6.036/6.862: Introduction to Machine Learning

Lecture: starts Tuesdays 9:35am (Boston time zone)

Course website: introml.odl.mit.edu

Who's talking? Prof. Tamara Broderick

Questions? discourse.odl.mit.edu ("Lecture 2" category)

Materials: Will all be available at course website

Last Time

I. Machine learning setup
II. Linear classifiers
III. Learning algorithms

Today's Plan

I. Perceptron algorithm
II. Harder and easier linear classification
III. Perceptron theorem

Recall: Classifiers

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- A linear classifier:

$h(x; \theta, \theta_0)$

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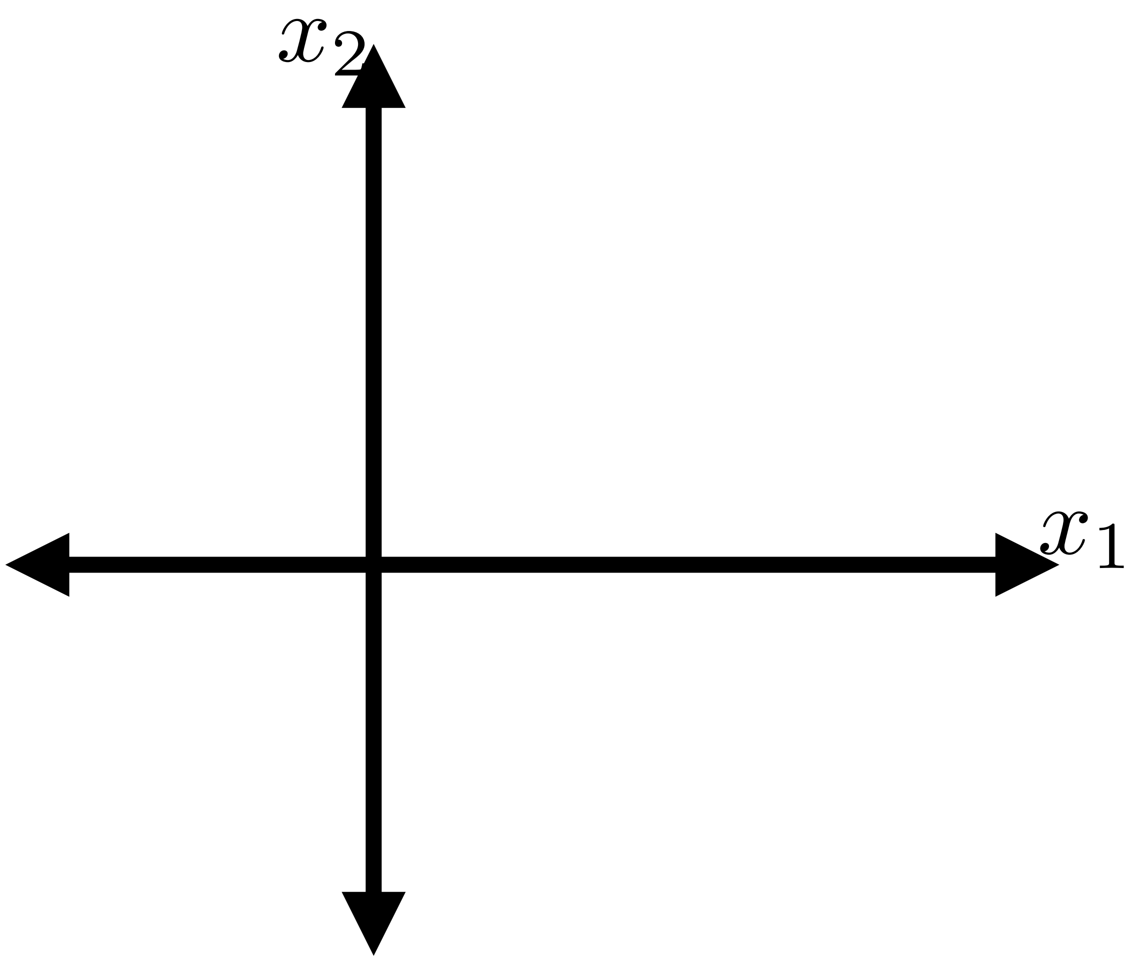

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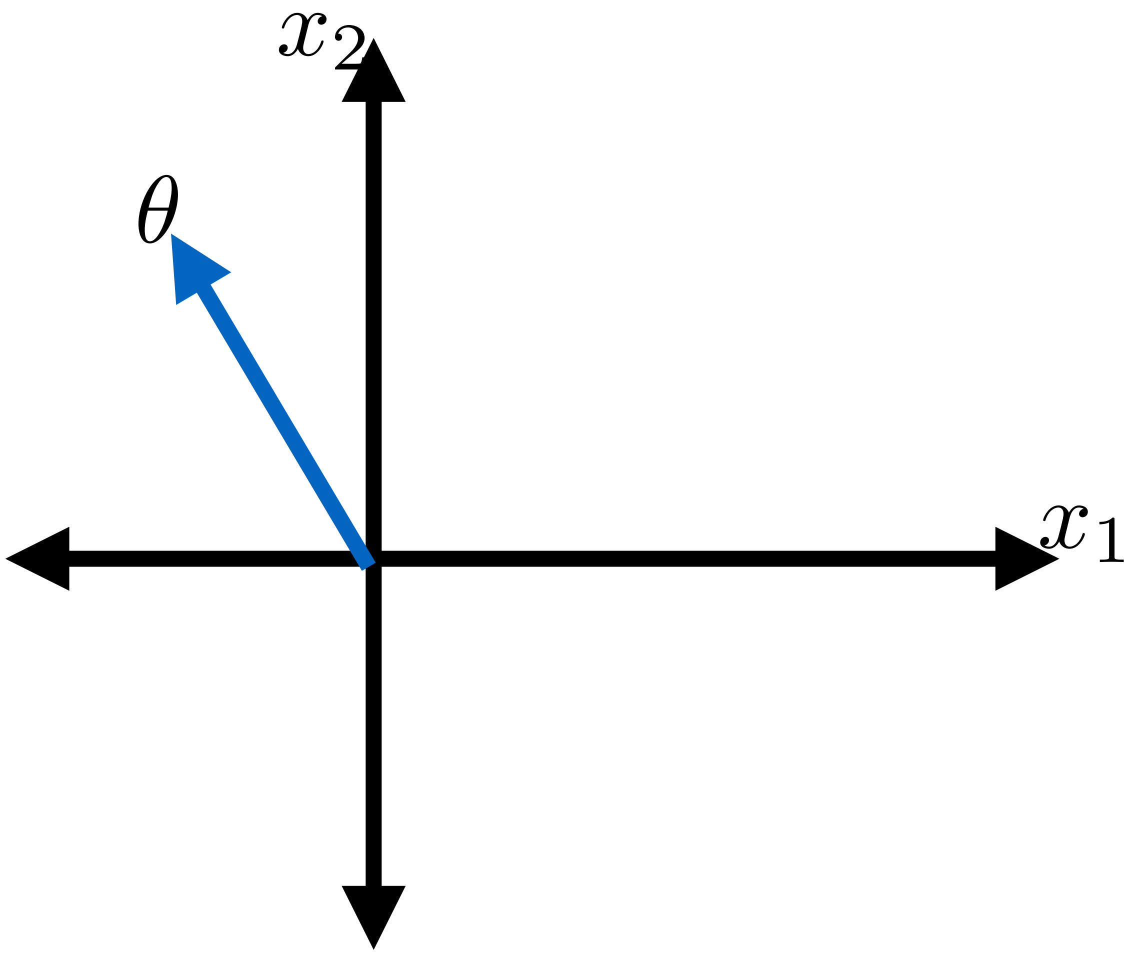

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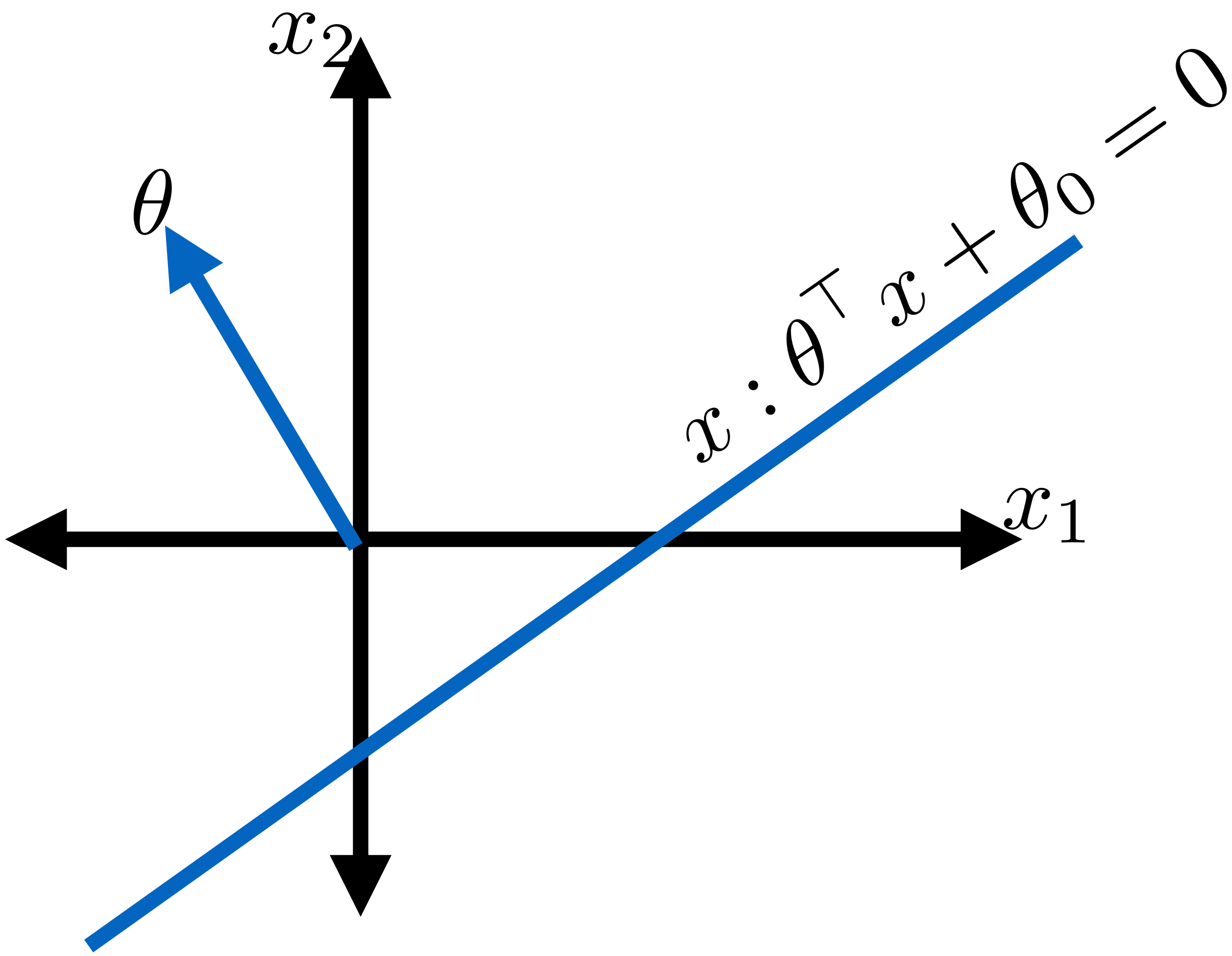

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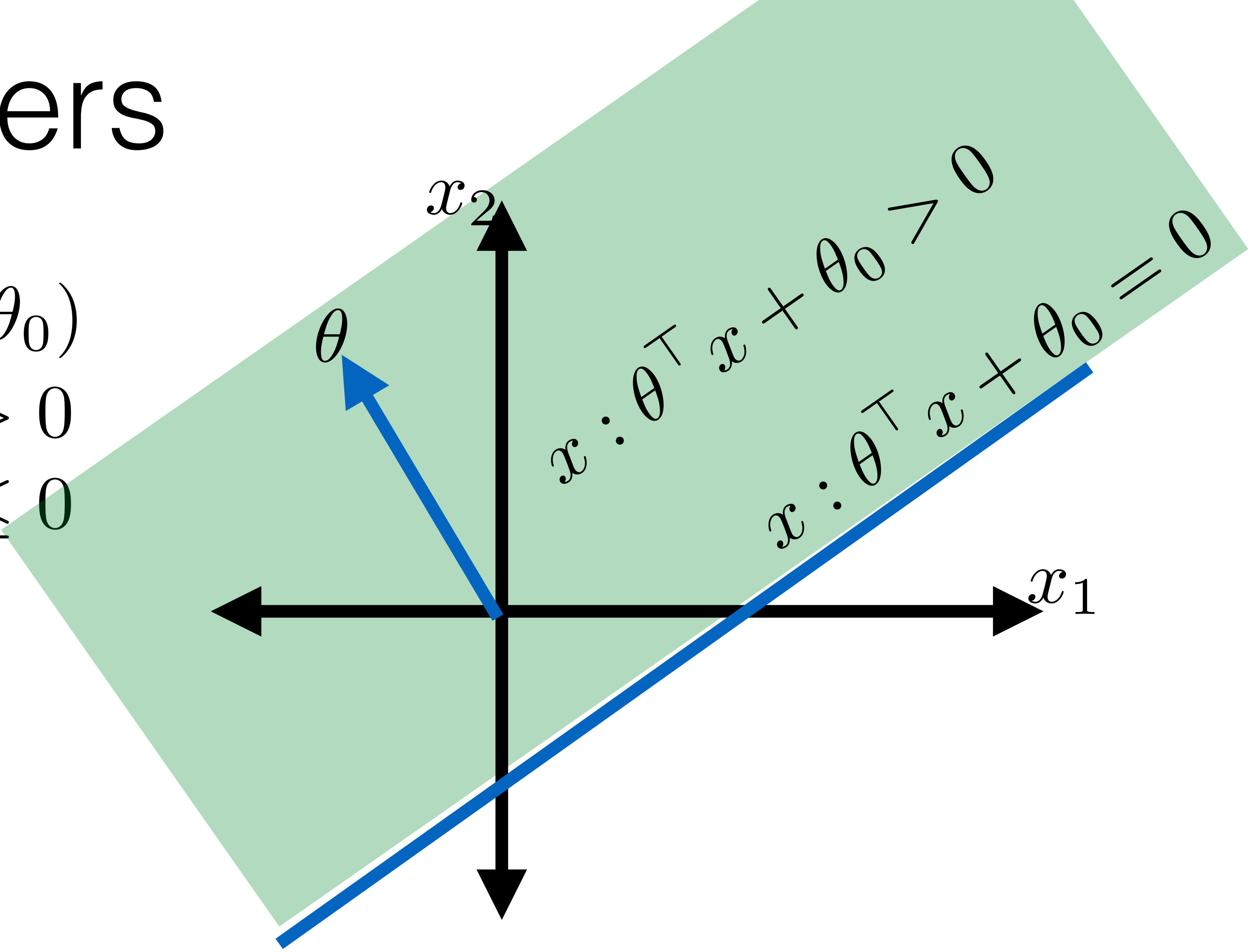

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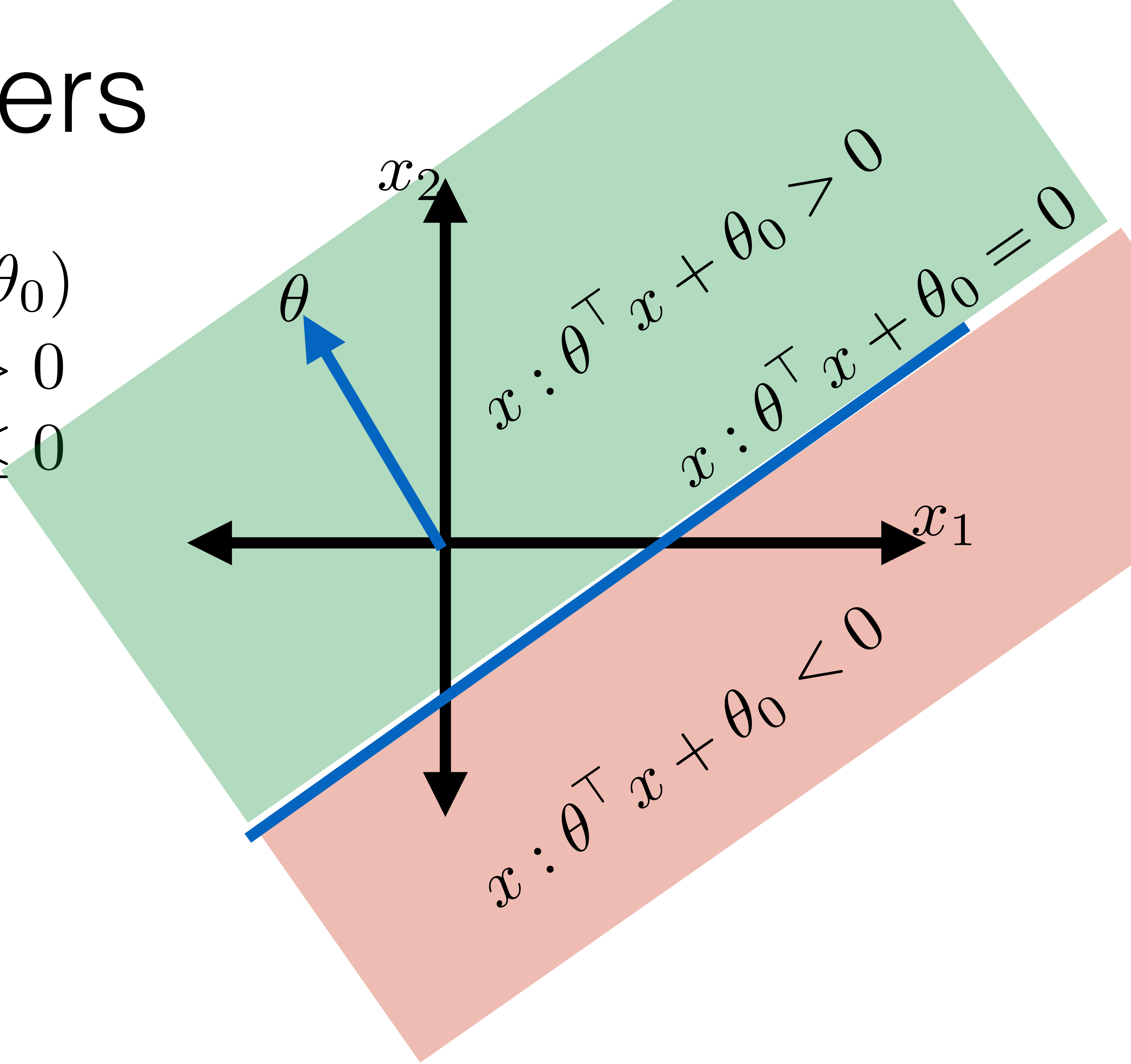

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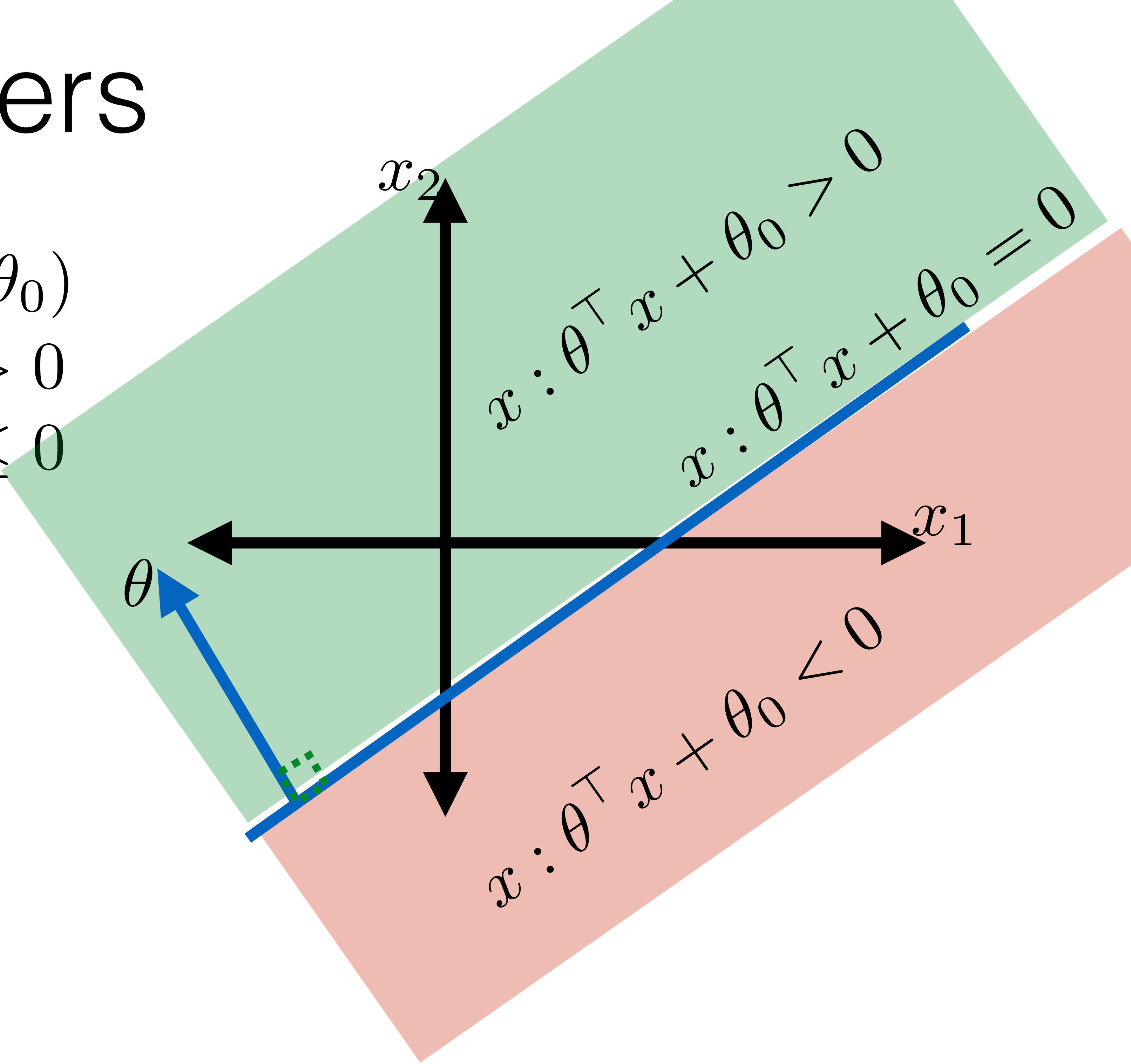

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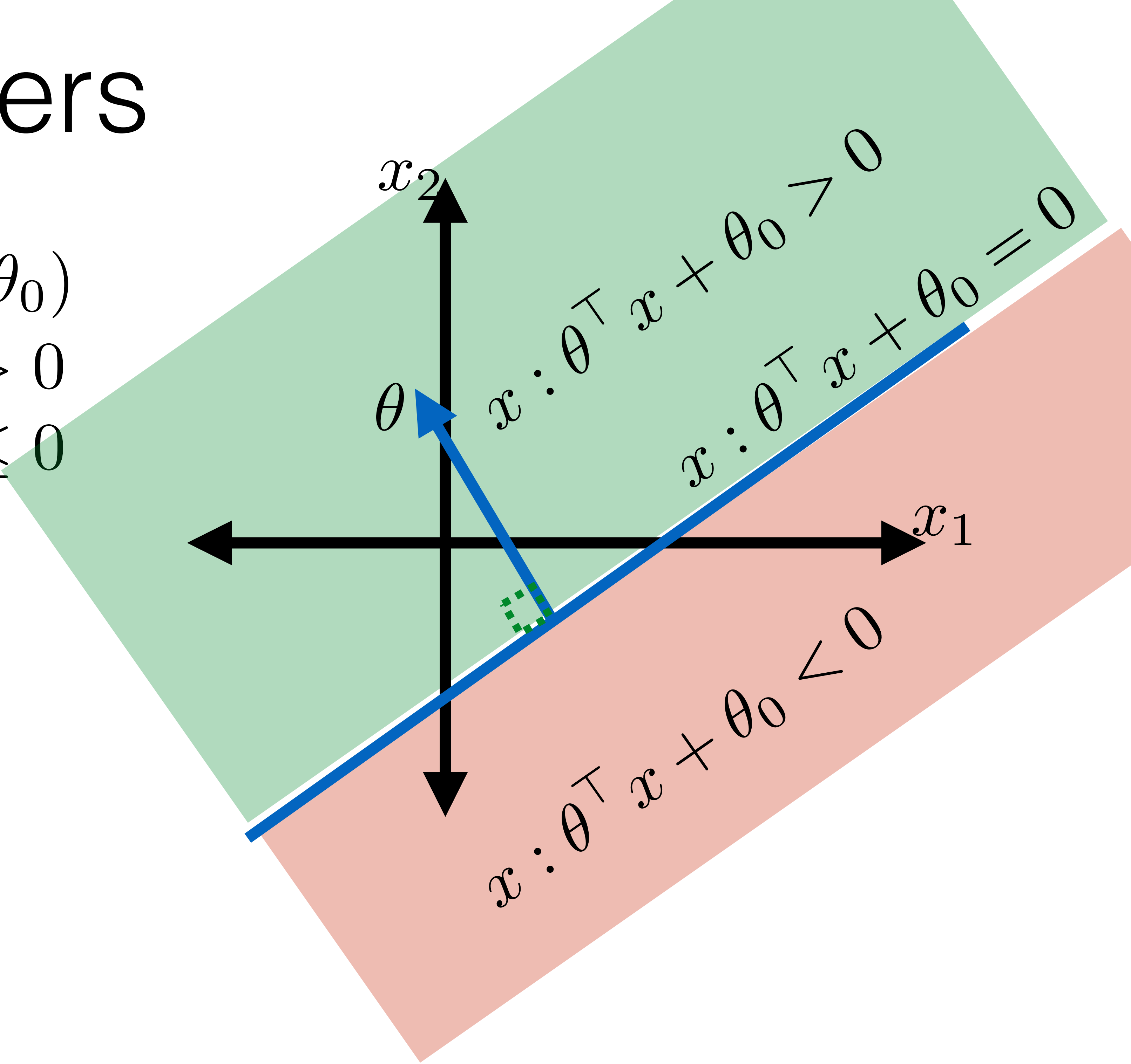

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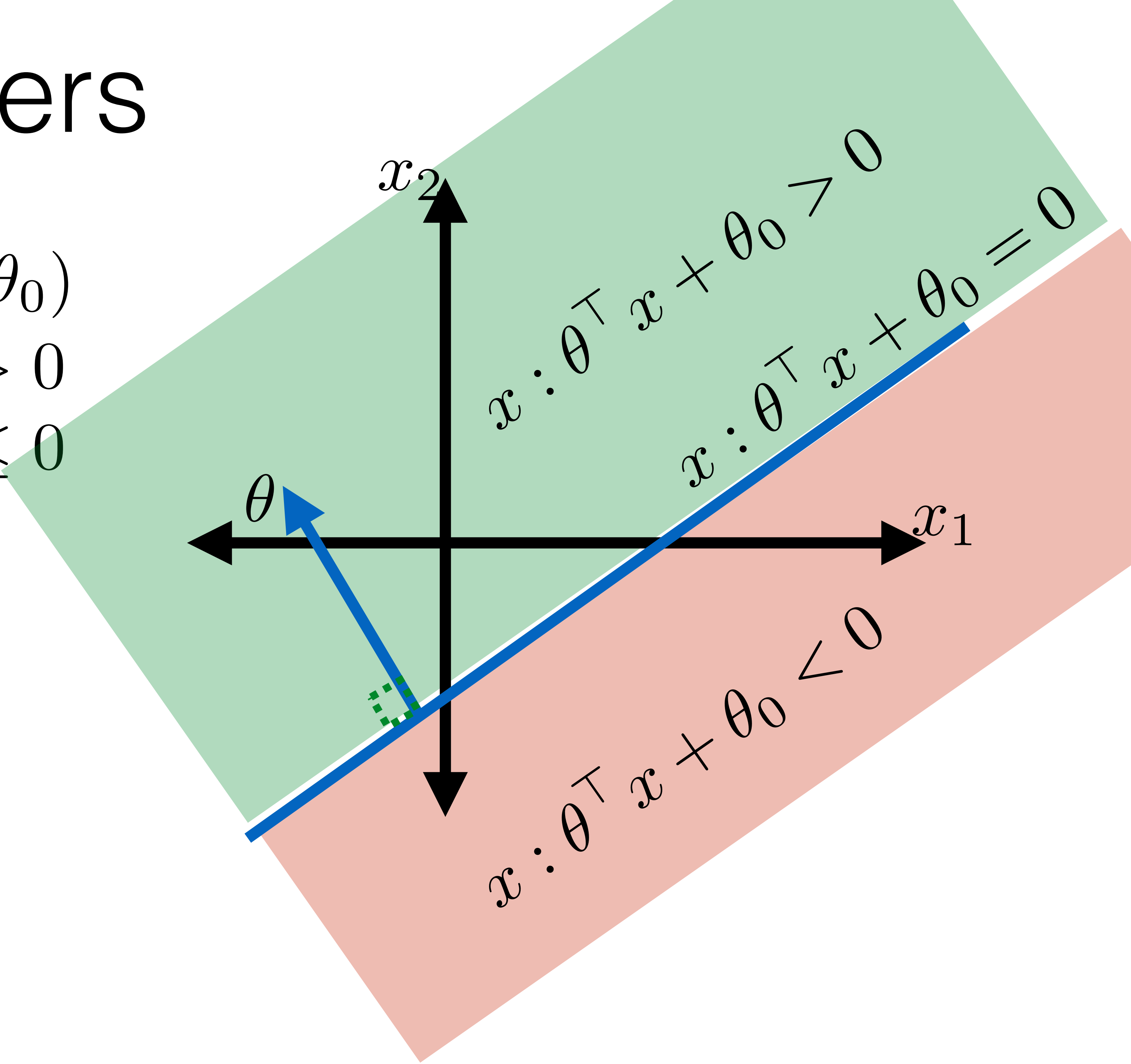

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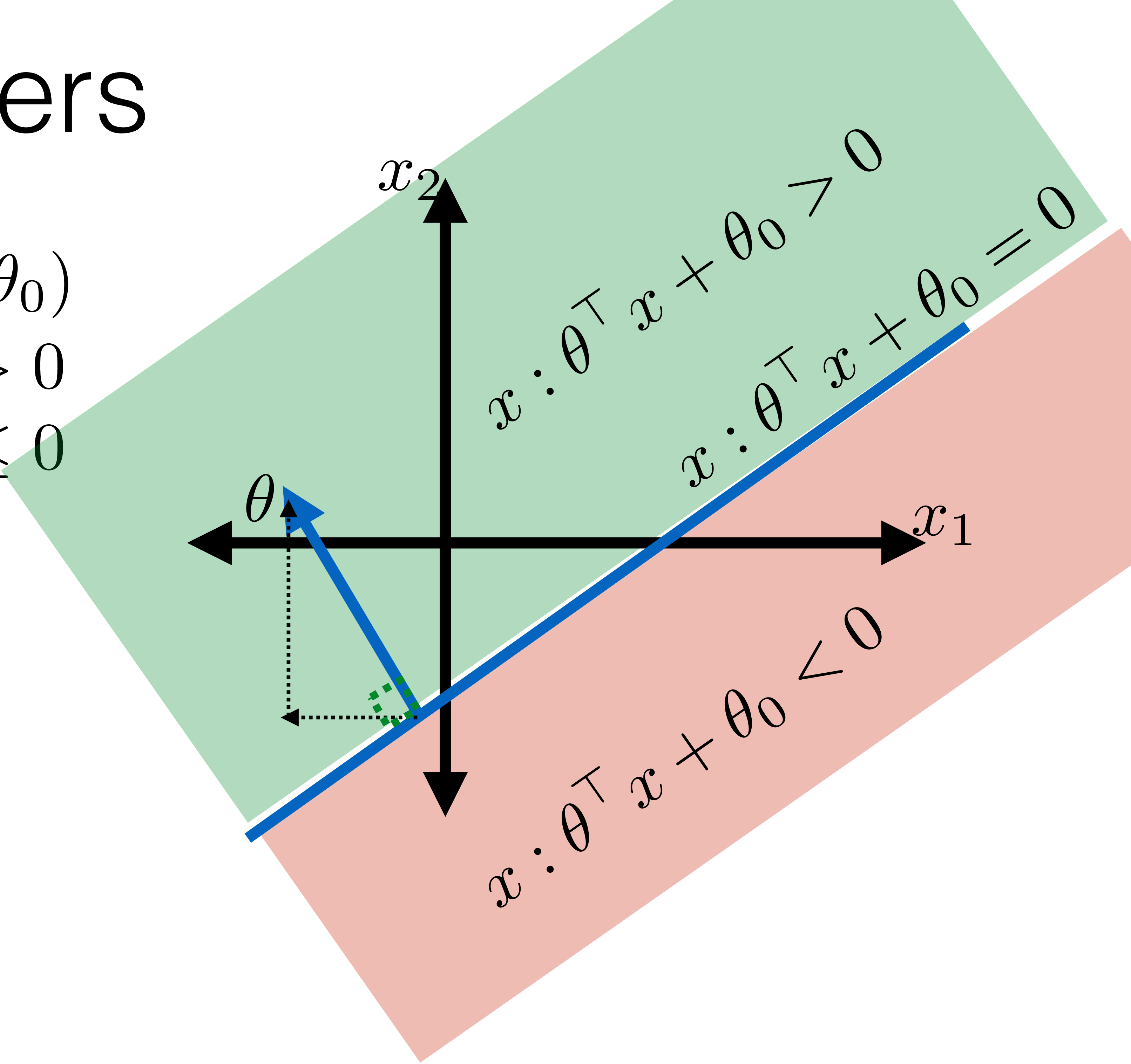

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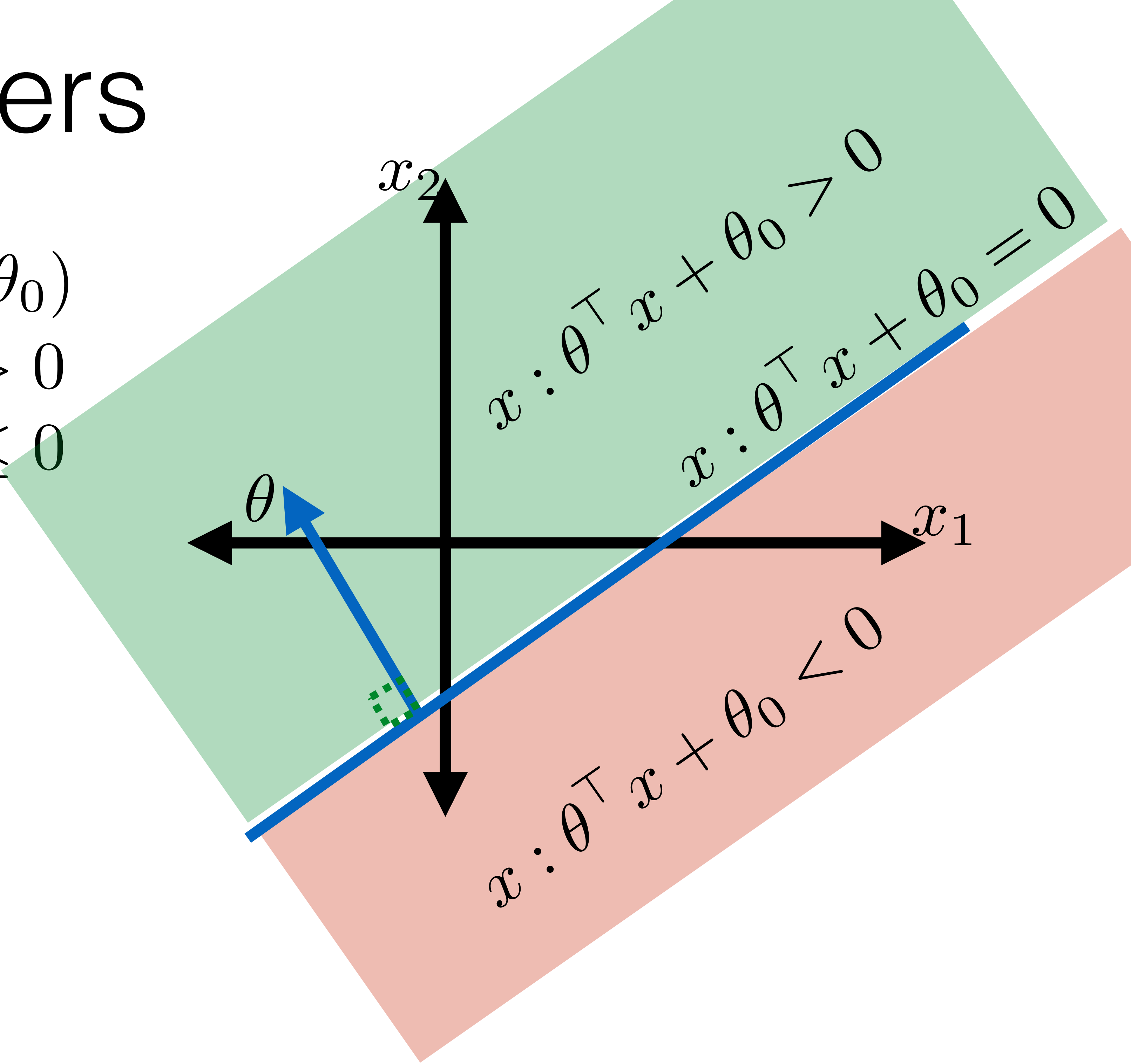

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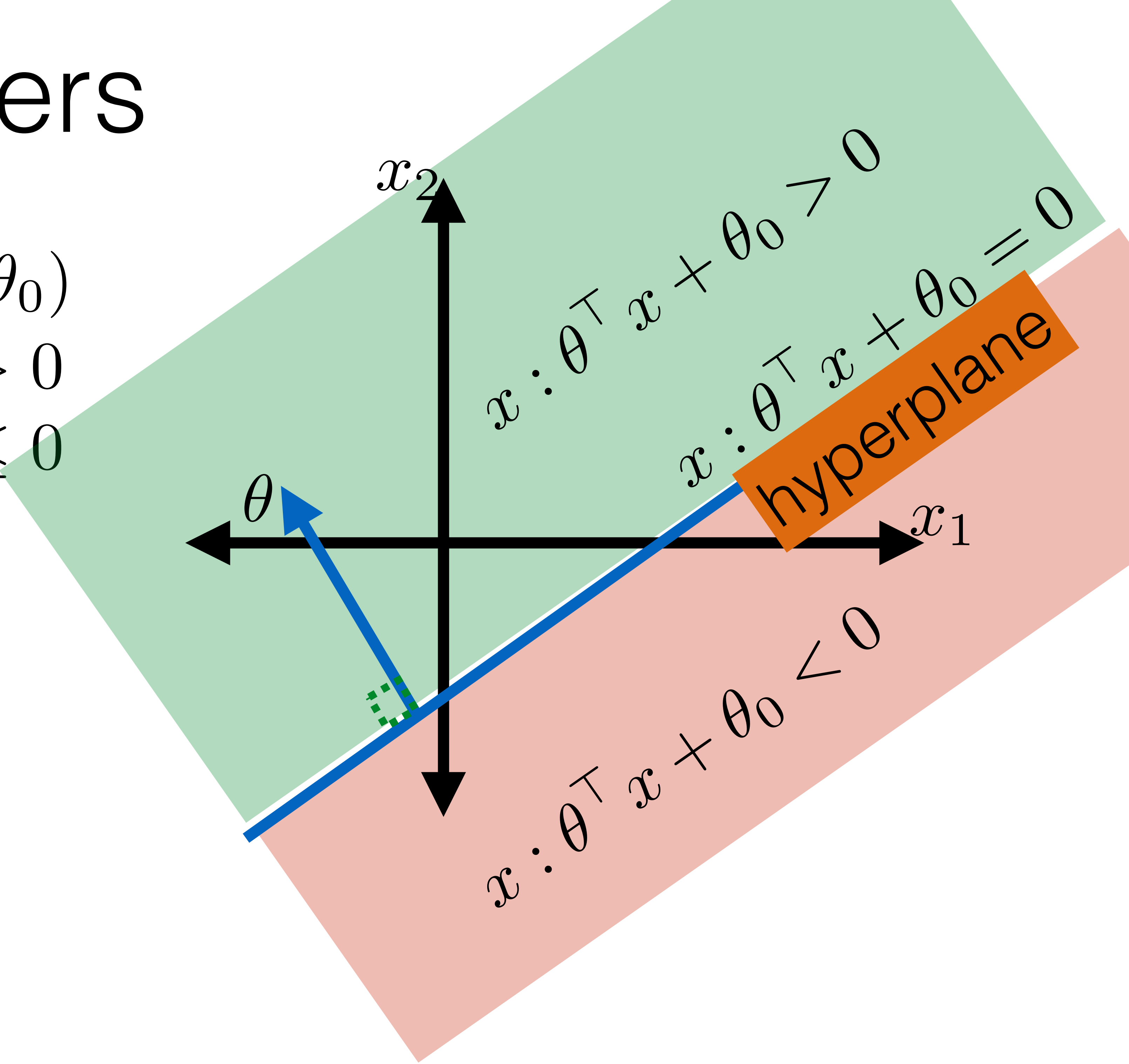

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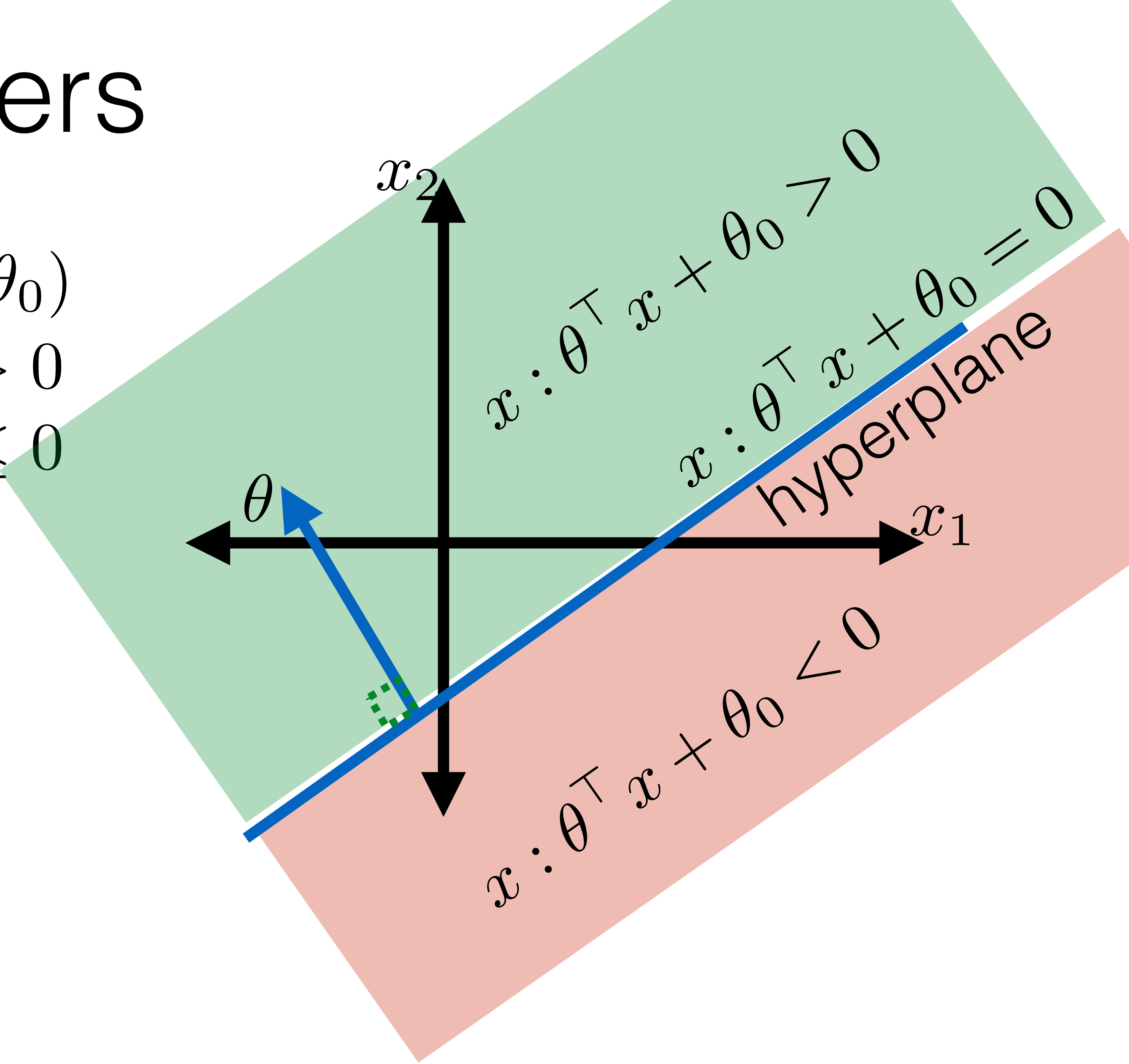

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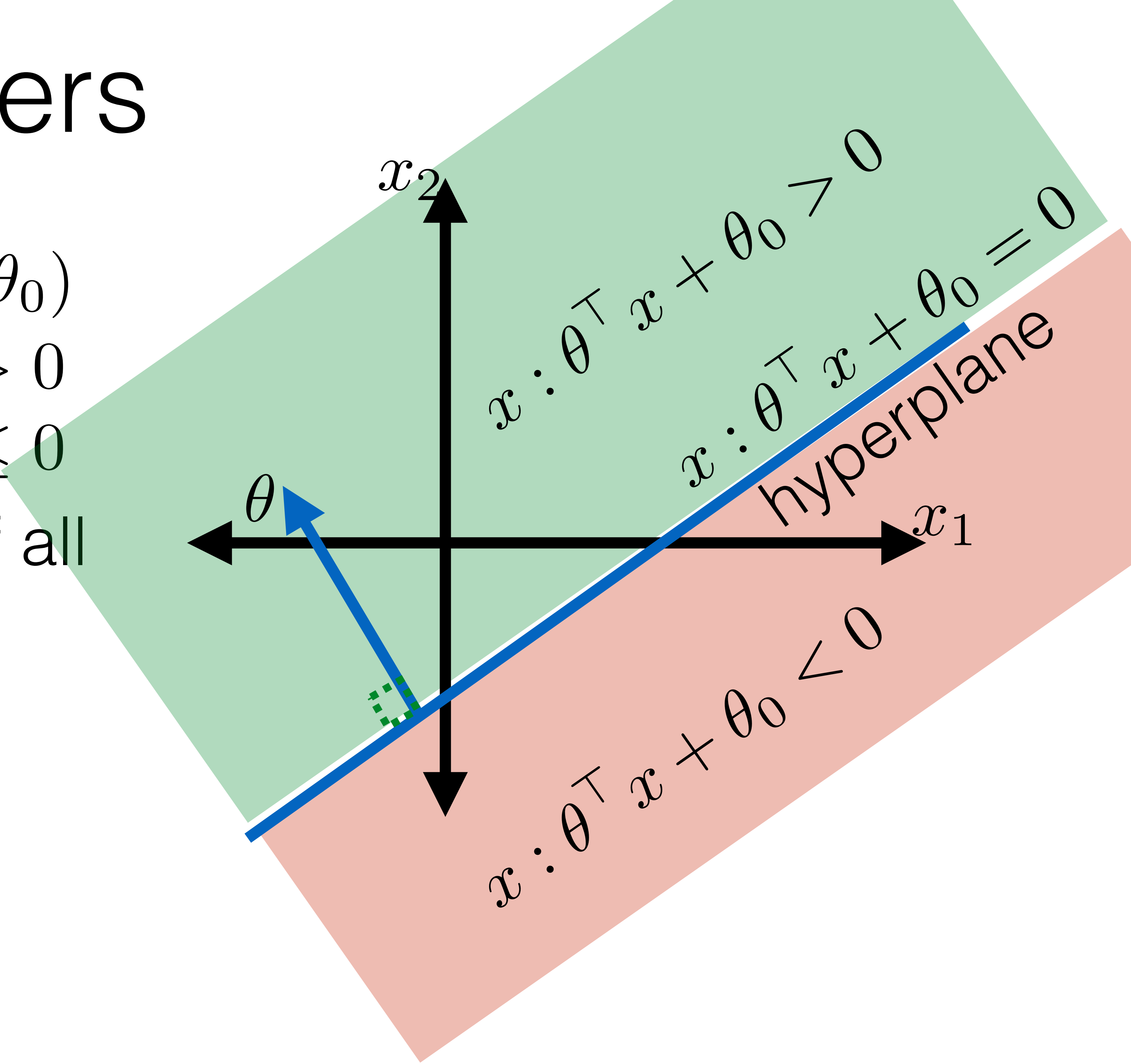

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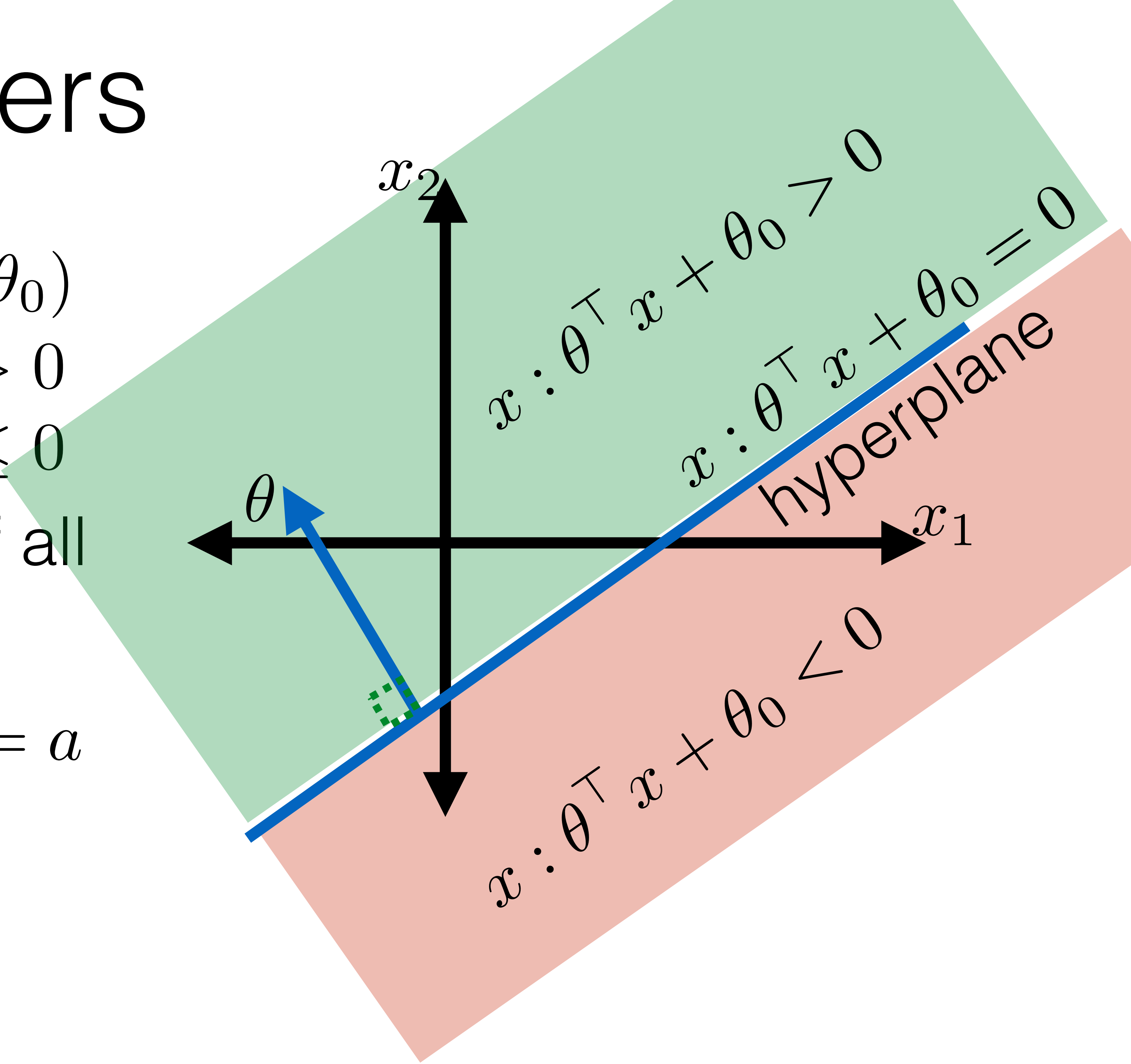
$$L(g,a) = \begin{cases} 0 \text{ if } g = a \\ 1 \text{ else} \end{cases}$$



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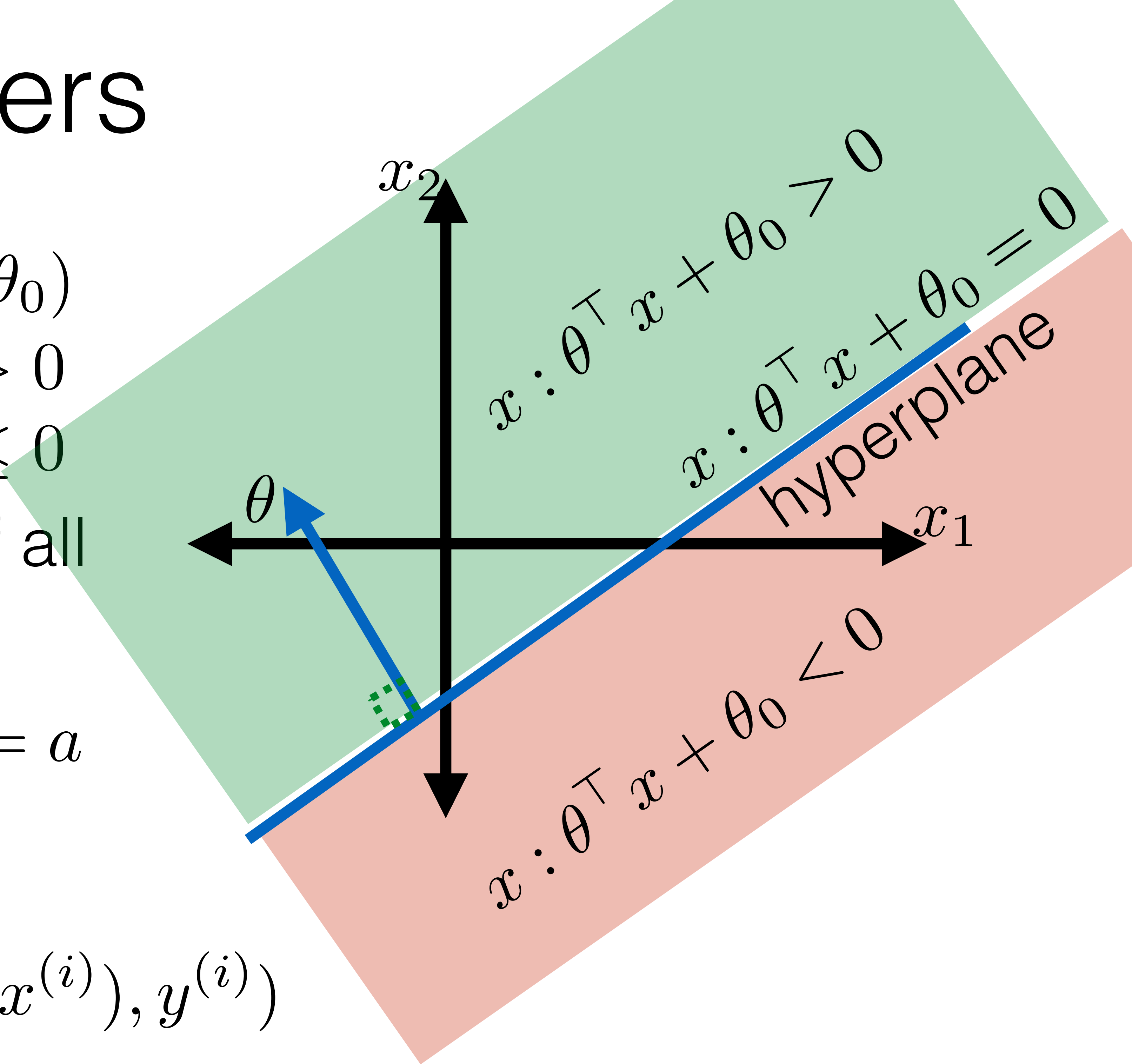
$$\mathcal{E}_n(h) = \frac{1}{n}\sum_{i=1}^{n} L(h(x^{(i)}), y^{(i)})$$



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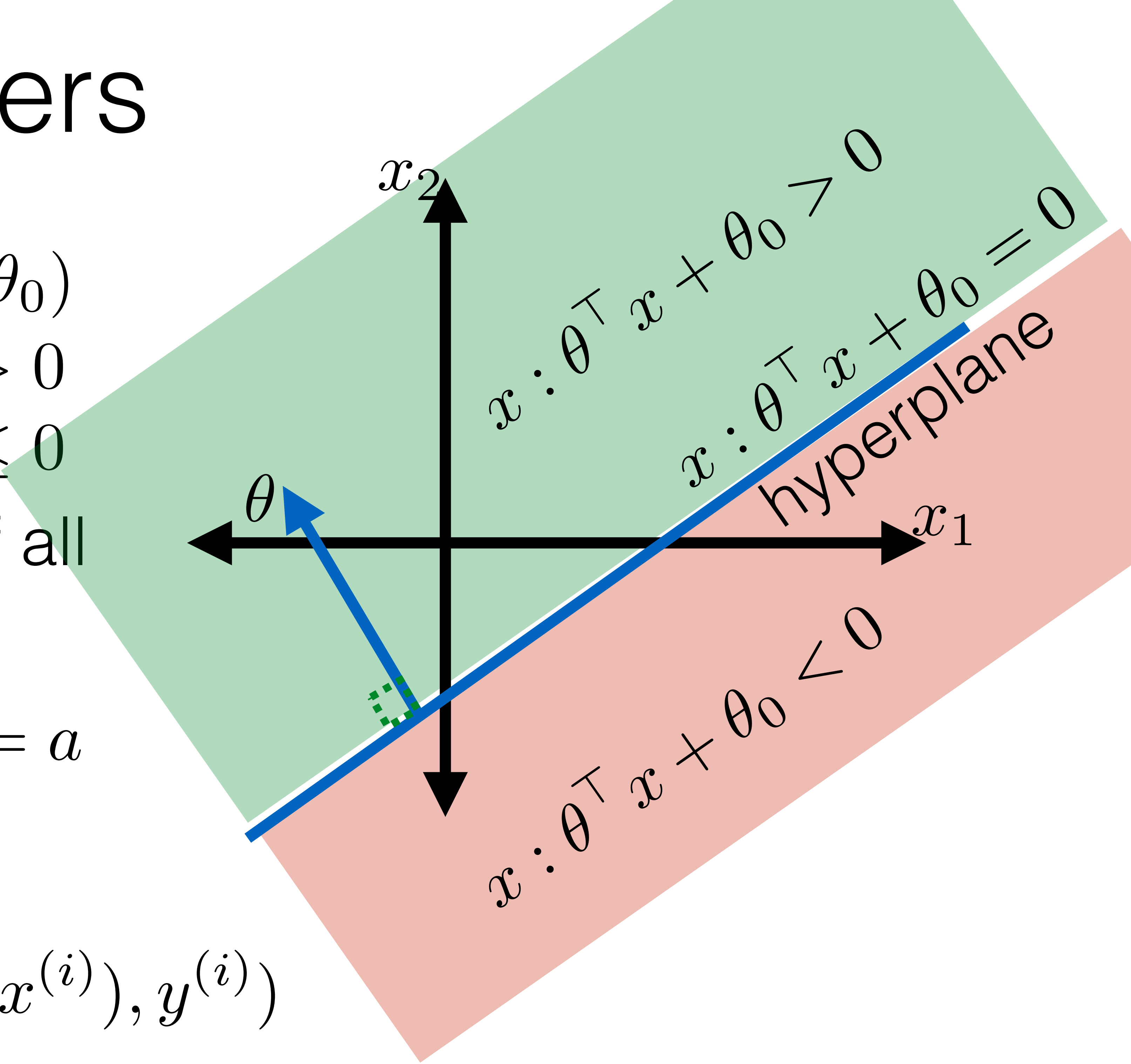

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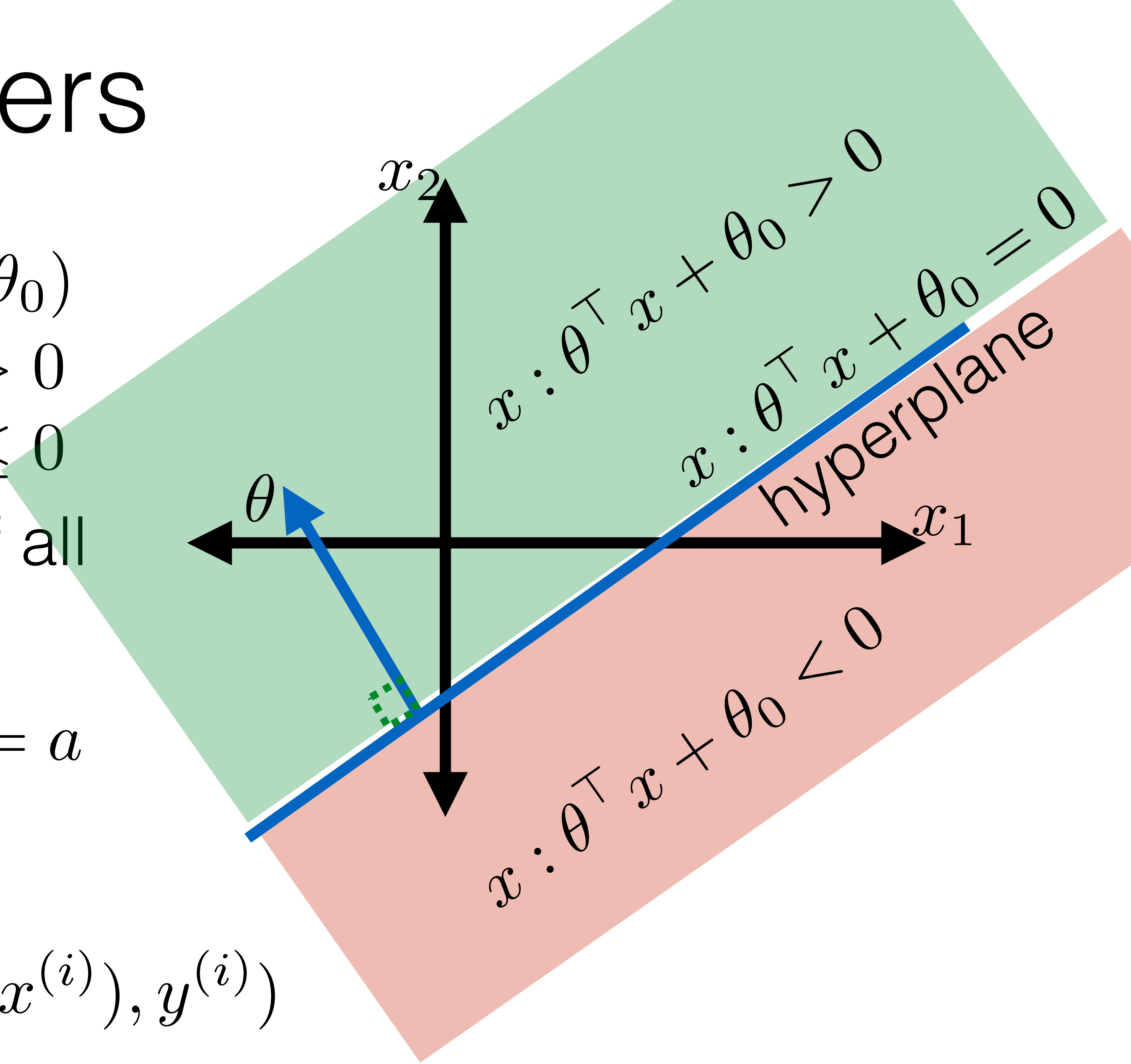

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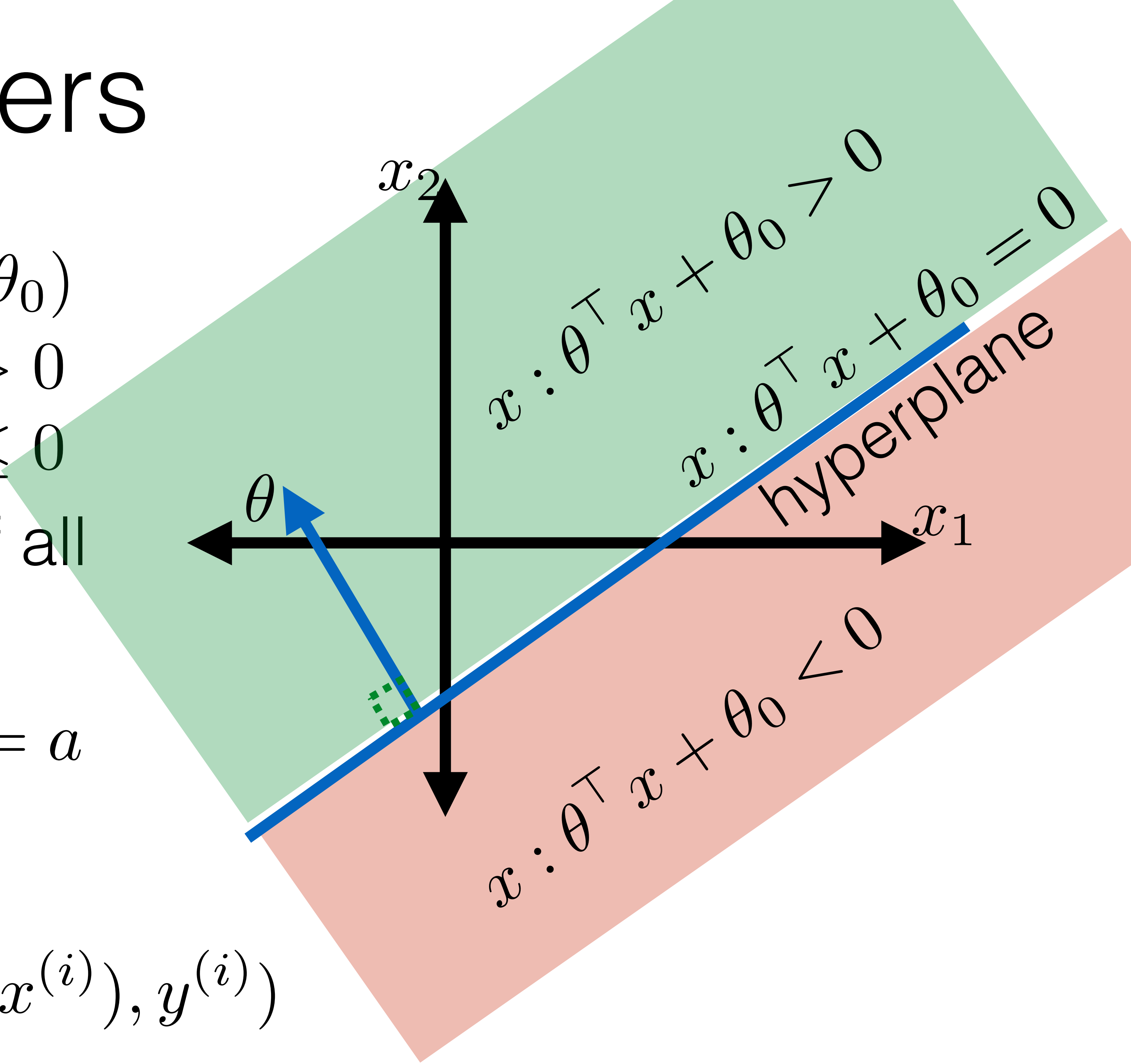

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[demo]

Perceptron Algorithm

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Perceptron Algorithm

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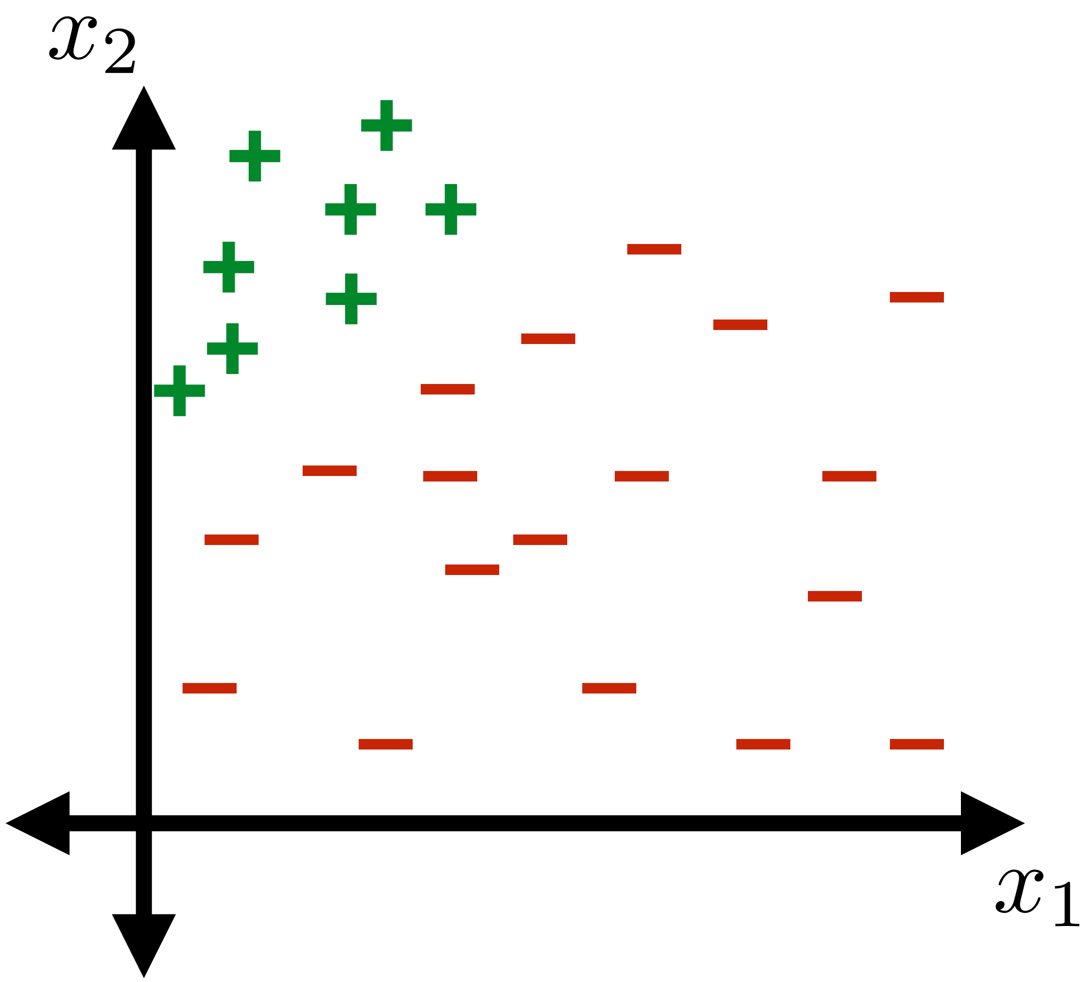

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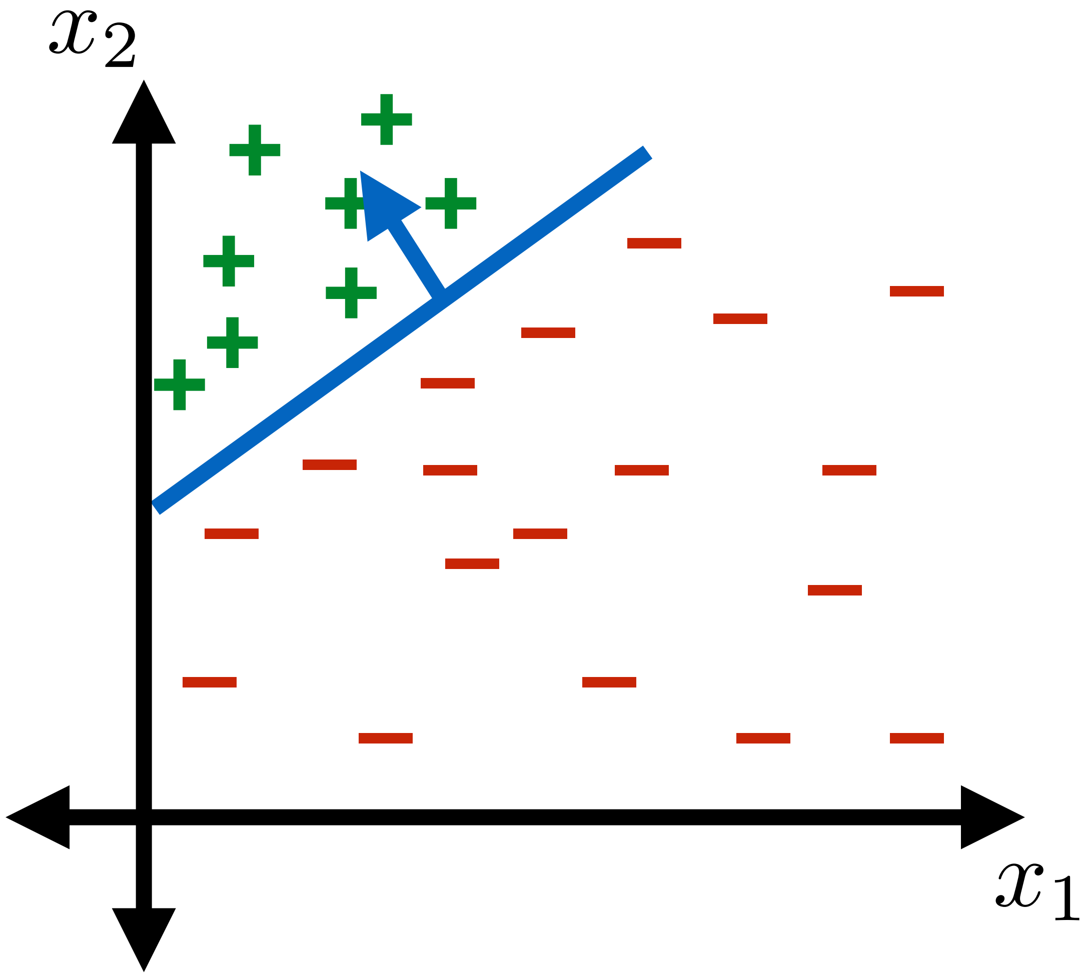

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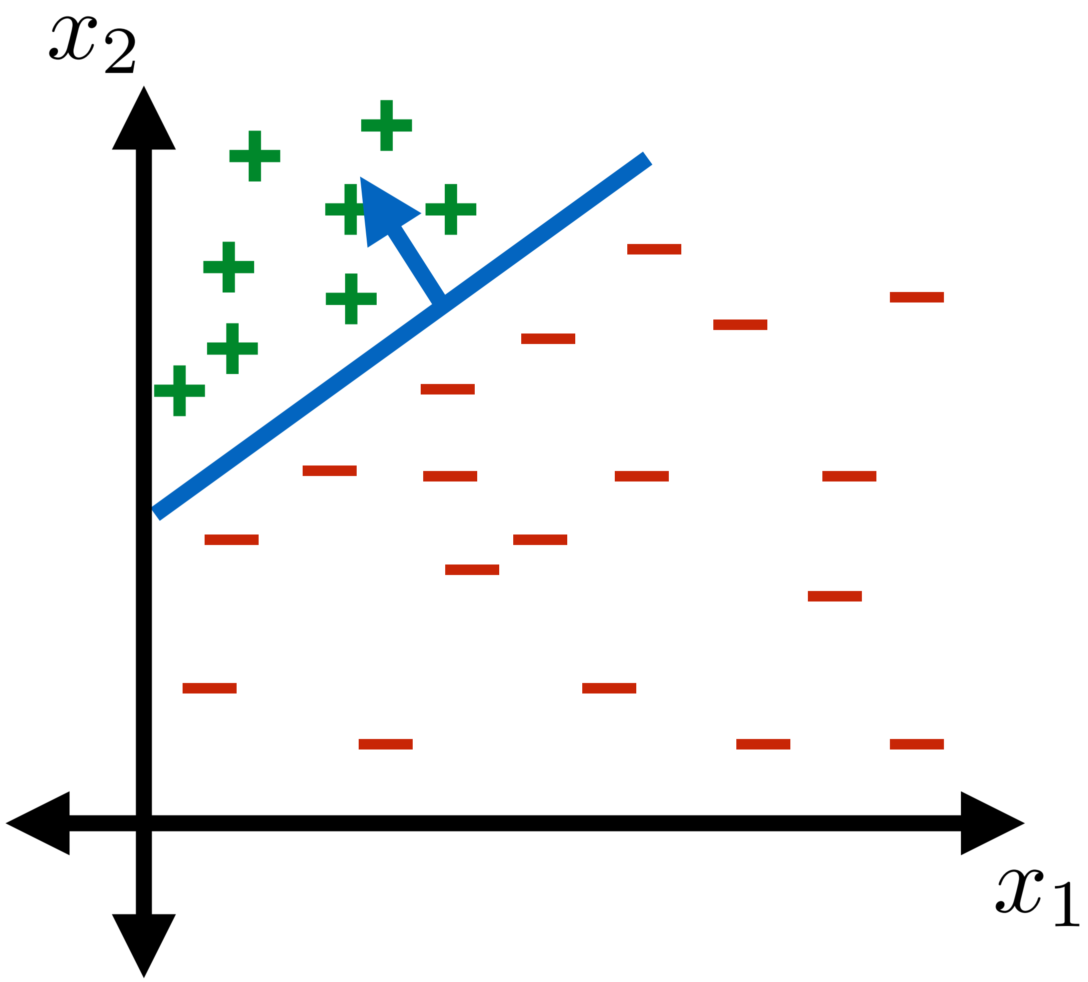

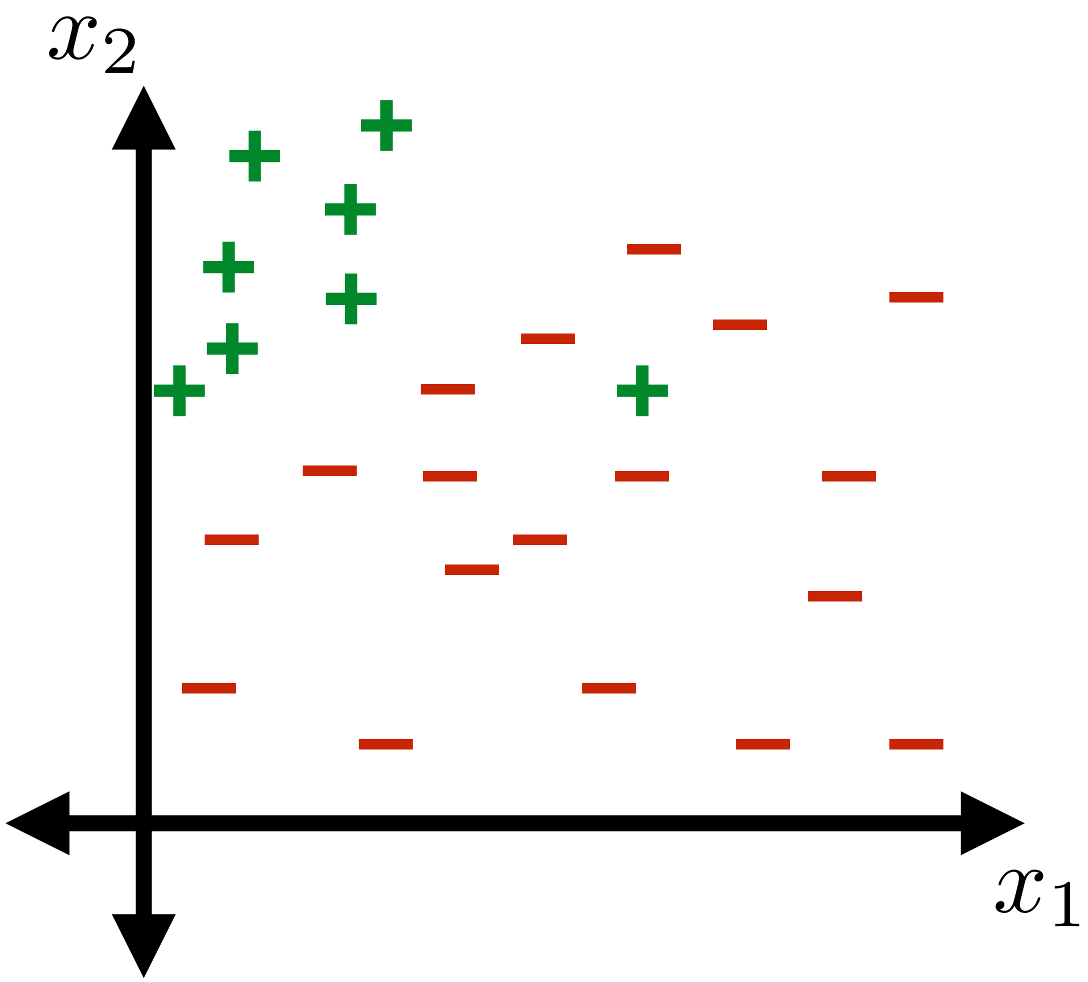

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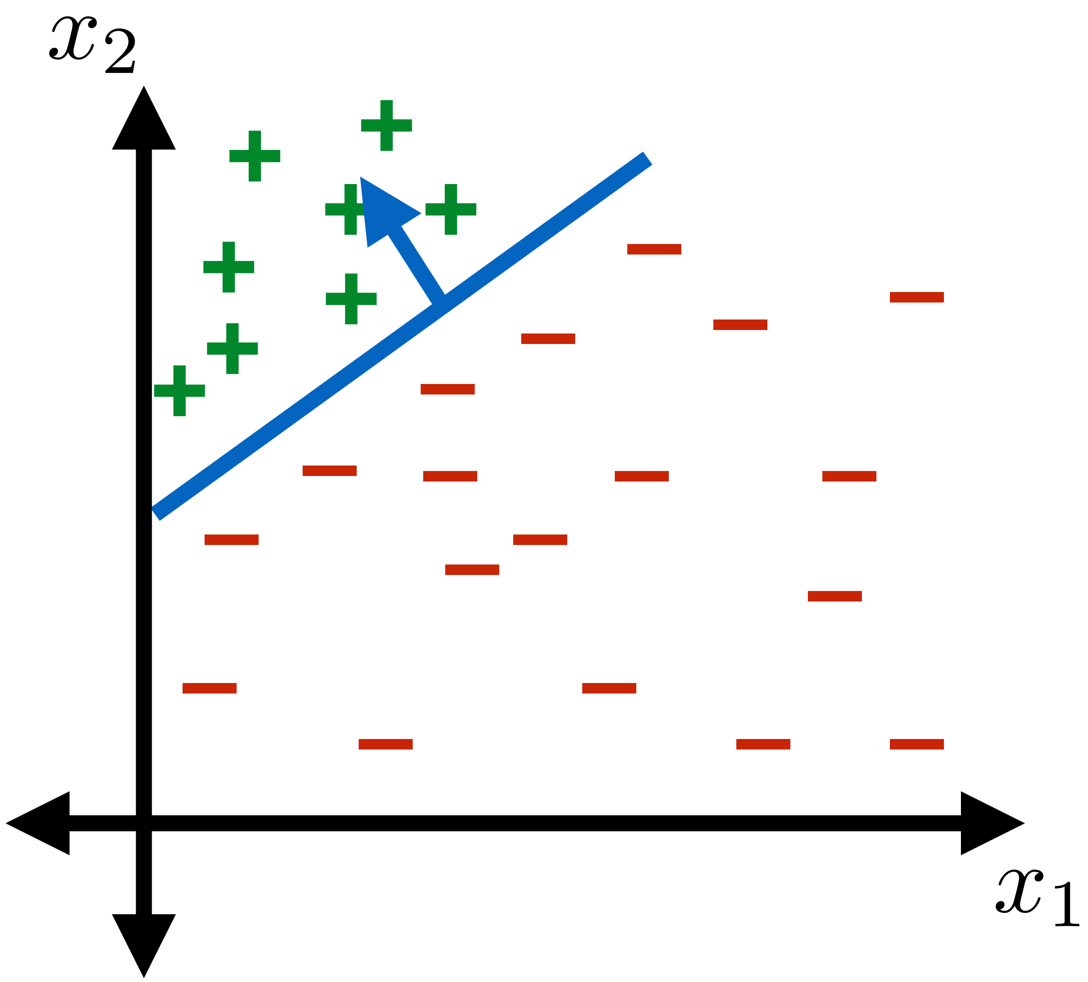

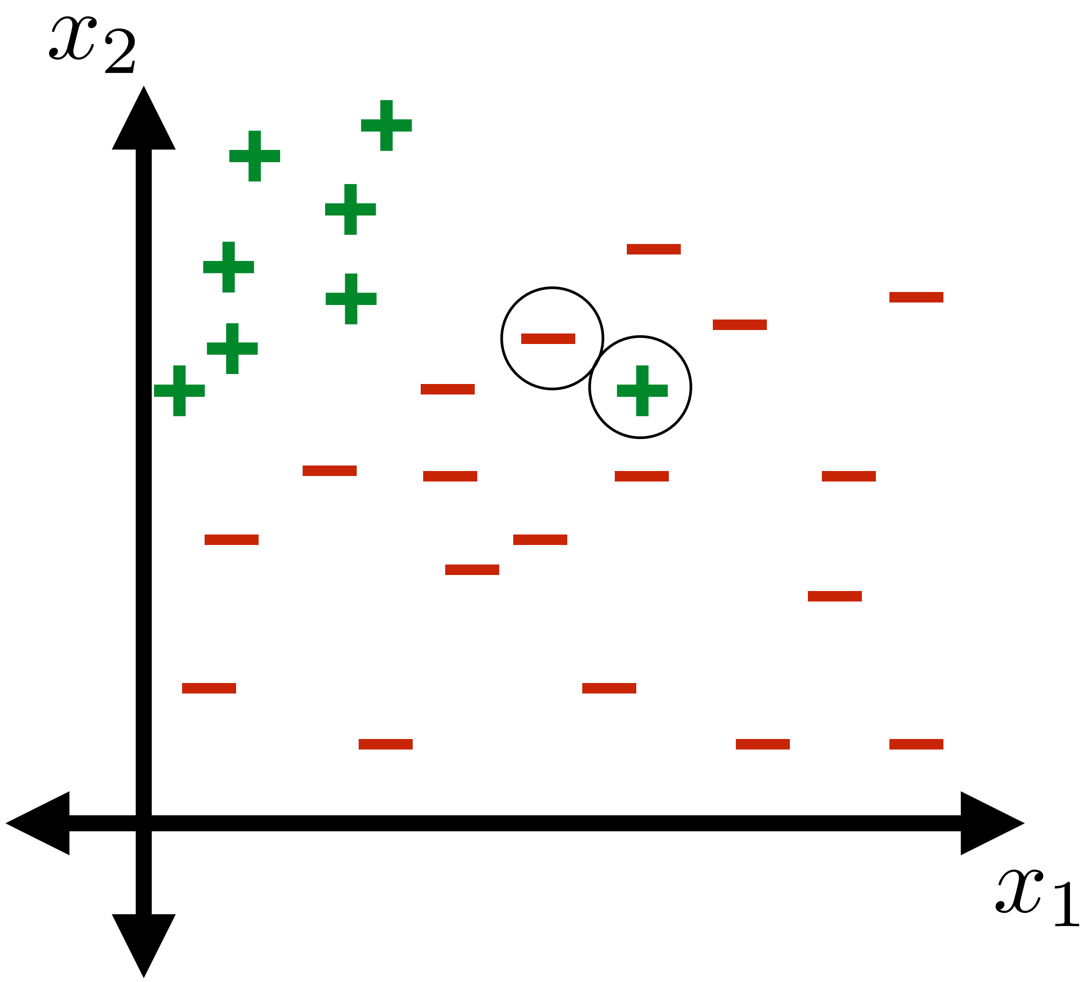

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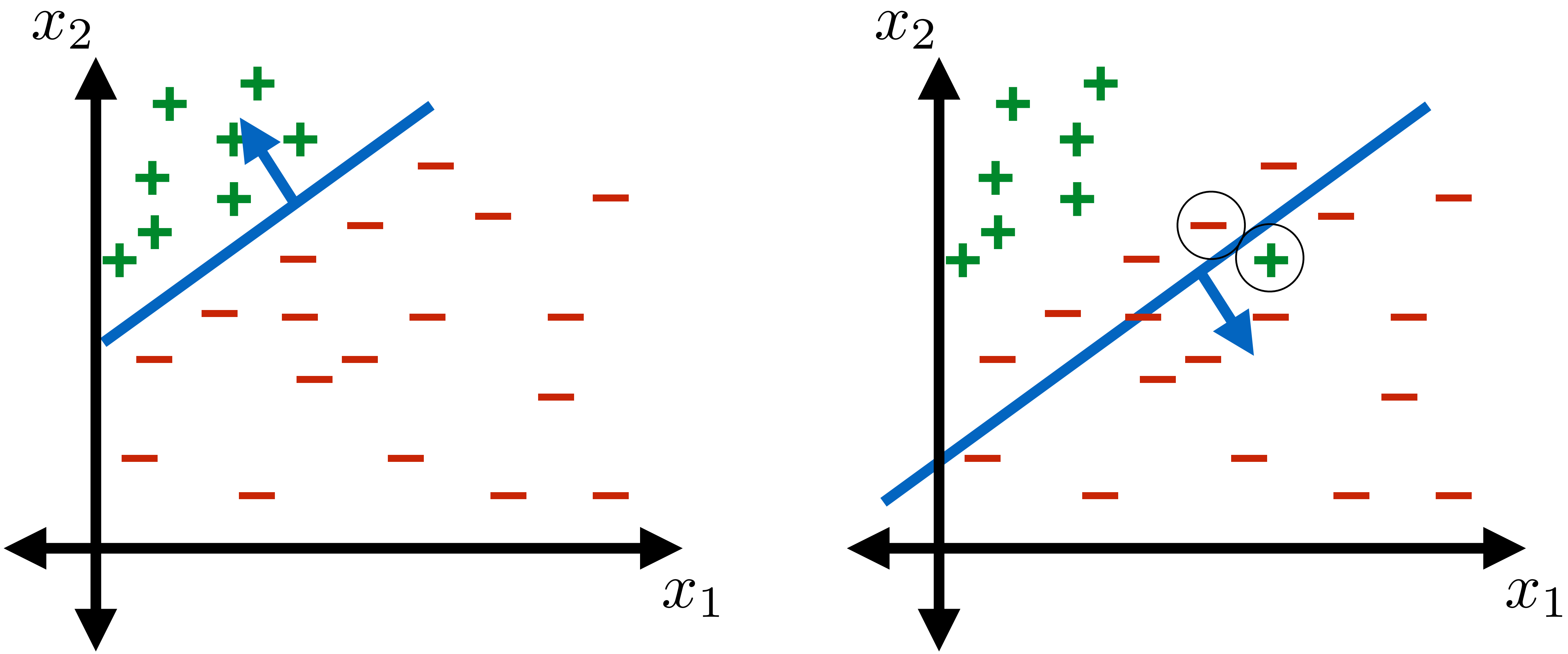

Let’s Talk About Classifier Quality

Math facts!

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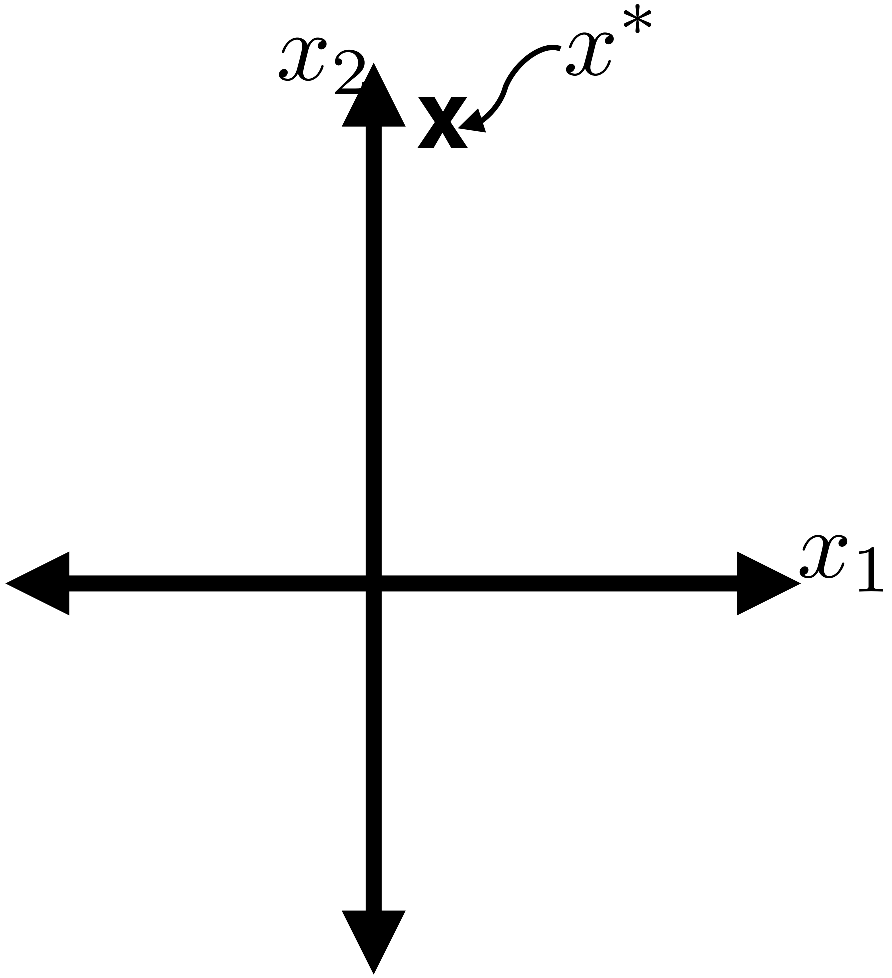

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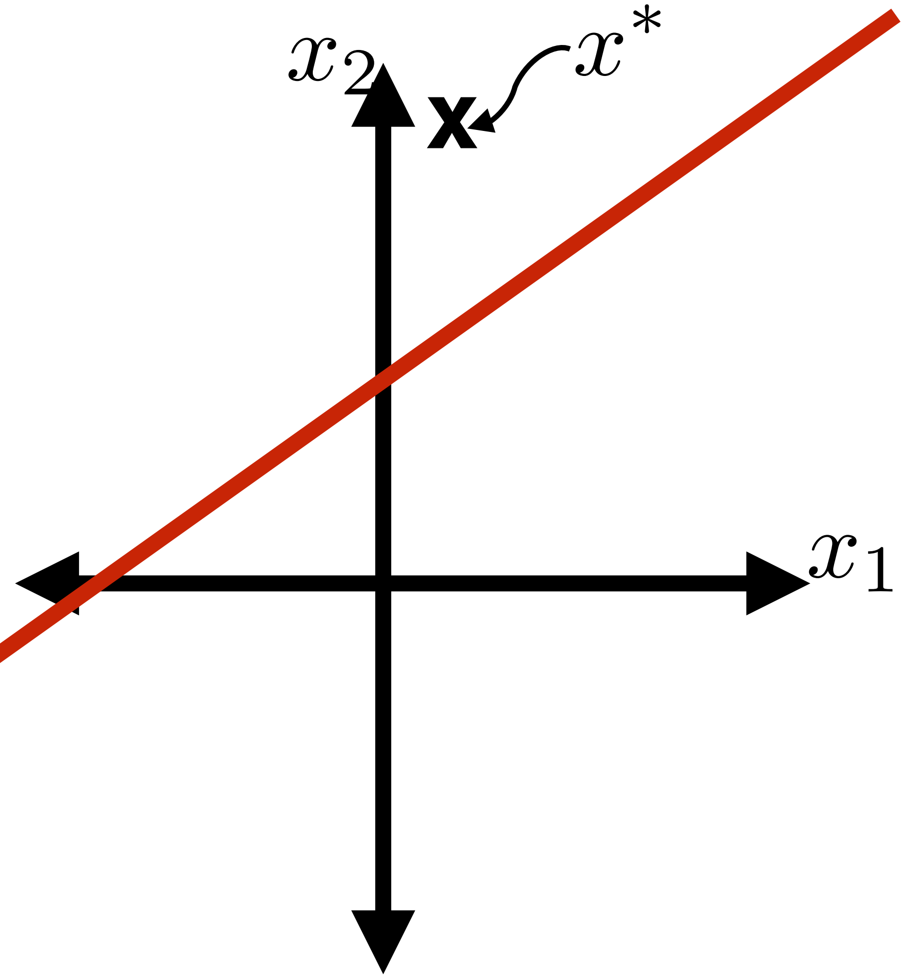

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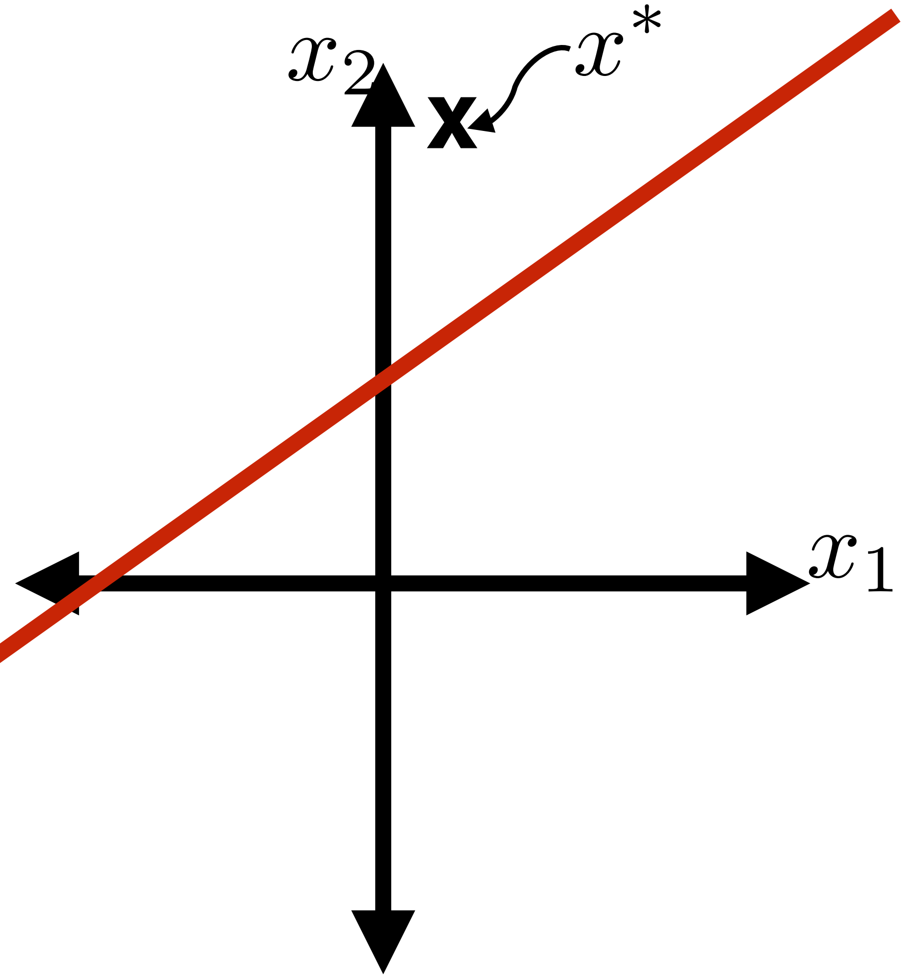

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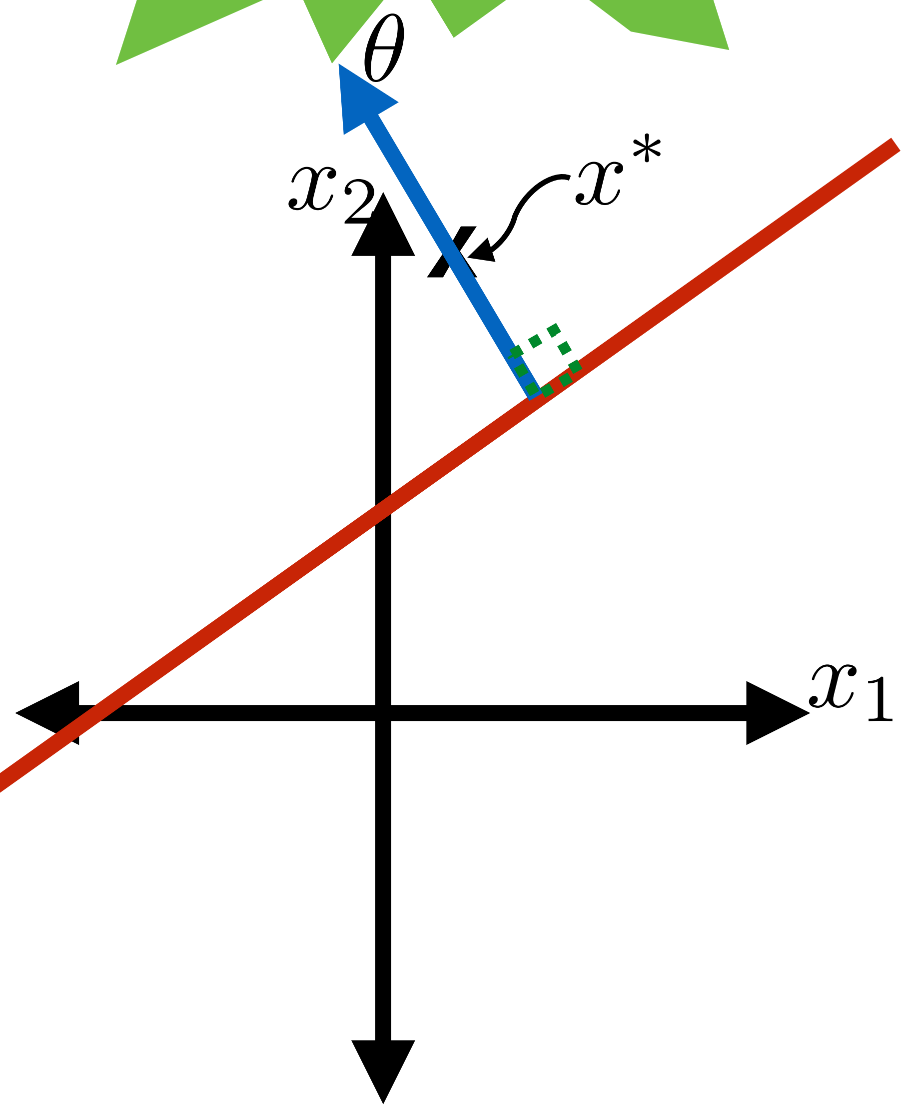

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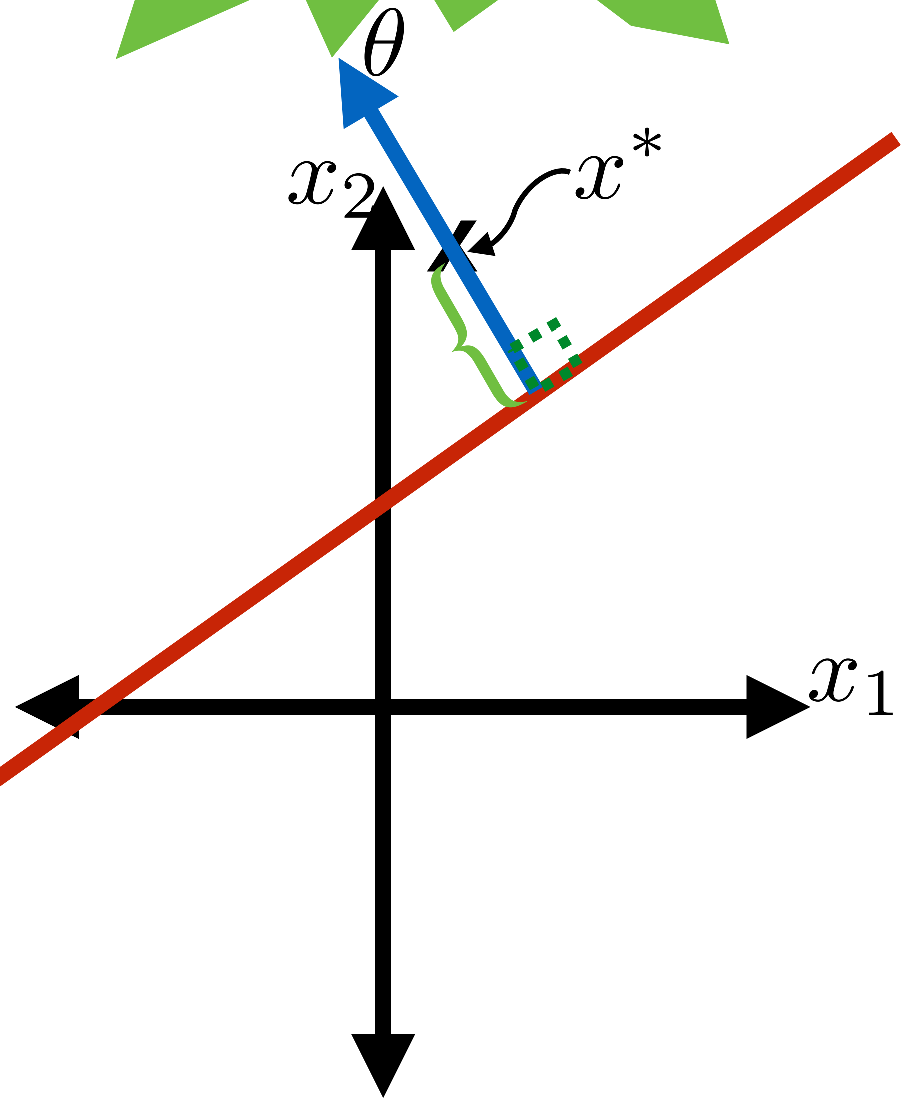

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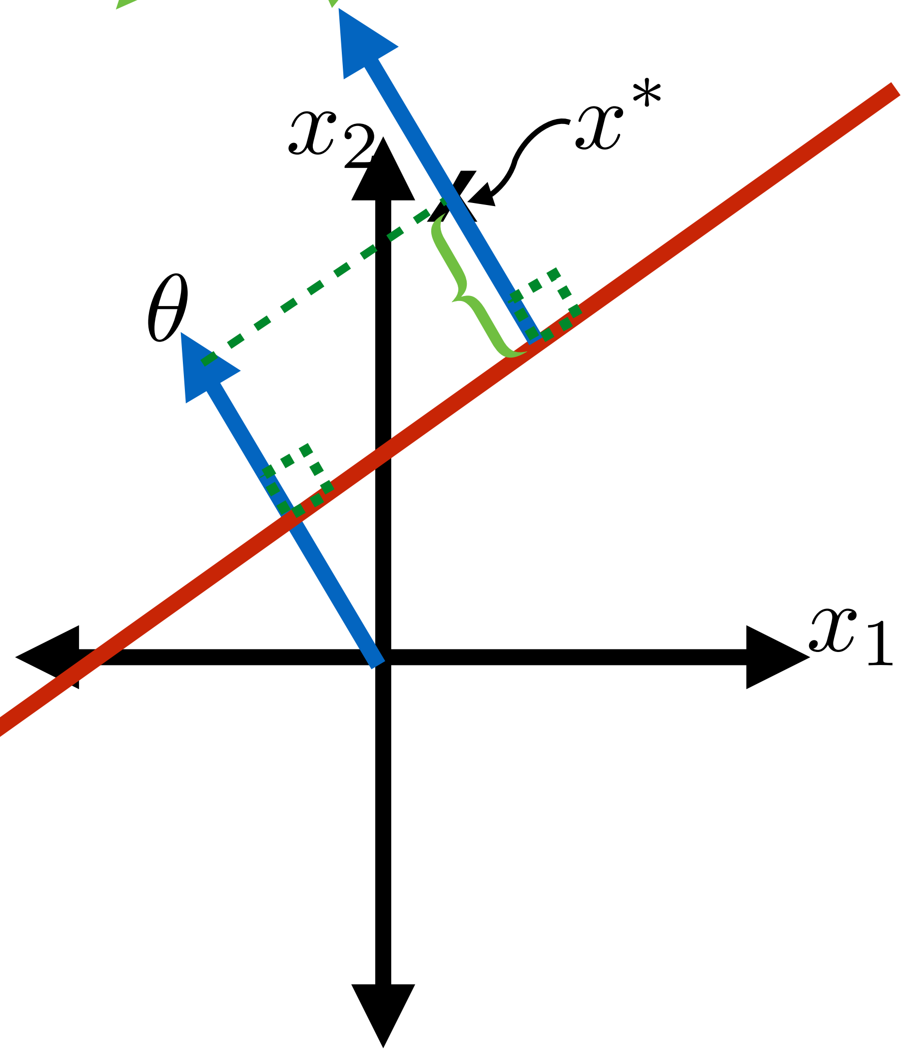

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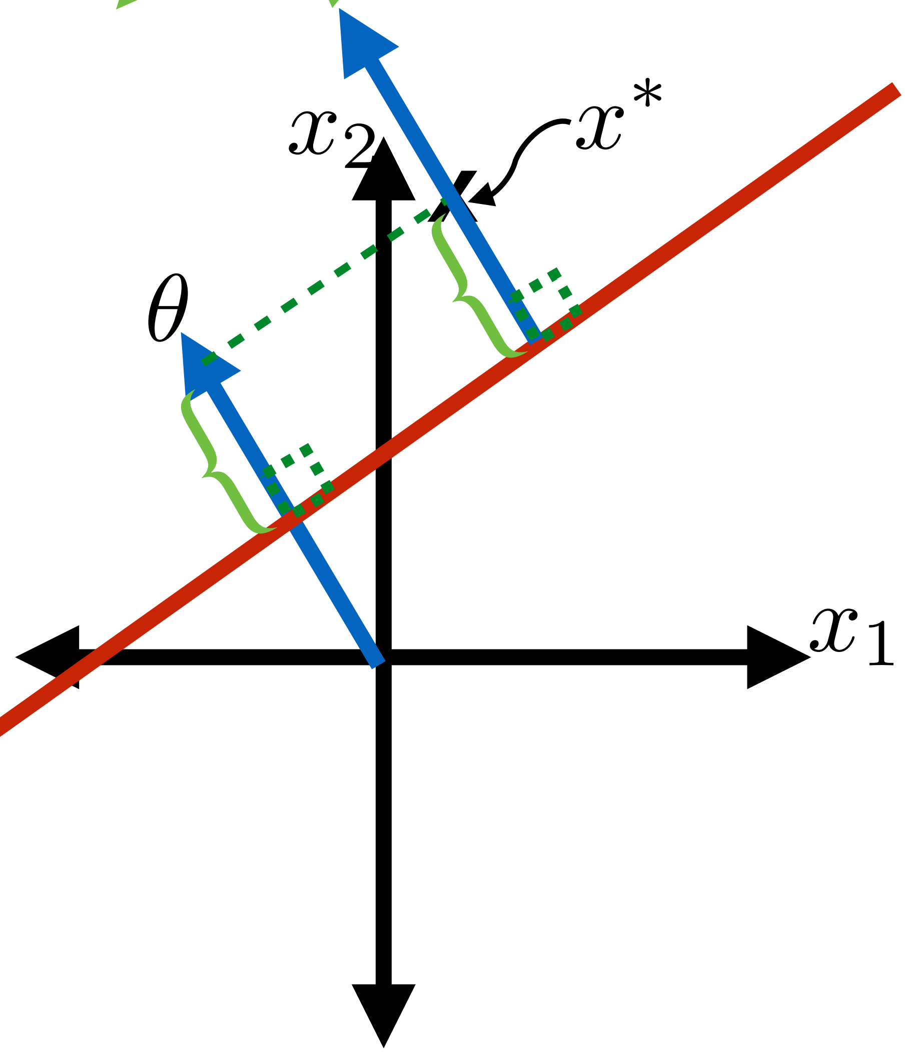

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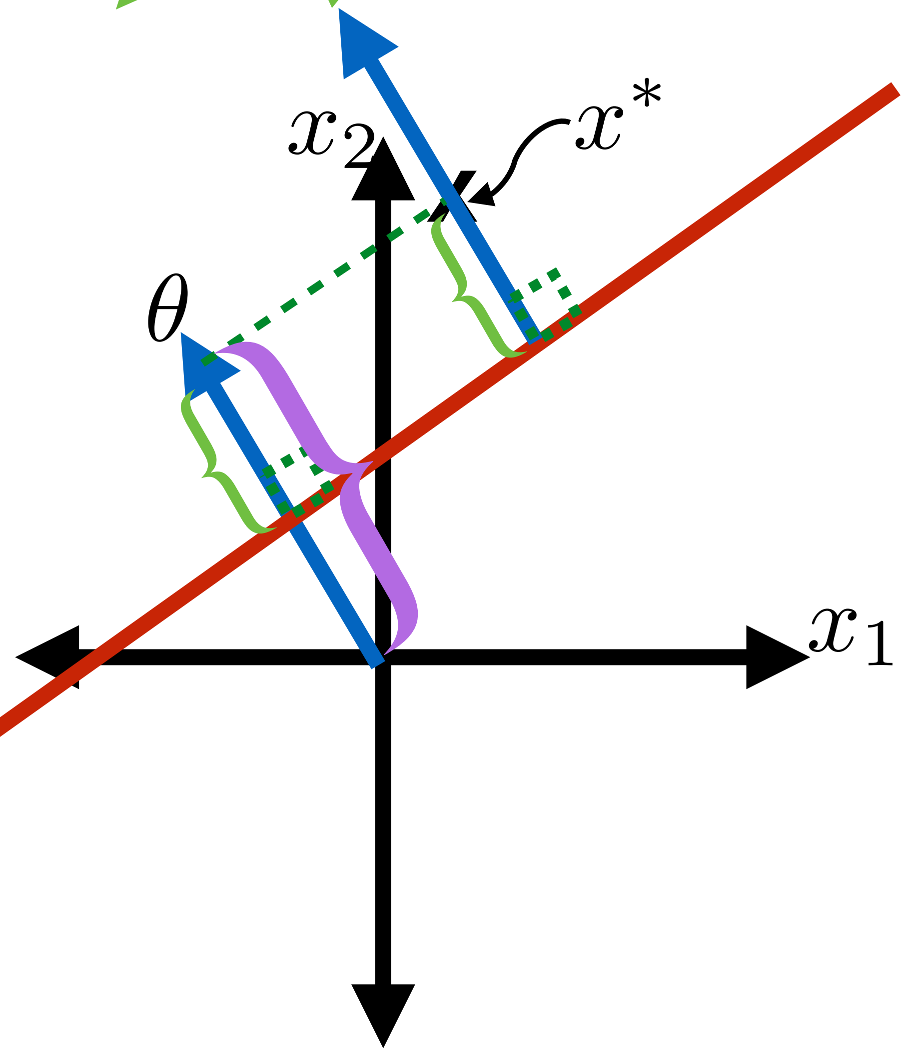

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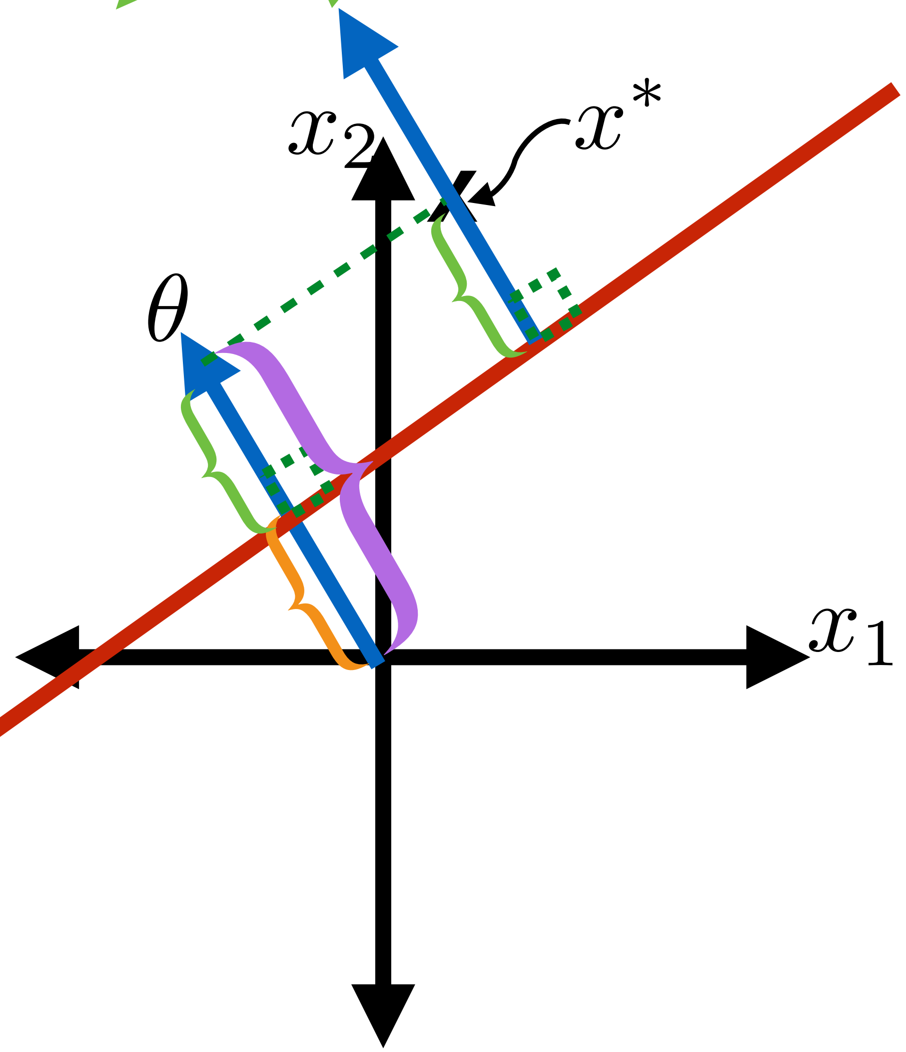

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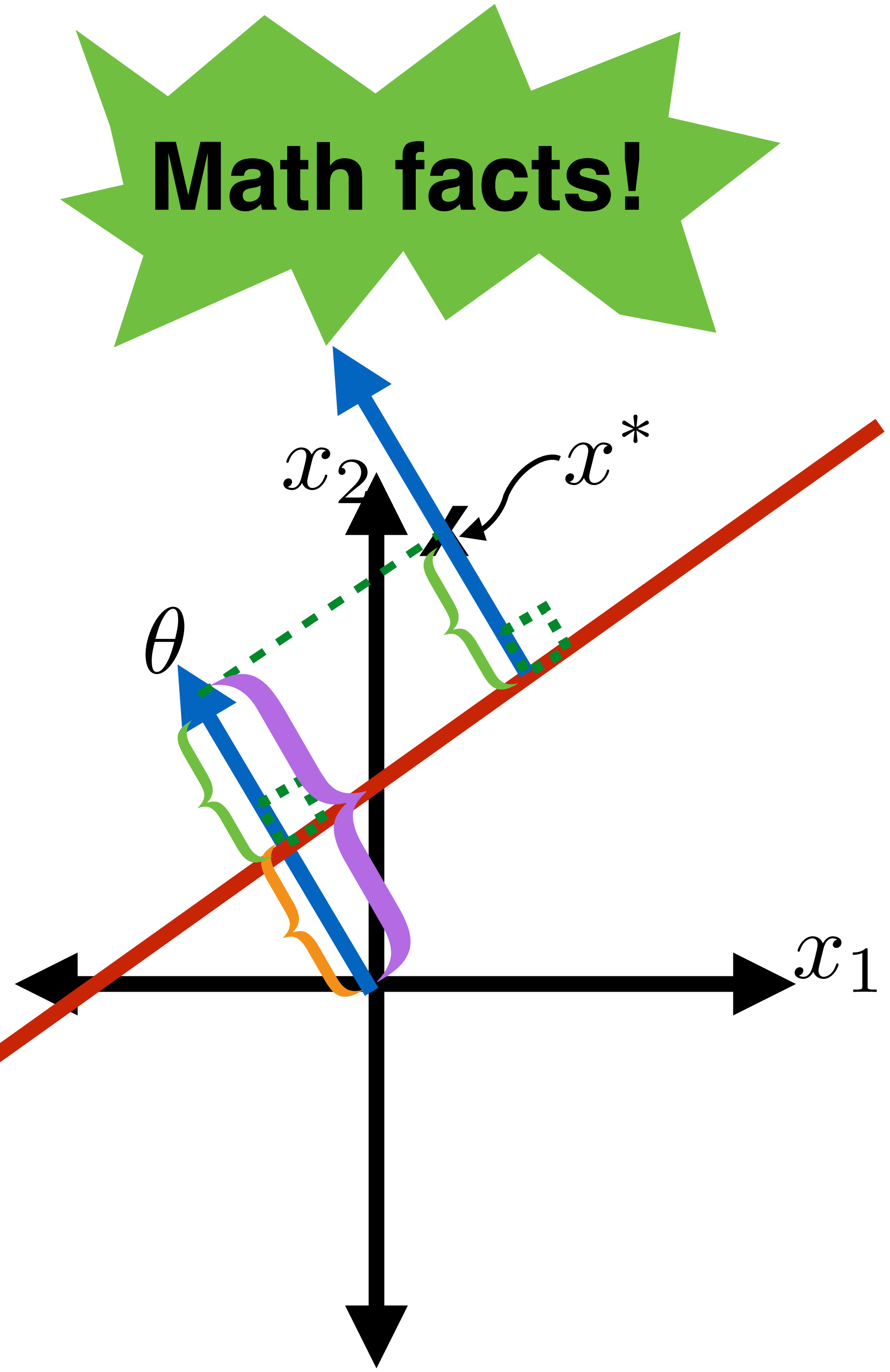

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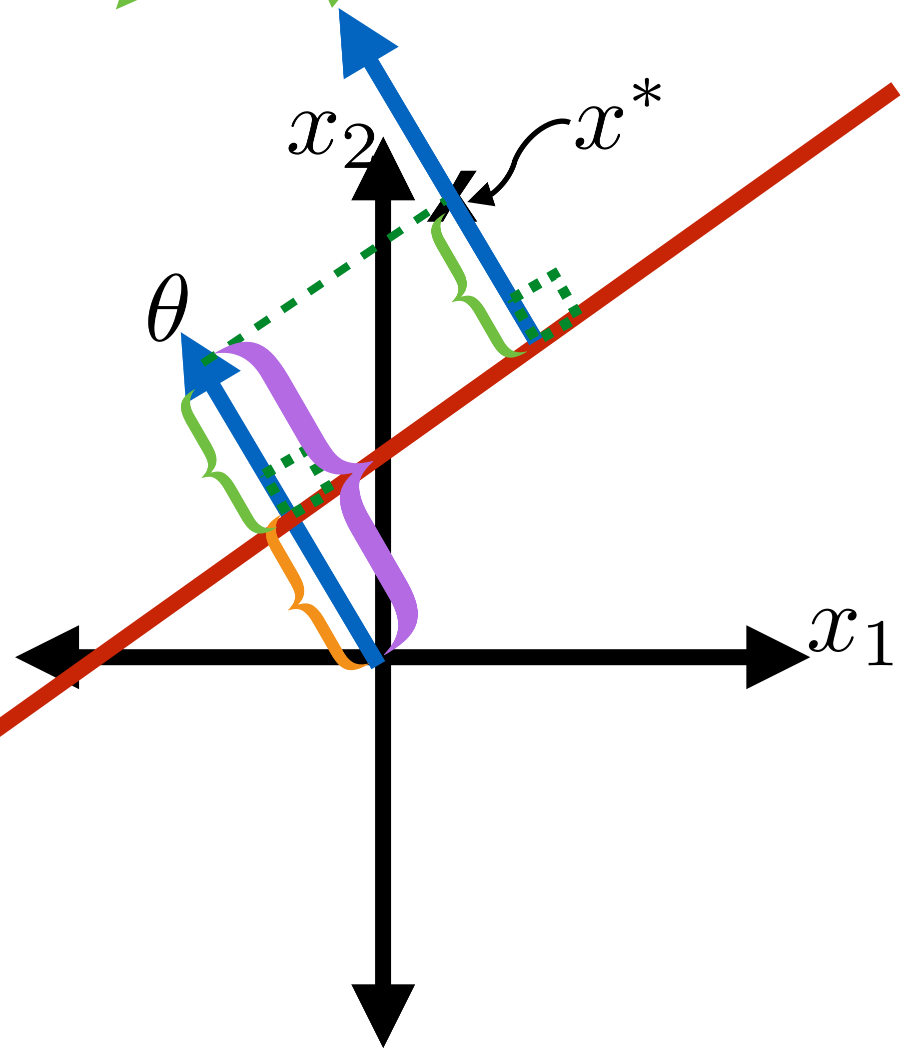

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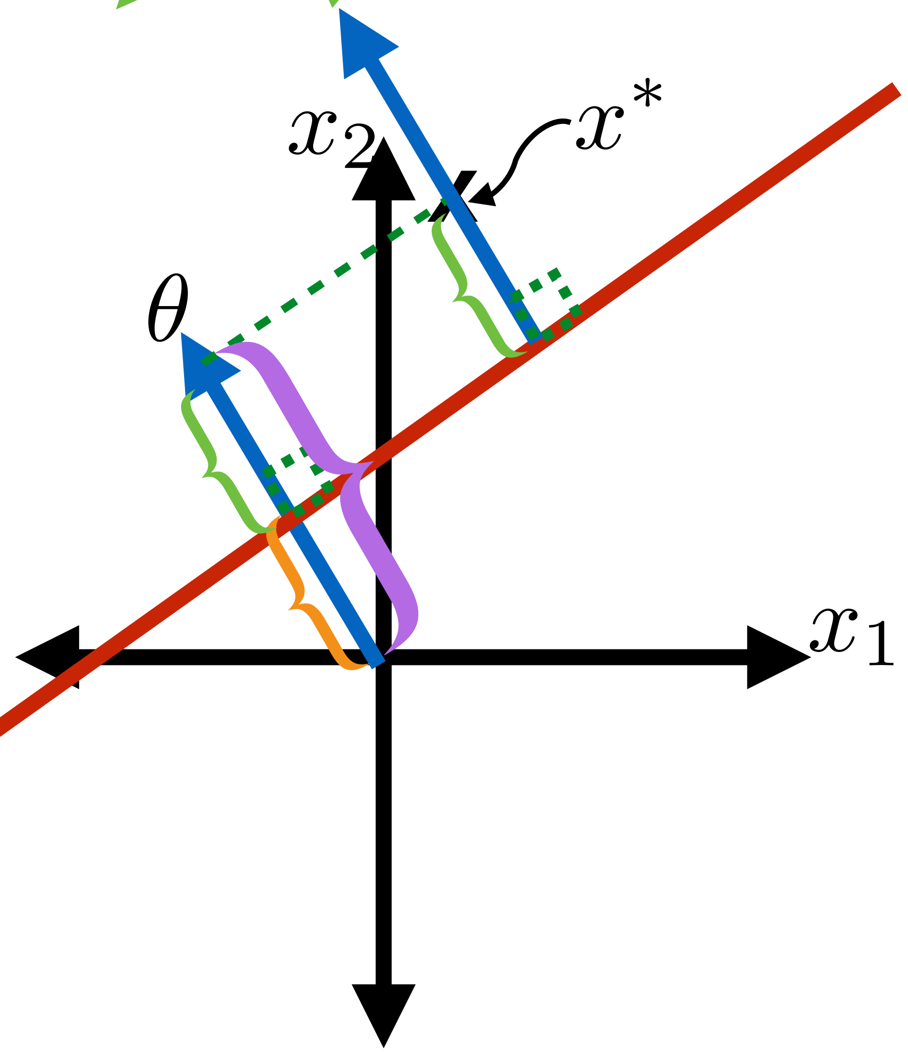

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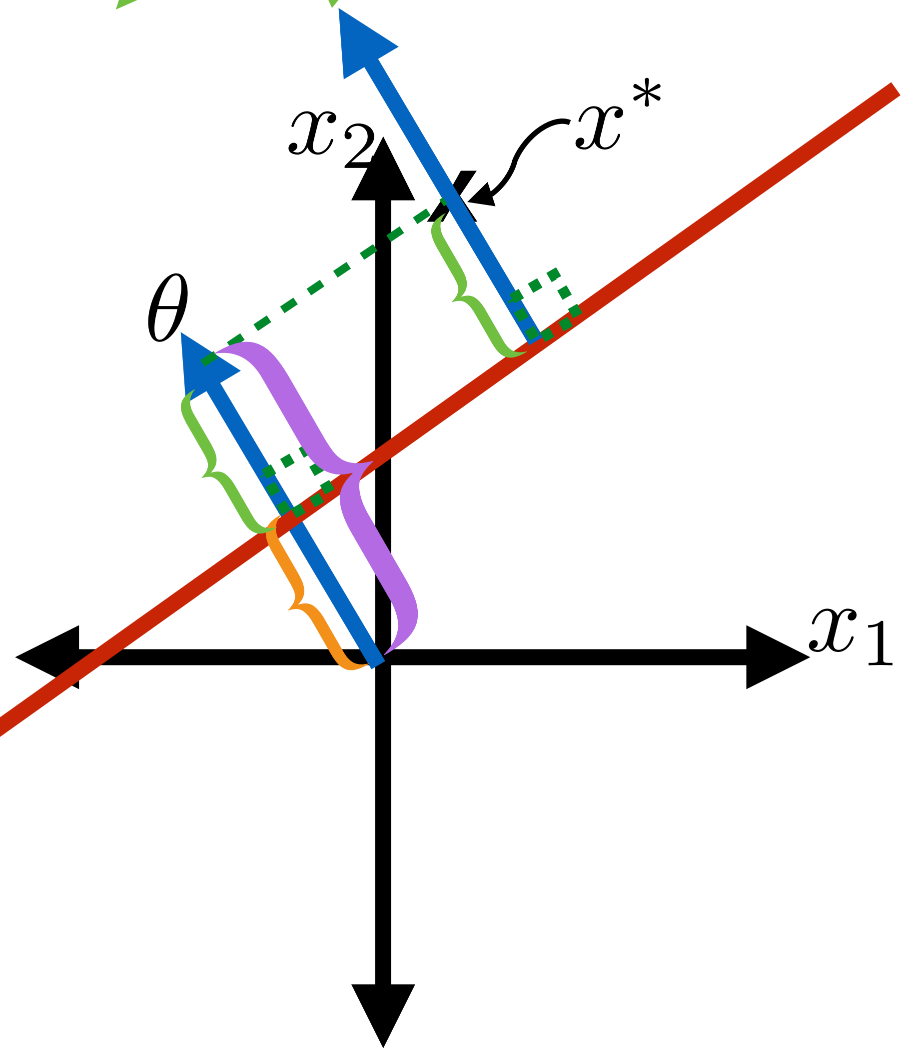

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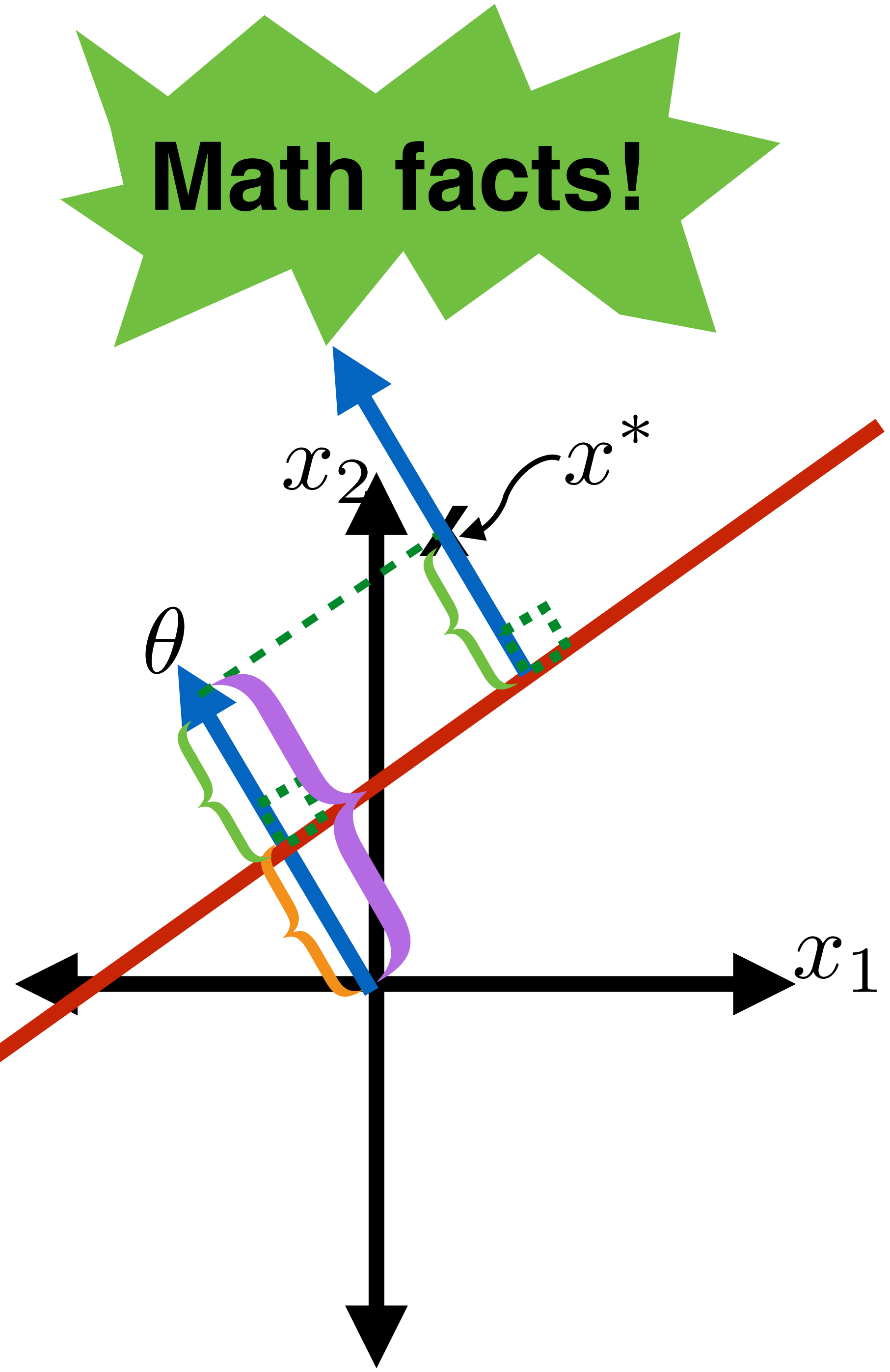

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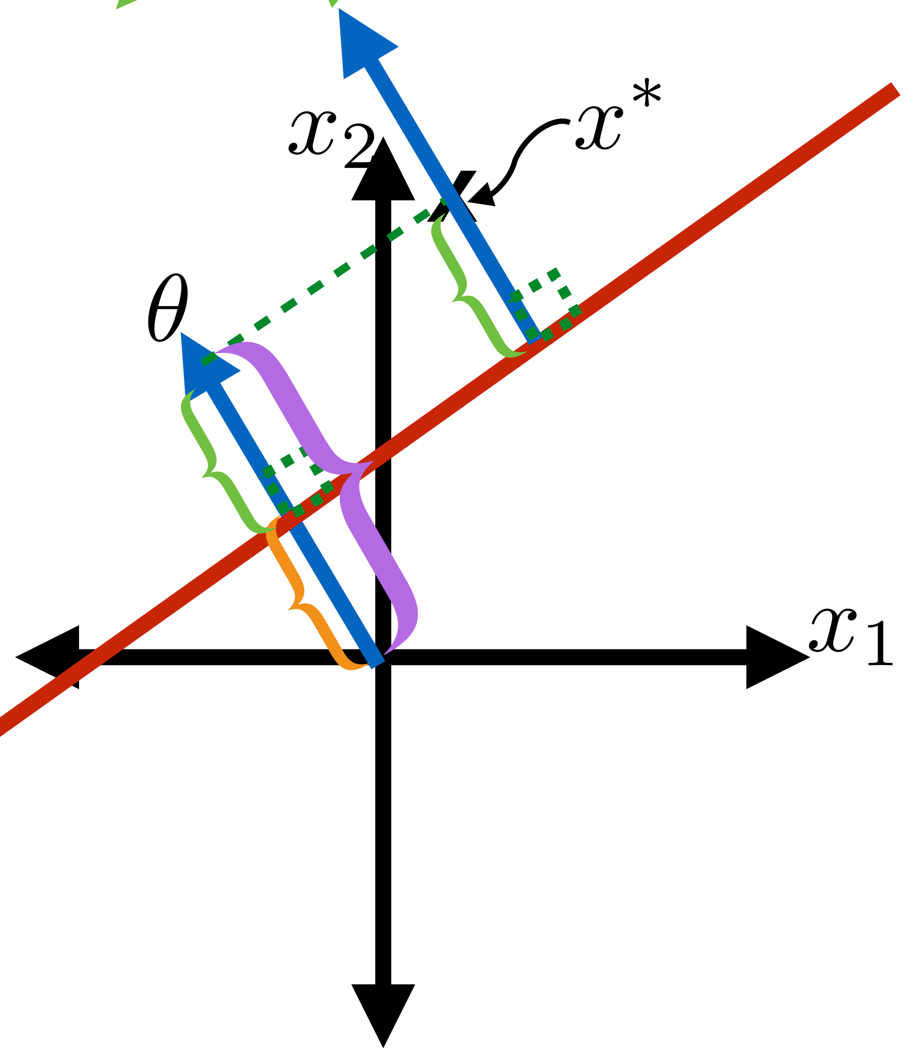

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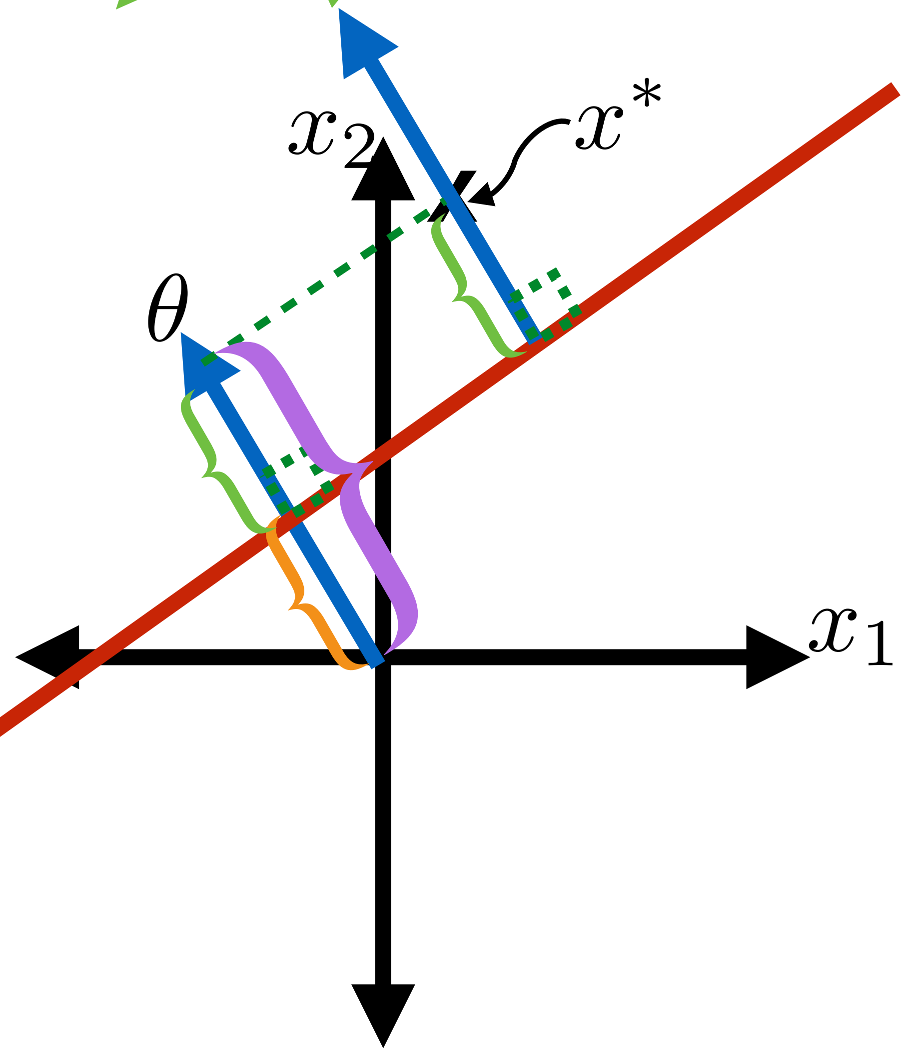

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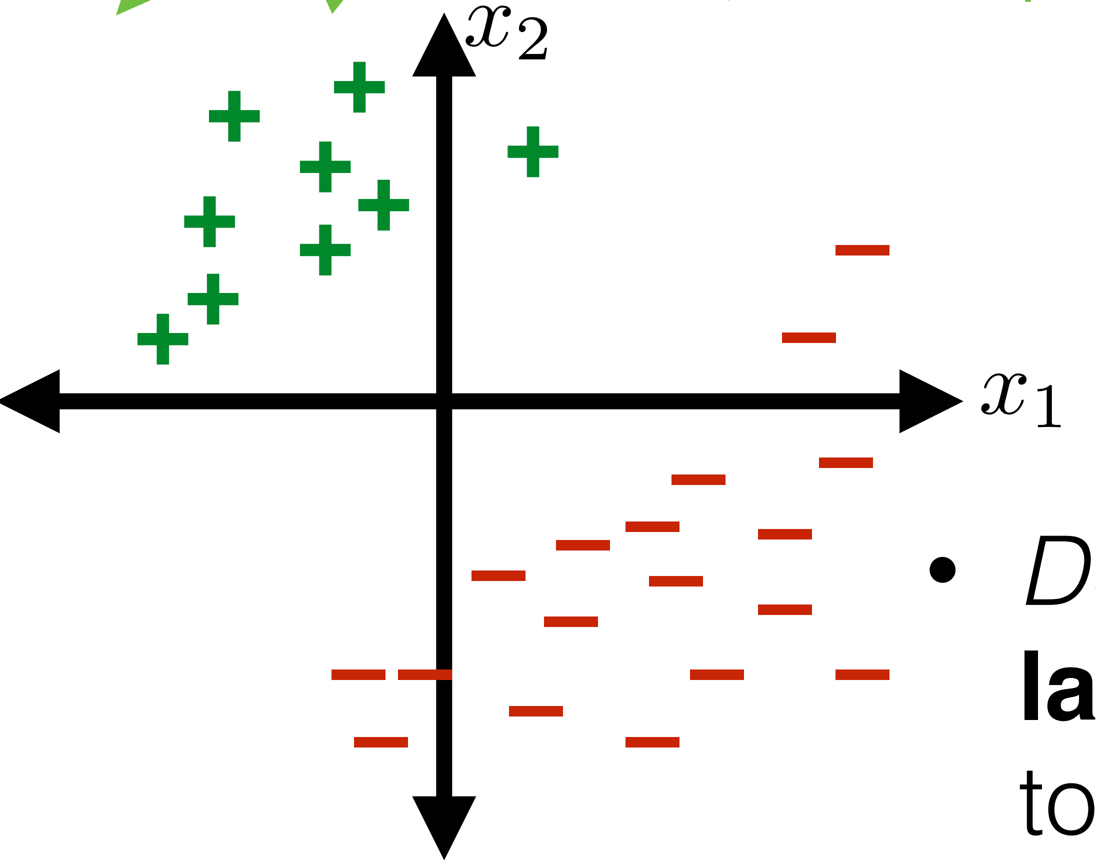

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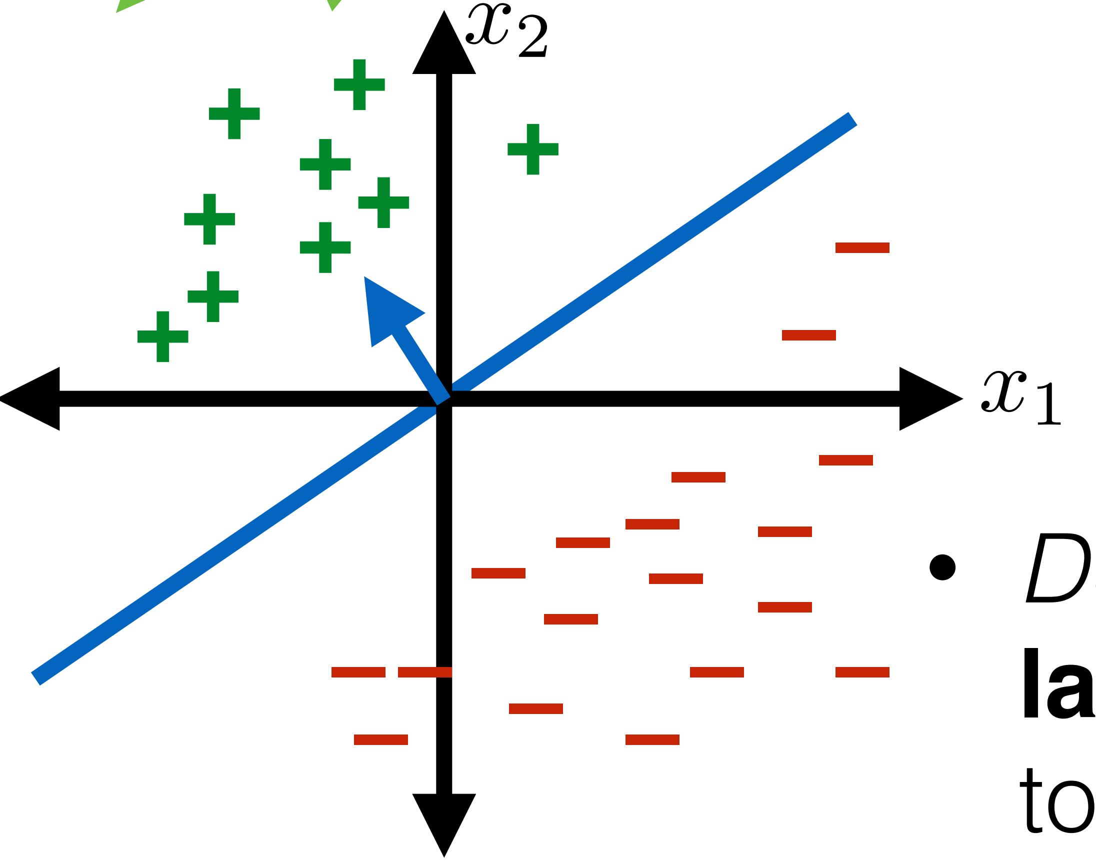

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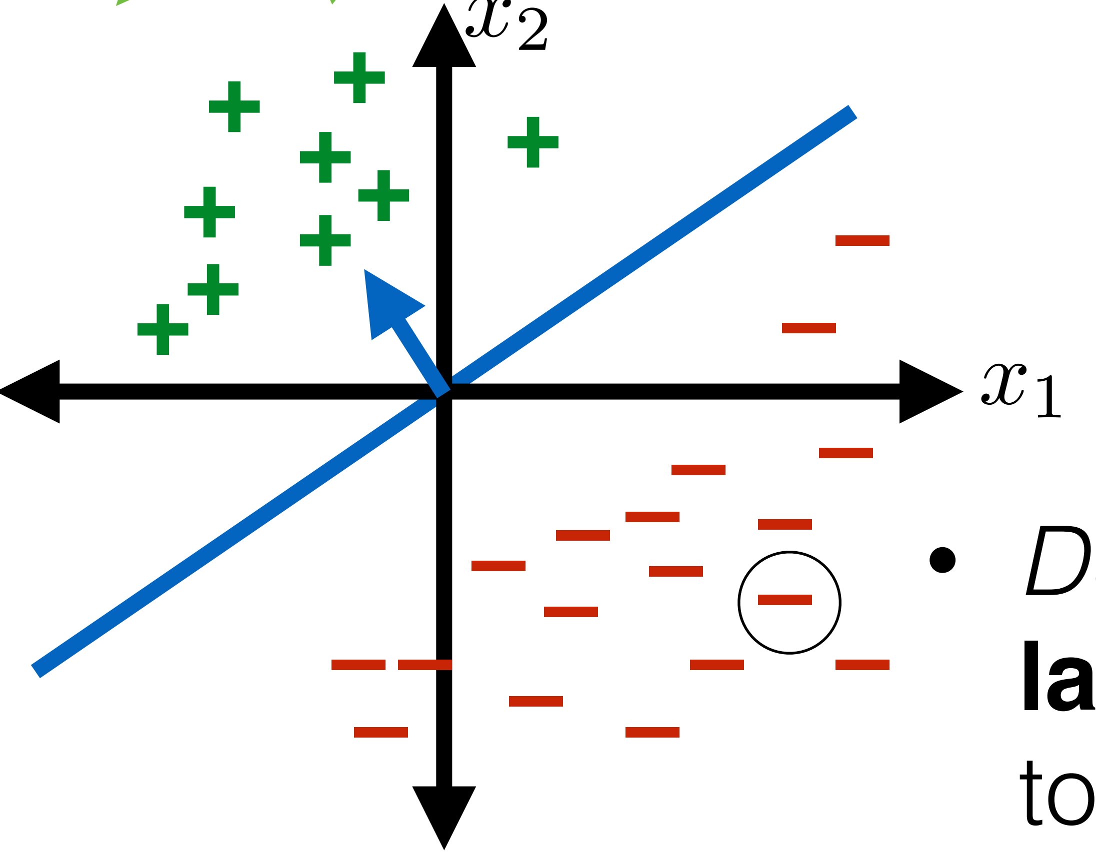

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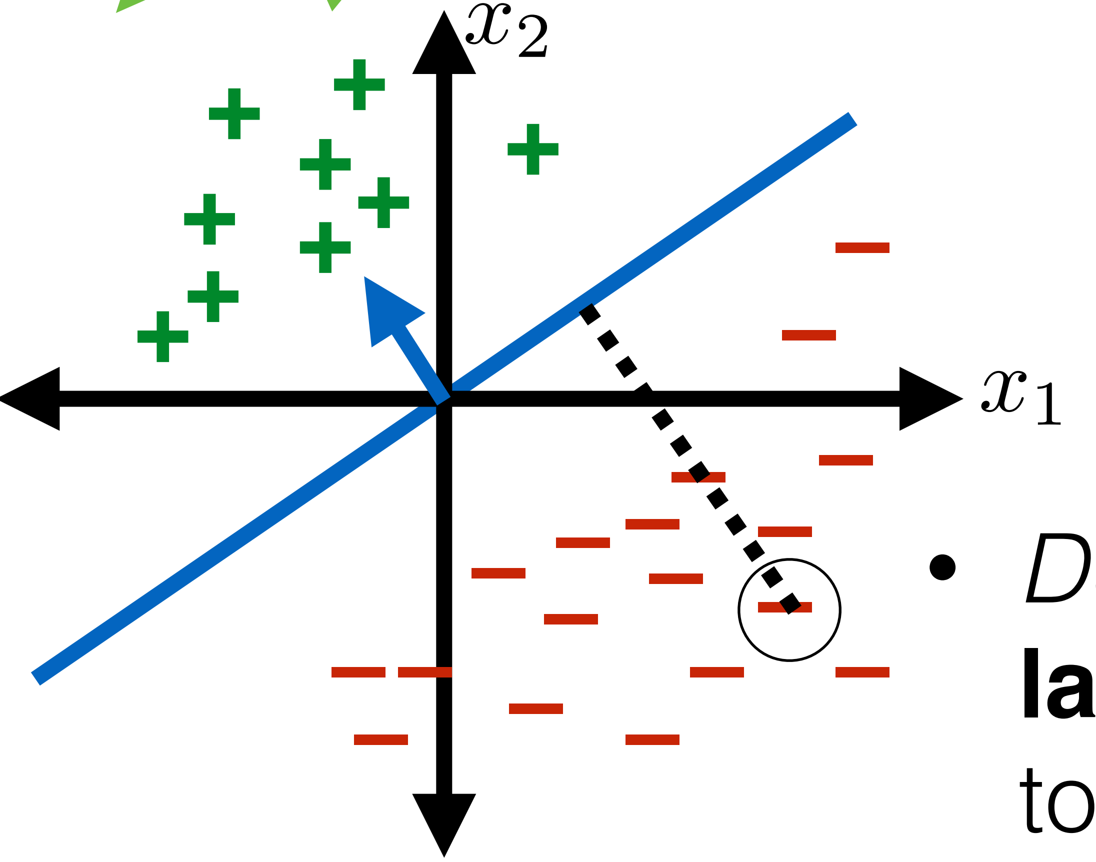

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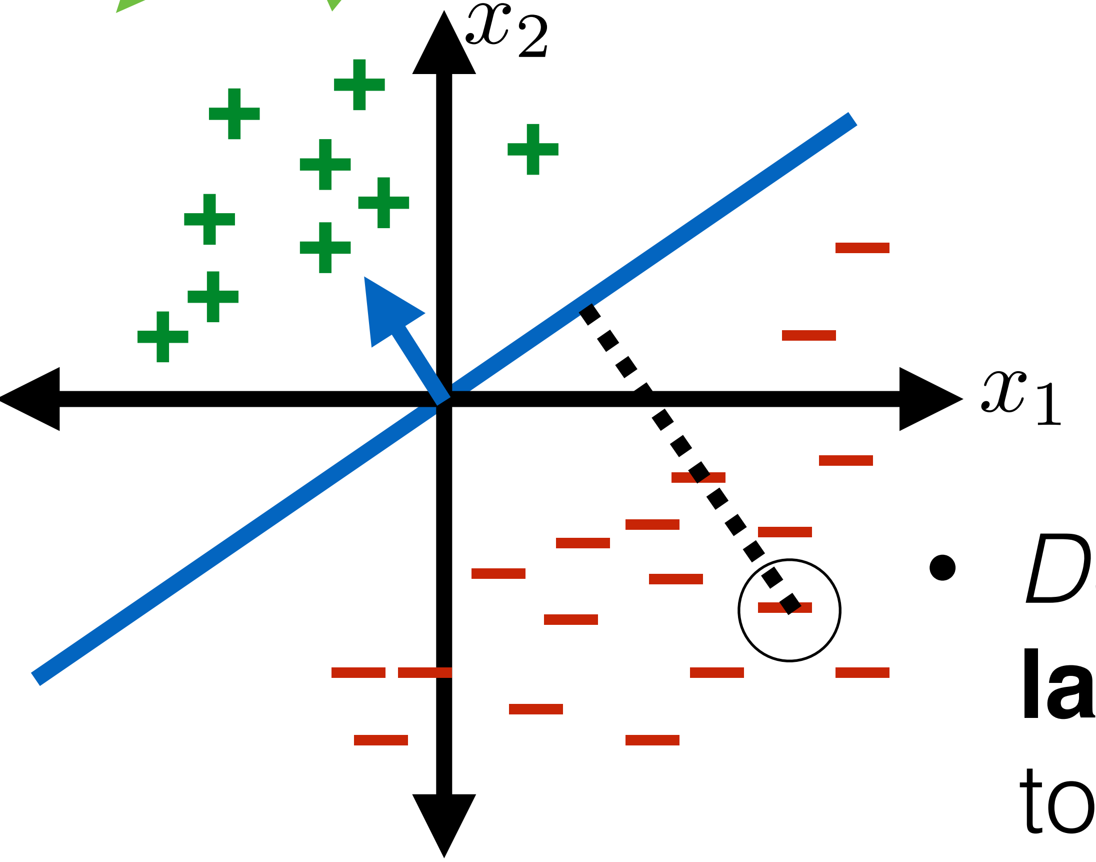

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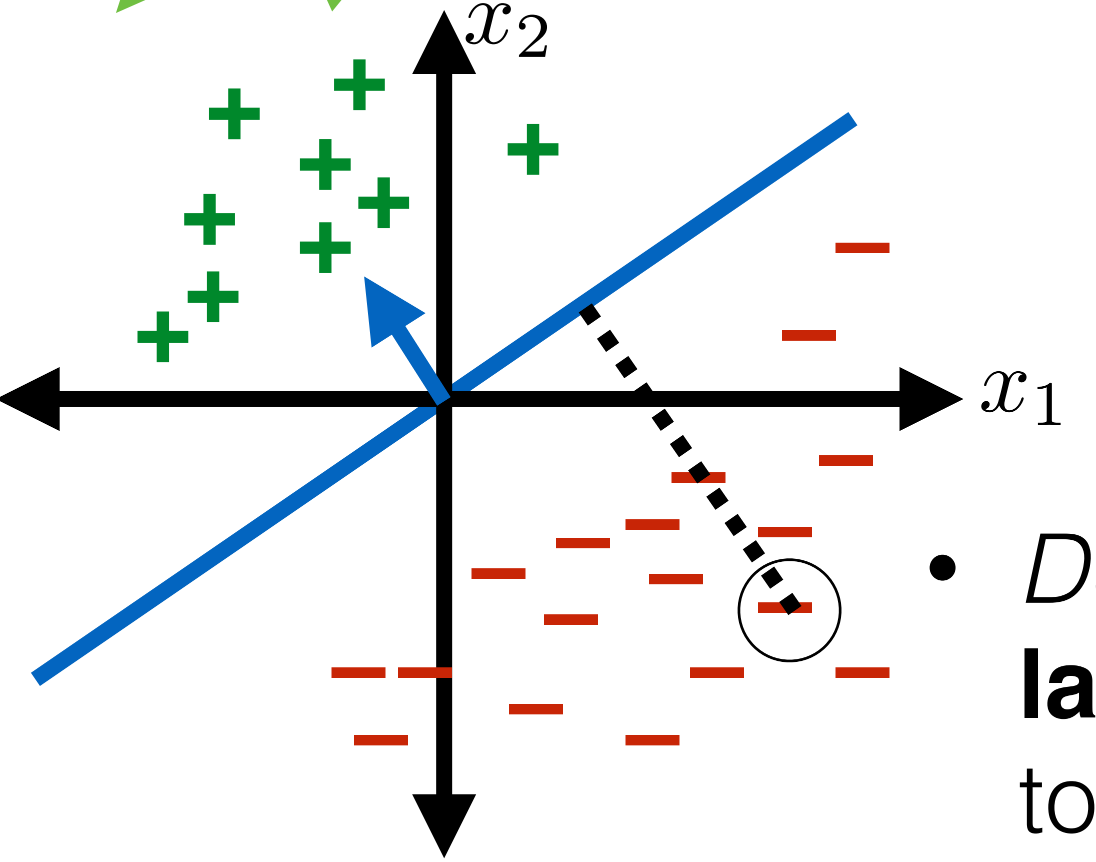

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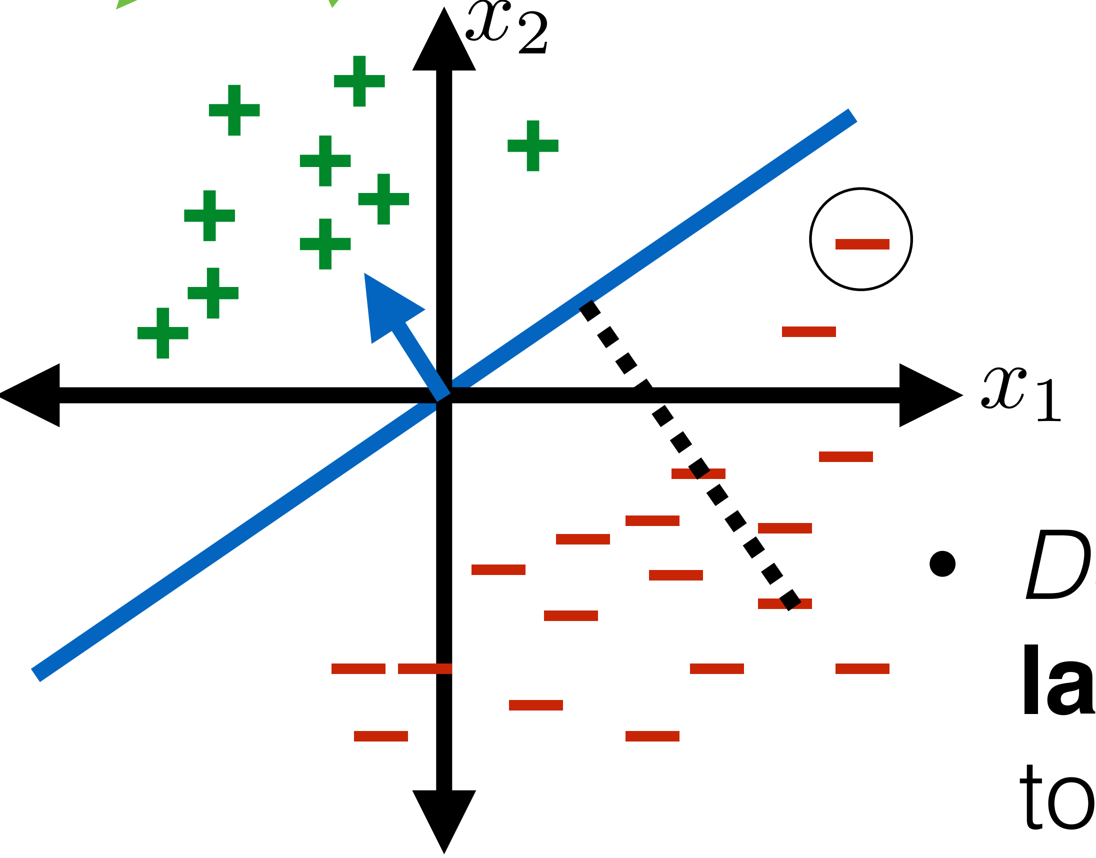

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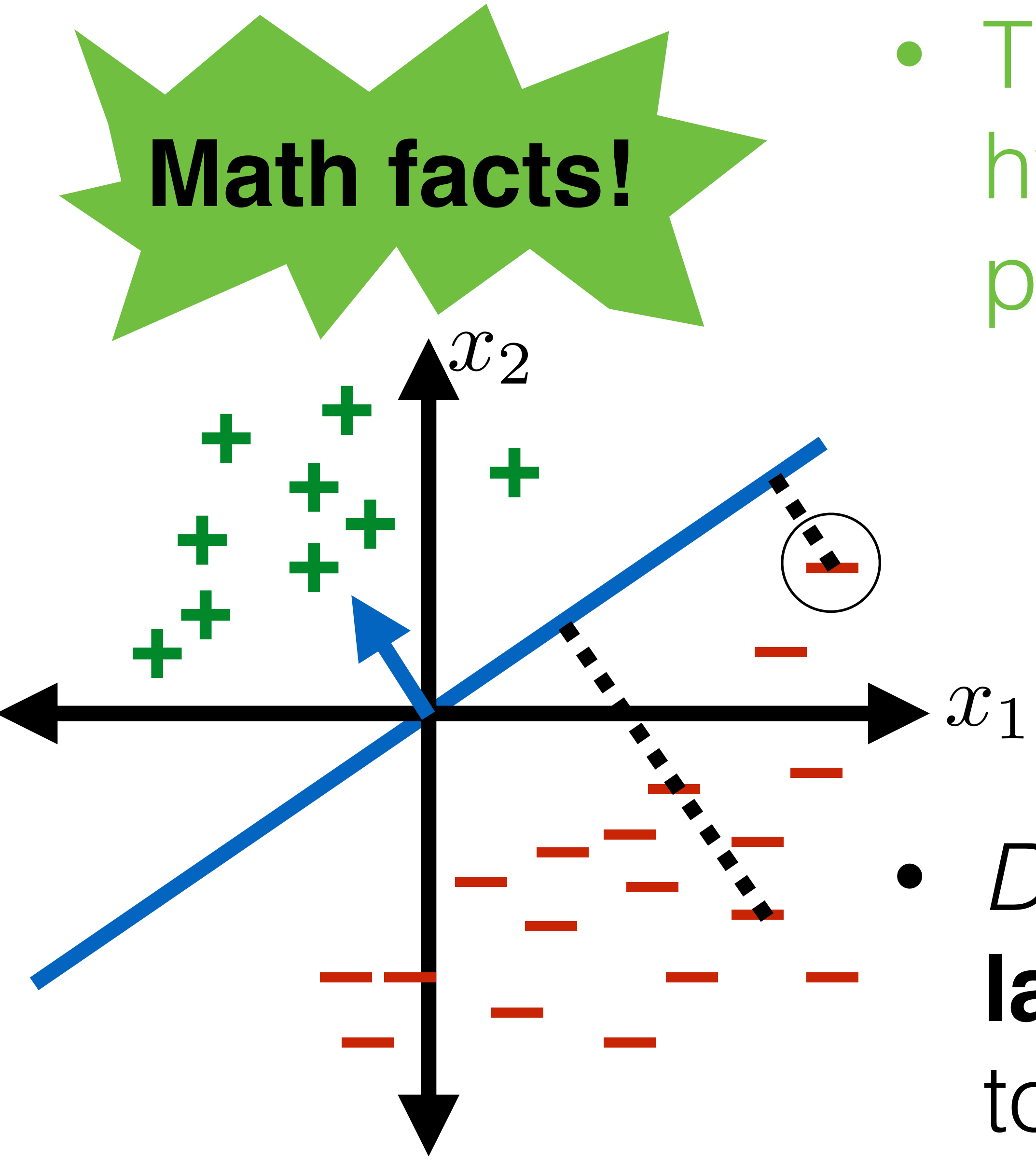

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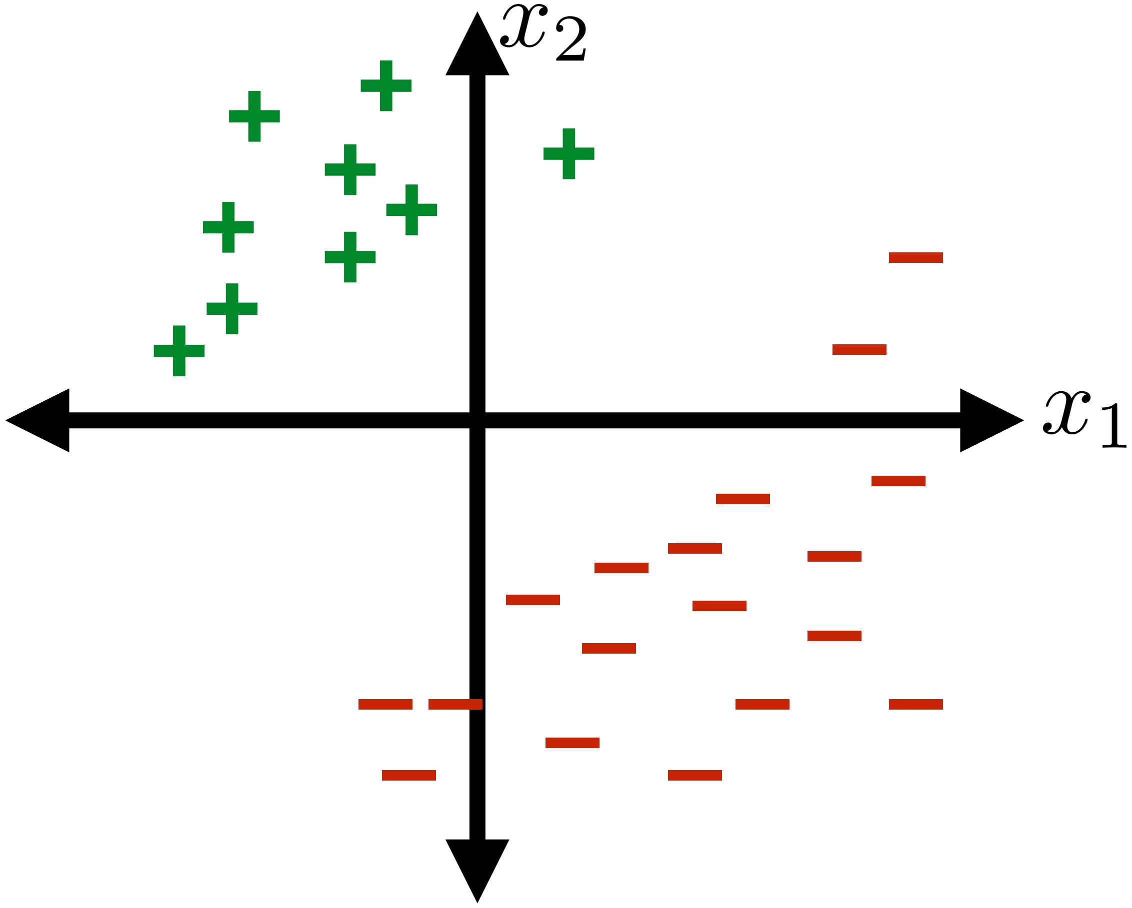

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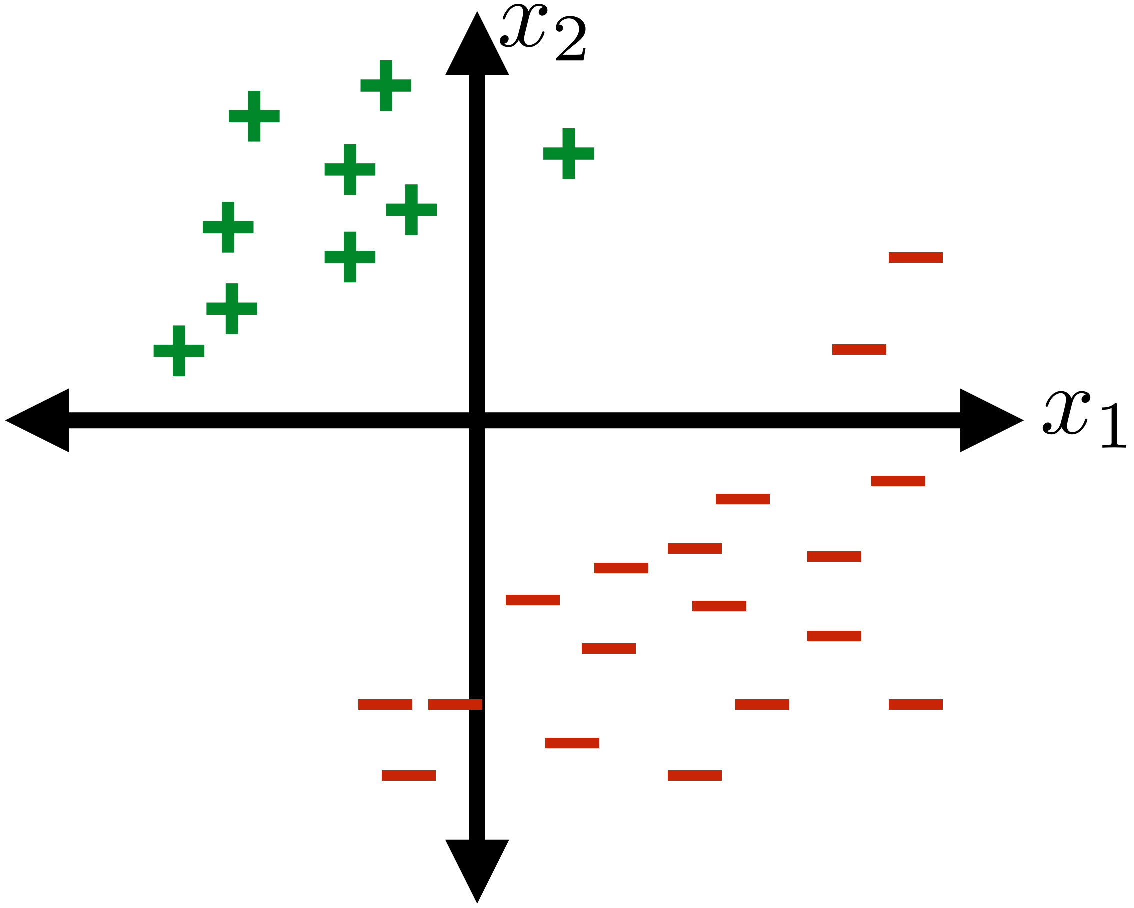

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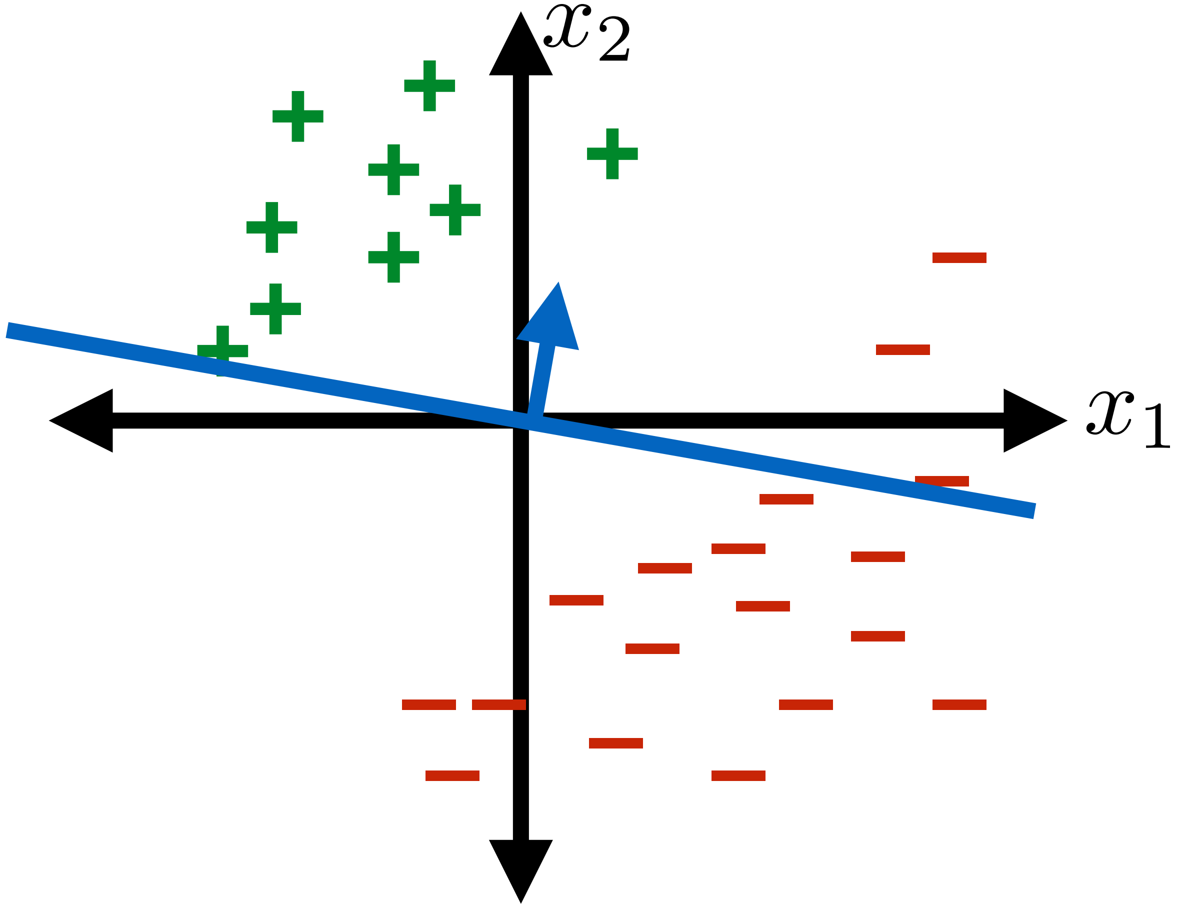

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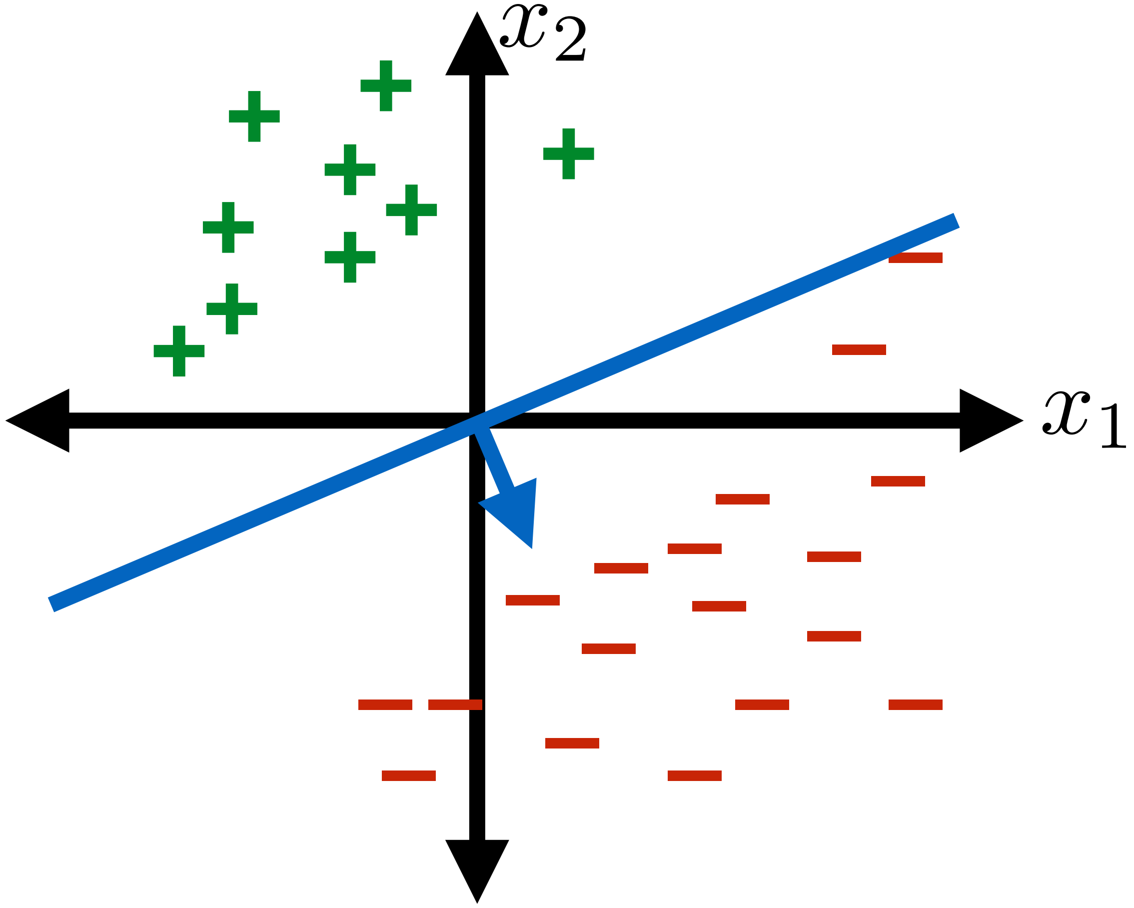

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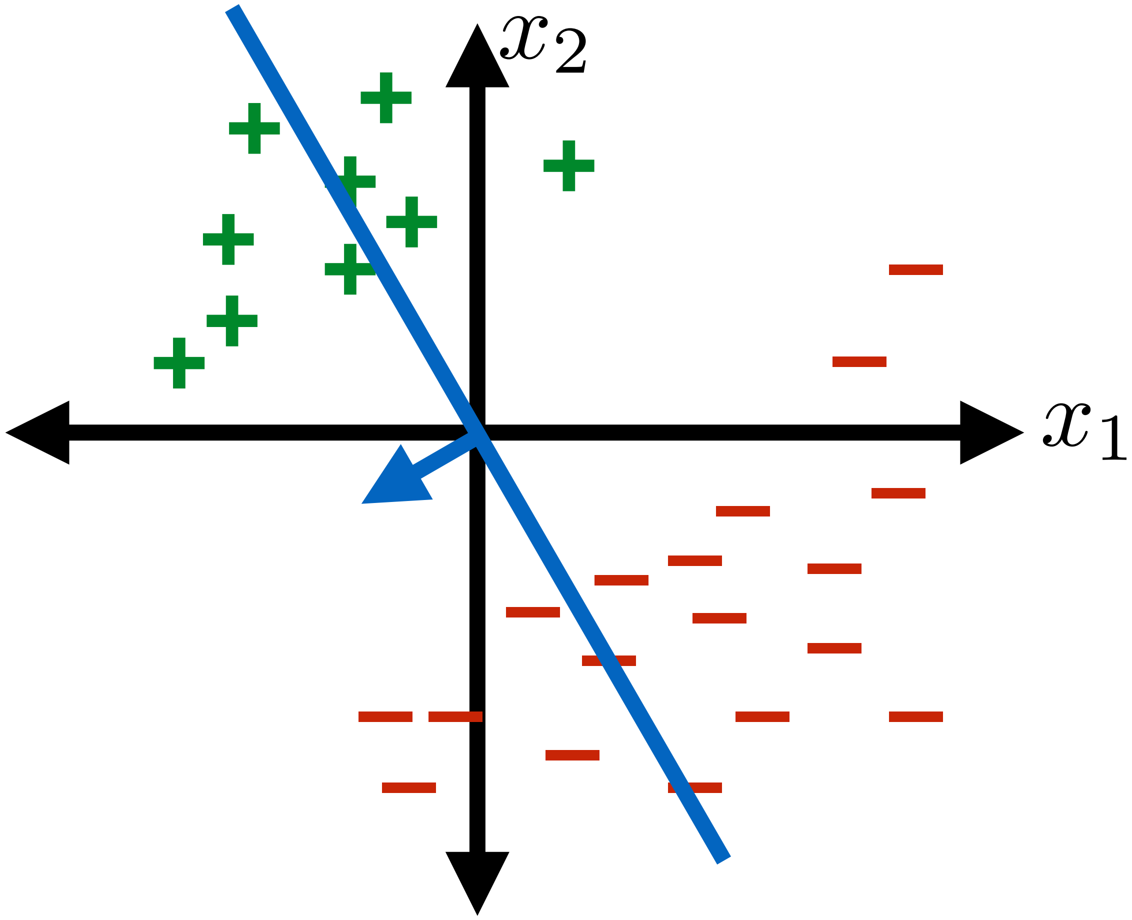

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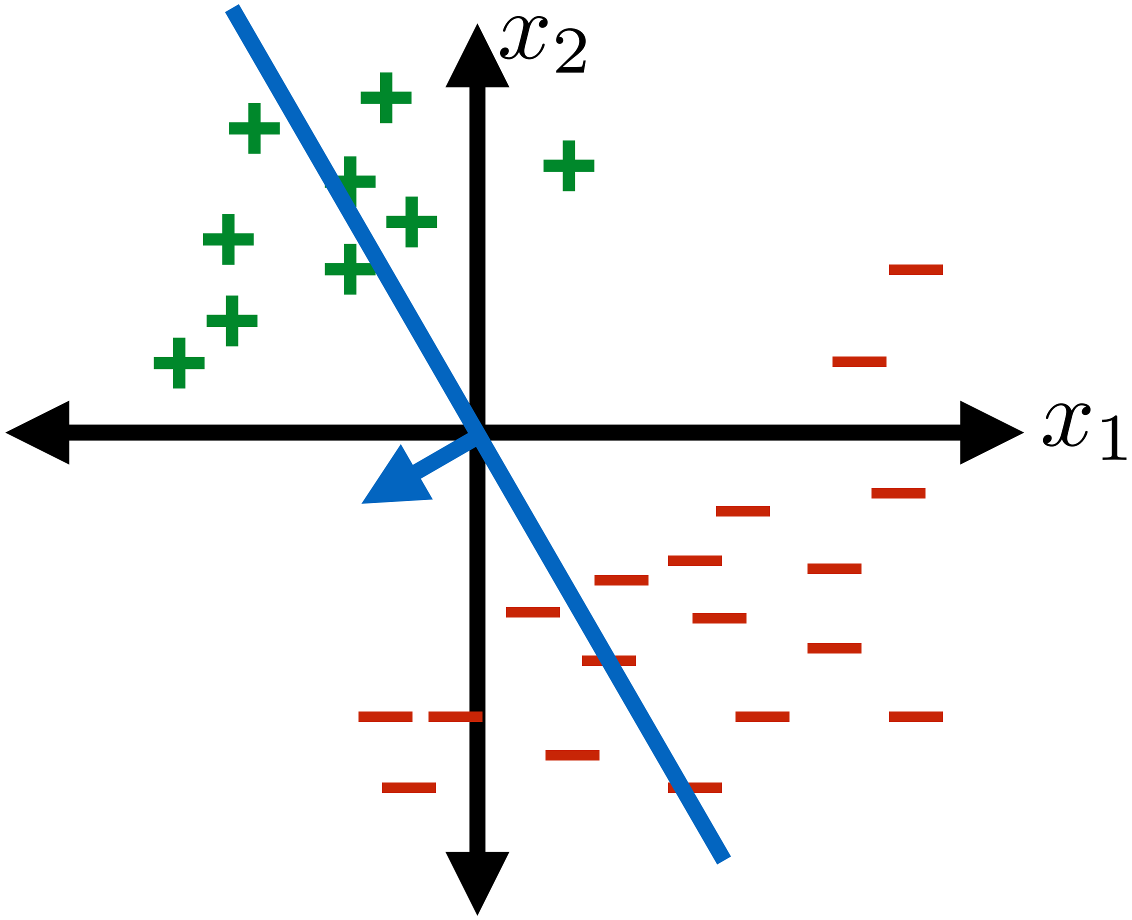

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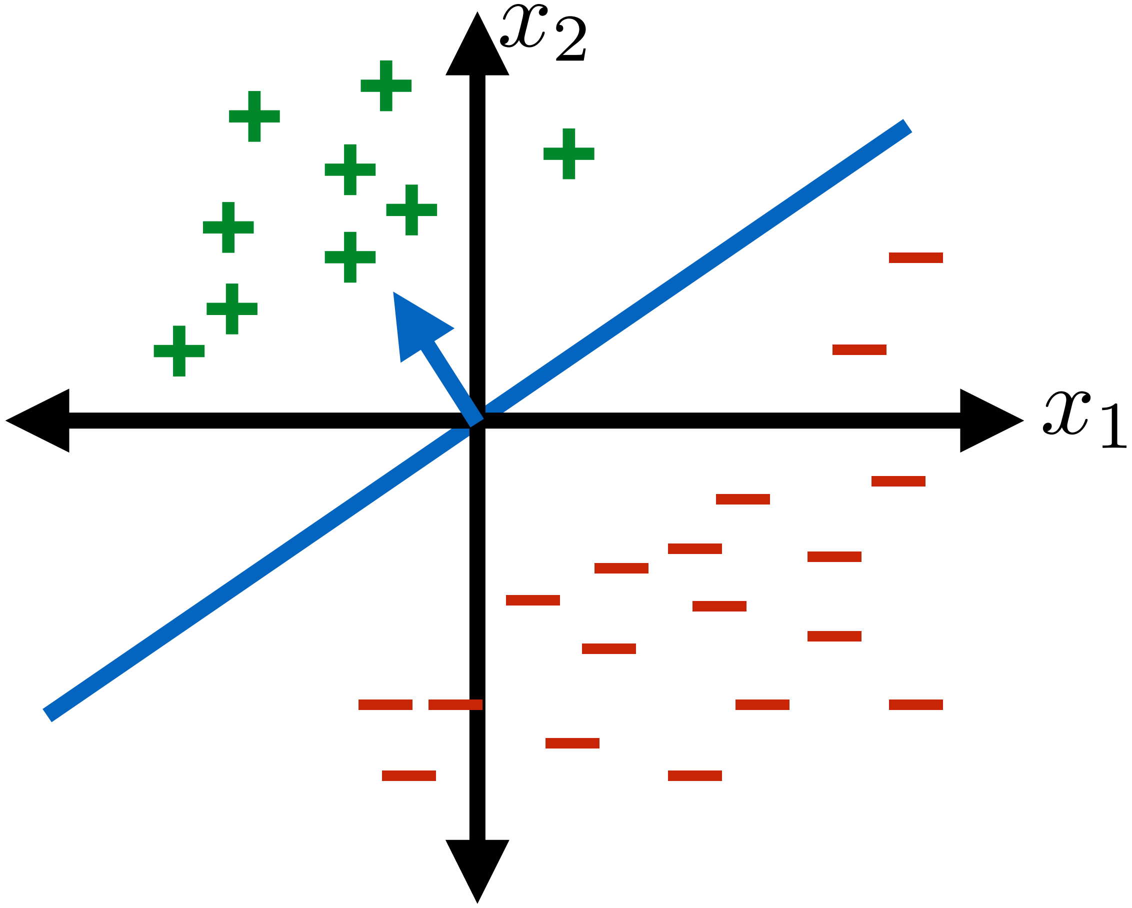

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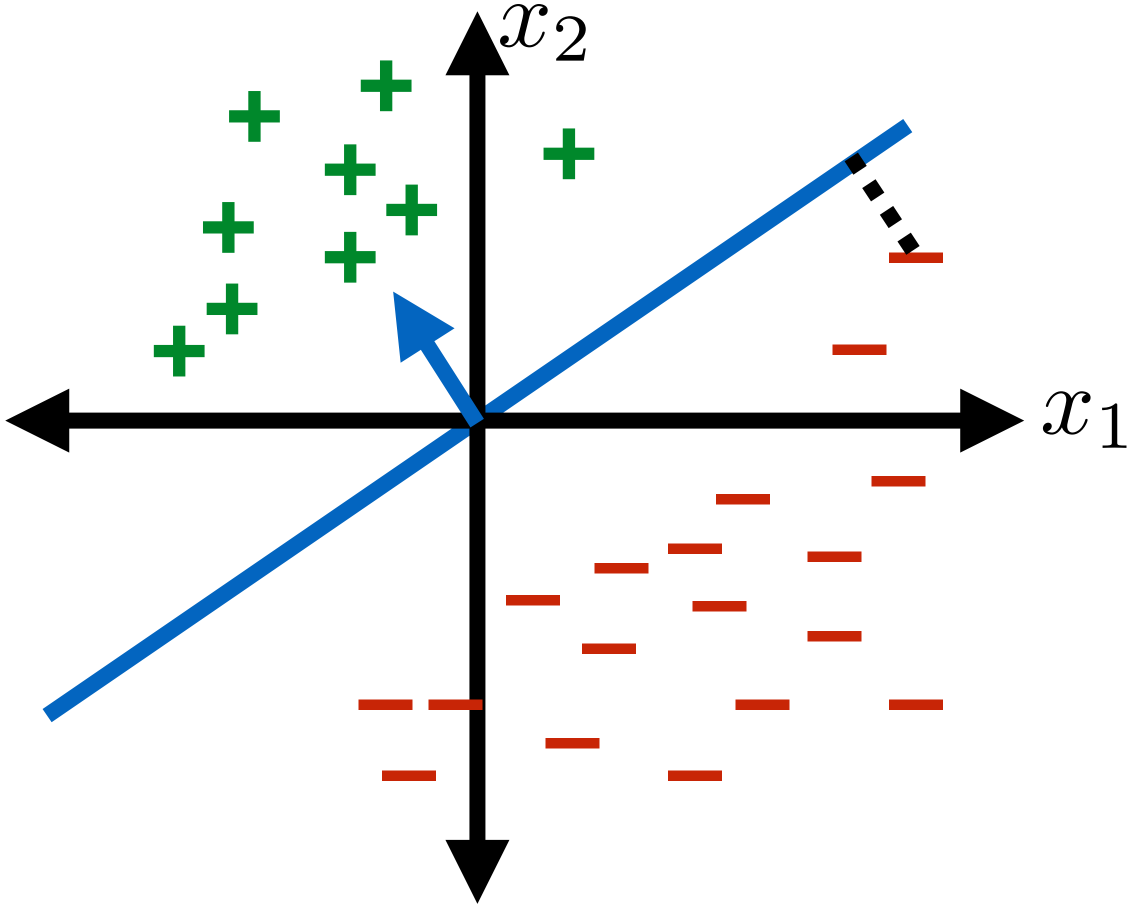

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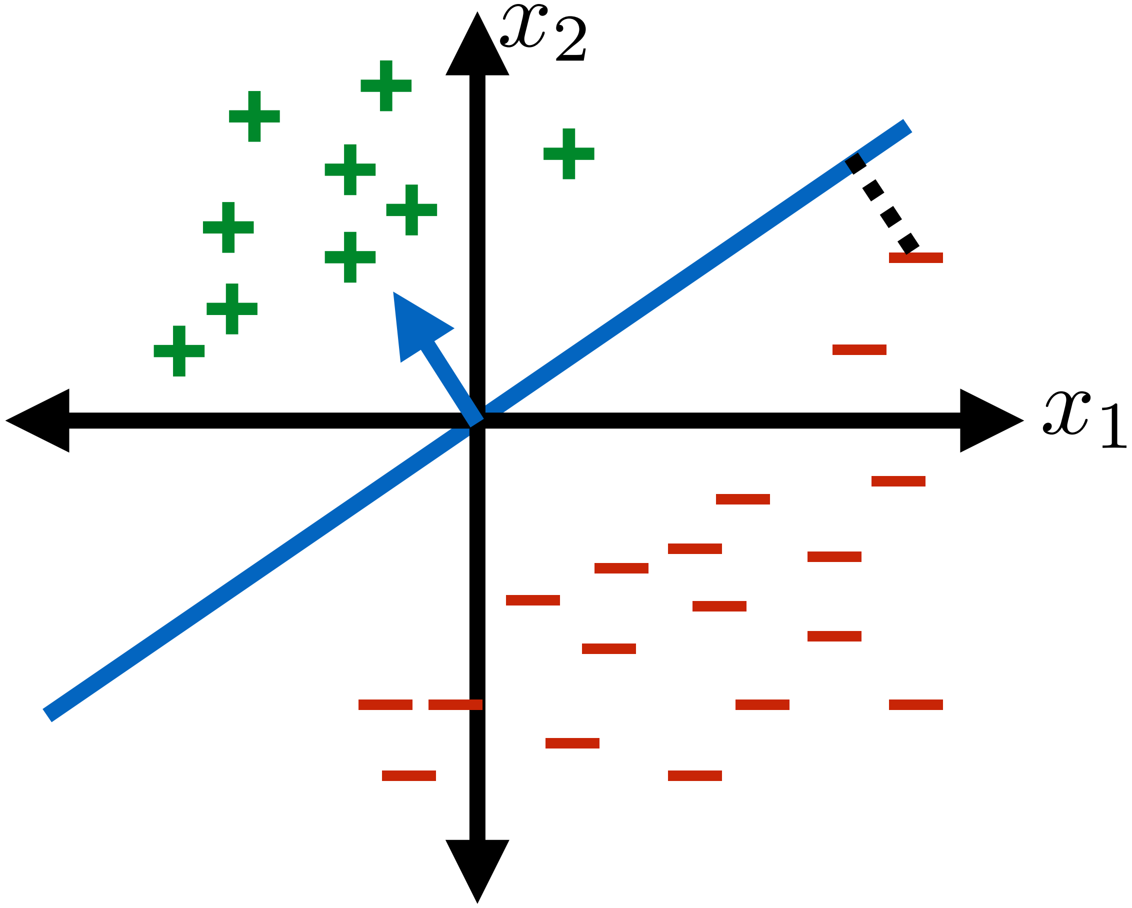

Theorem: Perceptron Performance

- **Assumptions**:
  A. Our hypothesis class = classifiers with separating hyperplanes that pass through the origin (i.e. $\theta_0 = 0$)
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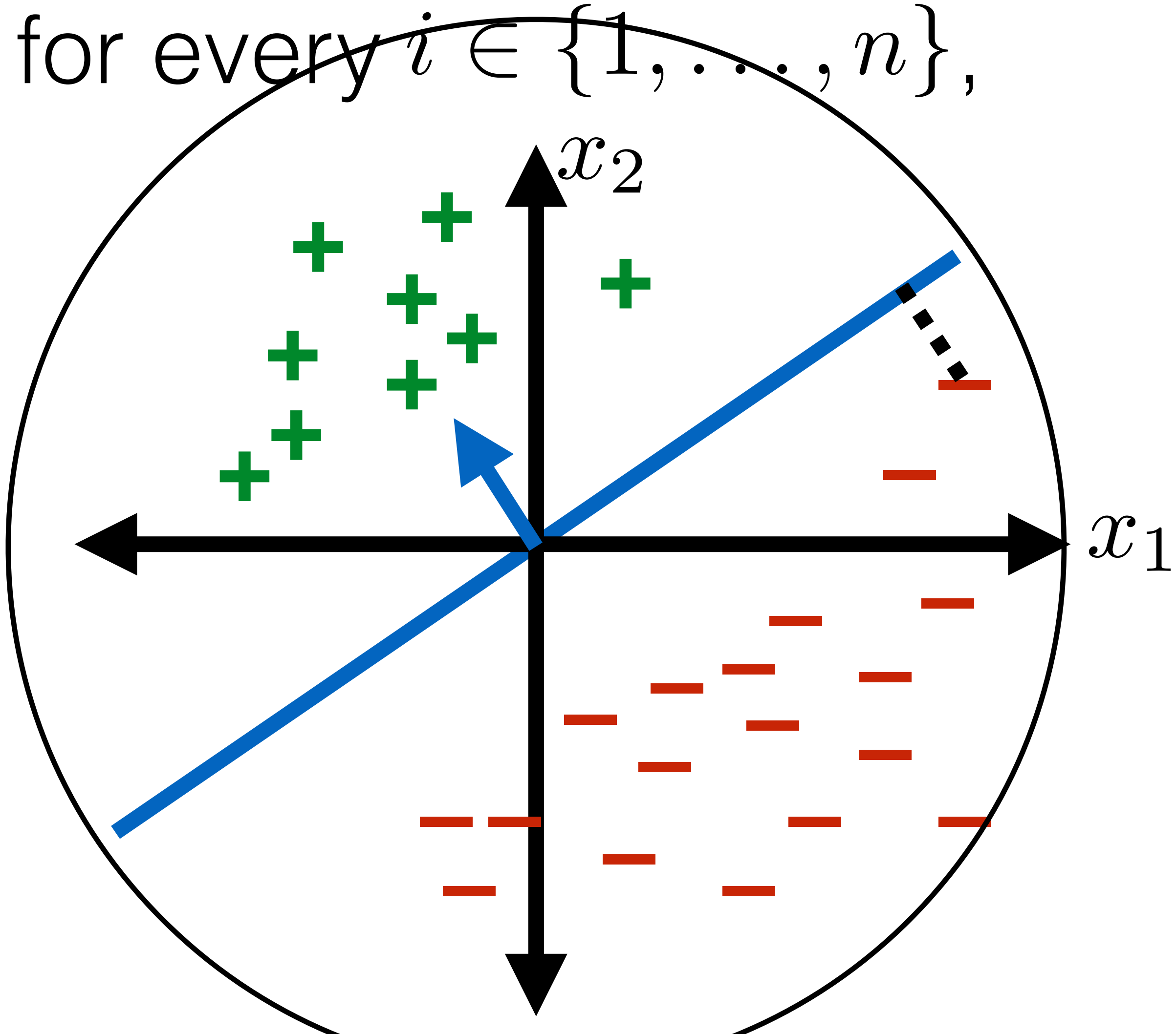

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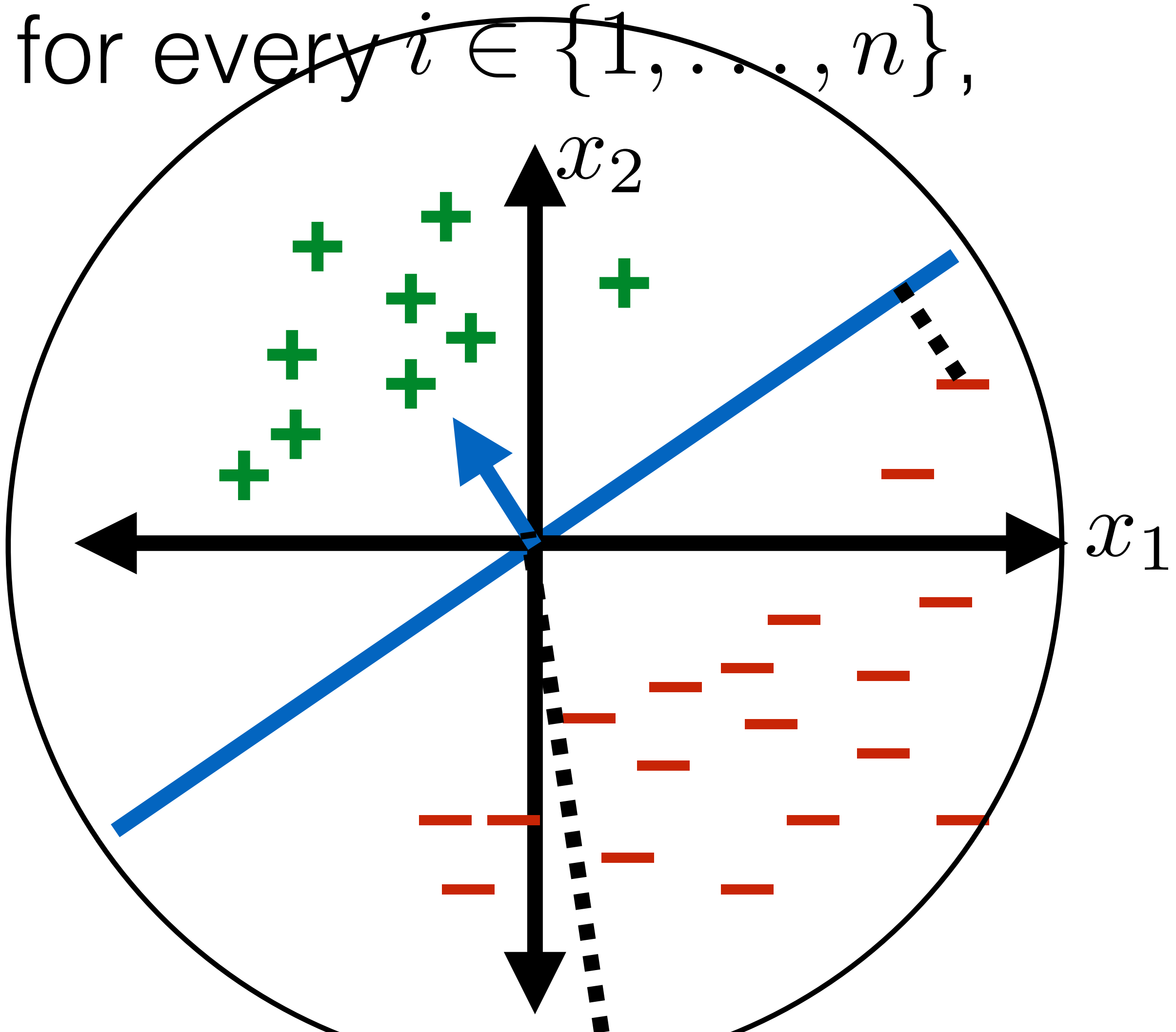

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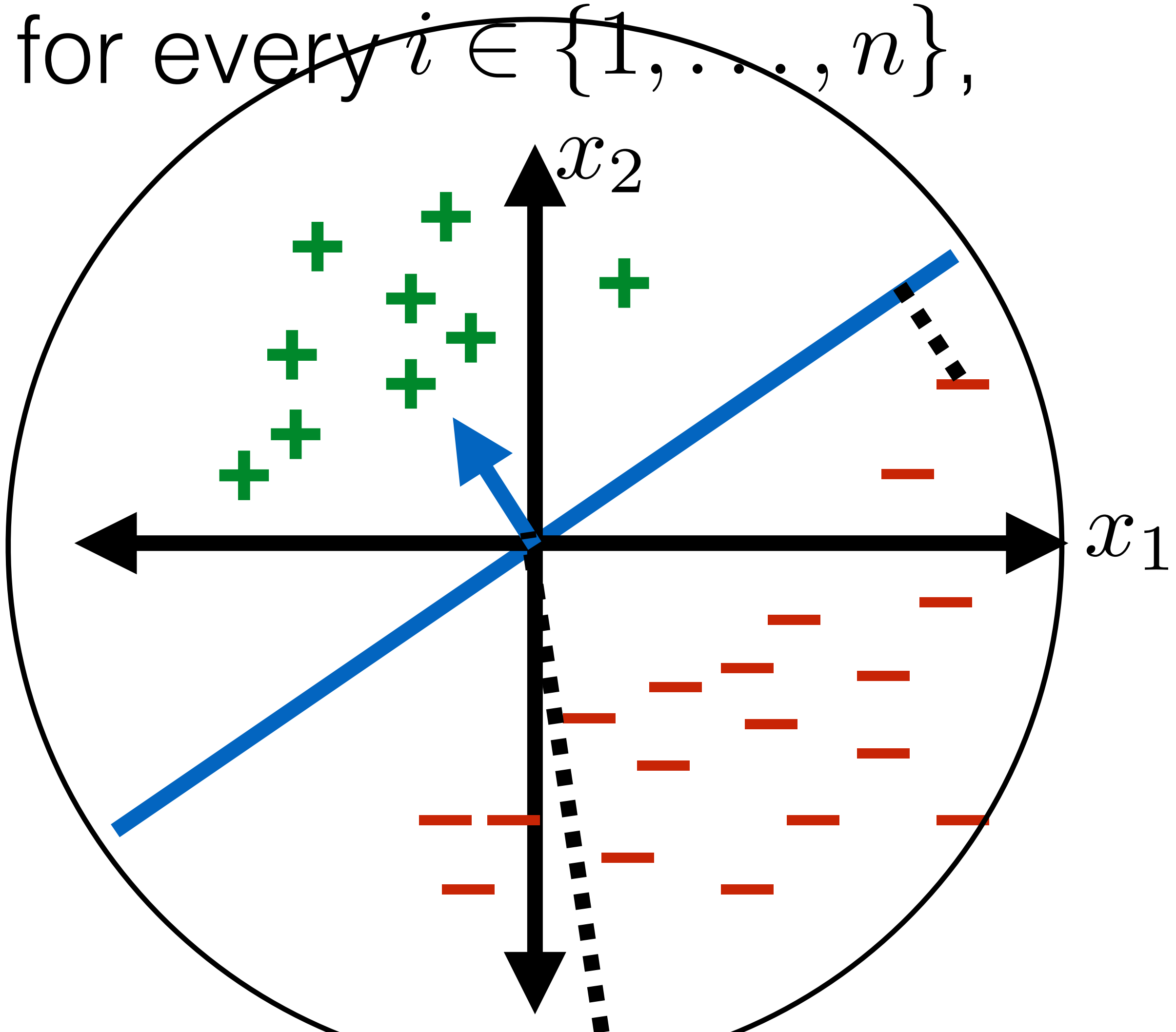
- **Conclusion**: Then the perceptron algorithm will make at most $(R/\gamma)^2$ updates to θ. Once it goes through a pass of *i* without changes, the training error of its hypothesis will be 0.



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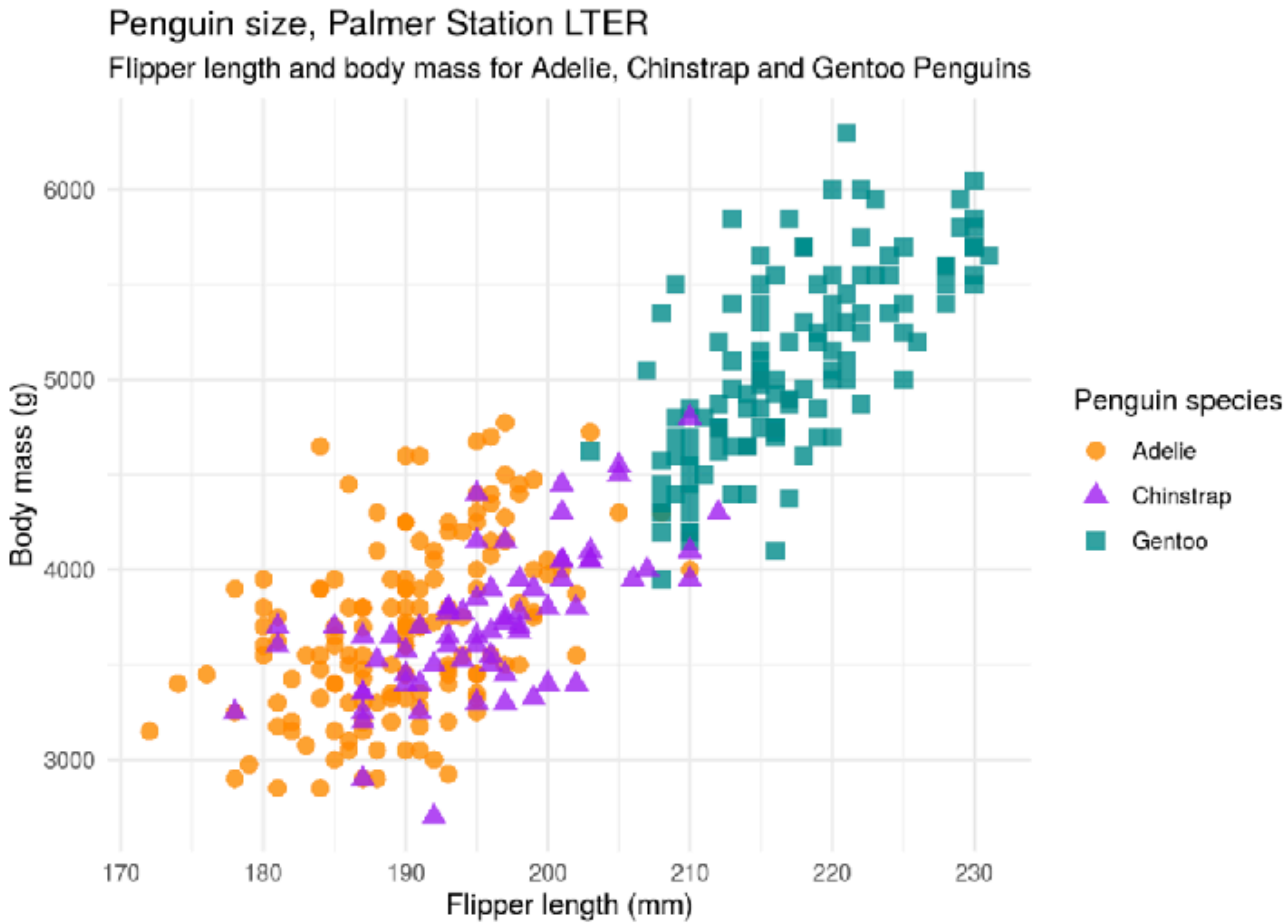
- Can first convert to "expanded" feature space, then apply theorem

Problem: data not linearly separable

- Typical real data sets aren't linearly separable



[https://towardsdatascience.com/penguins-dataset-overview-iris-alternative-9453bb8c8d95, https://github.com/allisonhorst/palmerpenguins]

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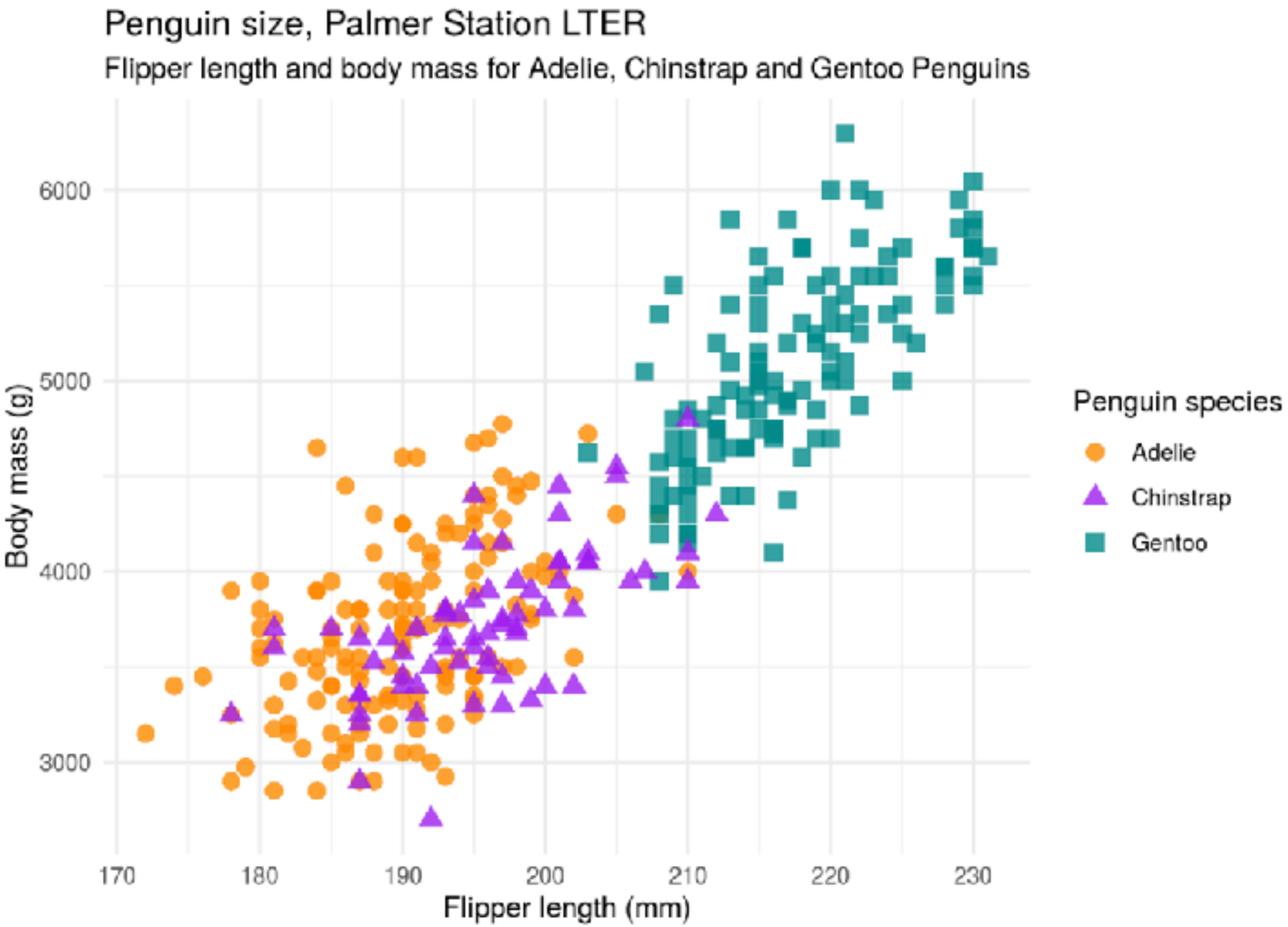

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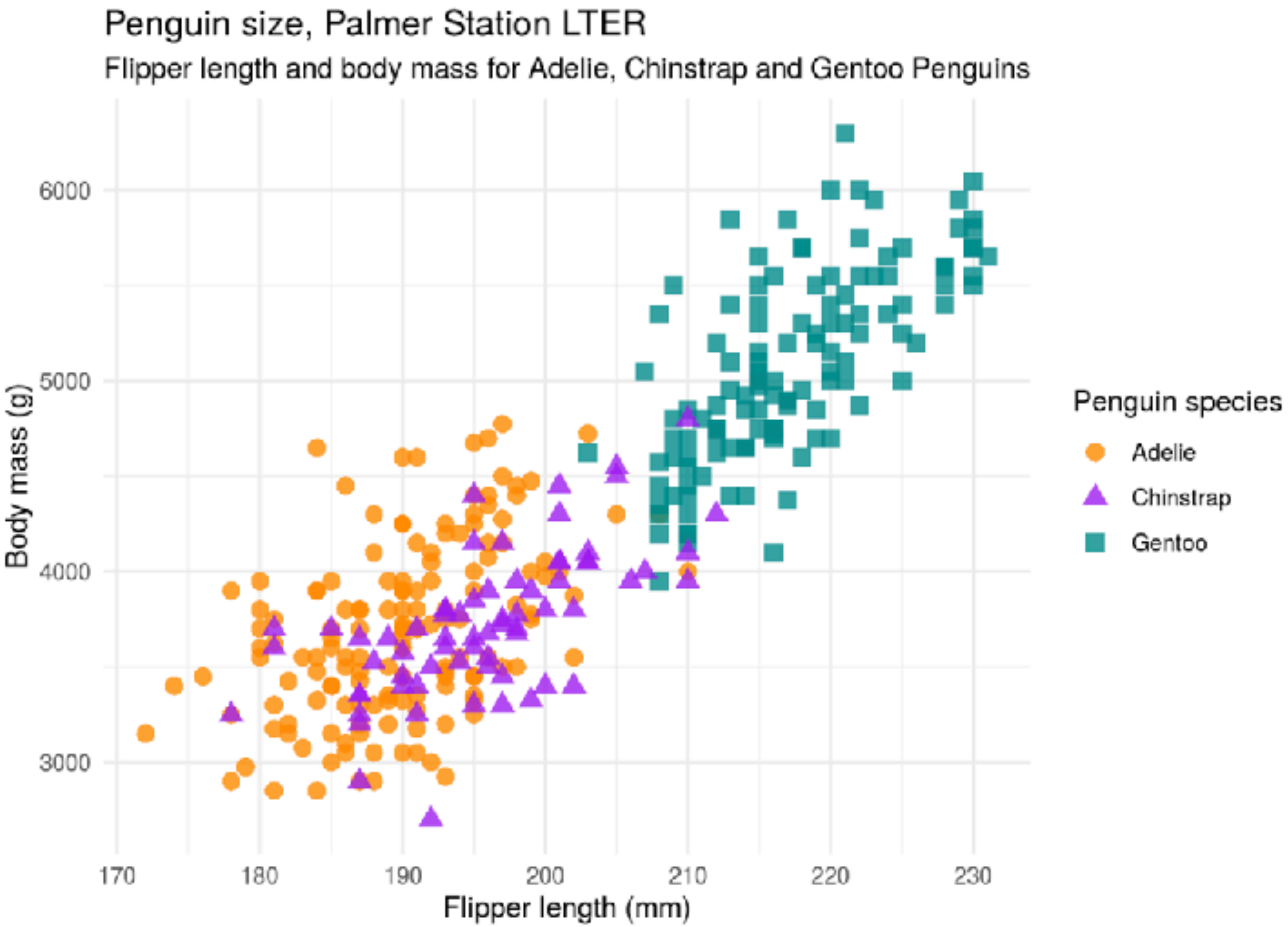

- What can we do?

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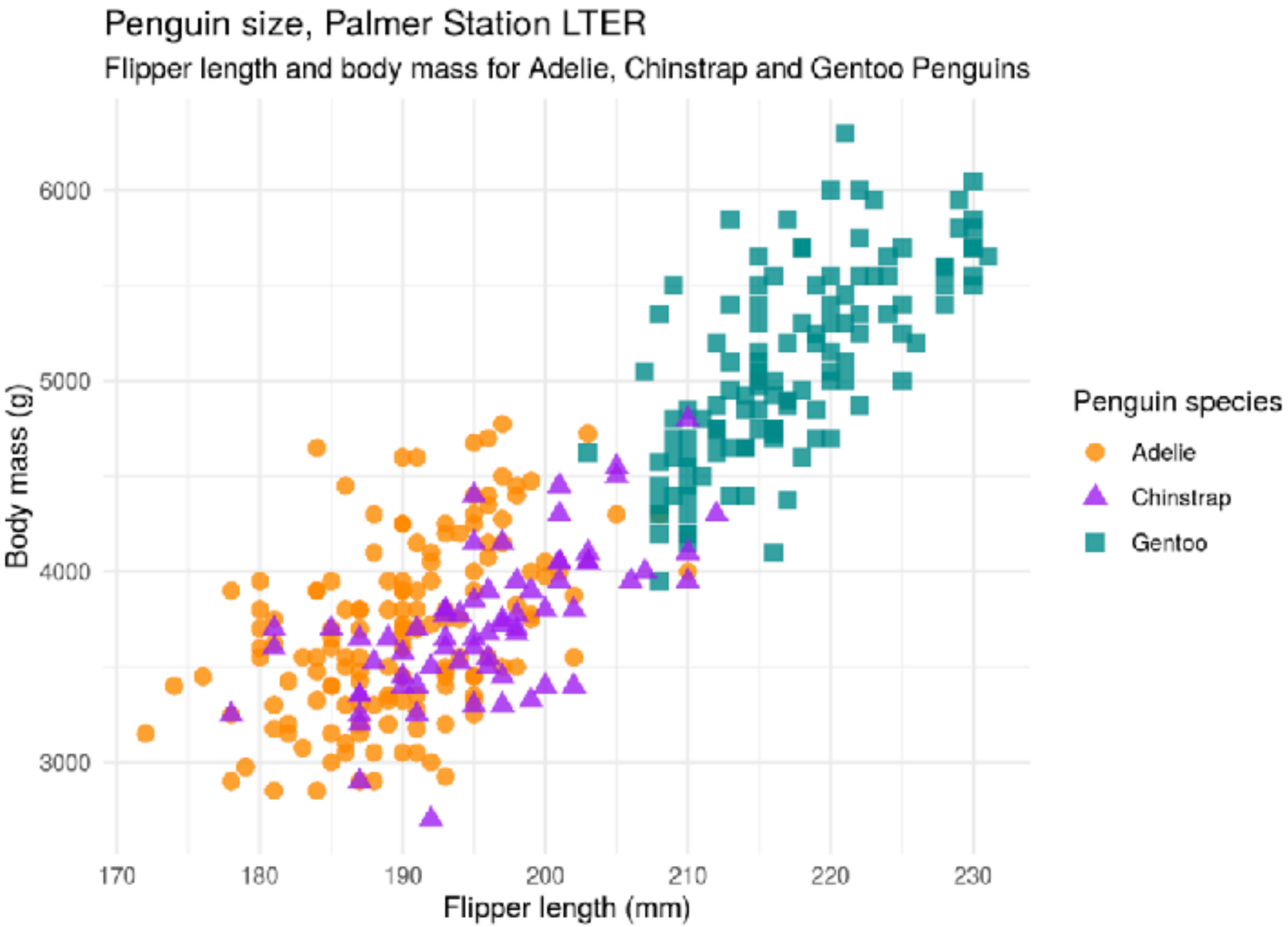

- What can we do? See upcoming lectures!

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Machine Learning Tasks

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- **Binary/two-class classification**

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- **Binary/two-class classification:** Learn a mapping: $\mathbb{R}^d \rightarrow \{-1, +1\}$

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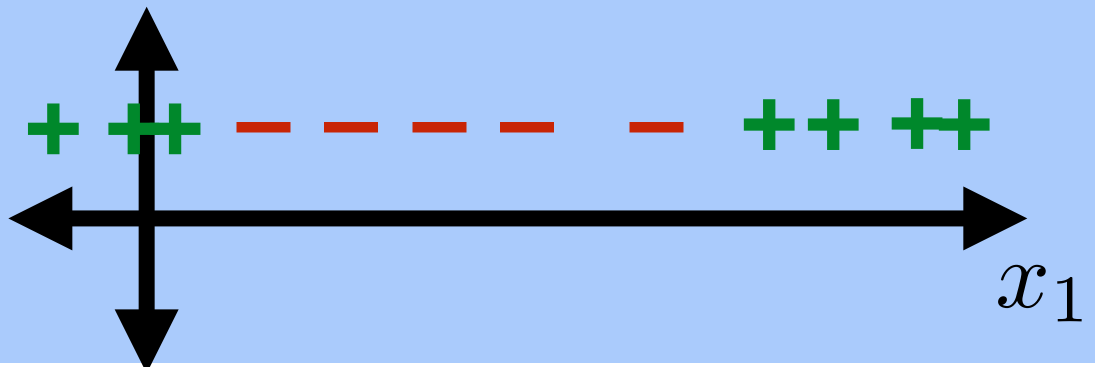

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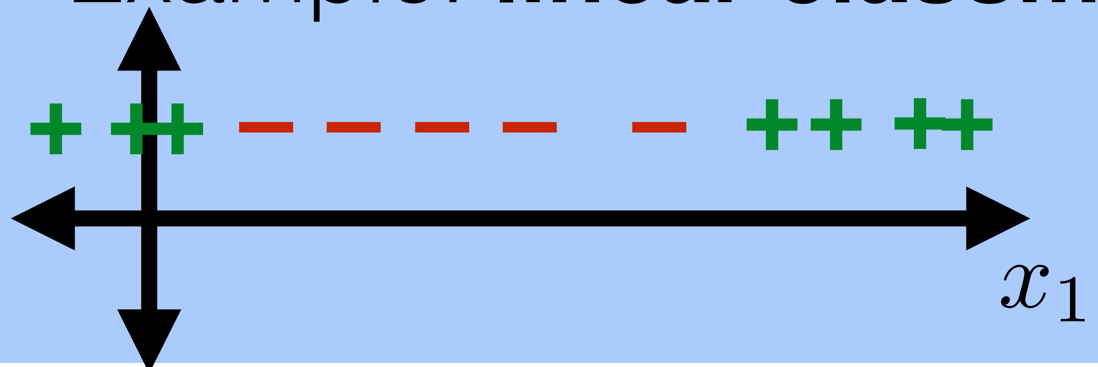

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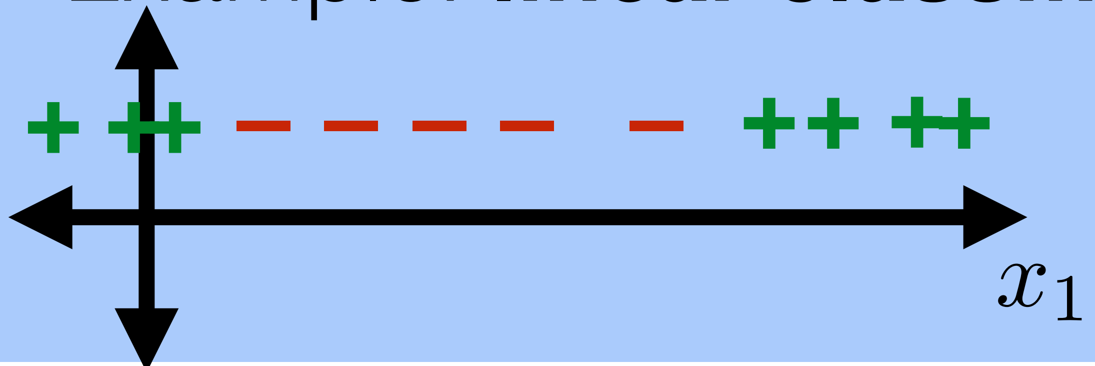

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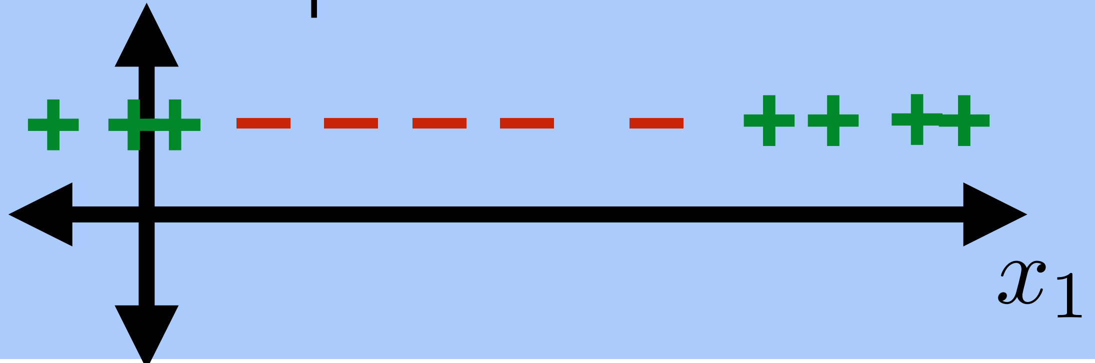

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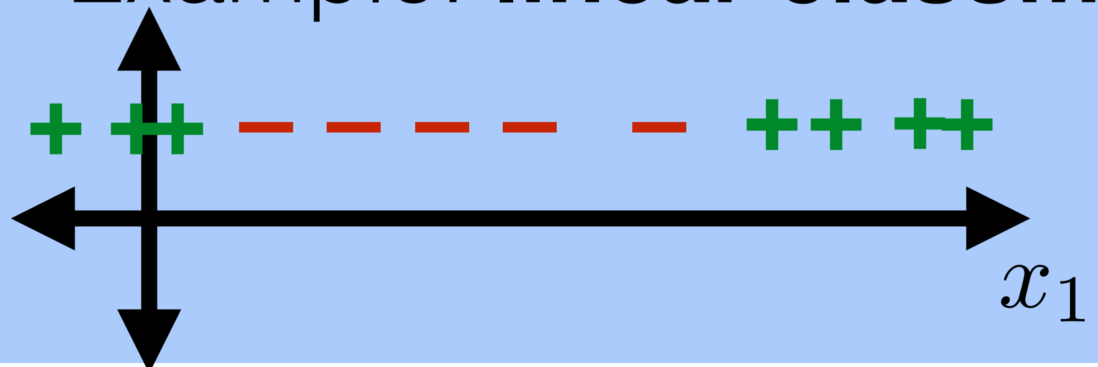

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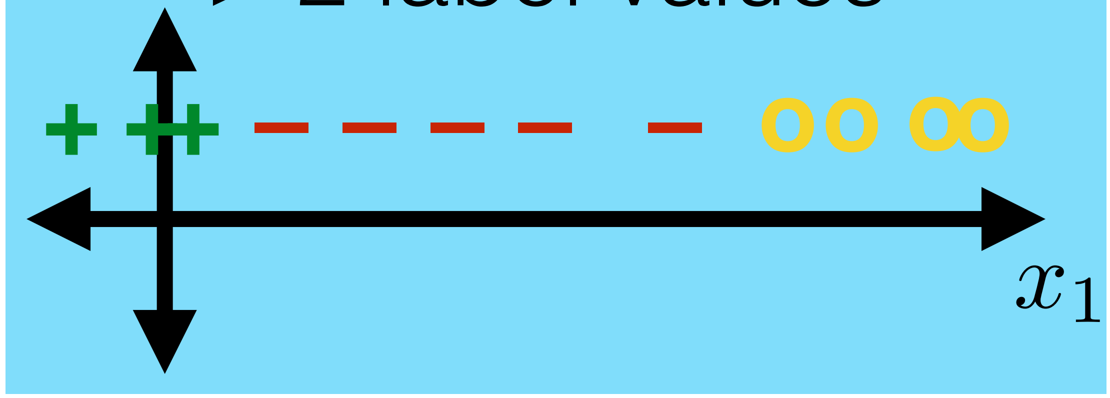

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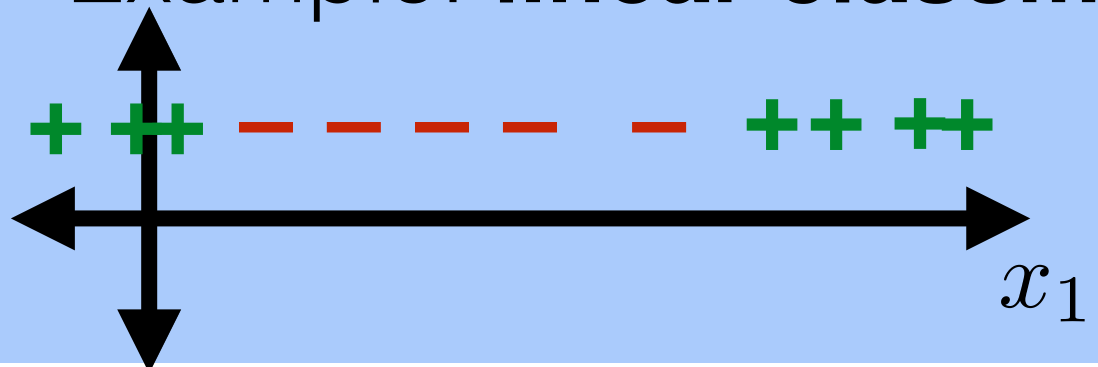

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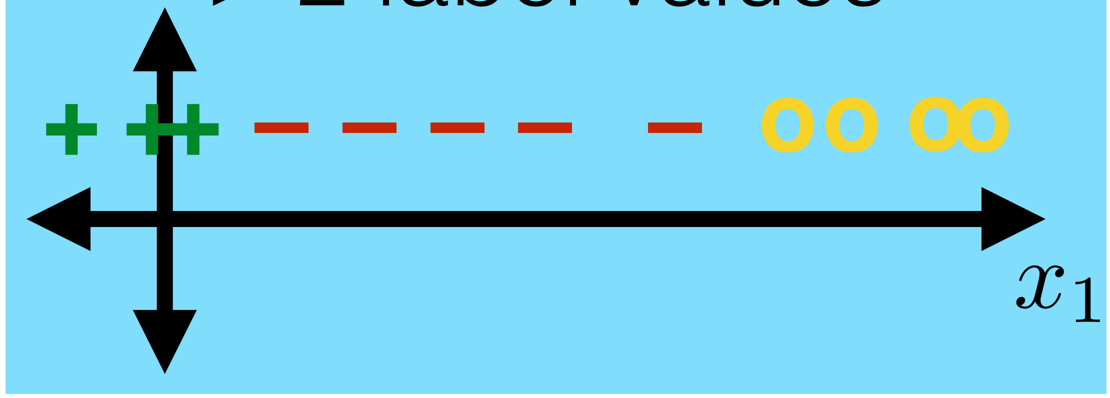

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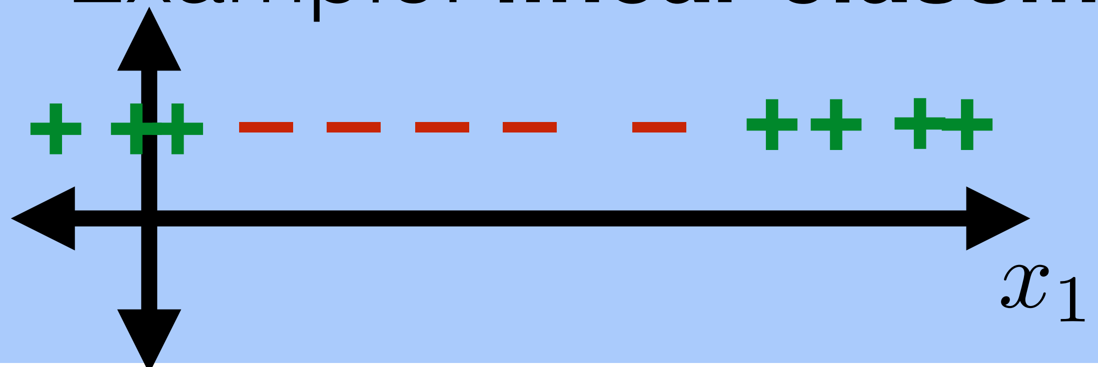

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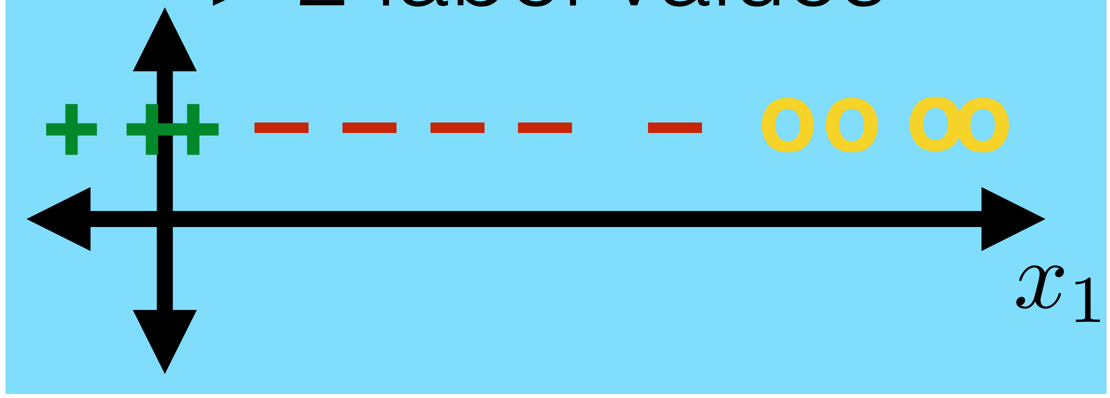

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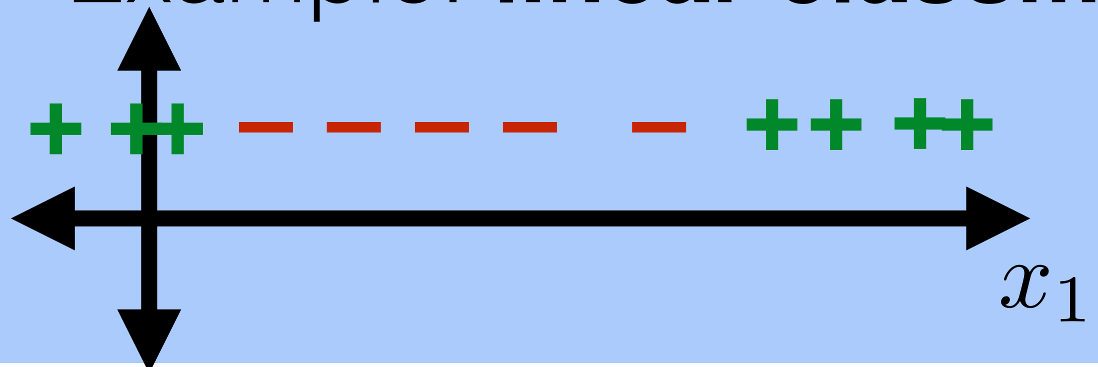

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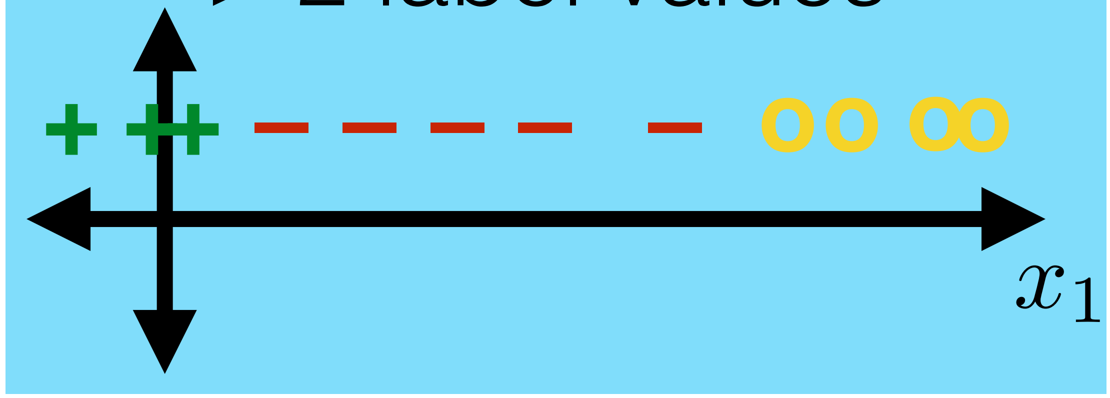

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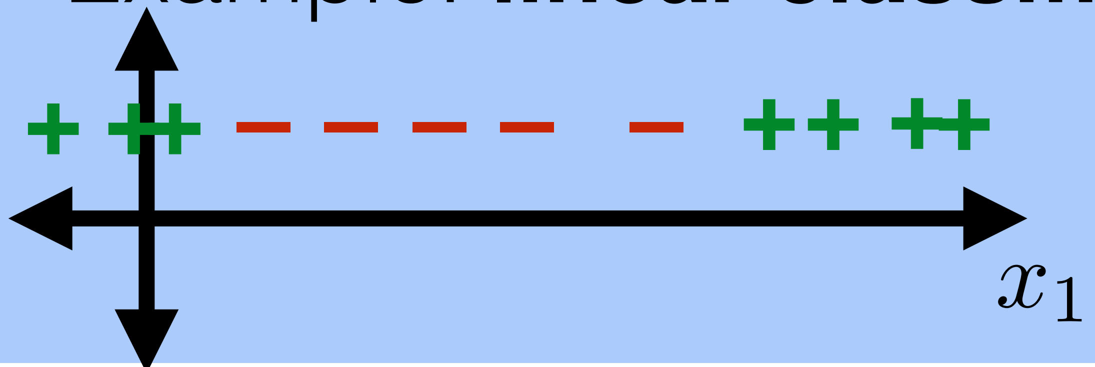

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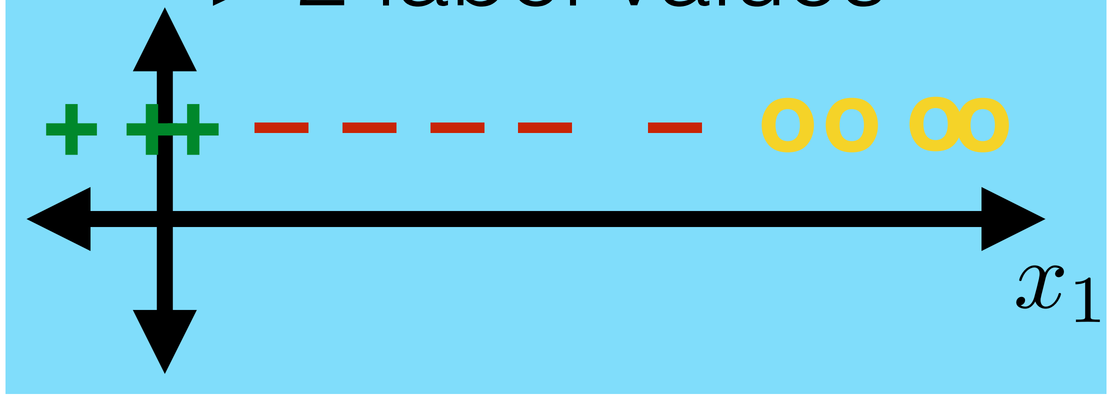

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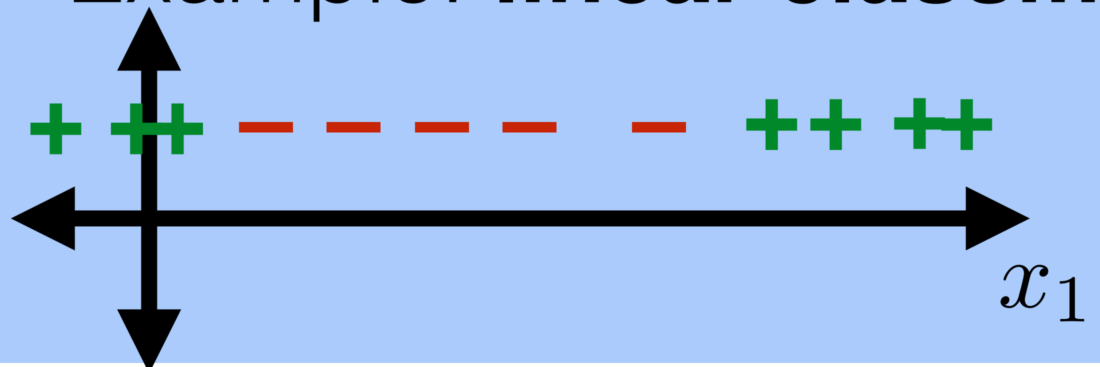

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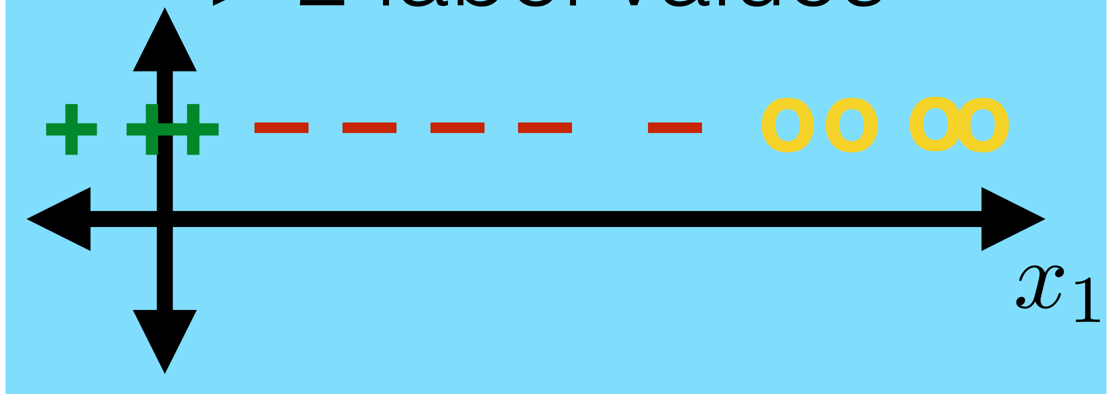

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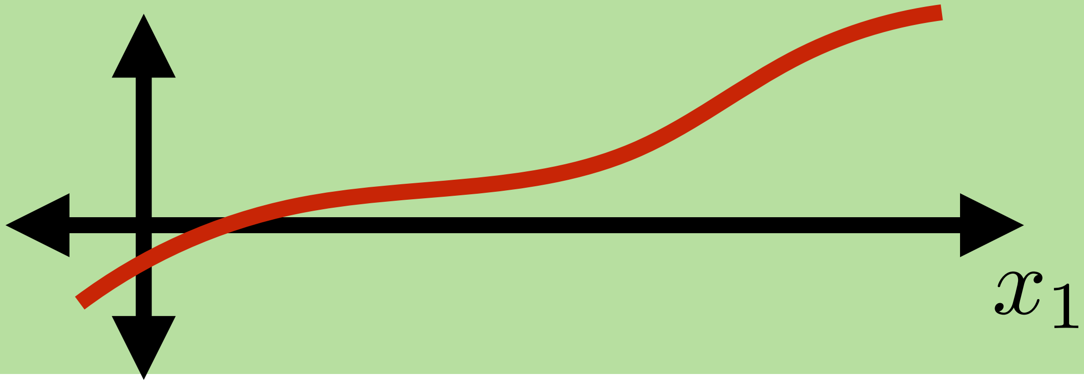

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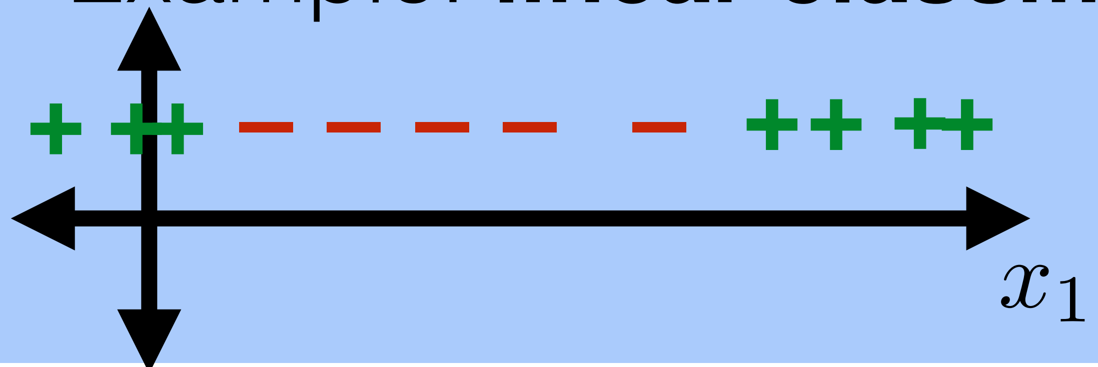

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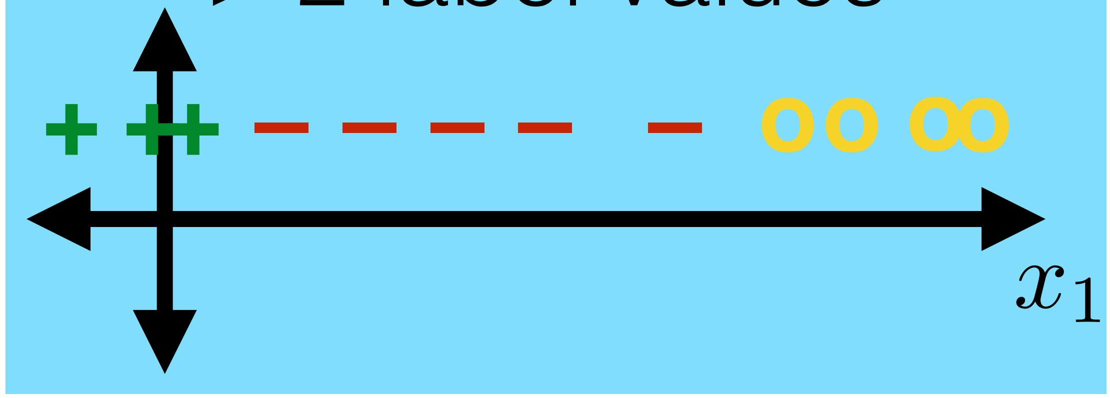

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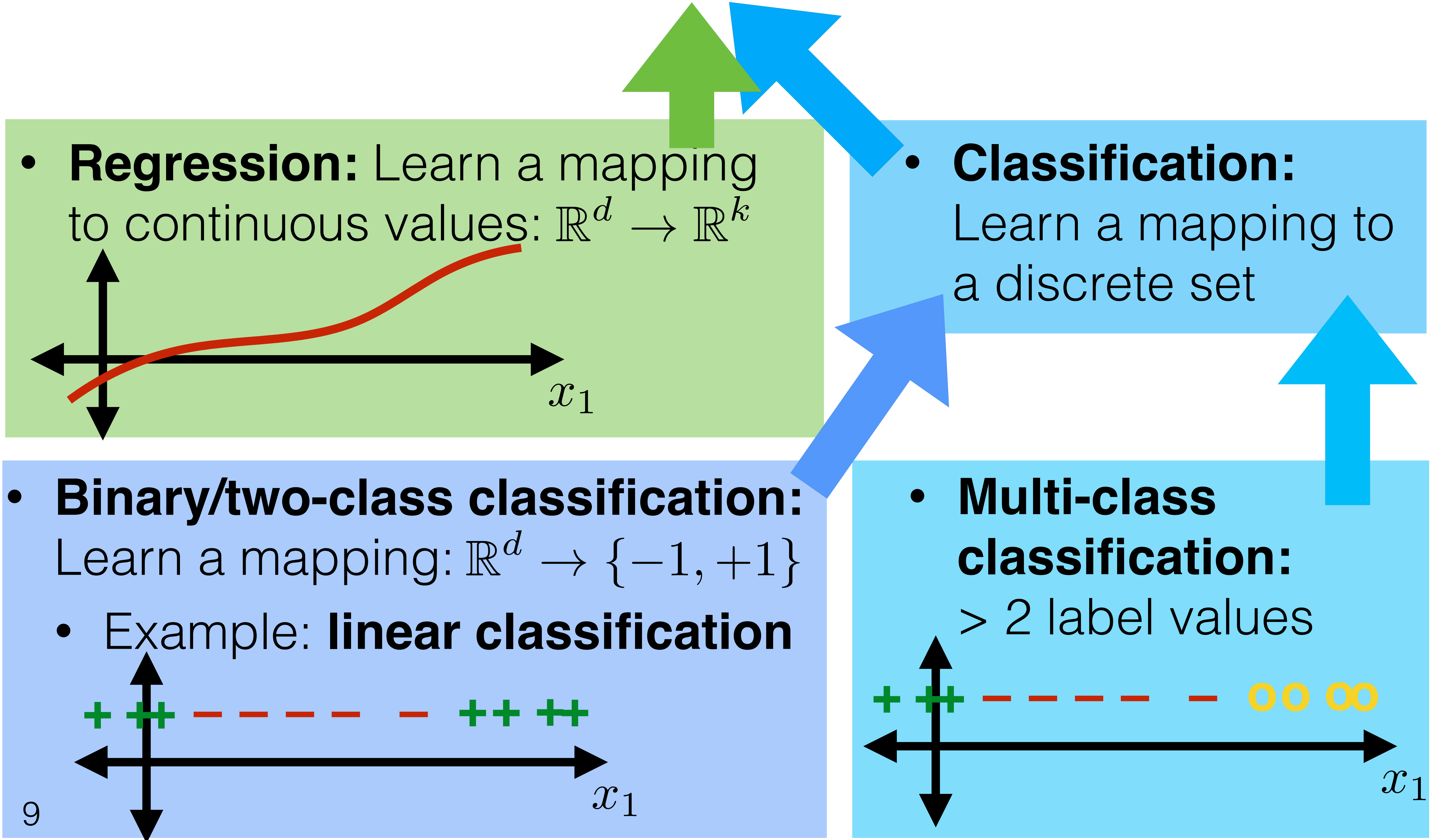

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- **Supervised learning**

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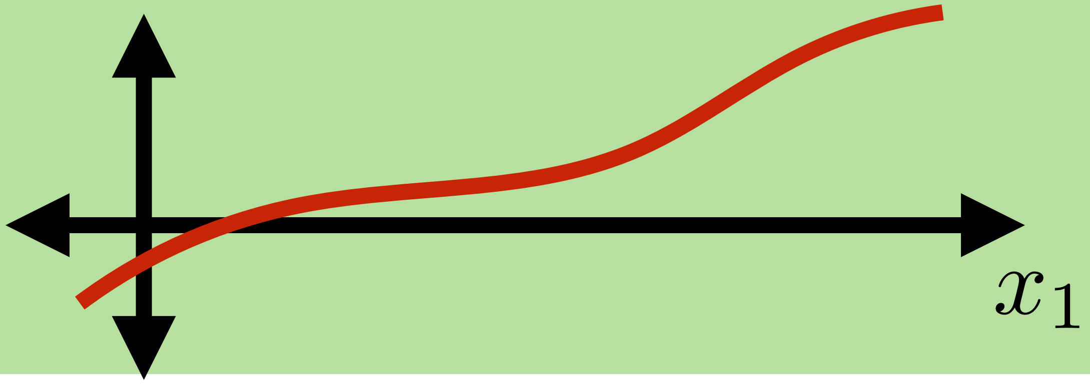

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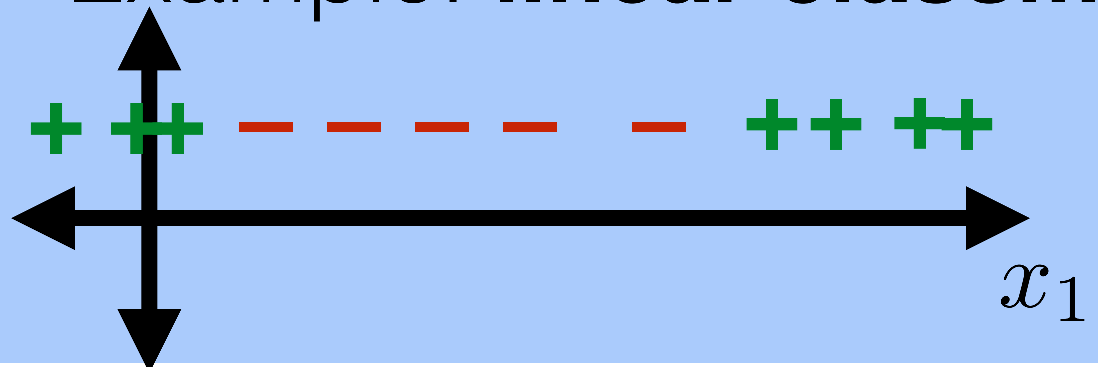

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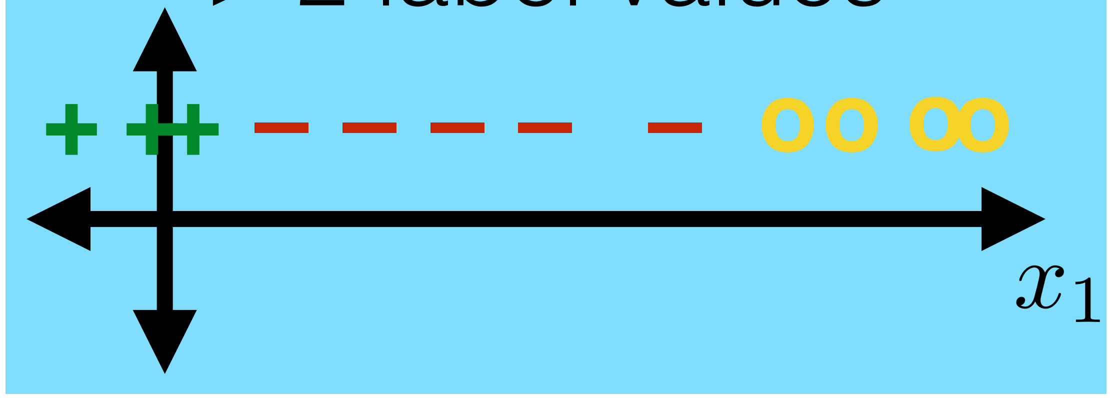

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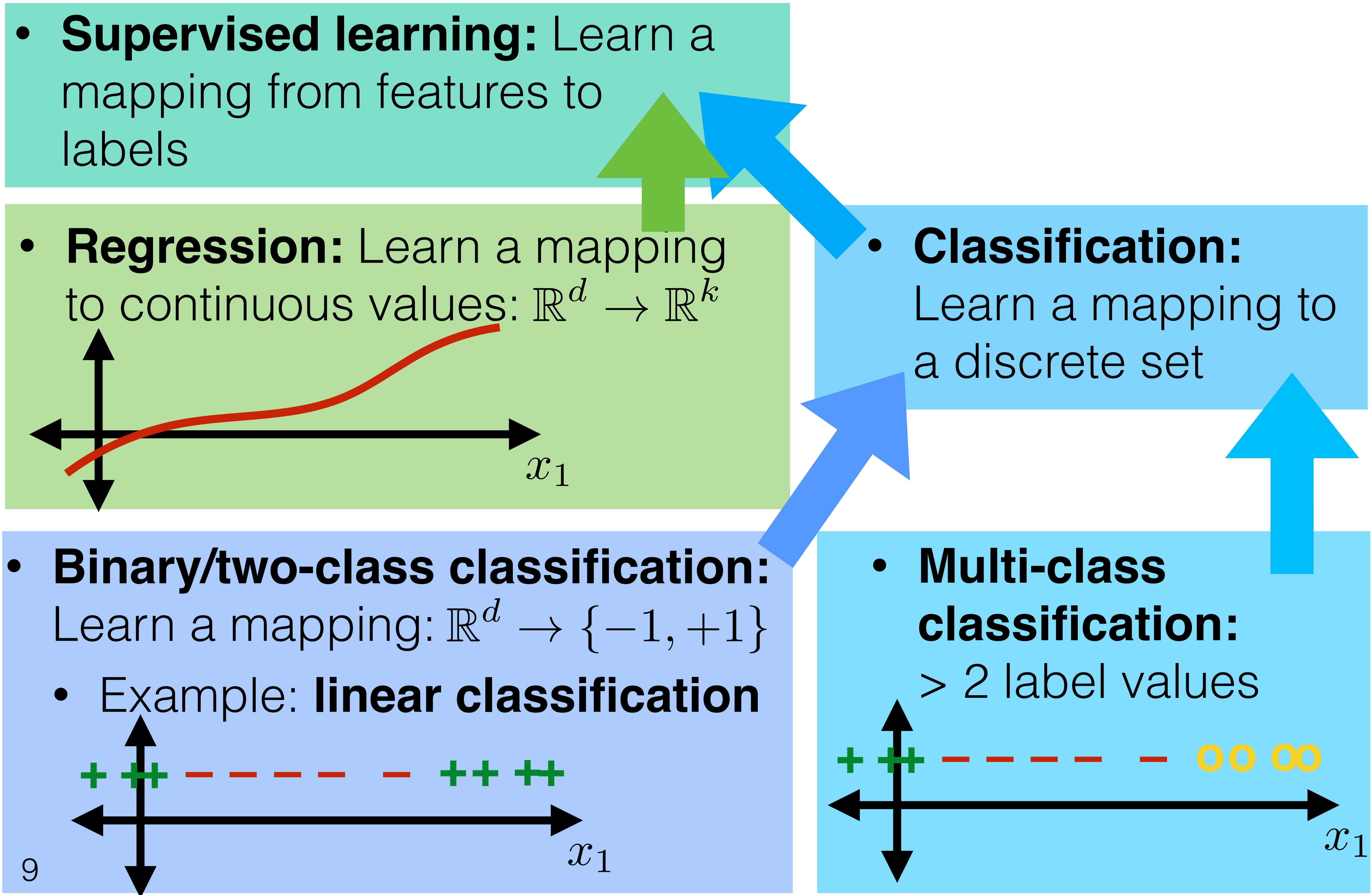

- **Supervised learning:** Learn a mapping from features to labels

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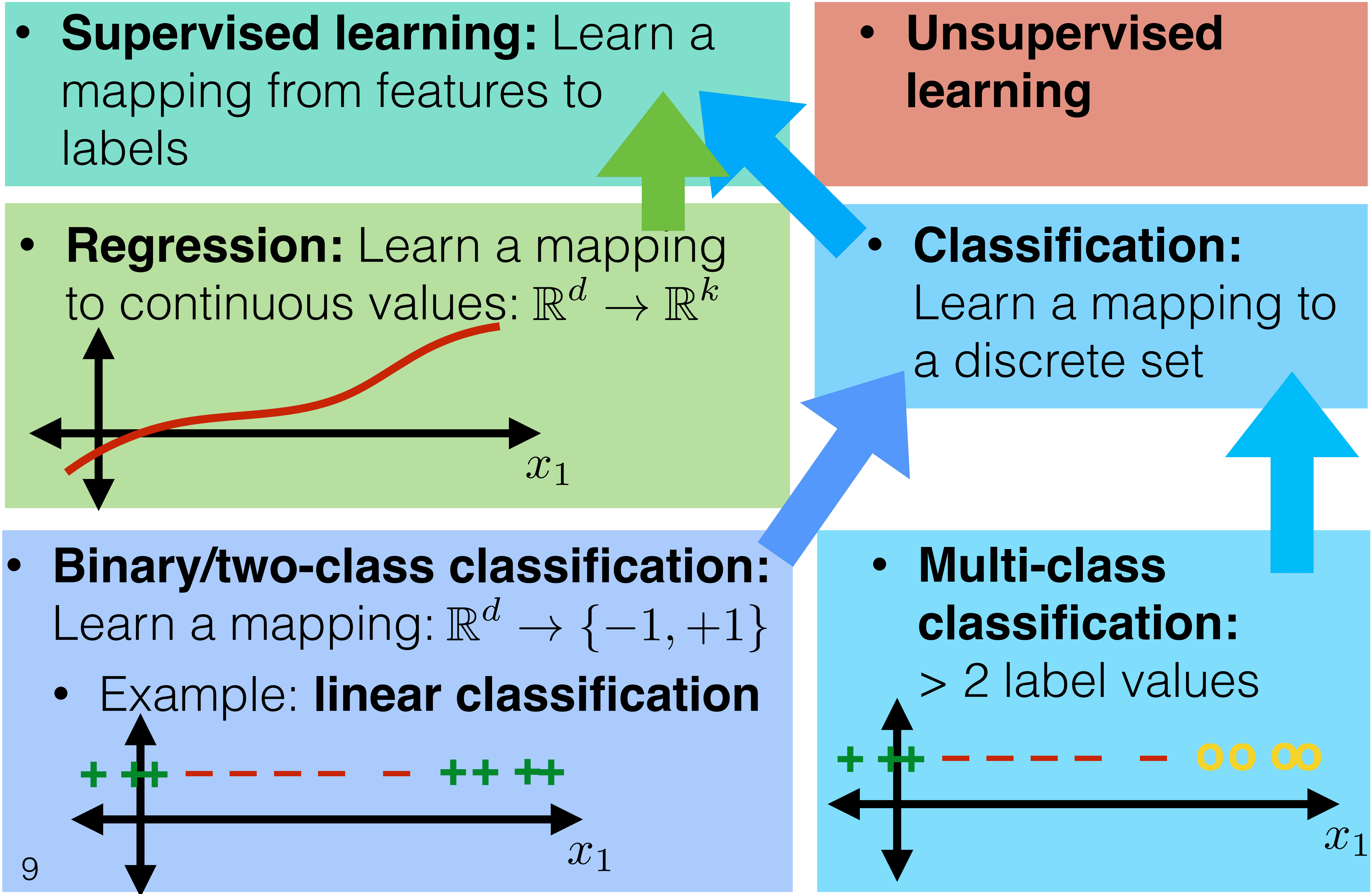

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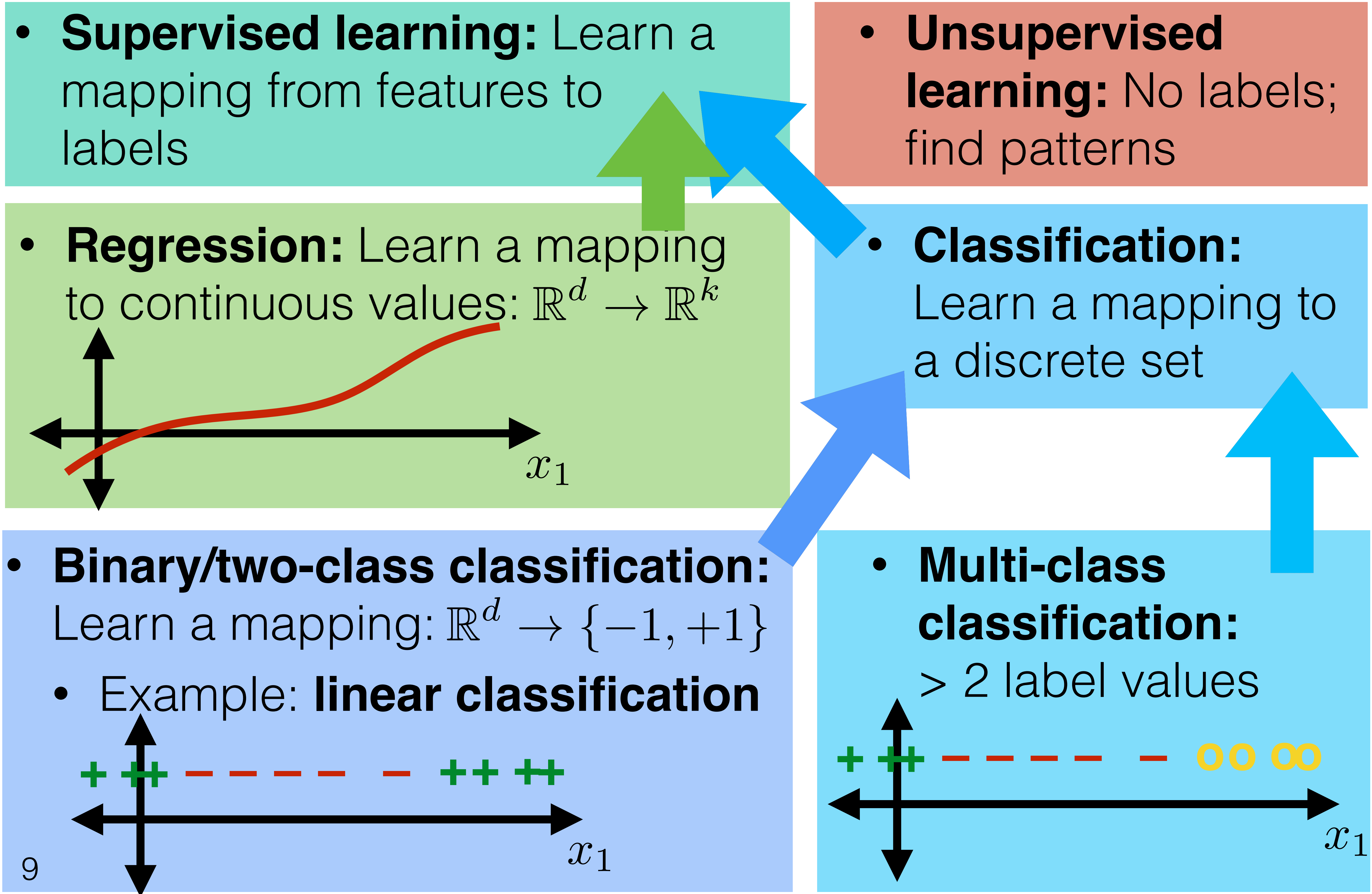

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