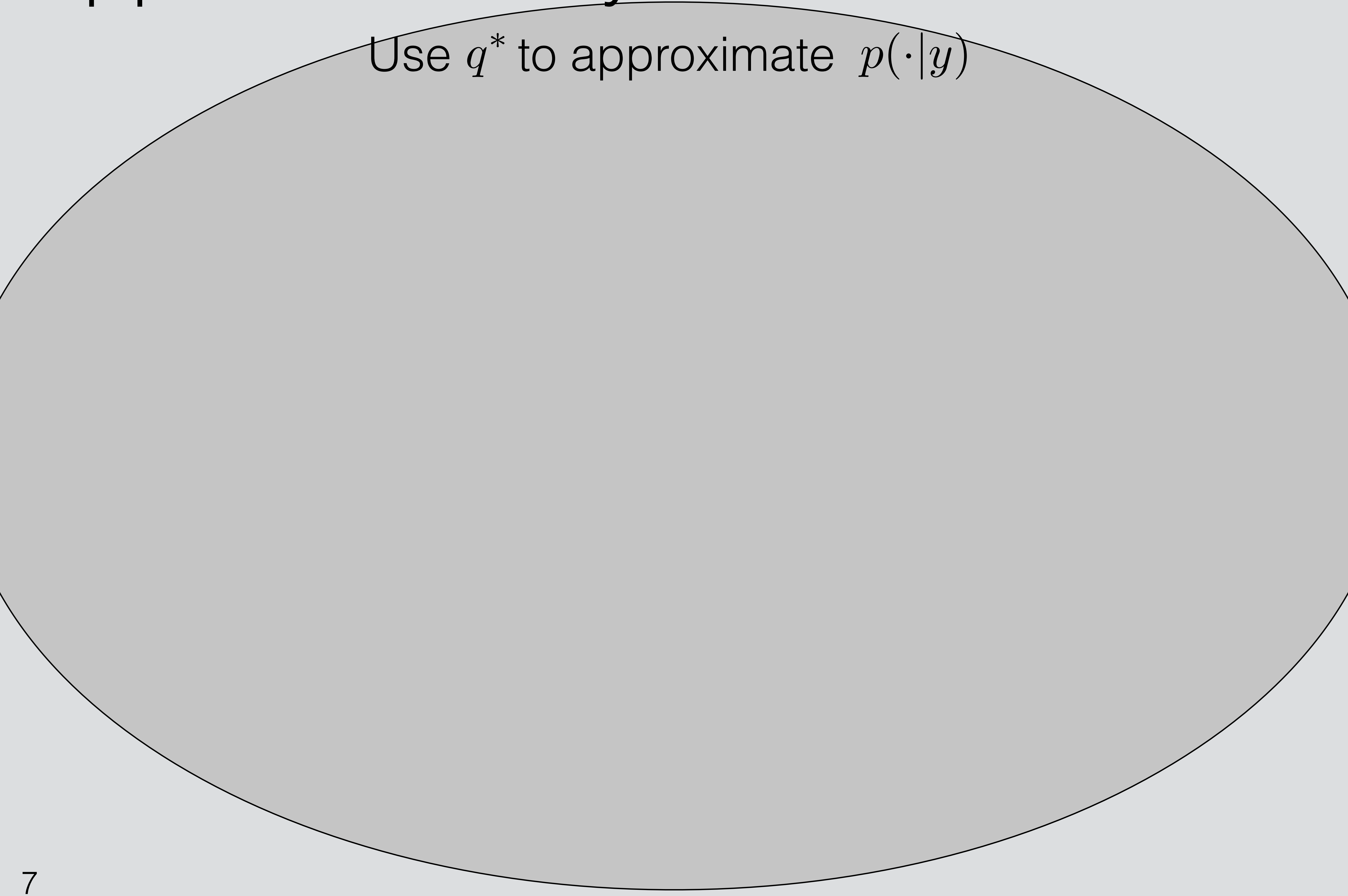

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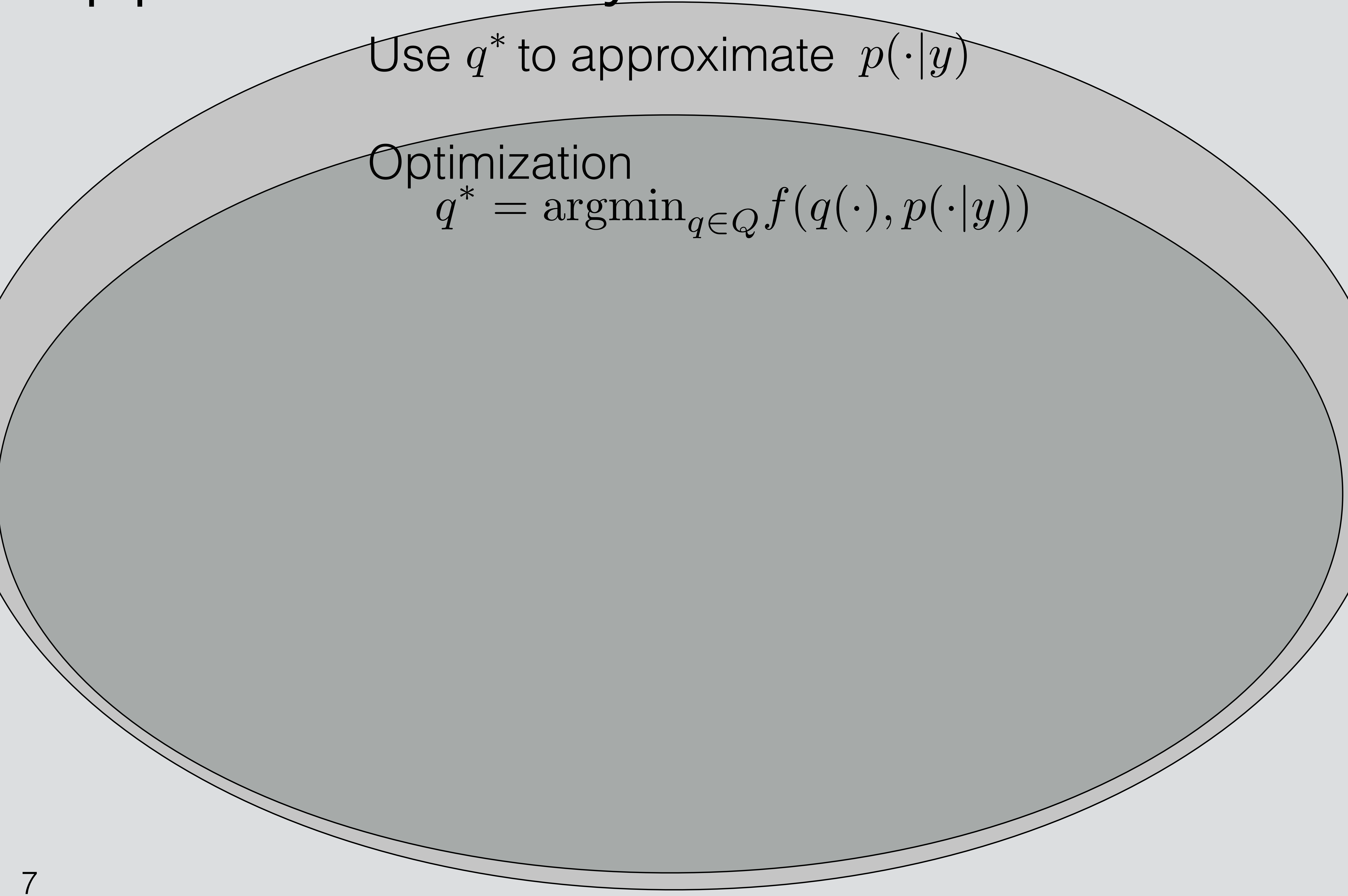

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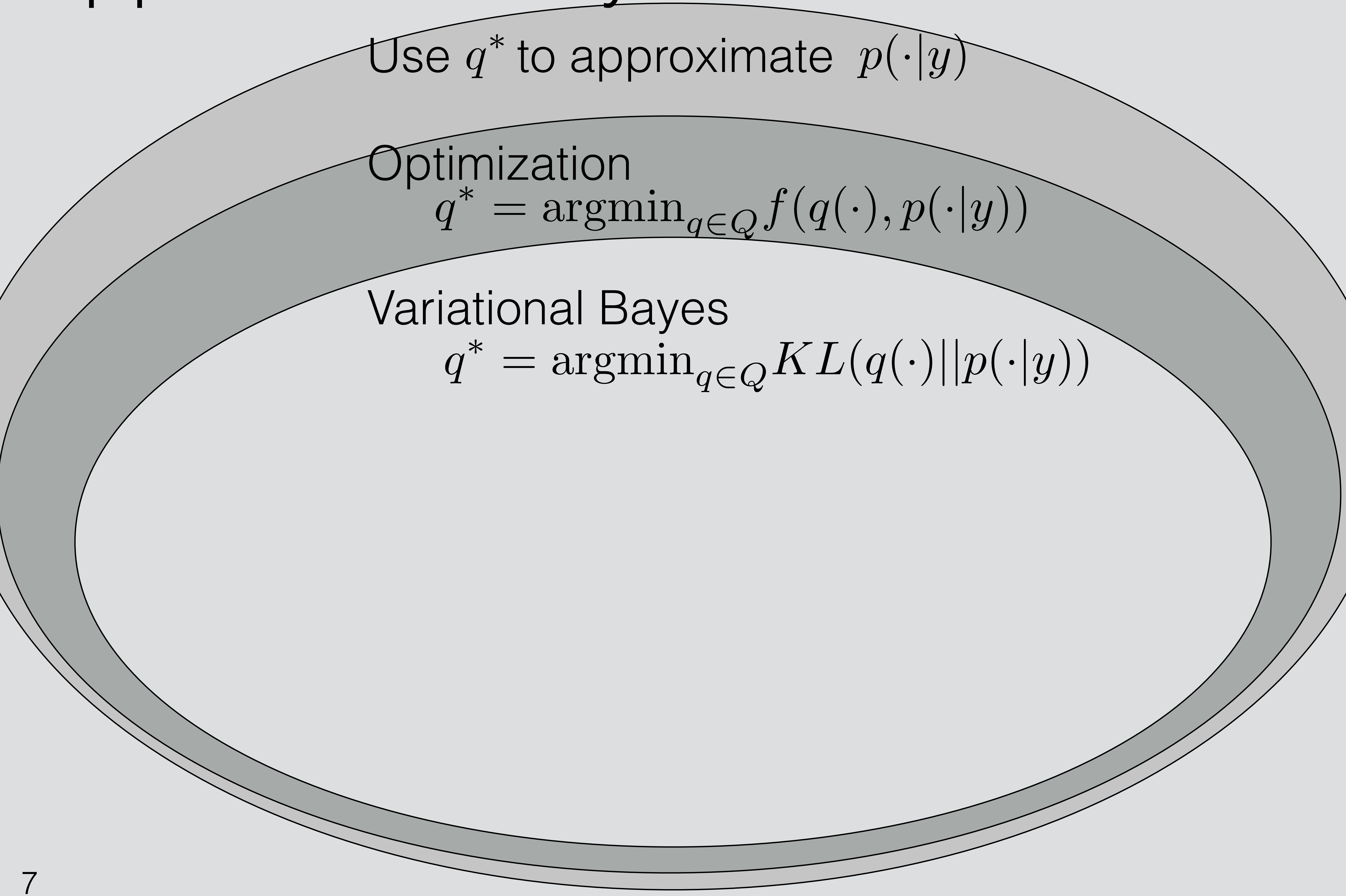

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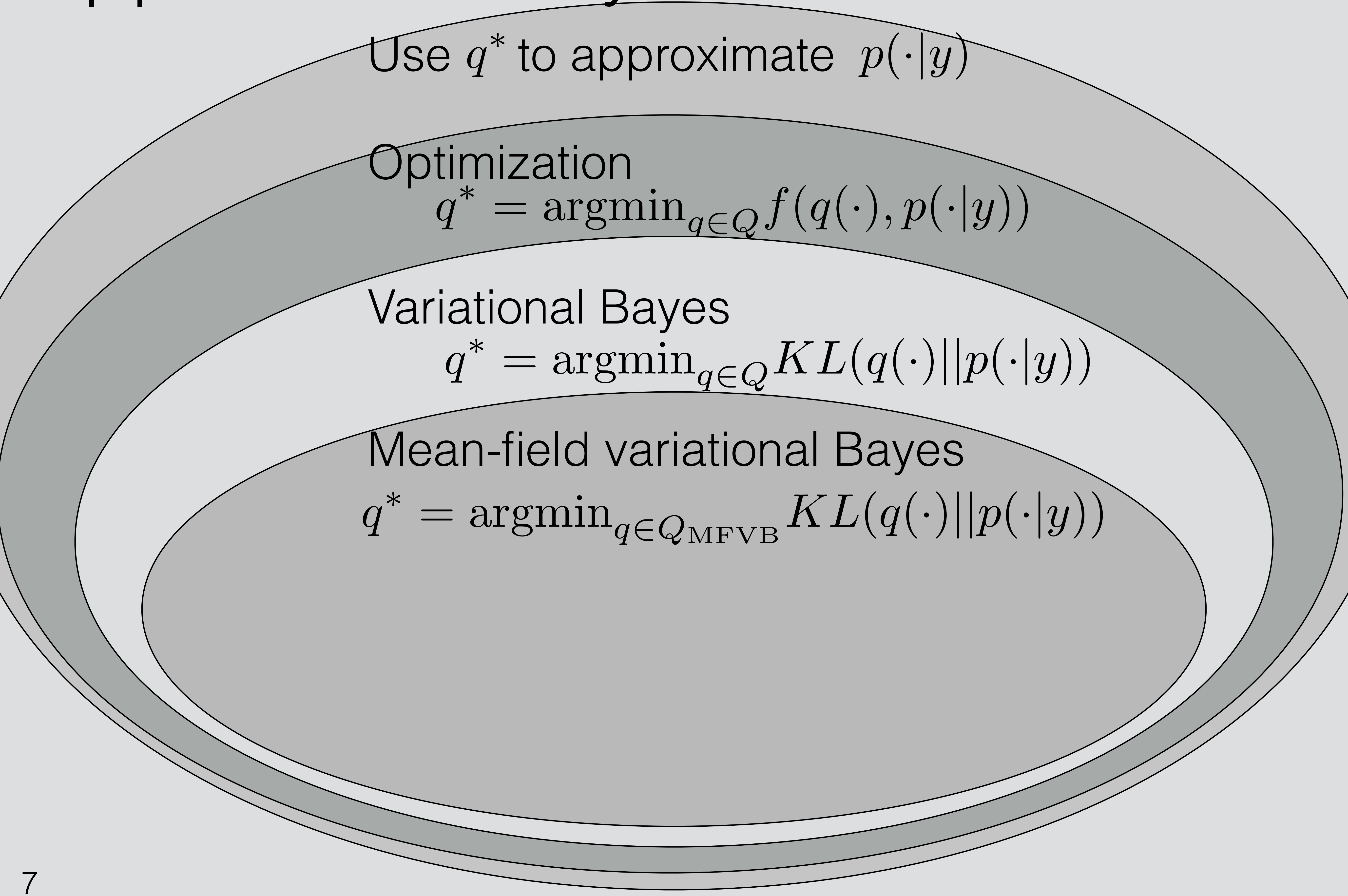

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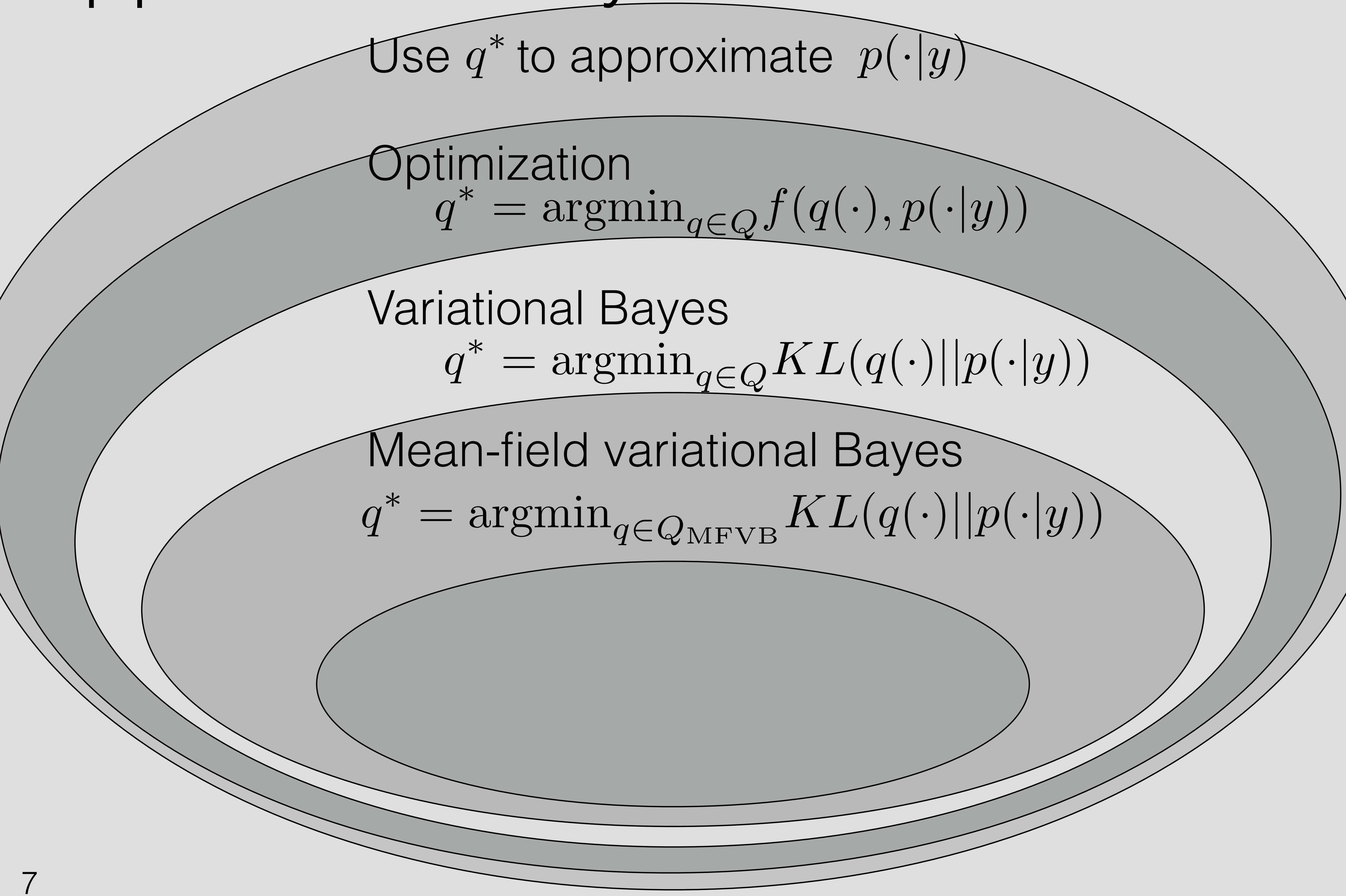

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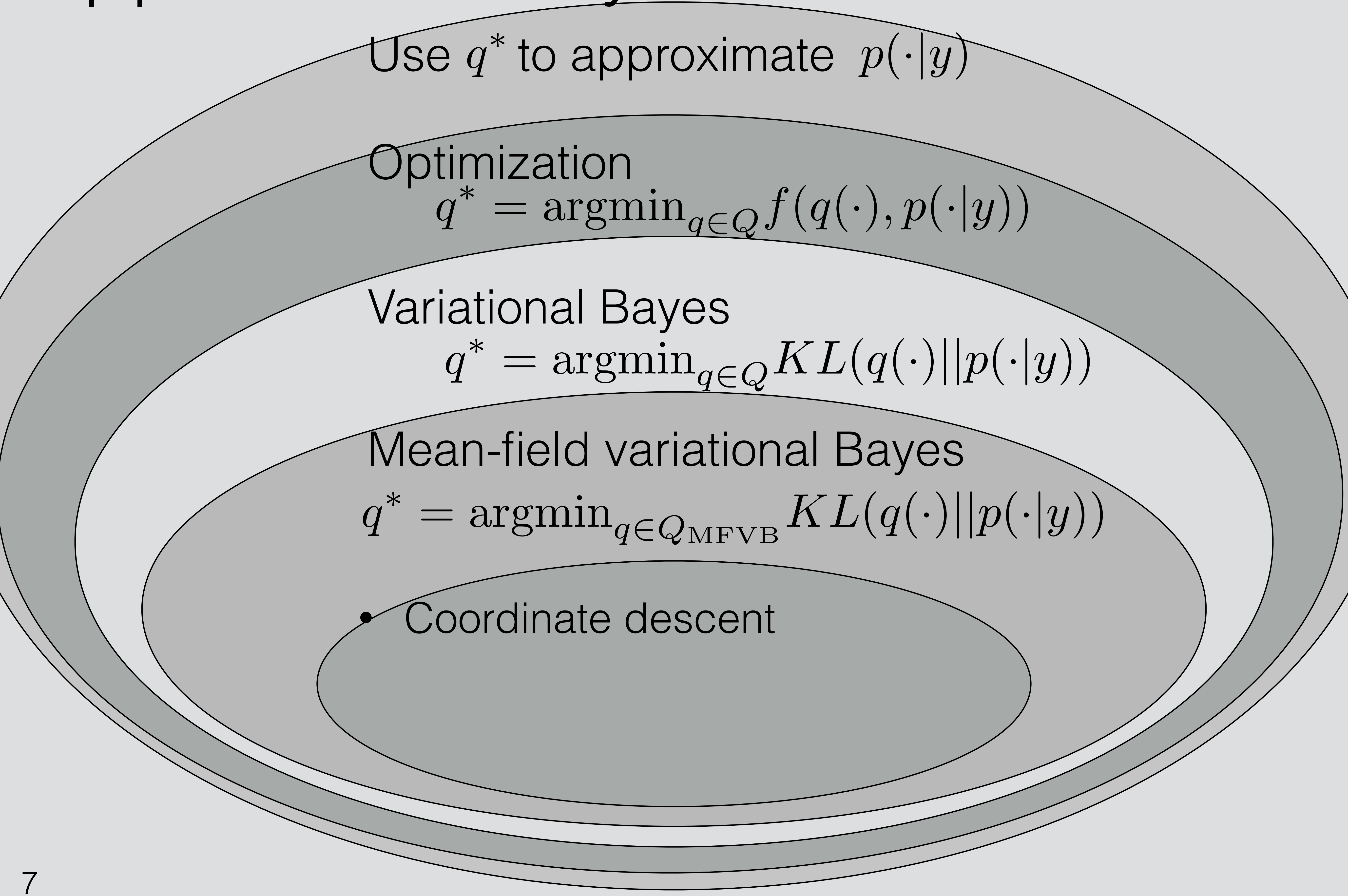

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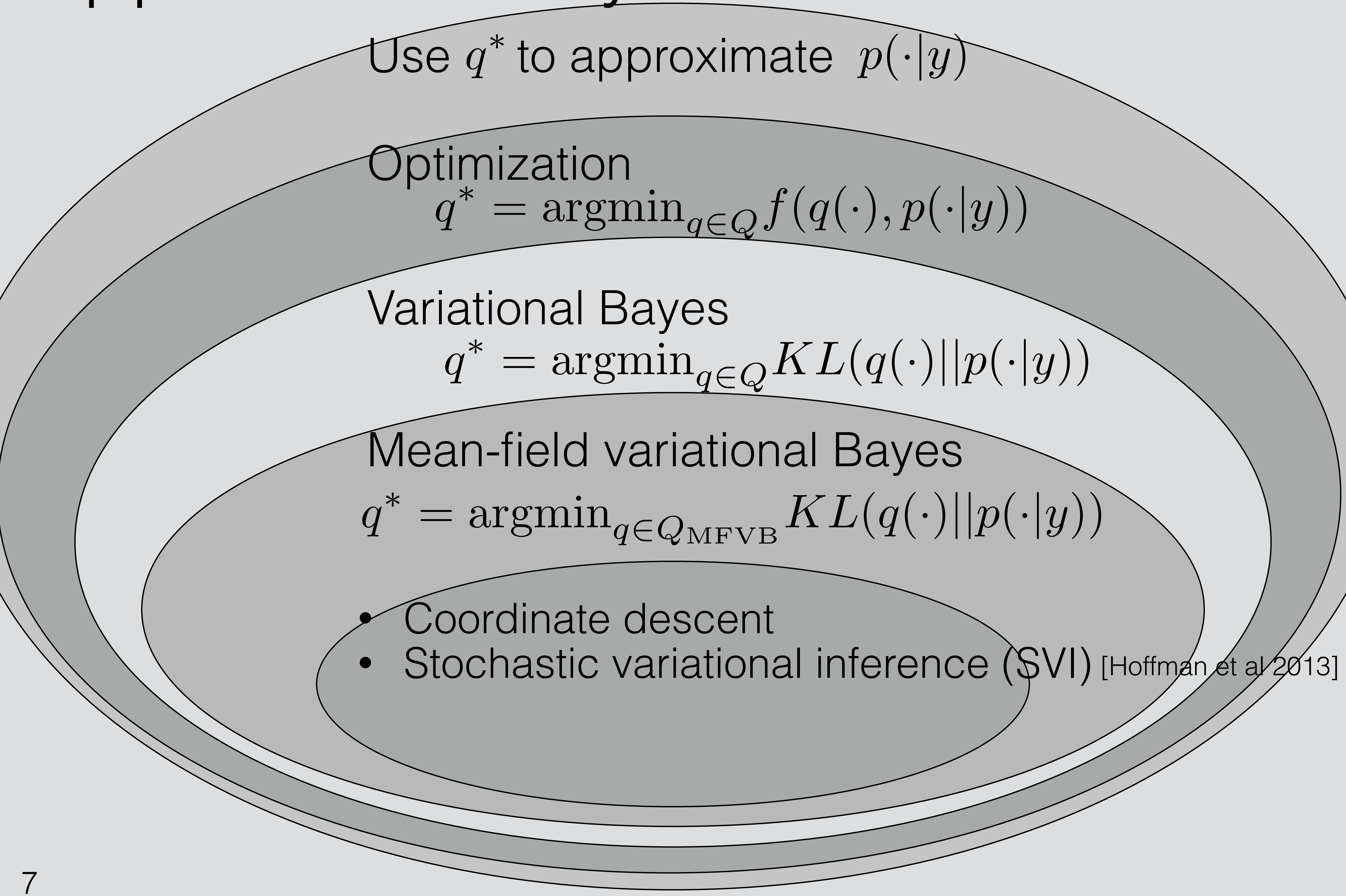

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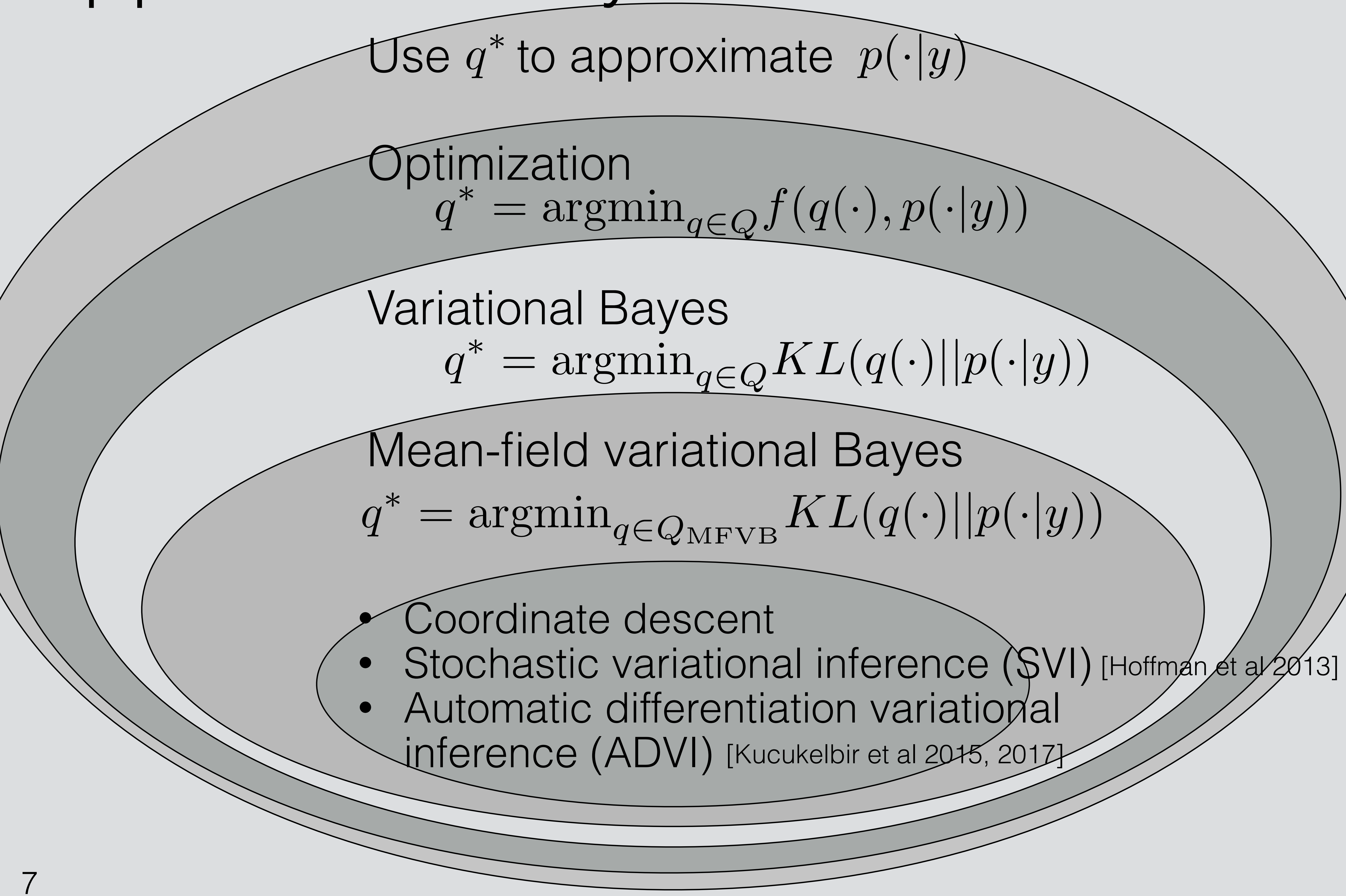

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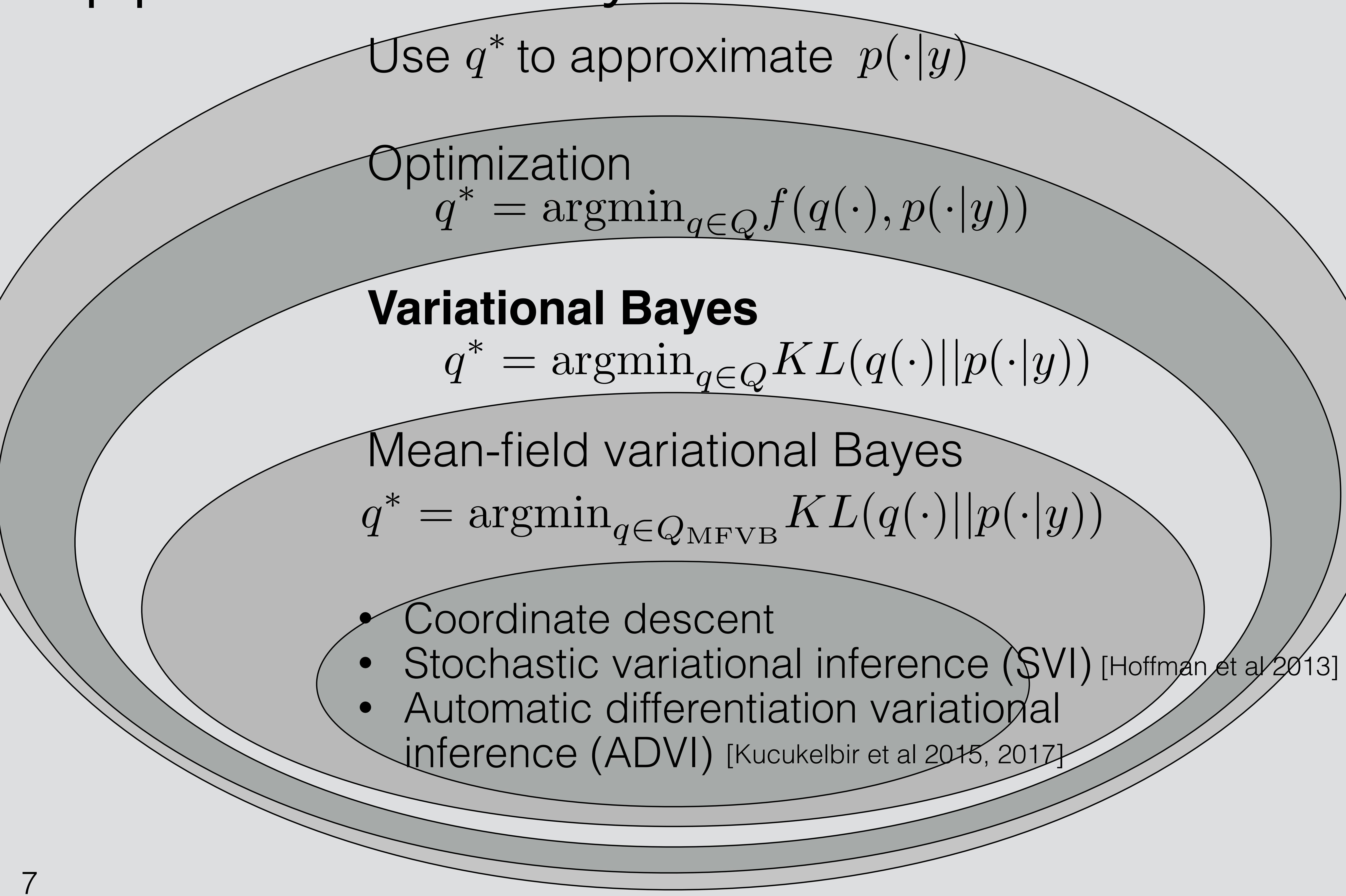

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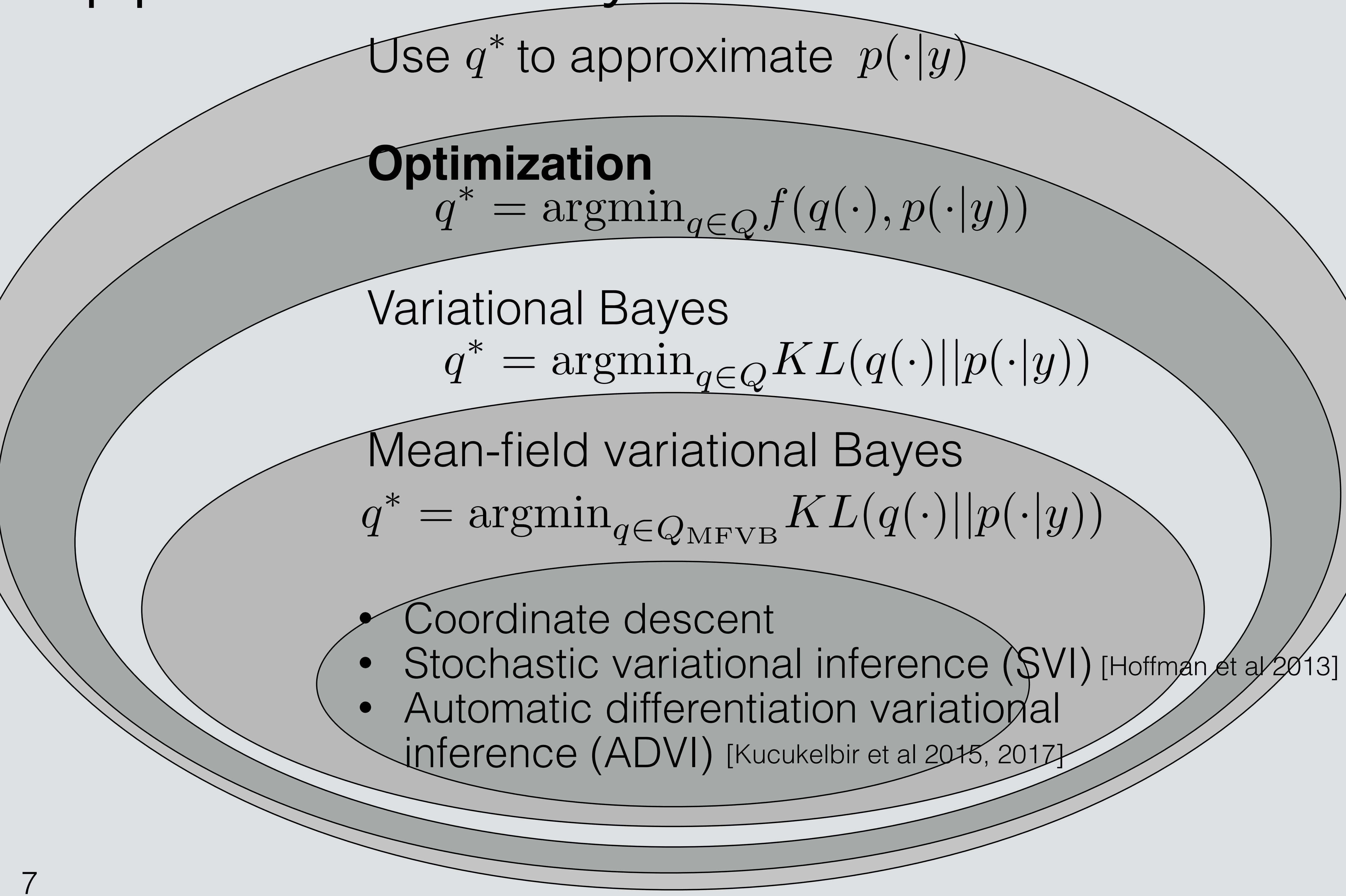

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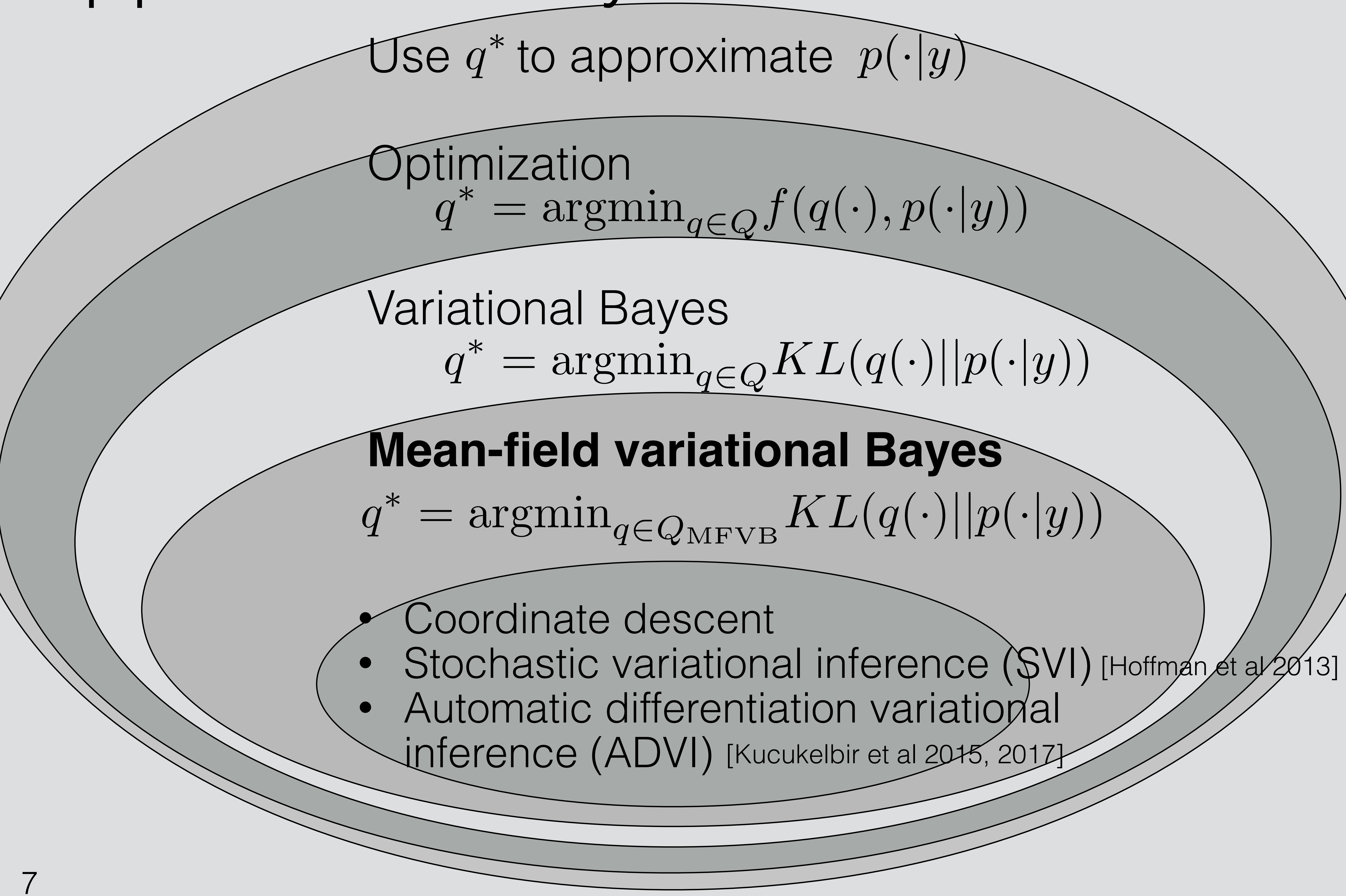

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Air pollution: Particulate matter

[Krongut 2020]

[MacKay 2003; Bishop 2006]

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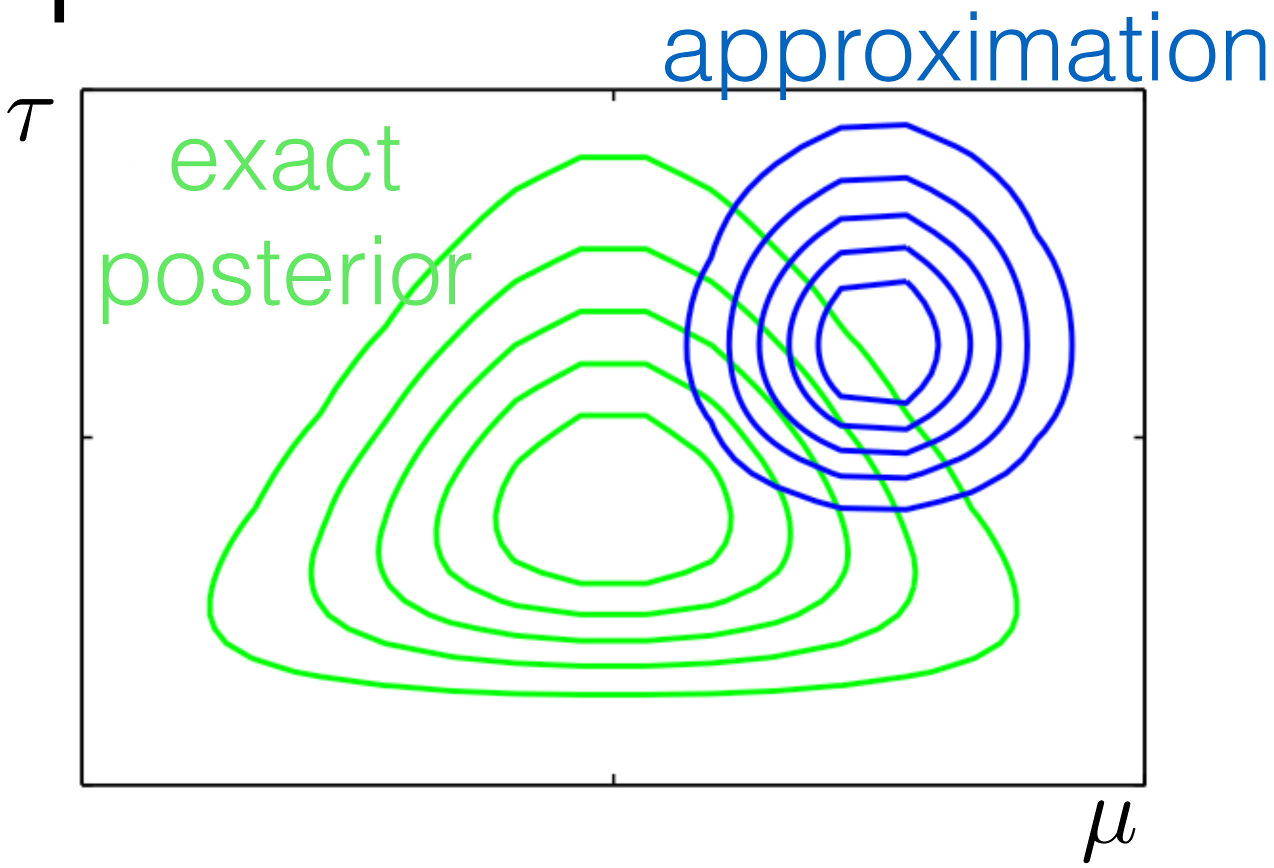

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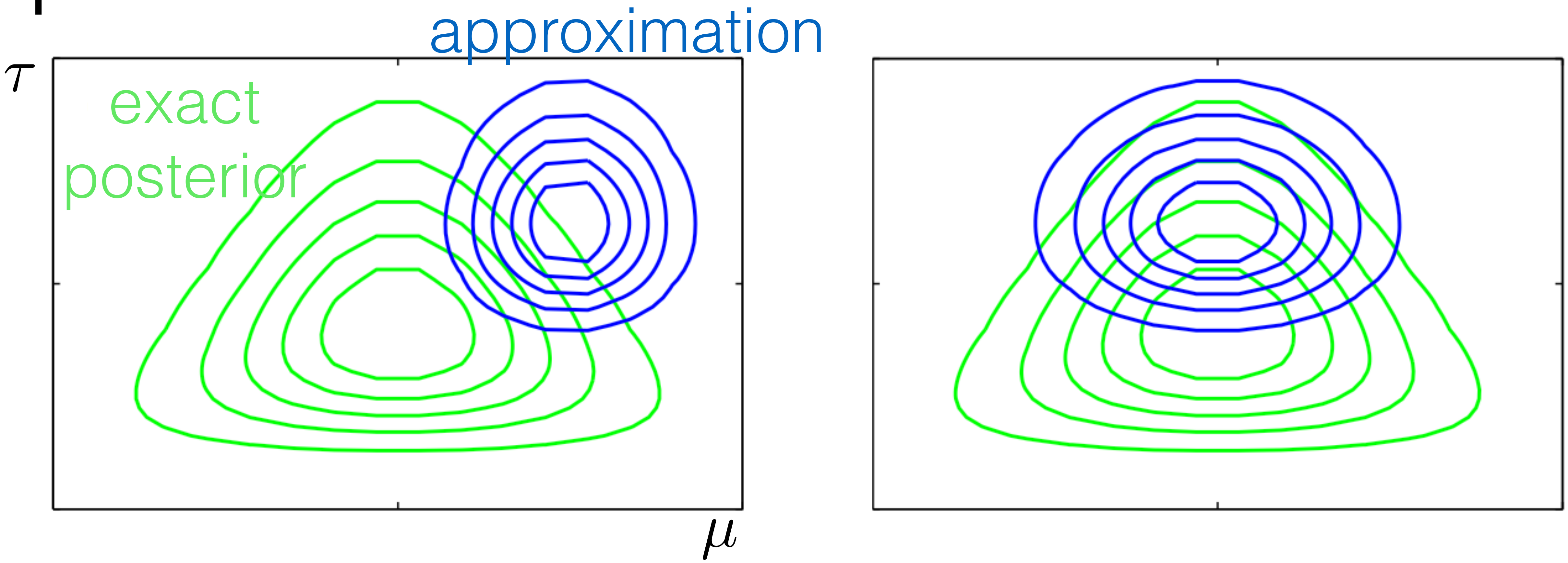

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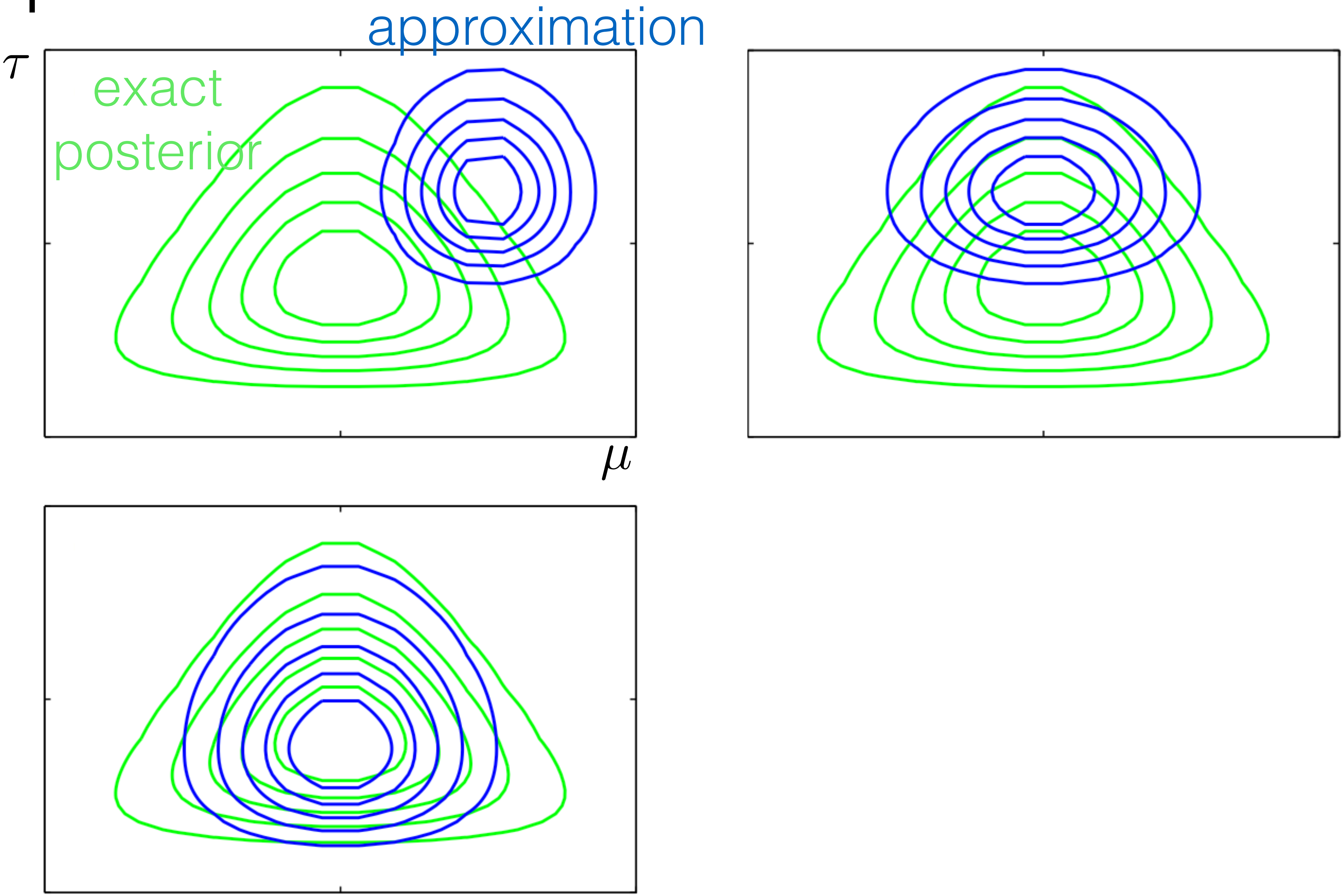

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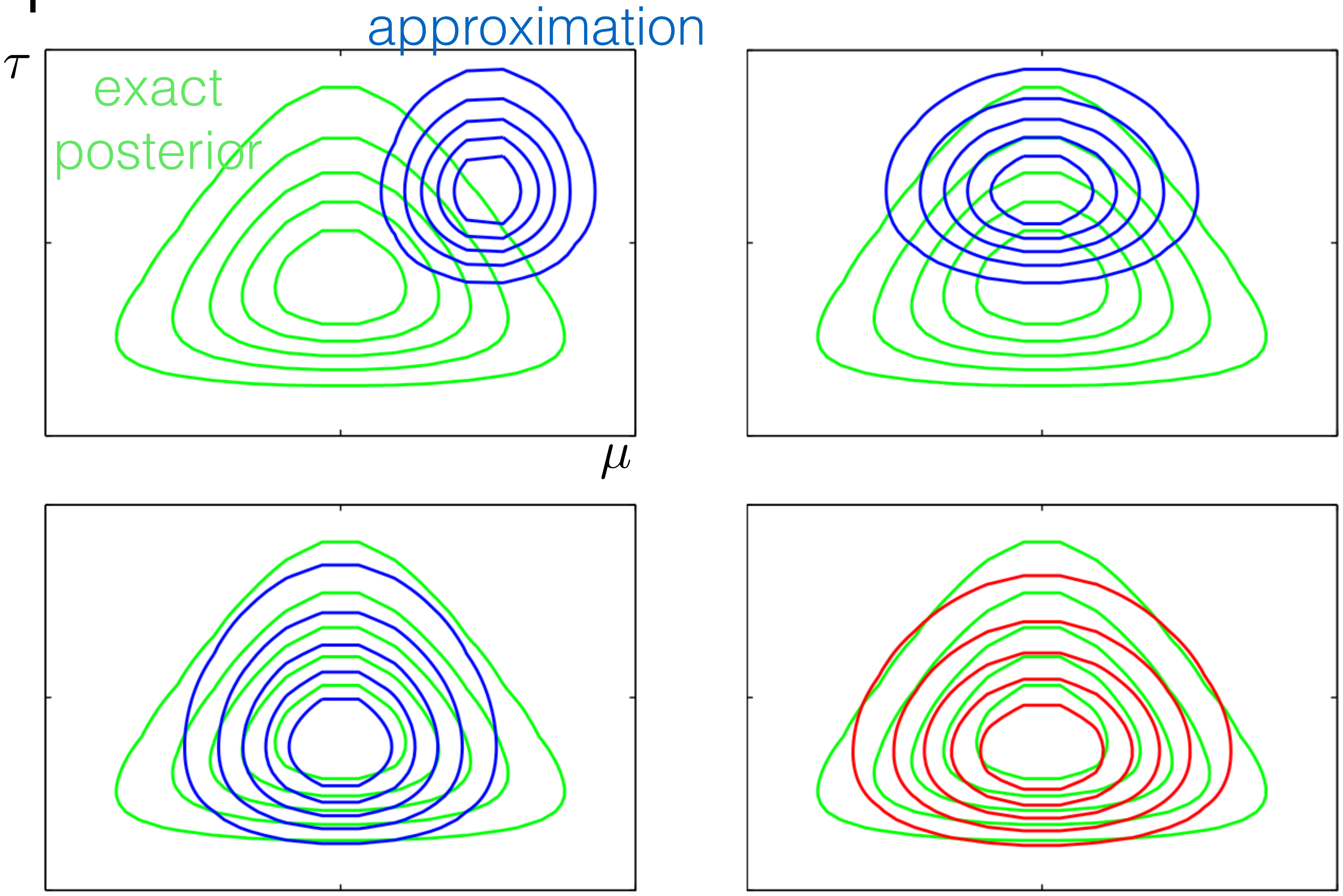

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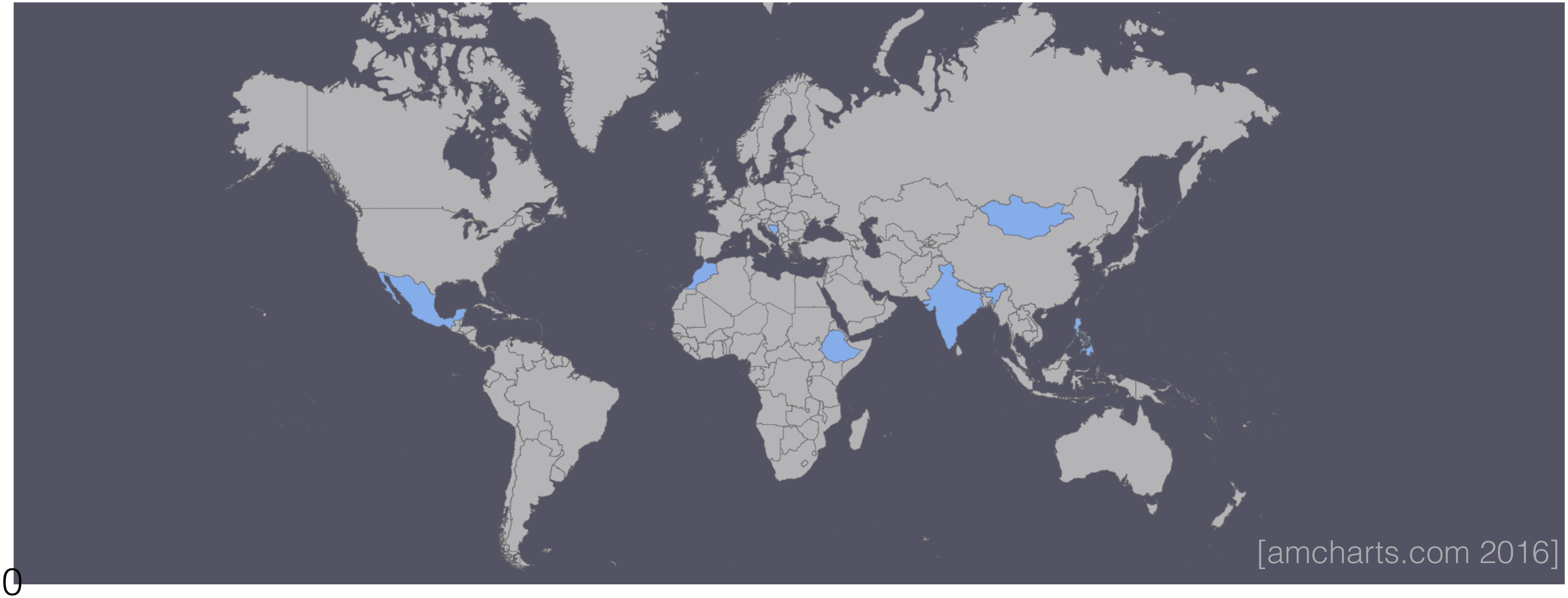

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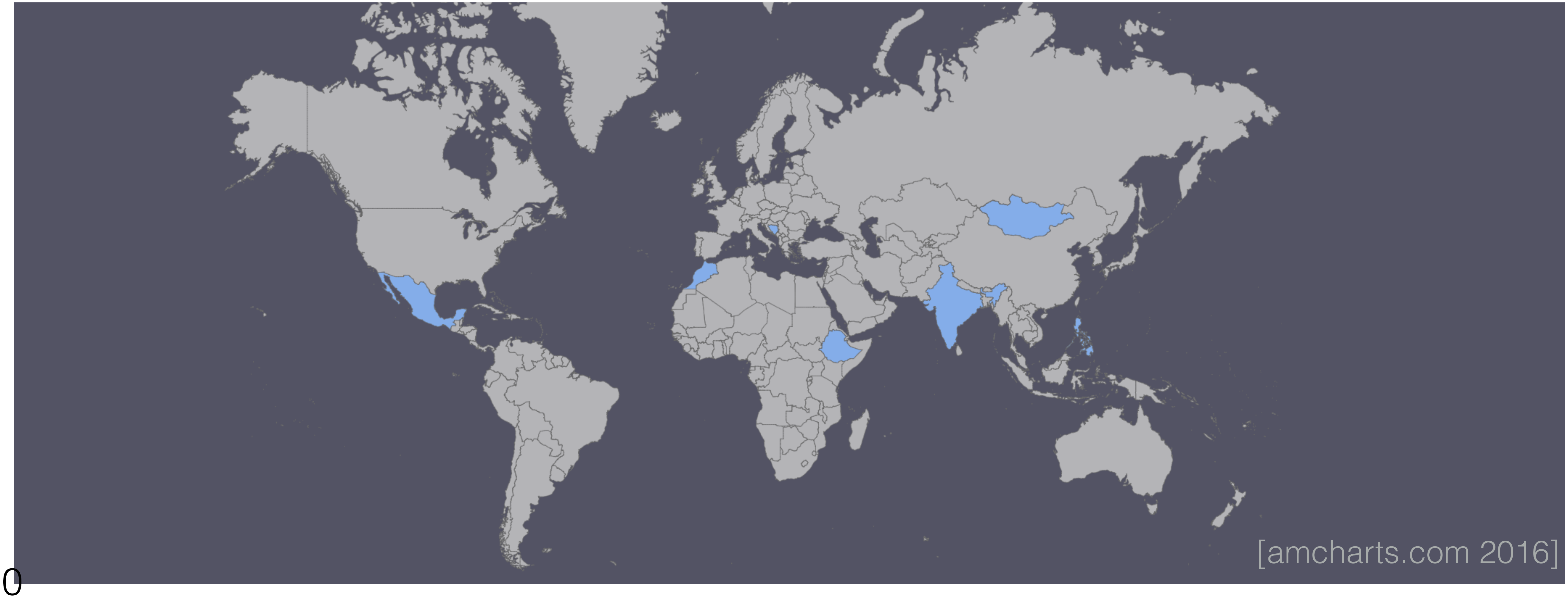

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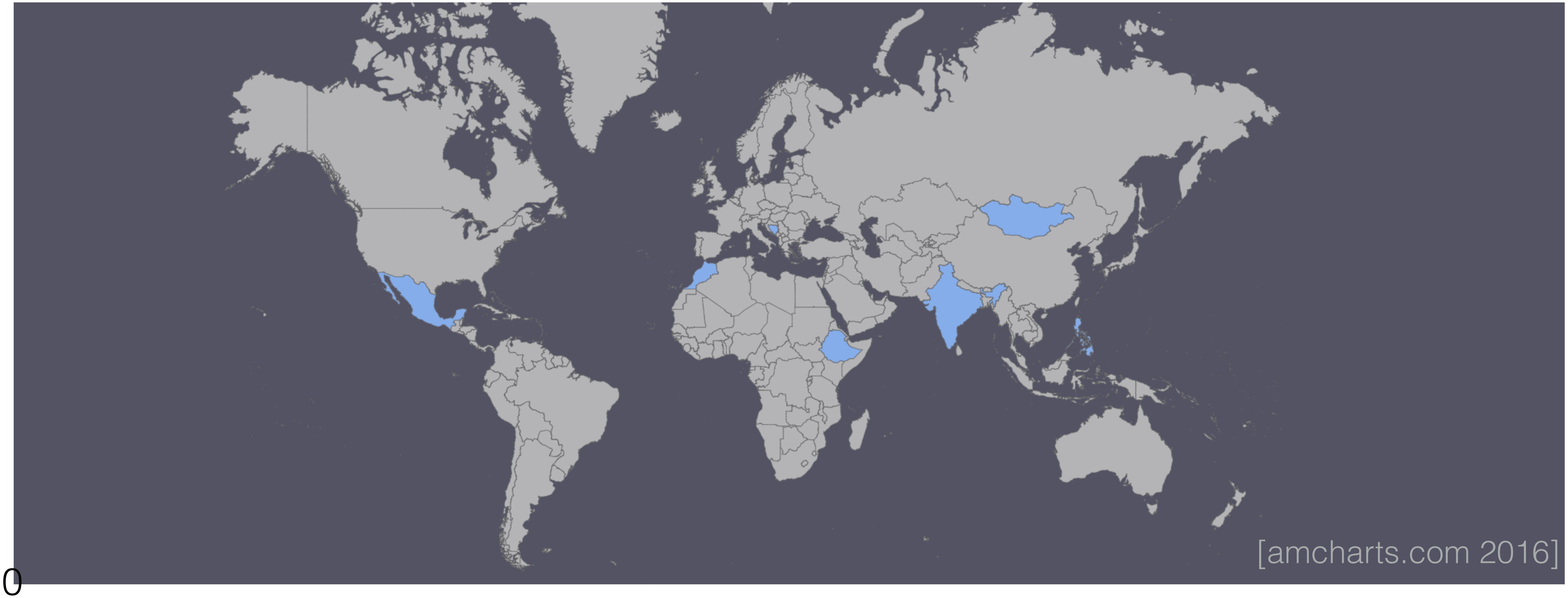

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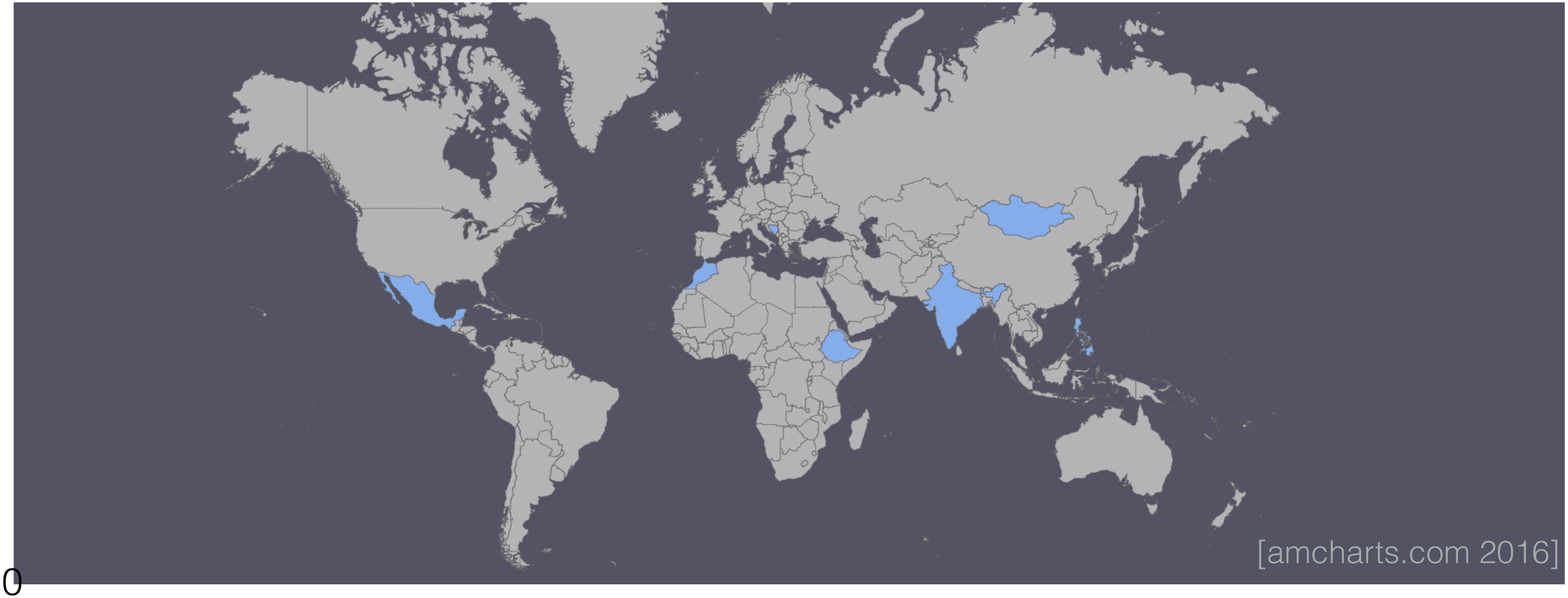

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- Profit of nth business at kth site:

1 if microcredit

profit

$$y_{kn} \overset{indep}{\sim} \mathcal{N}(\mu_k + T_{kn}\tau_k, \sigma_k^2)$$

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$$\sigma_k^{-2} \overset{iid}{\sim} \Gamma(a, b) \qquad C \sim \mathrm{Sep\&LKJ}(\eta, c, d)$$

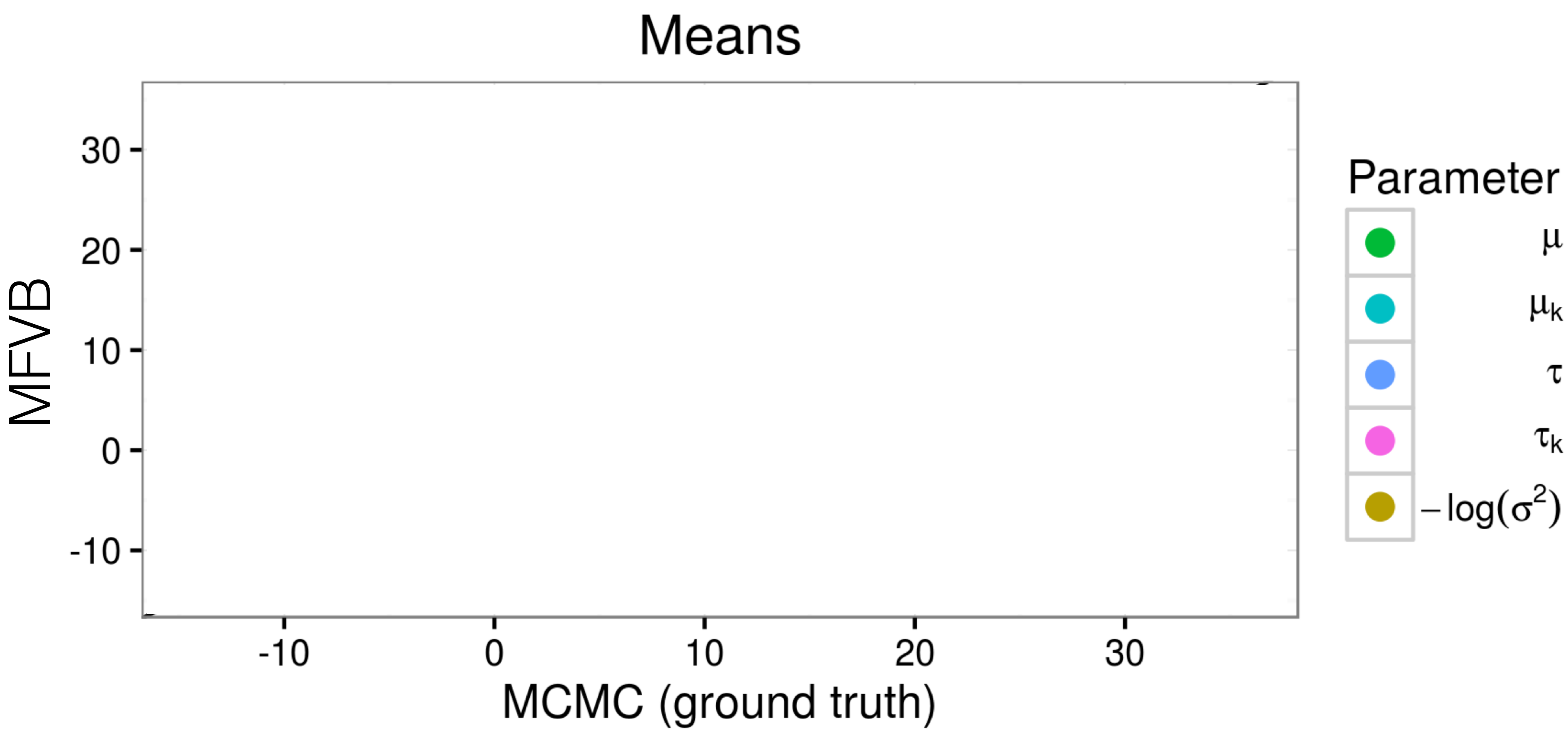
Microcredit

MFVB: Do we need to check the output?

Microcredit

MFVB: How will we know if it’s working?

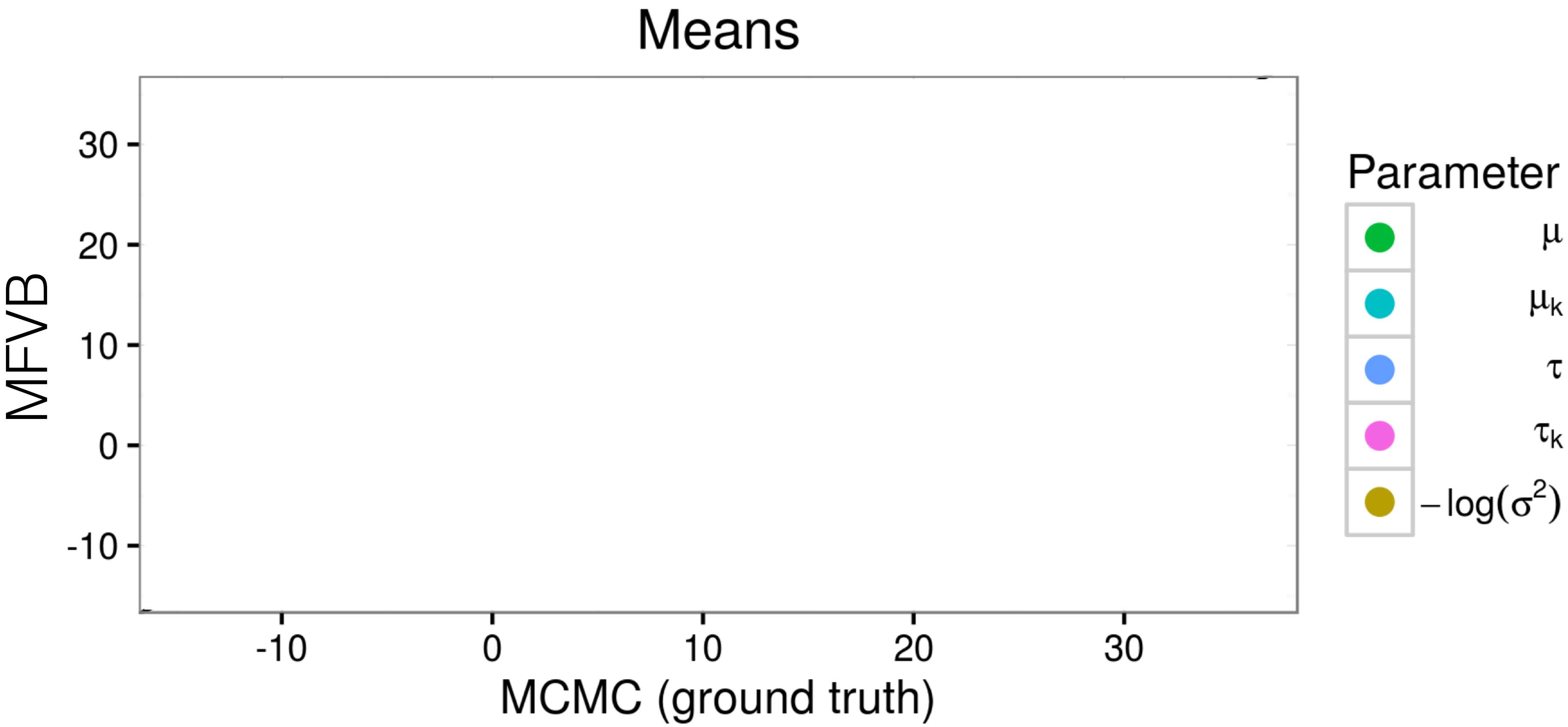
Microcredit



[Giordano, Broderick, Meager, Huggins, Jordan 2016]

Microcredit

- *One set* of 2500 MCMC draws: **45 minutes**



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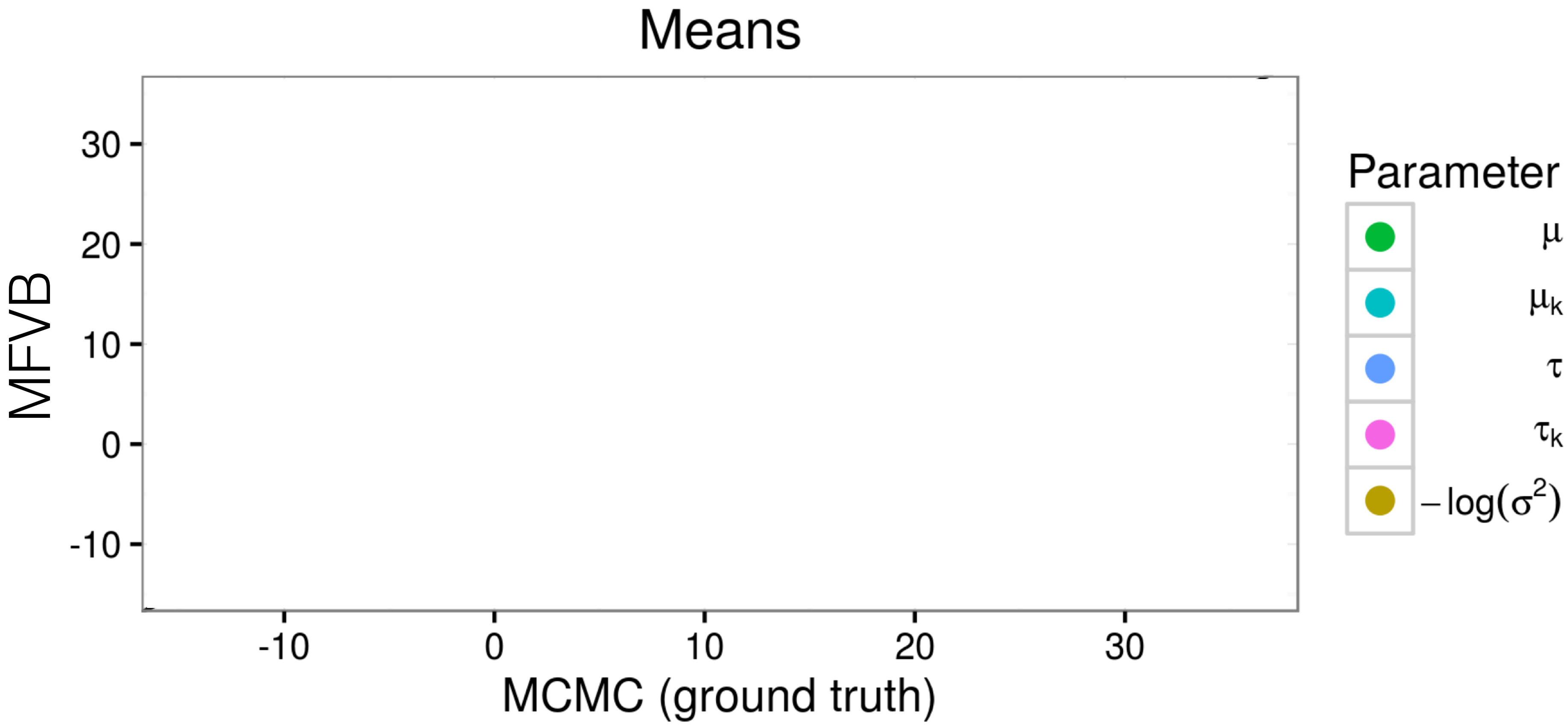

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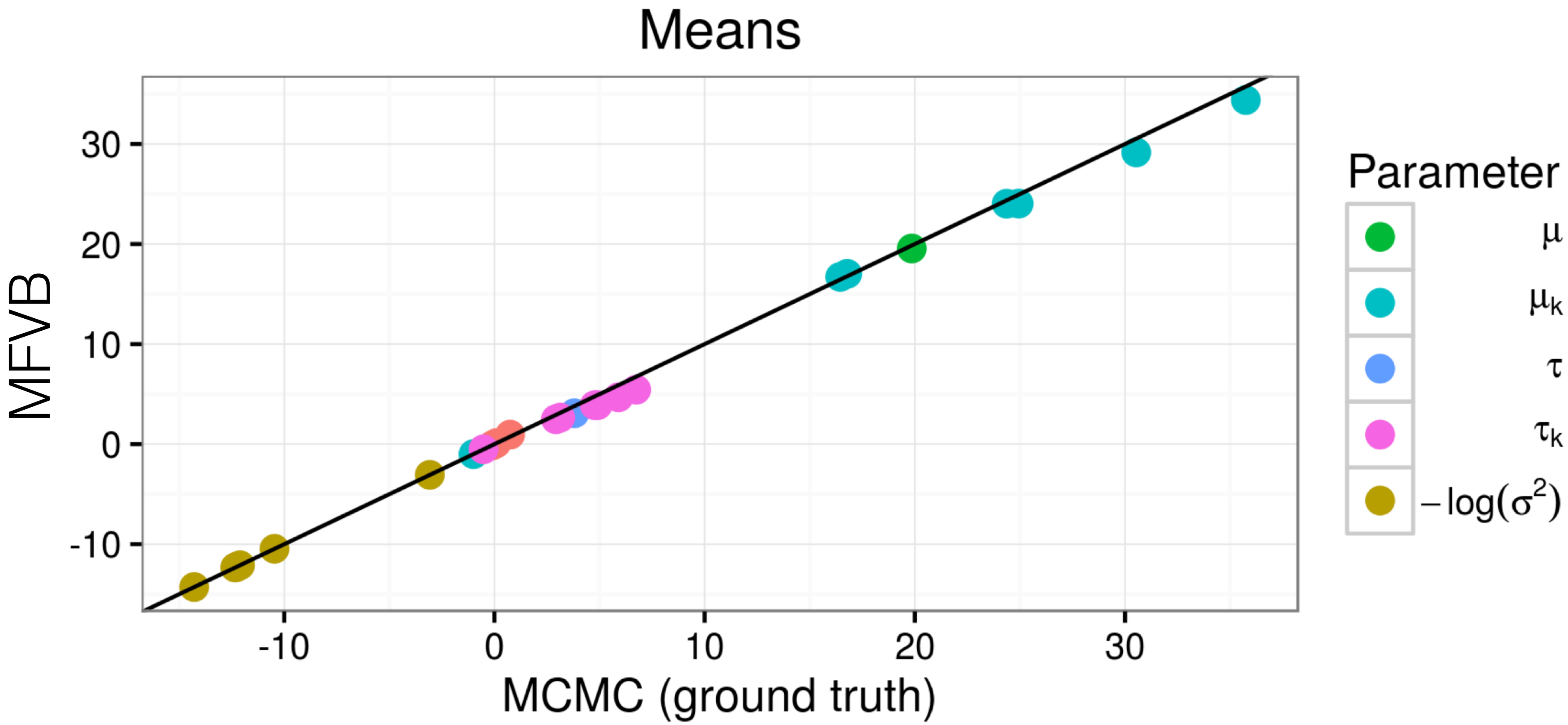

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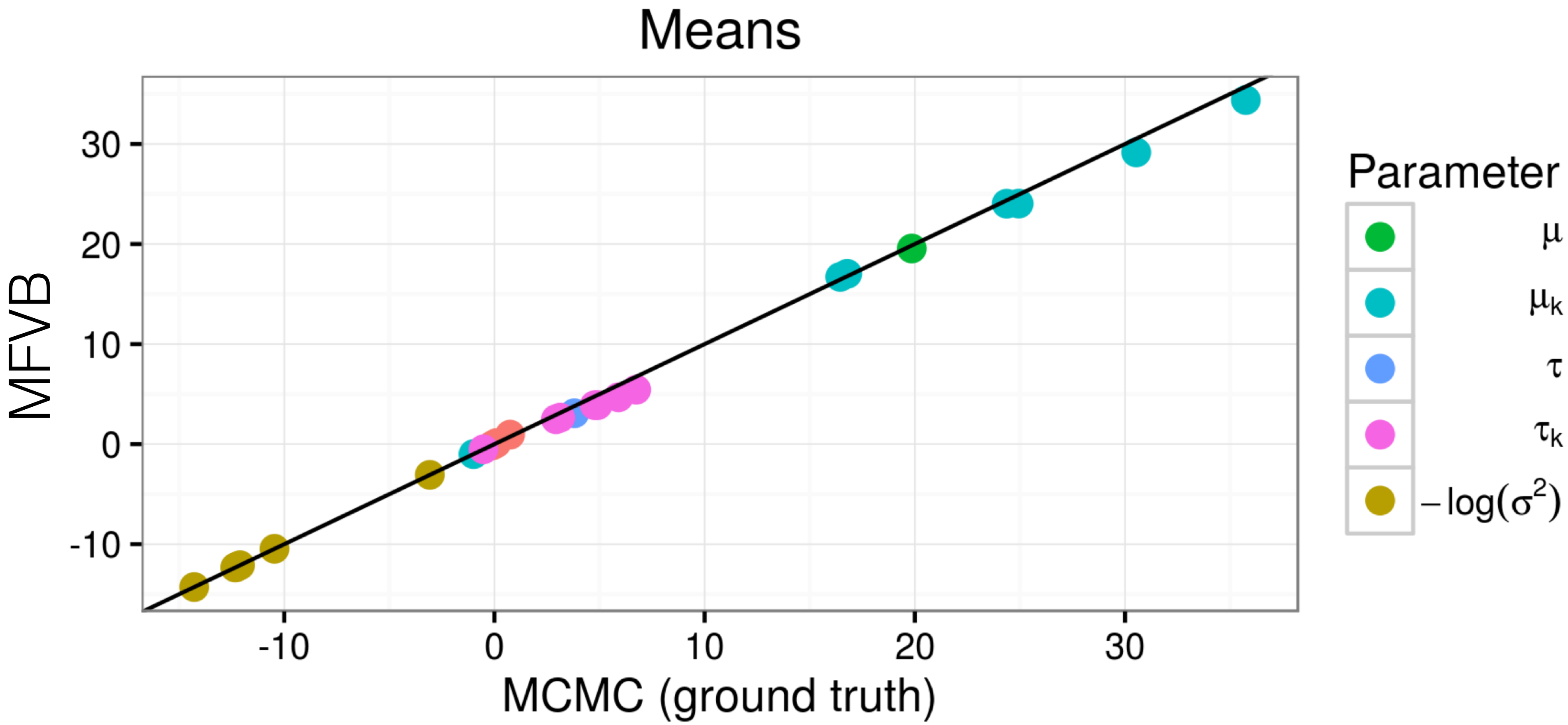

Criteo Online Ads Experiment

- Click-through conversion prediction
- Q: Will a customer (e.g.) buy a product after clicking?

[Giordano, Broderick, Meager, Huggins, Jordan 2016]

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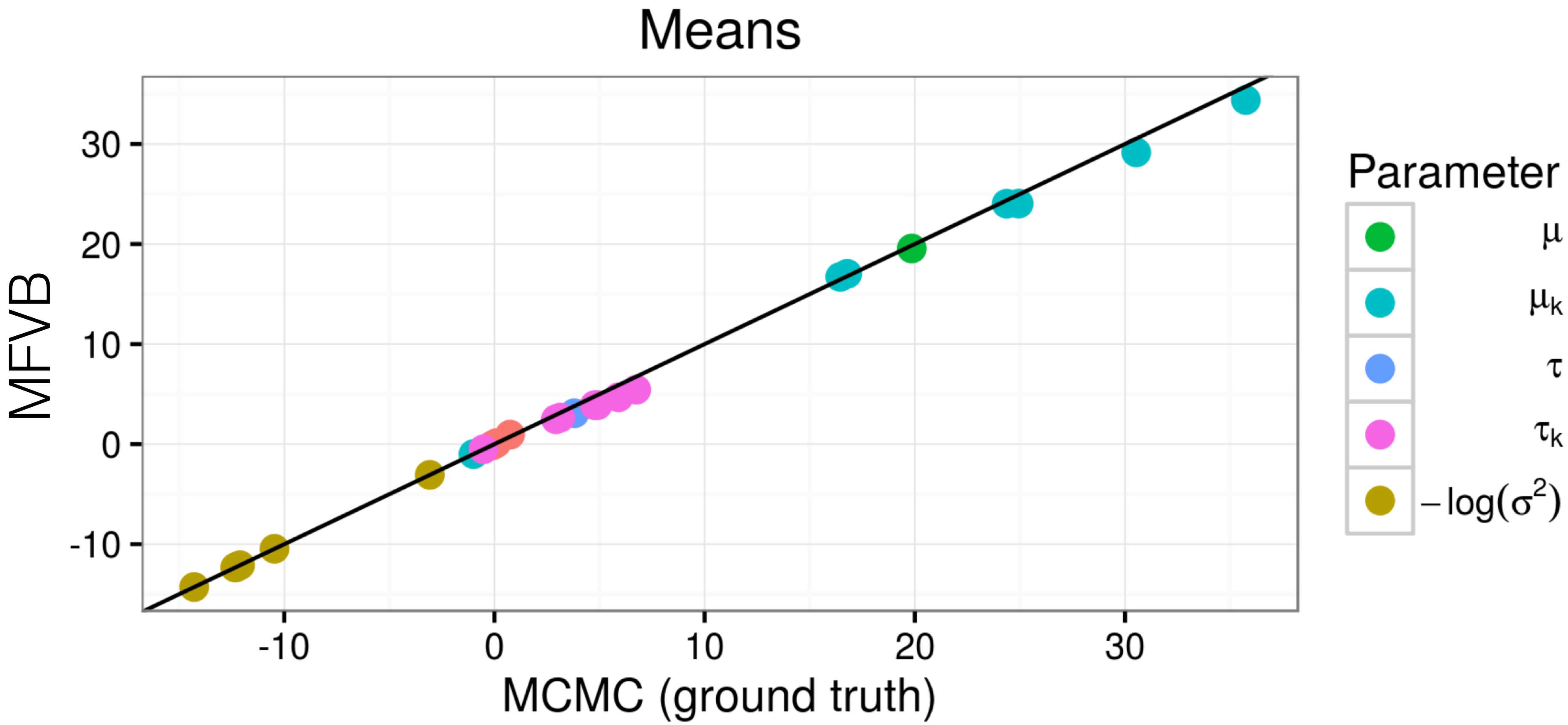

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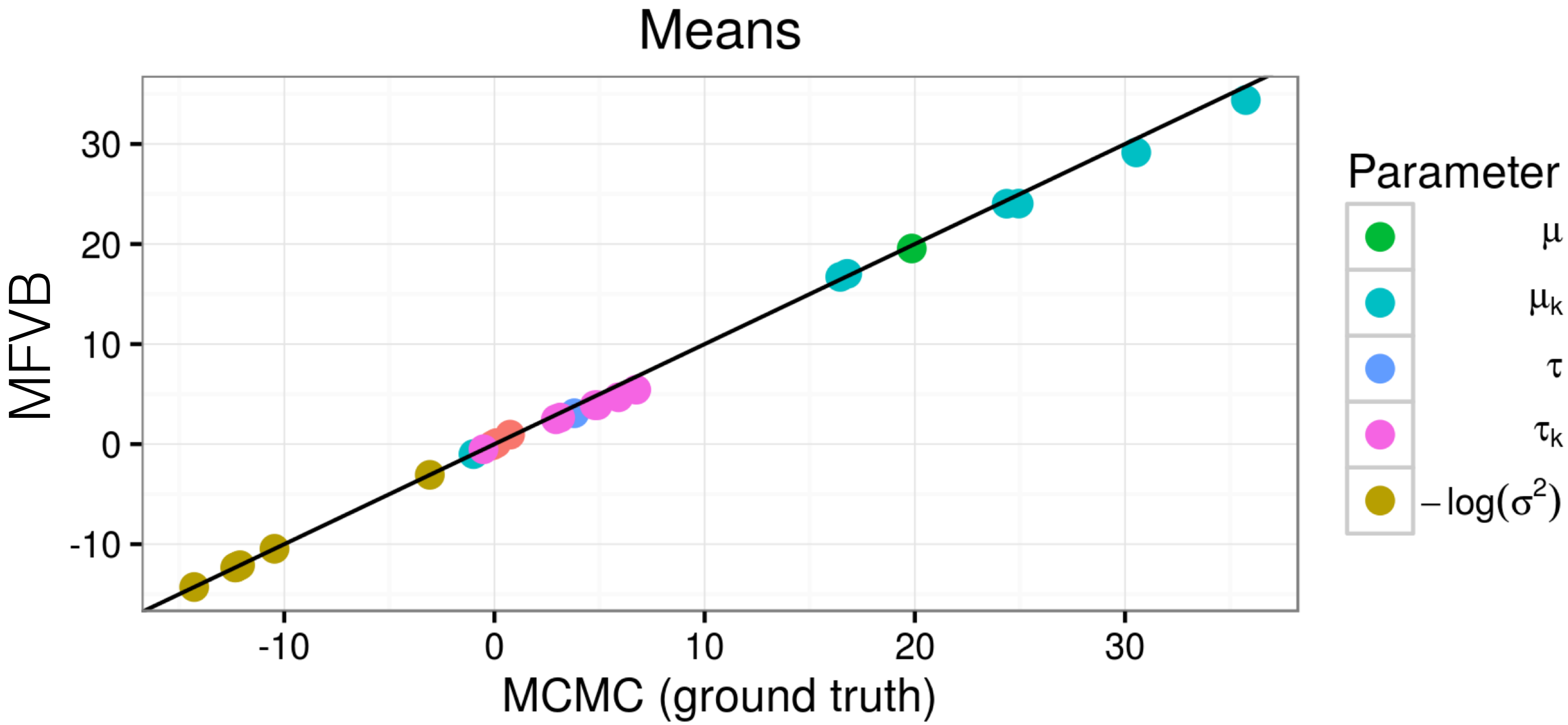

Criteo Online Ads Experiment

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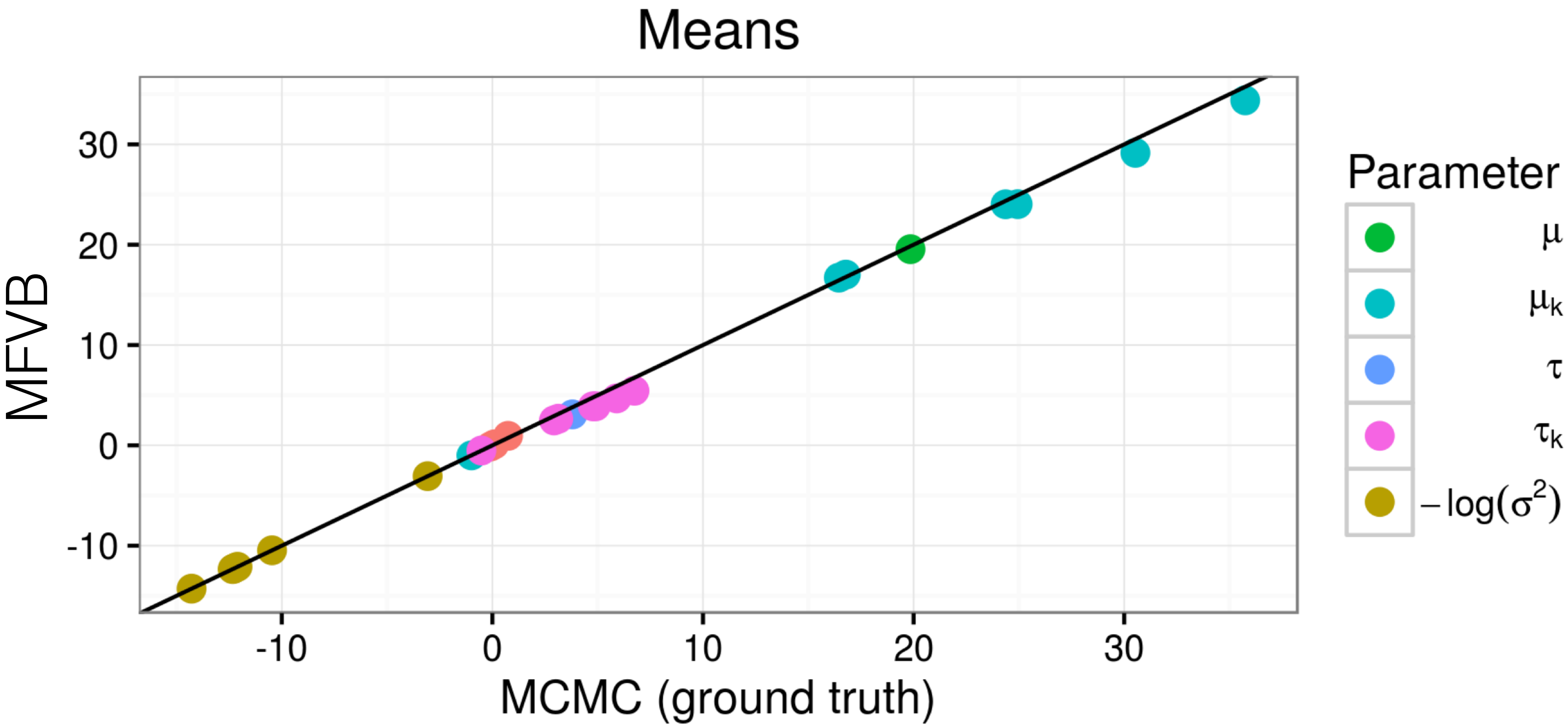


Criteo Online Ads Experiment

- Click-through conversion prediction
- Q: Will a customer (e.g.) buy a product after clicking?
- Q: How predictive of conversion are different features?
- Logistic GLMM; N = 61,895 subset to compare to MCMC

[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2018]

Criteo Online Ads Experiment

[Giordano, Broderick, Jordan 2018]

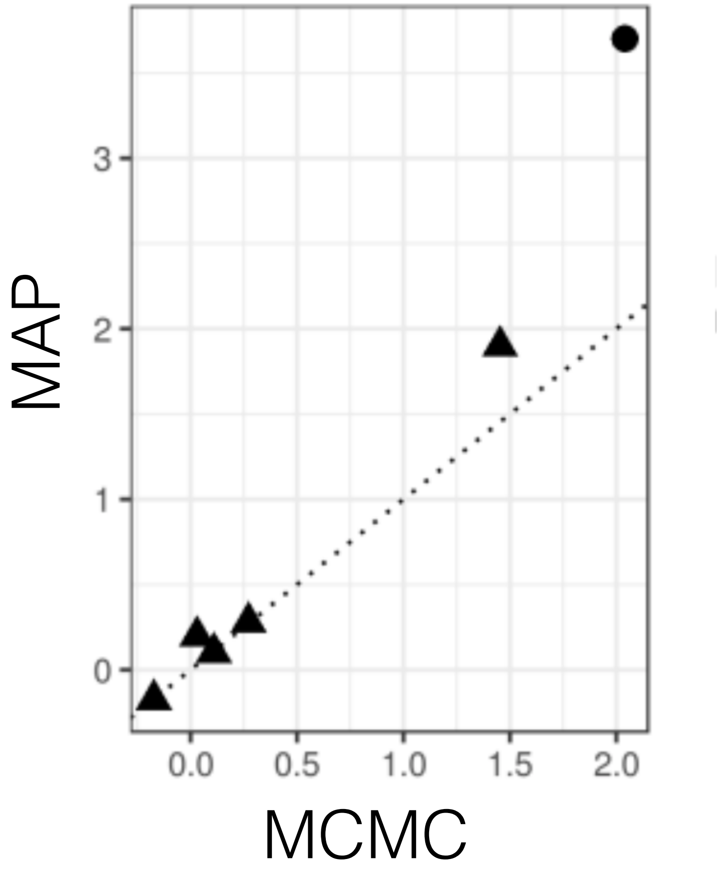
Criteo Online Ads Experiment

- MAP: **12 s**

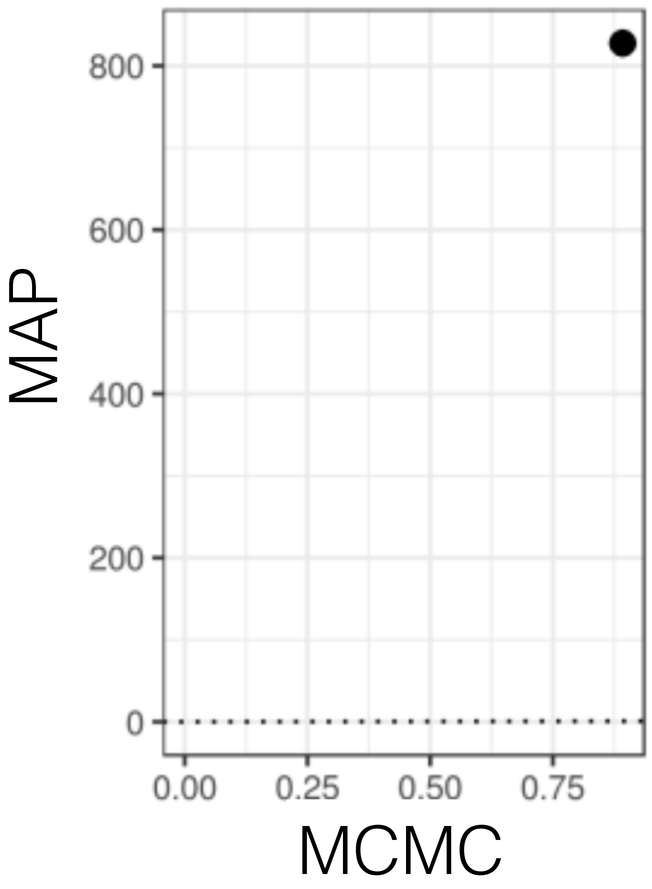
[Giordano, Broderick, Jordan 2018]

Criteo Online Ads Experiment

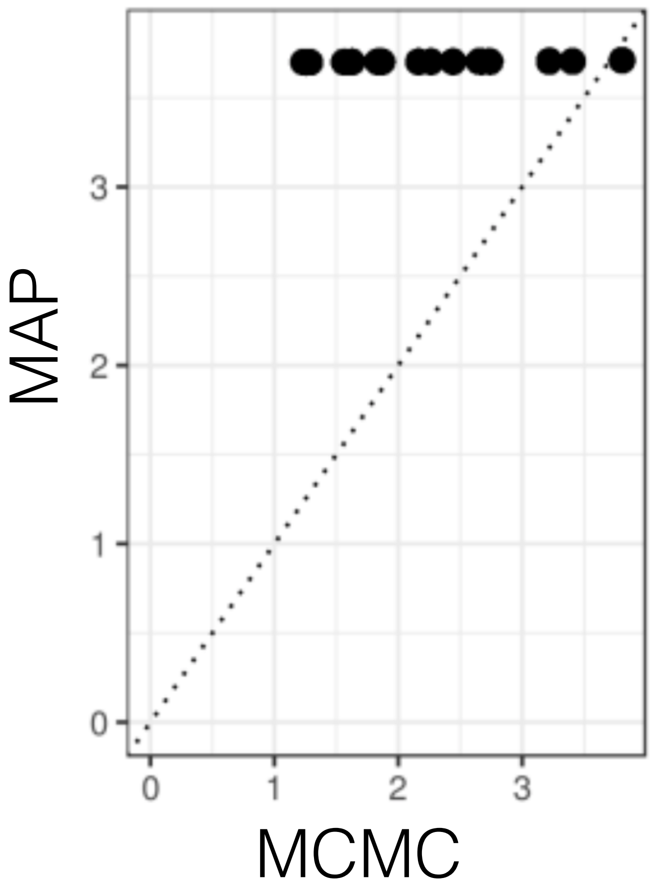
Global parameters (-τ)



Global parameter τ



Local parameters



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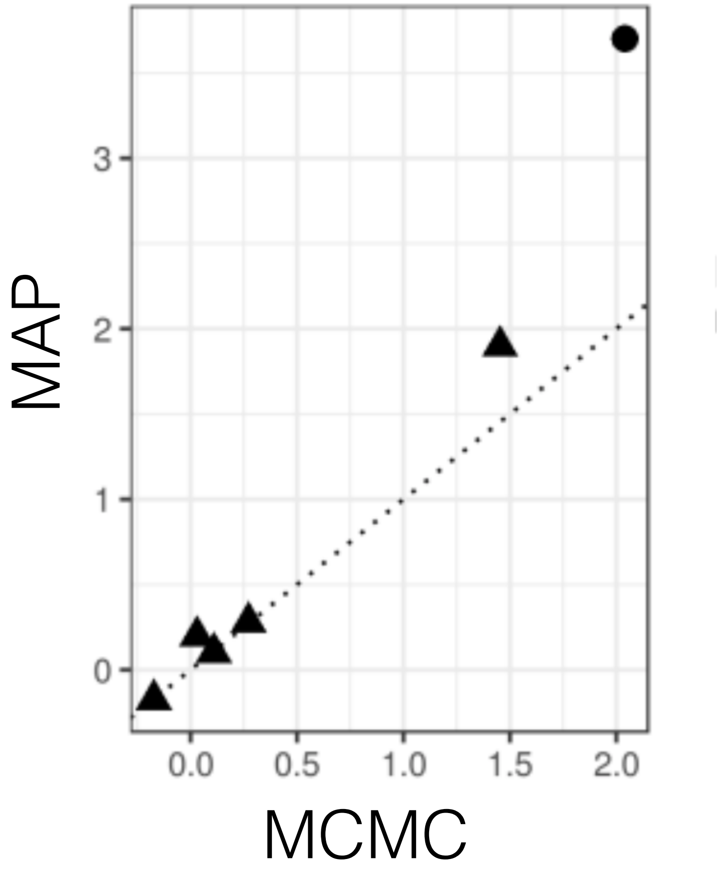

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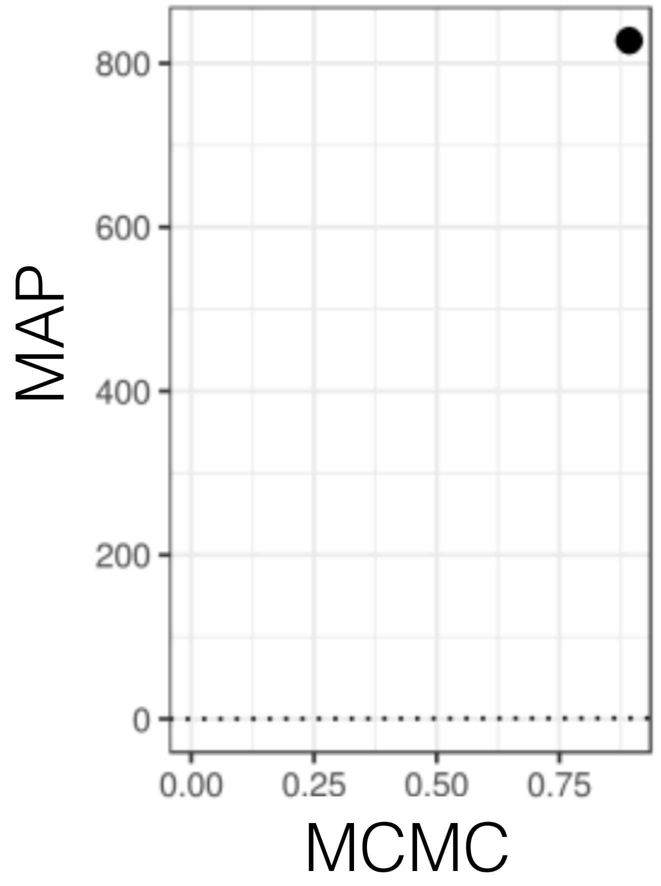

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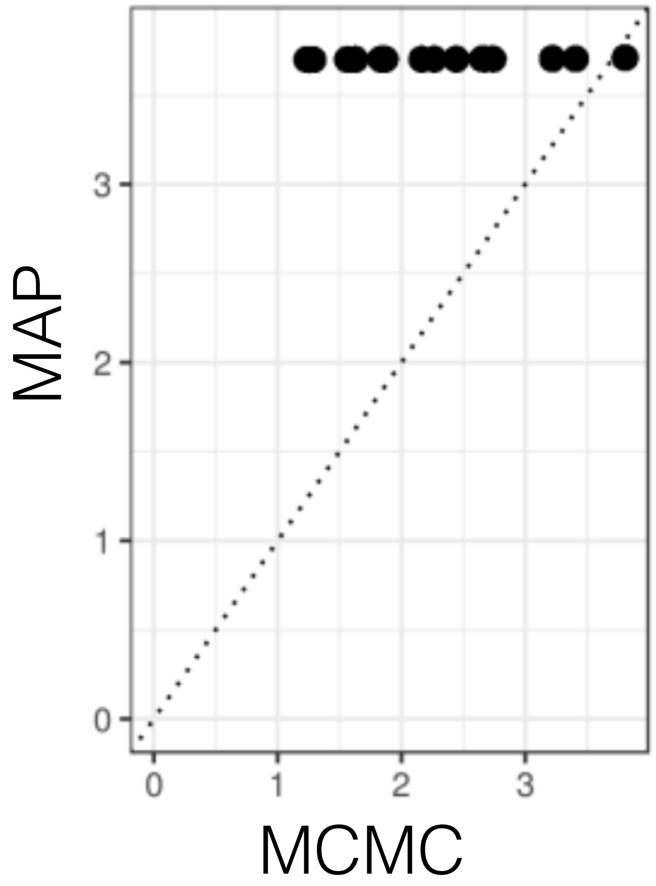

- MAP: **12 s**
- MFVB: **57 s**

[Giordano, Broderick, Jordan 2018]

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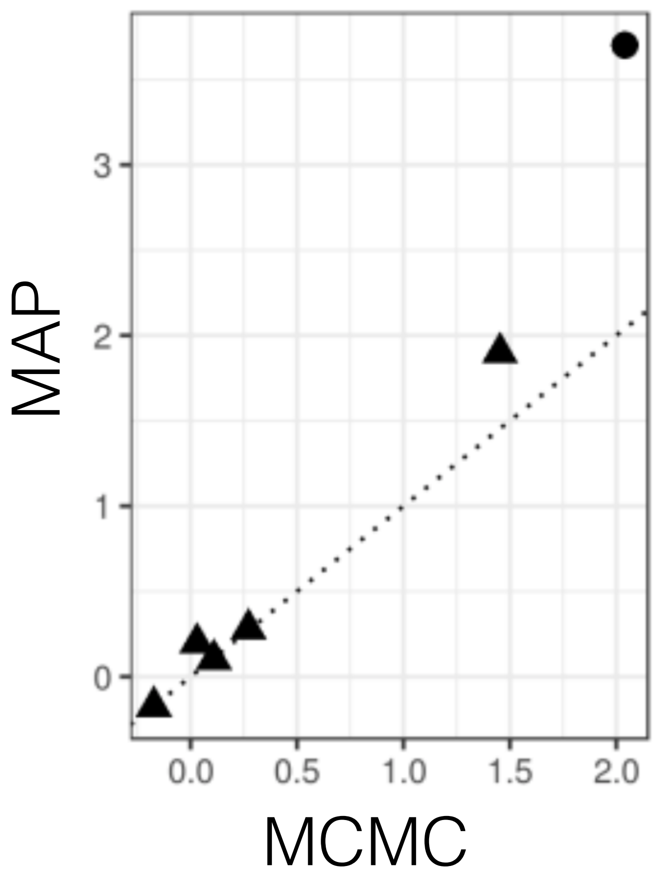

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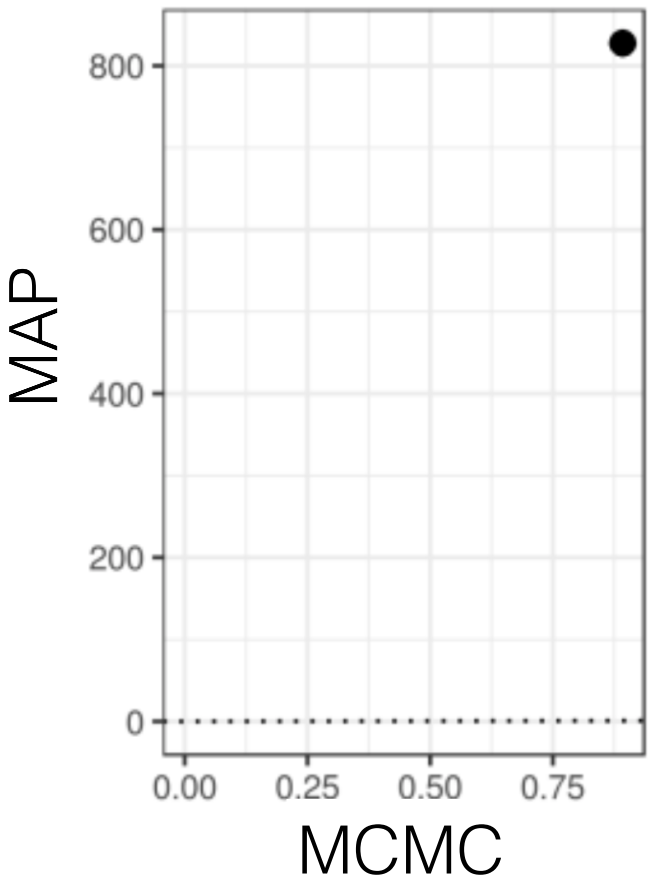

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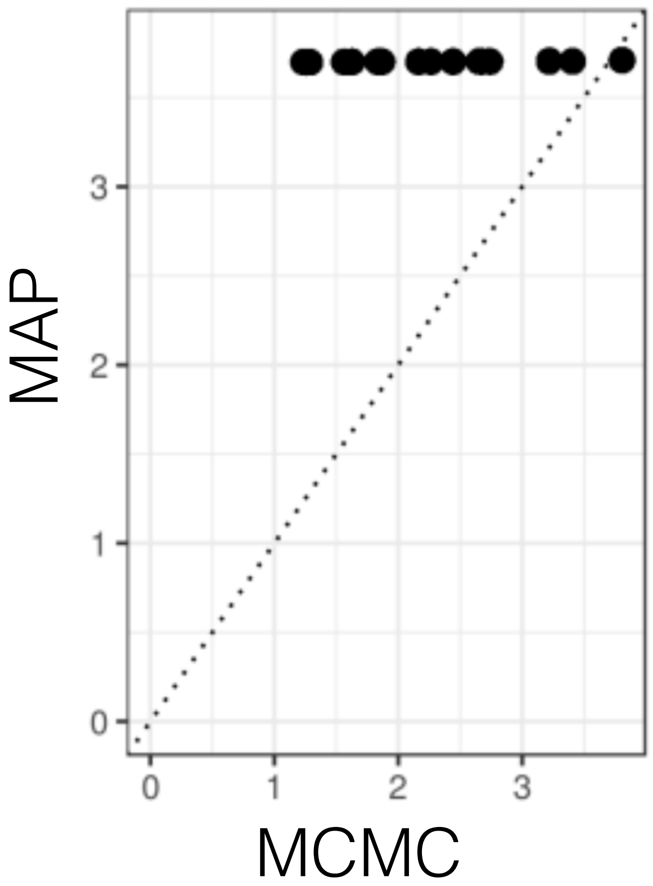

- MAP: **12 s**
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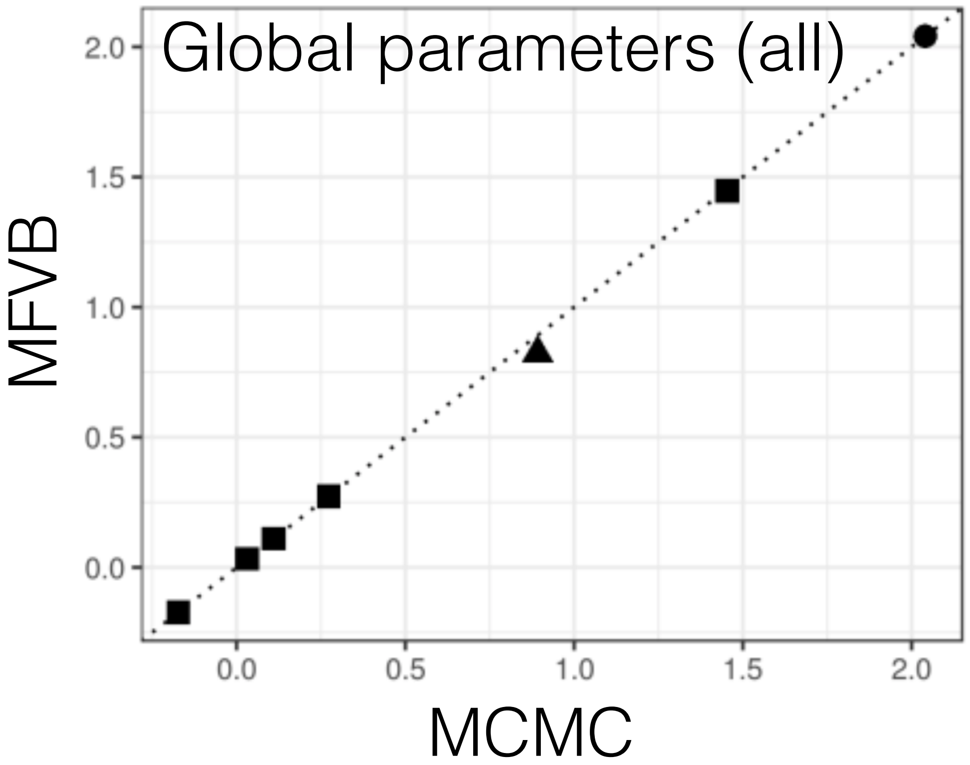
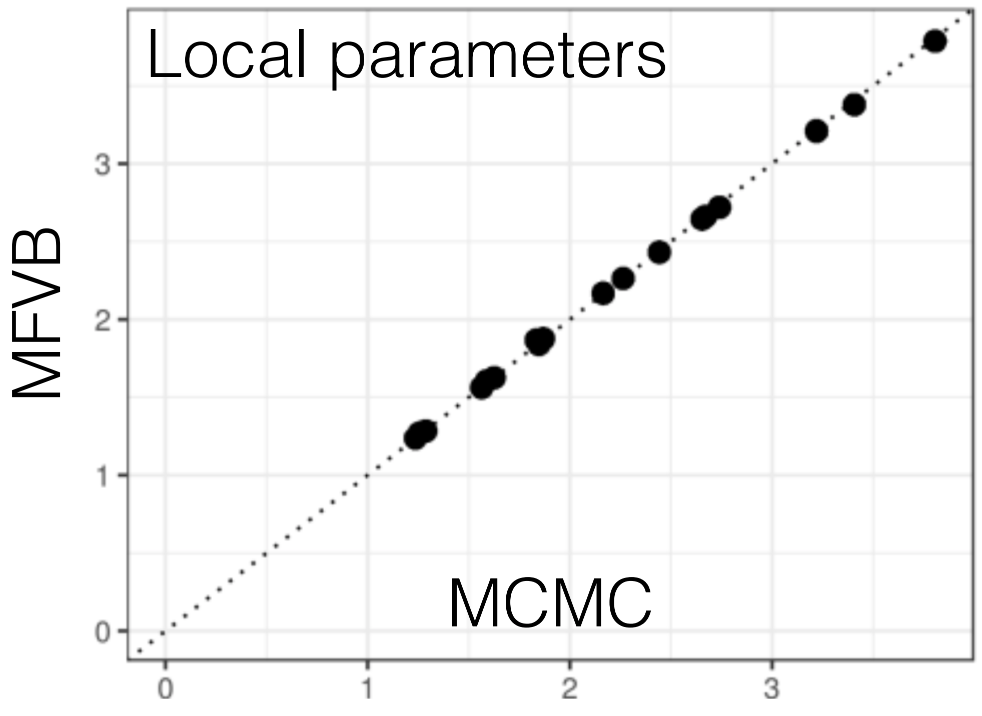

Global parameters (all)


Local parameters

[Giordano, Broderick, Jordan 2018]

Criteo Online Ads Experiment

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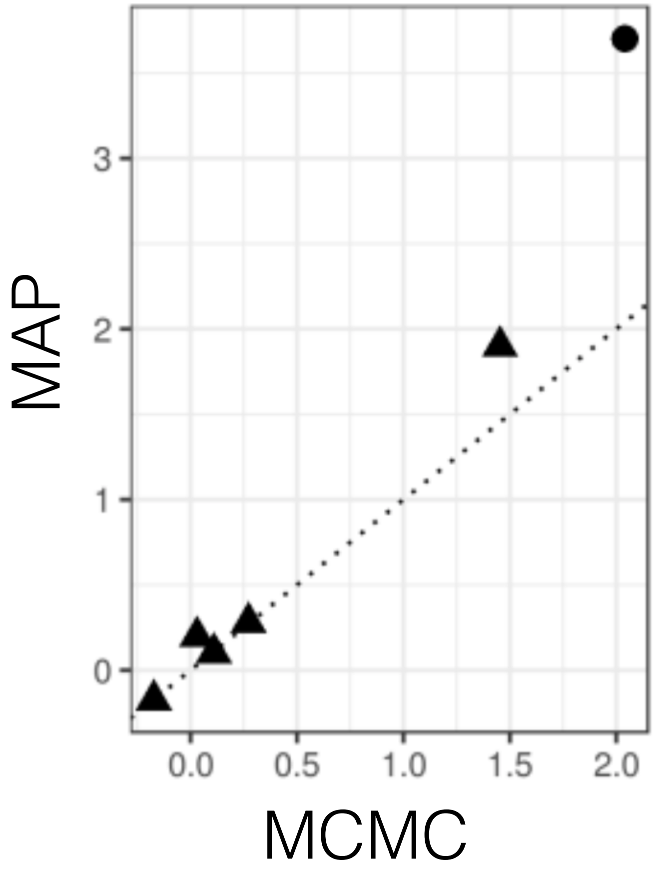

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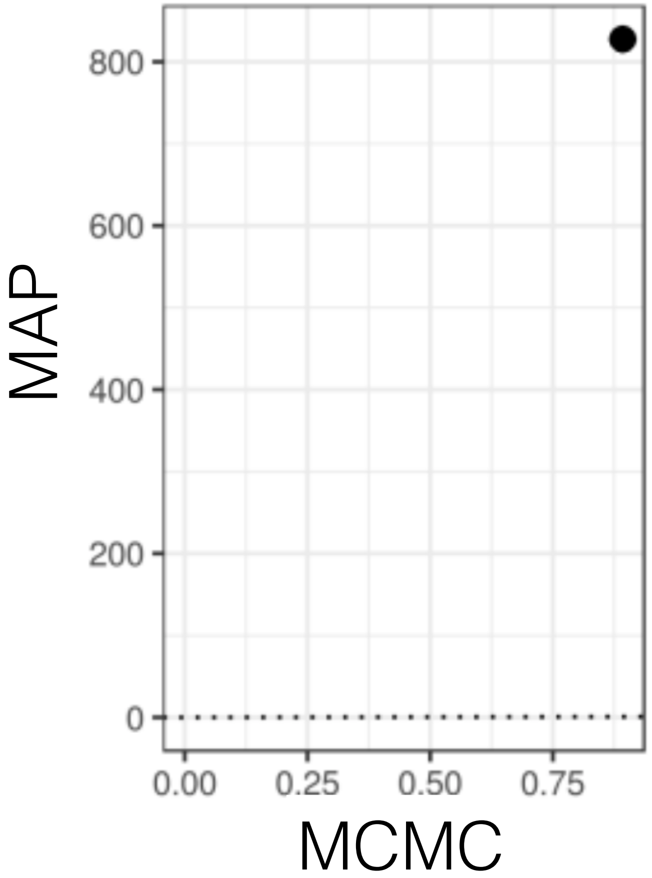

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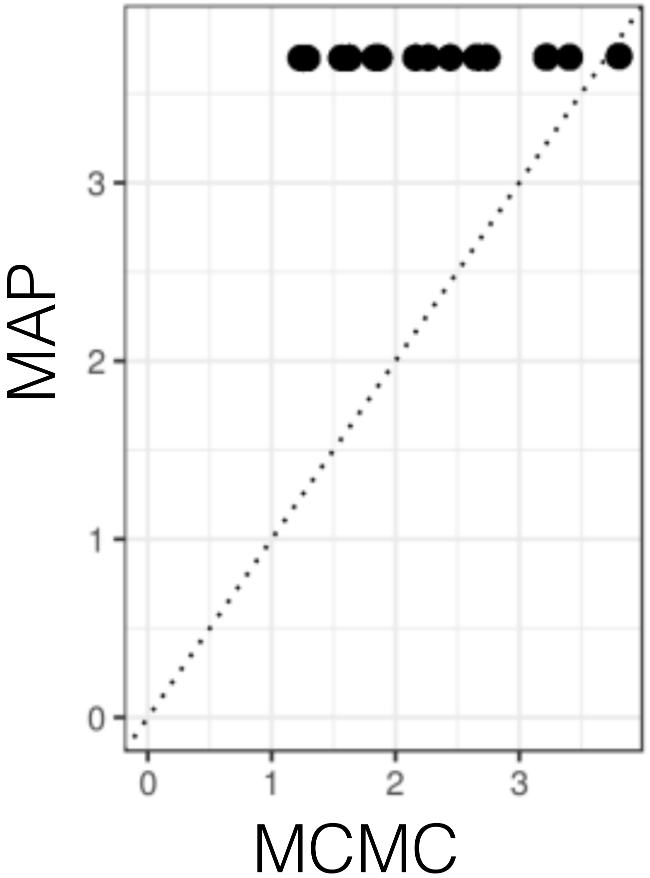
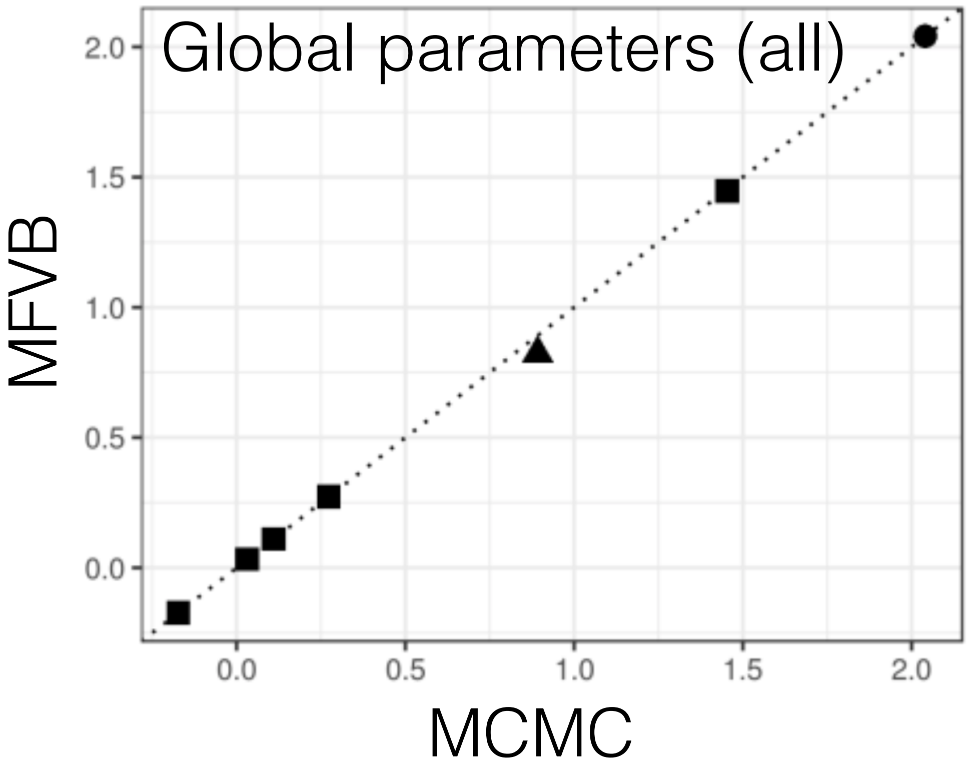
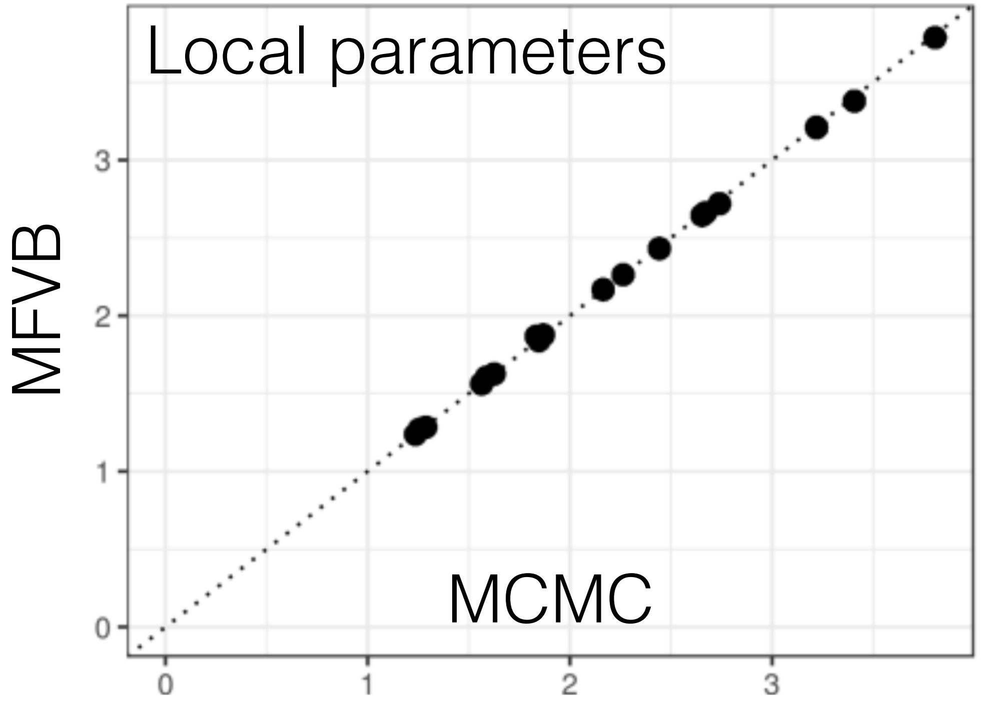


- MAP: **12 s**
- MFVB: **57 s**
- MCMC (5K samples): 21,066 s (**5.85 h**)





[Giordano, Broderick, Jordan 2018]

Why use MFVB?

- Topic discovery

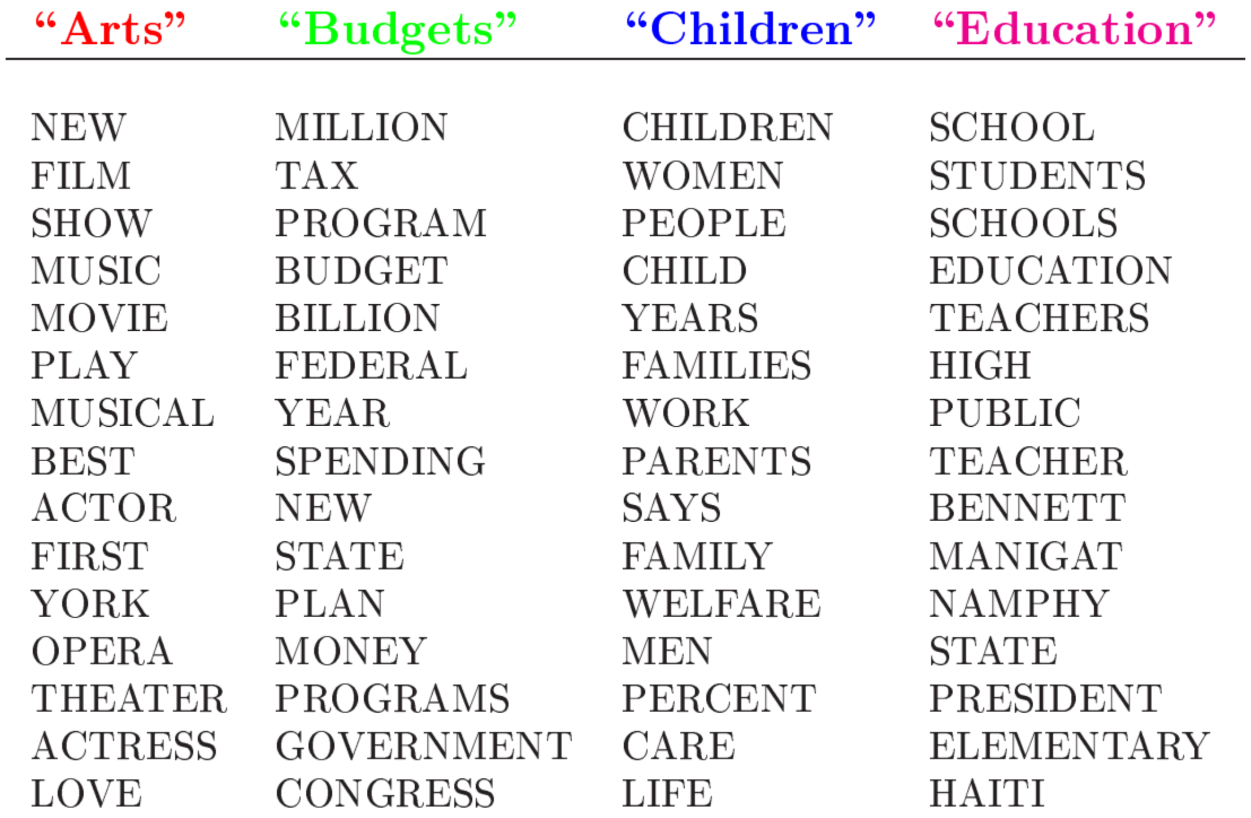
"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHER
ACTOR	NEW	SAYS	BENNETT
FIRST	STATE	FAMILY	MANIGAT
YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
THEATER	PROGRAMS	PERCENT	PRESIDENT
ACTRESS	GOVERNMENT	CARE	ELEMENTARY
LOVE	CONGRESS	LIFE	HAITI

The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.

[Blei et al 2003]

Why use MFVB?

- Topic discovery
  - Latent Dirichlet allocation (LDA)



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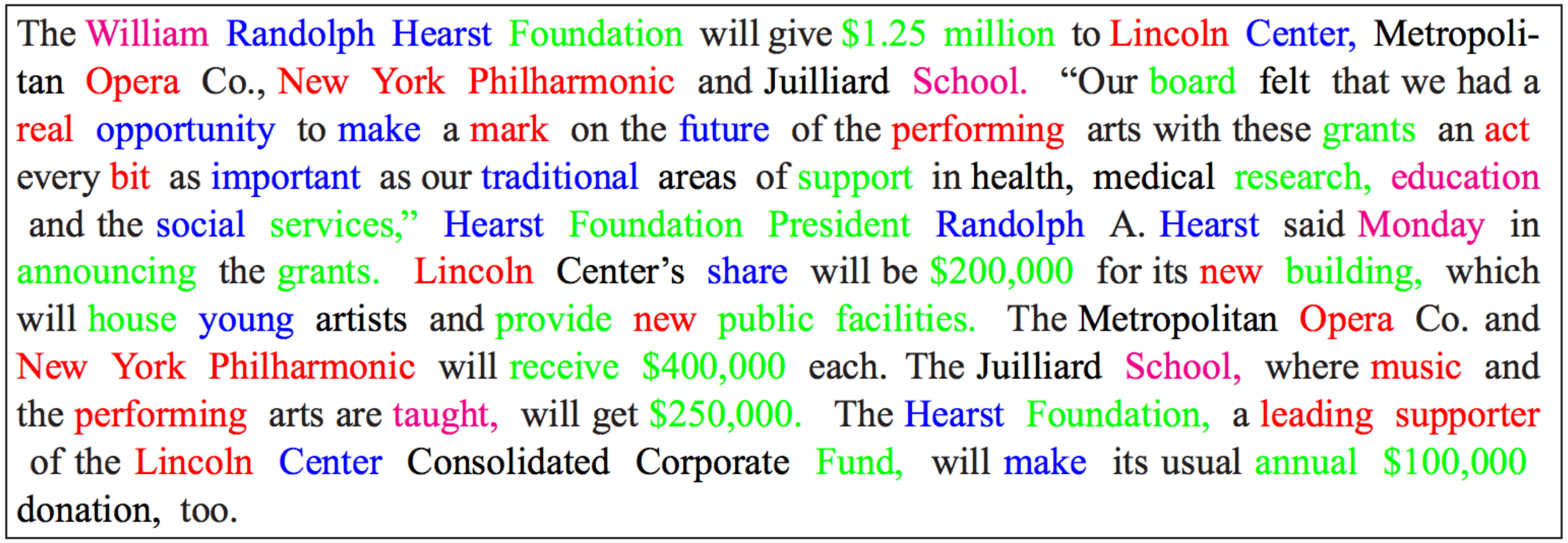

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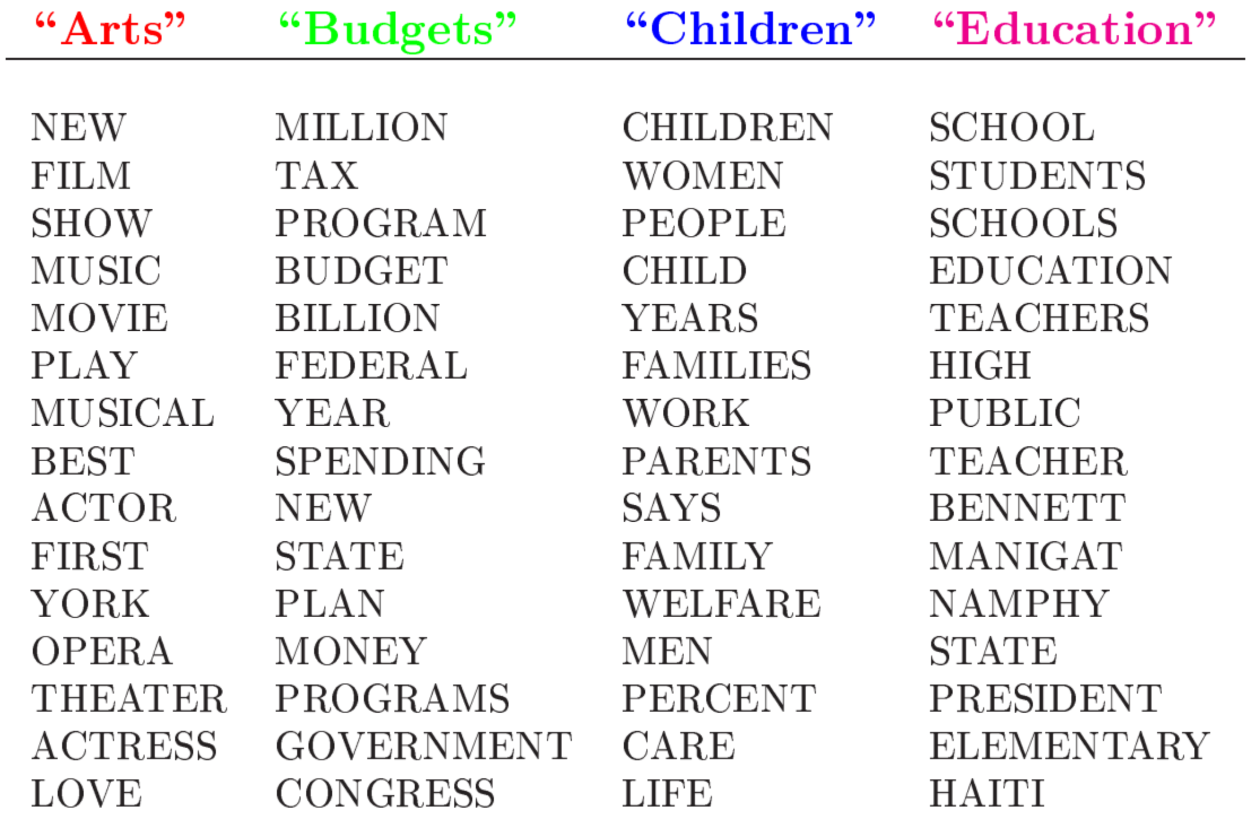

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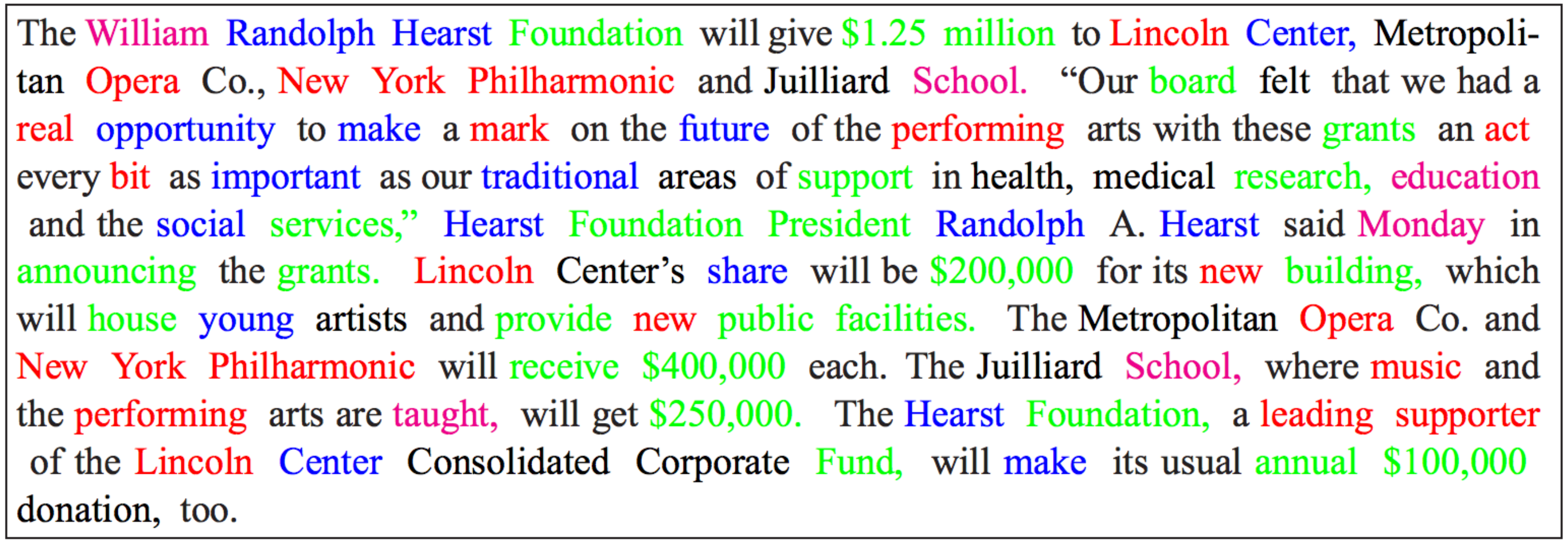

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Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use VB?
- When can we trust VB?
- Where do we go from here?

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What about uncertainty?

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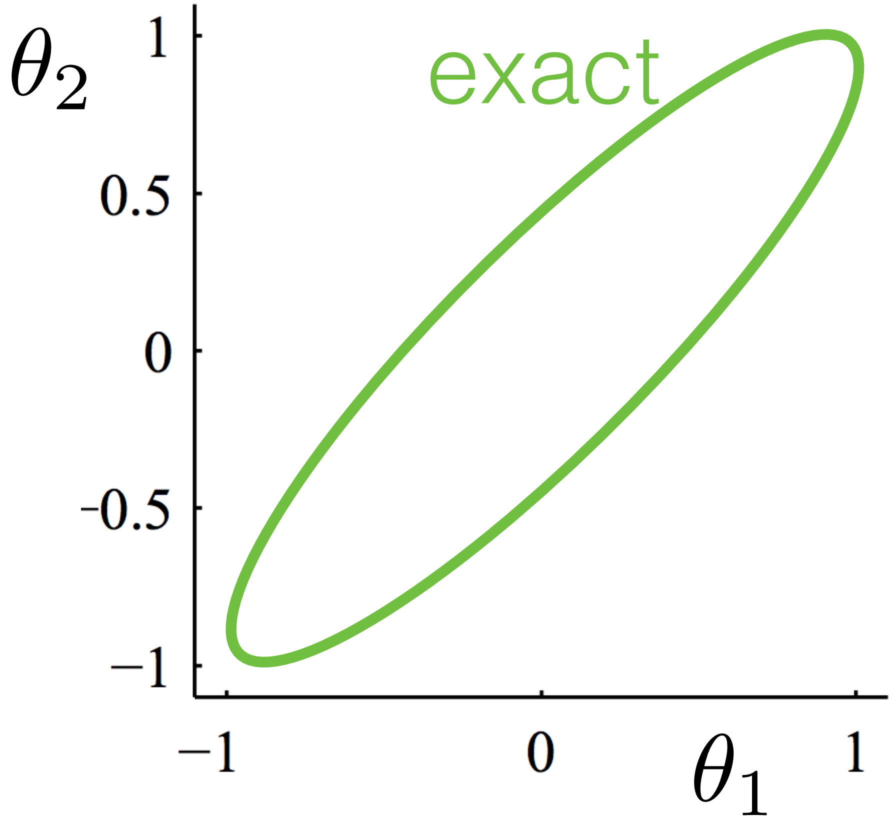

[Turner & Sahani 2011; MacKay 2003; Bishop 2006; Wang, Titterington 2004]

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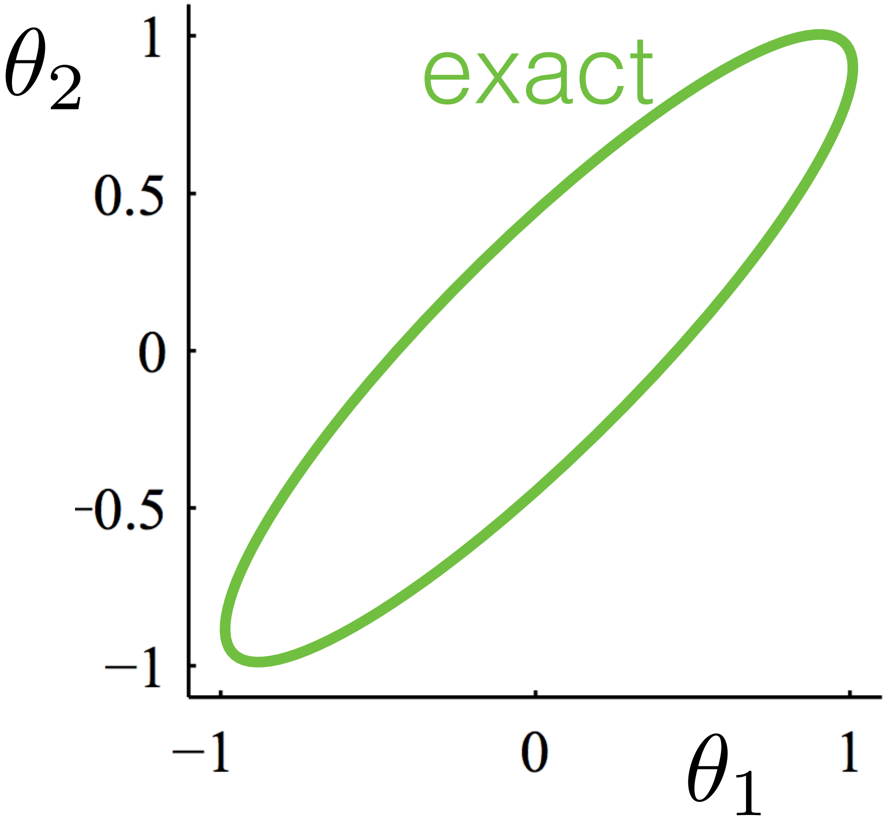

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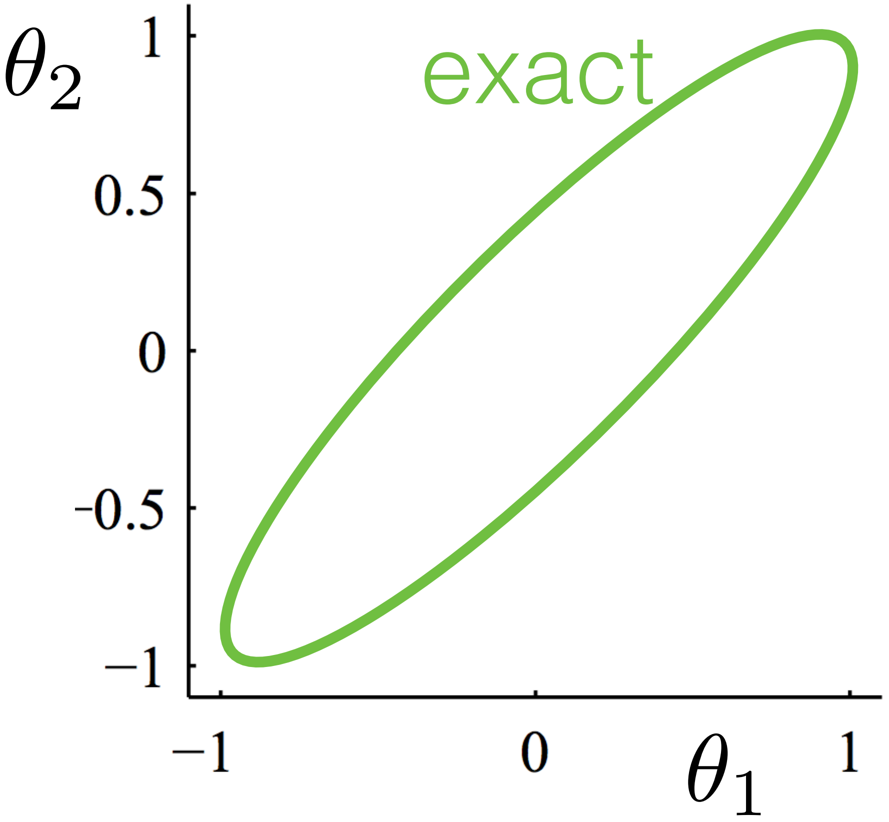

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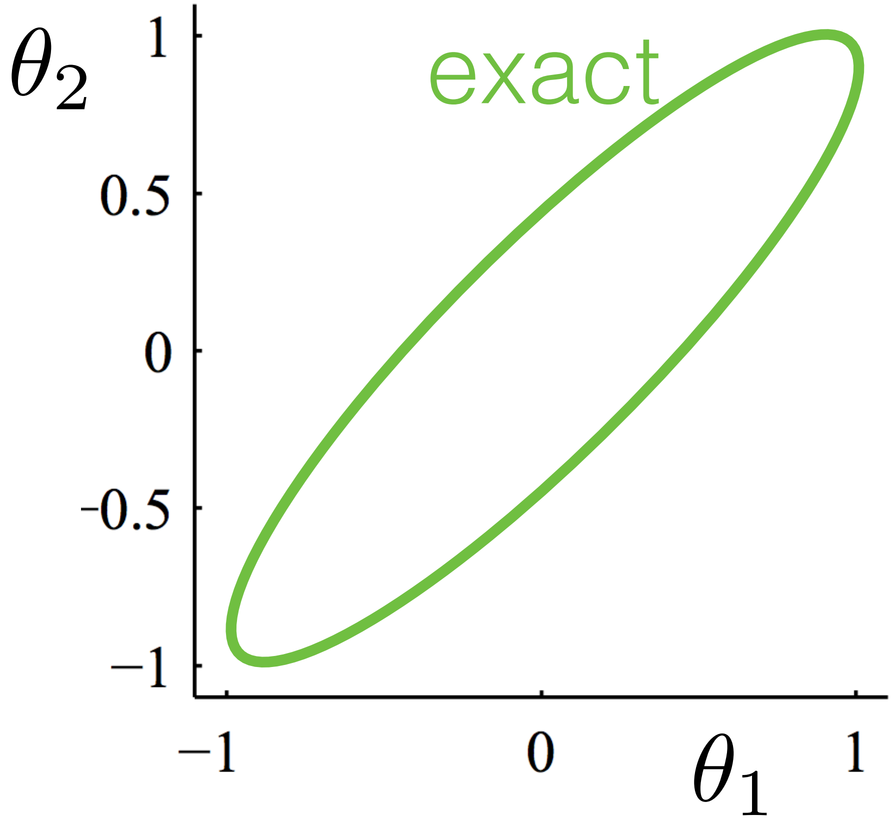

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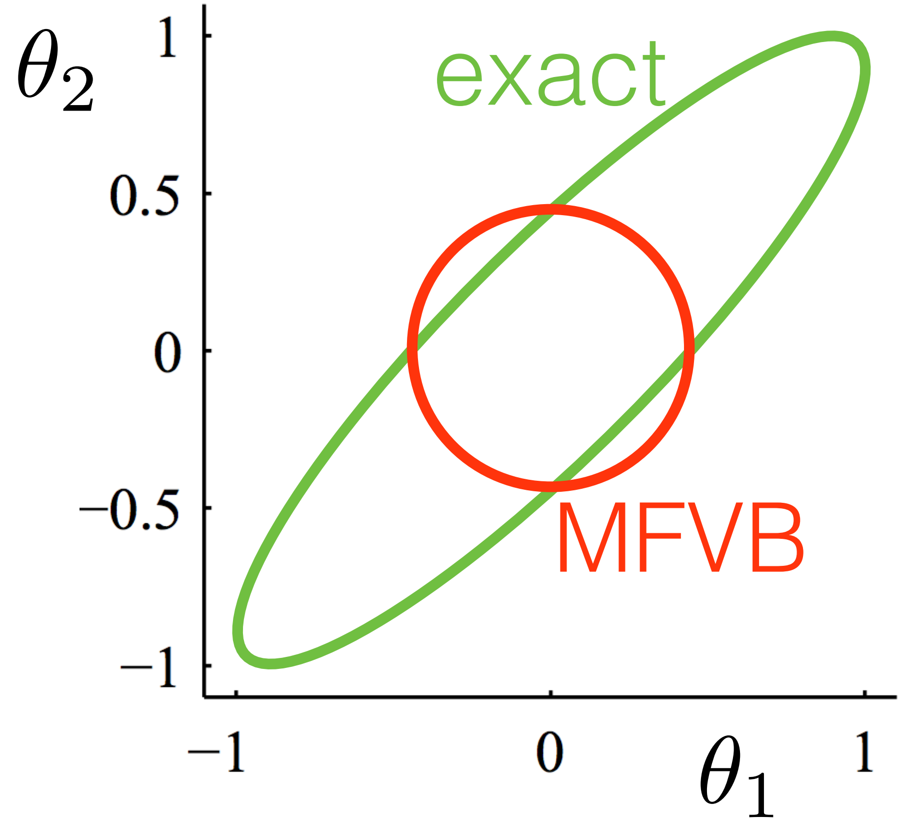

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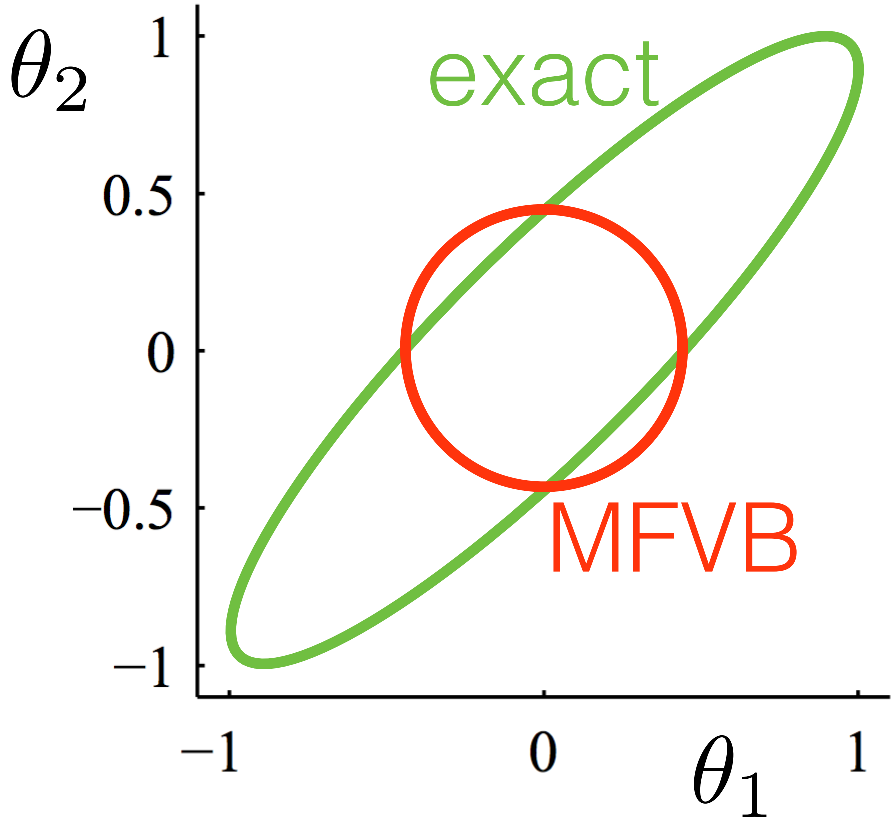

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- Underestimates variance (sometimes severely)

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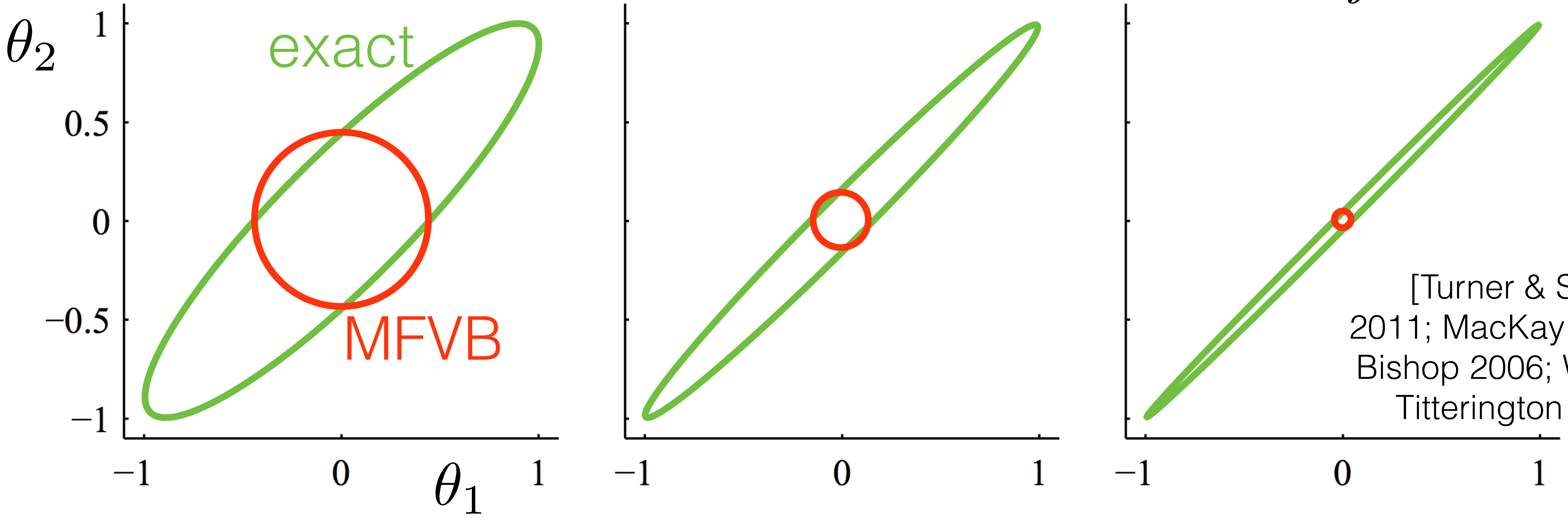

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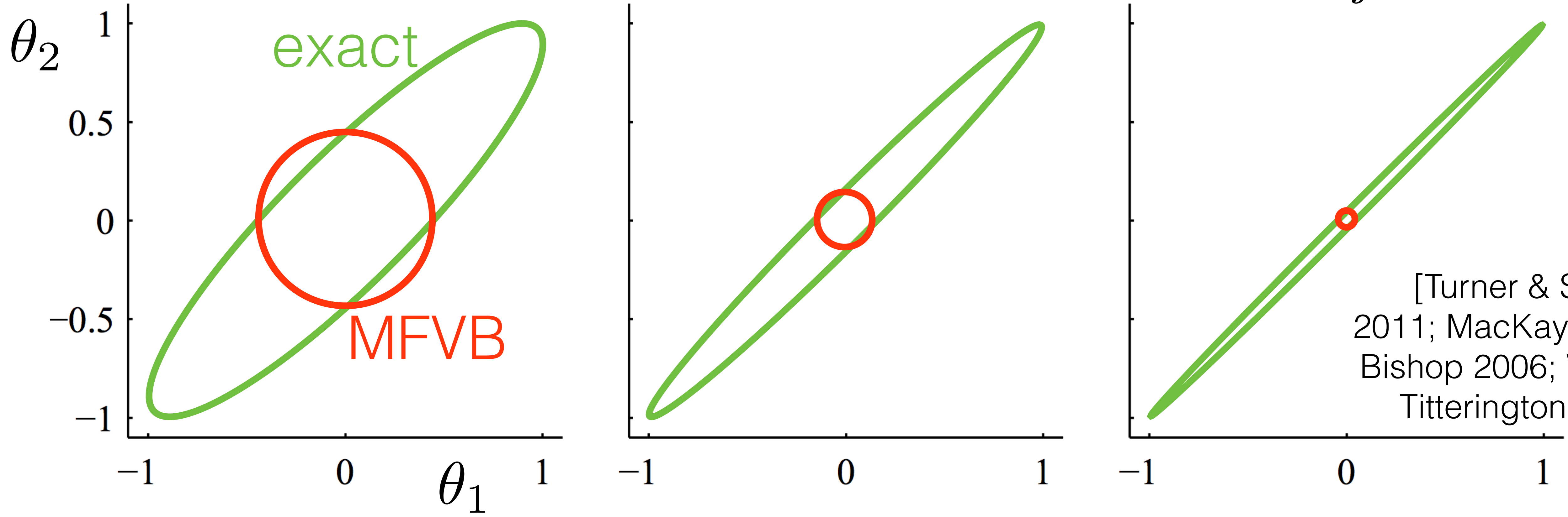
[Exercise: derive the MFVB-CA steps. Hint: use precision matrix.]

- Underestimates variance (sometimes severely)

What about uncertainty?

$$KL(q||p(\cdot|y)) = \int_\theta q(\theta) \log \frac{q(\theta)}{p(\theta|y)} d\theta \qquad q(\theta) = \prod_{j=1}^{J} q_j(\theta_j)$$



[Turner & Sahani 2011; MacKay 2003; Bishop 2006; Wang, Titterington 2004]

- Conjugate linear regression
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[Exercise: derive the MFVB-CA steps. Hint: use precision matrix.]

- Underestimates variance (sometimes severely)
- No covariance estimates

What about uncertainty?

- Microcredit

[Giordano, Broderick, Meager, Huggins, Jordan 2016]

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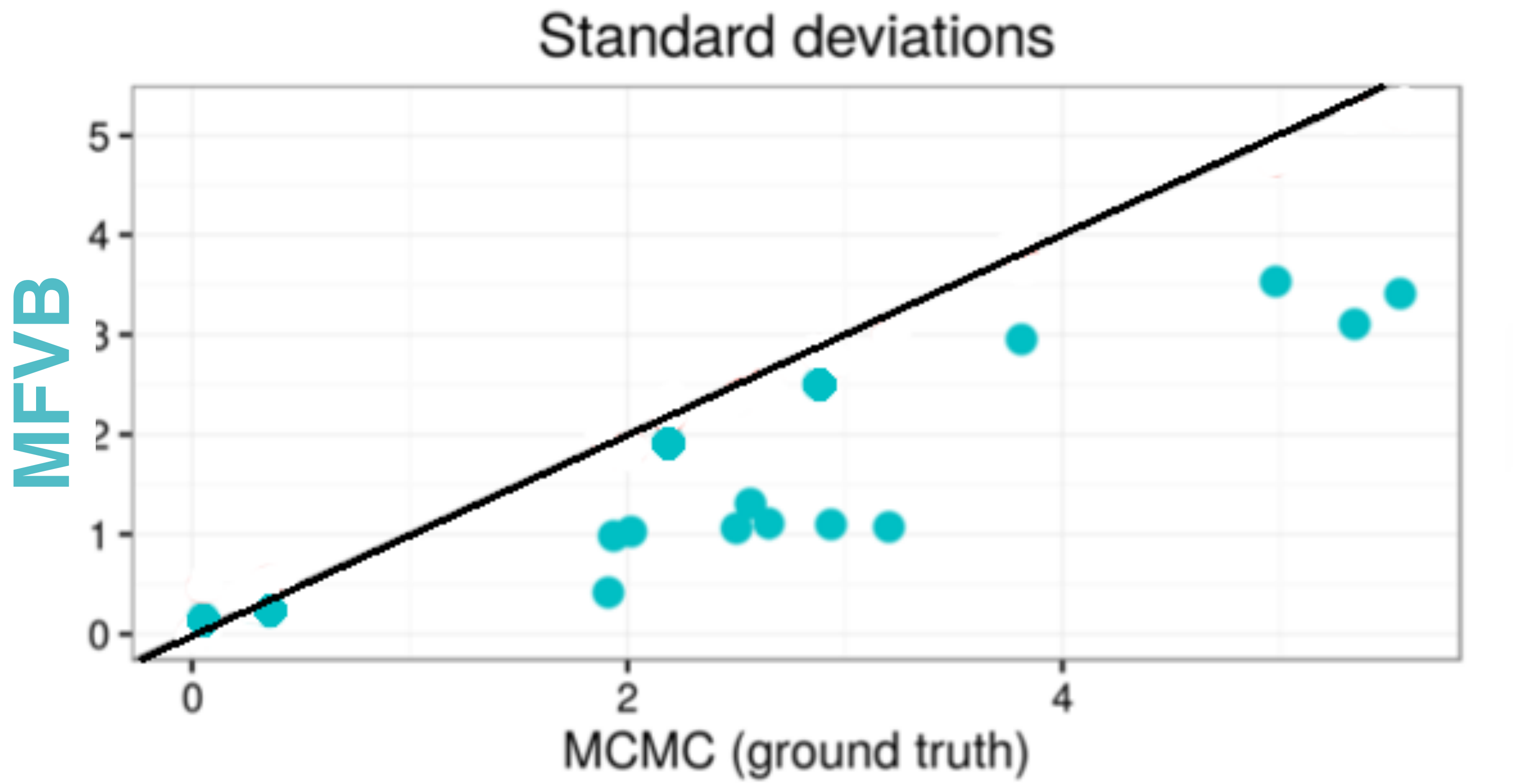

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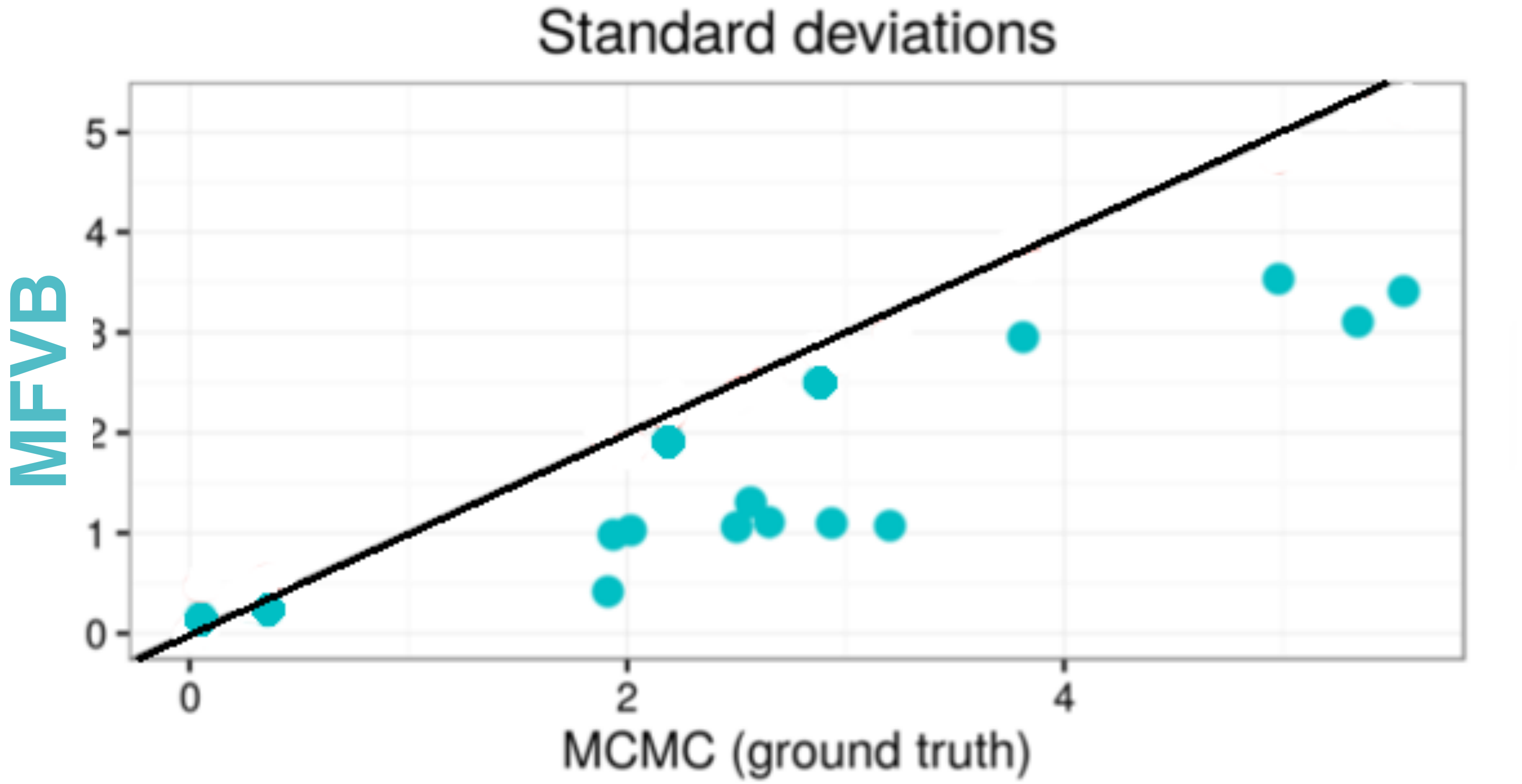
What about uncertainty?

- Microcredit effect
- τ mean: 3.08 USD PPP



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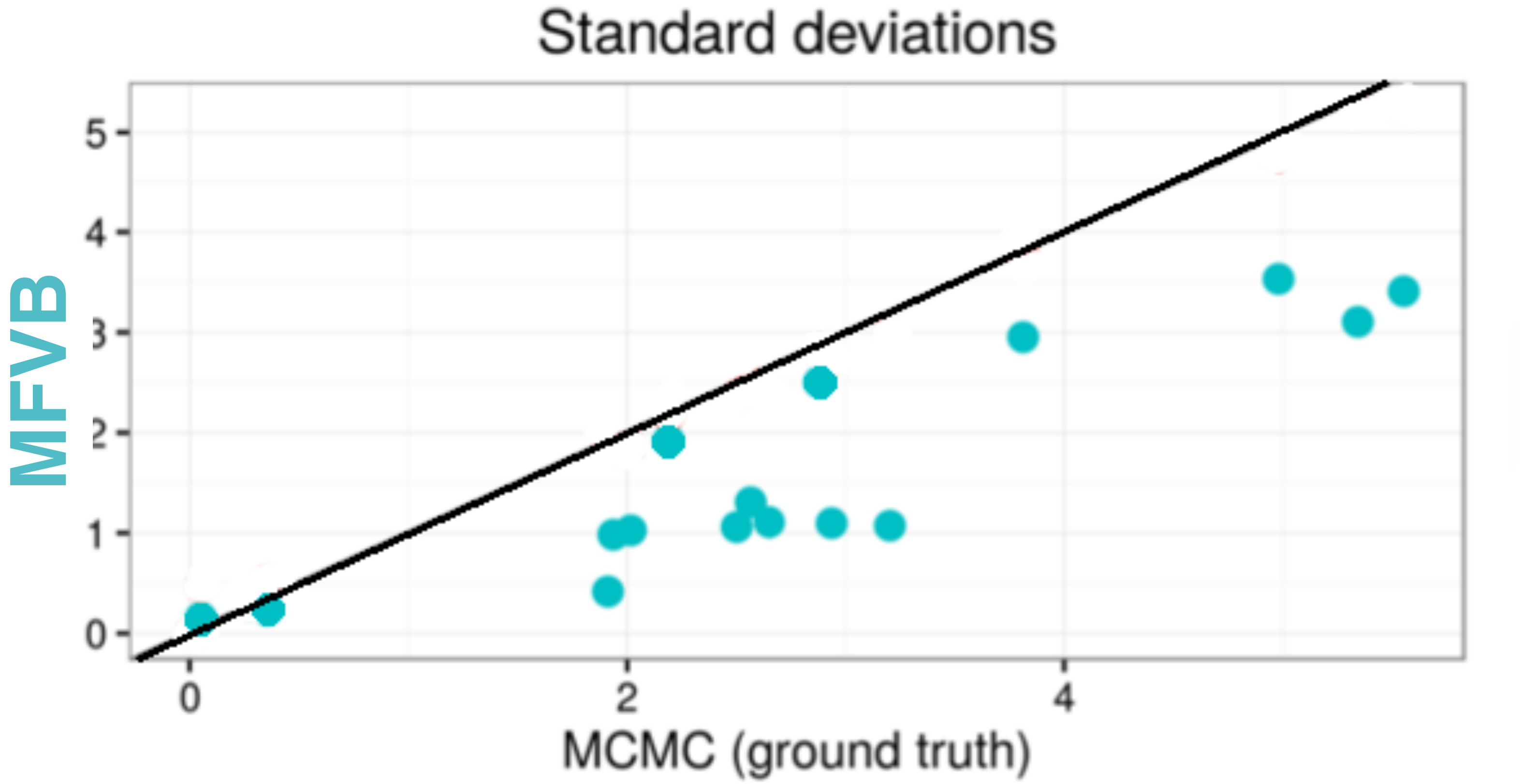

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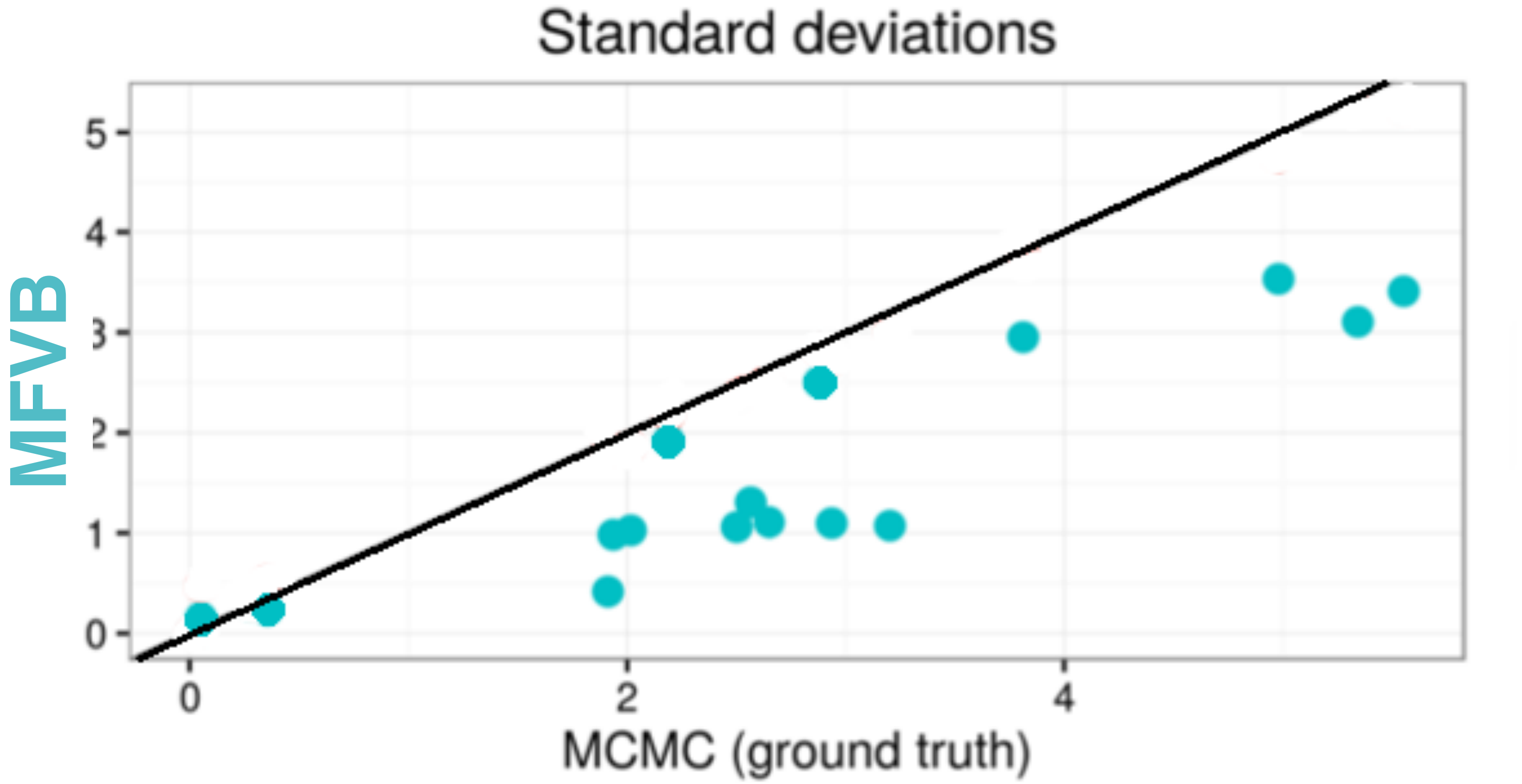

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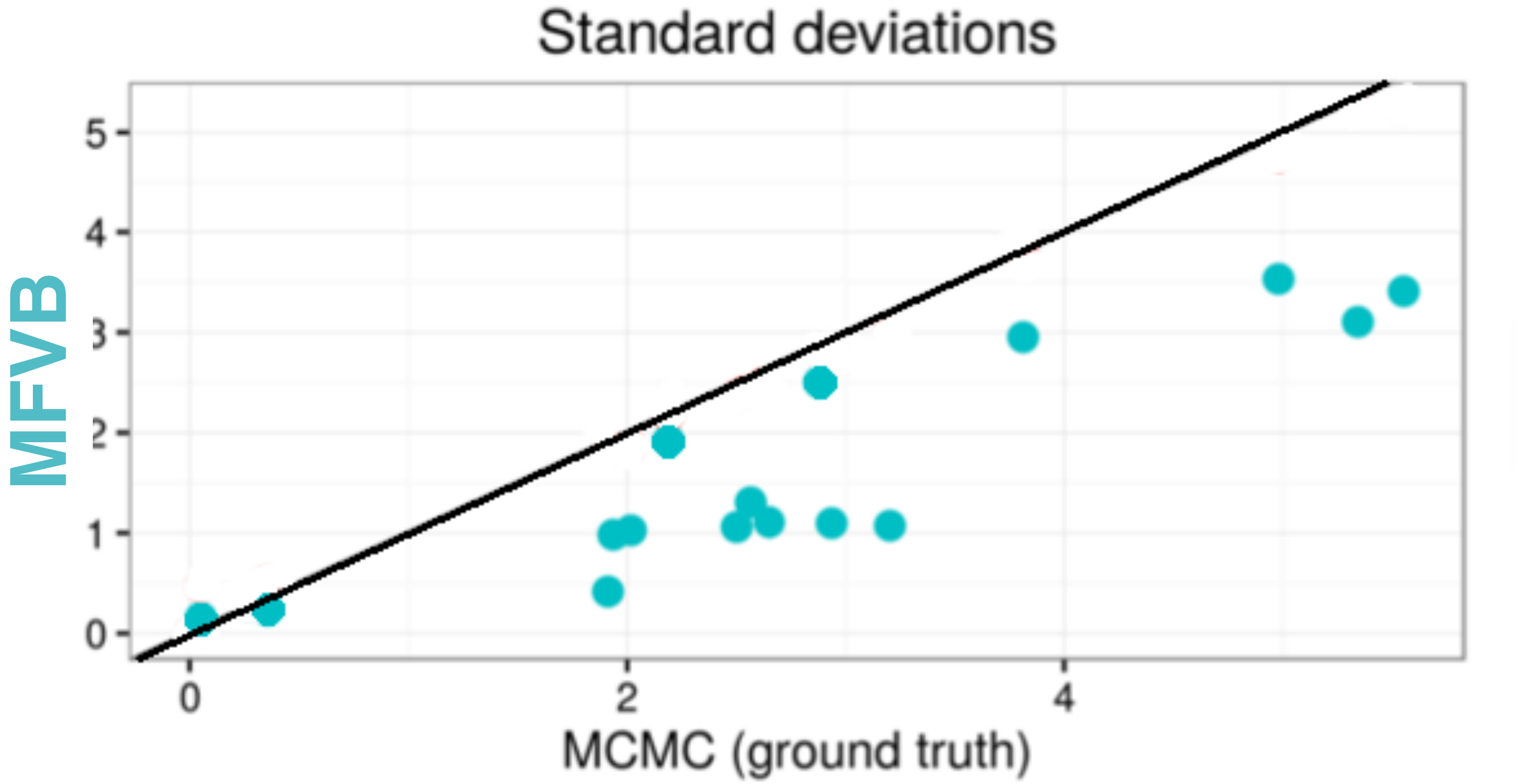
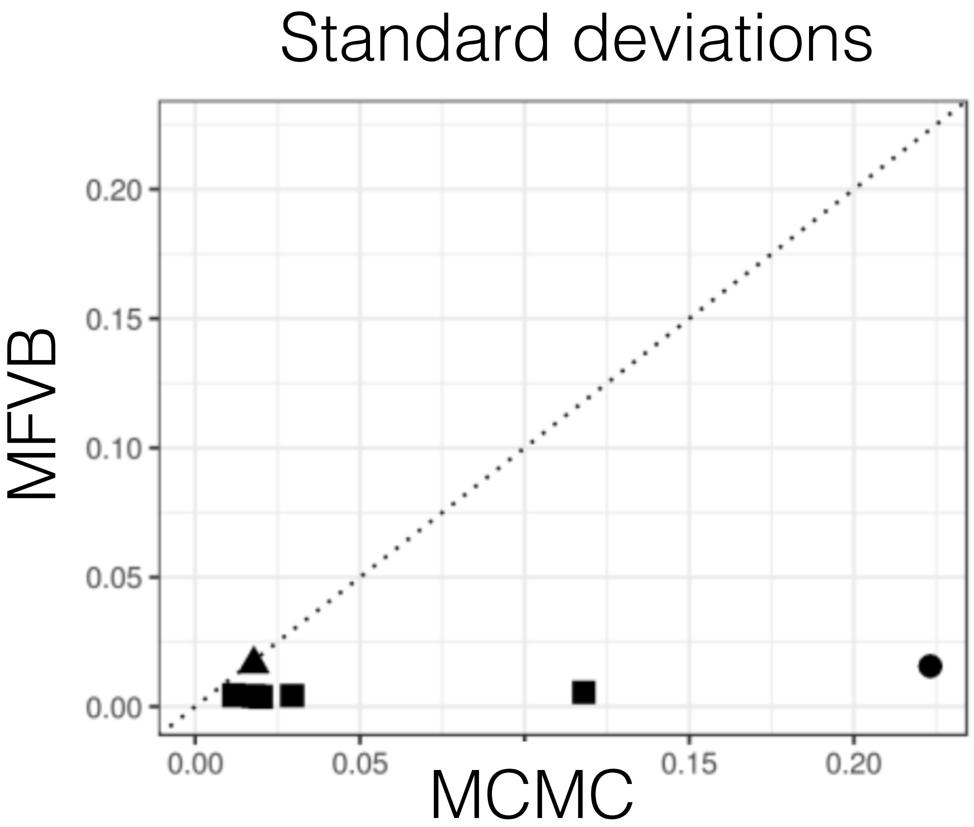


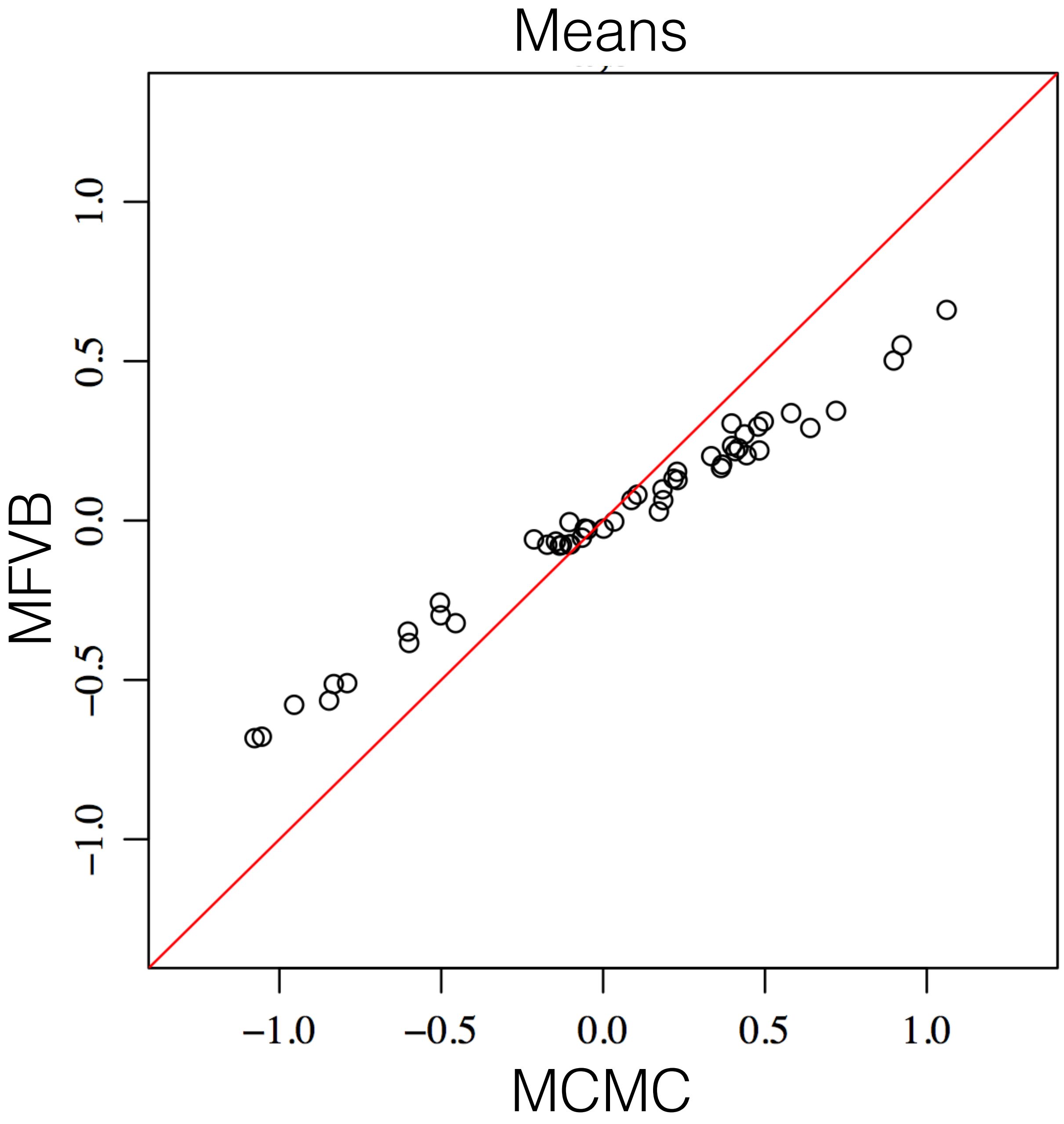
- Criteo online ads experiment



[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2018]

What about means?

- Model for relational data with covariates
- When 1000+ nodes, MCMC > 1 day [Fosdick 2013, Ch 4]



[Fosdick 2013, Ch 4, Fig 4.3]

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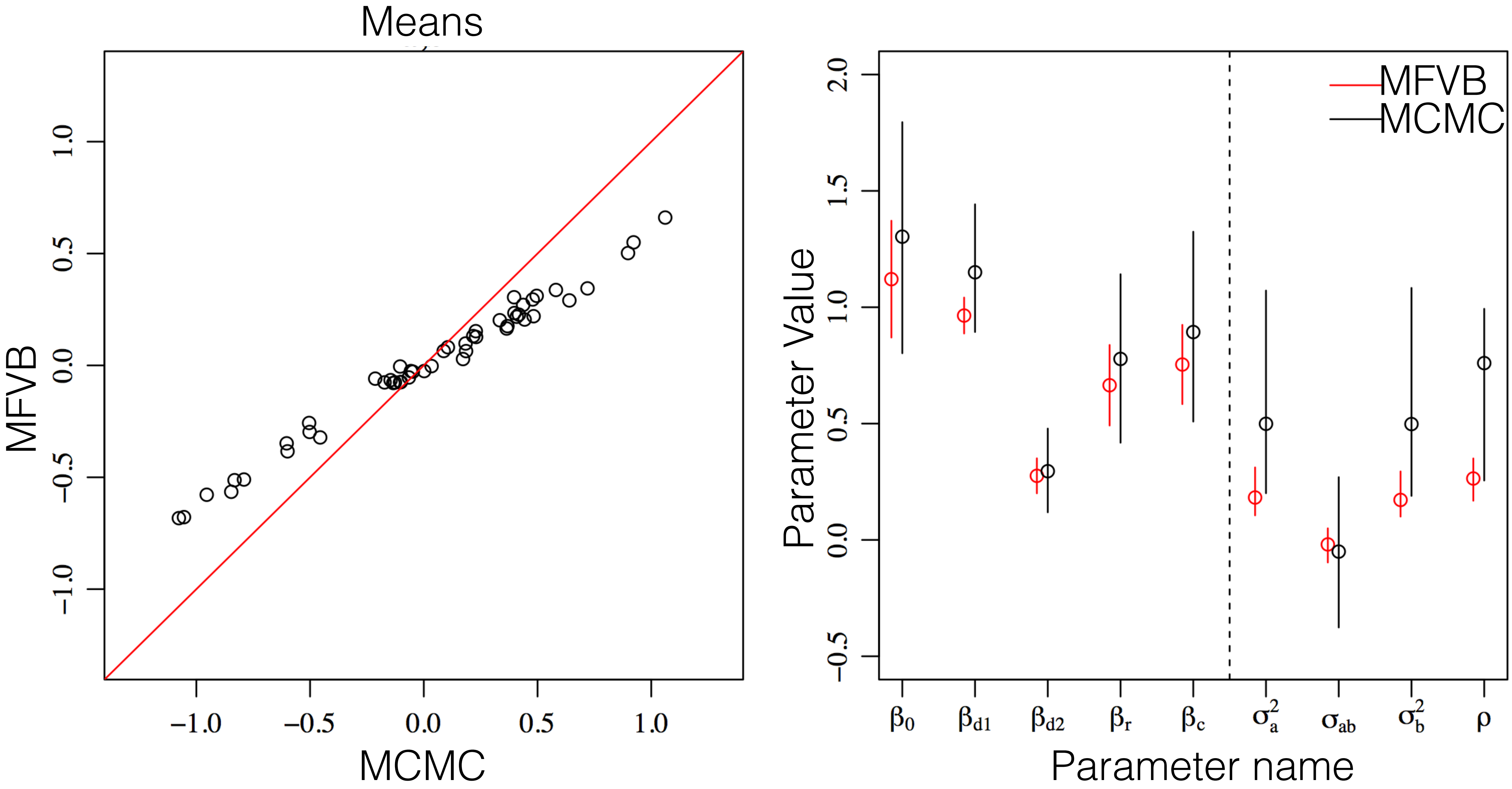

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Posterior means: revisited

- Want to predict college GPA y_n

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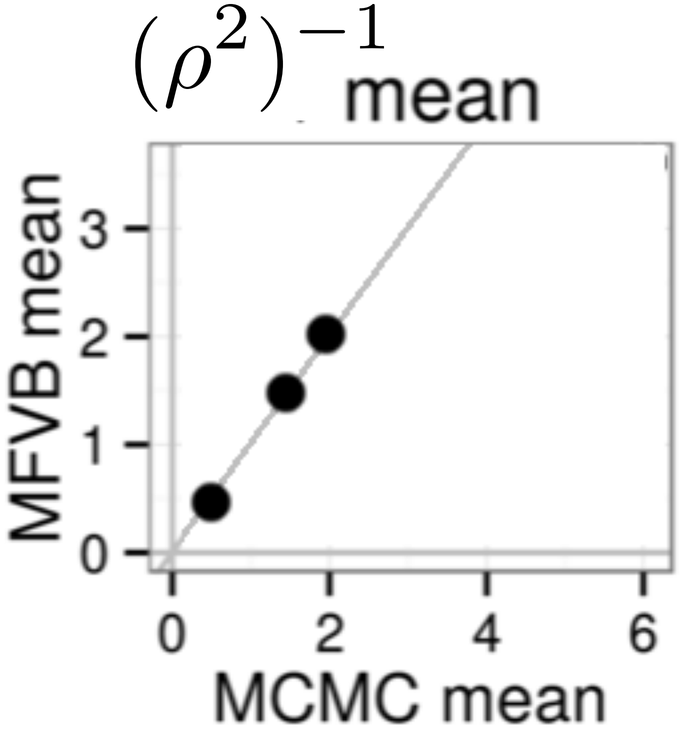

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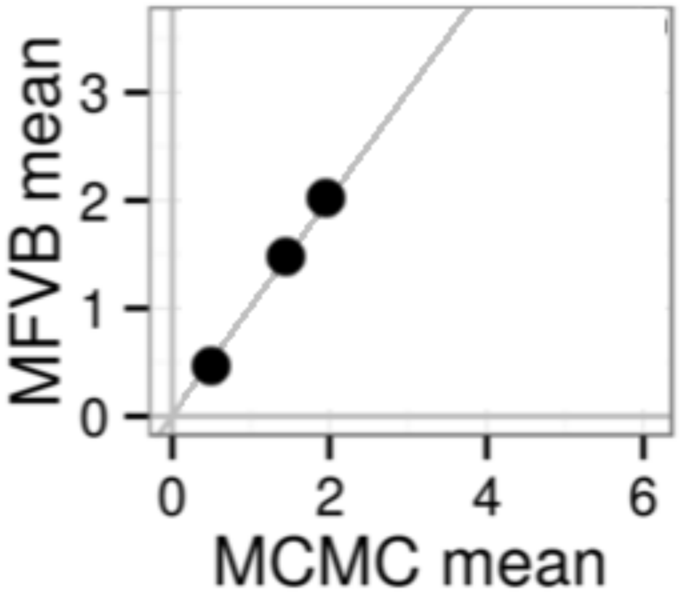

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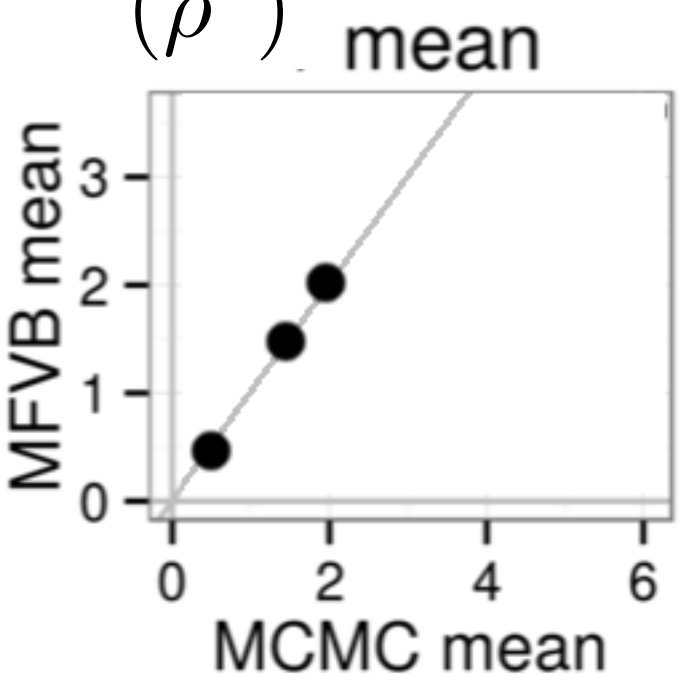

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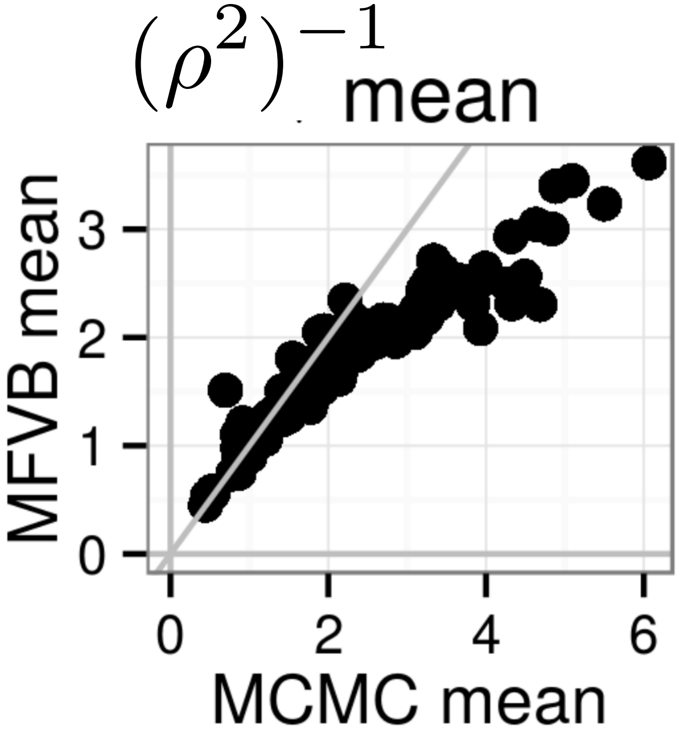
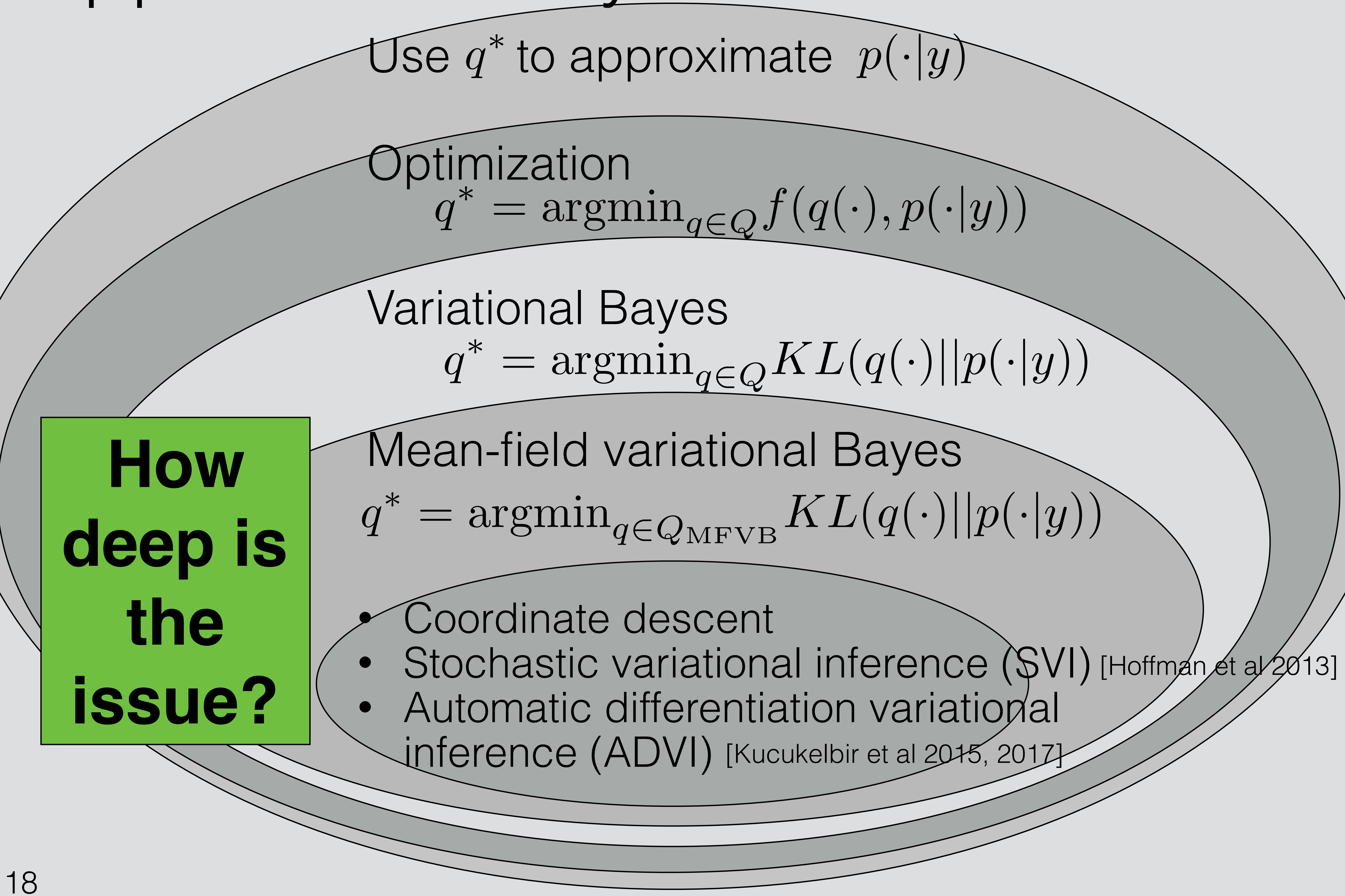


[Giordano, Broderick, Jordan 2015]

Approximate Bayesian inference



Use q^* to approximate $p(\cdot|y)$

Optimization

$$q^* = \text{argmin}_{q \in Q} f(q(\cdot), p(\cdot|y))$$

Variational Bayes

$$q^* = \text{argmin}_{q \in Q} KL(q(\cdot)||p(\cdot|y))$$

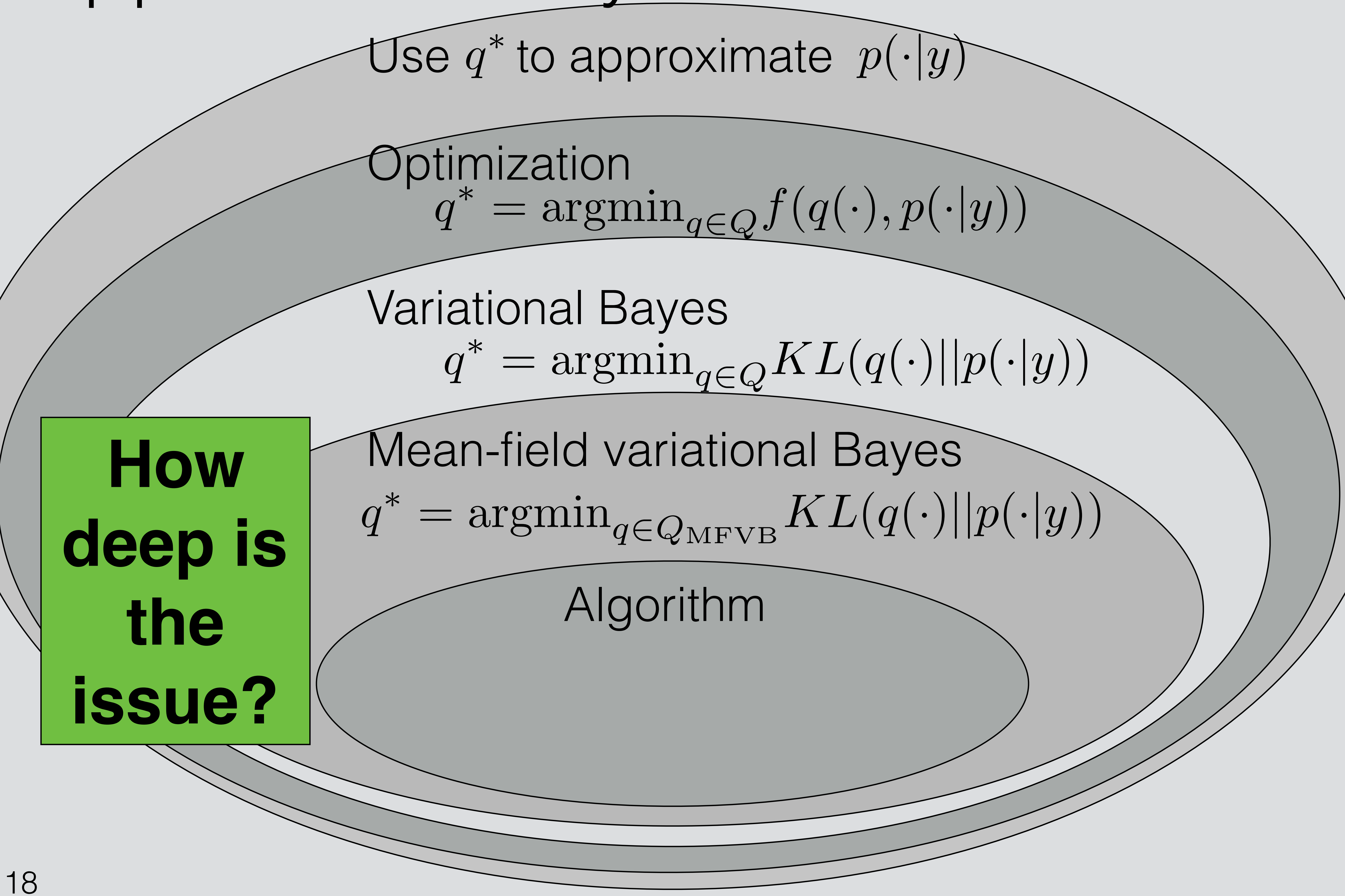
Mean-field variational Bayes

$$q^* = \text{argmin}_{q \in Q_{\text{MFVB}}} KL(q(\cdot)||p(\cdot|y))$$

- Coordinate descent
- Stochastic variational inference (SVI) [Hoffman et al 2013]
- Automatic differentiation variational inference (ADVI) [Kucukelbir et al 2015, 2017]

How deep is the issue?

Approximate Bayesian inference



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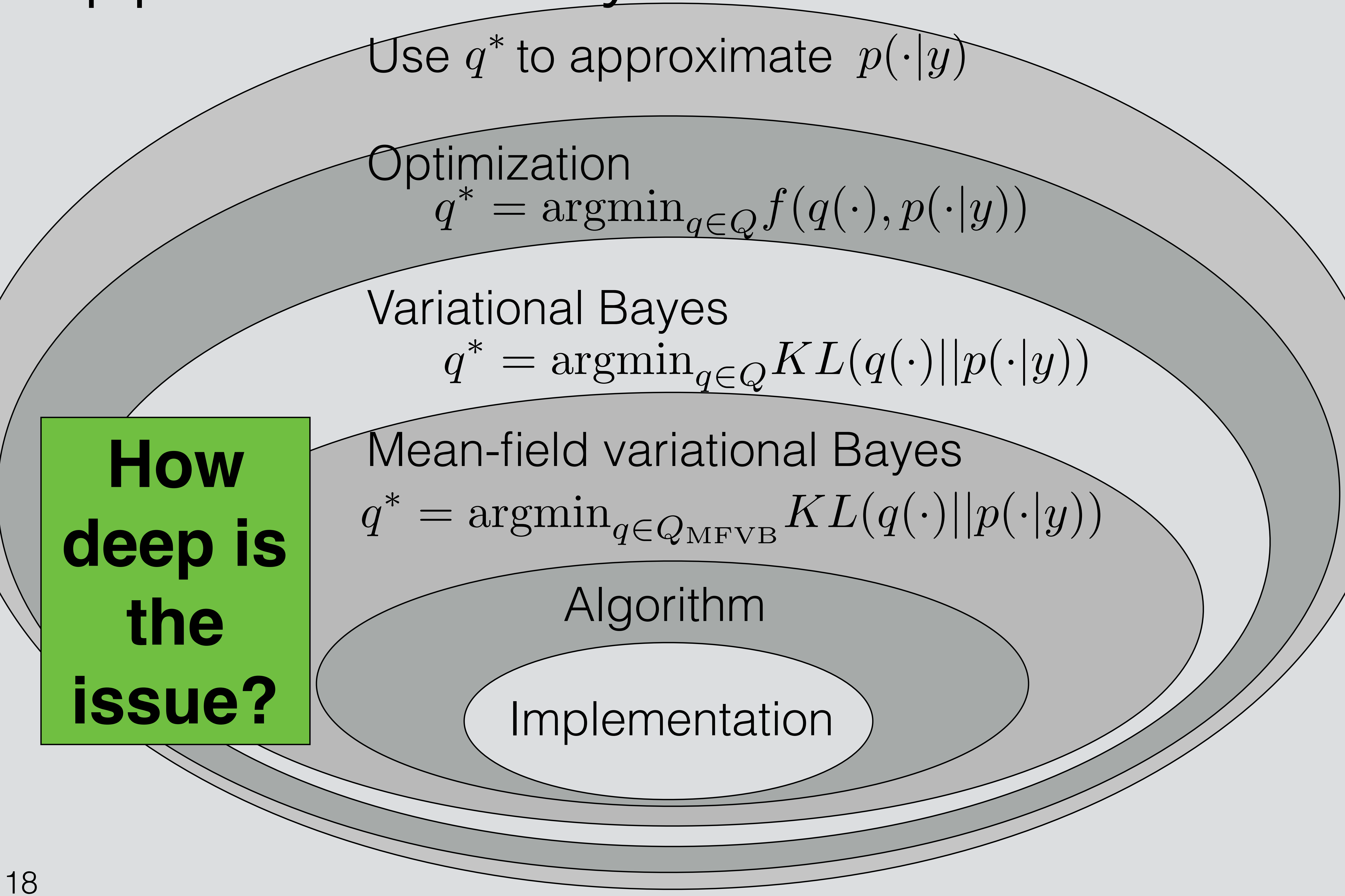

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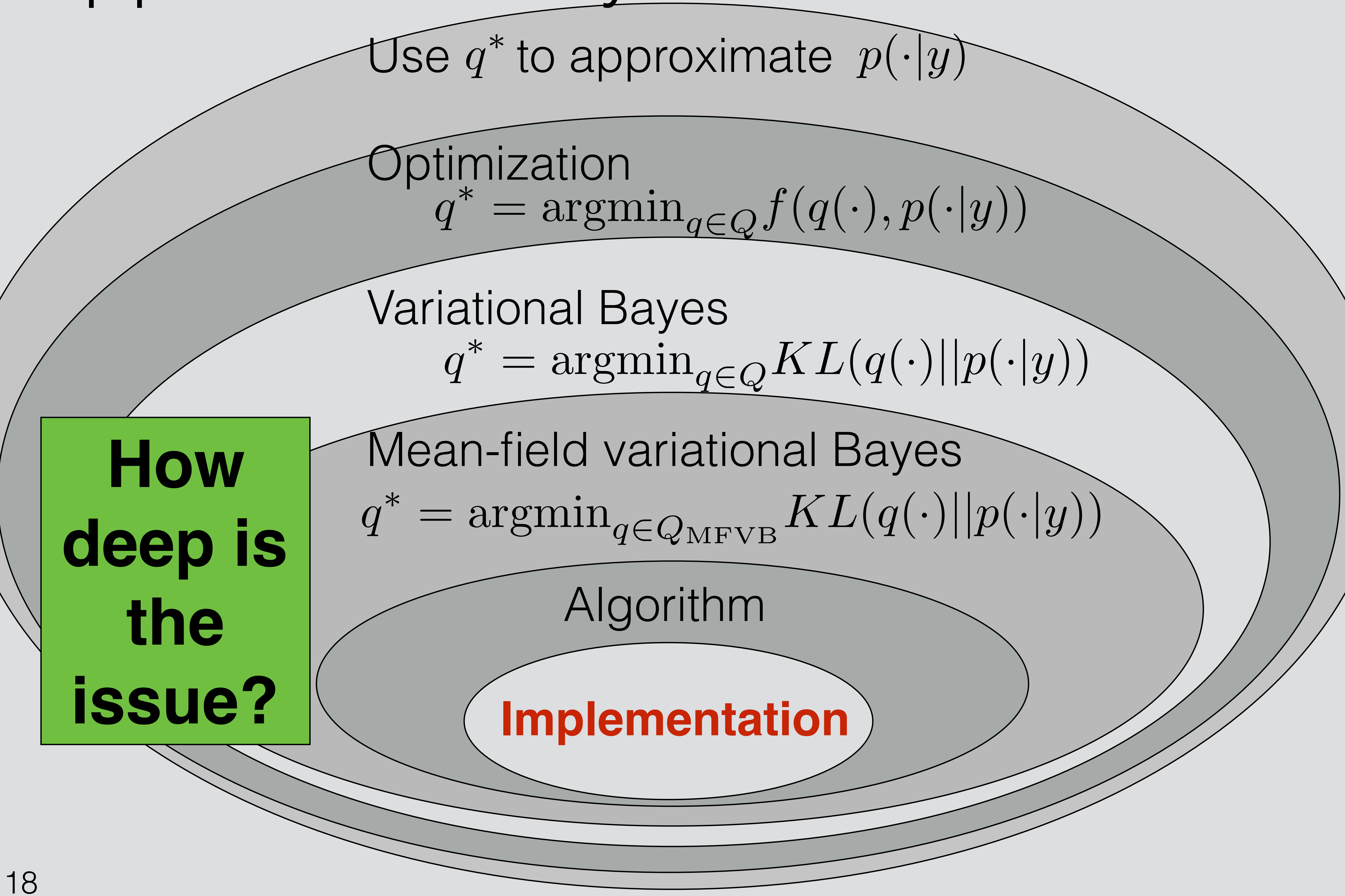

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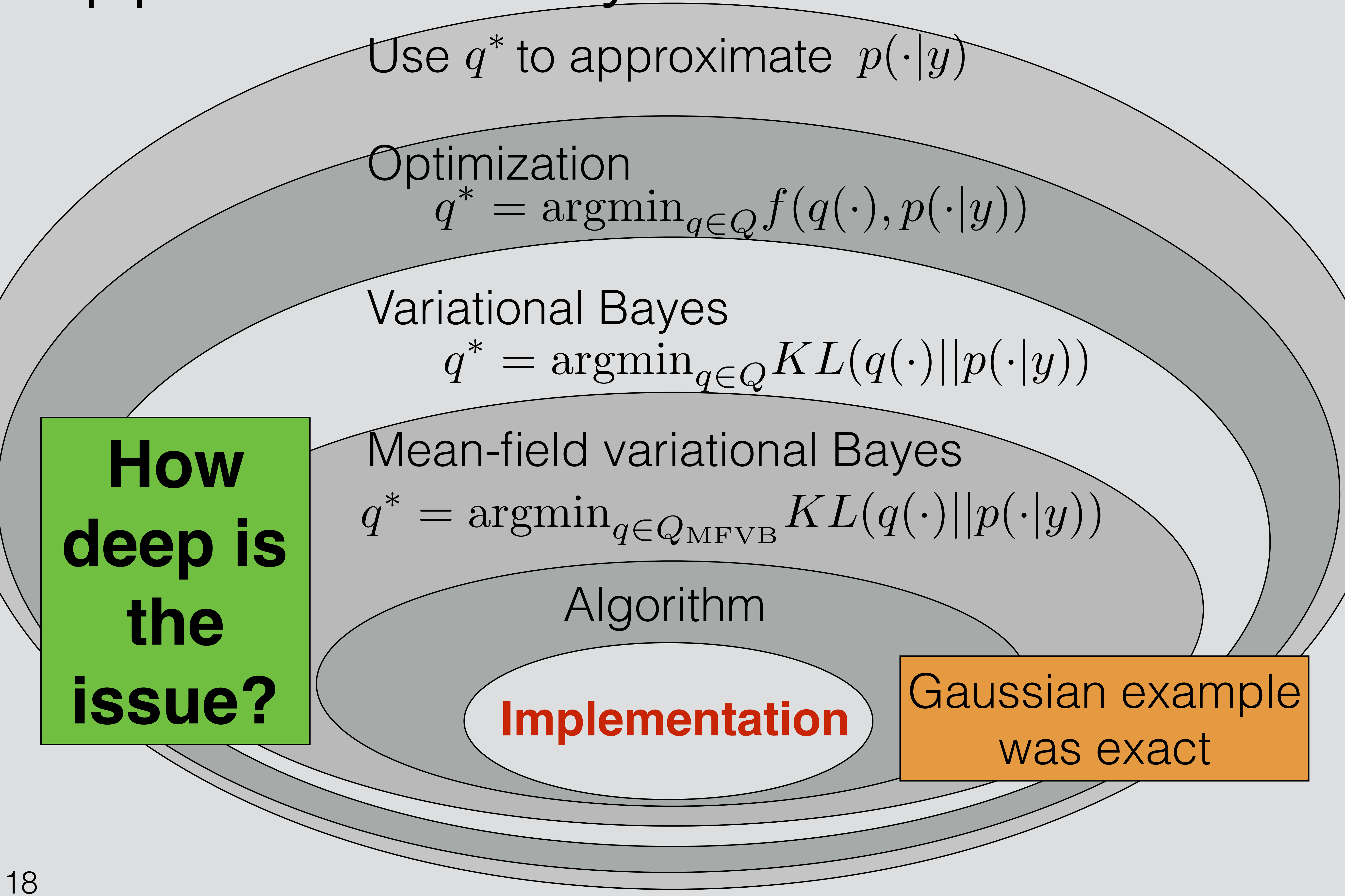

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Gaussian example was exact

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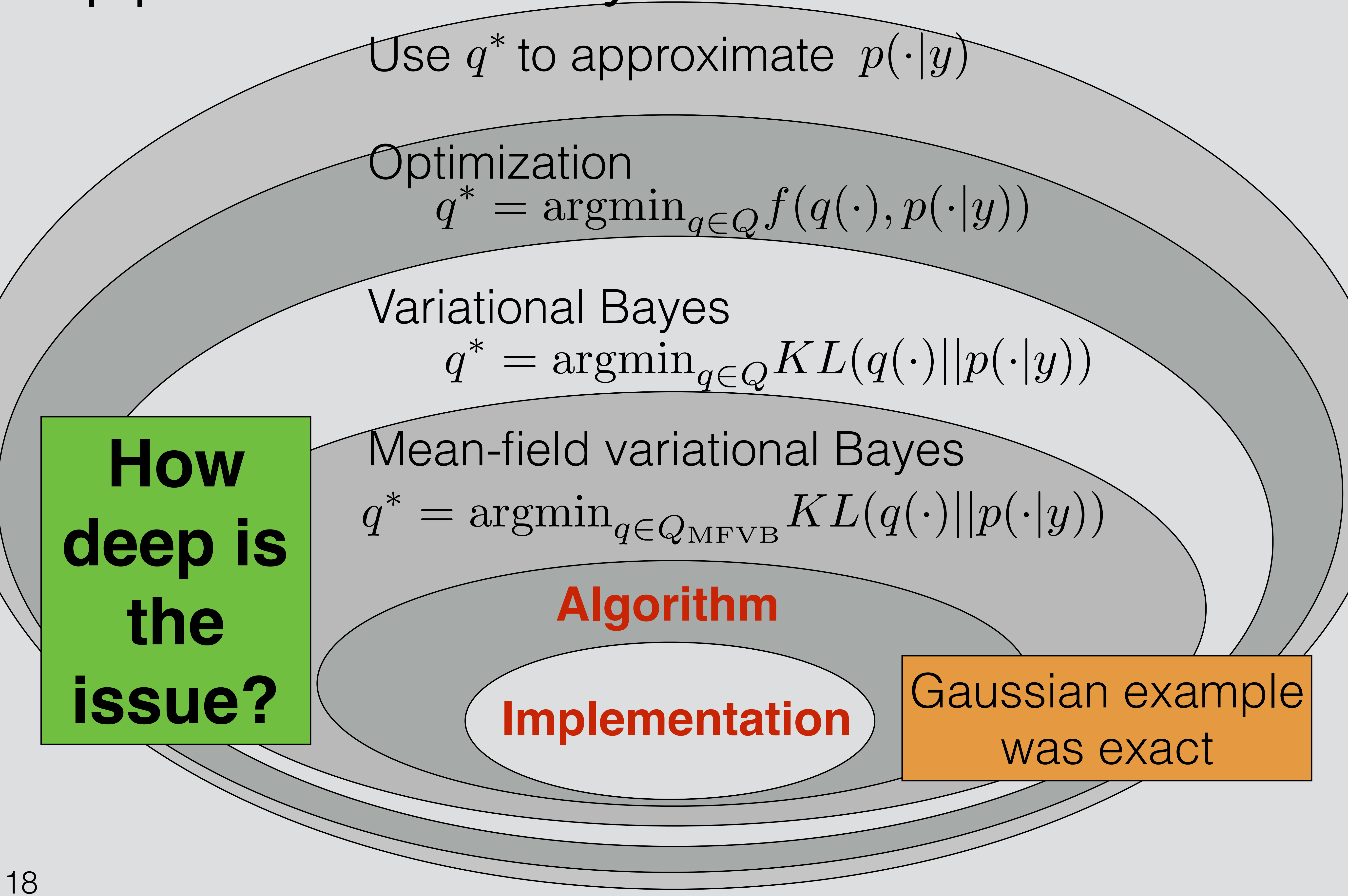

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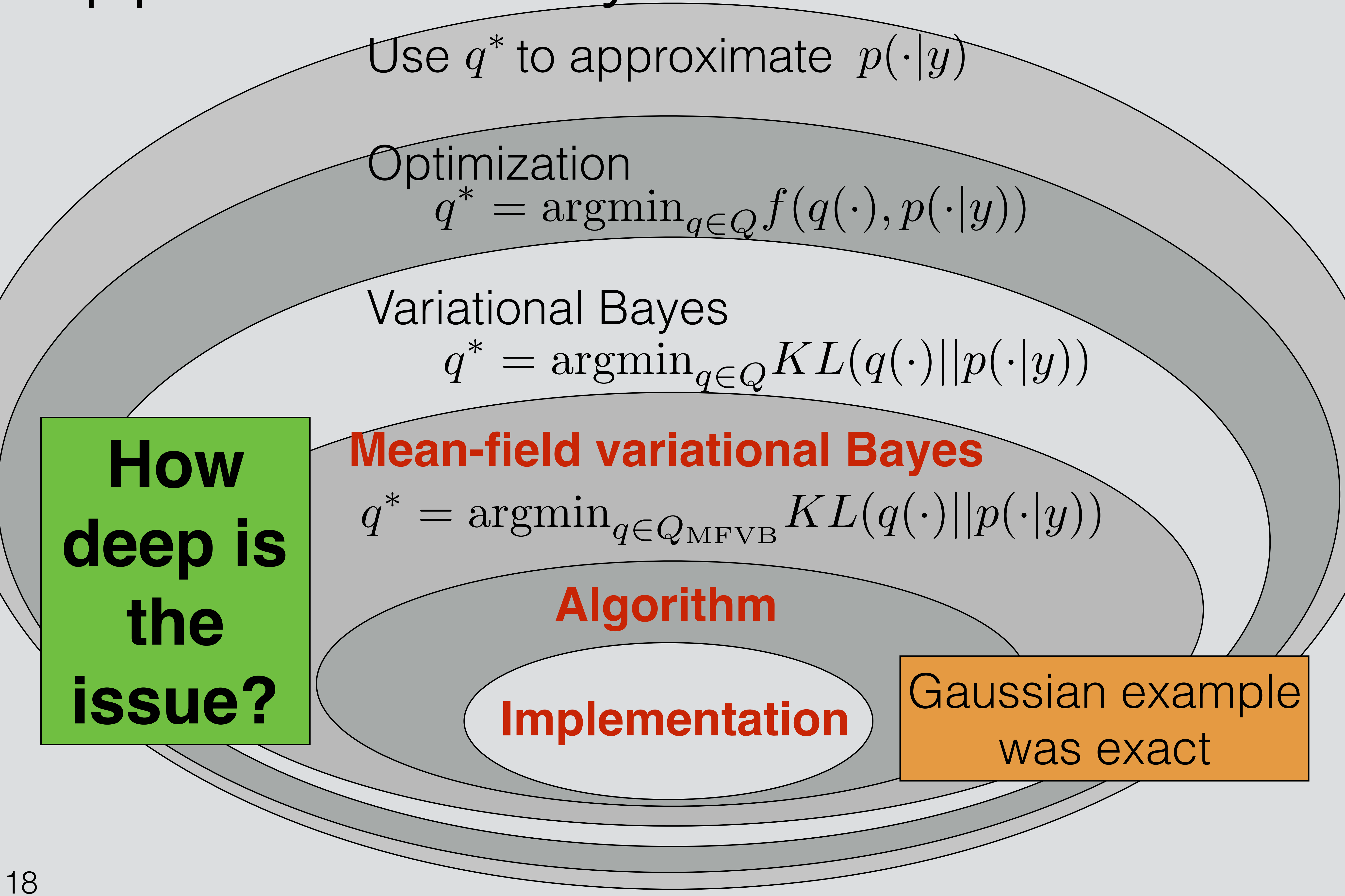

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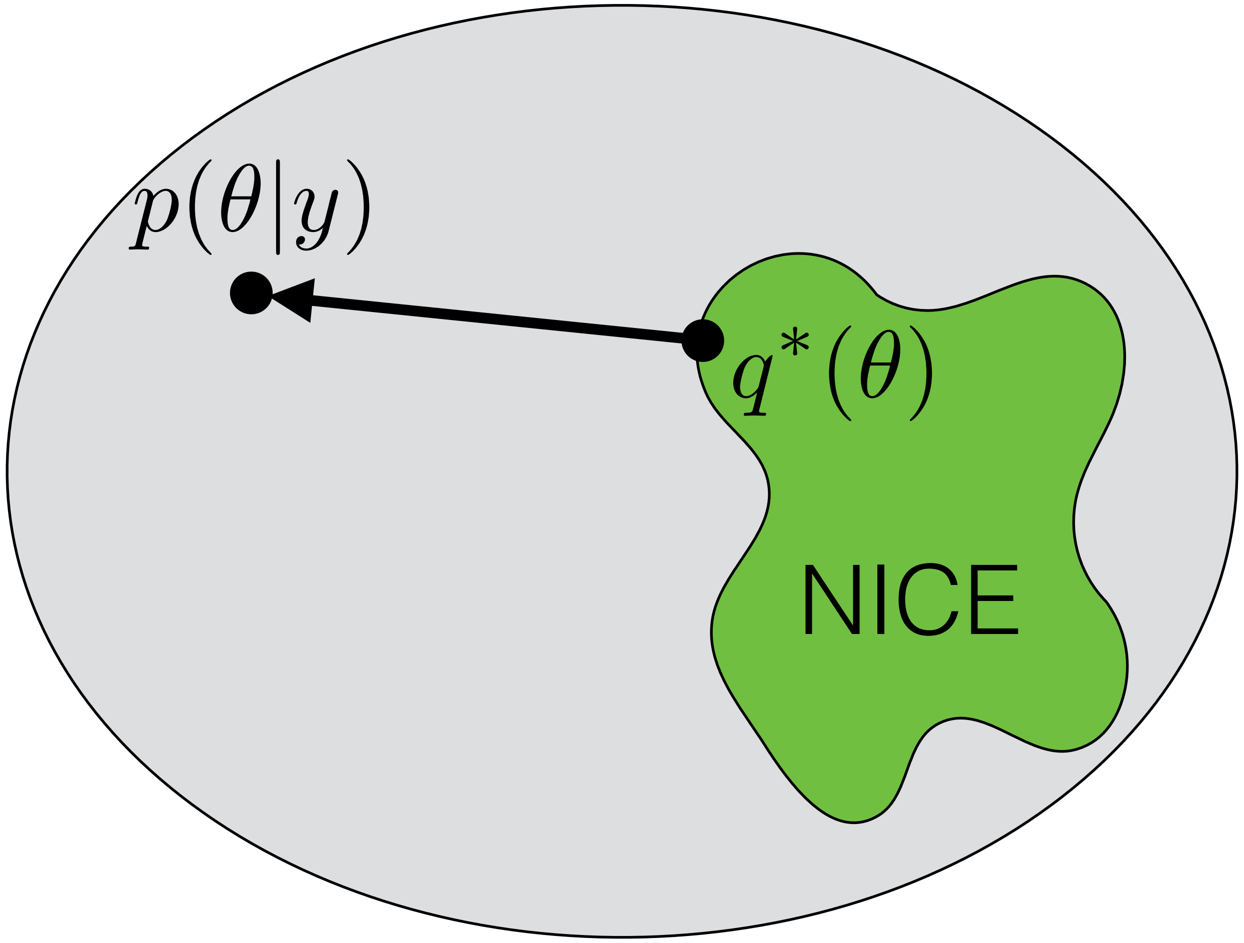

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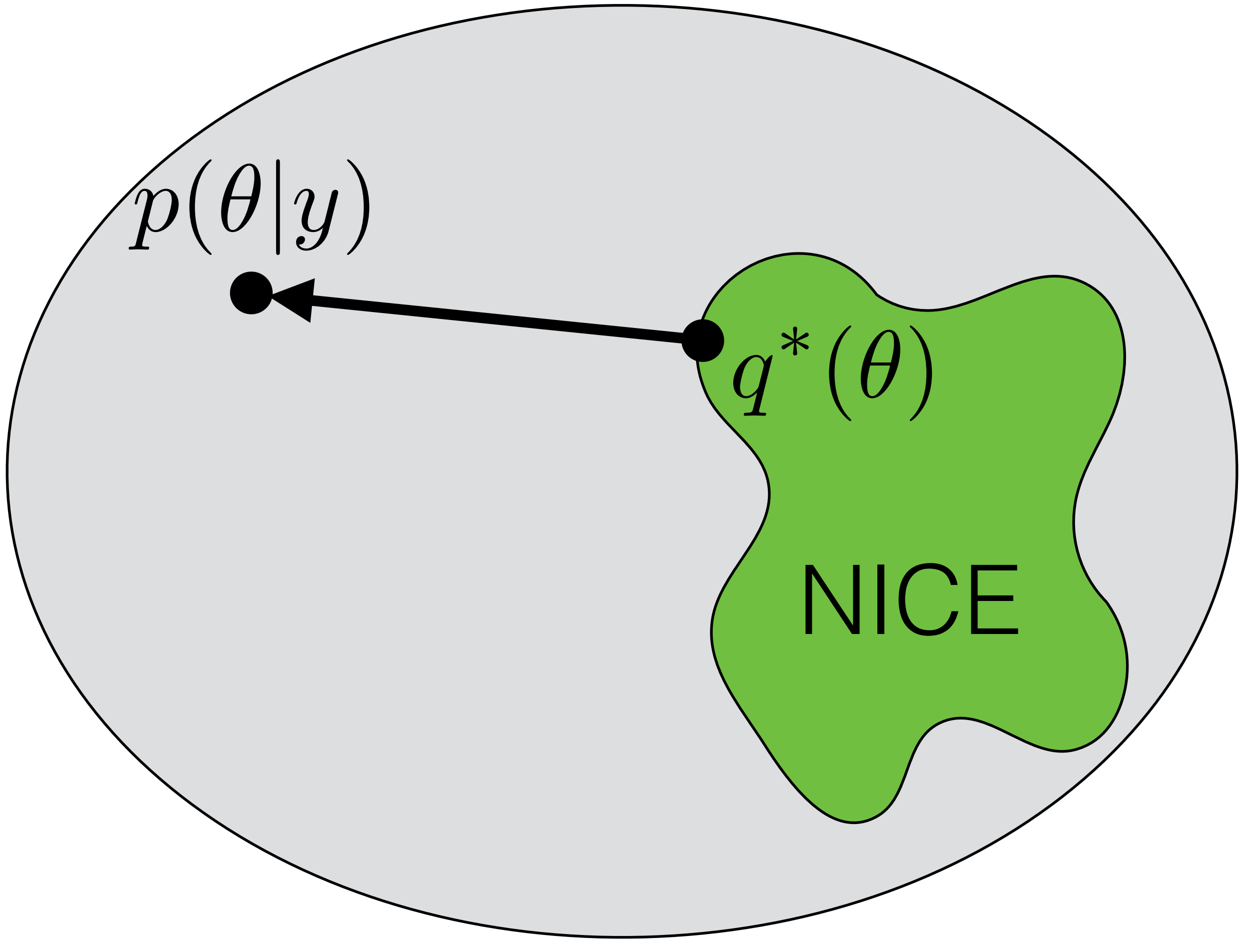

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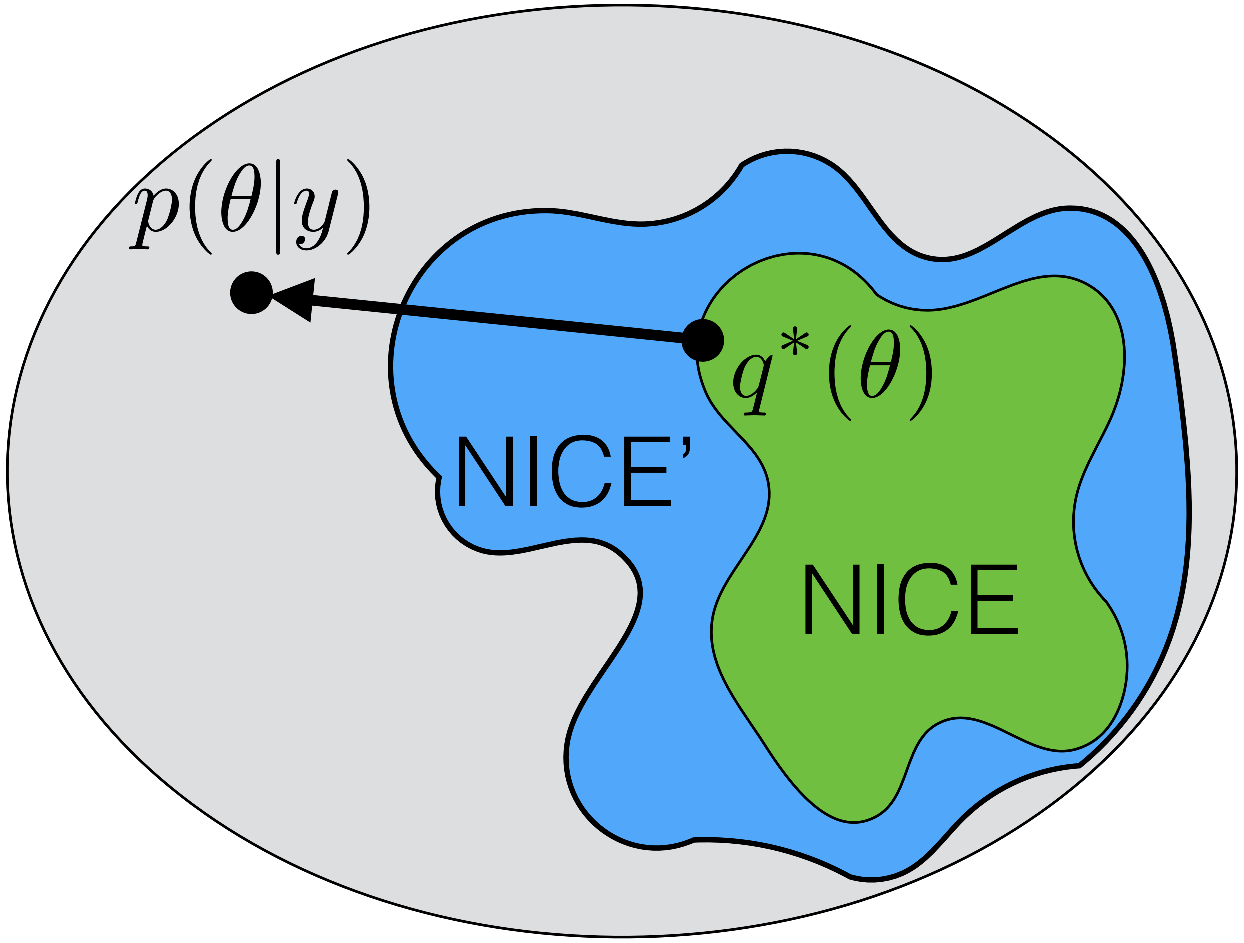

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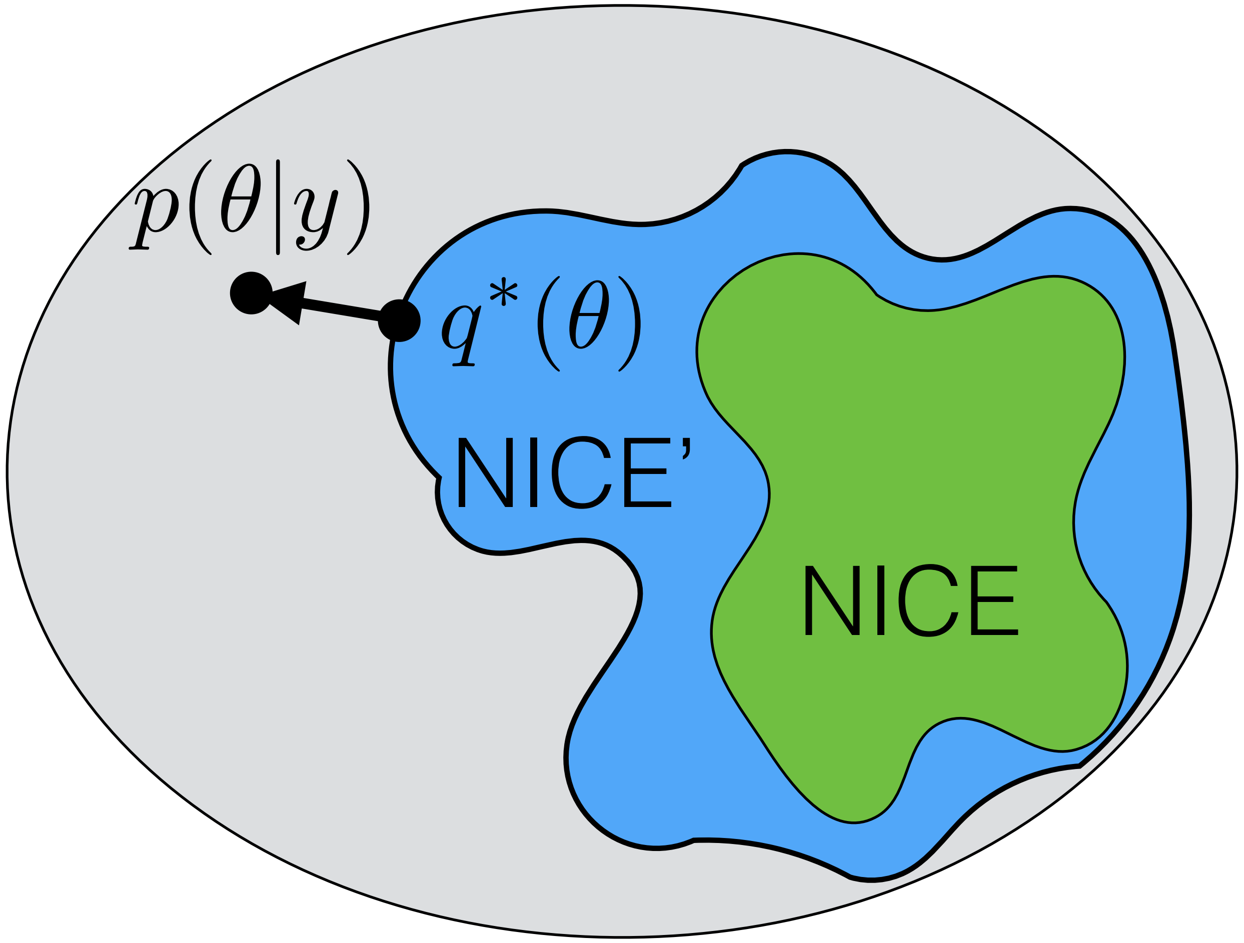

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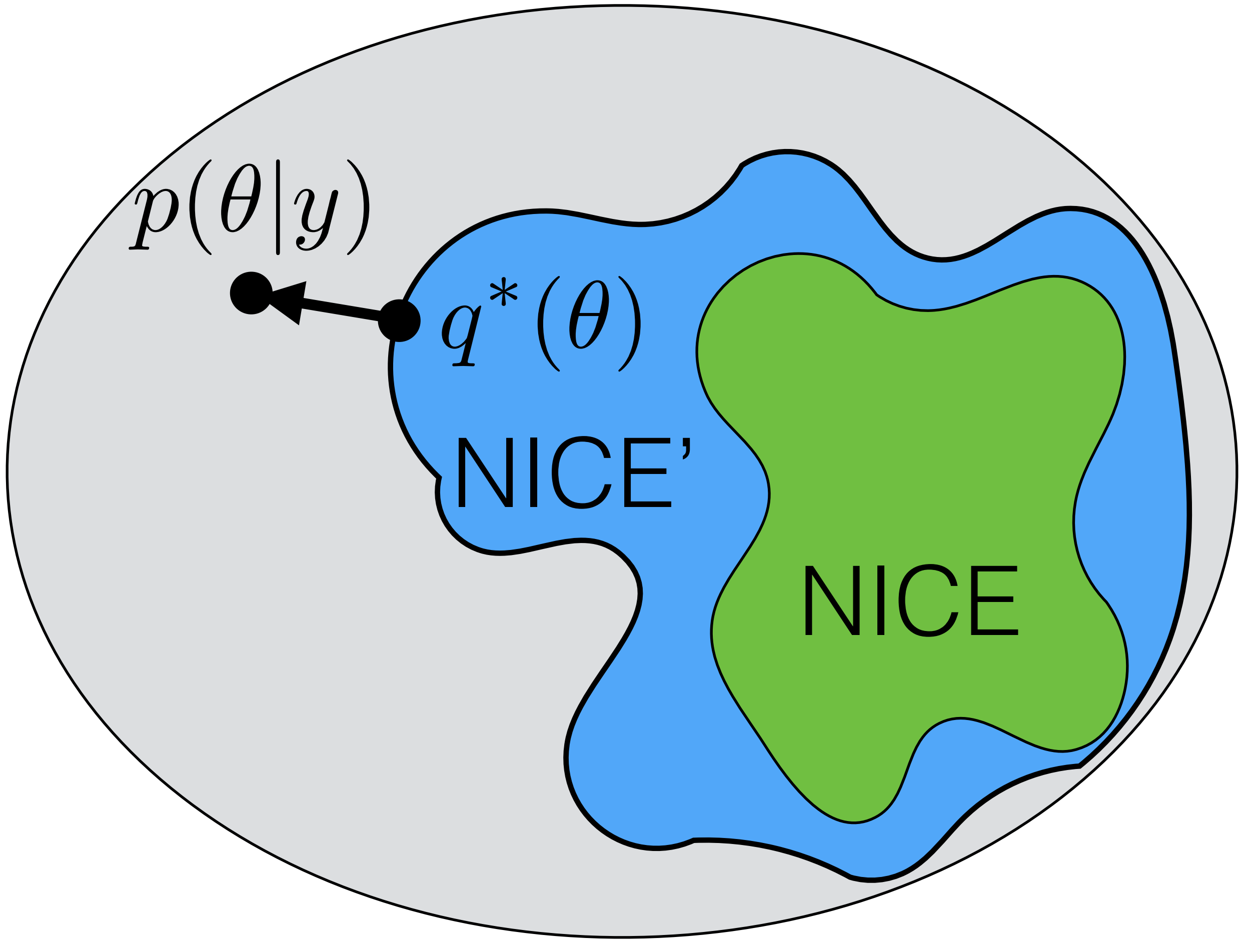

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- Exercise: Show, with a simple example, that a smaller KL does not imply better mean and variance estimates

[Baqué et al 2017; Huggins, Karsprzak, Campbell, Broderick 2019]

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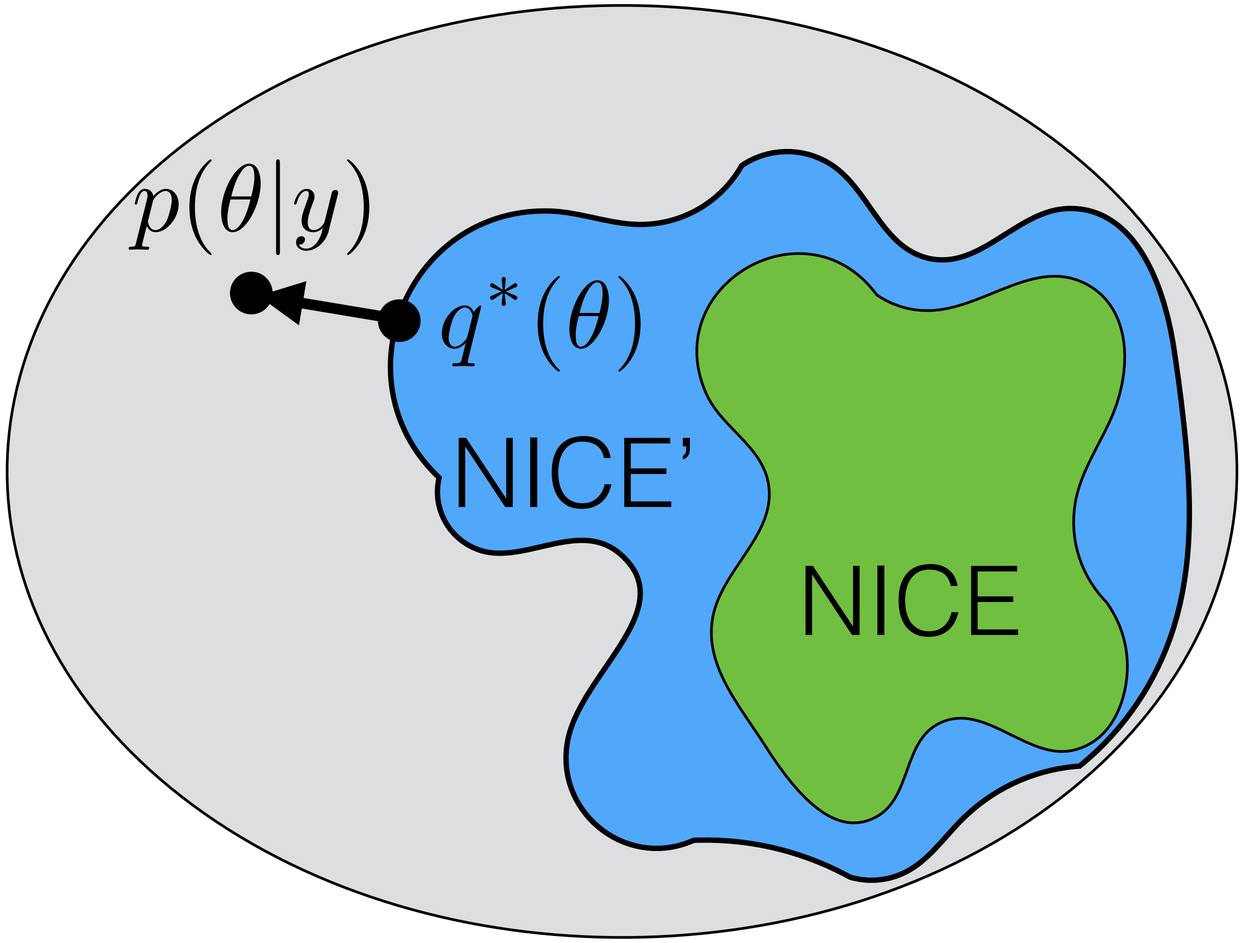

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- Exercise: Show, with a simple example, that a smaller KL does not imply better mean and variance estimates
- But how much worse can the estimates be? And could it have just been the implementation?

[Baqué et al 2017; Huggins, Karsprzak, Campbell, Broderick 2019]

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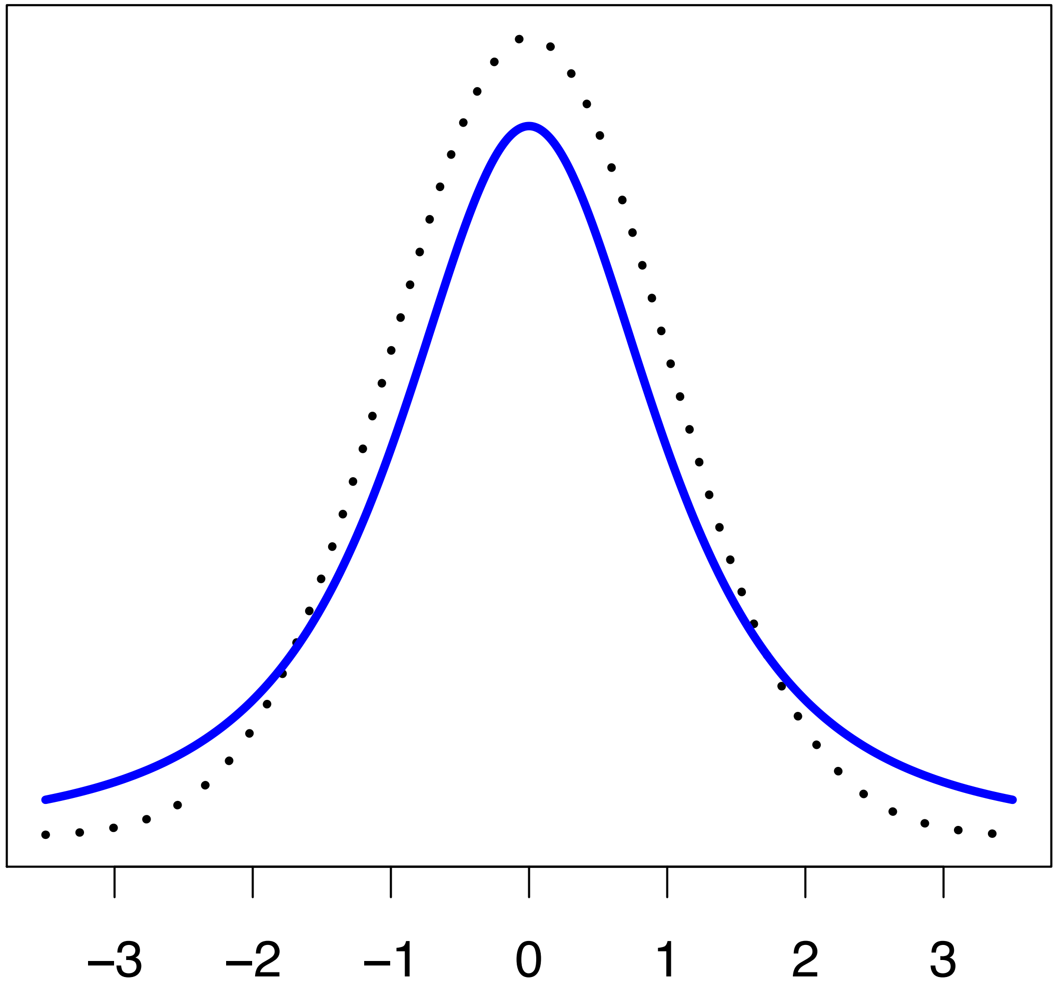

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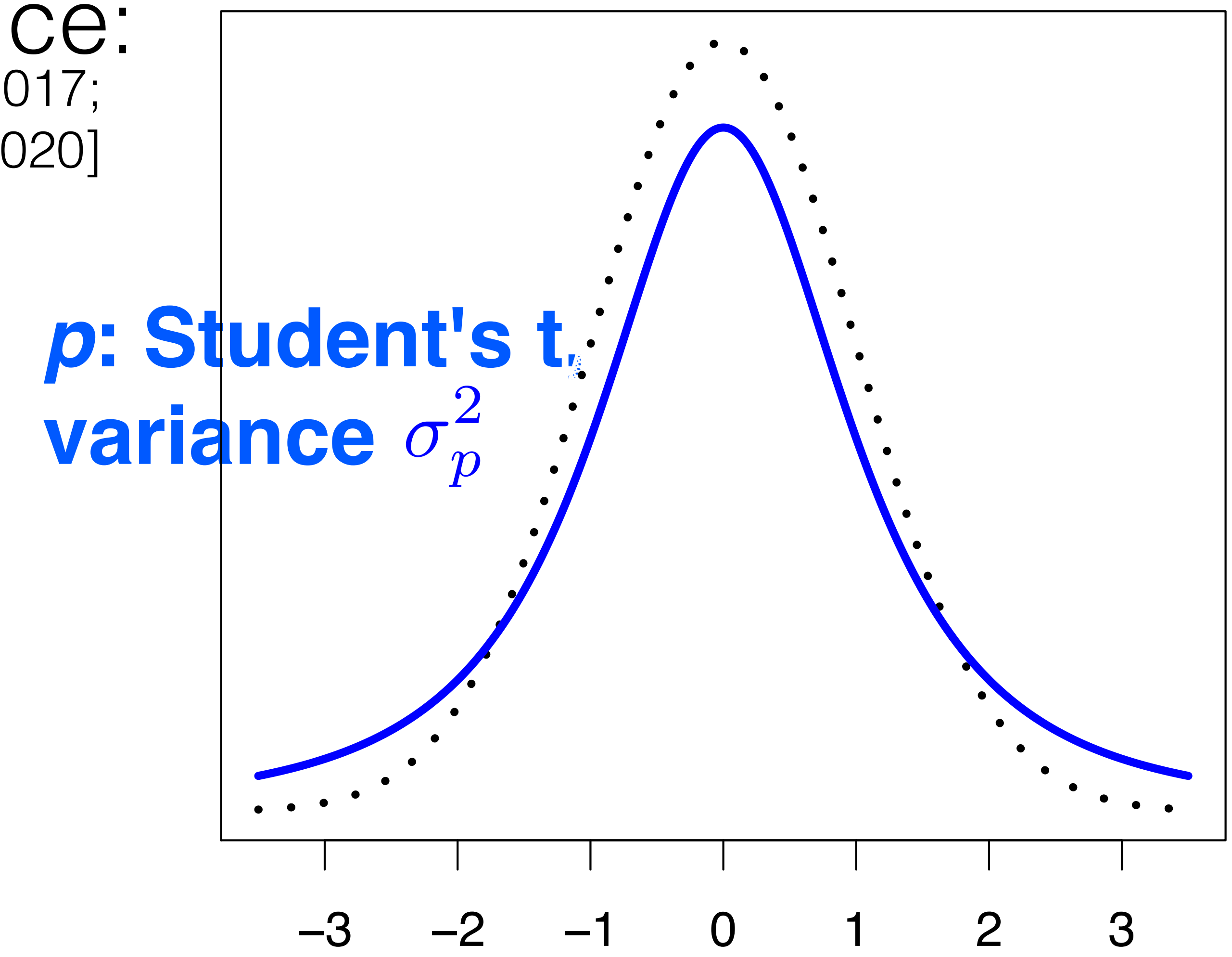

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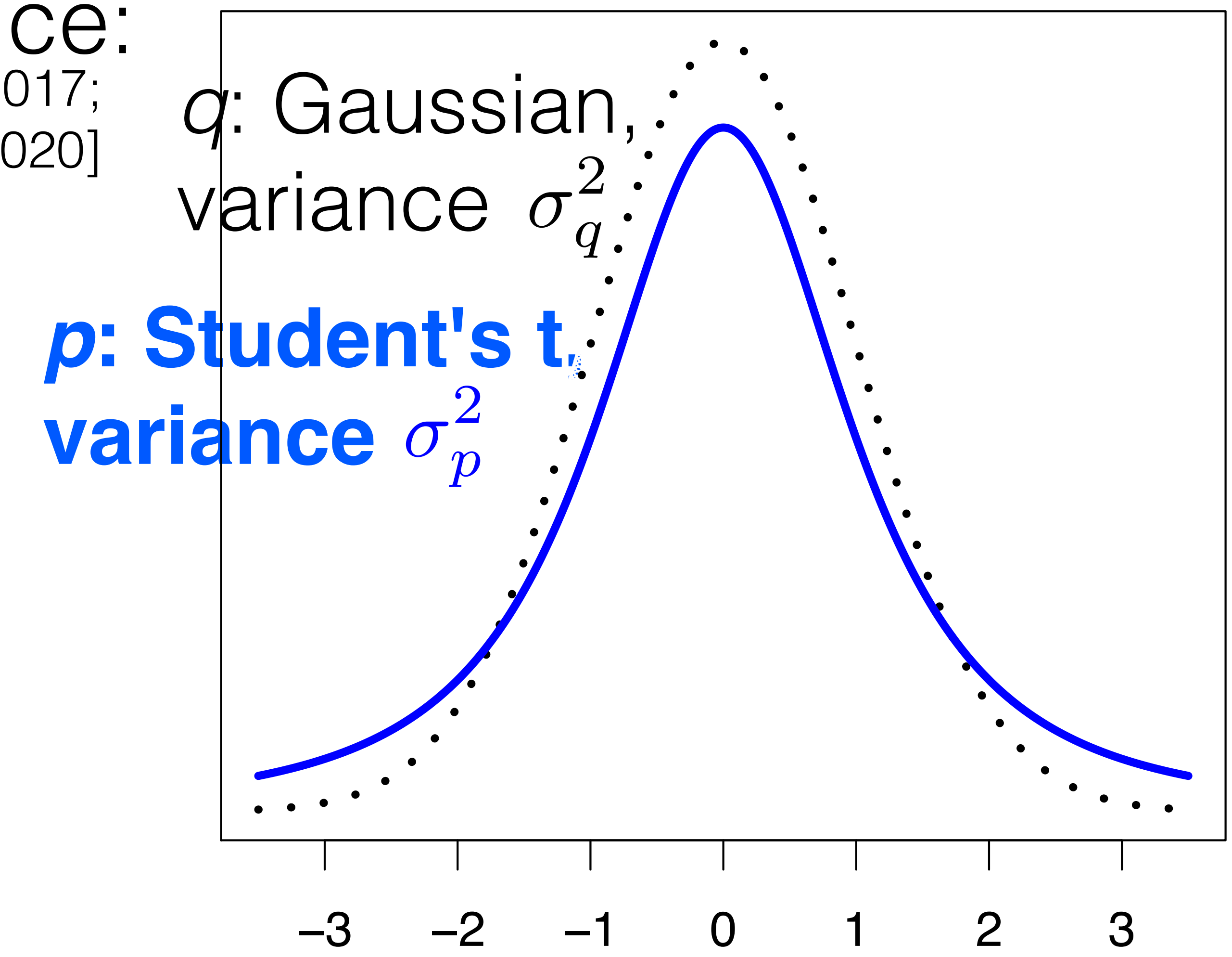

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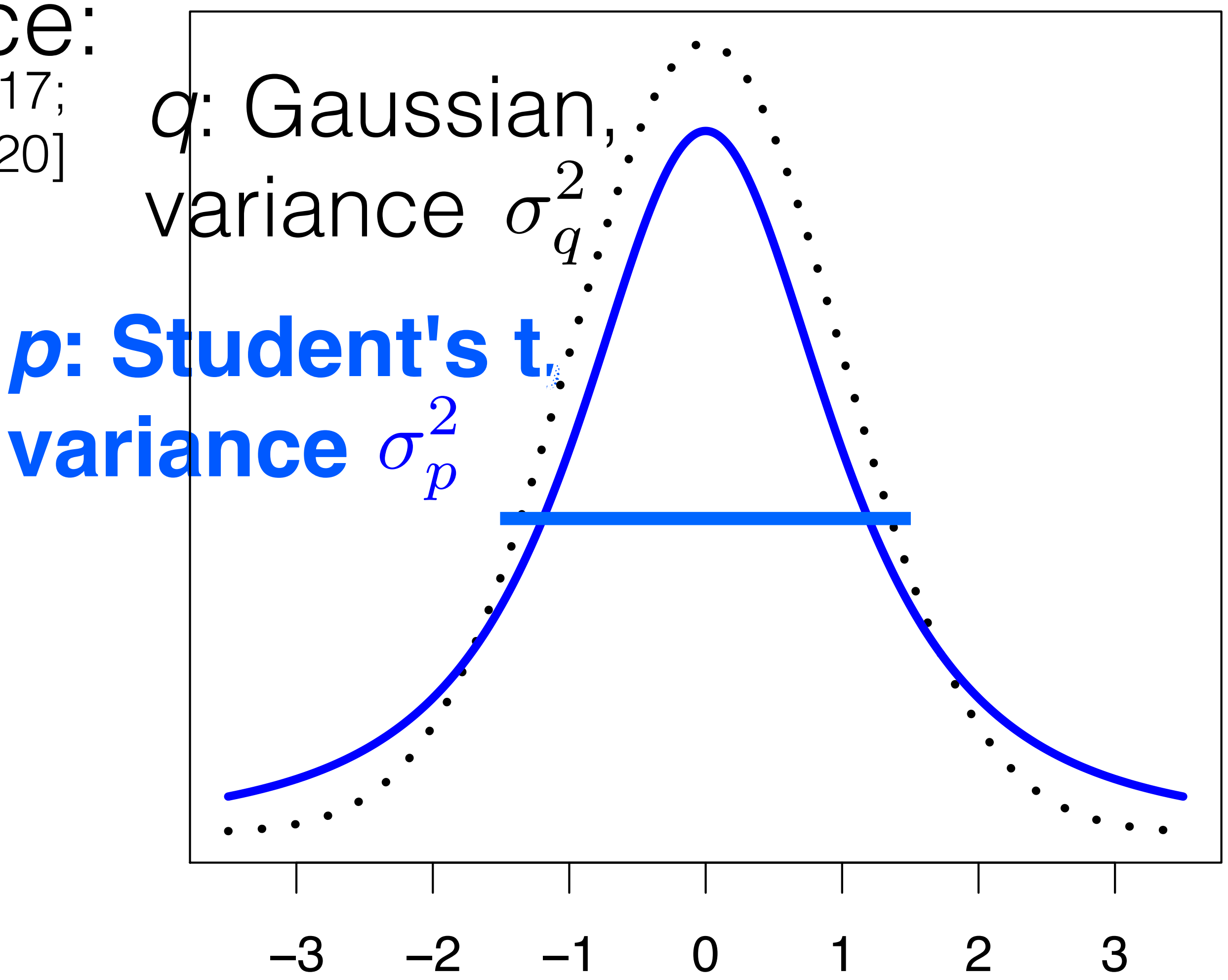

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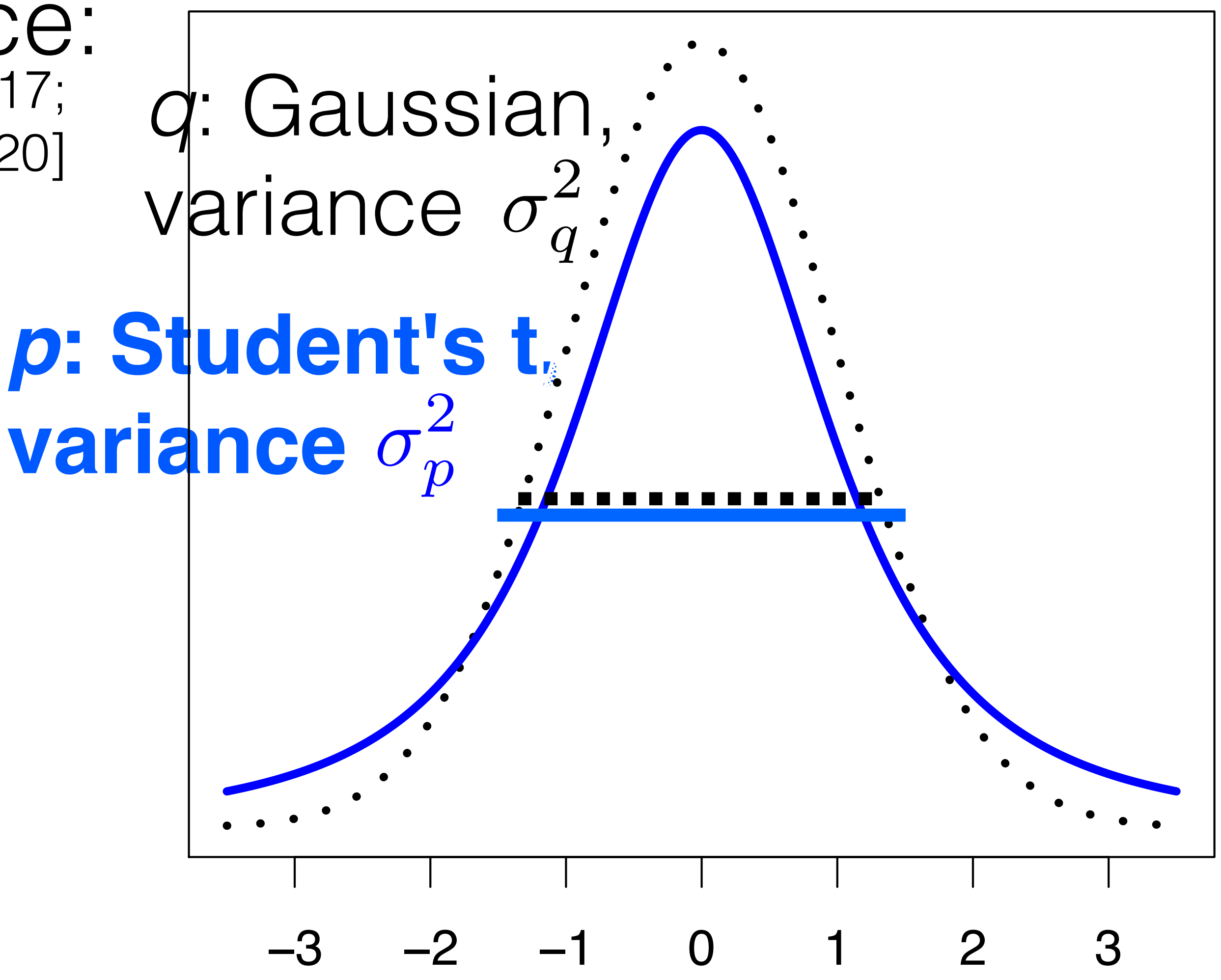

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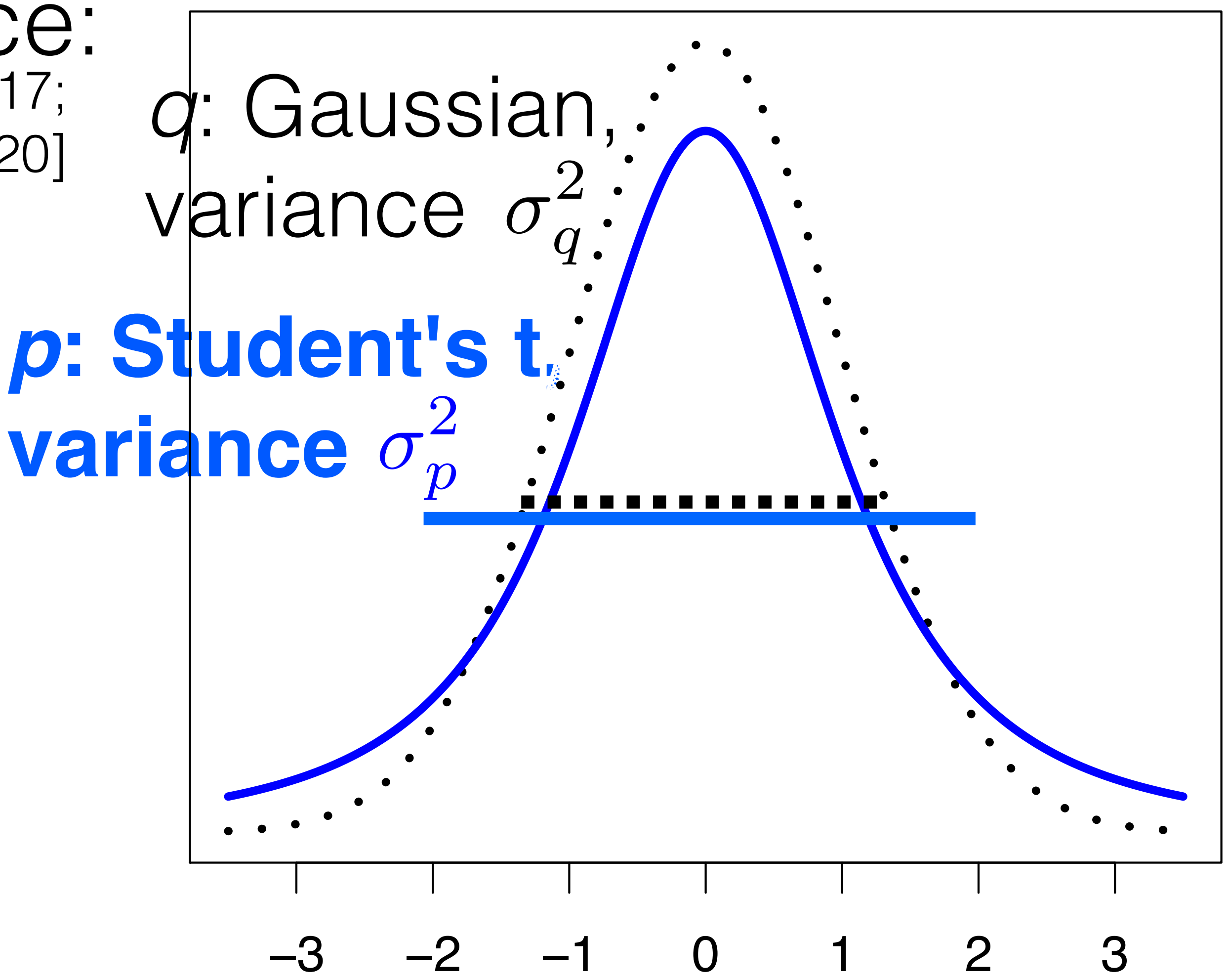

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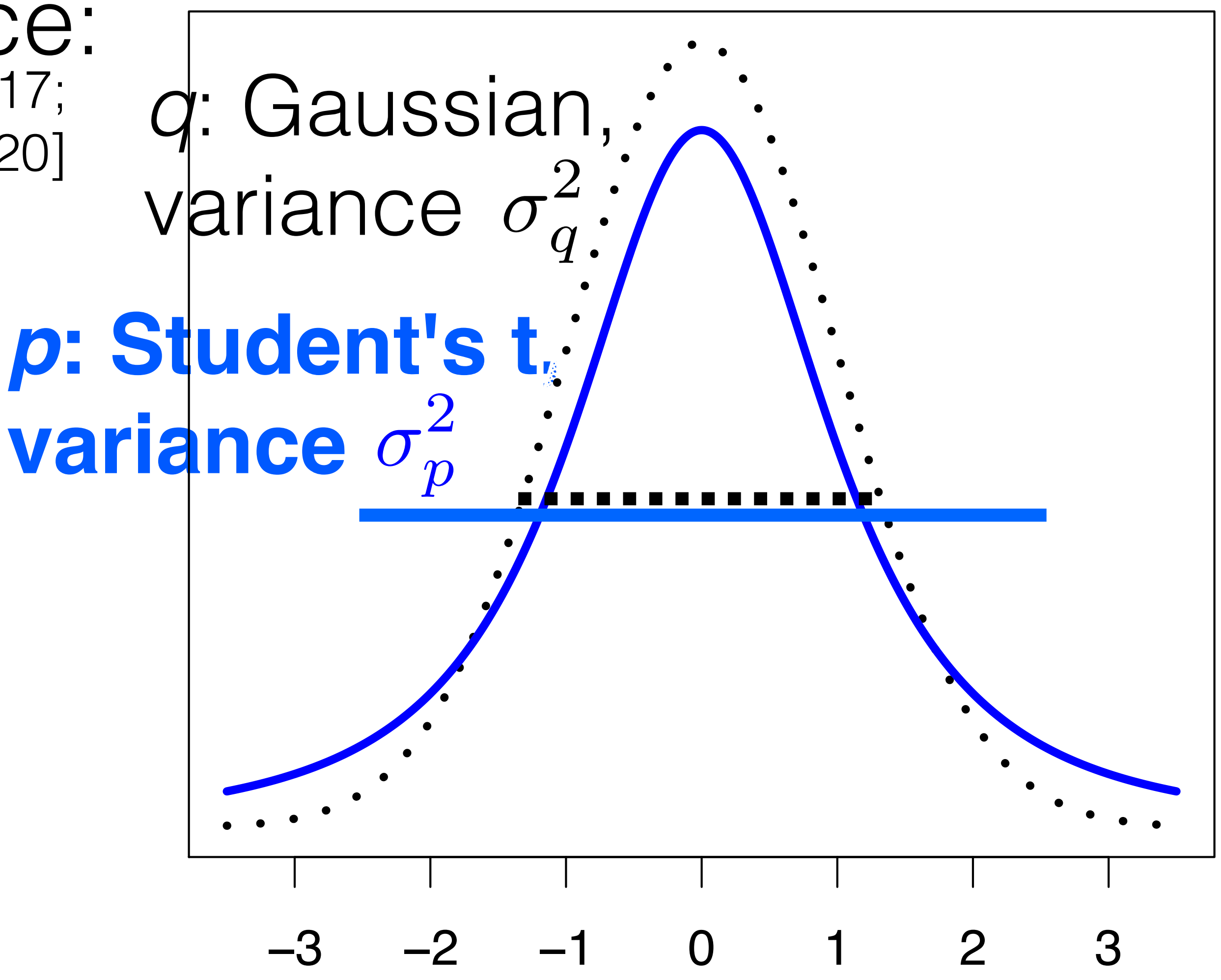

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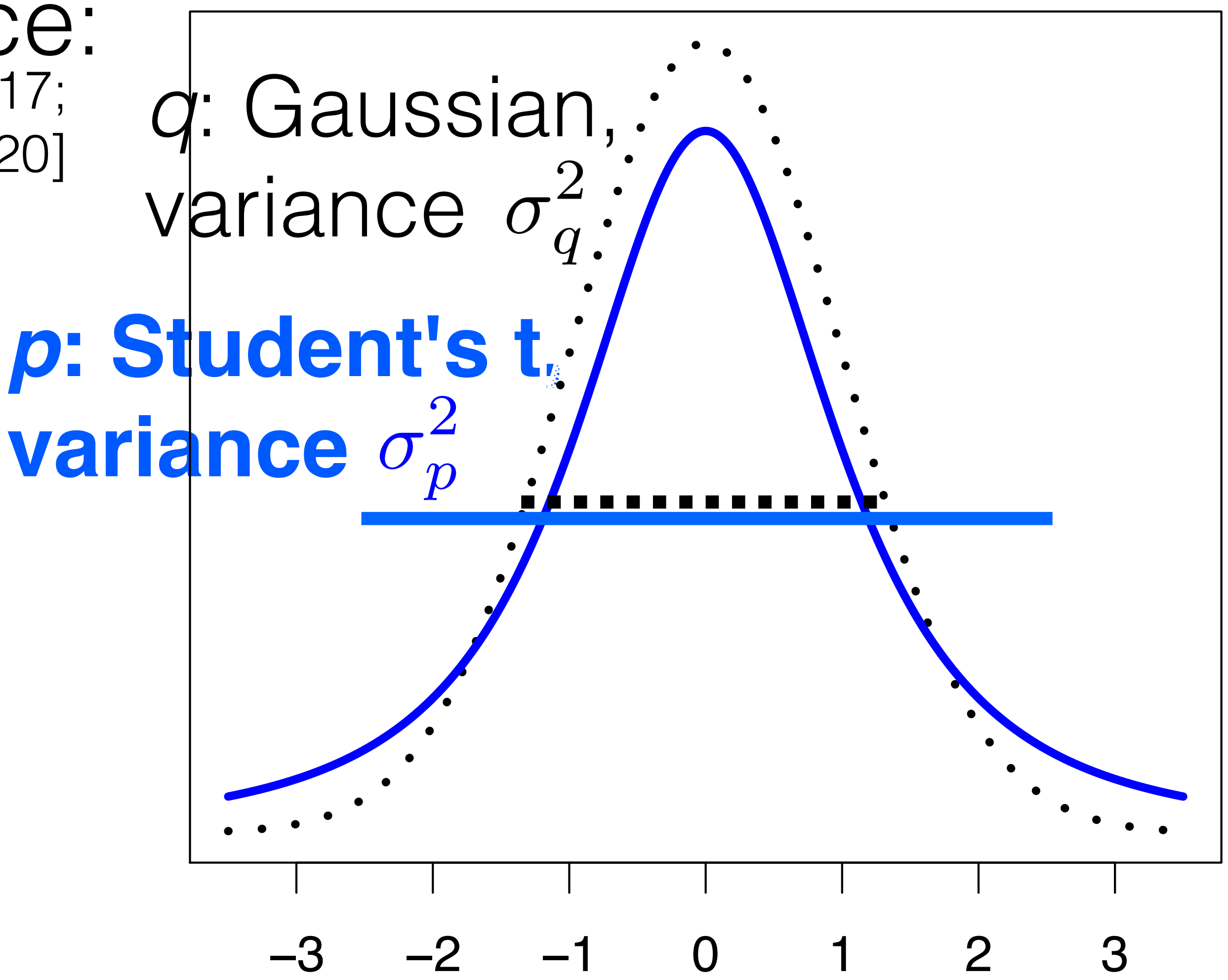

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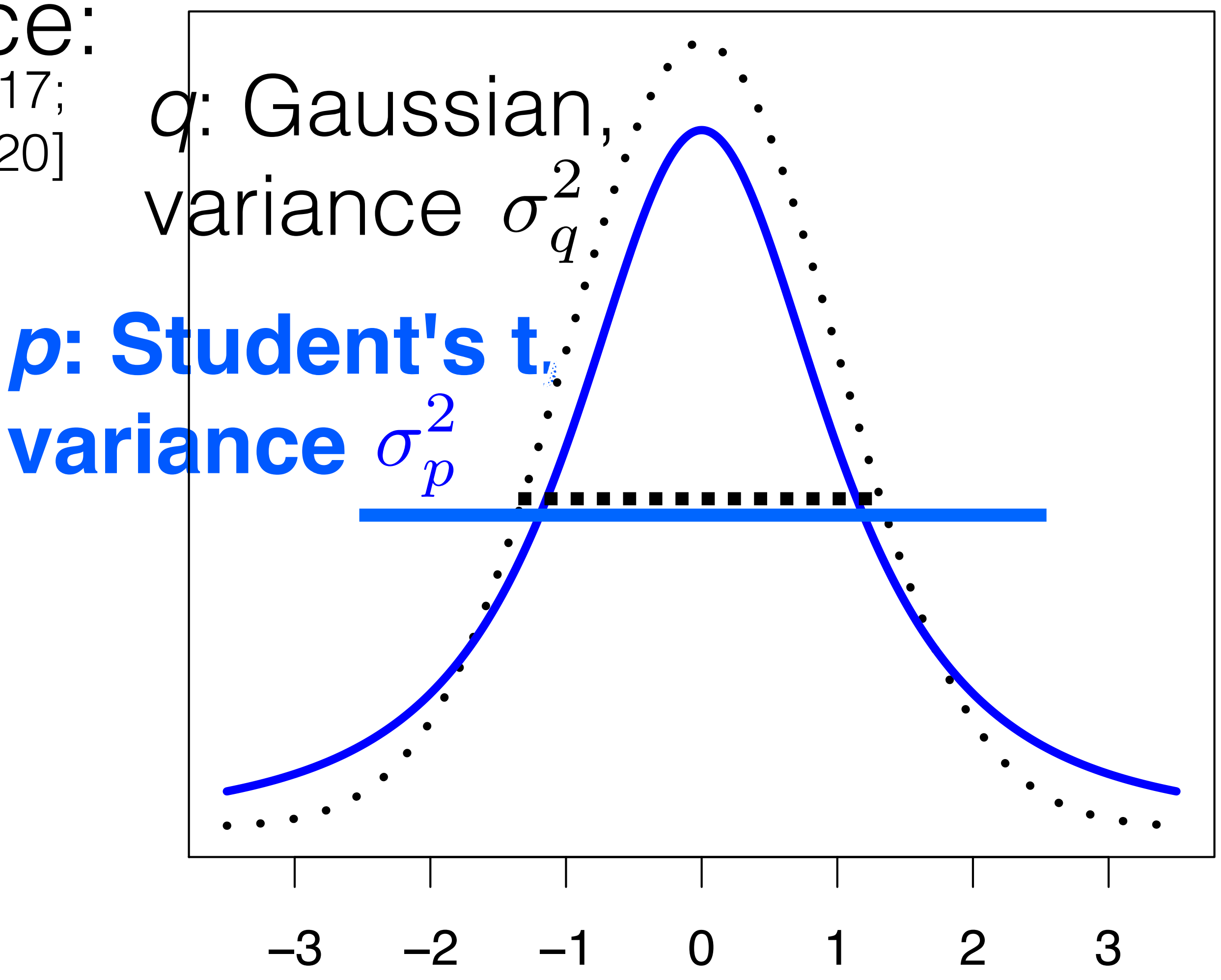

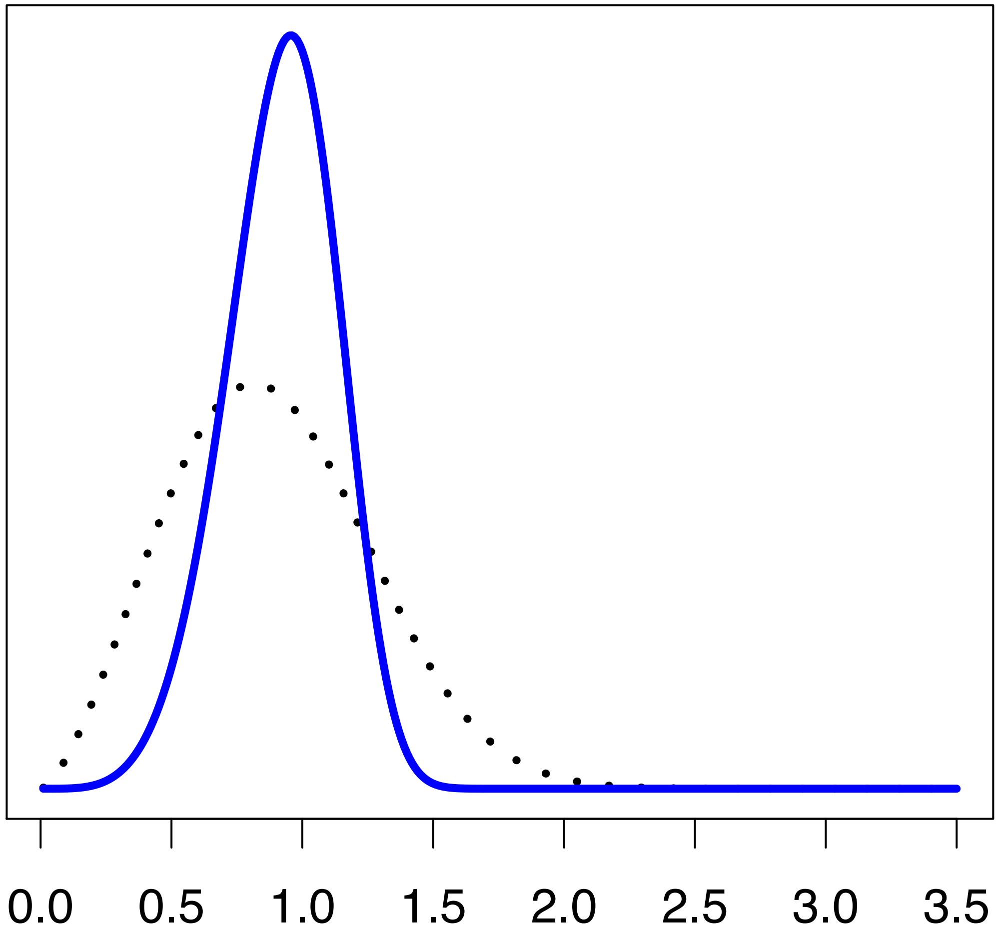

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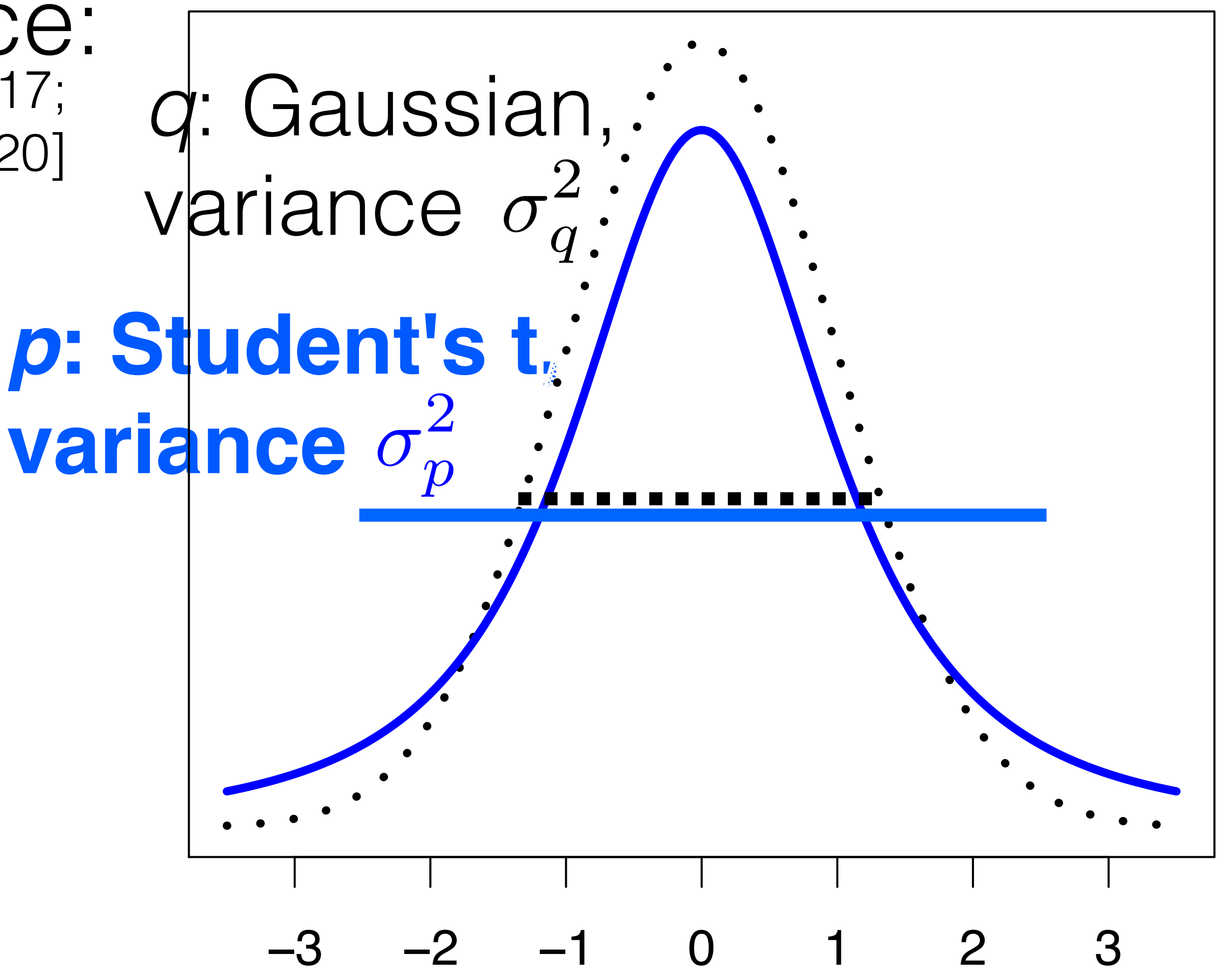

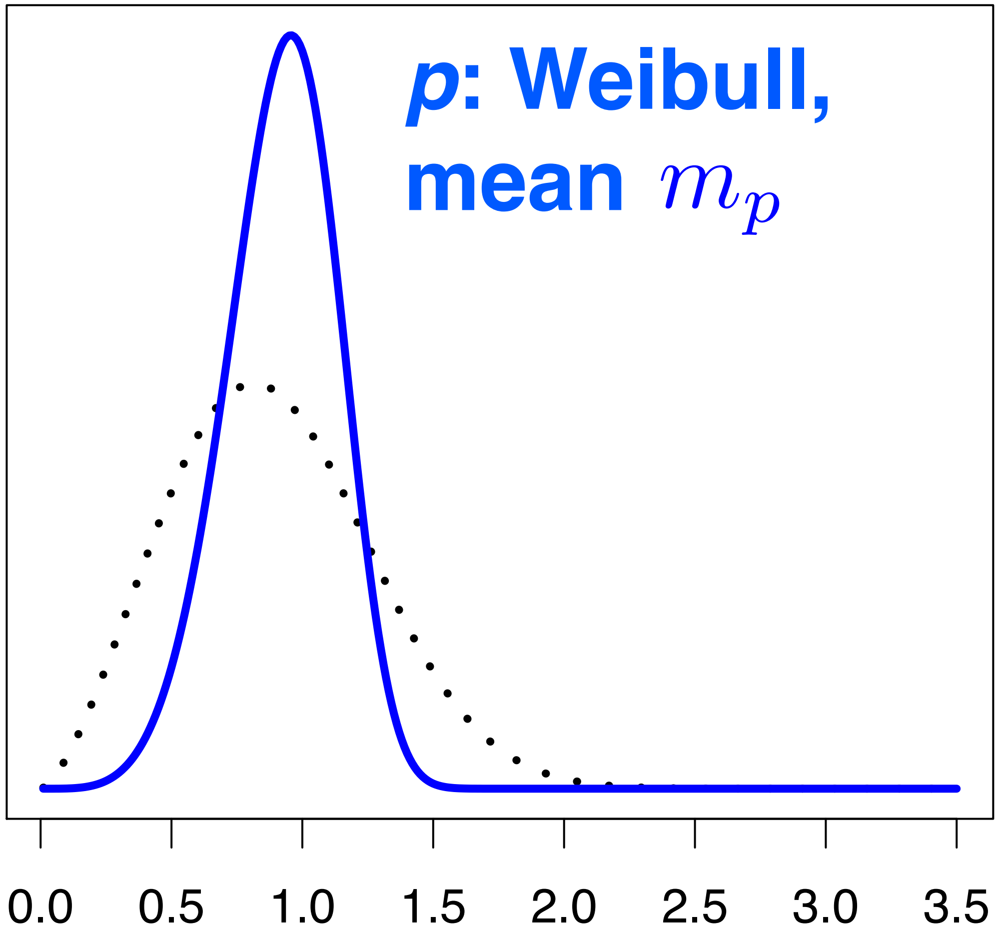

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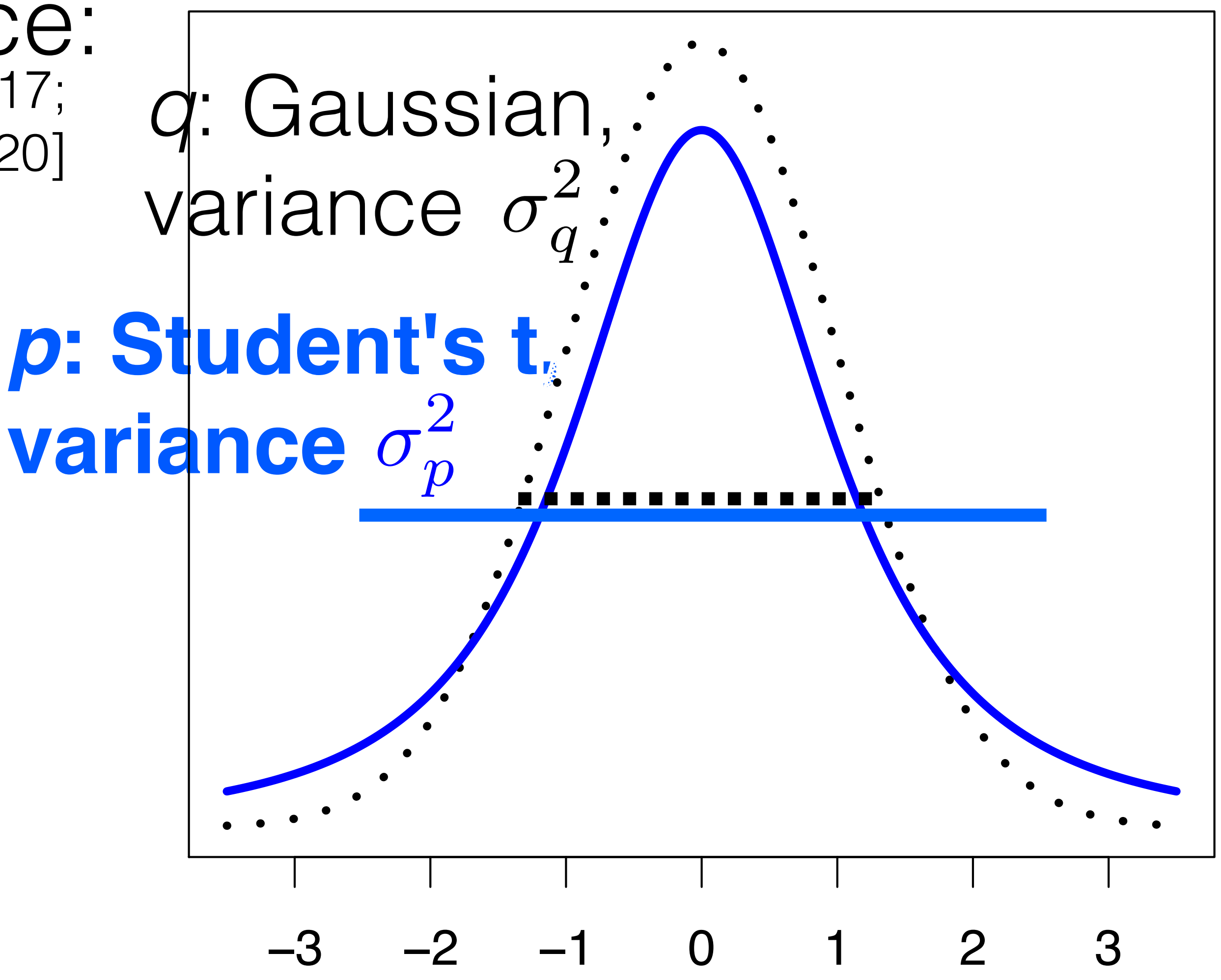

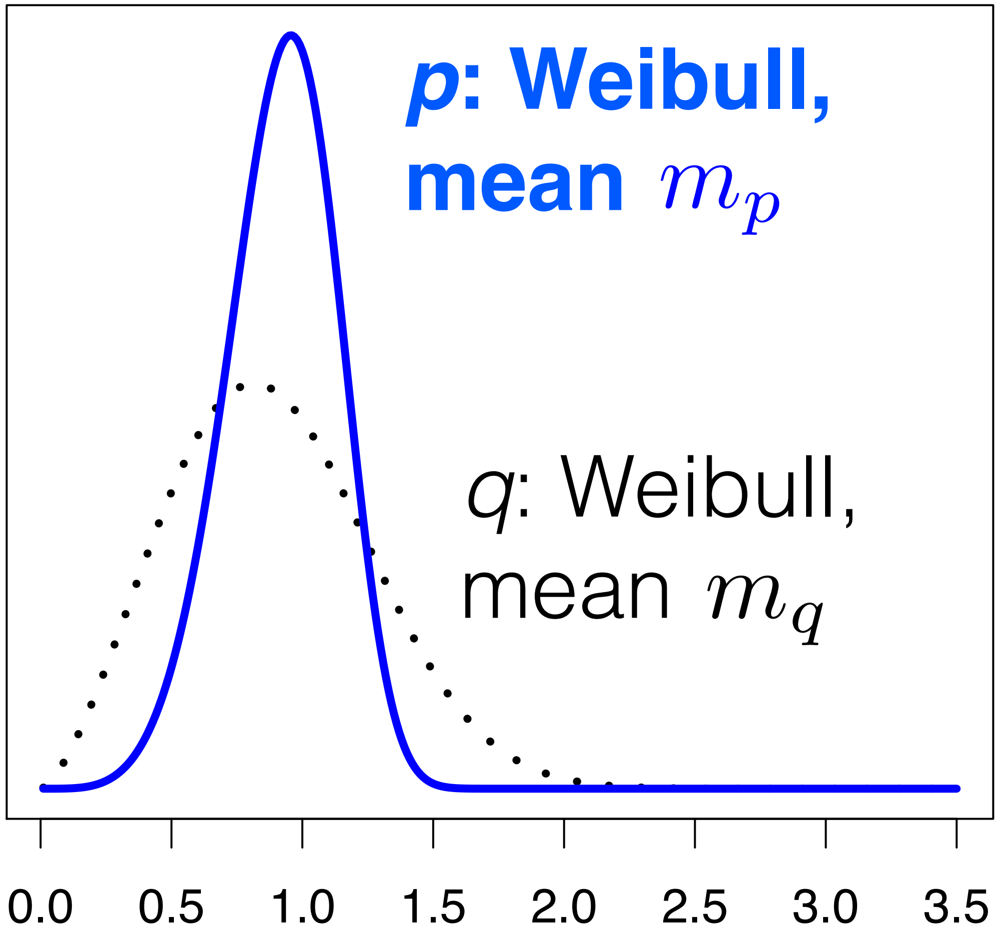

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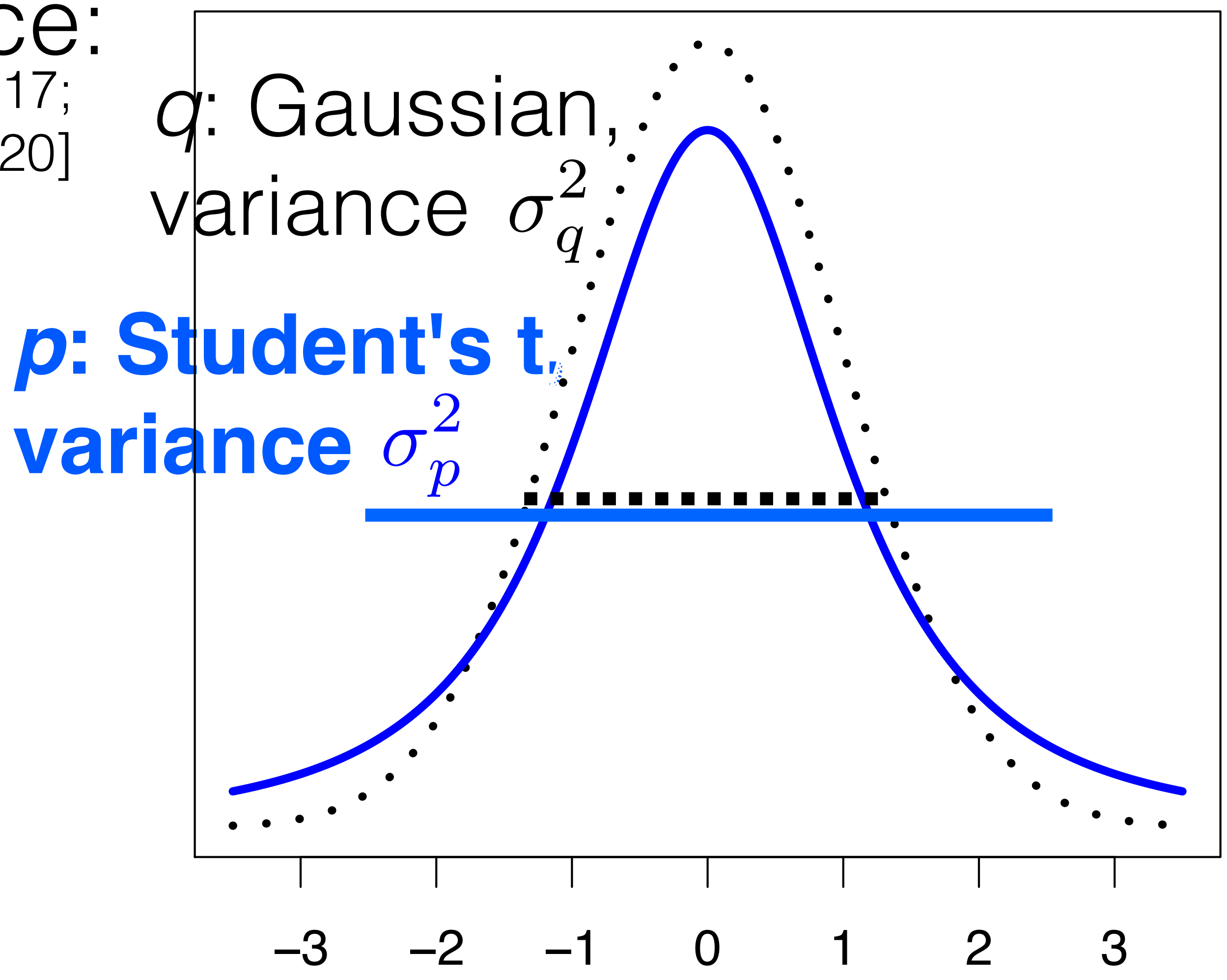

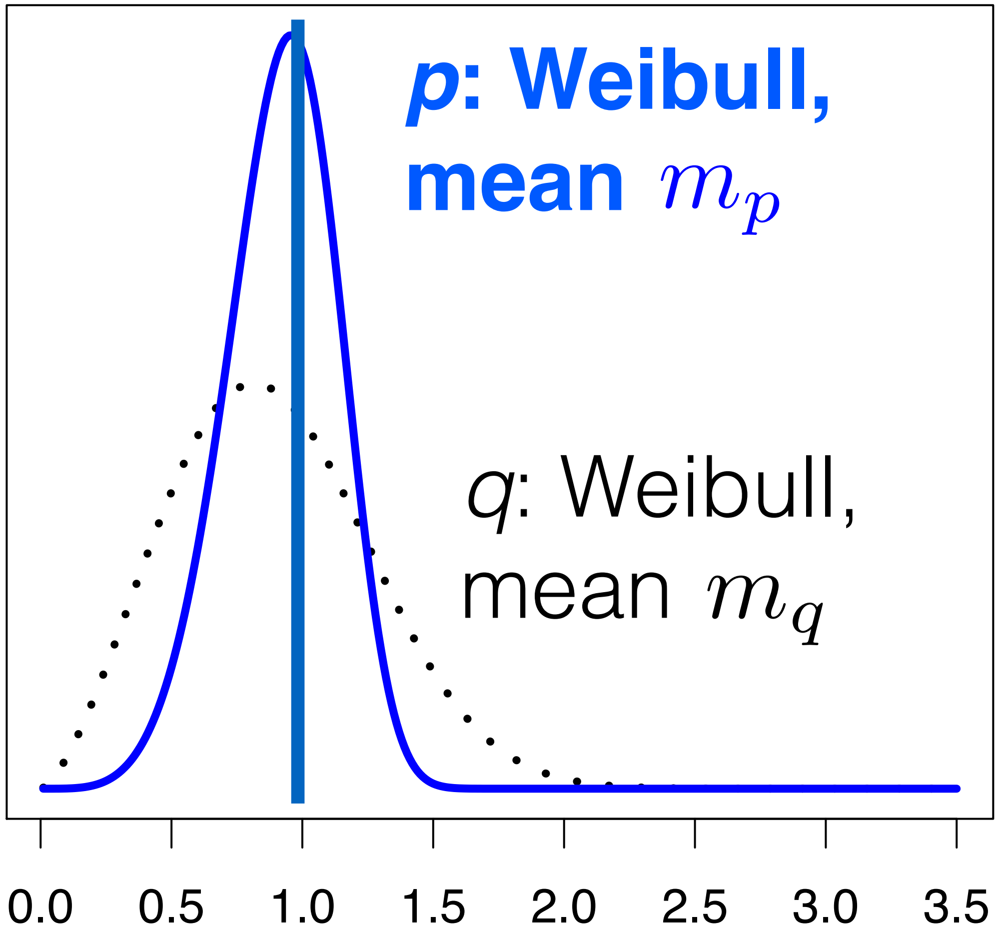

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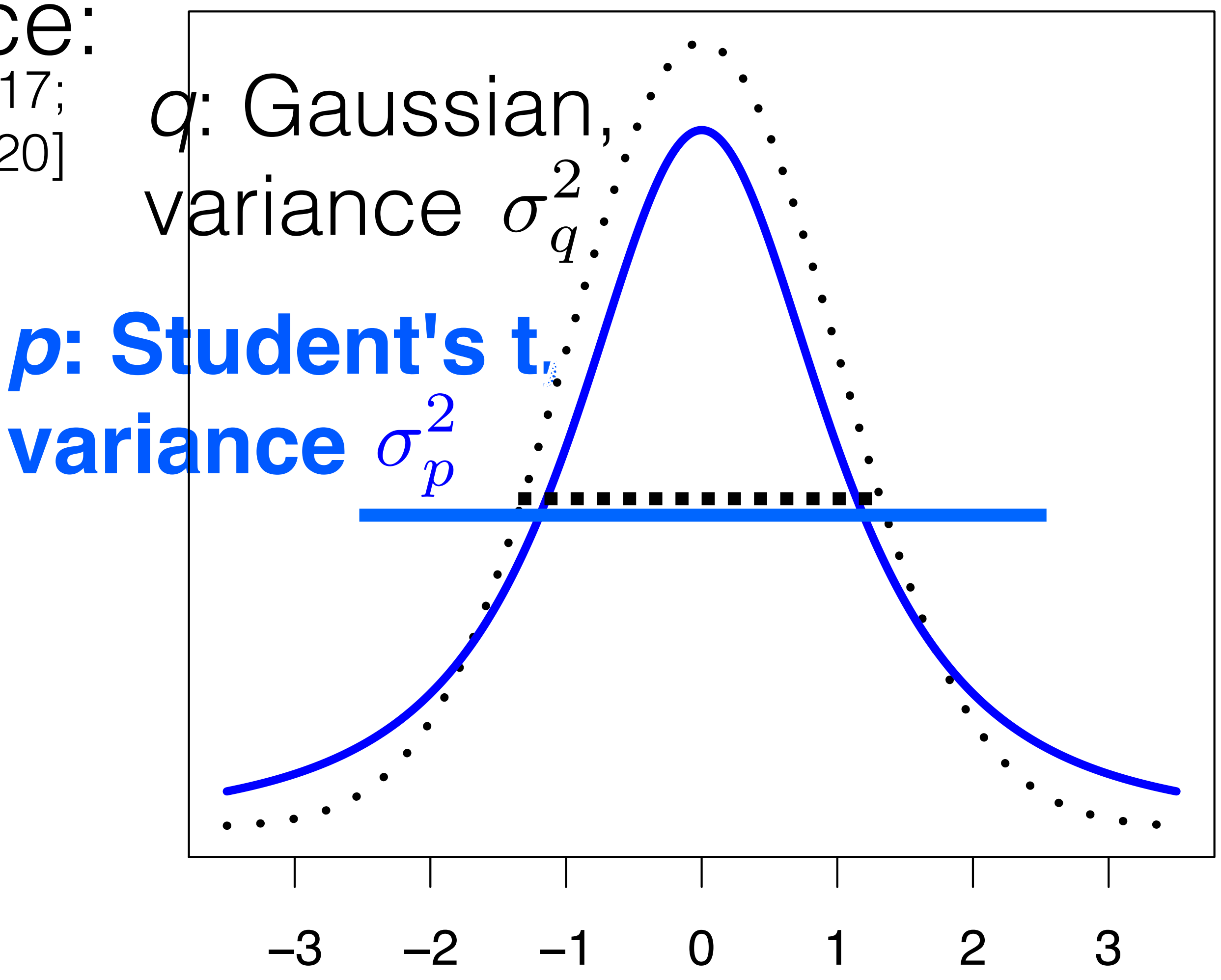

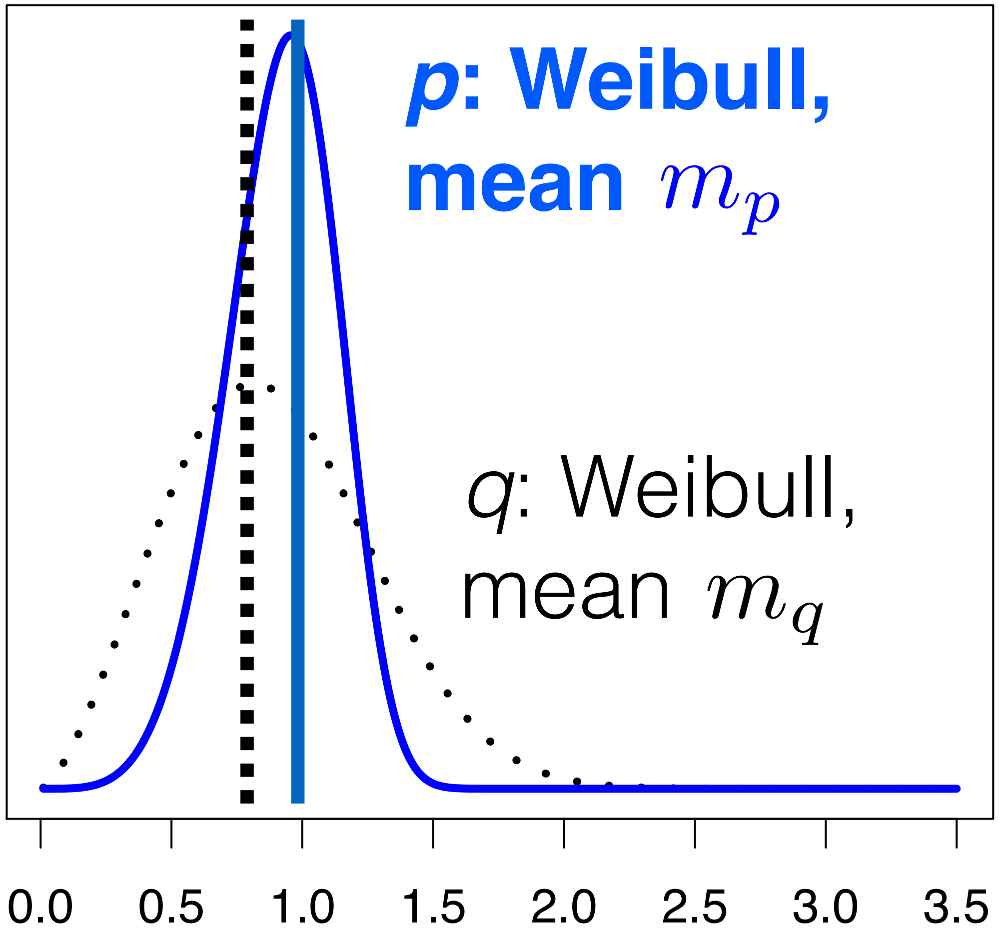

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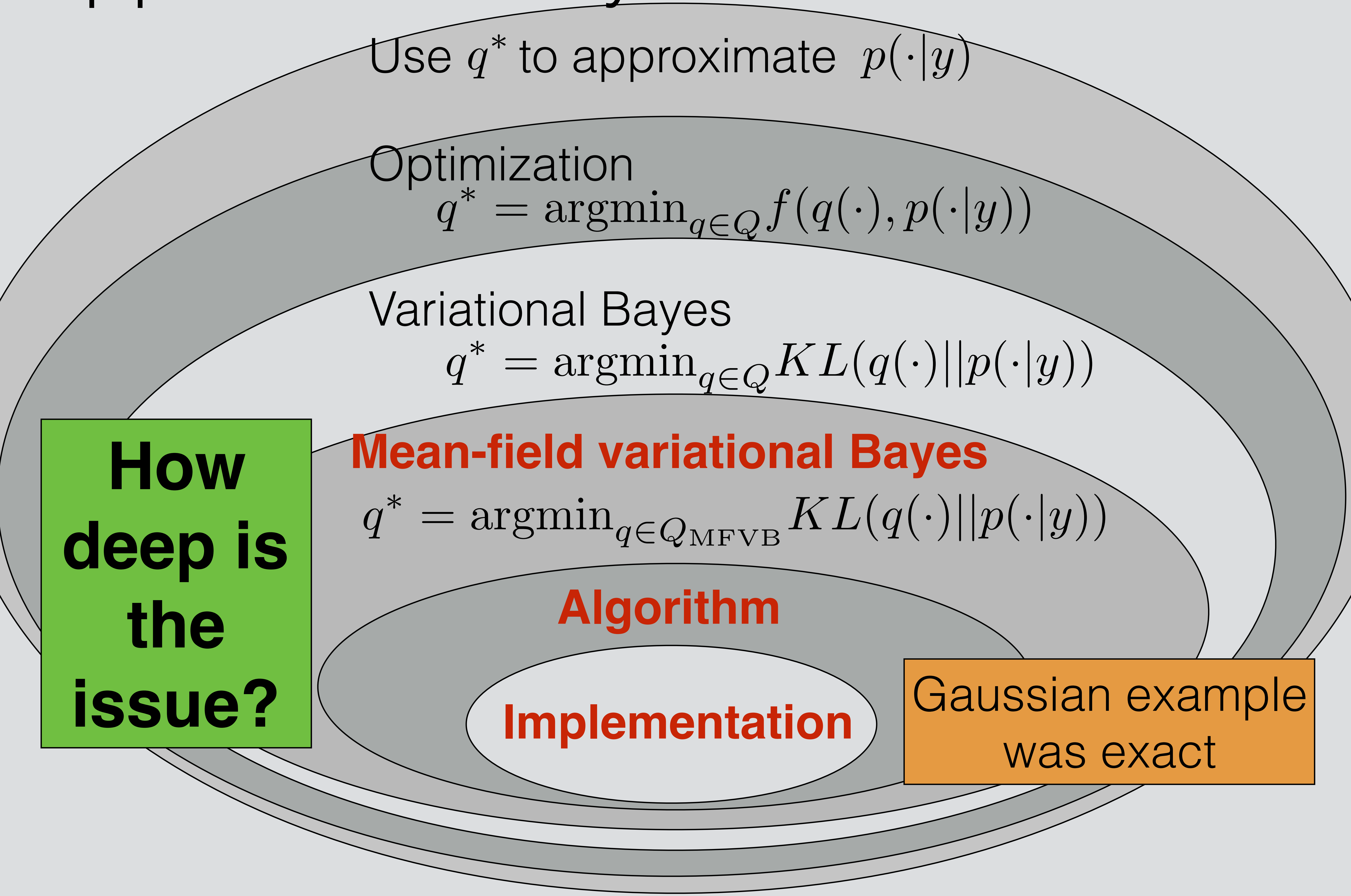

Use q^* to approximate $p(\cdot|y)$

Optimization

$$q^* = \mathrm{argmin}_{q \in Q} f(q(\cdot), p(\cdot|y))$$

Variational Bayes

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Mean-field variational Bayes

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Algorithm

Implementation

How deep is the issue?

Gaussian example was exact

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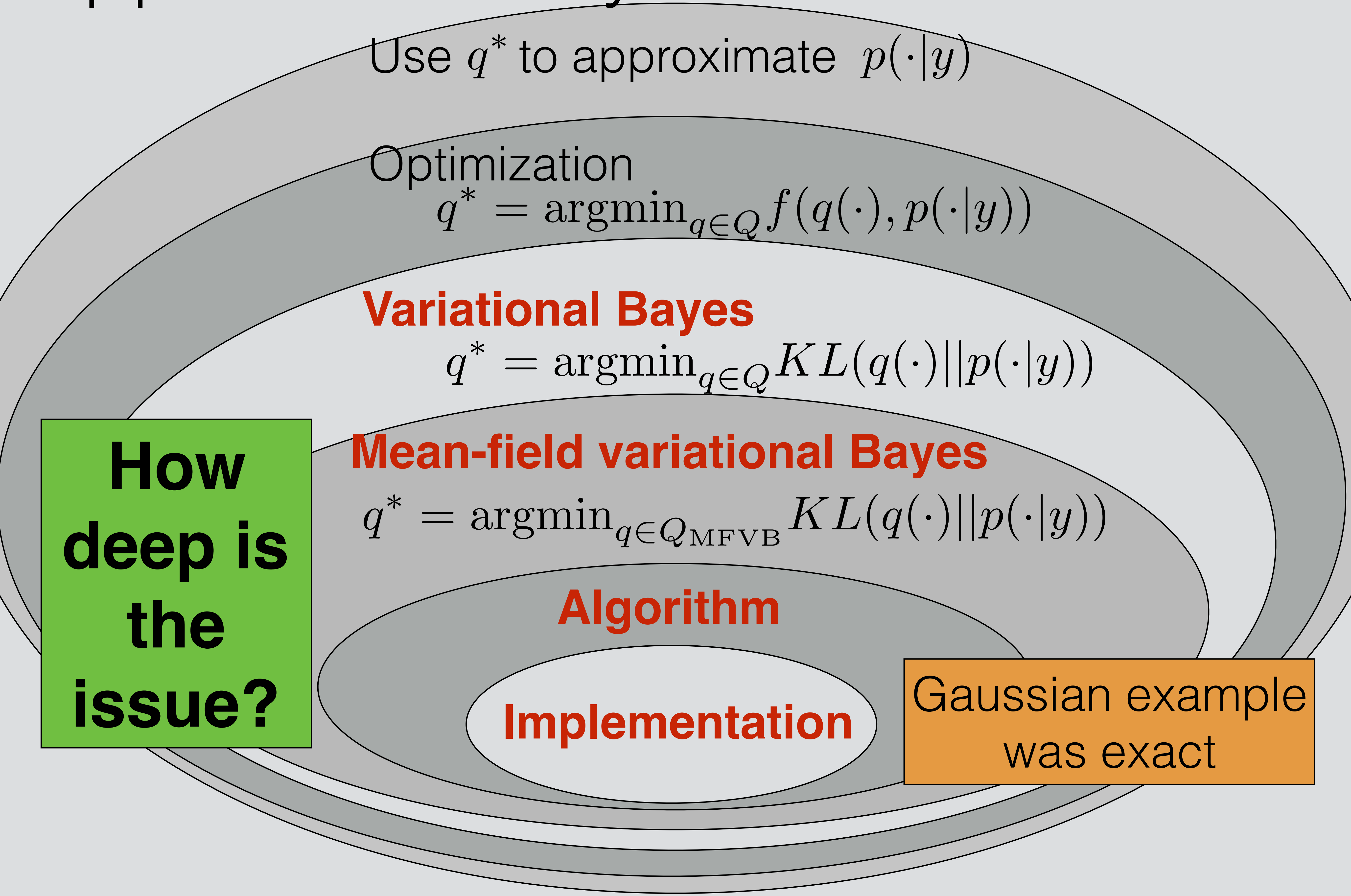

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- Why use VB?
- When can we trust VB?
- Where do we go from here?

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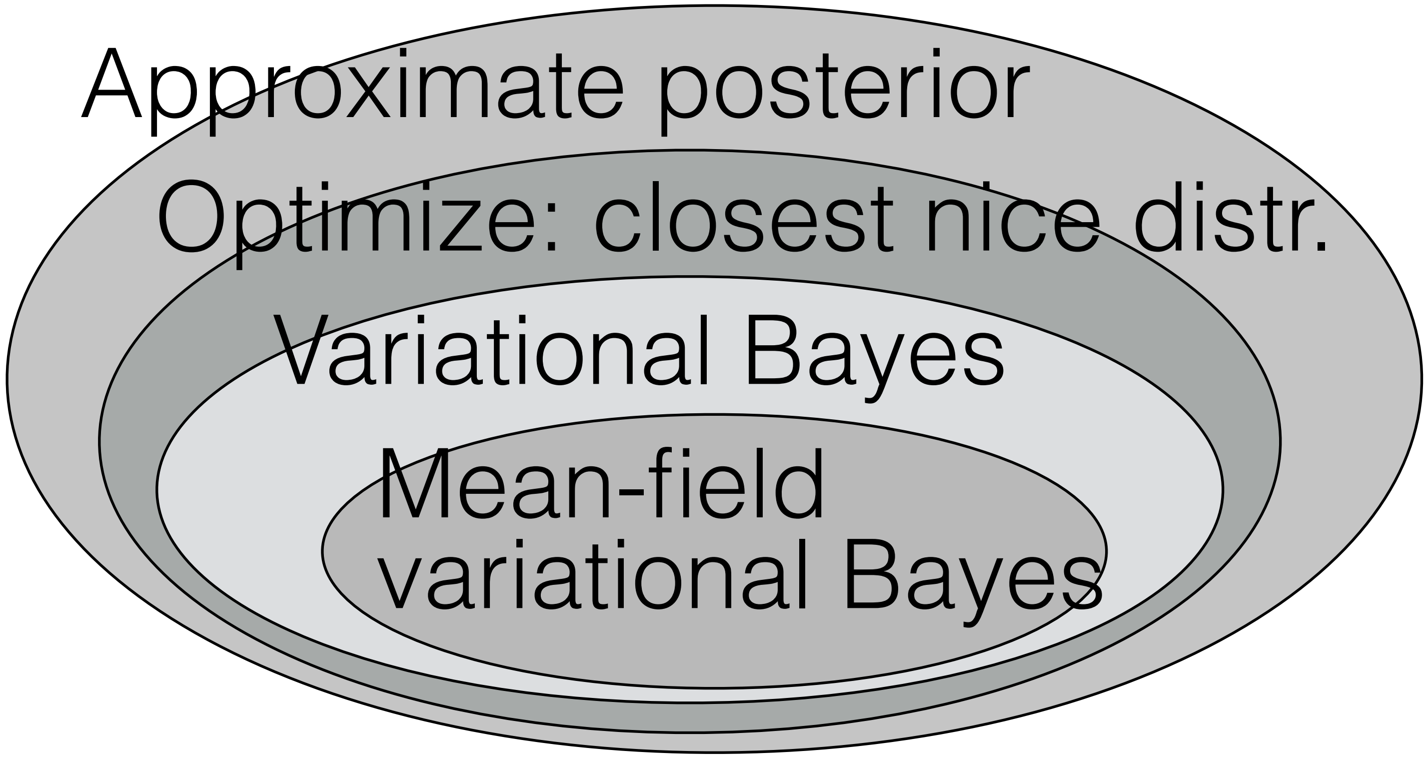

Approximate posterior

Optimize: closest nice distr.

Variational Bayes

Mean-field
variational Bayes

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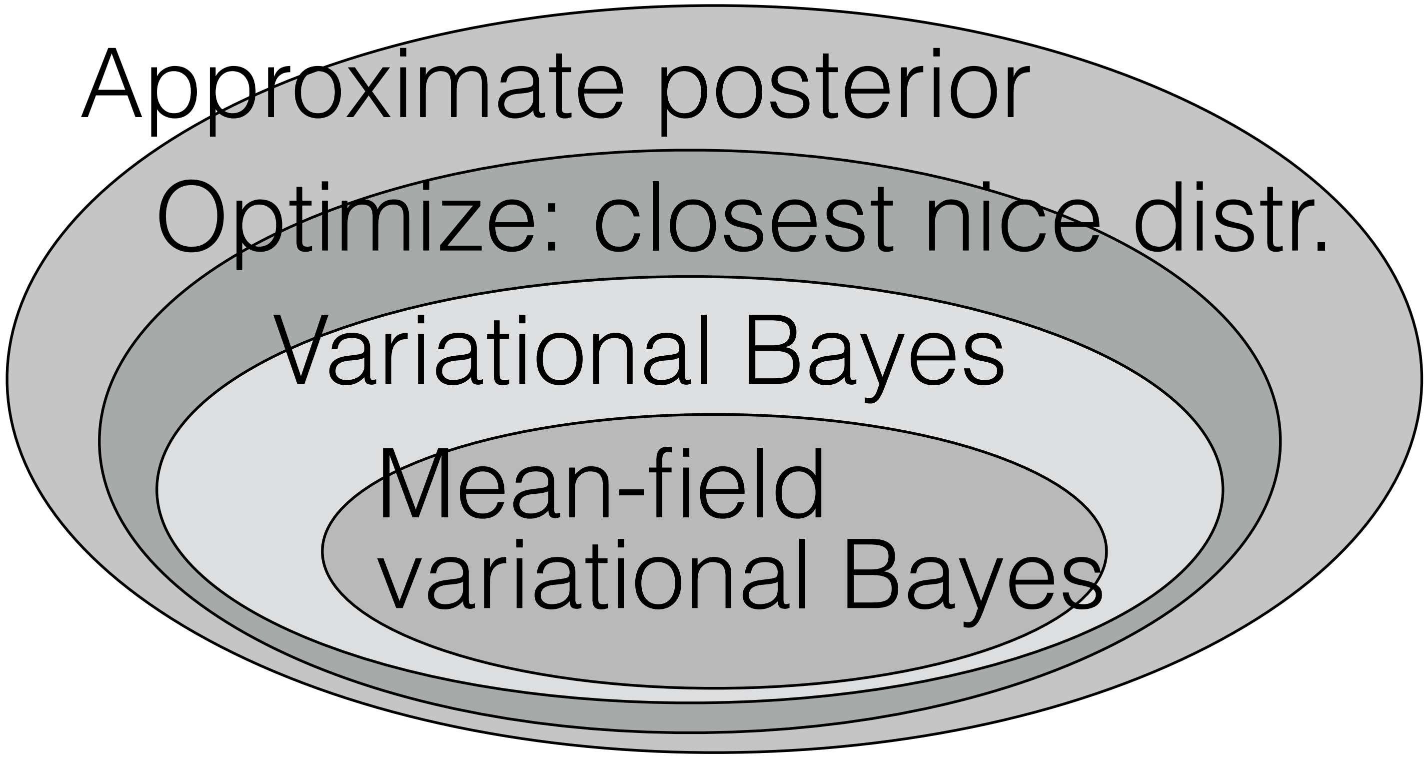

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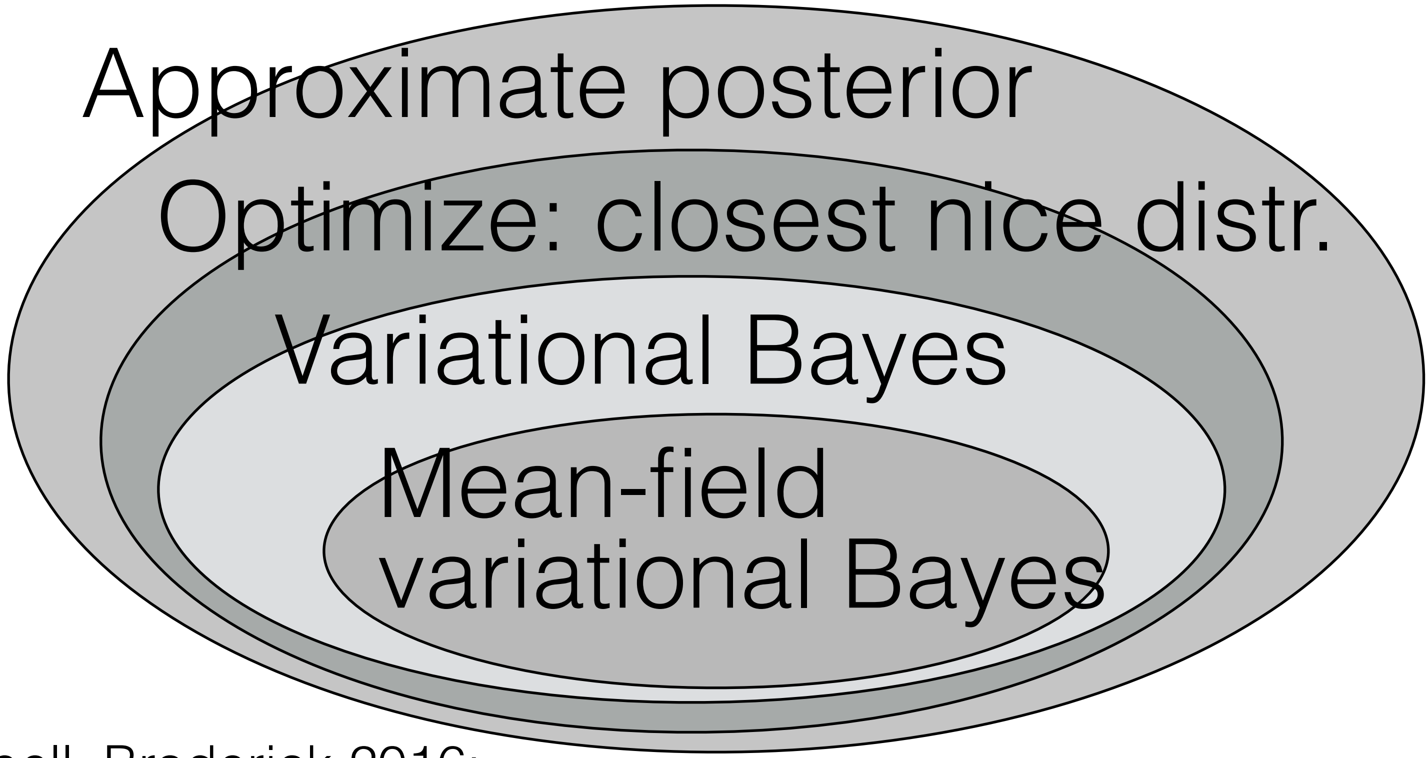

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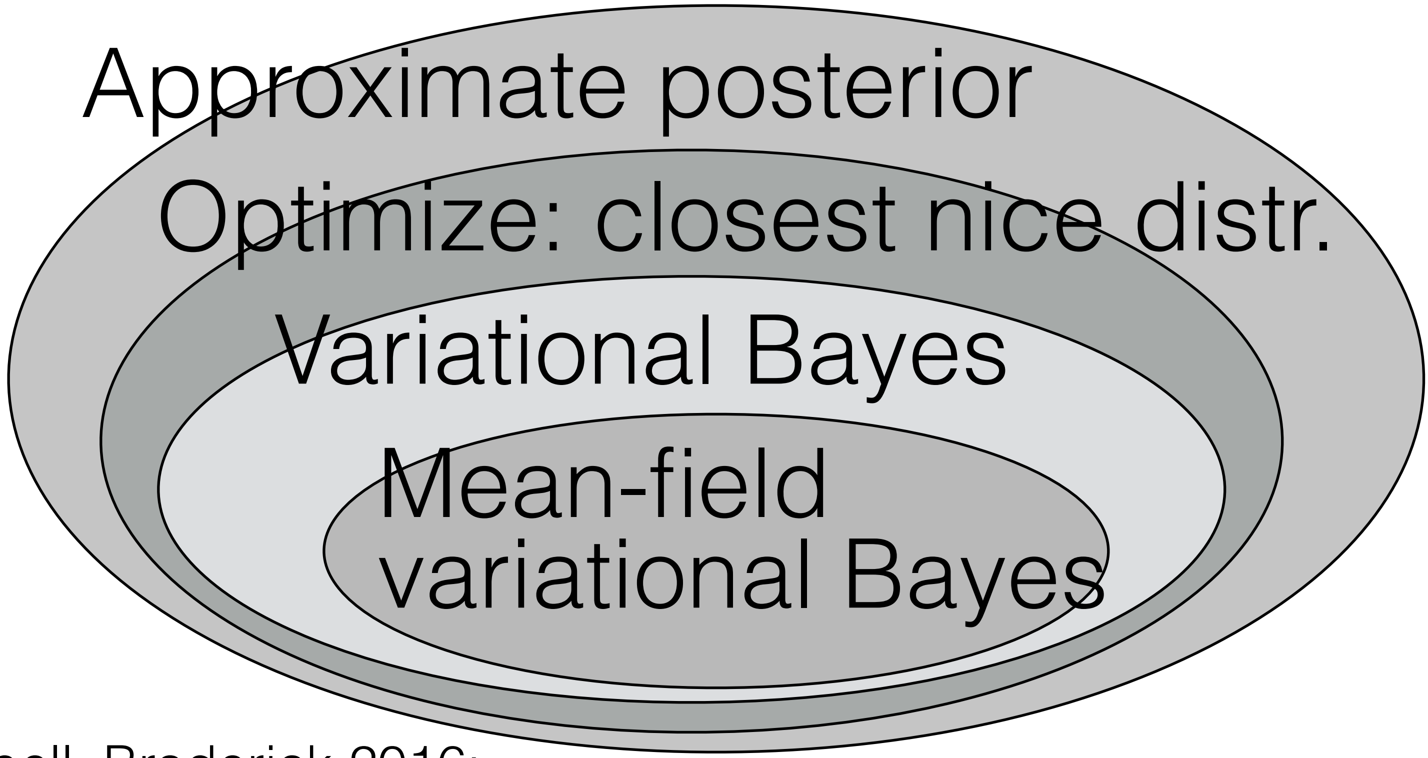

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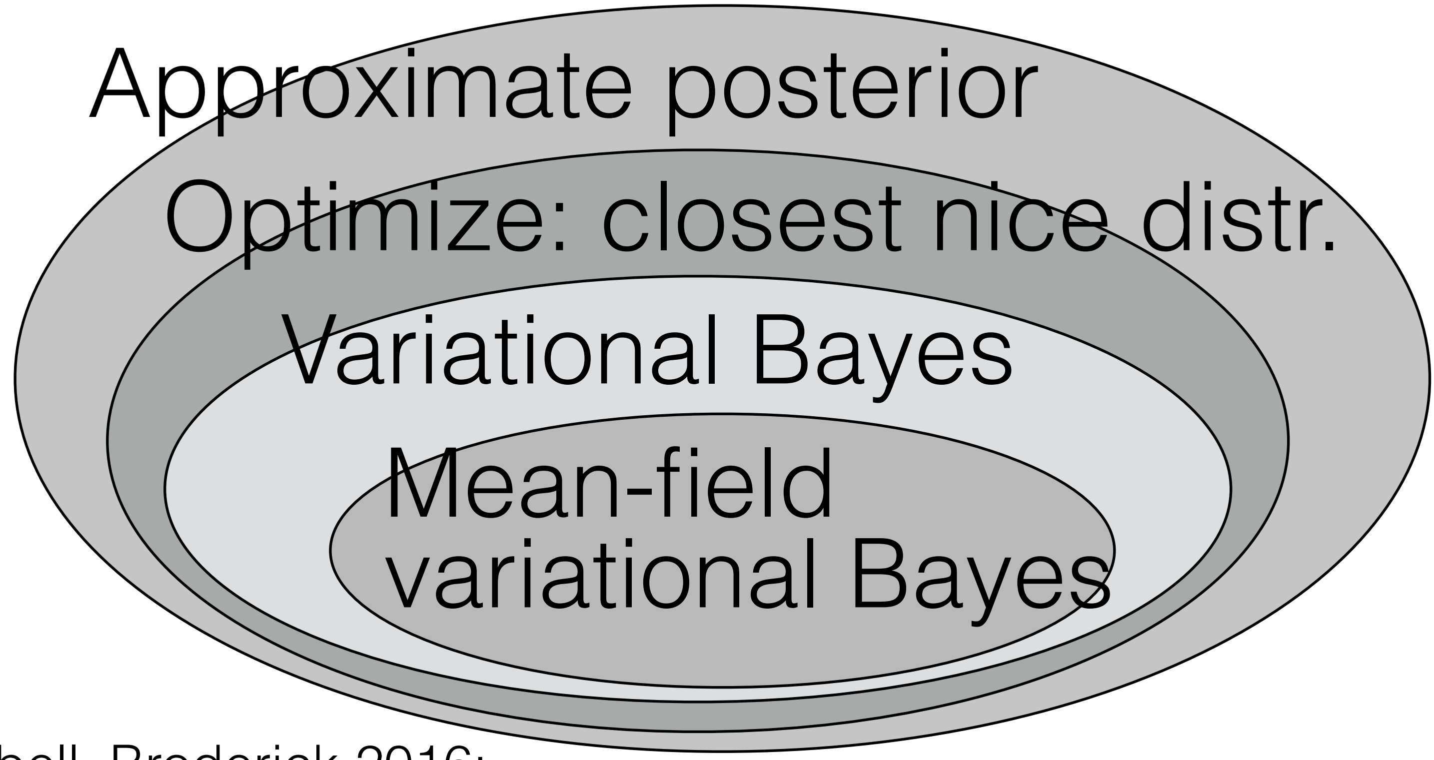

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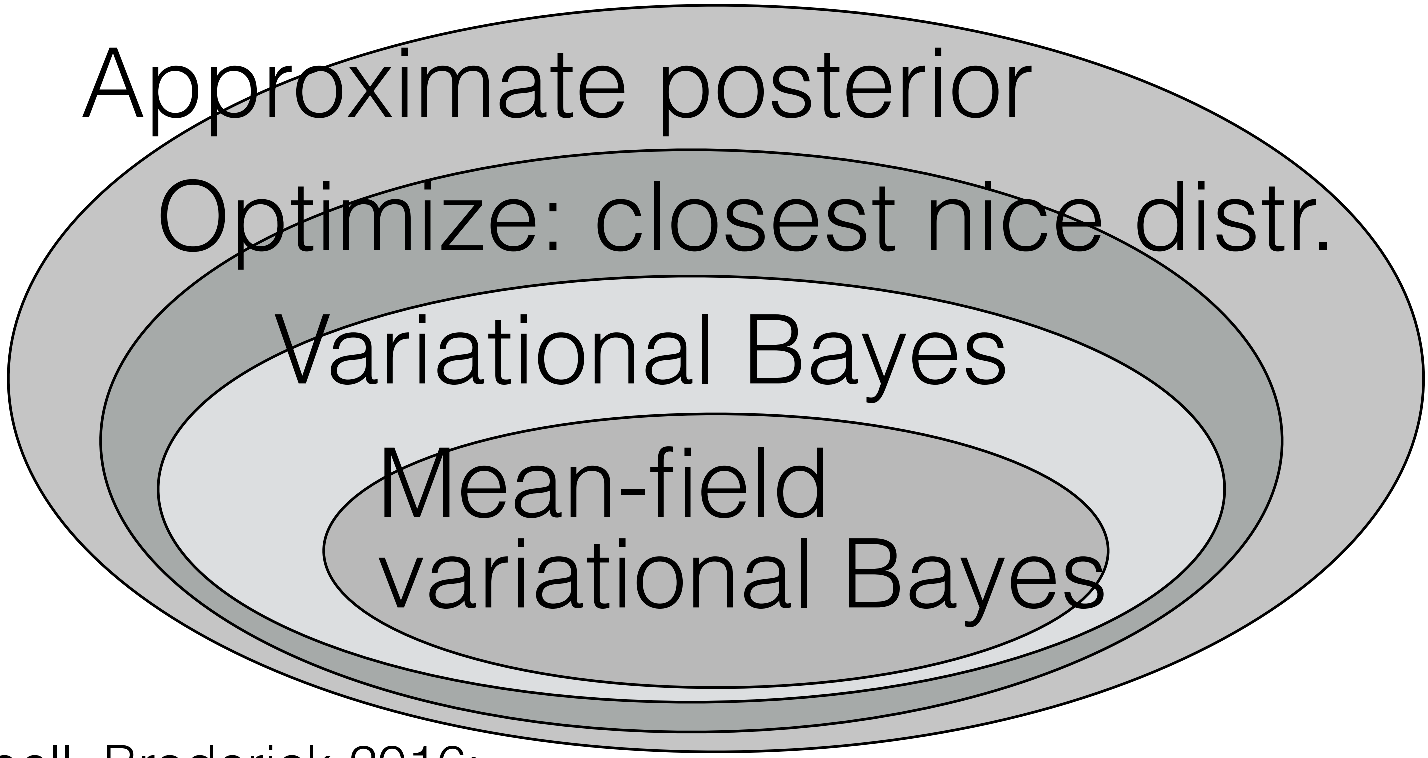

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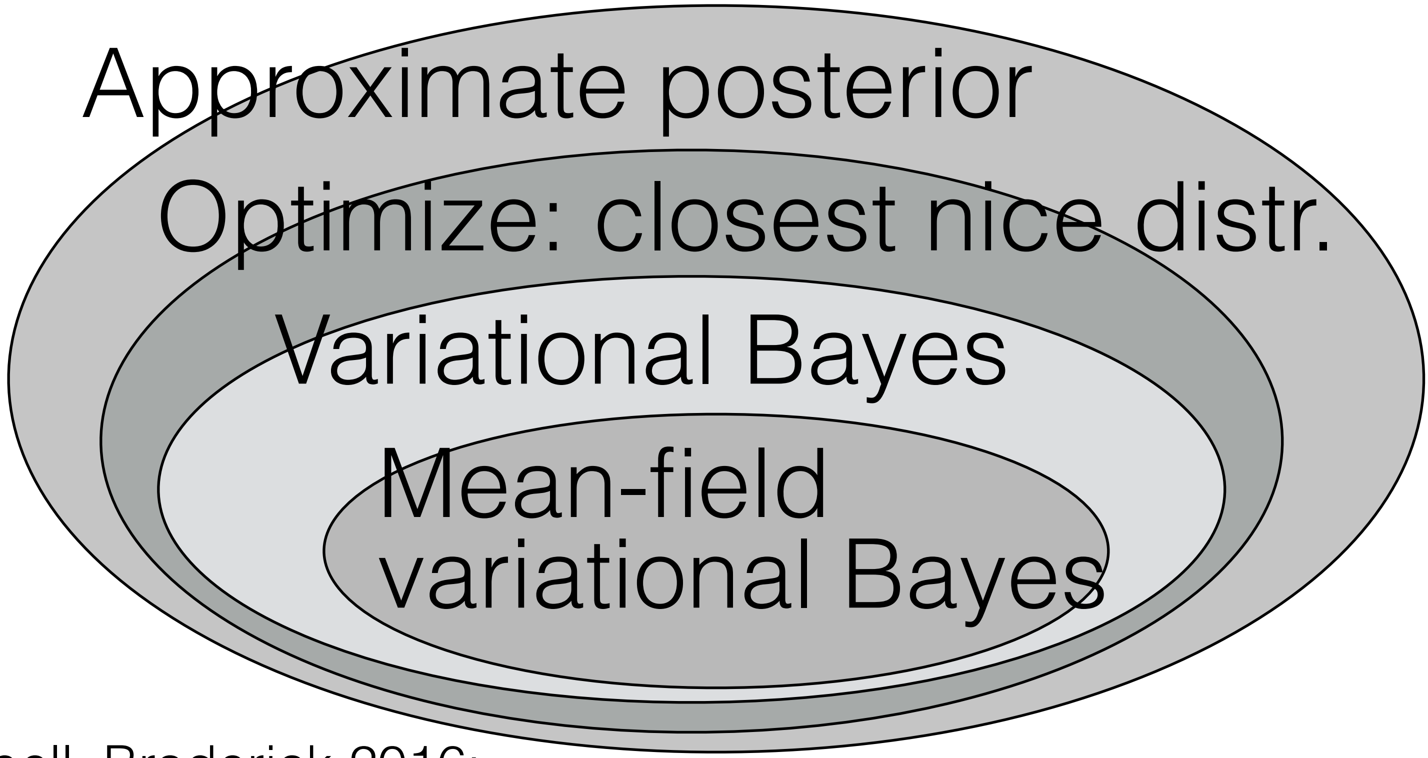

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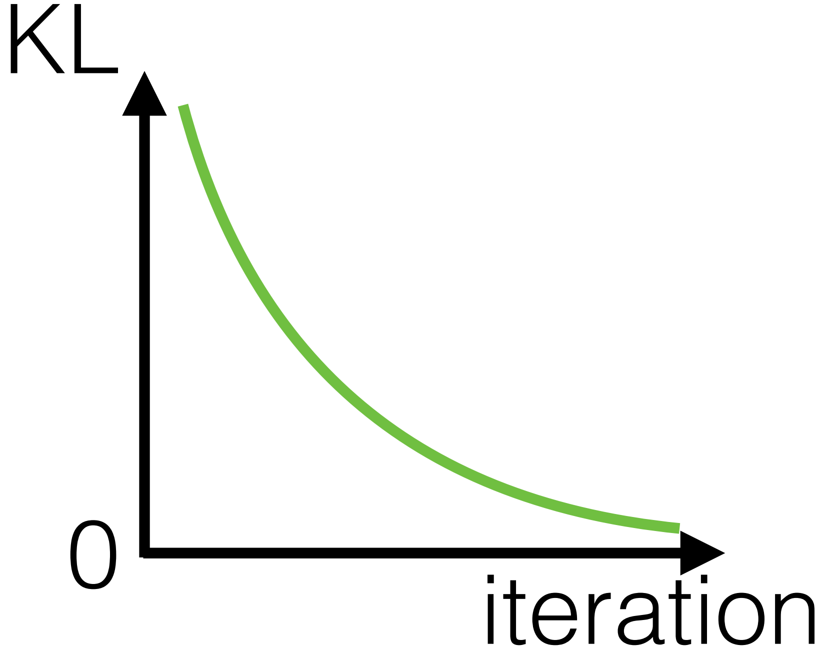

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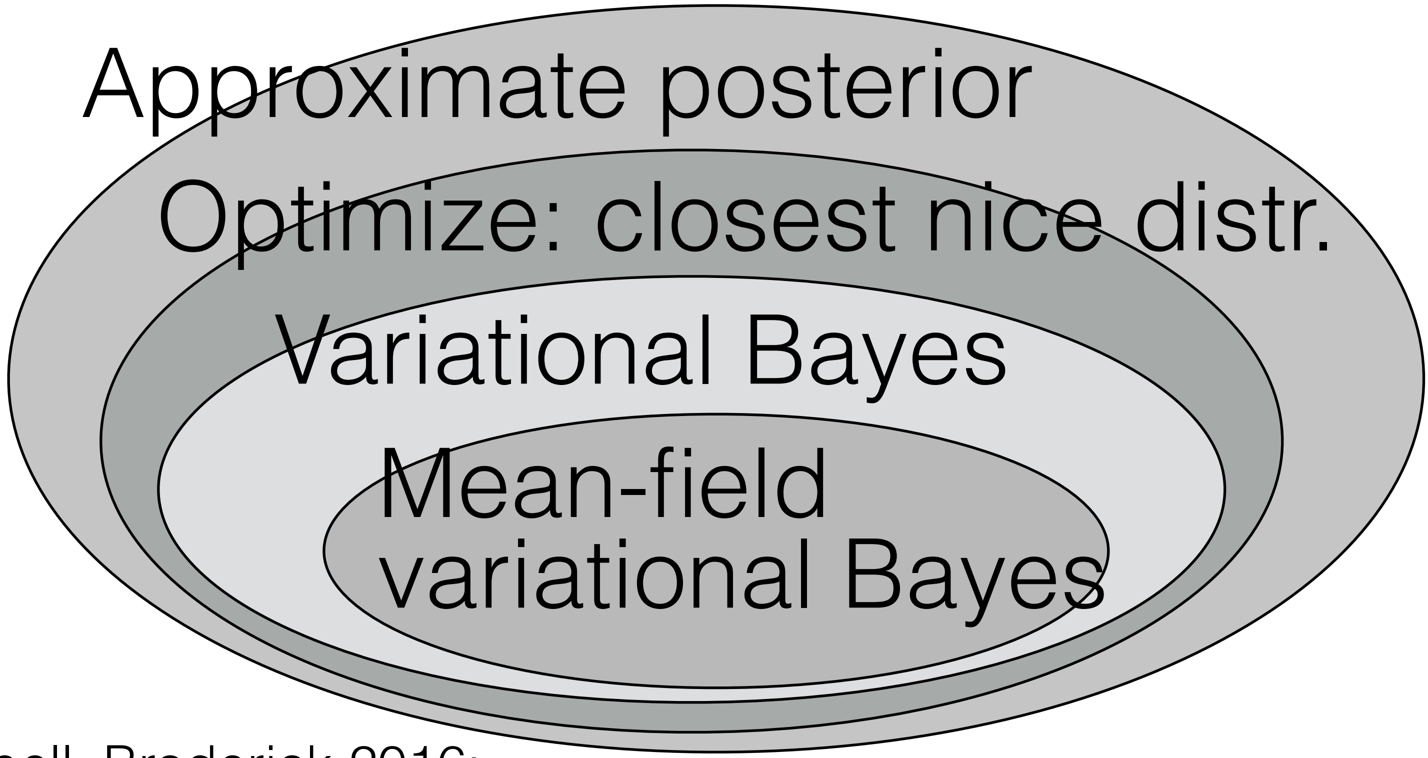

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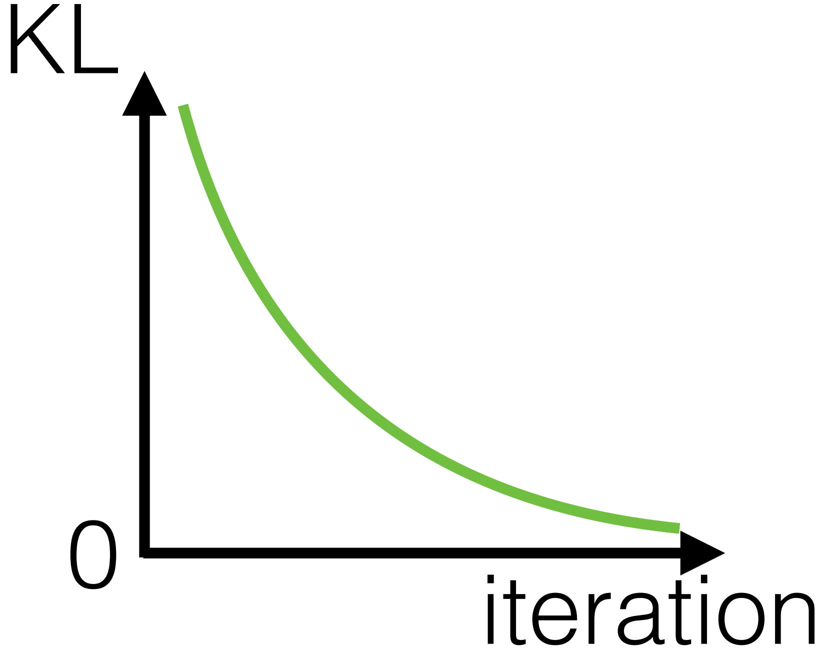

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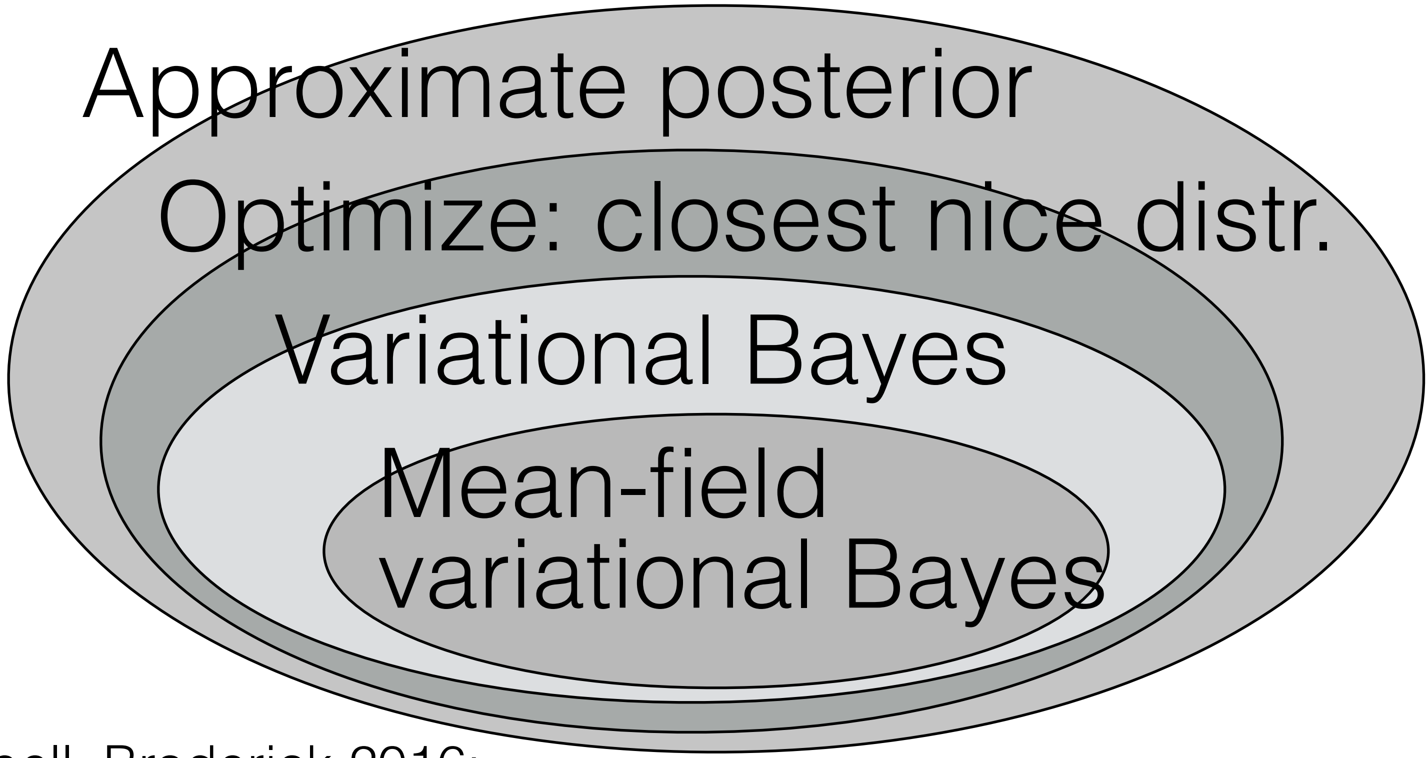

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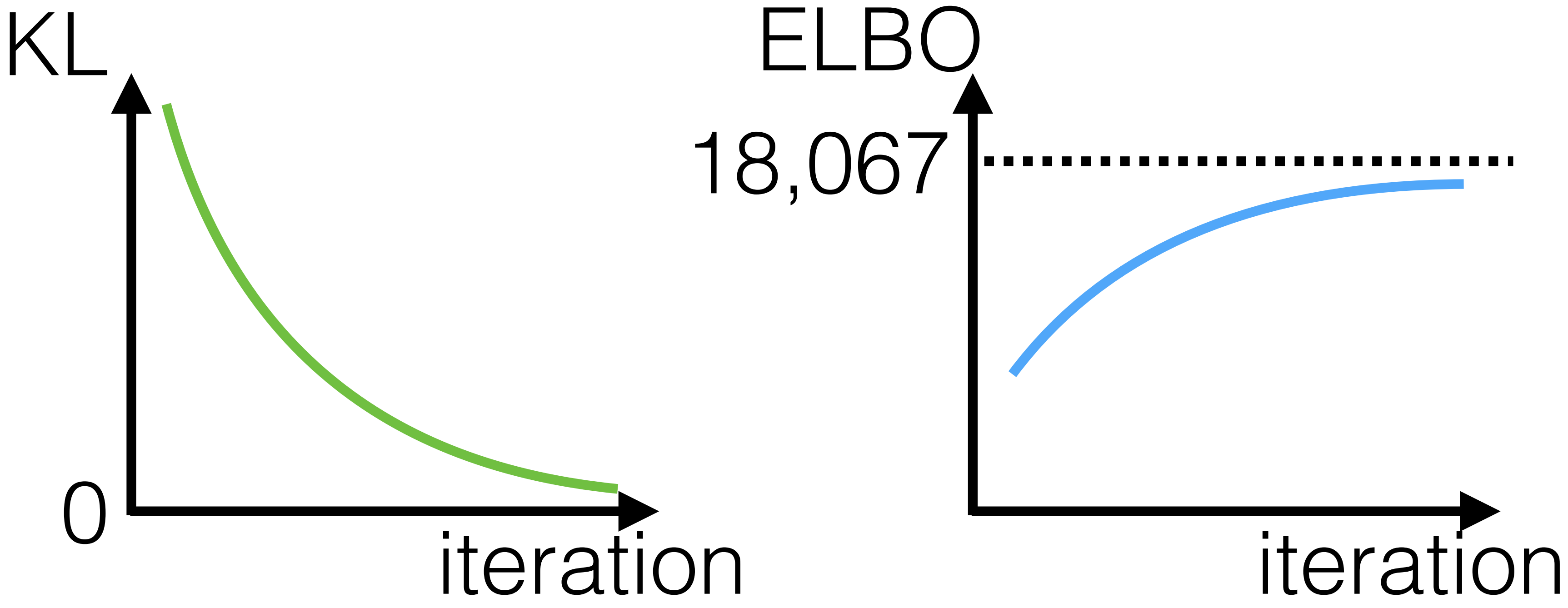

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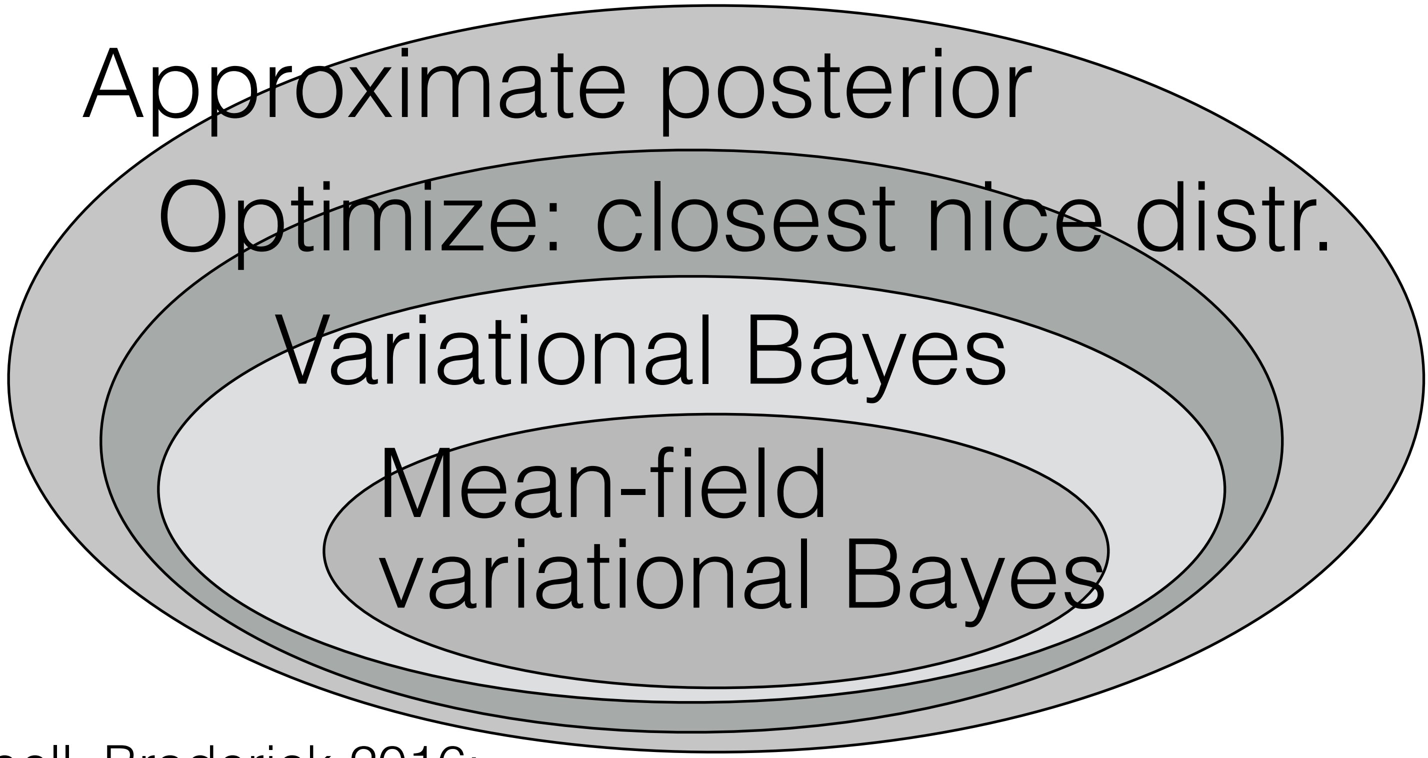

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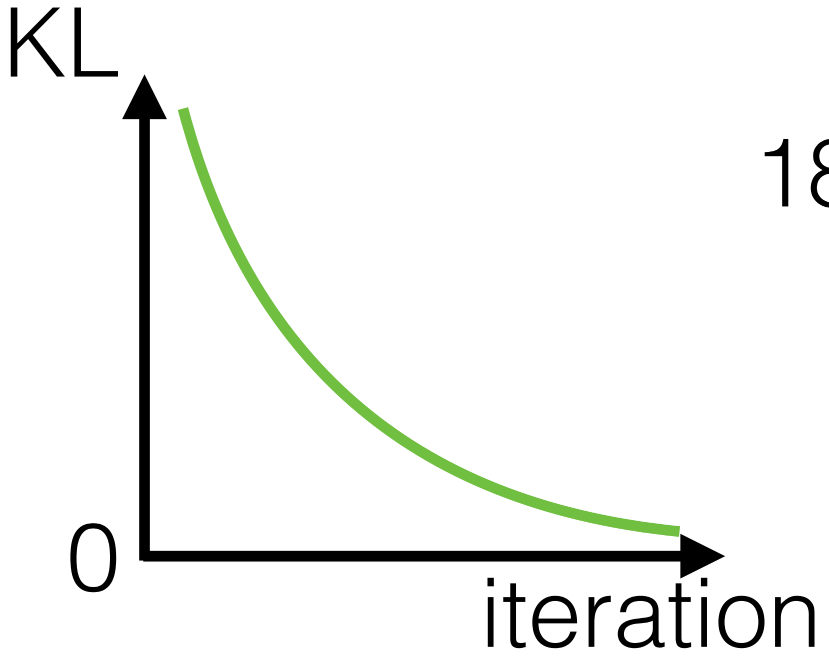

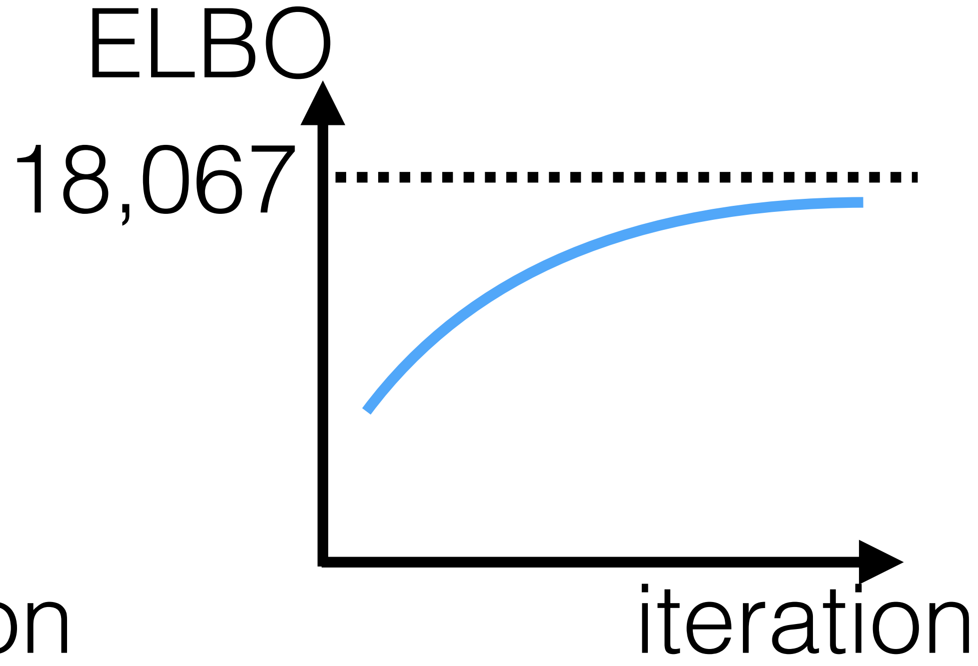

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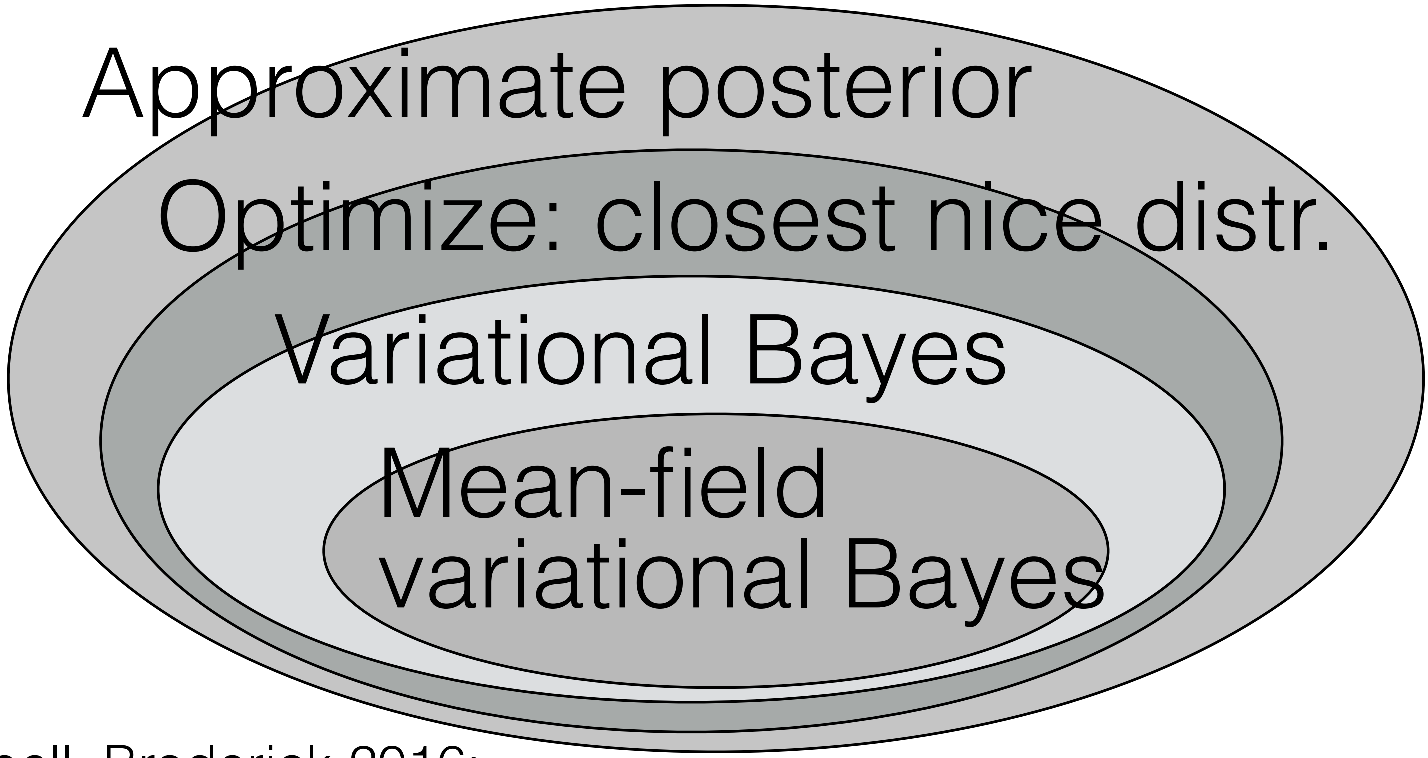
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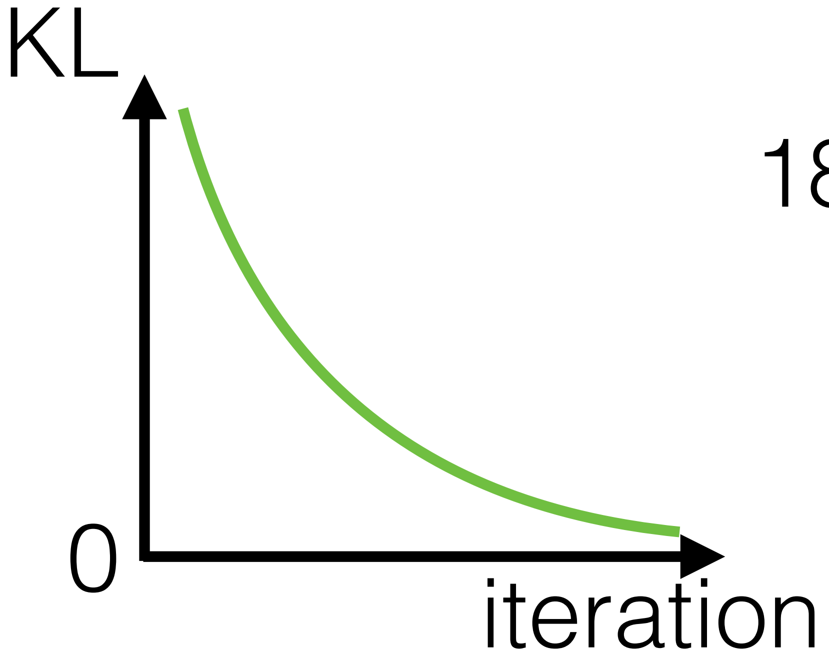

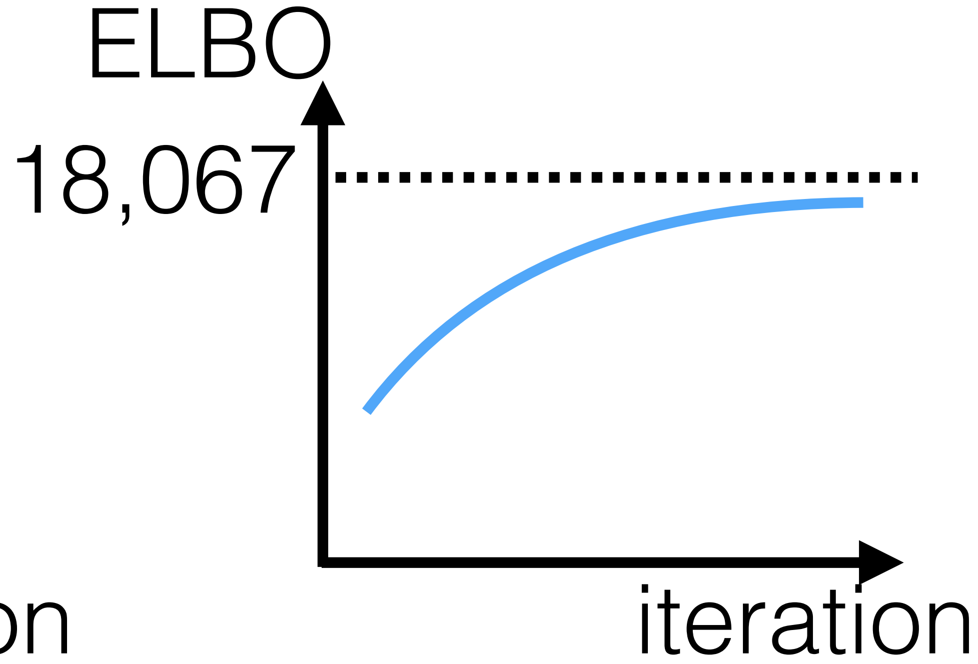

  - Diagnostics & workflow with theoretical guarantees

    “Validated Variational Inference via Practical Posterior Error Bounds” [Huggins, Kasprzak, Campbell, Broderick, 2020]

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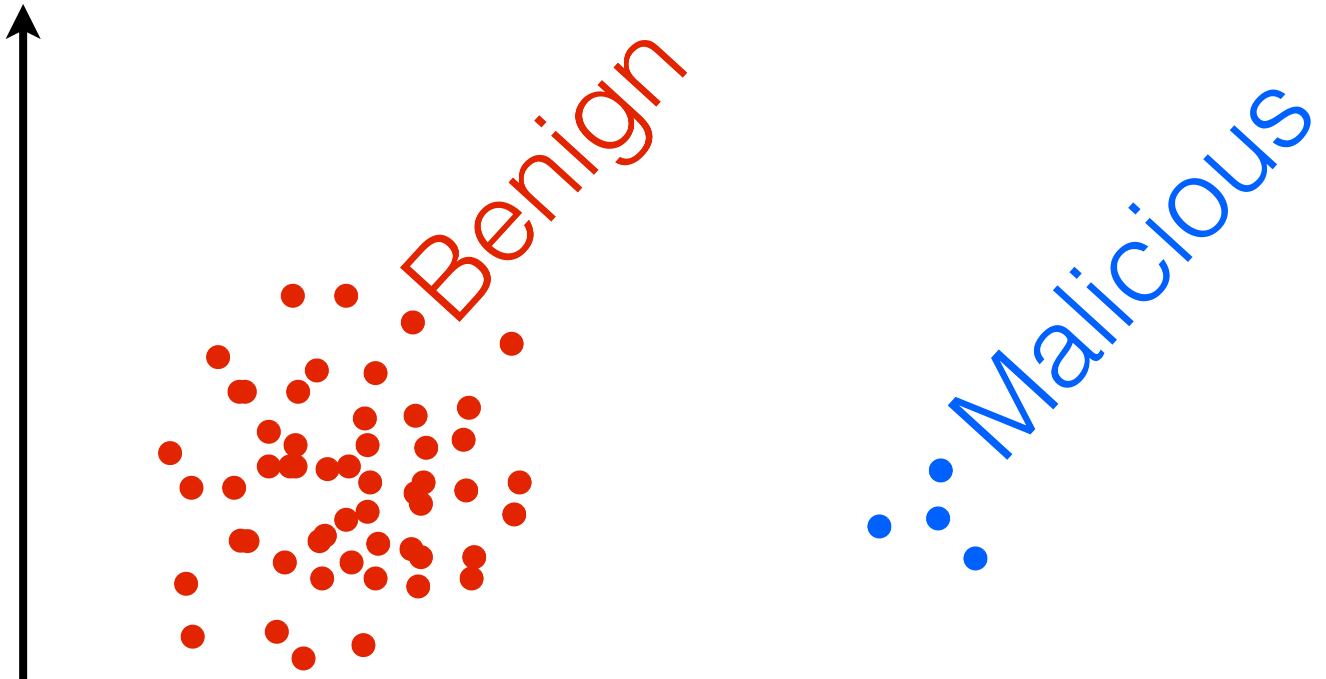

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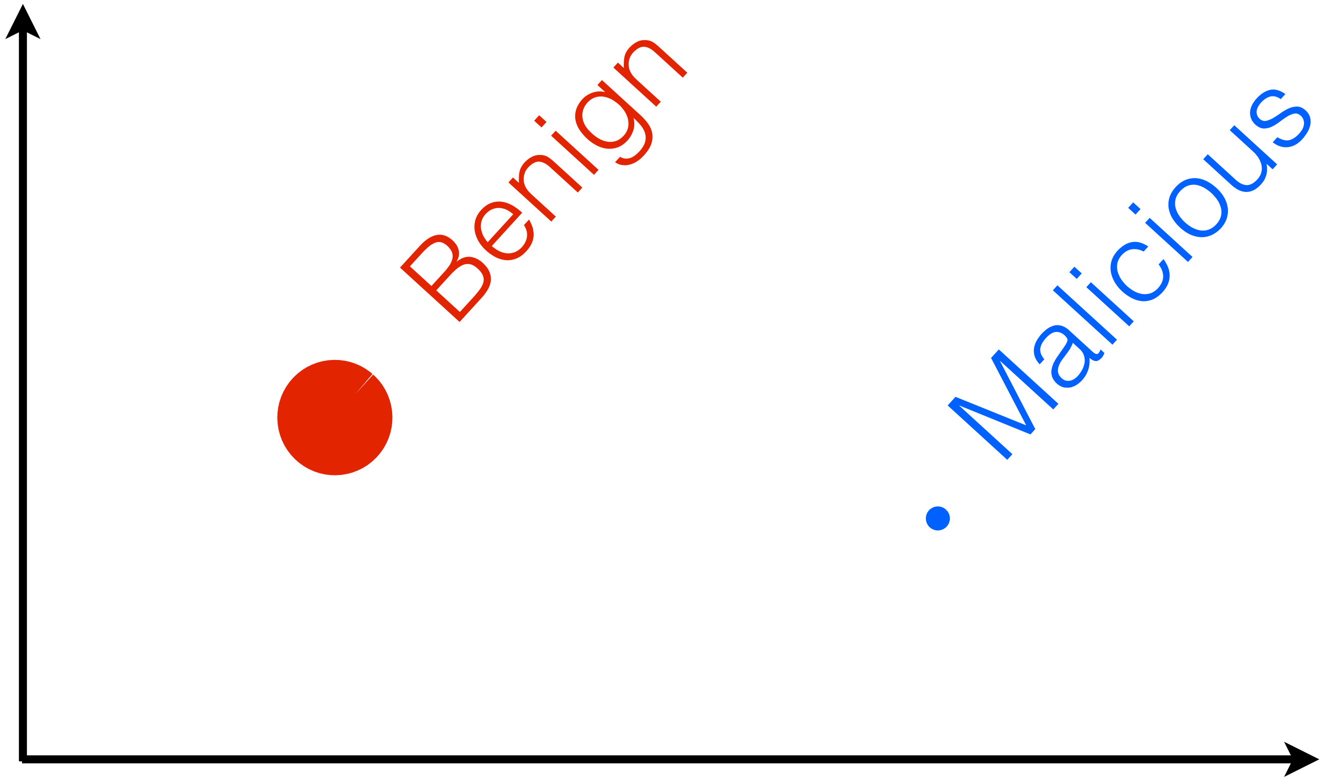

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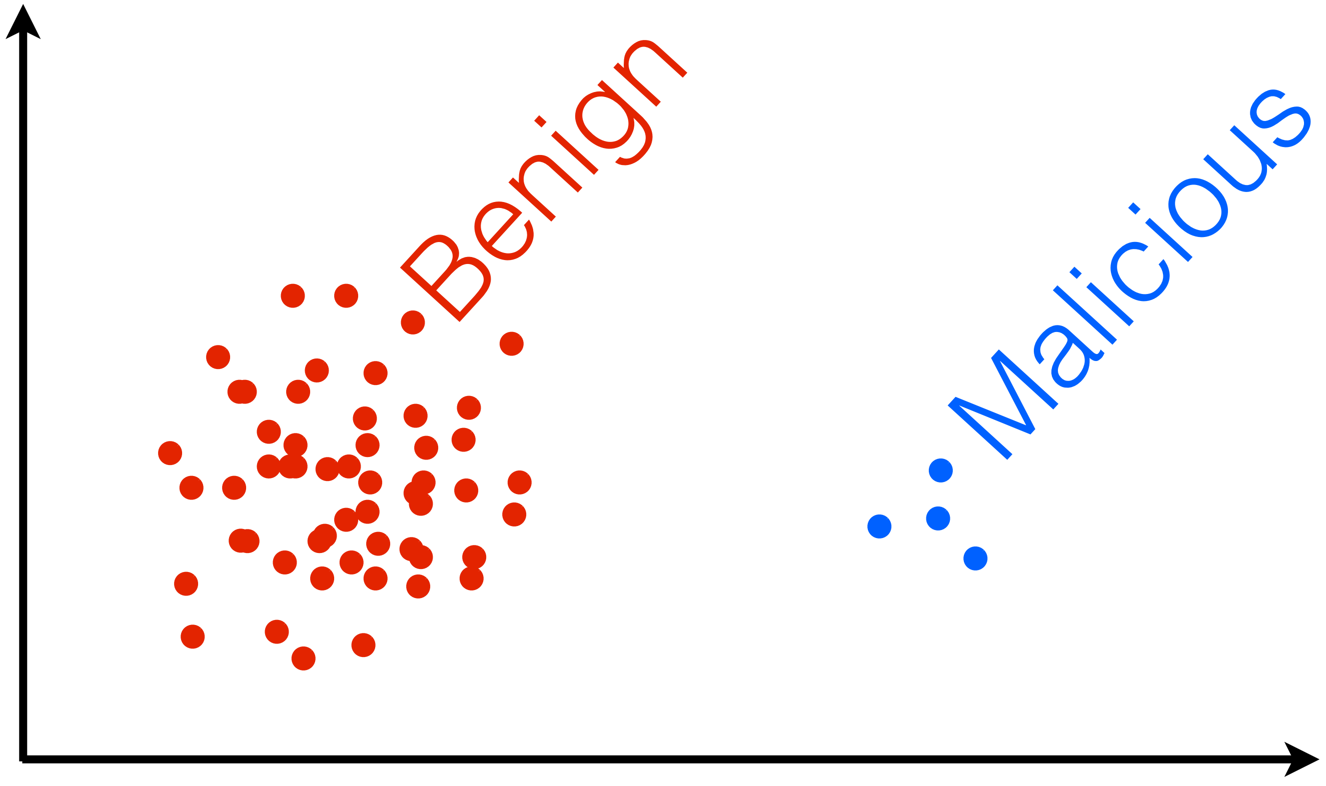

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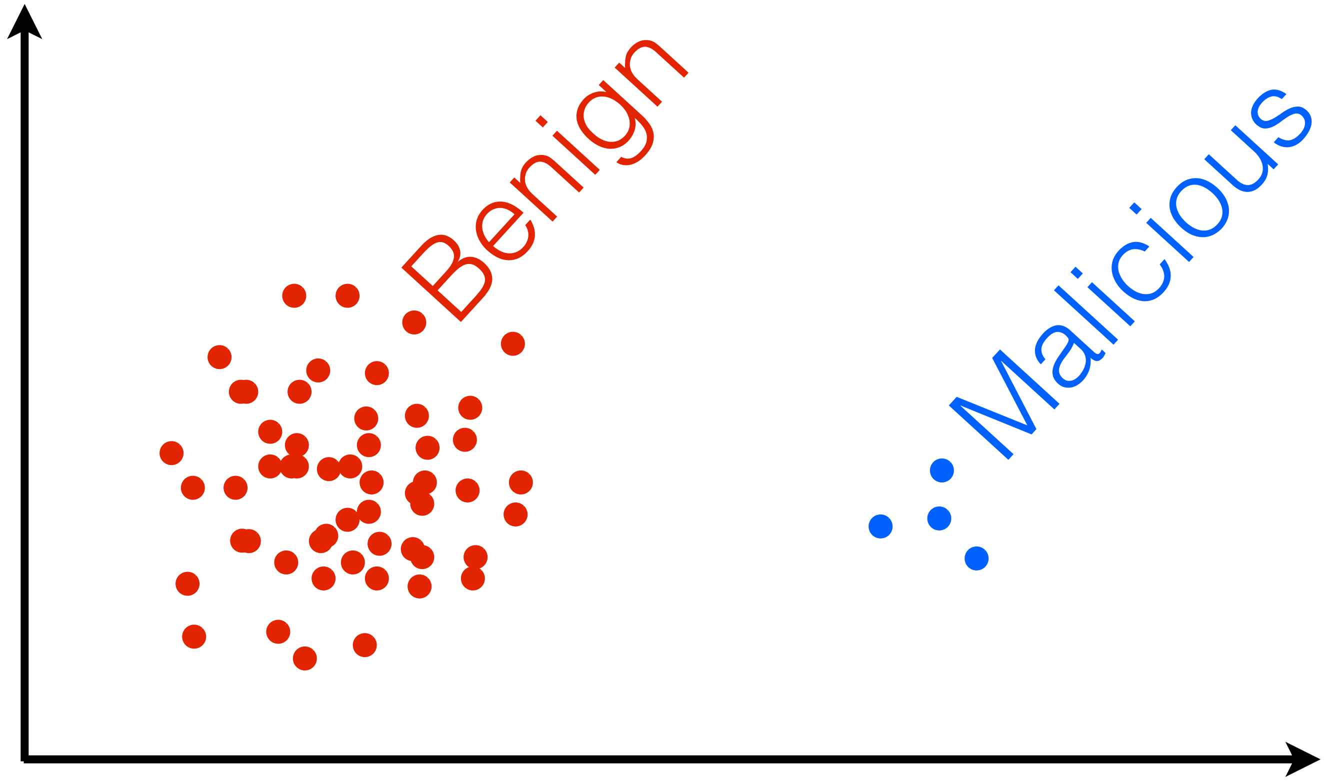

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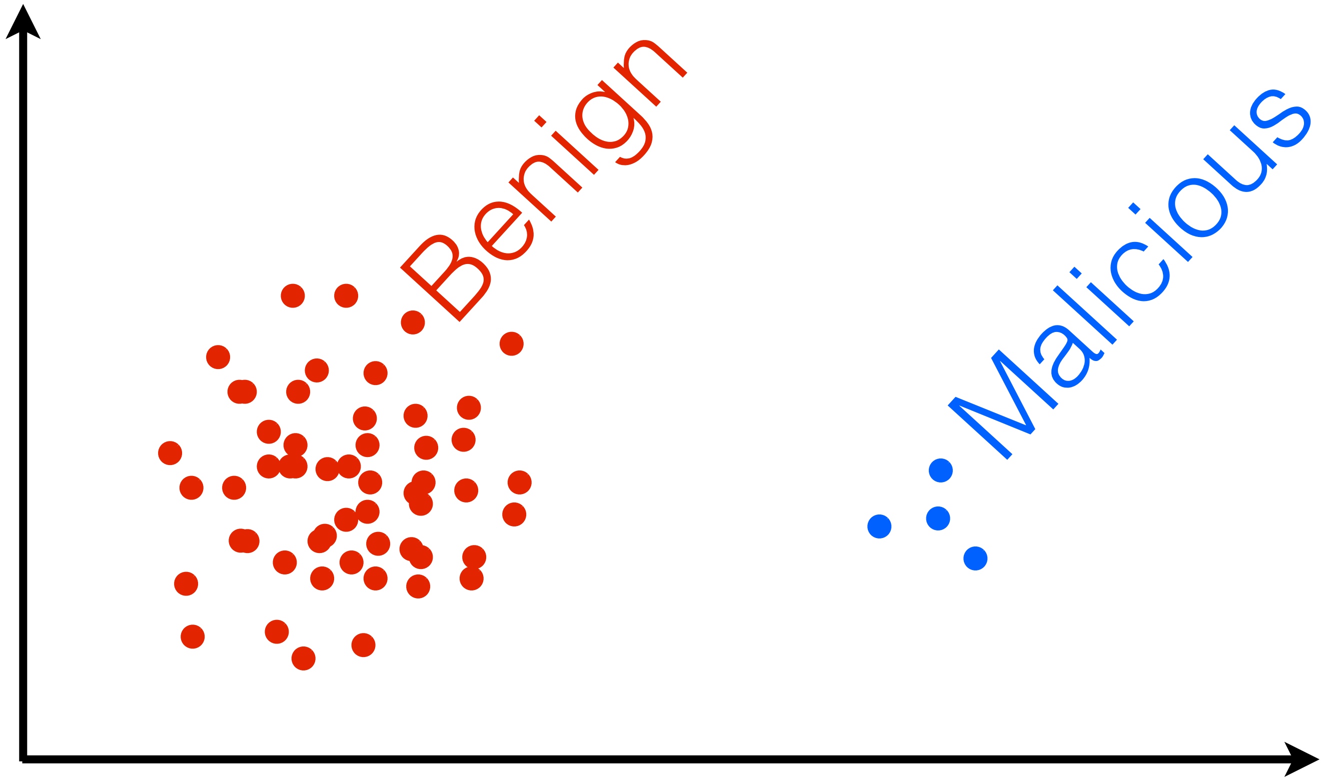

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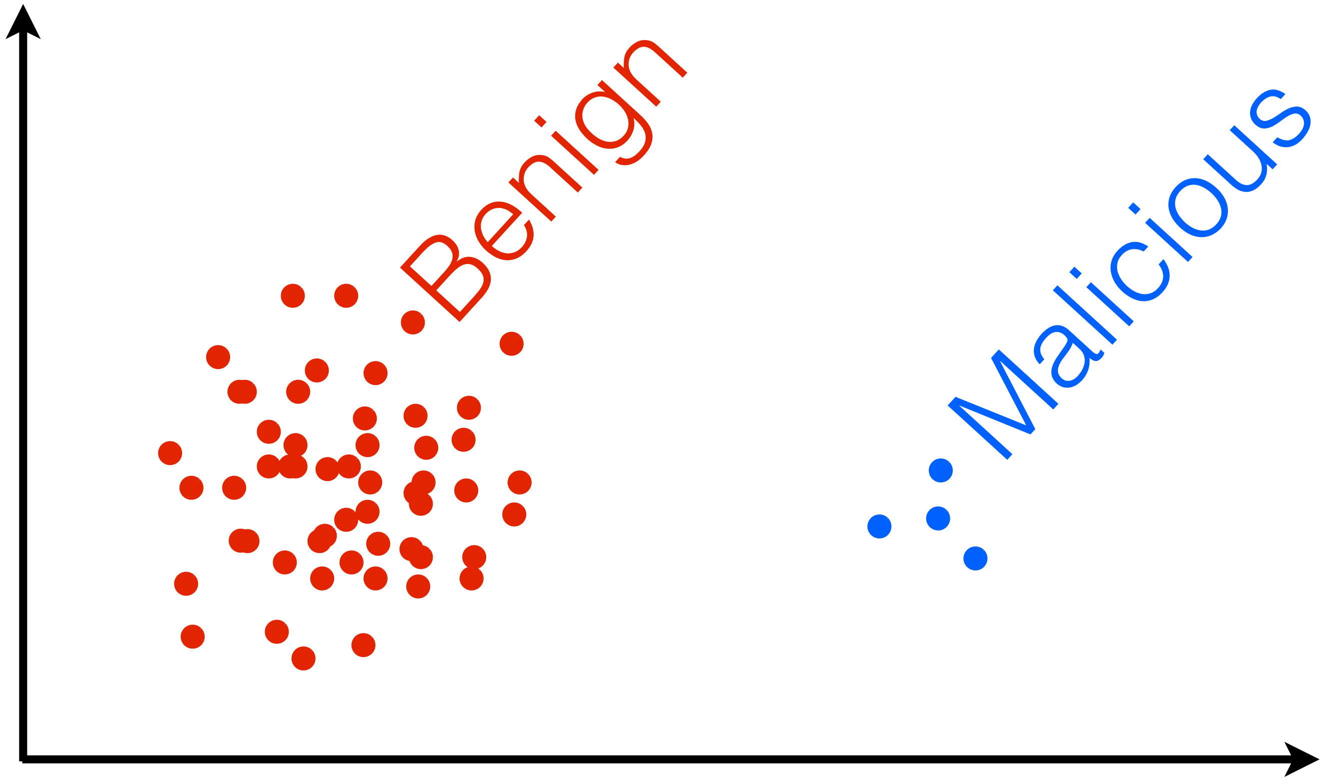

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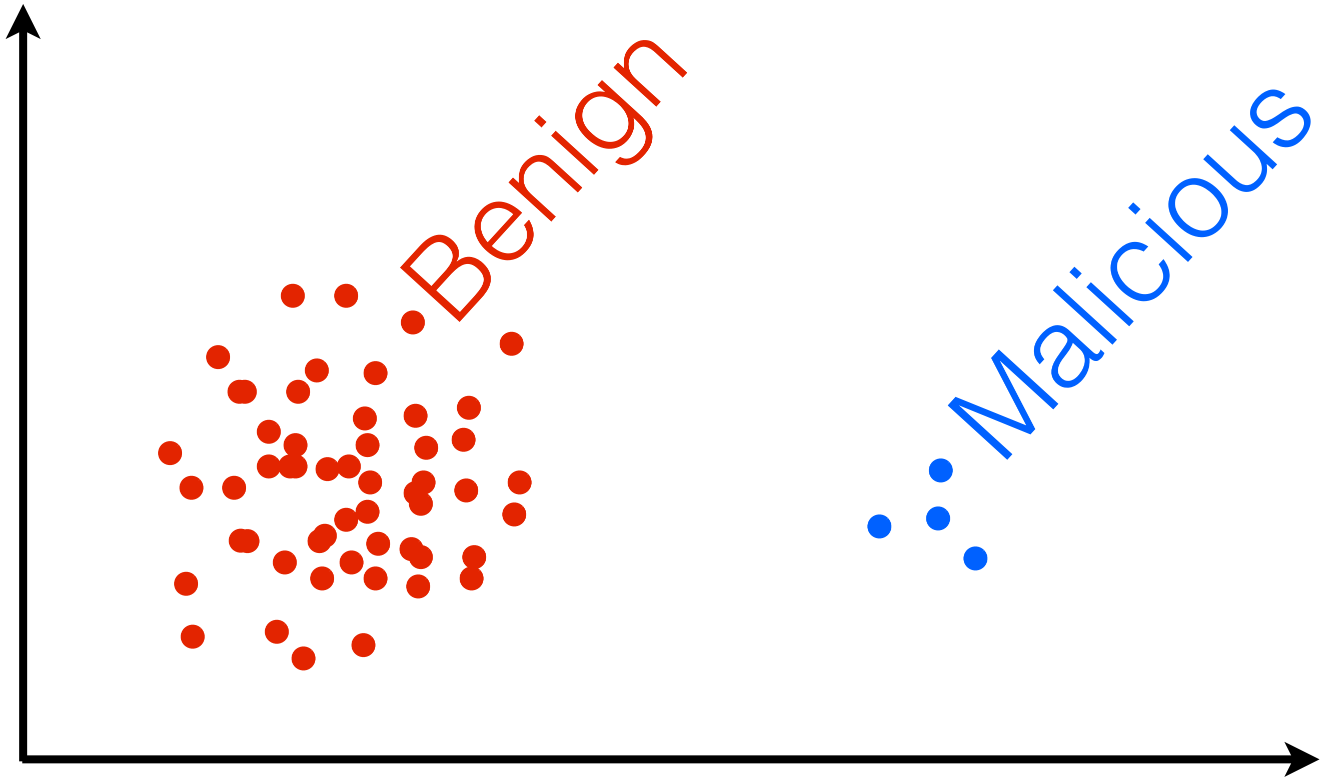

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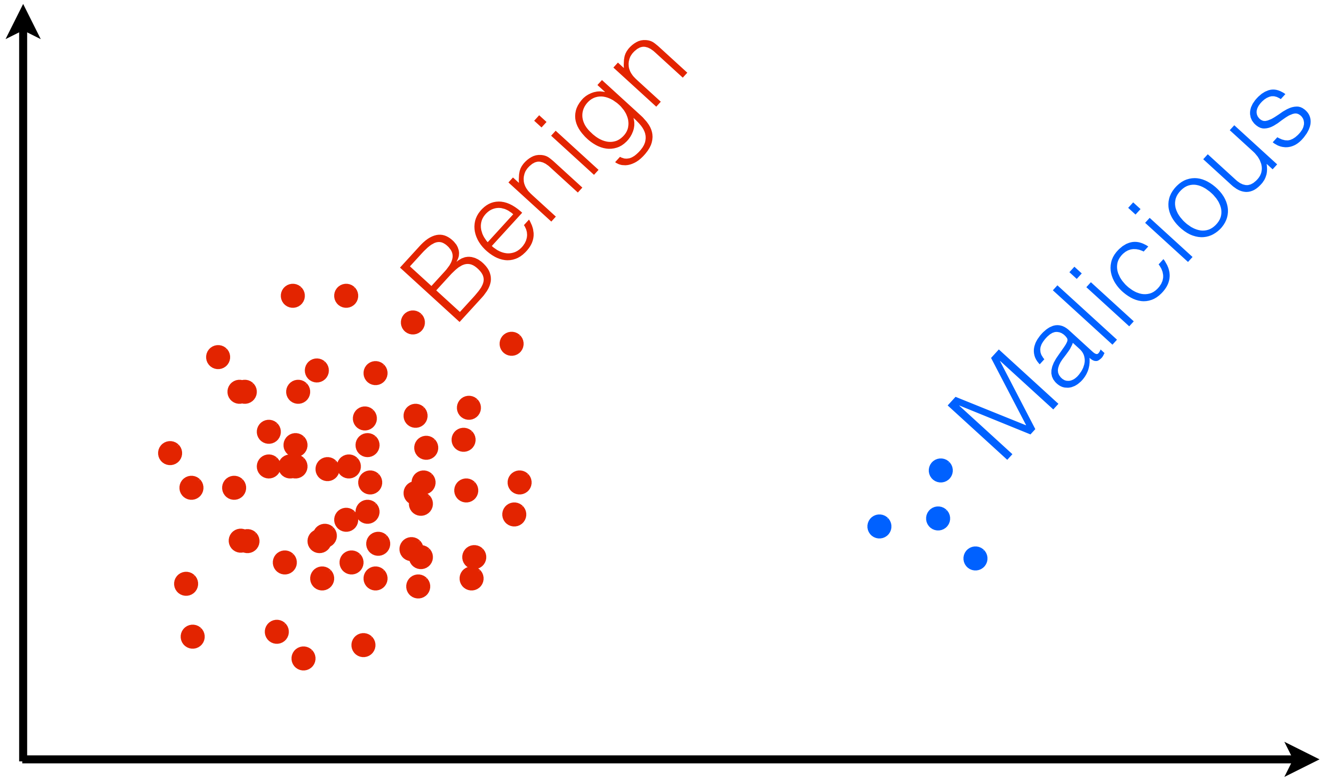

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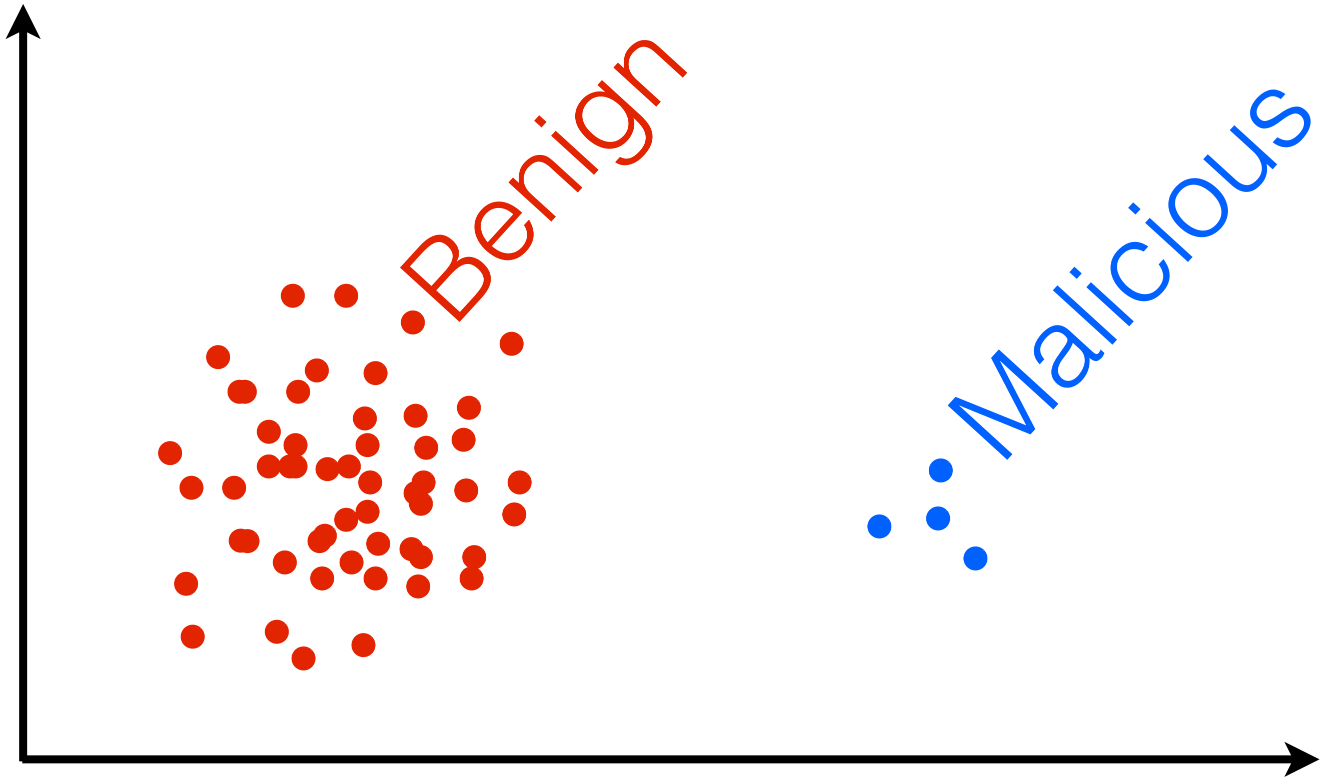

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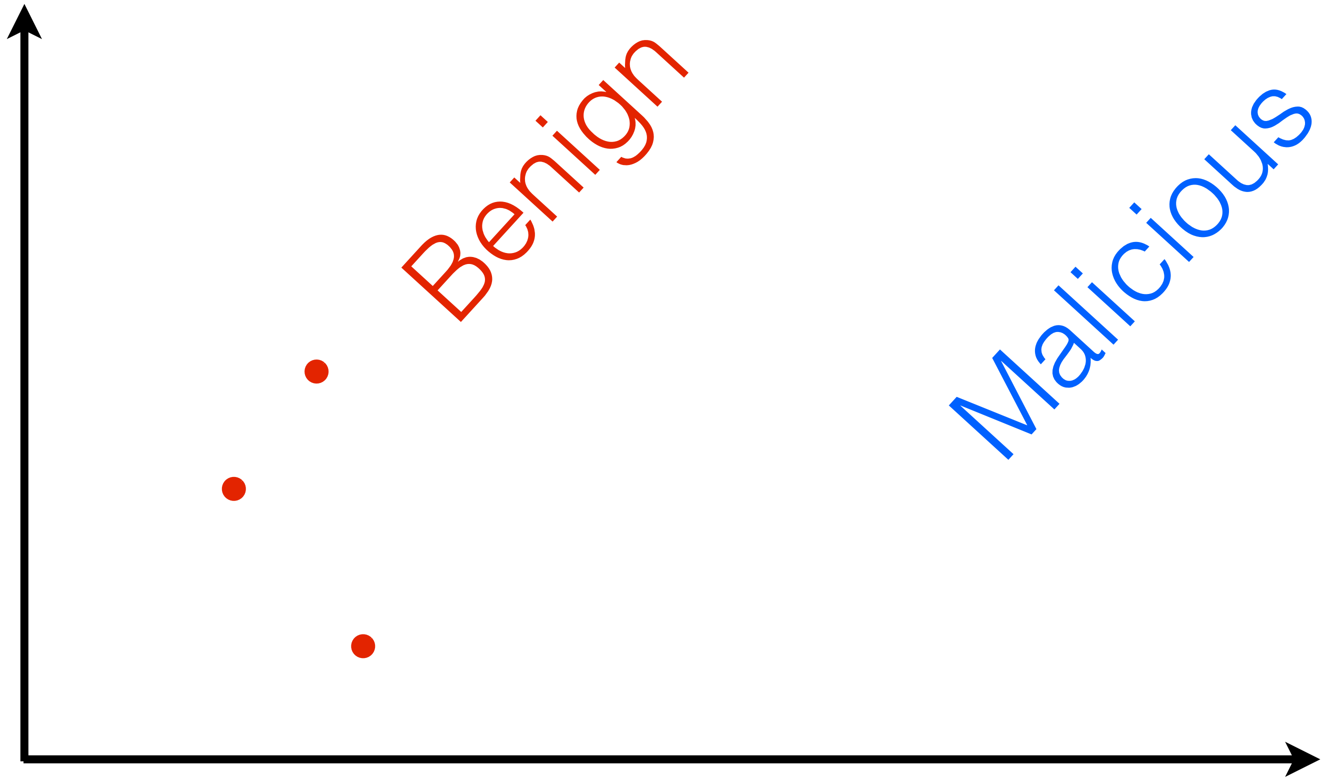

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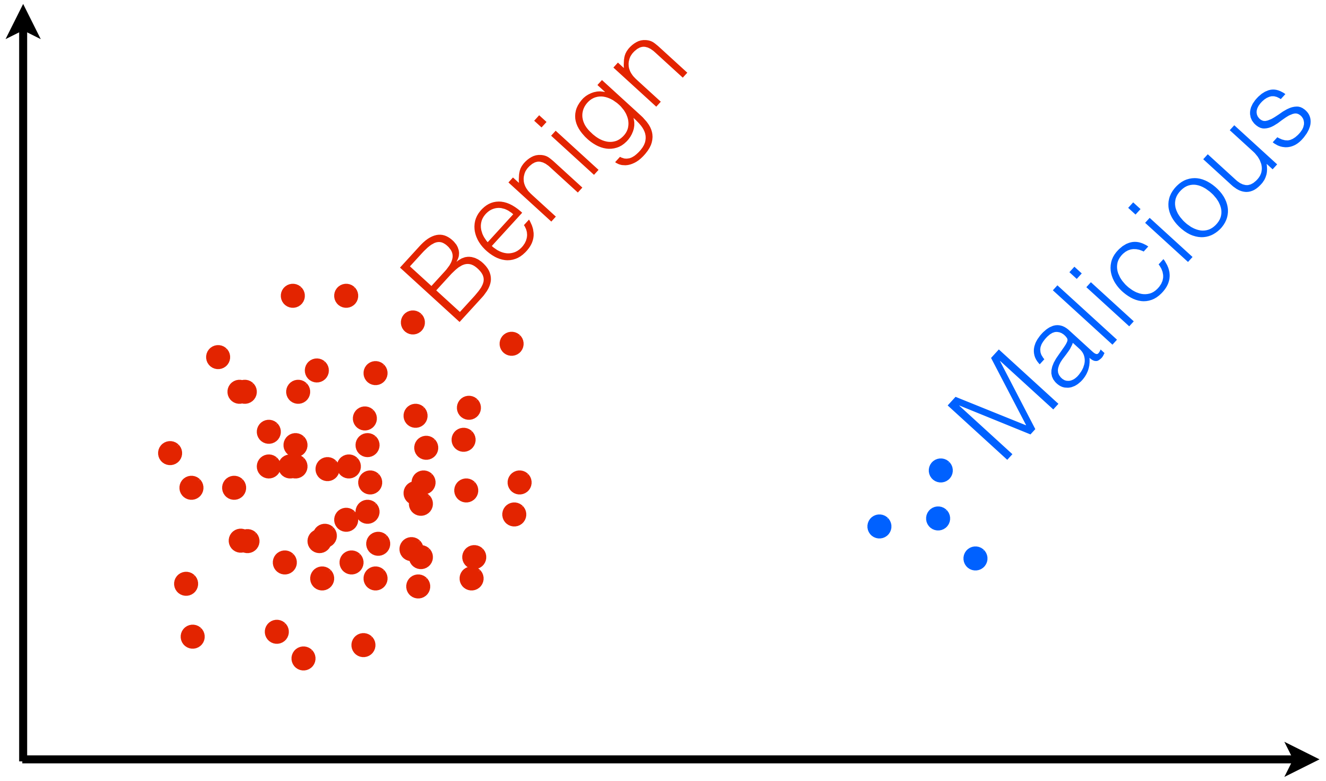
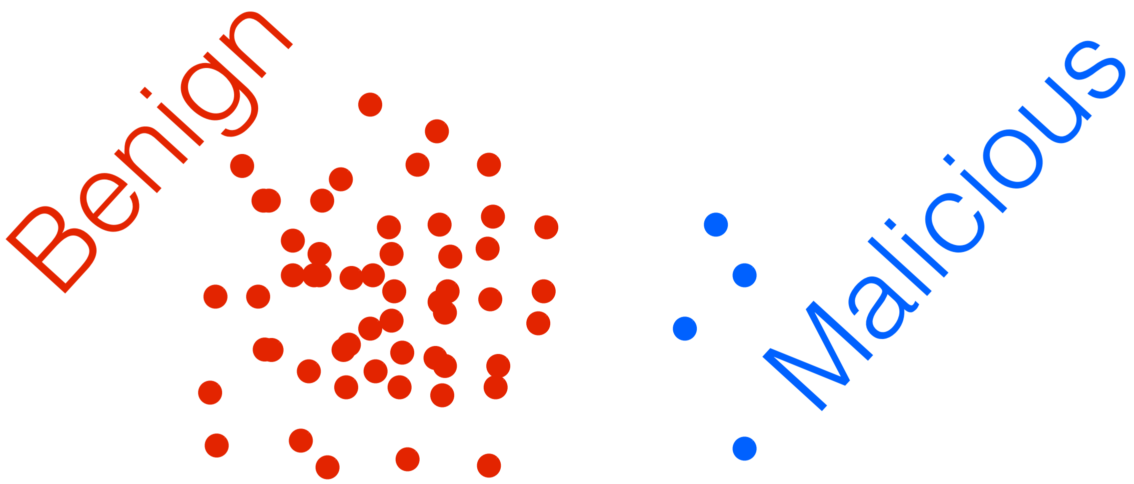
- Observe: redundancies can exist even if data isn’t “tall”
- Coresets: pre-process data to get a smaller, weighted data set



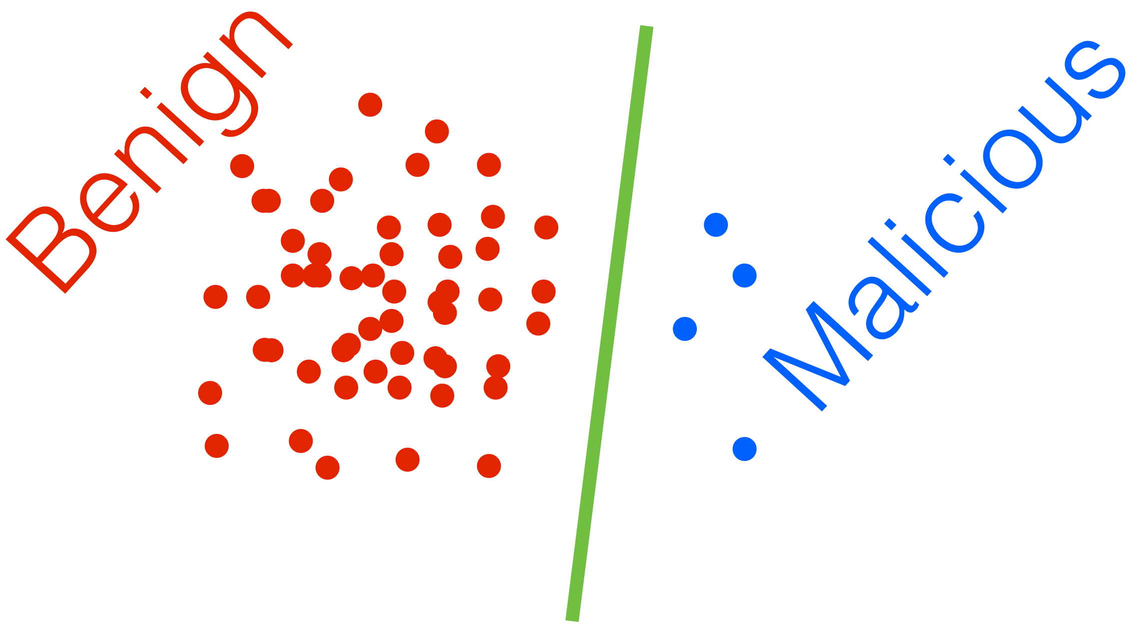
- Theoretical guarantees on quality
- How to develop **coresets for diverse tasks/geometries**?
- Previous heuristics: data squashing, big data GPs
- Compare to subsampling

[Bădoiu, Har-Peled, Indyk 2002; Agarwal et al 2005; Feldman & Langberg 2011; DuMouchel et al 1999; Madigan et al 2002; Huggins, Campbell, Broderick 2016; Campbell, Broderick 2019; Campbell, Broderick 2018; Agrawal, Campbell, Huggins, Broderick 2019]

Uniform subsampling



Uniform subsampling



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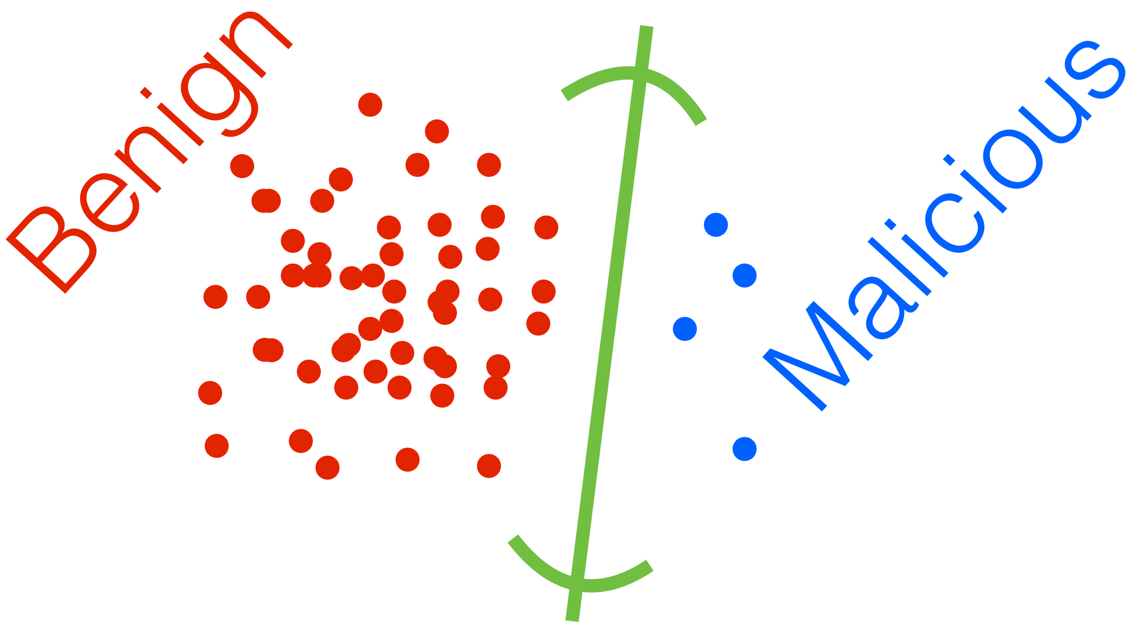
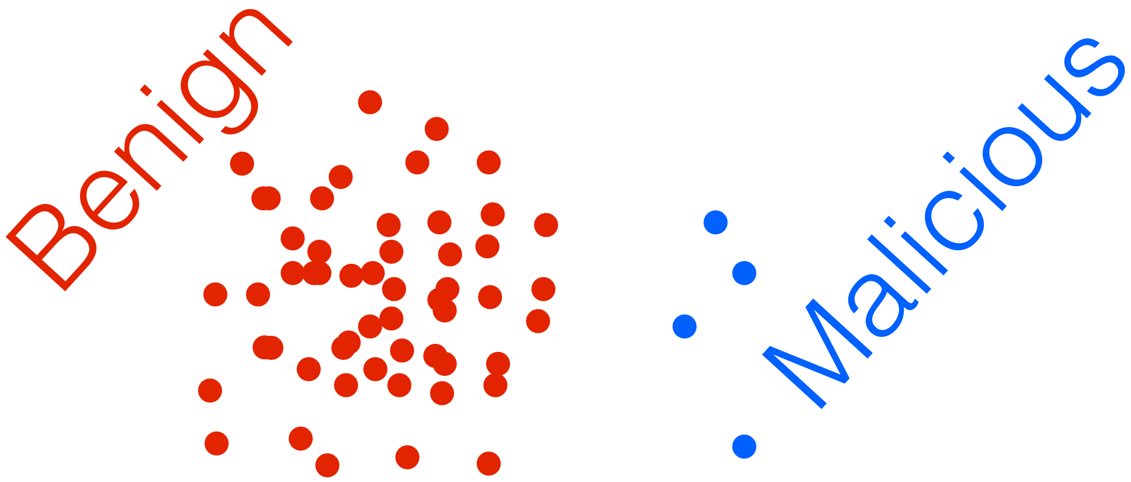


Uniform subsampling



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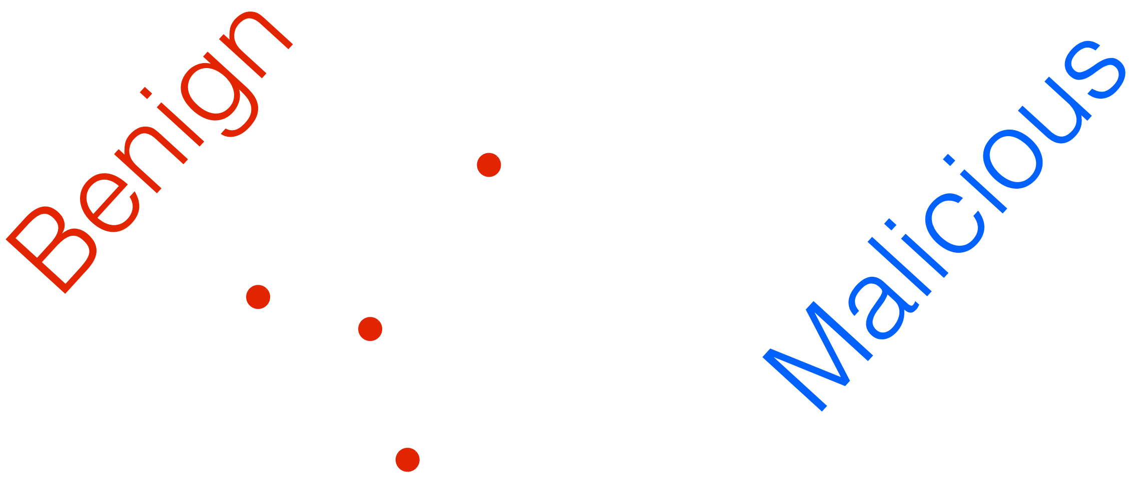

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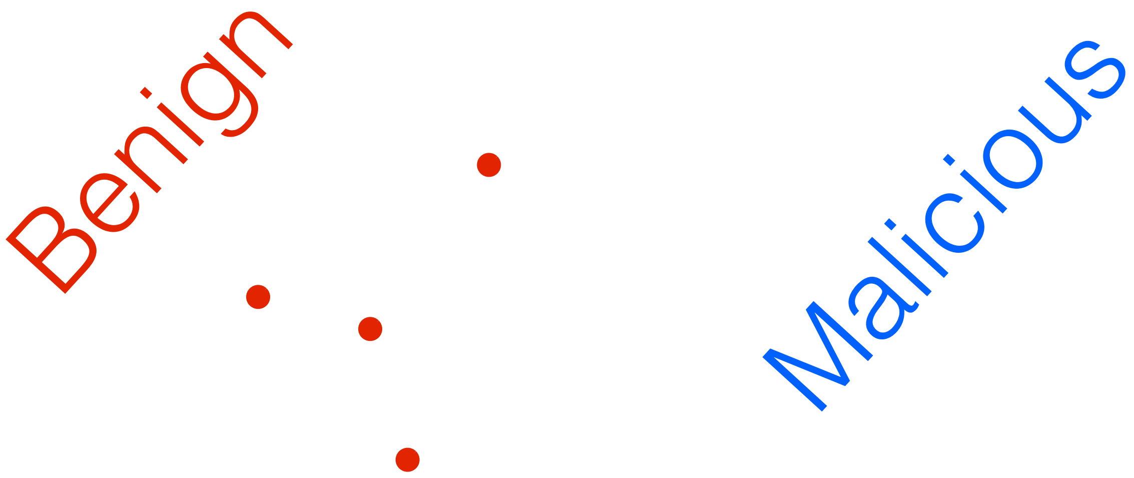
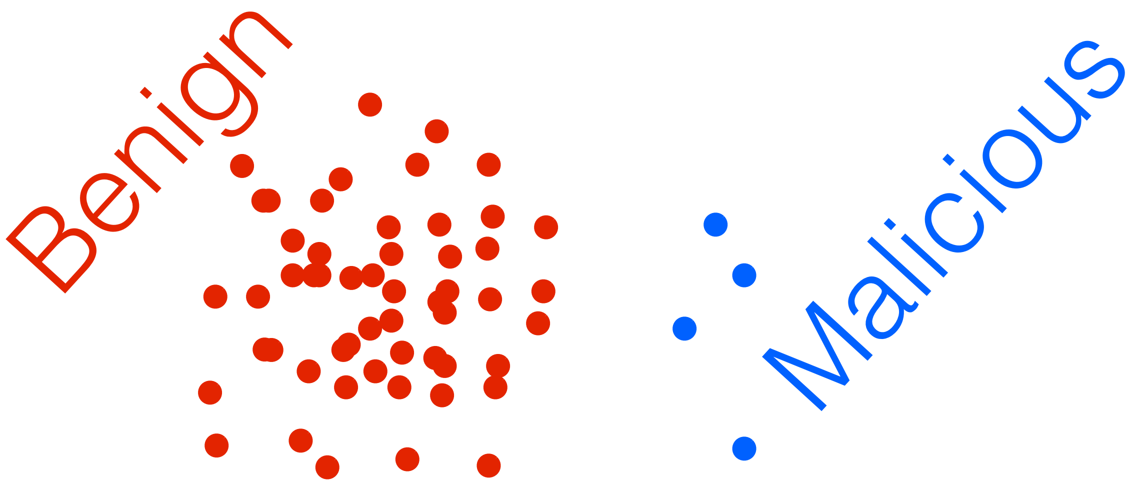


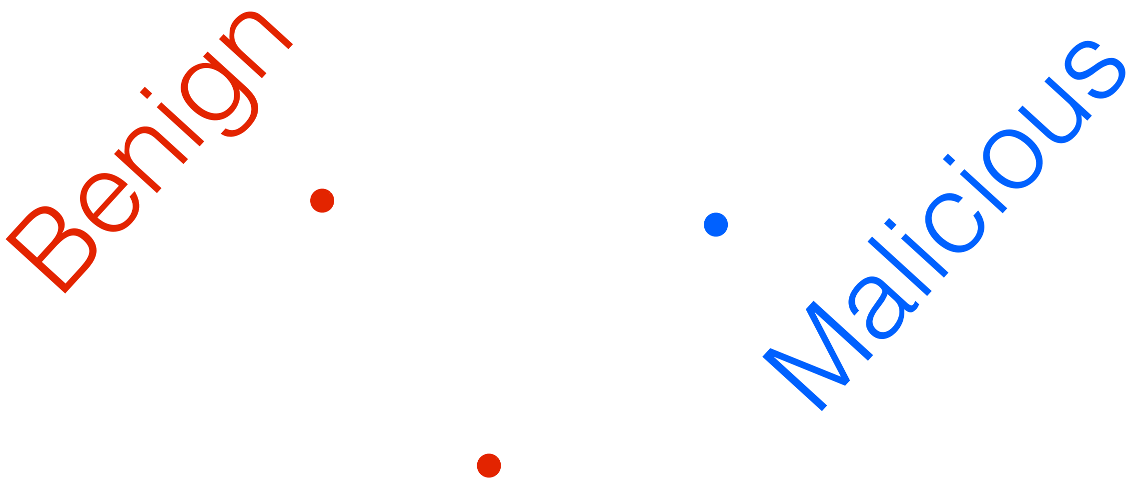
- Might miss important data

Uniform subsampling



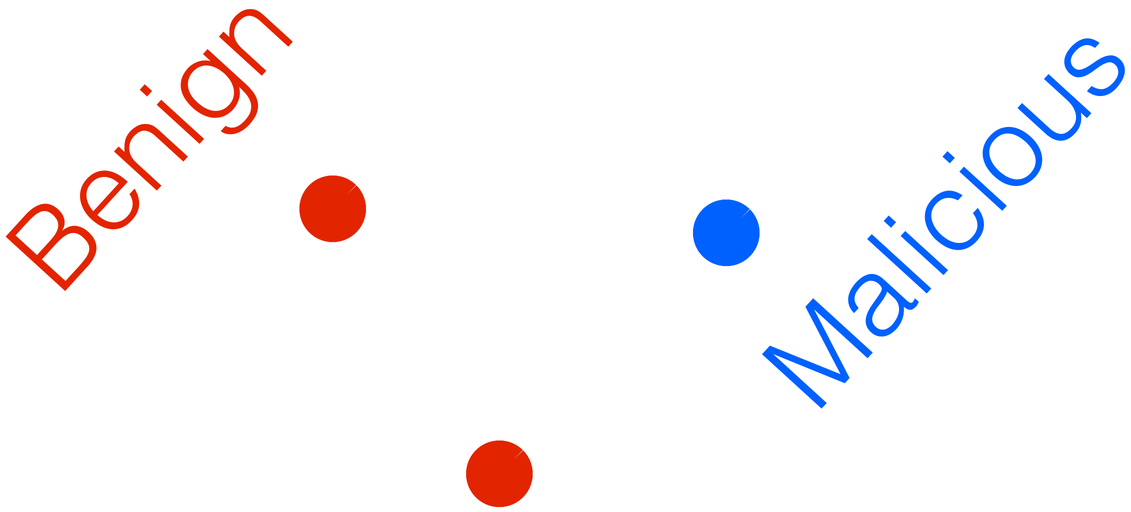
- Might miss important data

Uniform subsampling



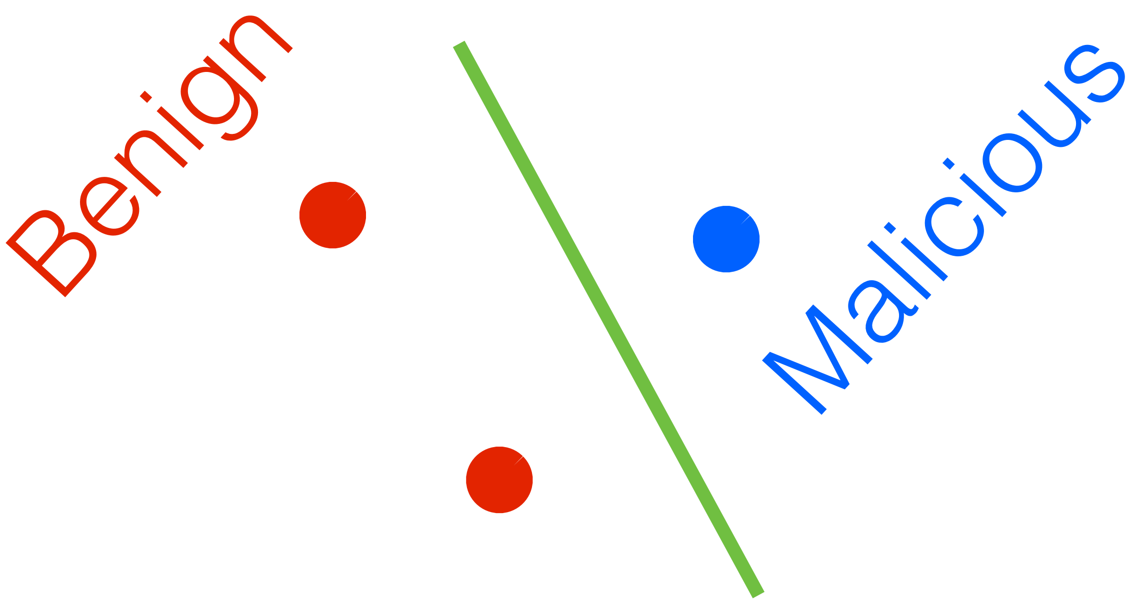
- Might miss important data

Uniform subsampling



- Might miss important data

Uniform subsampling



- Might miss important data

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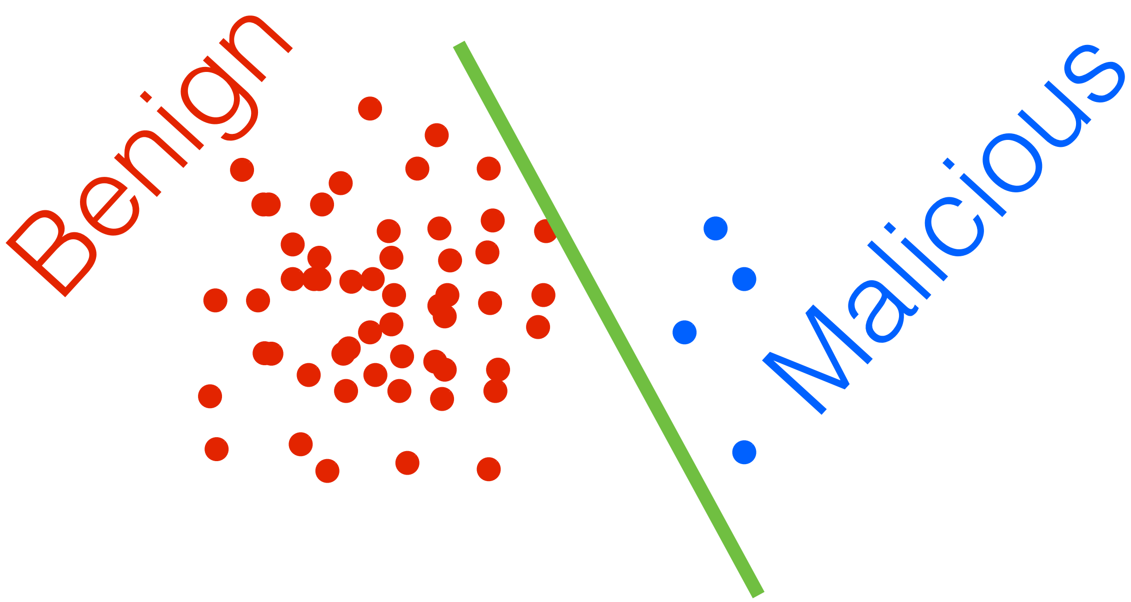
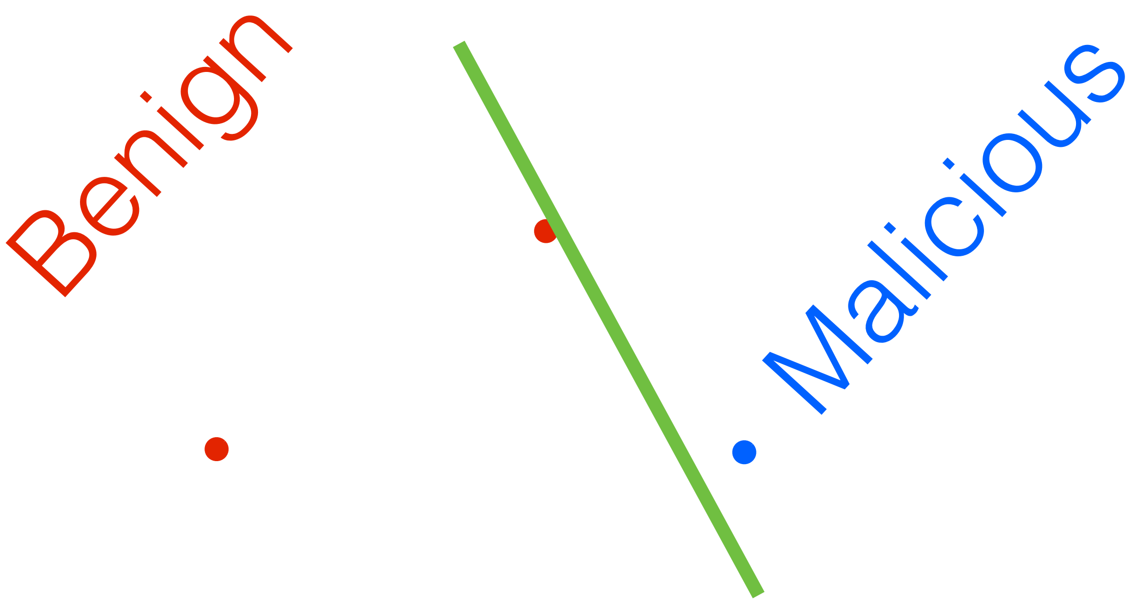


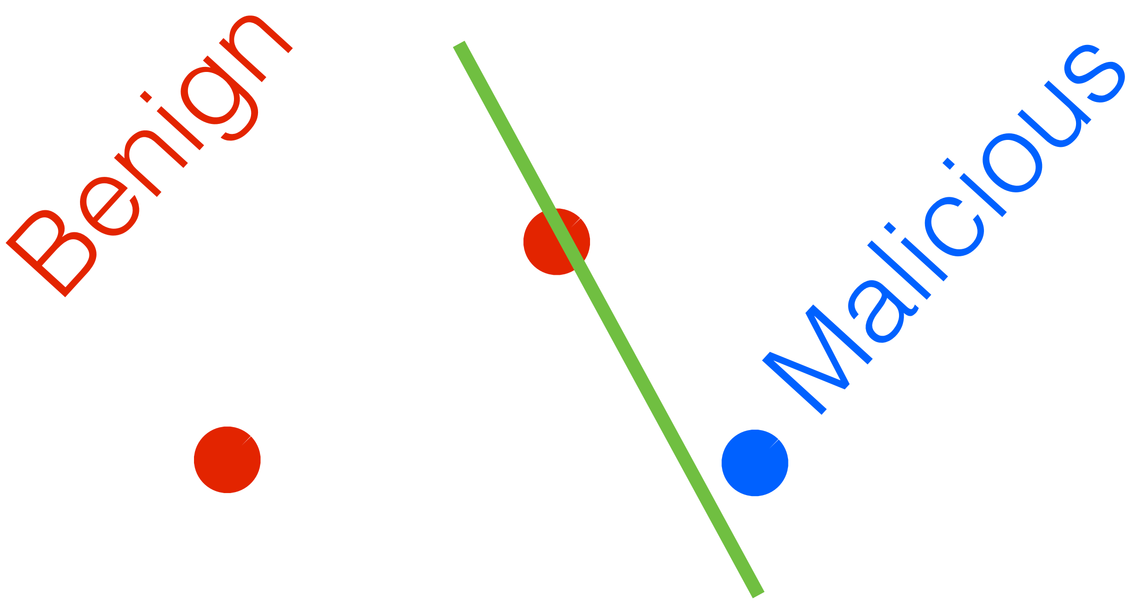
- Might miss important data

Uniform subsampling



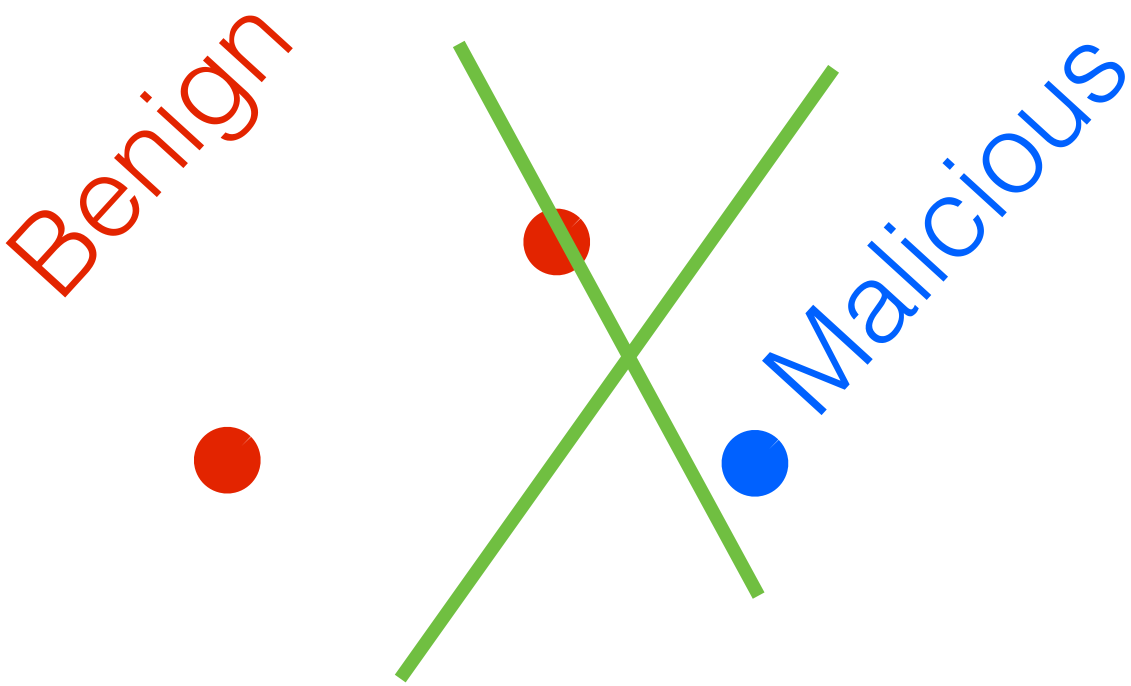
- Might miss important data

Uniform subsampling



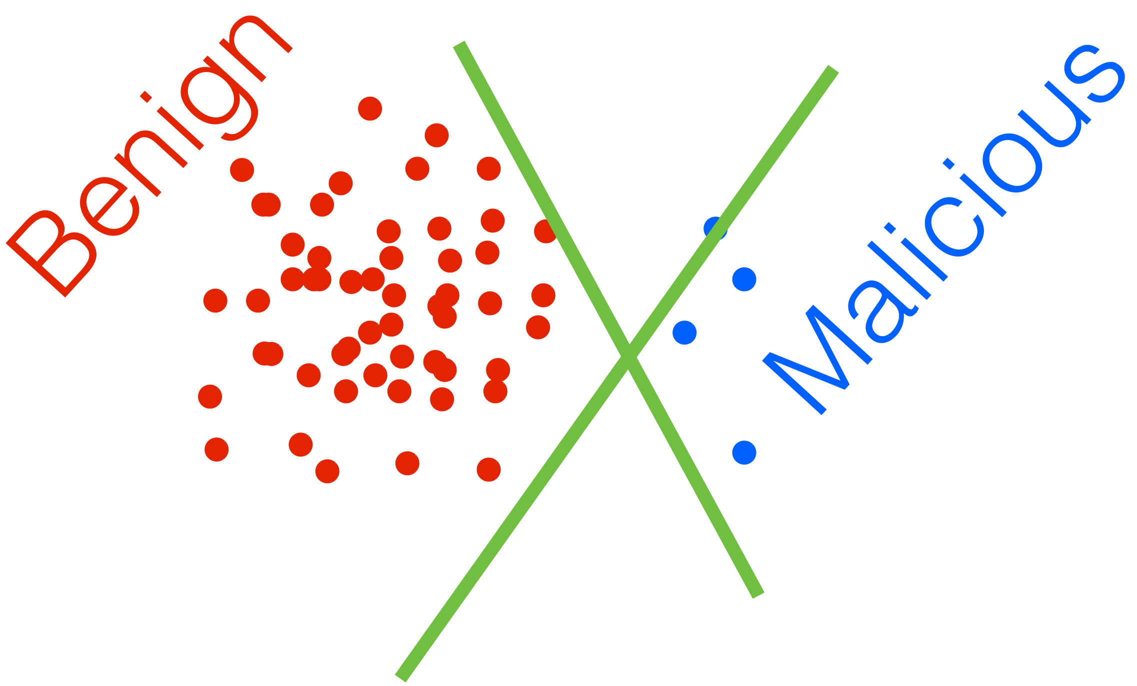
- Might miss important data

Uniform subsampling



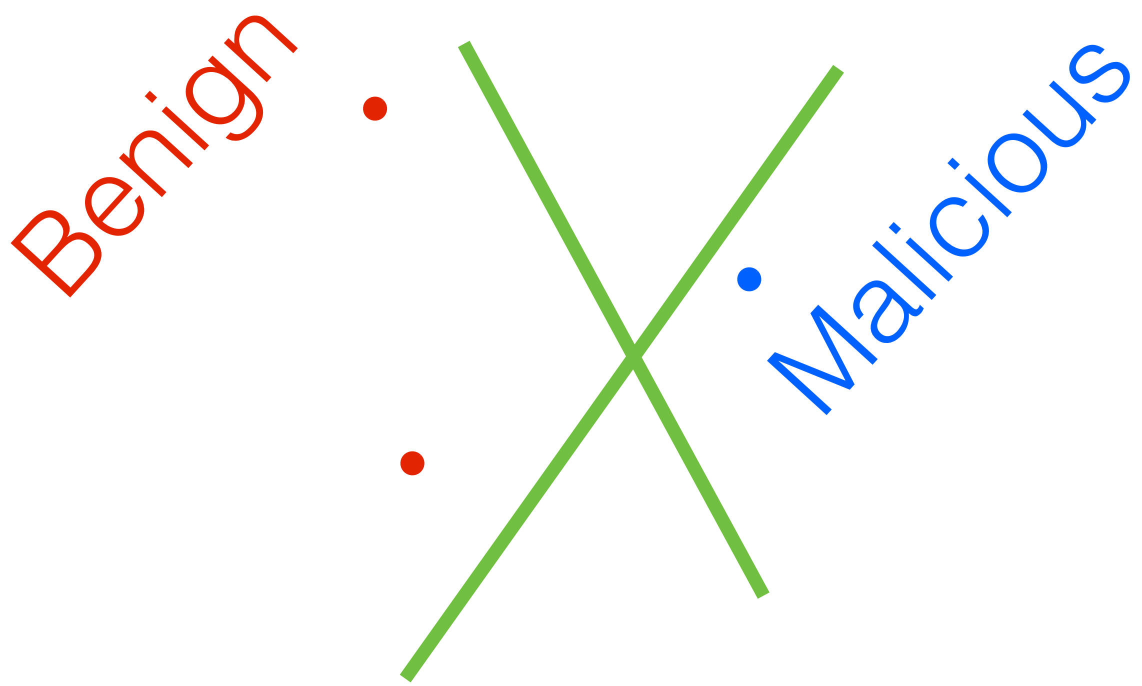
- Might miss important data

Uniform subsampling



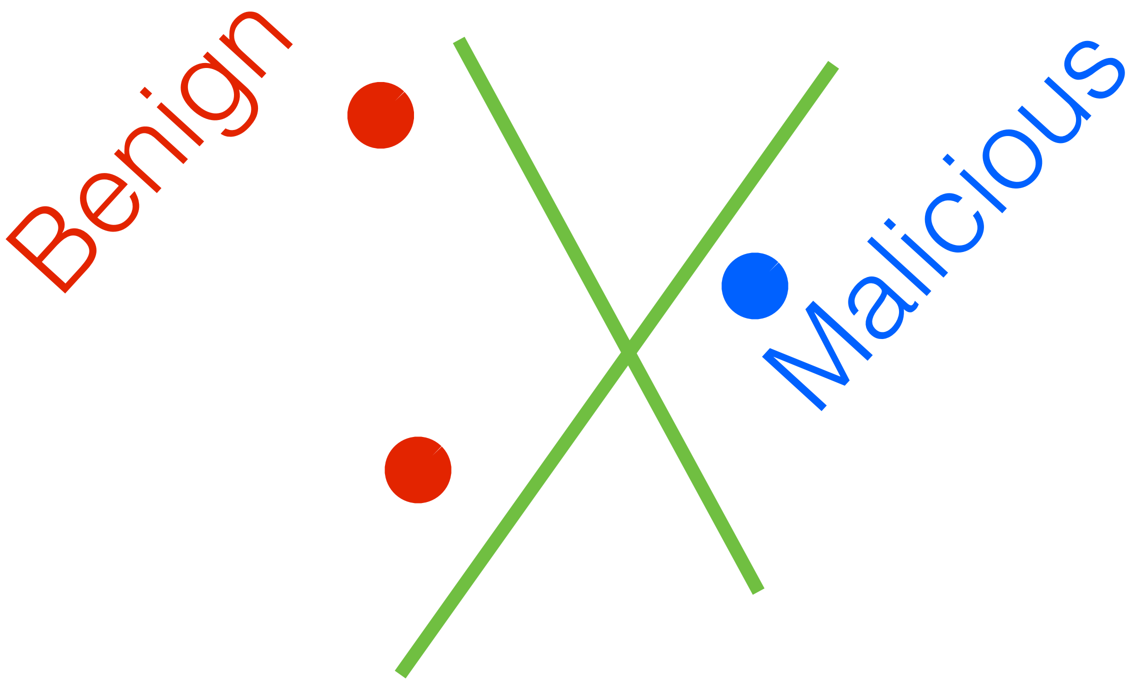
- Might miss important data

Uniform subsampling



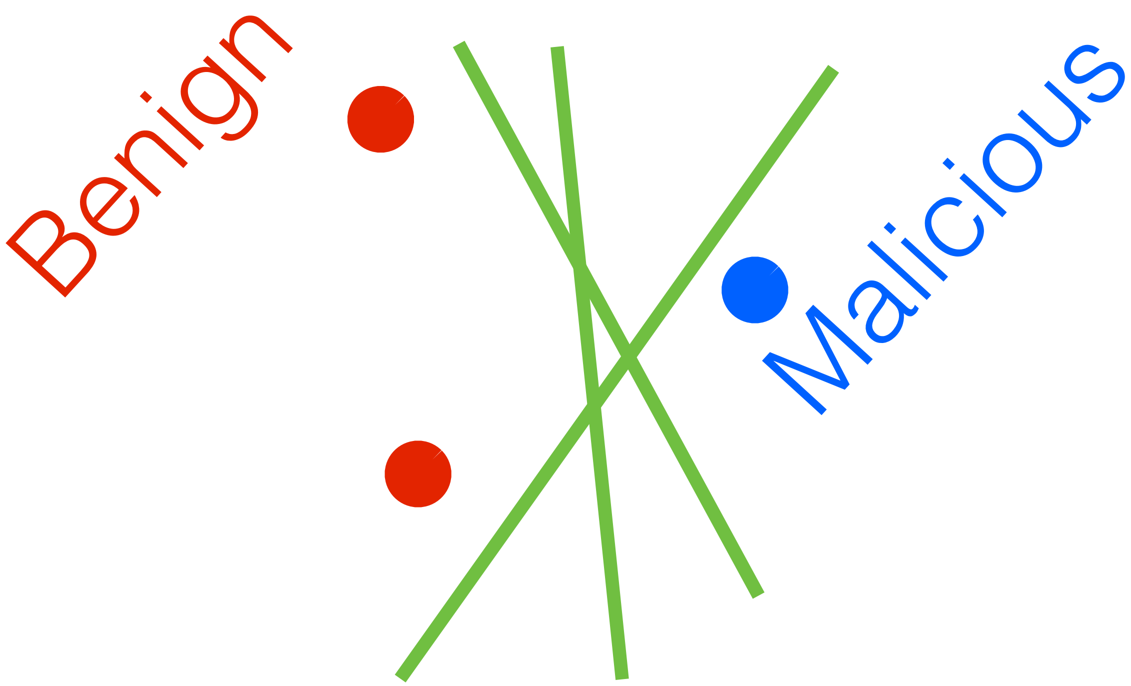
- Might miss important data

Uniform subsampling



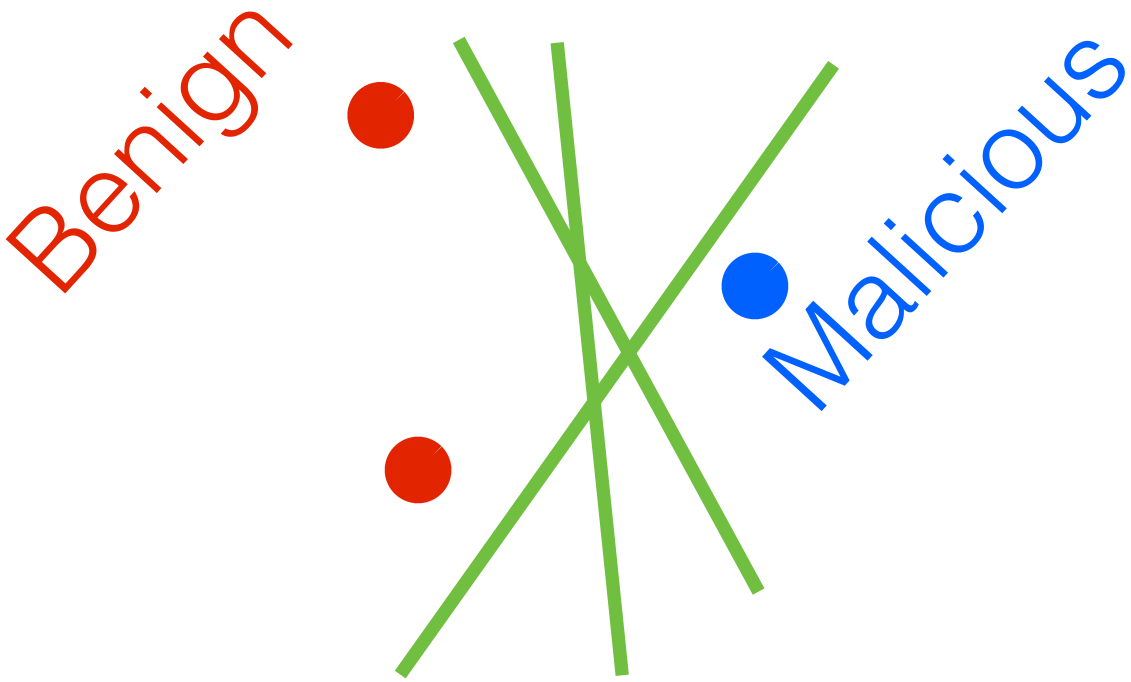
- Might miss important data

Uniform subsampling



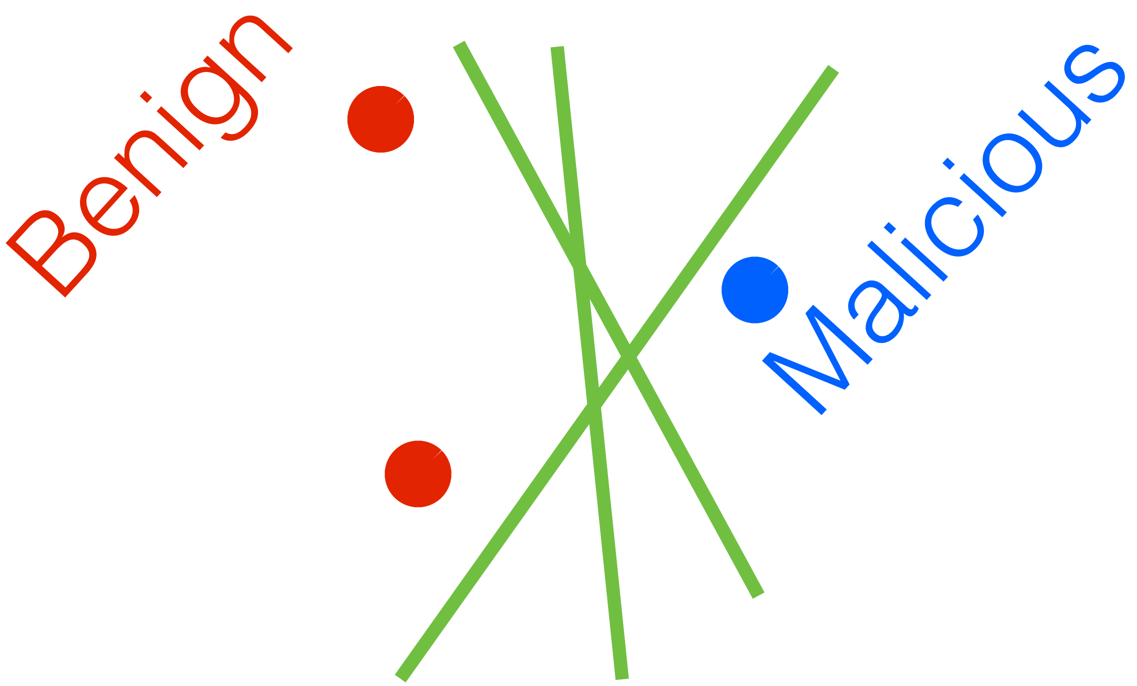
- Might miss important data

Uniform subsampling

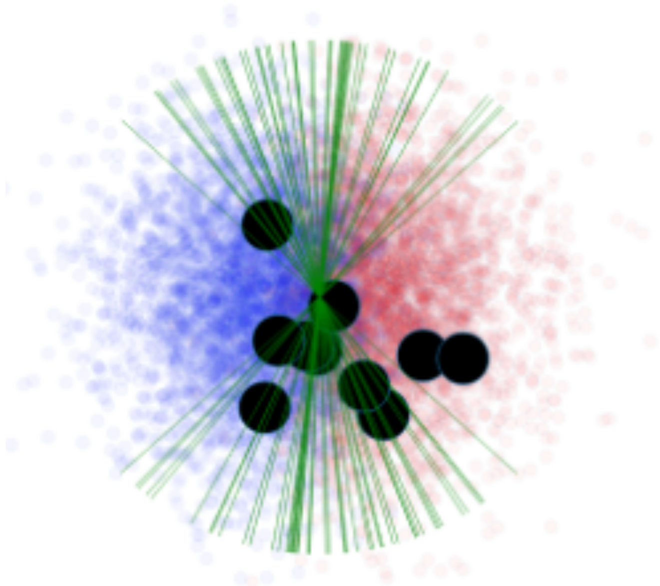


- Might miss important data
- Noisy estimates

Uniform subsampling



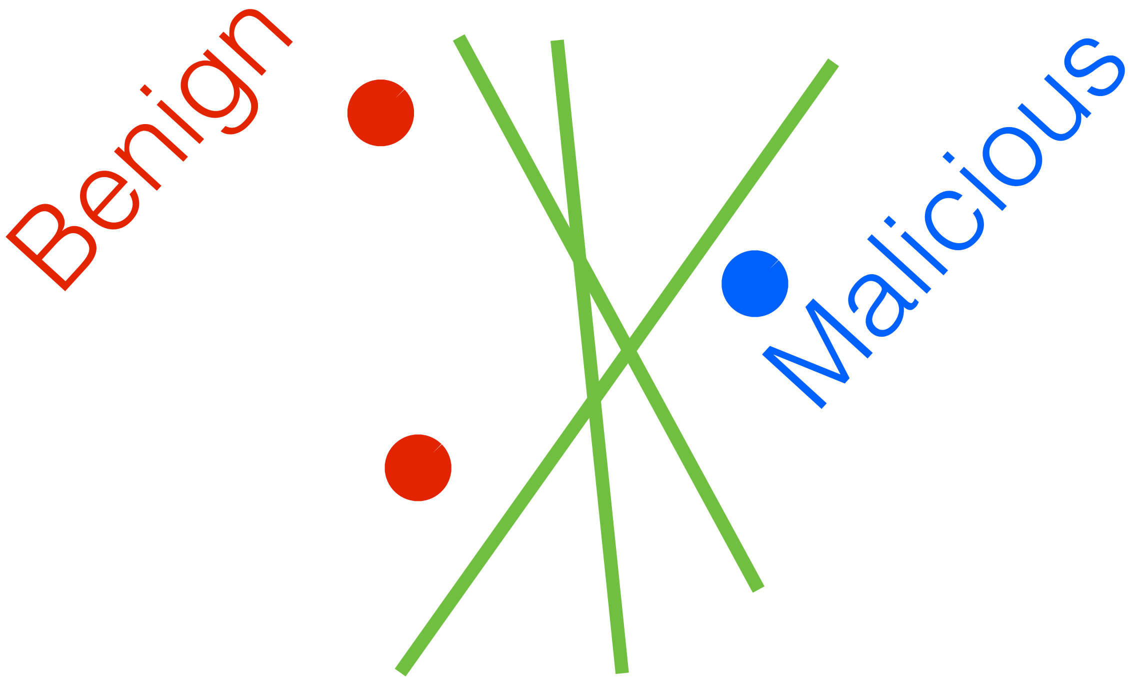
- Might miss important data
- Noisy estimates



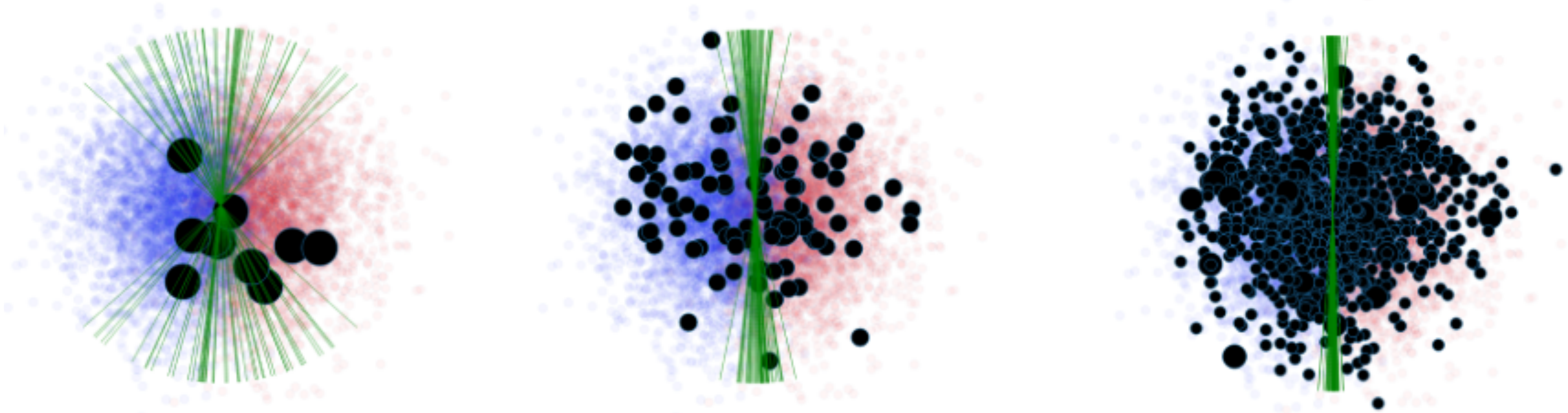
$M = 10$

[Campbell, Broderick 2018, 2019]

Uniform subsampling



- Might miss important data
- Noisy estimates

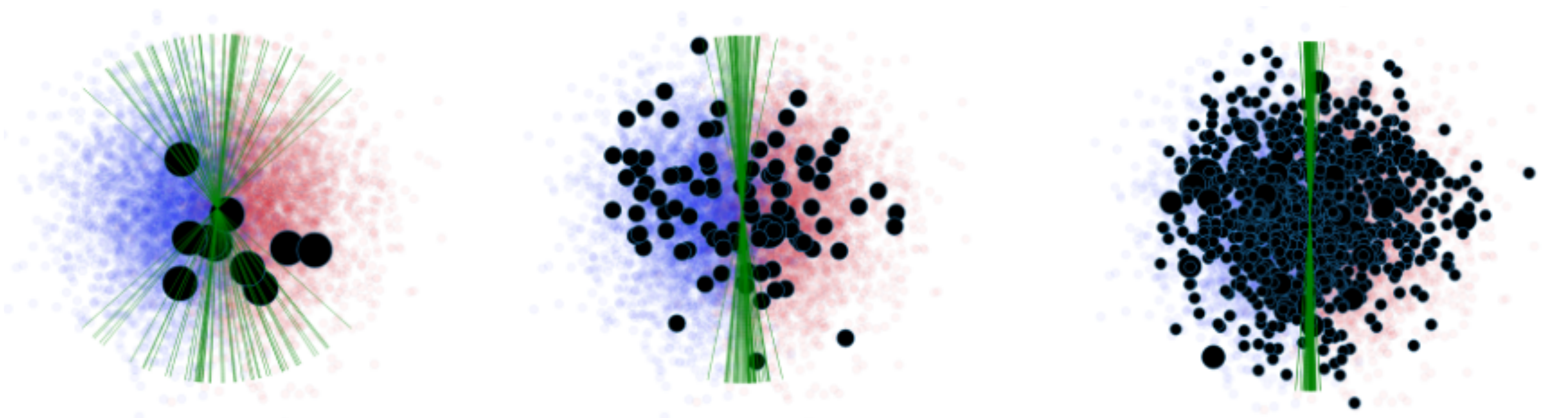


$M = 10$ $M = 100$ $M = 1000$

[Campbell, Broderick 2018, 2019]

Data summarization alternatives

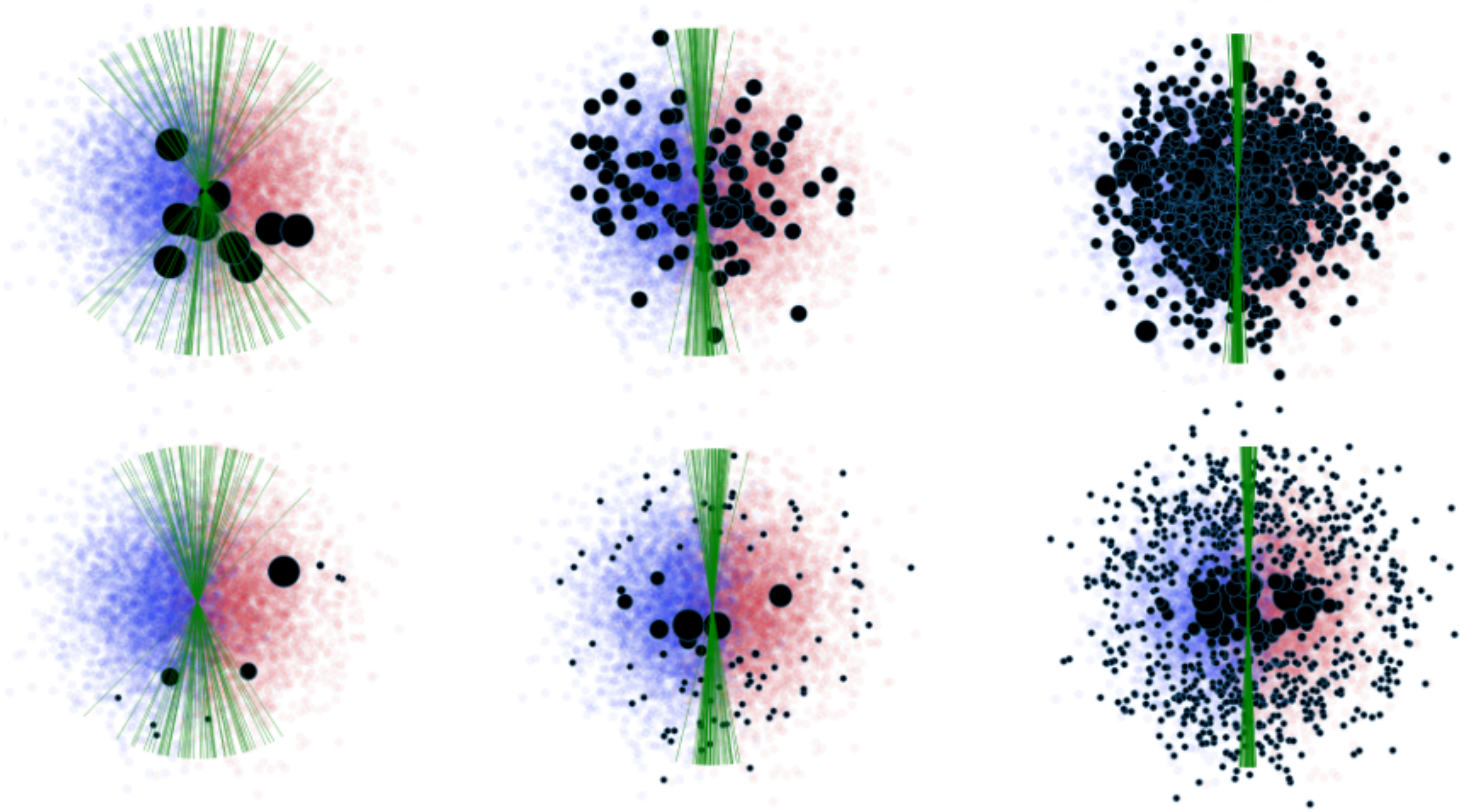
Uniform
subsampling



$M = 10$ $M = 100$ $M = 1000$

[Campbell, Broderick 2018, 2019]

Data summarization alternatives



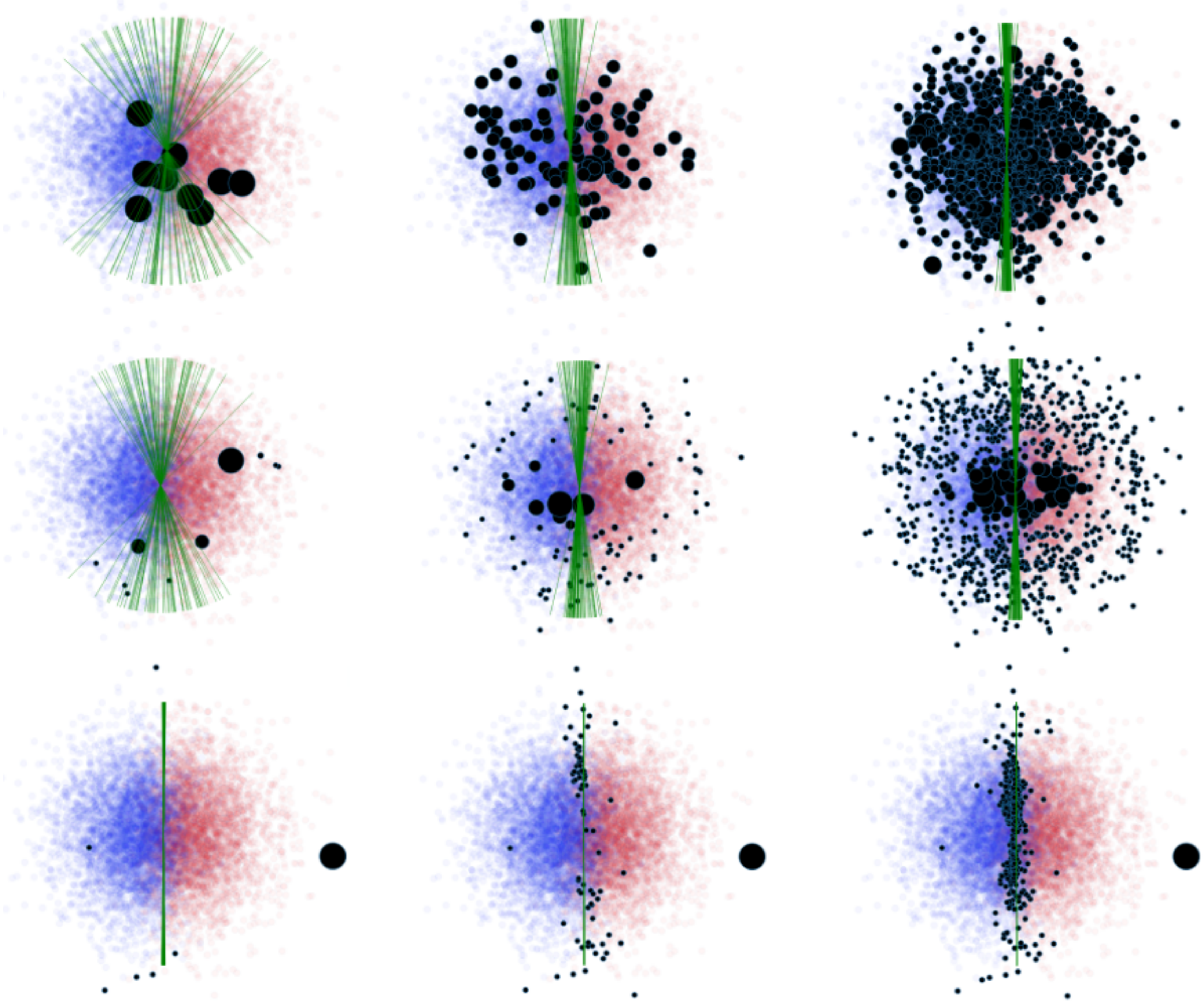
Uniform subsampling

Importance sampling

$M = 10$ $M = 100$ $M = 1000$

[Campbell, Broderick 2018, 2019]

Data summarization alternatives



Uniform subsampling

Importance sampling

Bayesian/Hilbert coresets

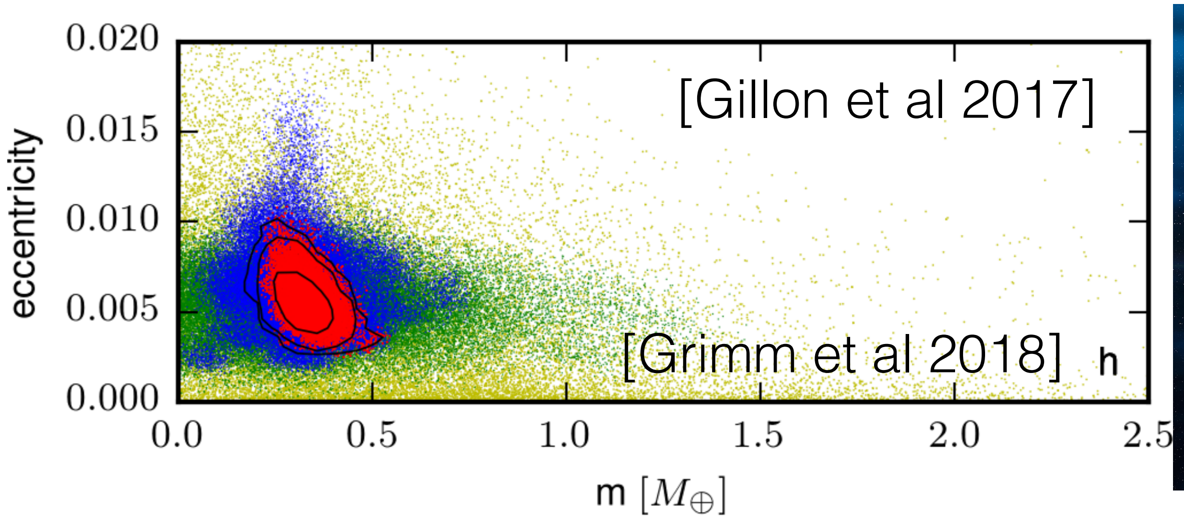
$M = 10$ $M = 100$ $M = 1000$

[Campbell, Broderick 2018, 2019]

Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use VB?
- When can we trust VB?
- Where do we go from here?

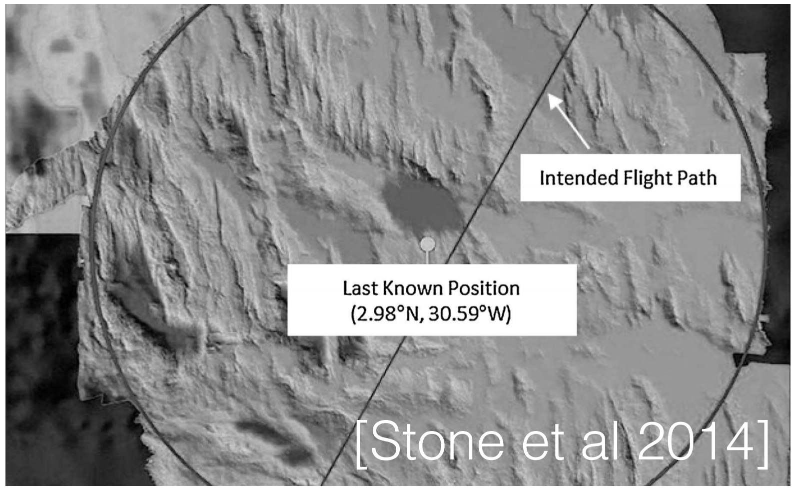
Bayesian inference



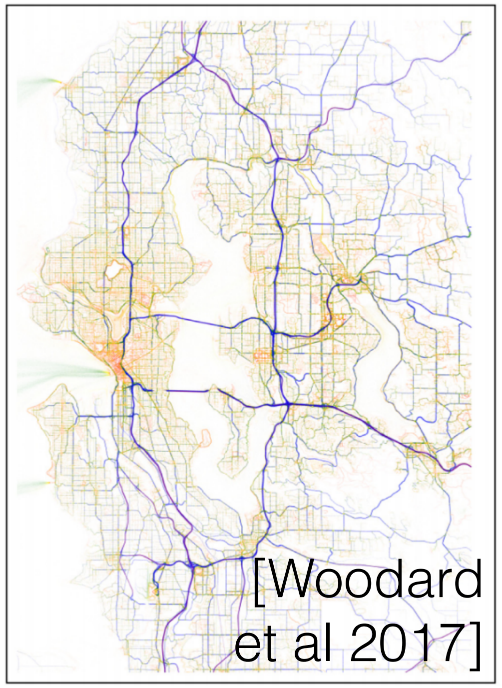
[Gillon et al 2017]

[Grimm et al 2018]

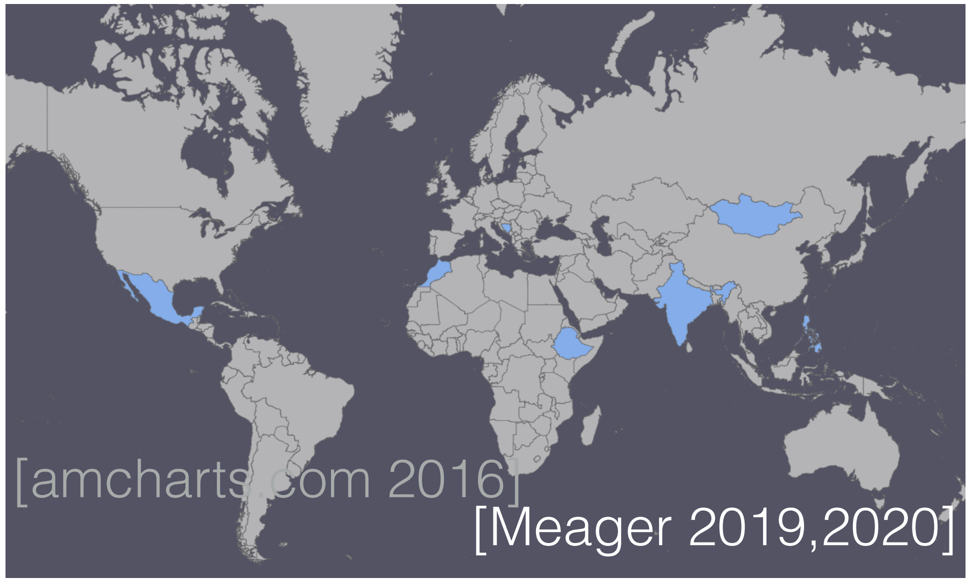
[ESO/ L. Calçada/ M. Kornmesser 2017] [Abbott et al 2016a,b]



[Stone et al 2014]



[Woodard et al 2017]



[amcharts.com 2016] [Meager 2019,2020]

[Kuikka et al 2014] [Baltic Salmon Fund]

[Chati, Balakrishnan 2017] [Julian Hertzog 2016]

[mc-stan.org]

- Goals: good point estimates, uncertainty estimates

- Challenge: speed (compute, user), reliable inference

What to read next

Textbooks and Reviews

- Bishop. *Pattern Recognition and Machine Learning*, Ch 10. 2006.
- Blei, Kucukelbir, McAuliffe. Variational inference: A review for statisticians, *JASA* 2016.
- MacKay. *Information Theory, Inference, and Learning Algorithms*, Ch 33. 2003.
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- RJ Giordano, T Broderick, and MI Jordan. Linear response methods for accurate covariance estimates from mean field variational Bayes. *NeurIPS* 2015.
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- J Huggins, M Kasprzak, T Campbell, T Broderick. Validated Variational Inference via Practical Posterior Error Bounds. ArXiv: 1910.04102. *AISTATS* 2020, to appear.
- T Campbell and T Broderick. Automated scalable Bayesian inference via Hilbert coresets. *JMLR* 2019.
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J Gorham and L Mackey. "Measuring sample quality with Stein's method." *NeurIPS* 2015.

J Gorham, and L Mackey. "Measuring sample quality with kernels." ArXiv:1703.01717 (2017).

PD Hoff. *A first course in Bayesian statistical methods*. Springer Science & Business Media, 2009.

MD Hoffman, DM Blei, C Wang, and J Paisley. "Stochastic variational inference." *The Journal of Machine Learning Research* 14.1 (2013): 1303-1347.

JH Huggins, T Campbell, and T Broderick. Coresets for scalable Bayesian logistic regression. *NeurIPS* 2016.

JH Huggins, RP Adams, and T Broderick. PASS-GLM: Polynomial approximate sufficient statistics for scalable Bayesian GLM inference. *NeurIPS* 2017.

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E. Xing. 2003. Slides “LOGOS: a modular Bayesian model for de novo motif detection.” Obtained from: https://www.cs.cmu.edu/~epxing/papers/Old_papers/slide_CSB03/CSB1.pdf Accessed: 2018.