

6.036/6.862: Introduction to Machine Learning

Lecture: starts Tuesdays 9:35am (Boston time zone)

Course website: introml.odl.mit.edu

Who's talking? Prof. Tamara Broderick

Questions? discourse.odl.mit.edu ("Lecture 3" category)

Materials: Will all be available at course website

Last Time(s)

- I. Linear classifiers
- II. Perceptron algorithm
- III. Linear separability
- IV. Perceptron theorem

Today's Plan

- I. A more-complete ML analysis
- II. Choosing good features
- III. Evaluation

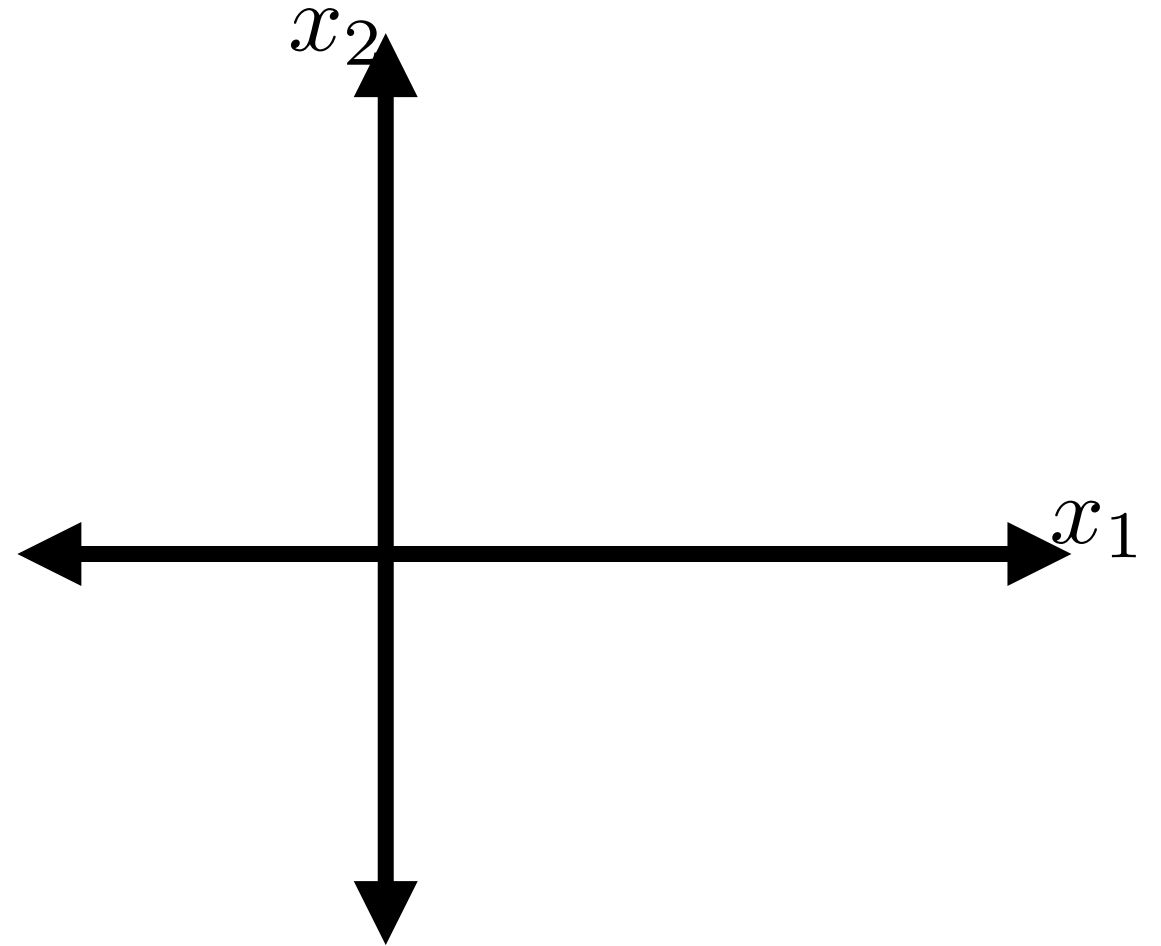
Recall

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- Linear classifier h

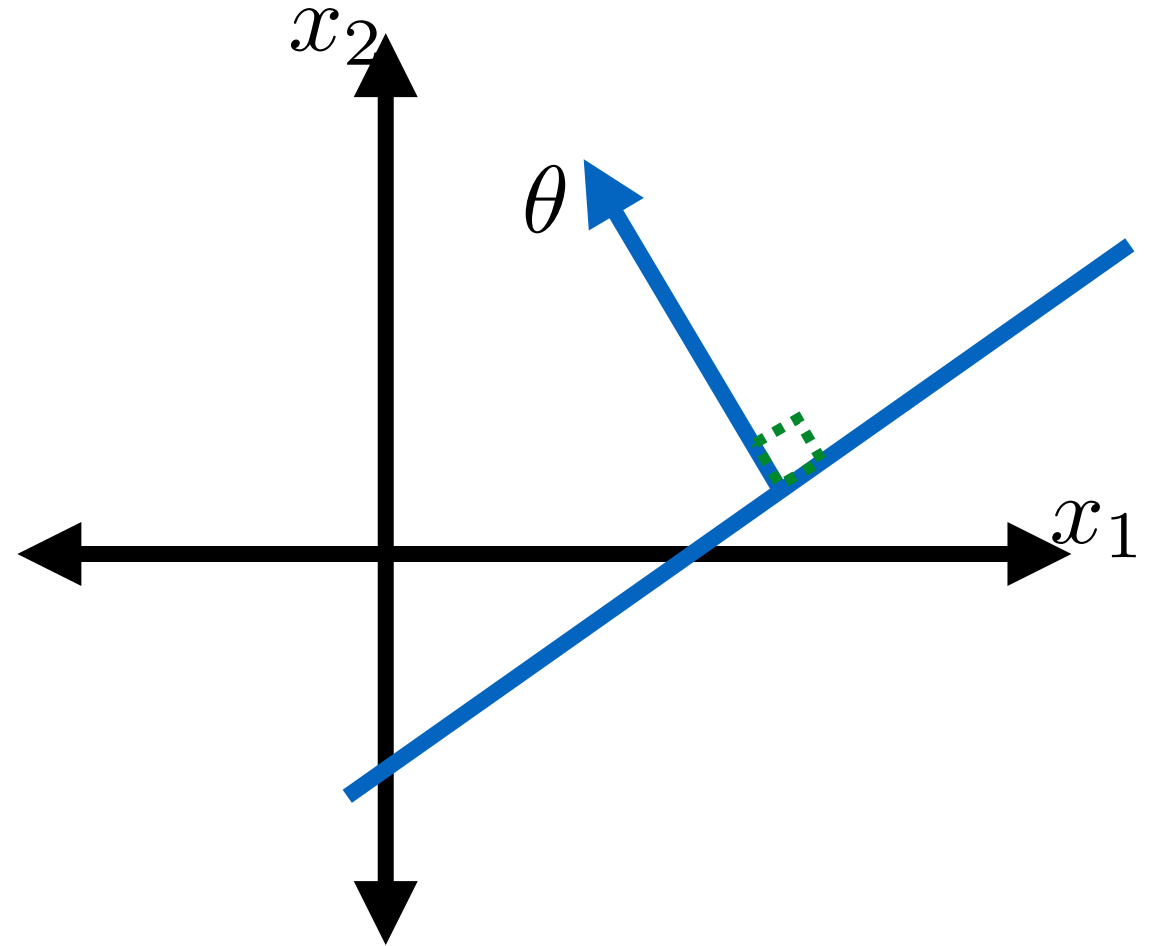
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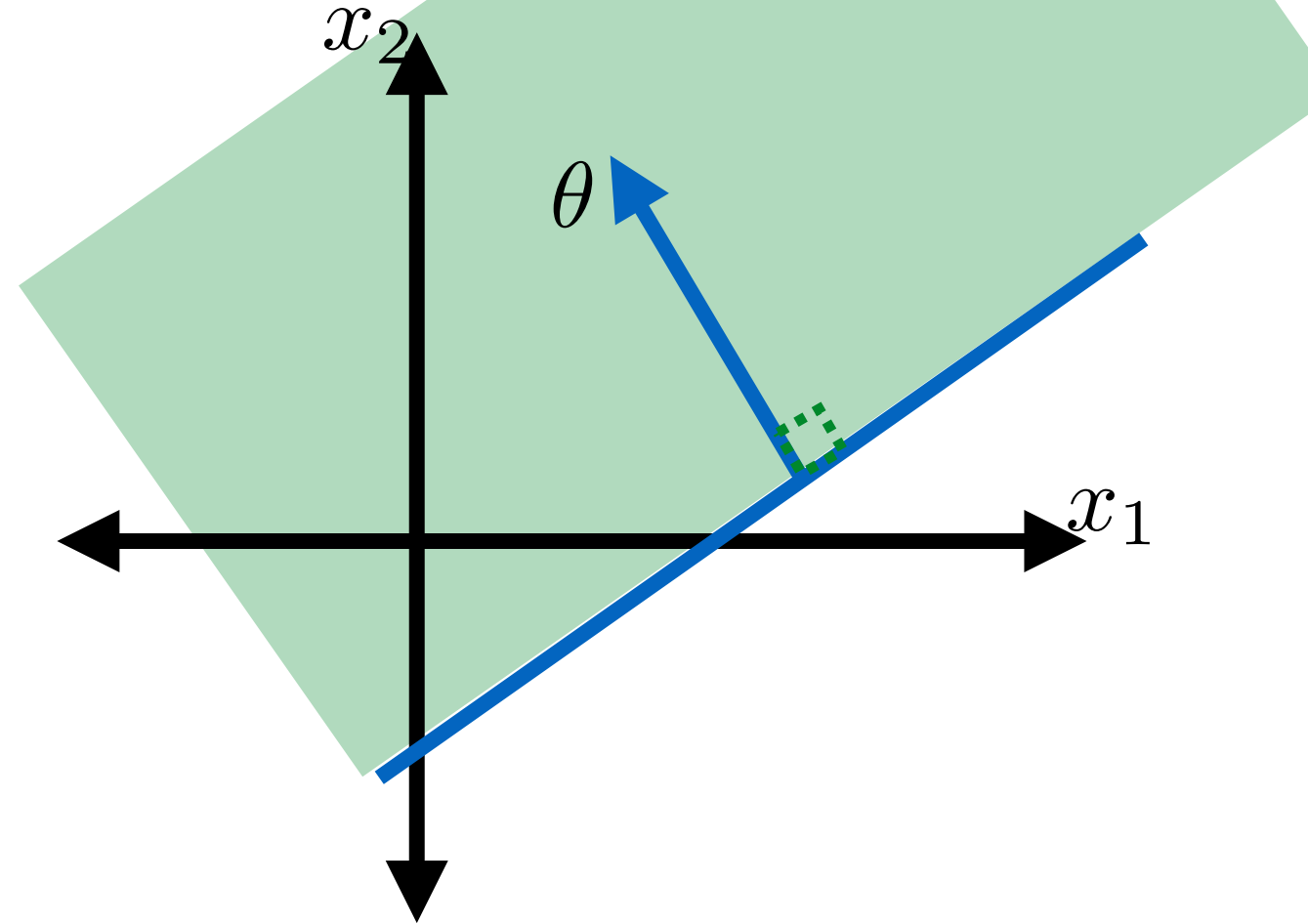
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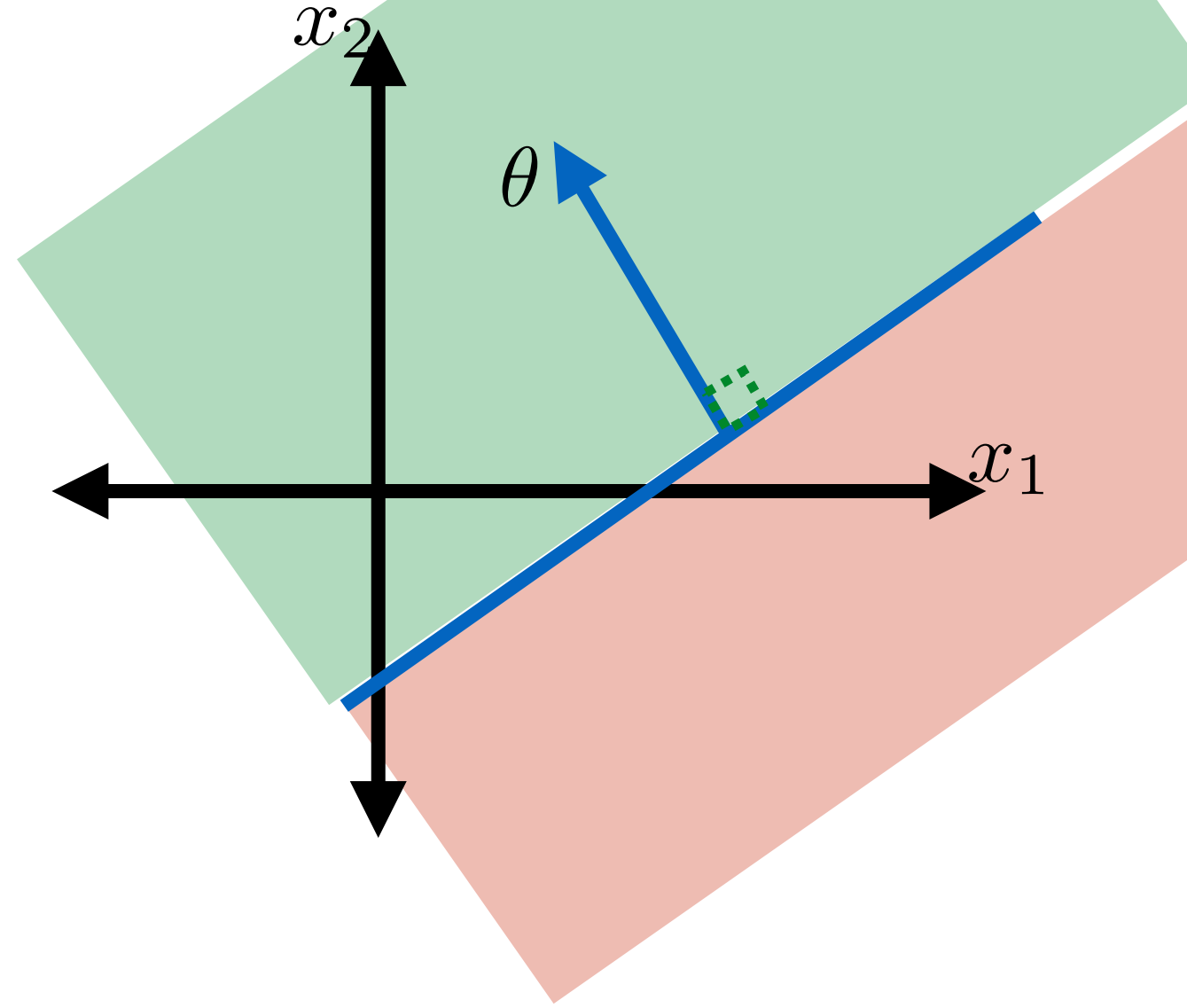
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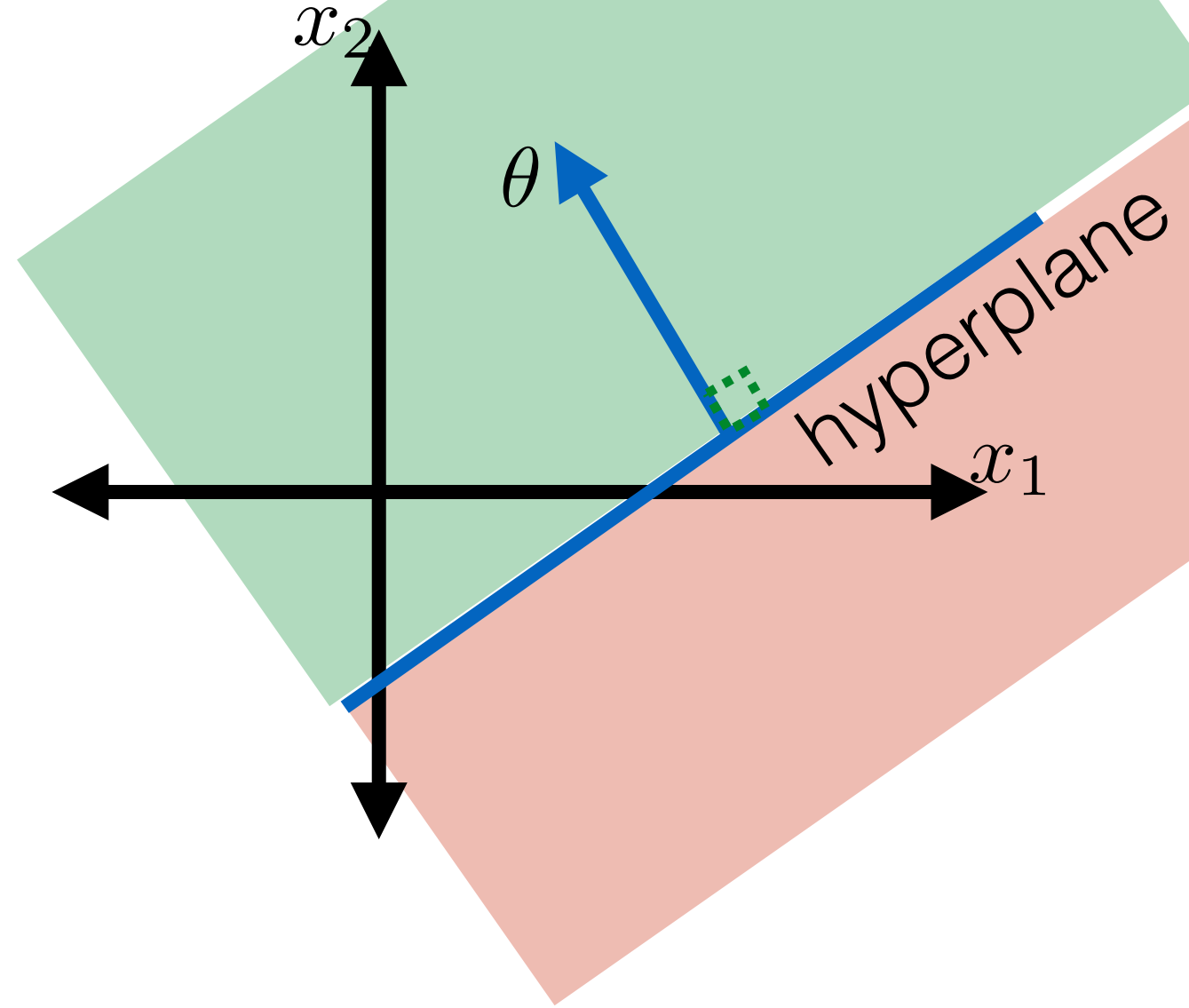
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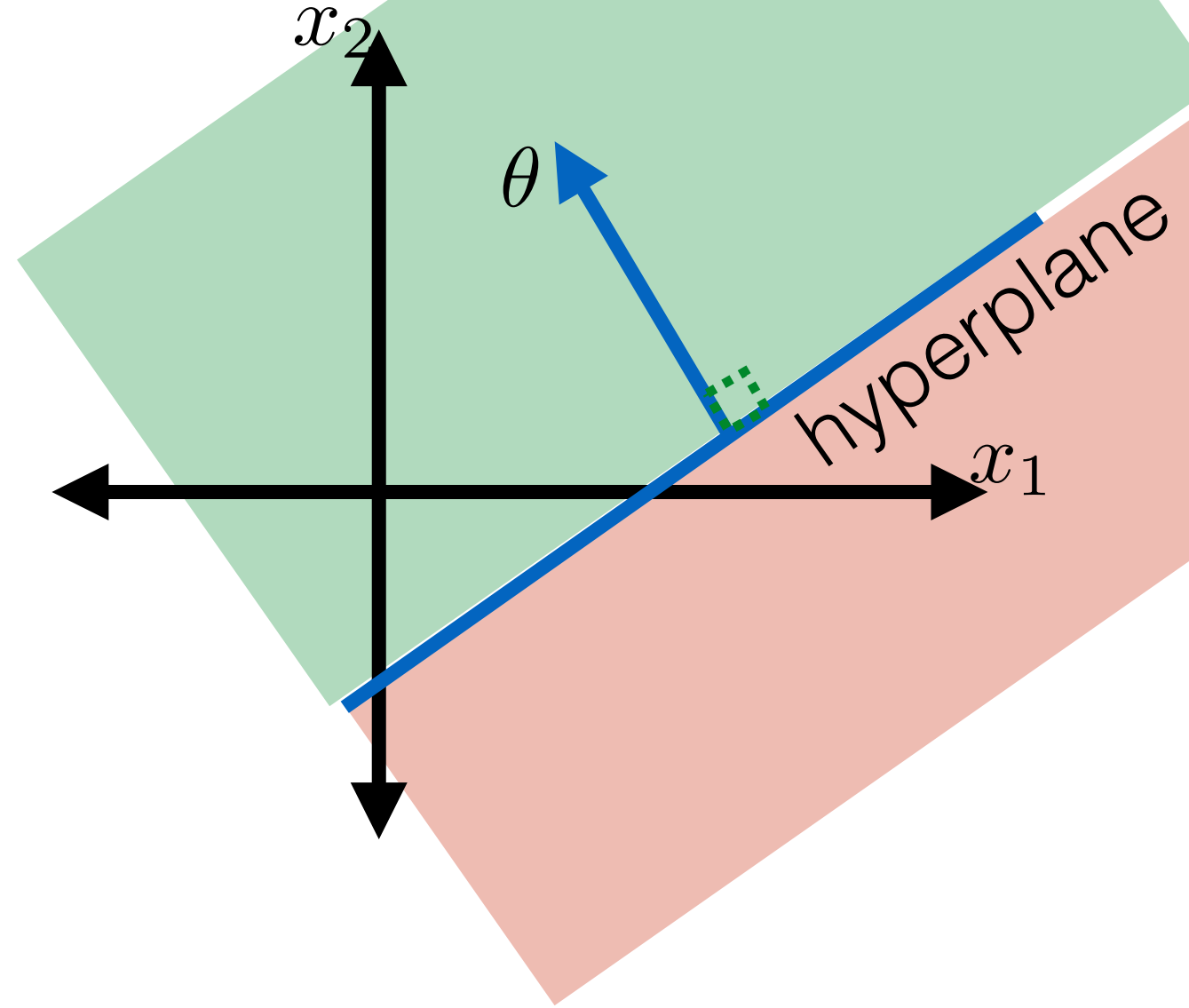
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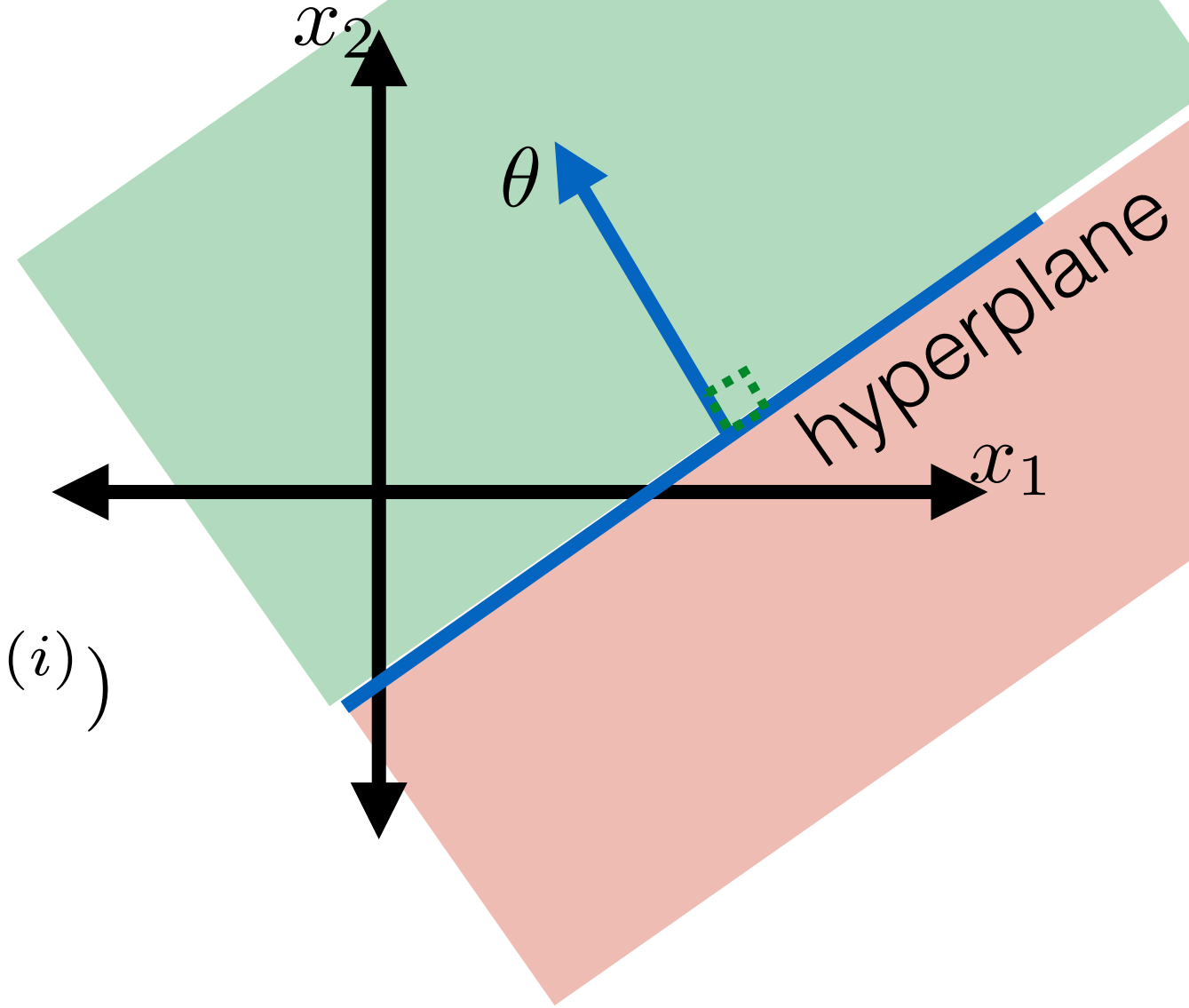
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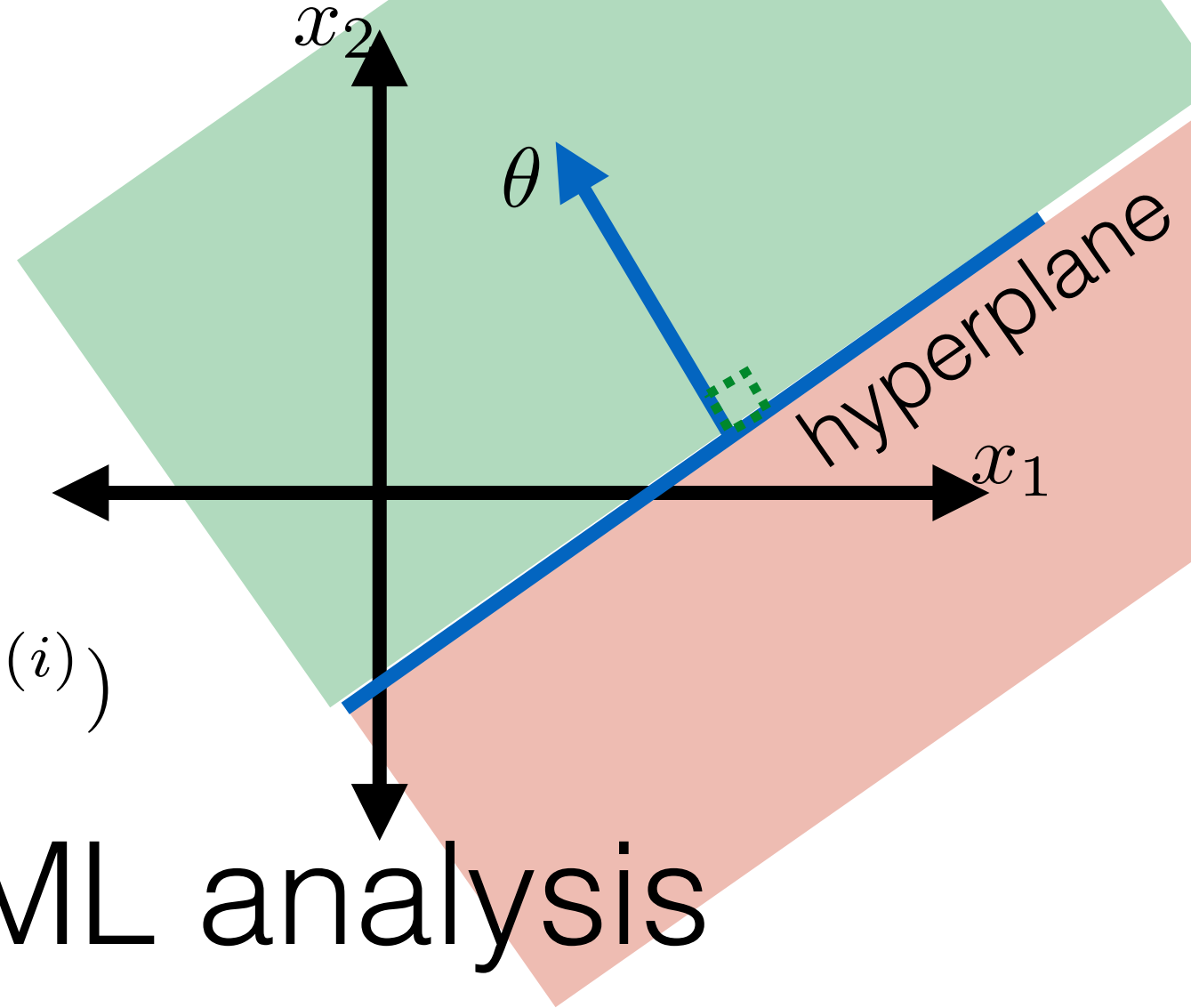
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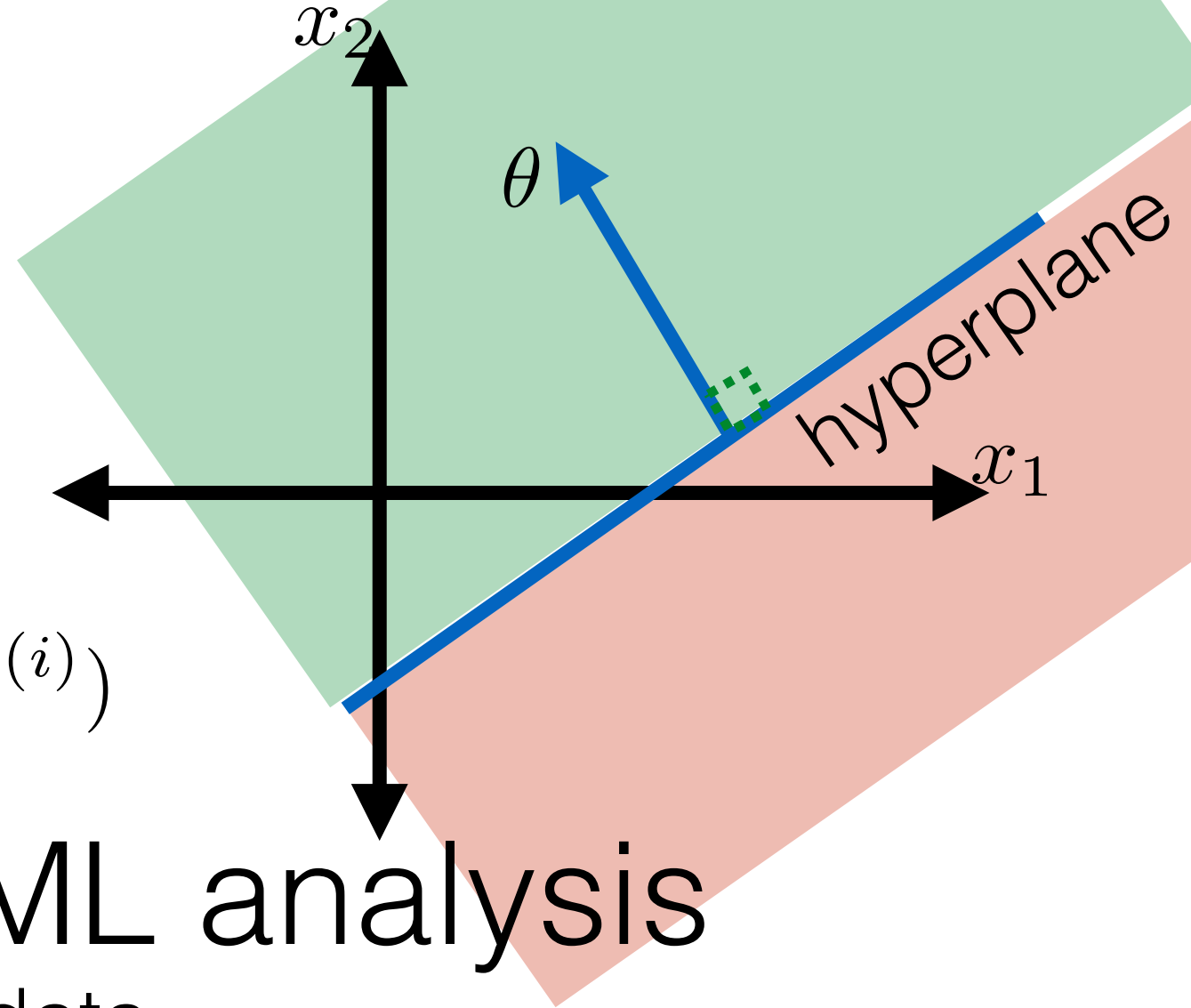


A more-complete ML analysis

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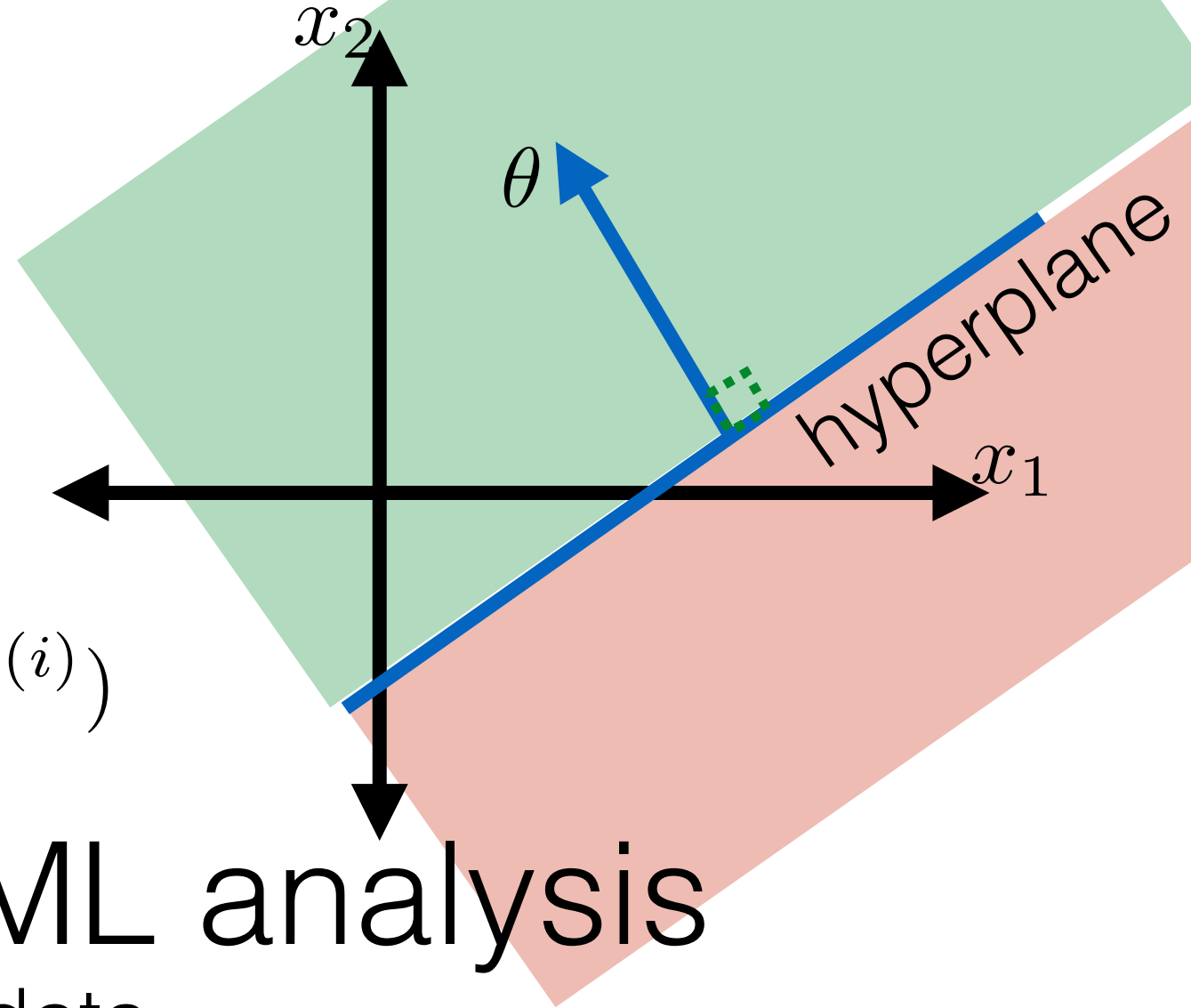
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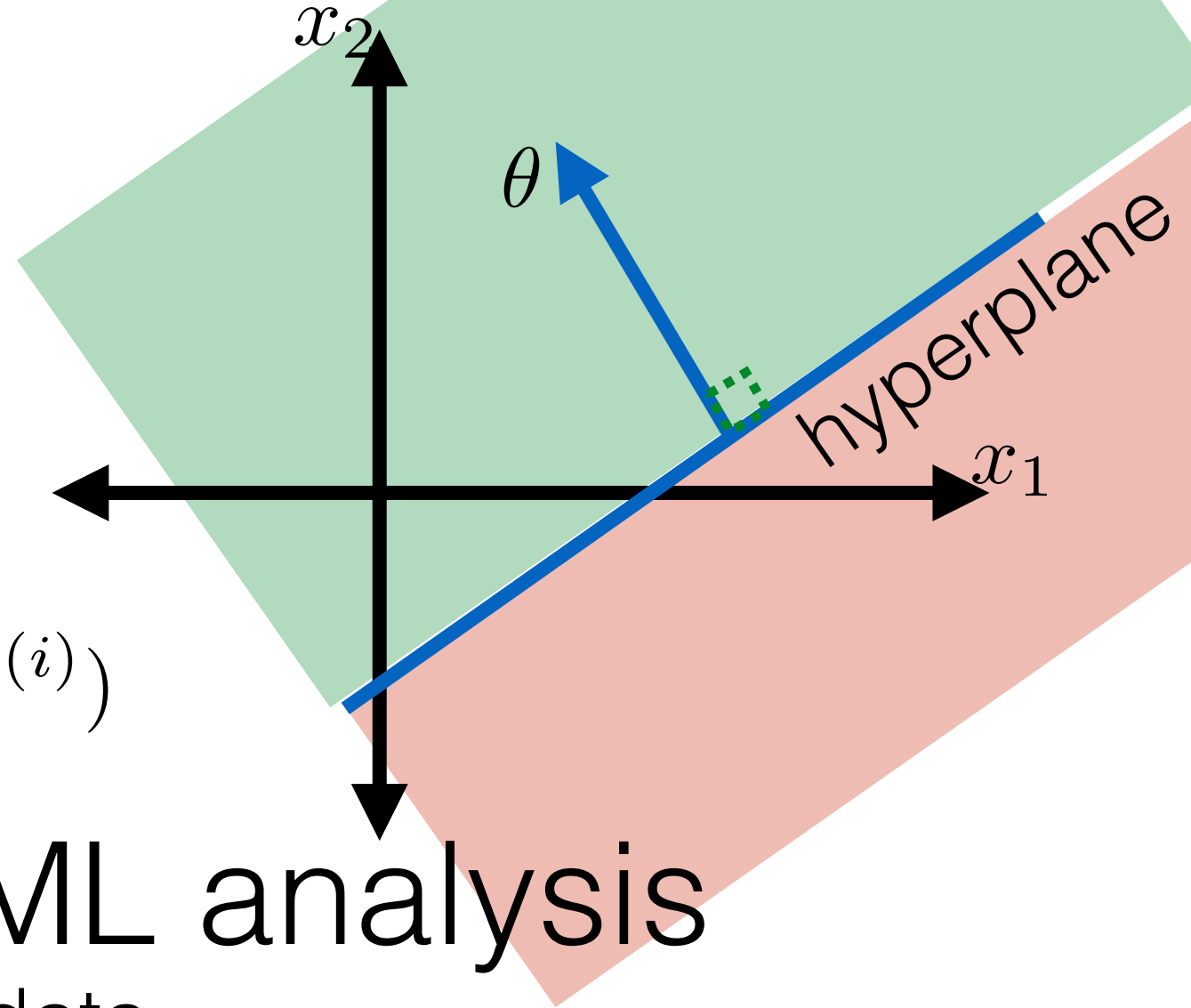
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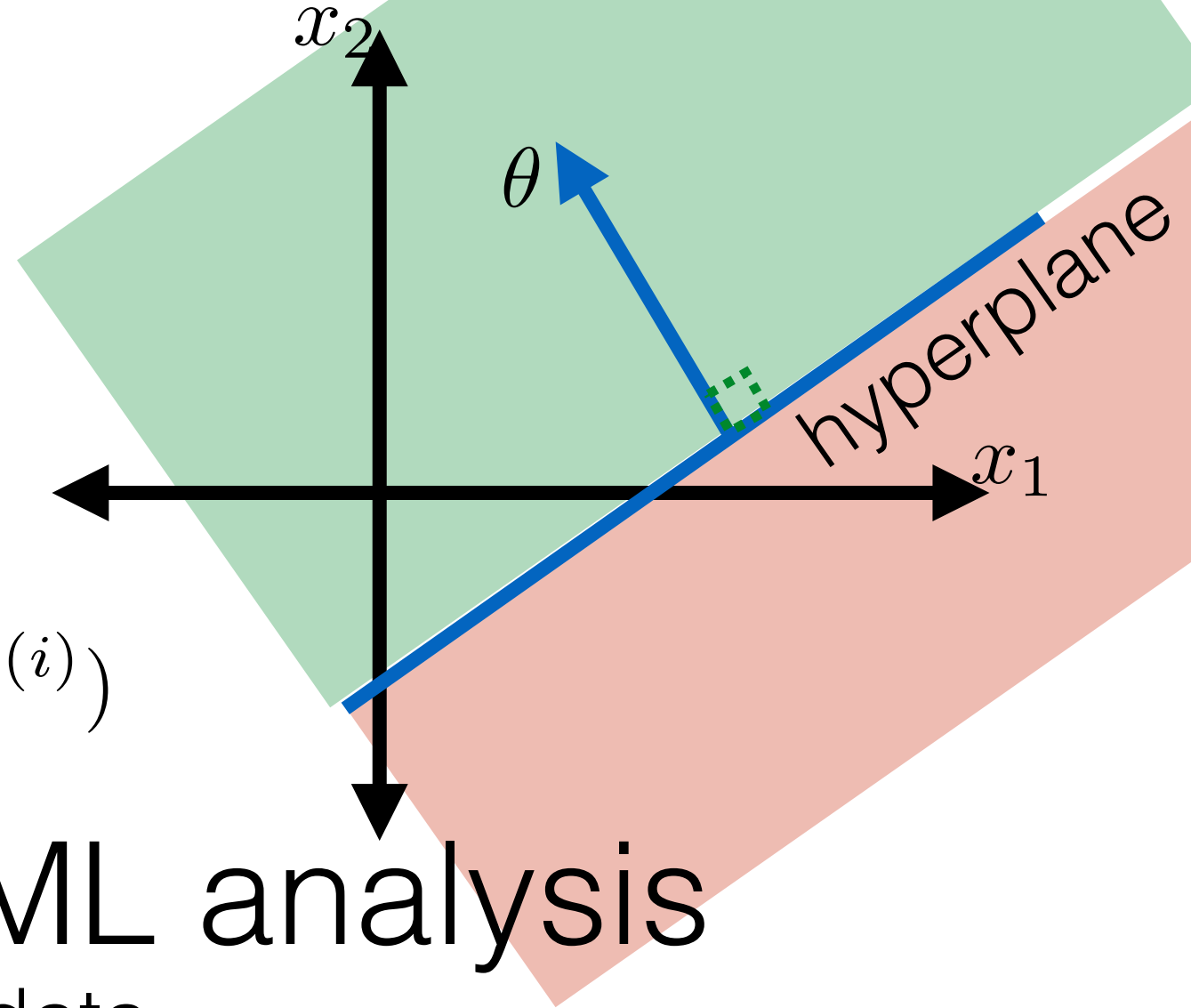
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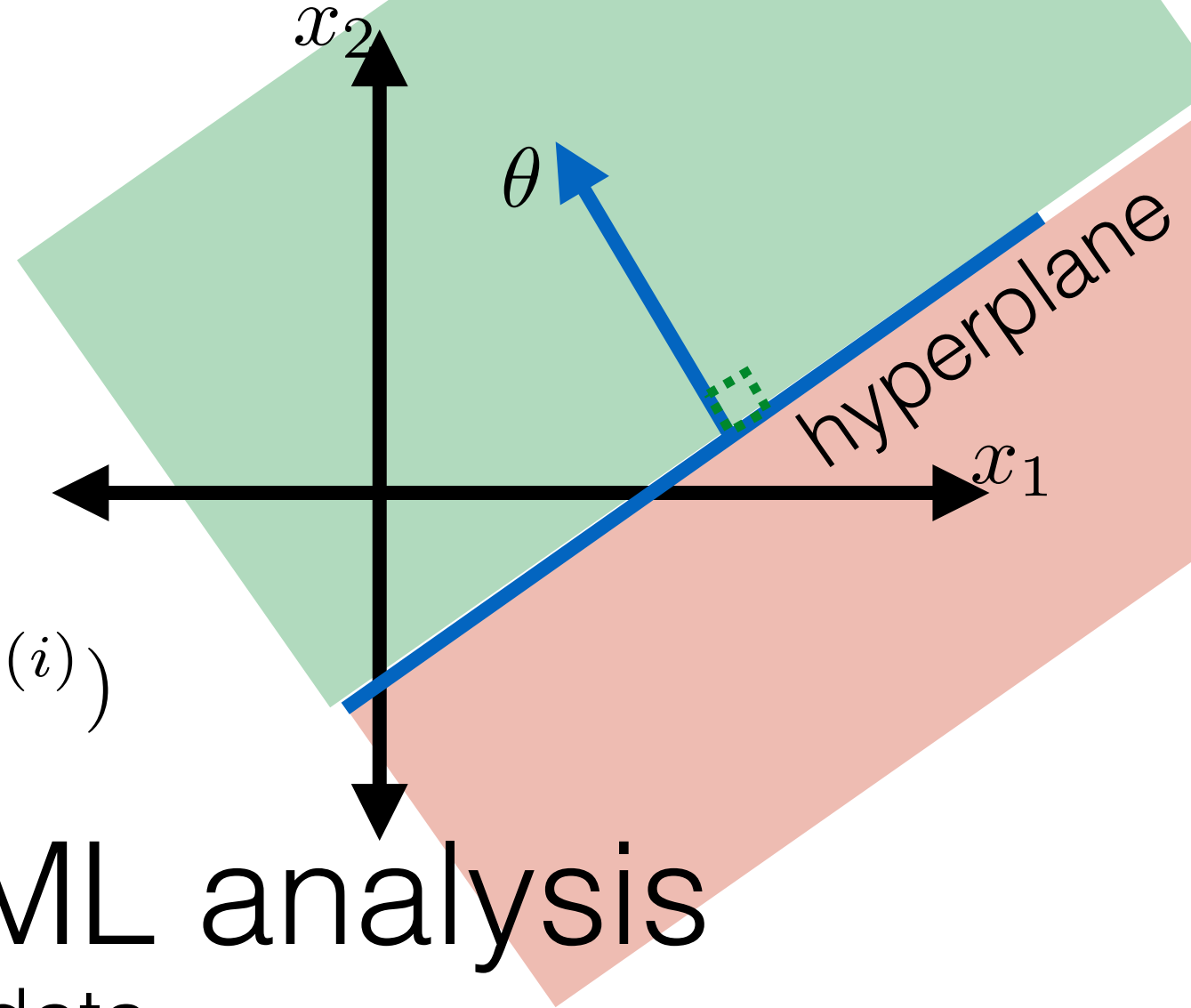
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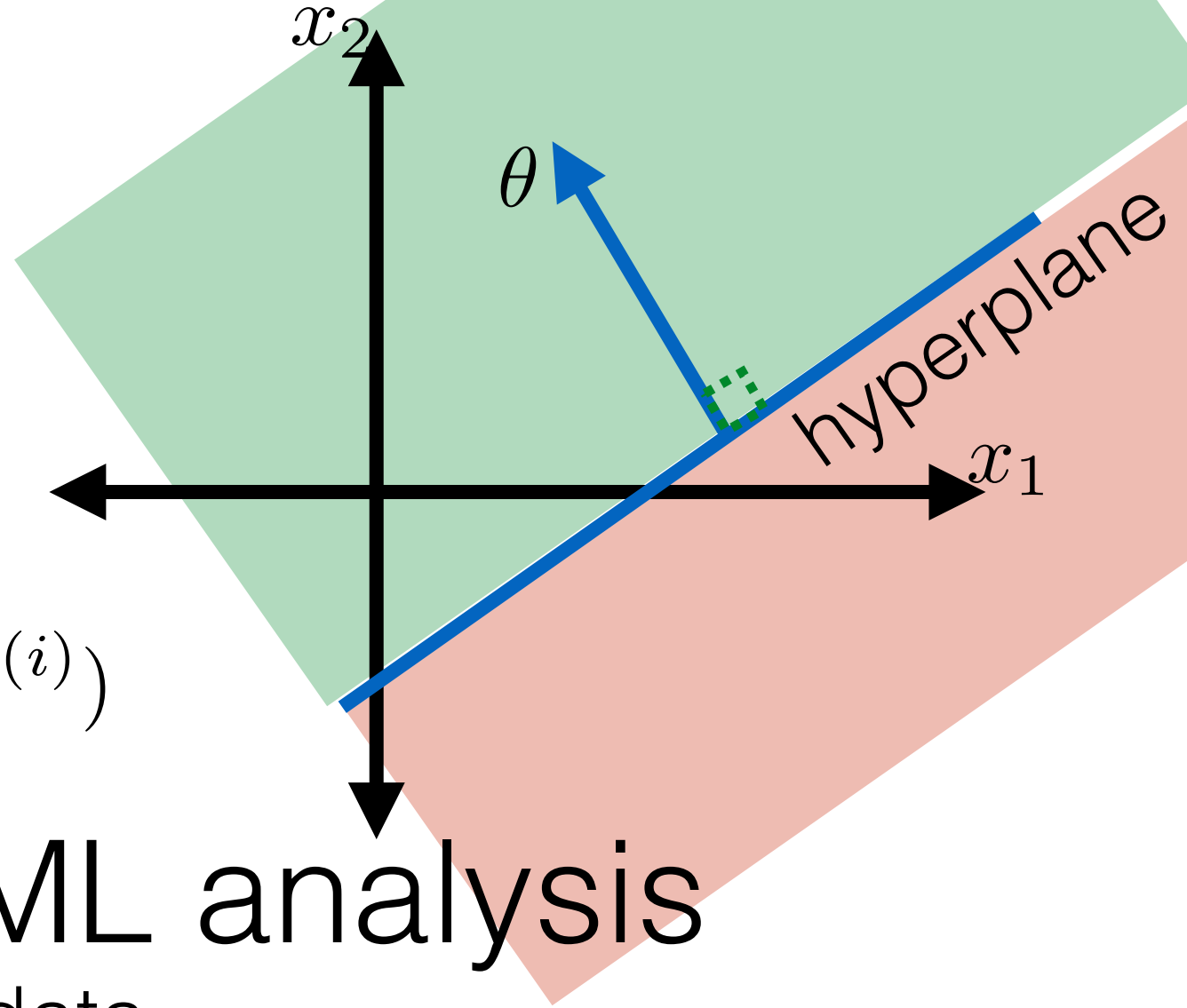
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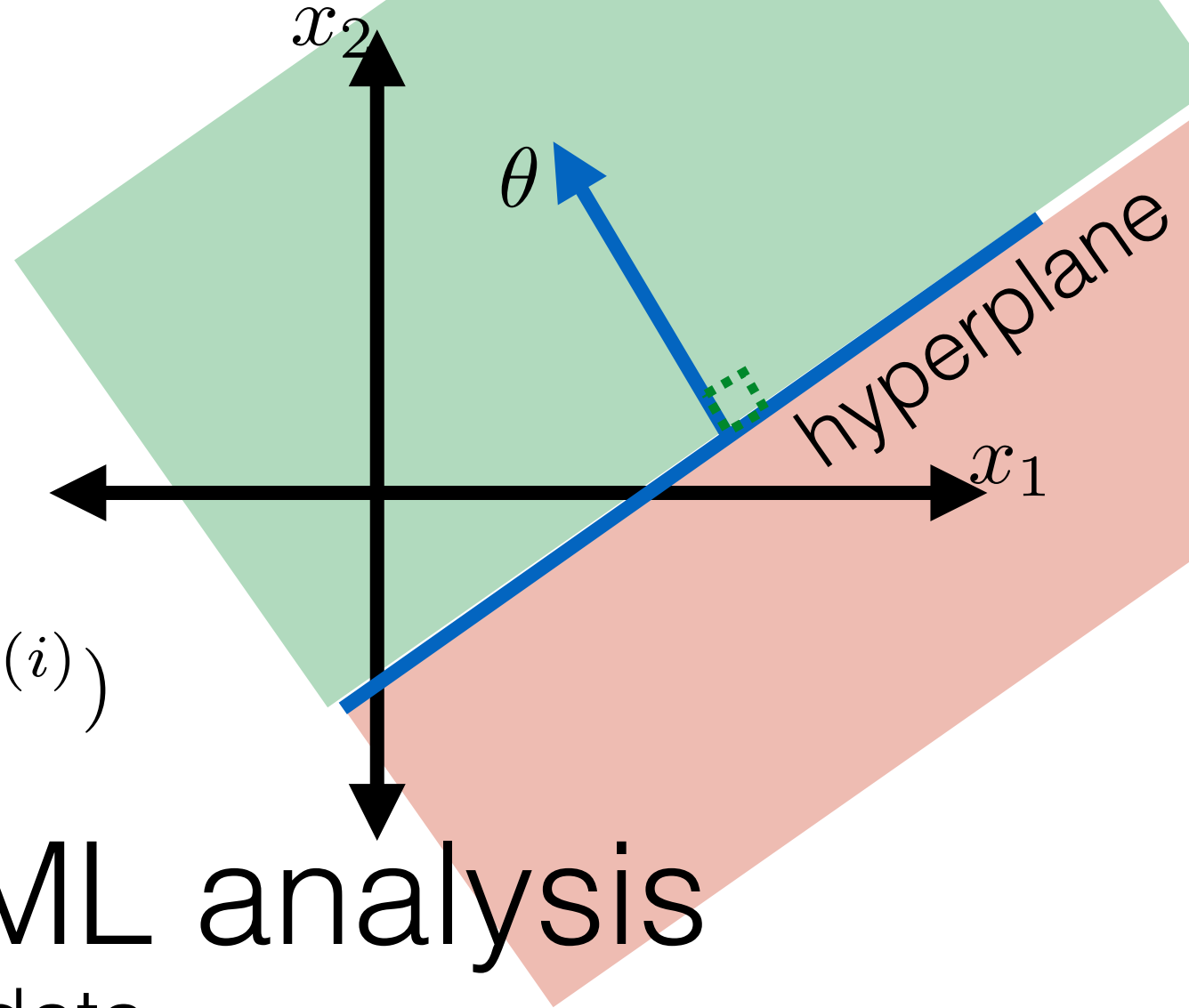
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4. Interpretation & evaluation

A machine learning (ML) analysis

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**has heart
disease?**

1

no

2

no

3

yes

4

no

Encode data in usable form

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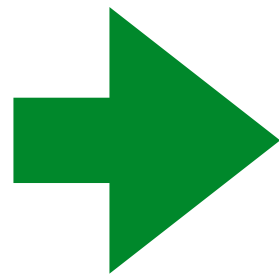
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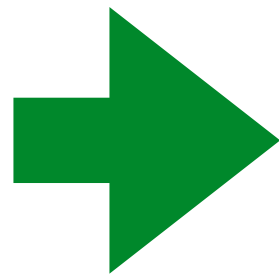


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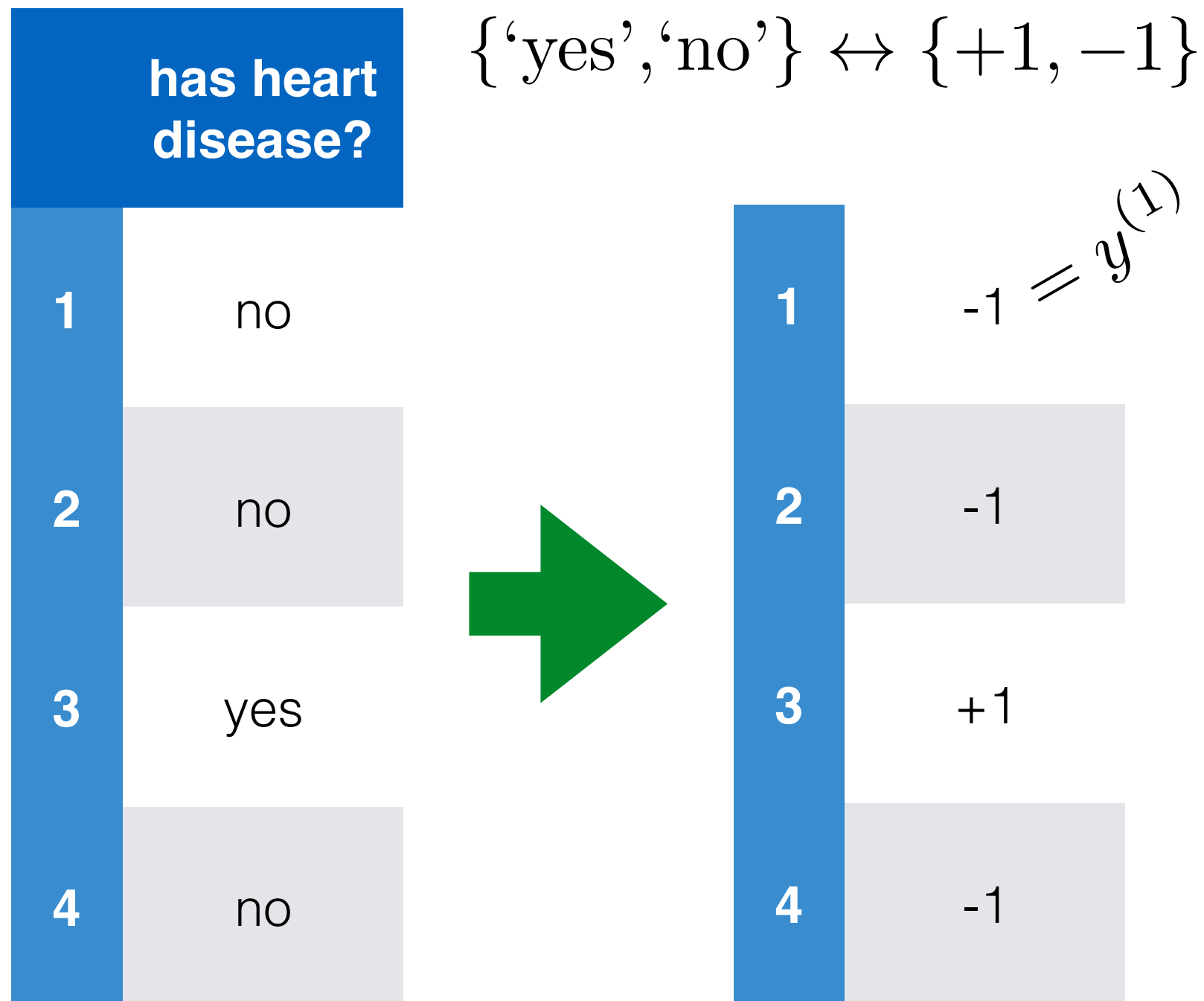
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1	-1
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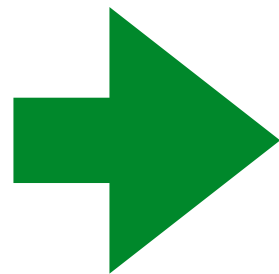


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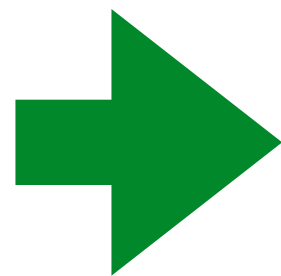
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- Save mapping to recover predictions of new points

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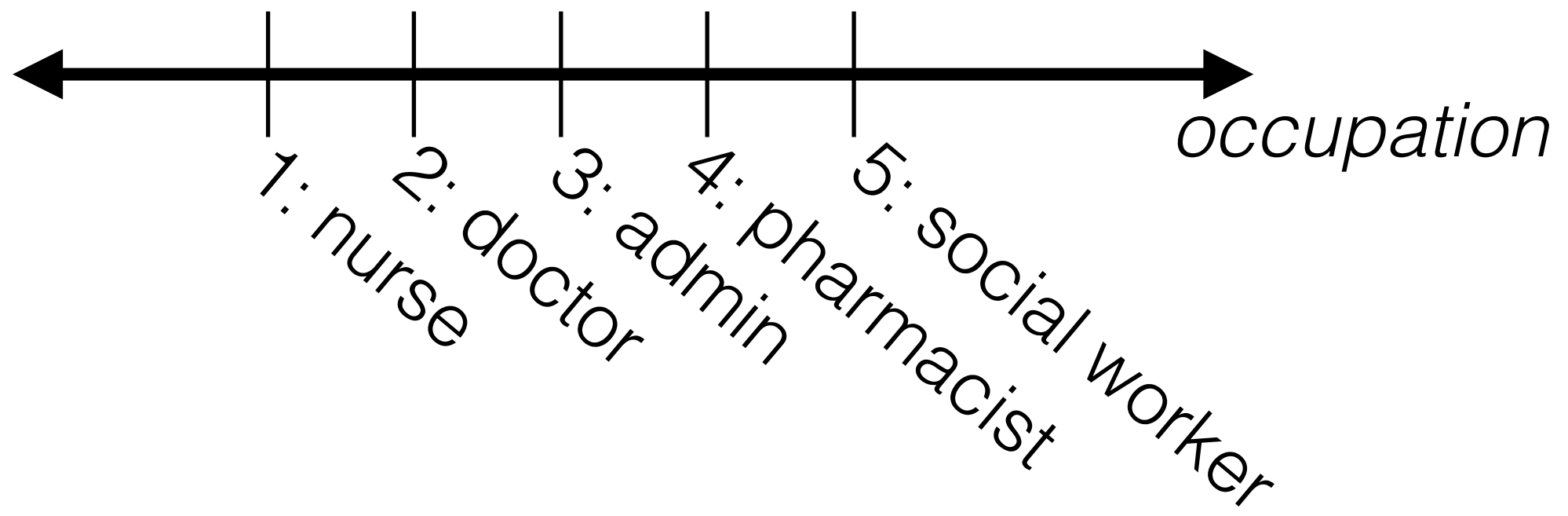
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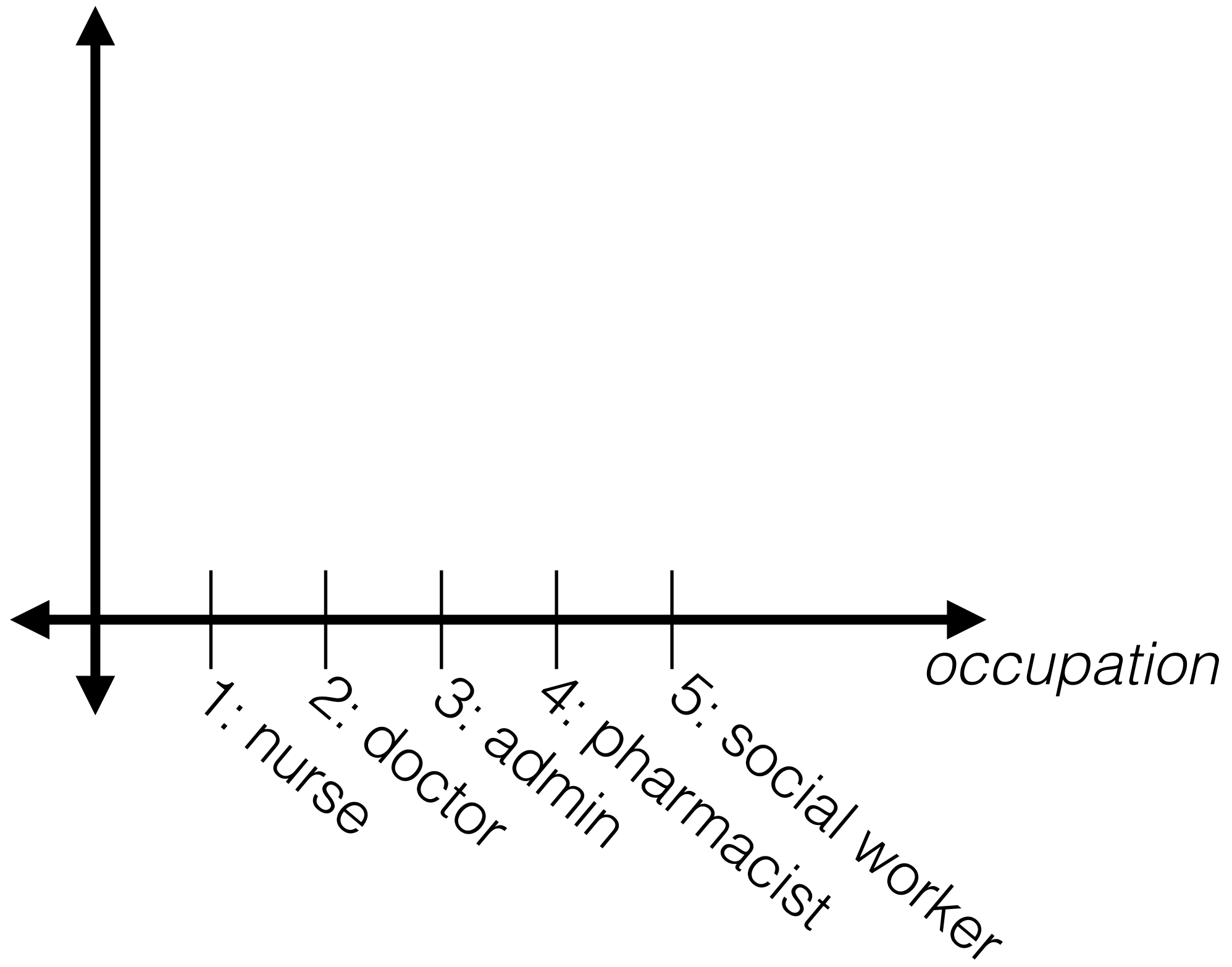
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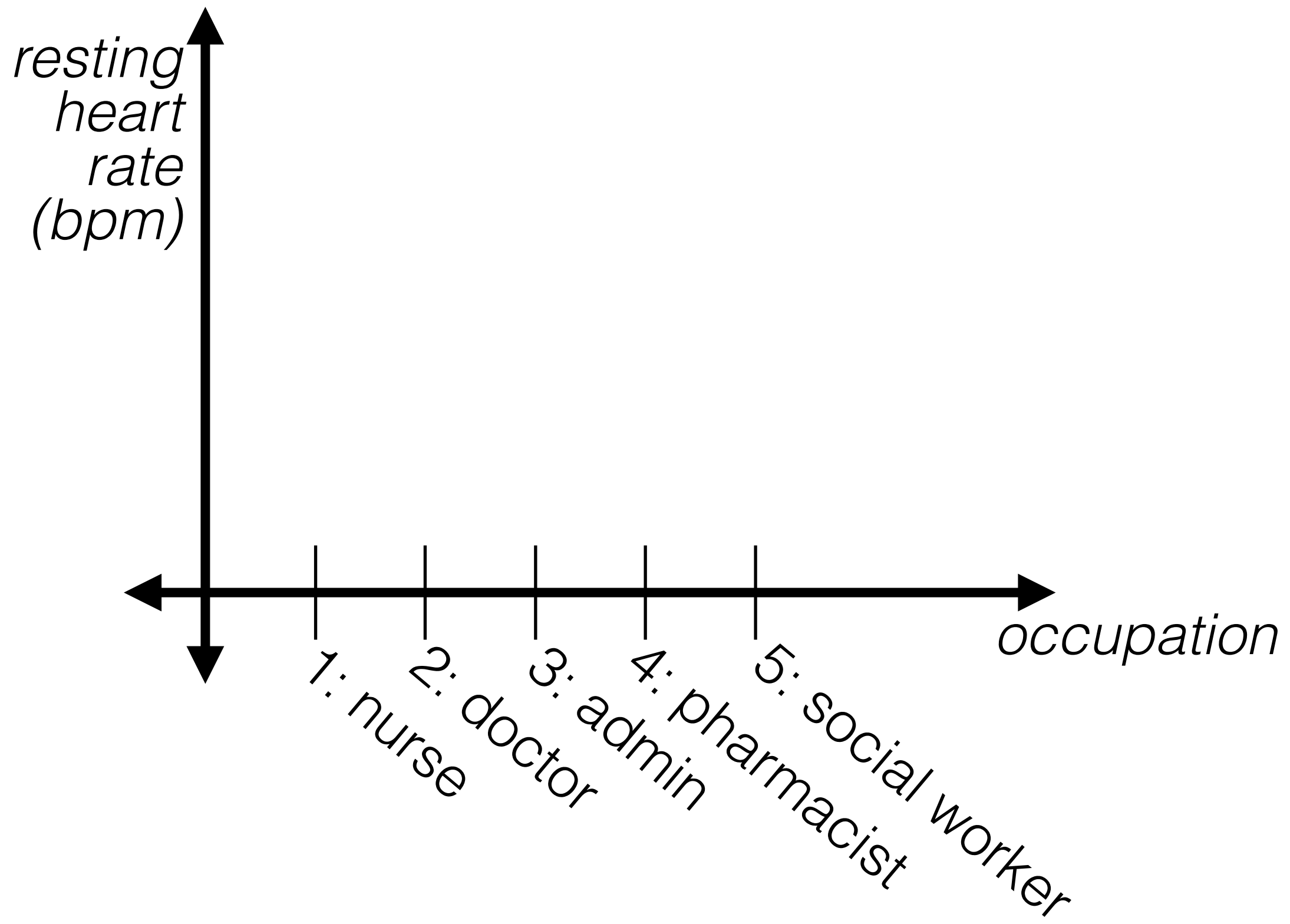
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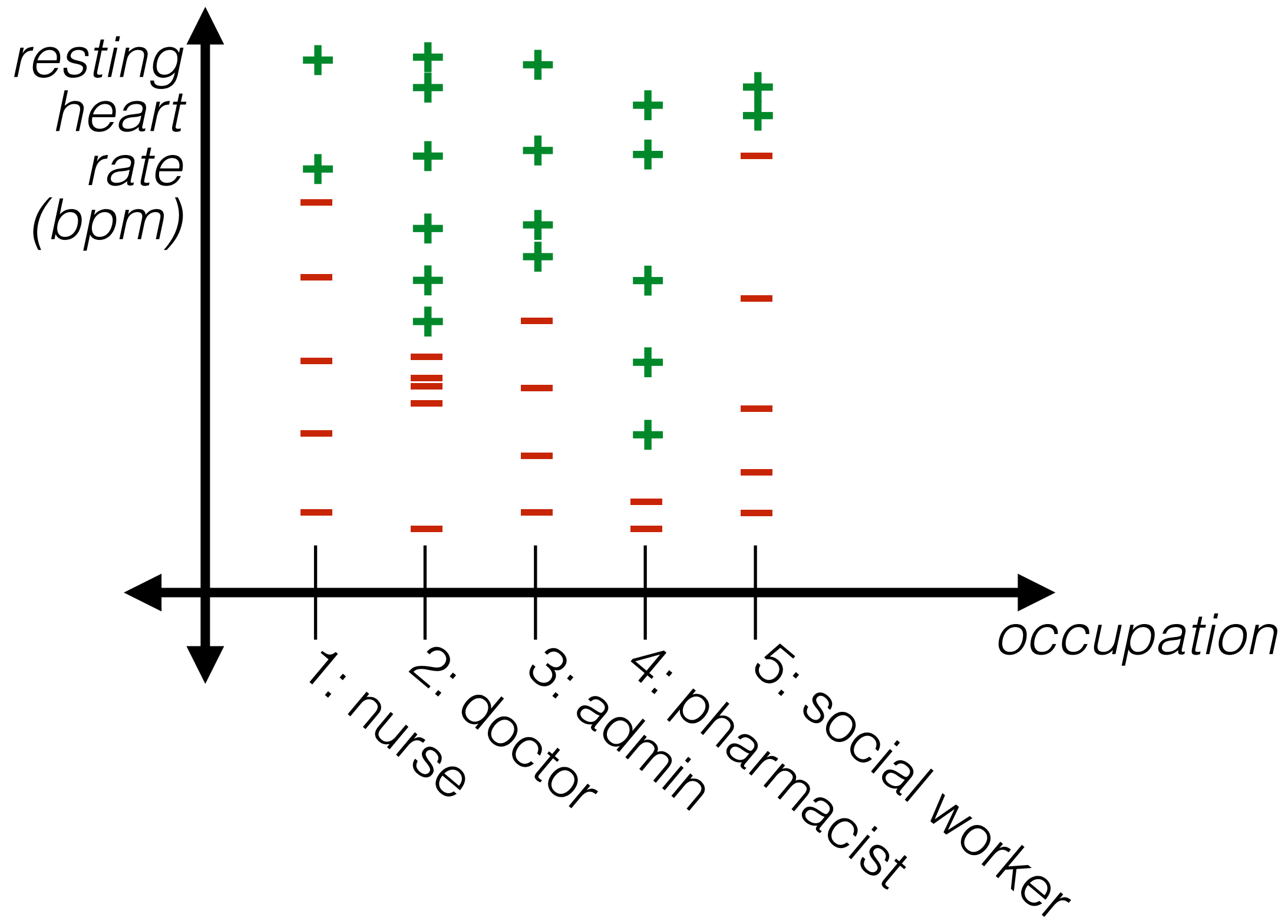
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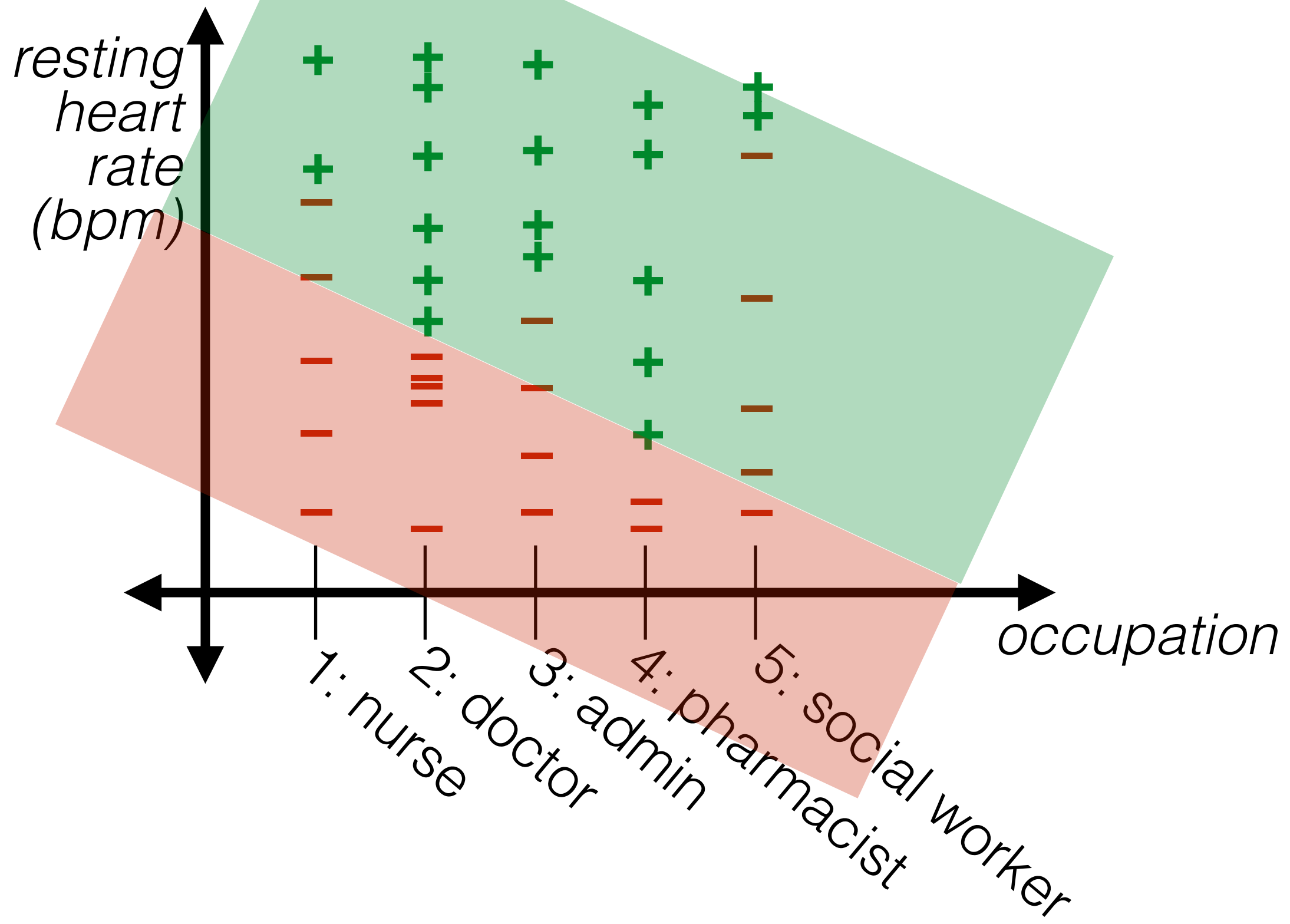
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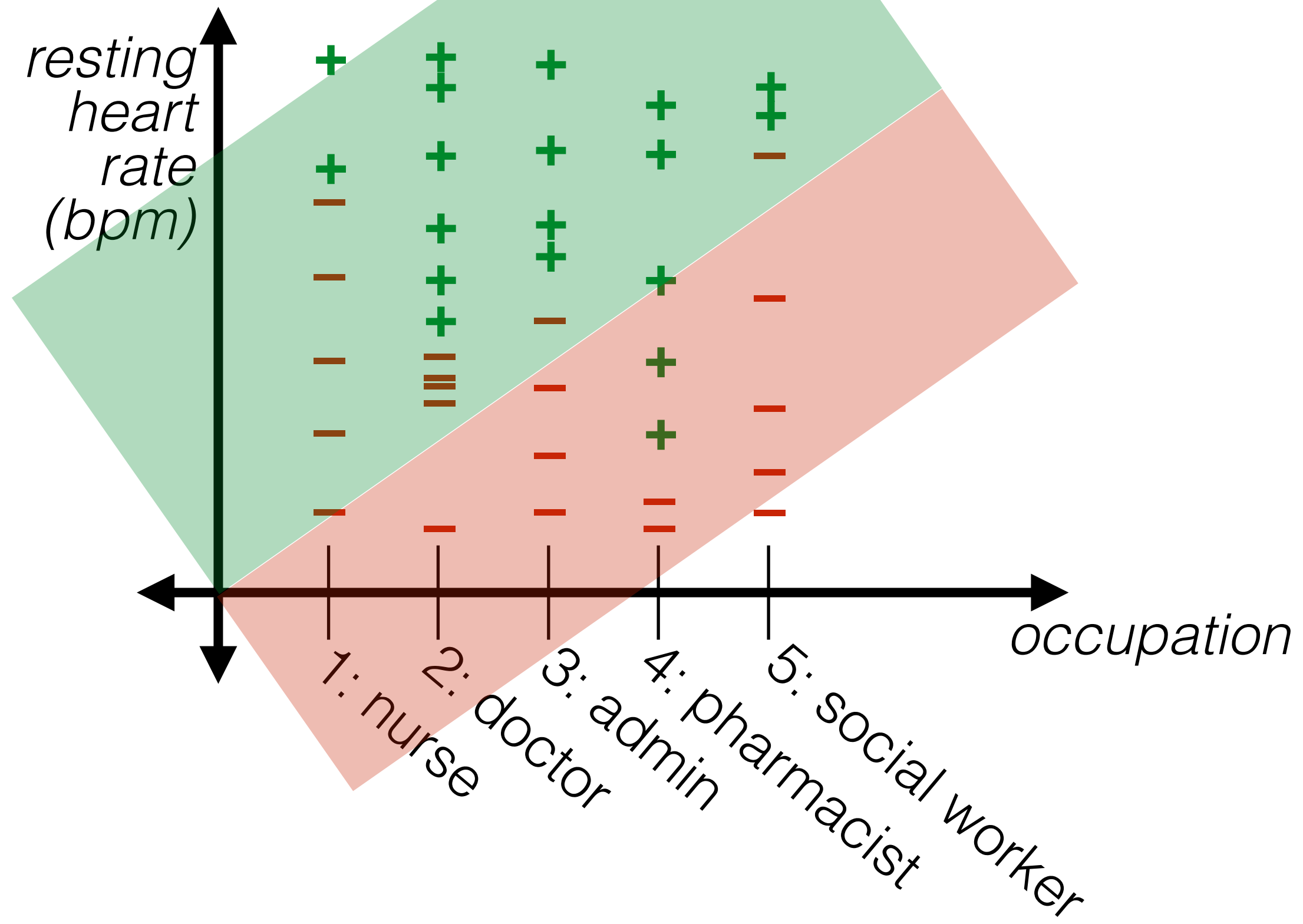
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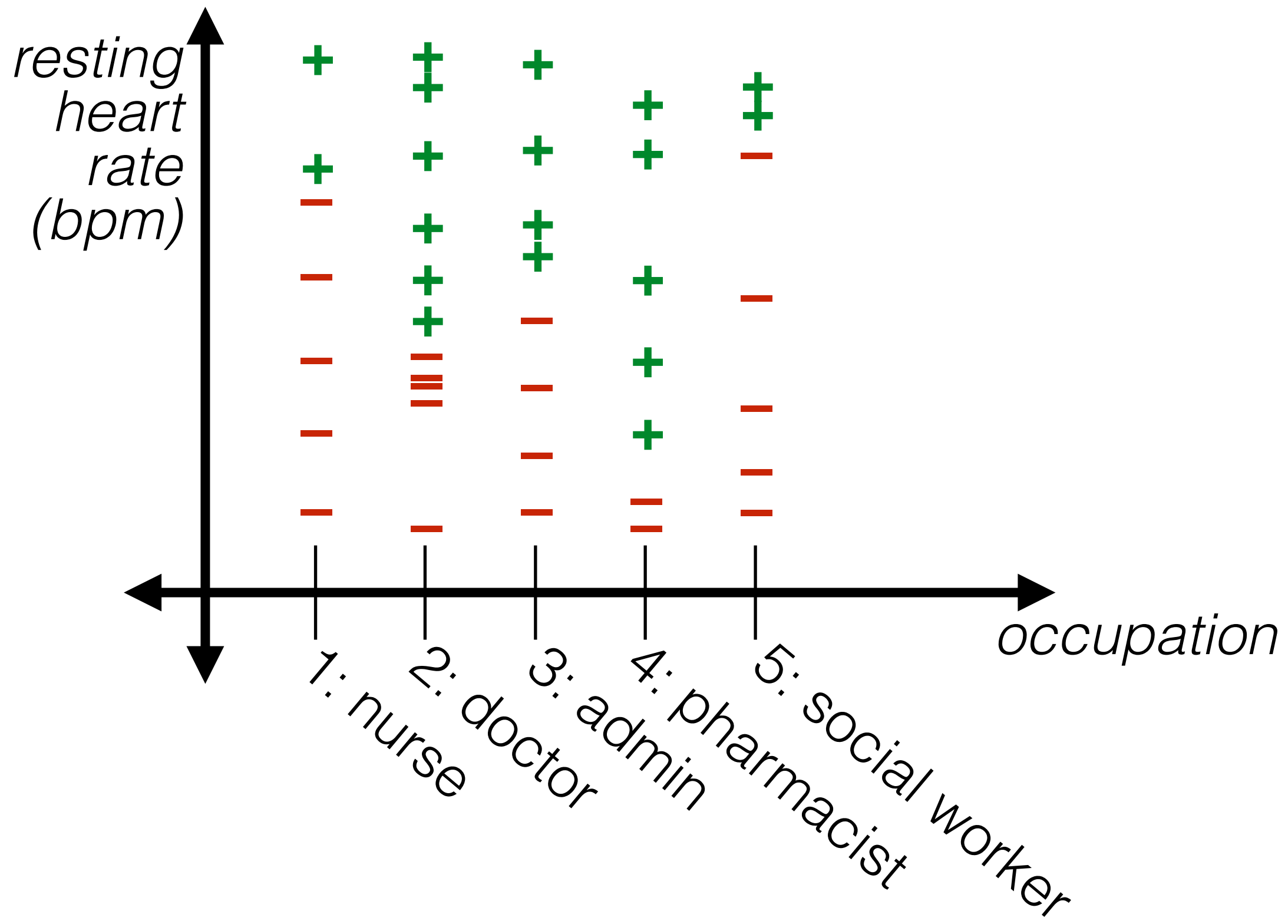
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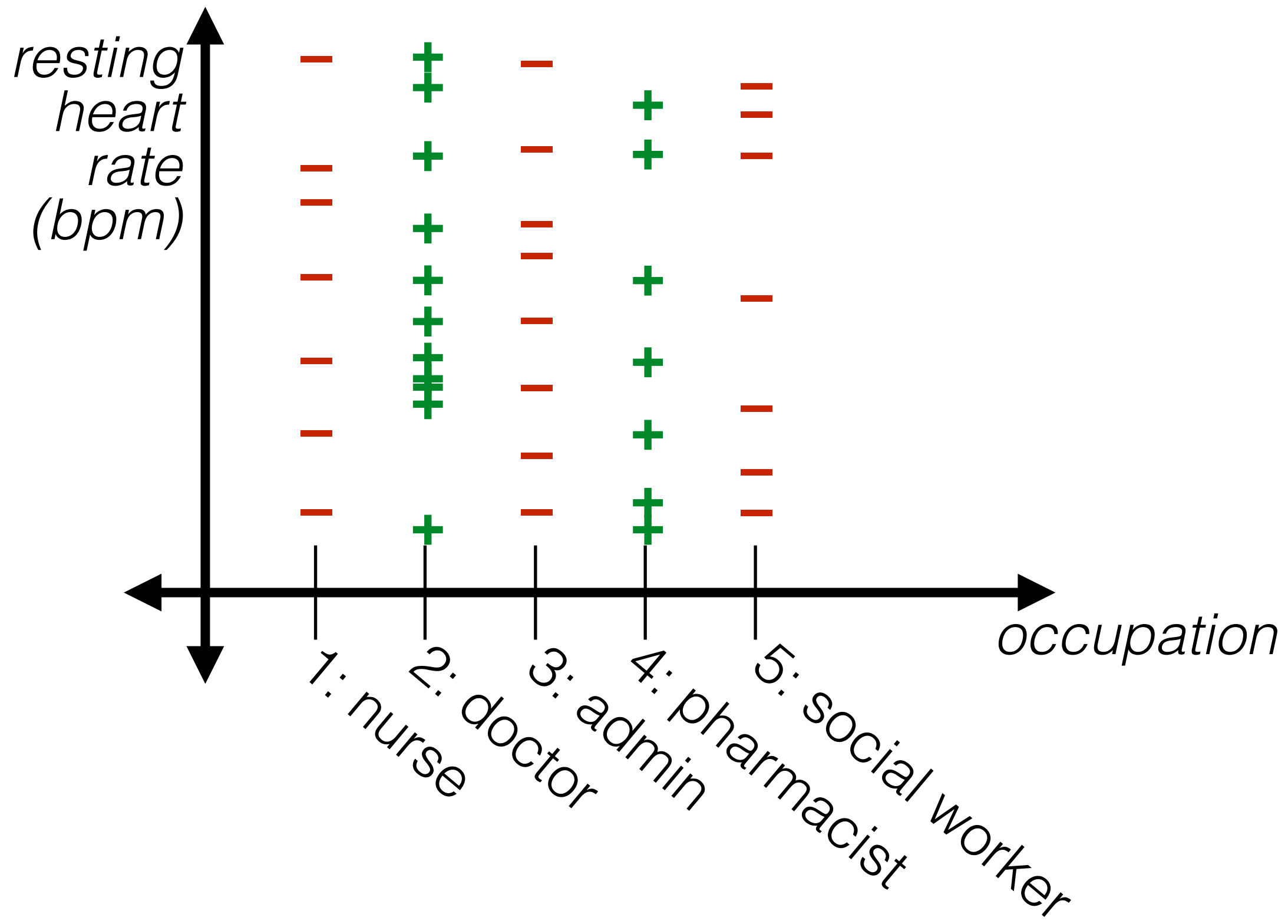
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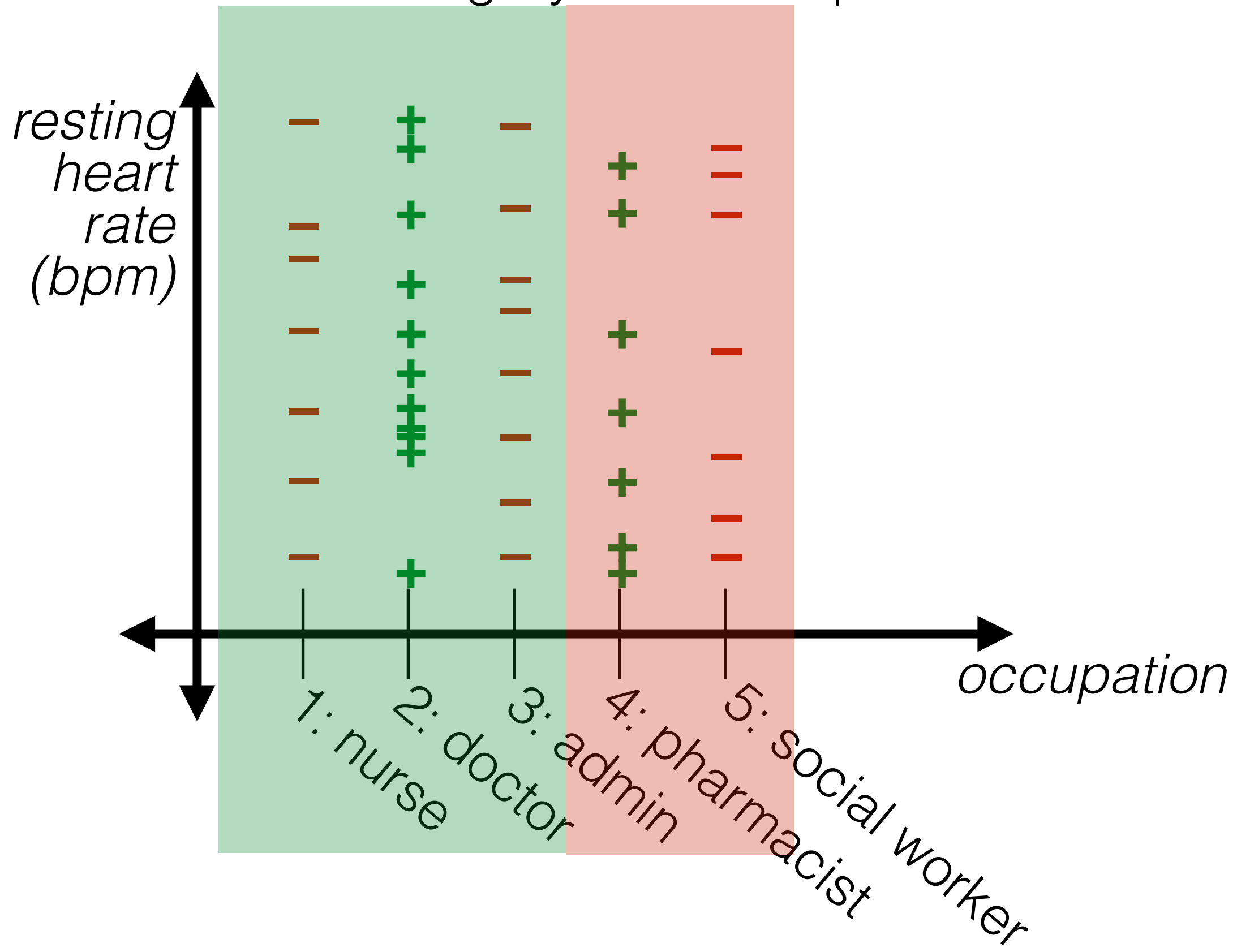
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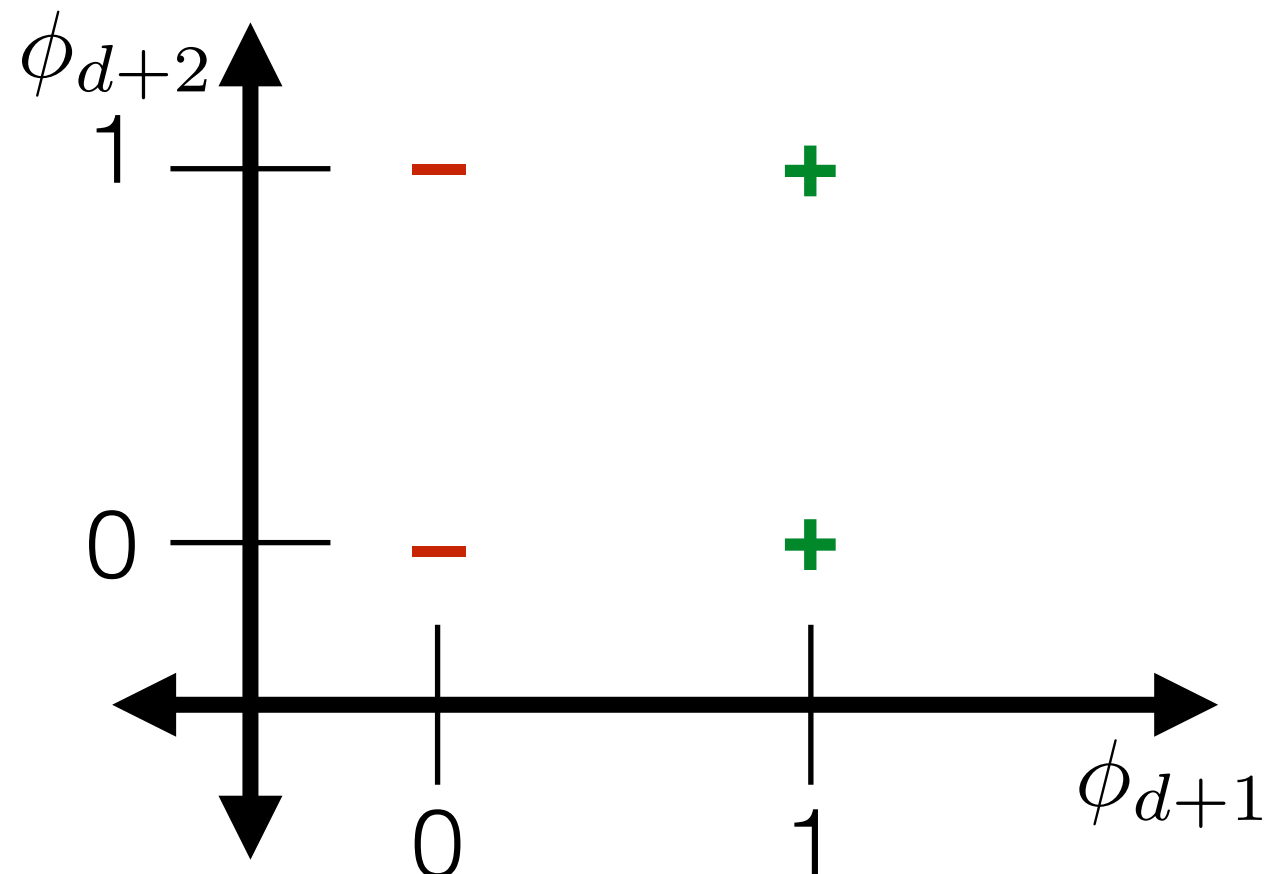
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	ϕ_d	ϕ_{d+1}	ϕ_{d+2}
nurse	0	0	0
admin	0	0	1
pharmacist	0	1	0
doctor	0	1	1
social worker	1	0	0

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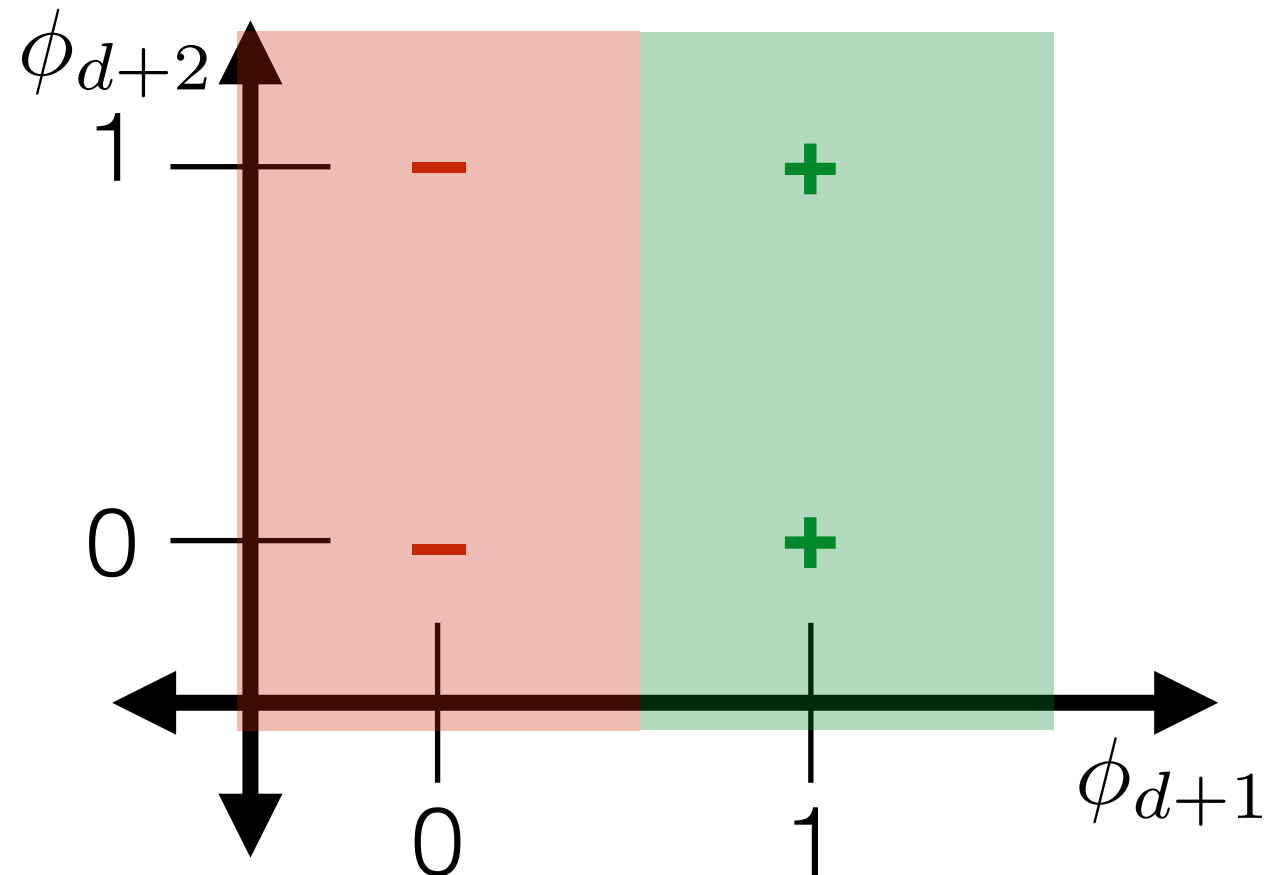
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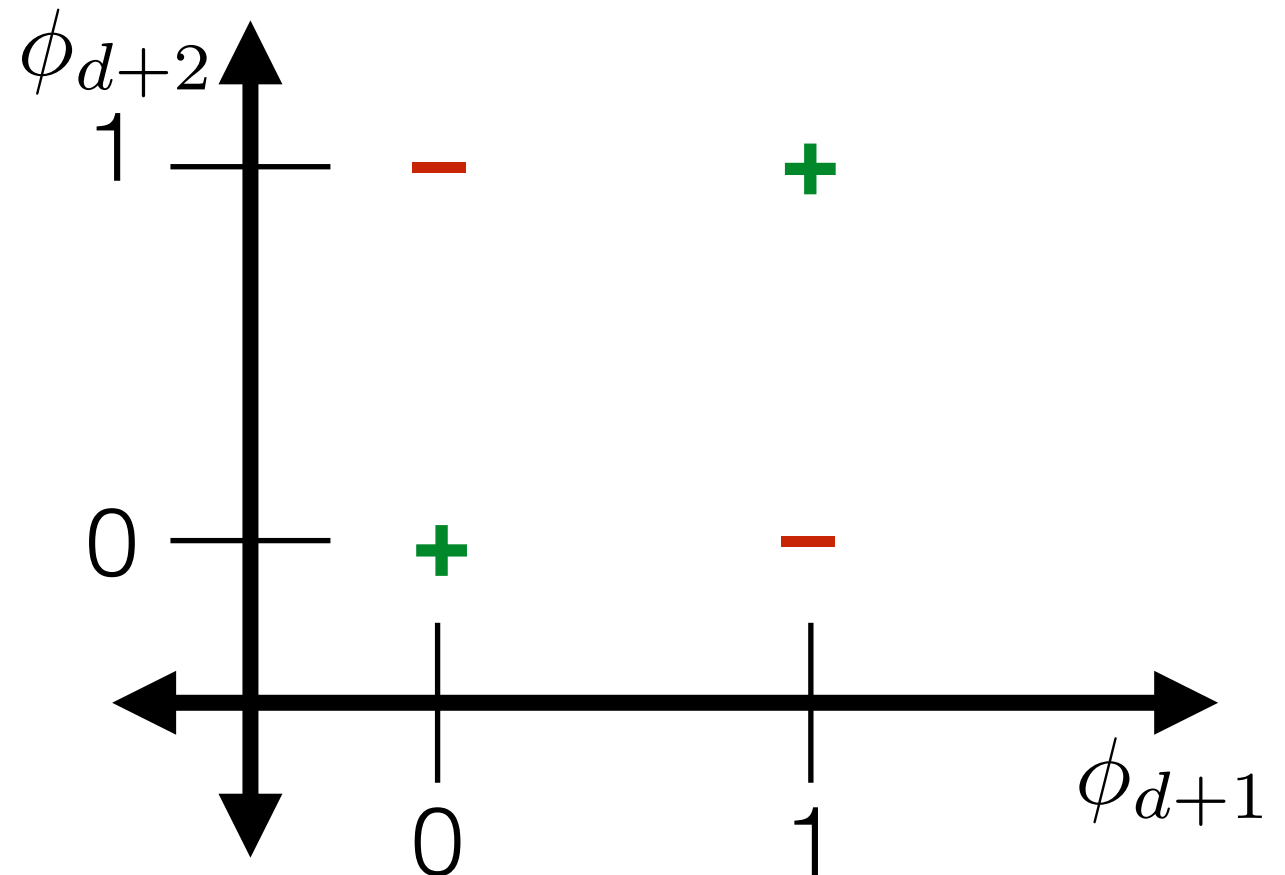
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- Idea: turn each category into own unique 0-1 feature

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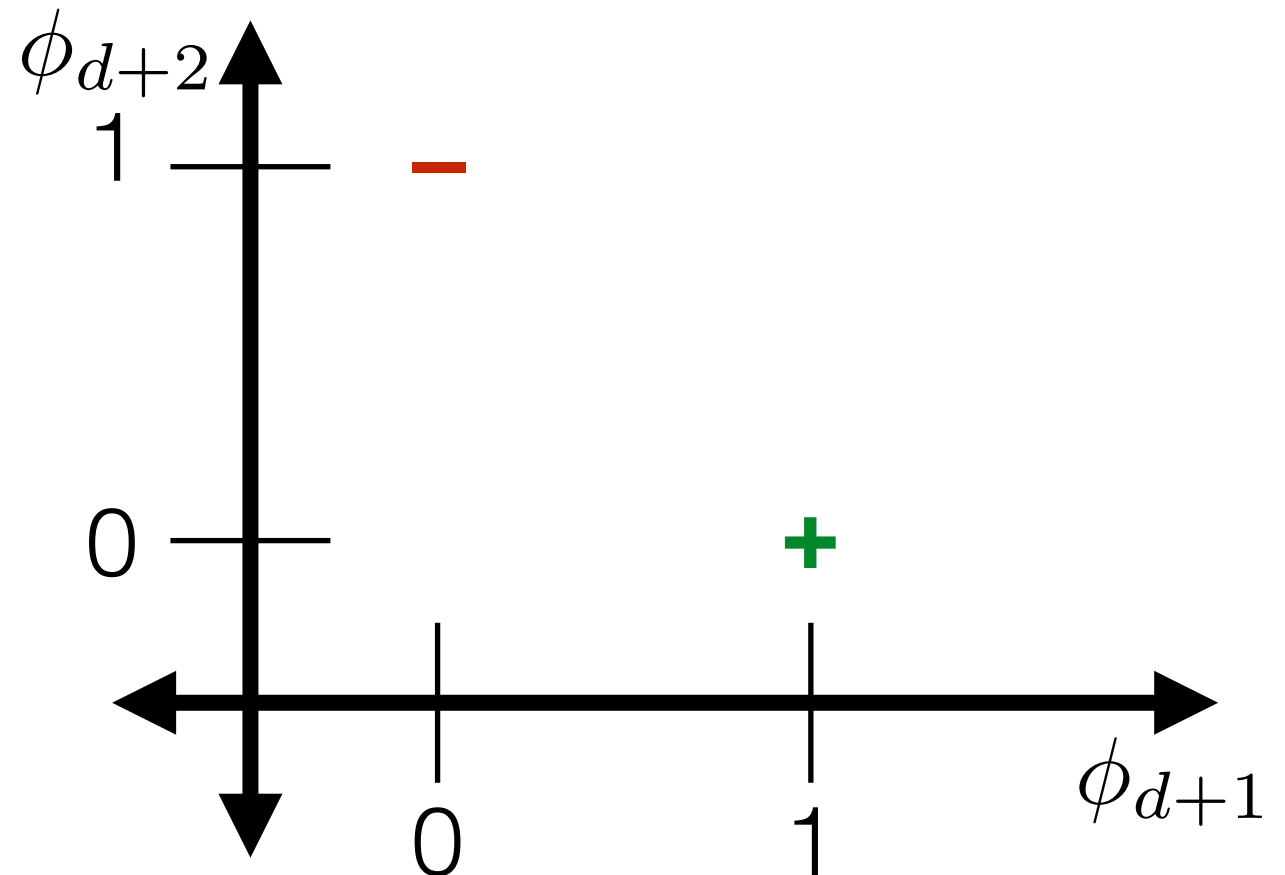
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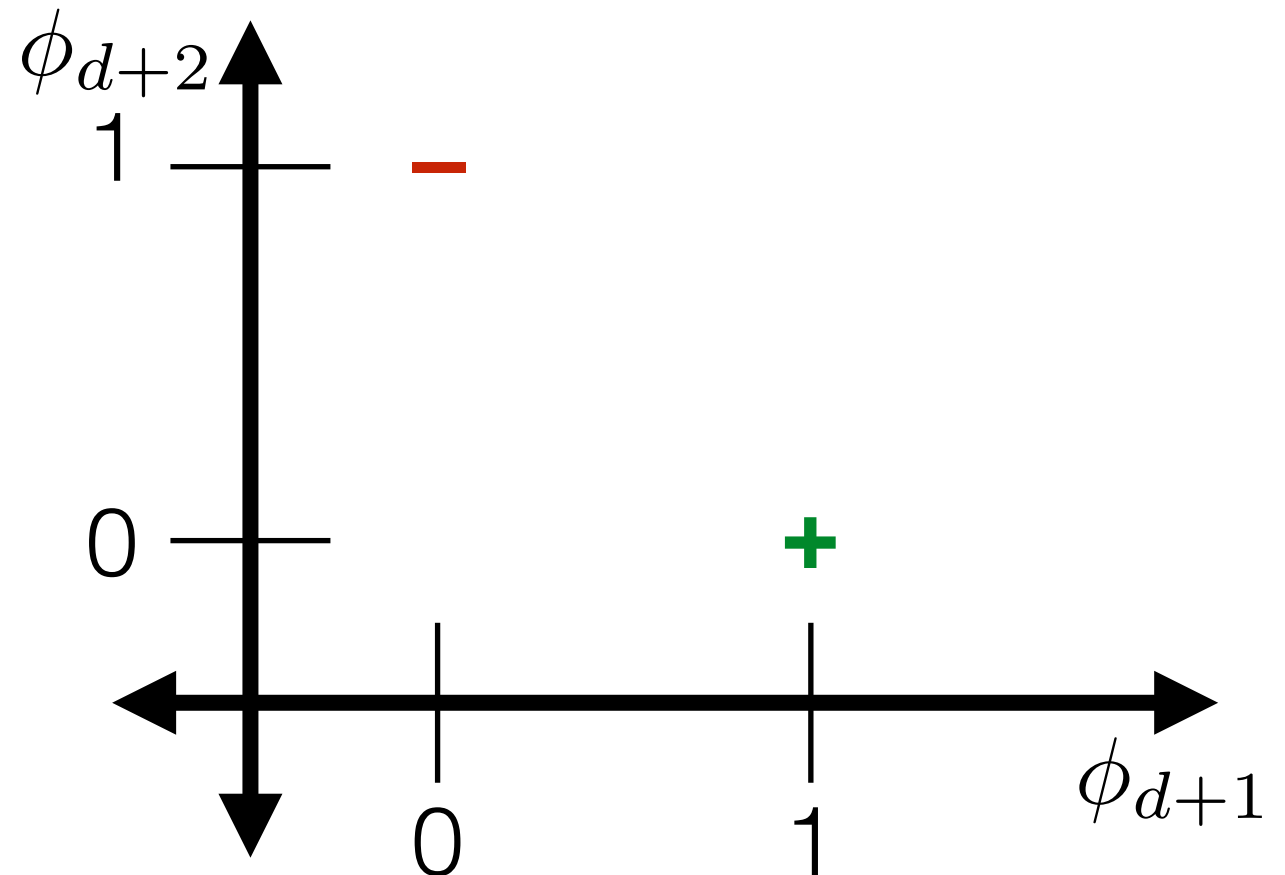


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pharmacist	0	0	1	0	0
doctor	0	0	0	1	0
social worker	0	0	0	0	1

- “one-hot encoding”



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- Identify the features and encode as real numbers

	resting heart rate (bpm)	pain?	job	medicines	age	family income (USD)
1	55	0	nurse	pain	40s	133000
2	71	0	admin	beta blockers, pain	20s	34000
3	89	1	nurse	beta blockers	50s	40000
4	67	0	doctor	none	50s	120000

Encode data in usable form

- Identify the features and encode as real numbers

	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	medicines	age	family income (USD)
1	55	0	1,0,0,0,0	pain	40s	133000
2	71	0	0,1,0,0,0	beta blockers, pain	20s	34000
3	89	1	1,0,0,0,0	beta blockers	50s	40000
4	67	0	0,0,0,1,0	none	50s	120000

Encode data in usable form

- Identify the features and encode as real numbers

	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	medicines	age	family income (USD)
1	55	0	1,0,0,0,0	pain	40s	133000
2	71	0	0,1,0,0,0	beta blockers, pain	20s	34000
3	89	1	1,0,0,0,0	beta blockers	50s	40000
4	67	0	0,0,0,1,0	none	50s	120000

Encode categorical data

pain
pain & beta blockers
beta blockers
no medications

Encode categorical data

- Should we use one-hot encoding?

pain
pain & beta blockers
beta blockers
no medications

Encode categorical data

- Should we use one-hot encoding?

	ϕ_d	ϕ_{d+1}	ϕ_{d+2}	ϕ_{d+3}
pain	1	0	0	0
pain & beta blockers	0	1	0	0
beta blockers	0	0	1	0
no medications	0	0	0	1

Encode categorical data

- Should we use one-hot encoding?

	ϕ_d	ϕ_{d+1}	ϕ_{d+2}	ϕ_{d+3}
pain	1	0	0	0
pain & beta blockers	0	1	0	0
beta blockers	0	0	1	0
no medications	0	0	0	1

- Idea: factored encoding

Encode categorical data

- Should we use one-hot encoding?

	ϕ_d	ϕ_{d+1}	ϕ_{d+2}	ϕ_{d+3}
pain	1	0	0	0
pain & beta blockers	0	1	0	0
beta blockers	0	0	1	0
no medications	0	0	0	1

- Idea: factored encoding

	ϕ_d	ϕ_{d+1}
pain	1	0
pain & beta blockers	1	1
beta blockers	0	1
no medications	0	0

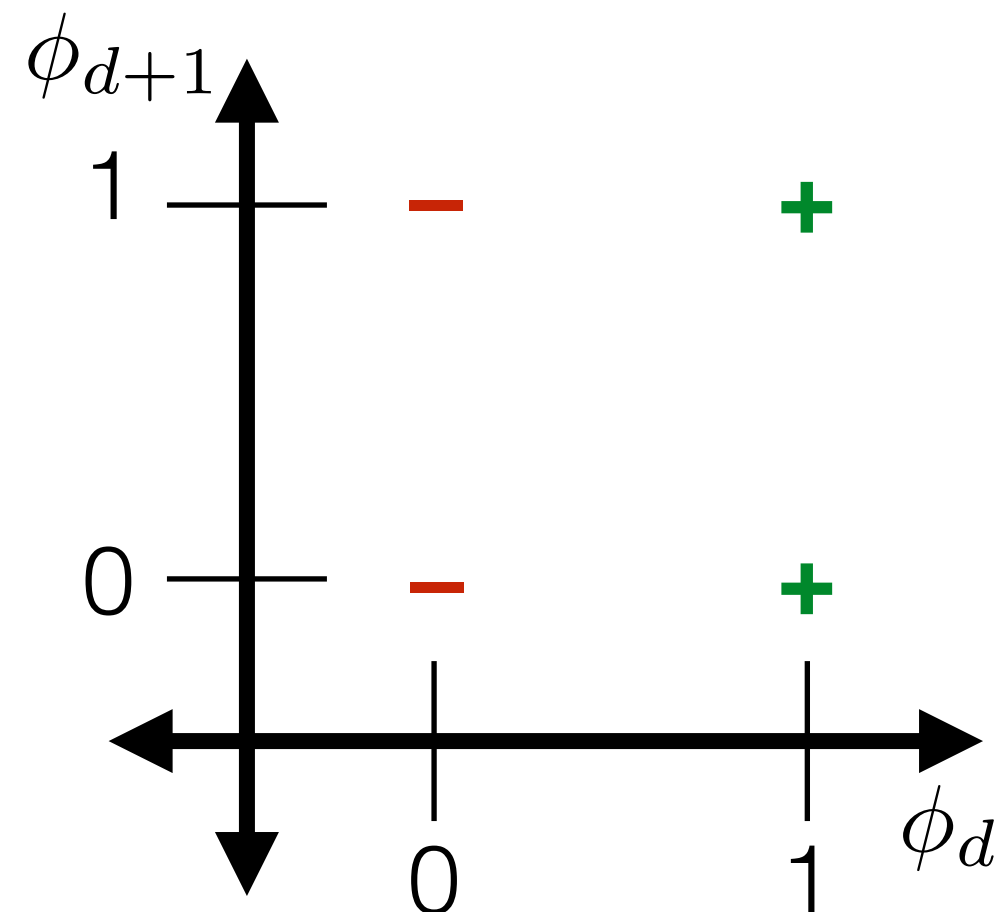
Encode categorical data

- Should we use one-hot encoding?

	ϕ_d	ϕ_{d+1}	ϕ_{d+2}	ϕ_{d+3}
pain	1	0	0	0
pain & beta blockers	0	1	0	0
beta blockers	0	0	1	0
no medications	0	0	0	1

- Idea: factored encoding

	ϕ_d	ϕ_{d+1}
pain	1	0
pain & beta blockers	1	1
beta blockers	0	1
no medications	0	0



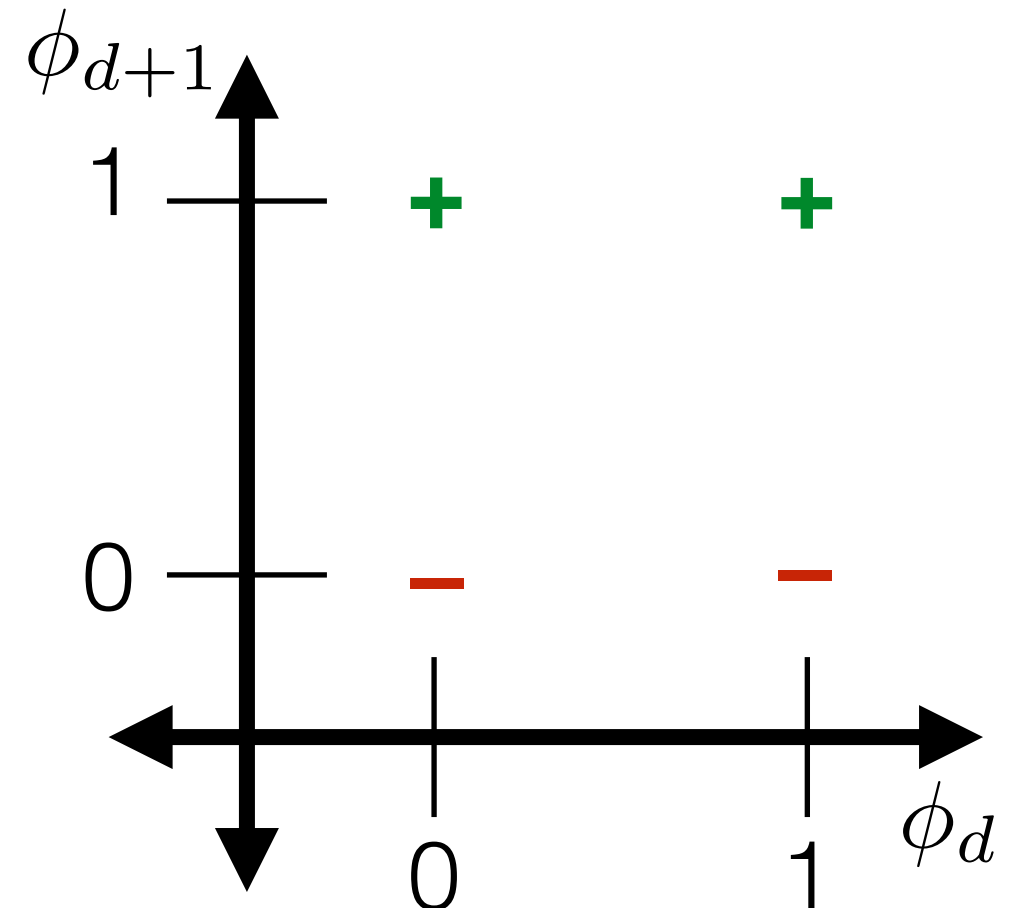
Encode categorical data

- Should we use one-hot encoding?

	ϕ_d	ϕ_{d+1}	ϕ_{d+2}	ϕ_{d+3}
pain	1	0	0	0
pain & beta blockers	0	1	0	0
beta blockers	0	0	1	0
no medications	0	0	0	1

- Idea: factored encoding

	ϕ_d	ϕ_{d+1}
pain	1	0
pain & beta blockers	1	1
beta blockers	0	1
no medications	0	0



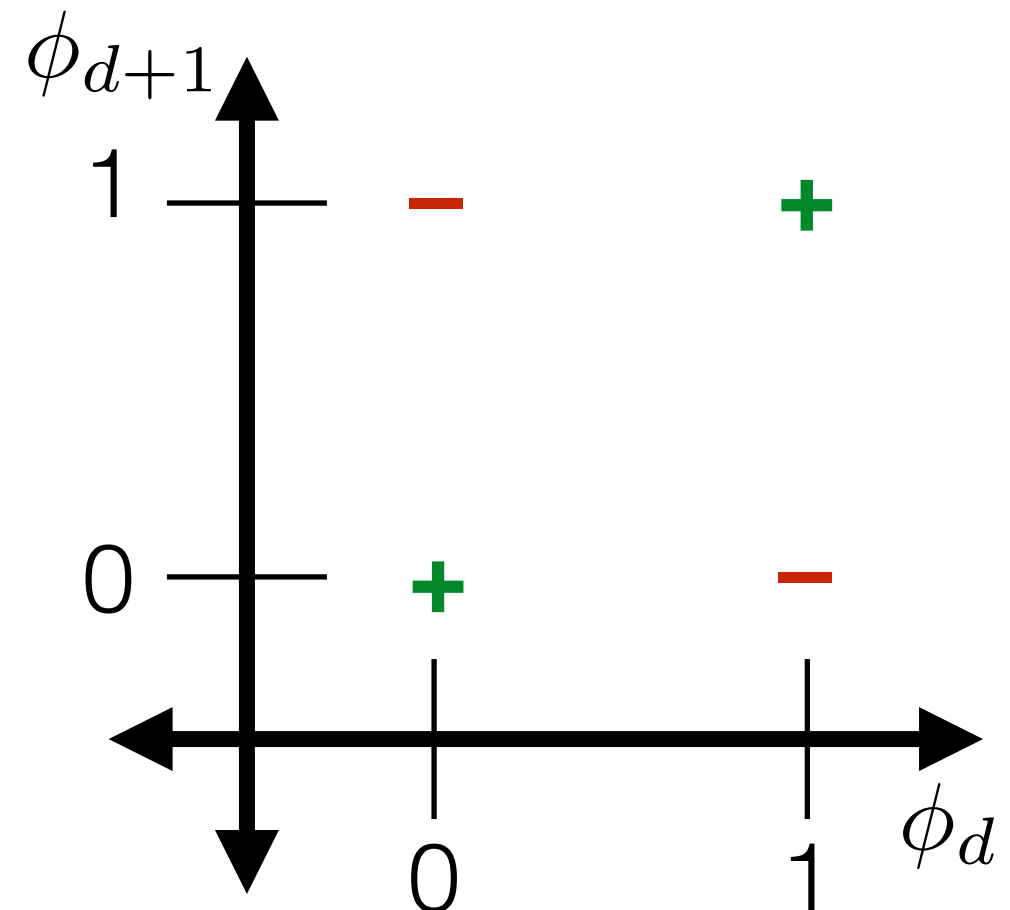
Encode categorical data

- Should we use one-hot encoding?

	ϕ_d	ϕ_{d+1}	ϕ_{d+2}	ϕ_{d+3}
pain	1	0	0	0
pain & beta blockers	0	1	0	0
beta blockers	0	0	1	0
no medications	0	0	0	1

- Idea: factored encoding

	ϕ_d	ϕ_{d+1}
pain	1	0
pain & beta blockers	1	1
beta blockers	0	1
no medications	0	0



Encode data in usable form

- Identify the features and encode as real numbers

	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	medicines	age	family income (USD)
1	55	0	1,0,0,0,0	pain	40s	133000
2	71	0	0,1,0,0,0	beta blockers, pain	20s	34000
3	89	1	1,0,0,0,0	beta blockers	50s	40000
4	67	0	0,0,0,1,0	none	50s	120000

Encode data in usable form

- Identify the features and encode as real numbers

	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	age	family income (USD)
1	55	0	1,0,0,0,0	1,0	40s	133000
2	71	0	0,1,0,0,0	1,1	20s	34000
3	89	1	1,0,0,0,0	0,1	50s	40000
4	67	0	0,0,0,1,0	0,0	50s	120000

Encode data in usable form

- Identify the features and encode as real numbers

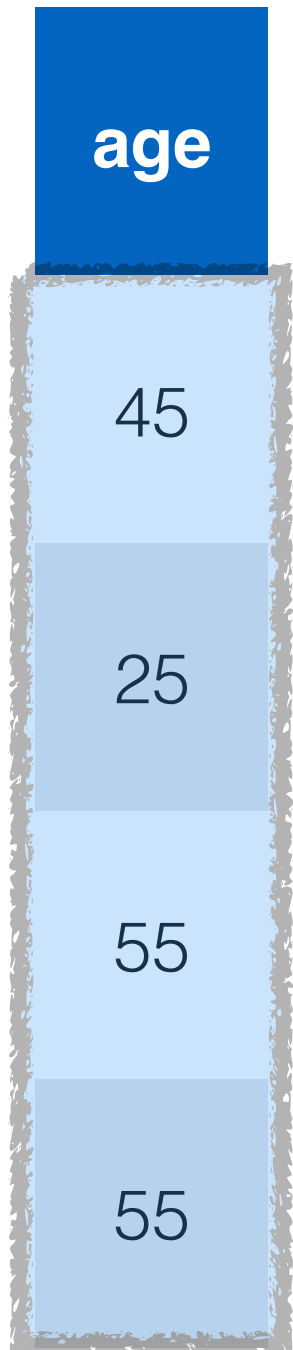
	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	age	family income (USD)
1	55	0	1,0,0,0,0	1,0	40s	133000
2	71	0	0,1,0,0,0	1,1	20s	34000
3	89	1	1,0,0,0,0	0,1	50s	40000
4	67	0	0,0,0,1,0	0,0	50s	120000

Encode data in usable form

- Identify the features and encode as real numbers

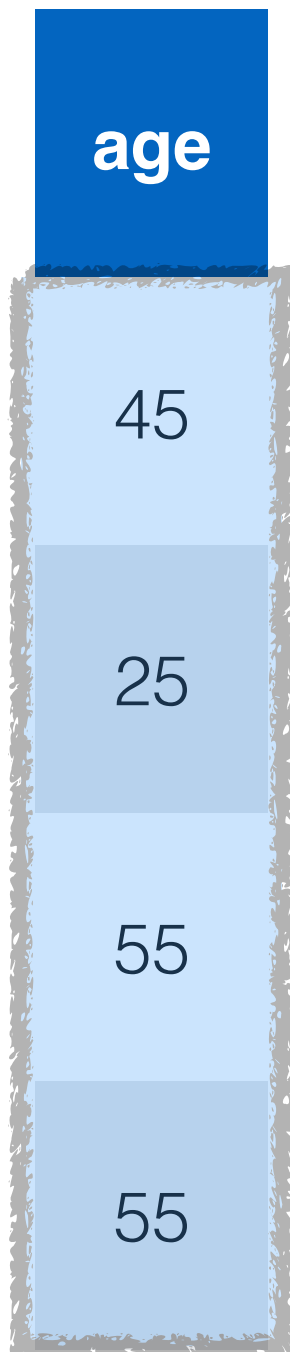
	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	age	family income (USD)
1	55	0	1,0,0,0,0	1,0	45	133000
2	71	0	0,1,0,0,0	1,1	25	34000
3	89	1	1,0,0,0,0	0,1	55	40000
4	67	0	0,0,0,1,0	0,0	55	120000

Using a representative # for a range



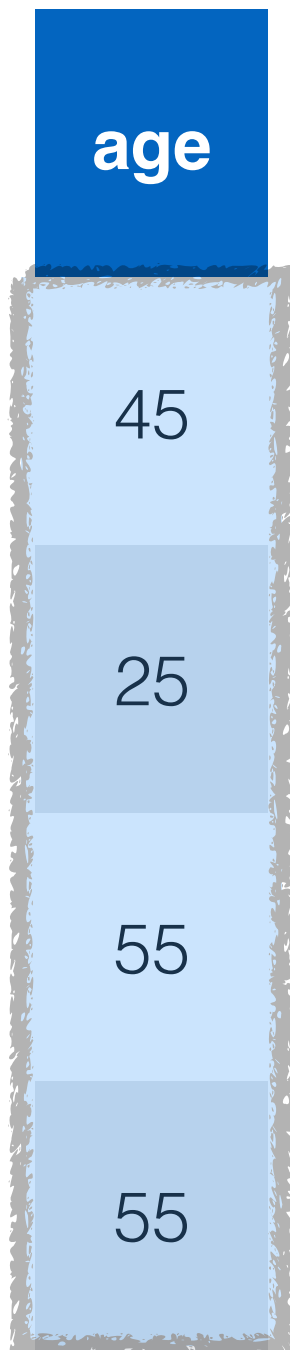
Using a representative # for a range

- Potential pitfall: level of detail might be treated as meaningful (by you or others using the data)



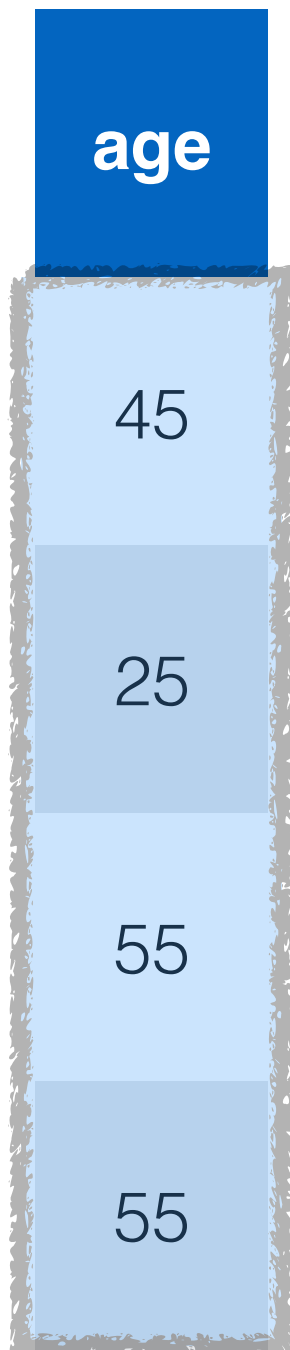
Using a representative # for a range

- Potential pitfall: level of detail might be treated as meaningful (by you or others using the data)



Using a representative # for a range

- Potential pitfall: level of detail might be treated as meaningful (by you or others using the data)
- A way to diagnose many problems: plot your data!



Encode data in usable form

- Identify the features and encode as real numbers

	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	age	family income (USD)
1	55	0	1,0,0,0,0	1,0	45	133000
2	71	0	0,1,0,0,0	1,1	25	34000
3	89	1	1,0,0,0,0	0,1	55	40000
4	67	0	0,0,0,1,0	0,0	55	120000

Encode data in usable form

- Identify the features and encode as real numbers

	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	decade	family income (USD)
1	55	0	1,0,0,0,0	1,0	4	133000
2	71	0	0,1,0,0,0	1,1	2	34000
3	89	1	1,0,0,0,0	0,1	5	40000
4	67	0	0,0,0,1,0	0,0	5	120000

Encode ordinal data

Encode ordinal data

- Numerical data: order on data values, and differences in value are meaningful

Encode ordinal data

- Numerical data: order on data values, and differences in value are meaningful
- Categorical data: no order on data values

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- Ordinal data: order on data values, but differences not meaningful

Encode ordinal data

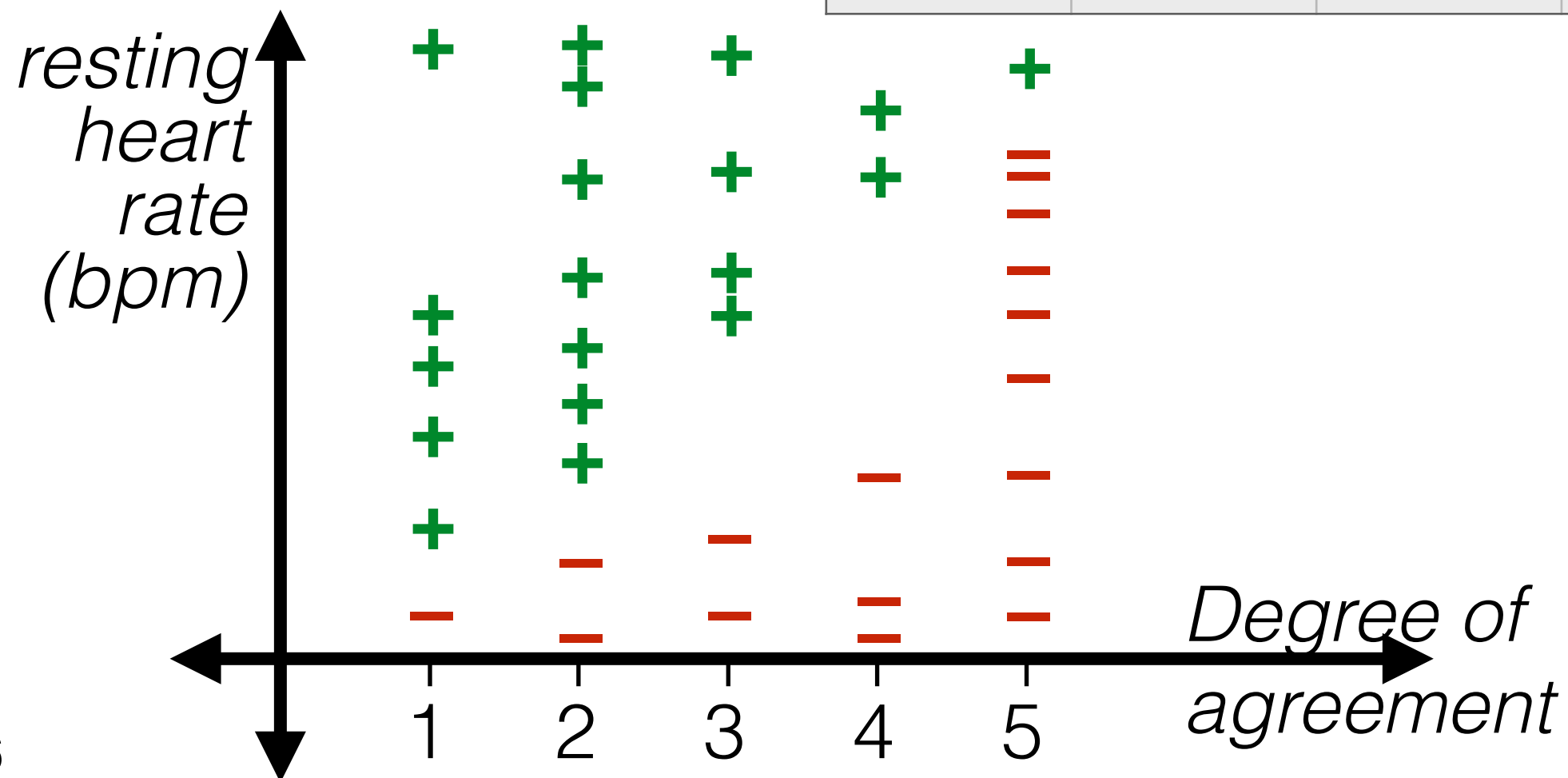
- Numerical data: order on data values, and differences in value are meaningful
- Categorical data: no order on data values
- Ordinal data: order on data values, but differences not meaningful
 - E.g. Likert scale:

Strongly disagree	Disagree	Neutral	Agree	Strongly agree
1	2	3	4	5

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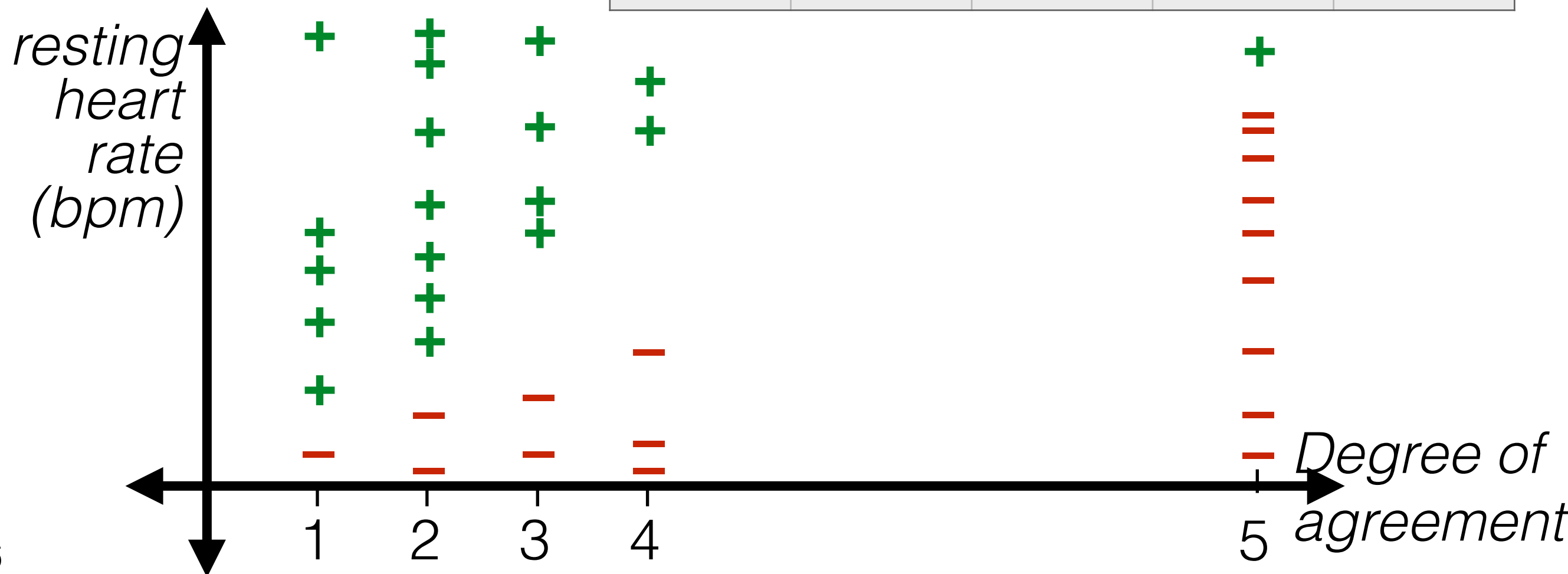
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
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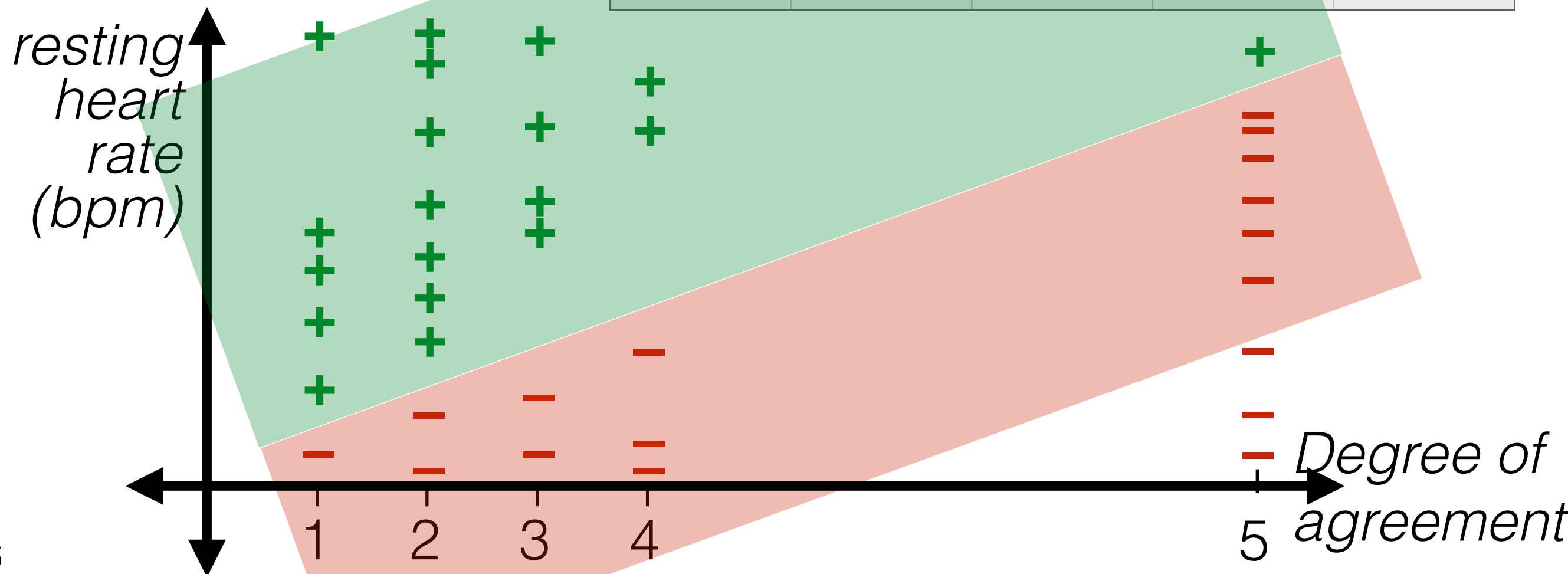
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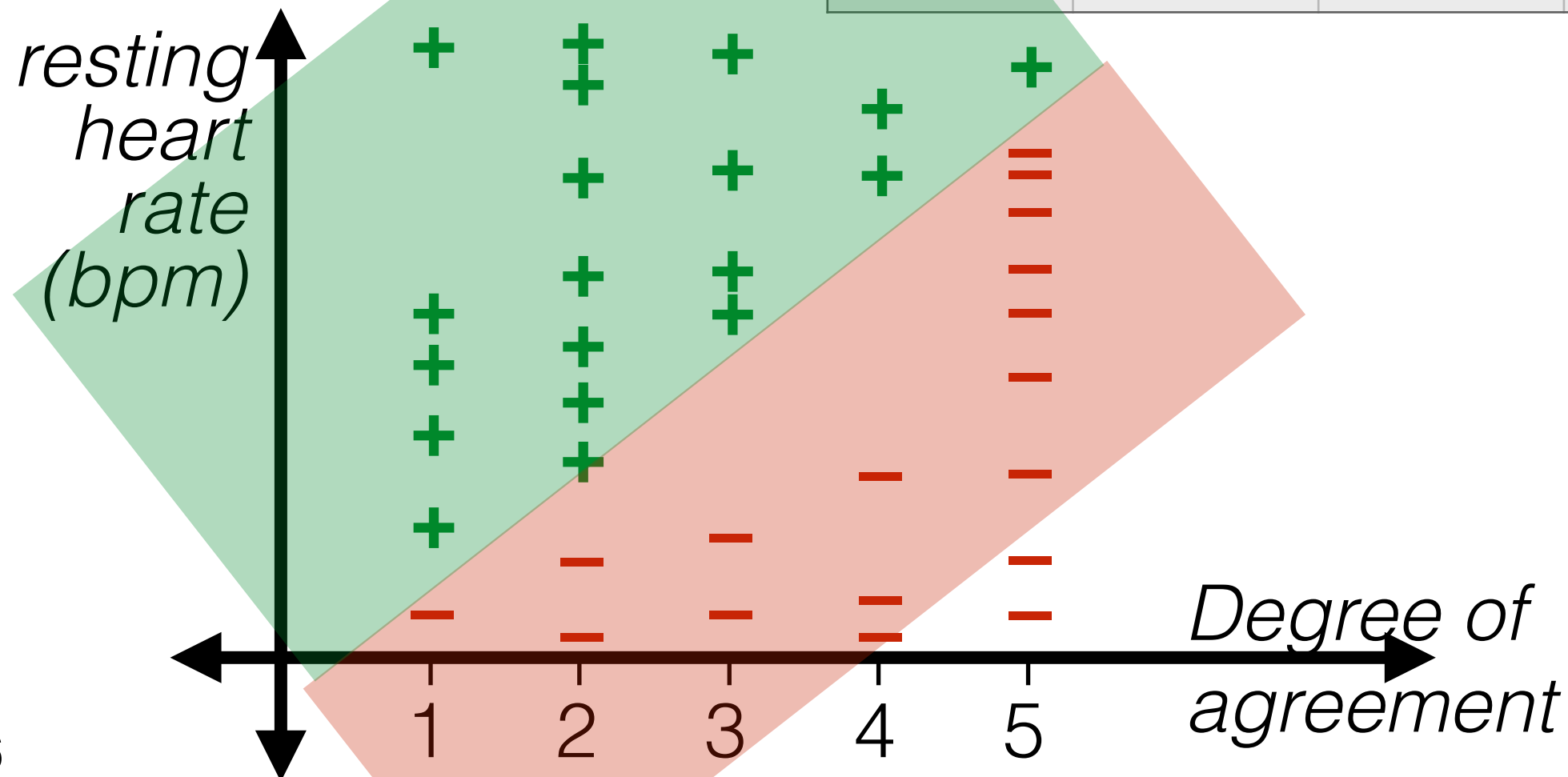
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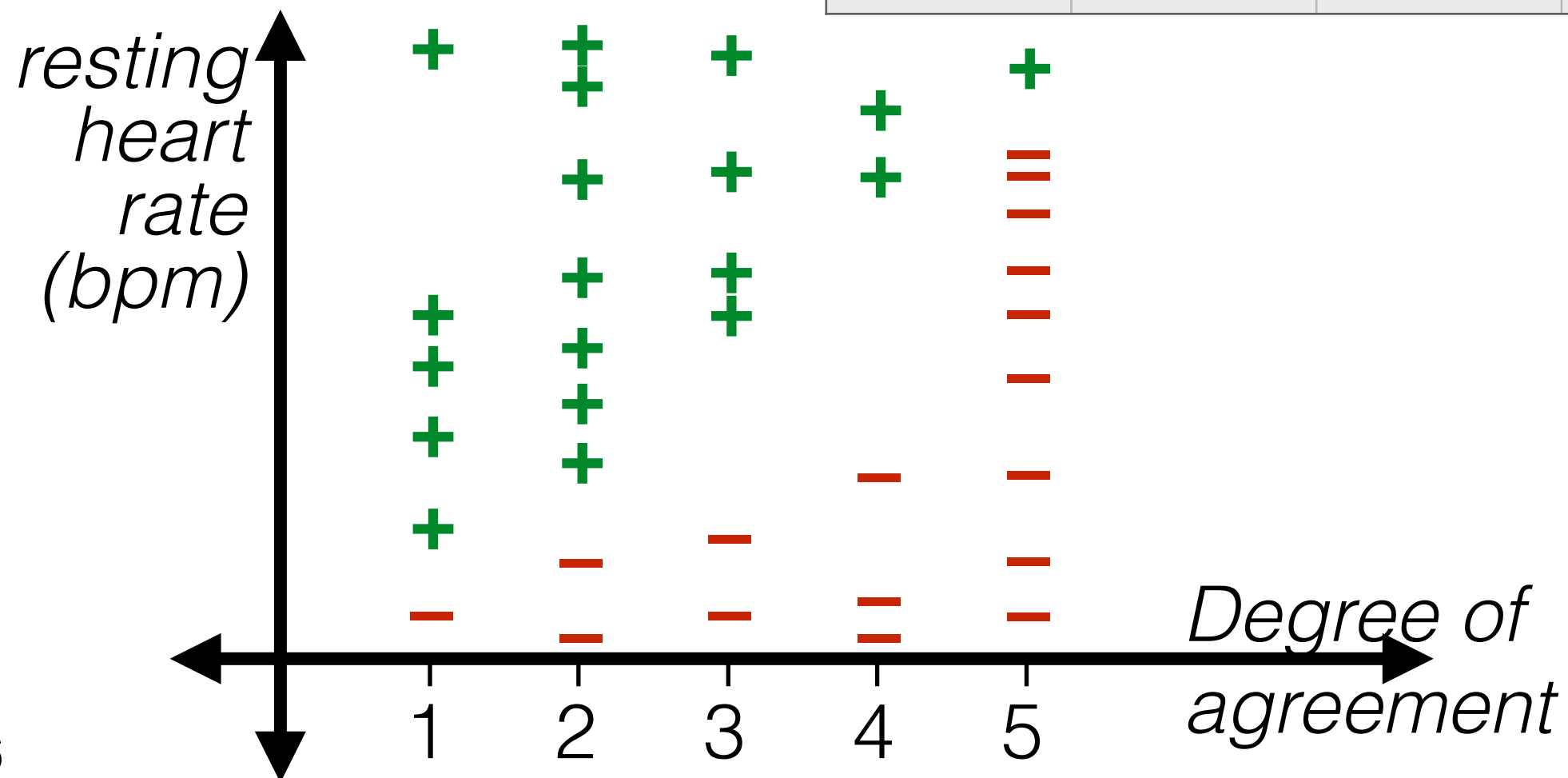
Strongly disagree	Disagree	Neutral	Agree	Strongly agree
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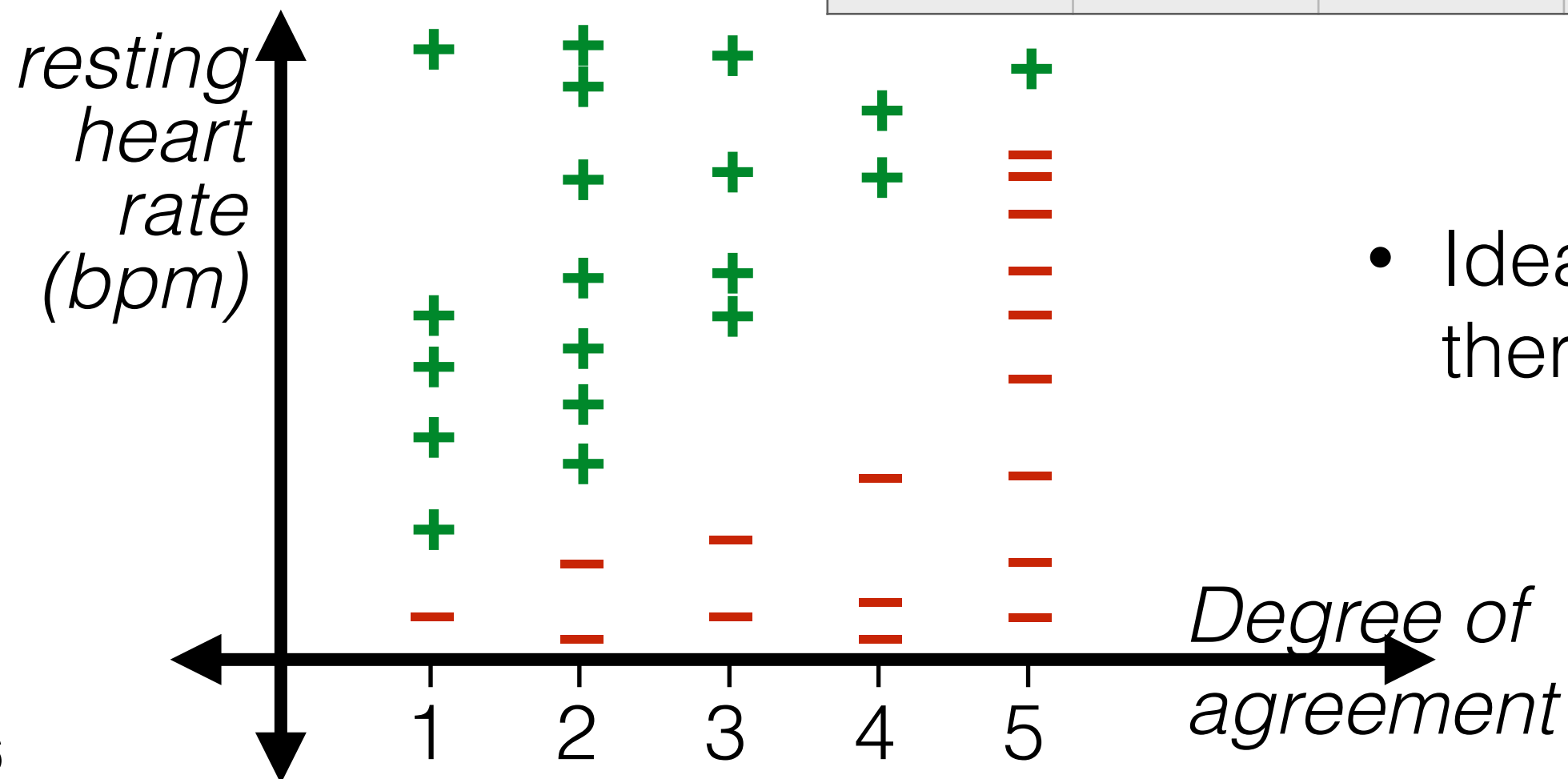
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1	2	3	4	5

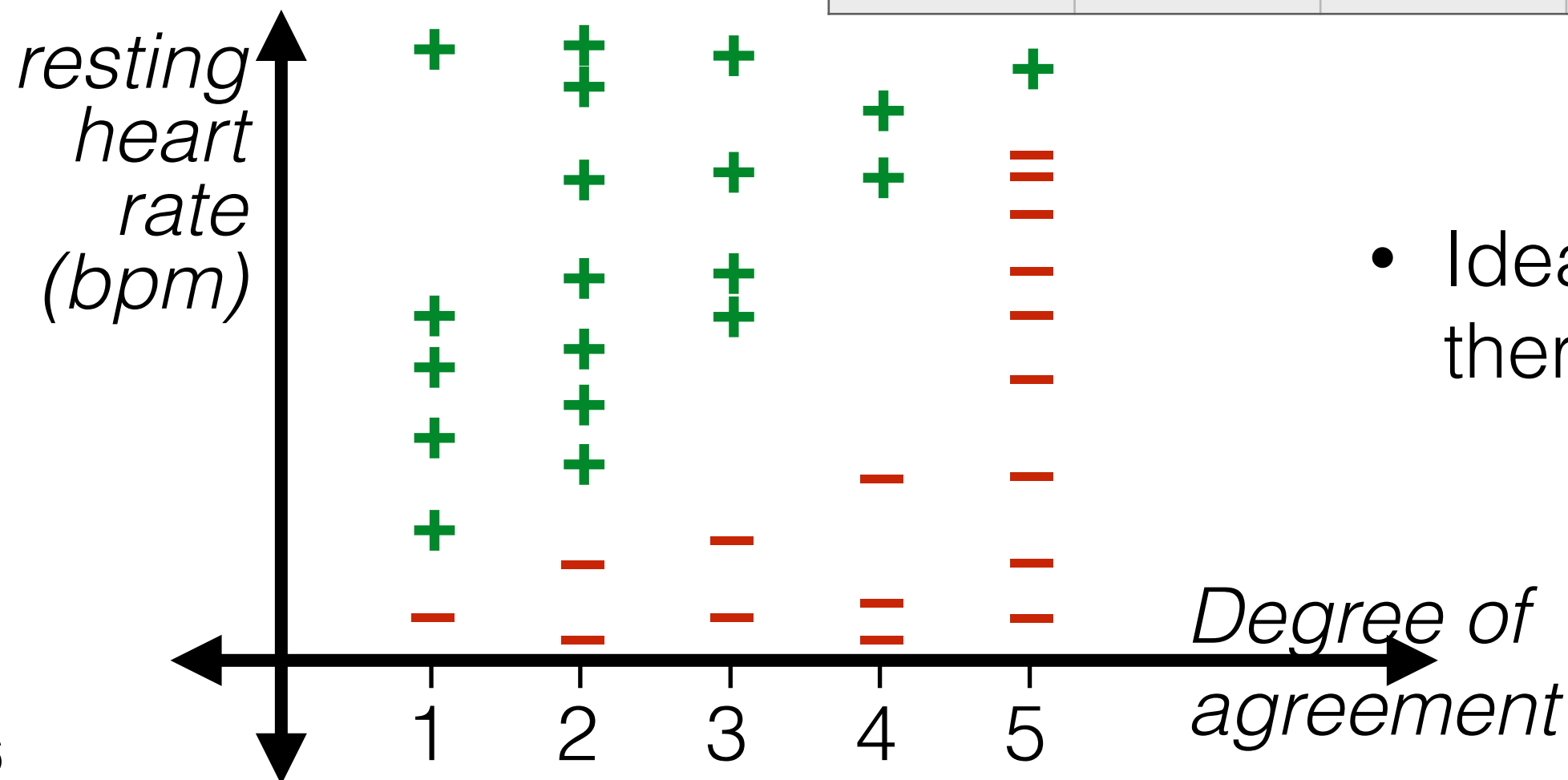


- Idea: Unary/ thermometer code

Encode ordinal data

- Numerical data: order on data values, and differences in value are meaningful
- Categorical data: no order on data values
- Ordinal data: order on data values, but differences not meaningful
 - E.g. Likert scale:

Strongly disagree	Disagree	Neutral	Agree	Strongly agree
1,0,0,0,0	1,1,0,0,0	1,1,1,0,0	1,1,1,1,0	1,1,1,1,1



- Idea: Unary/ thermometer code

Encode data in usable form

- Identify the features and encode as real numbers

	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	decade	family income (USD)
1	55	0	1,0,0,0,0	1,0	4	133000
2	71	0	0,1,0,0,0	1,1	2	34000
3	89	1	1,0,0,0,0	0,1	5	40000
4	67	0	0,0,0,1,0	0,0	5	120000

Encode data in usable form

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	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	decade	family income (USD)
1	55	0	1,0,0,0,0	1,0	4	133000
2	71	0	0,1,0,0,0	1,1	2	34000
3	89	1	1,0,0,0,0	0,1	5	40000
4	67	0	0,0,0,1,0	0,0	5	120000

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1	55	0	1,0,0,0,0	1,0	4	133000
2	71	0	0,1,0,0,0	1,1	2	34000
3	89	1	1,0,0,0,0	0,1	5	40000
4	67	0	0,0,0,1,0	0,0	5	120000

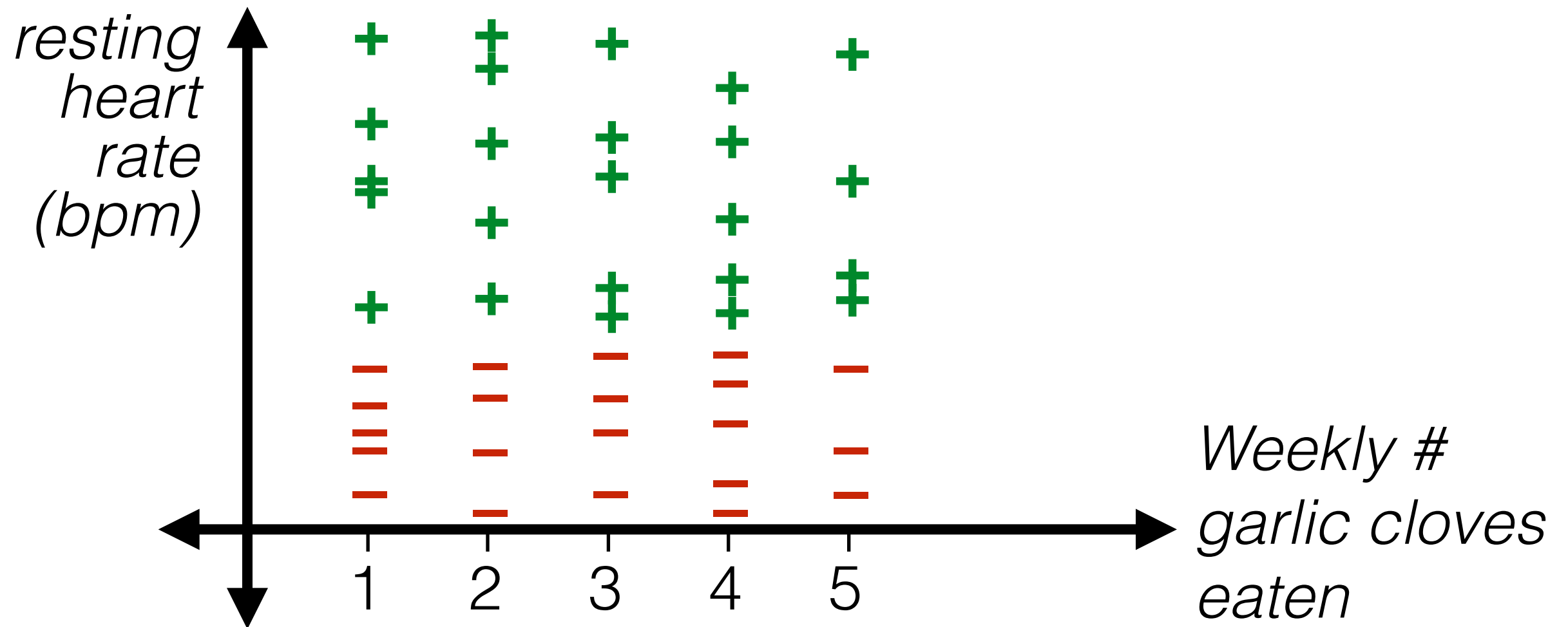
Encode numerical data

Encode numerical data

- A closer look at the output of a linear classifier

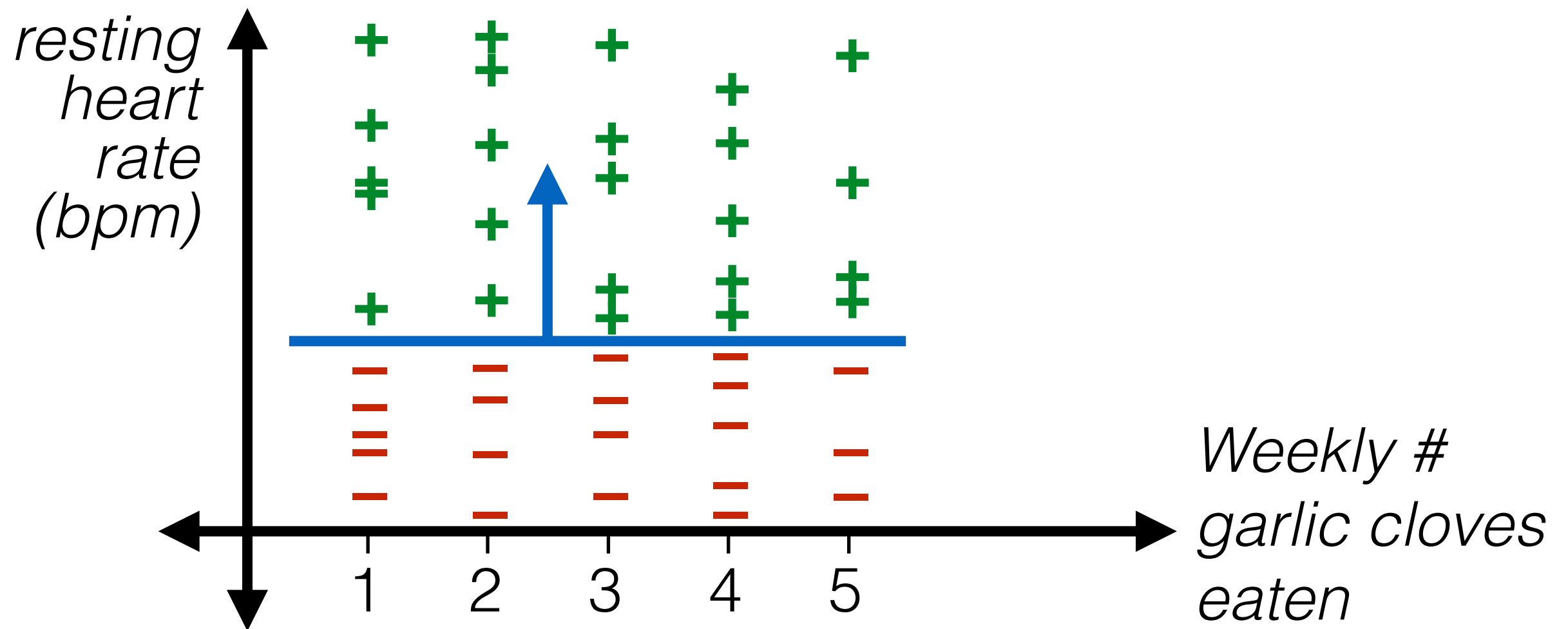
Encode numerical data

- A closer look at the output of a linear classifier



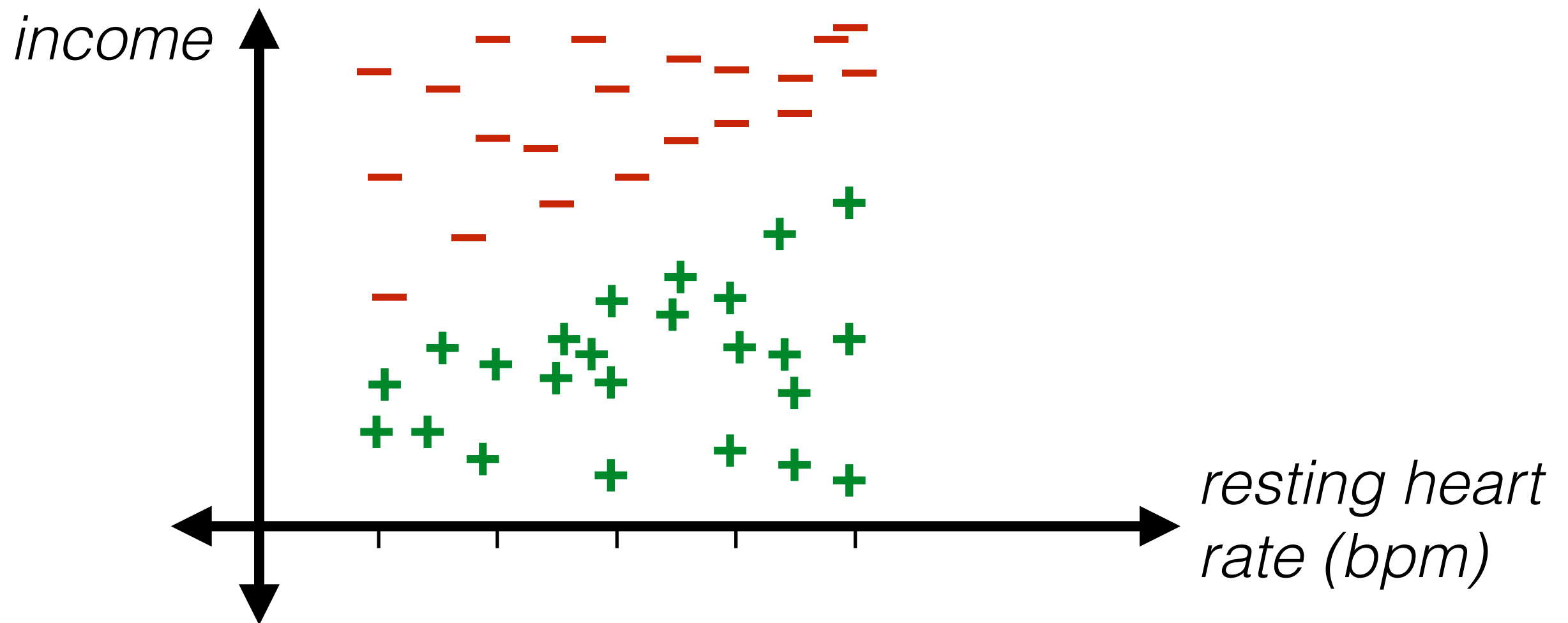
Encode numerical data

- A closer look at the output of a linear classifier



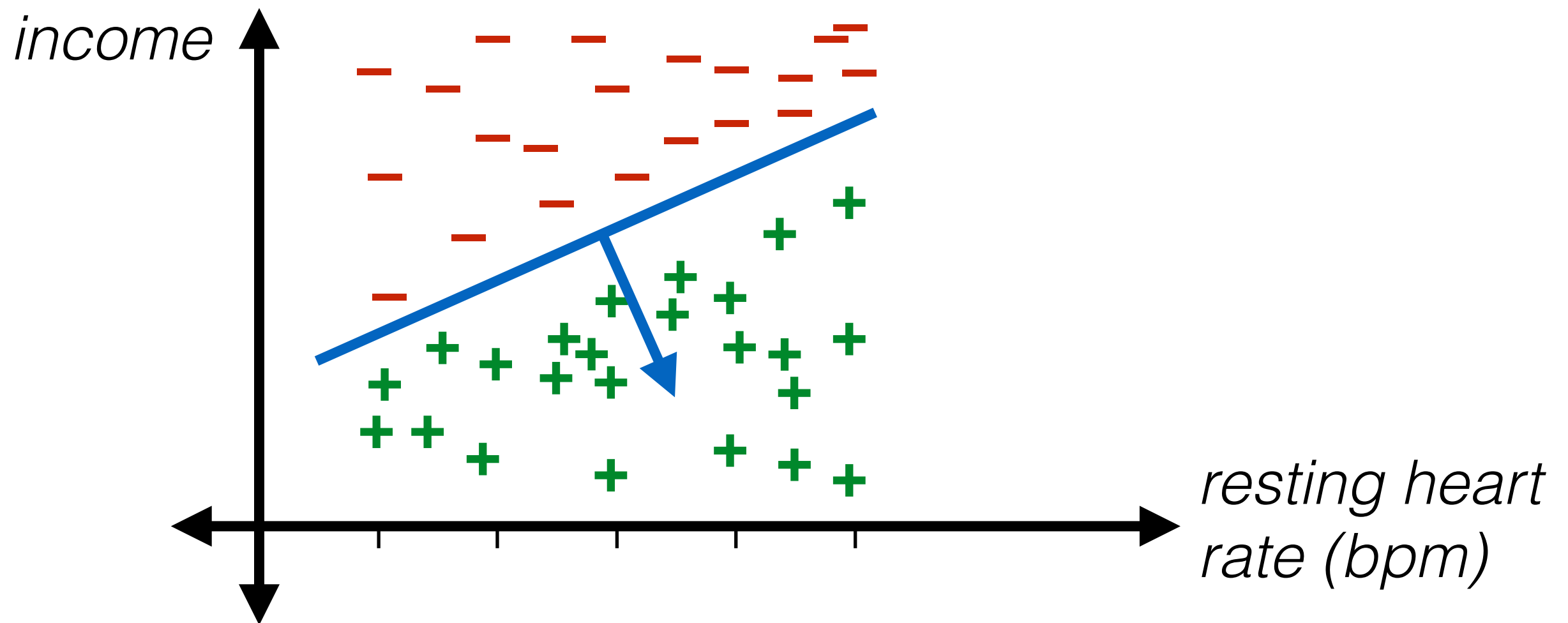
Encode numerical data

- A closer look at the output of a linear classifier



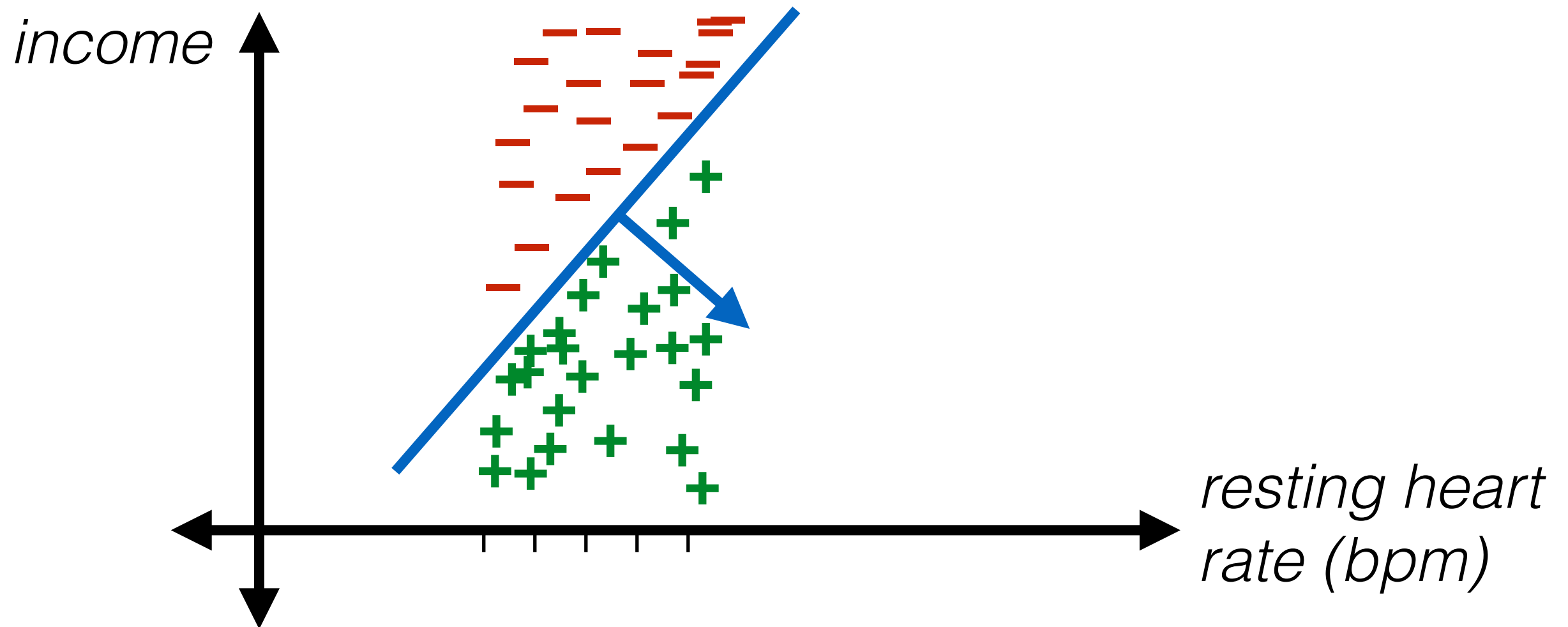
Encode numerical data

- A closer look at the output of a linear classifier



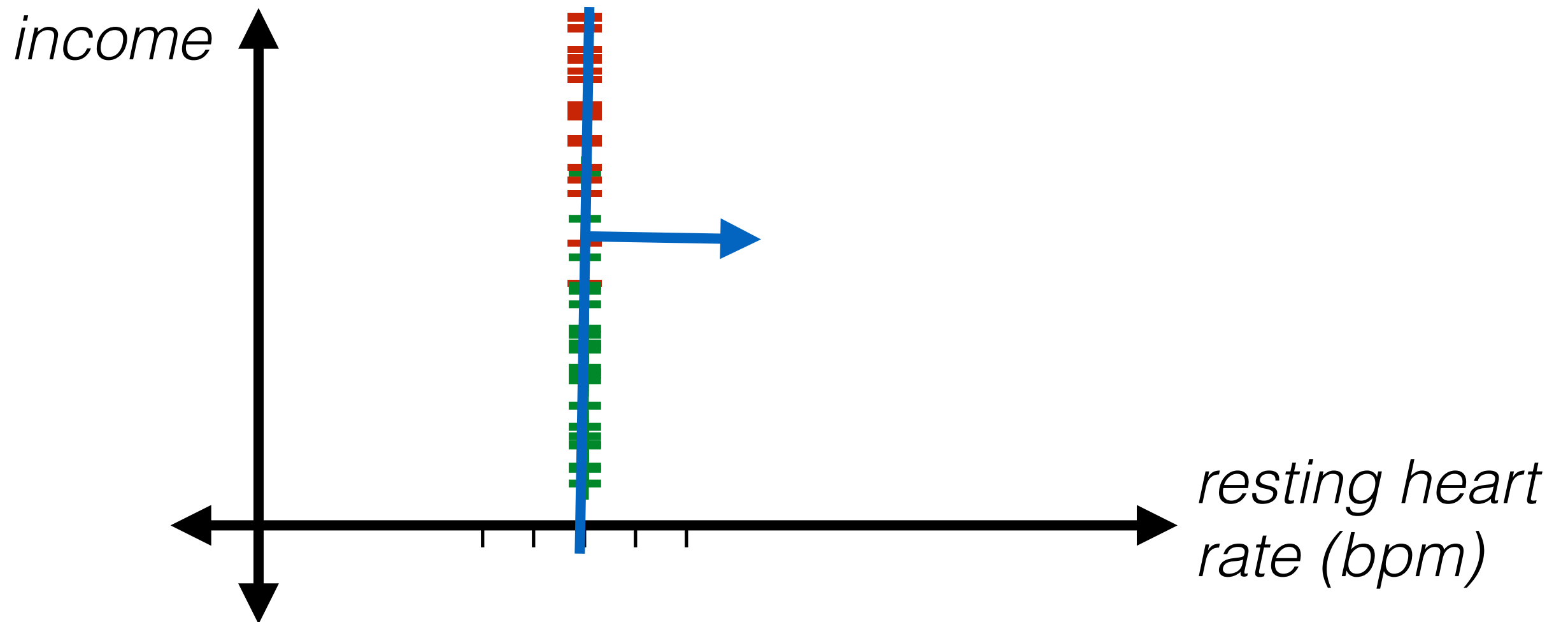
Encode numerical data

- A closer look at the output of a linear classifier



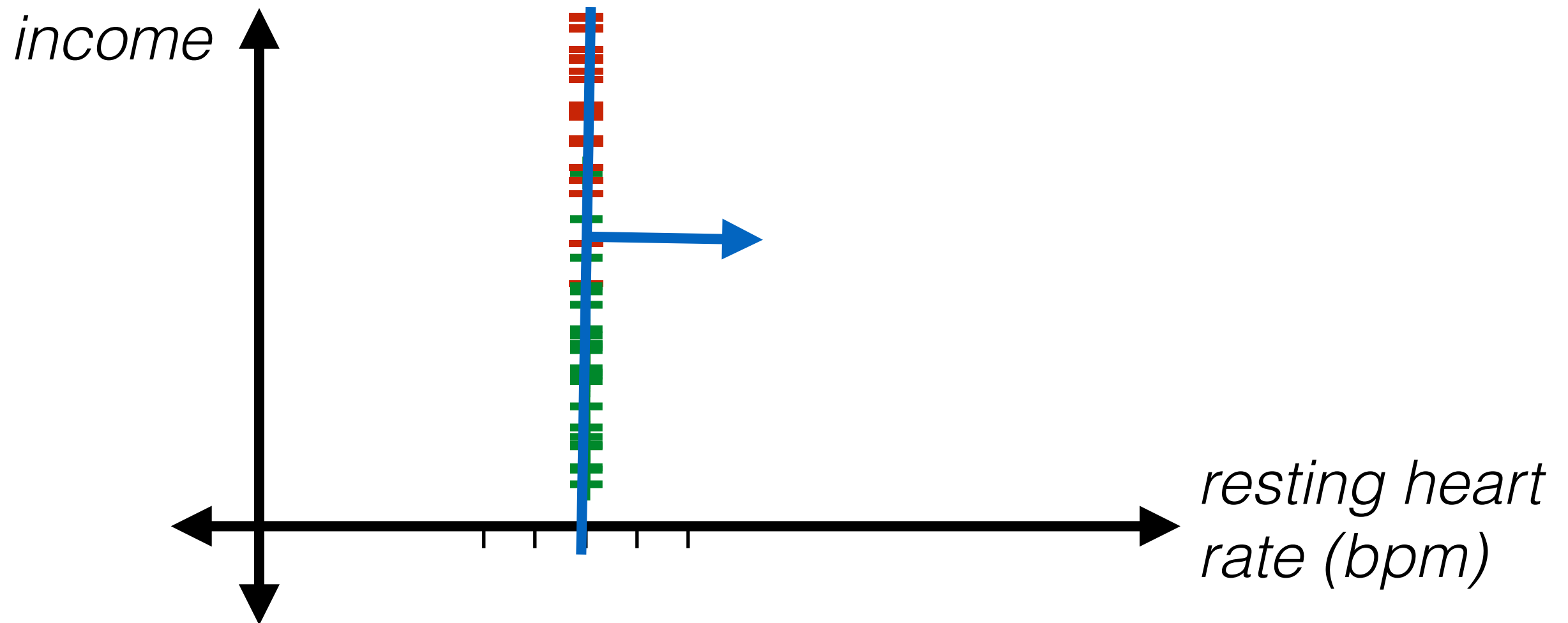
Encode numerical data

- A closer look at the output of a linear classifier



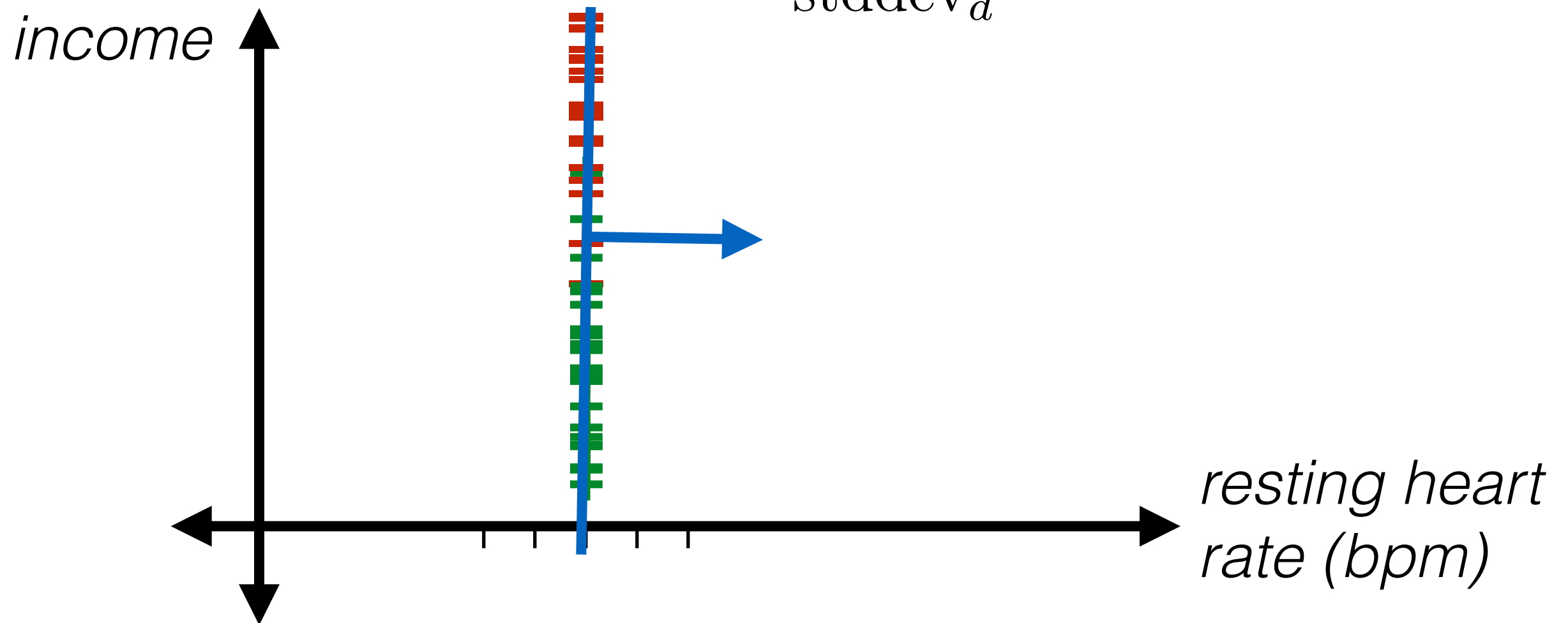
Encode numerical data

- A closer look at the output of a linear classifier
- Idea: standardize numerical data



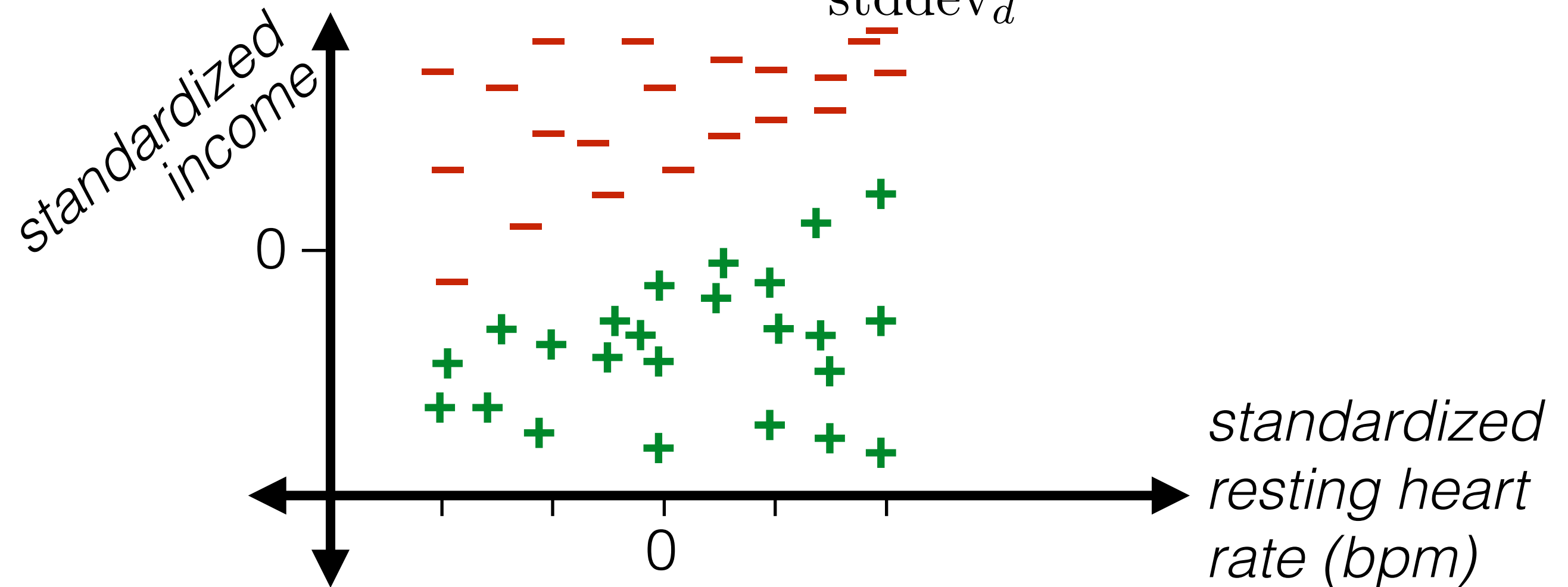
Encode numerical data

- A closer look at the output of a linear classifier
- Idea: standardize numerical data
 - For d th feature: $\phi_d^{(k)} = \frac{x_d^{(k)} - \text{mean}_d}{\text{stddev}_d}$



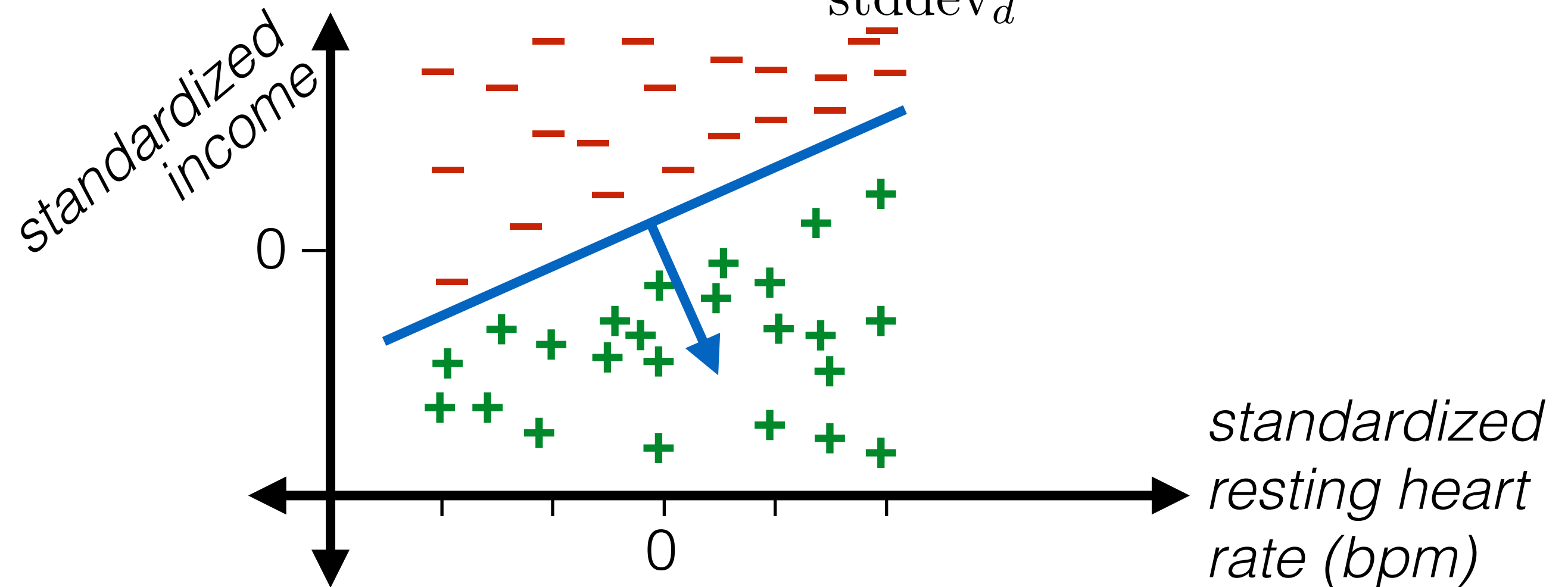
Encode numerical data

- A closer look at the output of a linear classifier
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 - For d th feature: $\phi_d^{(k)} = \frac{x_d^{(k)} - \text{mean}_d}{\text{stddev}_d}$



Encode numerical data

- A closer look at the output of a linear classifier
- Idea: standardize numerical data
 - For d th feature: $\phi_d^{(k)} = \frac{x_d^{(k)} - \text{mean}_d}{\text{stddev}_d}$



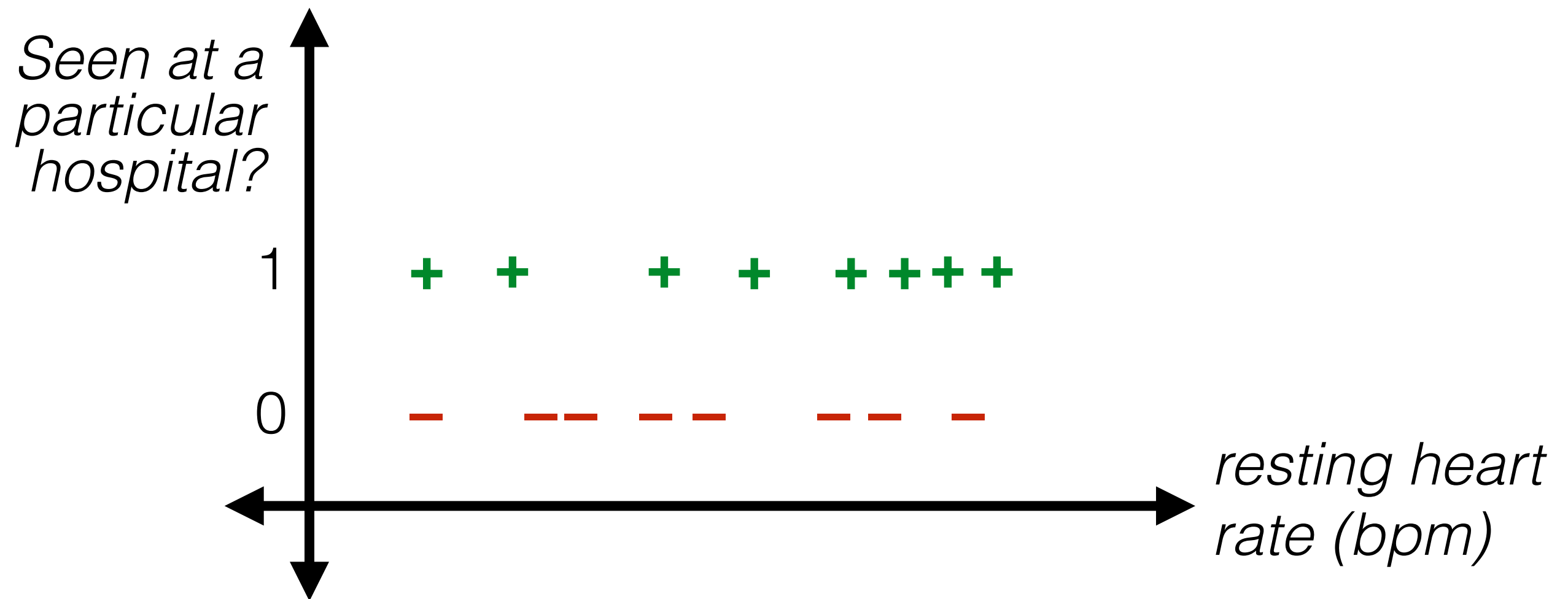
More benefits of plotting your data

More benefits of plotting your data

- And talking to experts

More benefits of plotting your data

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Encode data in usable form

- Identify the features and encode as real numbers

	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	decade	family income (USD)
1	55	0	1,0,0,0,0	1,0	4	133000
2	71	0	0,1,0,0,0	1,1	2	34000
3	89	1	1,0,0,0,0	0,1	5	40000
4	67	0	0,0,0,1,0	0,0	5	120000

Encode data in usable form

- Identify the features and encode as real numbers
- Standardize numerical features

	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	decade	family income (USD)
1	55	0	1,0,0,0,0	1,0	4	133000
2	71	0	0,1,0,0,0	1,1	2	34000
3	89	1	1,0,0,0,0	0,1	5	40000
4	67	0	0,0,0,1,0	0,0	5	120000

Encode data in usable form

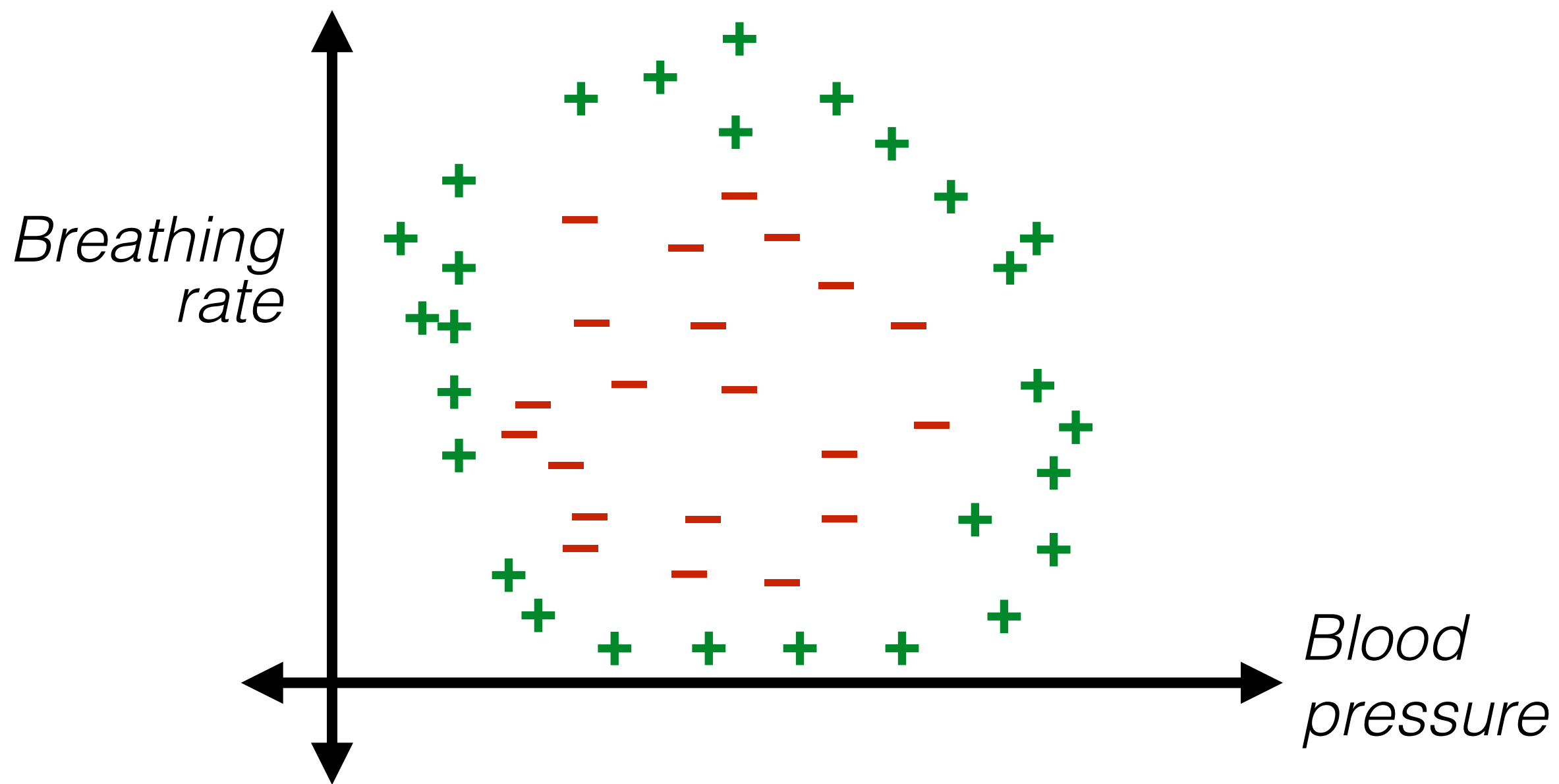
- Identify the features and encode as real numbers
- Standardize numerical features

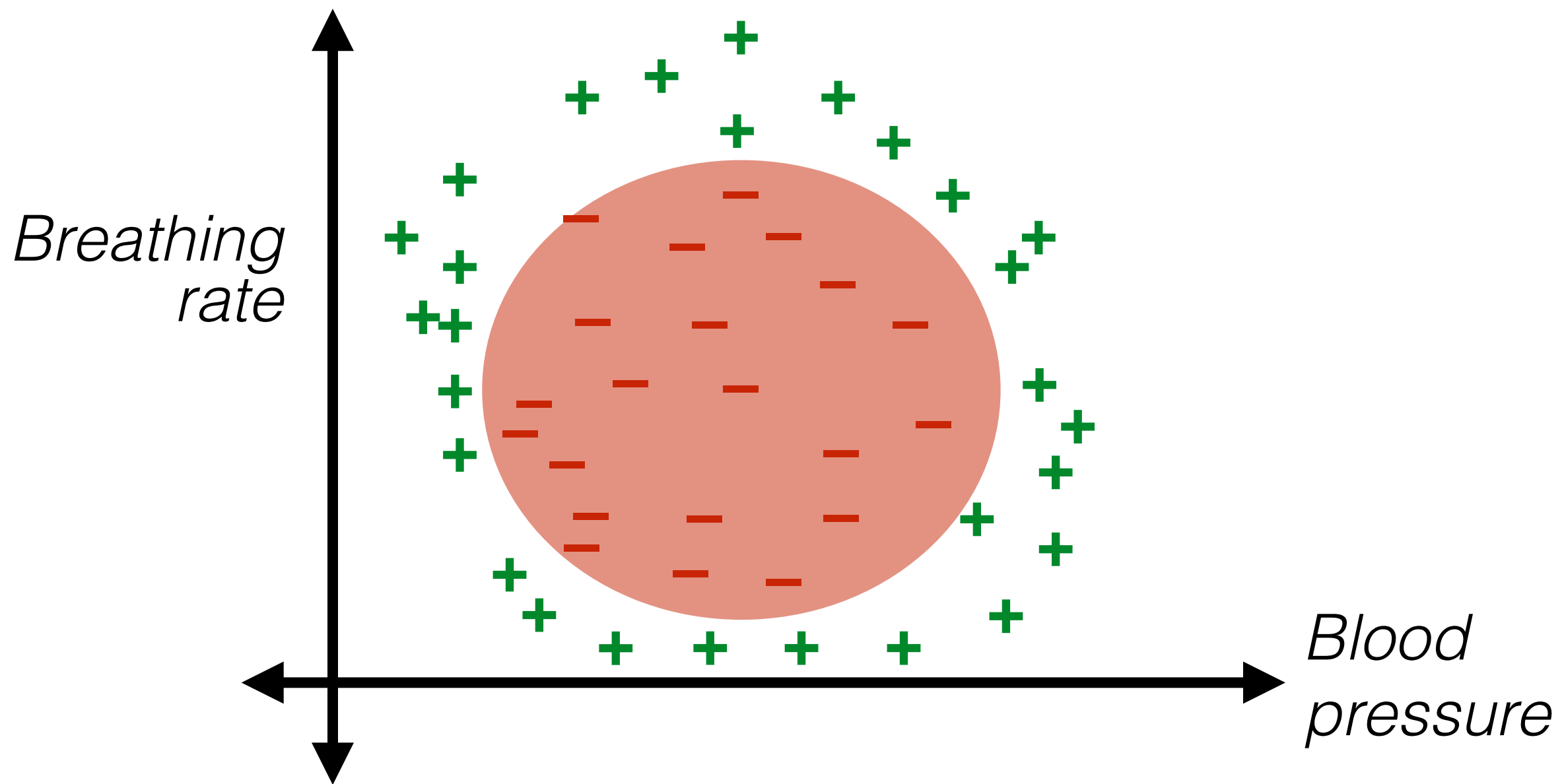
	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	decade	family income (USD)
1	55	0	1,0,0,0,0	1,0	4	133000
2	71	0	0,1,0,0,0	1,1	2	34000
3	89	1	1,0,0,0,0	0,1	5	40000
4	67	0	0,0,0,1,0	0,0	5	120000

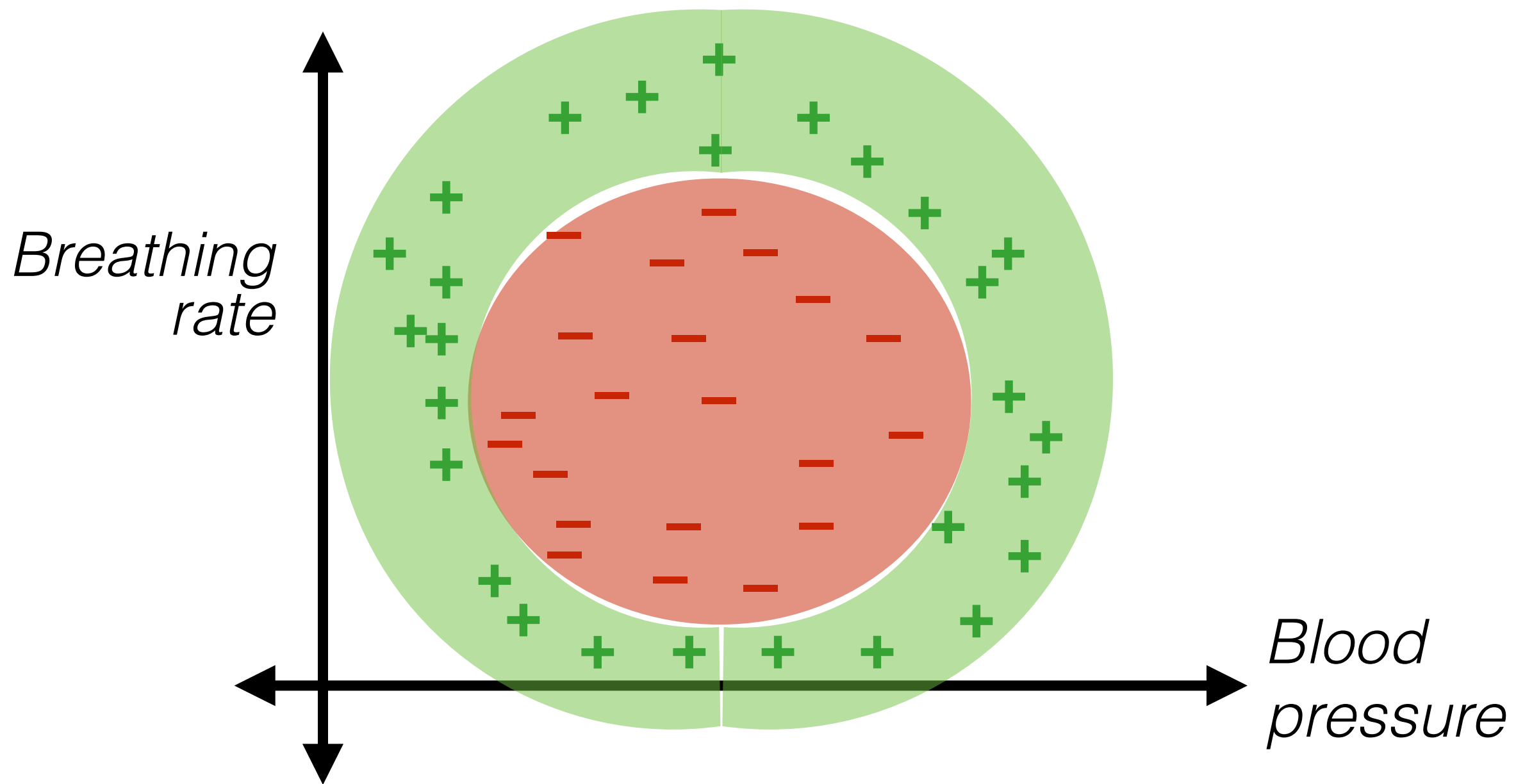
Encode data in usable form

- Identify the features and encode as real numbers
- Standardize numerical features

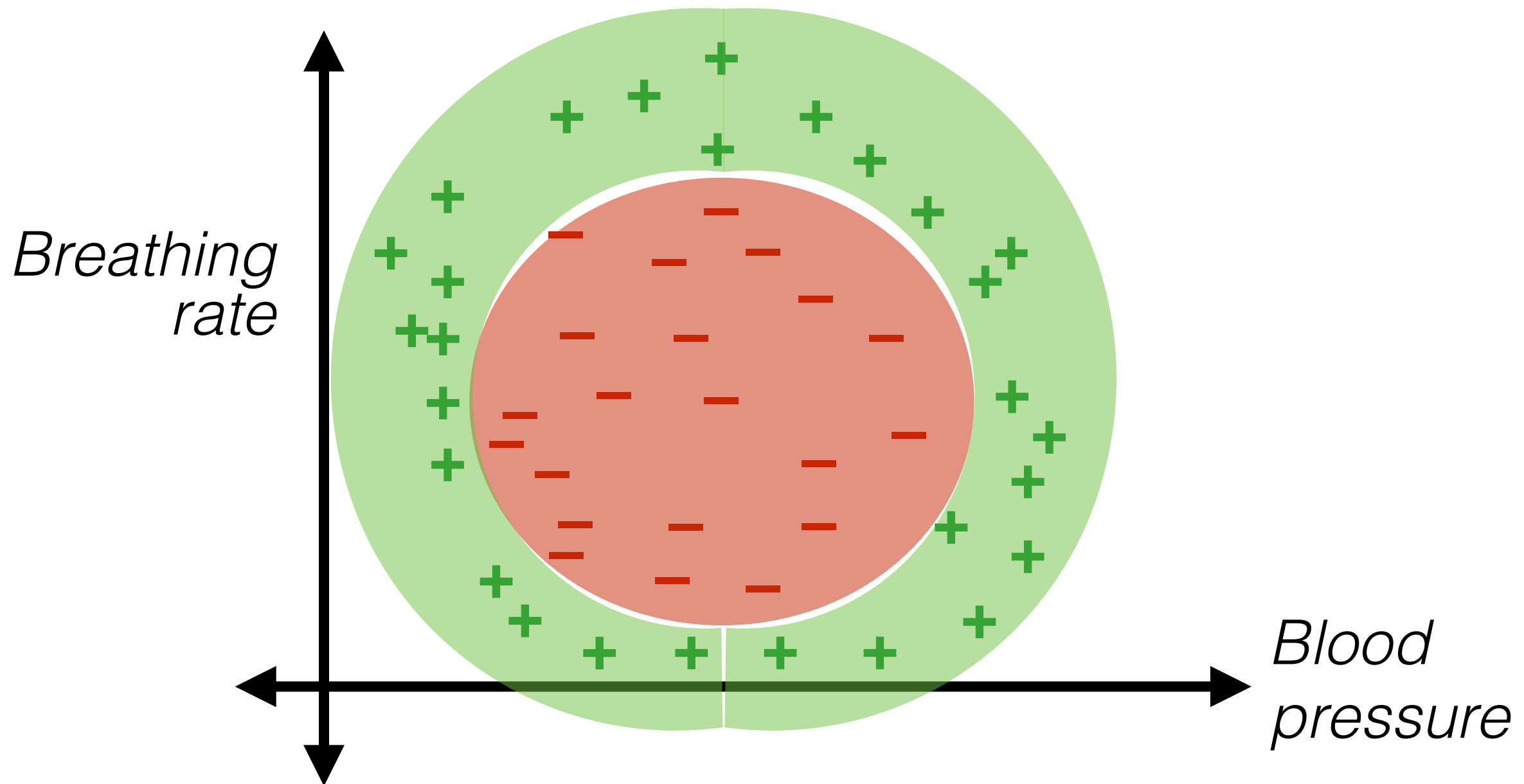
	resting heart rate (bpm)	pain?	j1,j2,j3,j4,j5	m1, m2	decade	family income (USD)
1	-1.5	0	1,0,0,0,0	1,0	1	2.075
2	0.1	0	0,1,0,0,0	1,1	-1	-0.4
3	1.9	1	1,0,0,0,0	0,1	2	-0.25
4	-0.3	0	0,0,0,1,0	0,0	2	1.75





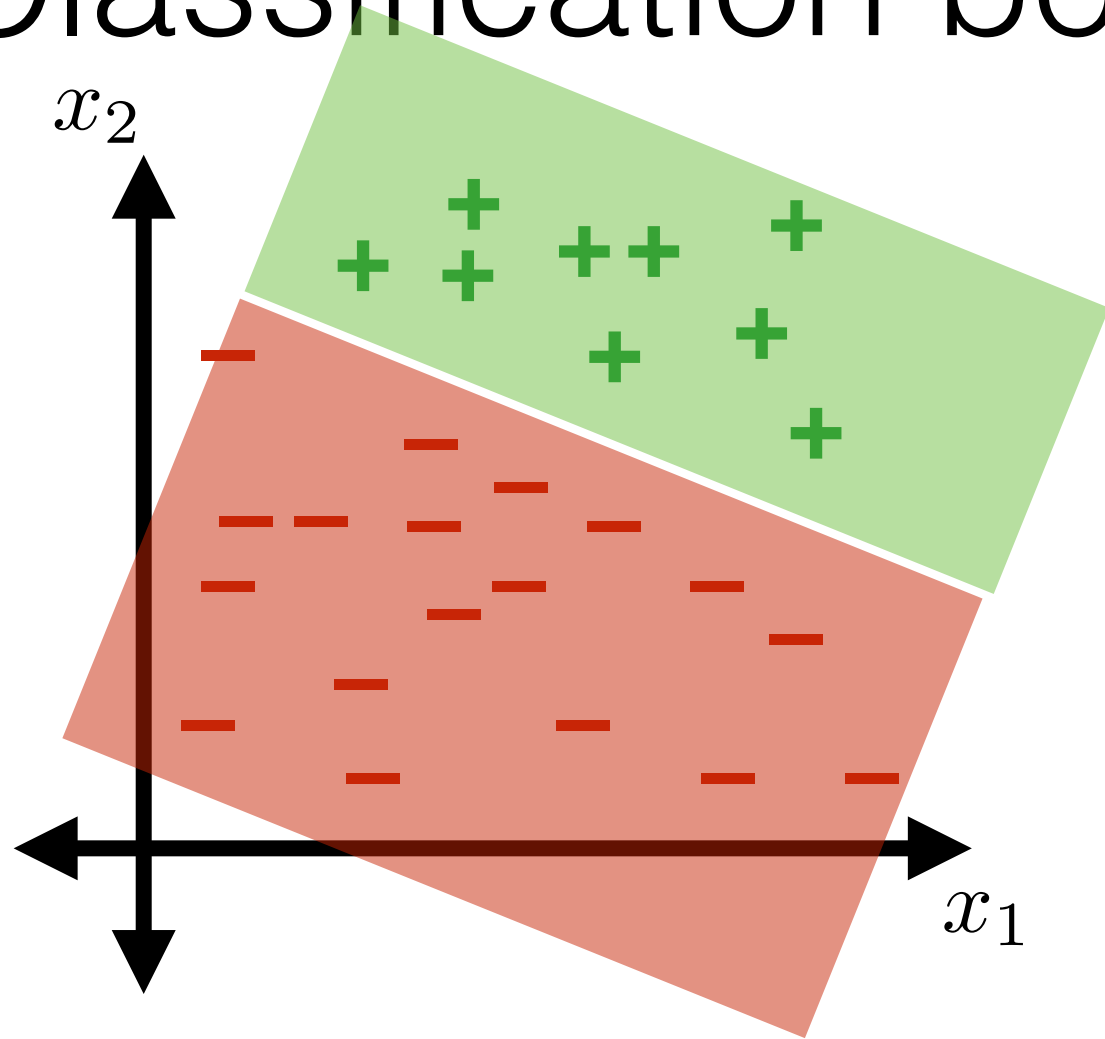


Nonlinear boundaries

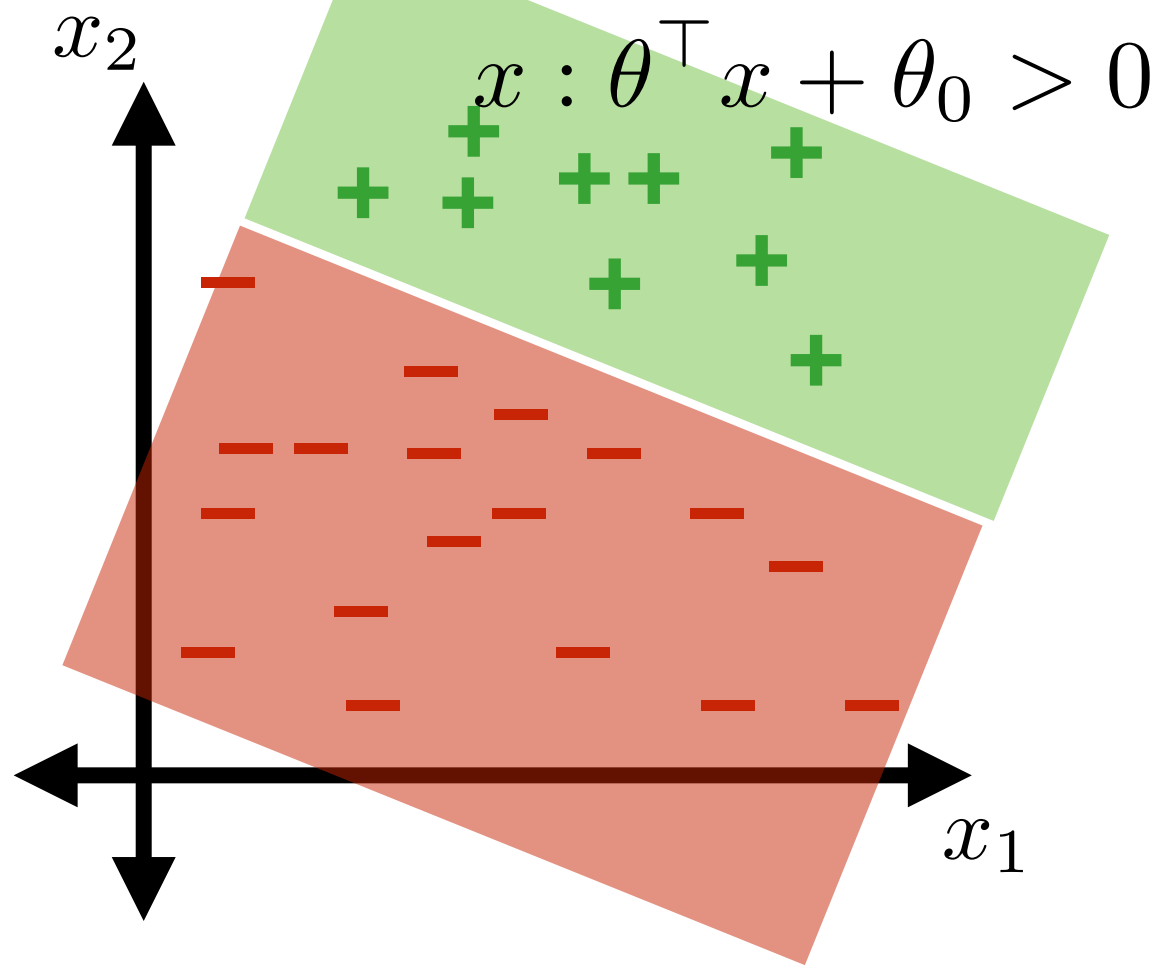


Classification boundaries

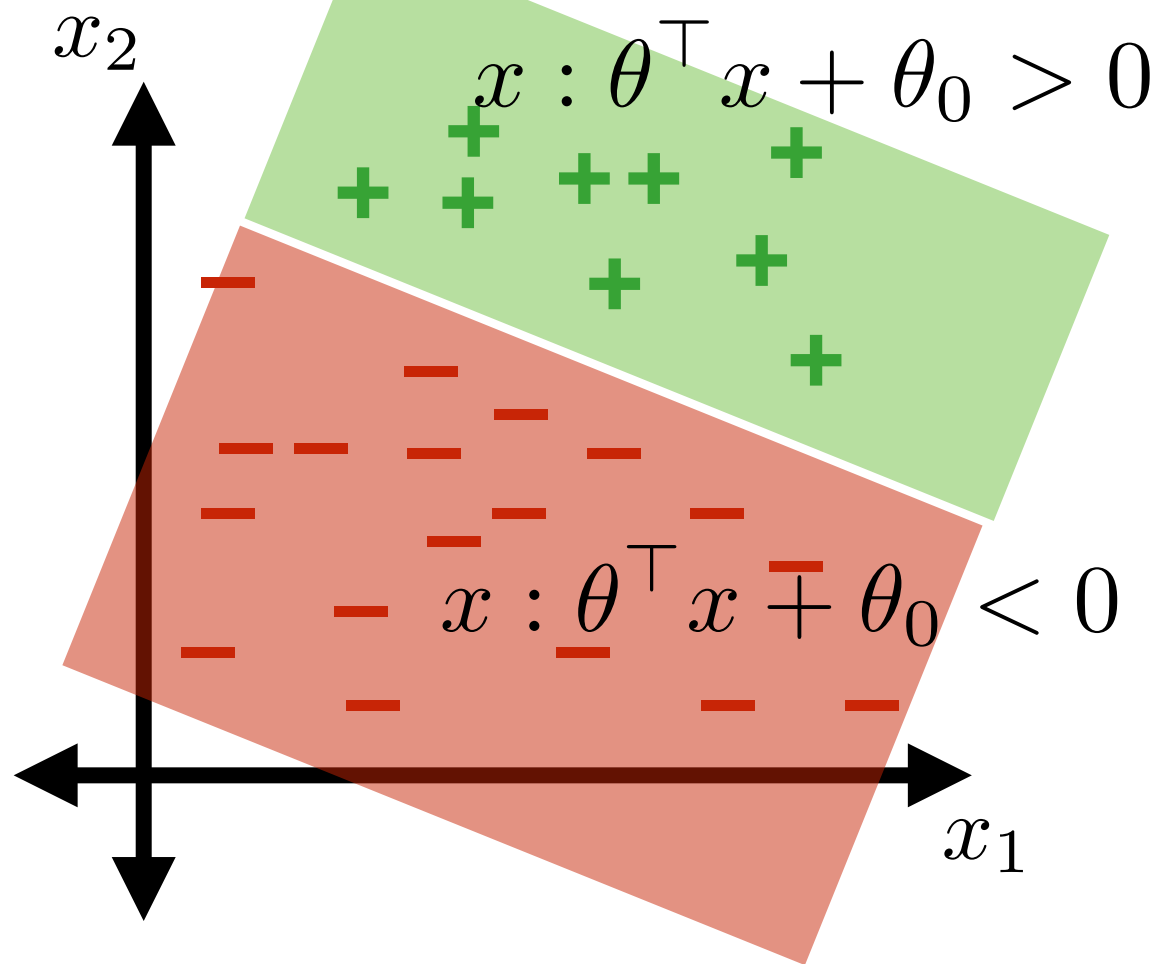
Classification boundaries



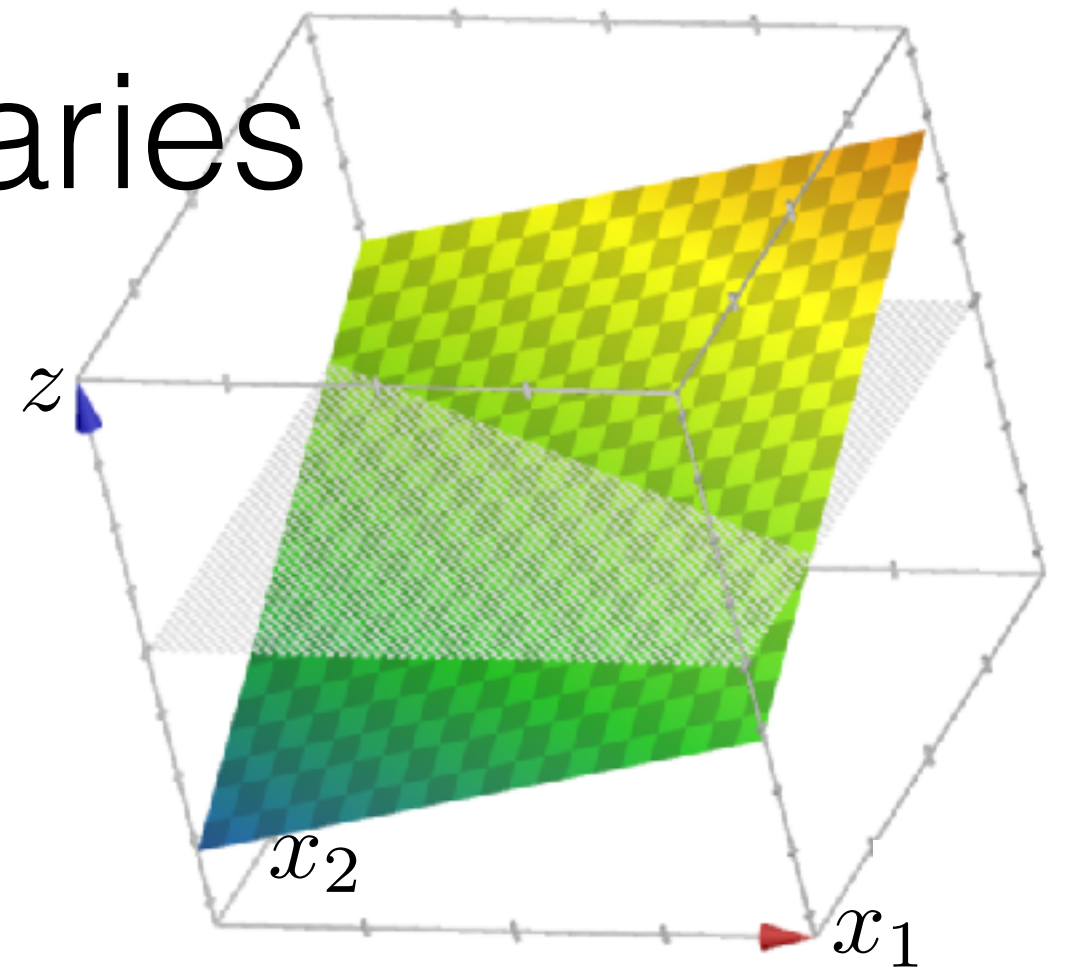
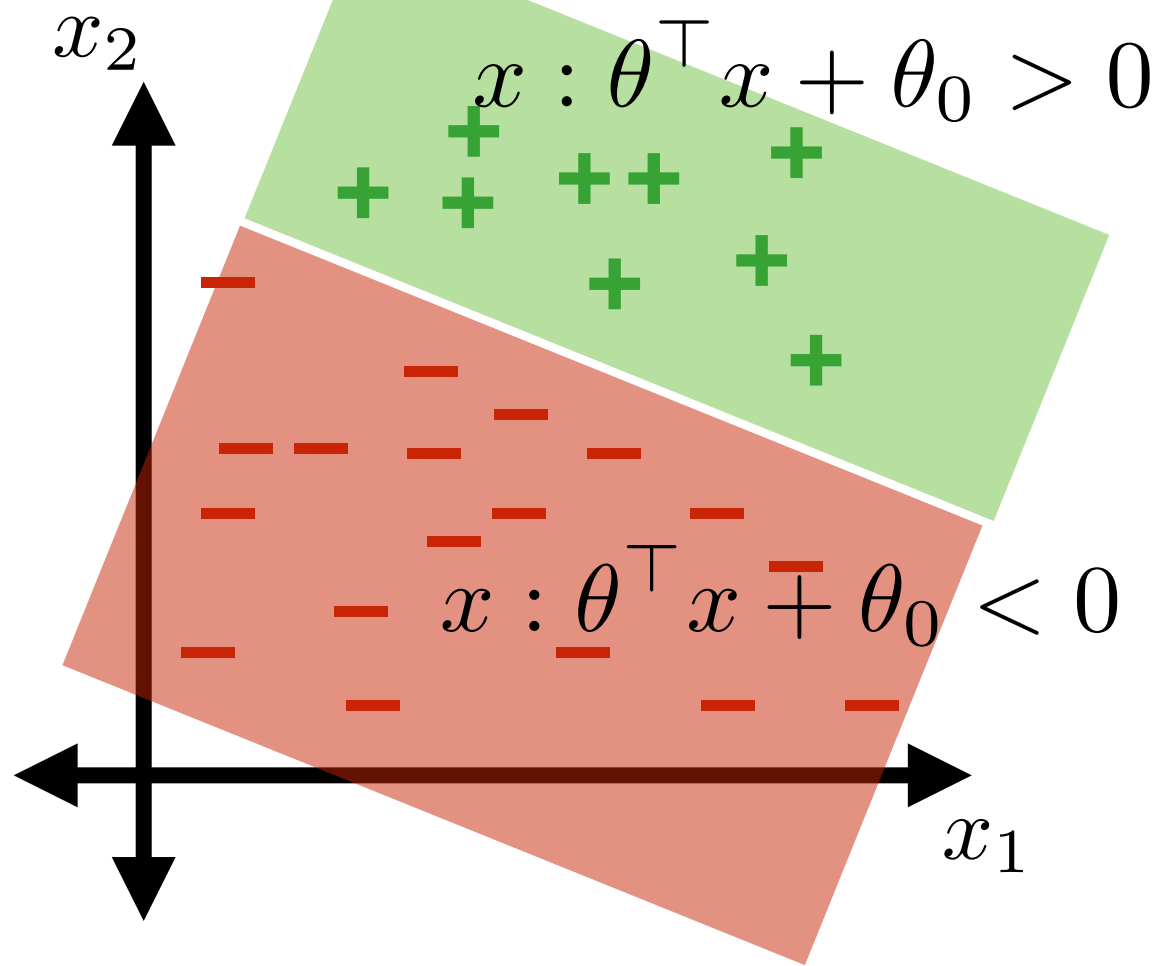
Classification boundaries



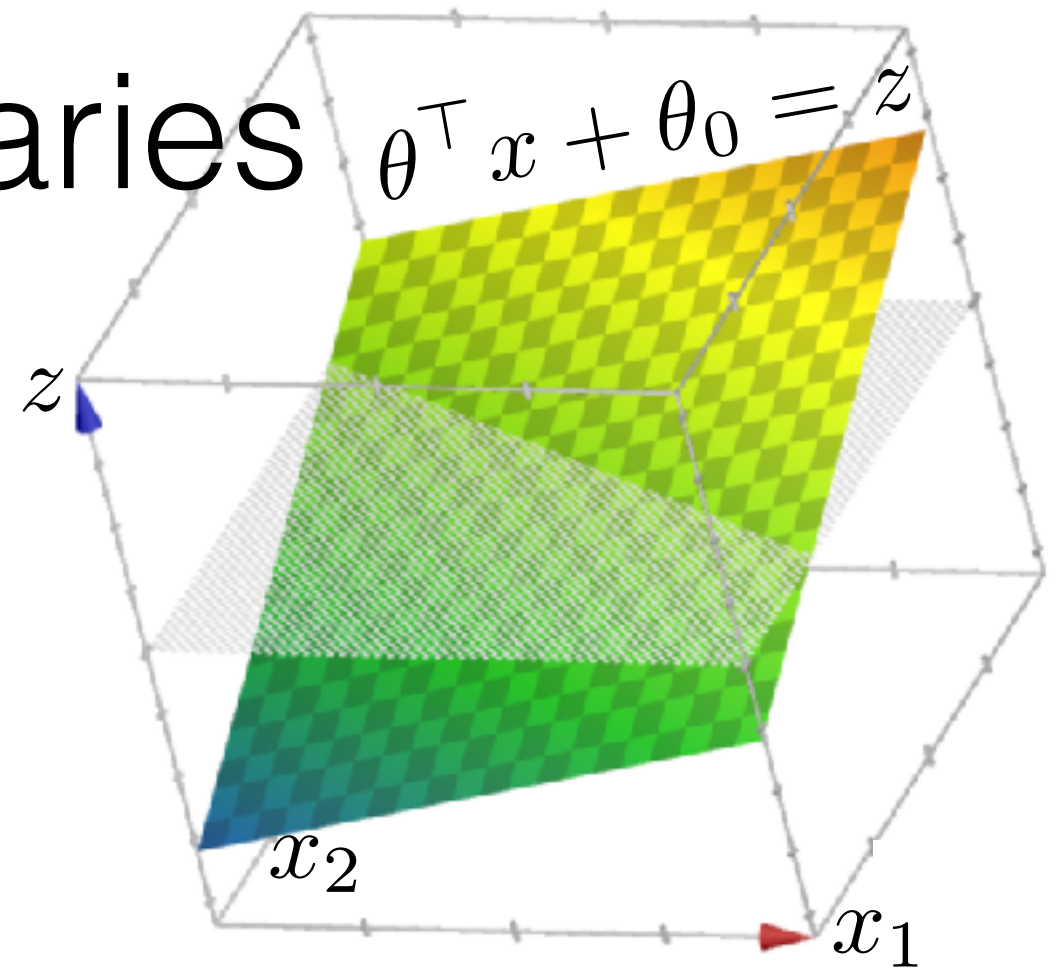
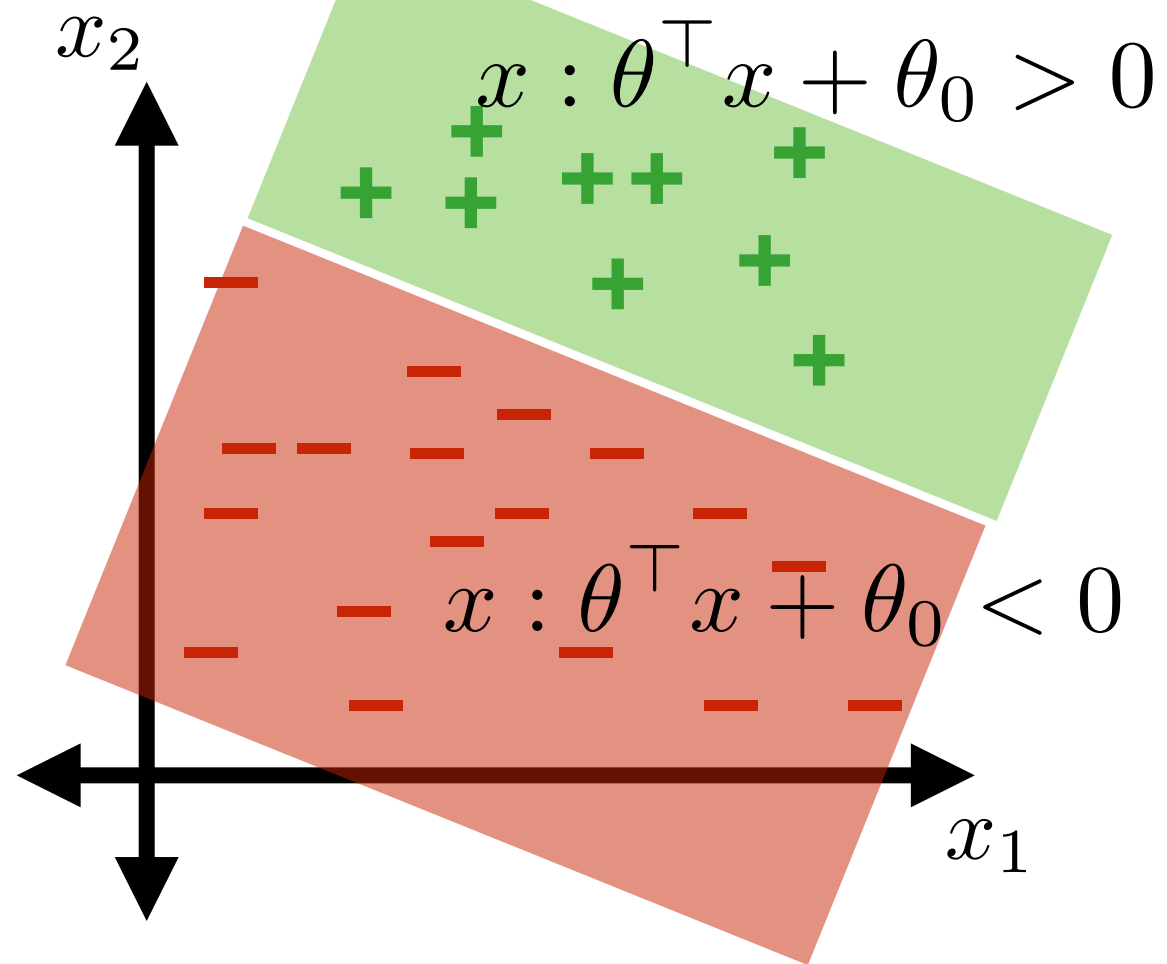
Classification boundaries



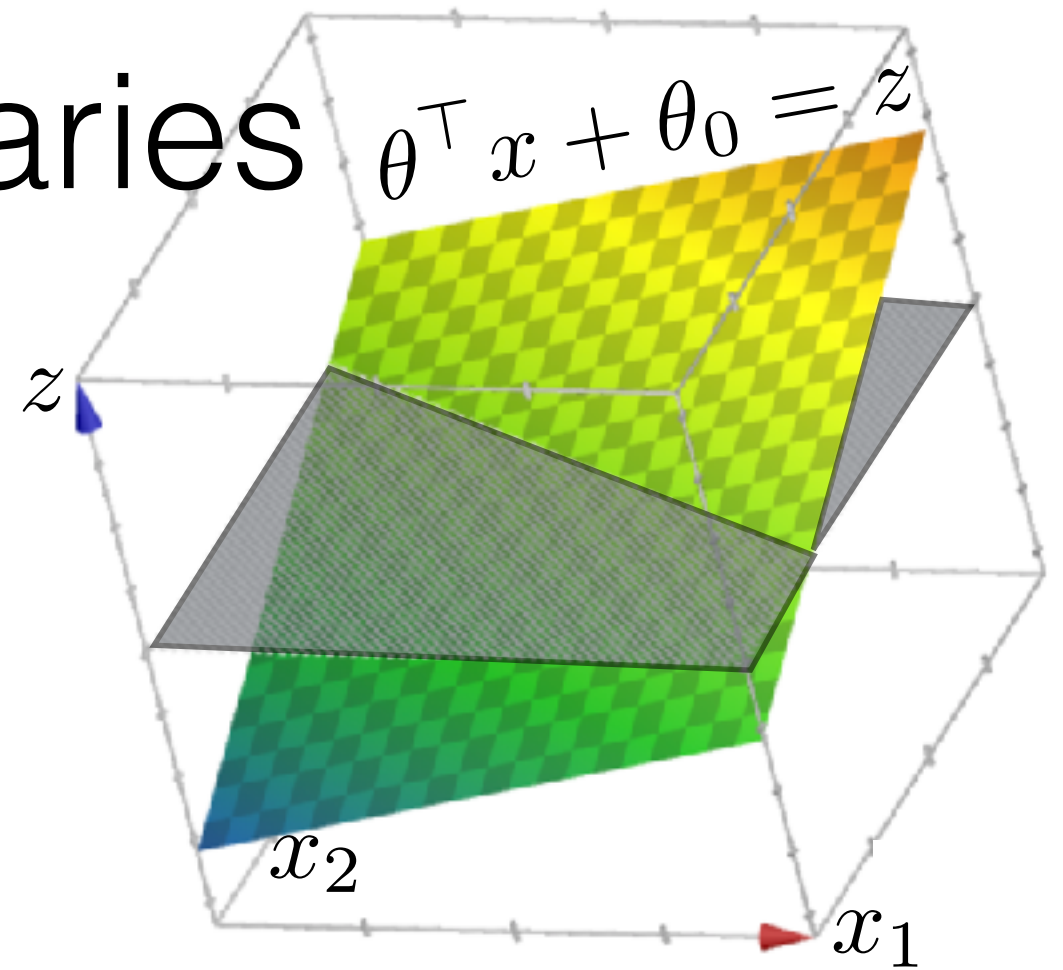
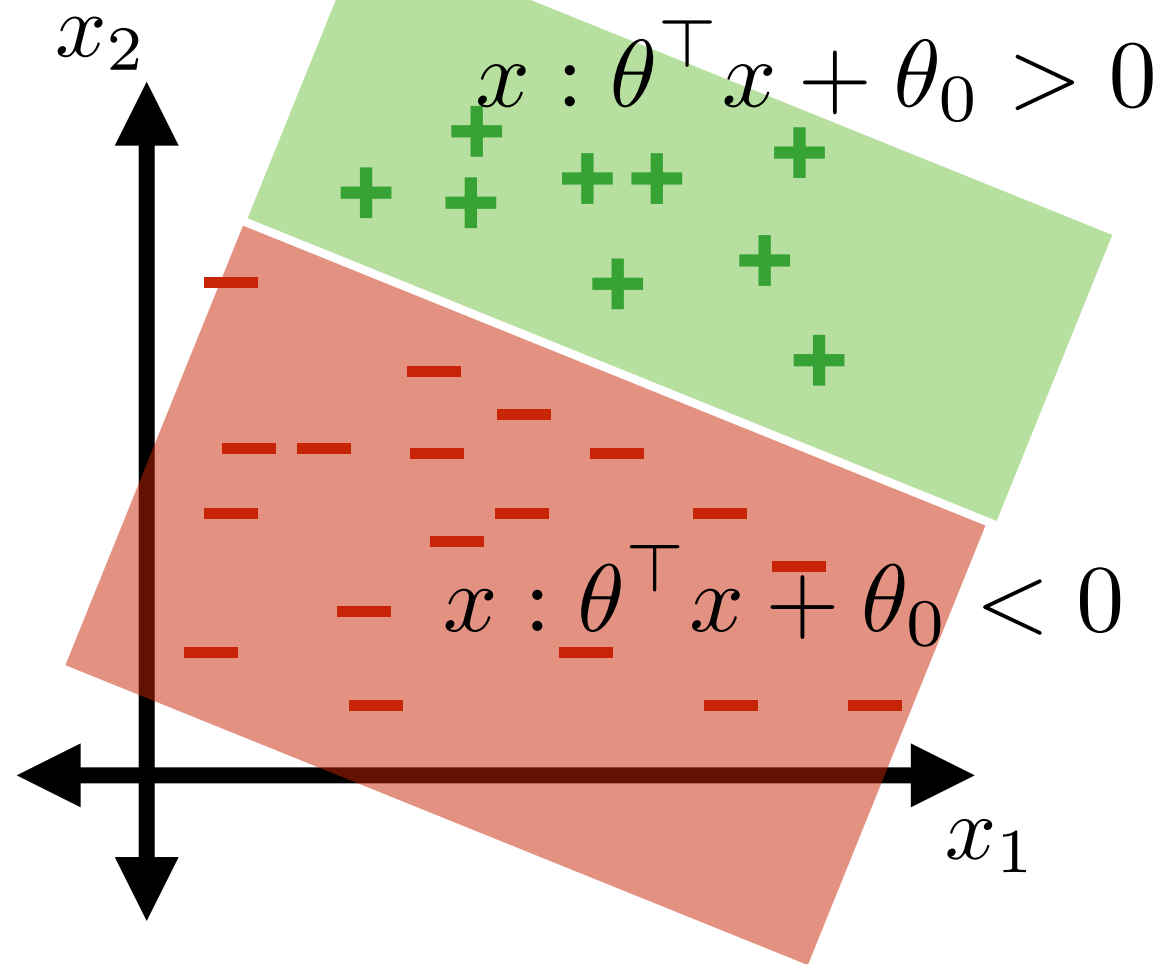
Classification boundaries



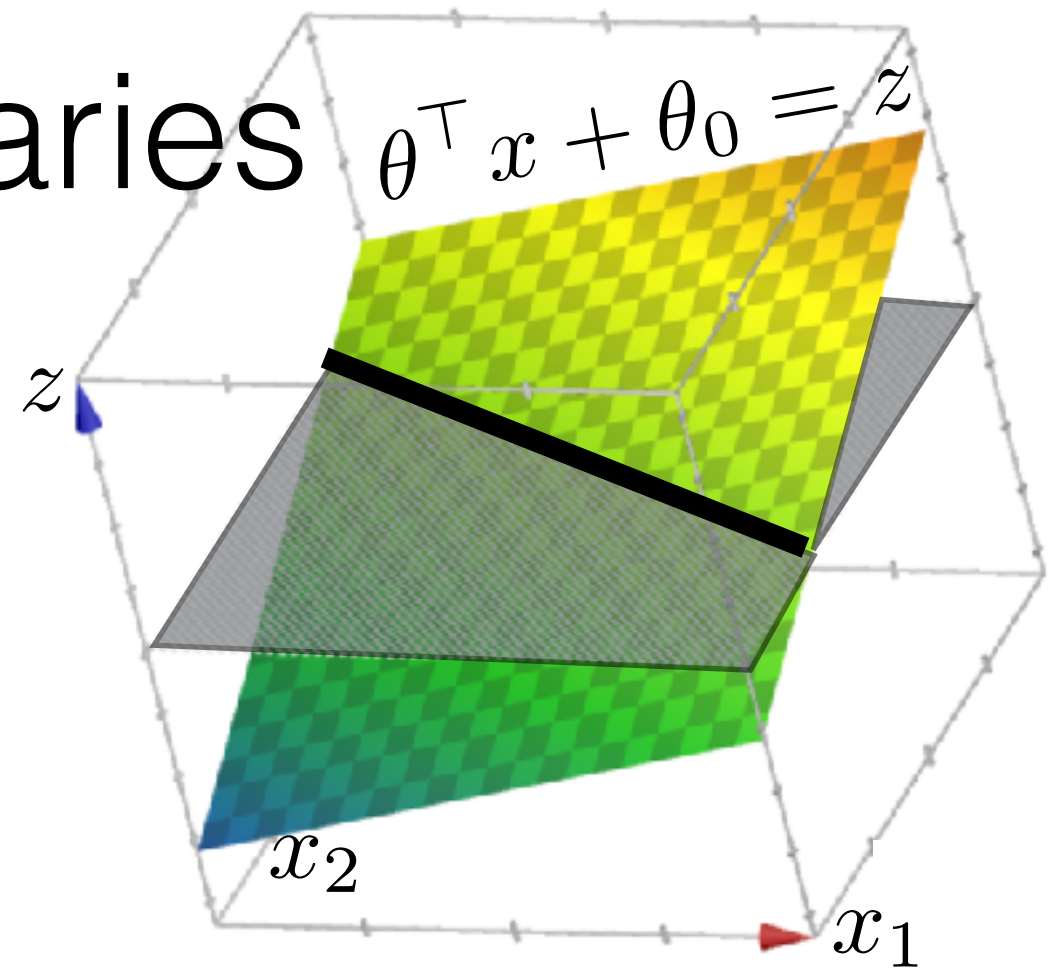
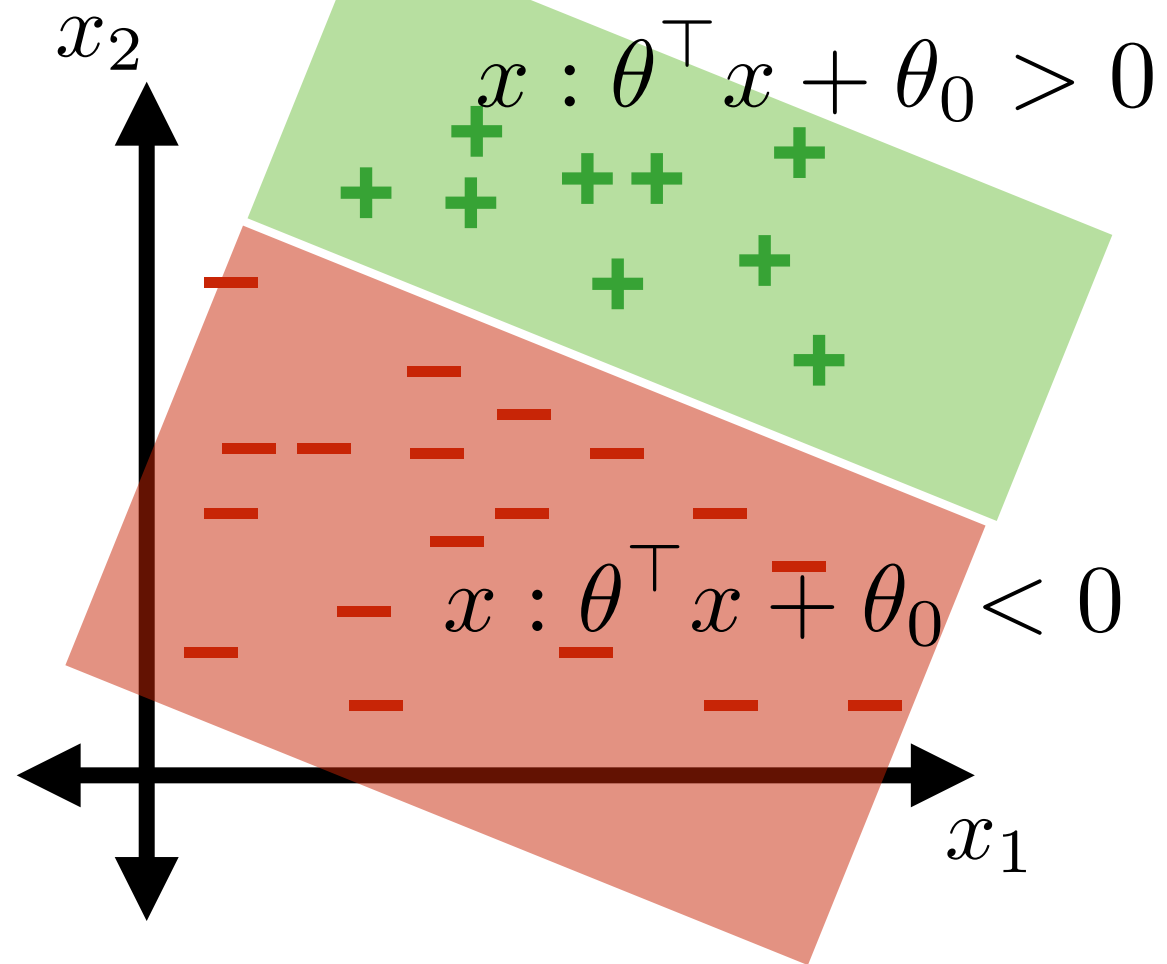
Classification boundaries



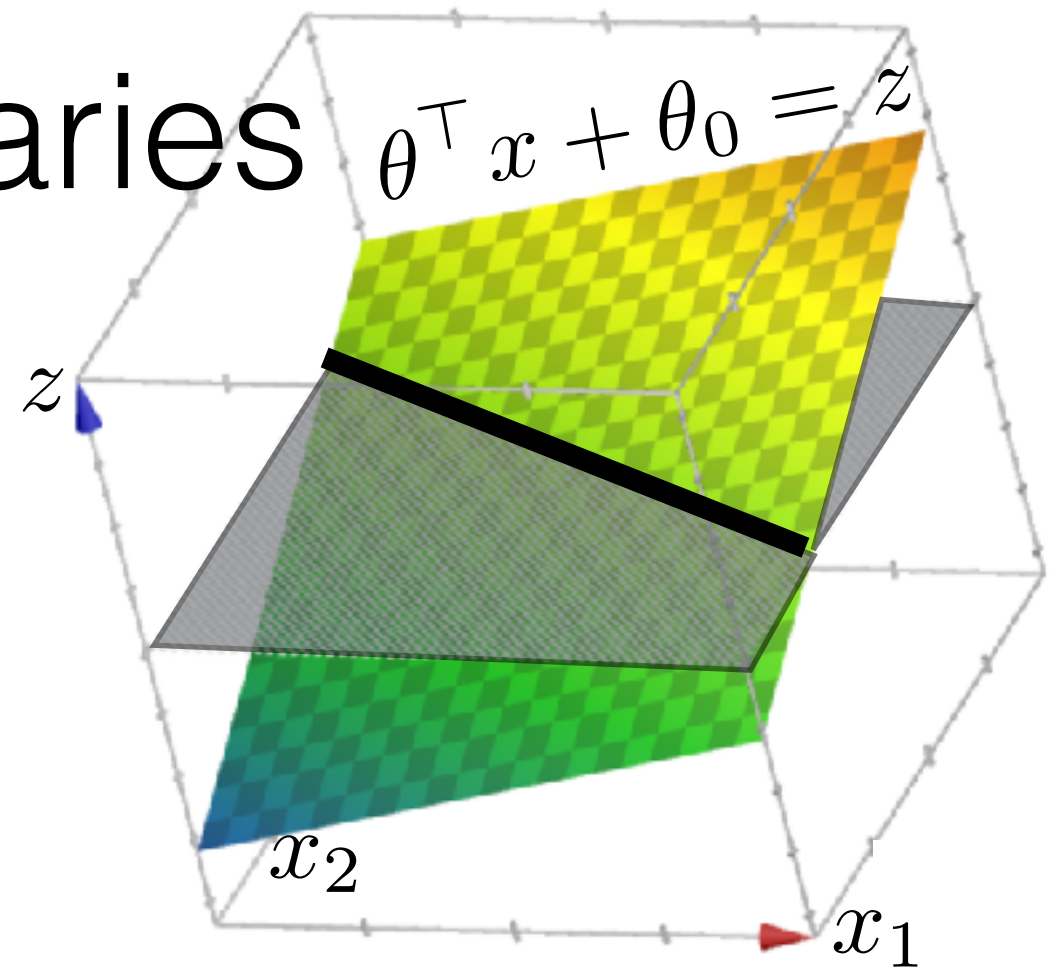
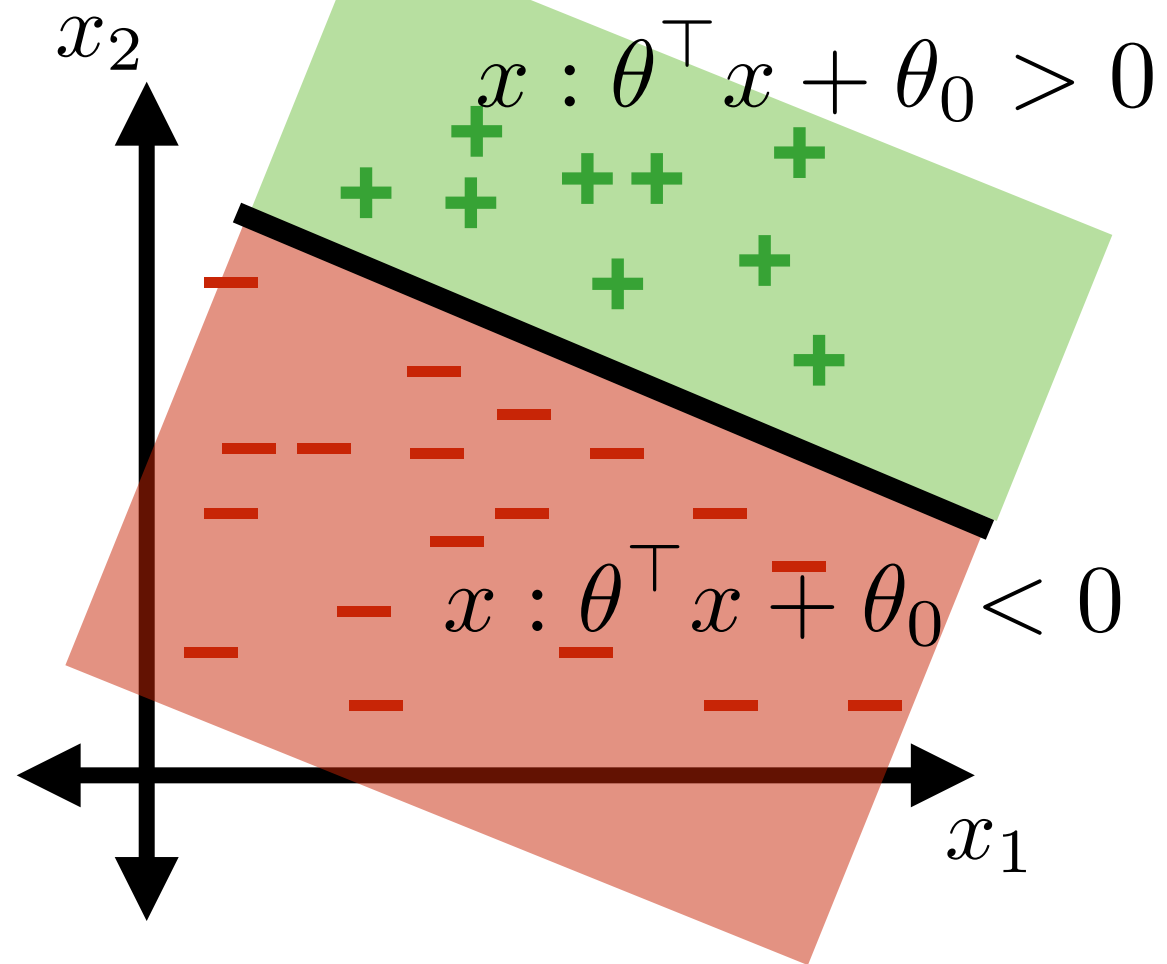
Classification boundaries



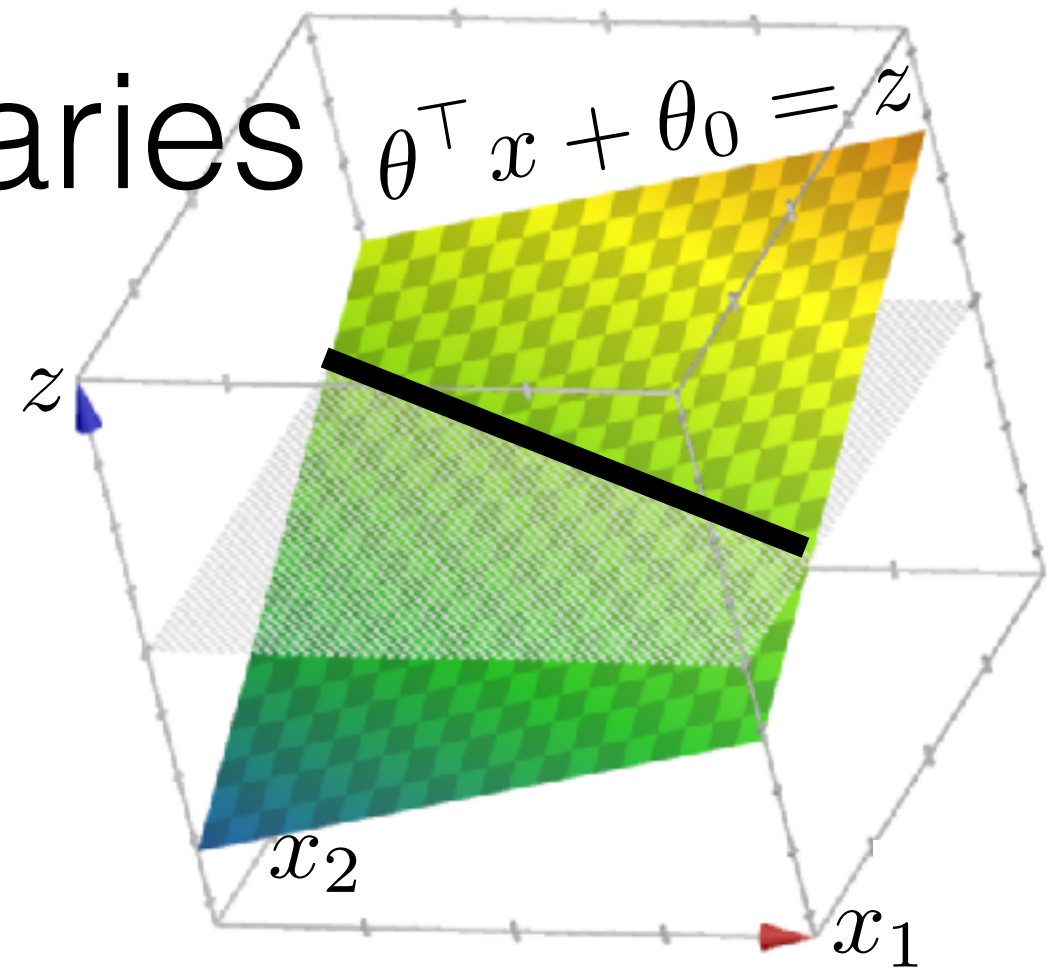
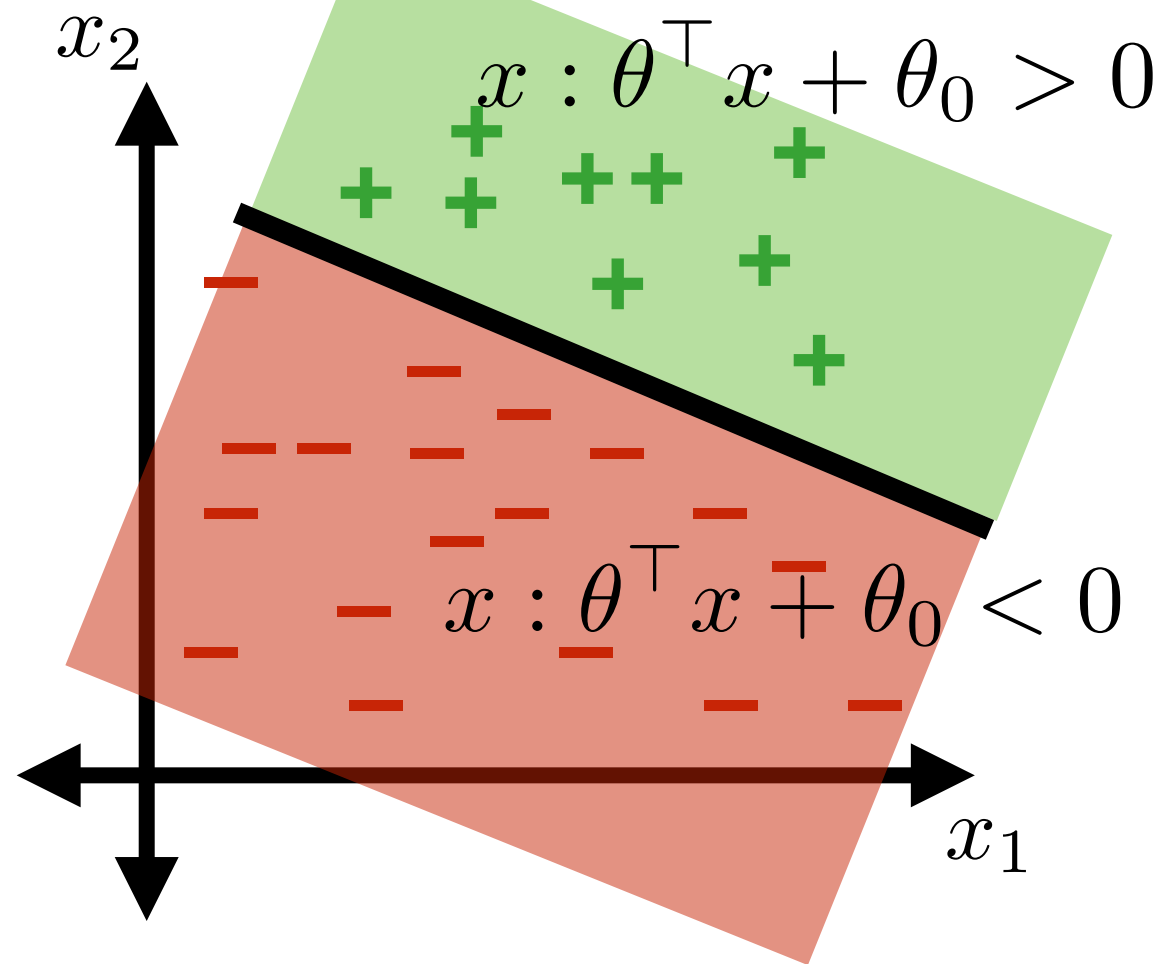
Classification boundaries



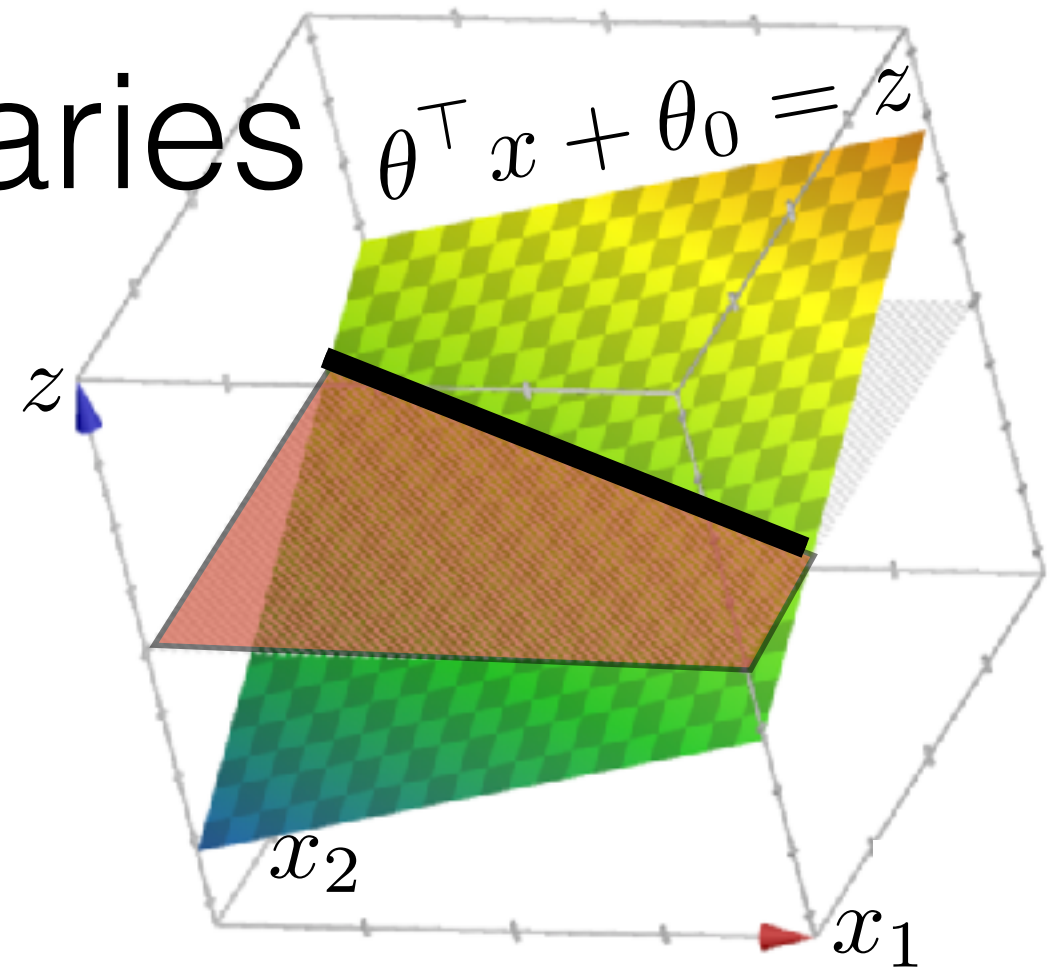
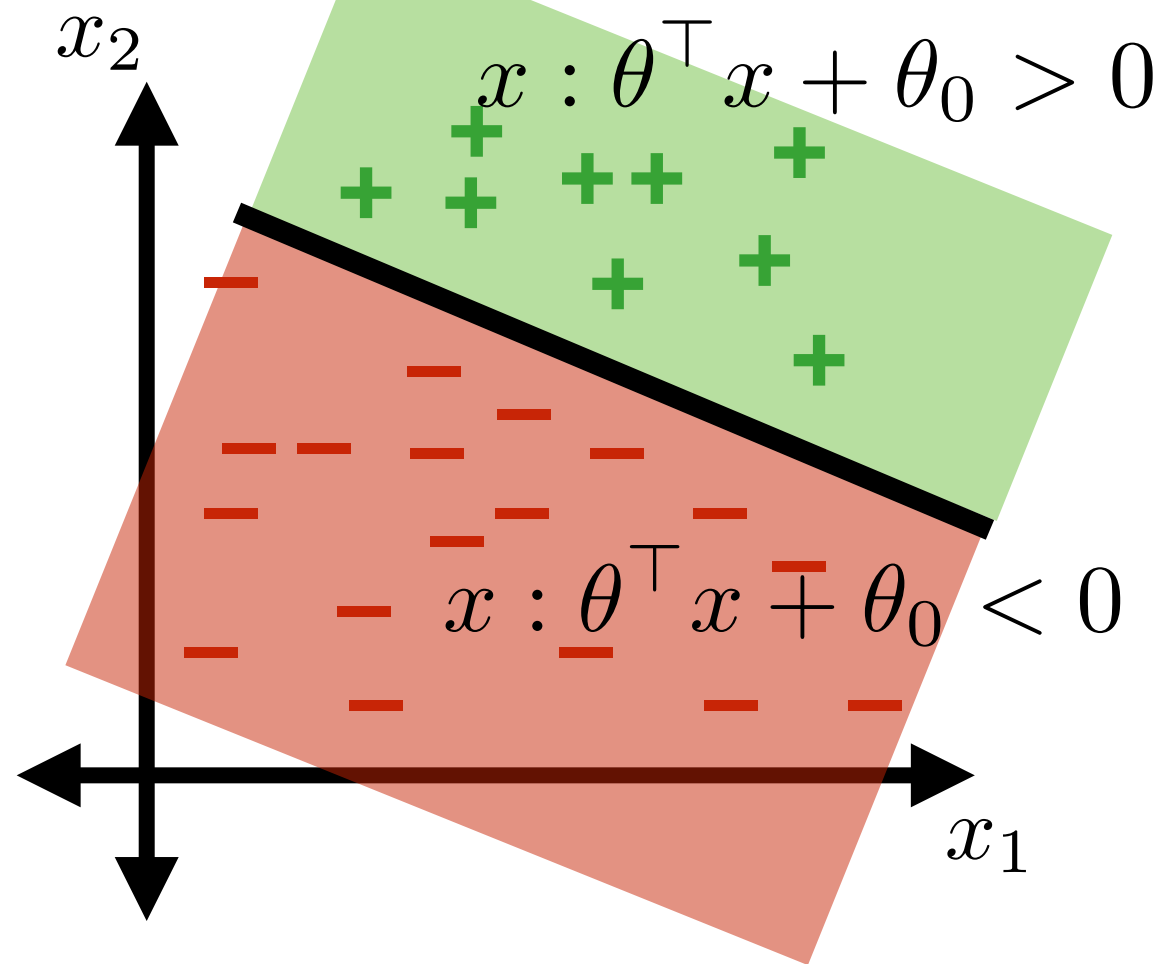
Classification boundaries



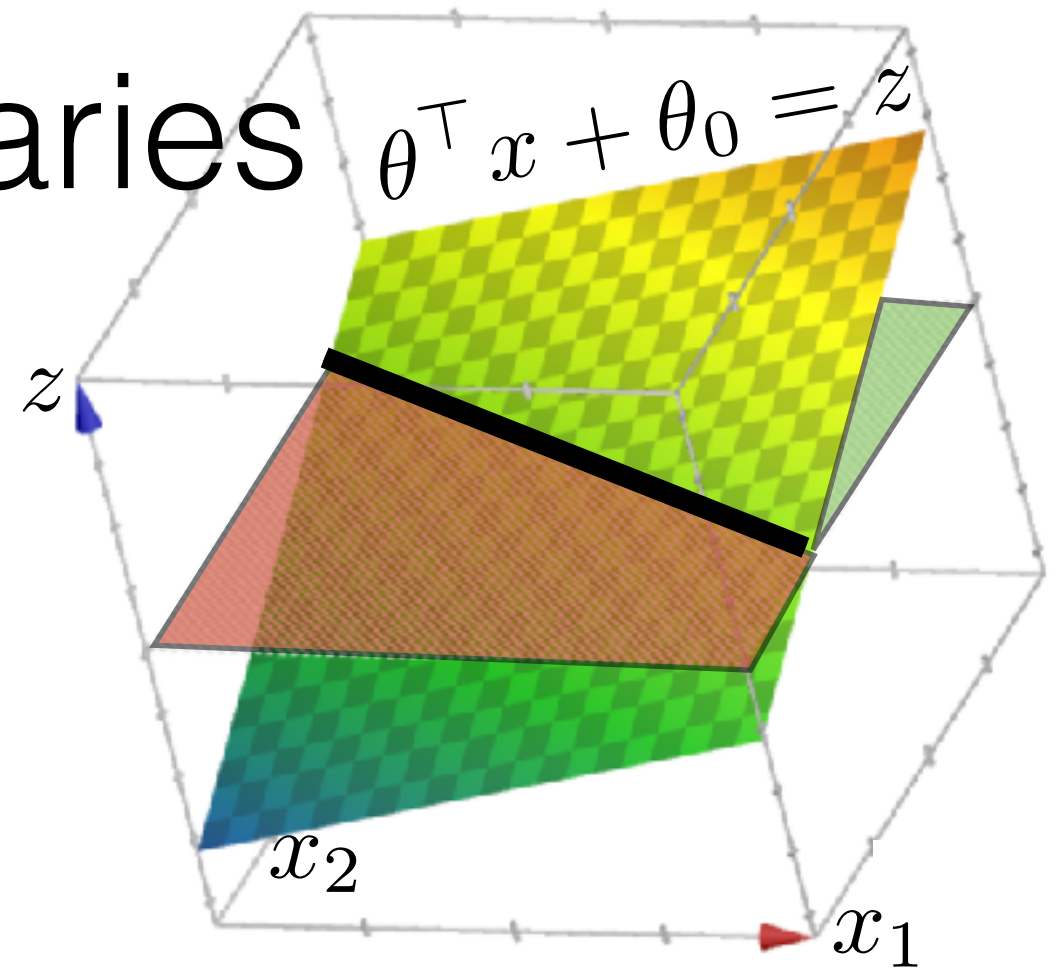
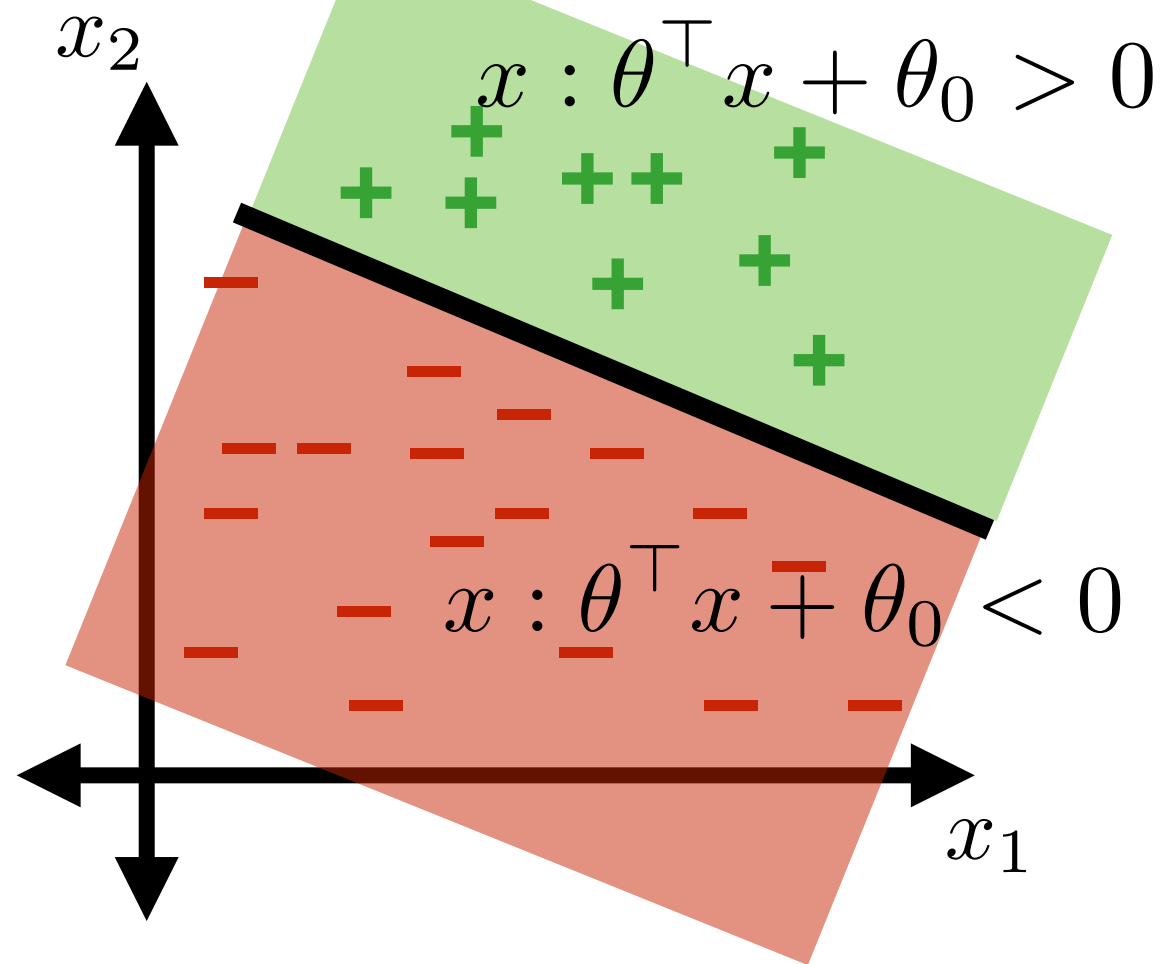
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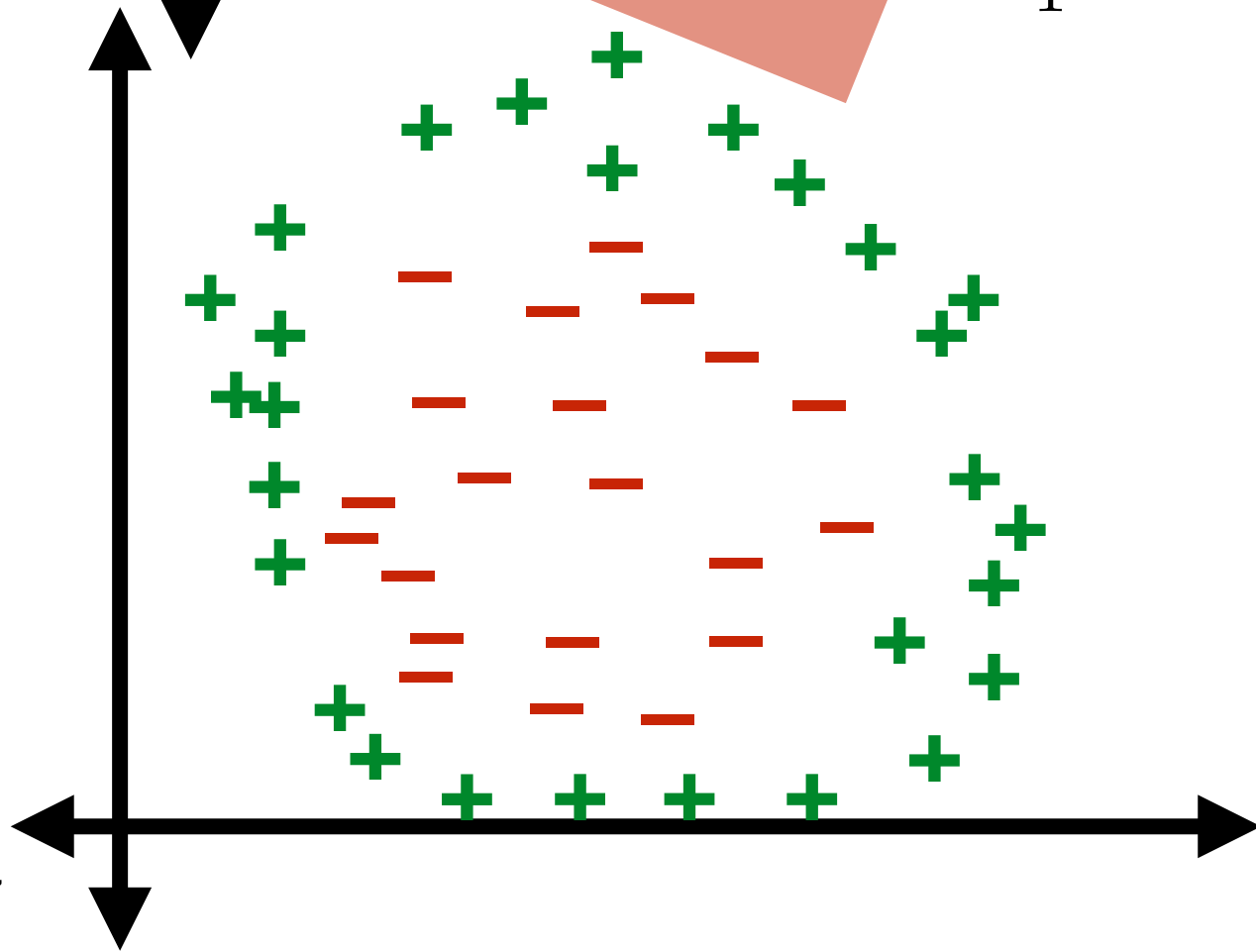
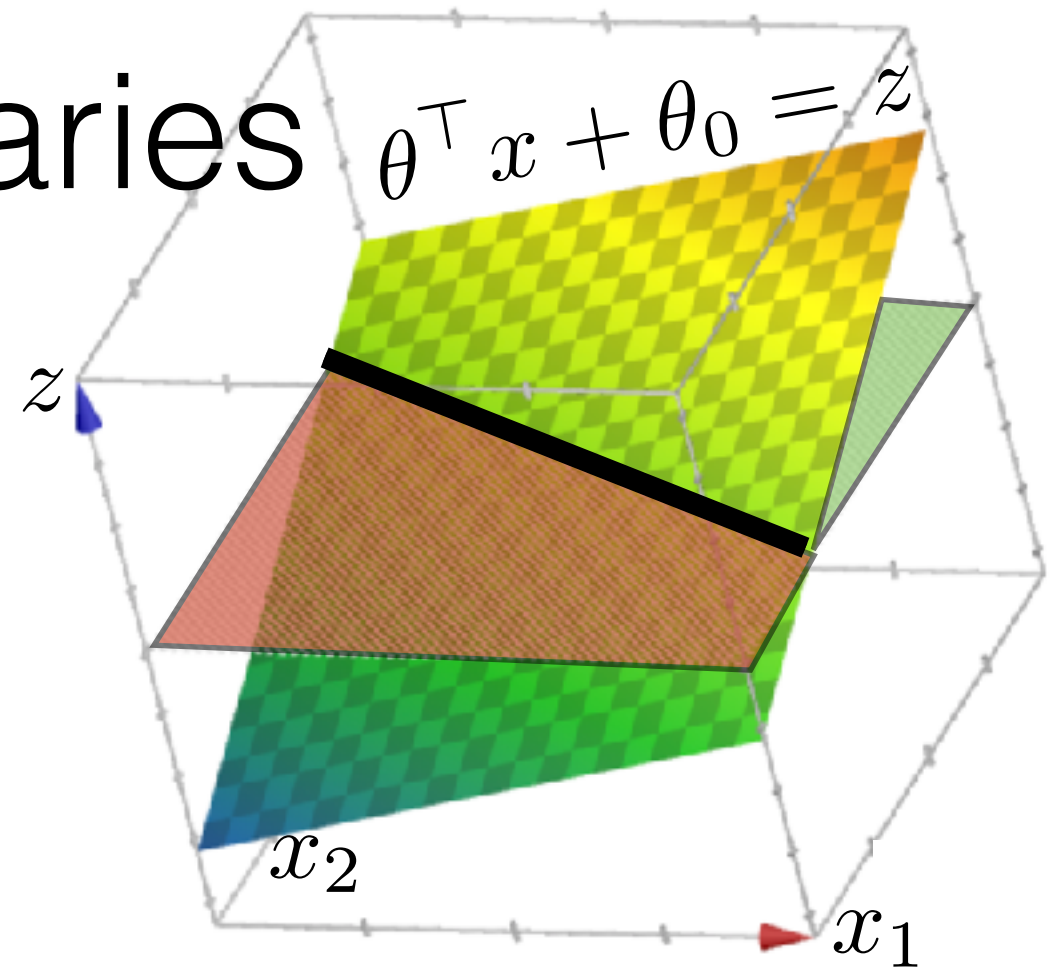
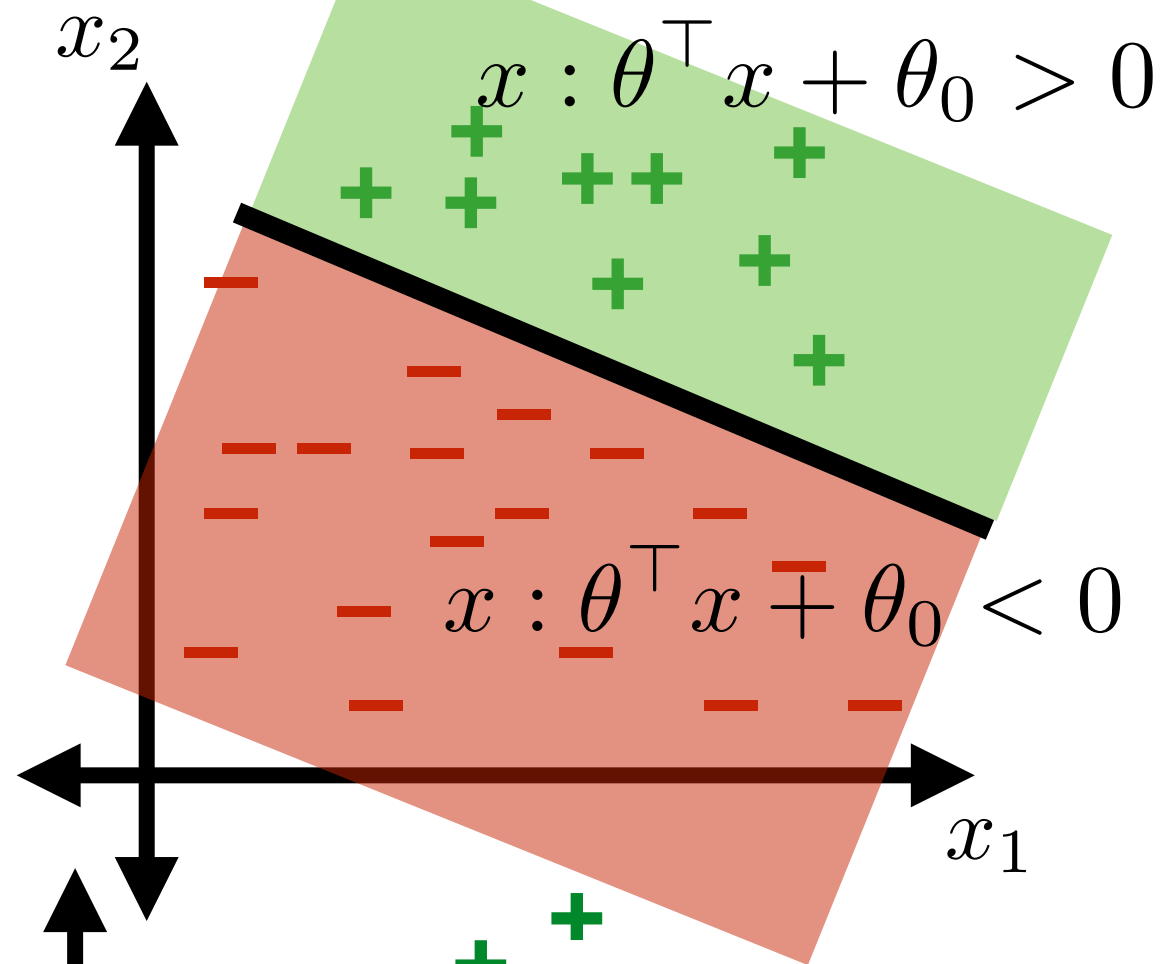
Classification boundaries



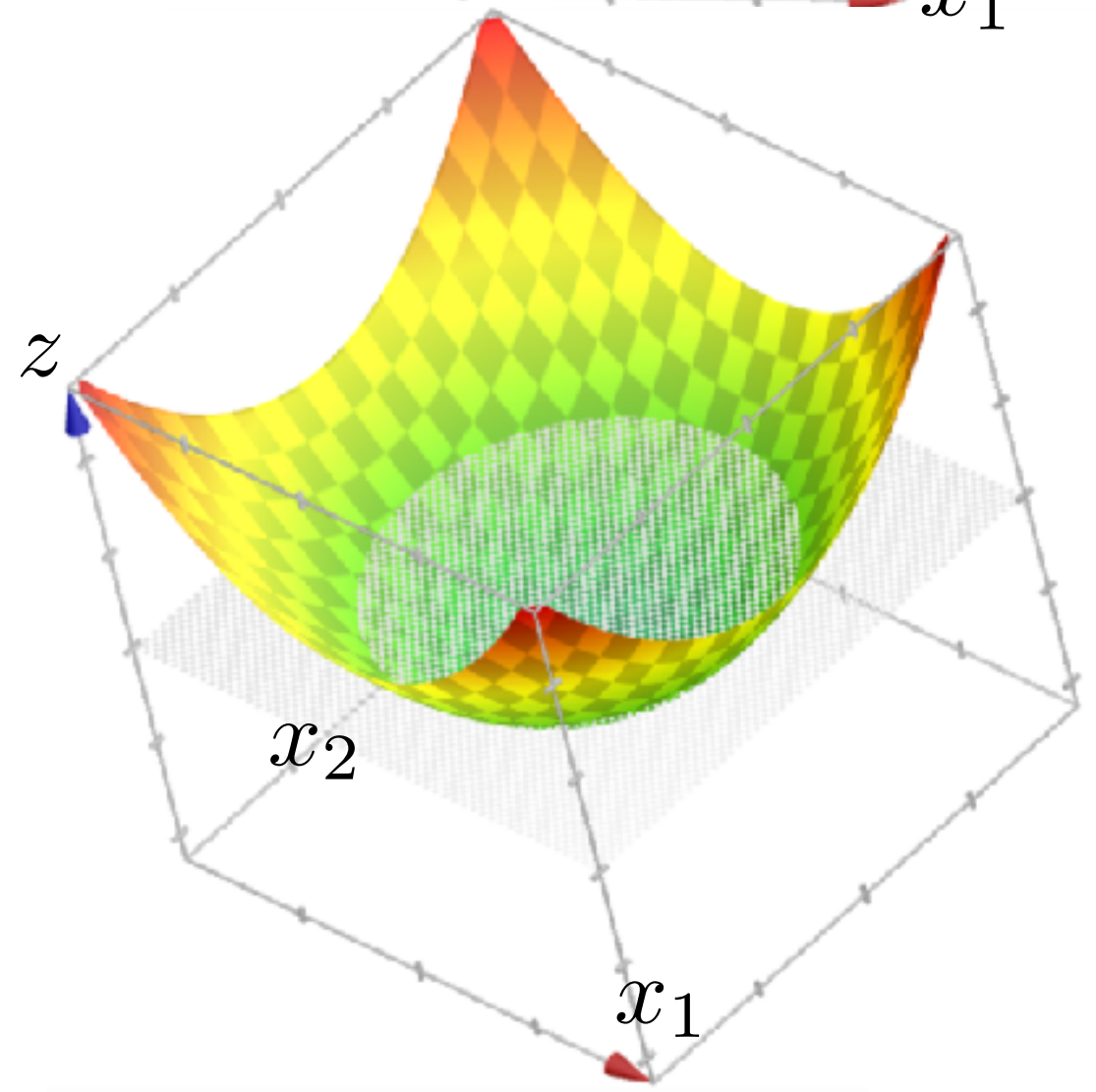
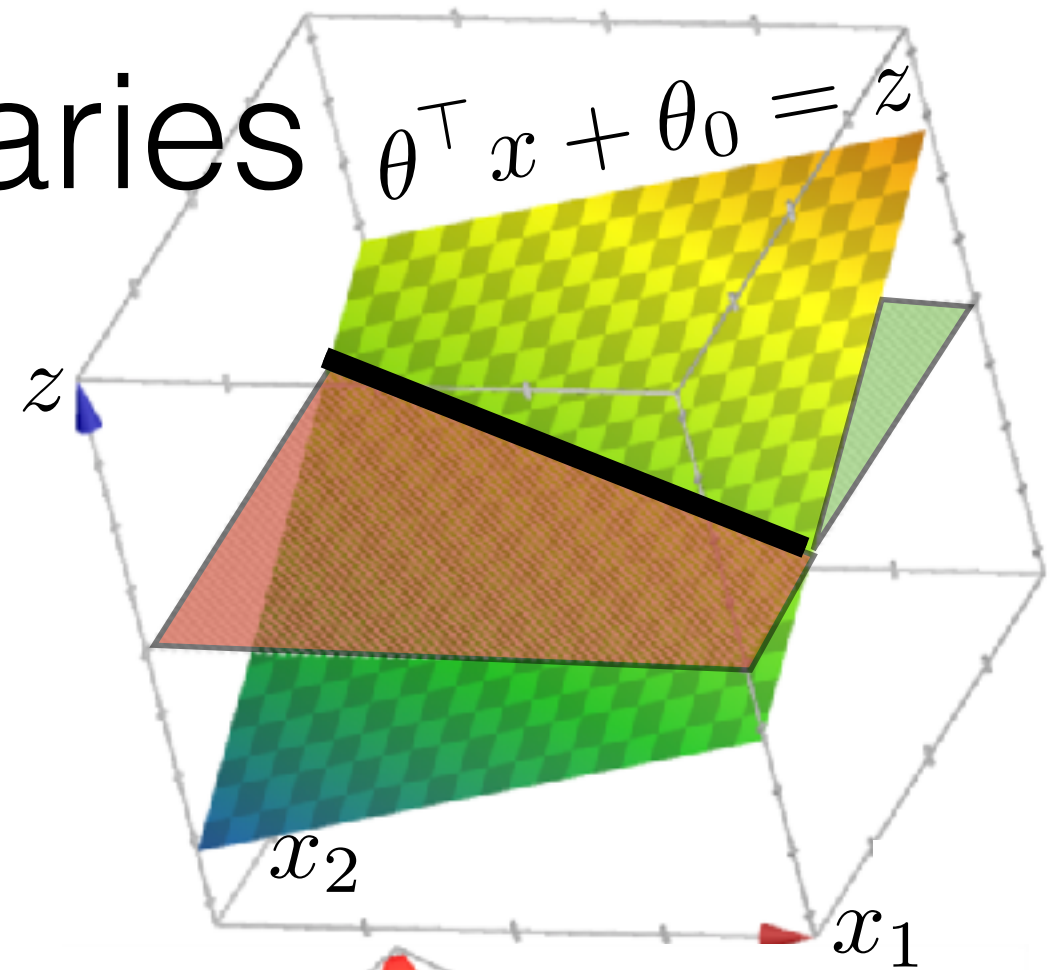
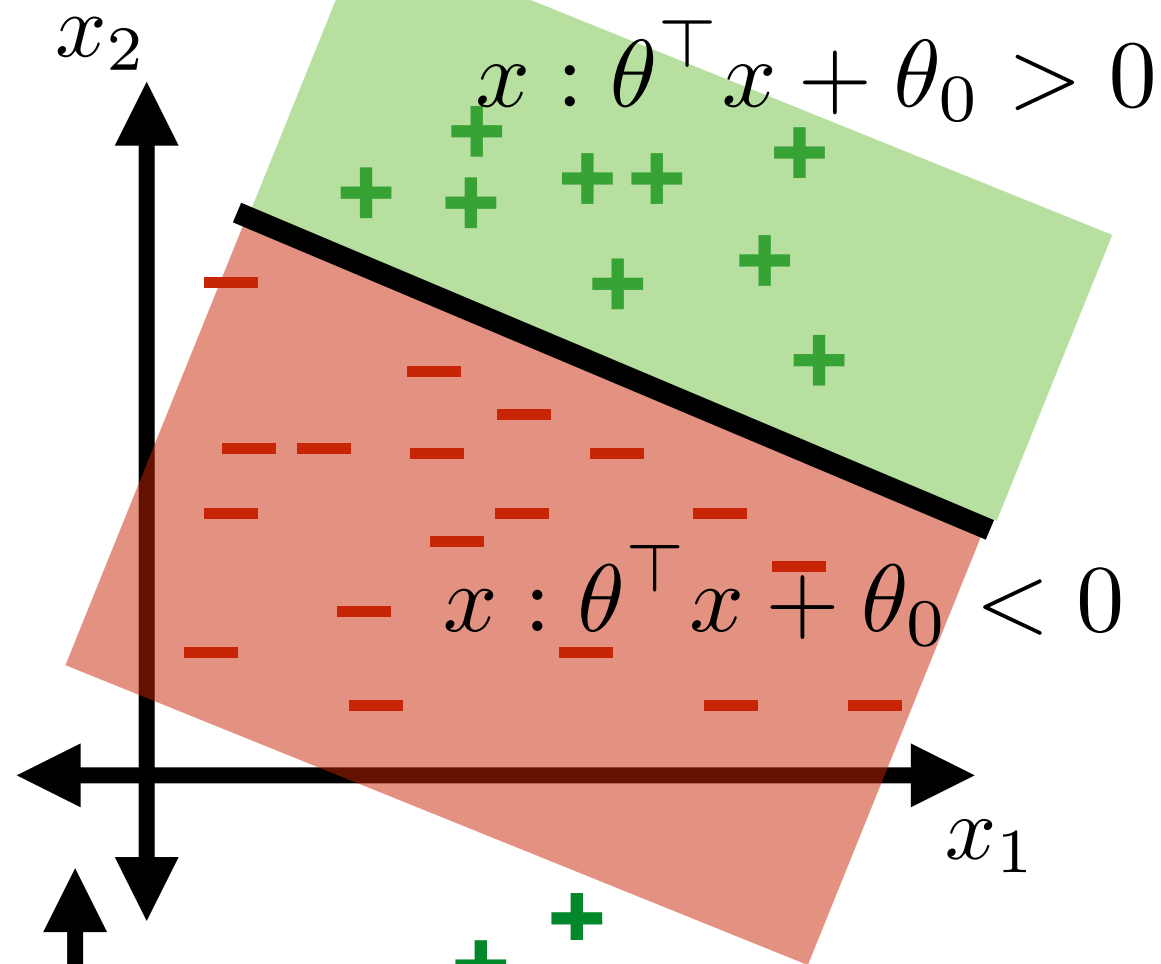
Classification boundaries



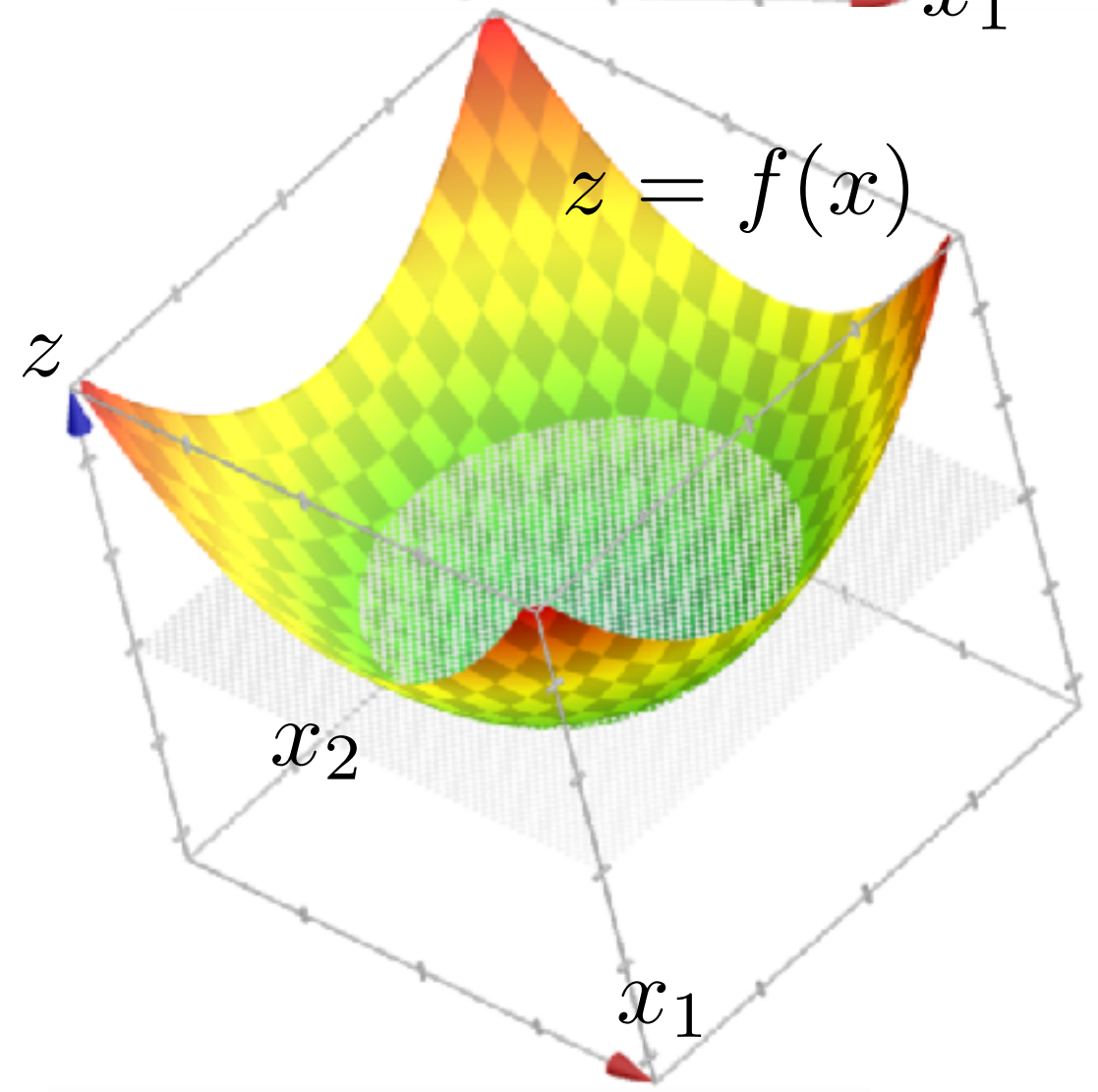
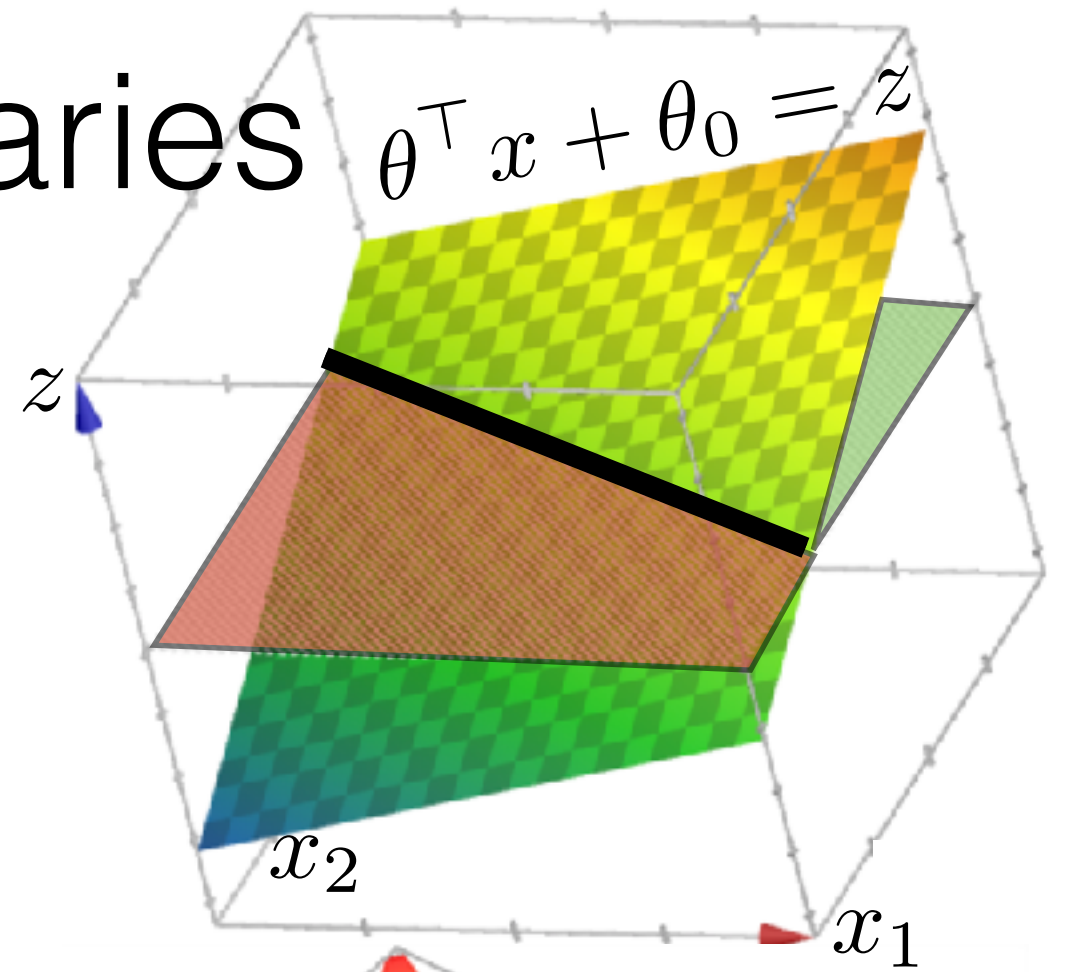
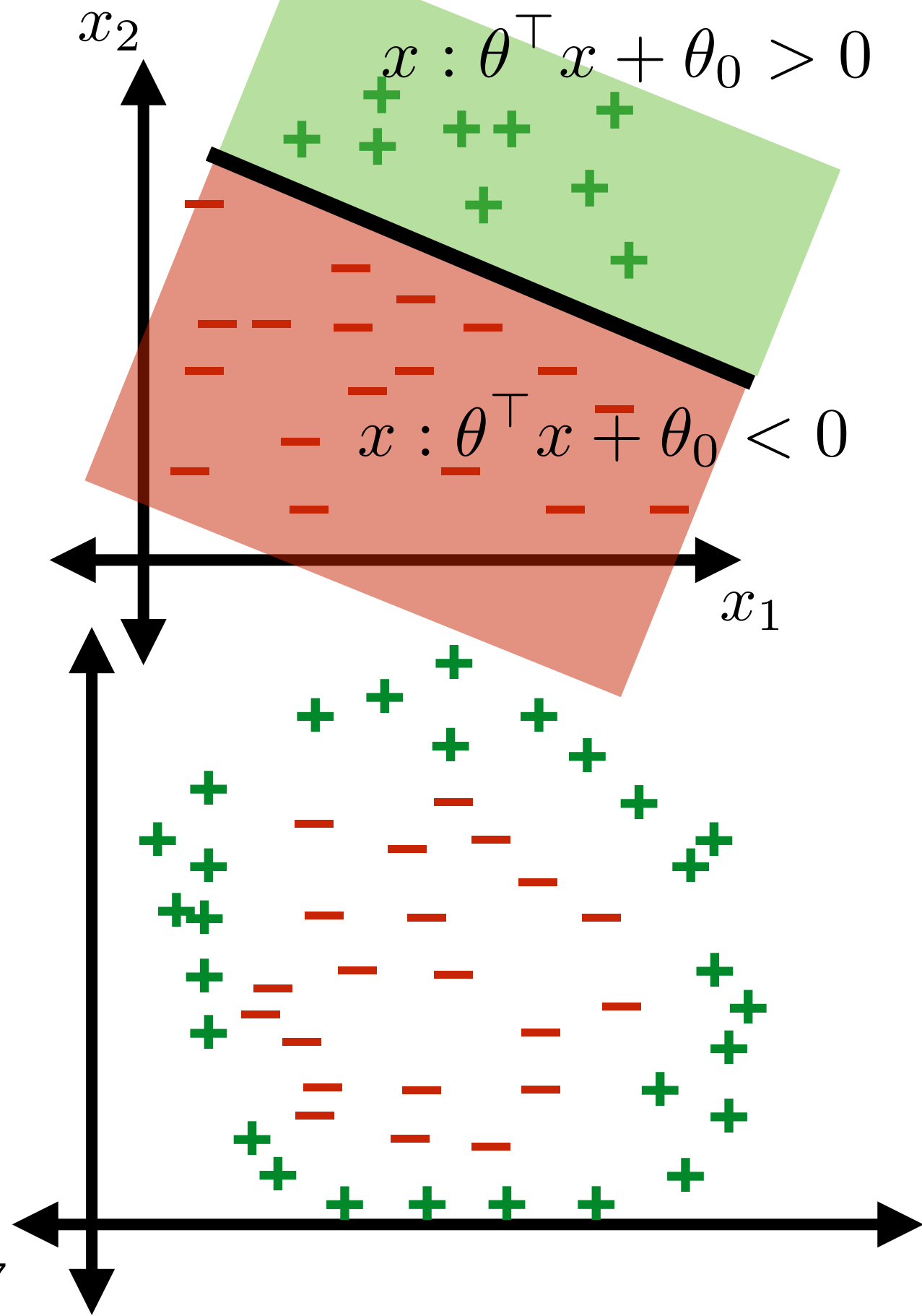
Classification boundaries



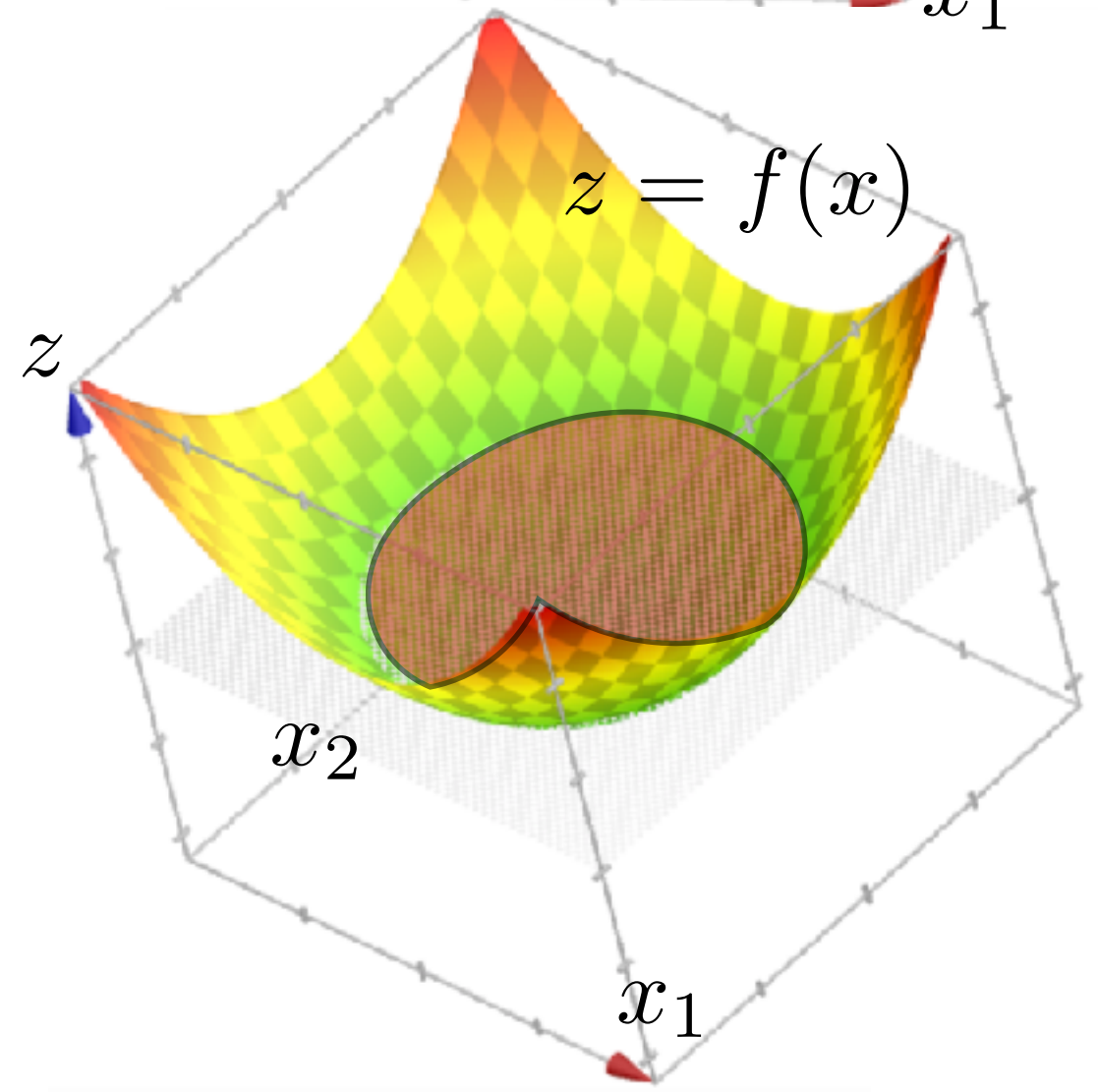
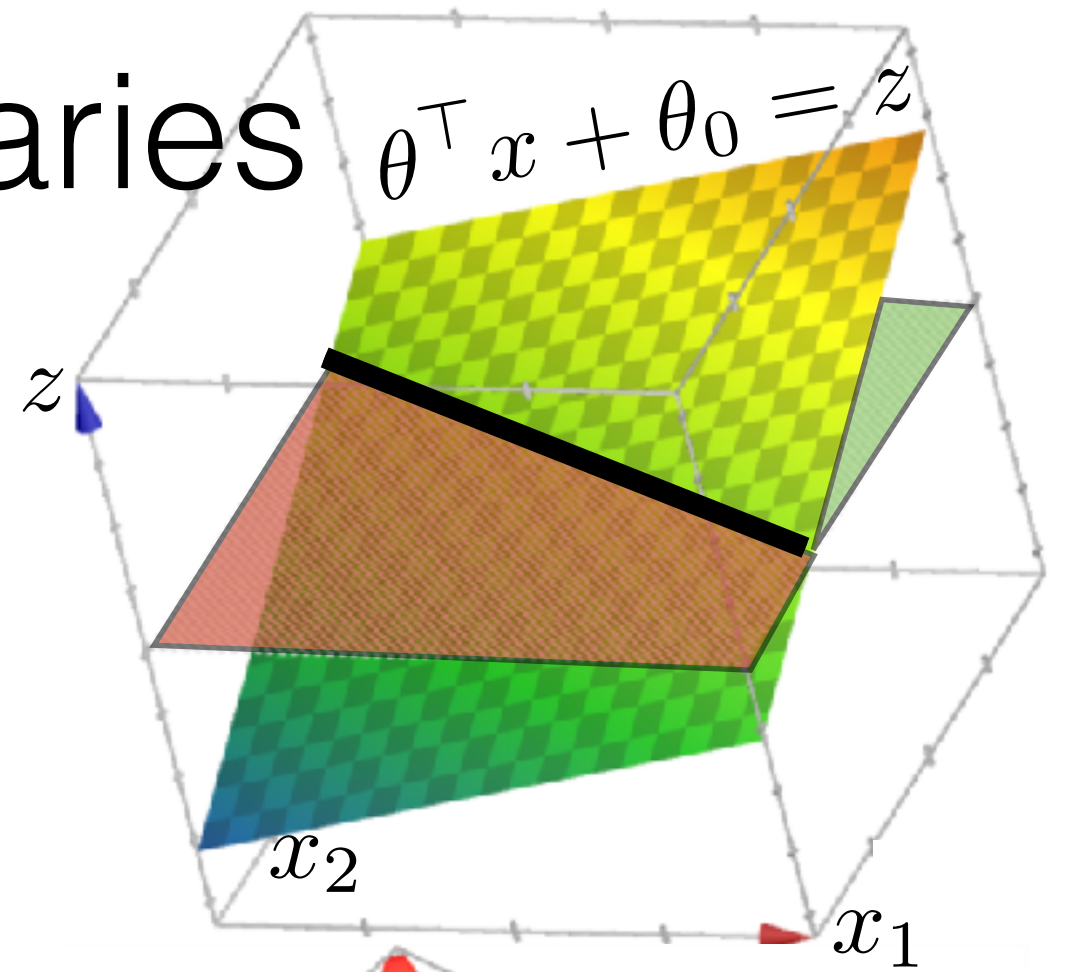
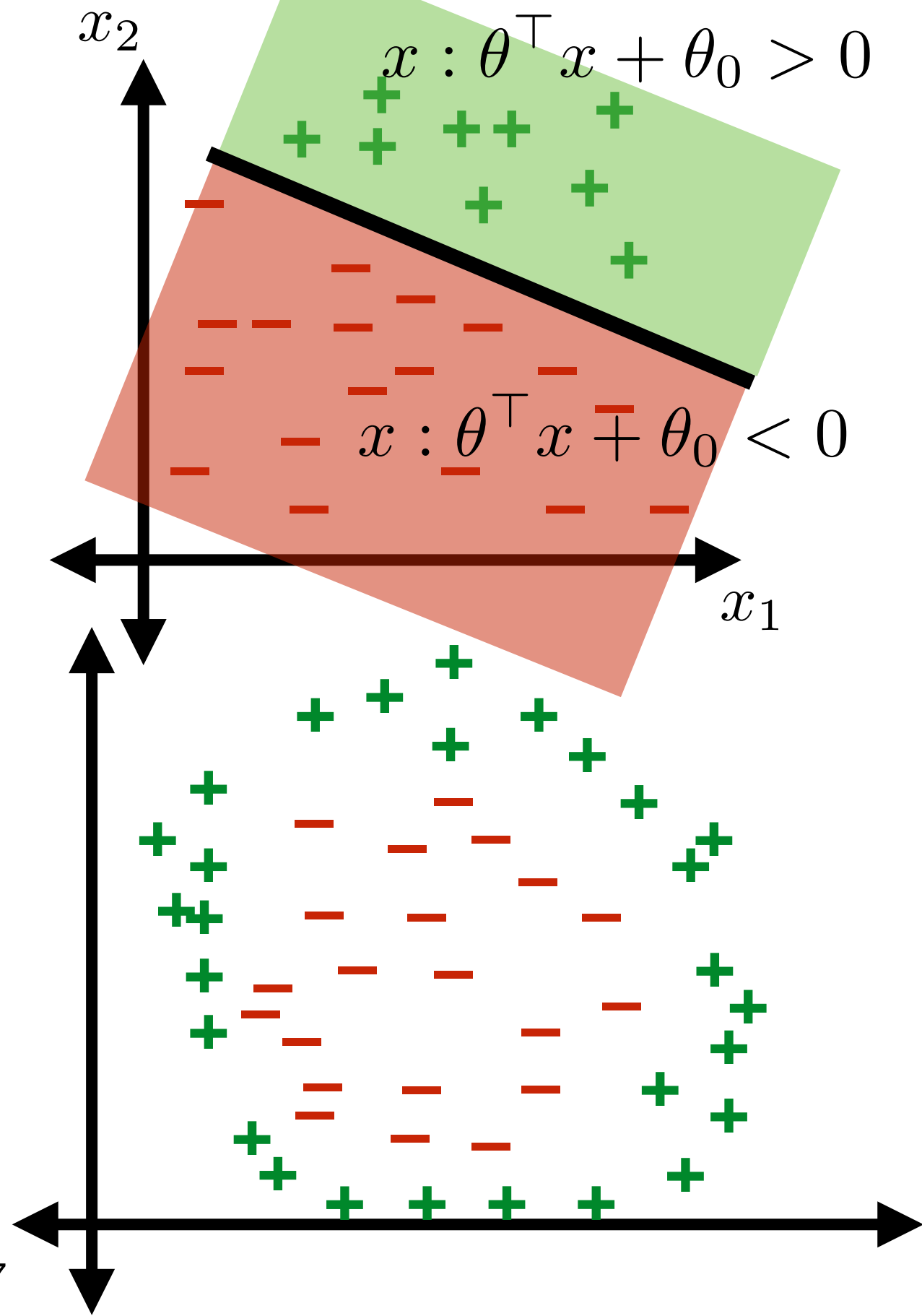
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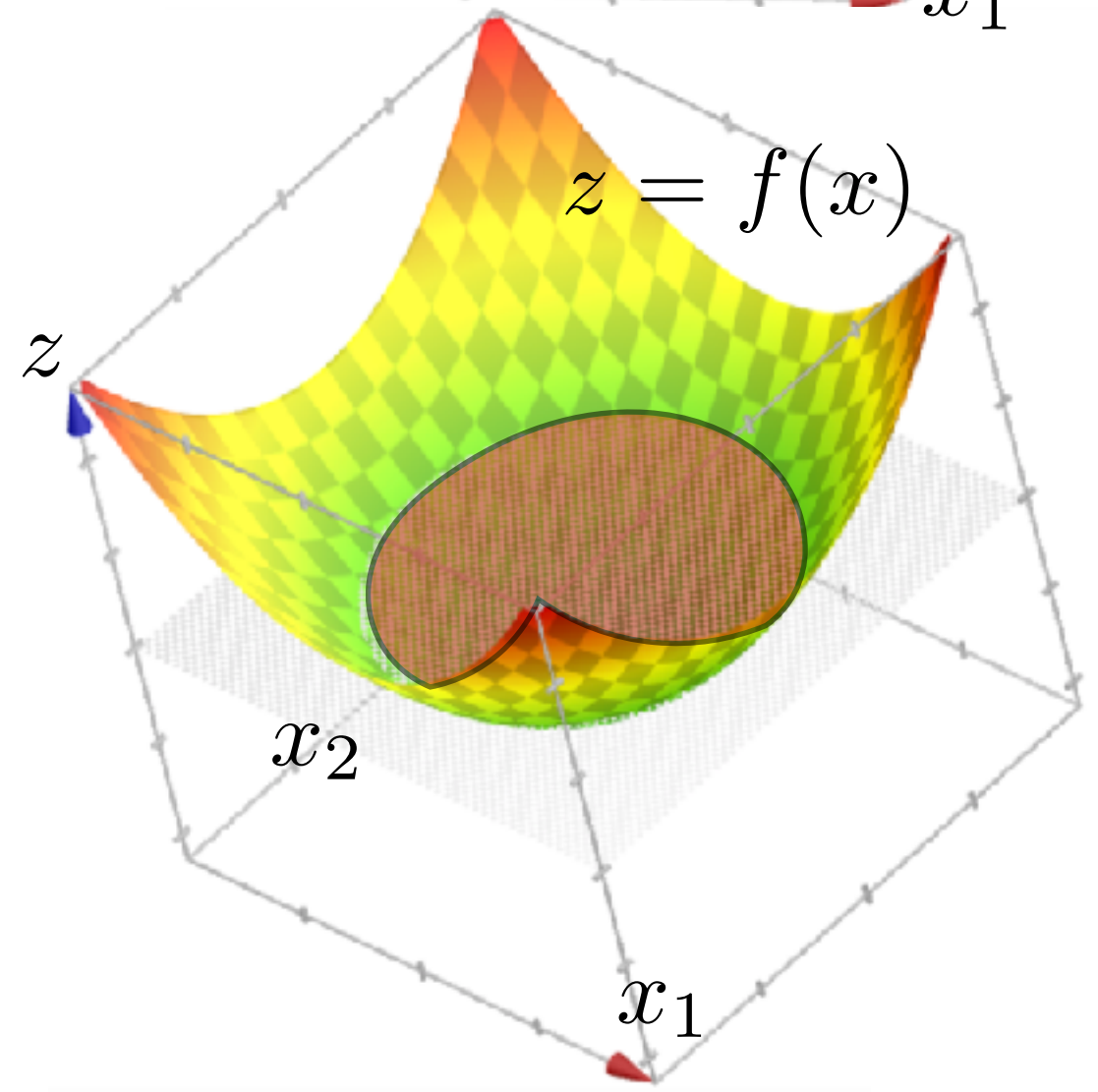
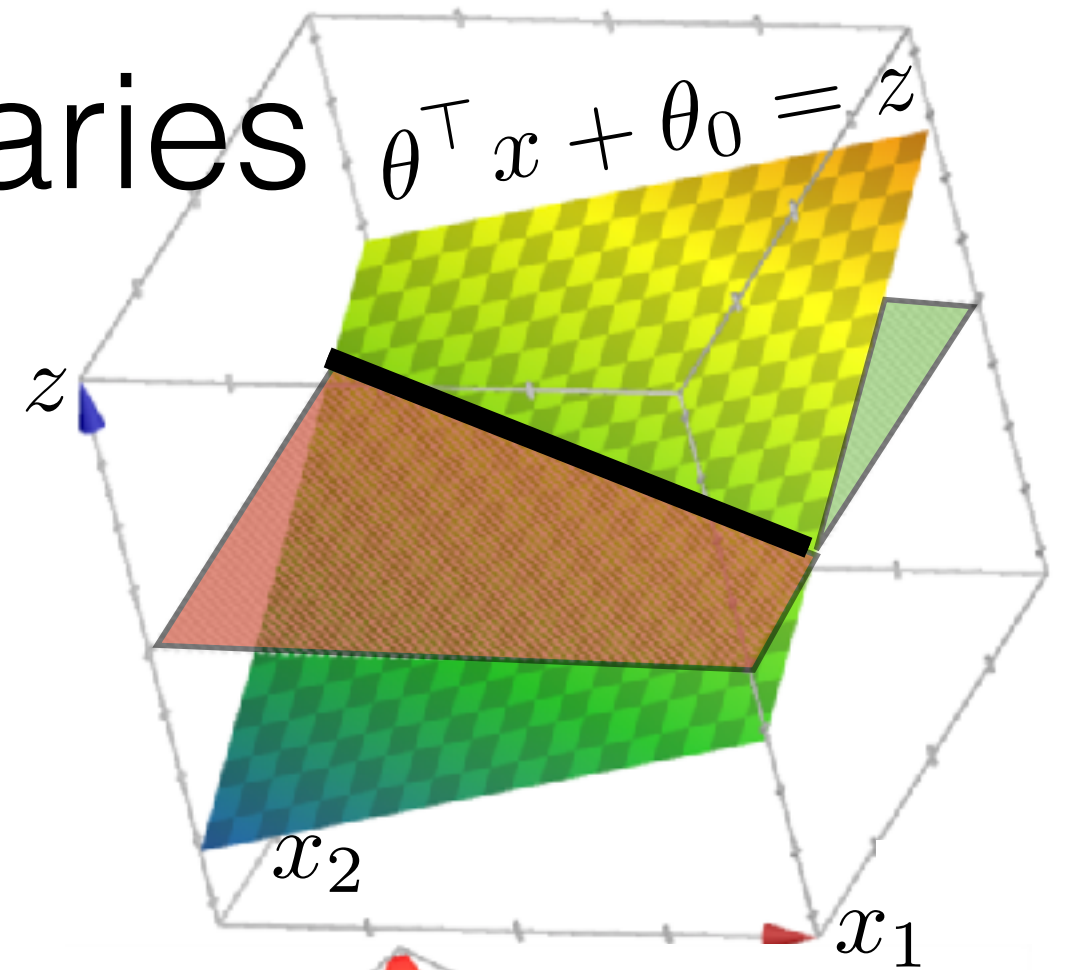
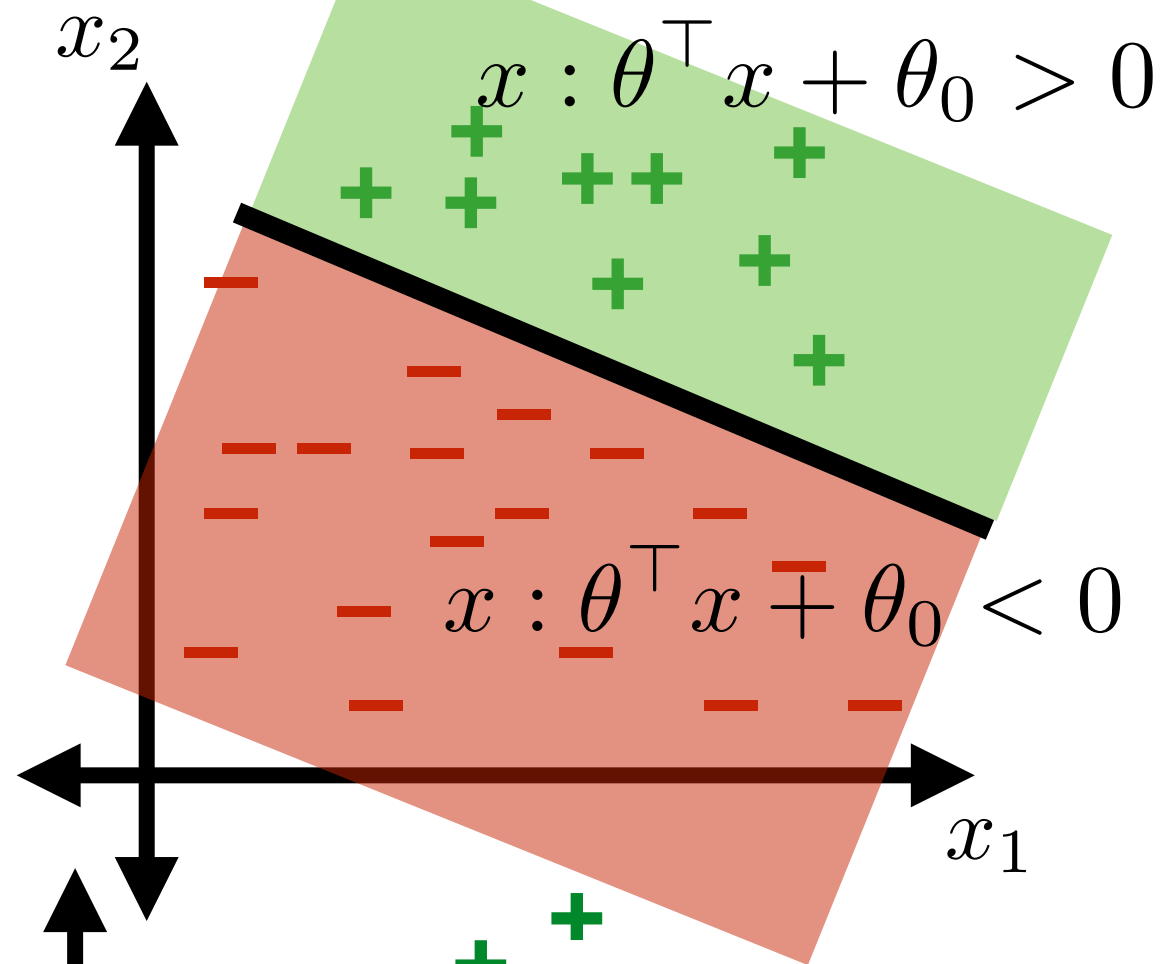
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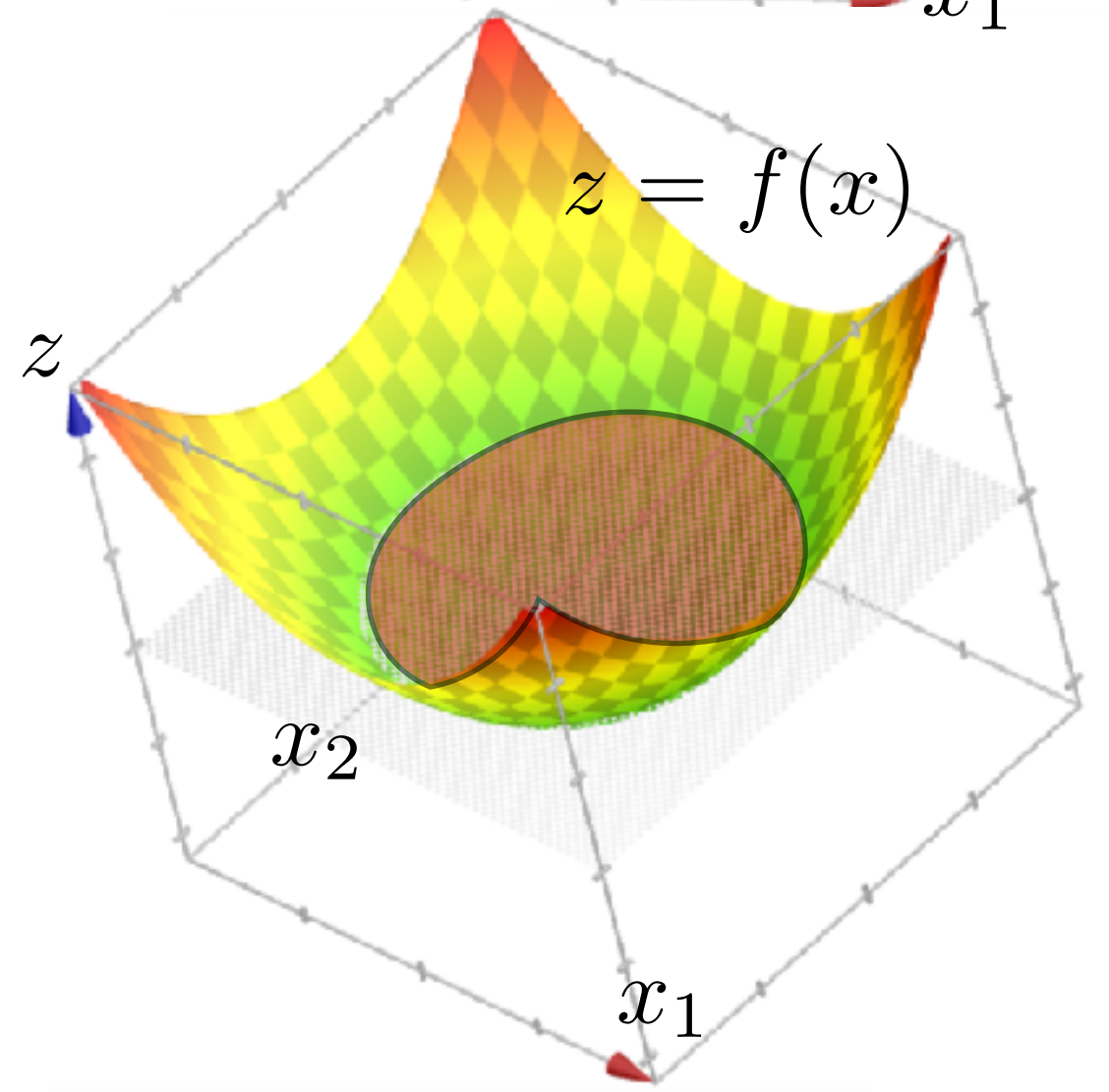
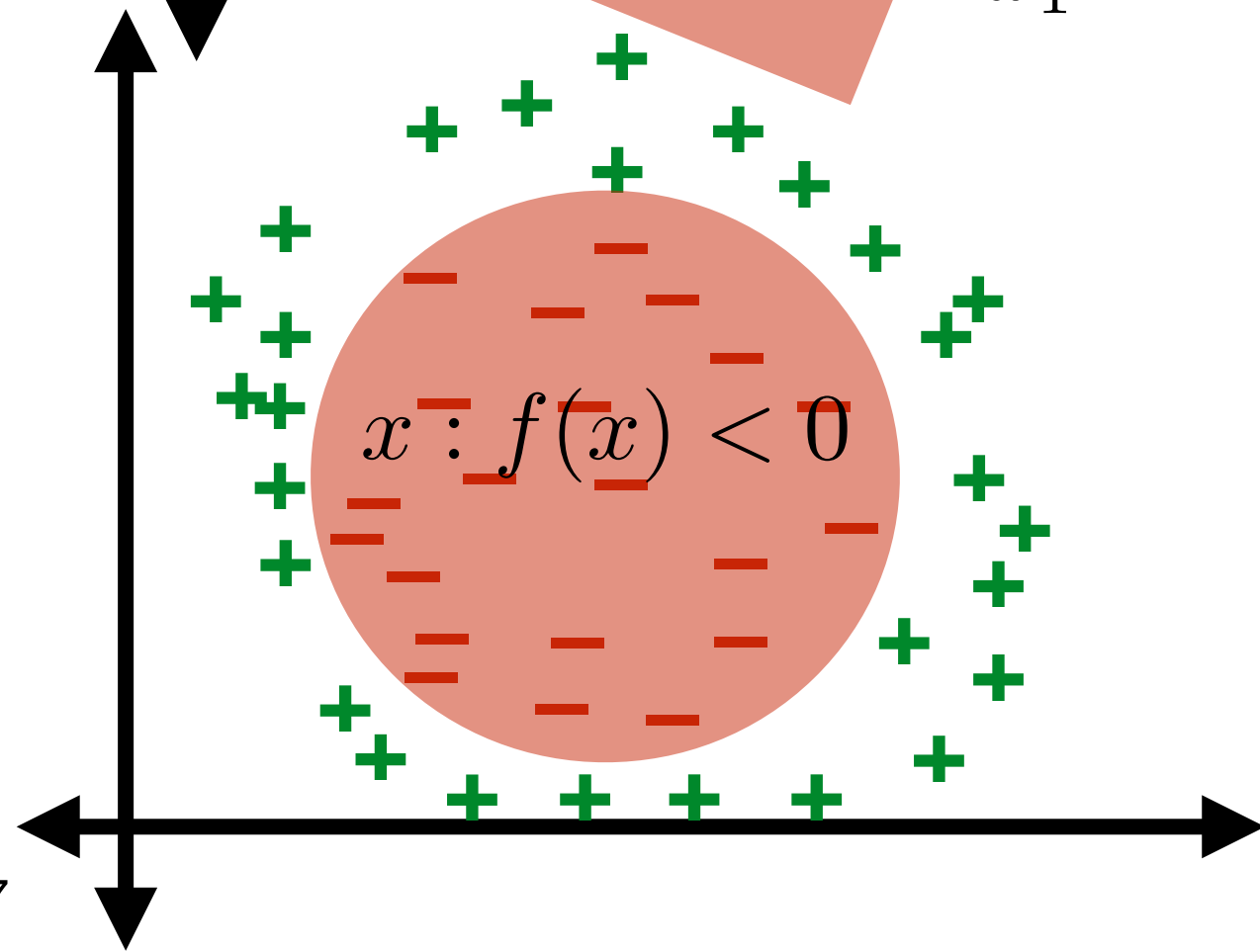
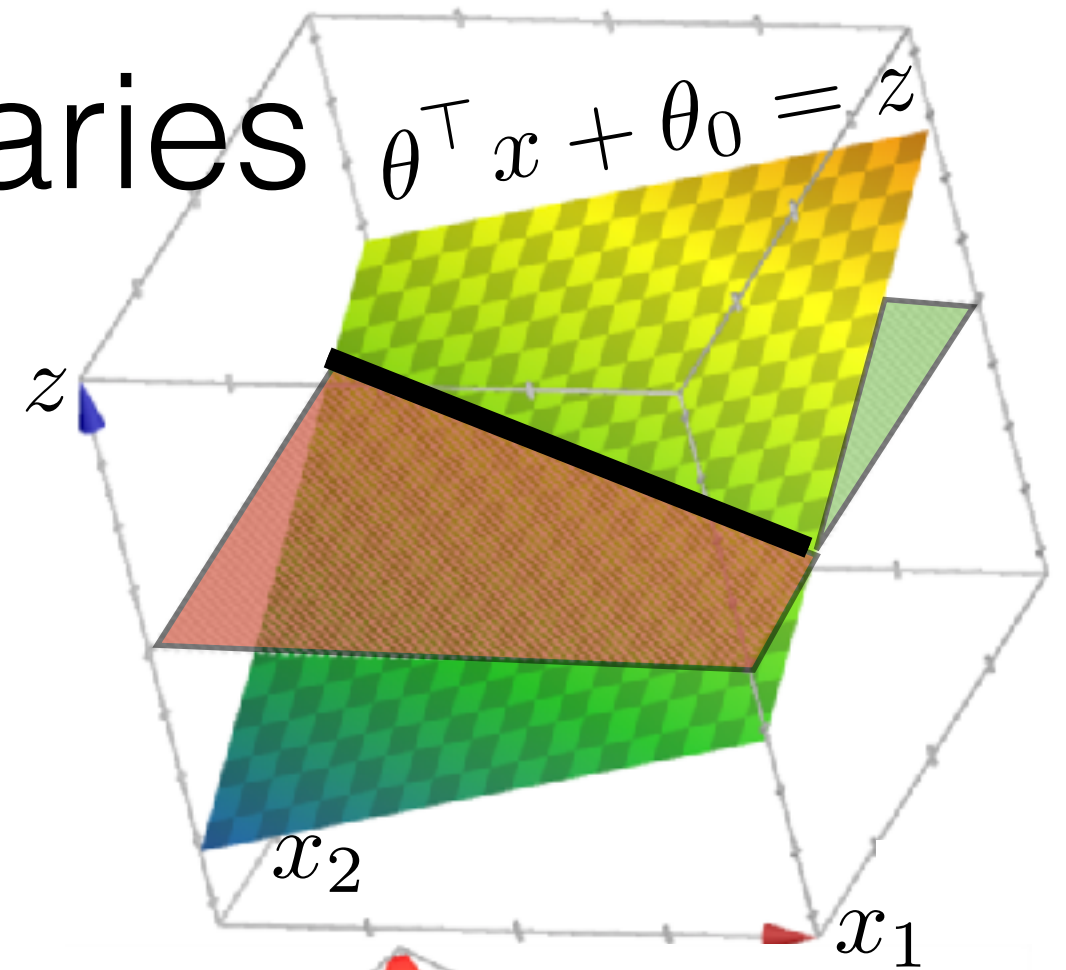
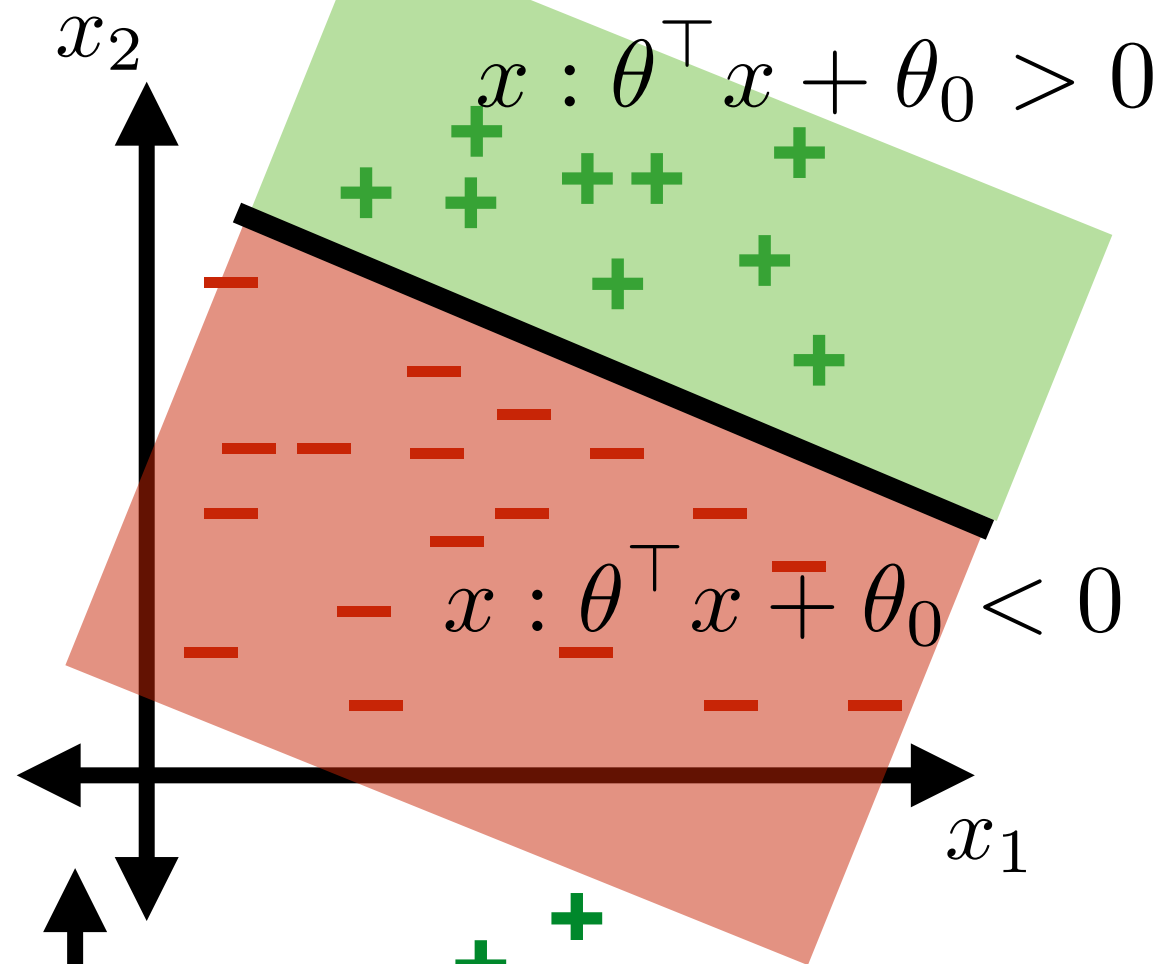
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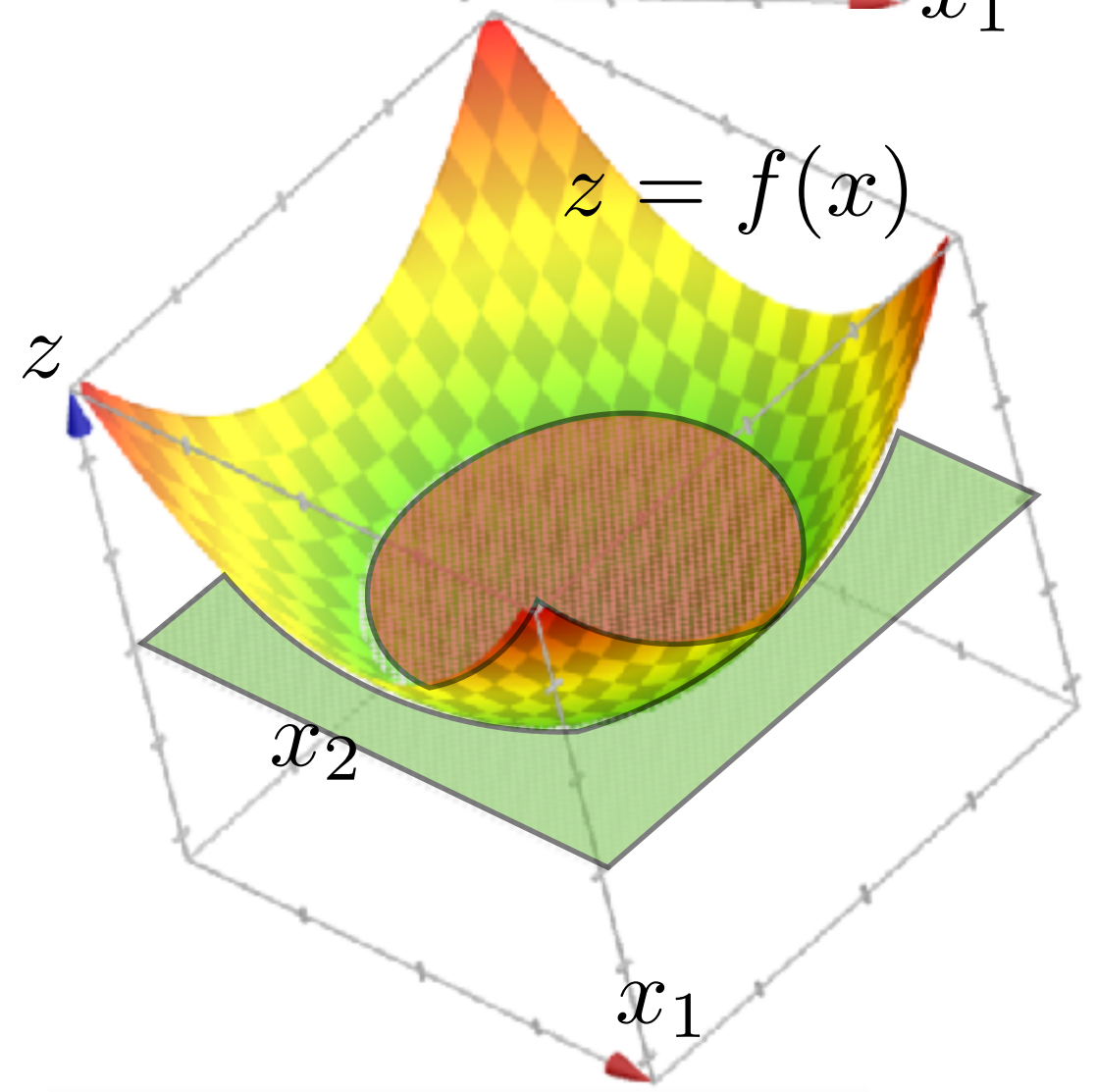
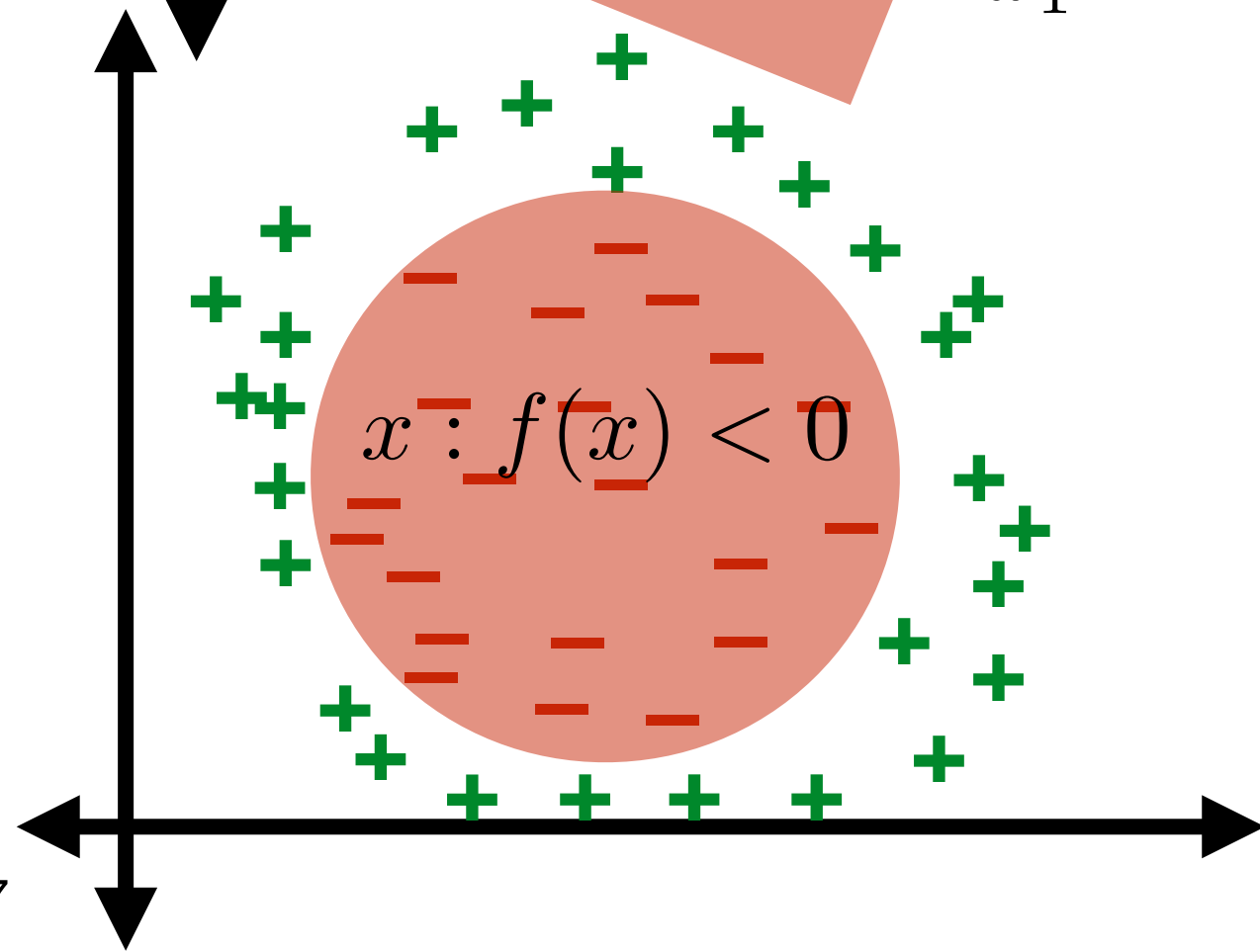
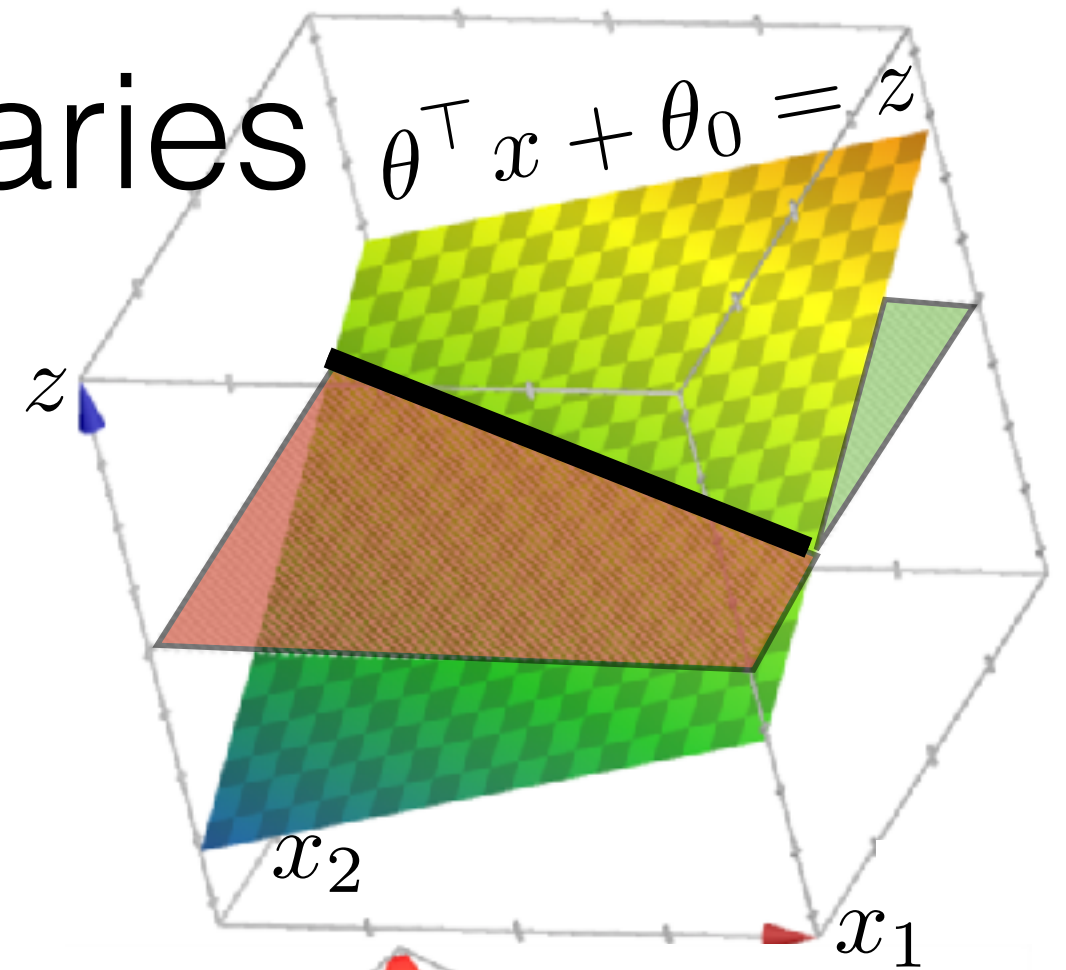
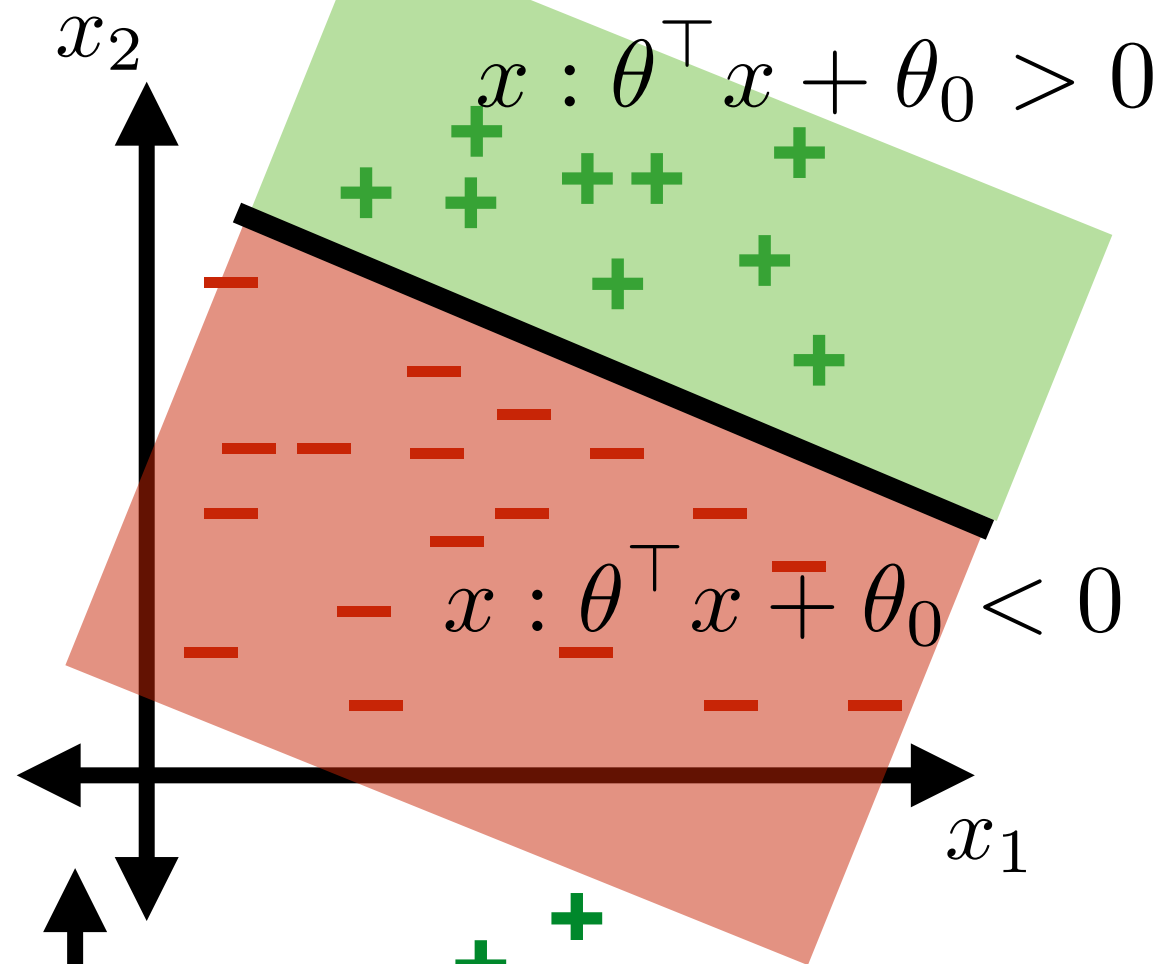
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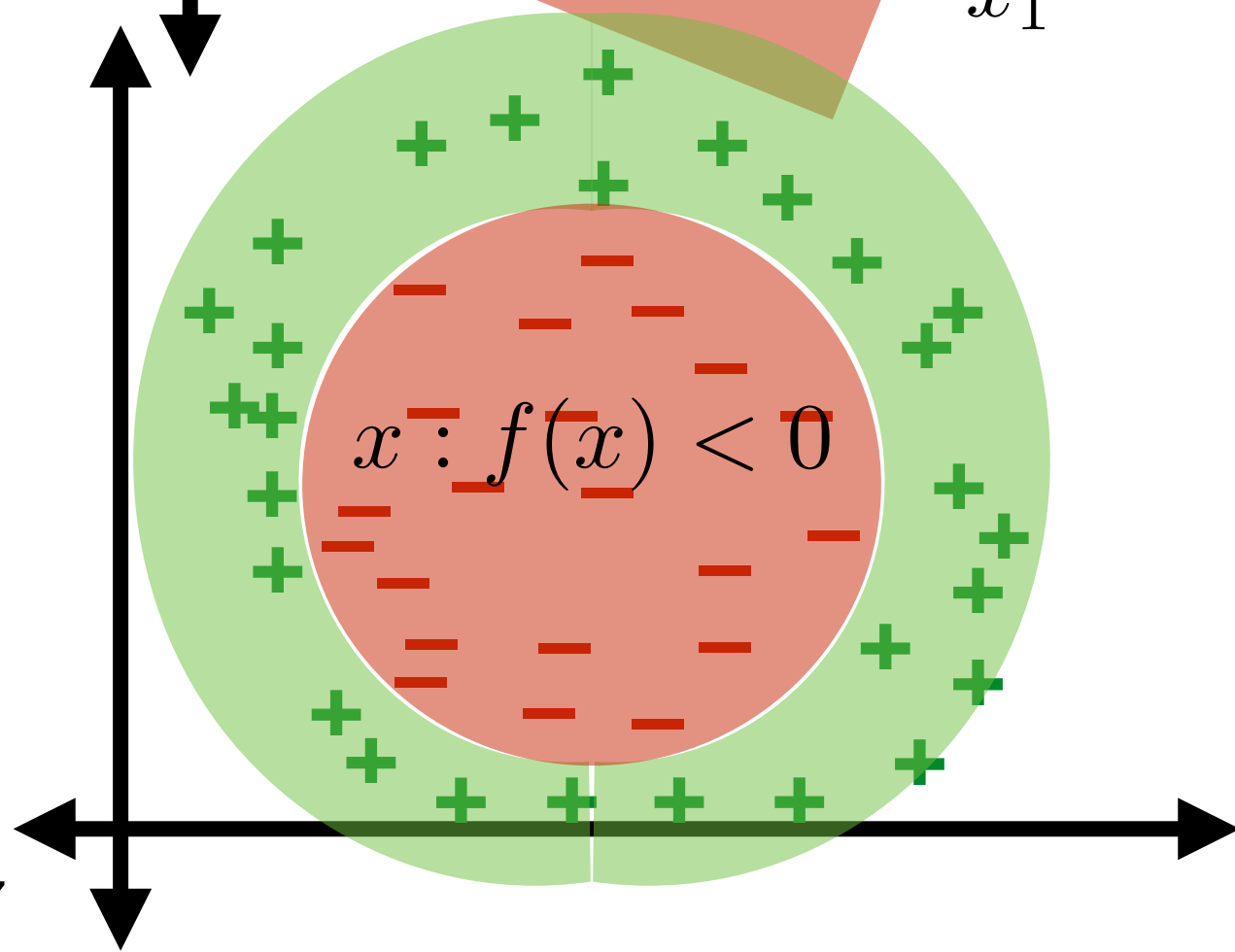
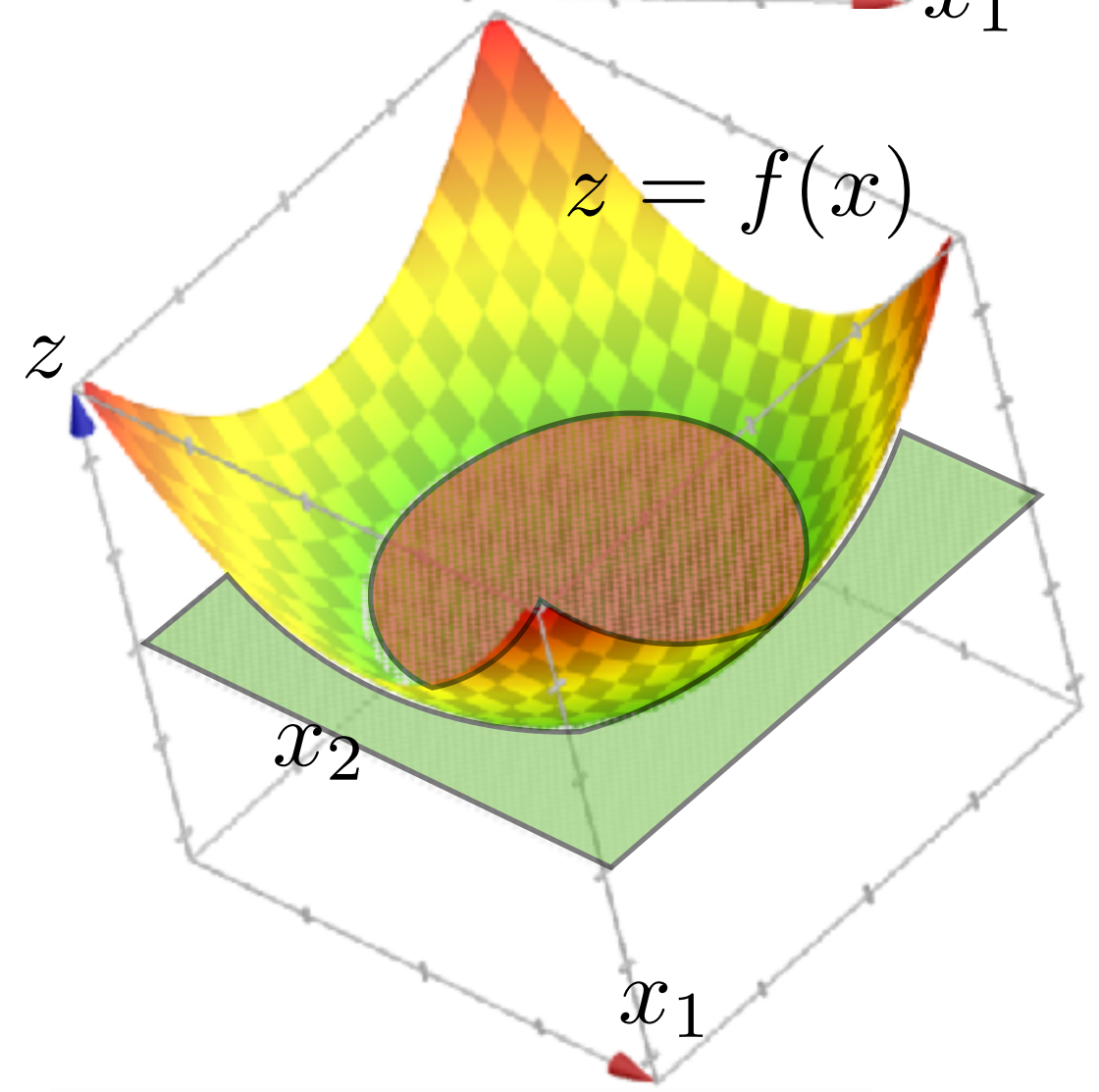
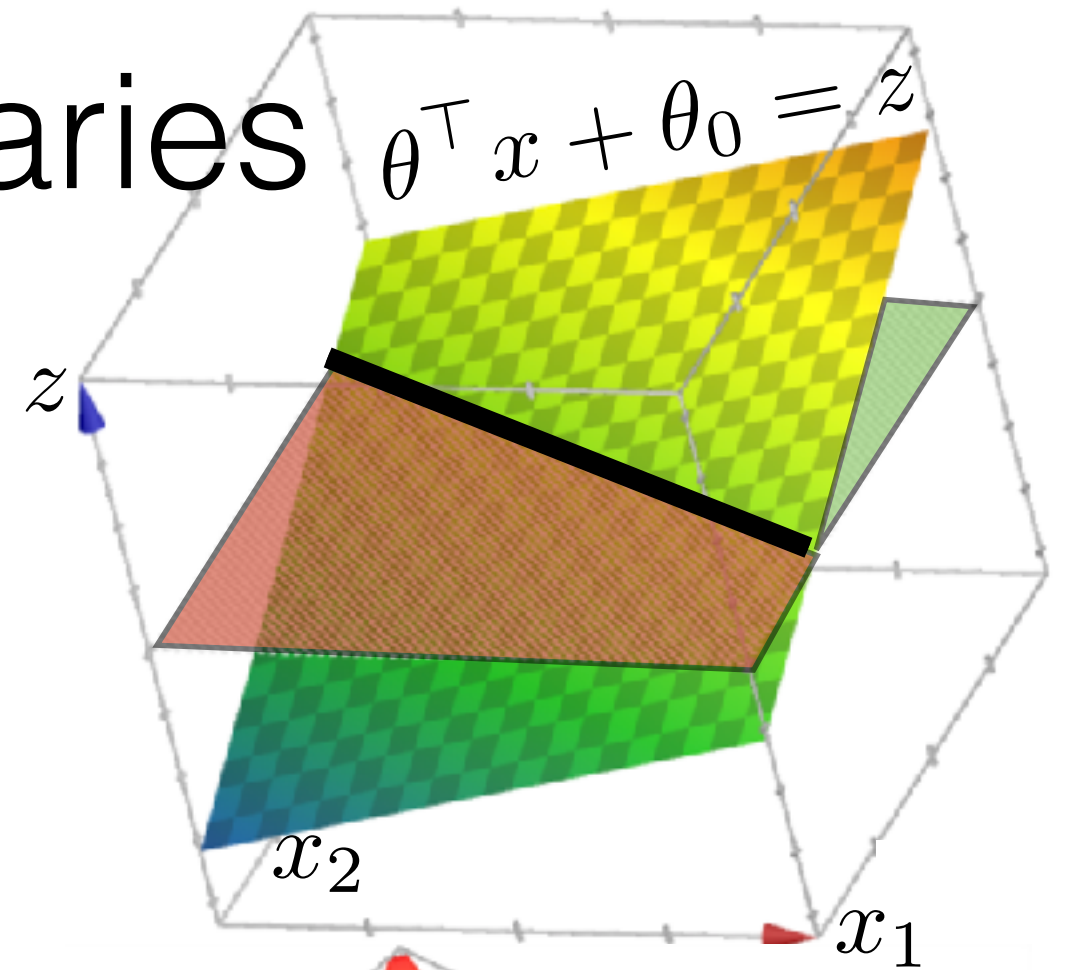
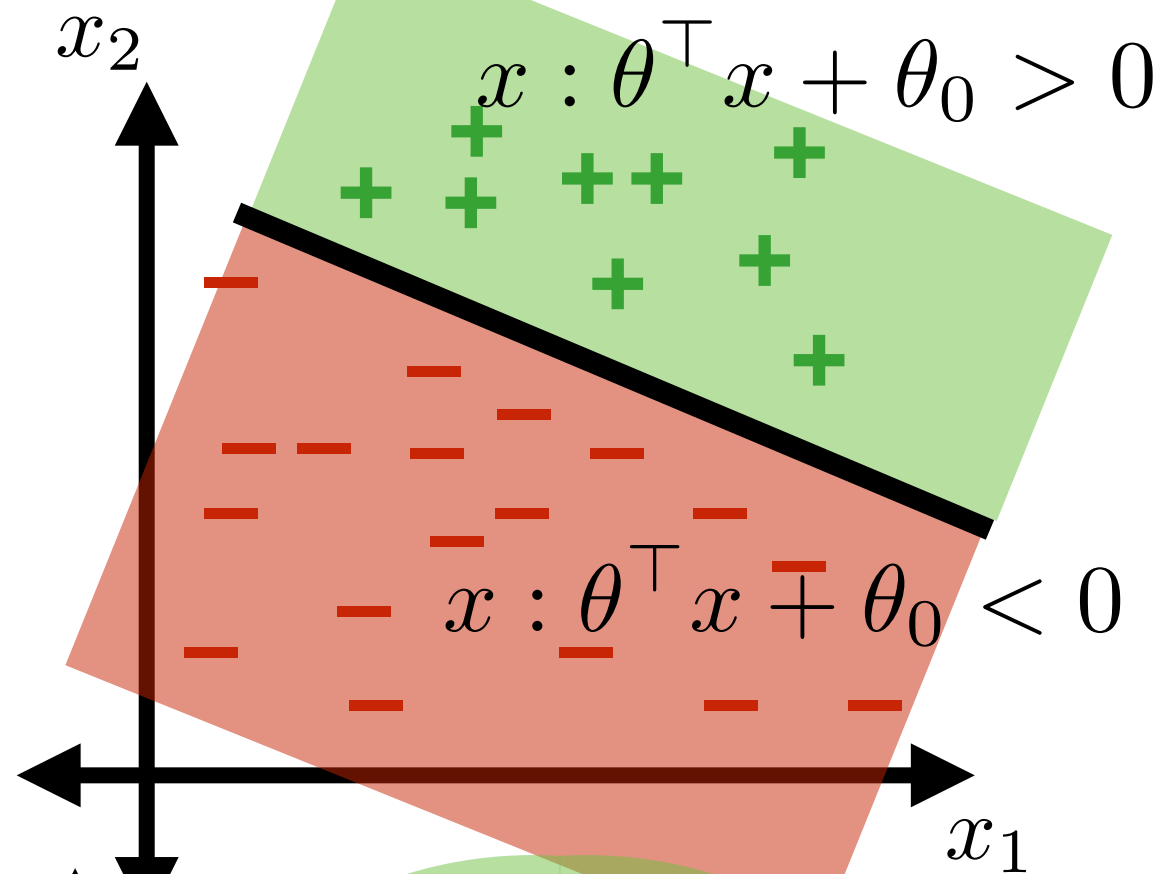
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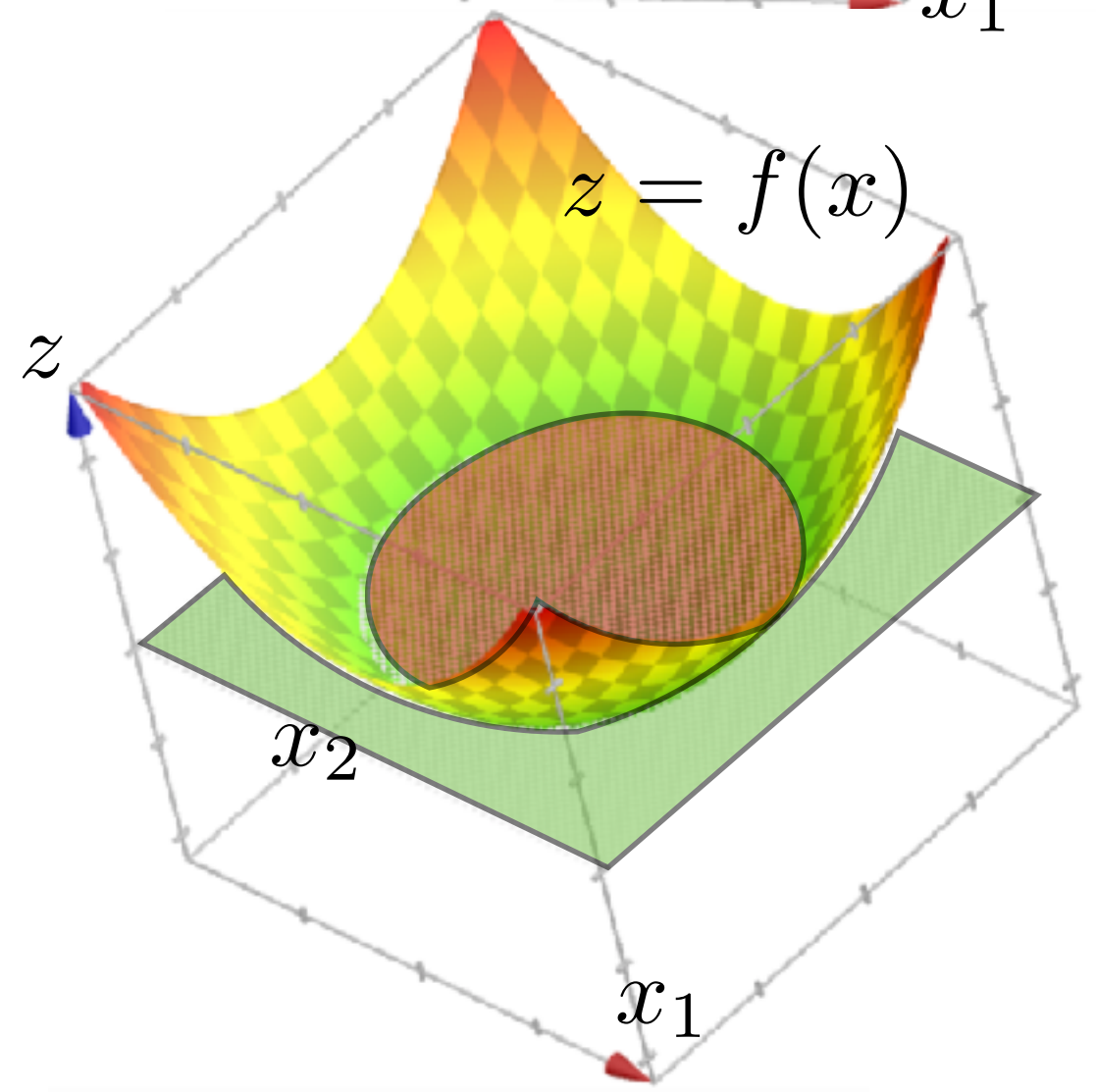
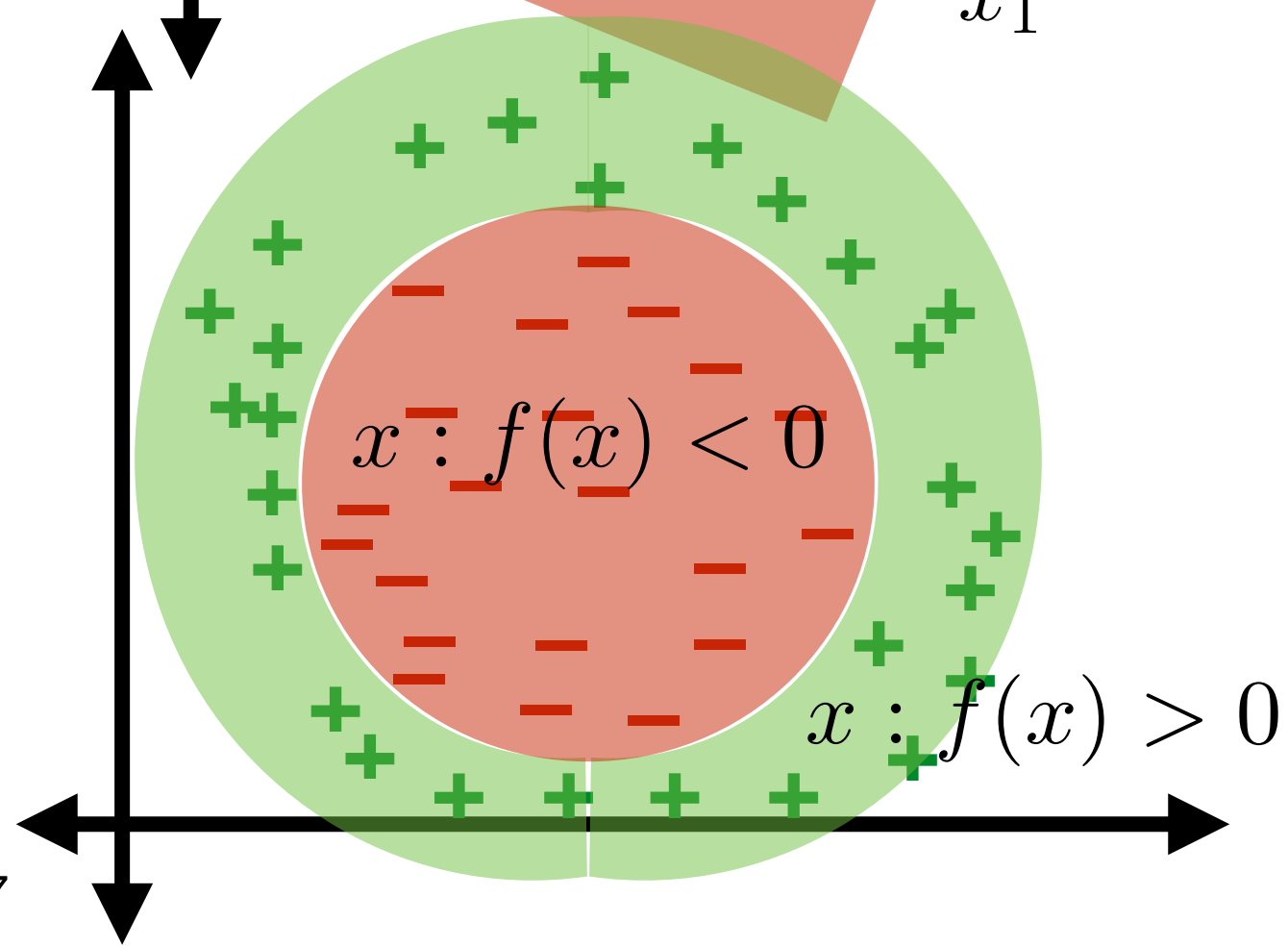
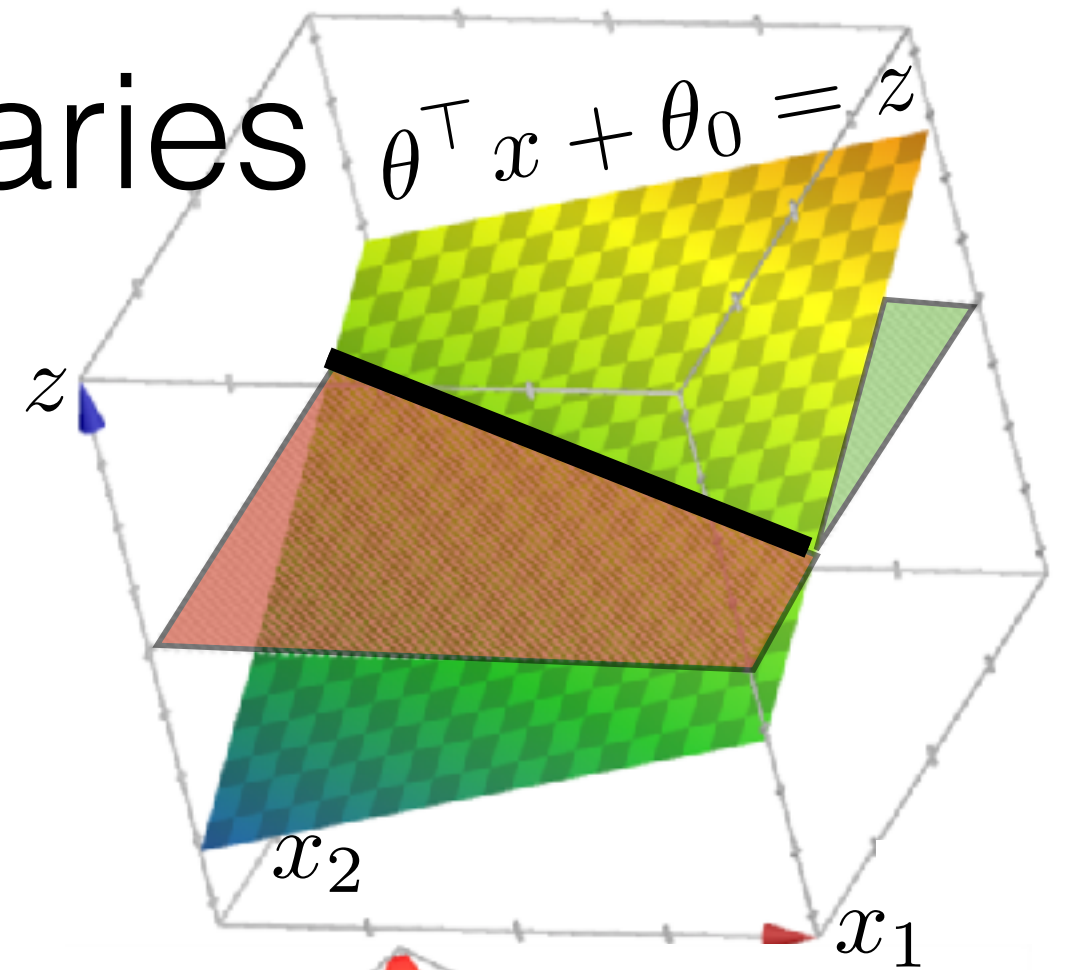
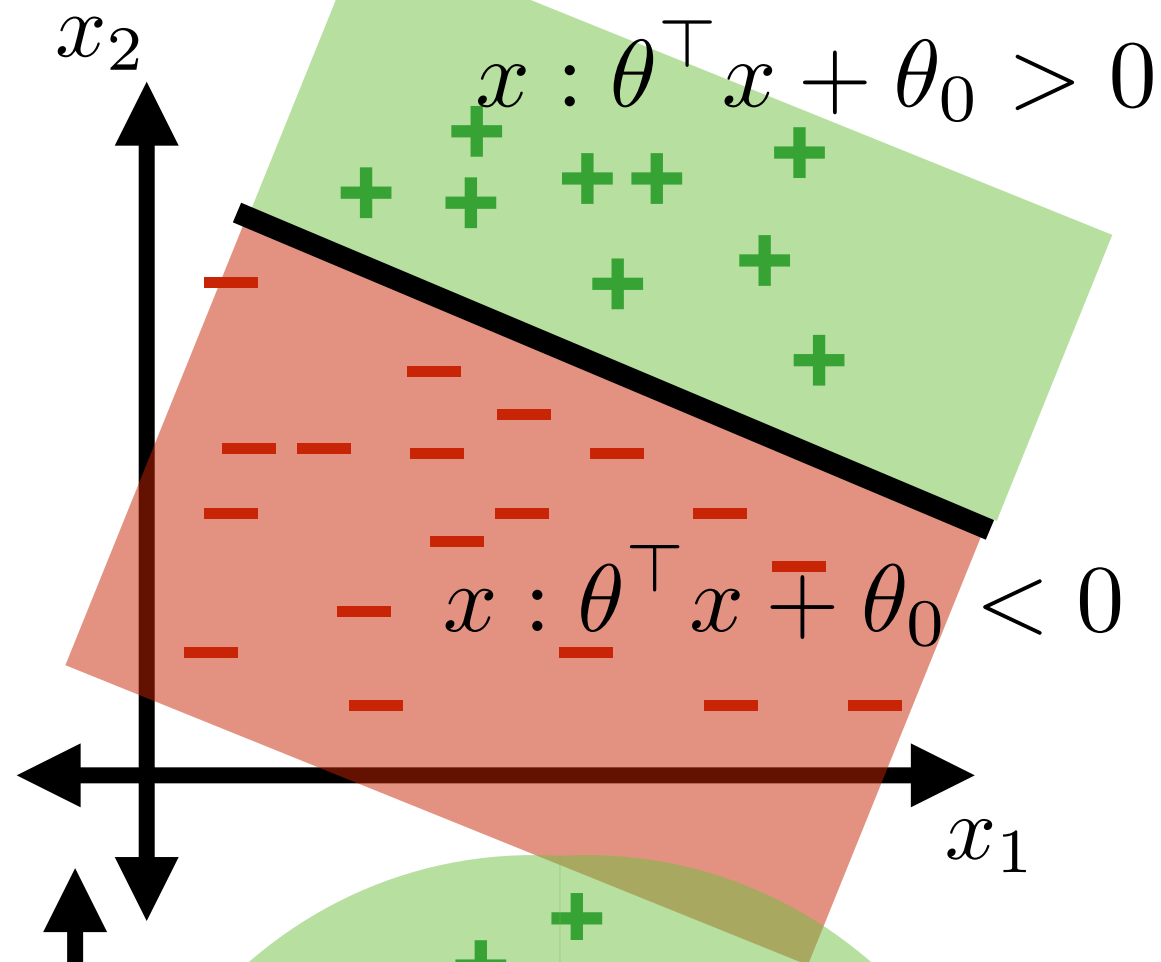
Classification boundaries



Classification boundaries



Classification boundaries



Nonlinear boundaries

Nonlinear boundaries

- Idea: can approximate a smooth function with a k th order Taylor polynomial (e.g. around 0)

Nonlinear boundaries

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order (k)	terms when $d=1$	terms for general d
0		
1		
2		
3		

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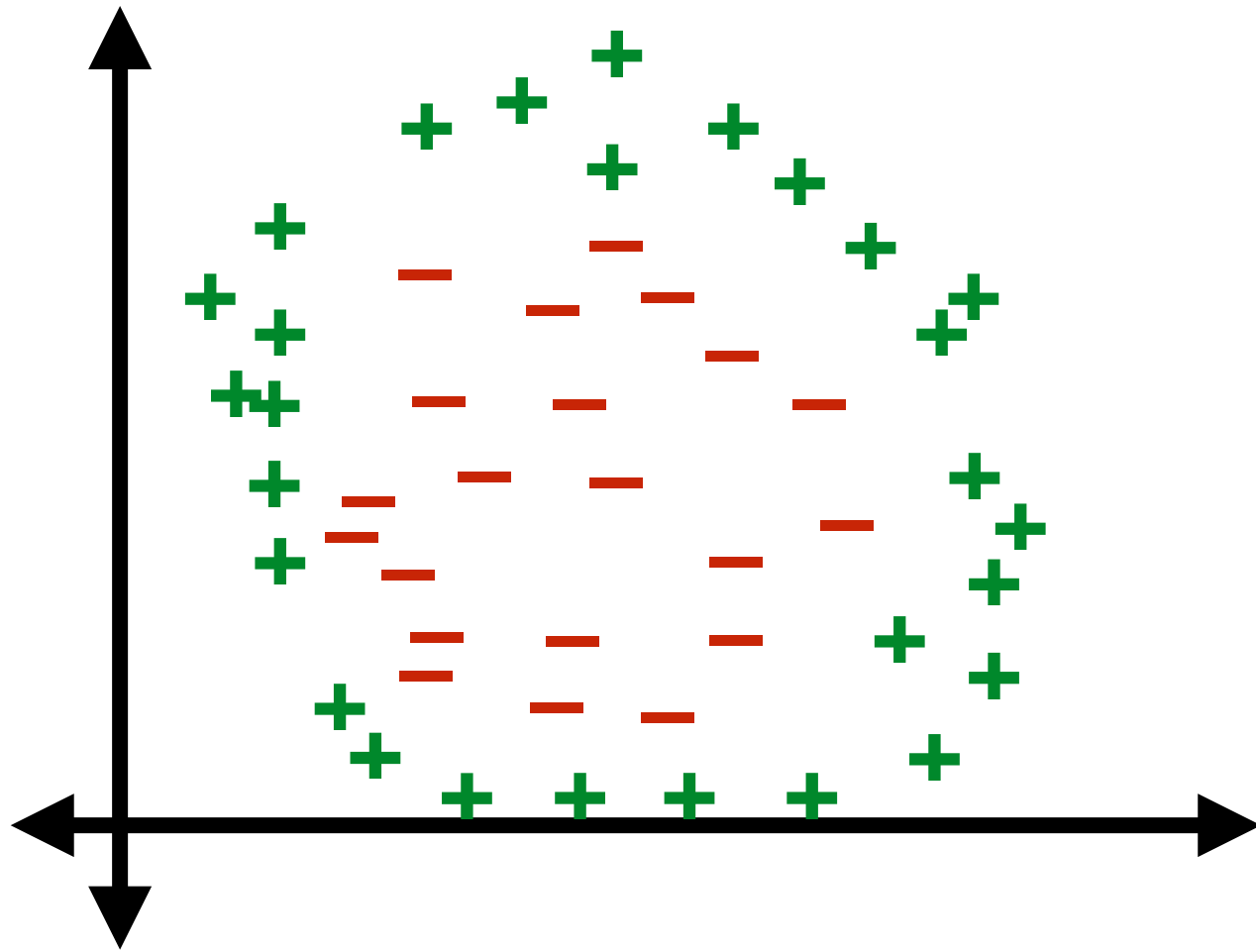
Nonlinear boundaries

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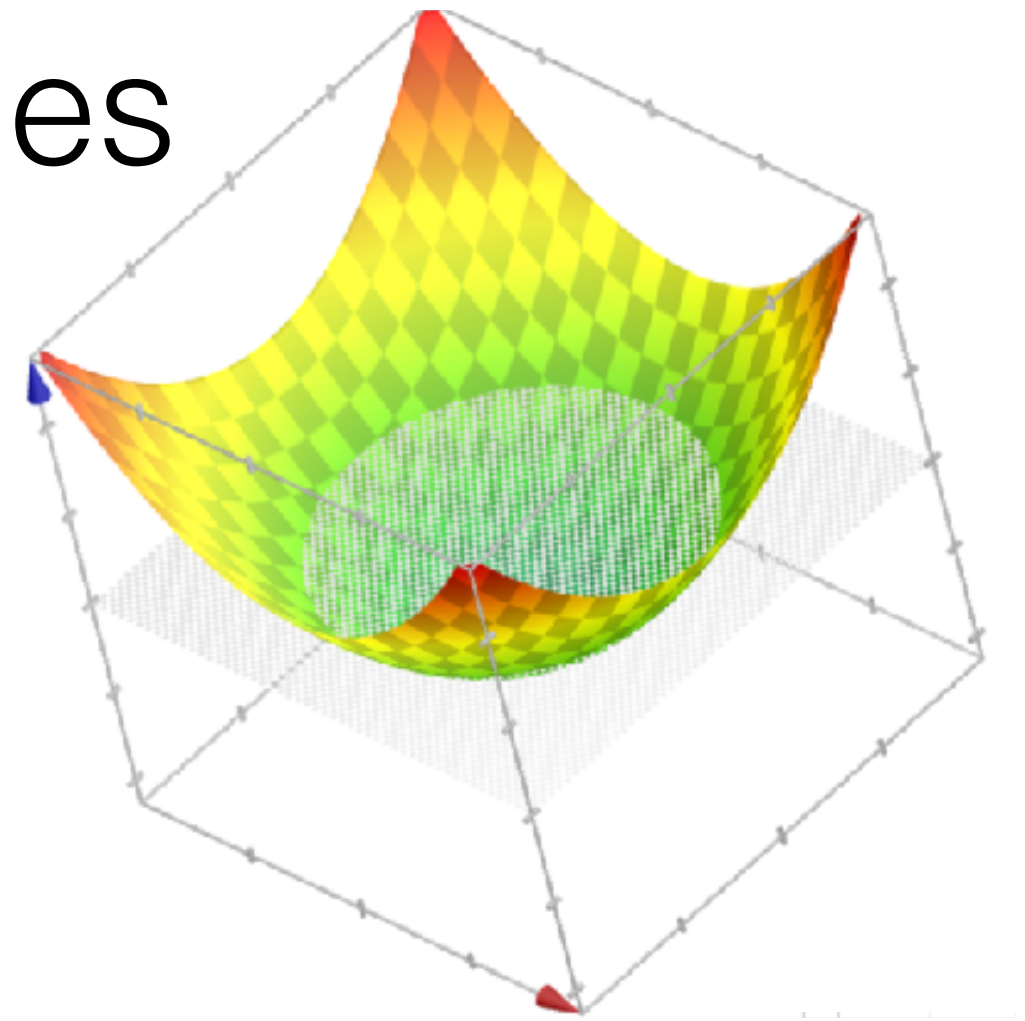
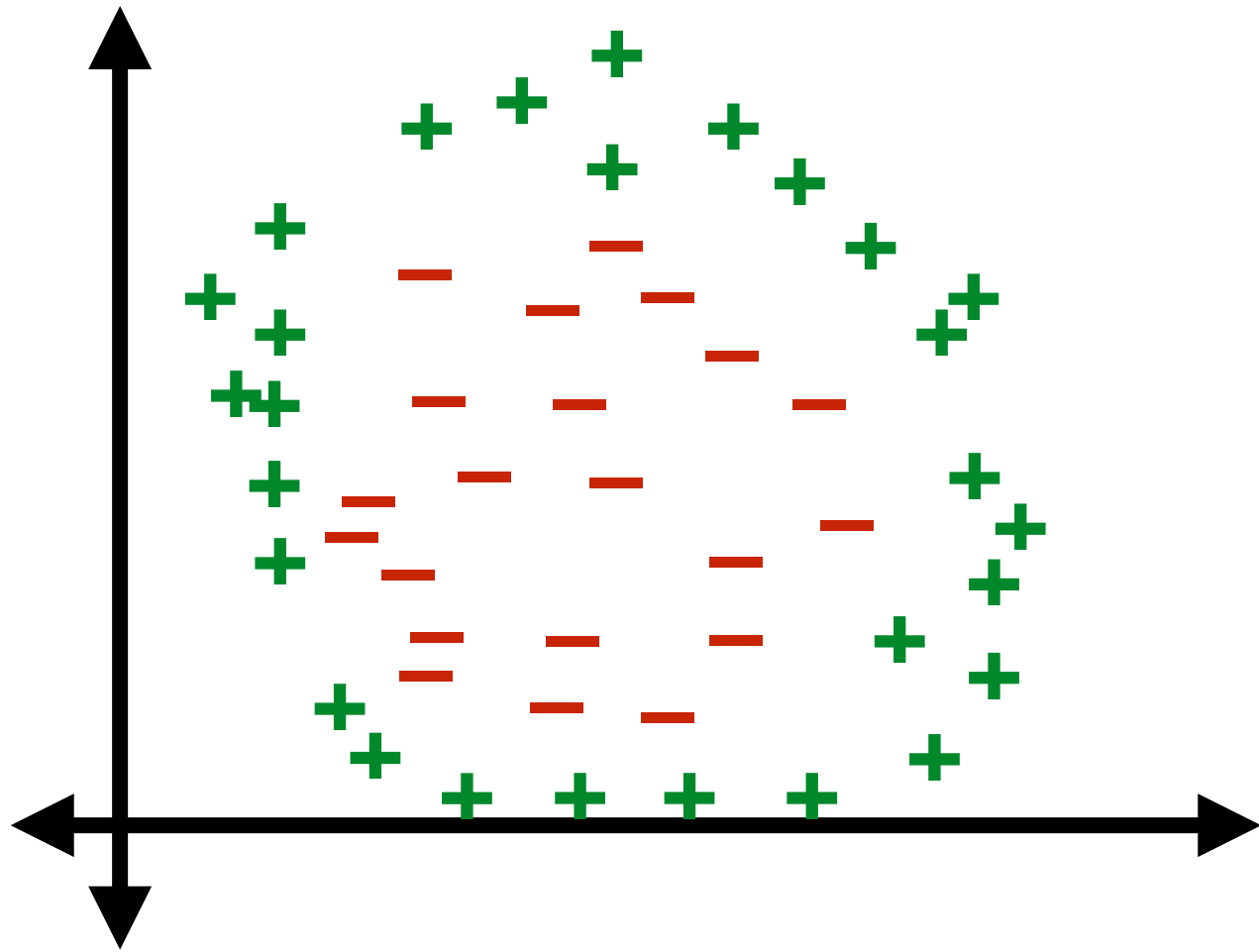
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0	$[1]$	$[1]$
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Nonlinear boundaries

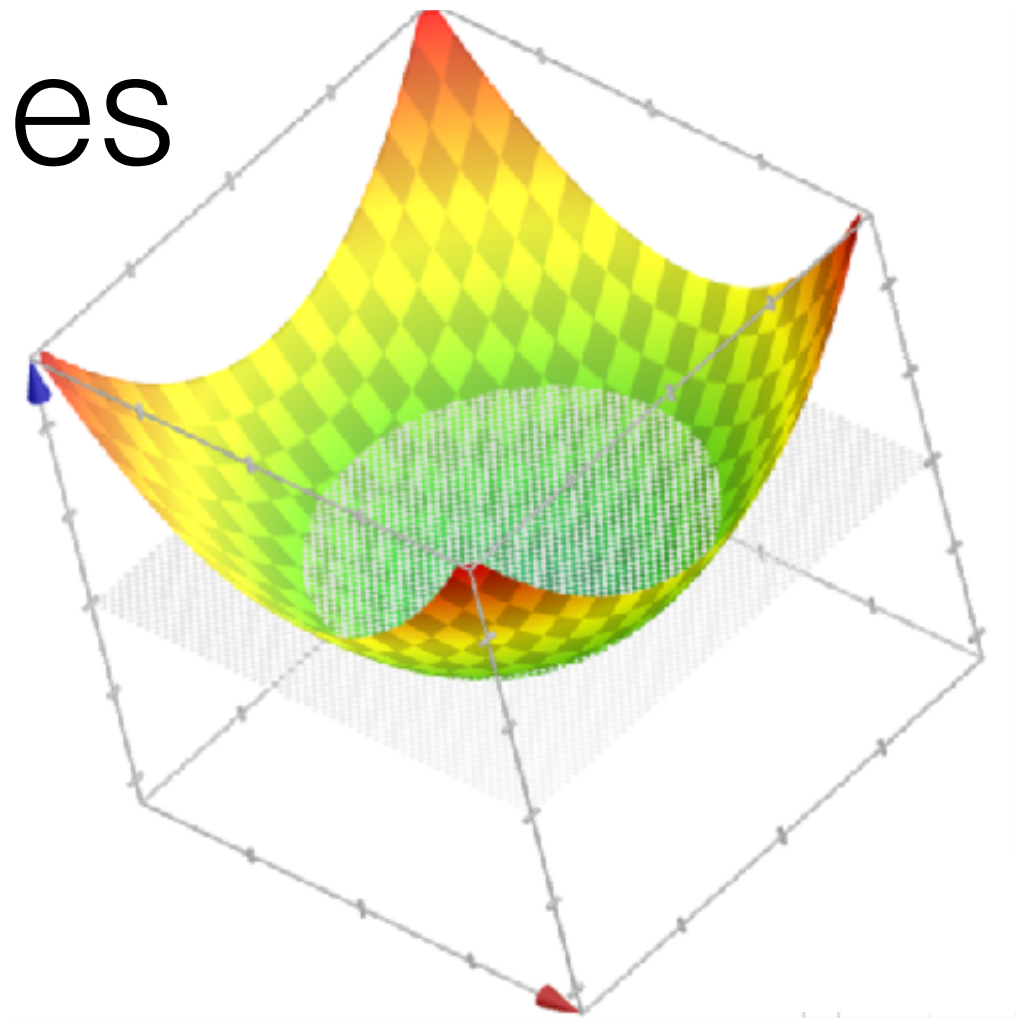
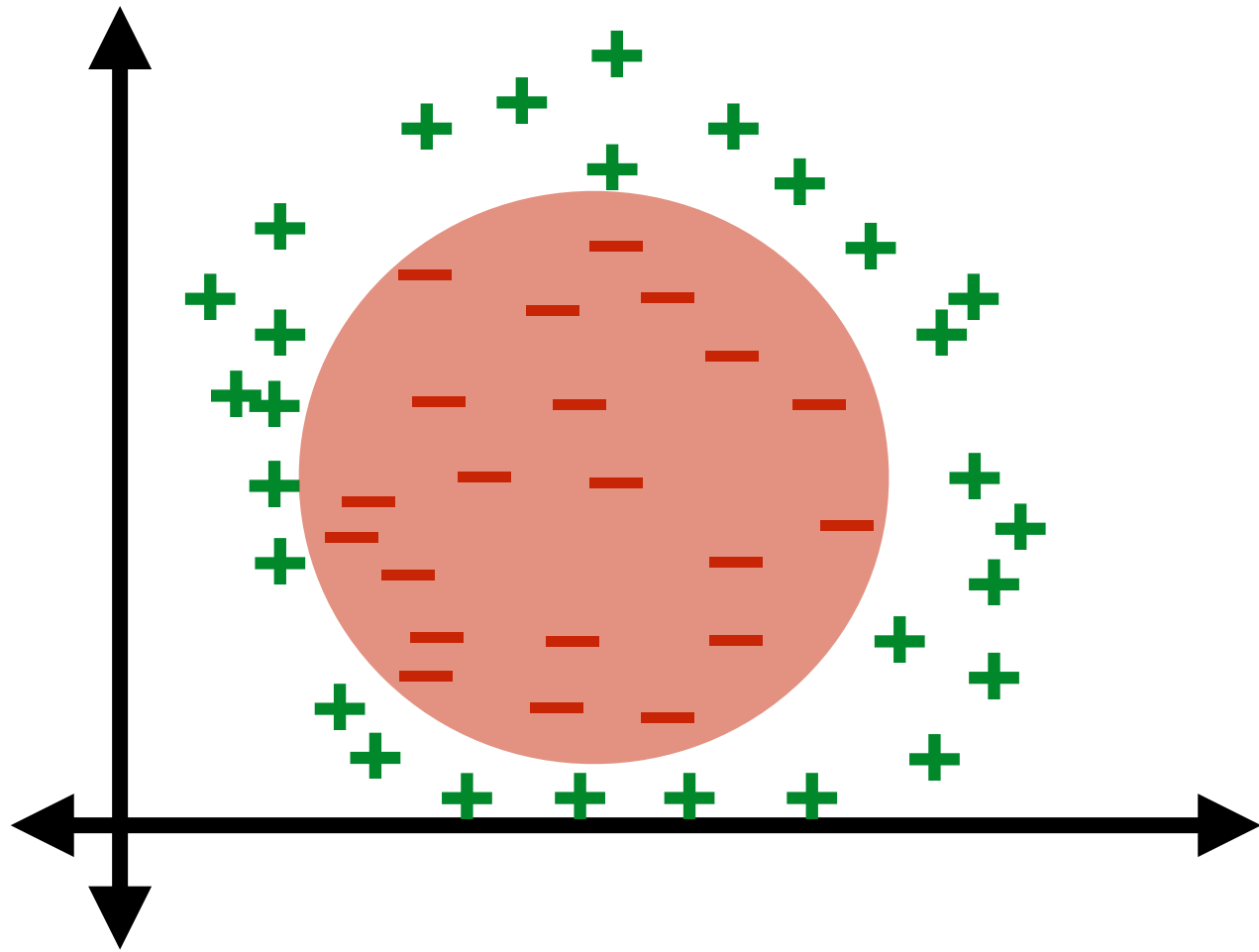
Nonlinear boundaries



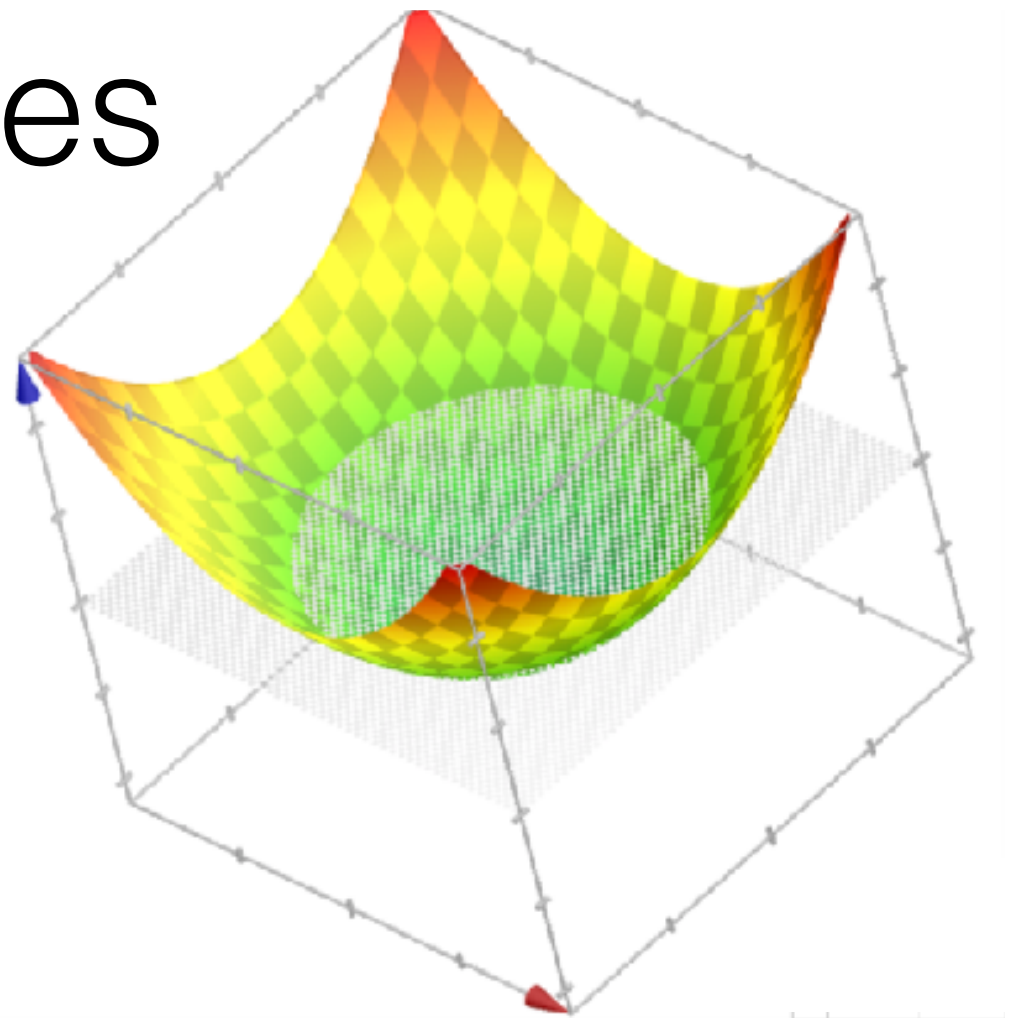
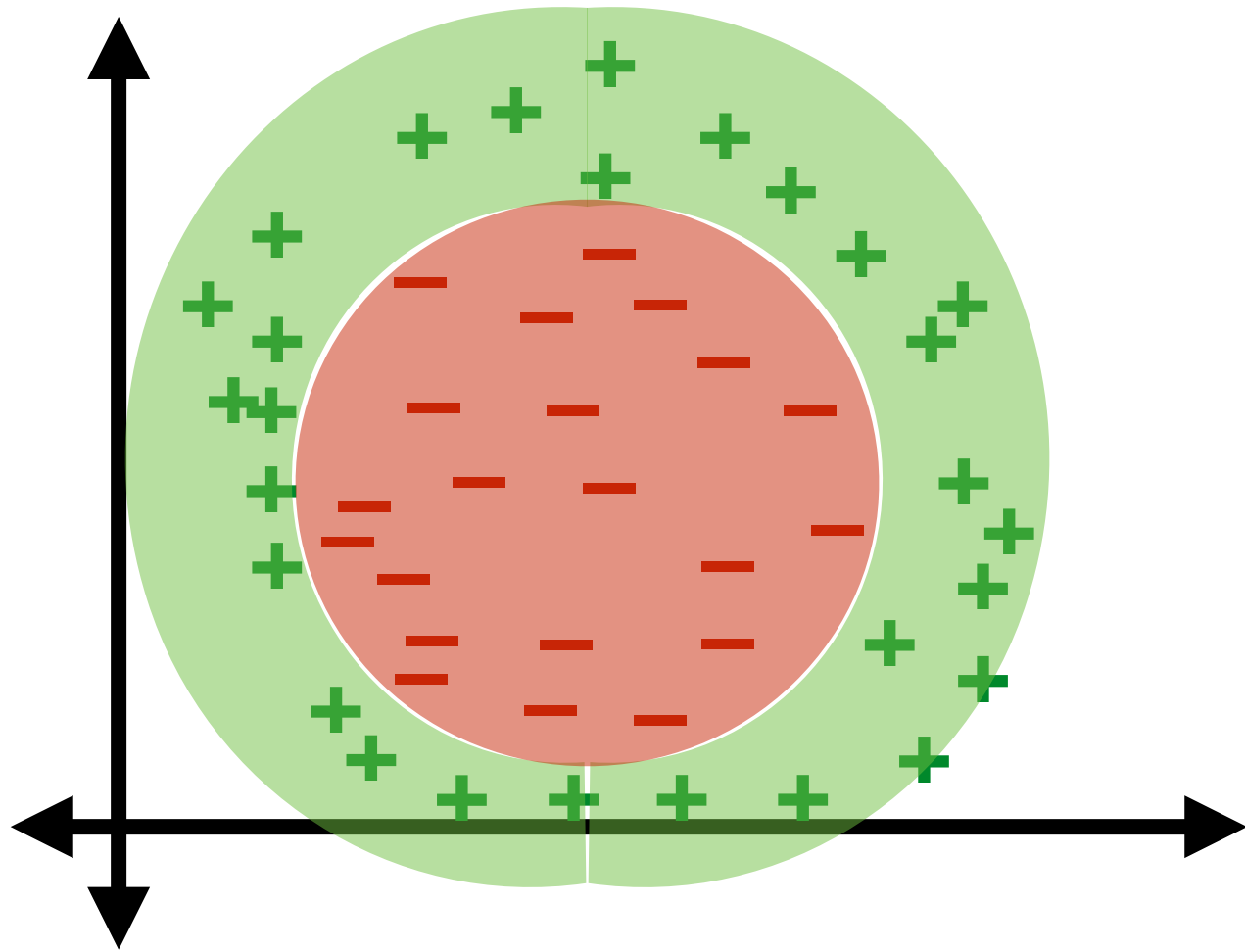
Nonlinear boundaries



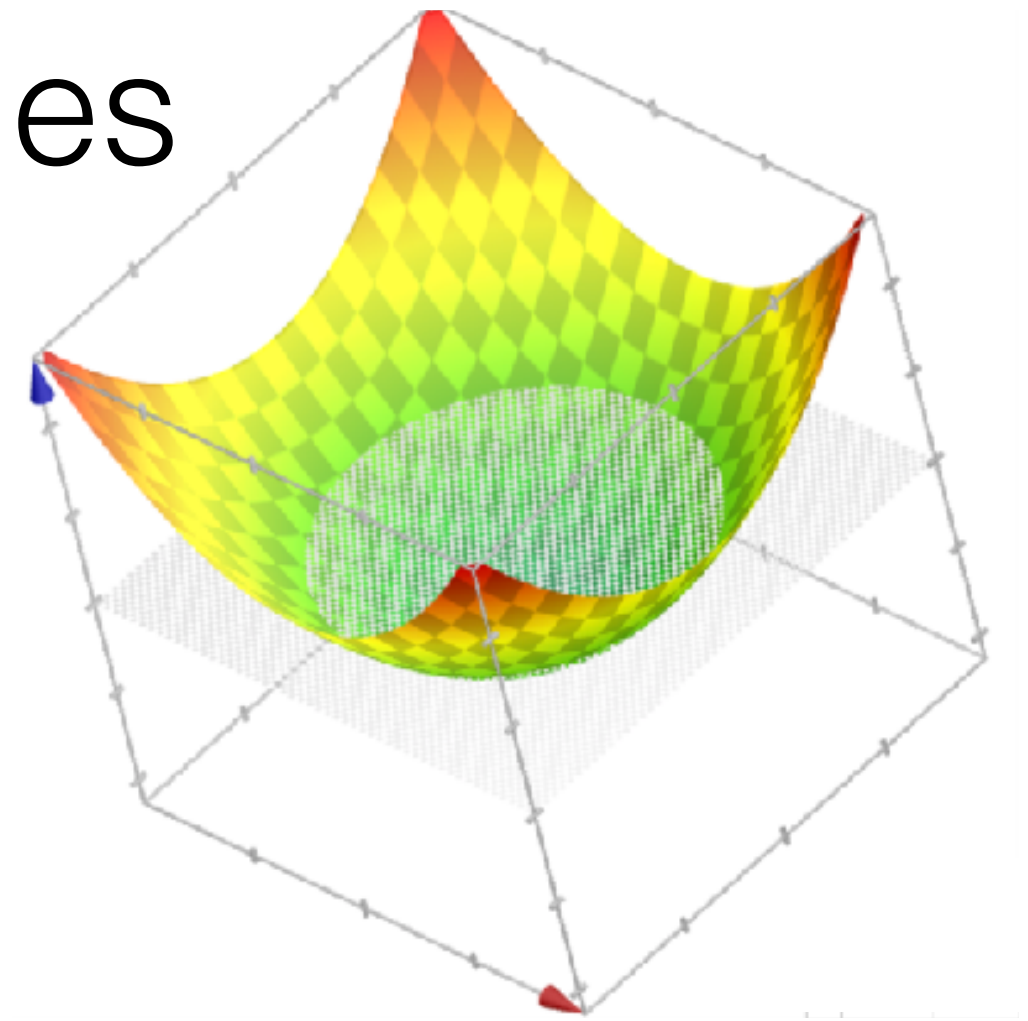
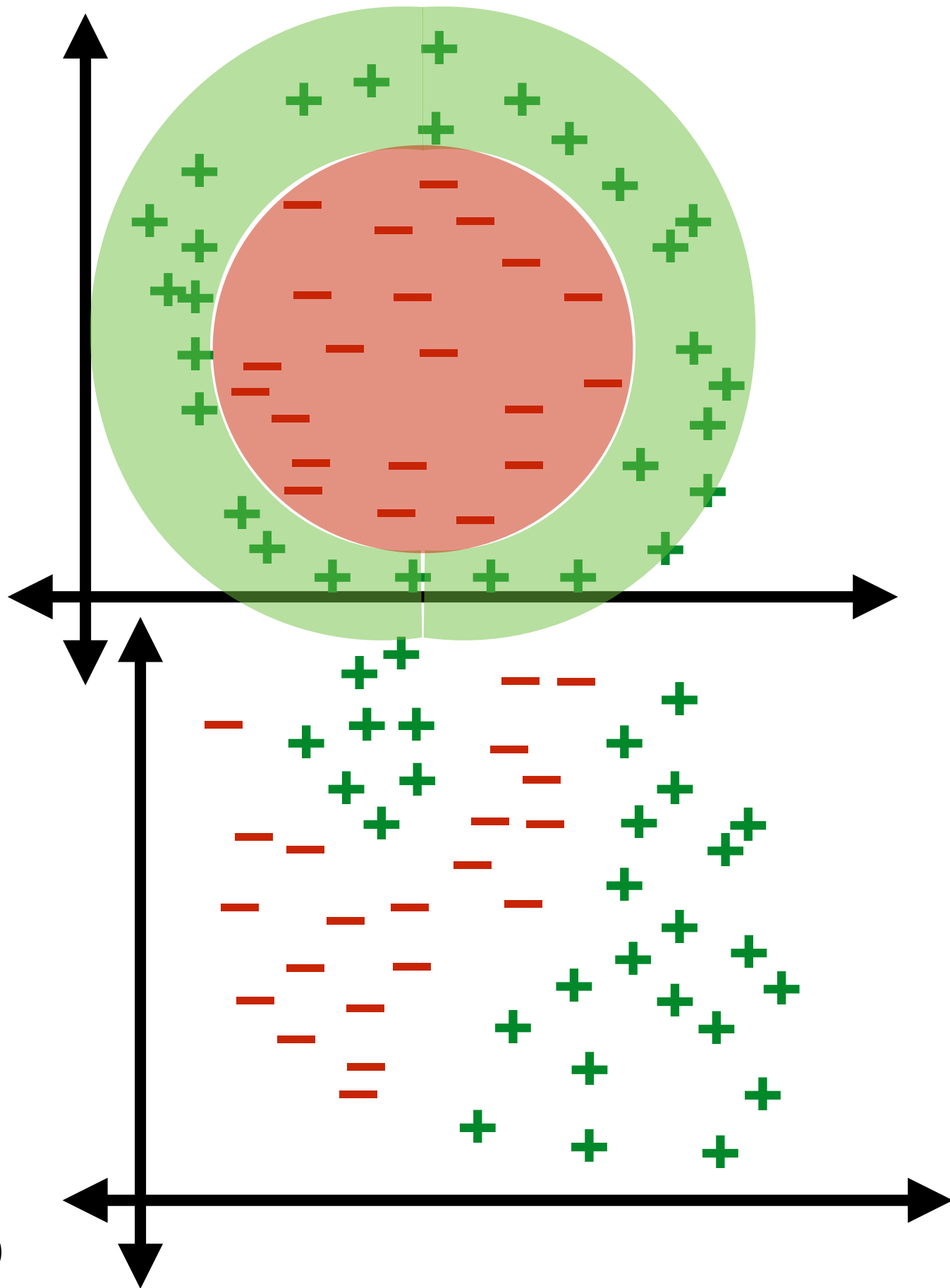
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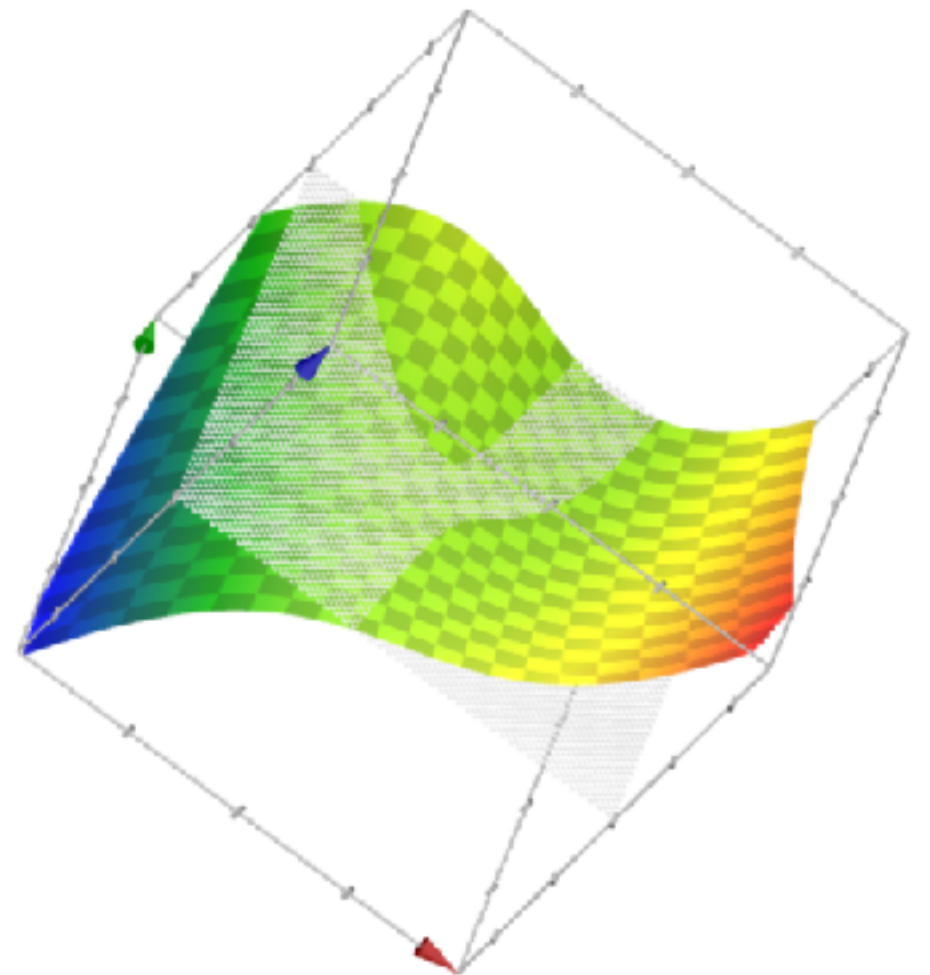
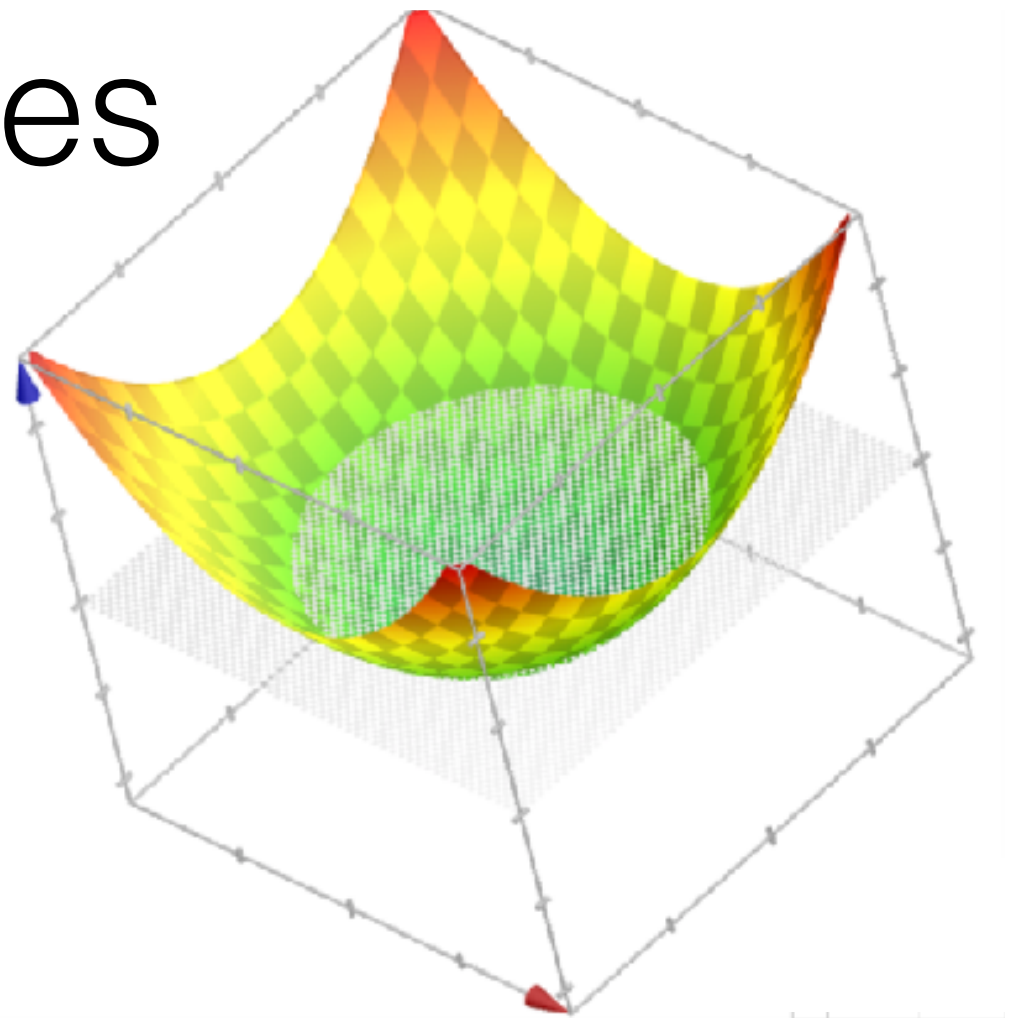
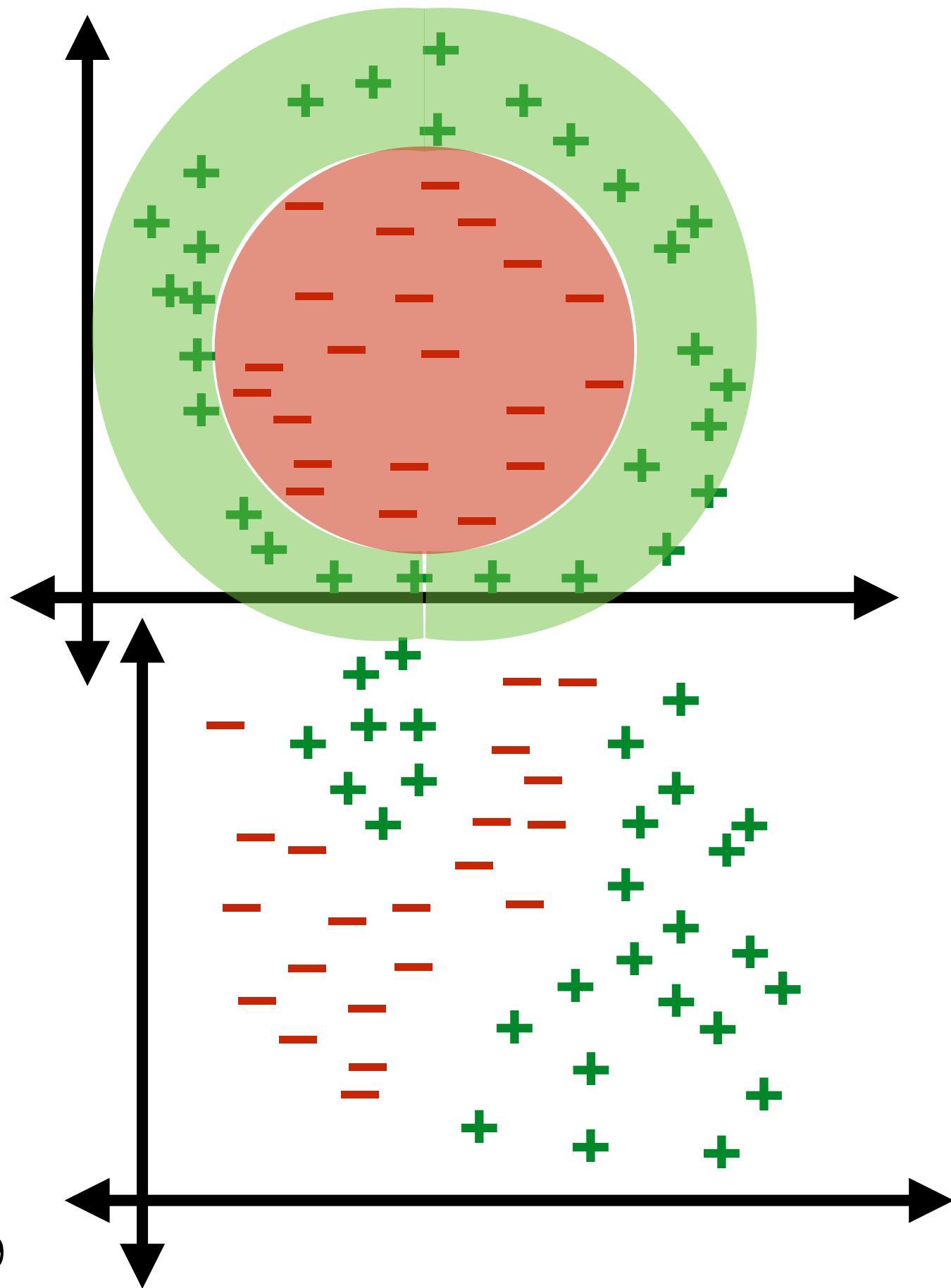
Nonlinear boundaries



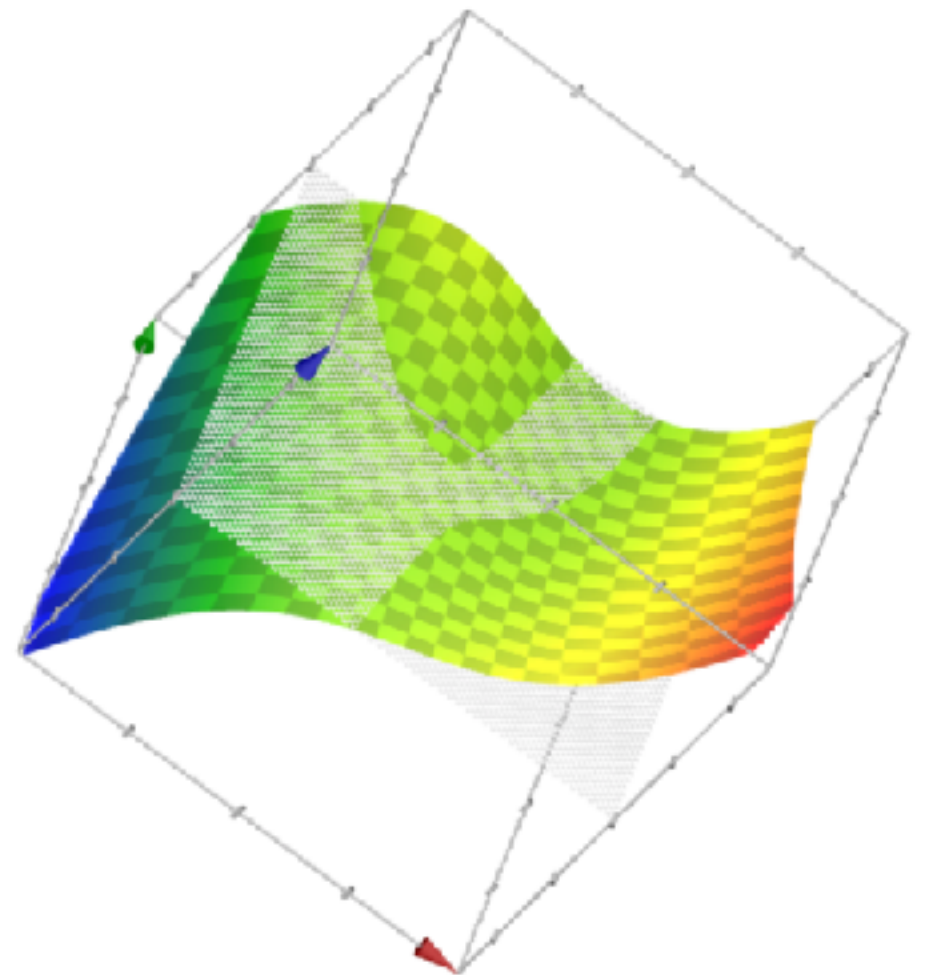
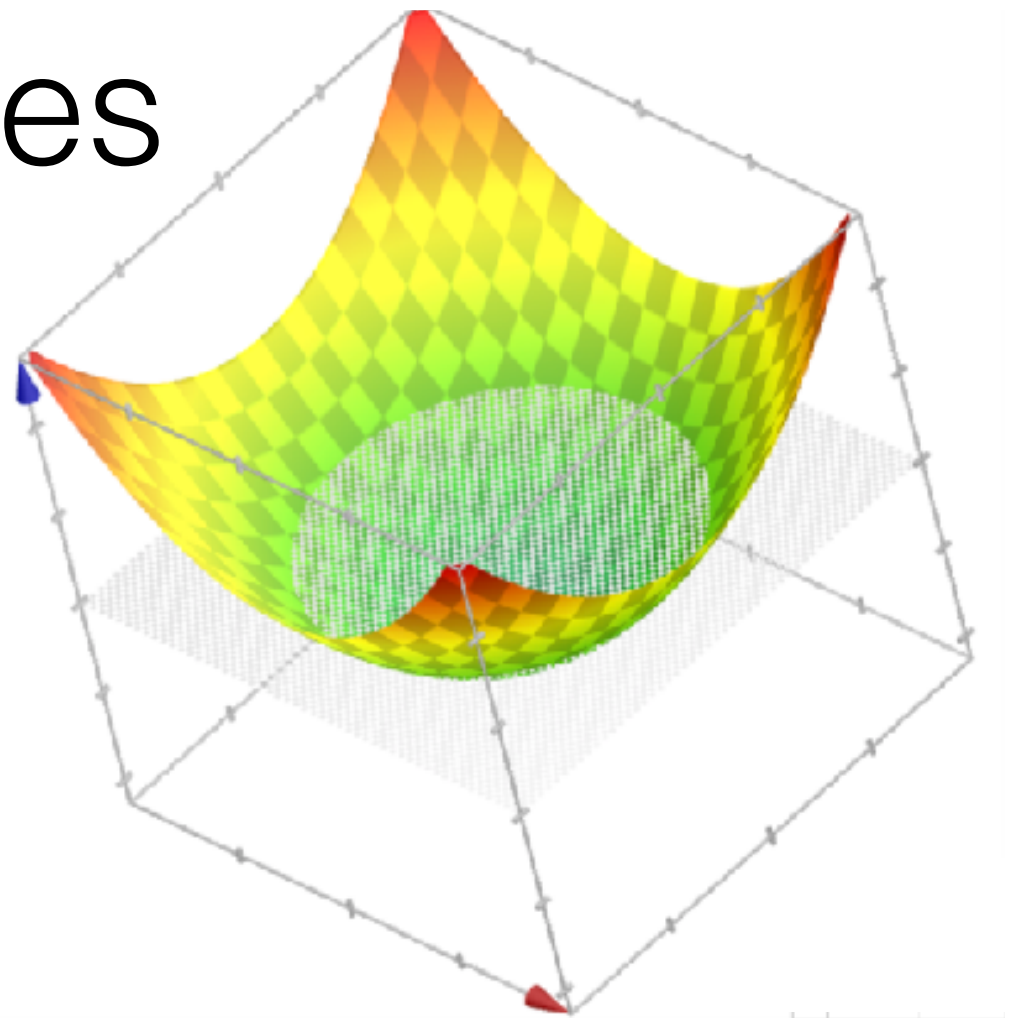
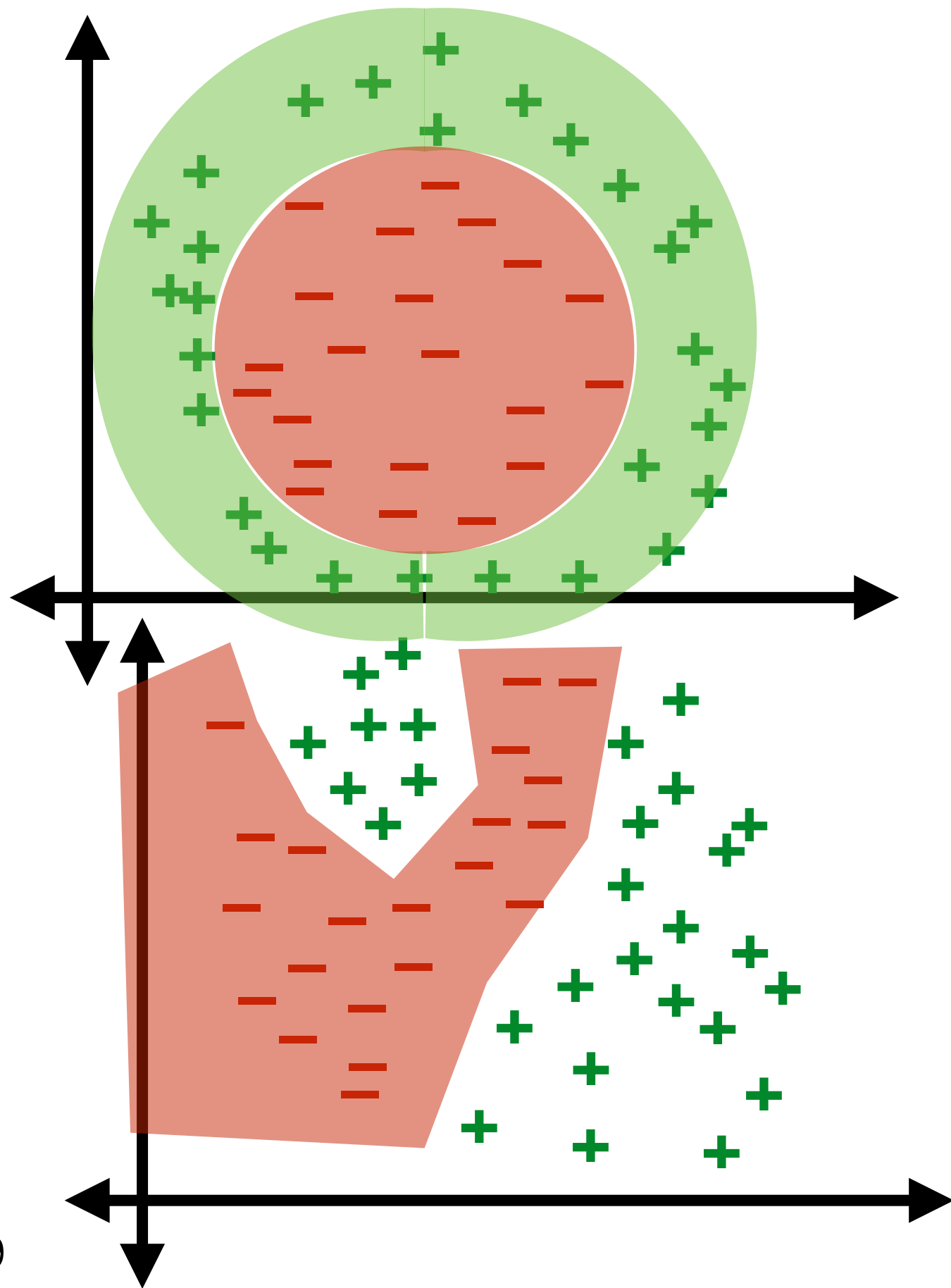
Nonlinear boundaries



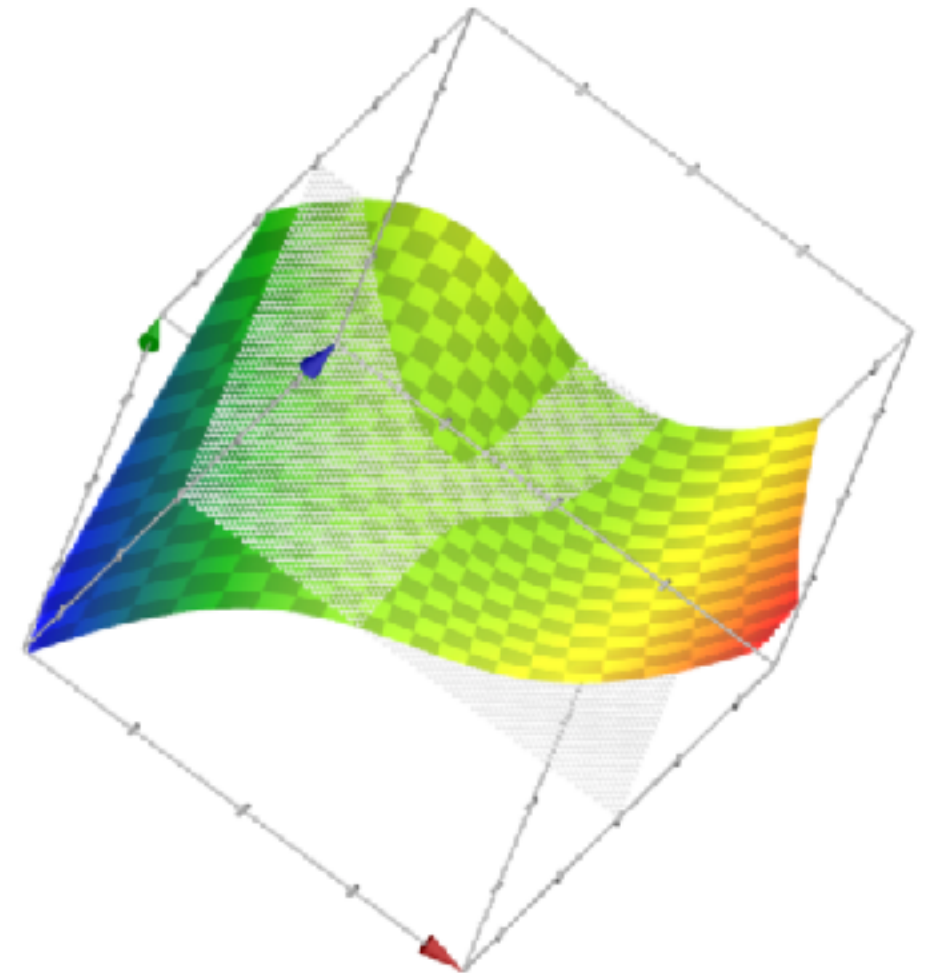
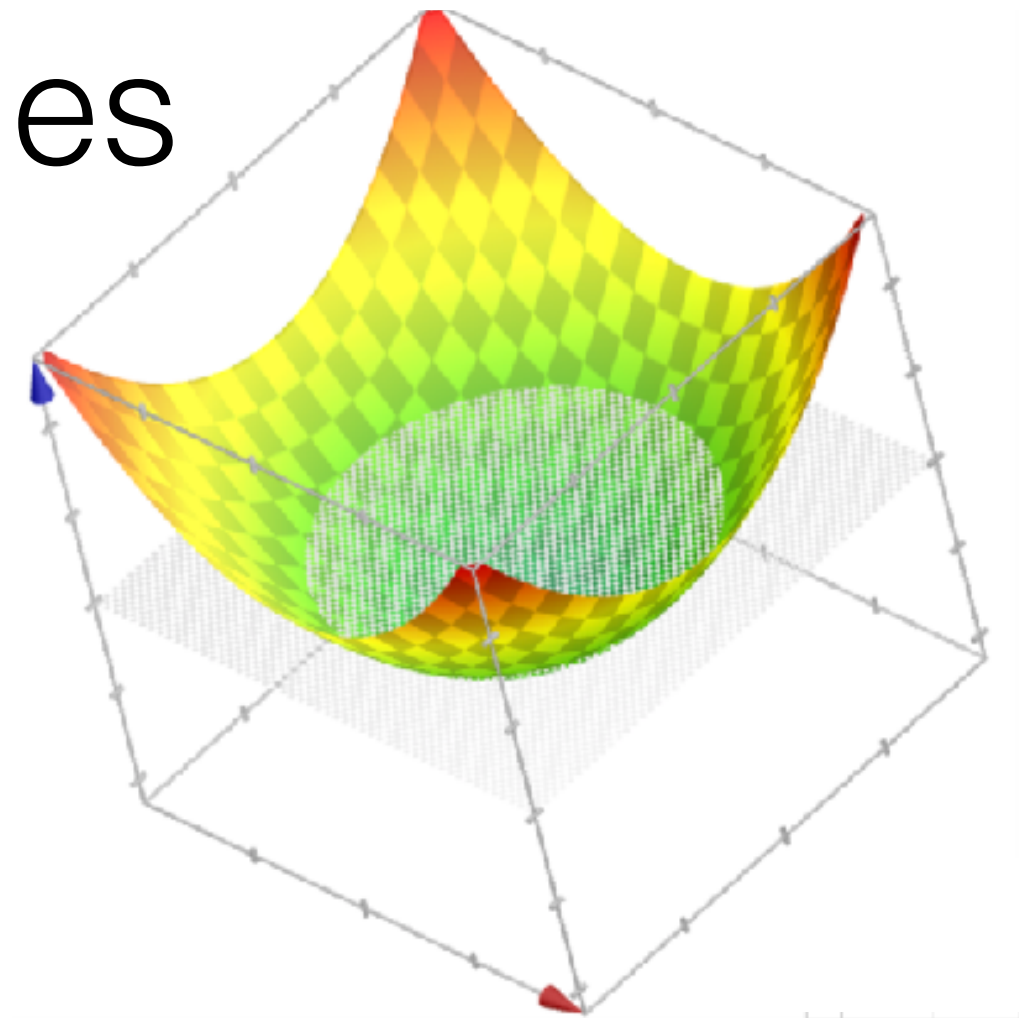
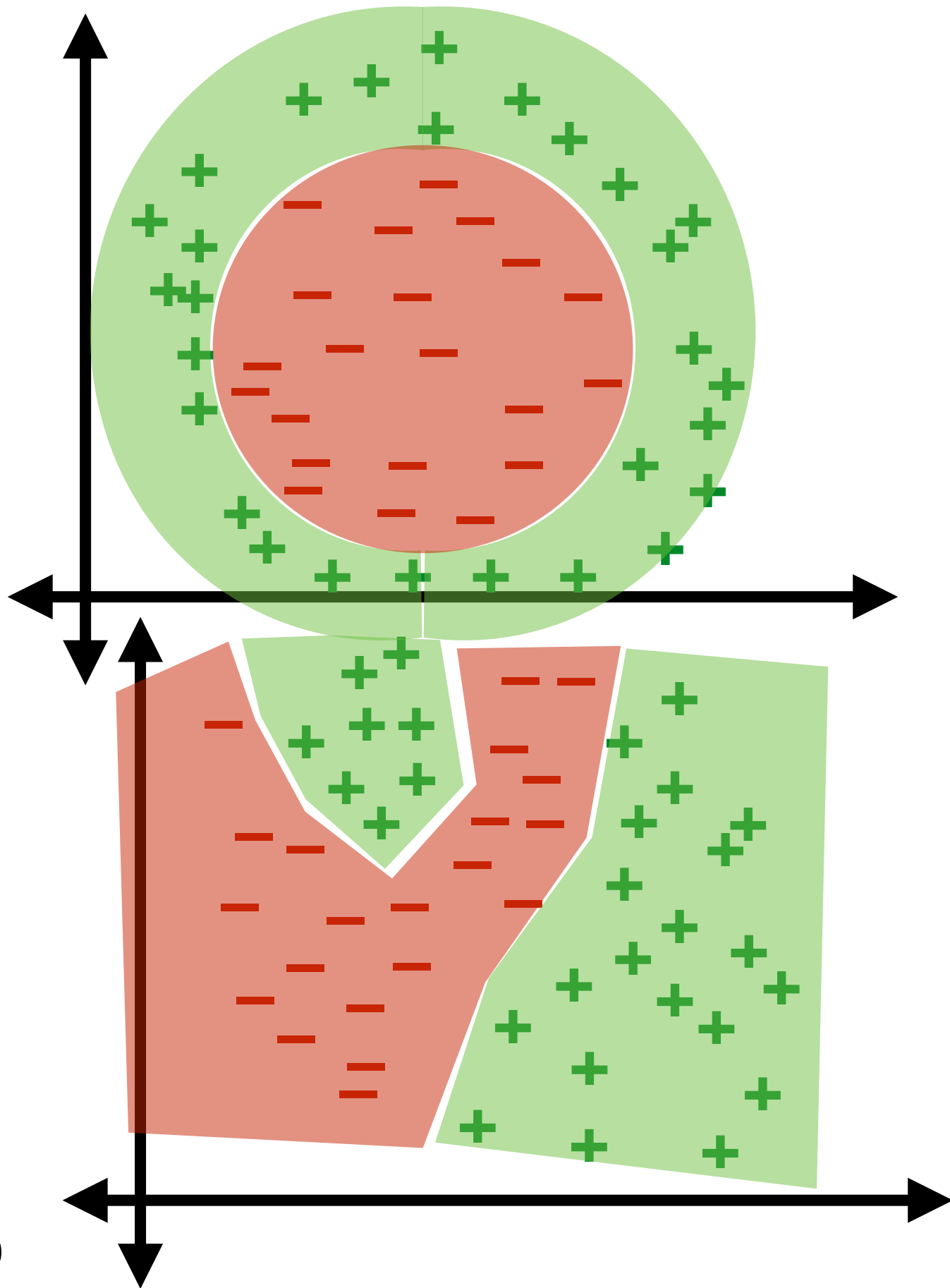
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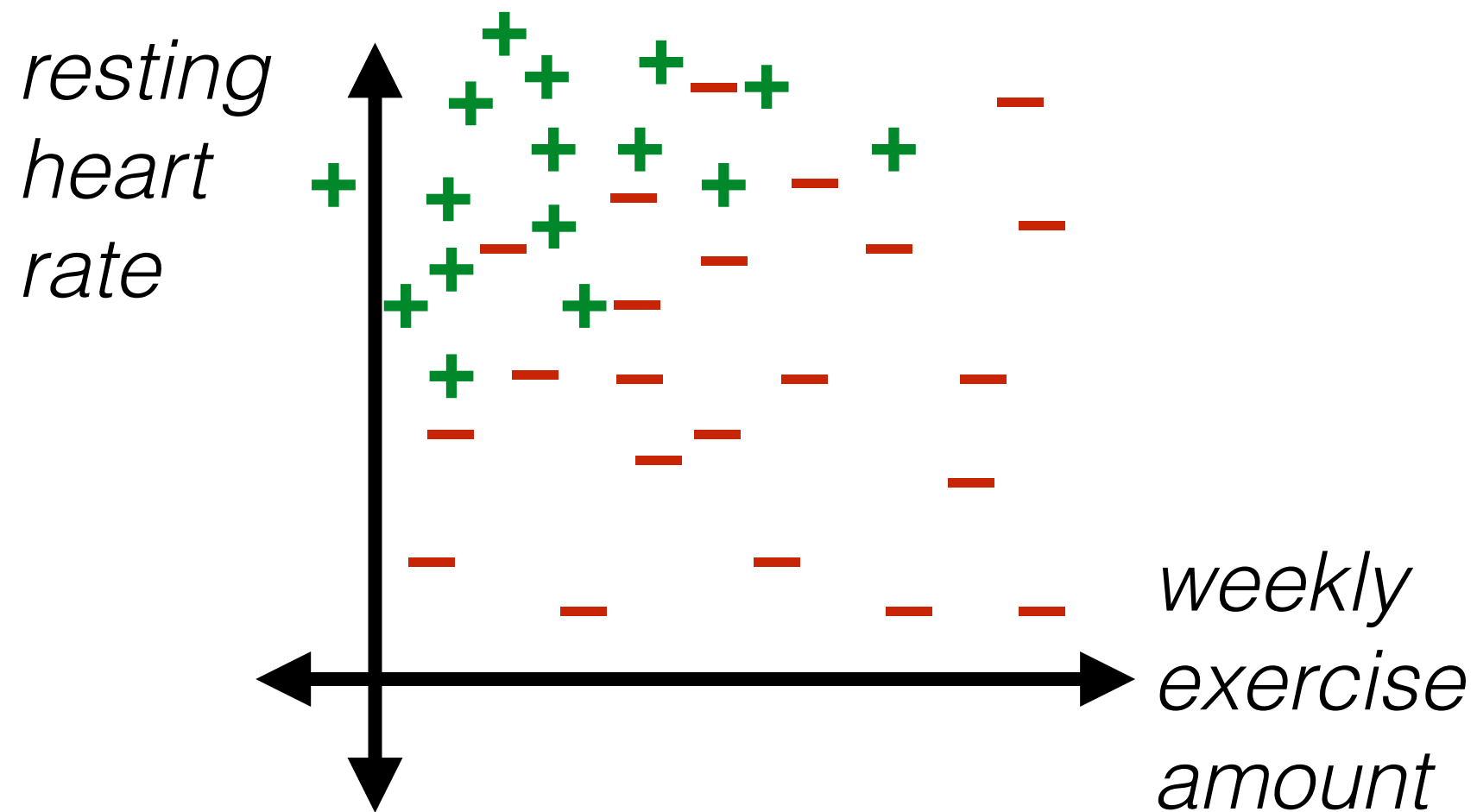
Nonlinear boundaries



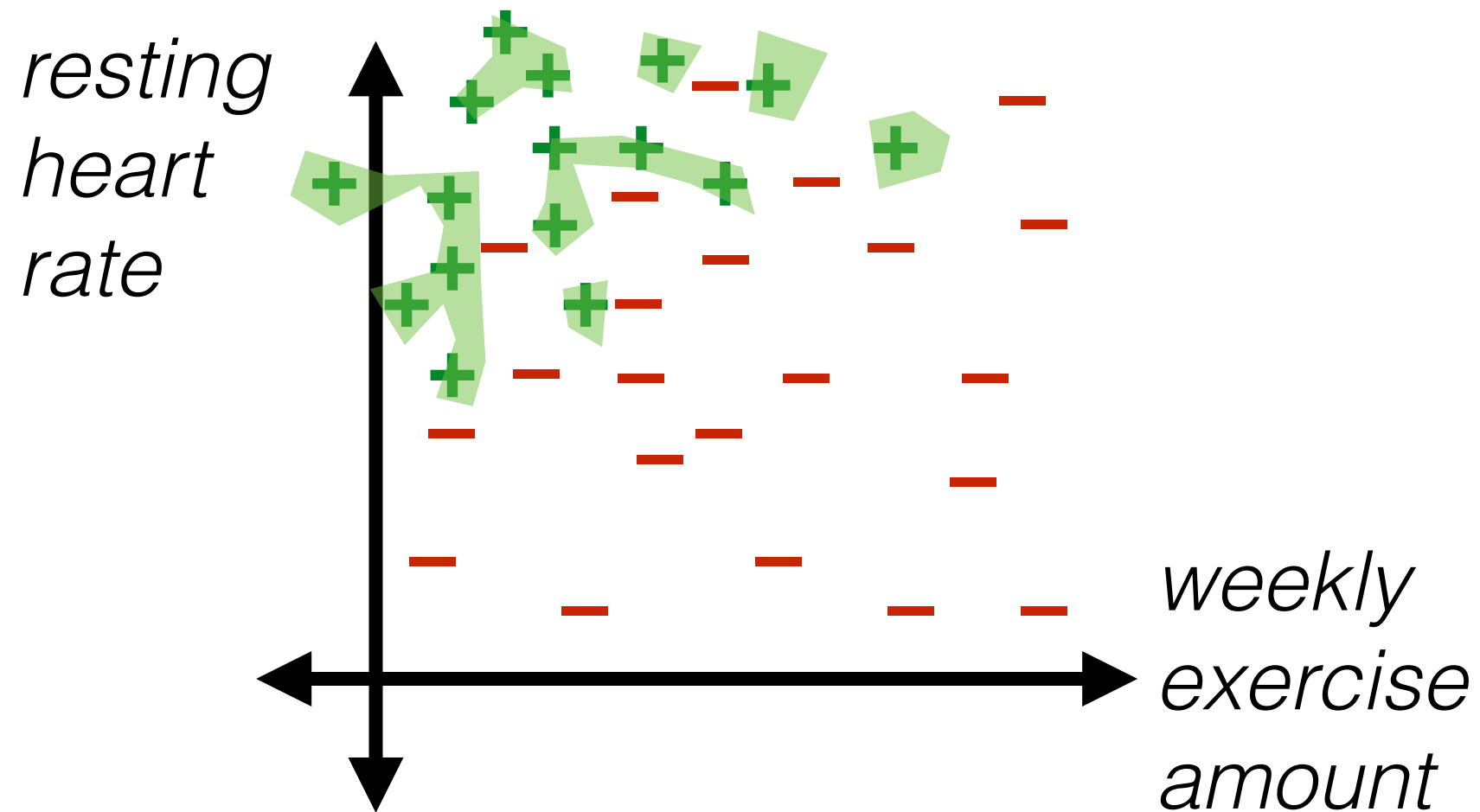
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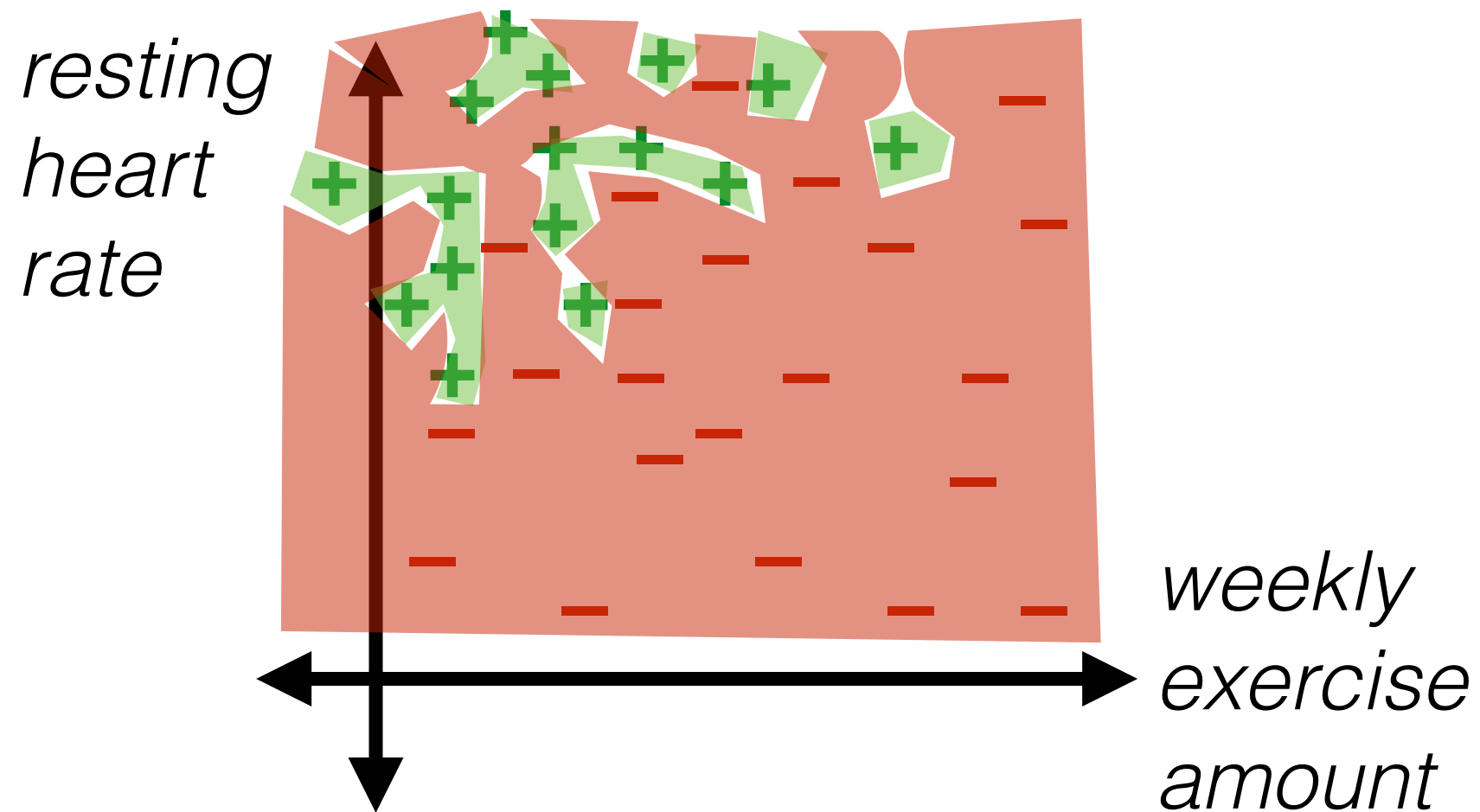
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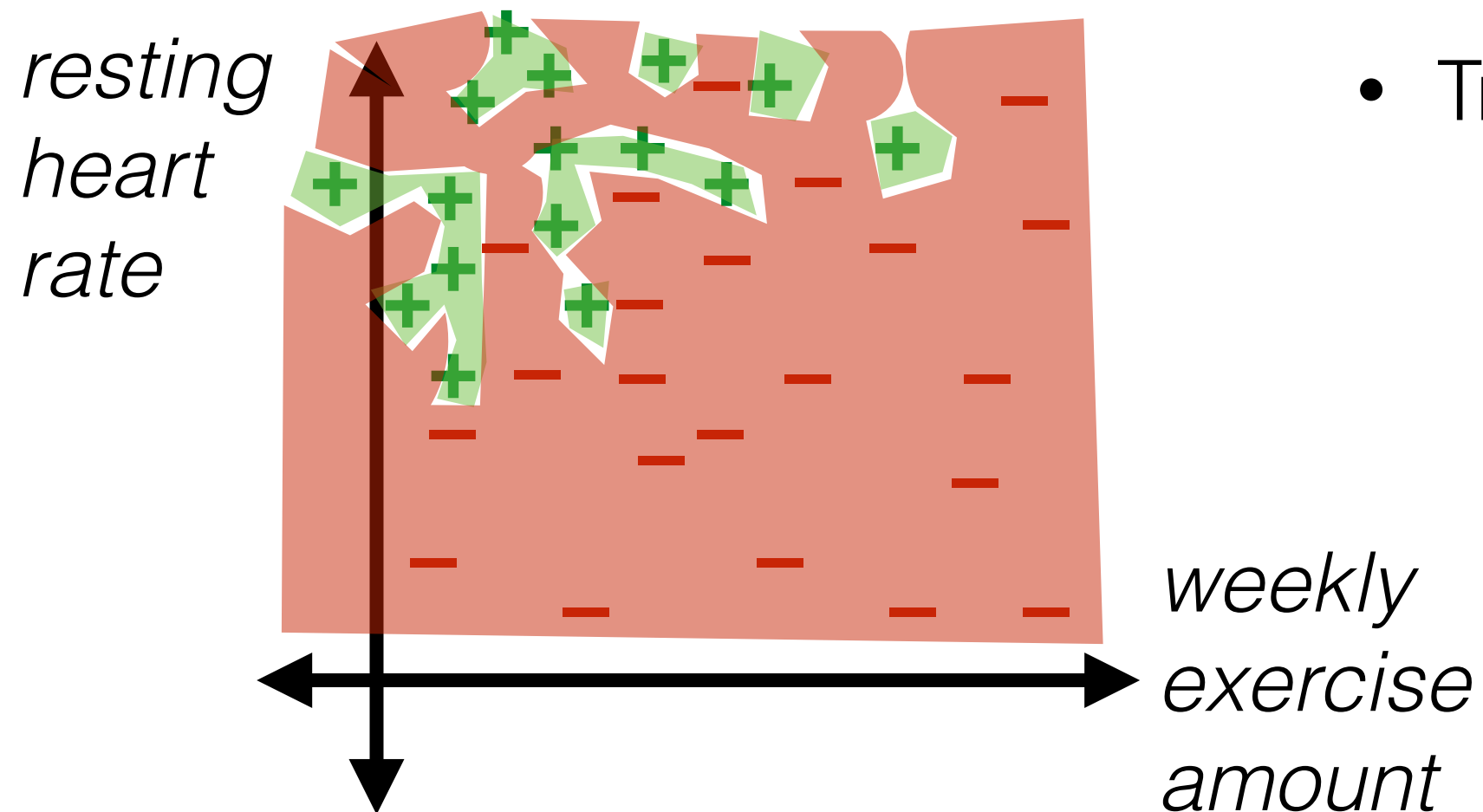
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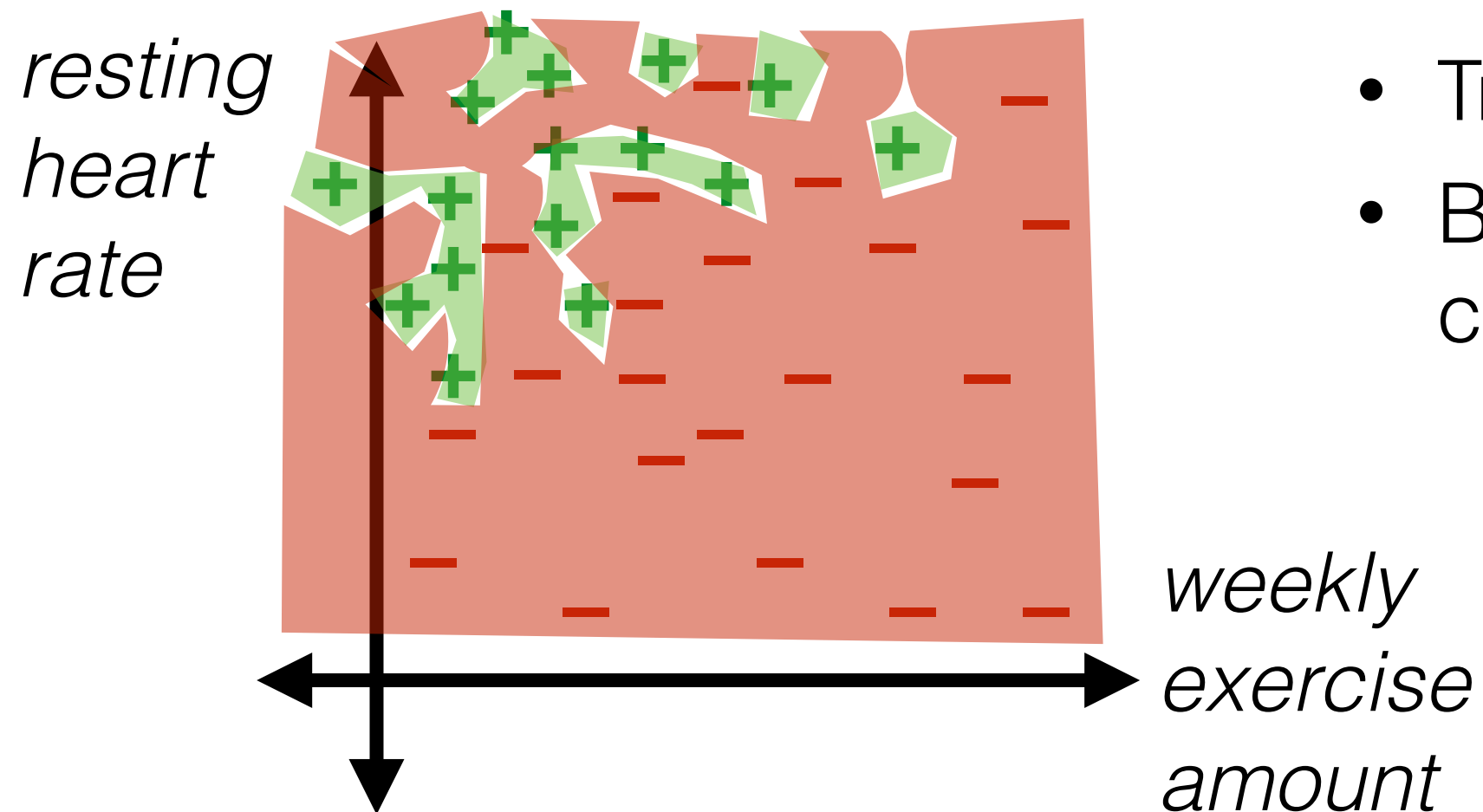


Nonlinear boundaries



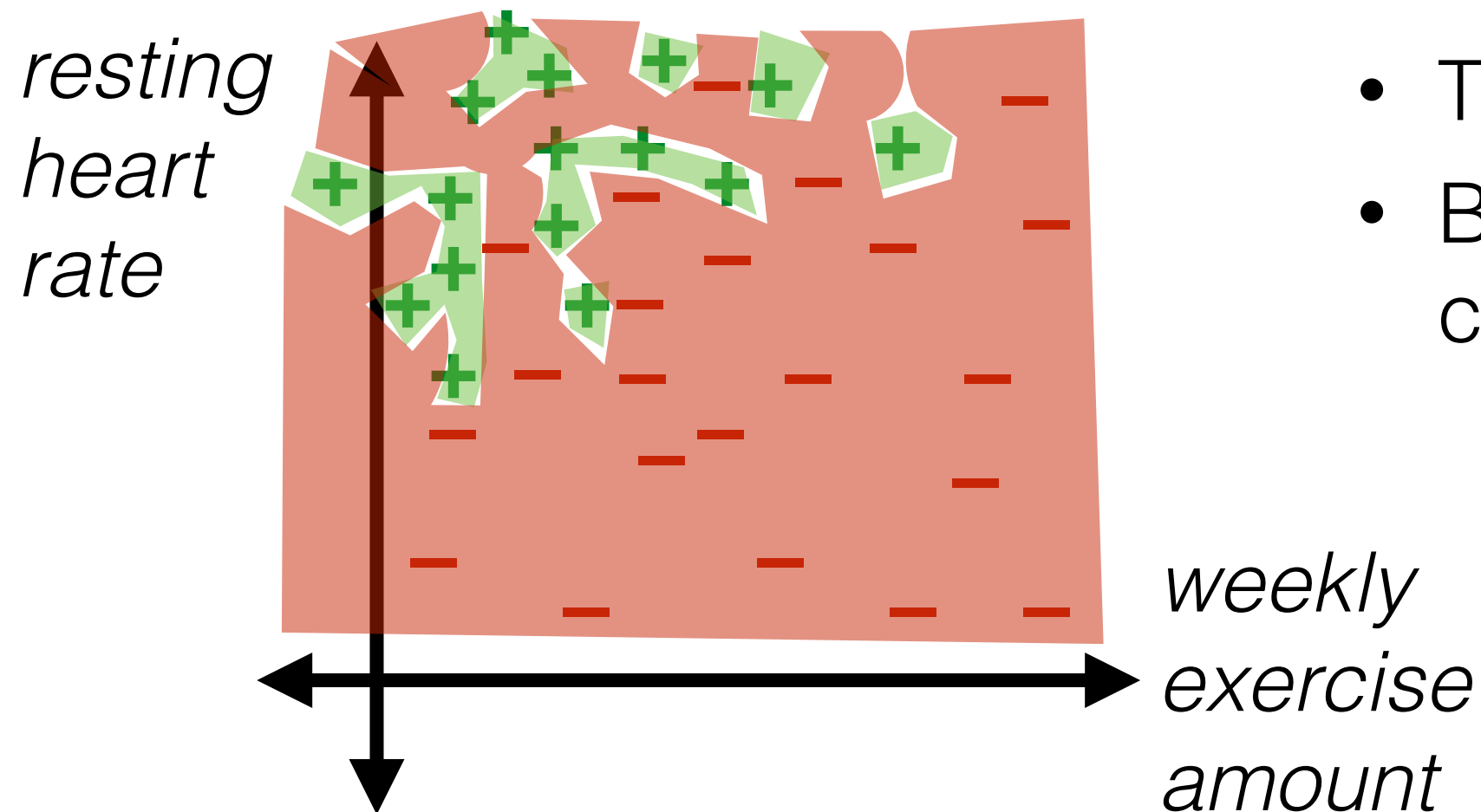
- Training error is 0!

Nonlinear boundaries



- Training error is 0!
- But seems like our classifier is overfitting

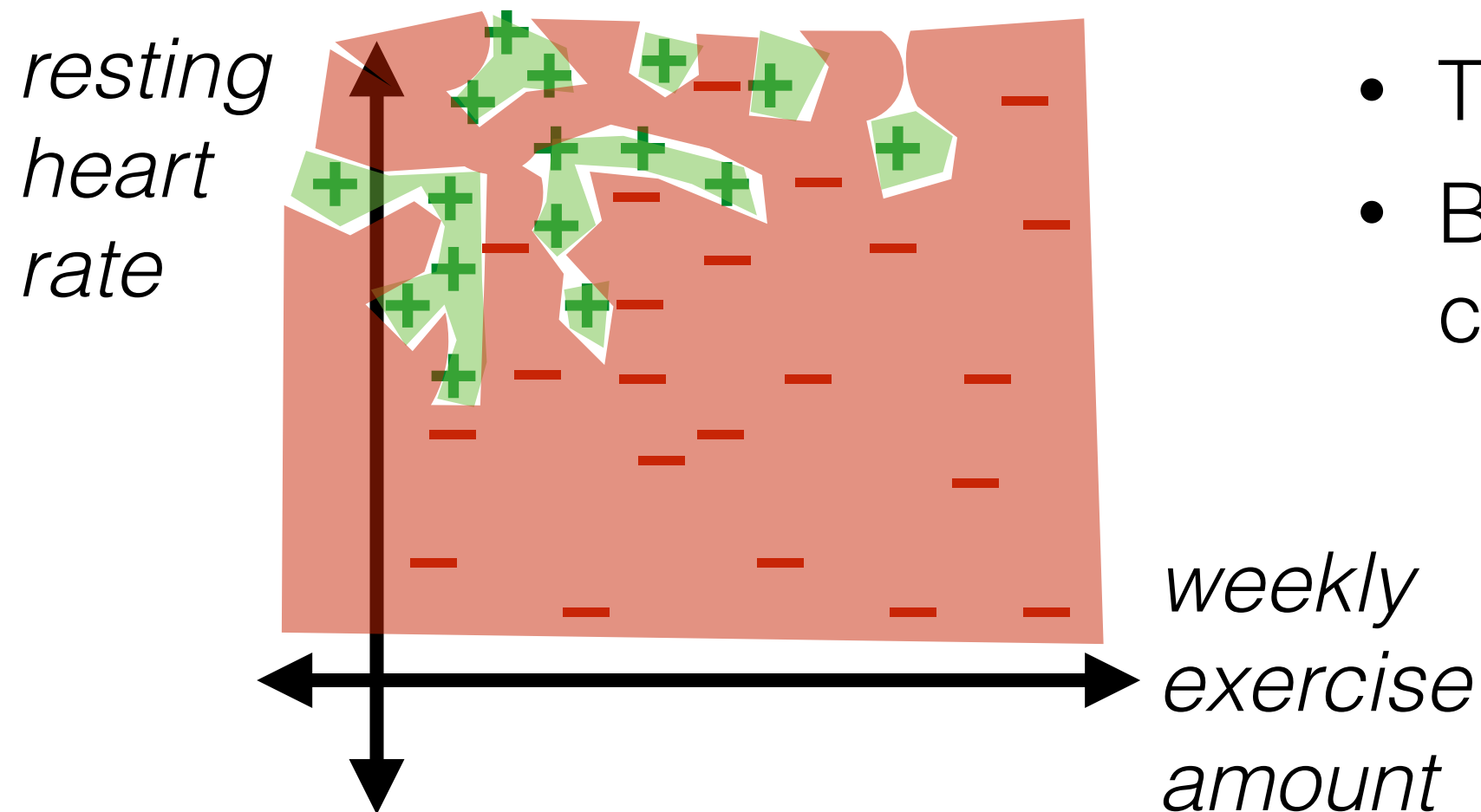
Nonlinear boundaries



- Training error is 0!
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- How can we detect overfitting?

Nonlinear boundaries



- Training error is 0!
- But seems like our classifier is overfitting

- How can we detect overfitting?
- How can we avoid overfitting?

Evaluation of a learning algorithm

- How good is our learning algorithm on data like ours?


Evaluation of a learning algorithm

- How good is our learning algorithm on data like ours?
- Idea: use full data for training and then report training error


Evaluation of a learning algorithm

- How good is our learning algorithm on data like ours?
- Idea: use full data for training and then report training error
- Idea: reserve some data for testing

Evaluation of a learning algorithm

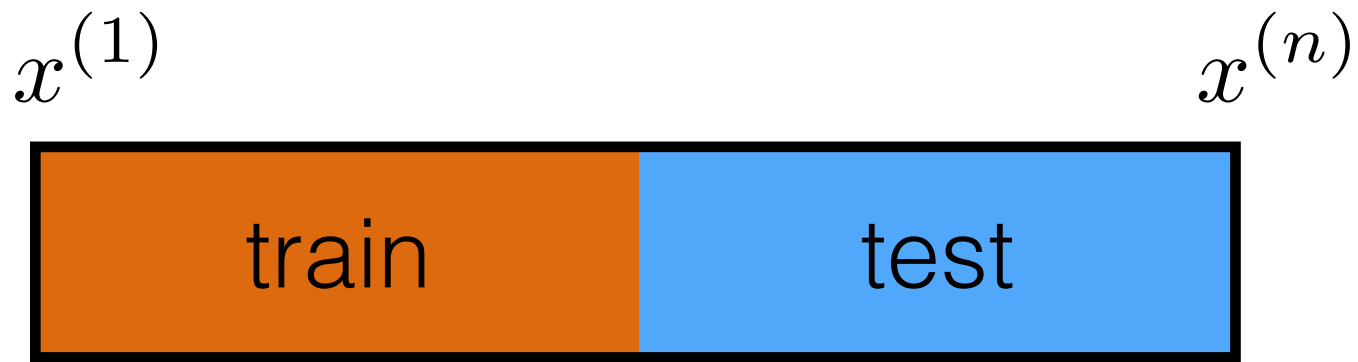
- How good is our learning algorithm on data like ours?  $x^{(1)}$ $x^{(n)}$
- Idea: use full data for training and then report training error
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Evaluation of a learning algorithm

- How good is our learning algorithm on data like ours?
A horizontal bar with a black border, divided into two sections. The left section is orange and labeled 'train'. The right section is blue and labeled 'test'. Above the left end of the bar is the label $x^{(1)}$ and above the right end is the label $x^{(n)}$.
- Idea: use full data for training and then report training error
- Idea: reserve some data for testing

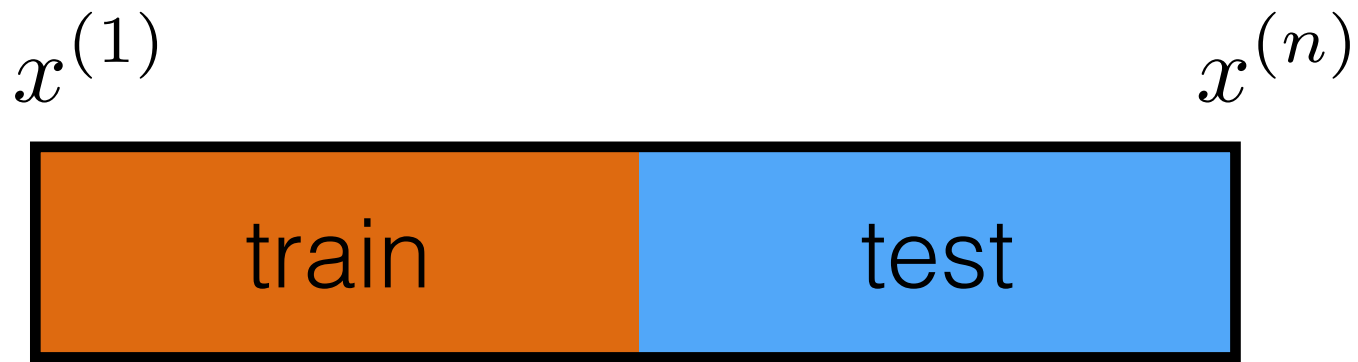
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Evaluation of a learning algorithm

- How good is our learning algorithm on data like ours?
- Idea: use full data for training and then report training error
- Idea: reserve some data for testing
 - More training data: closer to training on full data



Evaluation of a learning algorithm

- How good is our learning algorithm on data like ours?

$x^{(1)}$

train

test

$x^{(n)}$
- Idea: use full data for training and then report training error
- Idea: reserve some data for testing
 - More training data: closer to training on full data
 - More testing data: less noisy estimate of performance

Evaluation of a learning algorithm

- How good is our learning algorithm on data like ours?

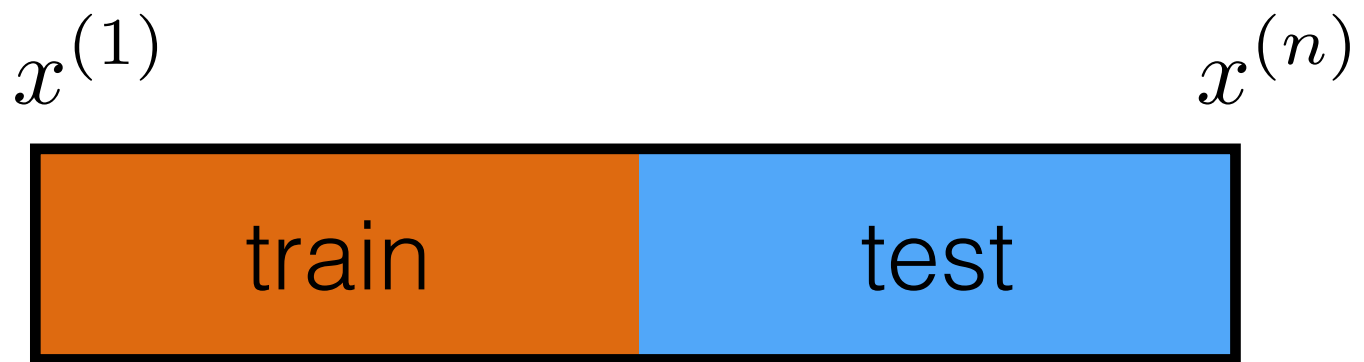
$x^{(1)}$

train

test

$x^{(n)}$
- Idea: use full data for training and then report training error
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 - More training data: closer to training on full data
 - More testing data: less noisy estimate of performance
 - Only one classifier might not be representative

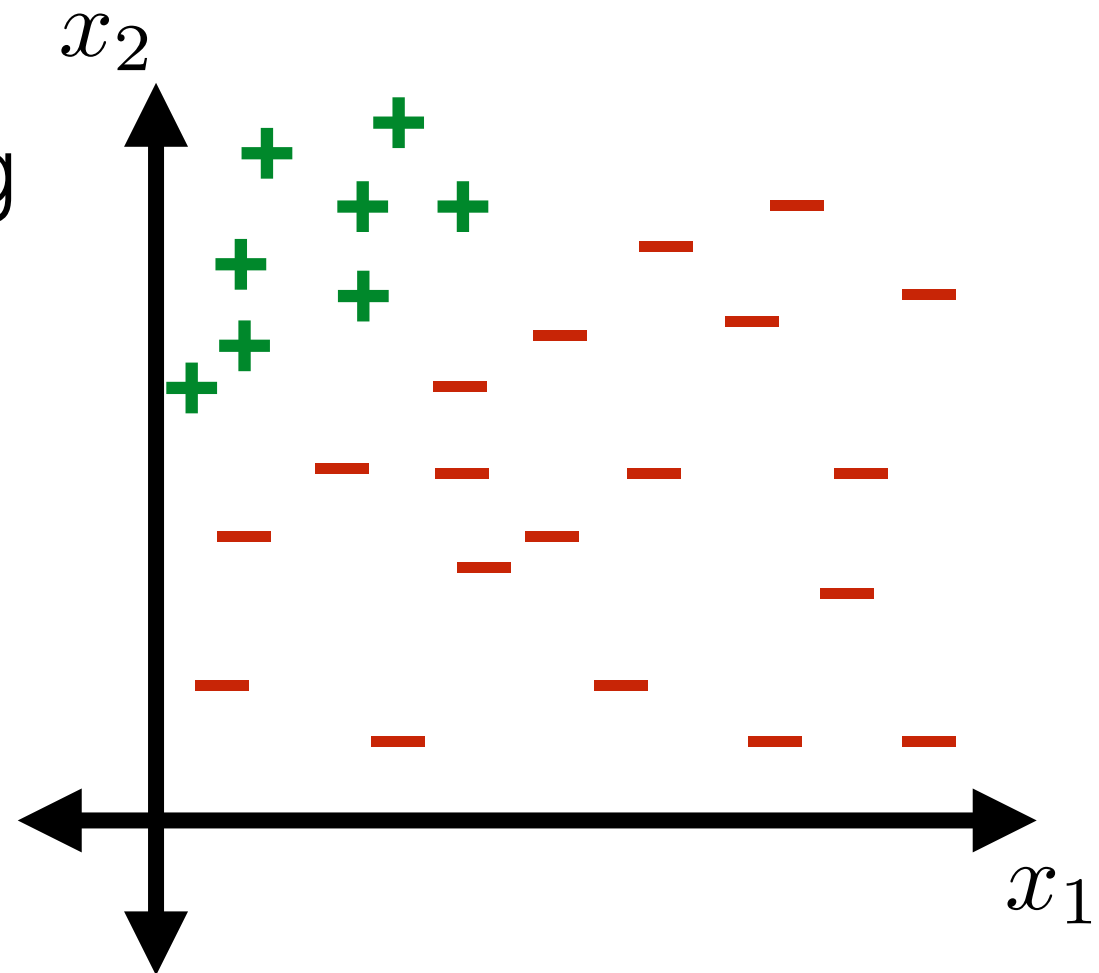
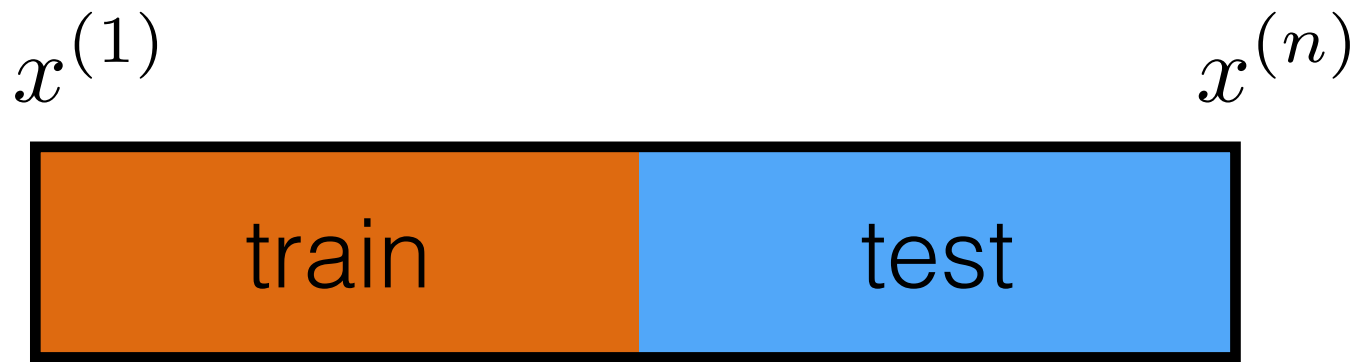
Evaluation of a learning algorithm

- How good is our learning algorithm on data like ours?
- 

The diagram illustrates a data split for training and testing. A horizontal bar is divided into two equal-width sections. The left section is orange and labeled 'train'. The right section is blue and labeled 'test'. Above the 'train' section, the label $x^{(1)}$ is positioned. Above the 'test' section, the label $x^{(n)}$ is positioned.
- Idea: use full data for training and then report training error
 - Idea: reserve some data for testing
 - More training data: closer to training on full data
 - More testing data: less noisy estimate of performance
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 - Good idea to shuffle order of data

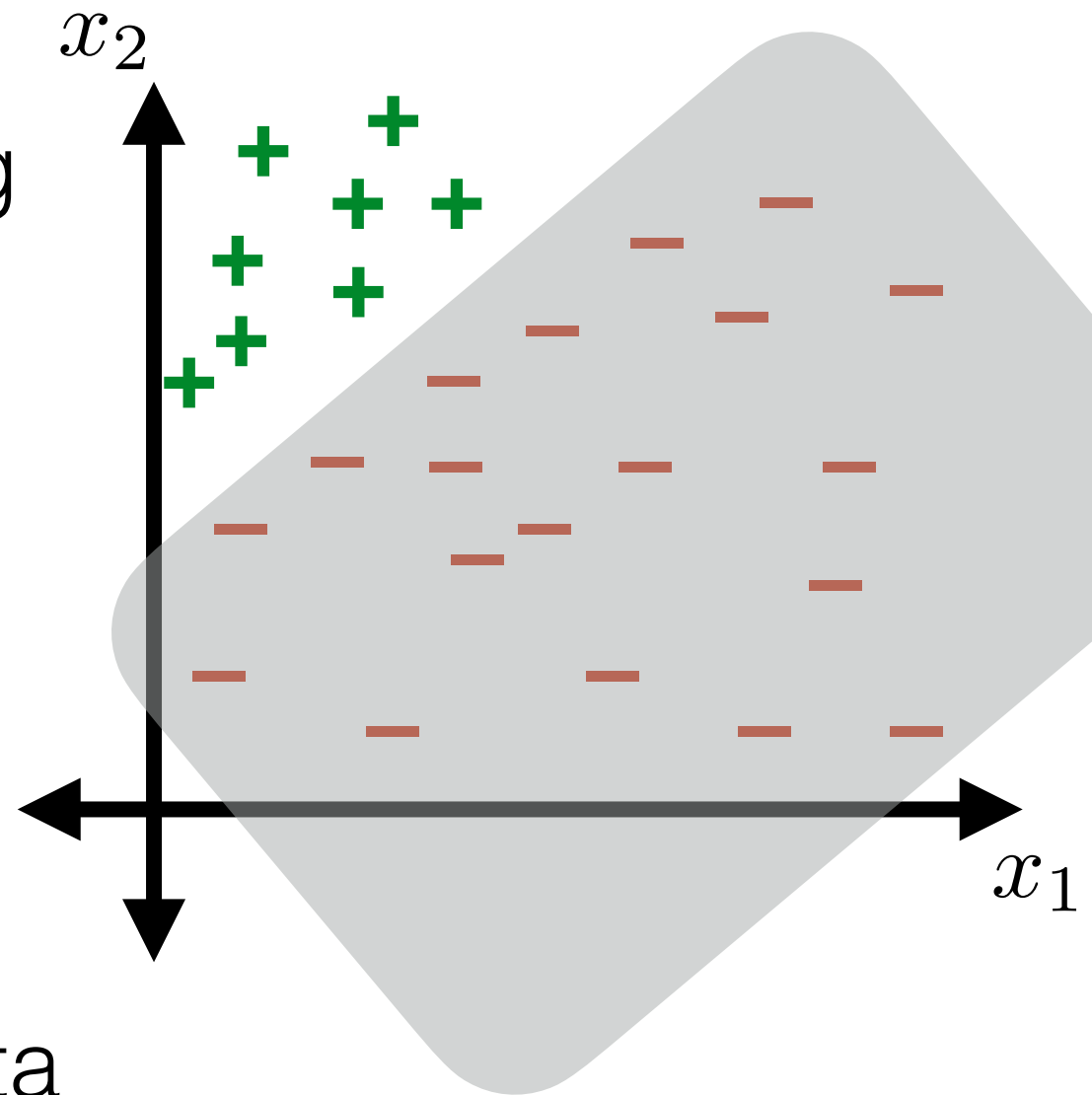
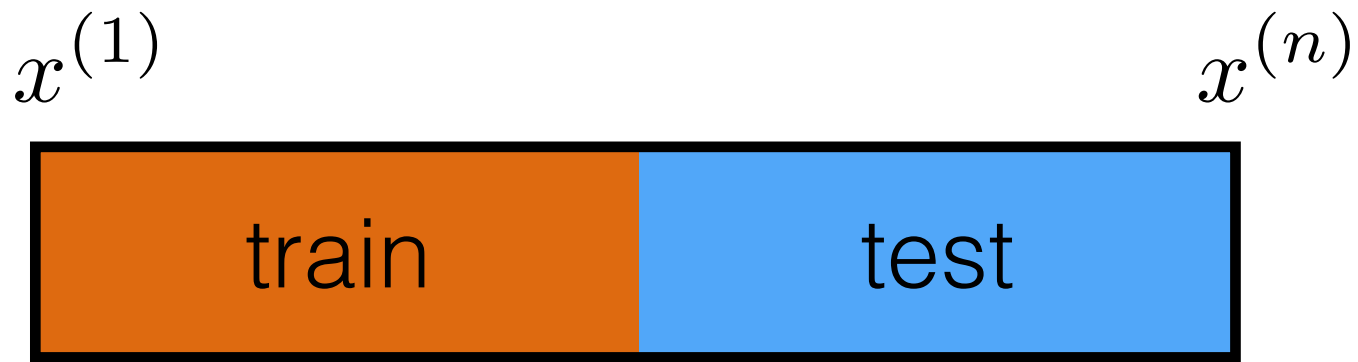
Evaluation of a learning algorithm

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Evaluation of a learning algorithm

- How good is our learning algorithm on data like ours?
- Idea: use full data for training and then report training error
- Idea: reserve some data for testing
 - More training data: closer to training on full data
 - More testing data: less noisy estimate of performance
 - Only one classifier might not be representative
 - Good idea to shuffle order of data



Evaluation of a learning algorithm

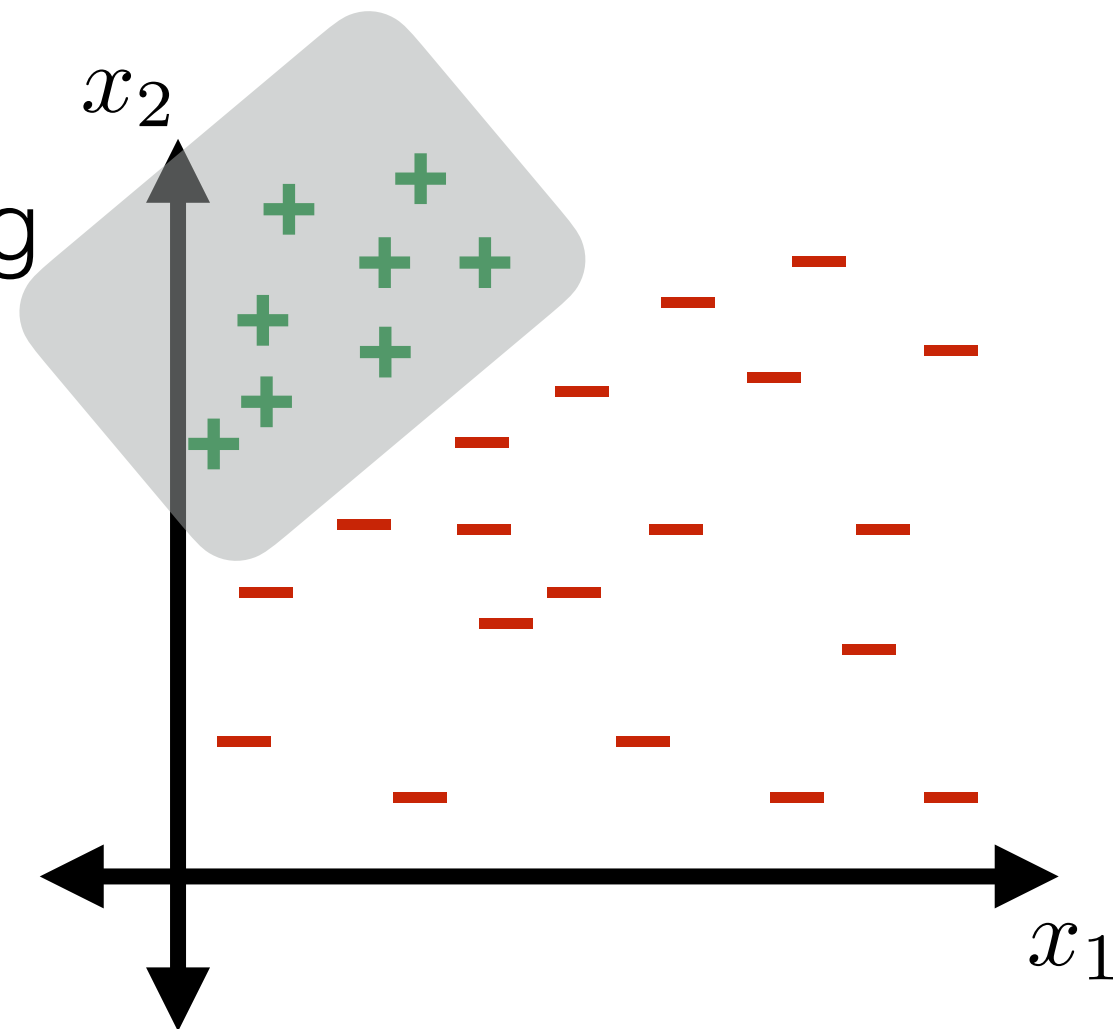
- How good is our learning algorithm on data like ours?

$x^{(1)}$

train

test

$x^{(n)}$
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Cross-validate (\mathcal{D}_n , k)

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Divide \mathcal{D}_n into k chunks $\mathcal{D}_{n,1}, \dots, \mathcal{D}_{n,k}$ (of roughly equal size)

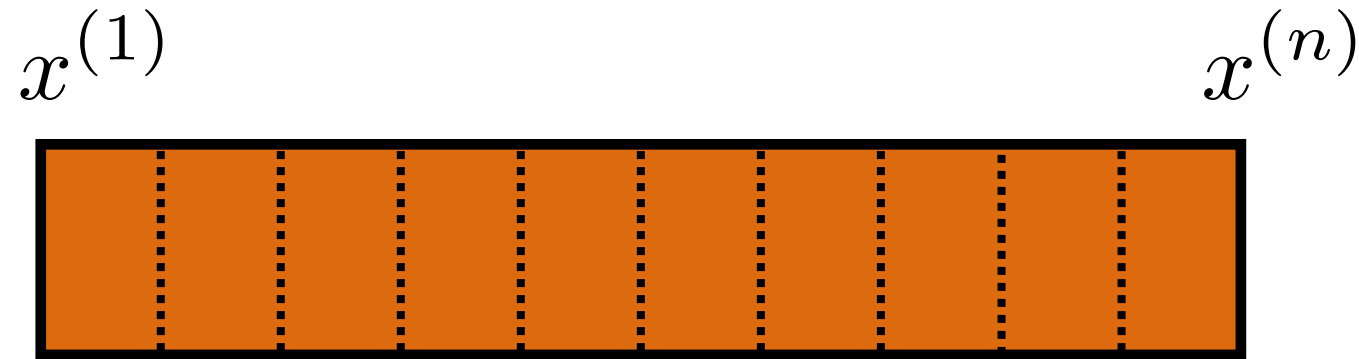
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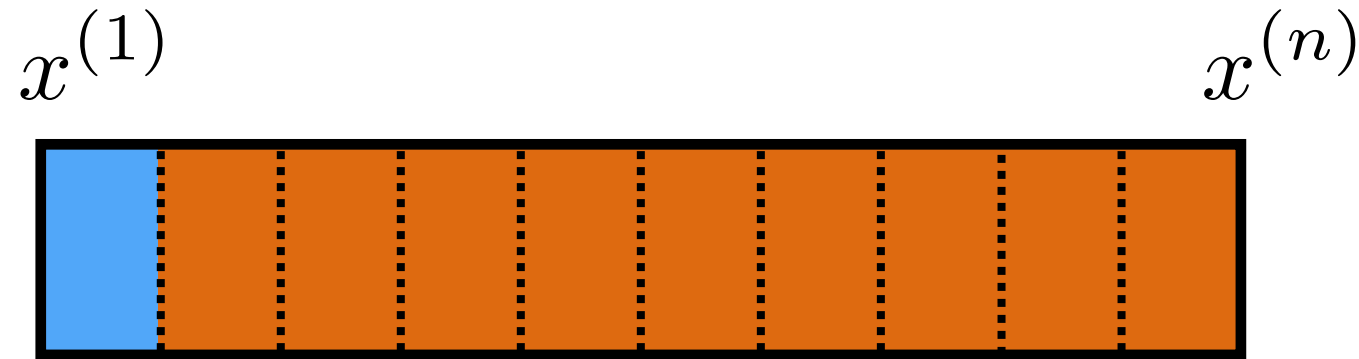
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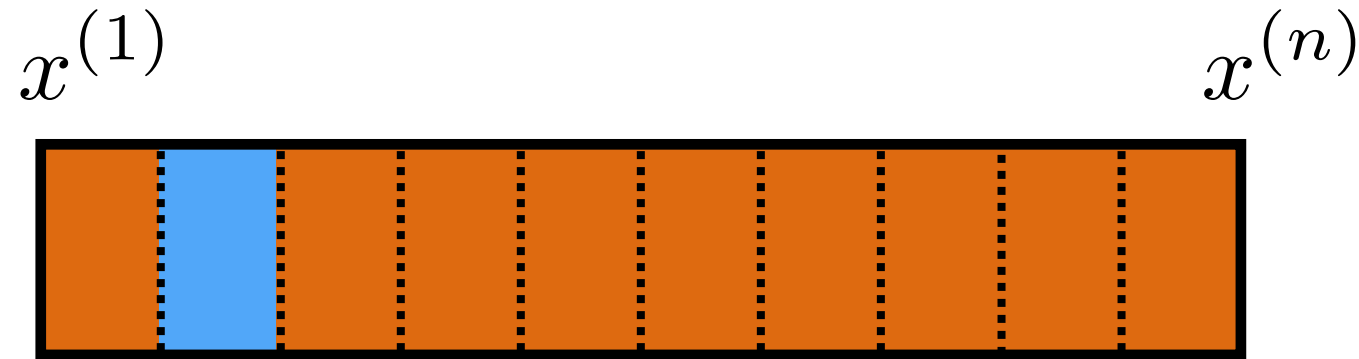


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for $i = 1$ to k

Evaluation of a learning algorithm

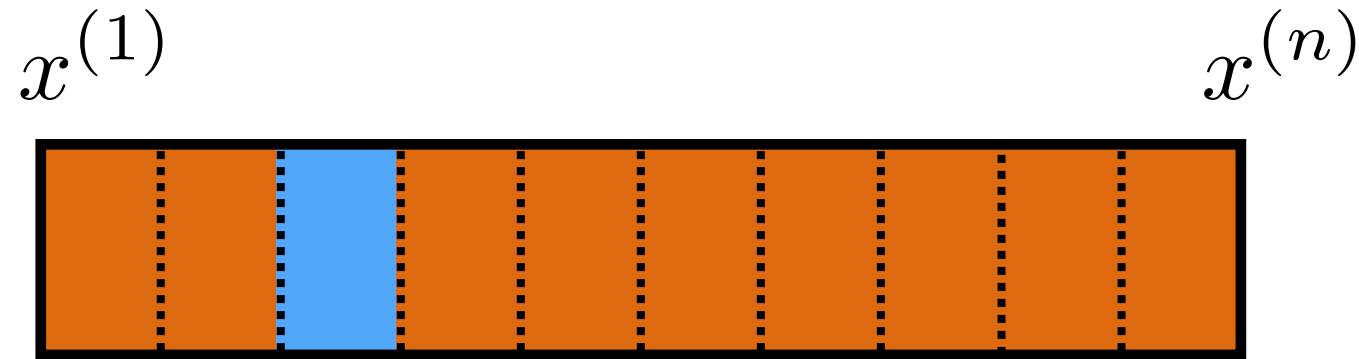


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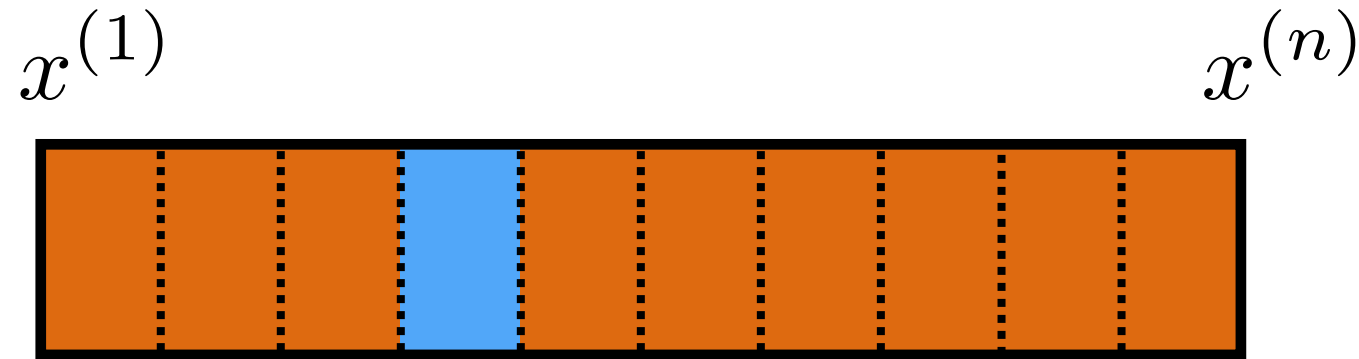


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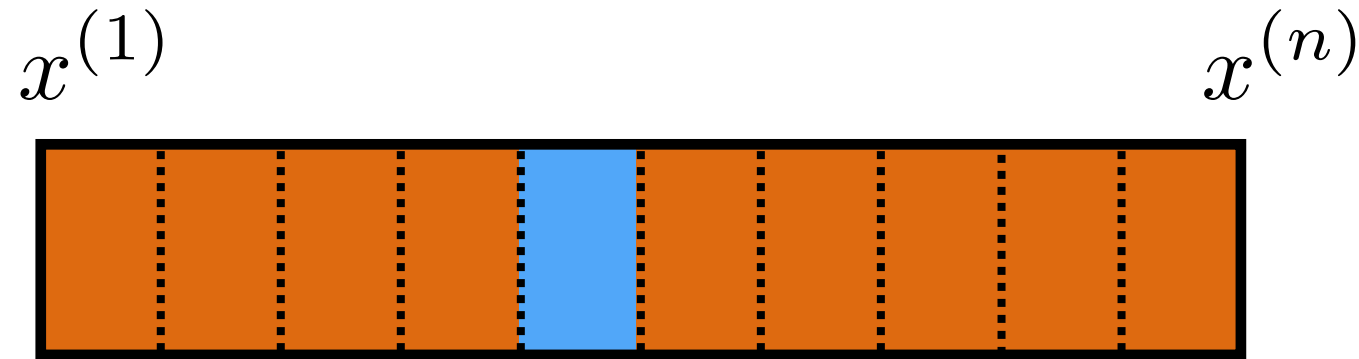


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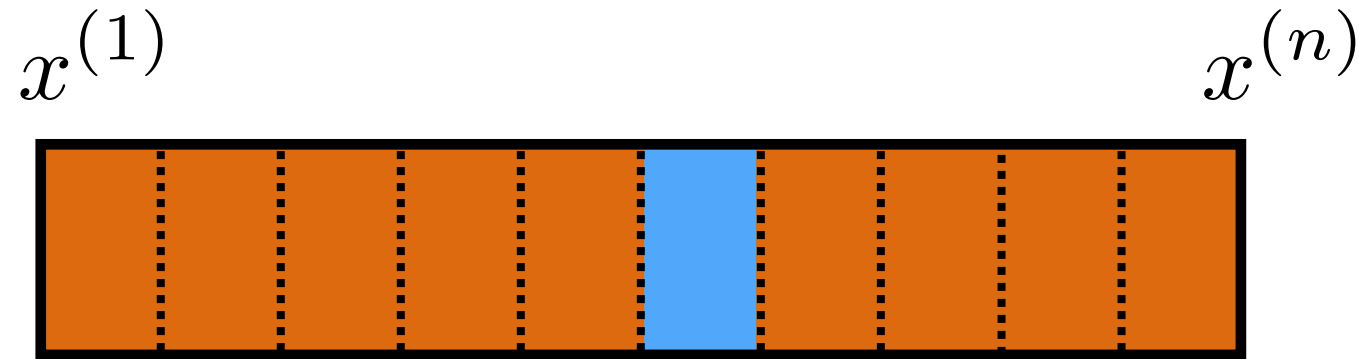


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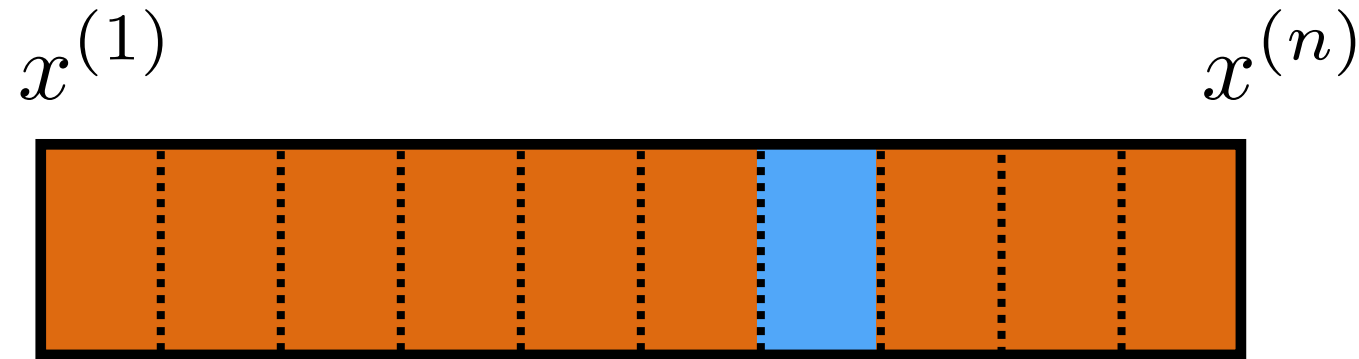


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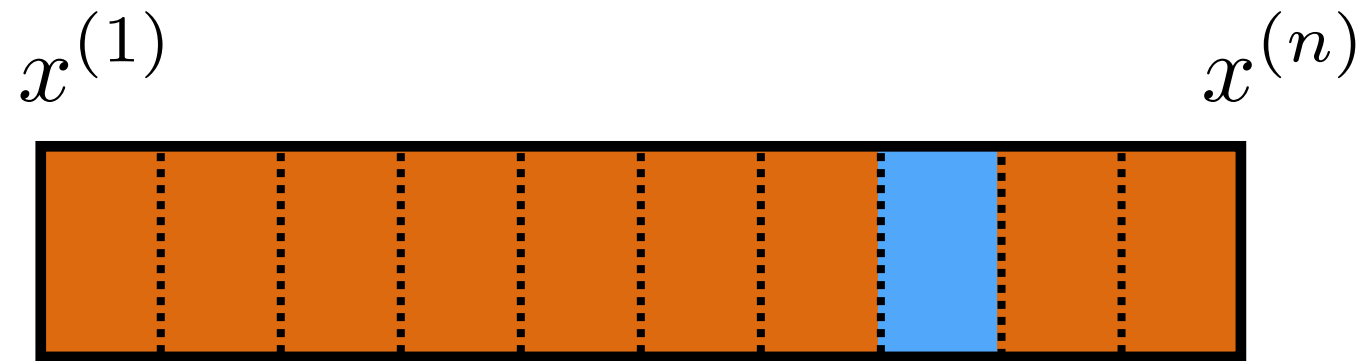


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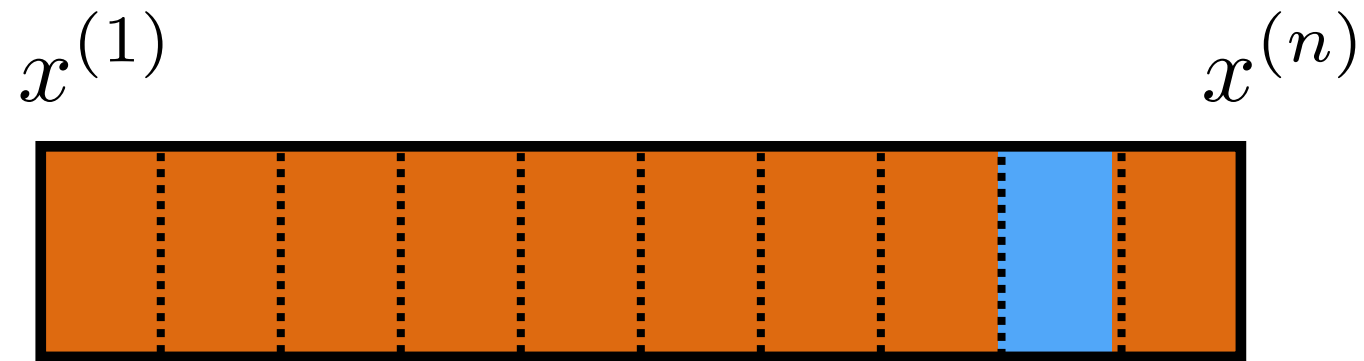


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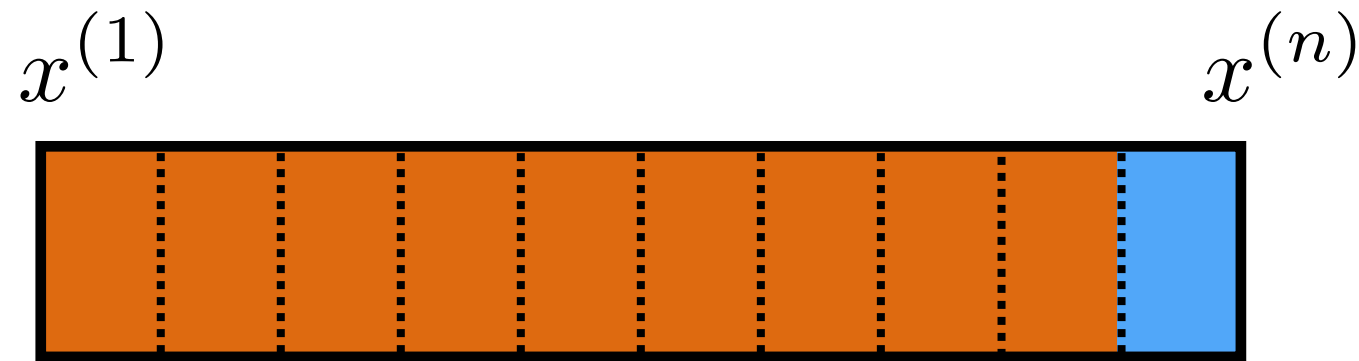


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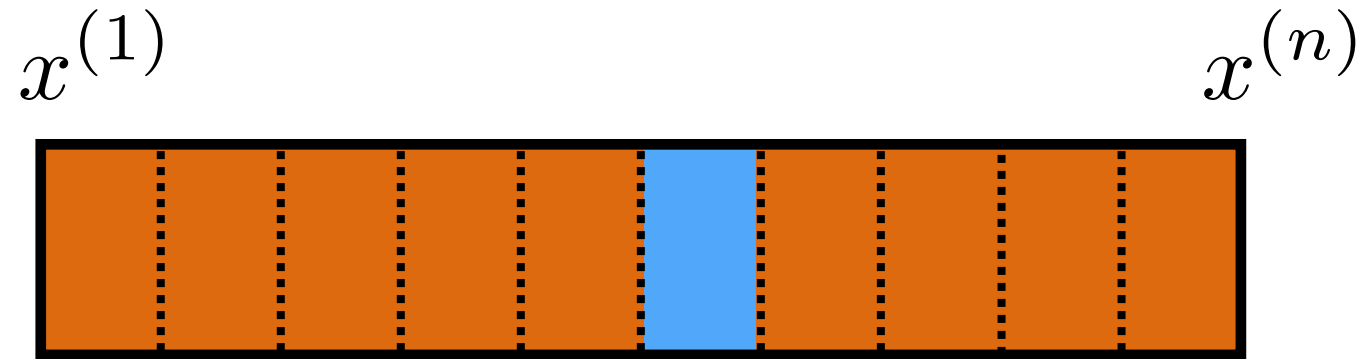


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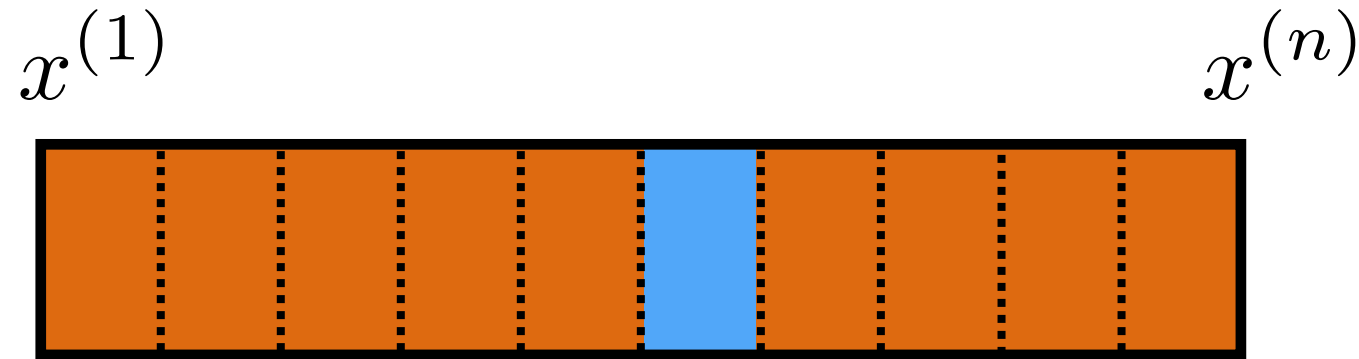


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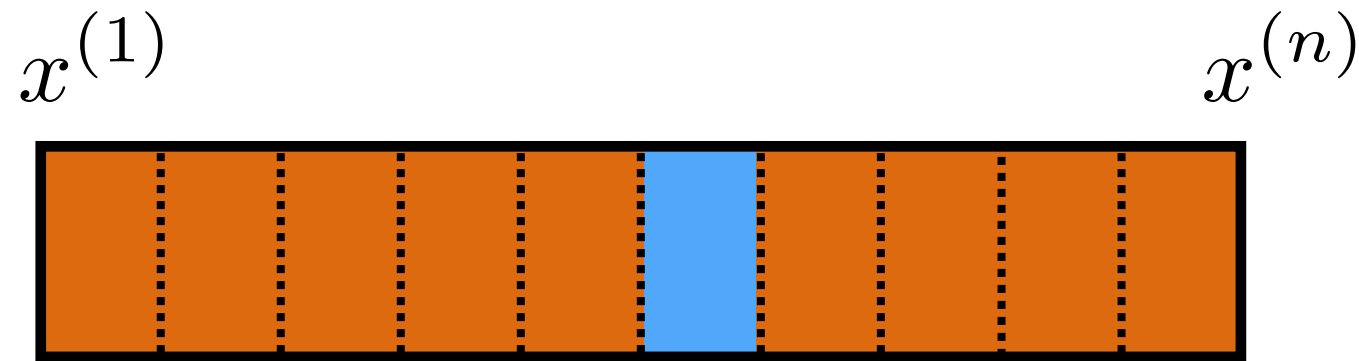
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Evaluation of a learning algorithm



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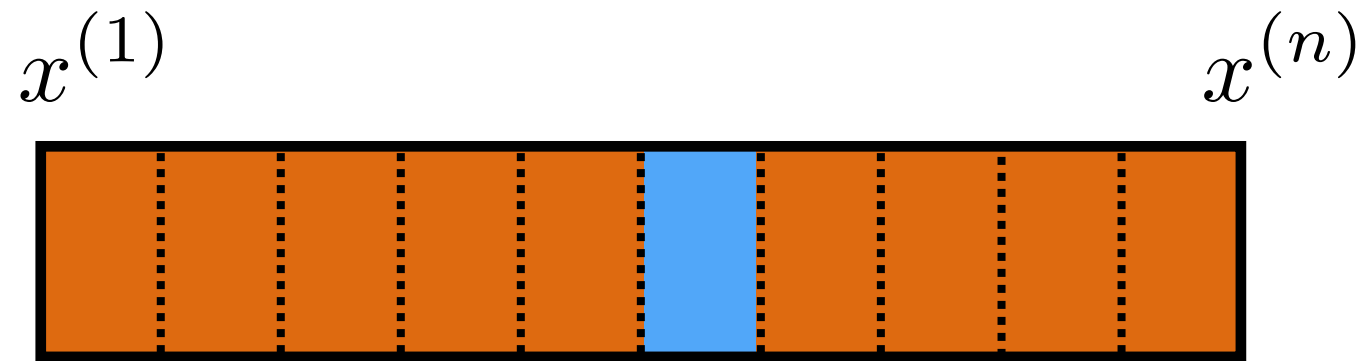
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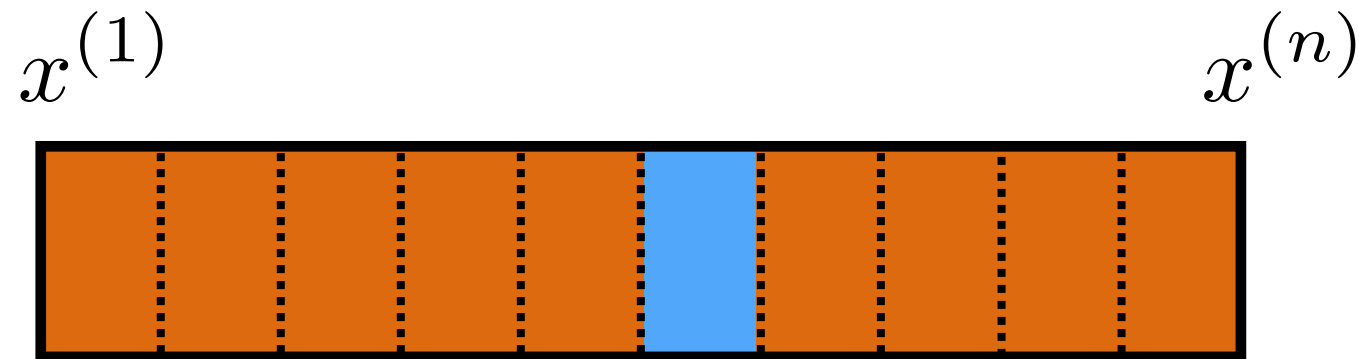
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- Again, good idea to shuffle order of data first