Variational Bayes and beyond: Foundations of scalable Bayesian inference

Tamara Broderick
Associate Professor
MIT

http://www.tamarabroderick.com/tutorials.html

Bayesian inference

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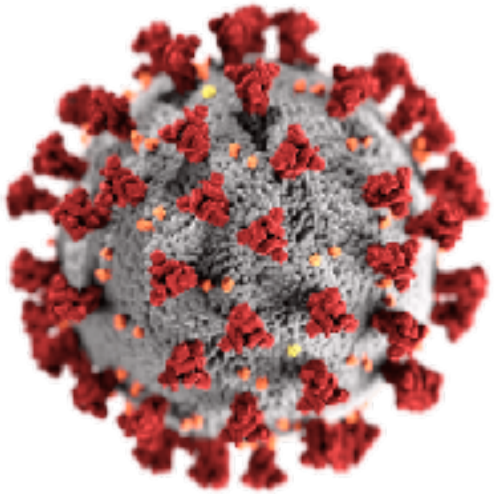

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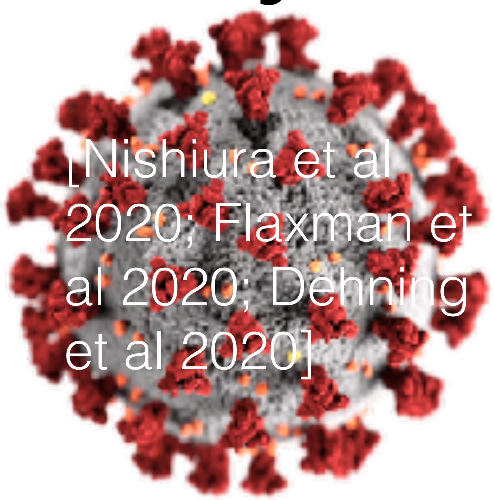

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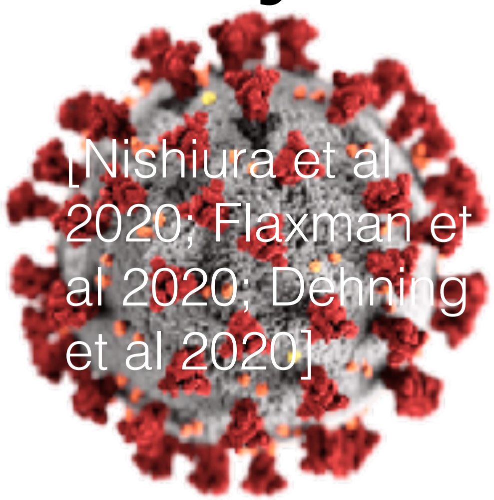

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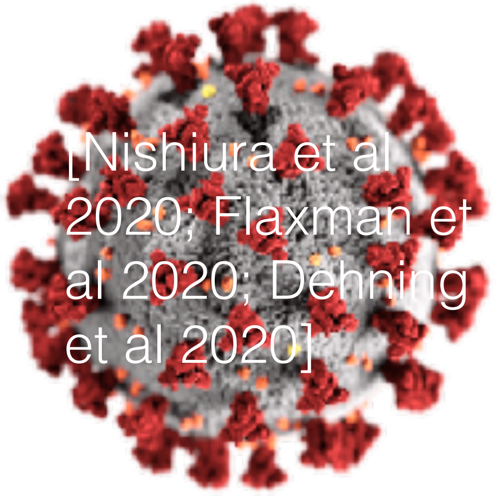

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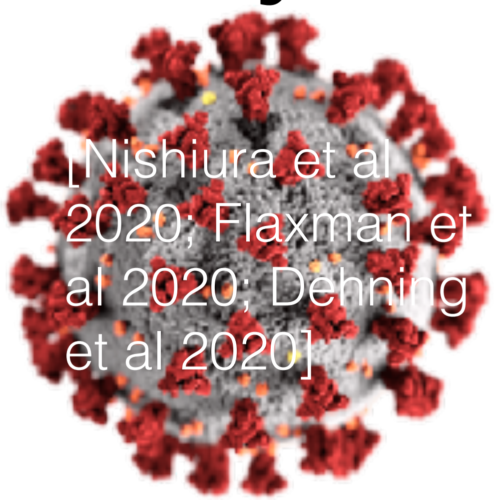

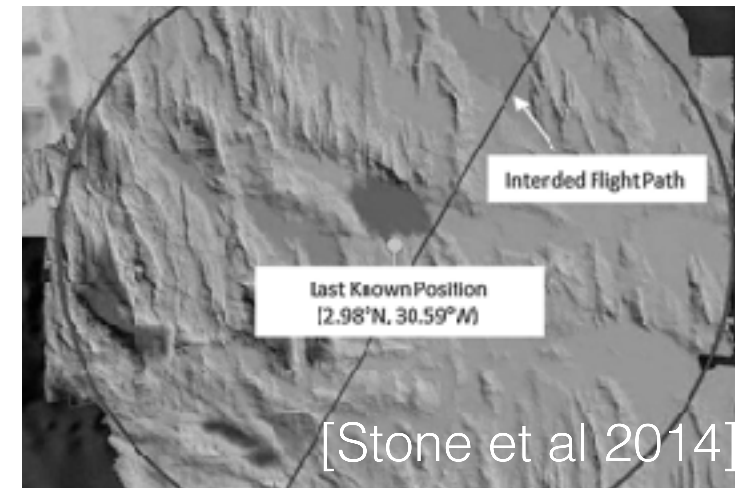

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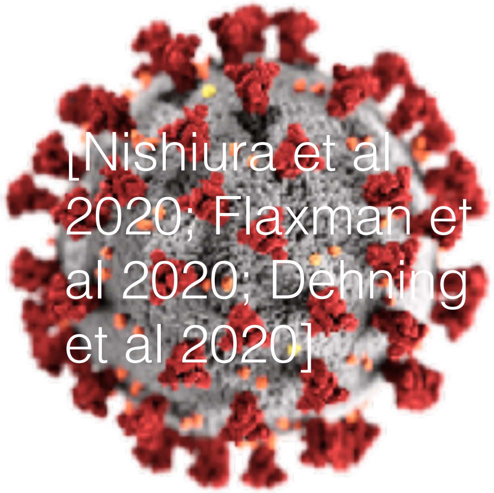

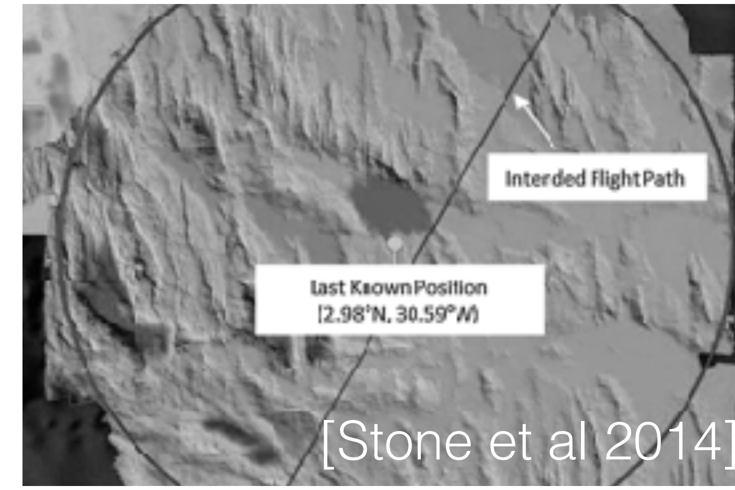

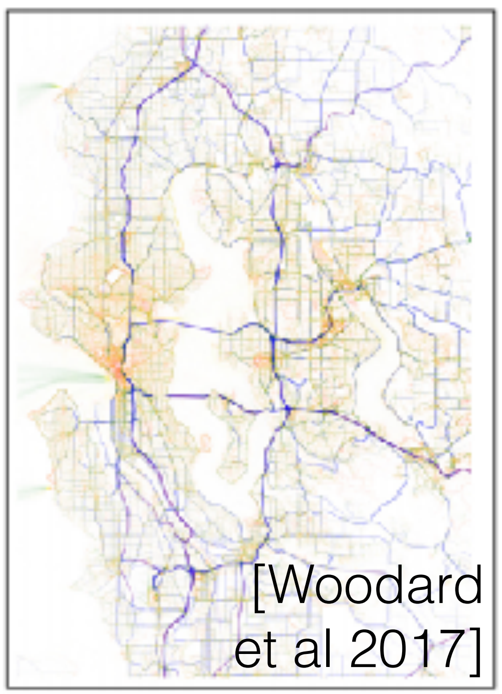

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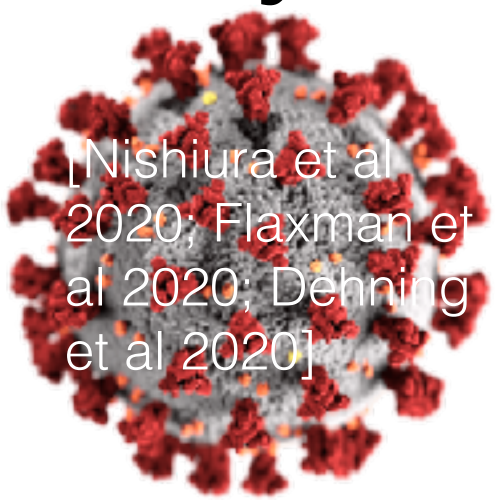
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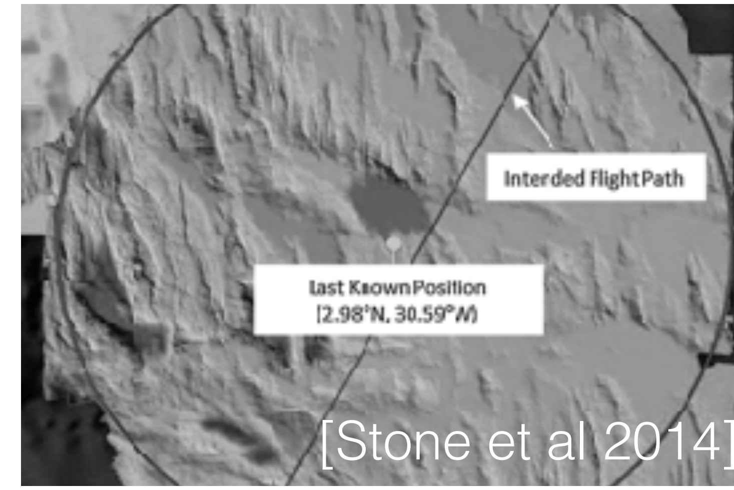

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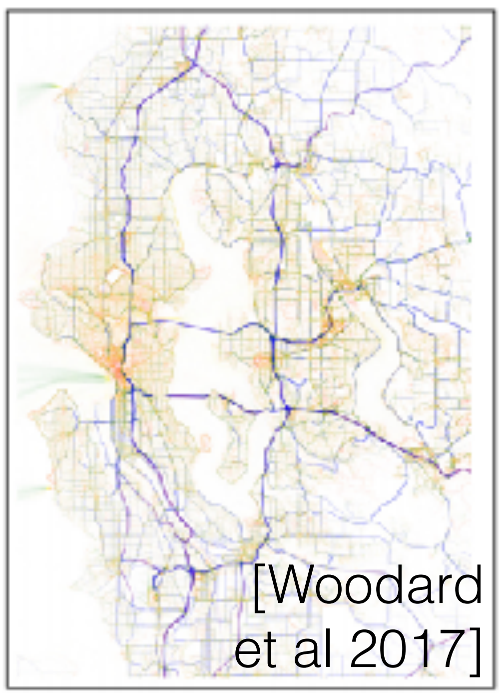
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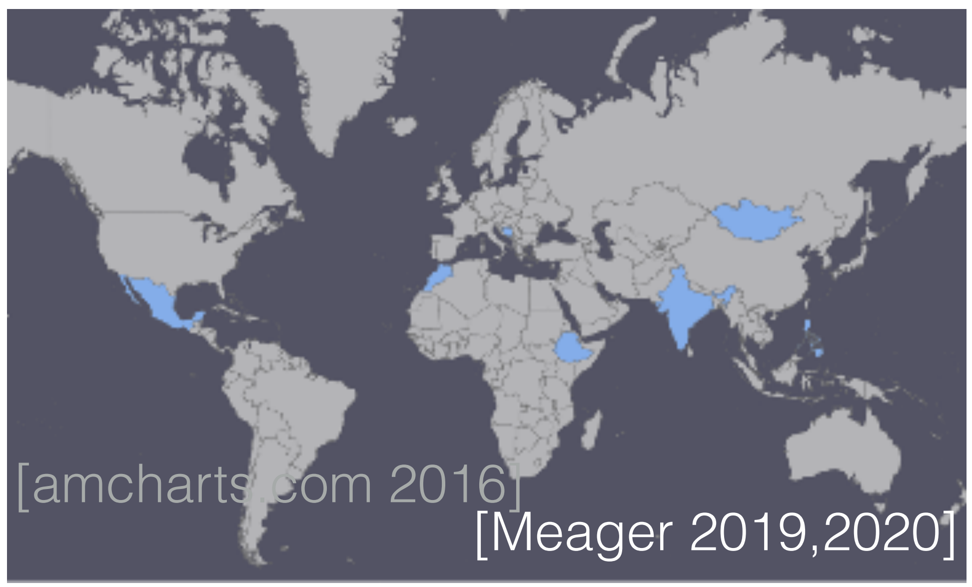
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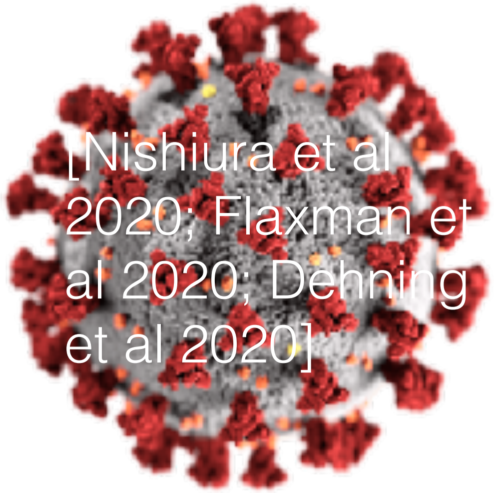
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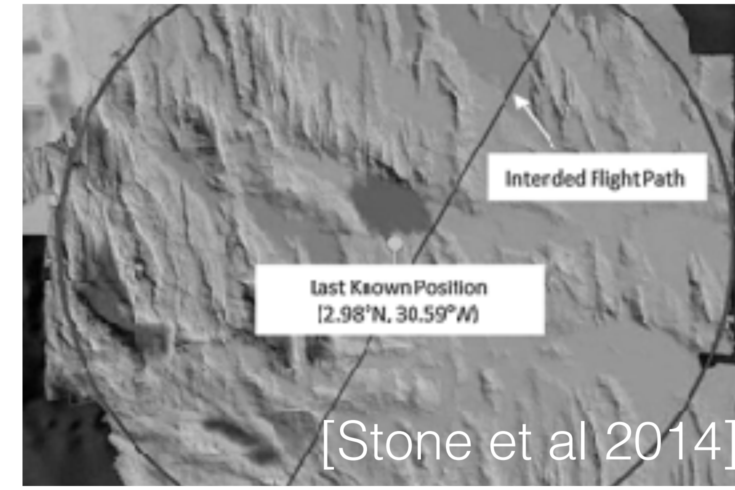

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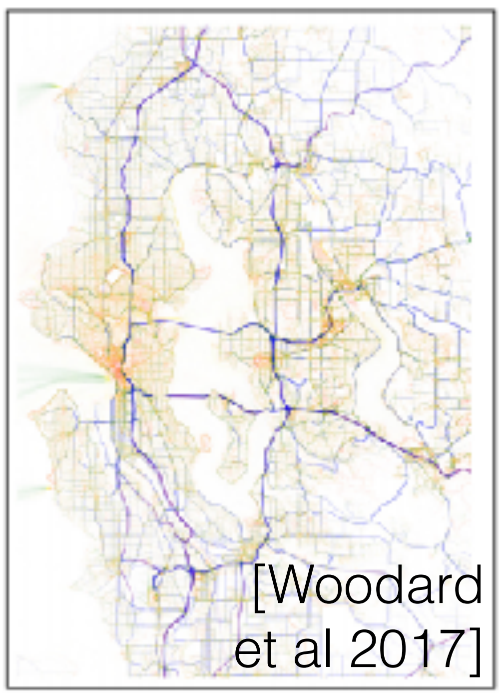
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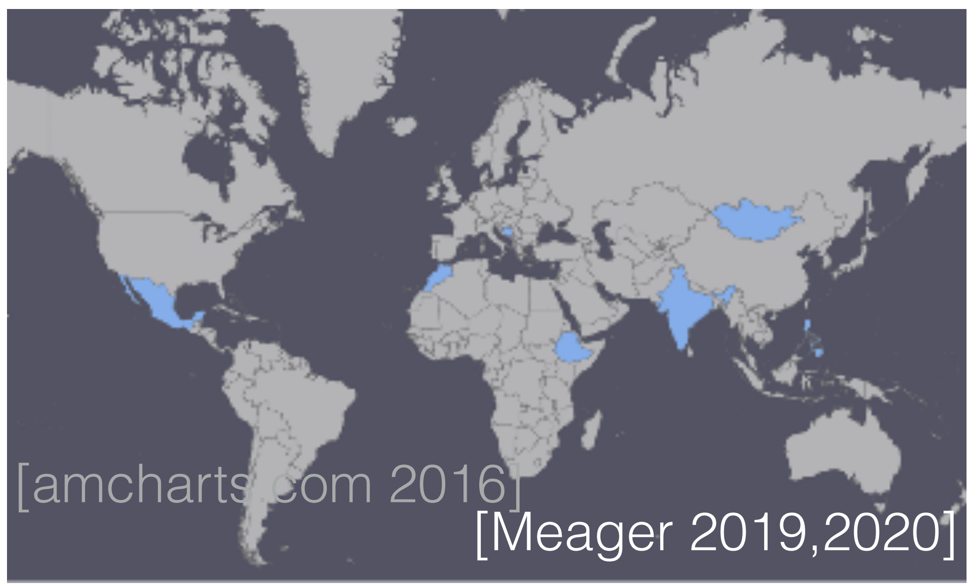
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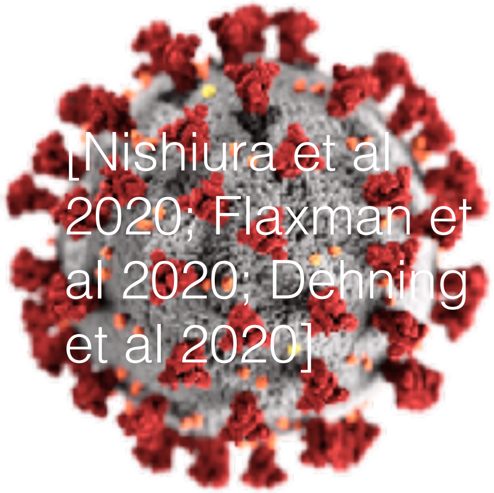

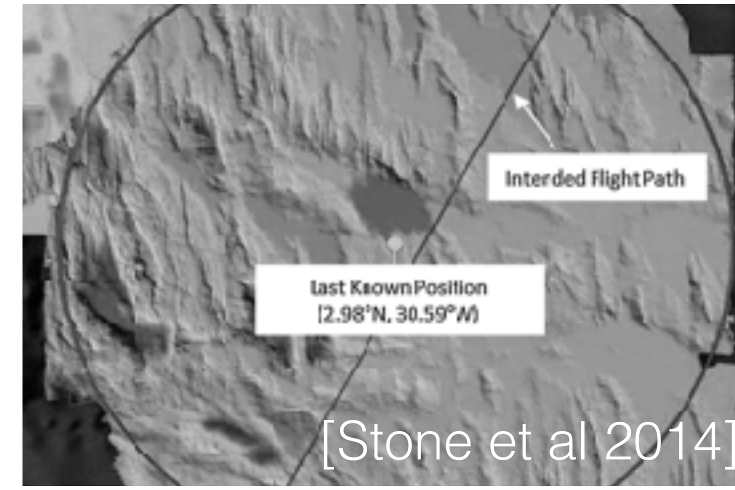

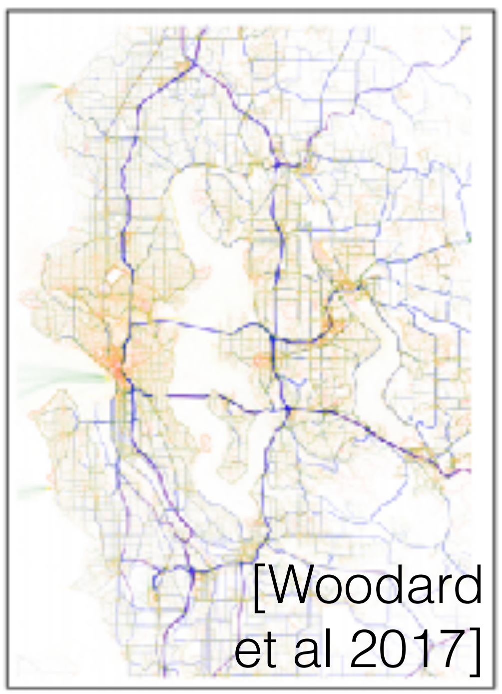

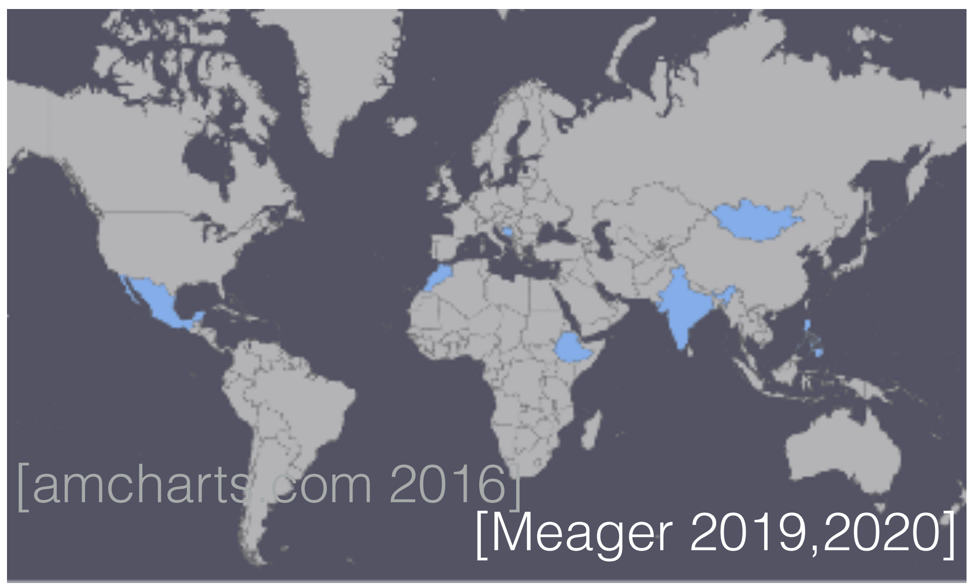

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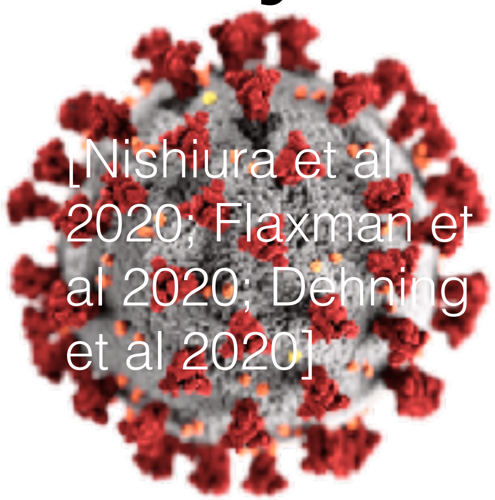
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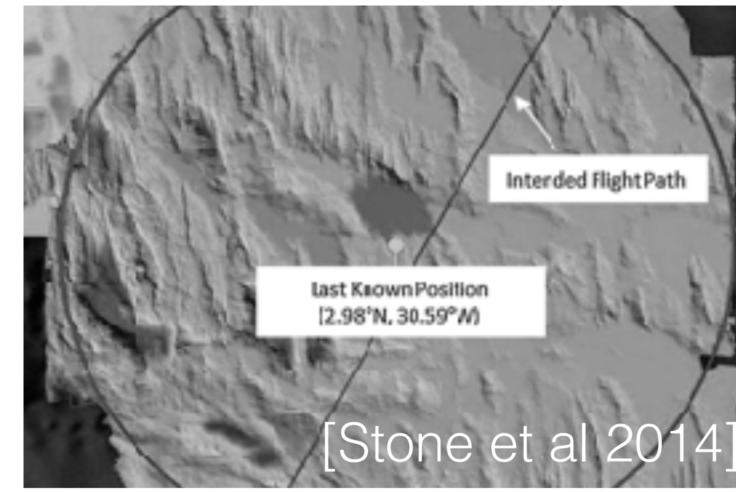

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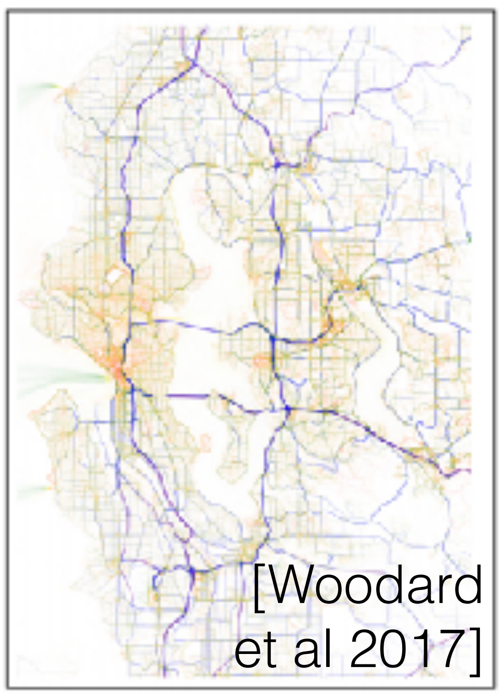
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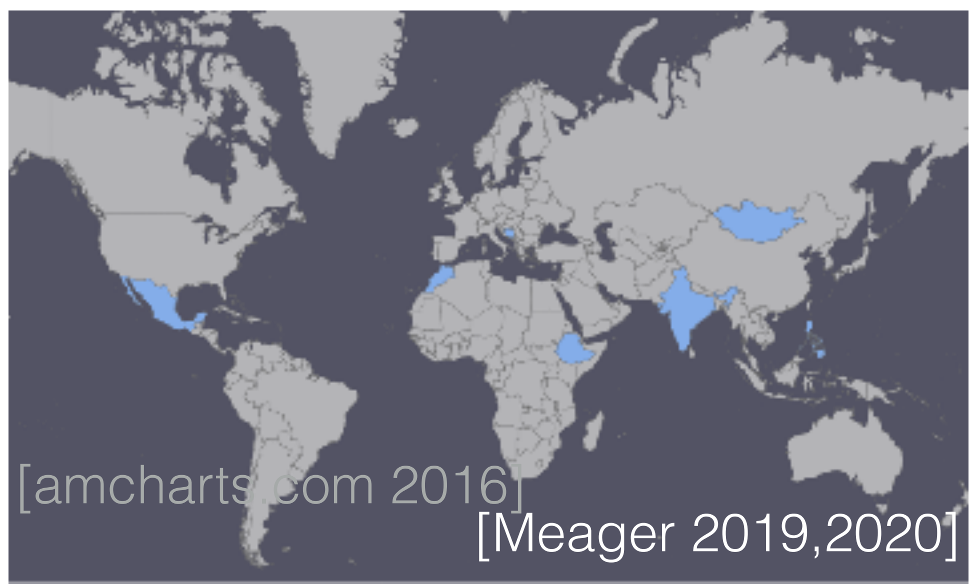
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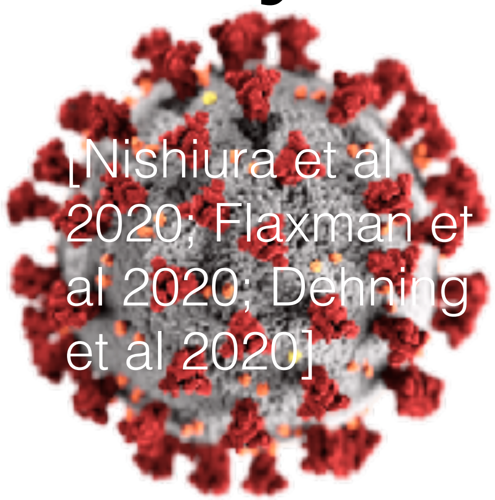

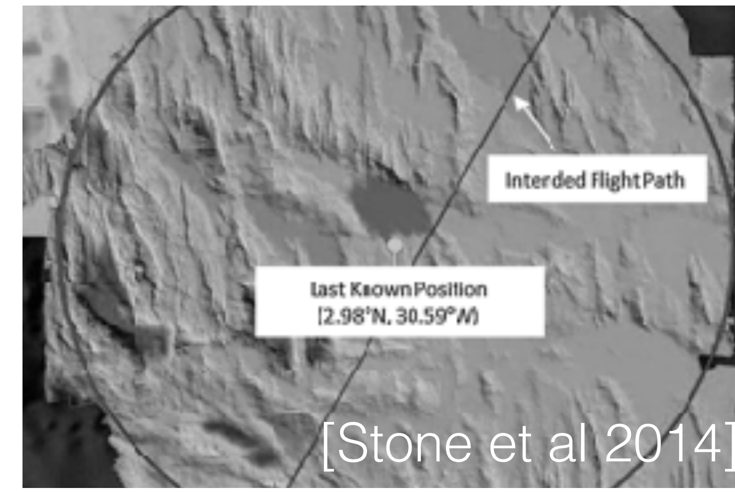

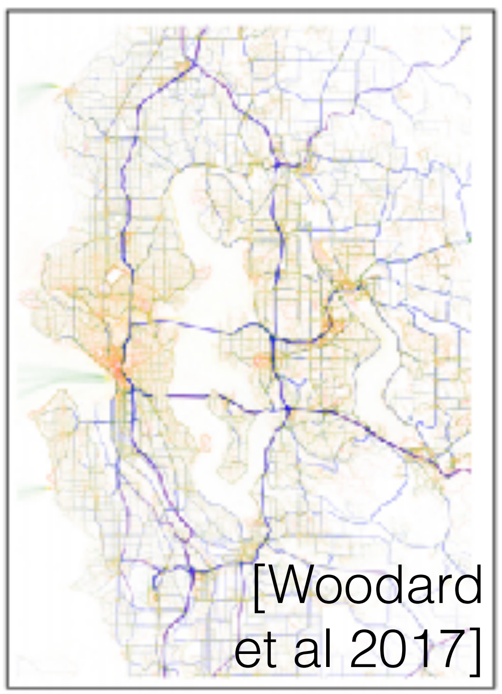

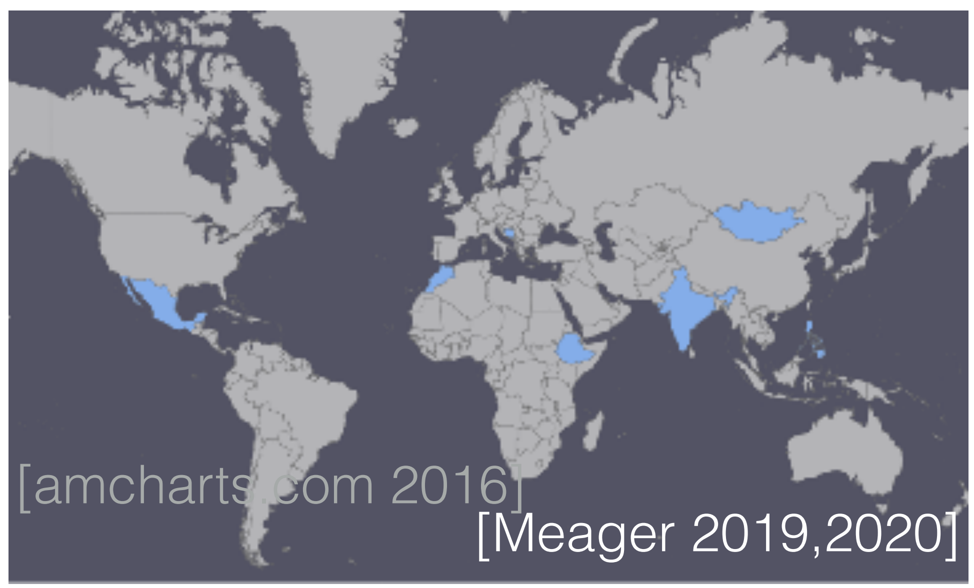

- Goals: good point estimates, uncertainty estimates
  - More: interpretable, modular, expert info

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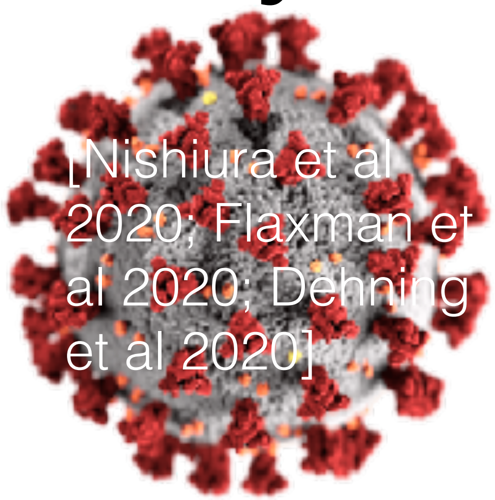

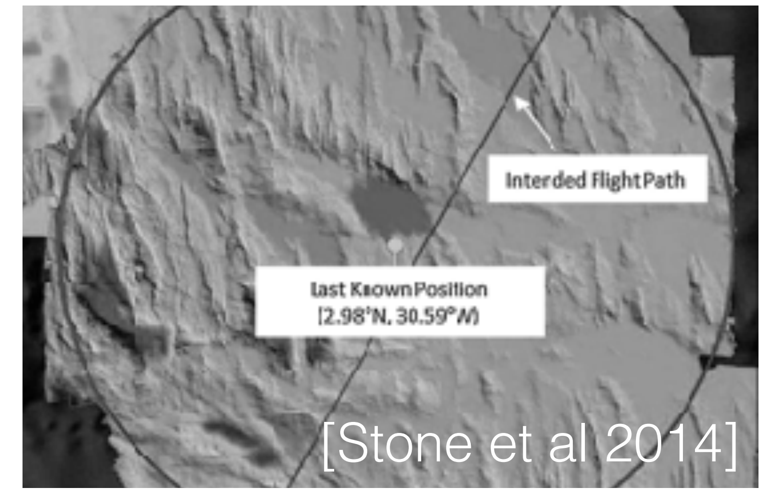

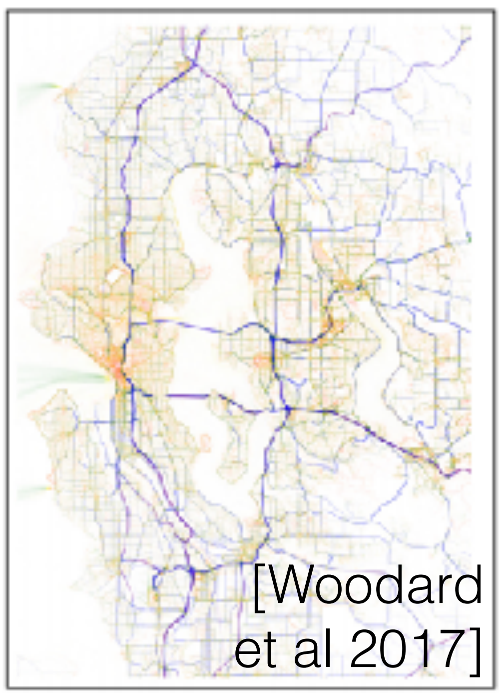

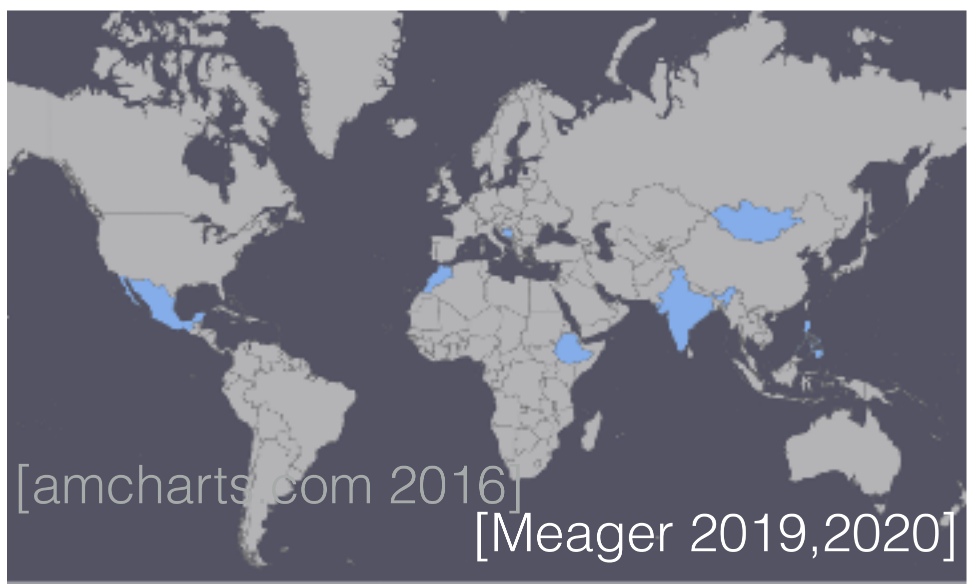

[mc-stan.org]

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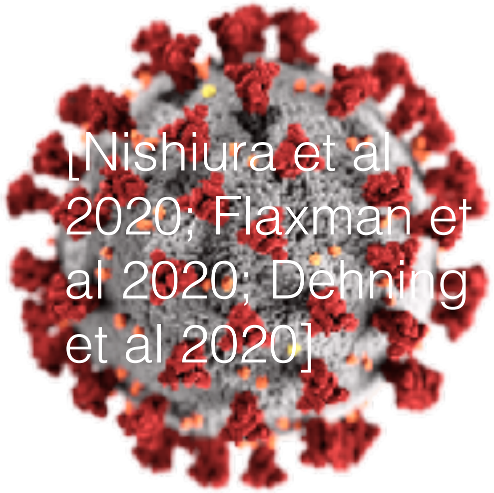

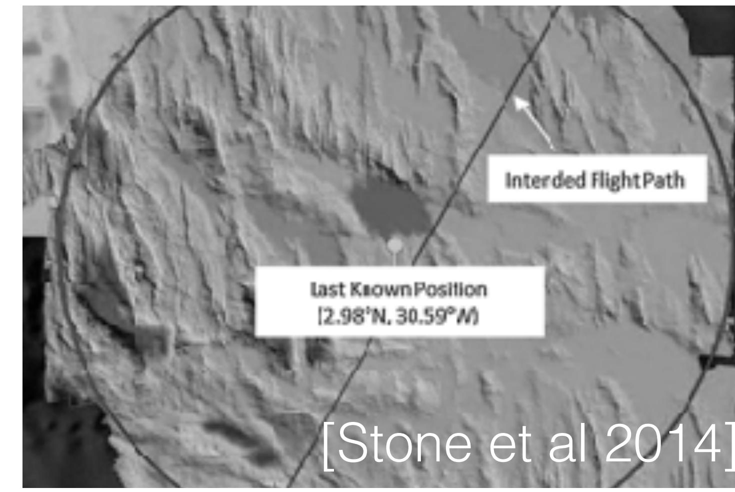

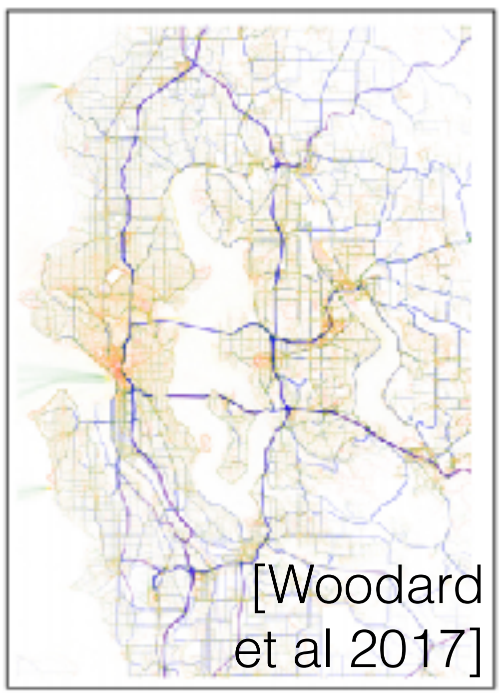

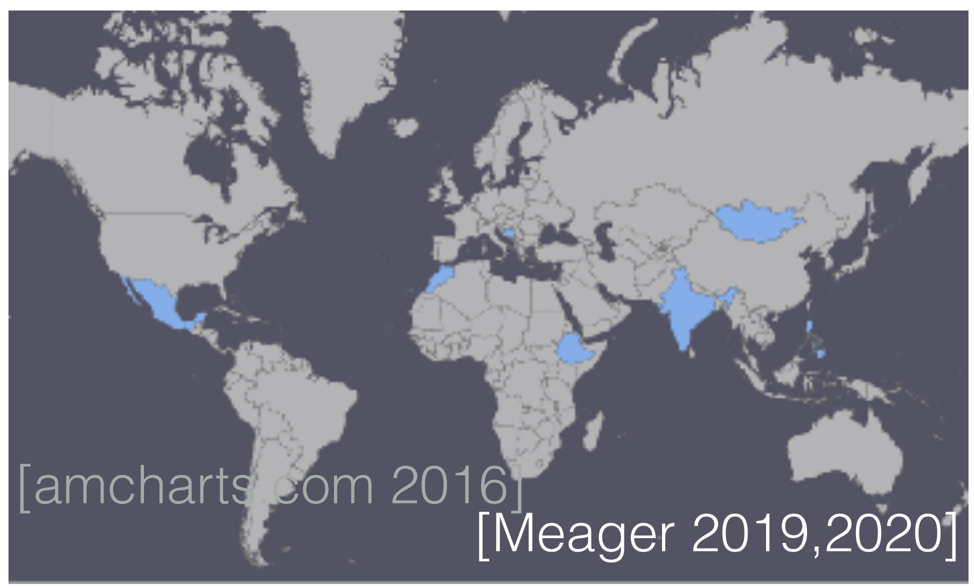

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  - More: interpretable, modular, expert info
- Challenge: speed (compute, user), reliable inference

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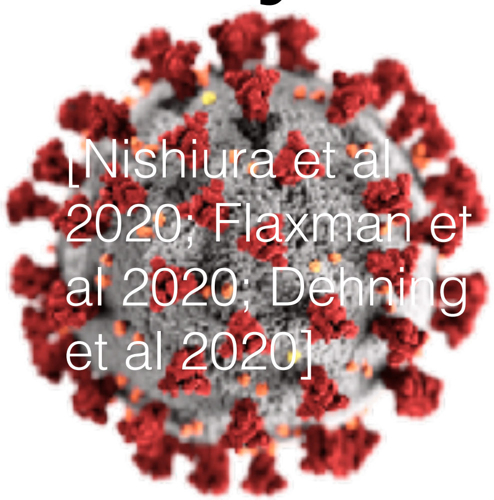
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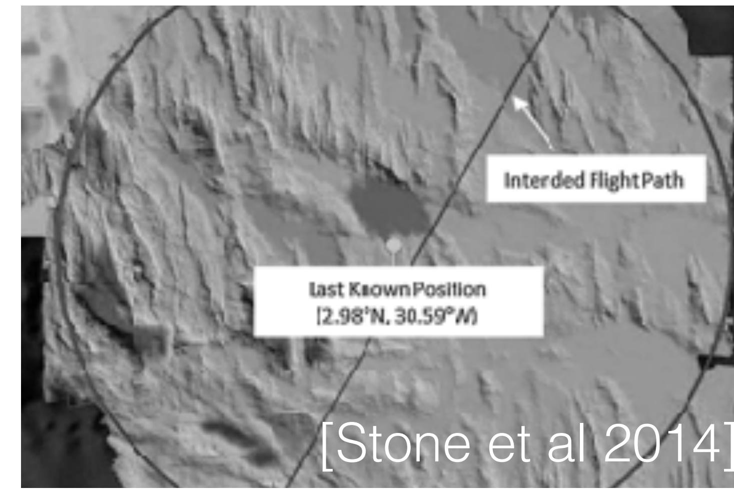

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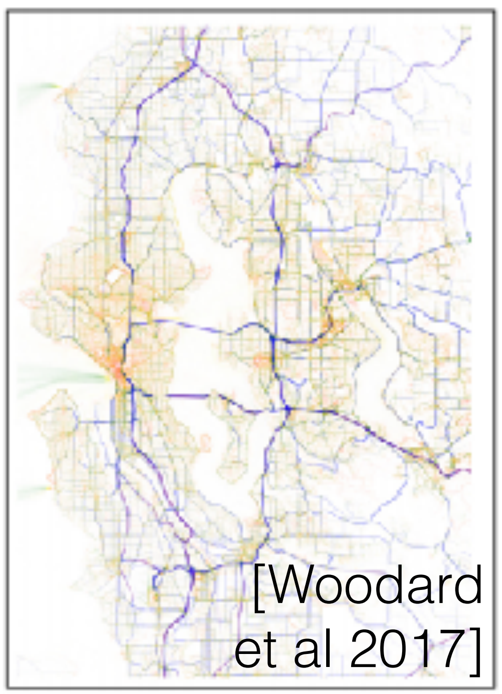
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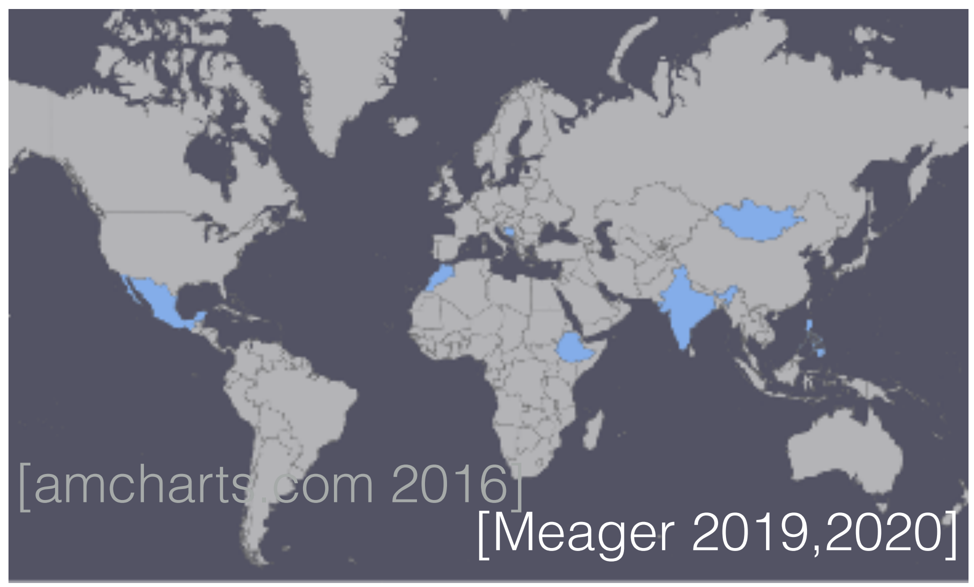
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- Goals: good point estimates, uncertainty estimates
  - More: interpretable, modular, expert info
- Challenge: speed (compute, user), reliable inference
- Uncertainty doesn't have to disappear in large data sets

Variational Bayes

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- Modern problems: often large data, large dimensions

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[Blei et al 2003]

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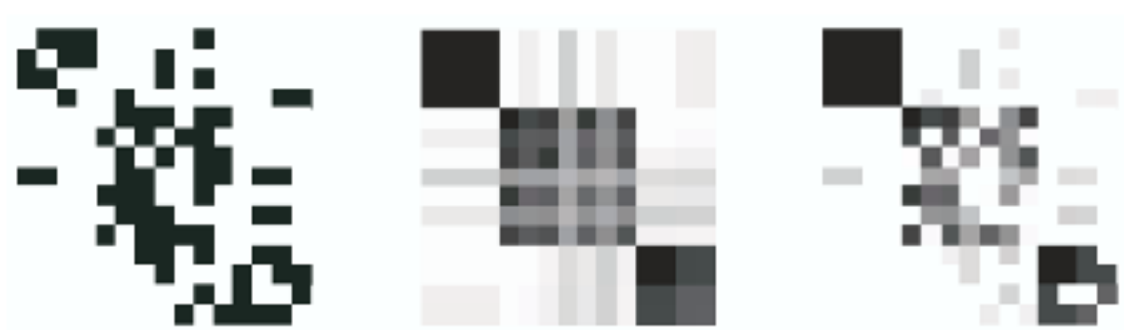

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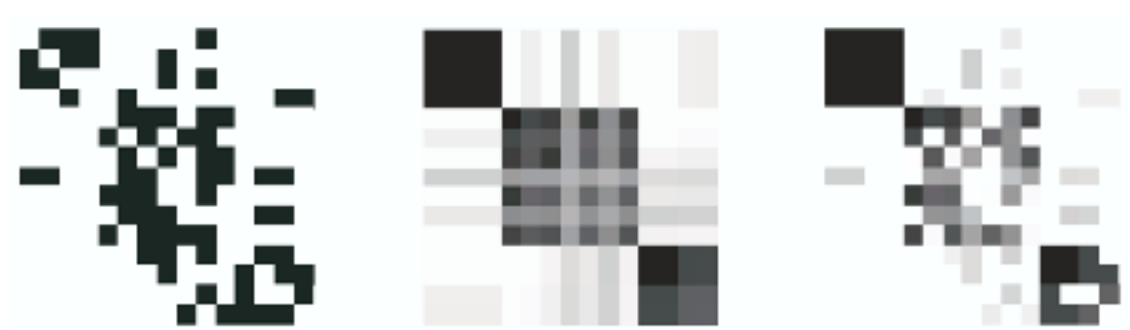

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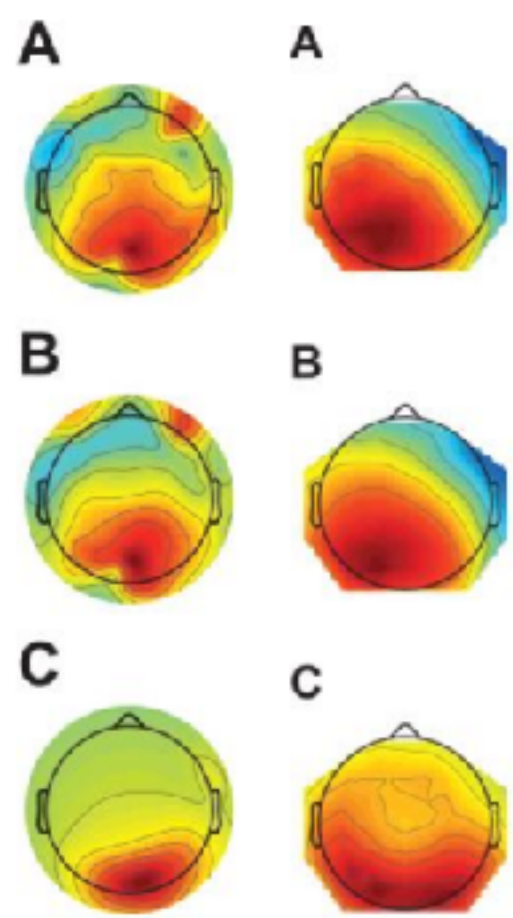

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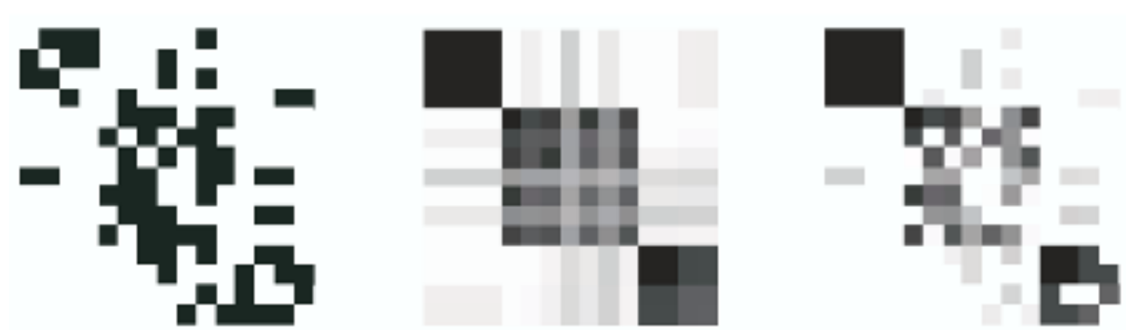

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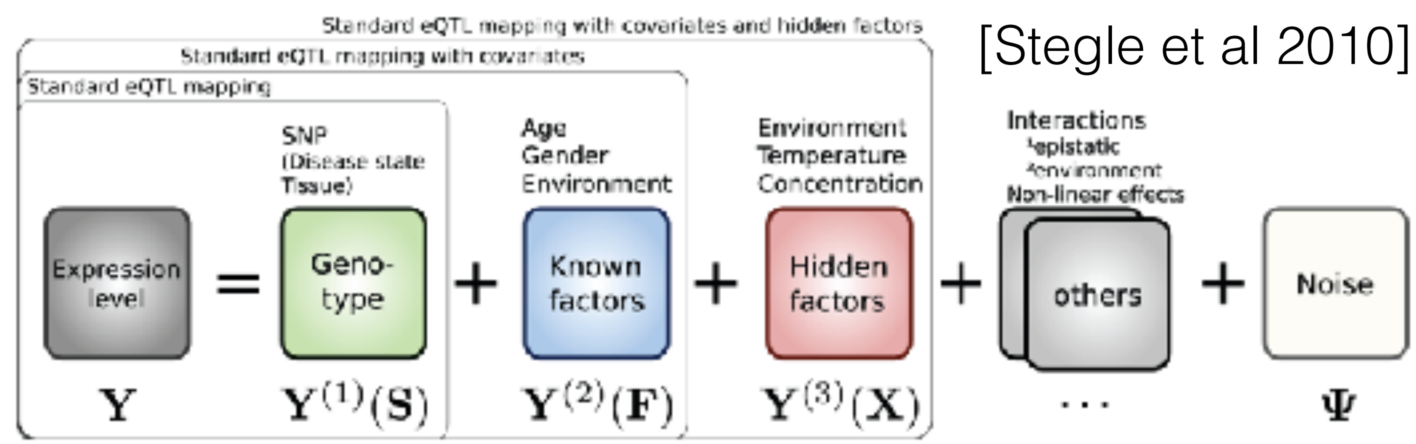

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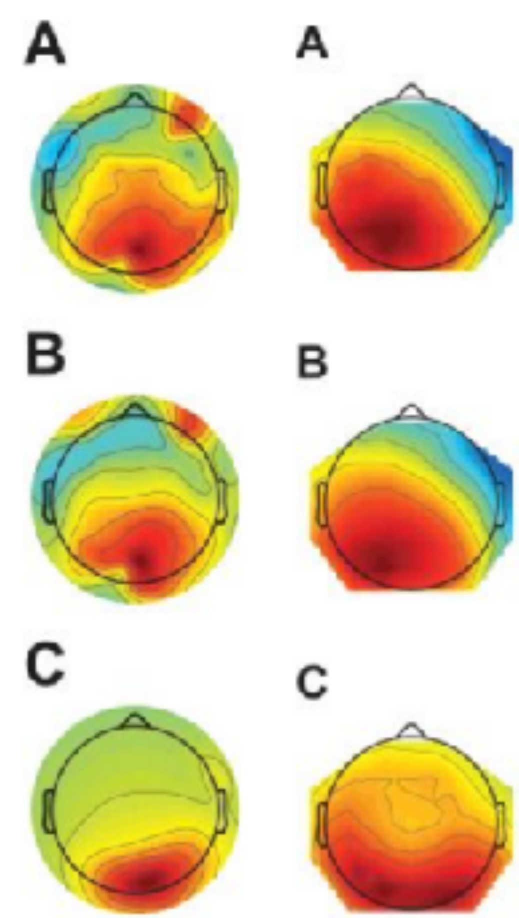

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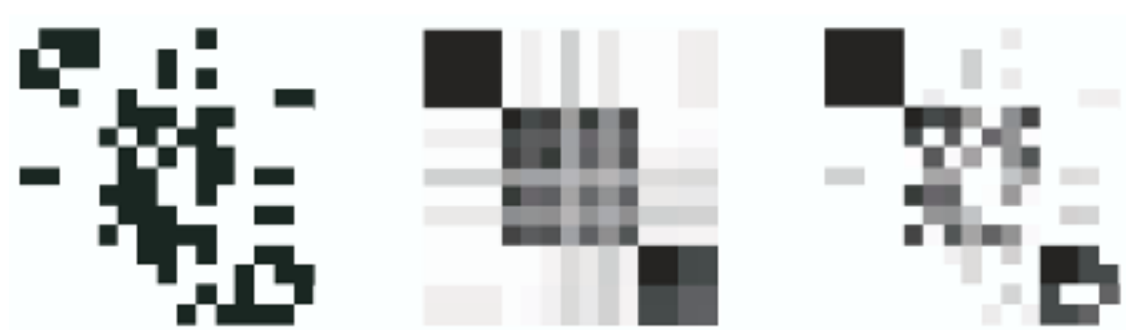
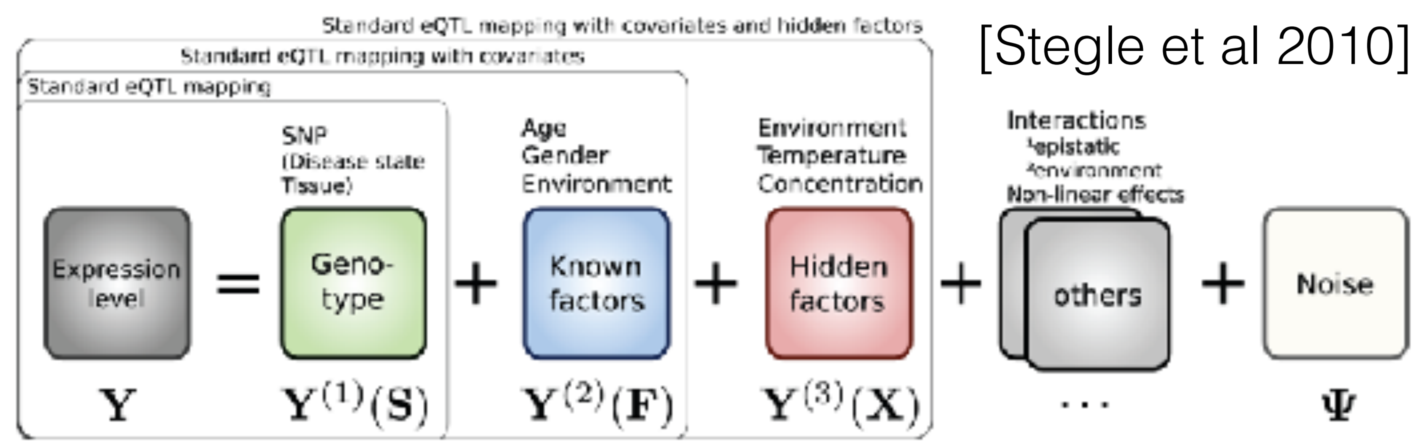
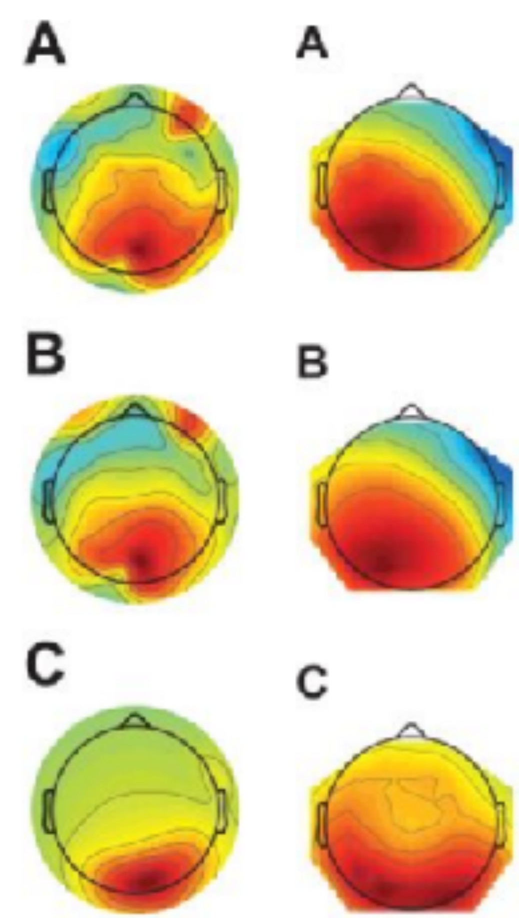
The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.



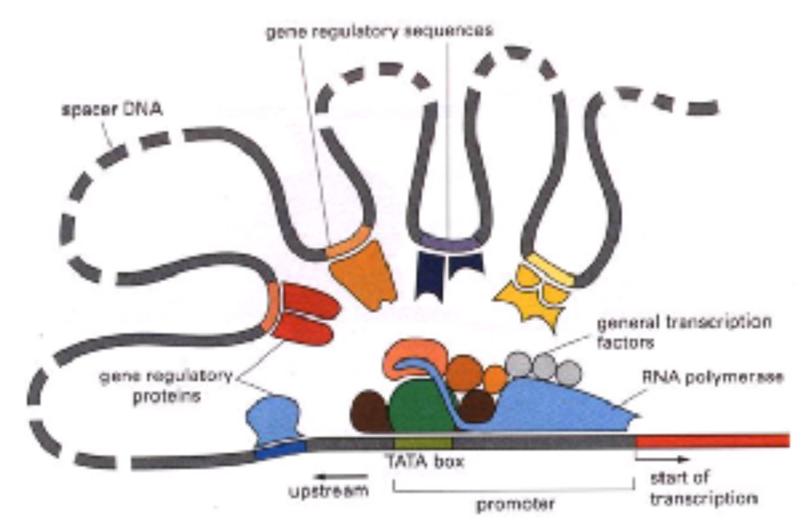
[Airoldi et al 2008]



[Stegle et al 2010]



[Gershman et al 2014]



[Xing et al 2004]
[Xing 2003]

[Blei et al 2018]

Roadmap

- Bayes & Approximate Bayes review

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- What is:
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  - Mean-field variational Bayes (MFVB)

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Bayesian inference

Bayesian inference

parameters

θ

Bayesian inference

parameters

$$p(\theta)$$

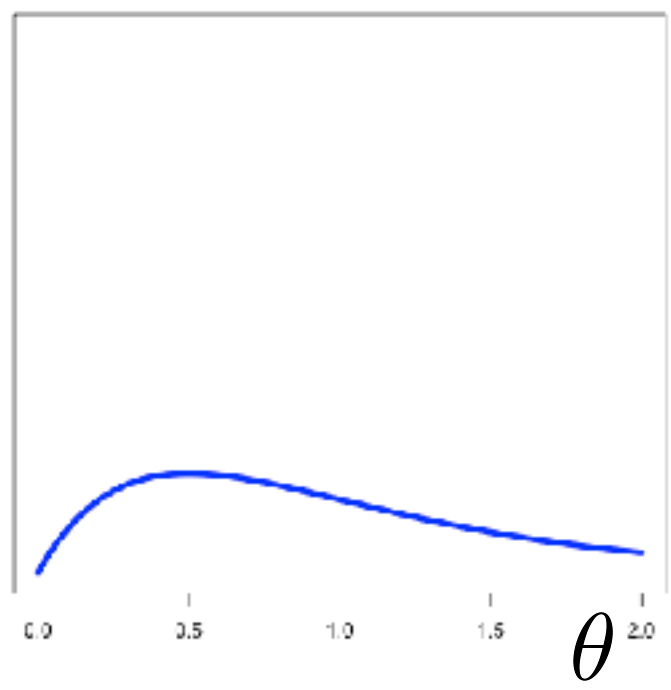
prior

Bayesian inference

parameters

$p(\theta)$

prior

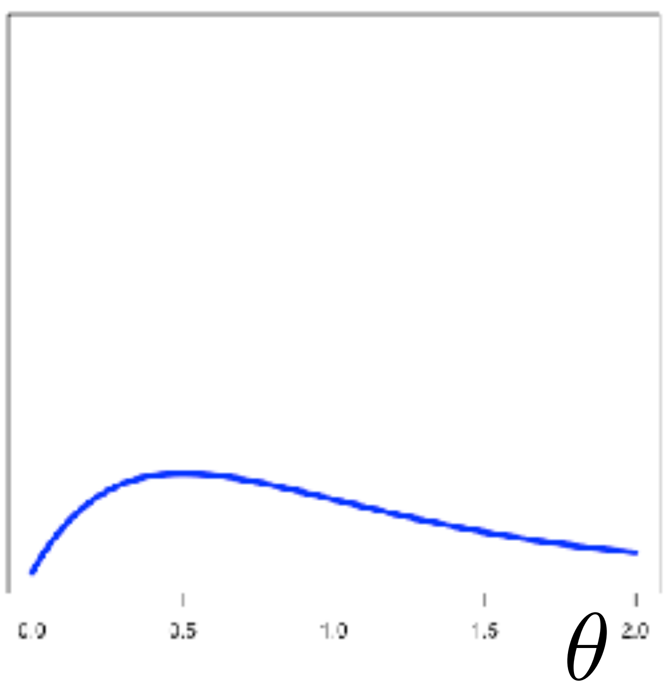


Bayesian inference

parameters

$$p(y_{1:N}|\theta)p(\theta)$$

likelihood prior



Bayesian inference

data → parameters

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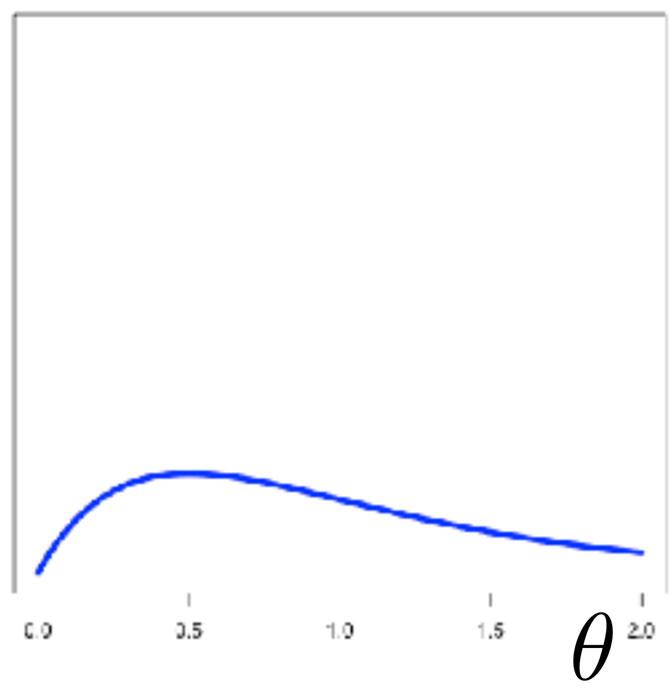
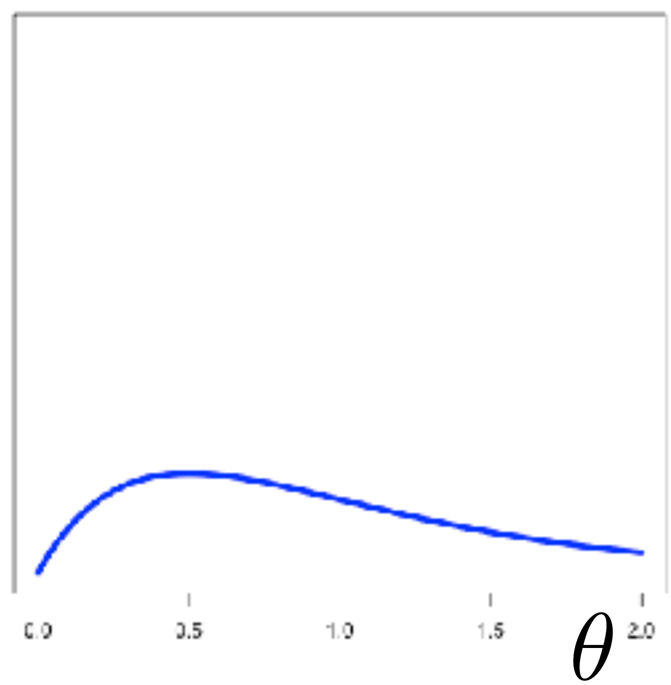


Bayesian inference

data → parameters

$$p(\theta|y_{1:N}) \propto_\theta p(y_{1:N}|\theta)p(\theta)$$

posterior likelihood prior



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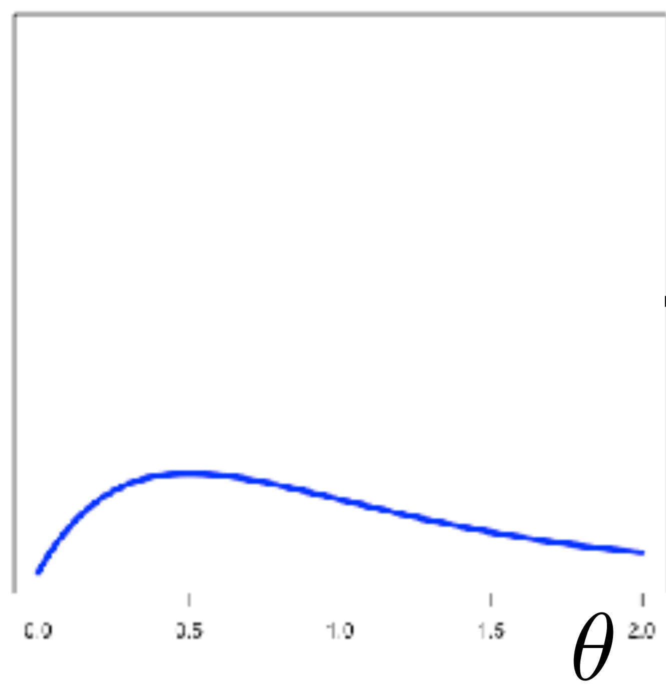

Bayes Theorem

Bayesian inference

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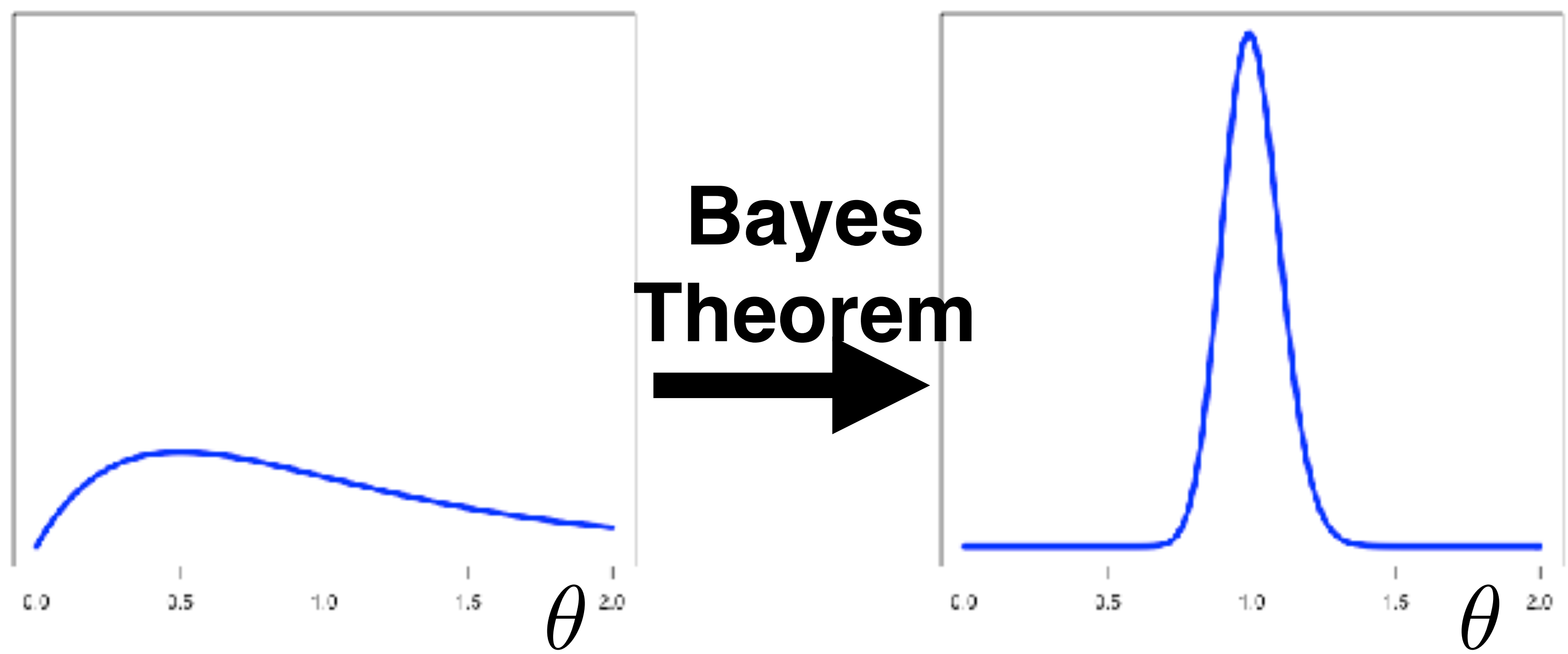

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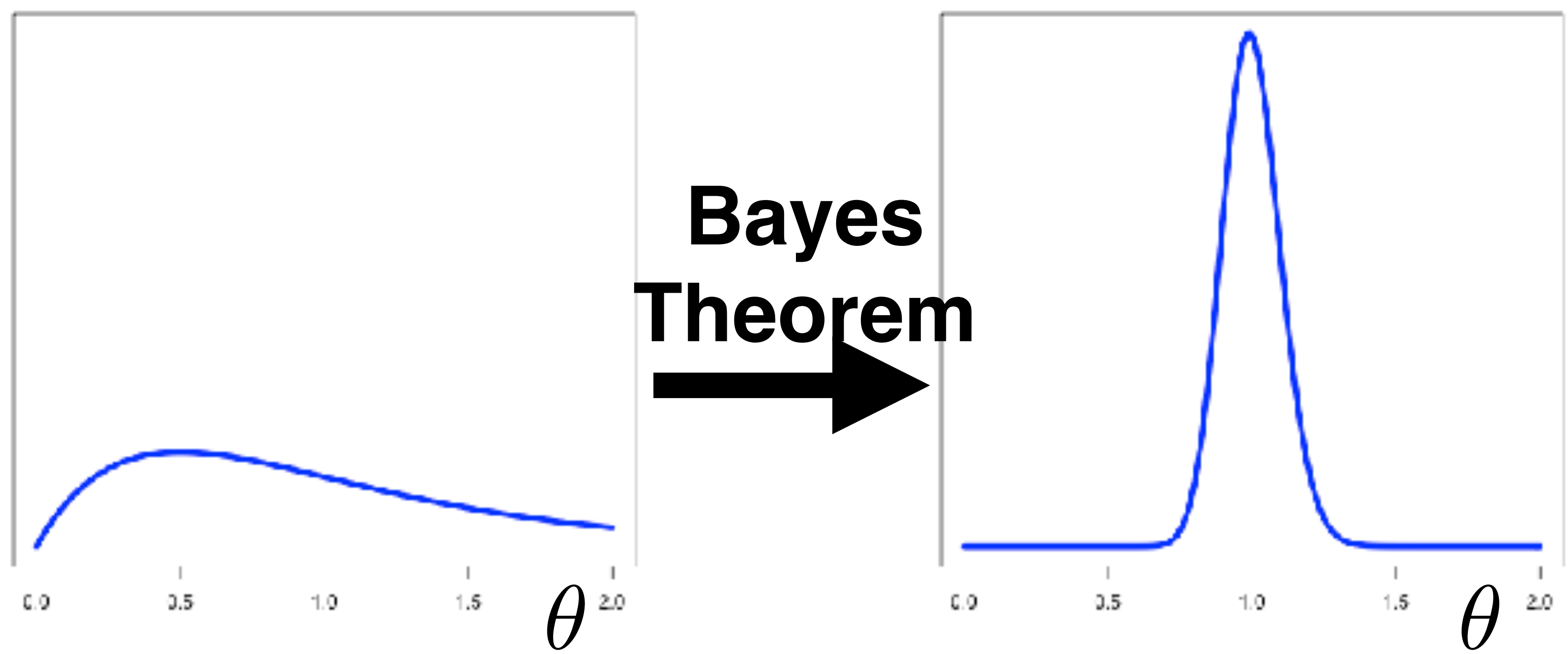

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Bayesian inference

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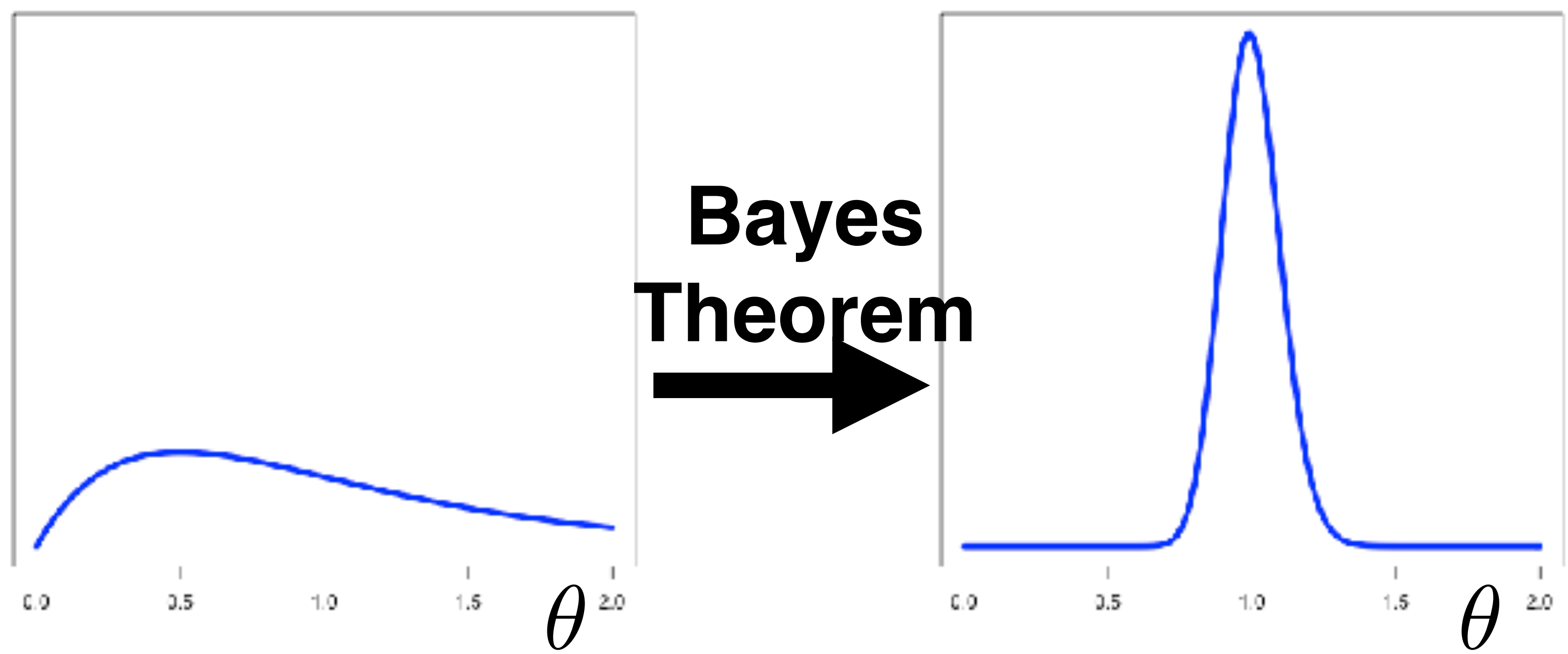

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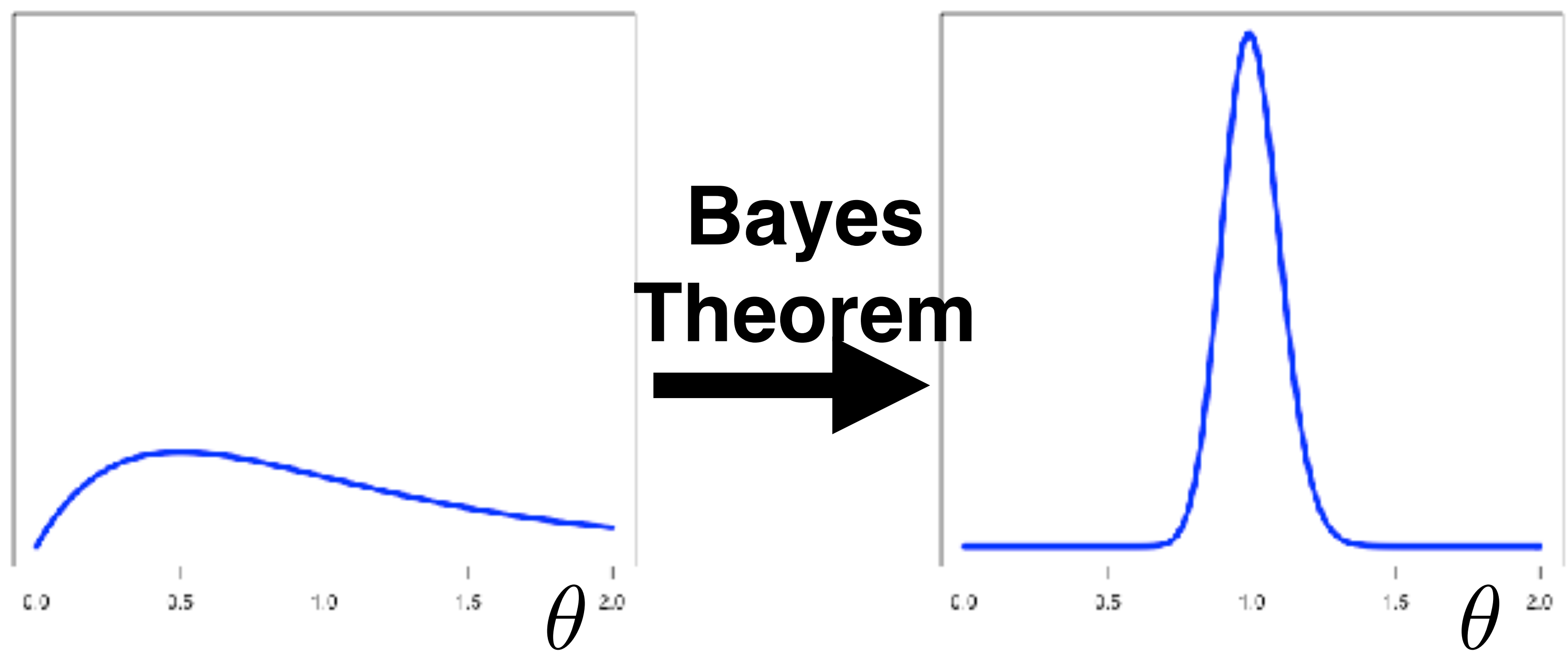

1. Build a model: choose prior & choose likelihood
2. Compute the posterior
3. Report a summary, e.g. posterior means and (co)variances

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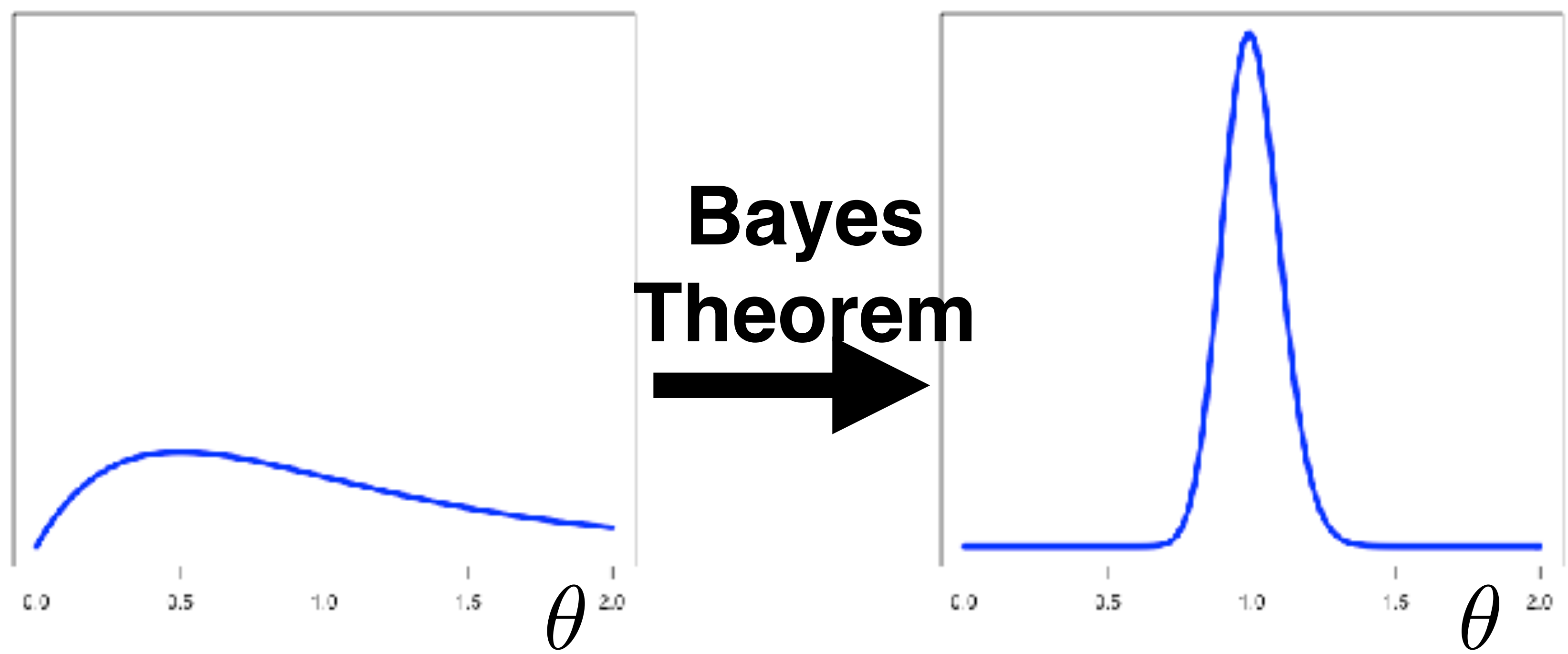

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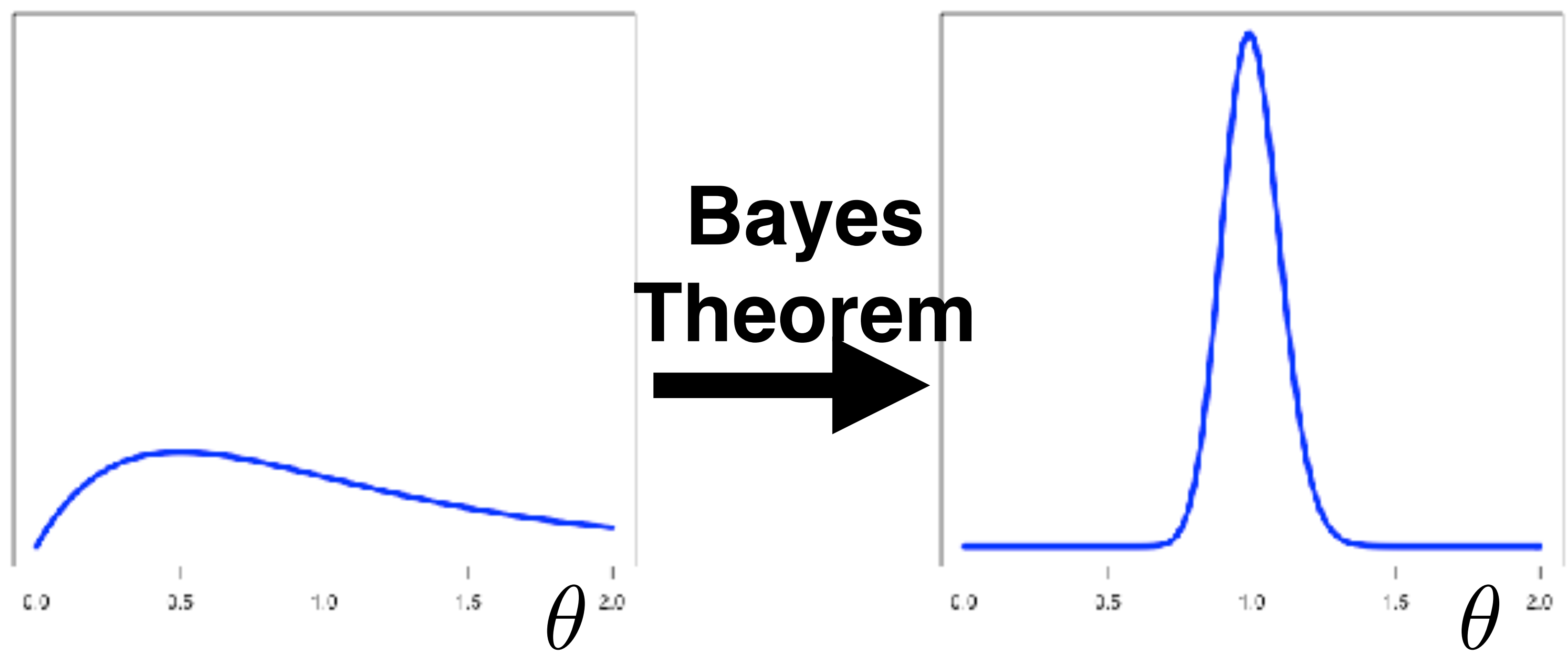

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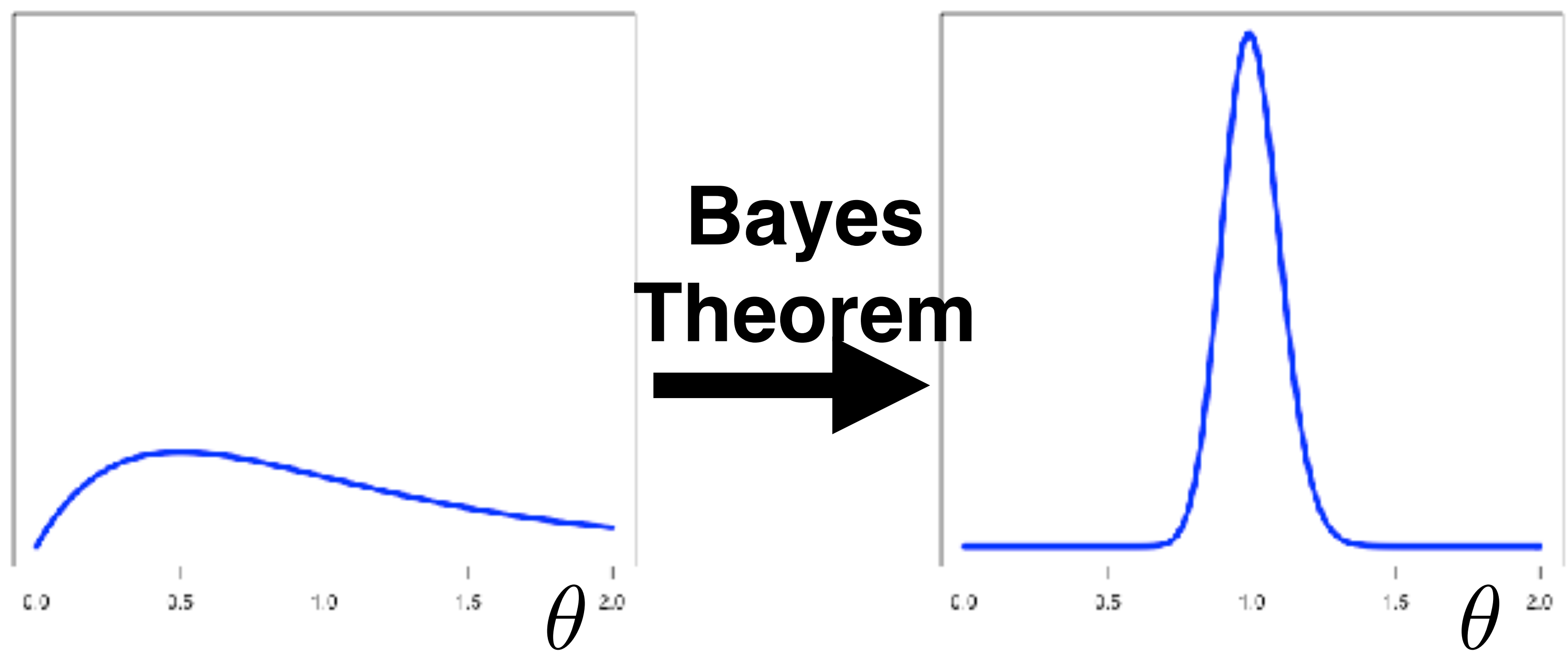

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Bayesian inference

data parameters

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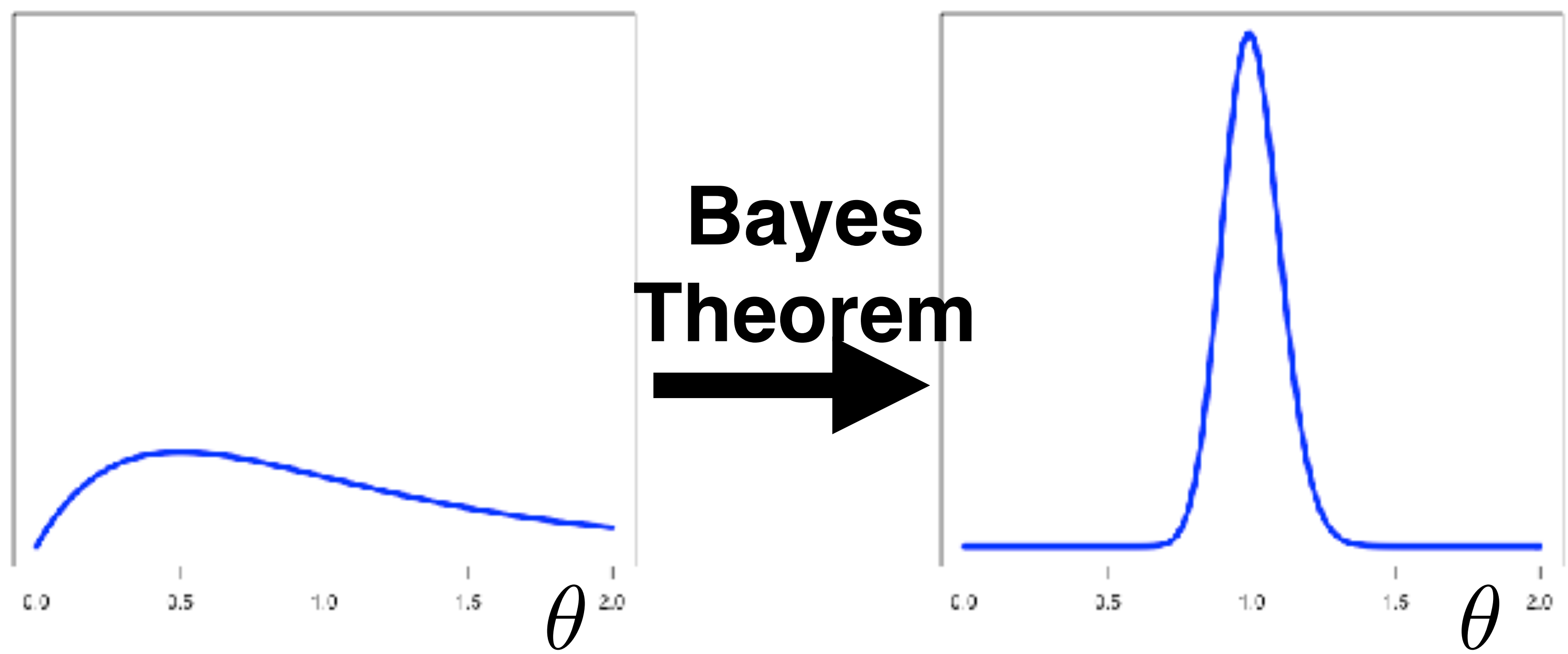
posterior likelihood prior



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Bayesian inference

data → $y_{1:N}$, parameters → θ

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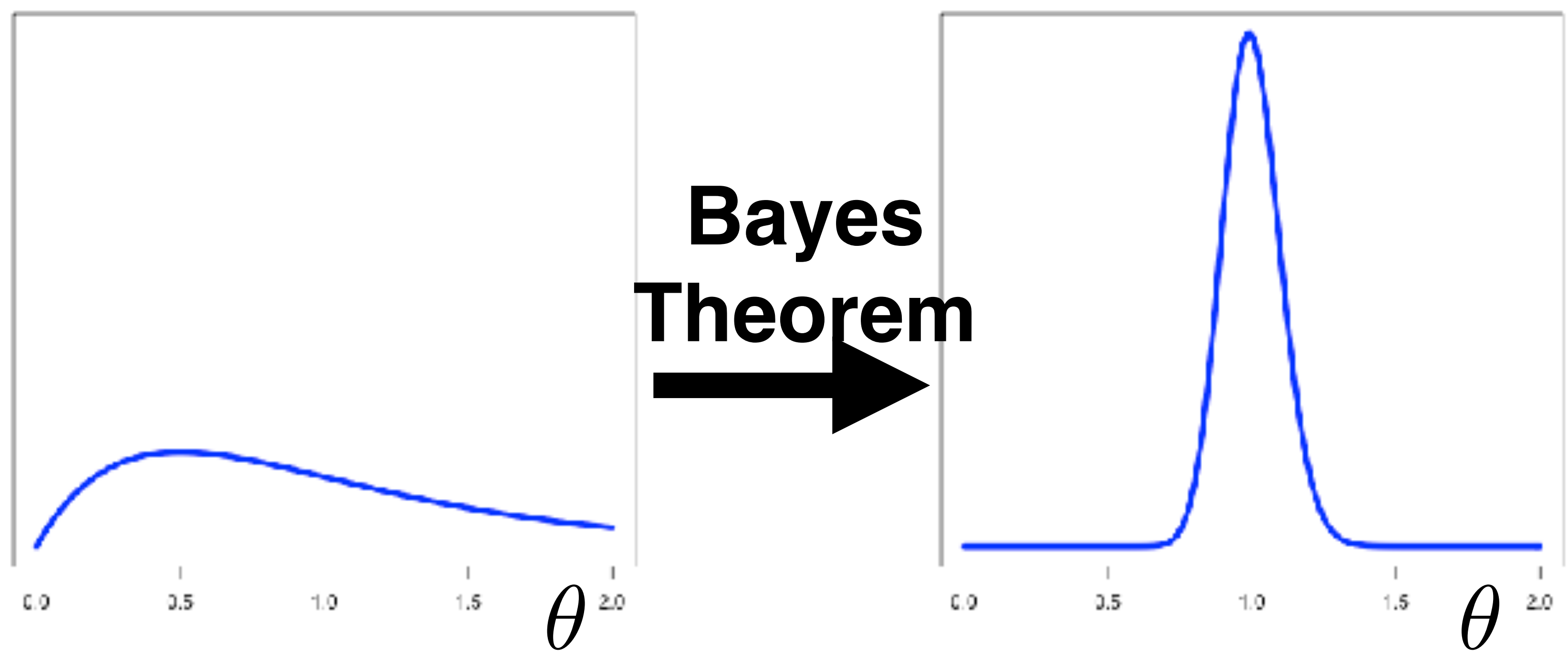
posterior likelihood prior evidence



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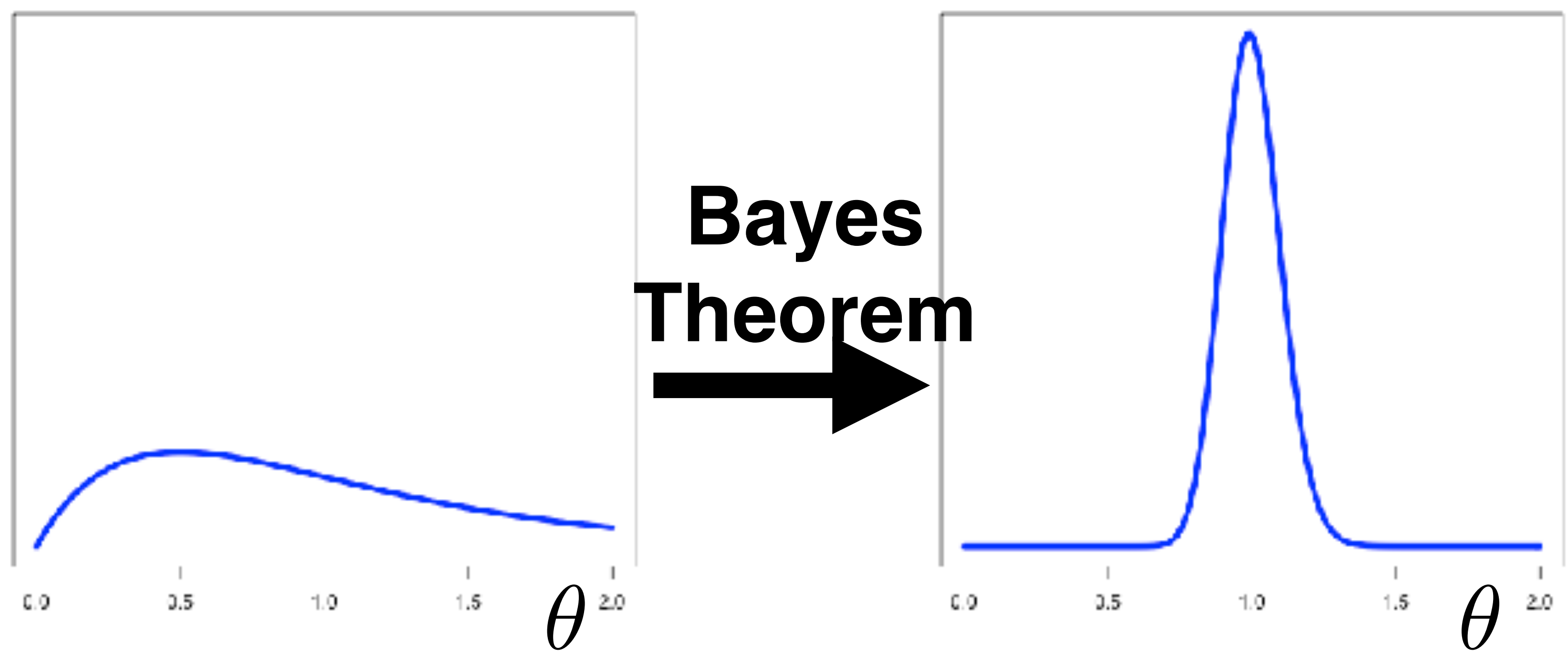
data → $y_{1:N}$, parameters → θ

$$p(\theta|y_{1:N}) = p(y_{1:N}|\theta)p(\theta)/\int p(y_{1:N},\theta)d\theta$$

posterior | likelihood | prior | evidence



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- Gold standard: Markov Chain Monte Carlo (MCMC)

[Bardenet, Doucet, Holmes 2017]

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Instead: an optimization approach

- Approximate posterior with q^*

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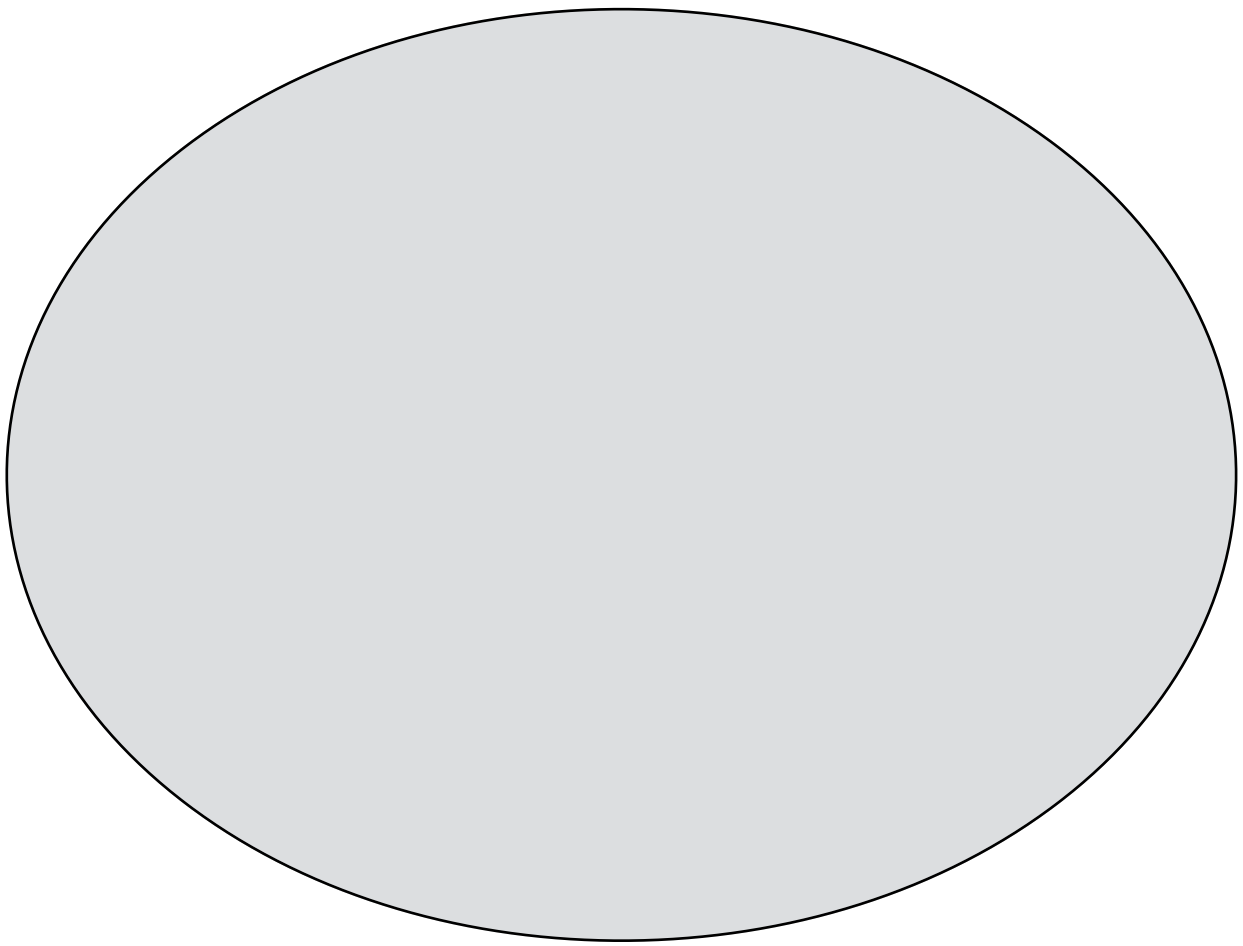

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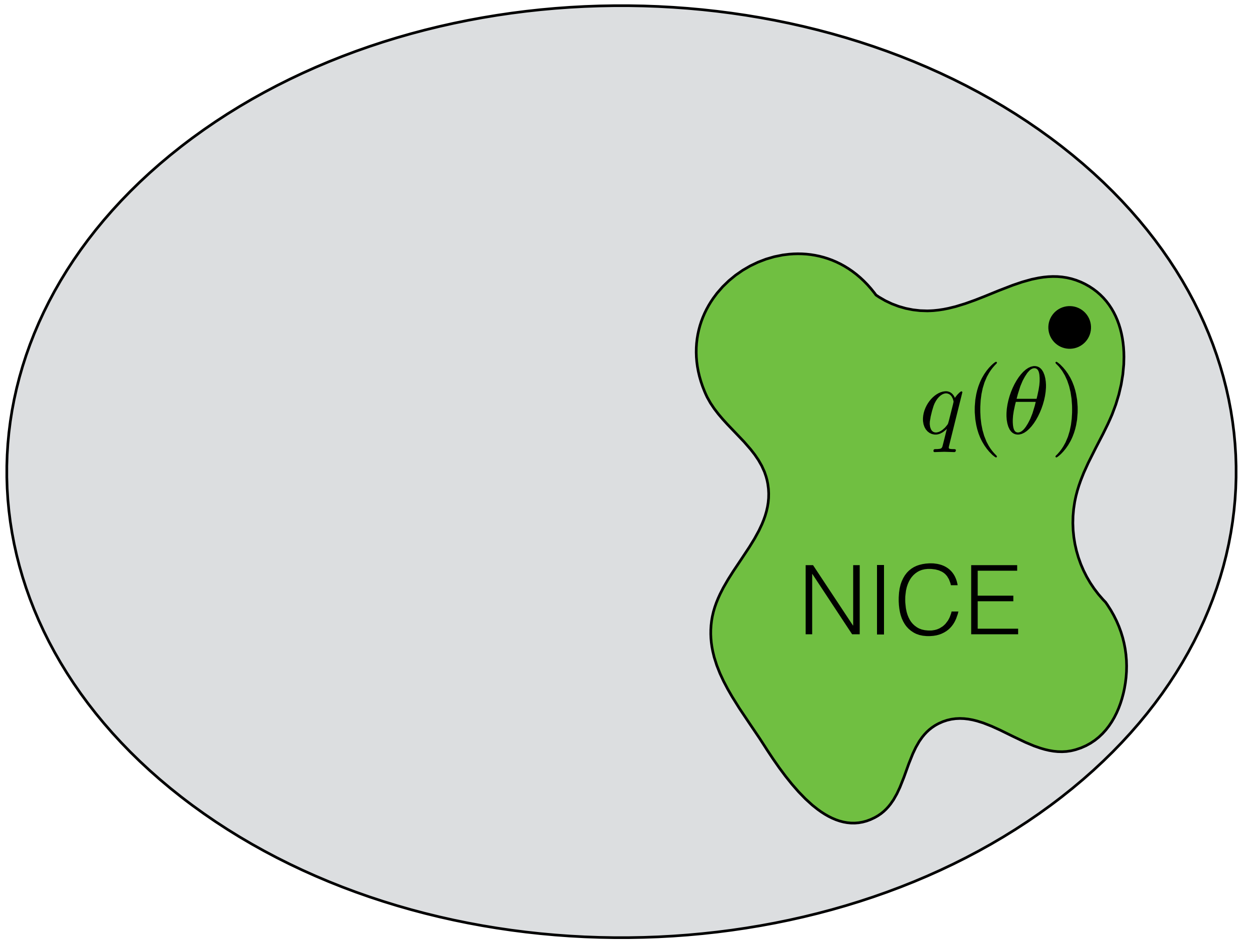

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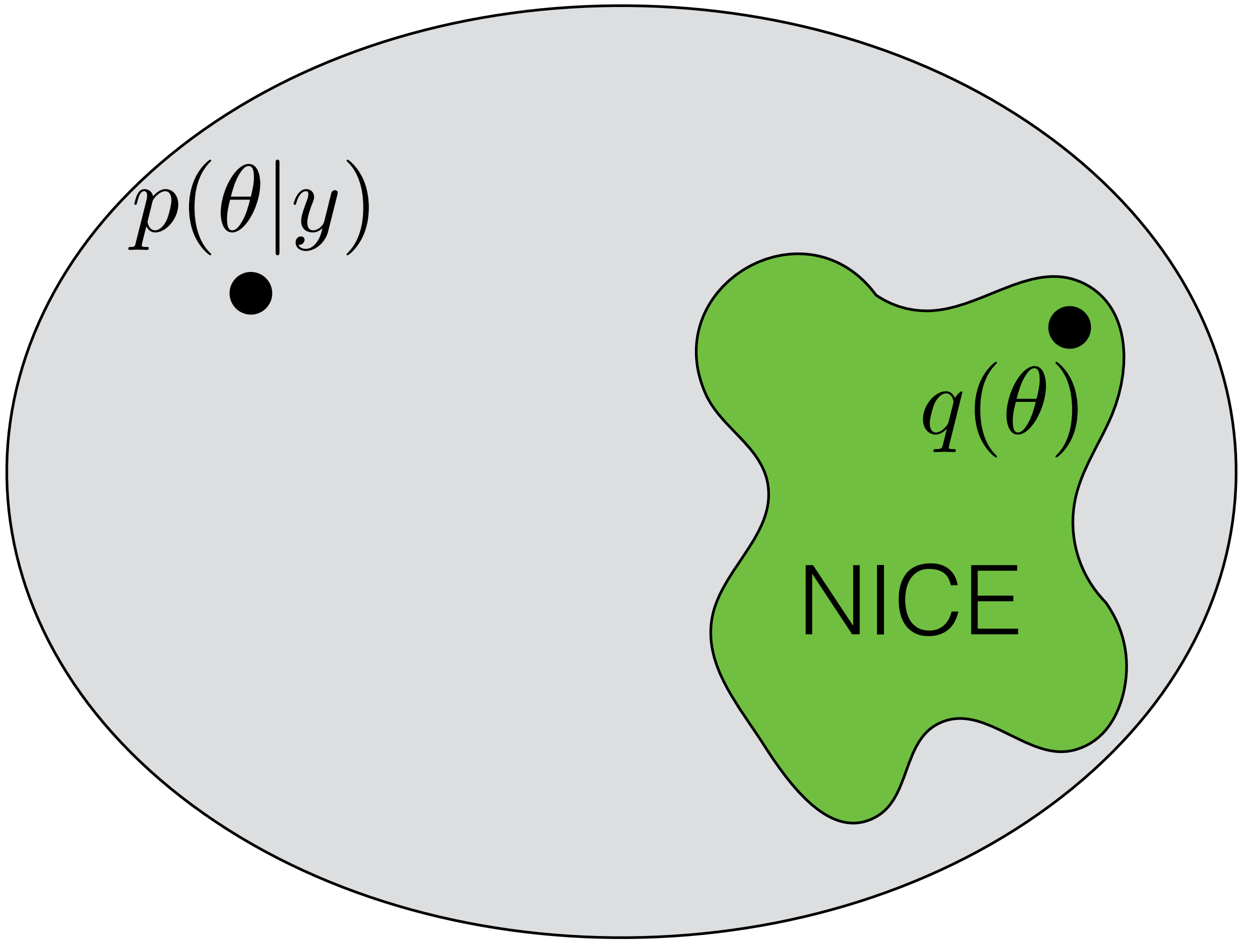

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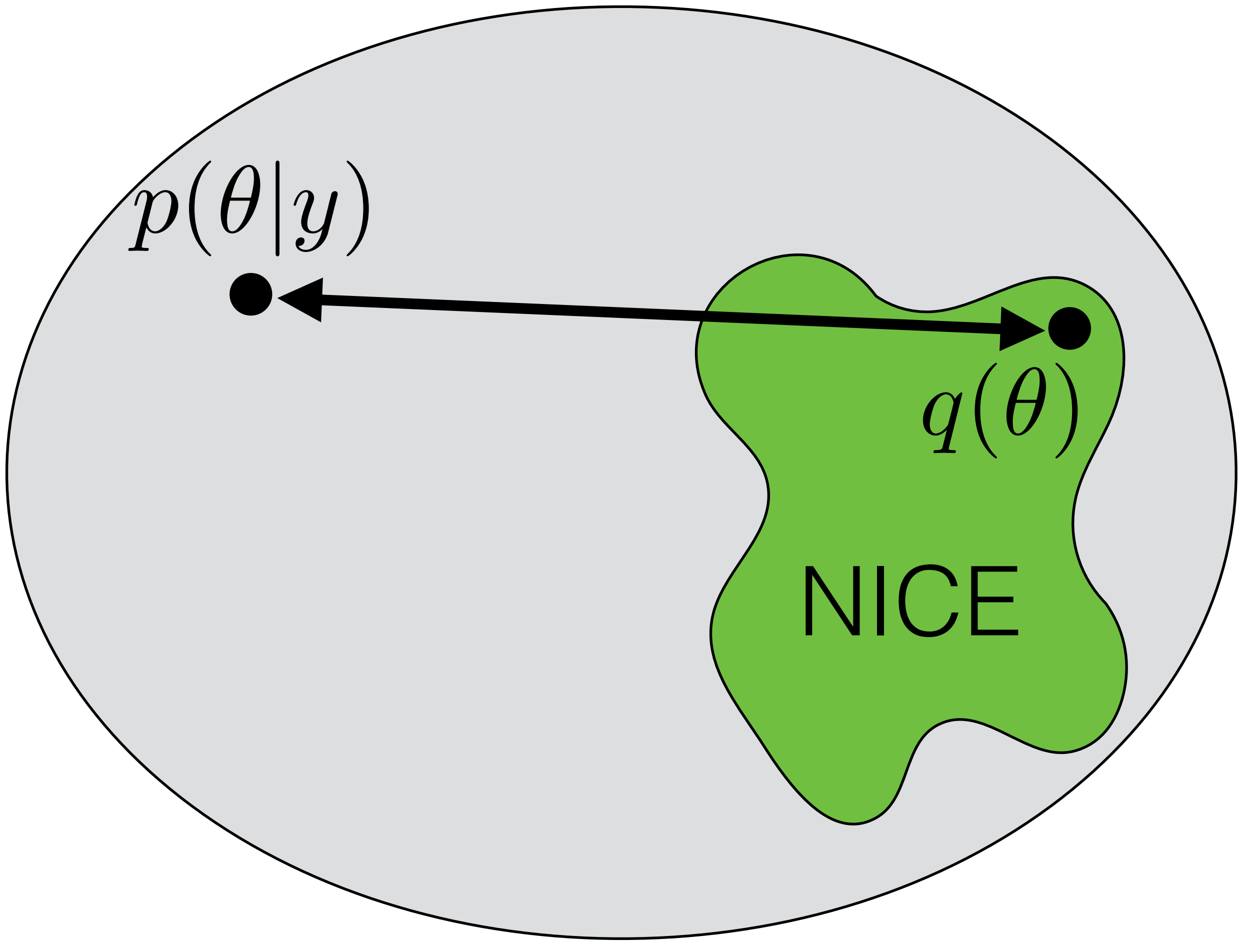

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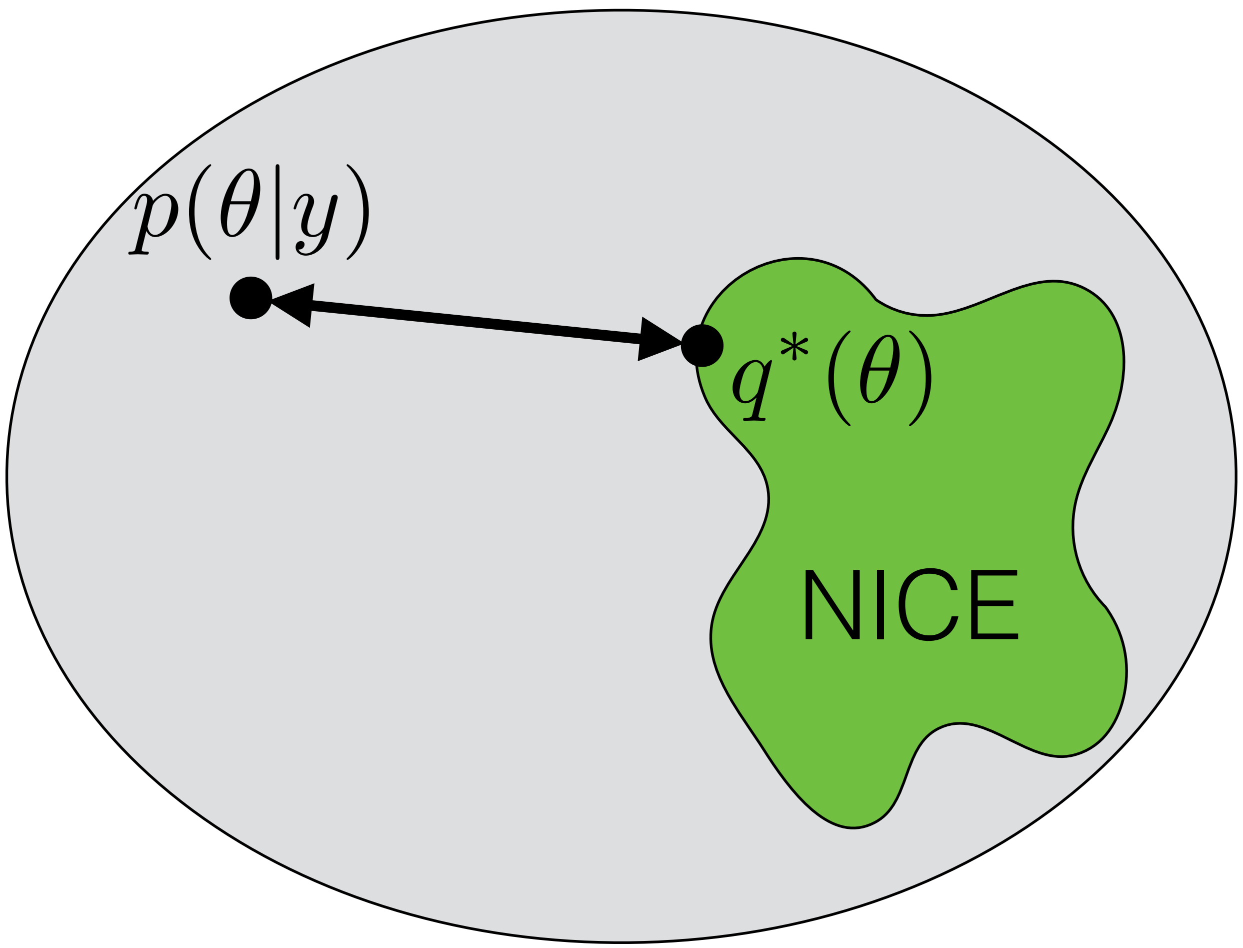

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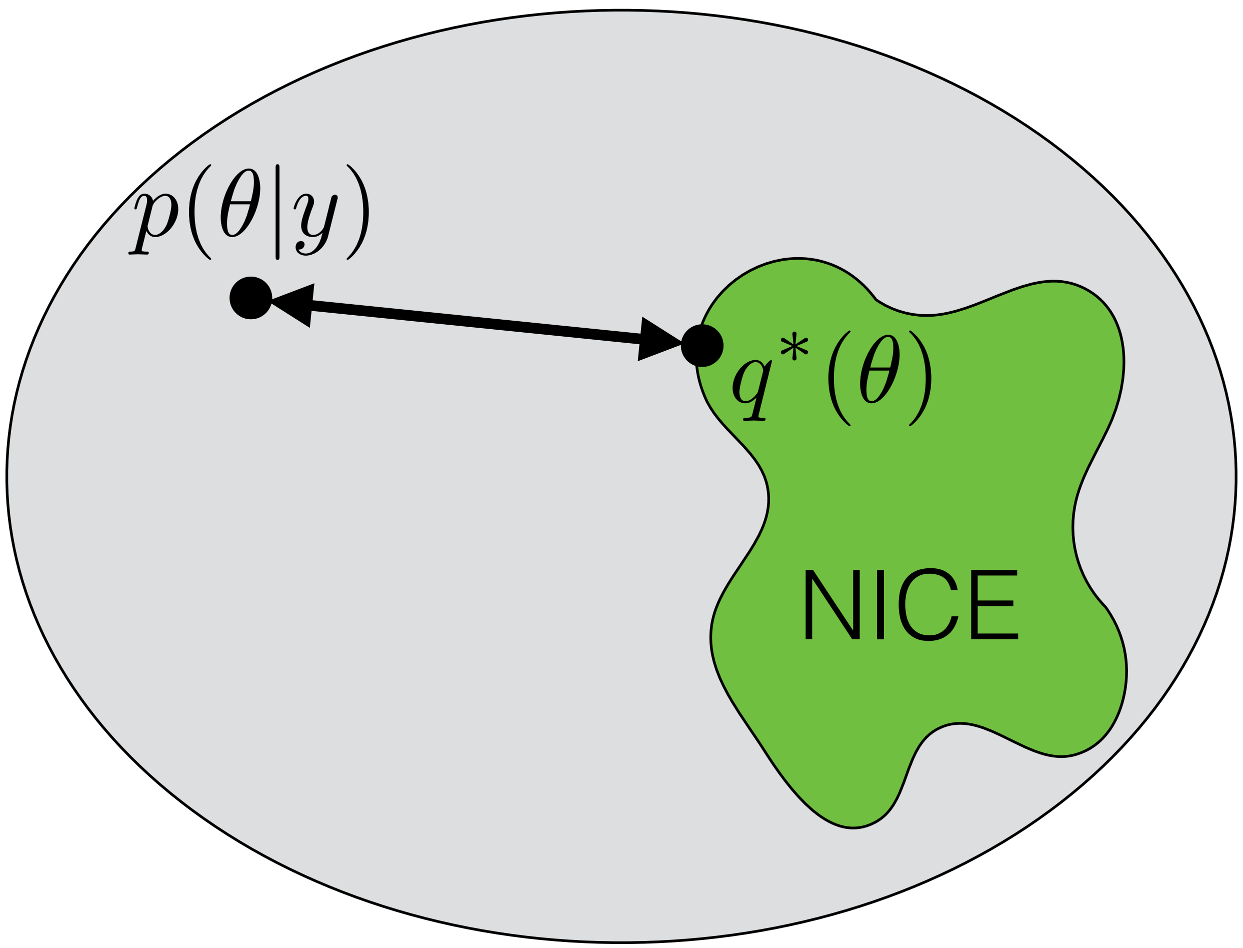

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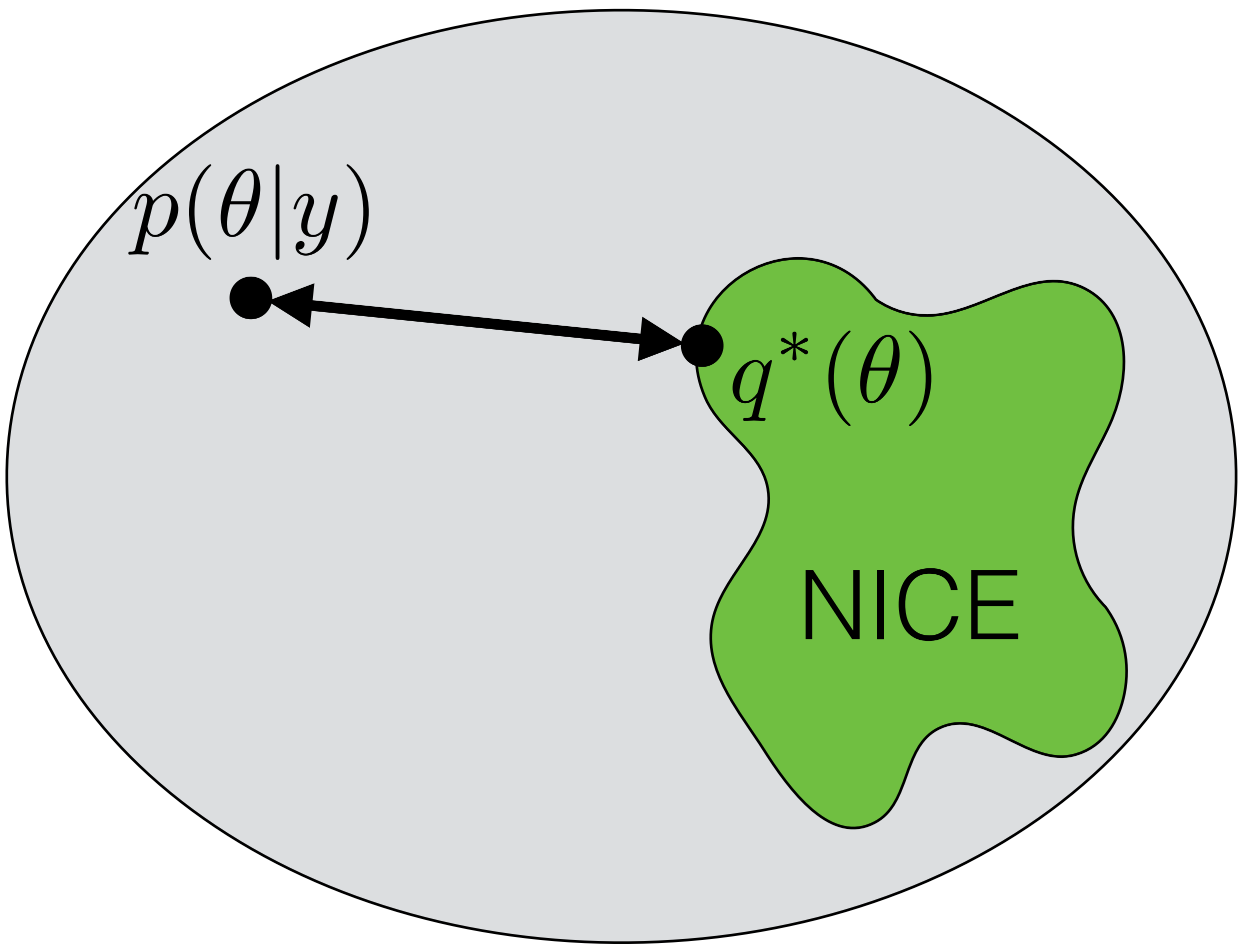

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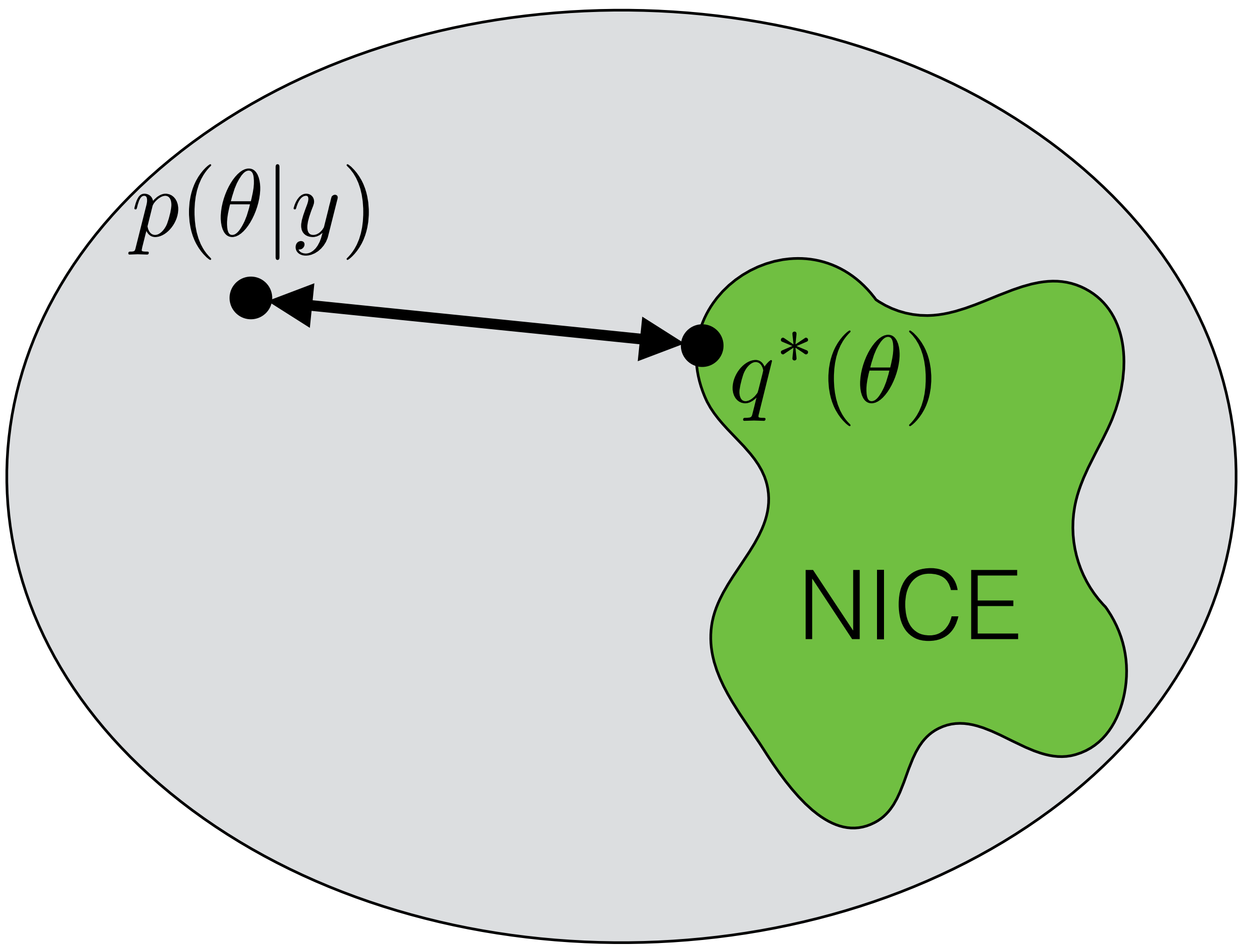

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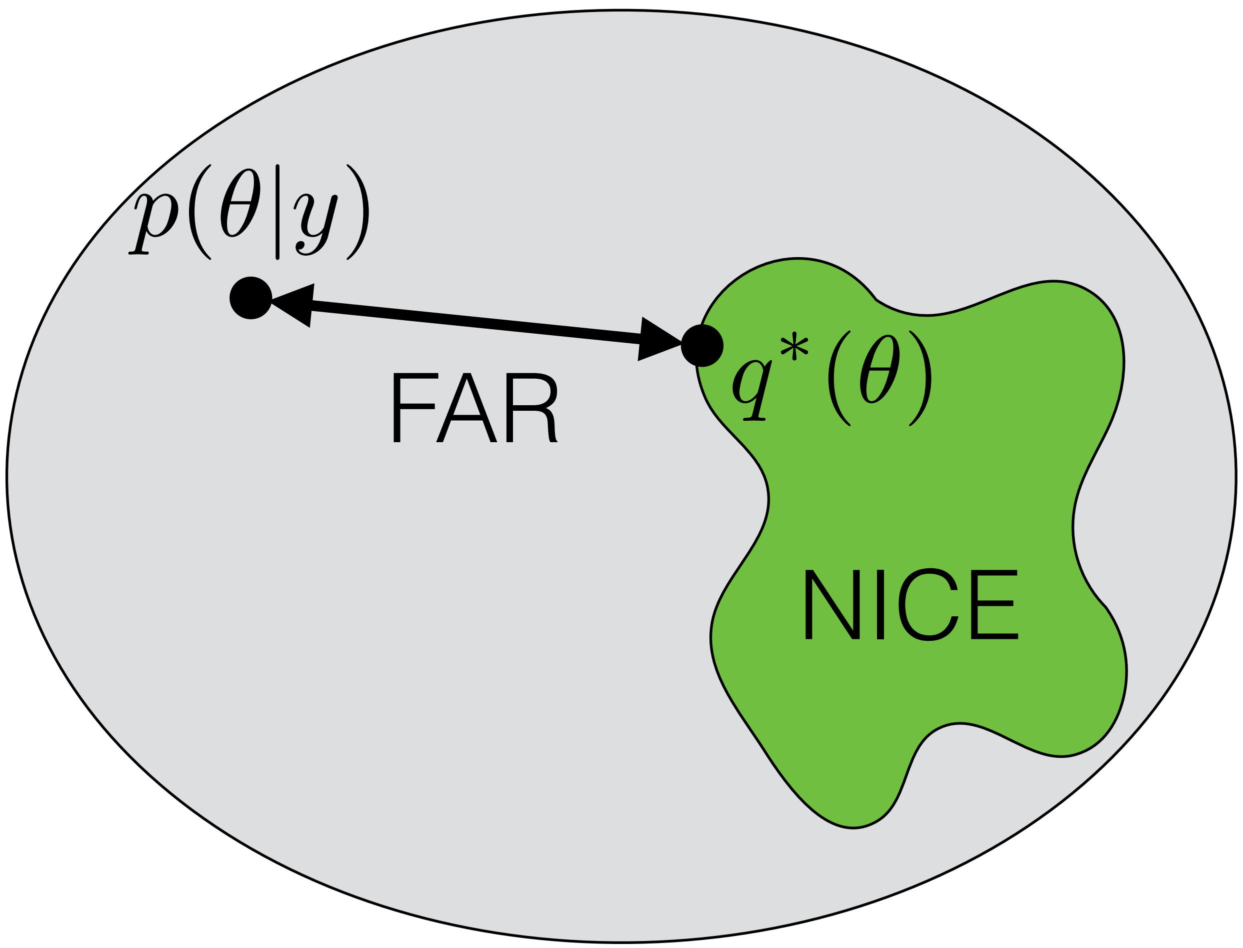

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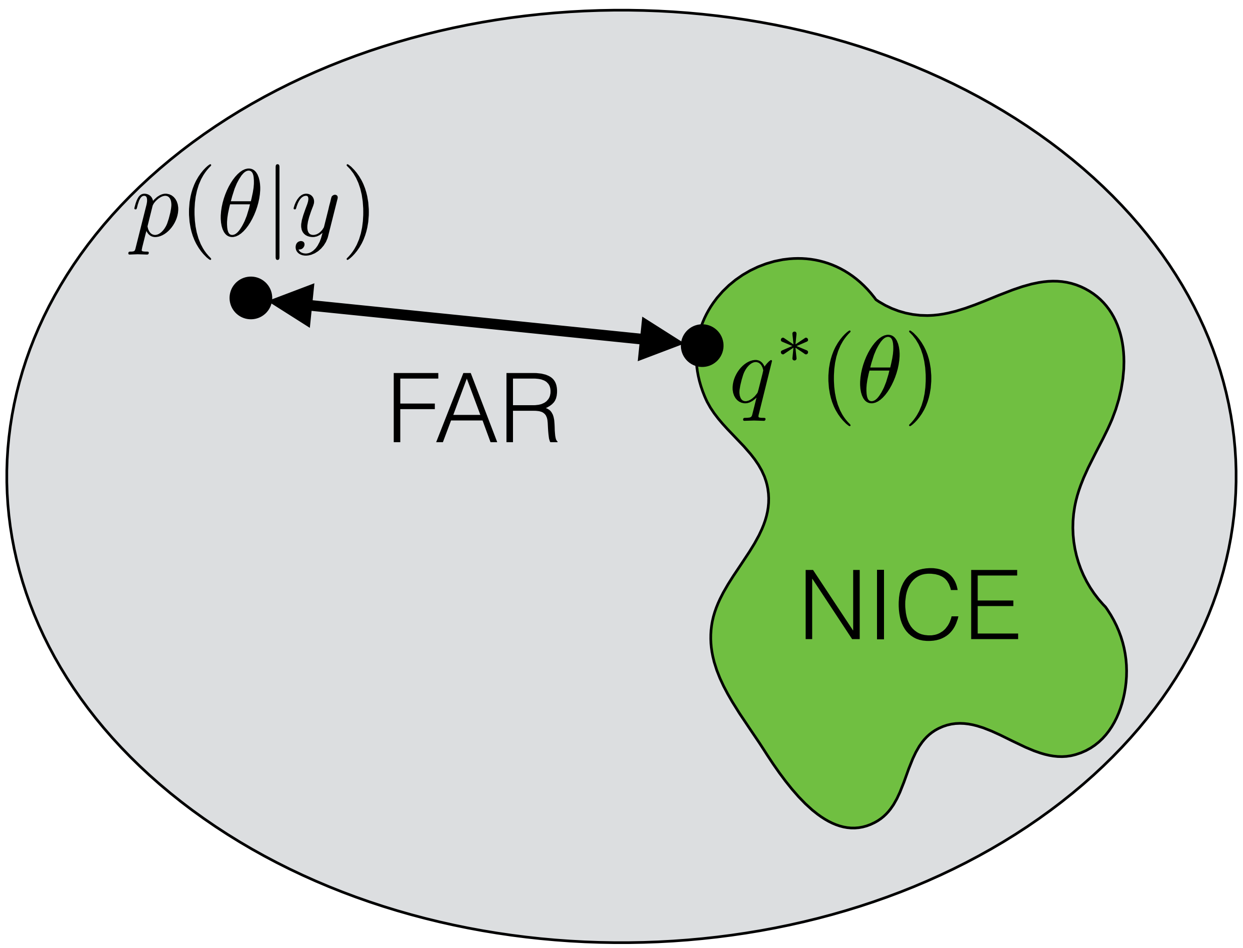

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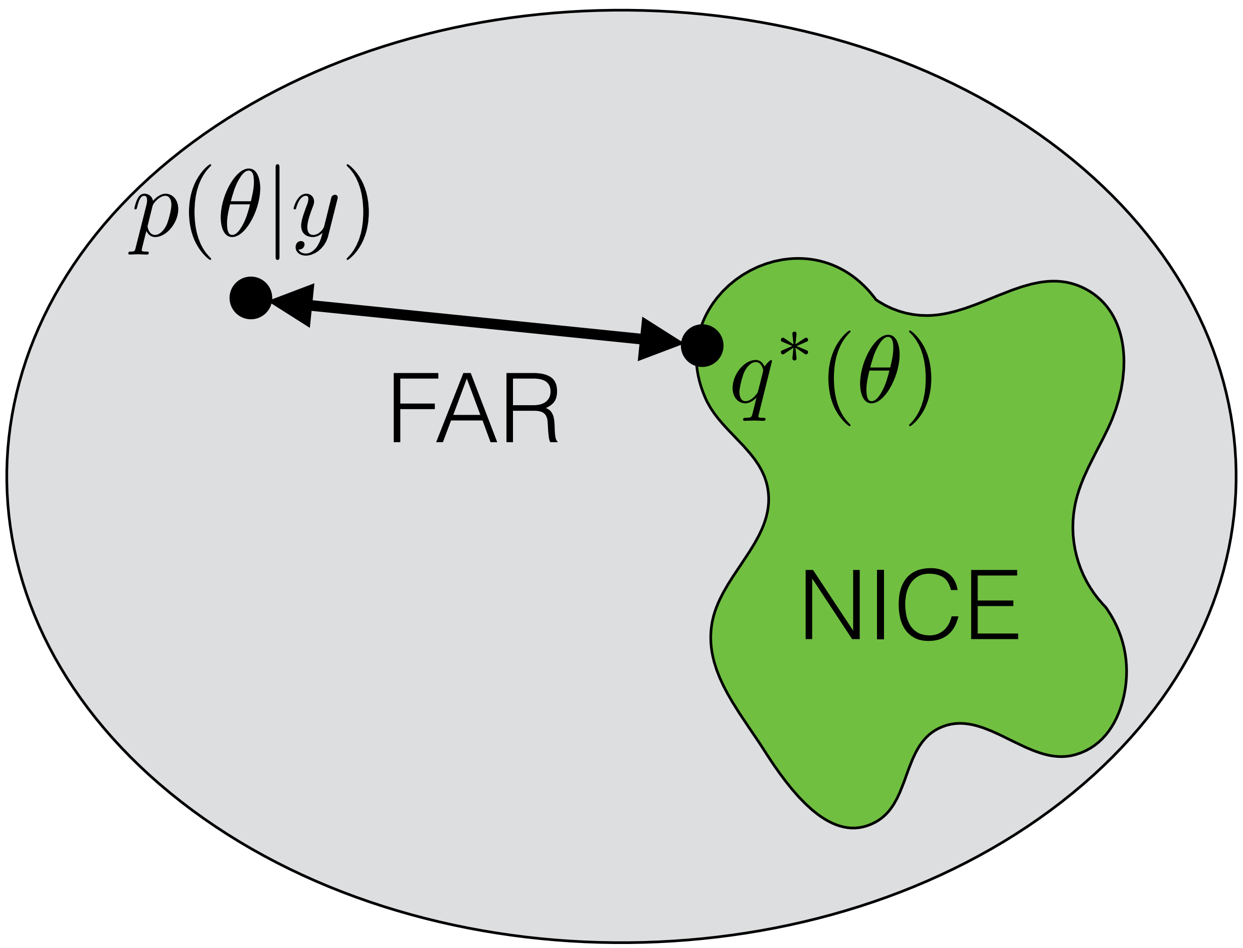

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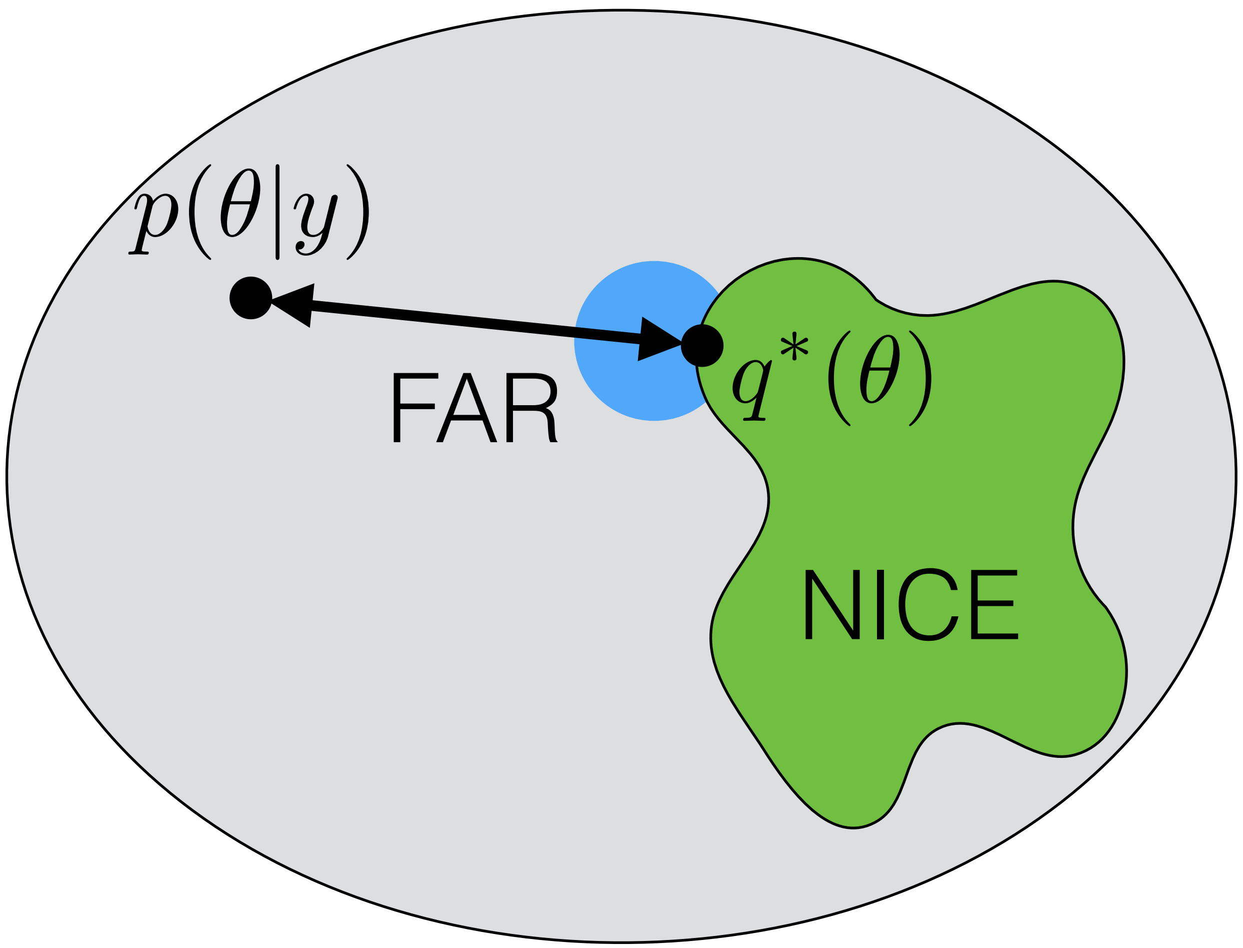

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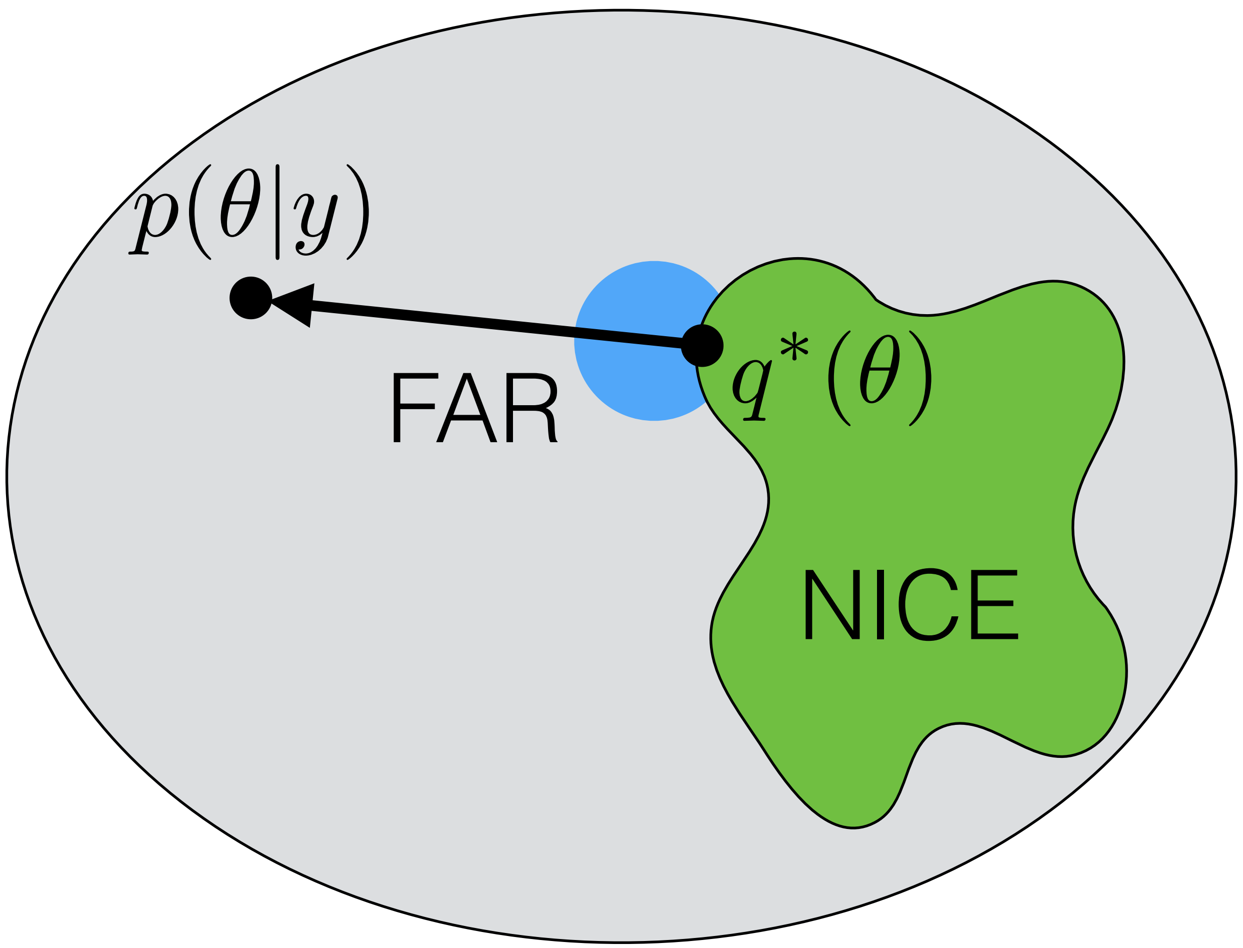

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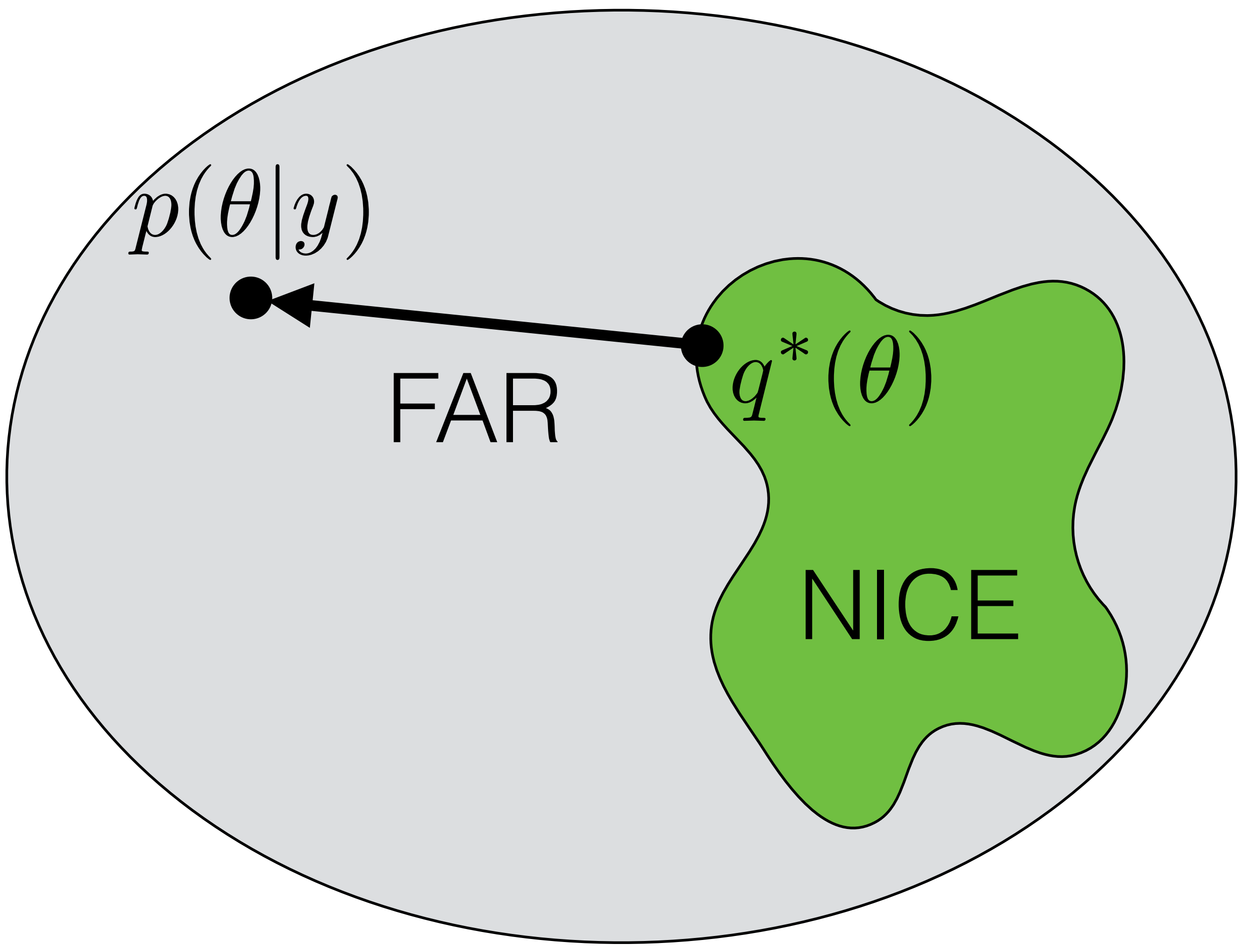

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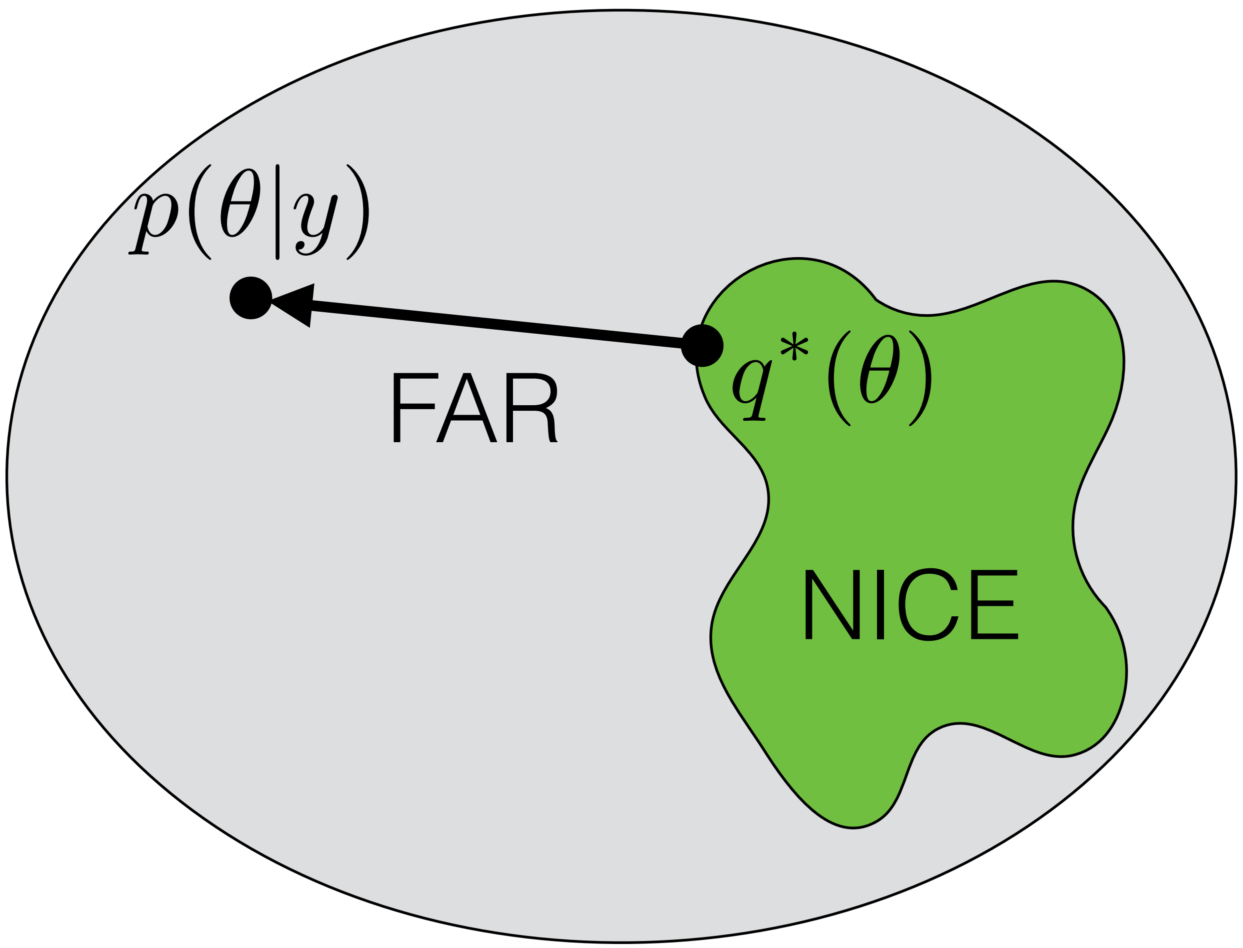

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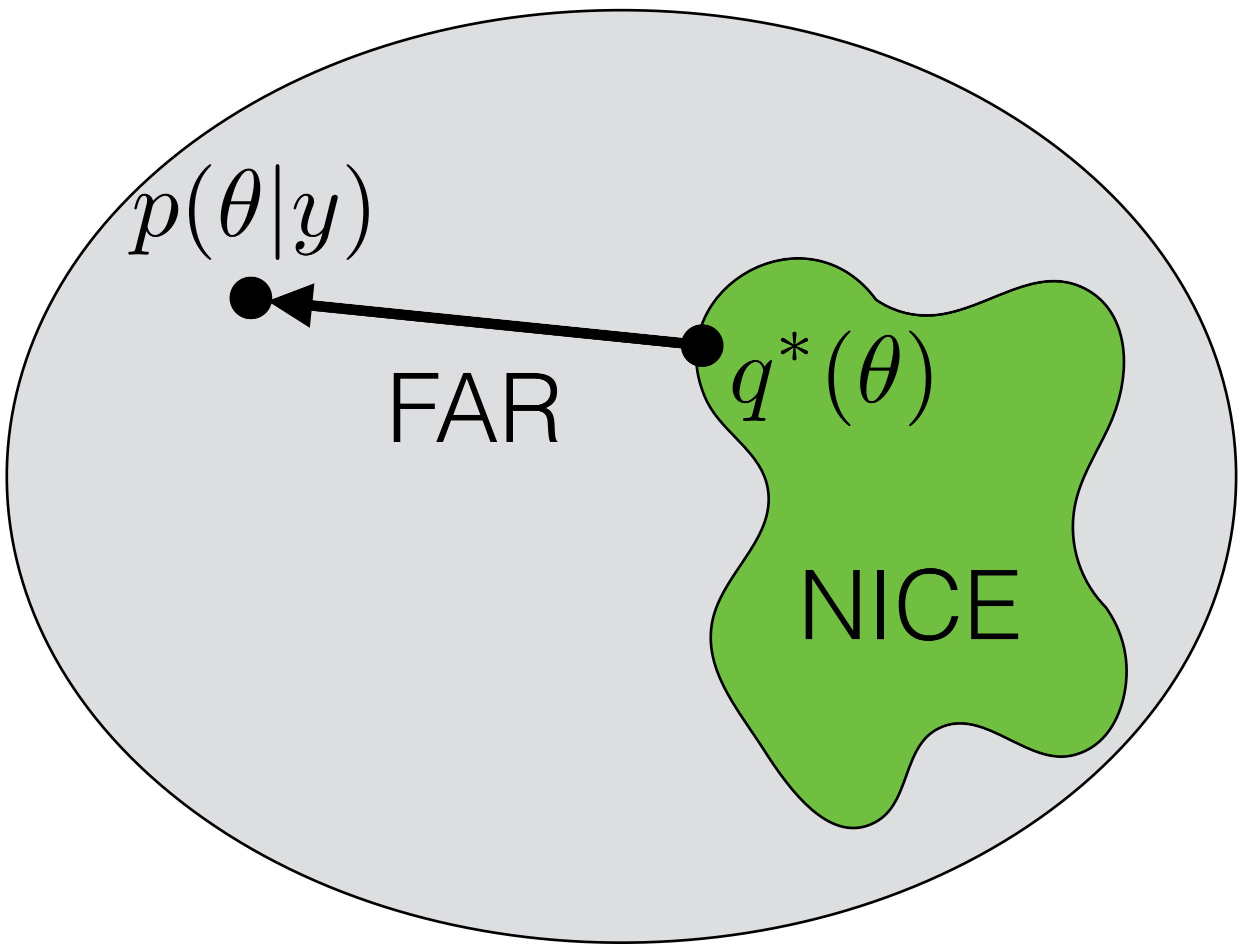

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Approximate Bayesian Inference

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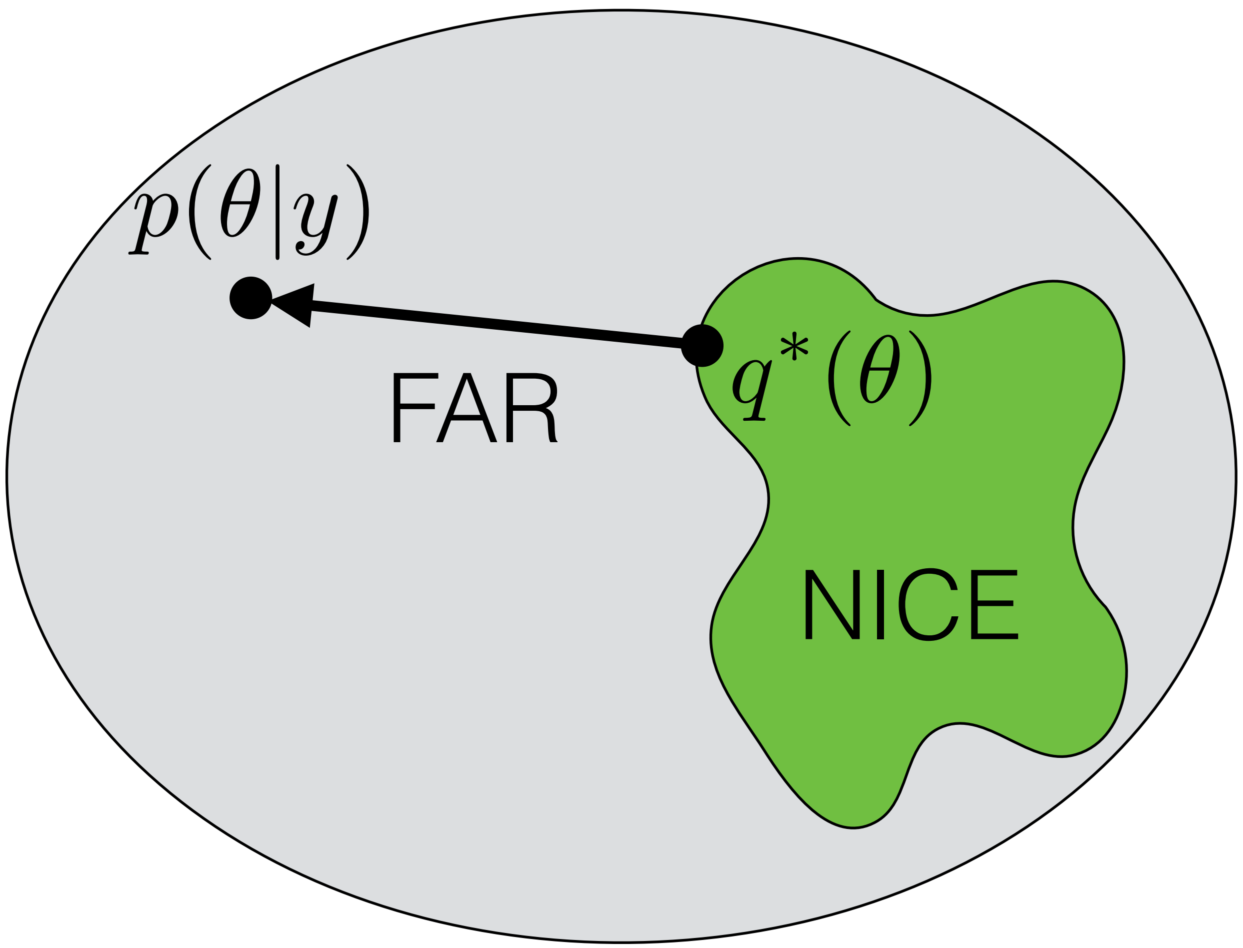

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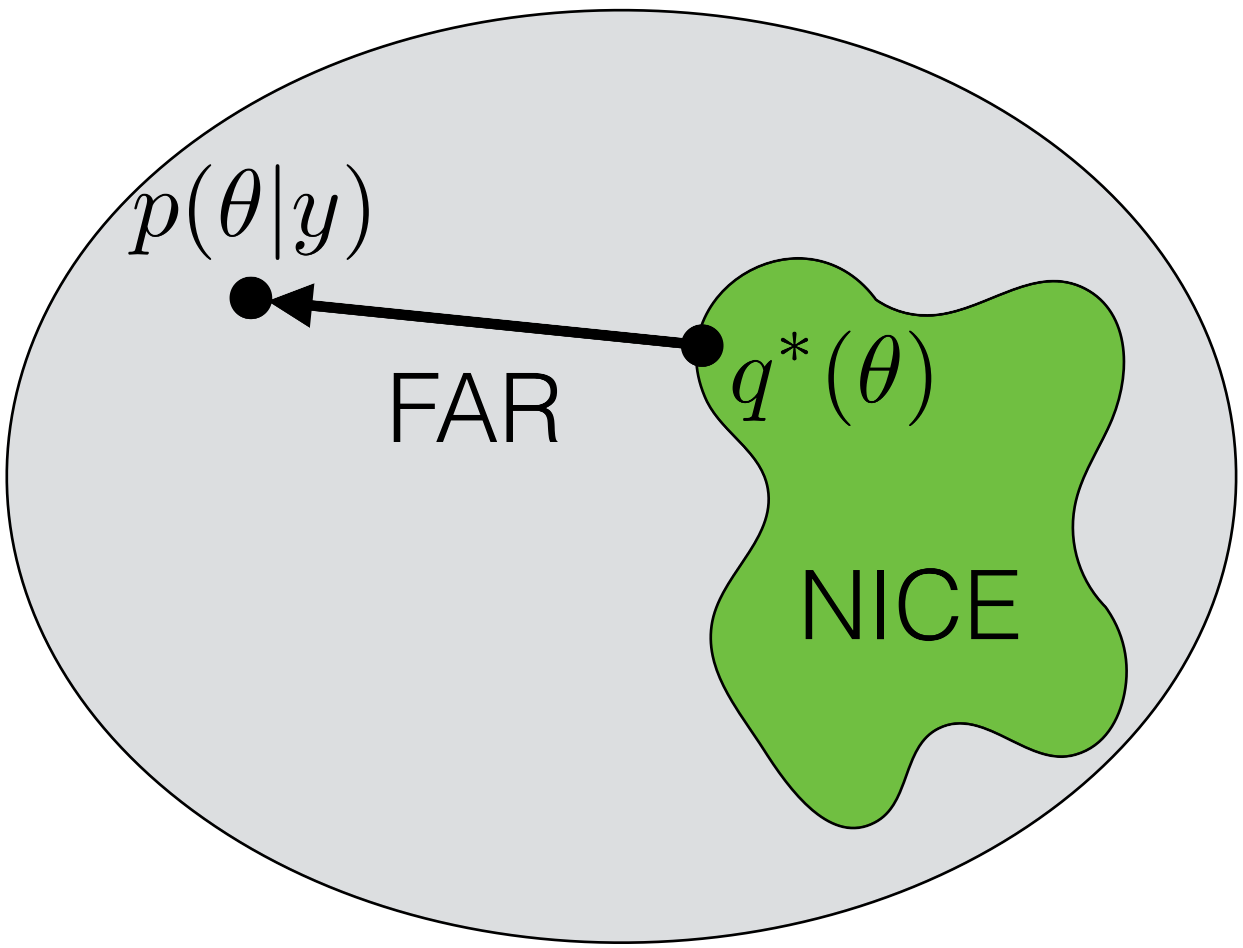

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$$KL(q(\cdot)||p(\cdot|y))$$

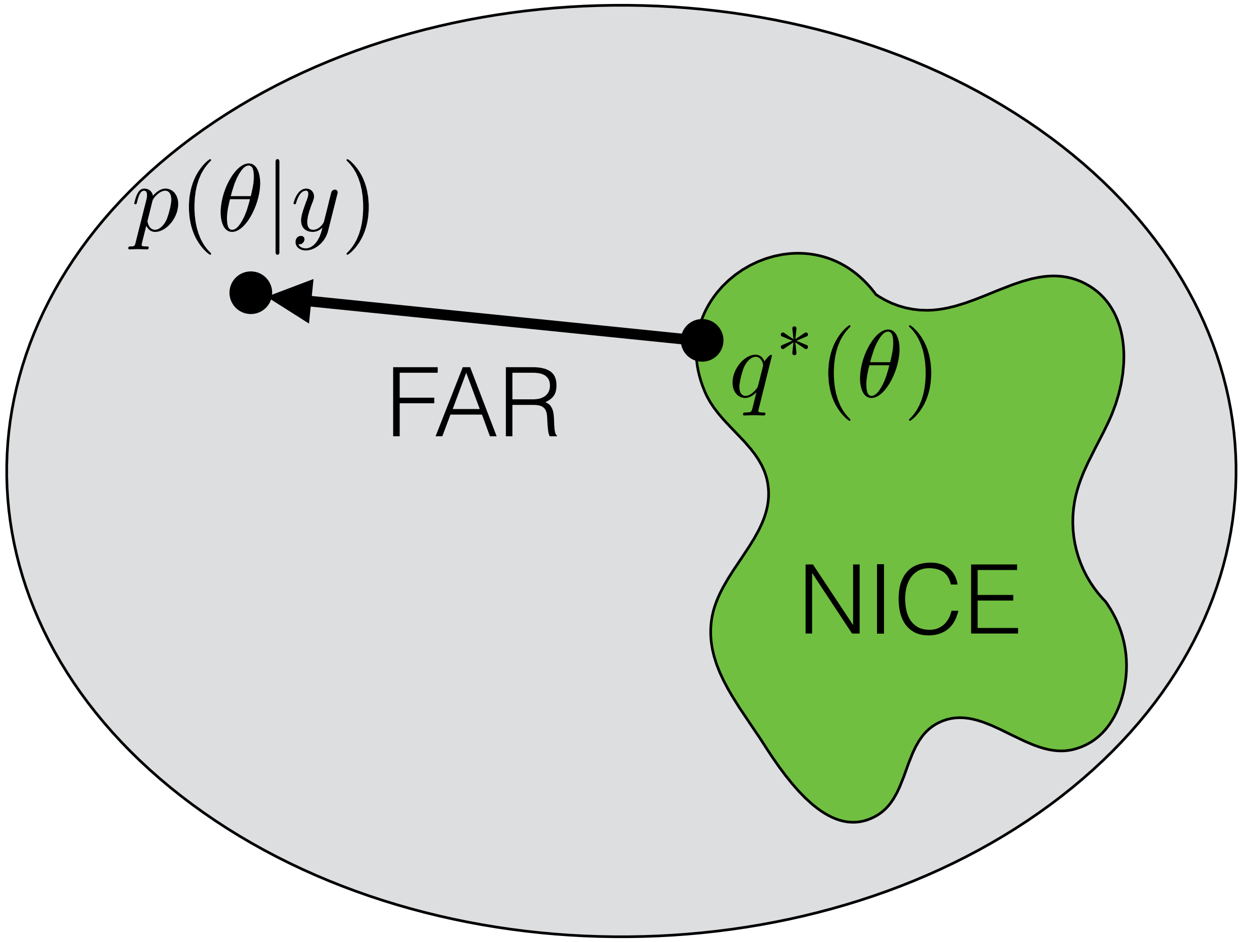
- VB practical success: point estimates and prediction, fast, streaming, distributed (3.6M Wikipedia, 350K Nature)

[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

Why KL?

- Variational Bayes

$$q^* = \text{argmin}_{q \in Q} \text{KL}\left(q(\cdot) || p(\cdot|y)\right)$$



Why KL?

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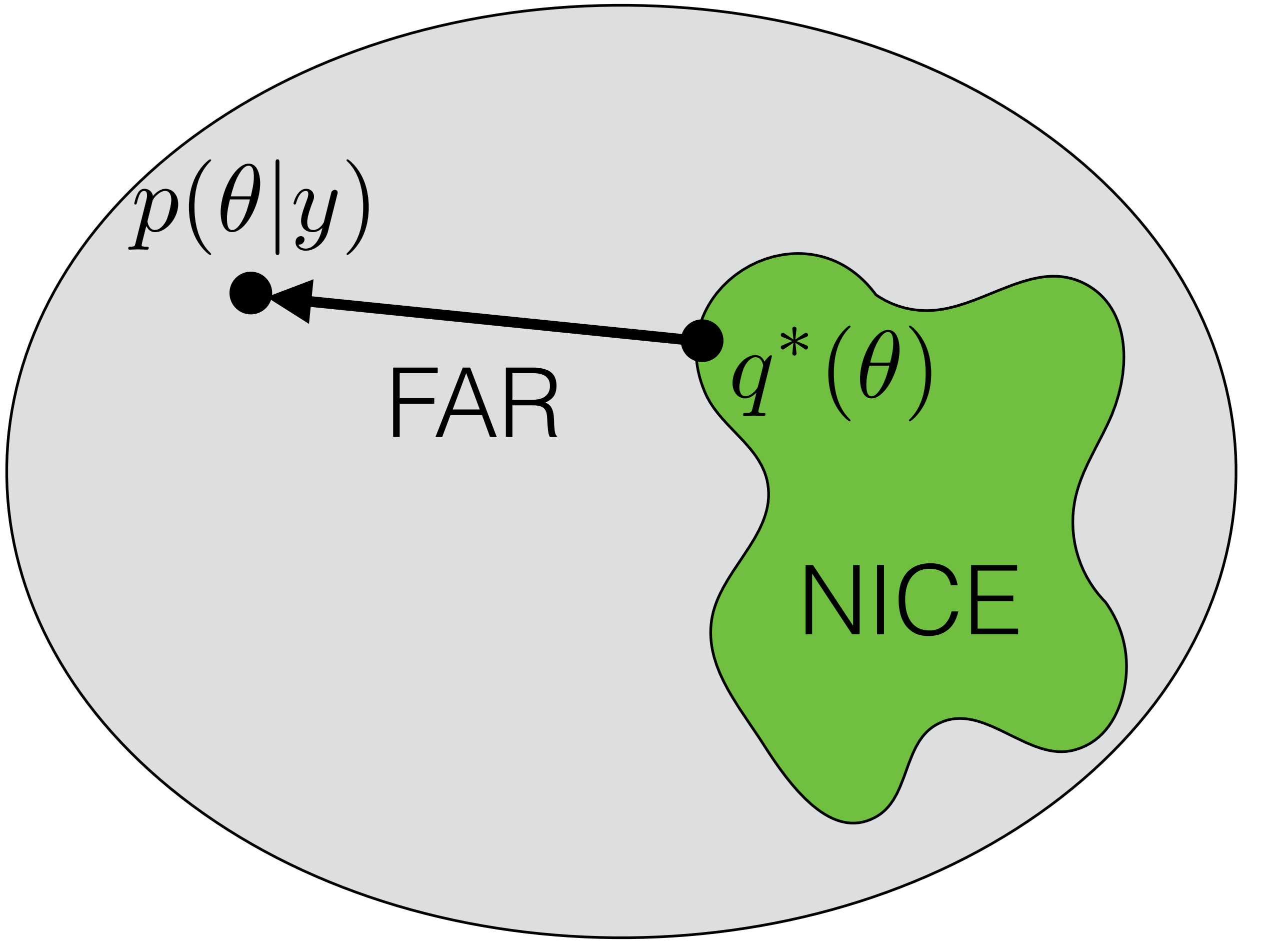
$$q^* = \text{argmin}_{q \in Q} \text{KL}\left(q(\cdot)||p(\cdot|y)\right)$$

$$\text{KL}\left(q(\cdot)||p(\cdot|y)\right) := \int q(\theta) \log \frac{q(\theta)}{p(\theta|y)} d\theta$$



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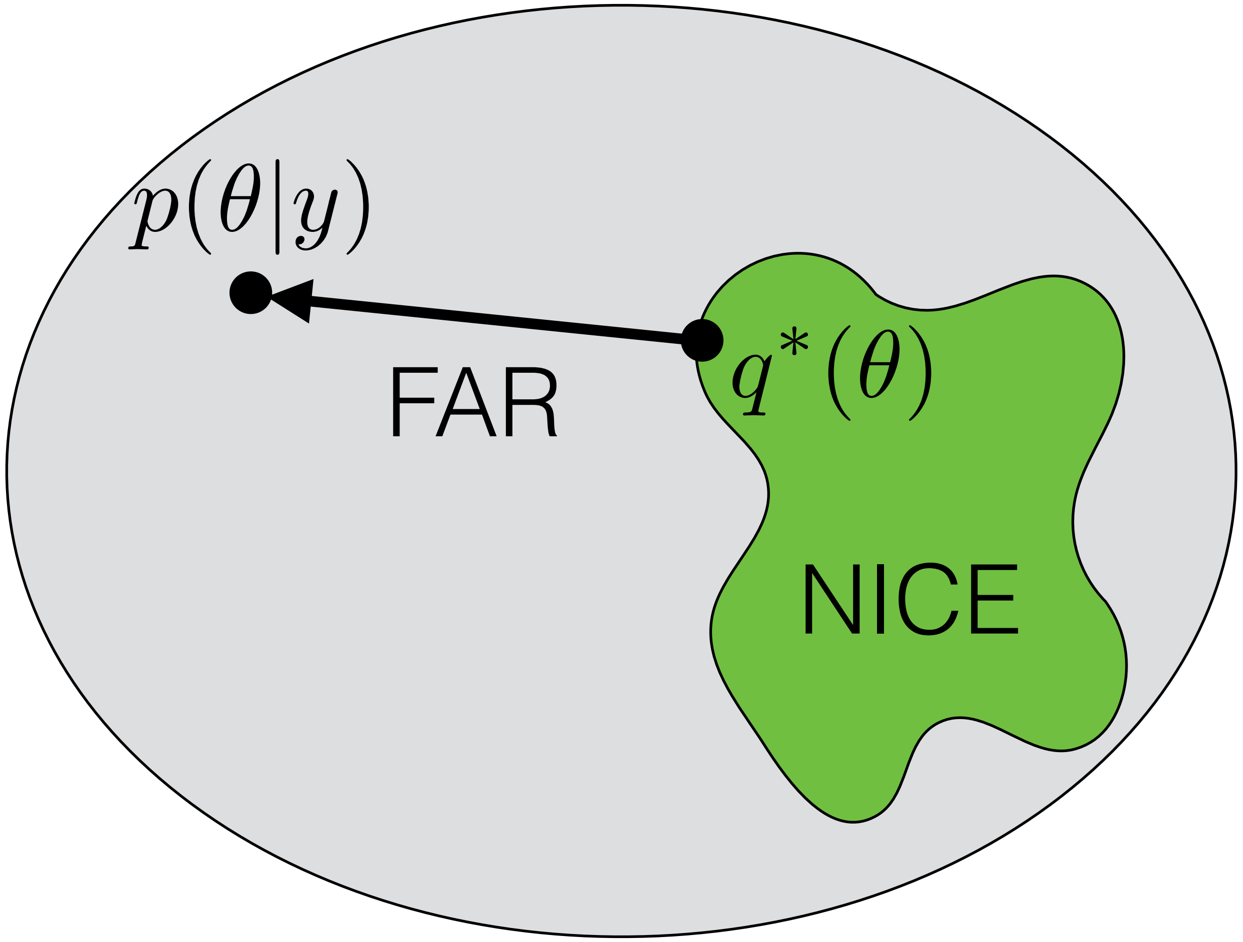

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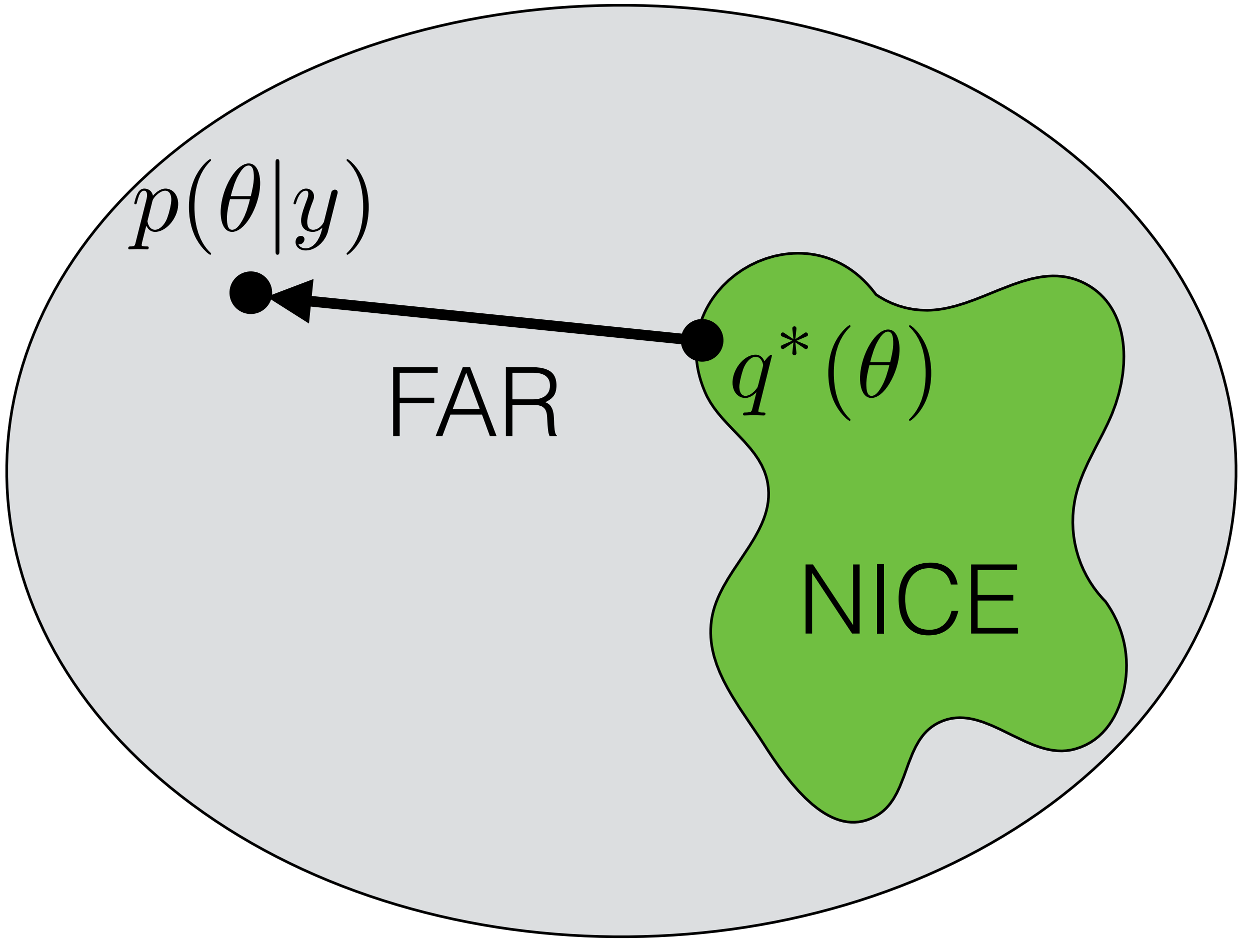

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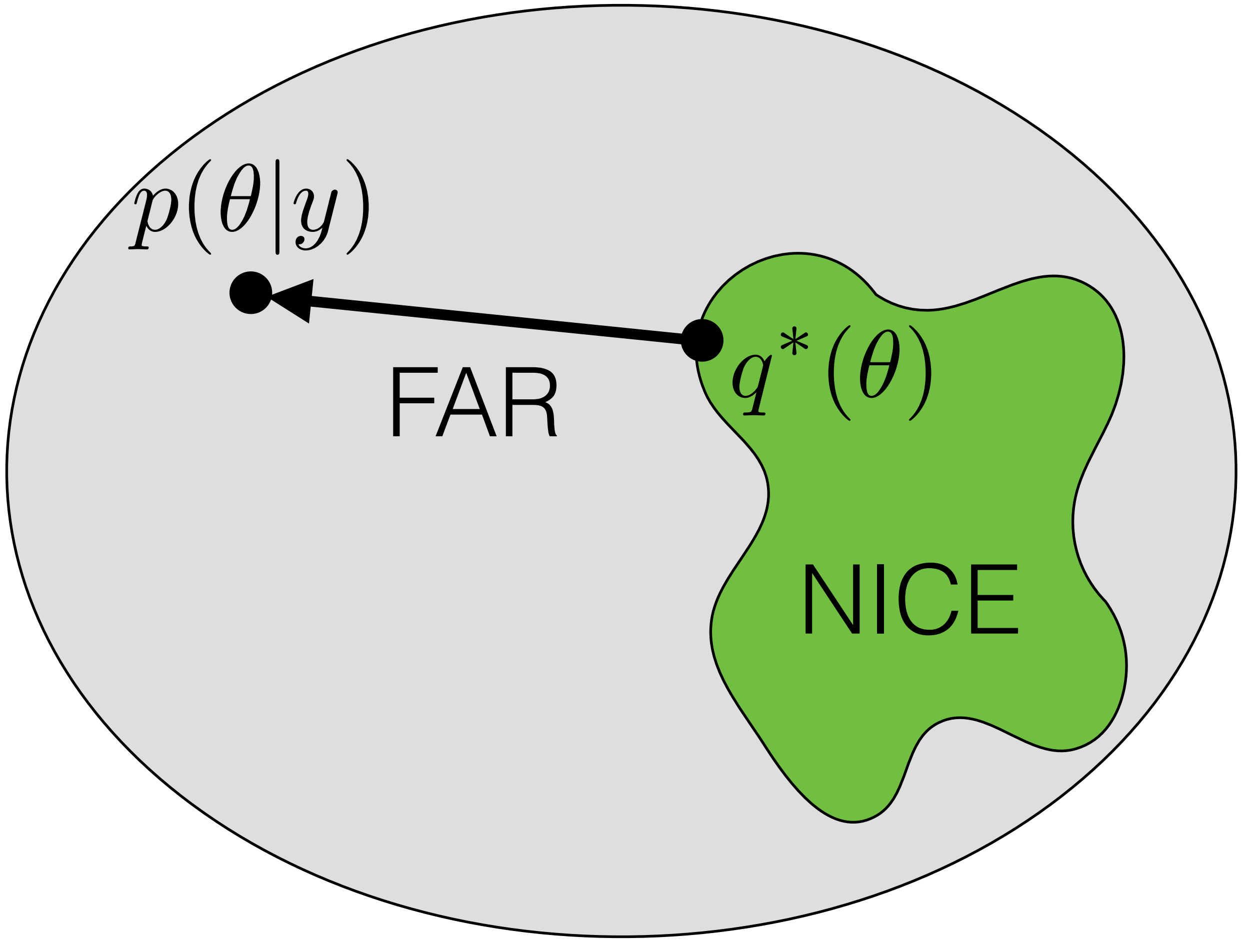

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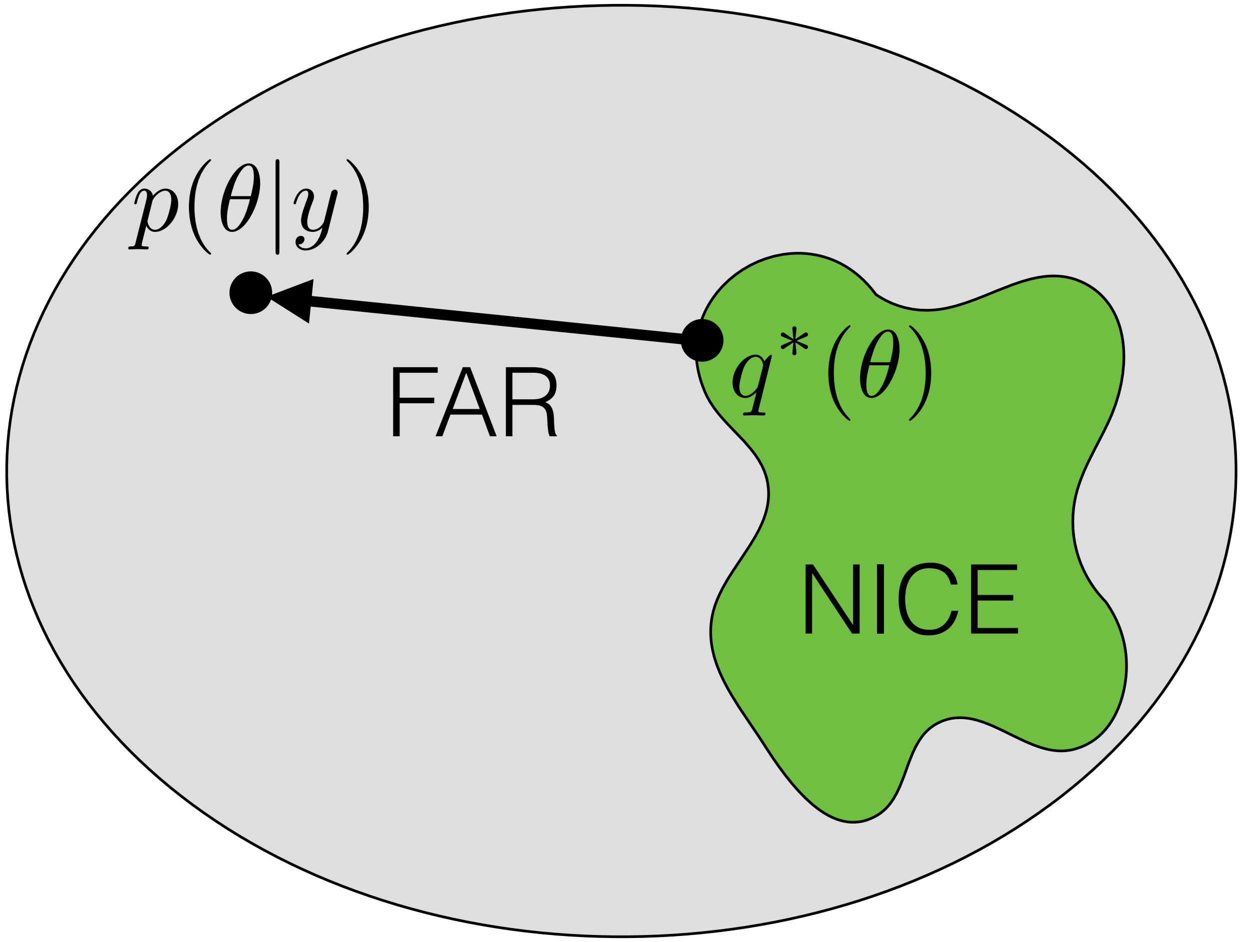

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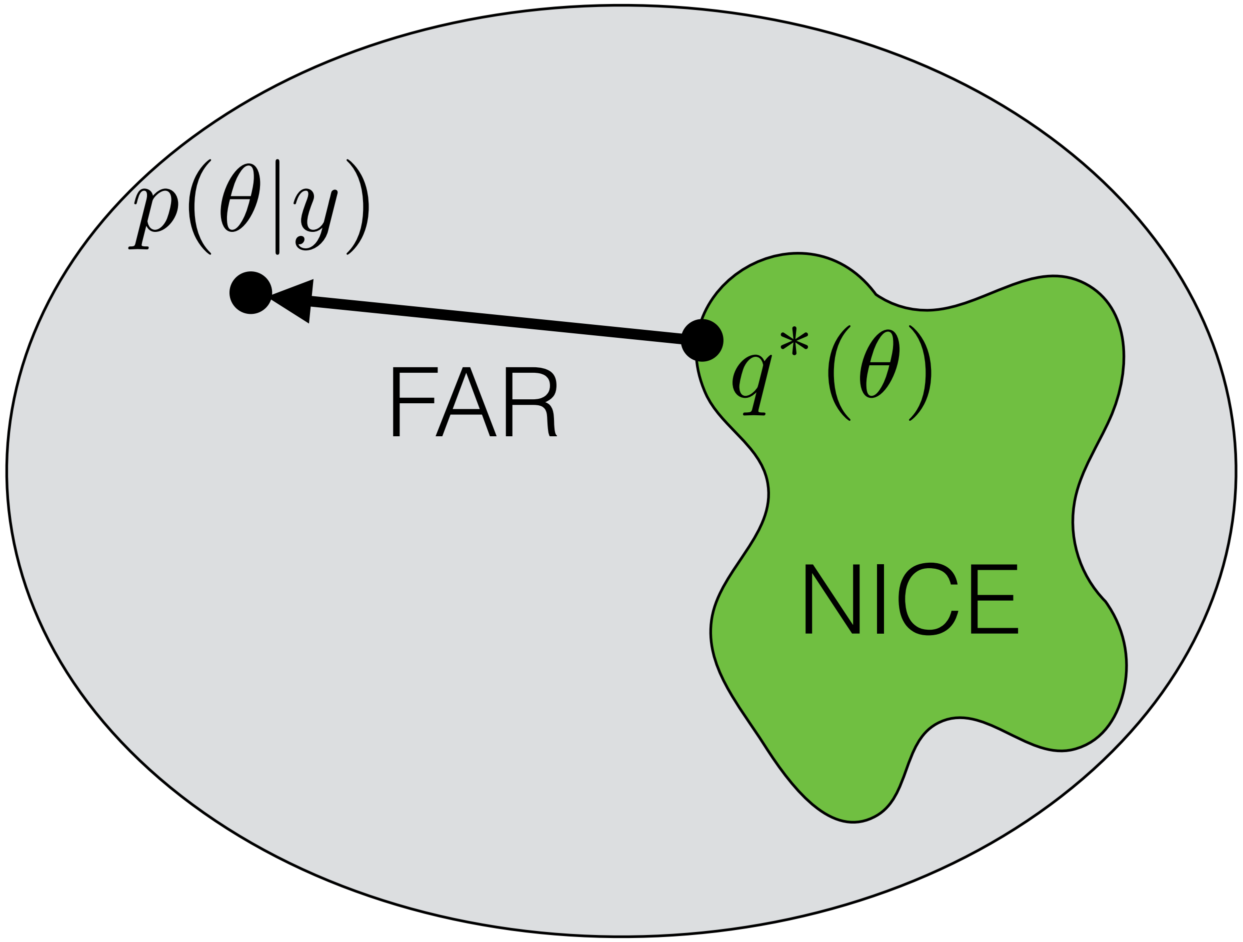

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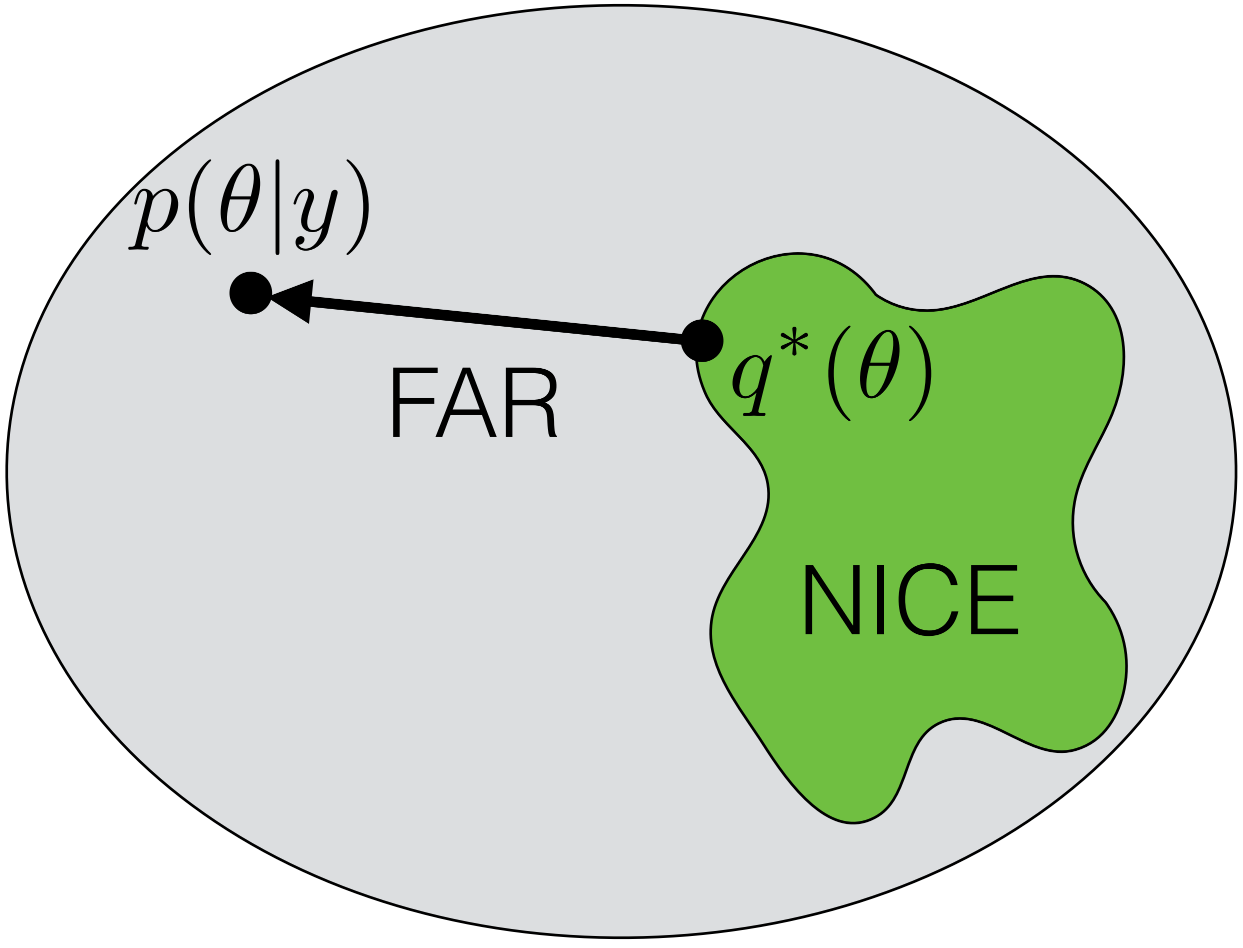

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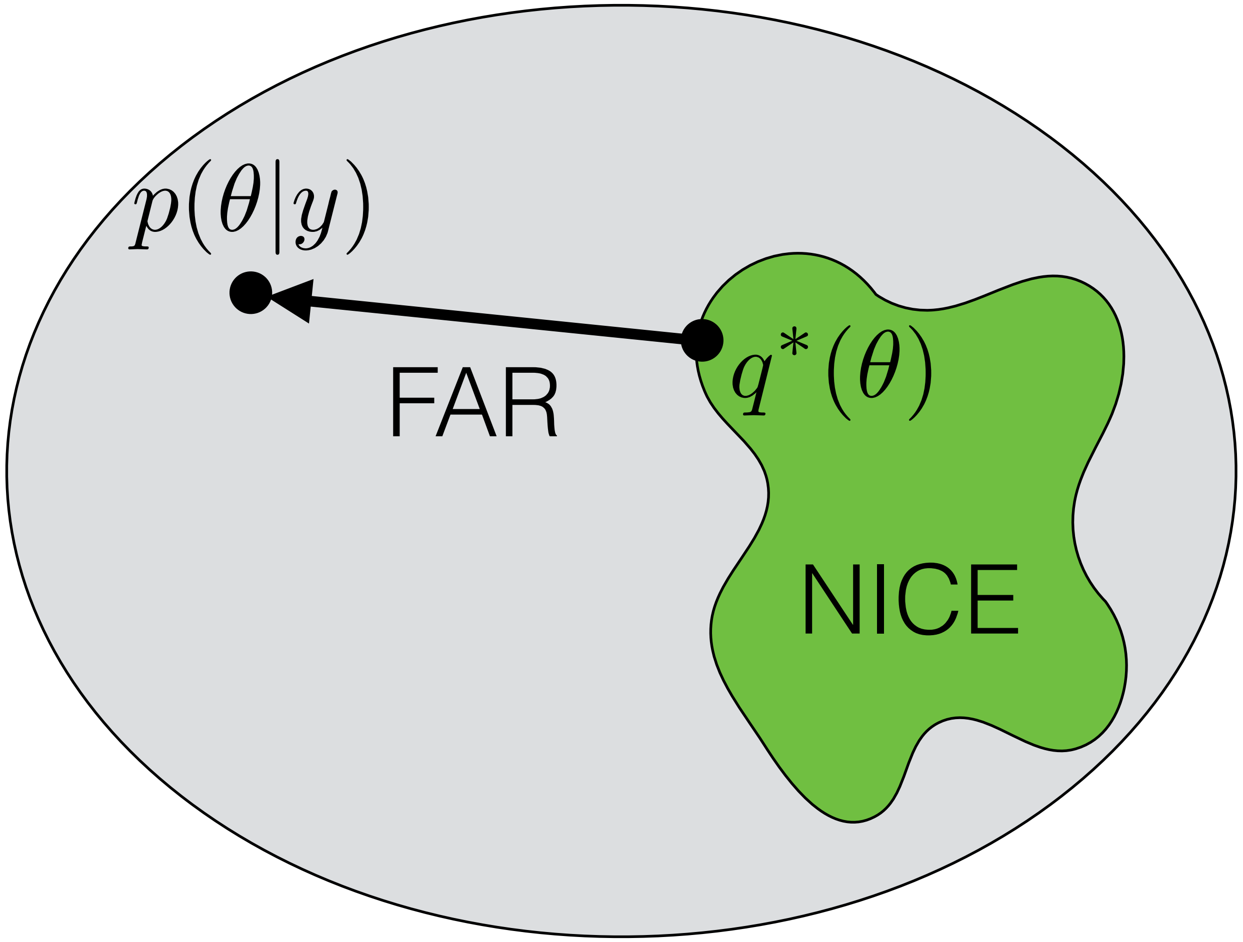

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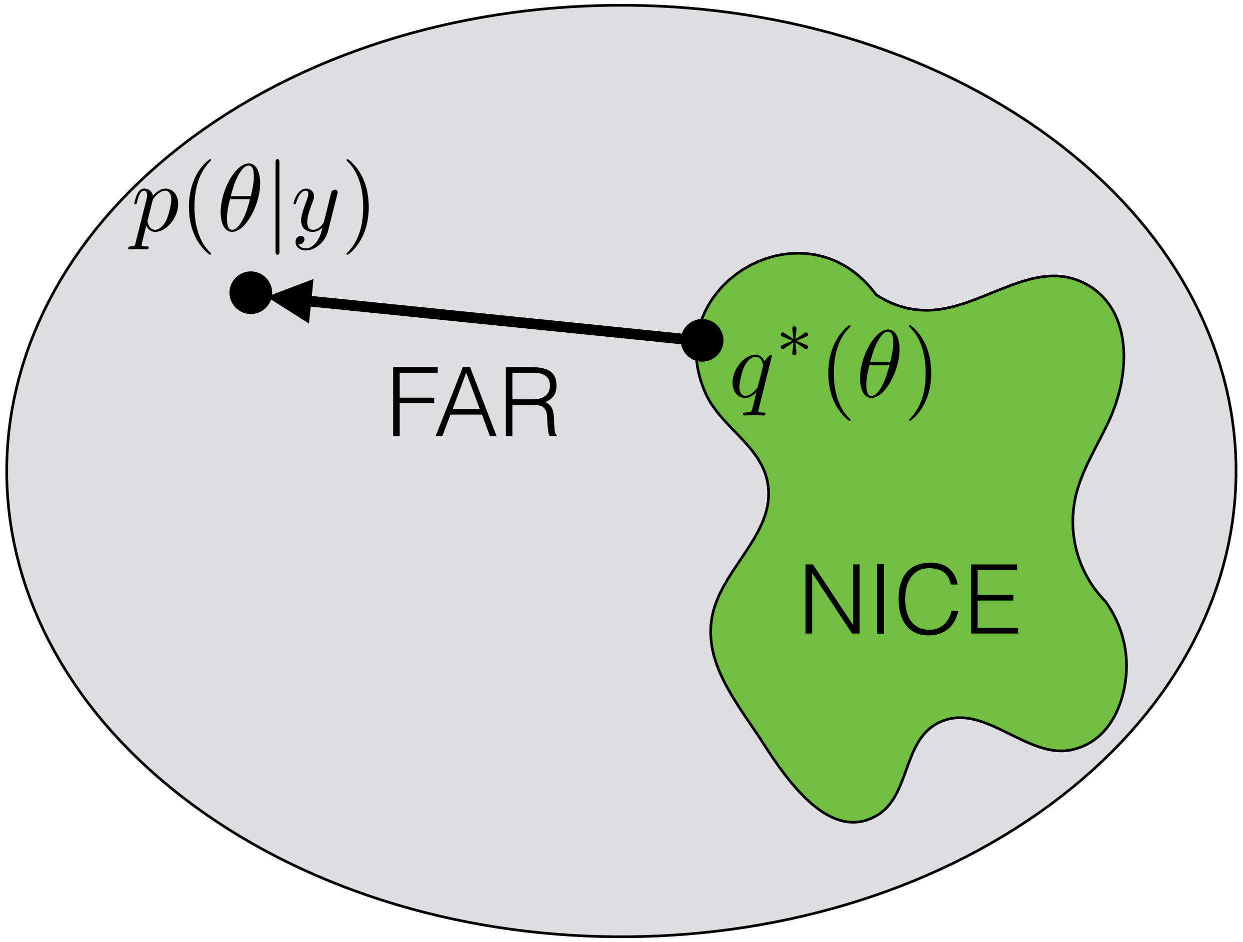

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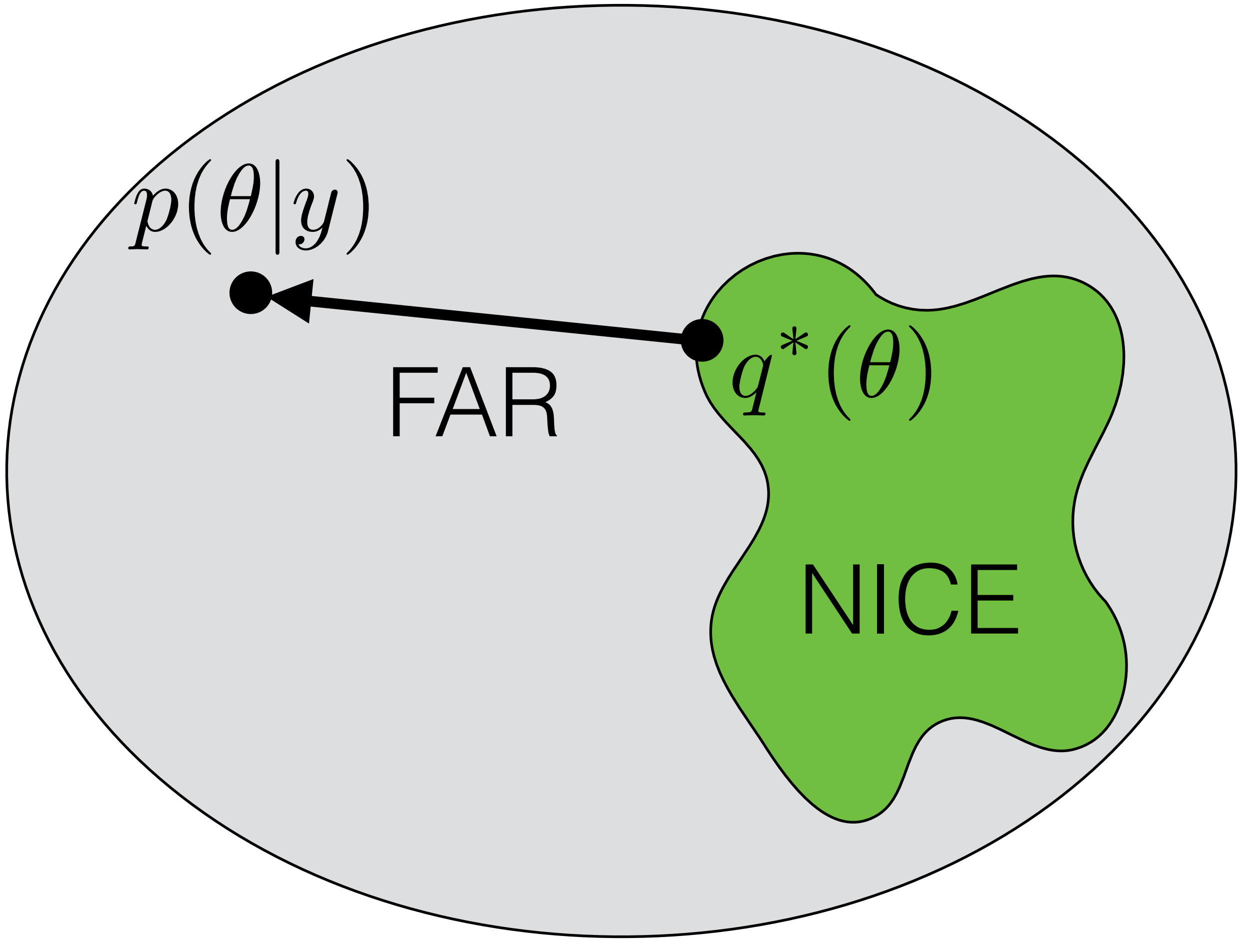

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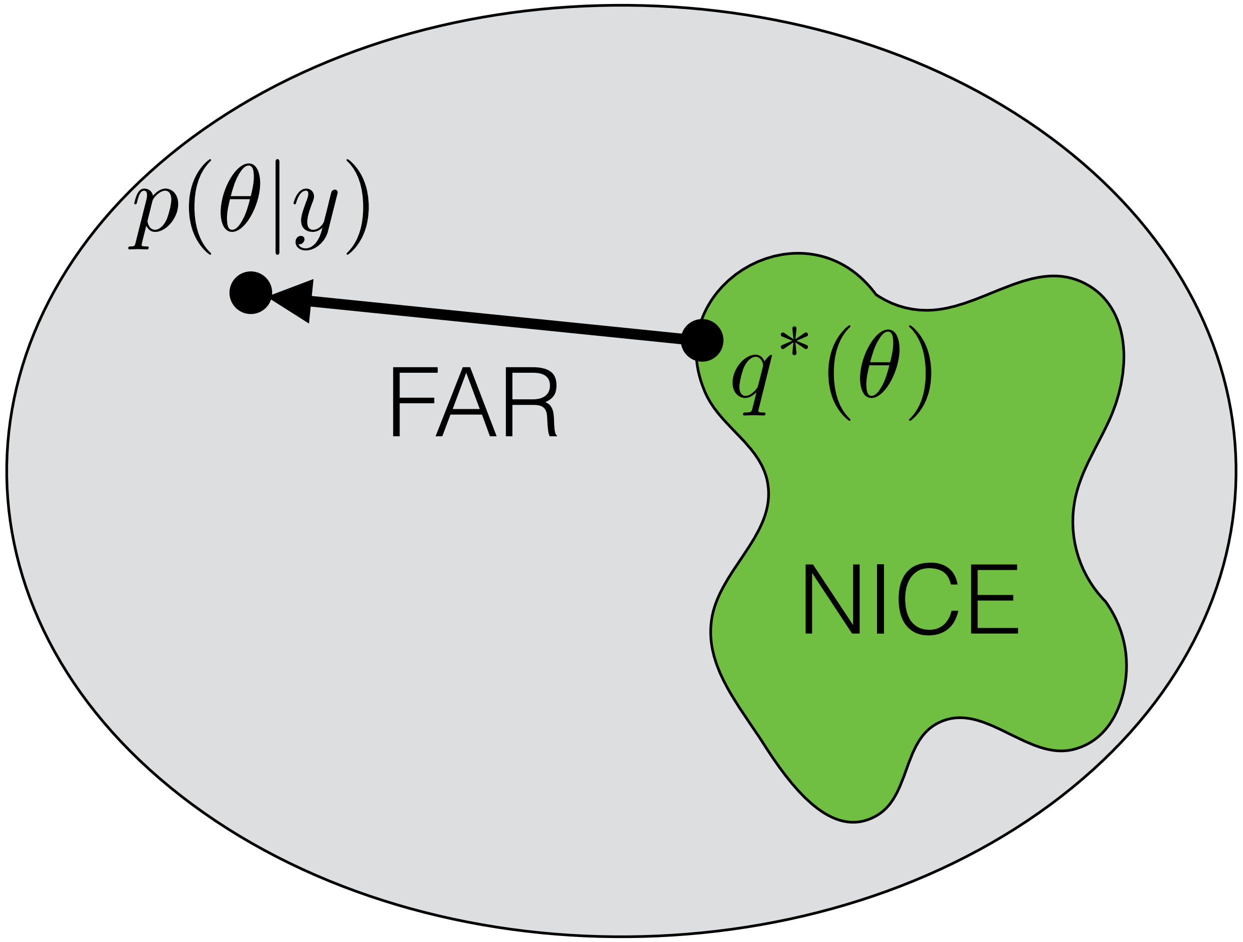

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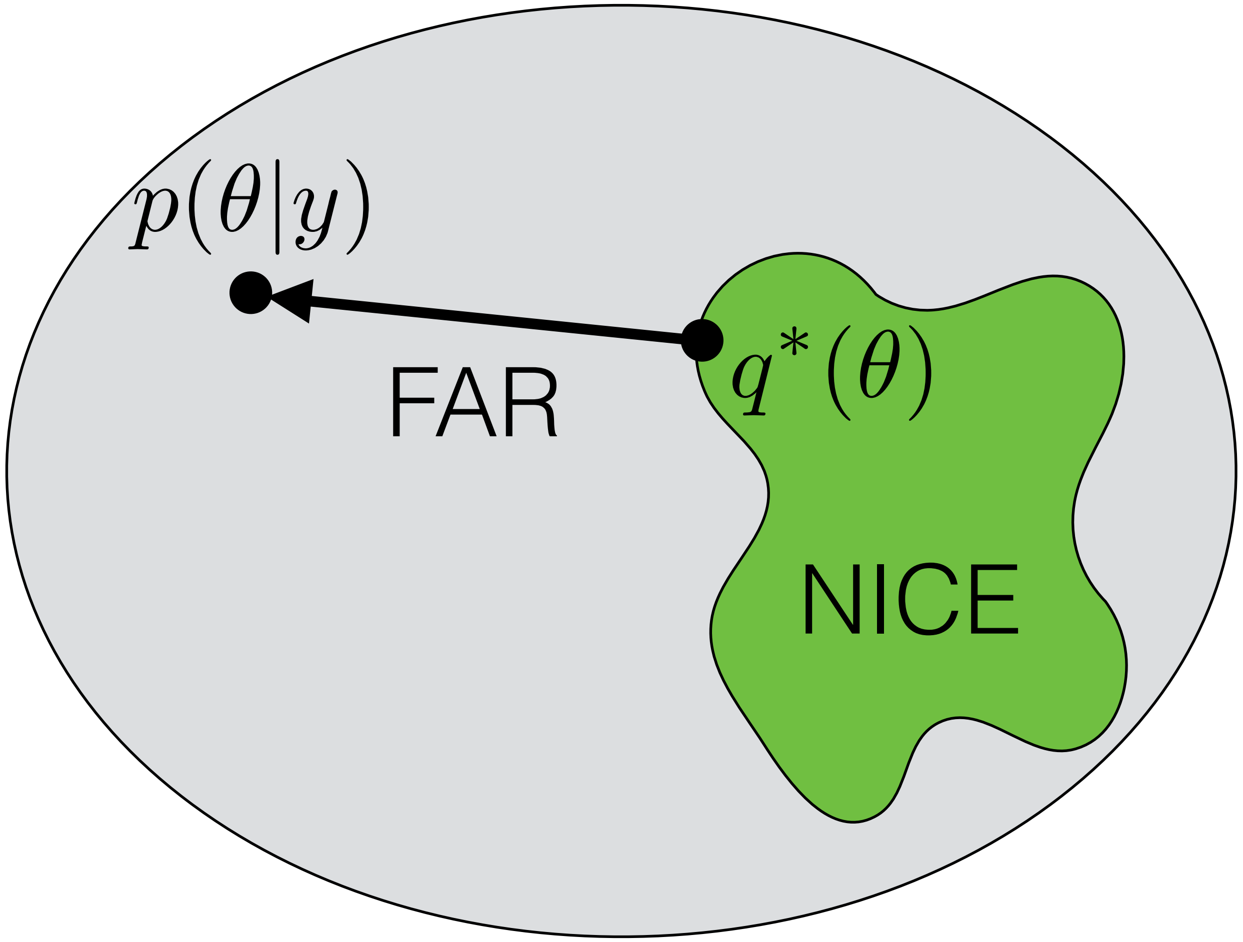

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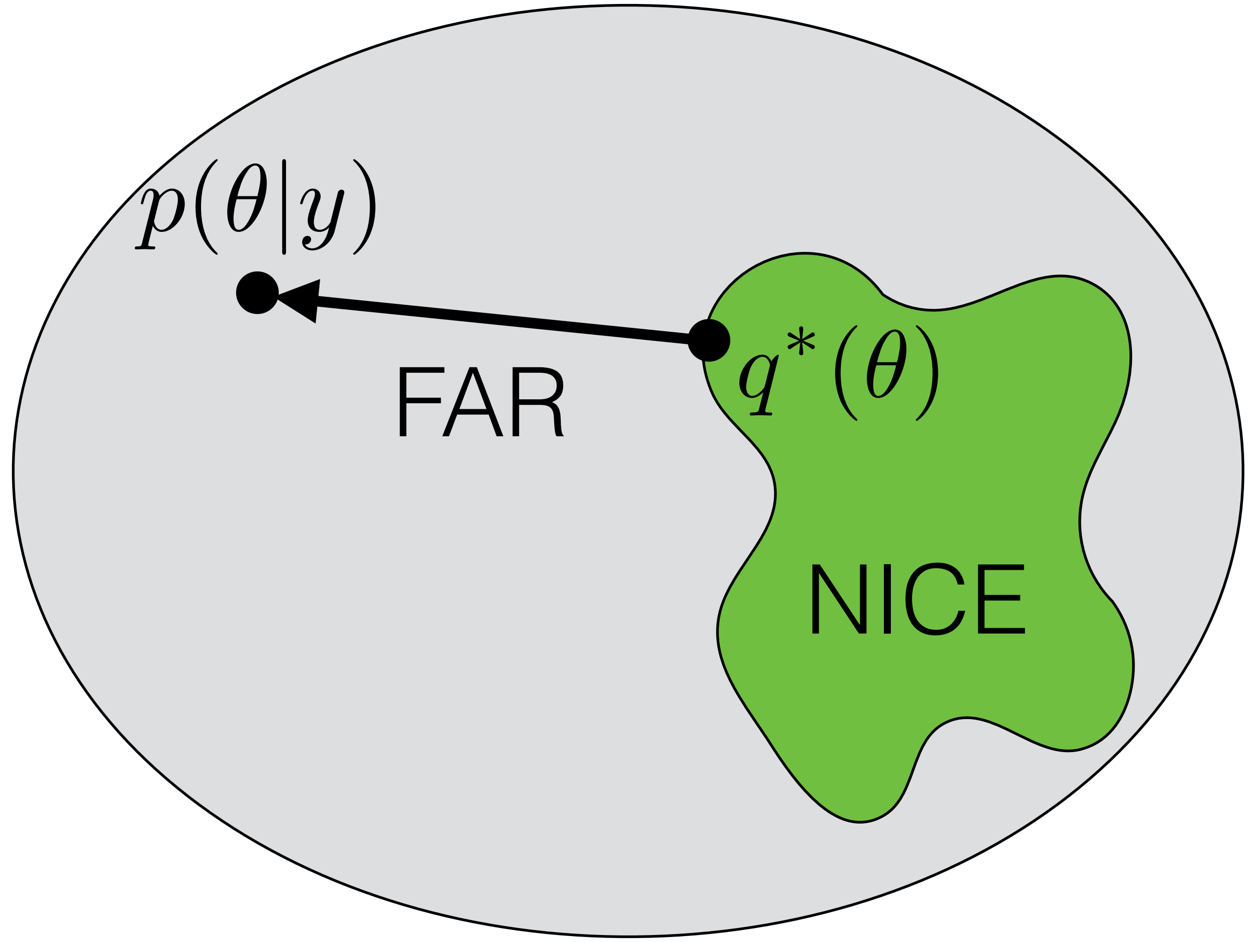

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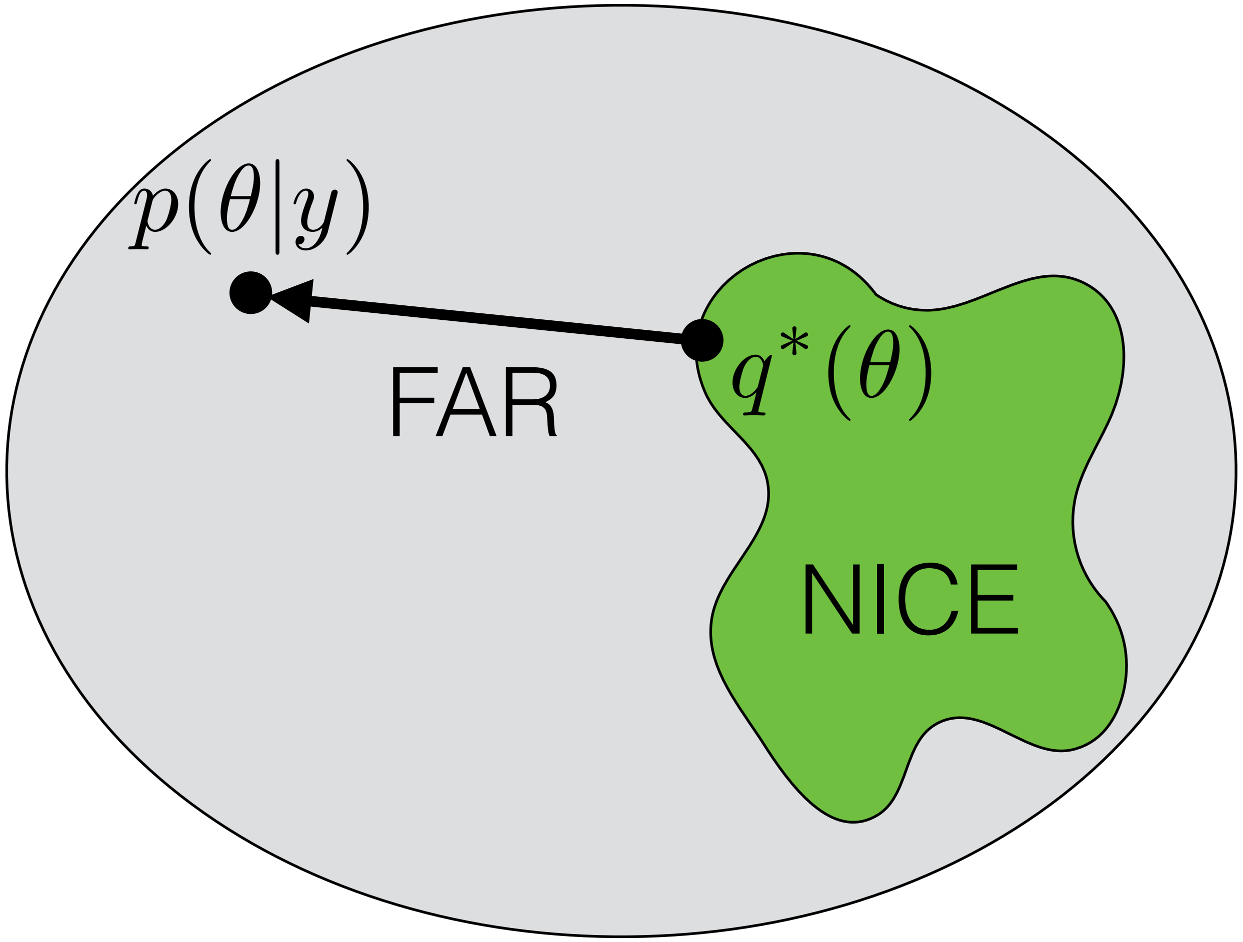

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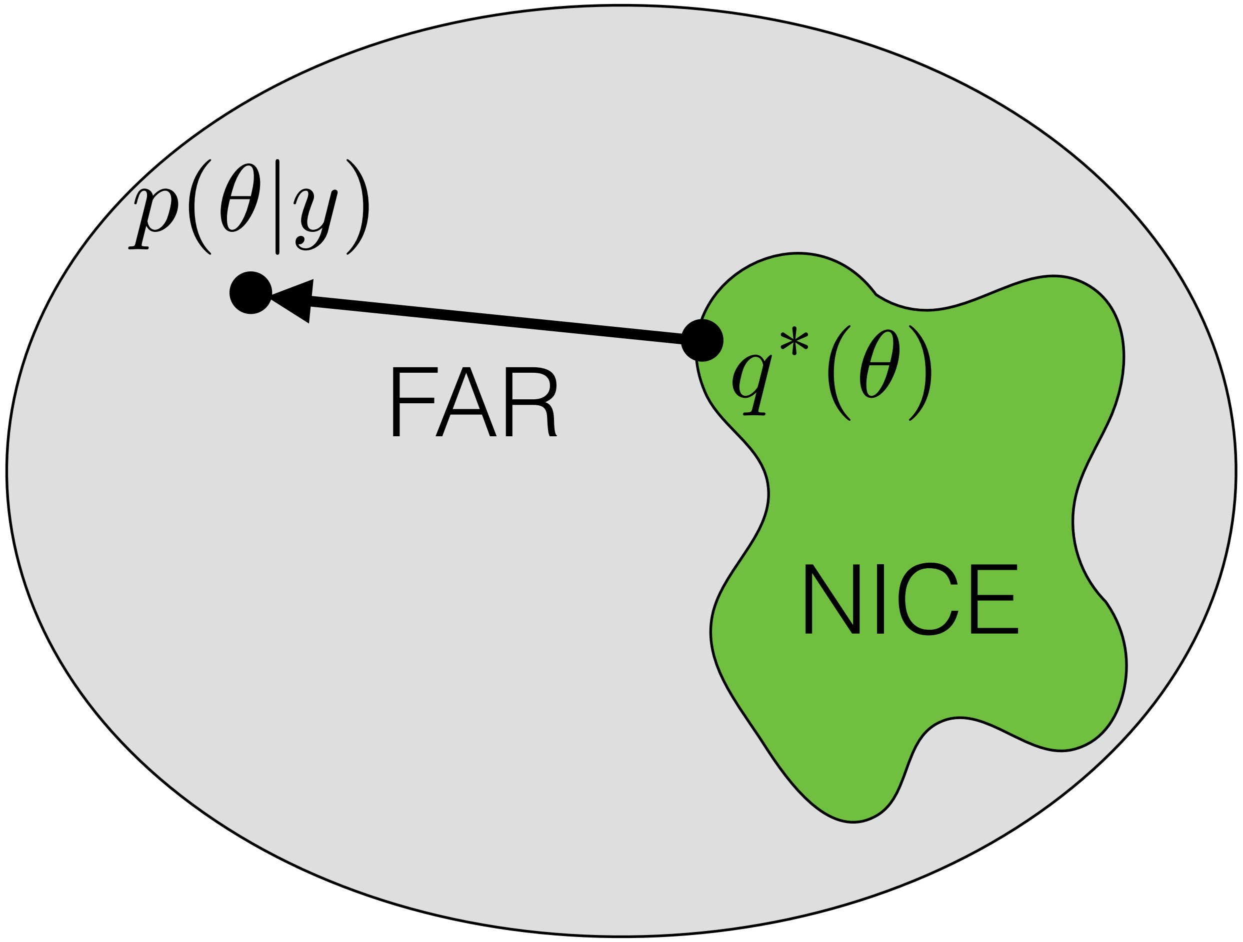

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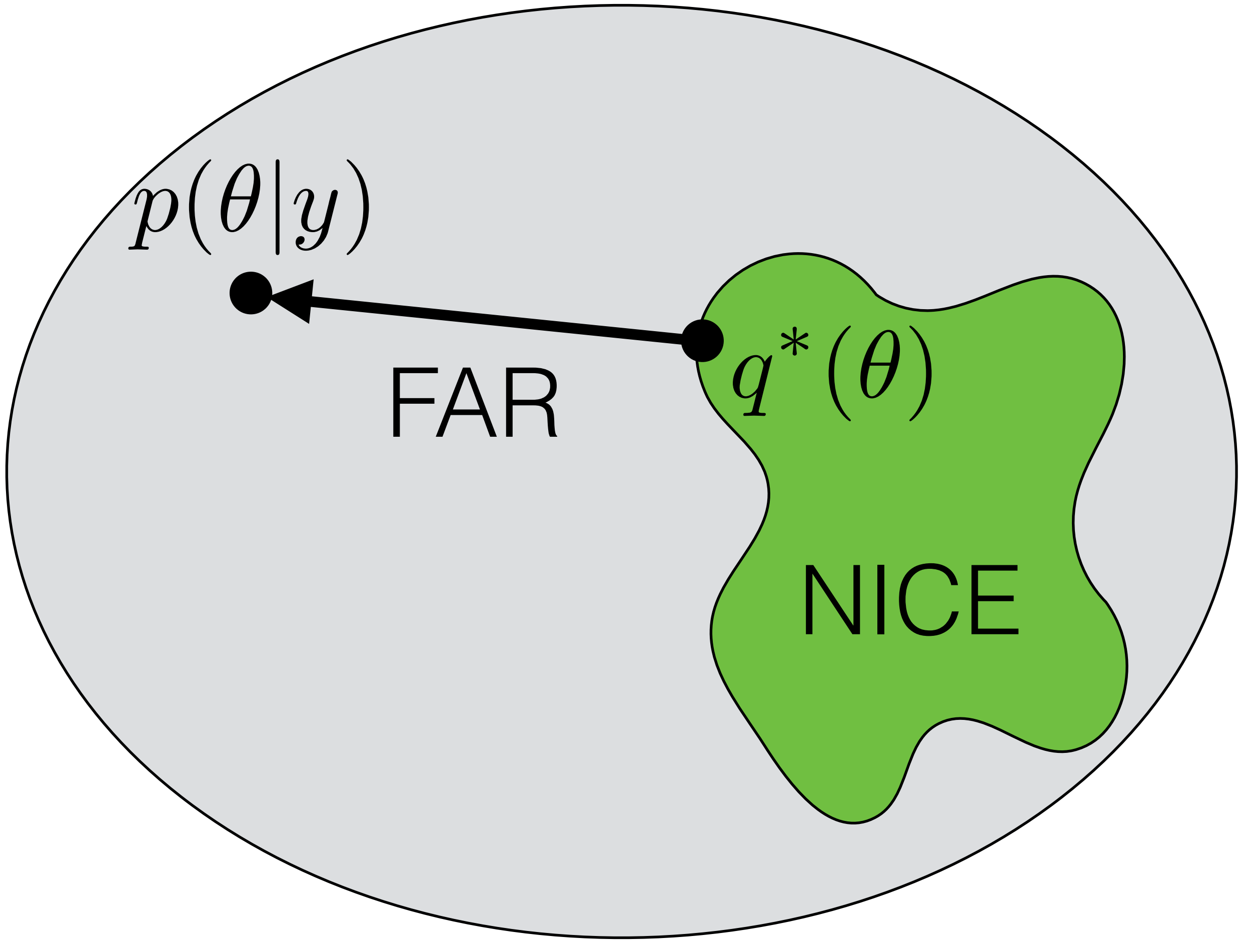

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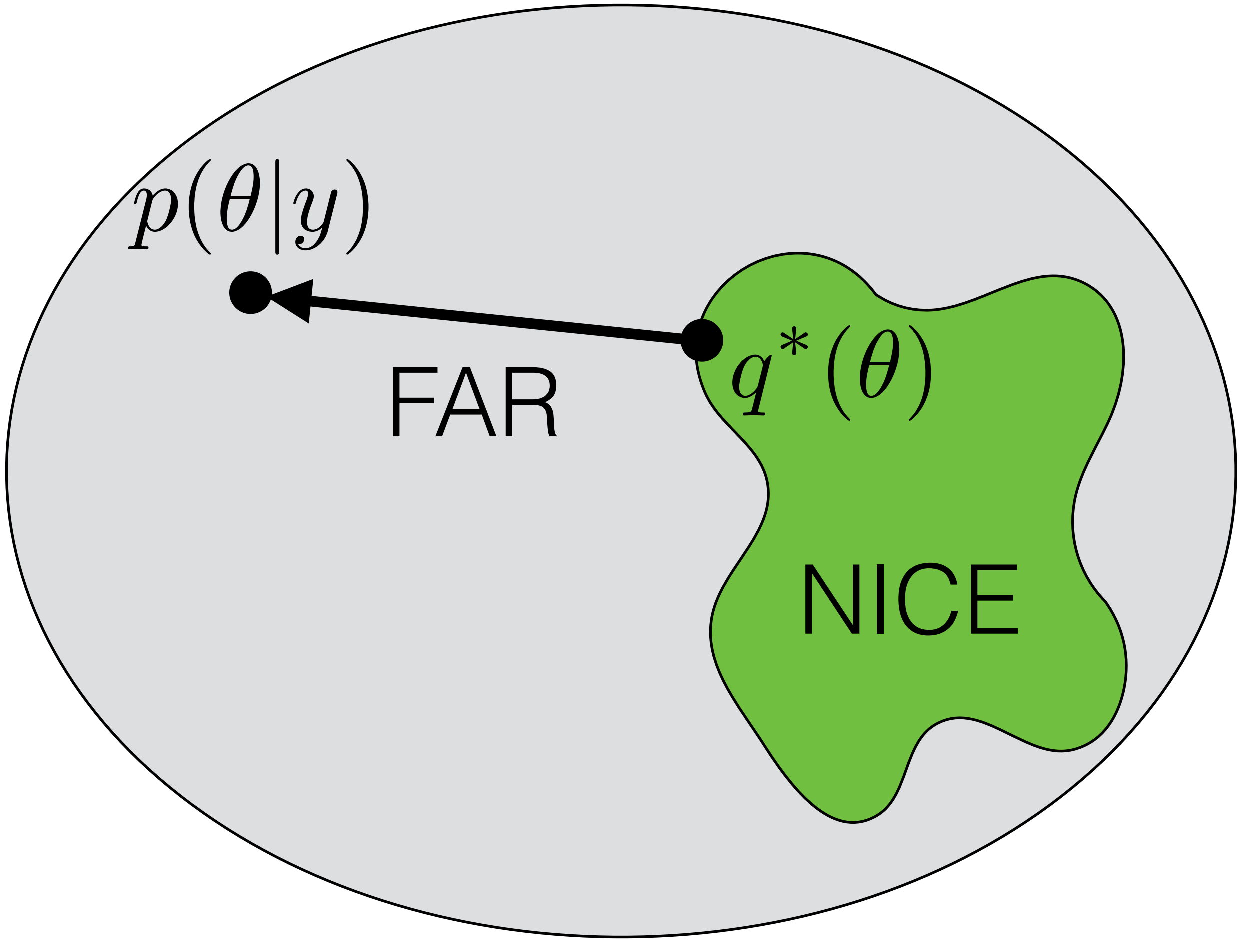

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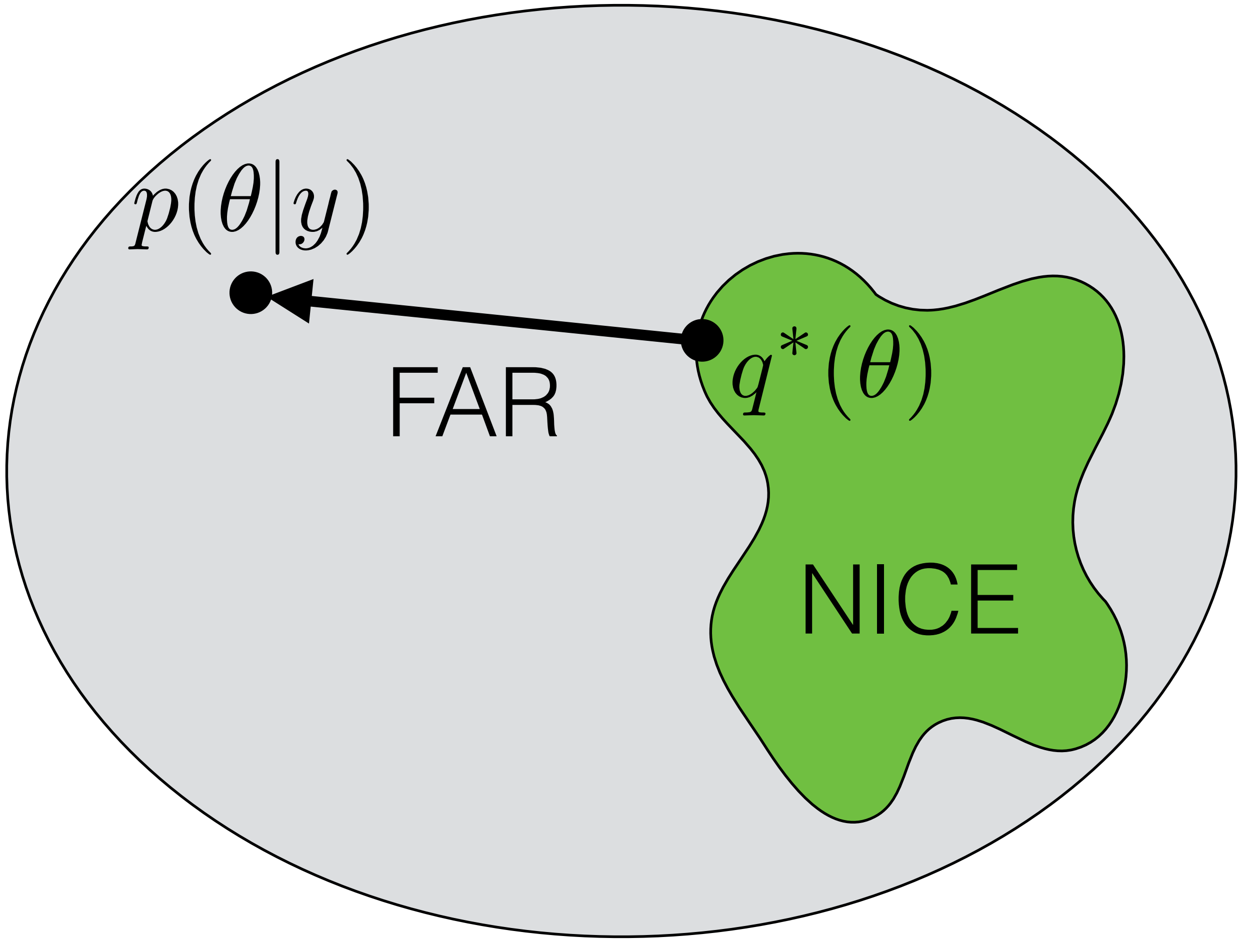

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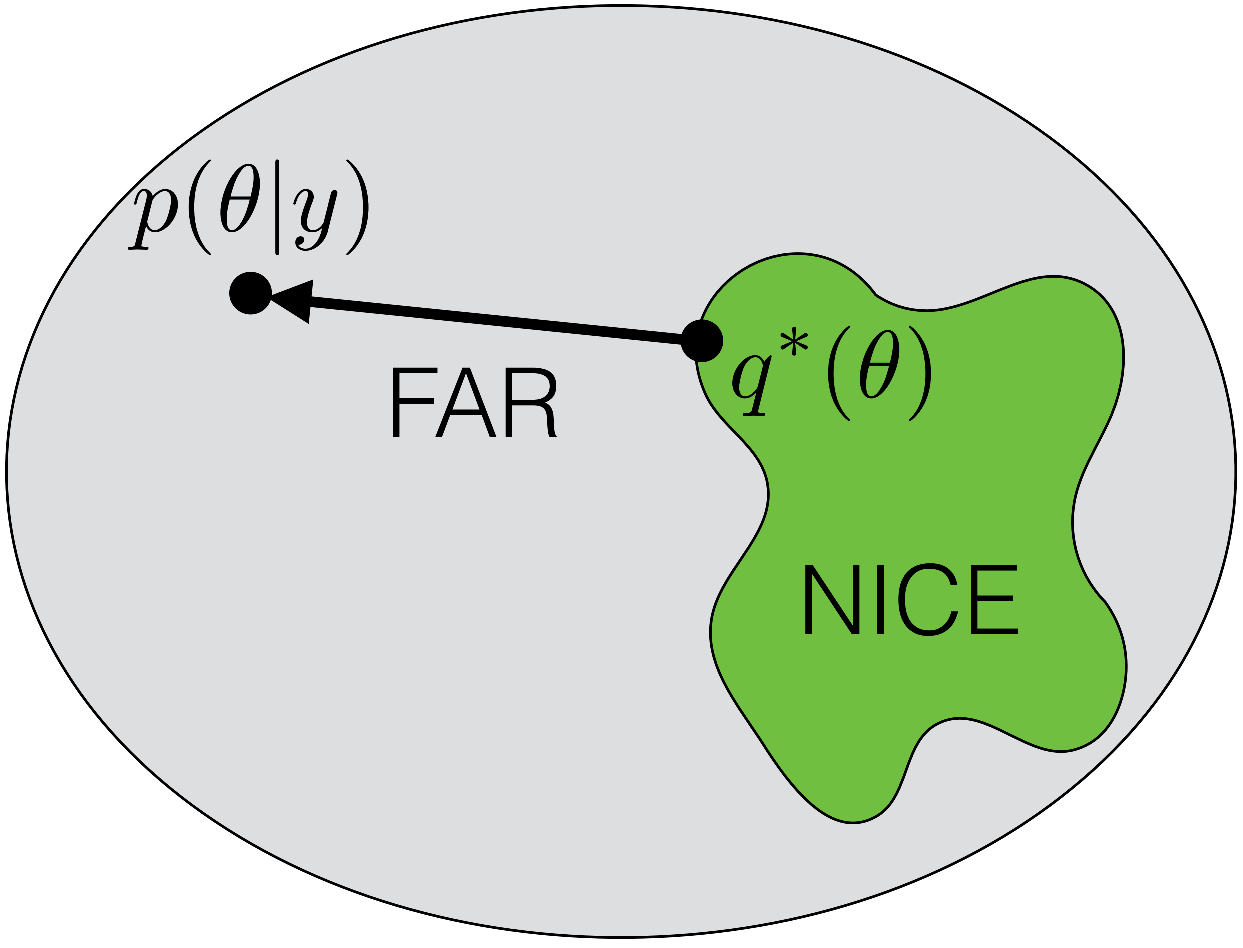

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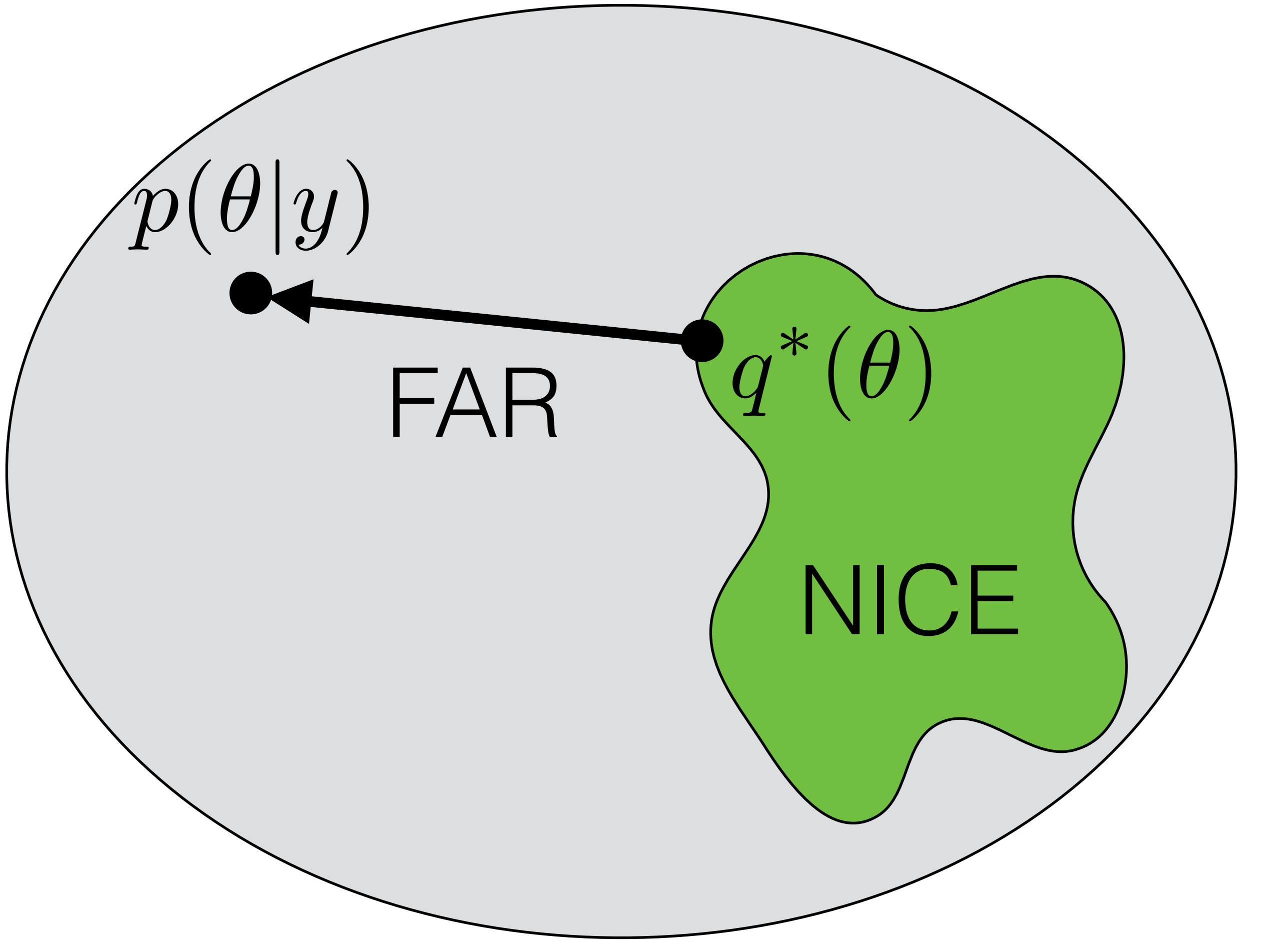

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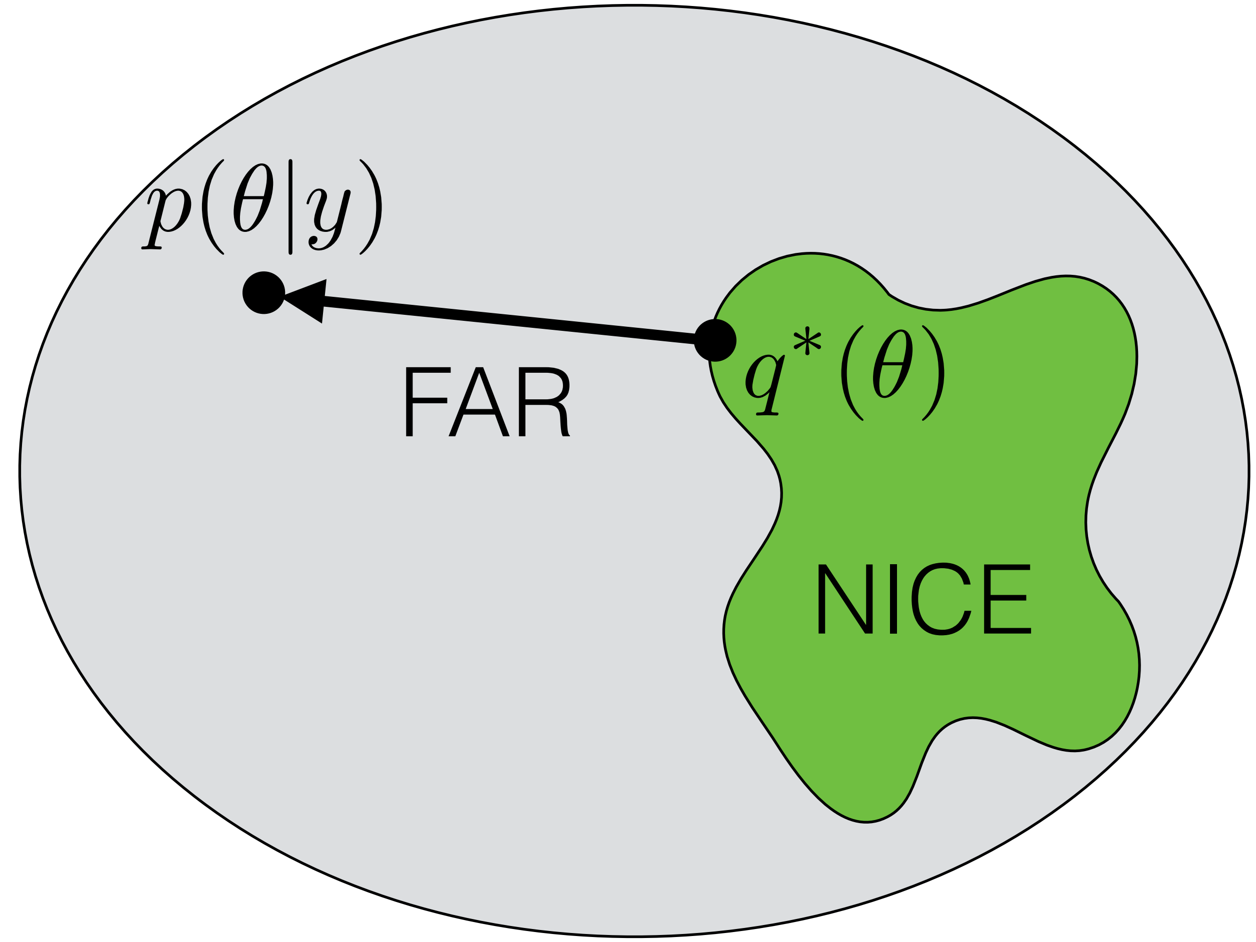

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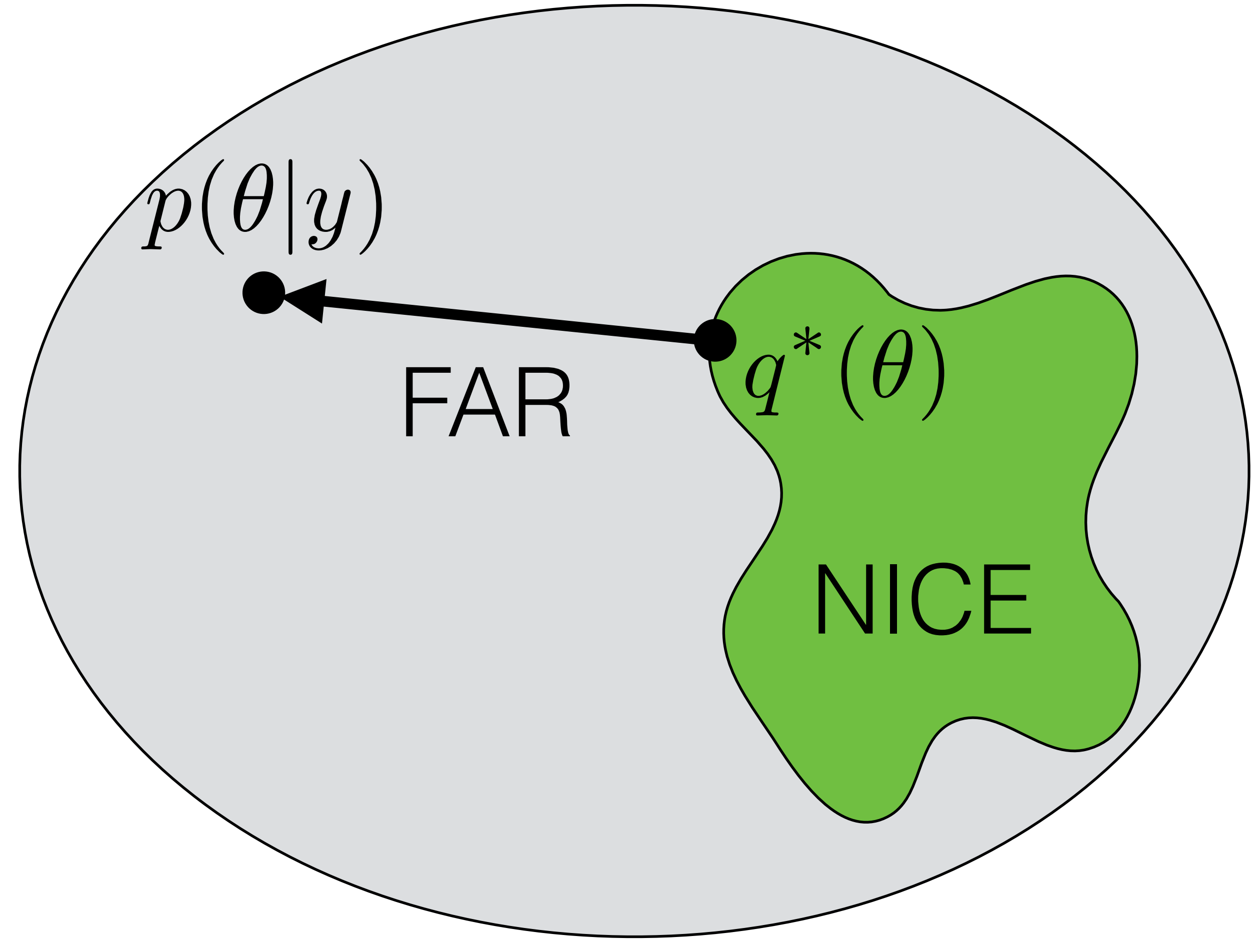

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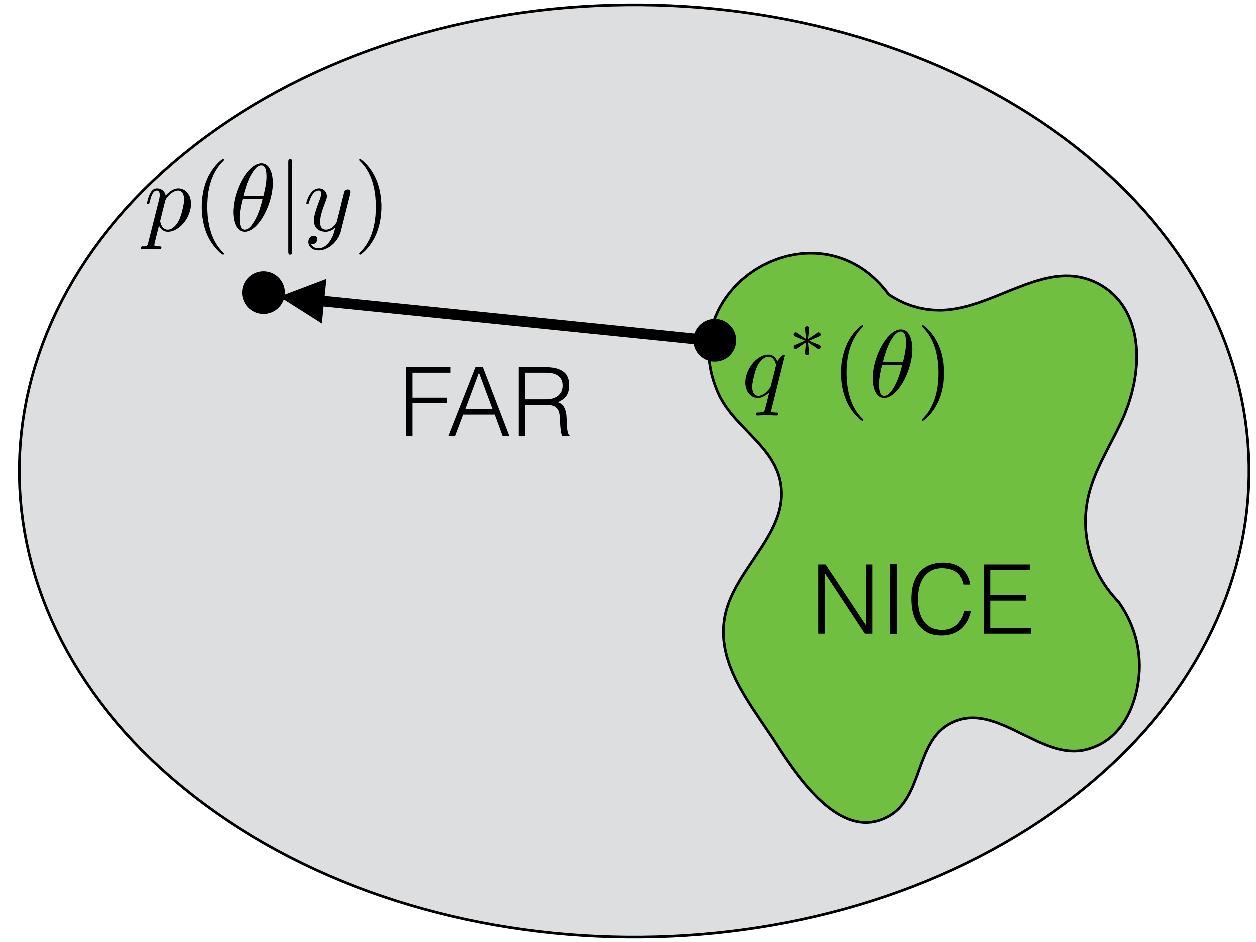

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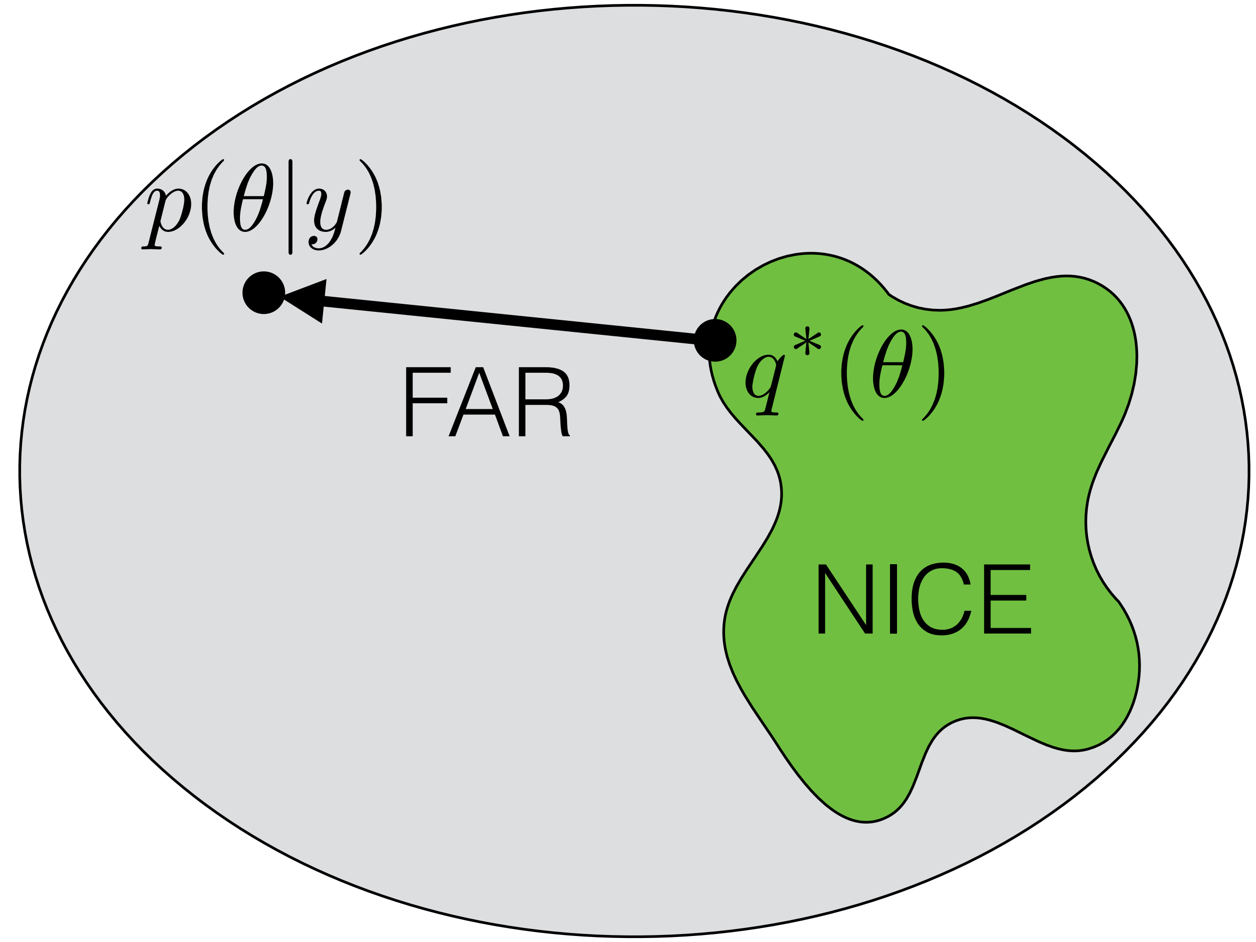

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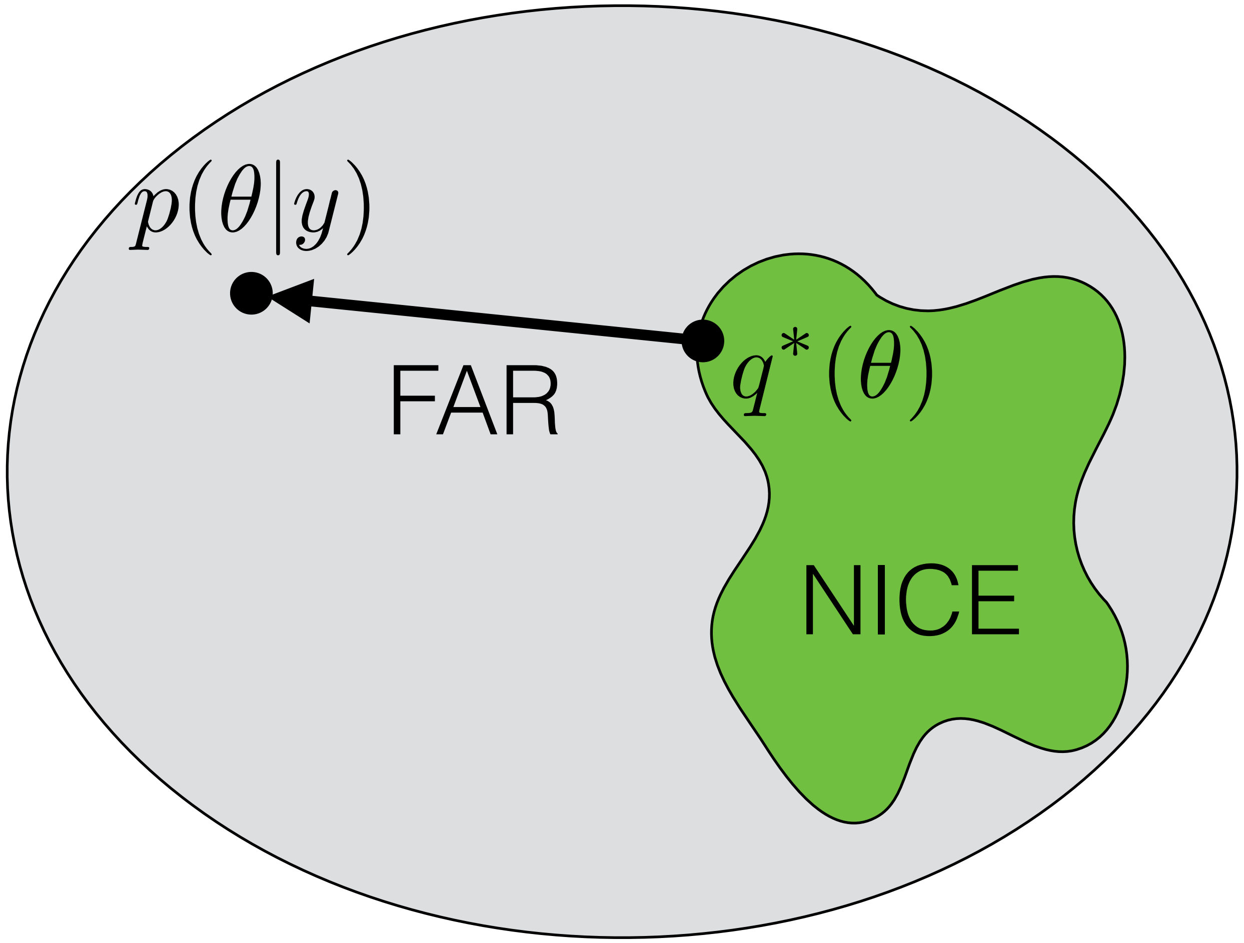

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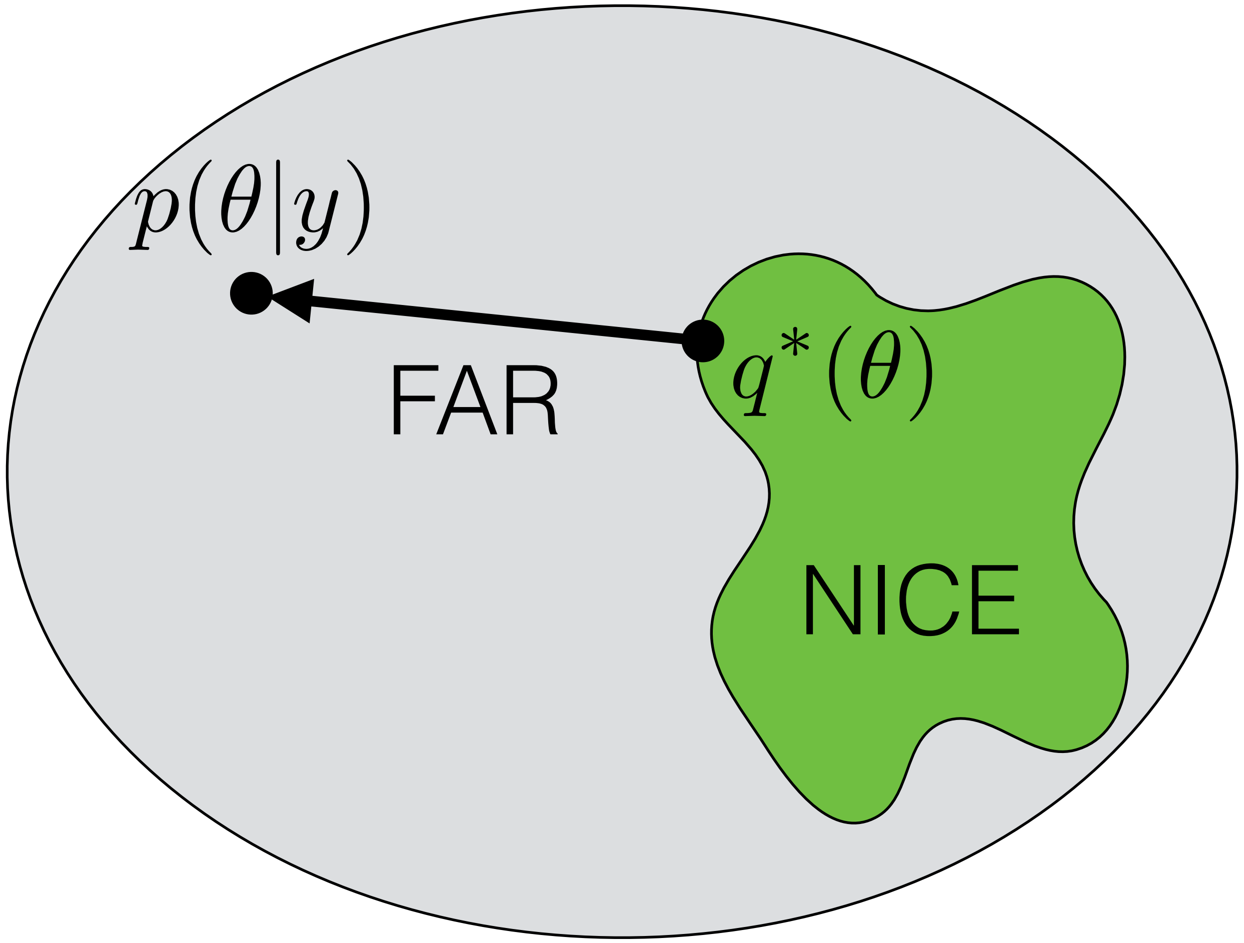

Choose "NICE" distributions

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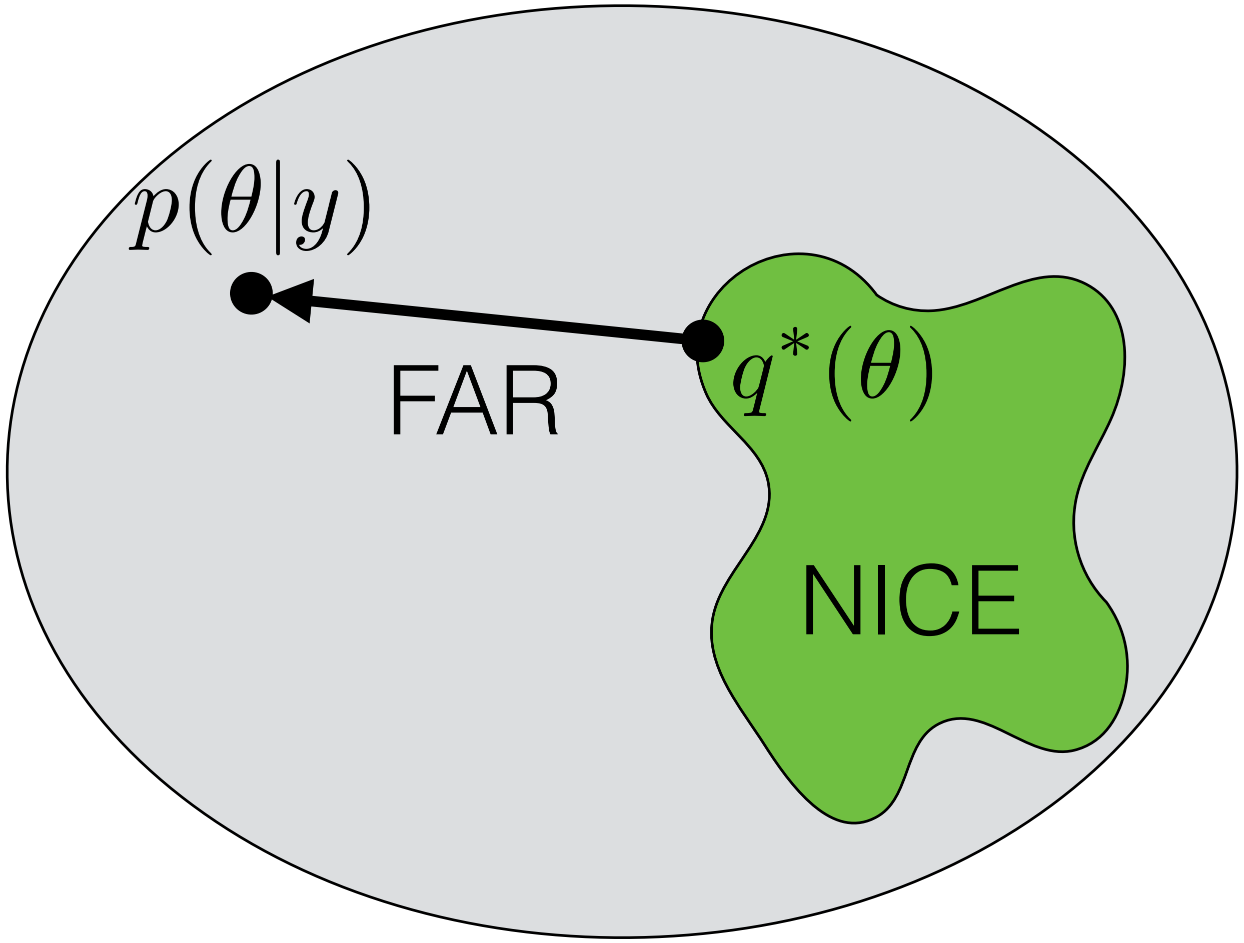

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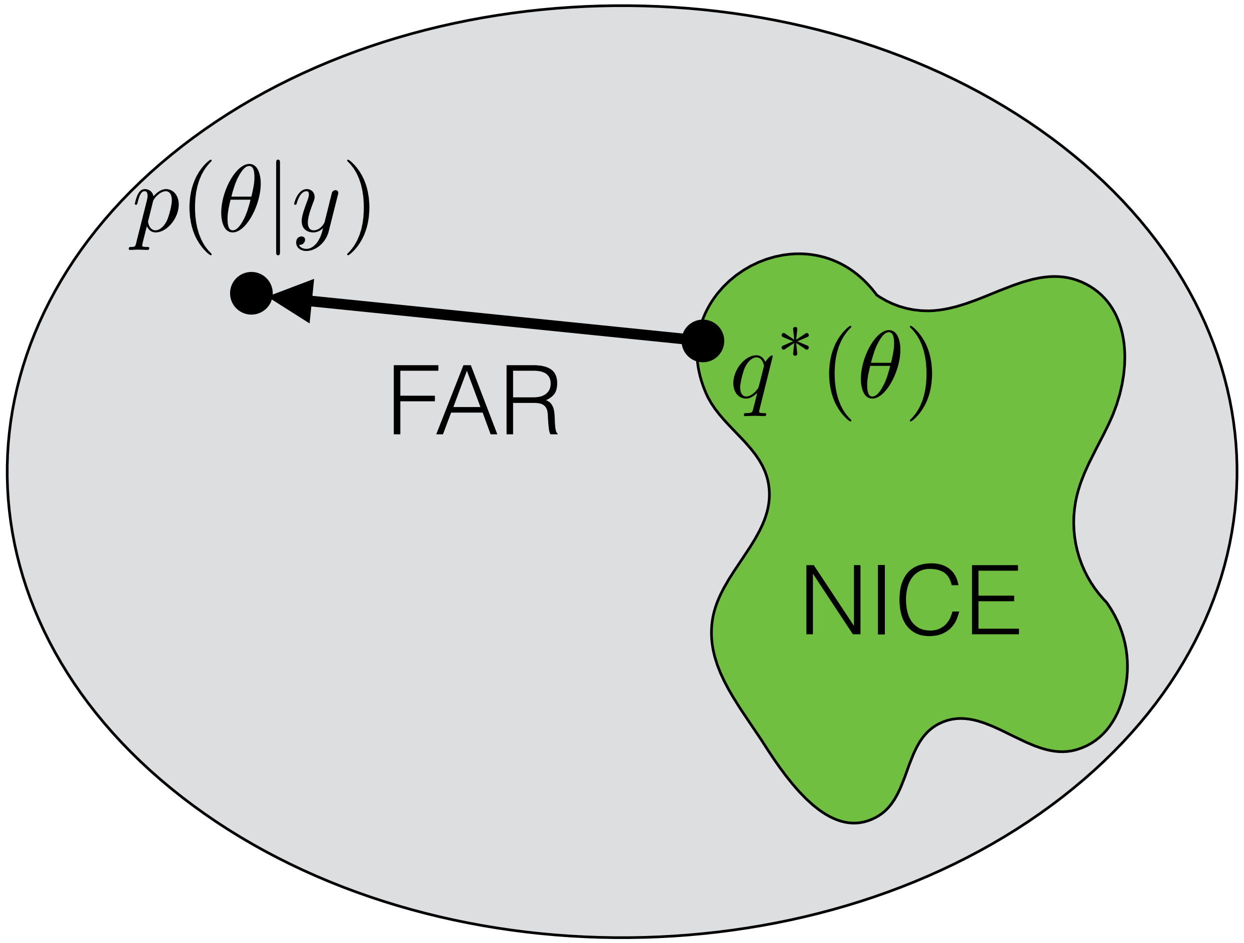

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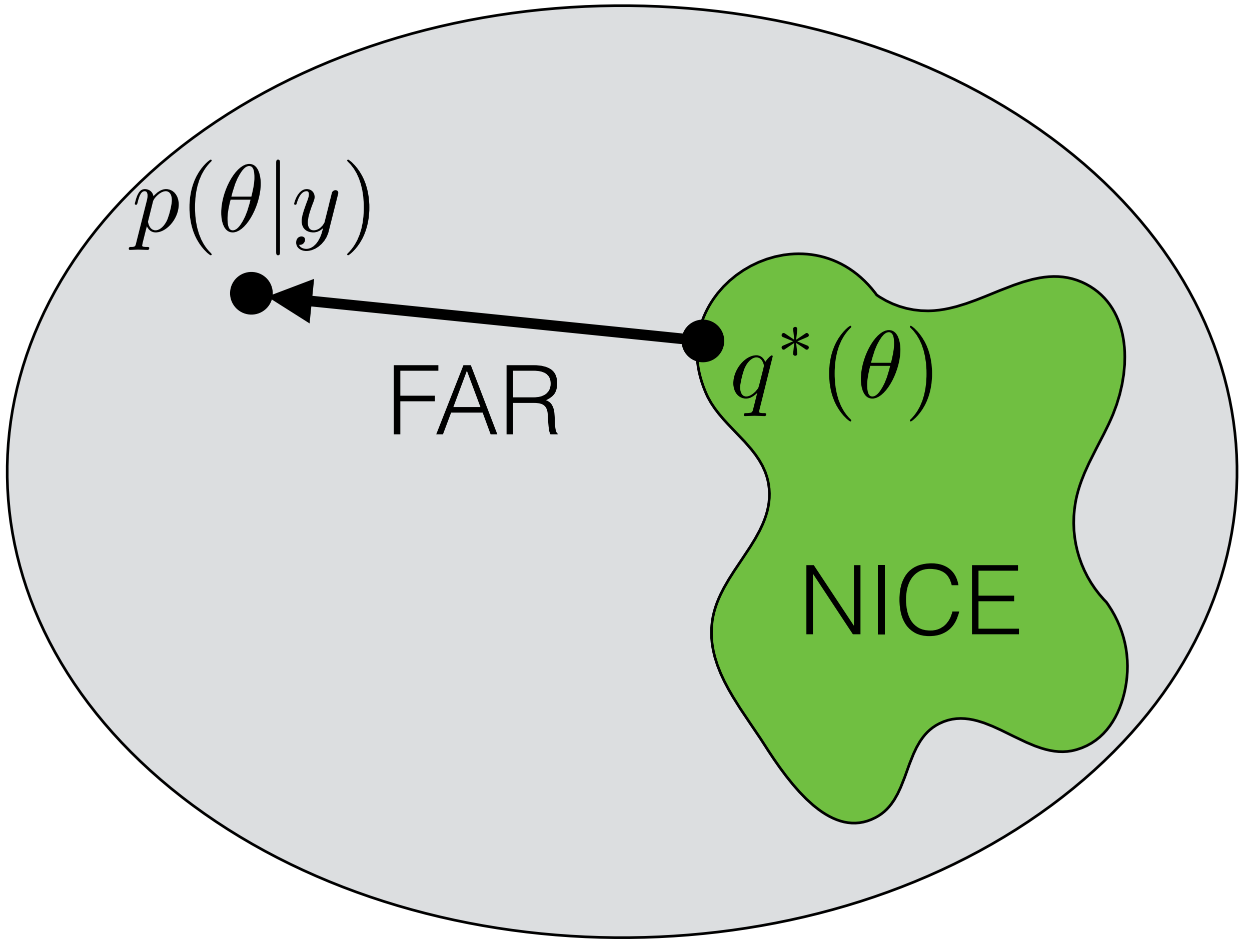

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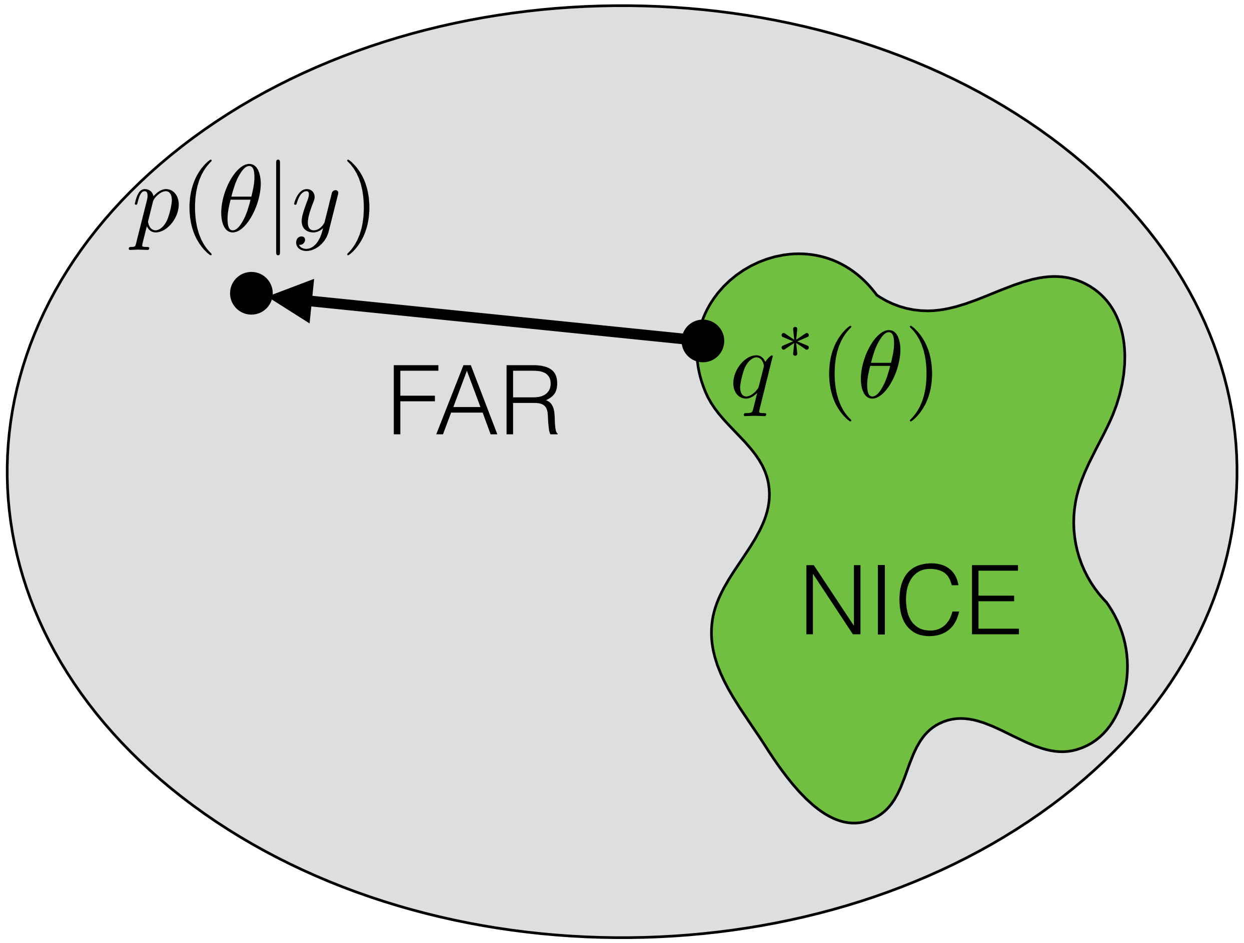

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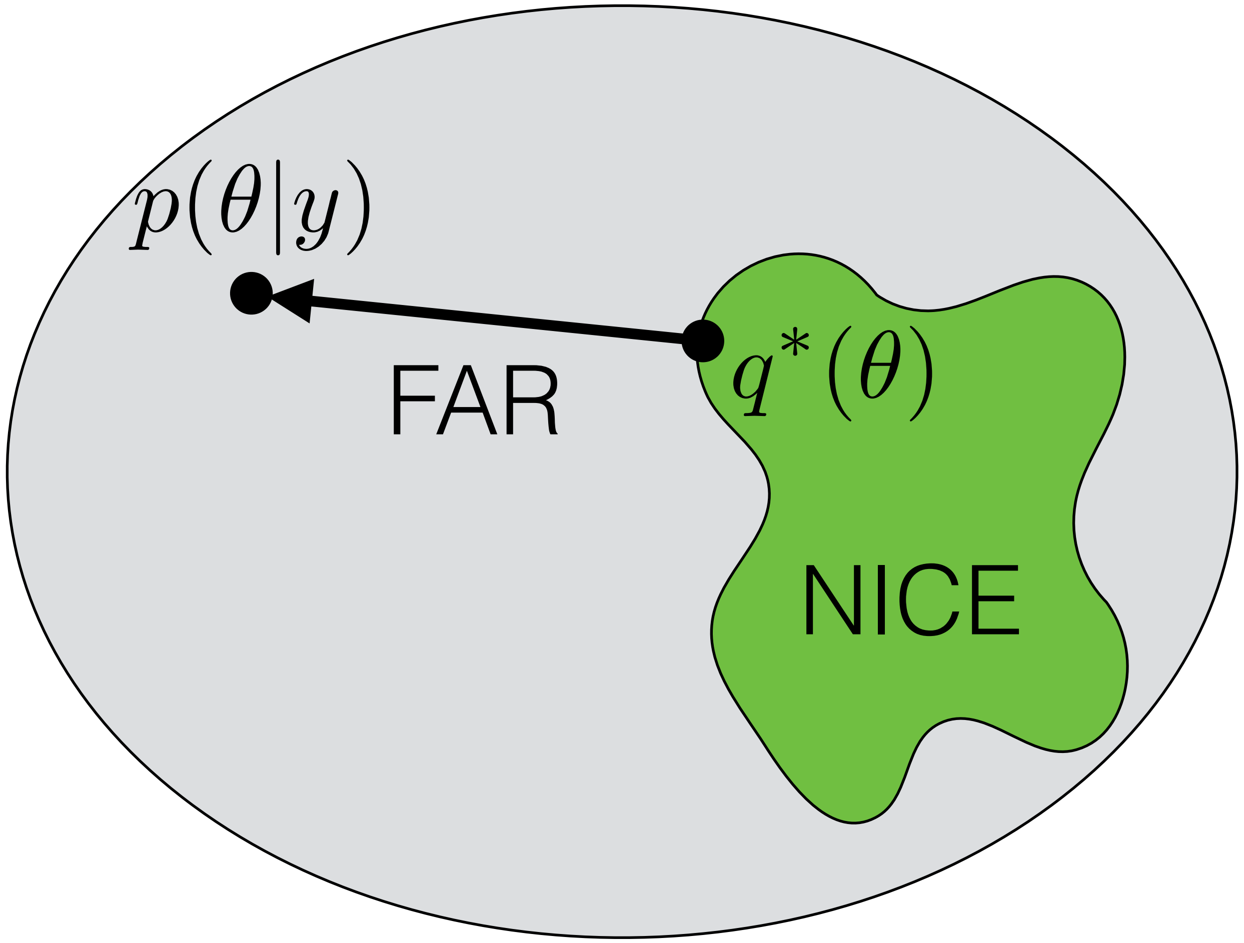

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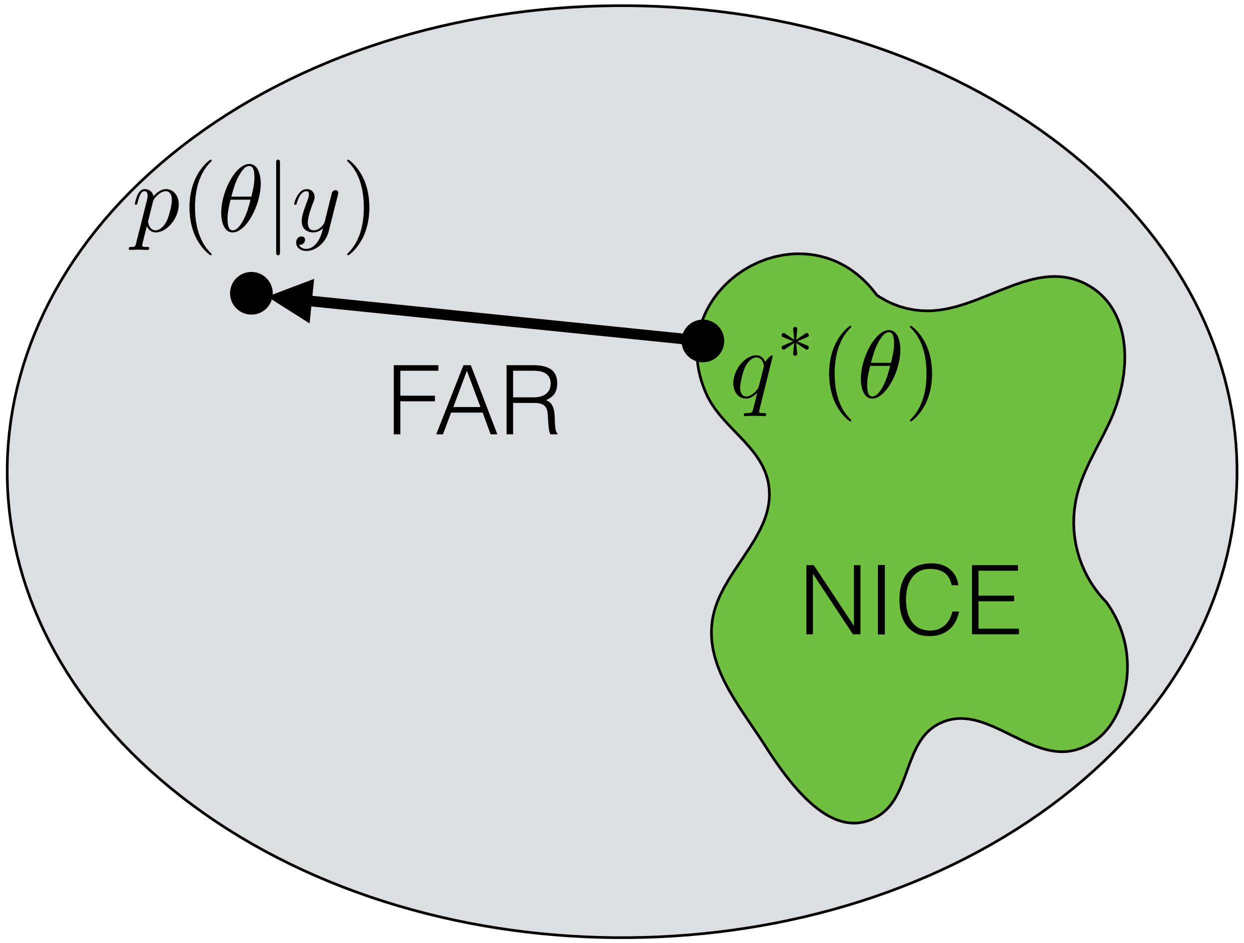

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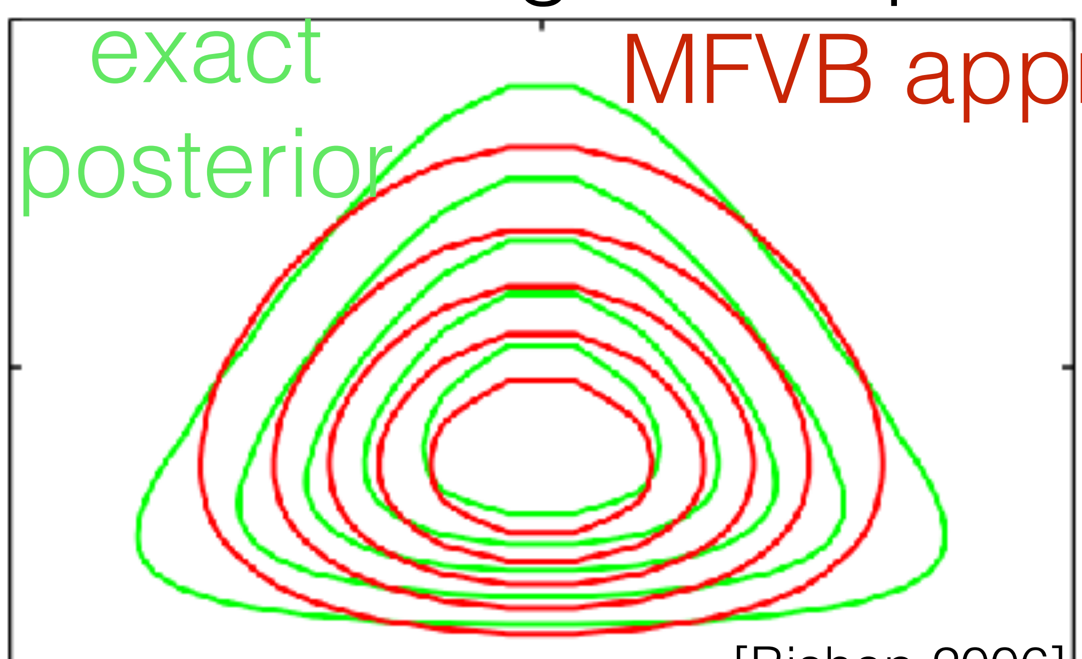

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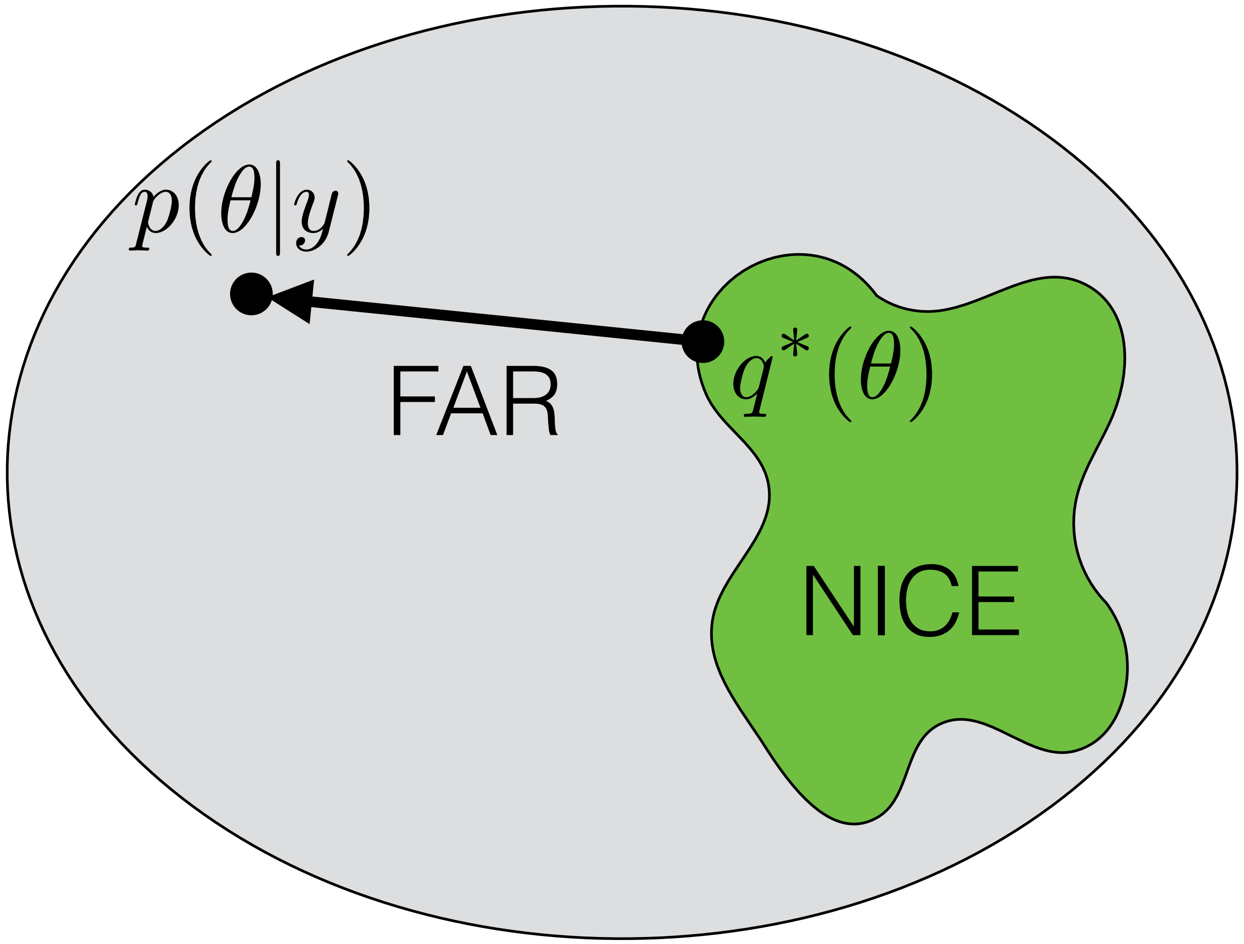

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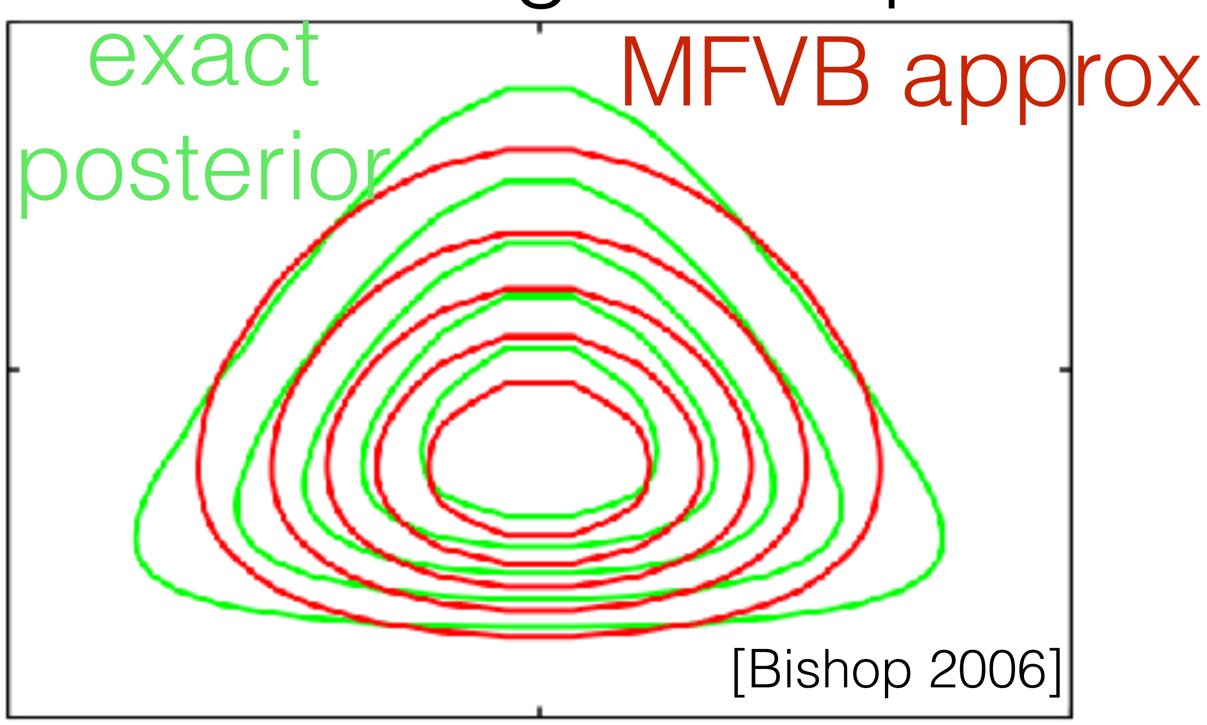
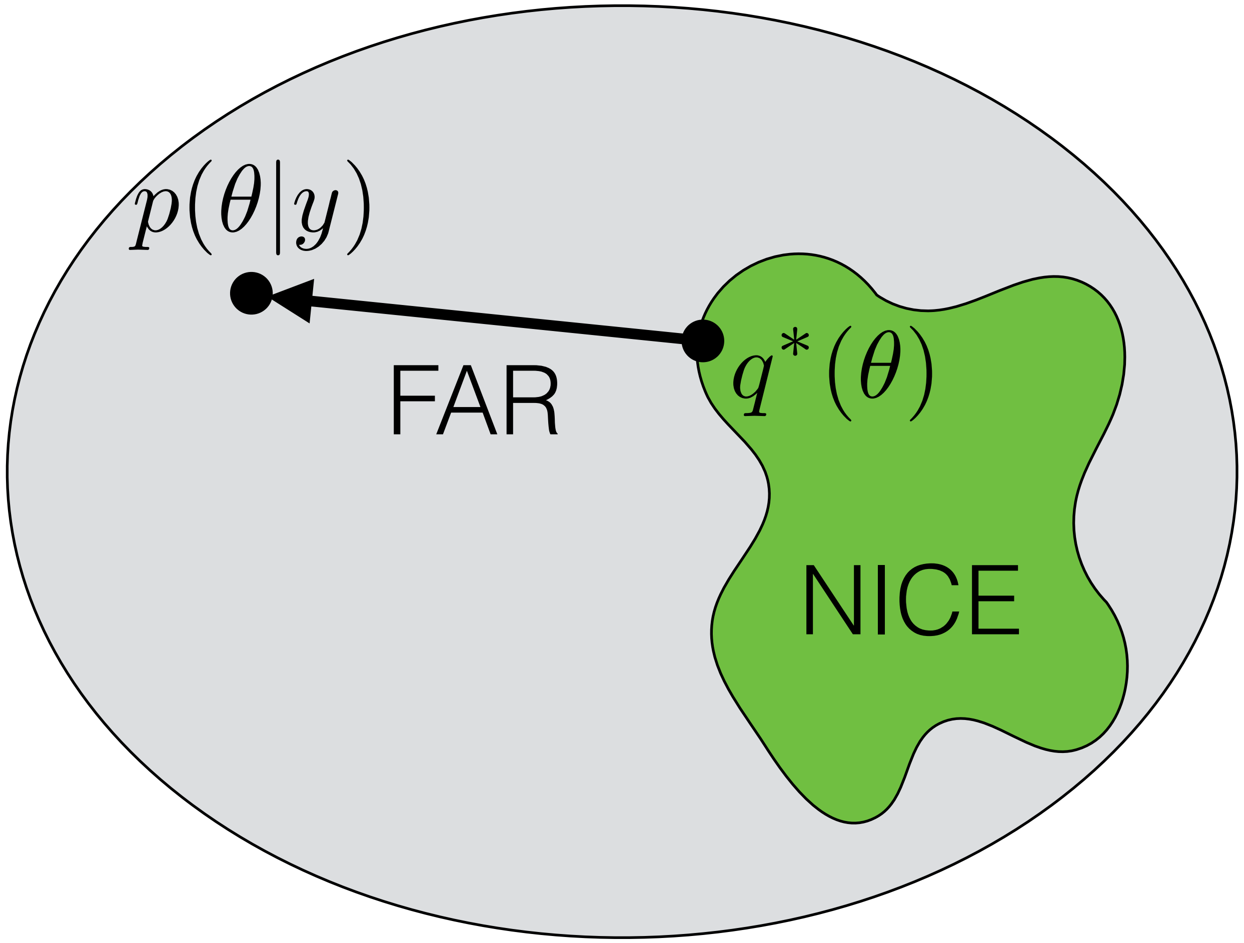
Now we have an optimization problem; how to solve it?



Variational Bayes

$$q^* = \text{argmin}_{q \in Q} \text{KL}\left(q(\cdot) || p(\cdot|y)\right)$$



Choose "NICE" distributions

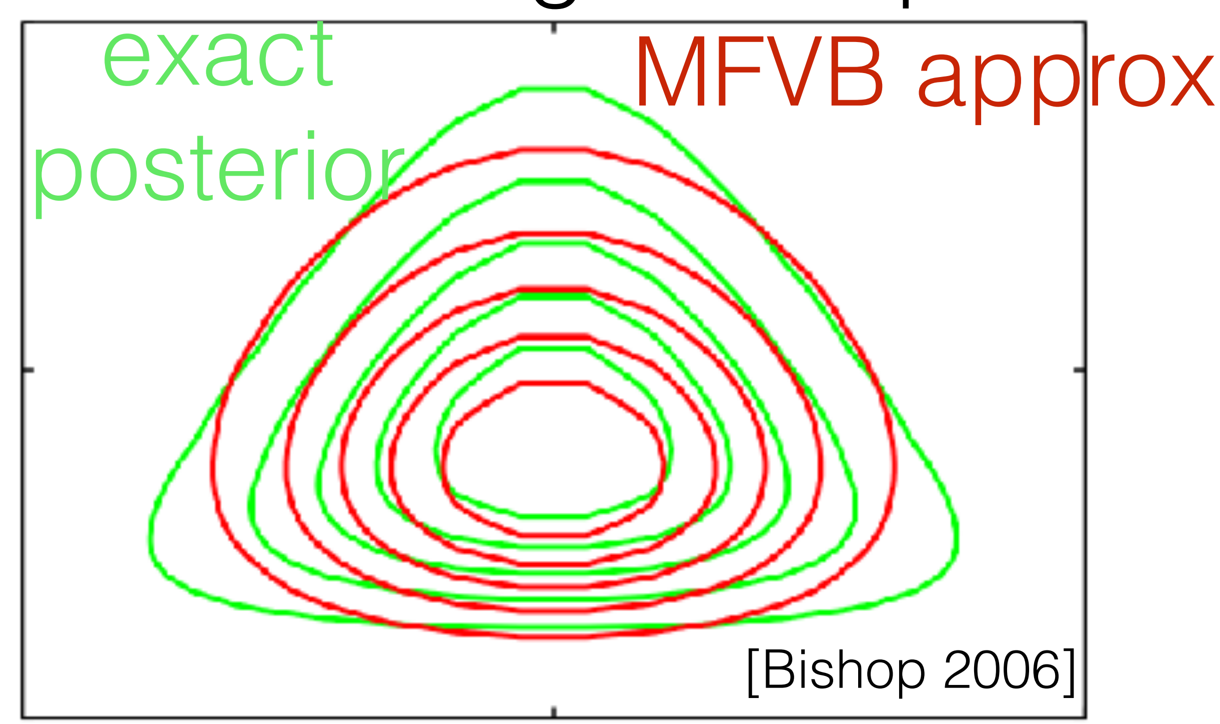
- Mean-field variational Bayes (MFVB)

$$Q_{MFVB} := \left\{ q : q(\theta) = \prod_{j=1}^{J} q_j(\theta_j) \right\}$$

- Often also exponential family
- *Not* a modeling assumption

Now we have an optimization problem; how to solve it?

- *One* option: Coordinate descent in $q_1, \ldots, q_J$



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Use q^* to approximate $p(\cdot|y)$

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$$q^* = \mathrm{argmin}_{q \in Q} f(q(\cdot), p(\cdot|y))$$

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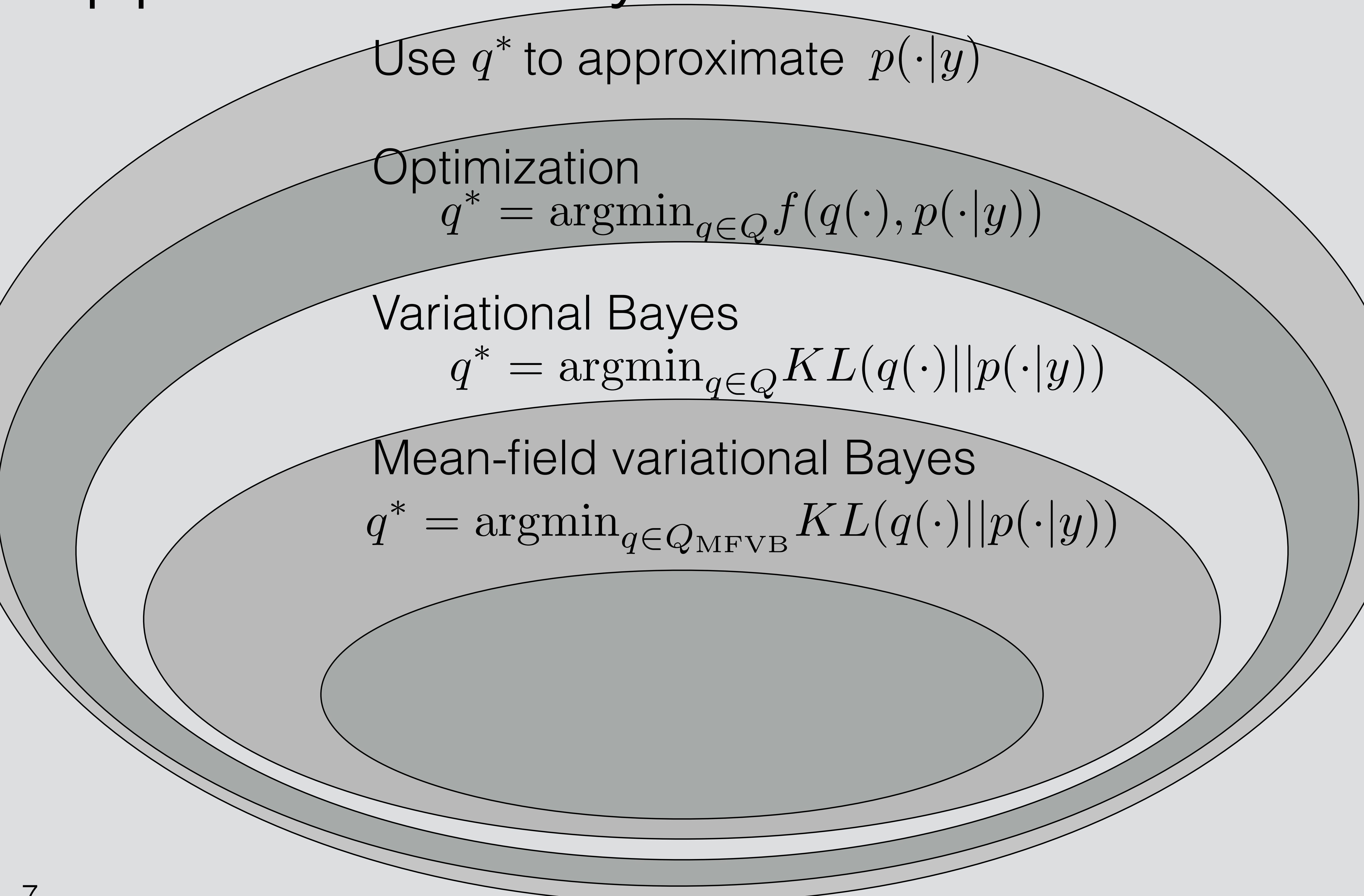

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  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use VB?
- When can we trust VB?
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What to read next

Textbooks and Reviews

- Bishop. *Pattern Recognition and Machine Learning*, Ch 10. 2006.
- Blei, Kucukelbir, McAuliffe. Variational inference: A review for statisticians, *JASA* 2016.
- MacKay. *Information Theory, Inference, and Learning Algorithms*, Ch 33. 2003.
- Murphy. *Machine Learning: A Probabilistic Perspective*, Ch 21. 2012.
- Ormerod, Wand. Explaining variational approximations. *Amer Stat* 2010.
- Turner, Sahani. Two problems with variational expectation maximisation for time-series models. In *Bayesian Time Series Models*, 2011.
- Wainwright, Jordan. Graphical models, exponential families, and variational inference. *Foundations and Trends in Machine Learning*, 2008.

Our Experiments

- RJ Giordano, T Broderick, and MI Jordan. Linear response methods for accurate covariance estimates from mean field variational Bayes. *NeurIPS* 2015.
- RJ Giordano, T Broderick, R Meager, JH Huggins, and MI Jordan. Fast robustness quantification with variational Bayes. *ICML Data4Good Workshop* 2016.
- RJ Giordano, T Broderick, and MI Jordan. Covariances, robustness, and variational Bayes. *JMLR* 2018.
- J Huggins, M Kasprzak, T Campbell, T Broderick. Validated Variational Inference via Practical Posterior Error Bounds. ArXiv: 1910.04102. *AISTATS* 2020, to appear.
- T Campbell and T Broderick. Automated scalable Bayesian inference via Hilbert coresets. *JMLR* 2019.
- T Campbell and T Broderick. Bayesian Coreset Construction via Greedy Iterative Geodesic Ascent. *ICML* 2018.

References

Full references at end of final slides