6.036/6.862: Introduction to Machine Learning

Lecture: starts Tuesdays 9:35am (Boston time zone)

Course website: introml.odl.mit.edu

Who's talking? Prof. Tamara Broderick

Questions? discourse.odl.mit.edu ("Lecture 4" category)

Materials: Will all be available at course website

Last Time(s)

I. Linear classifiers
II. Perceptron algorithm
III. A more-complete ML analysis

Today's Plan

I. Linear logistic classification/logistic regression
II. Gradient descent

Recall

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- Perceptron struggles with data that's not linearly separable

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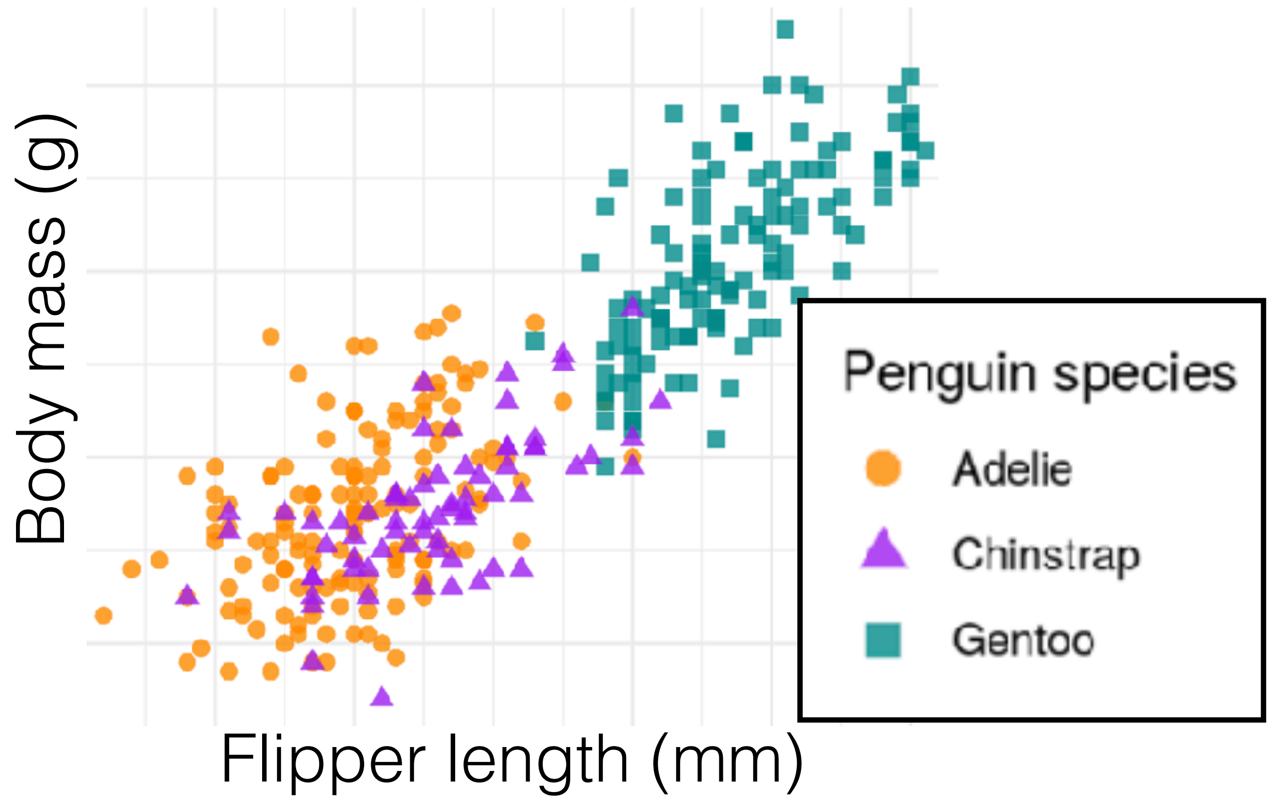

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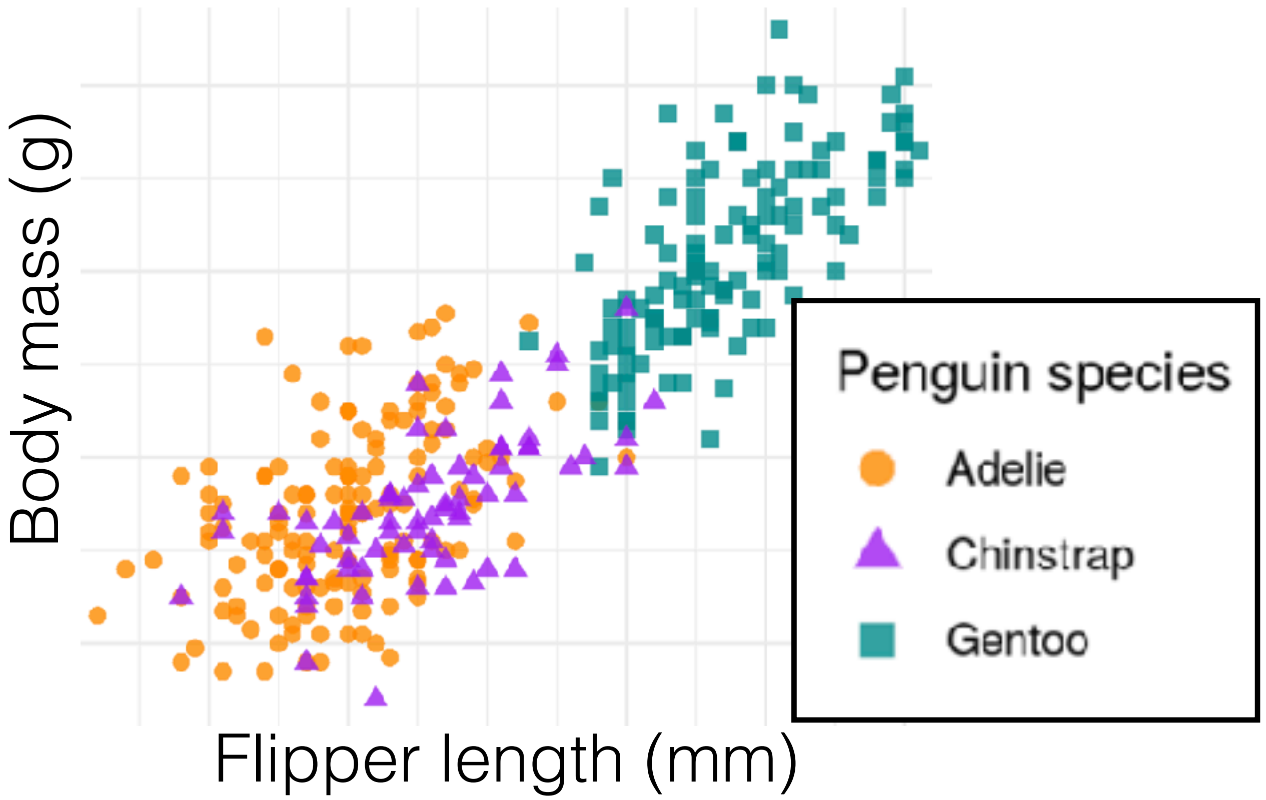

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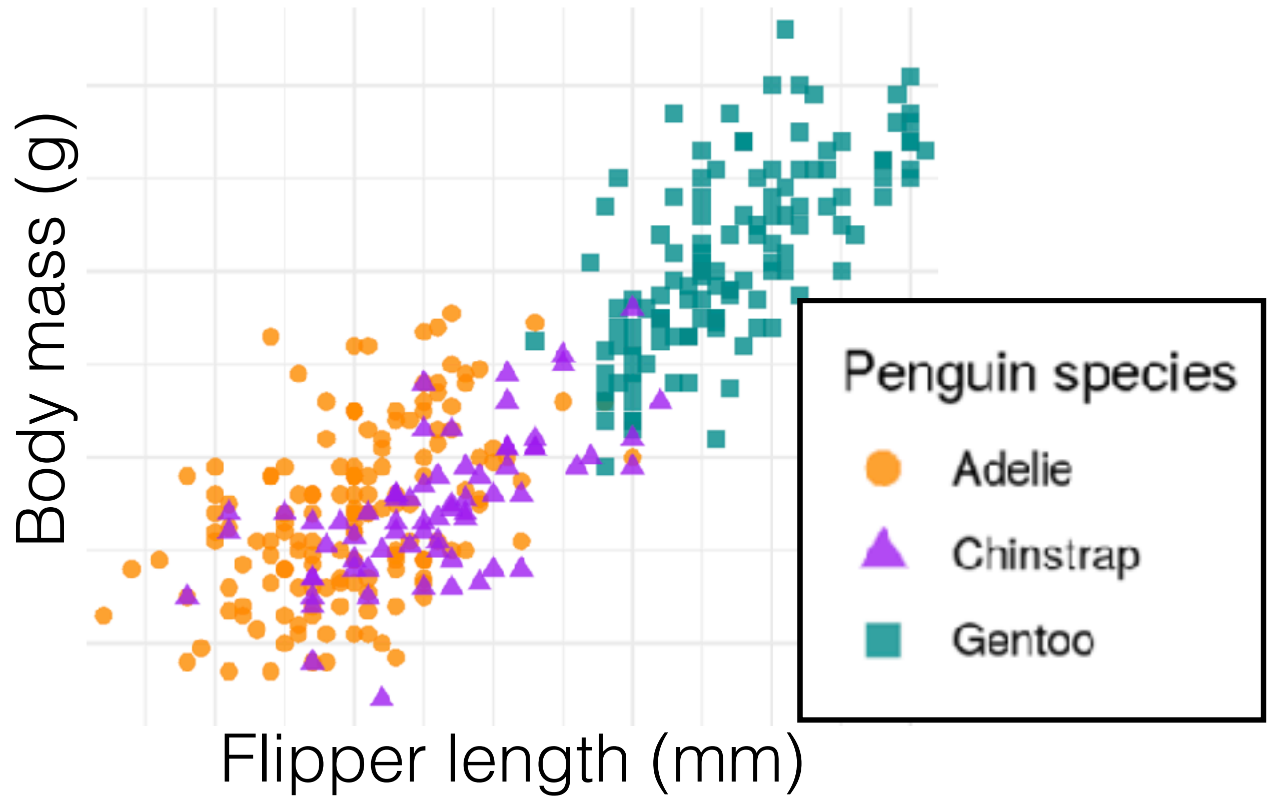

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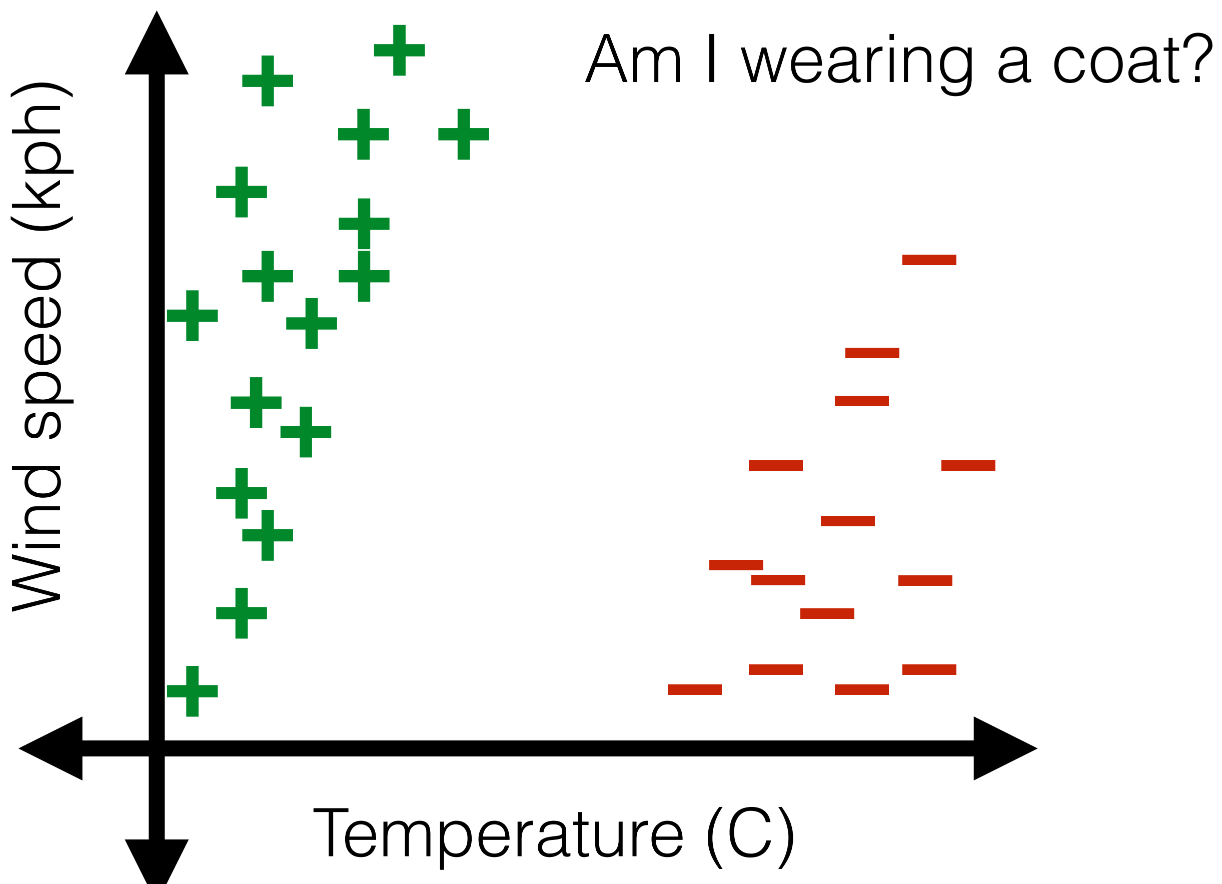

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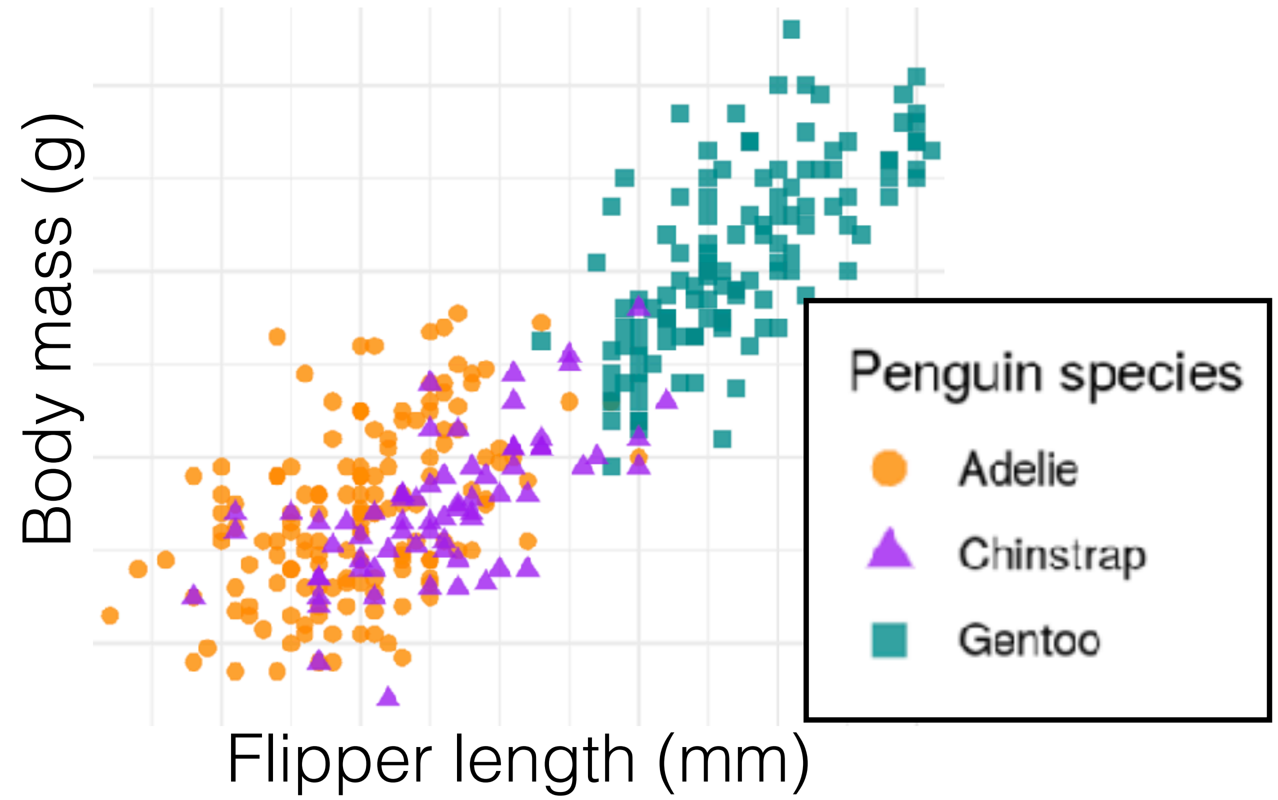

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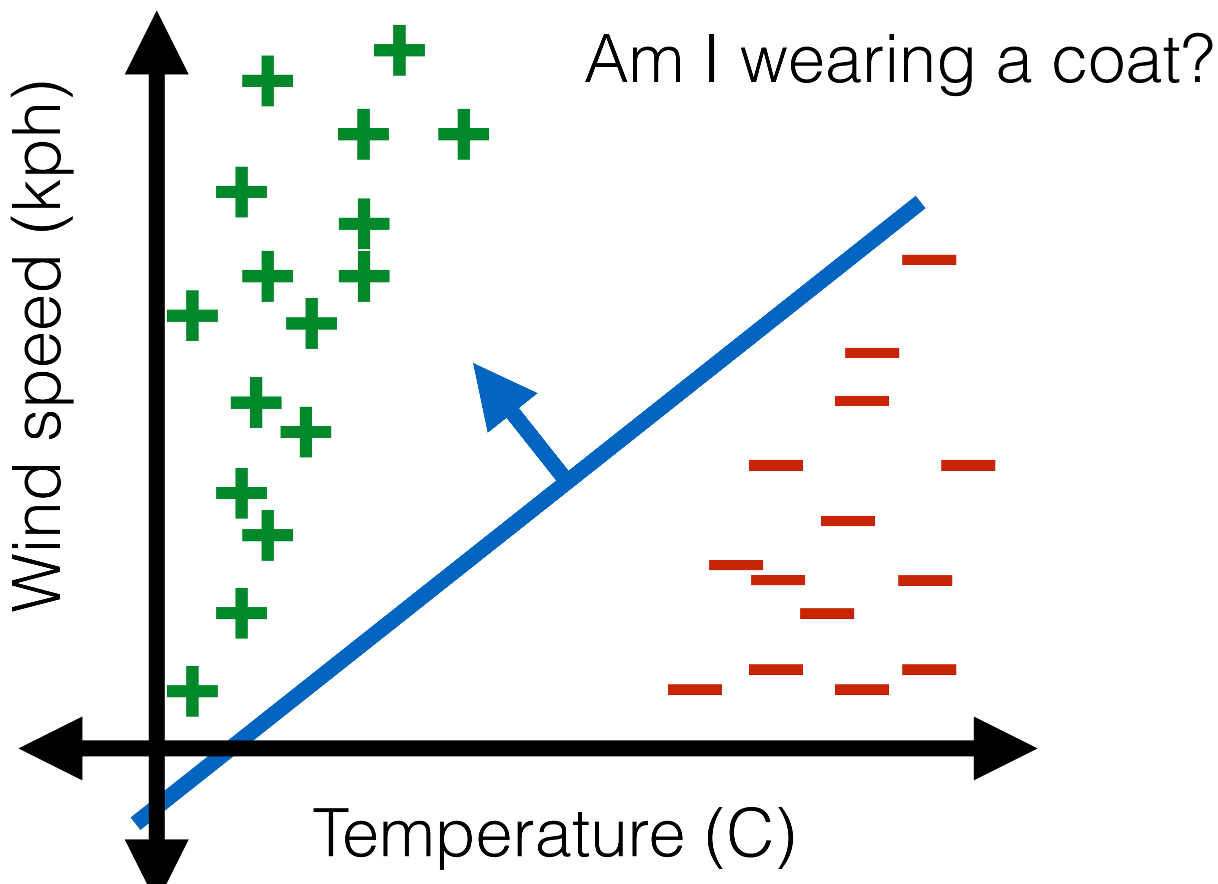

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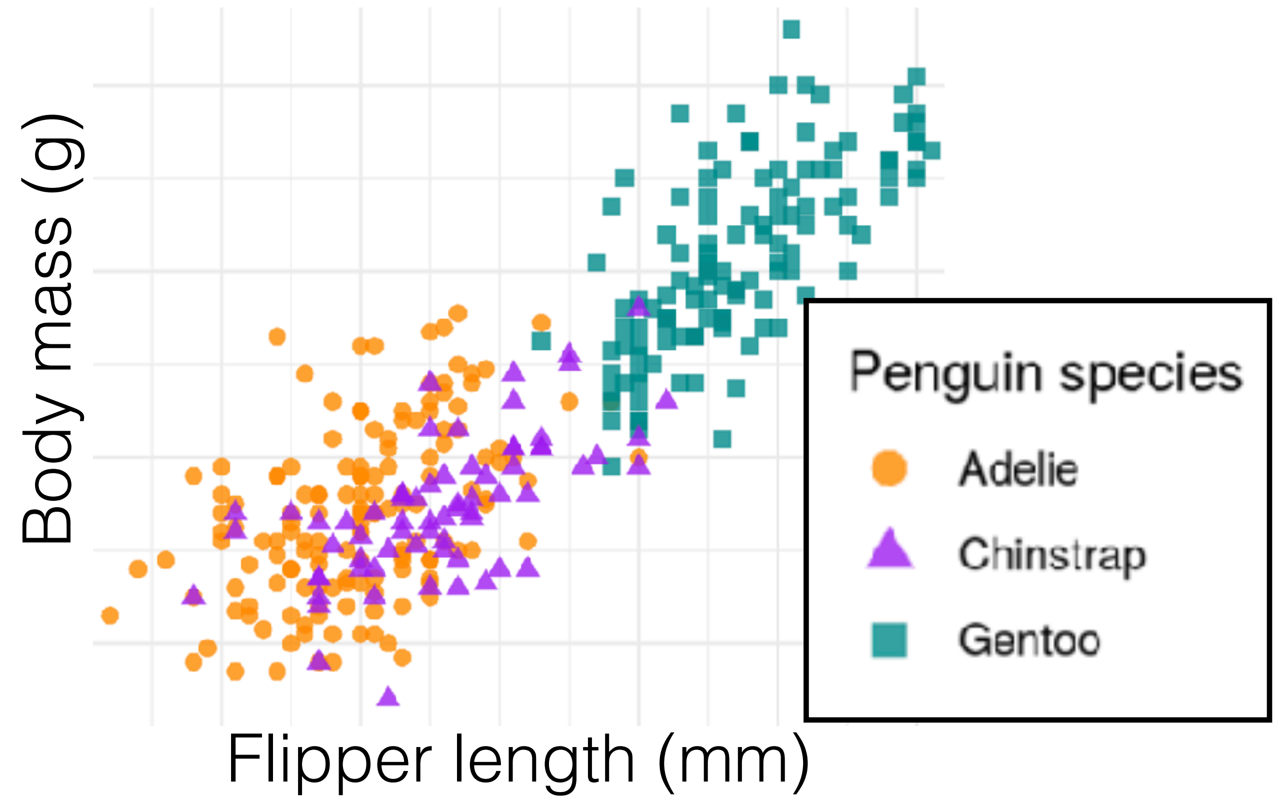

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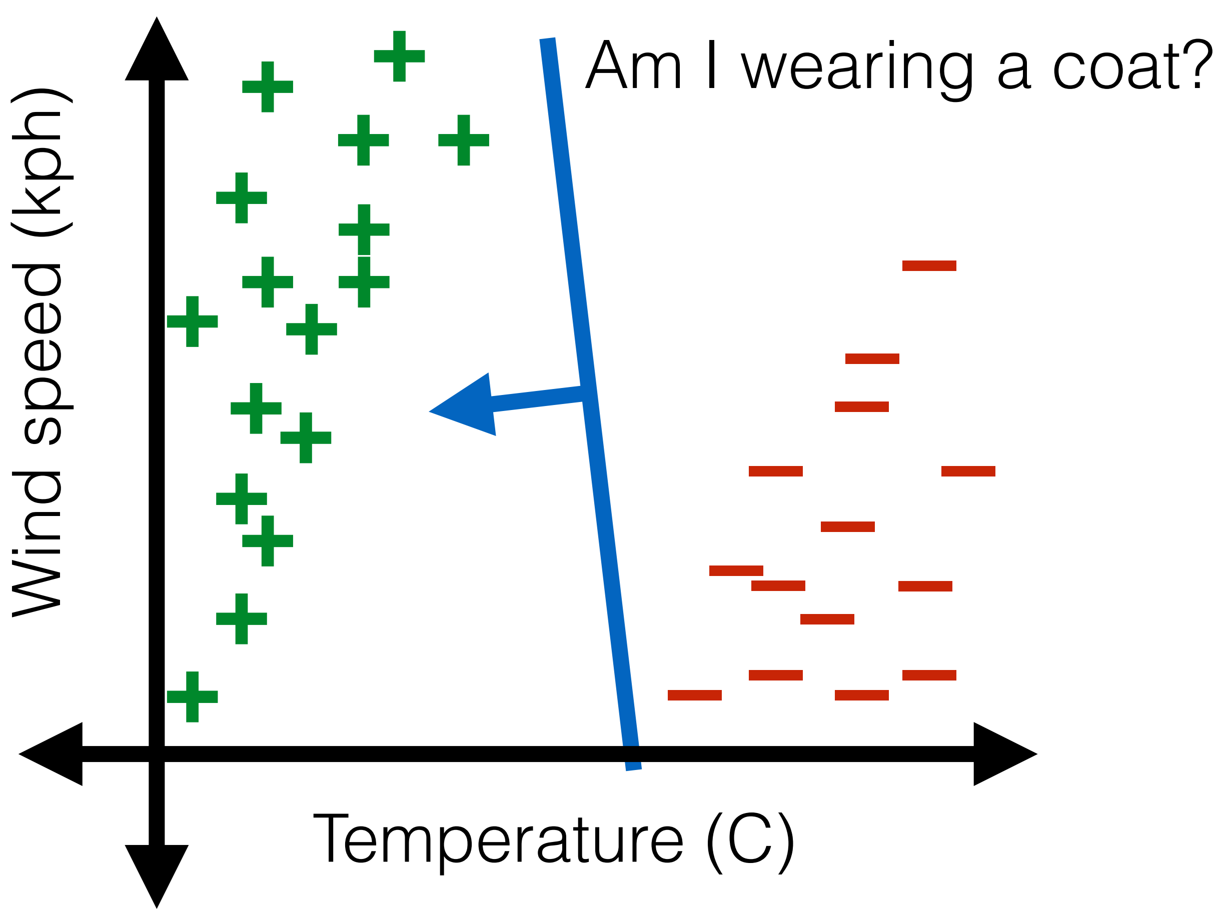

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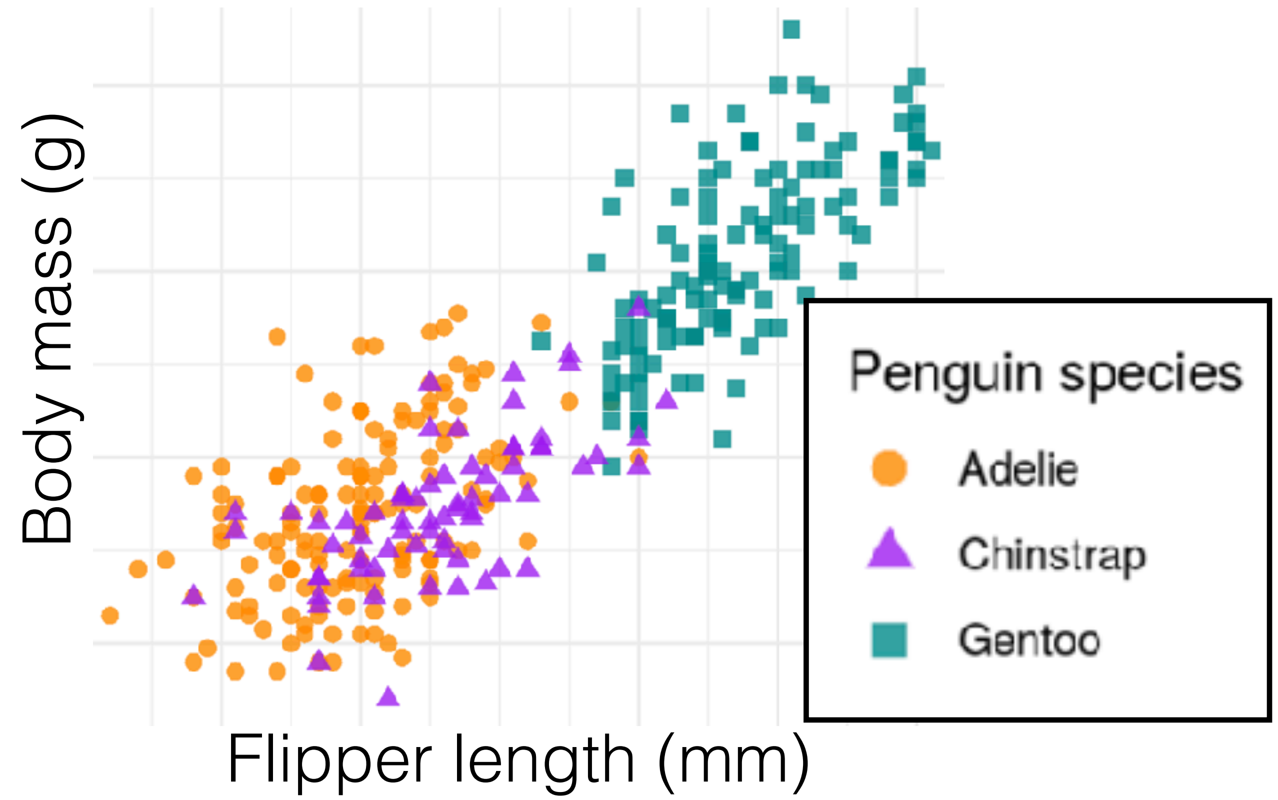

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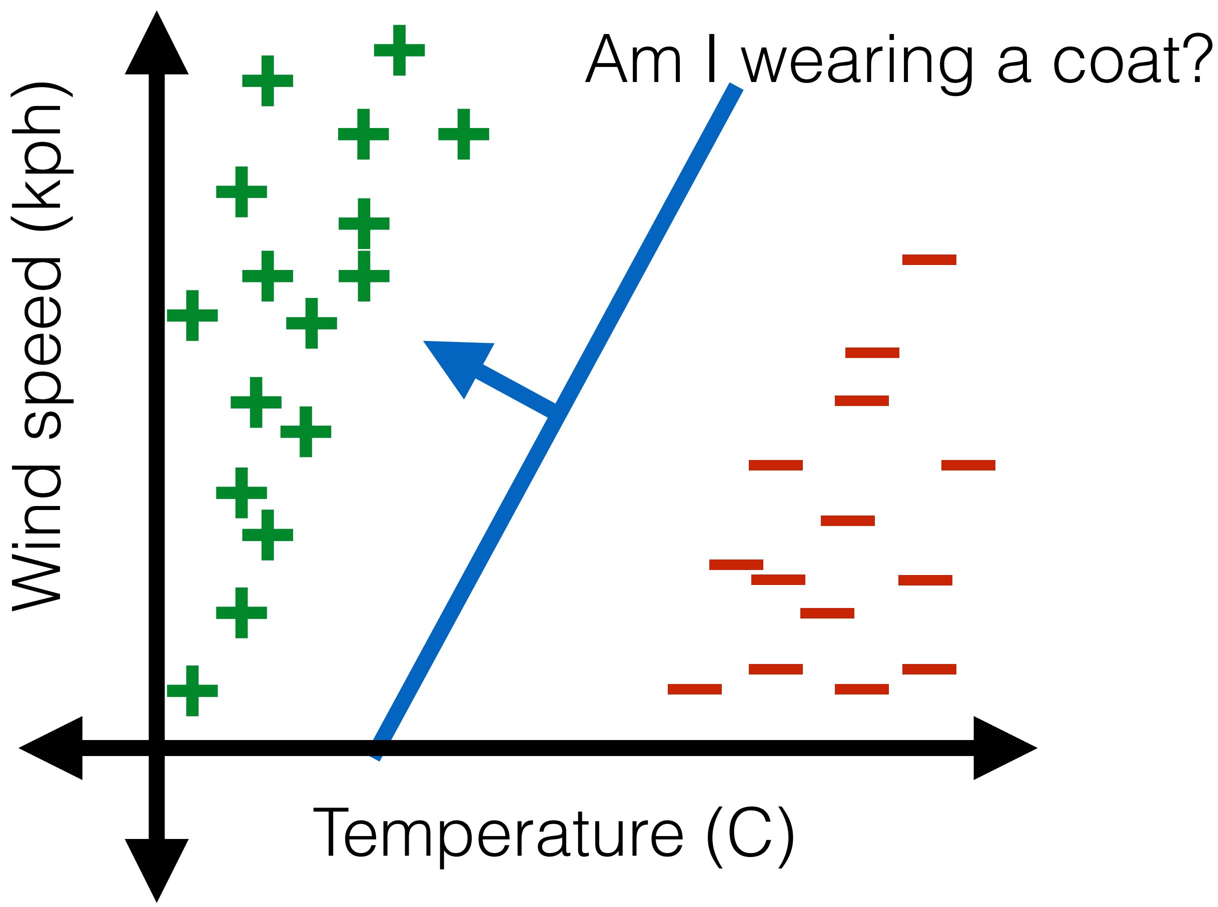

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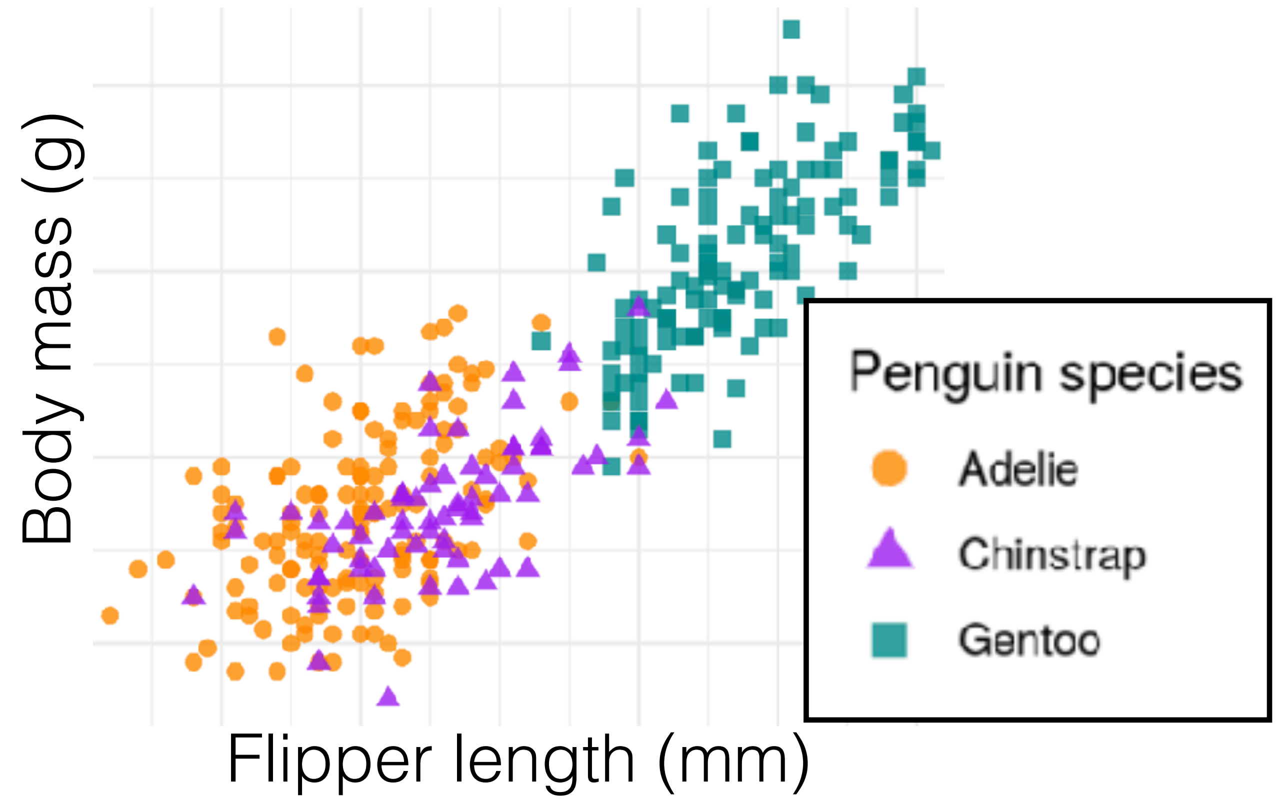

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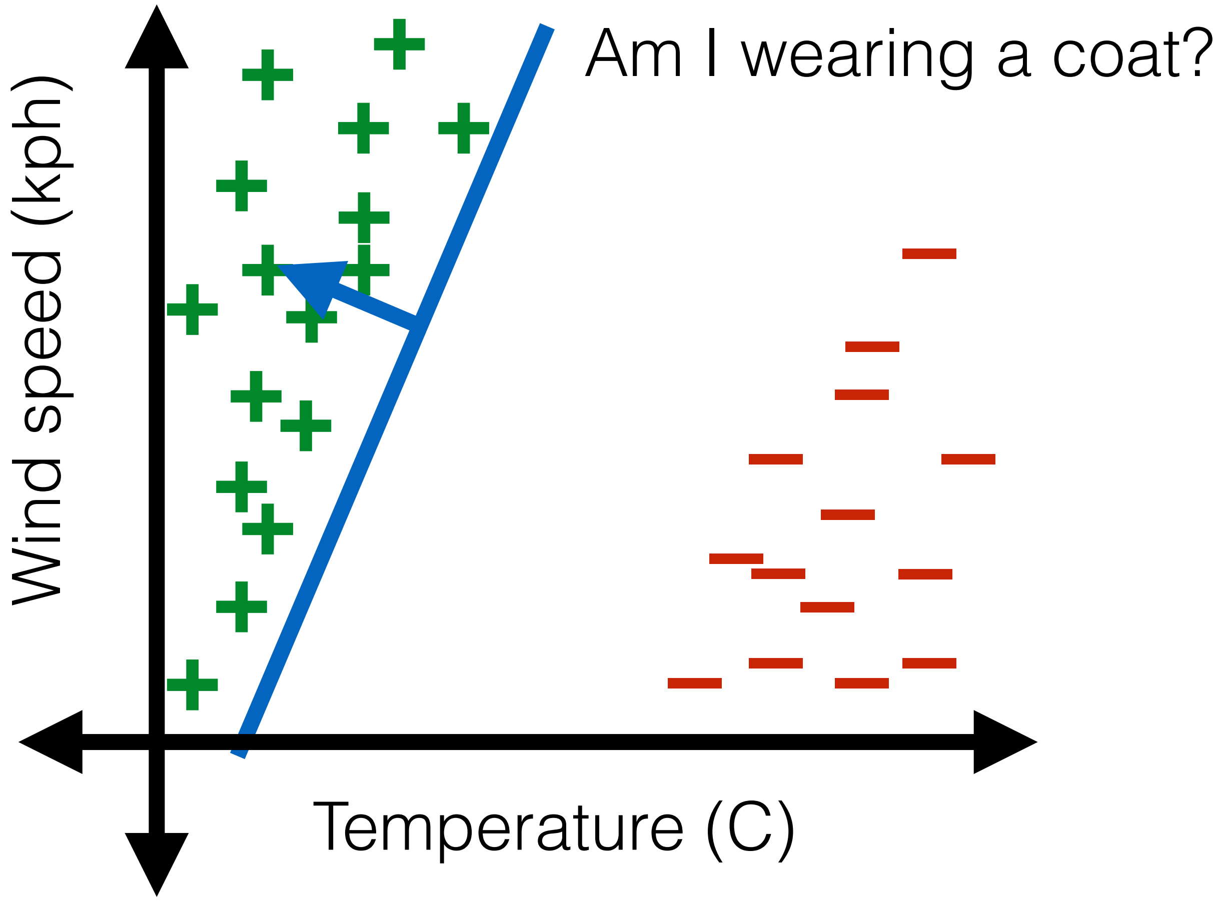

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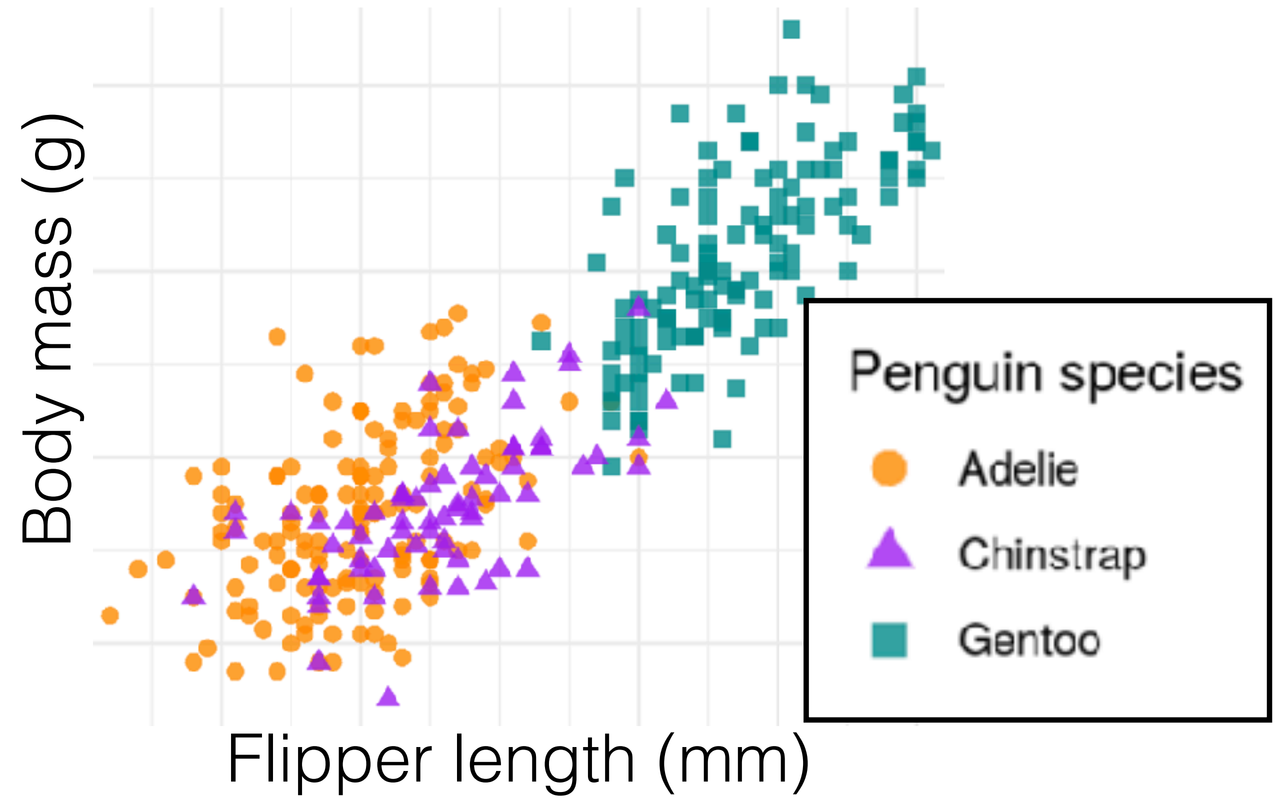

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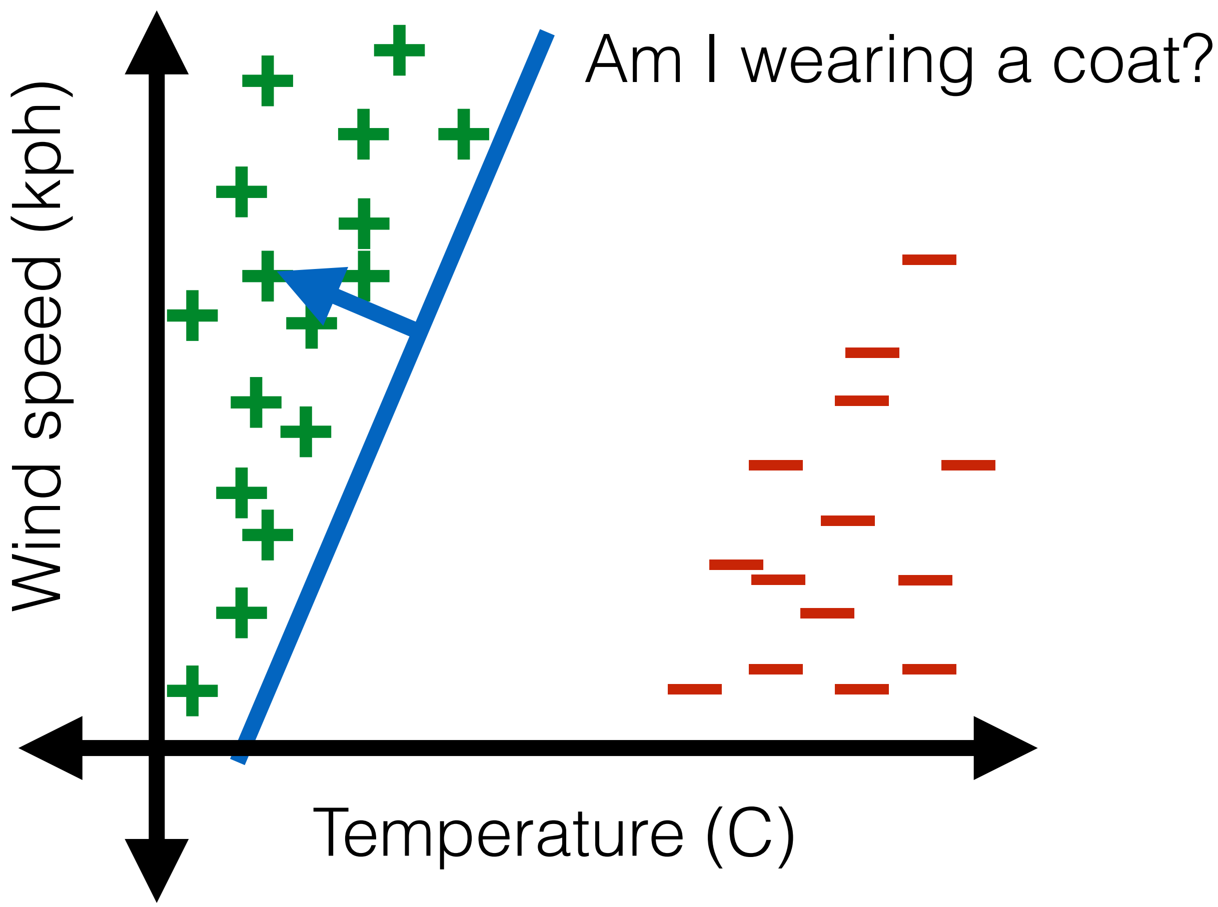

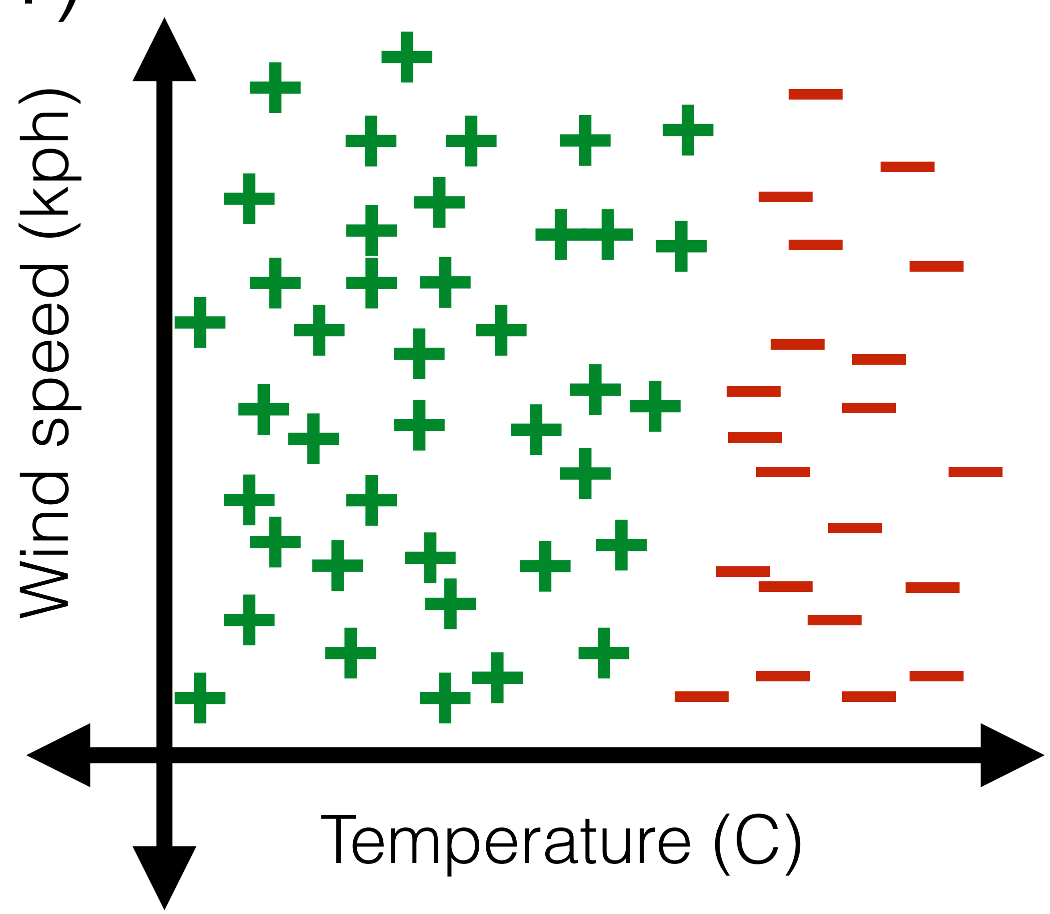

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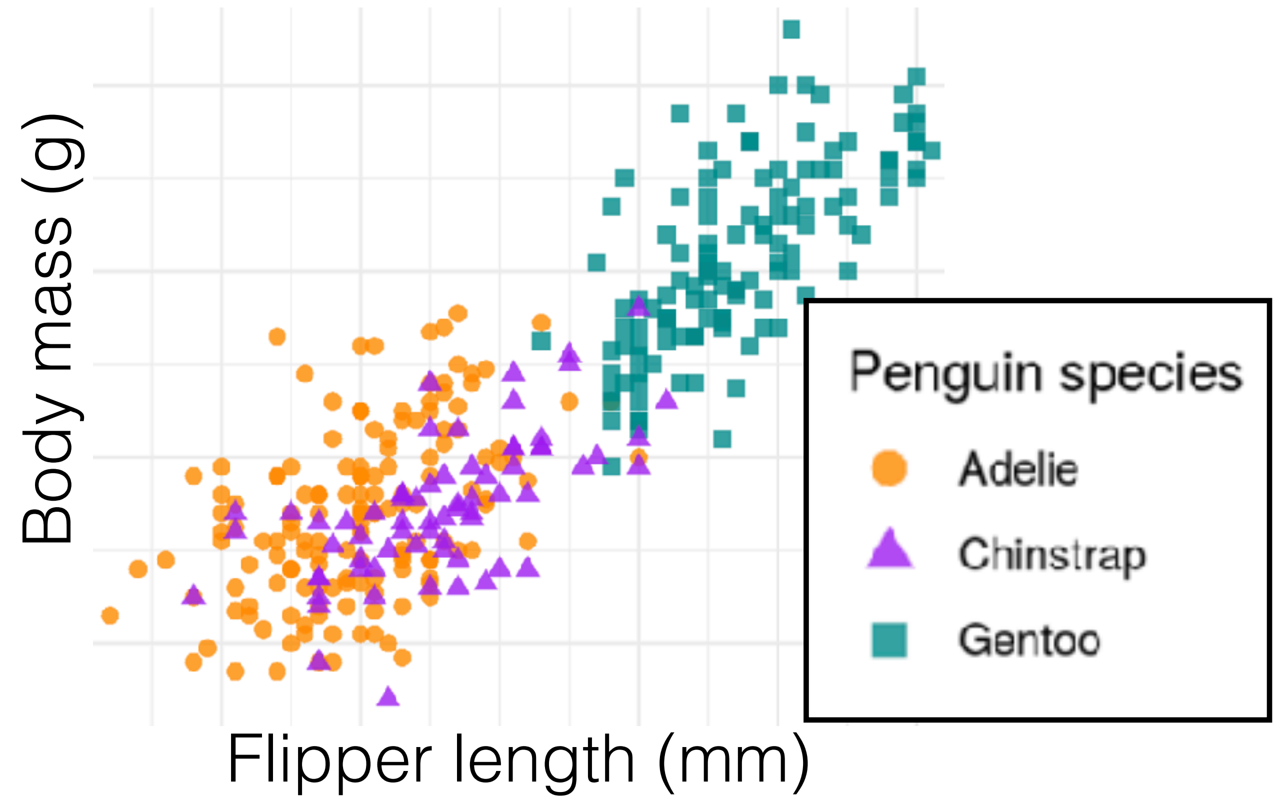

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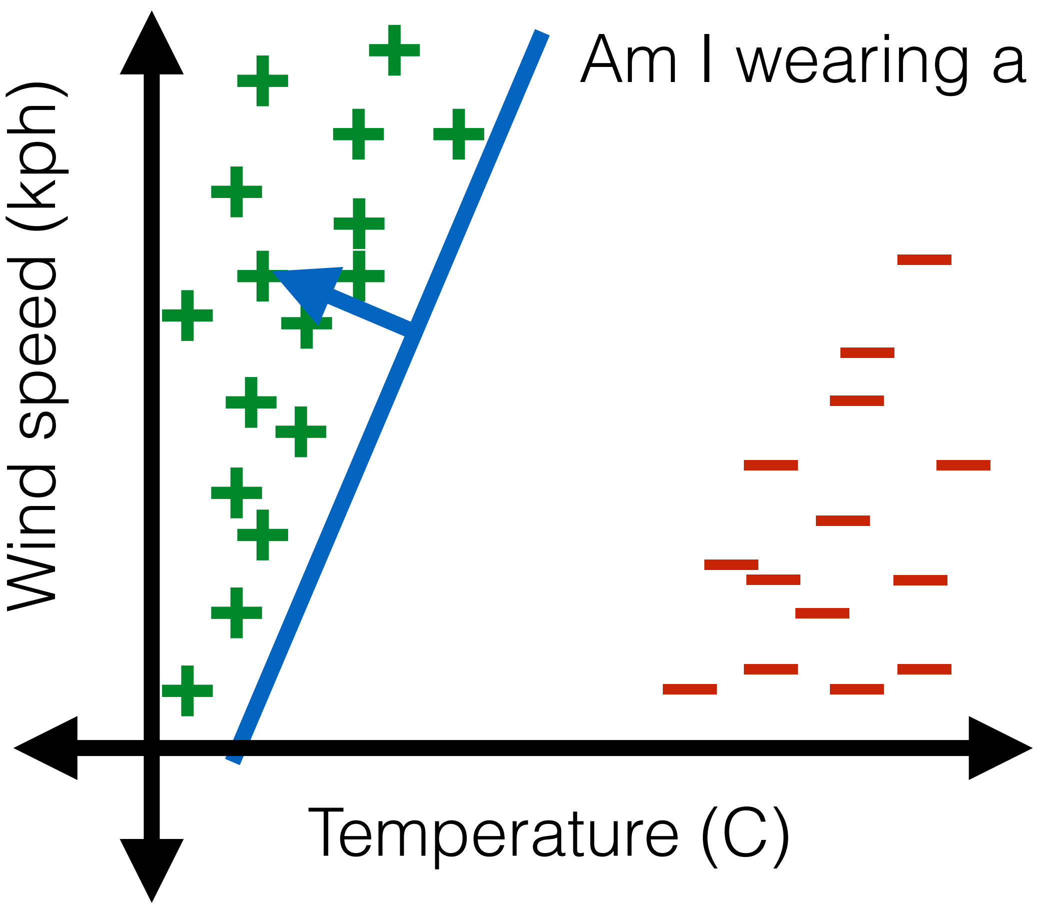

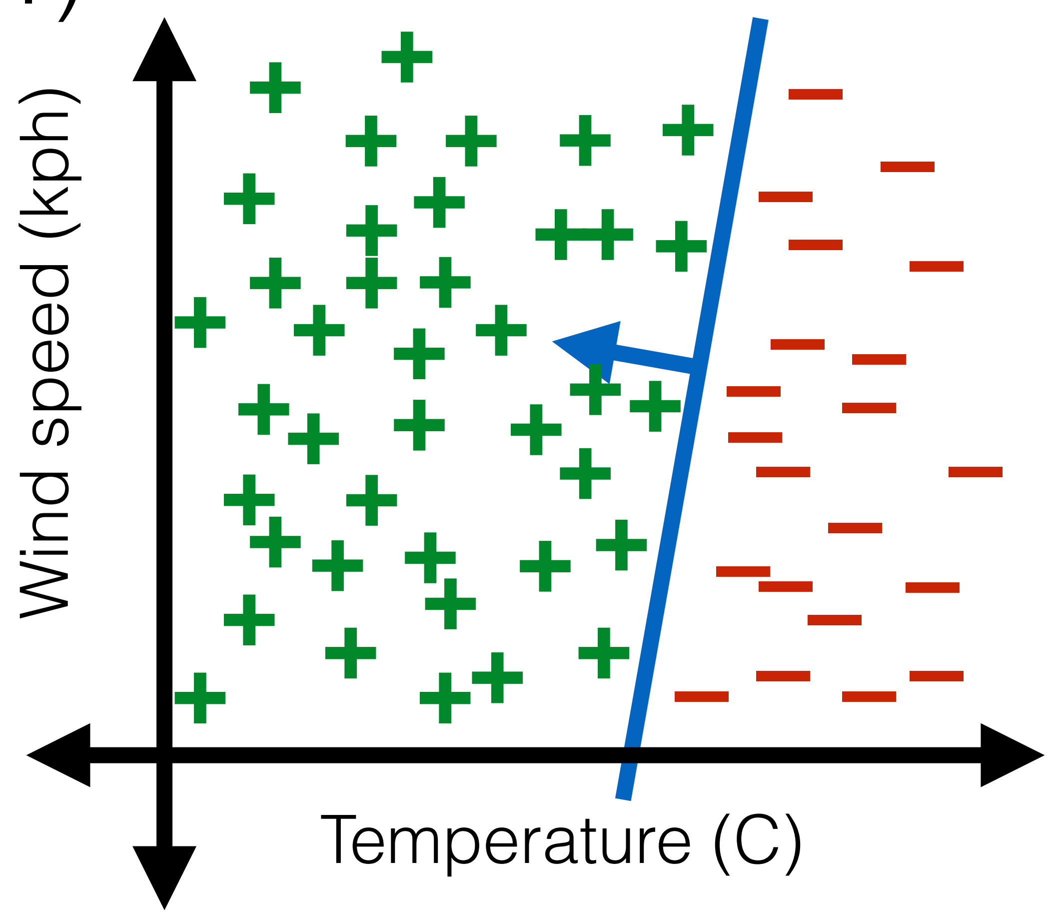

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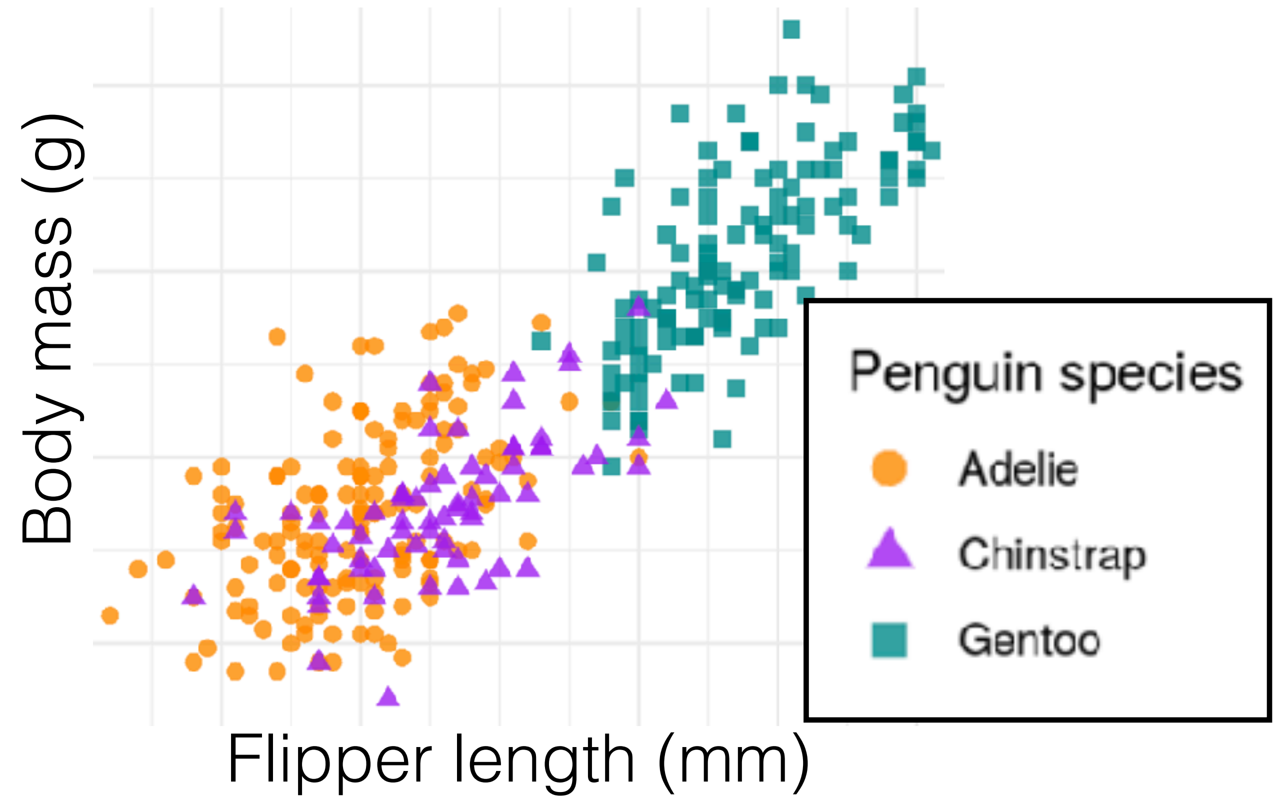

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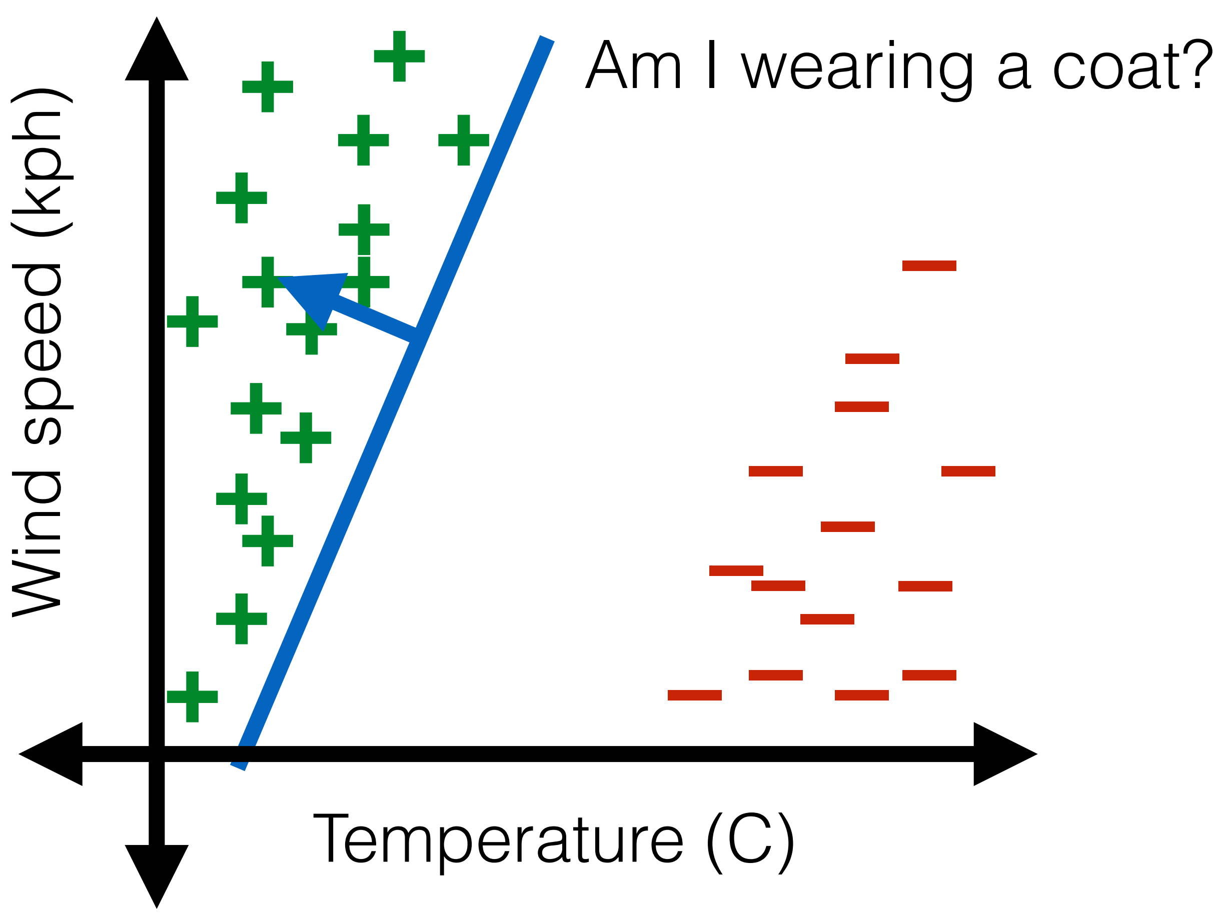

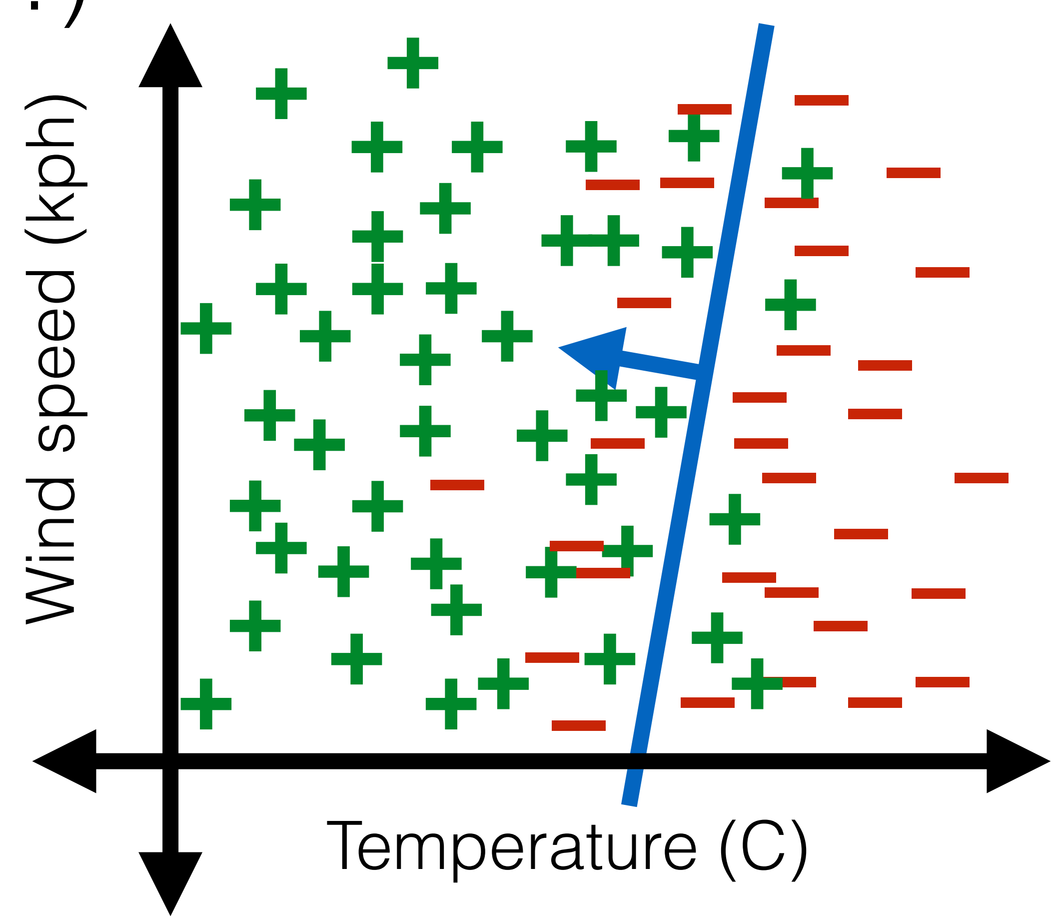

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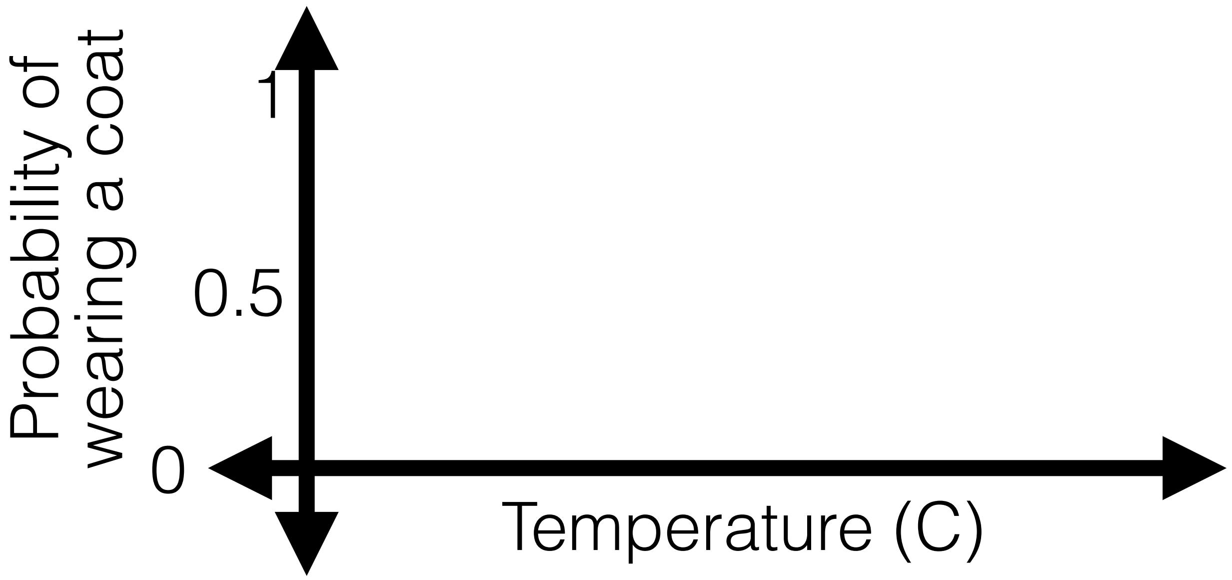

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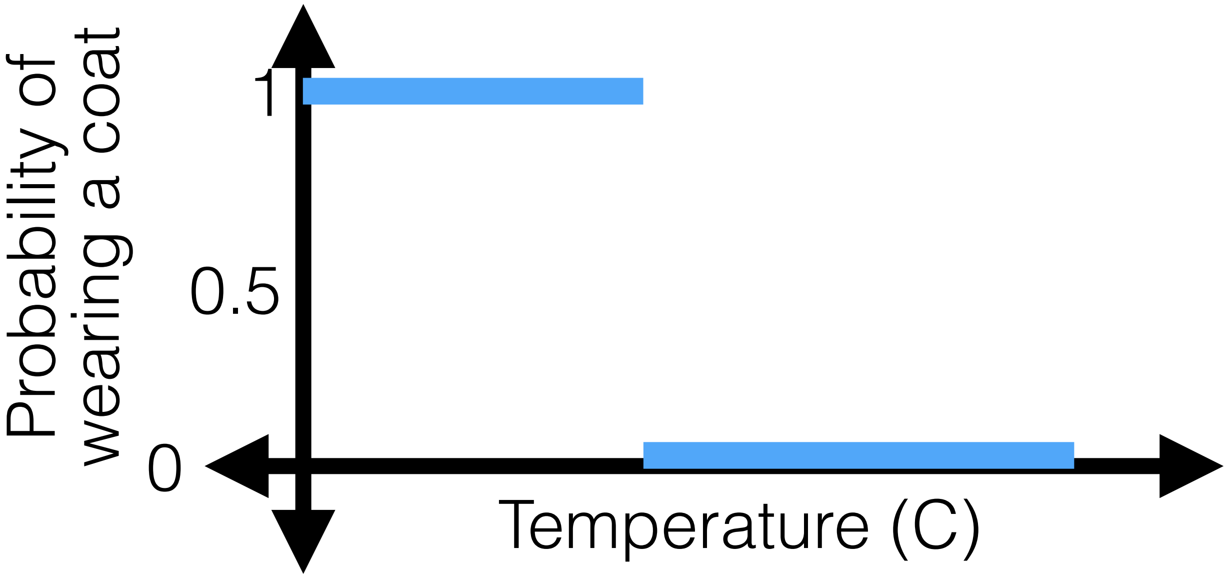

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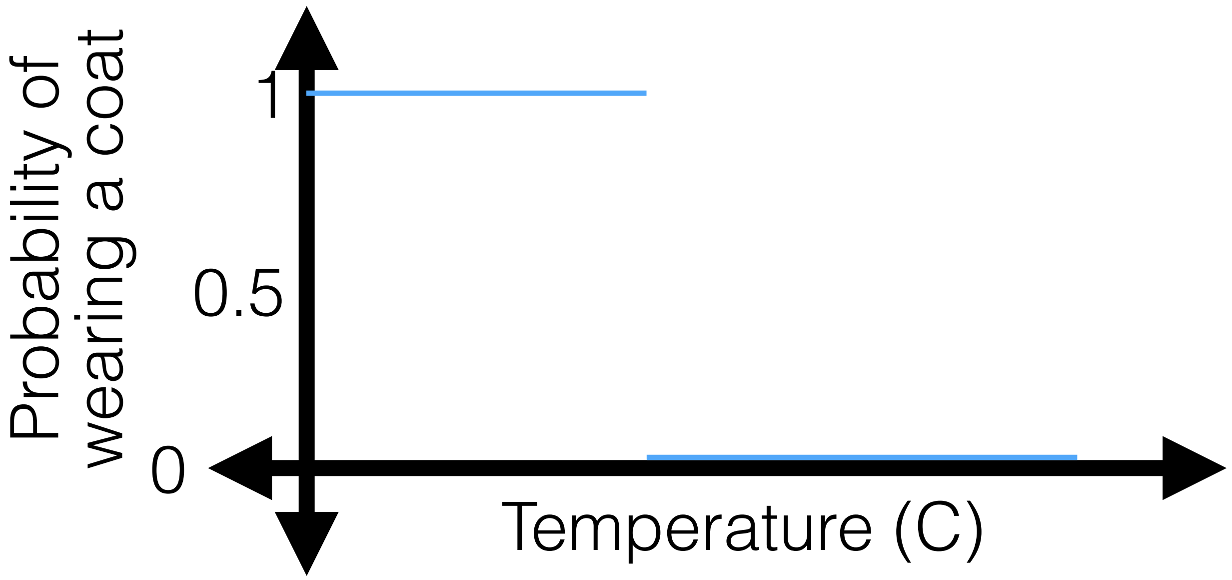

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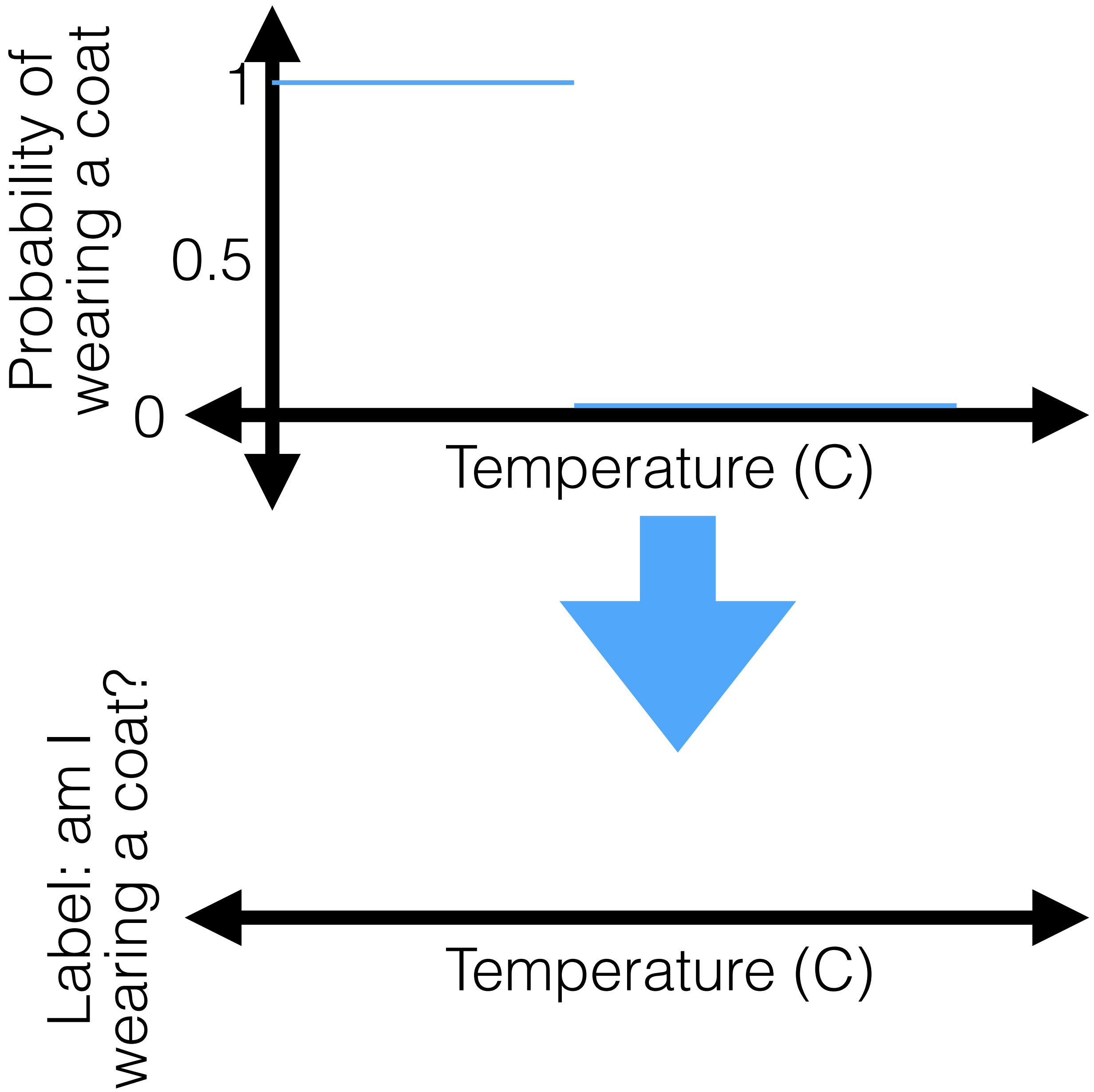

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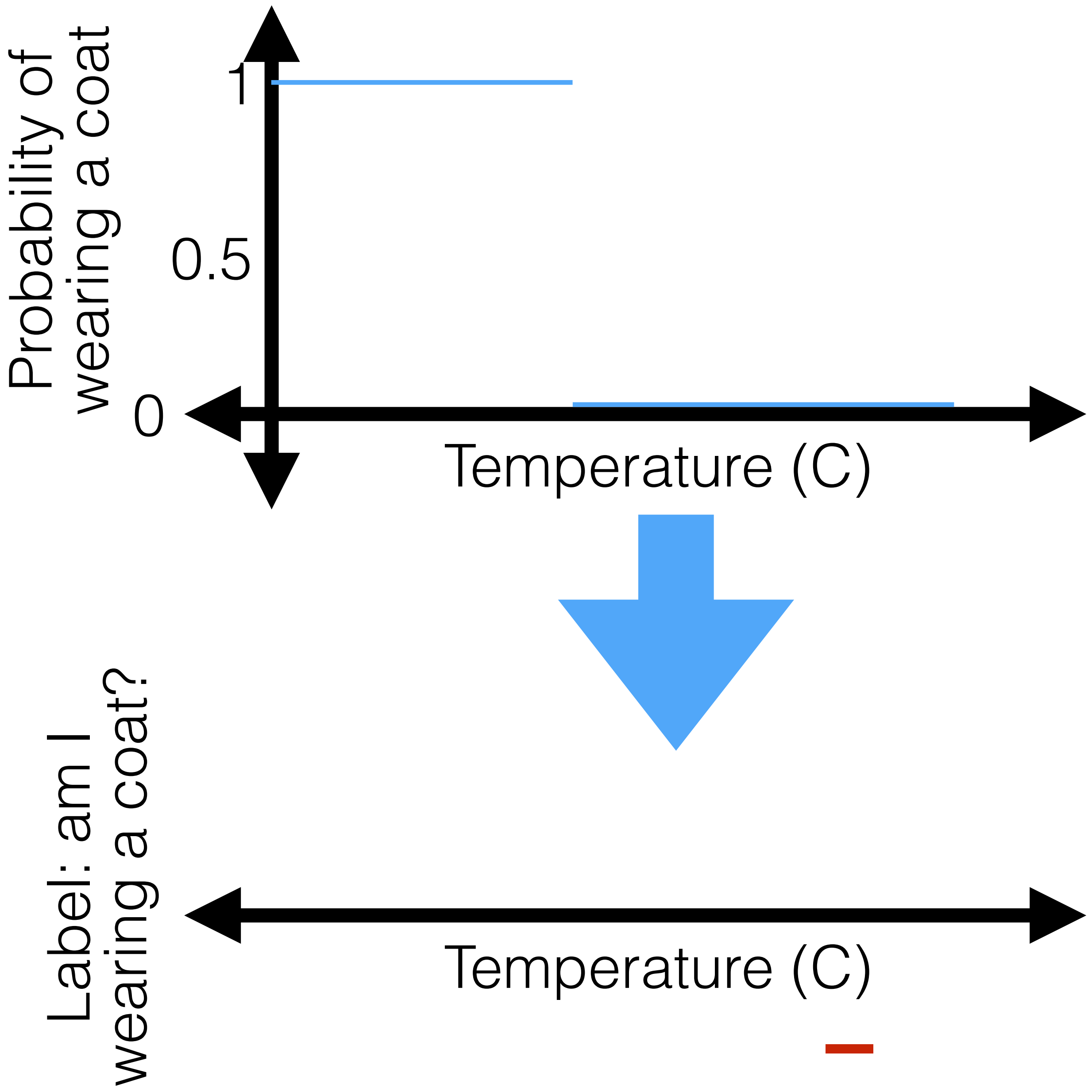

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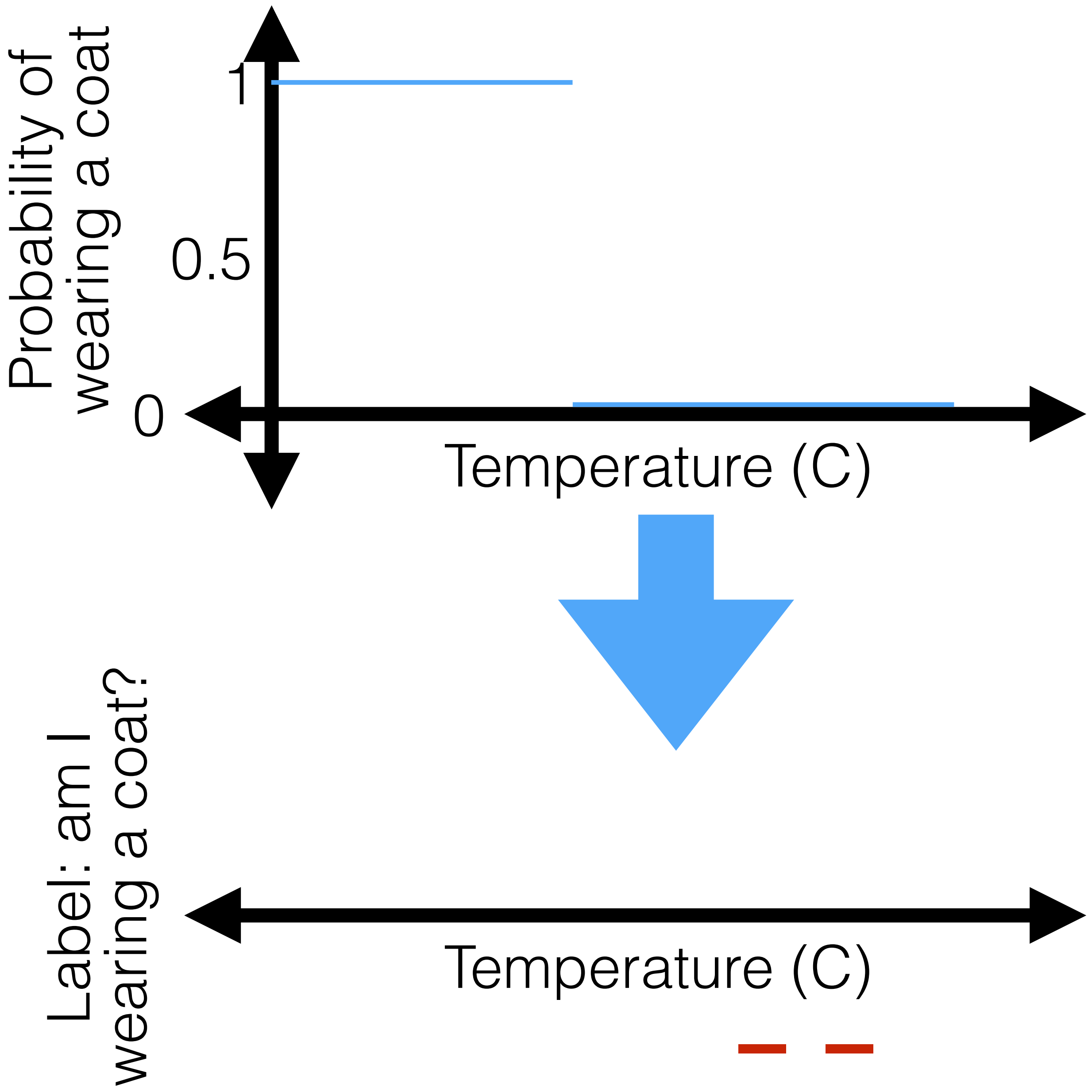

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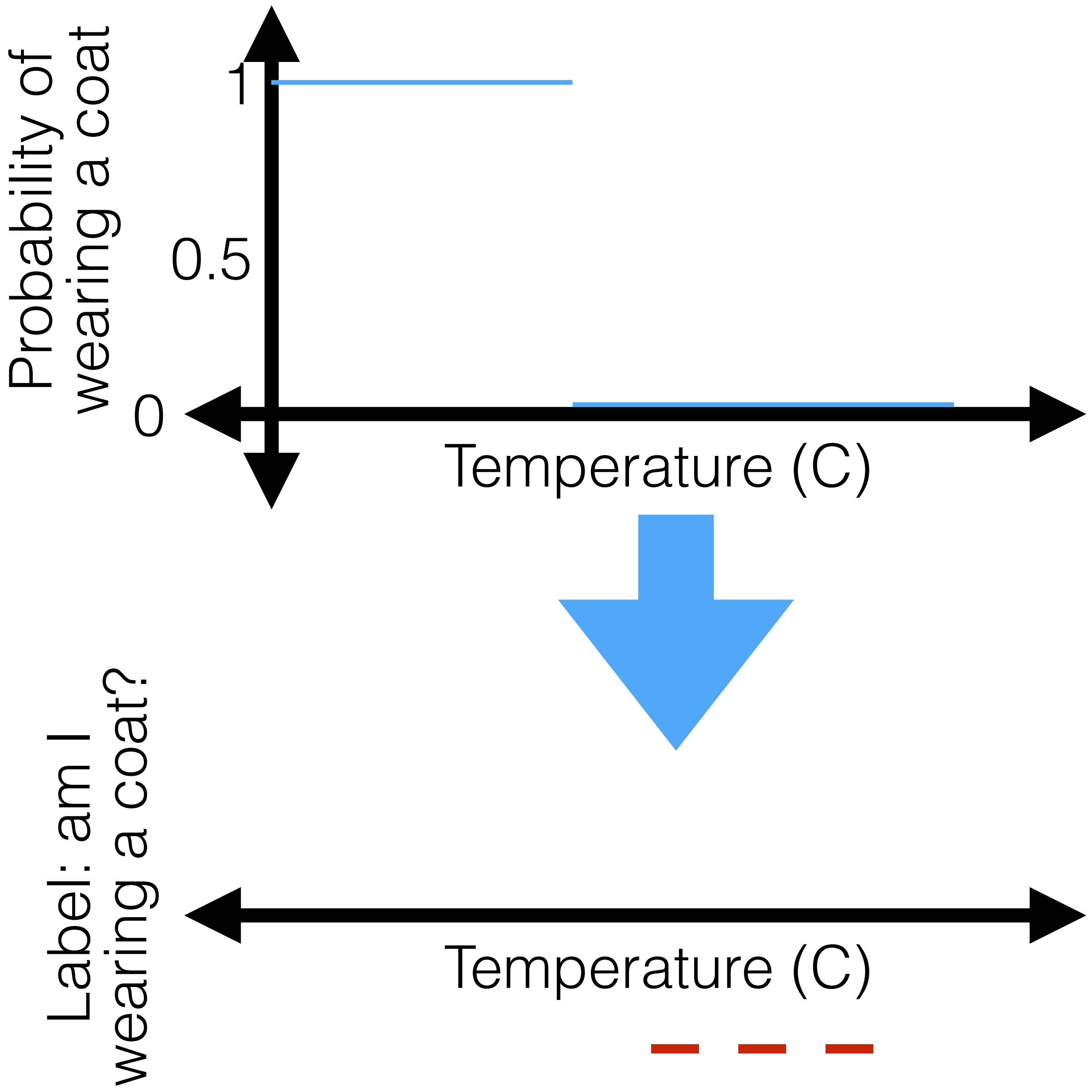

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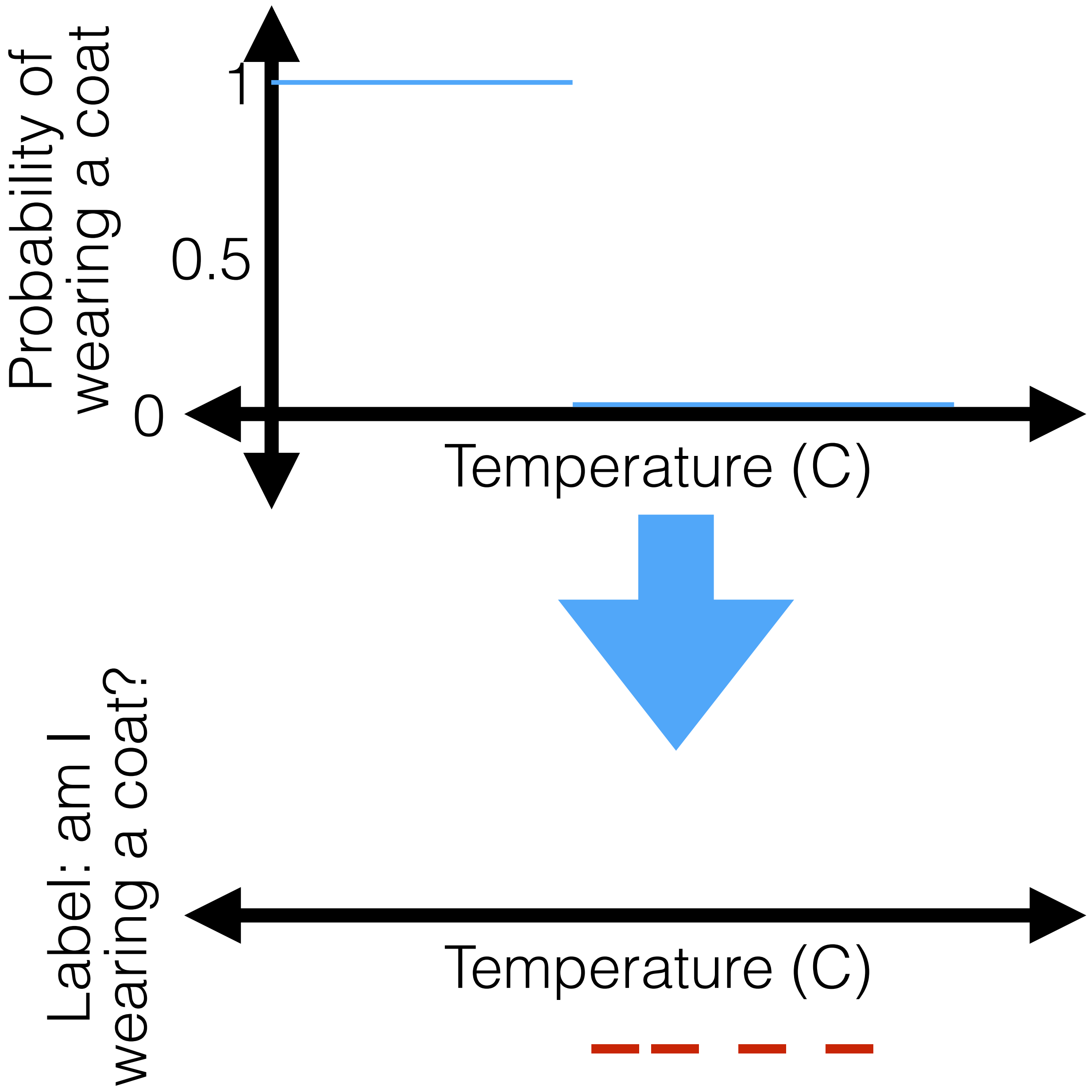

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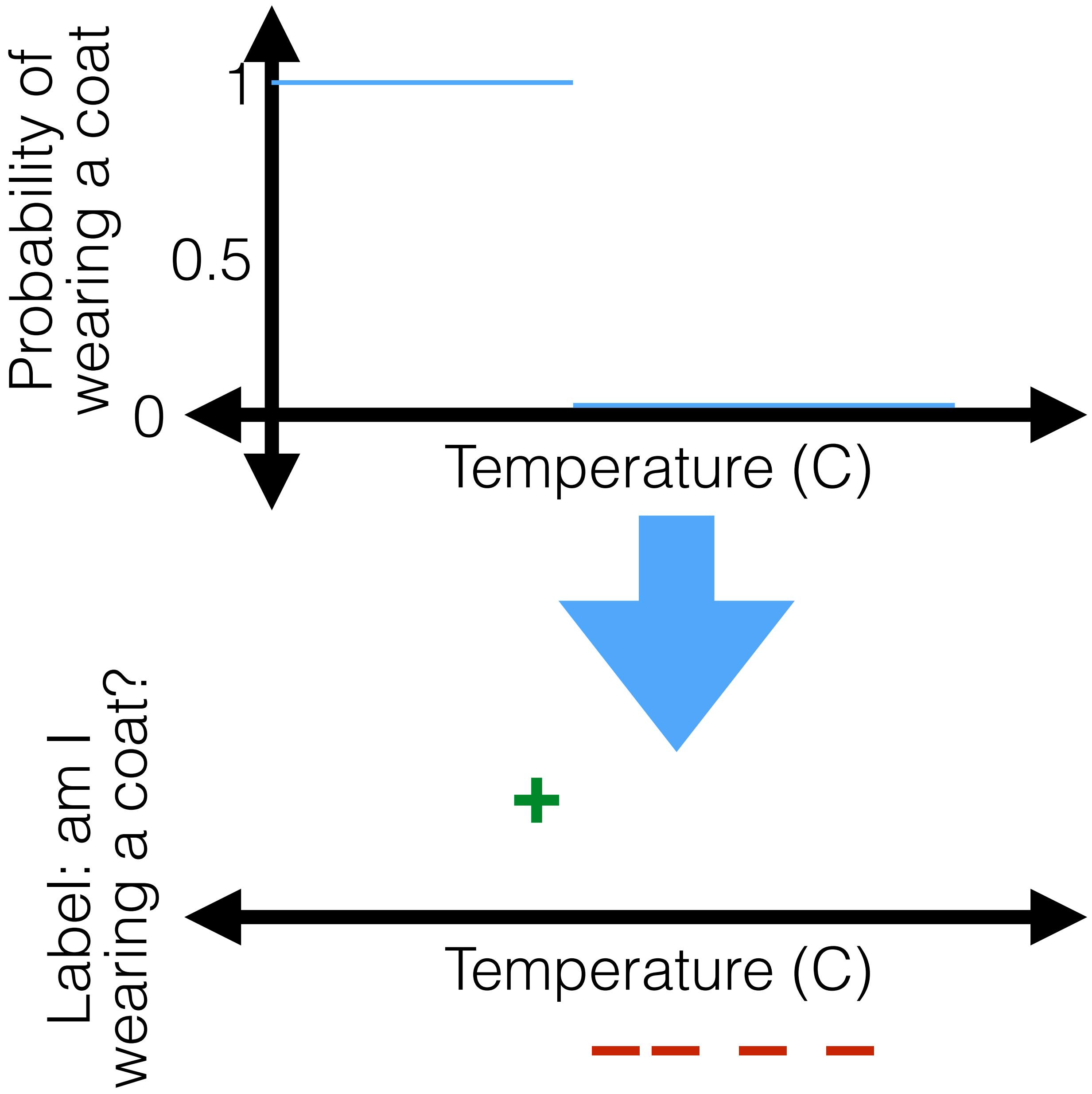

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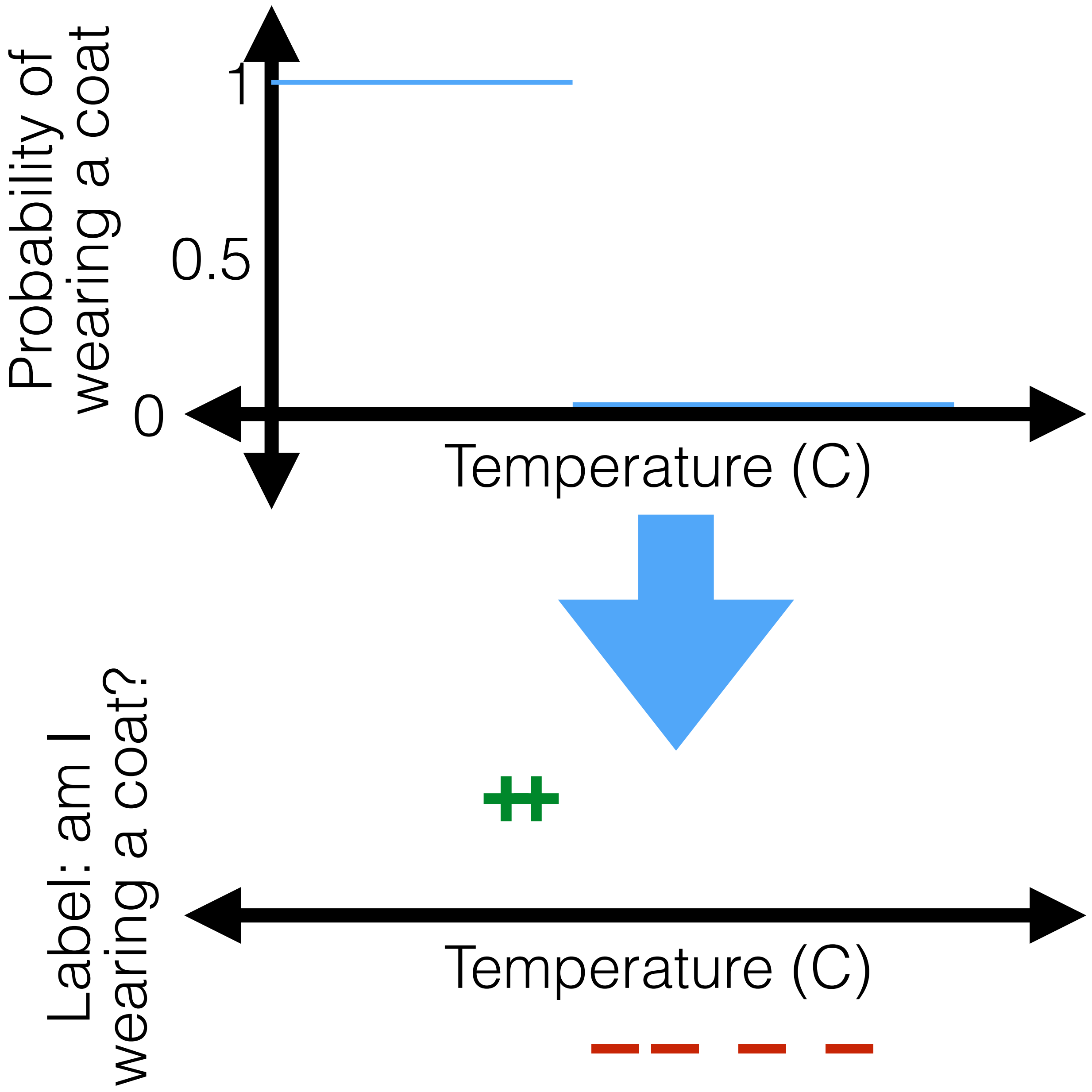

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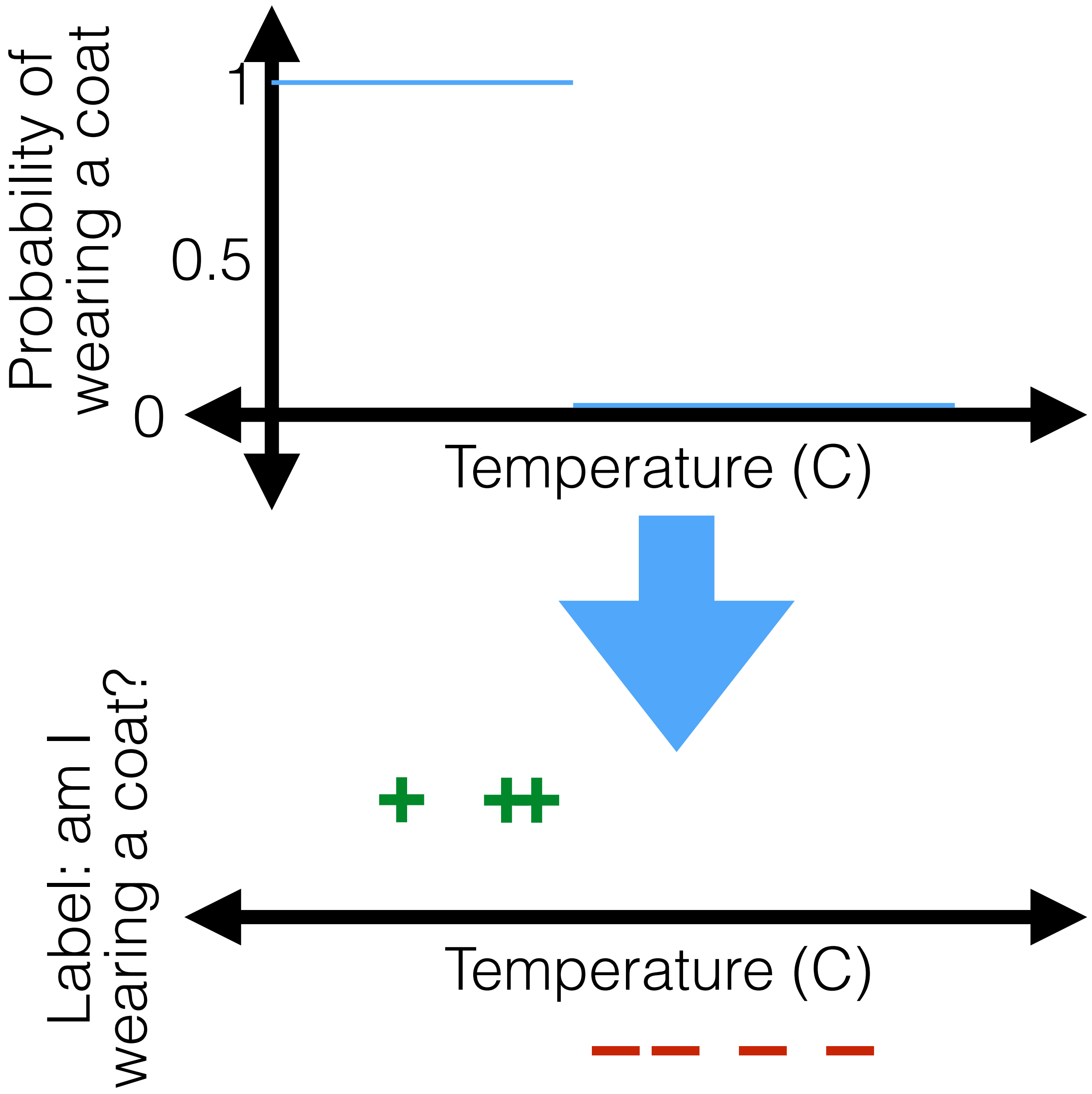

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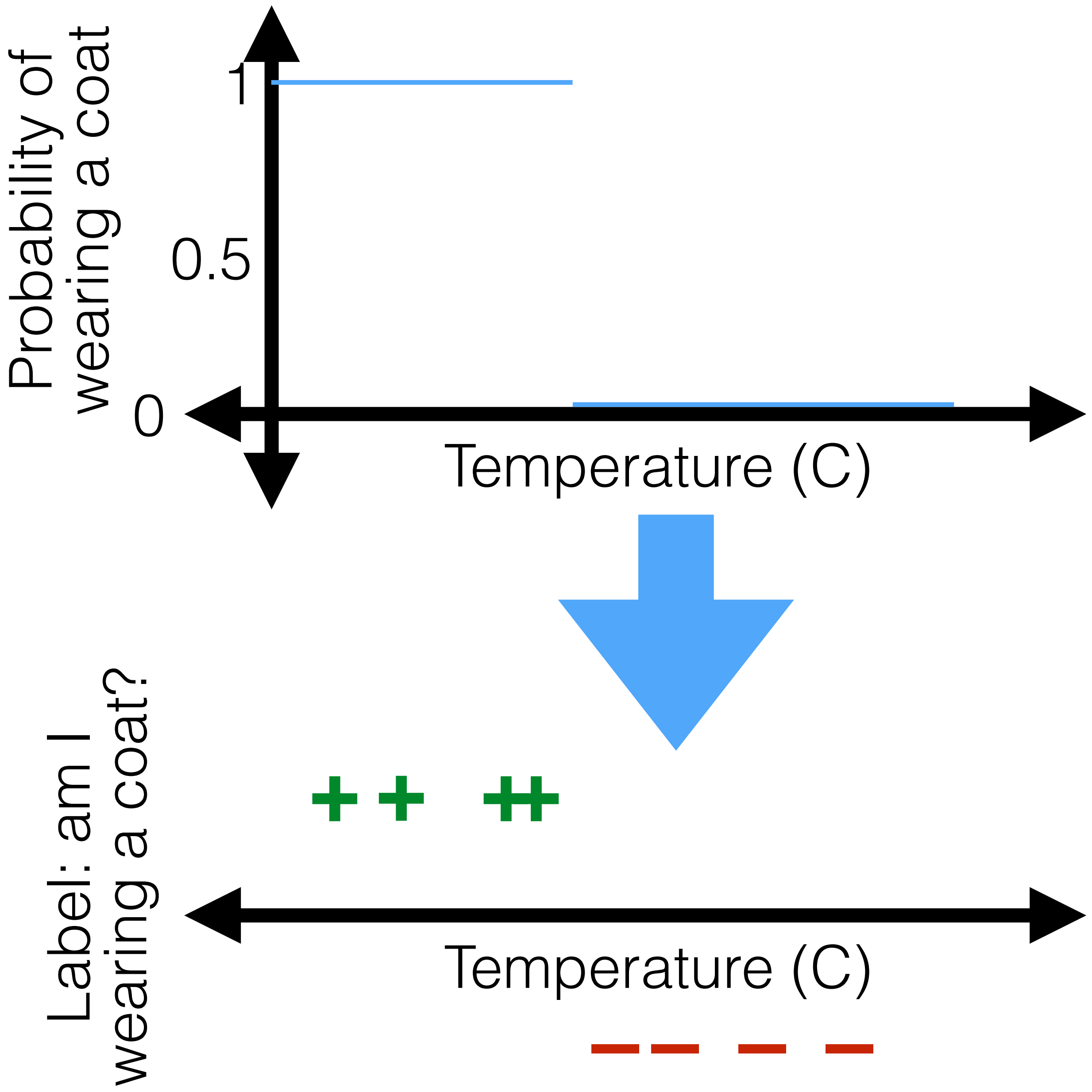

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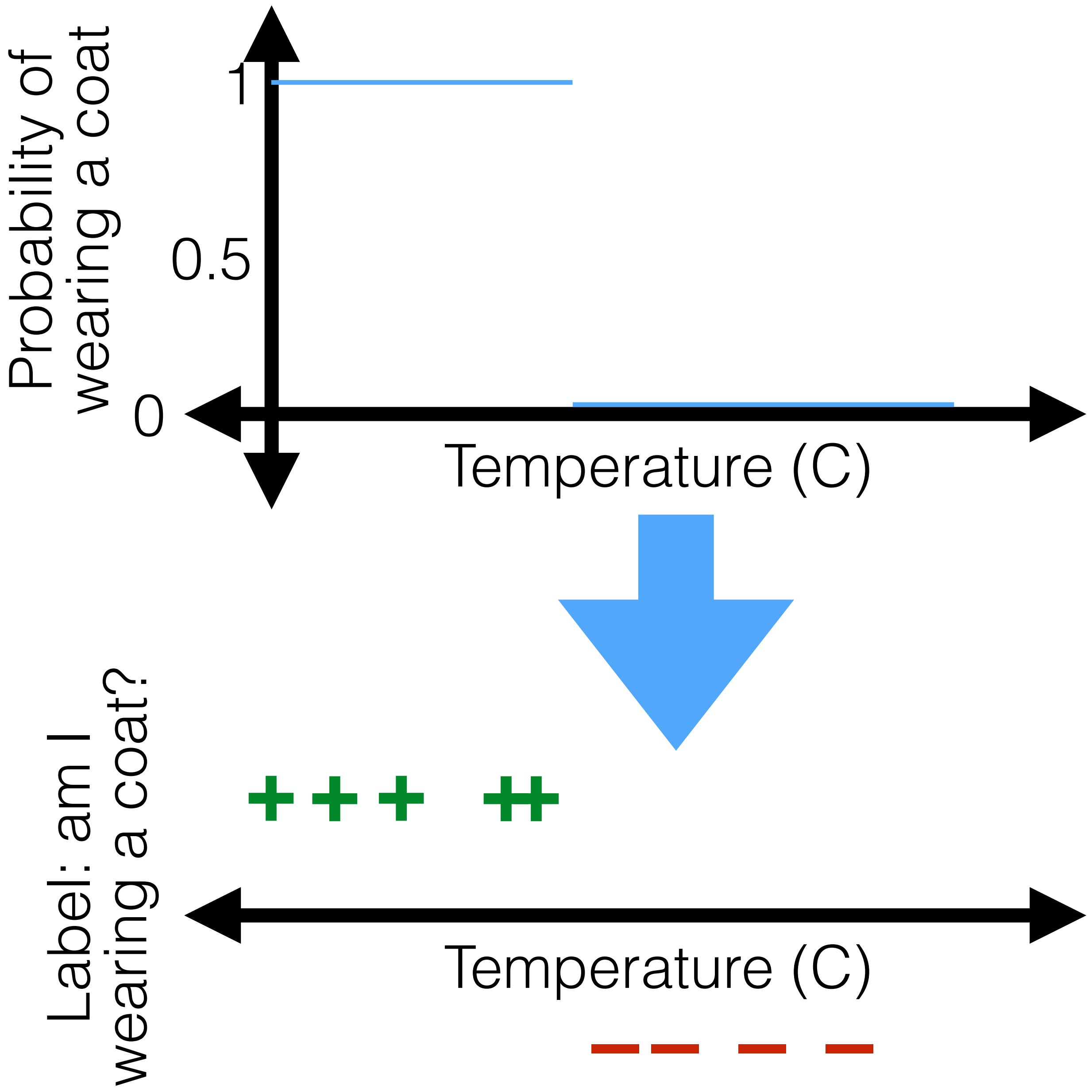

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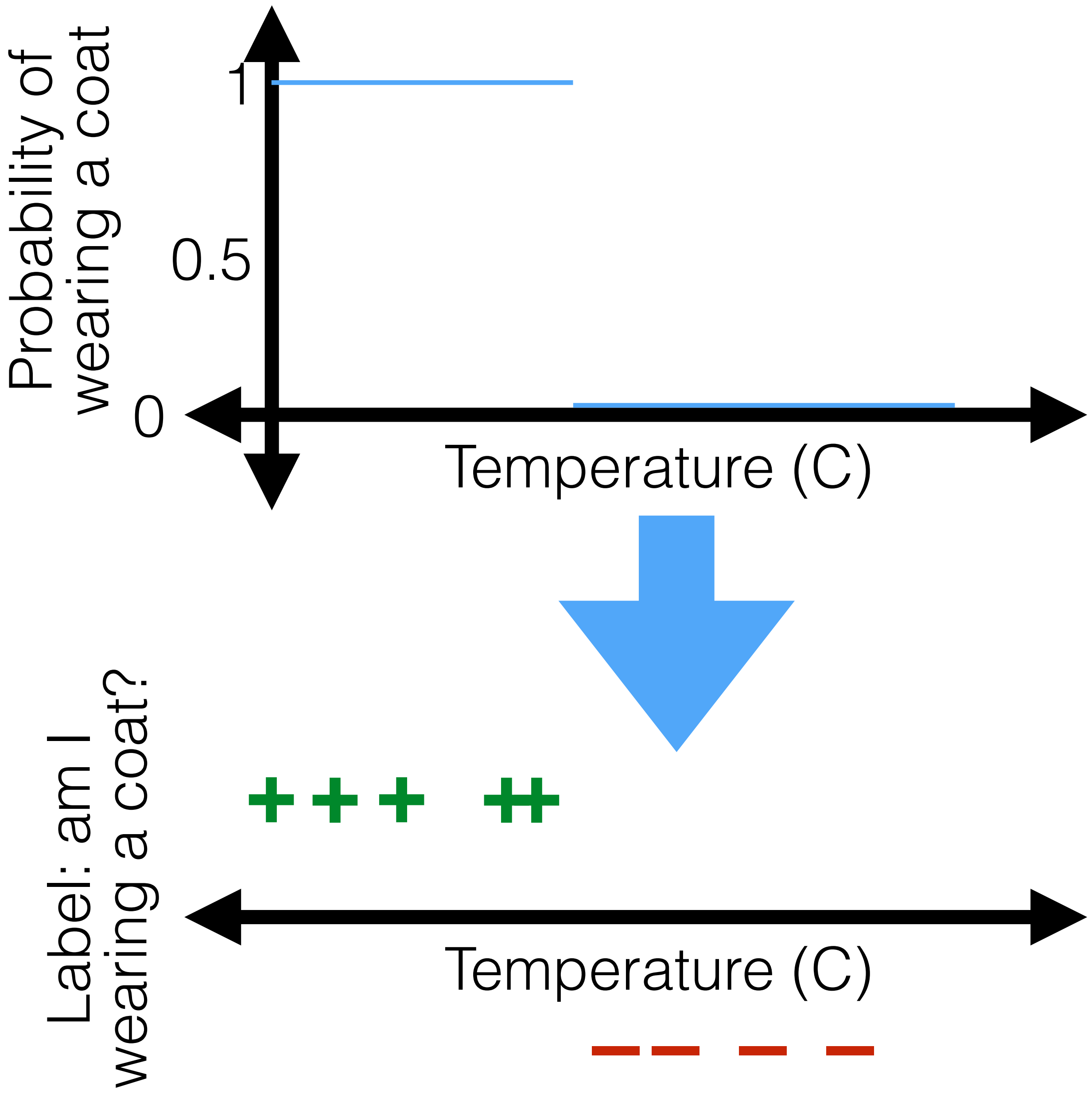

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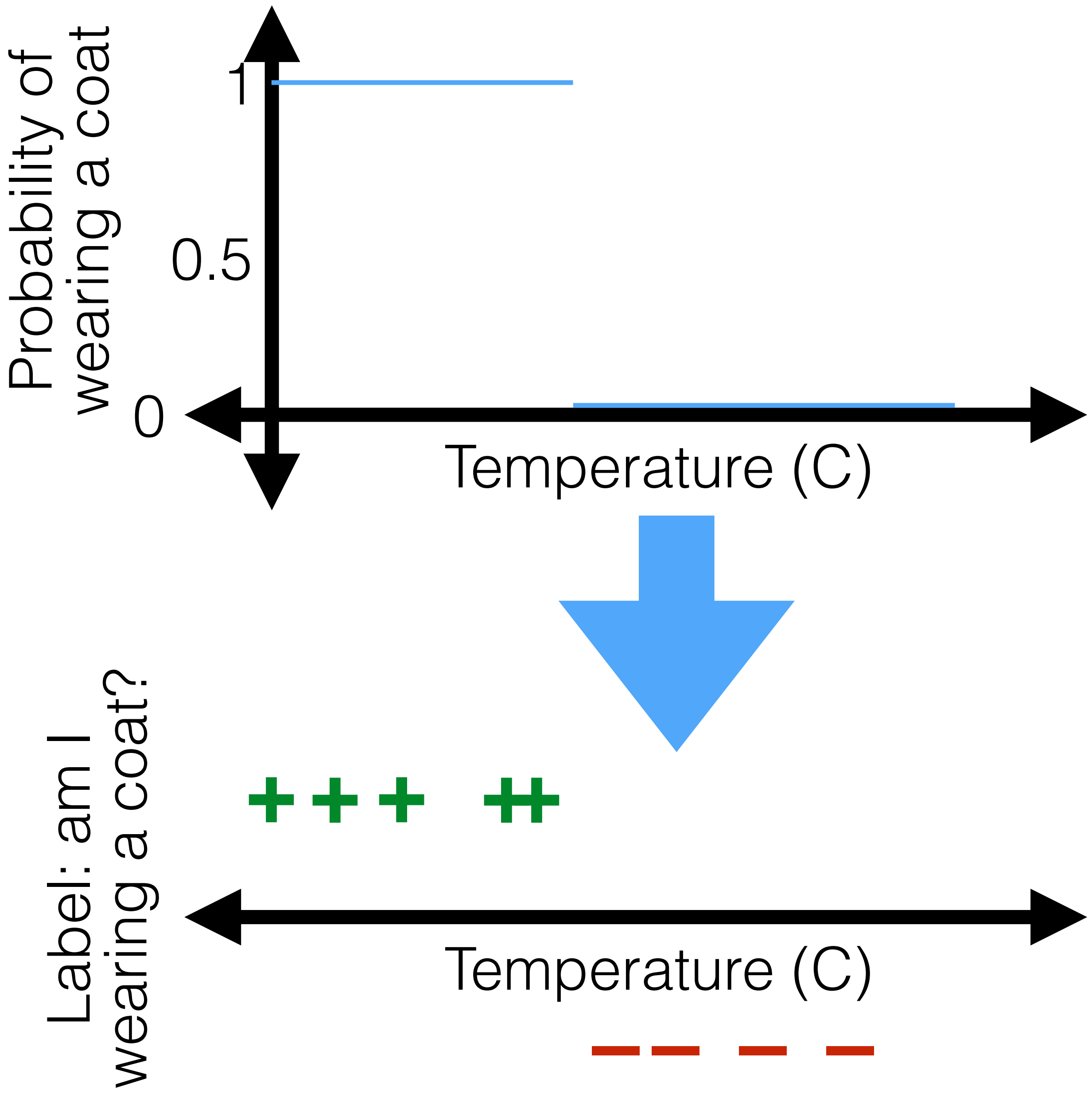

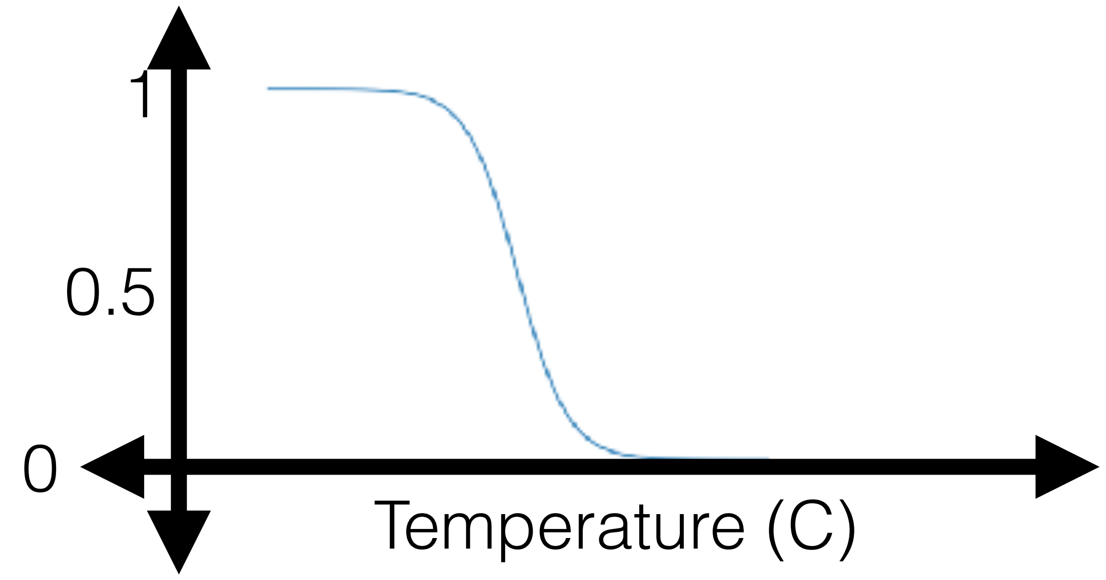

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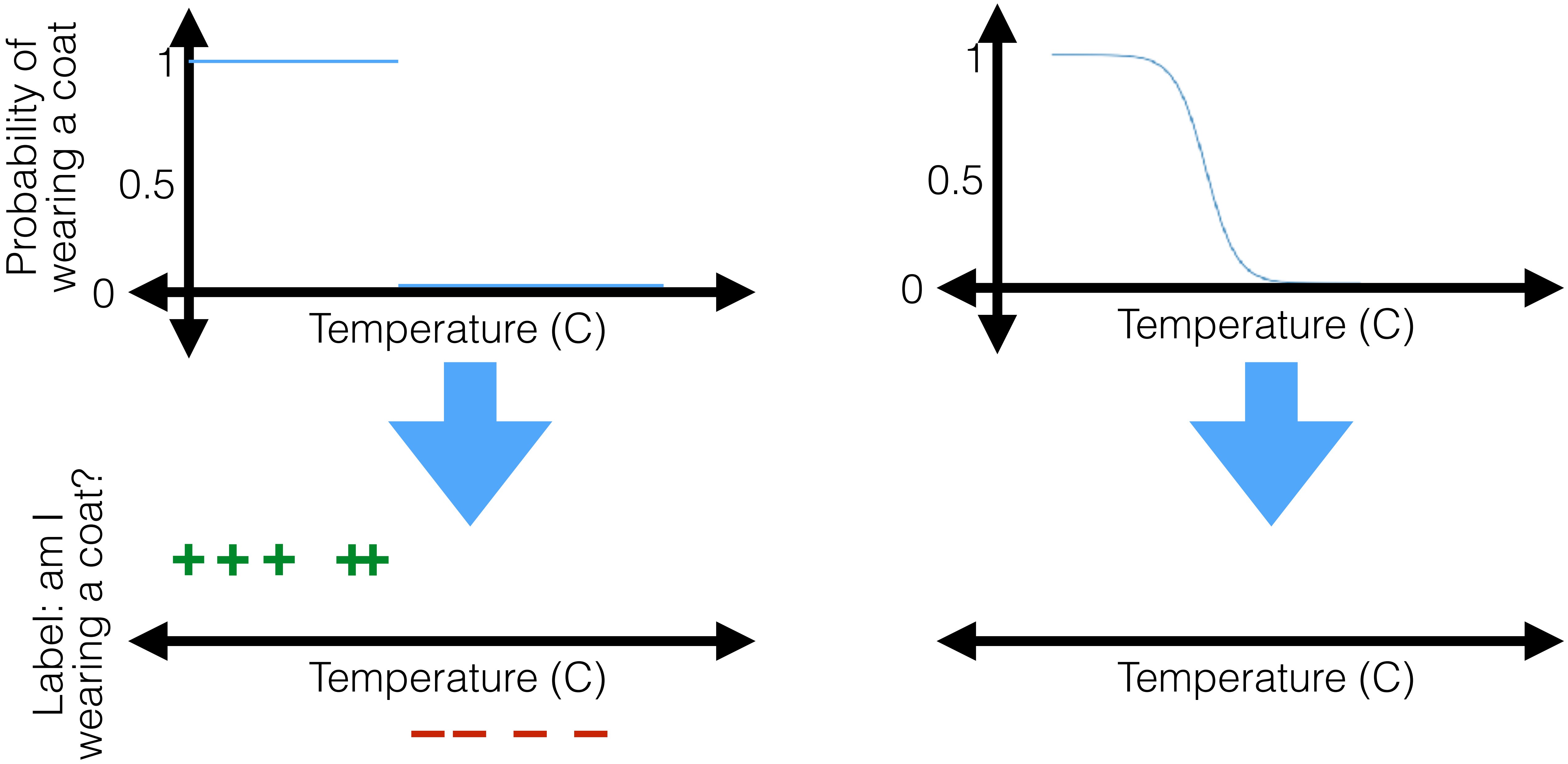

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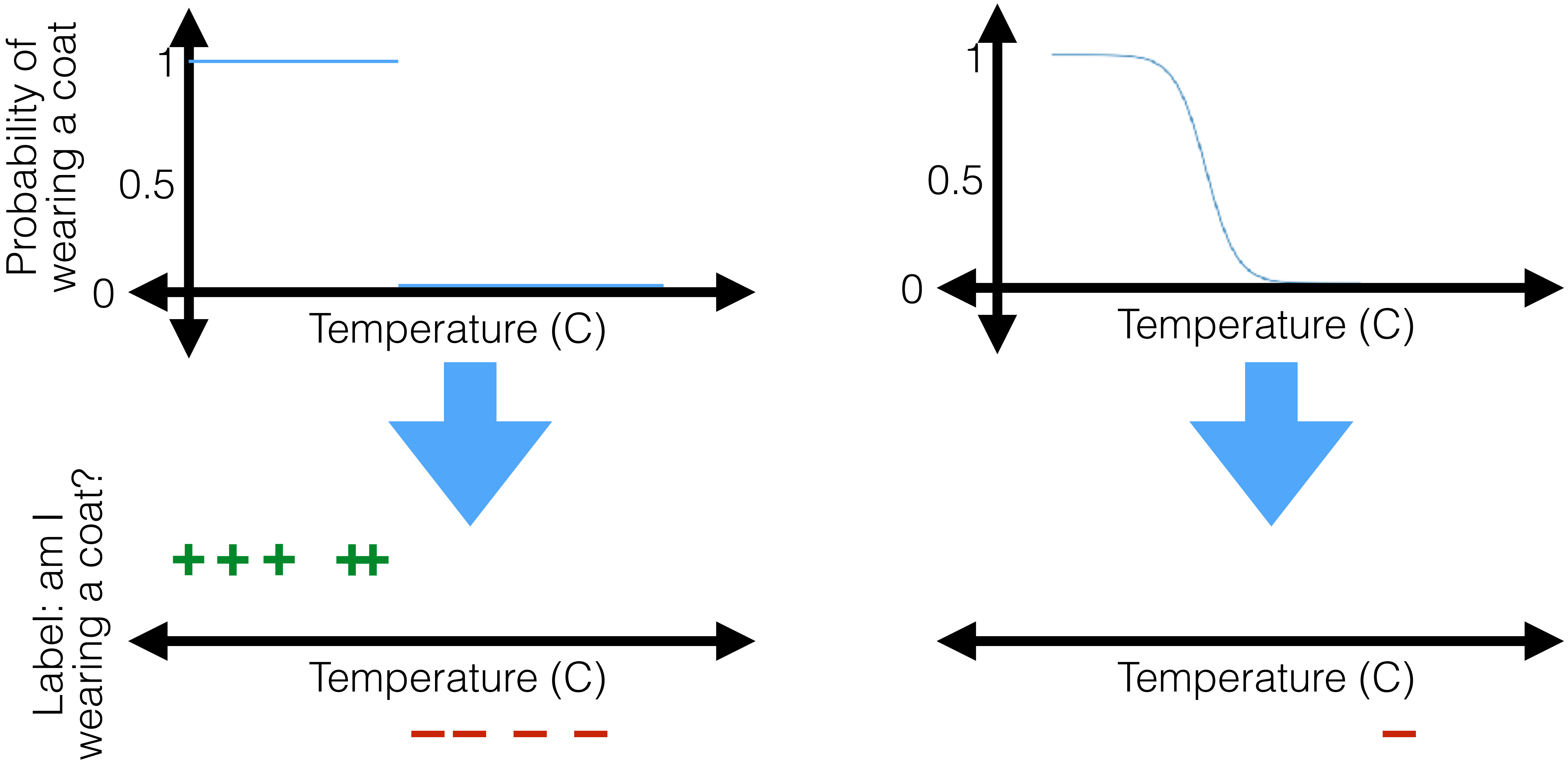

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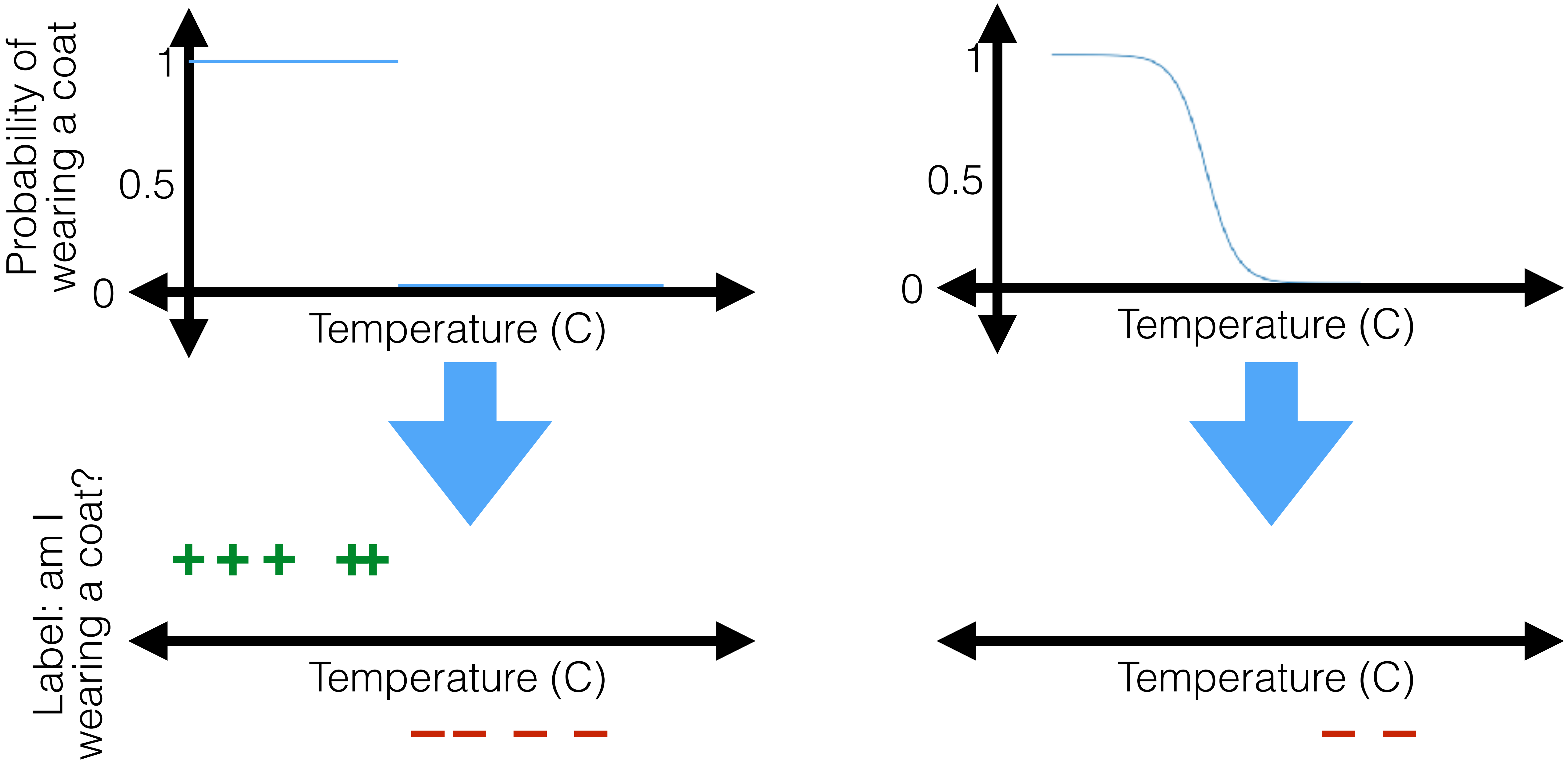

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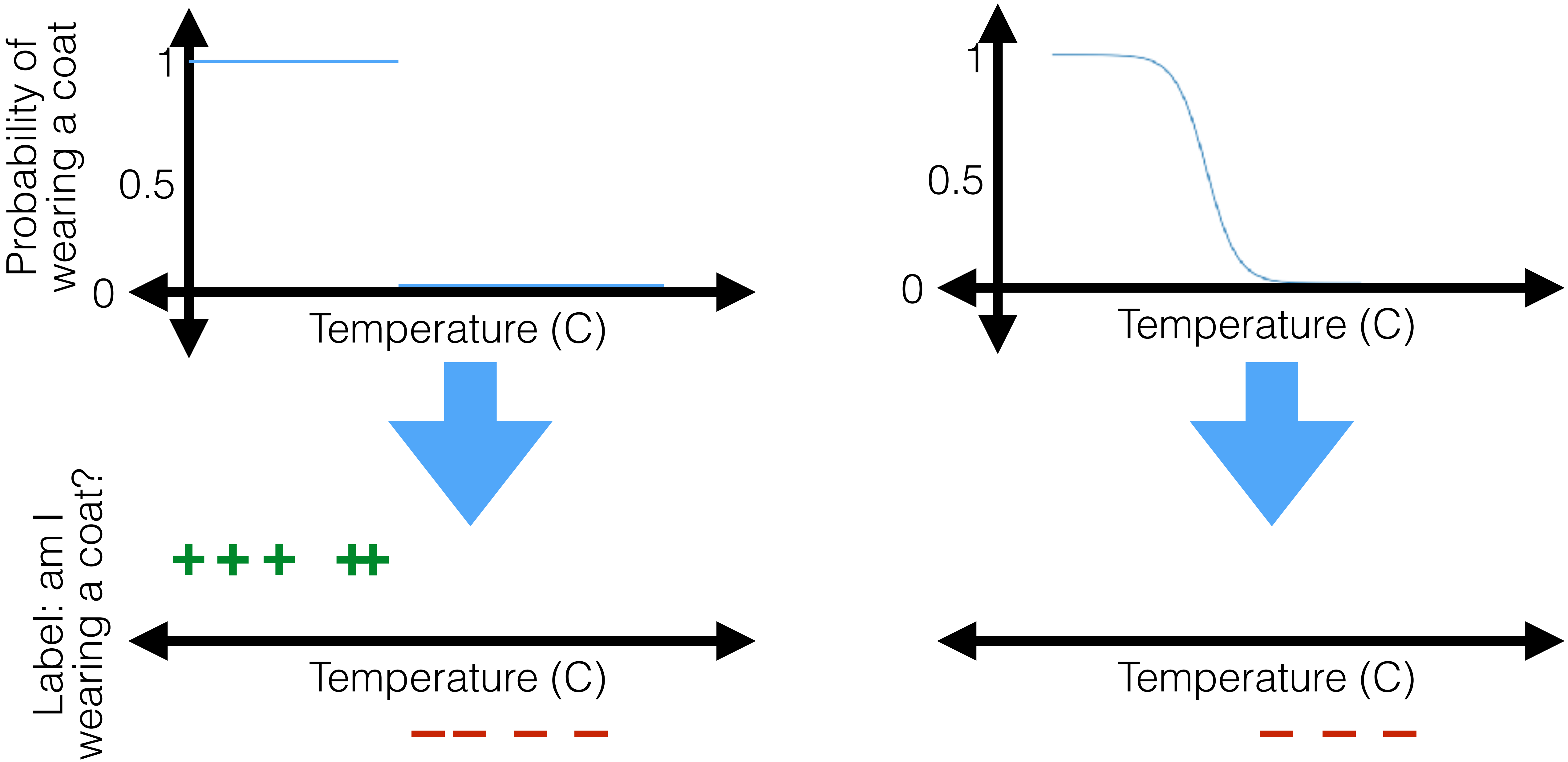

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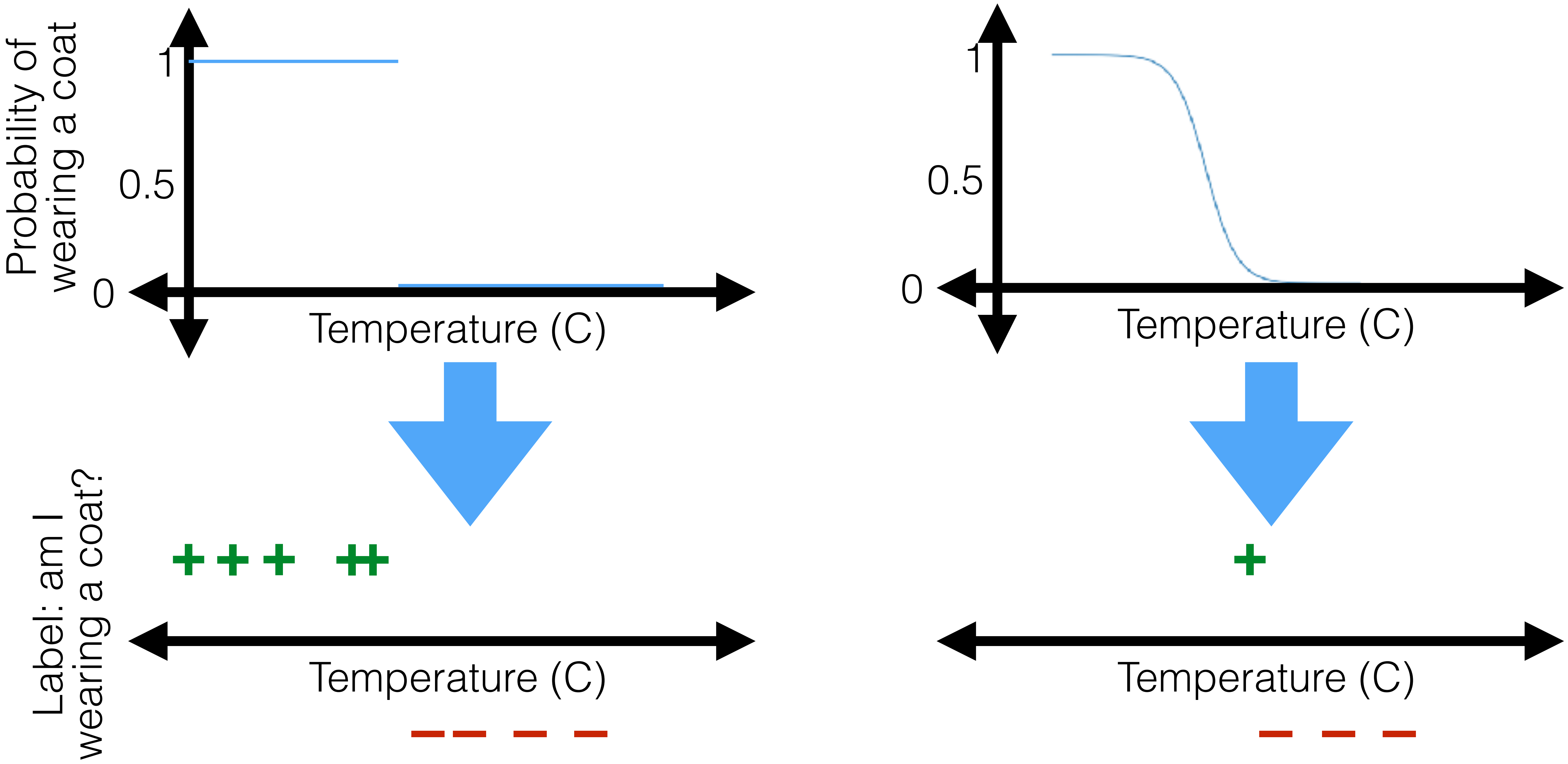

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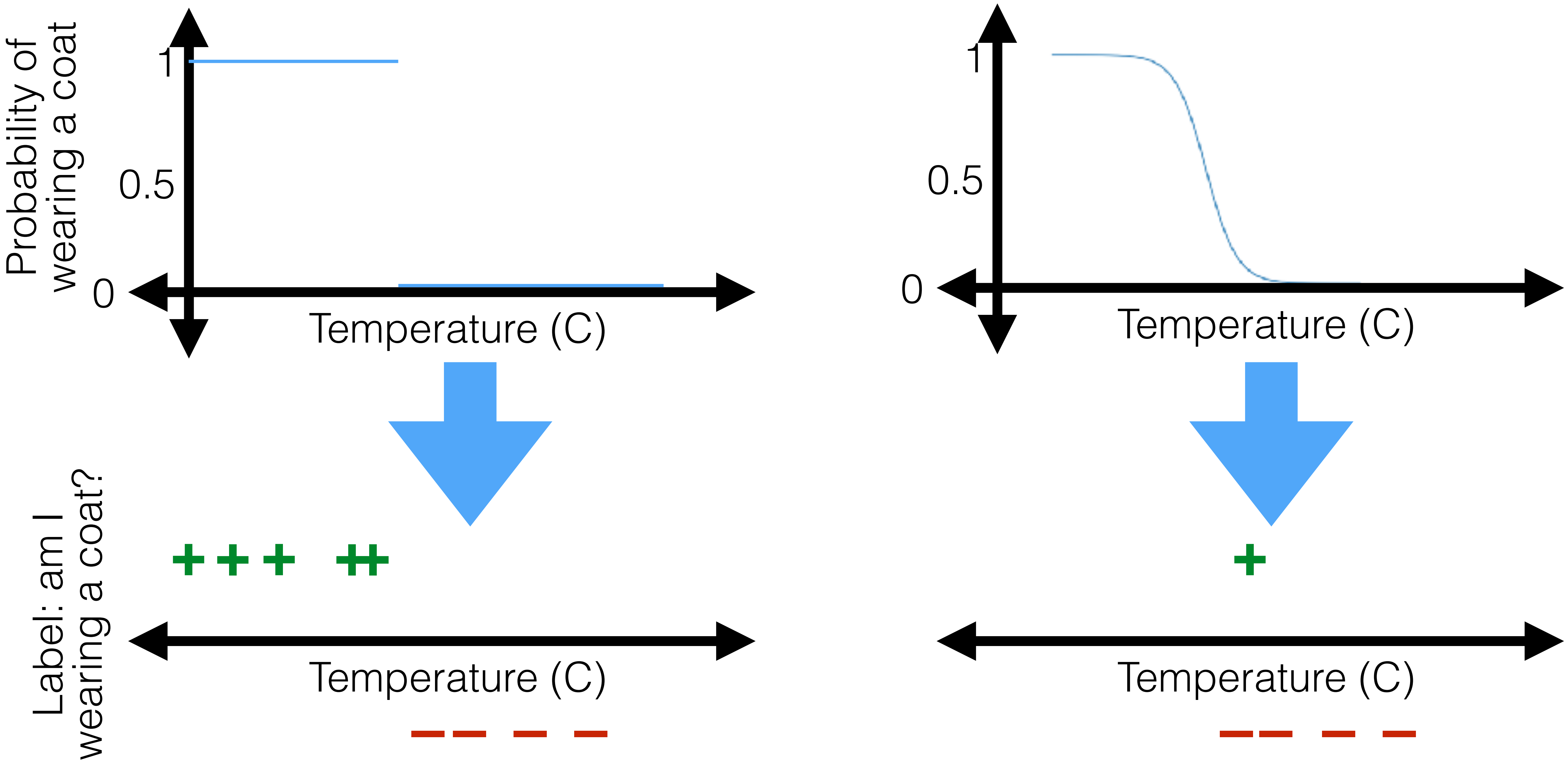

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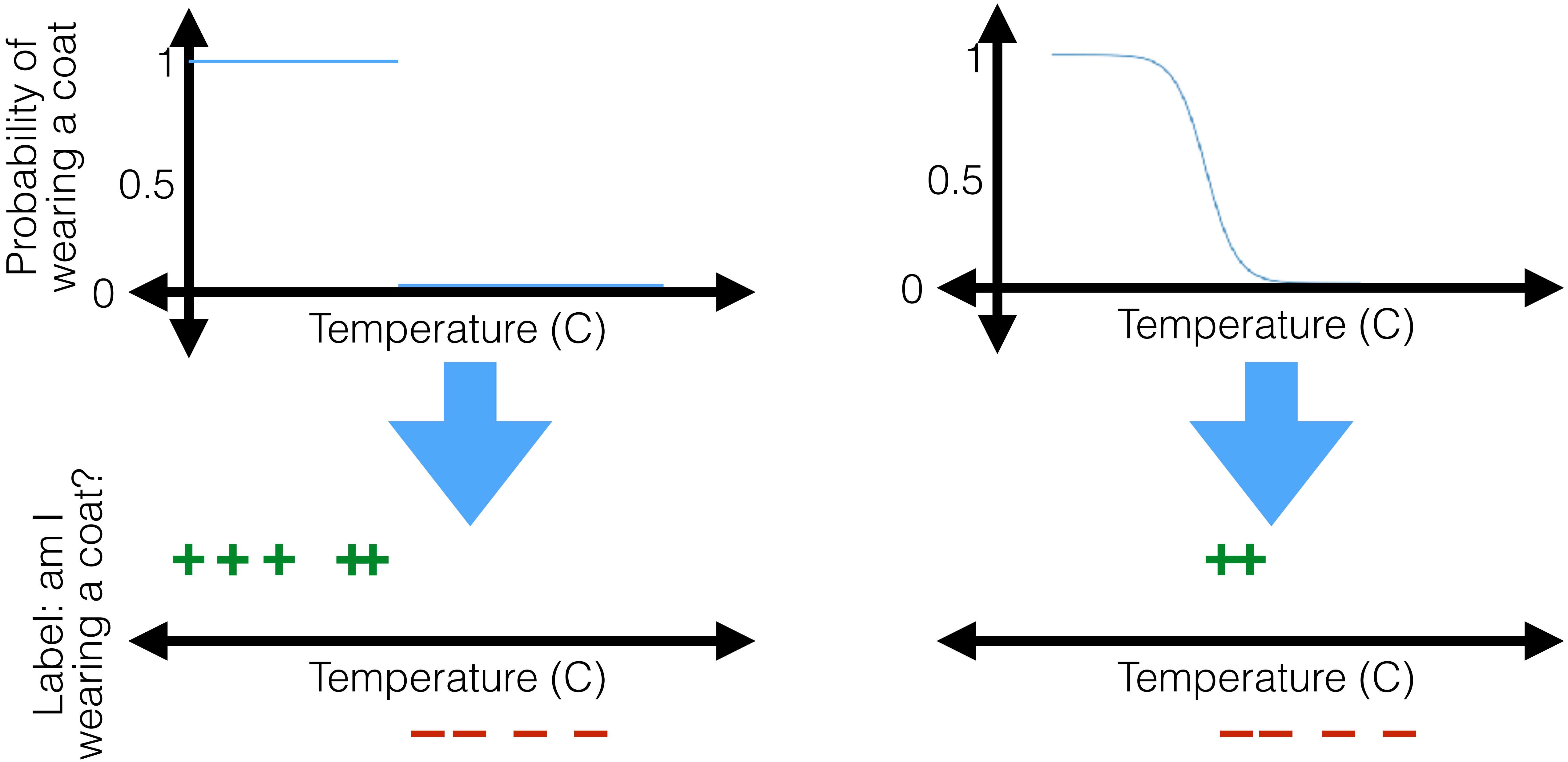

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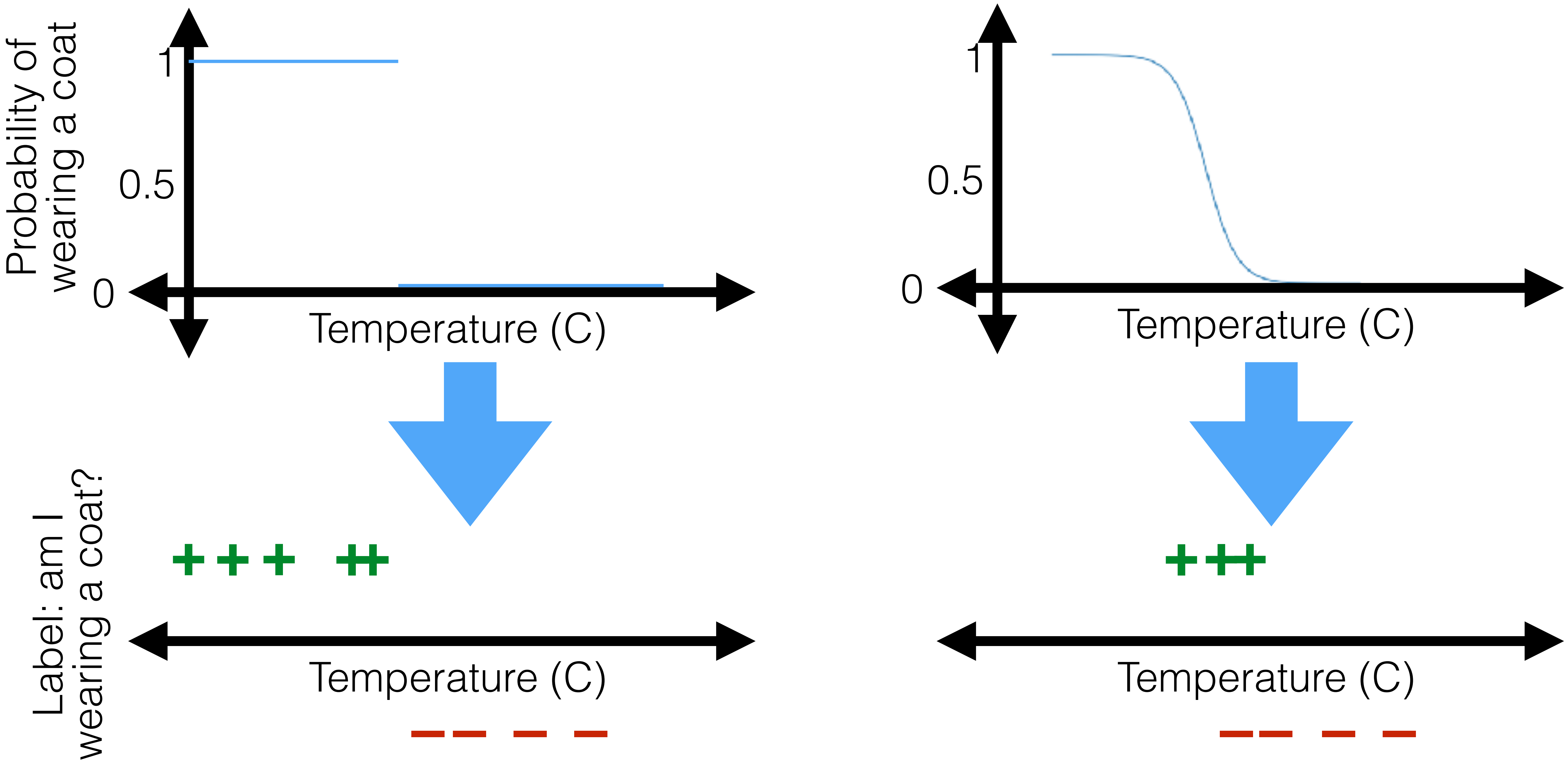

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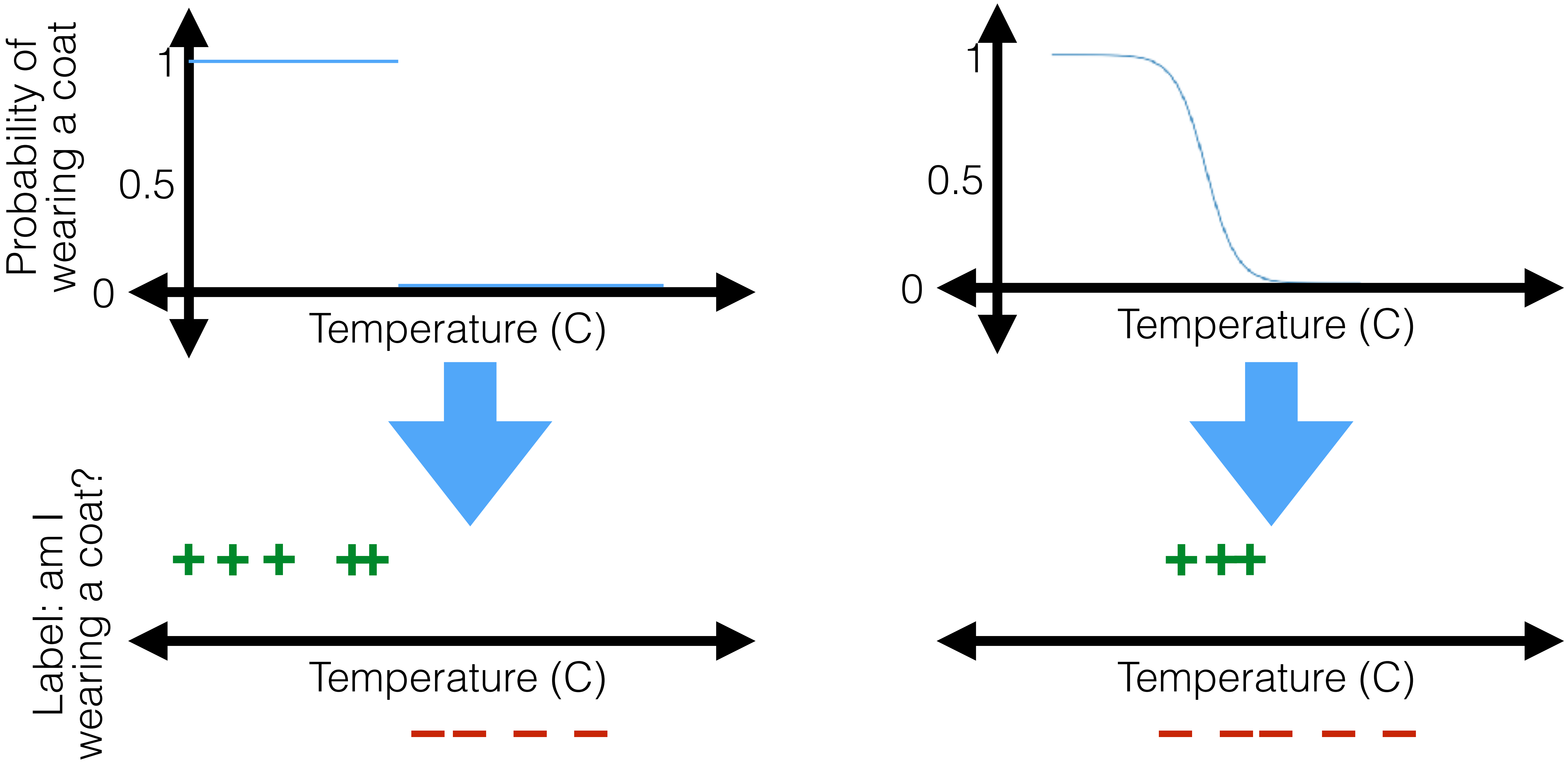

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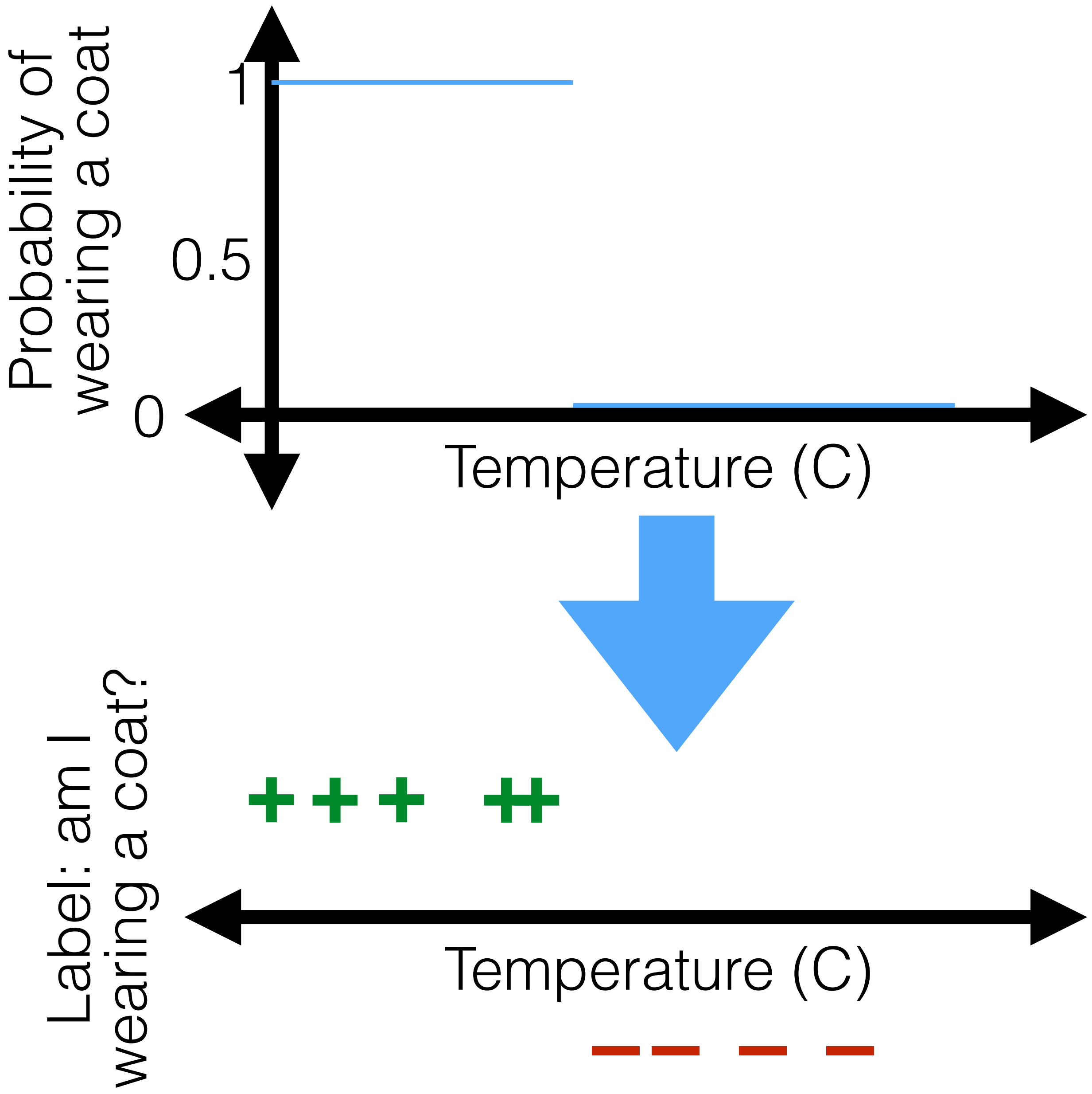

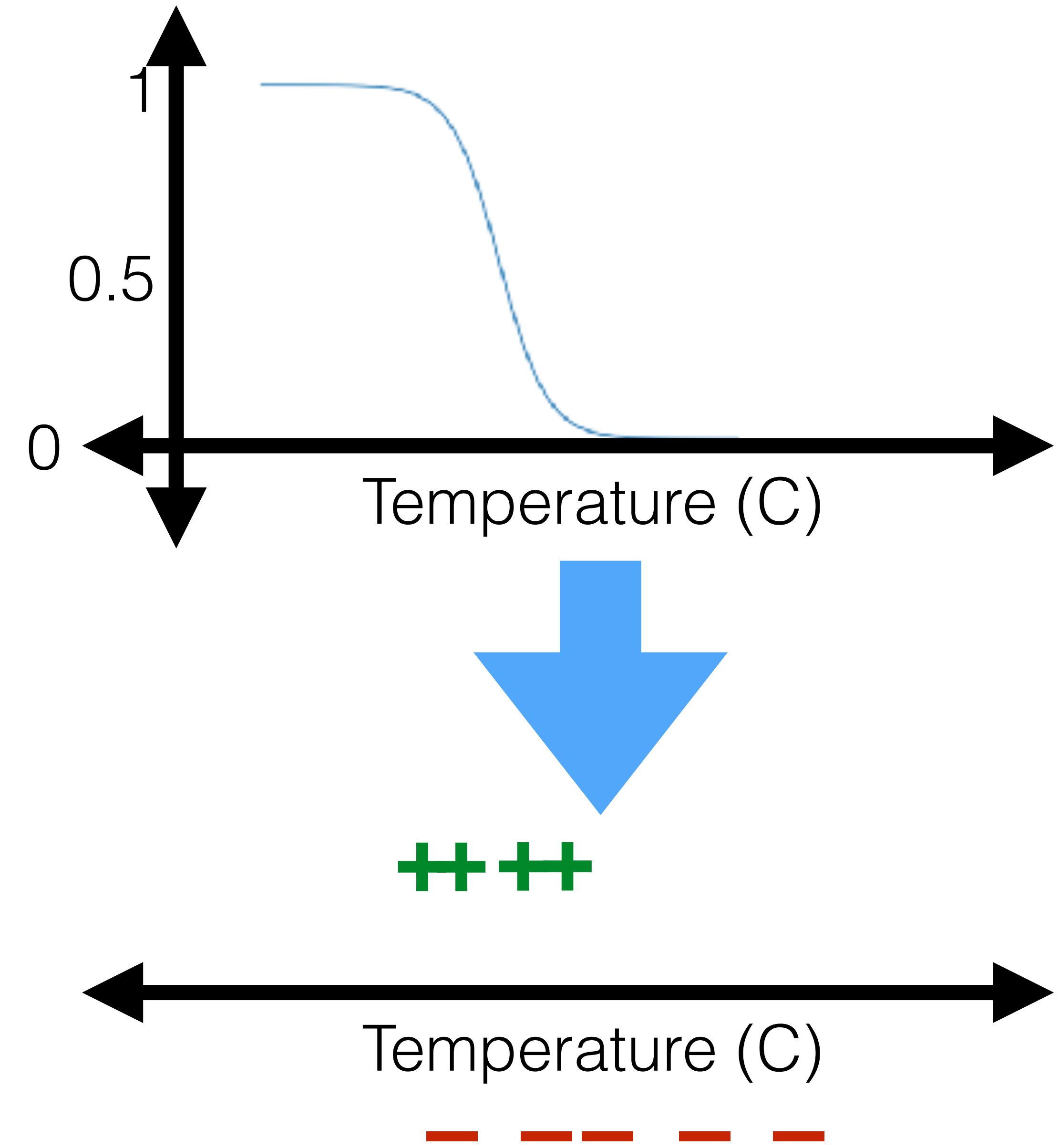

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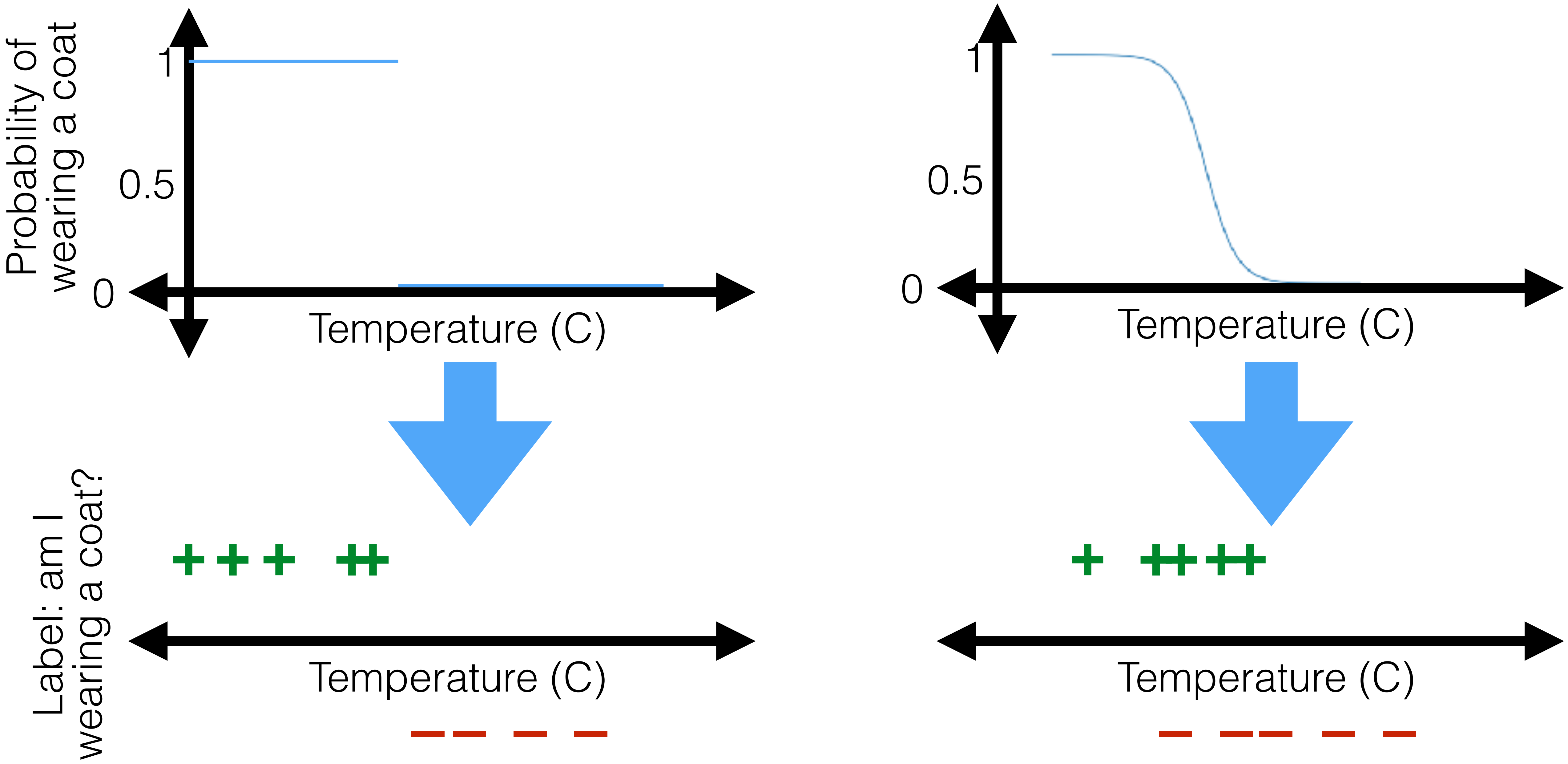

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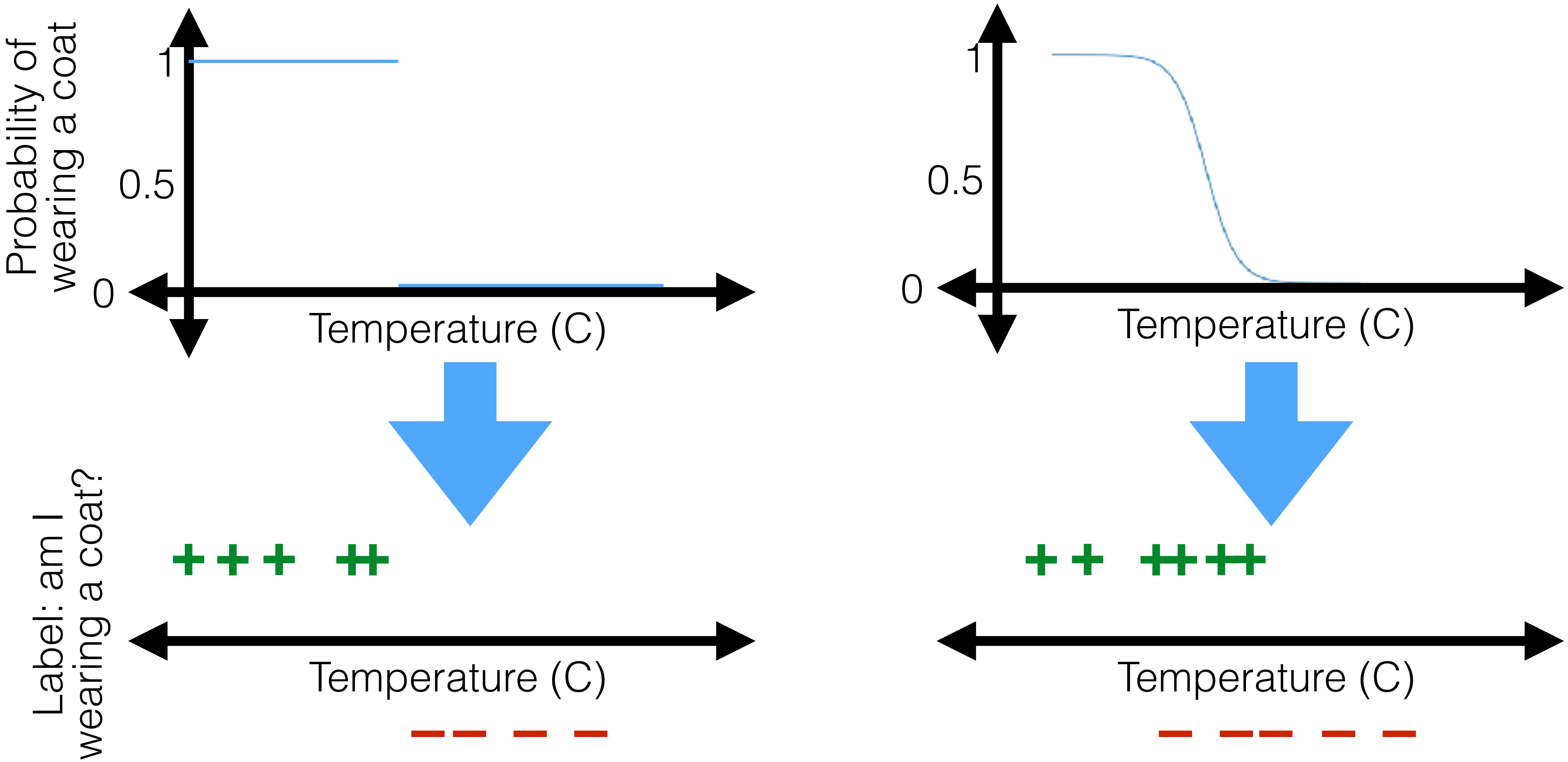

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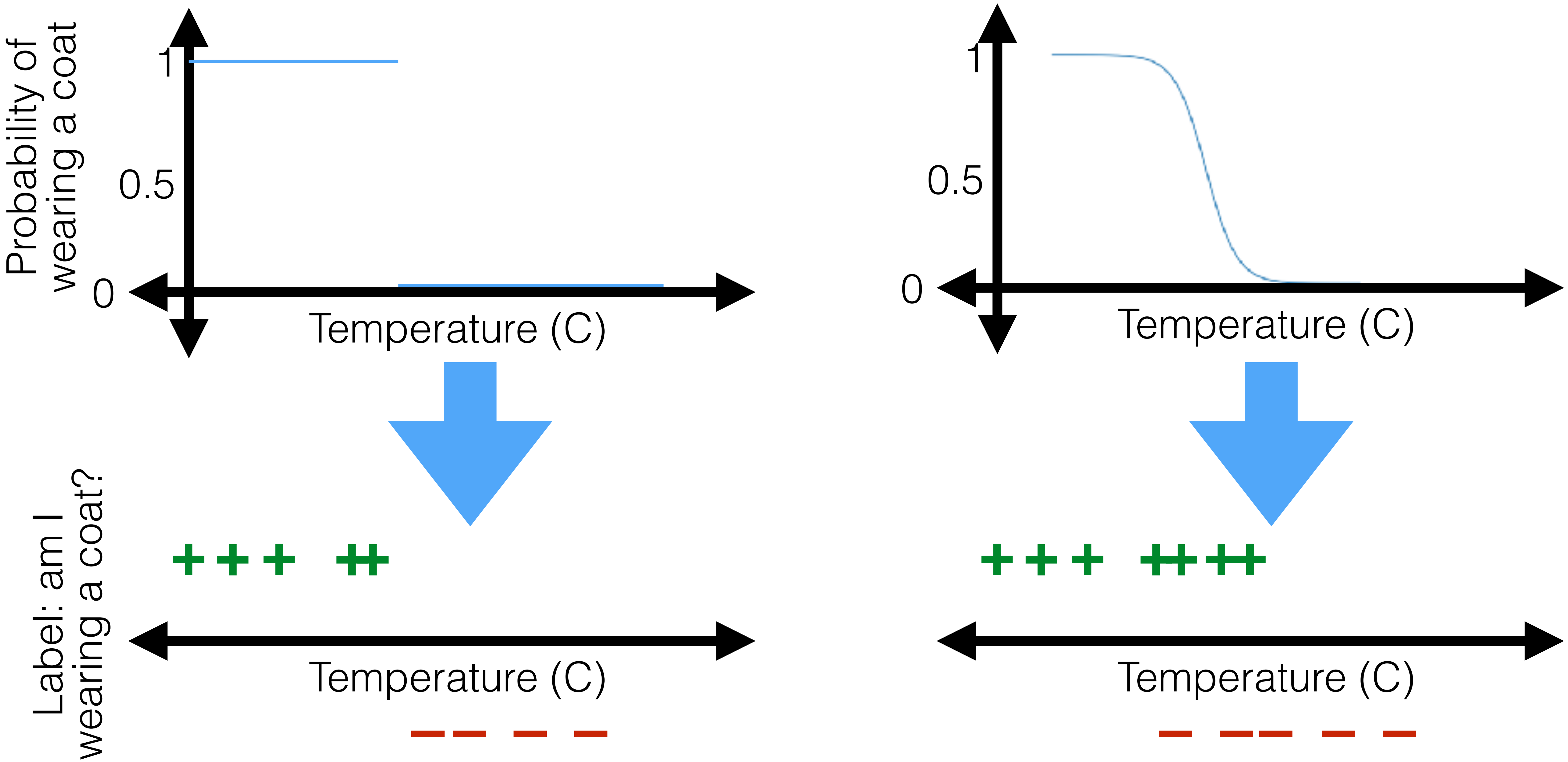

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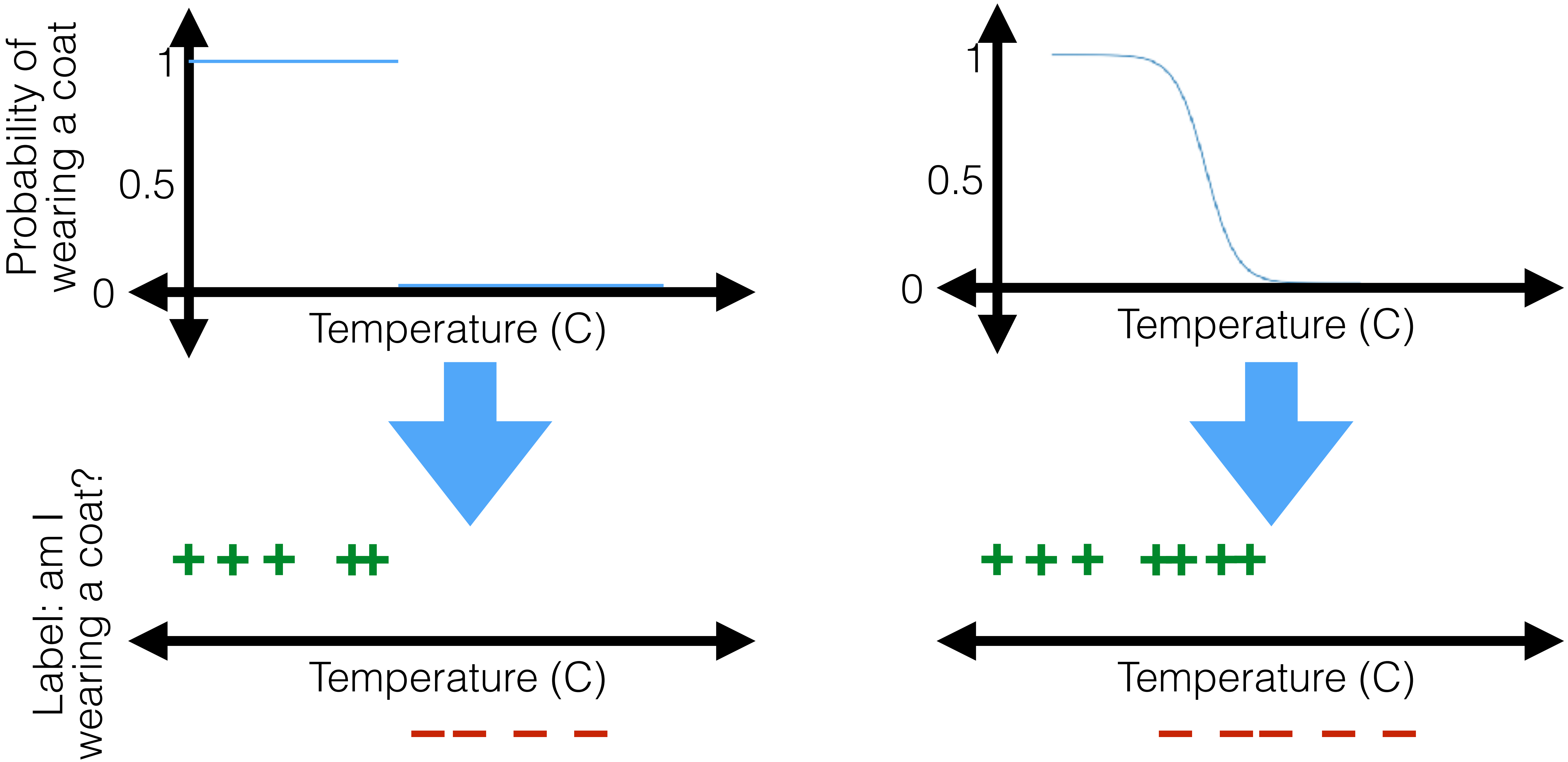

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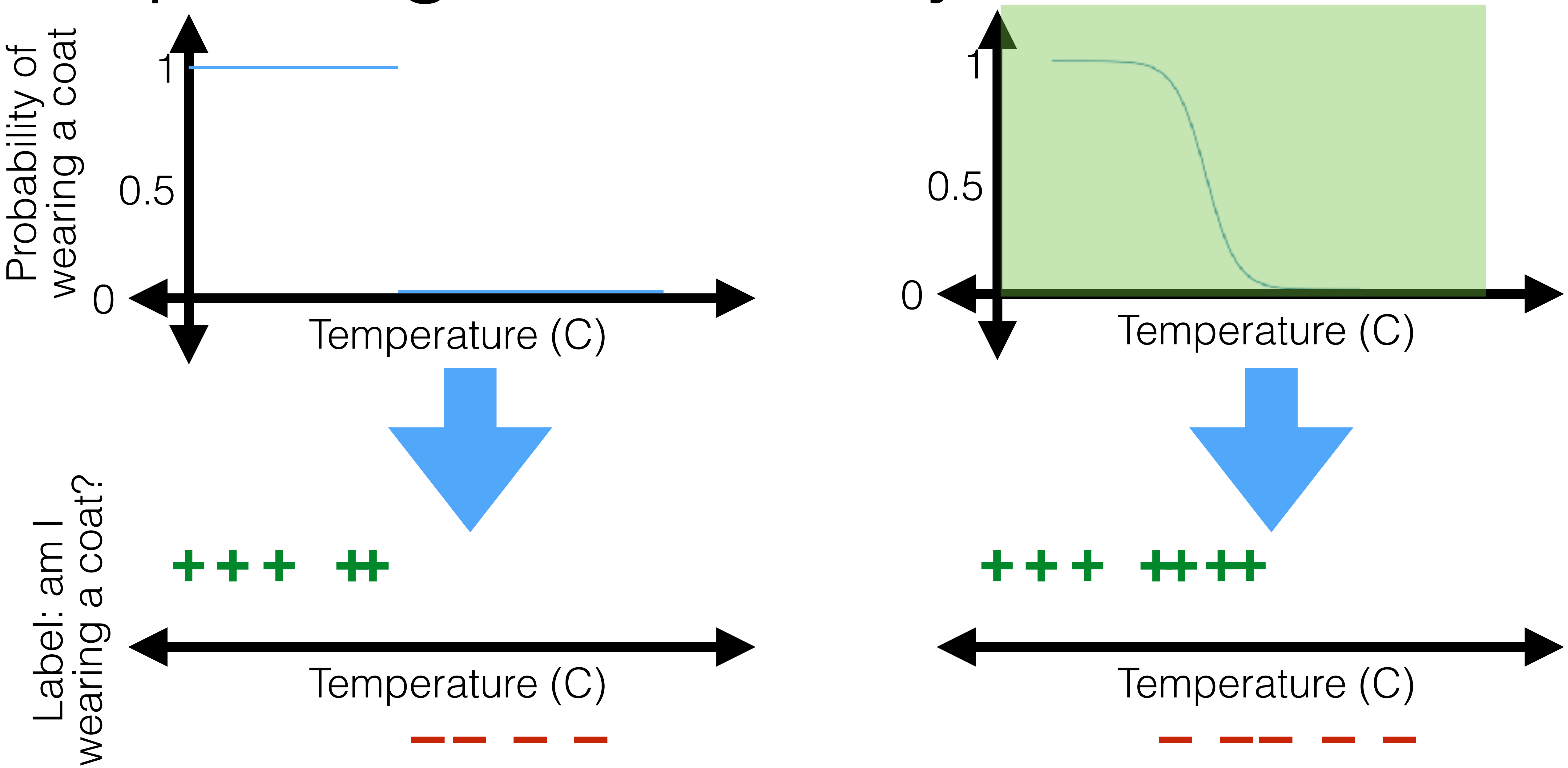

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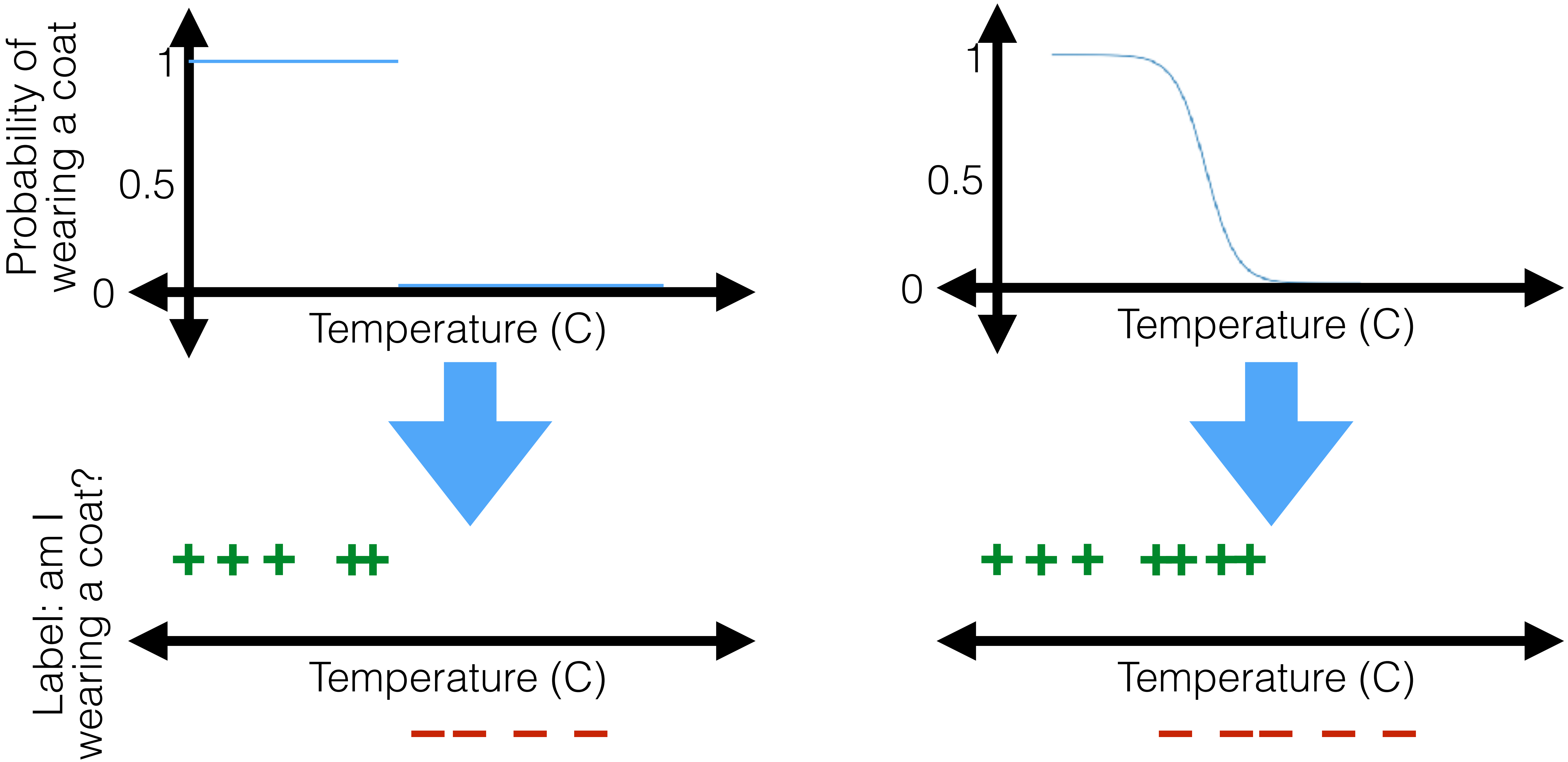

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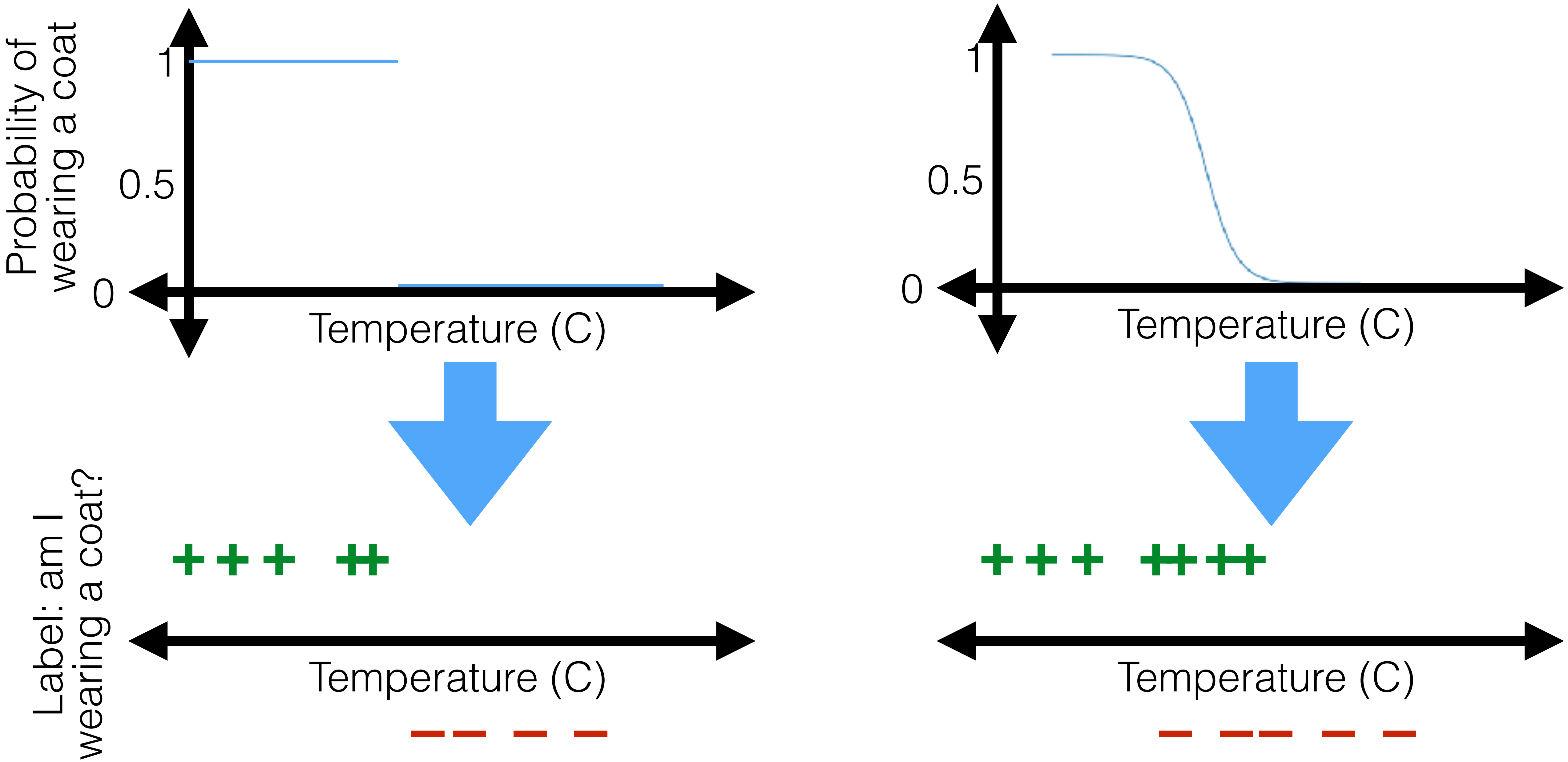

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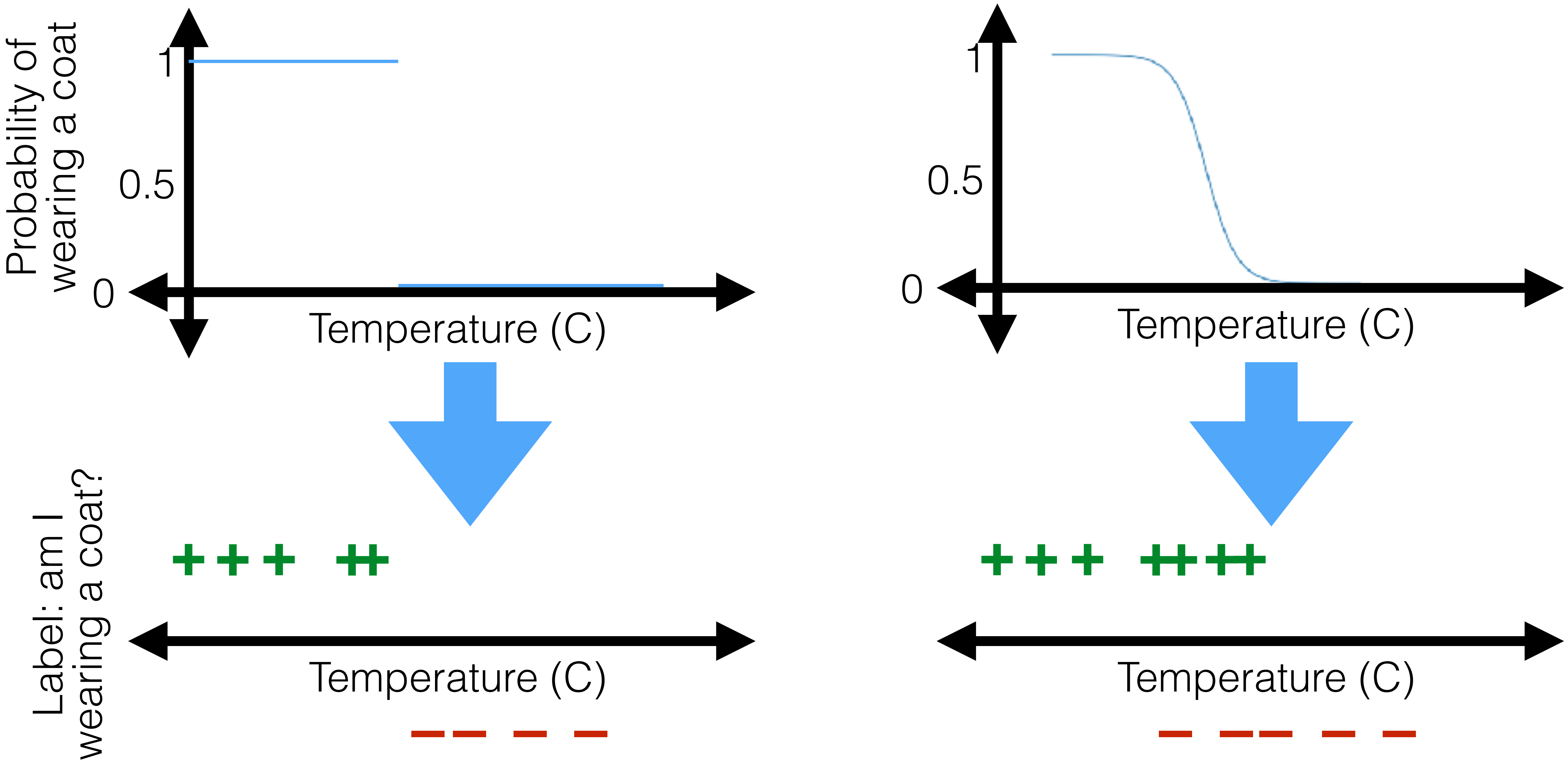

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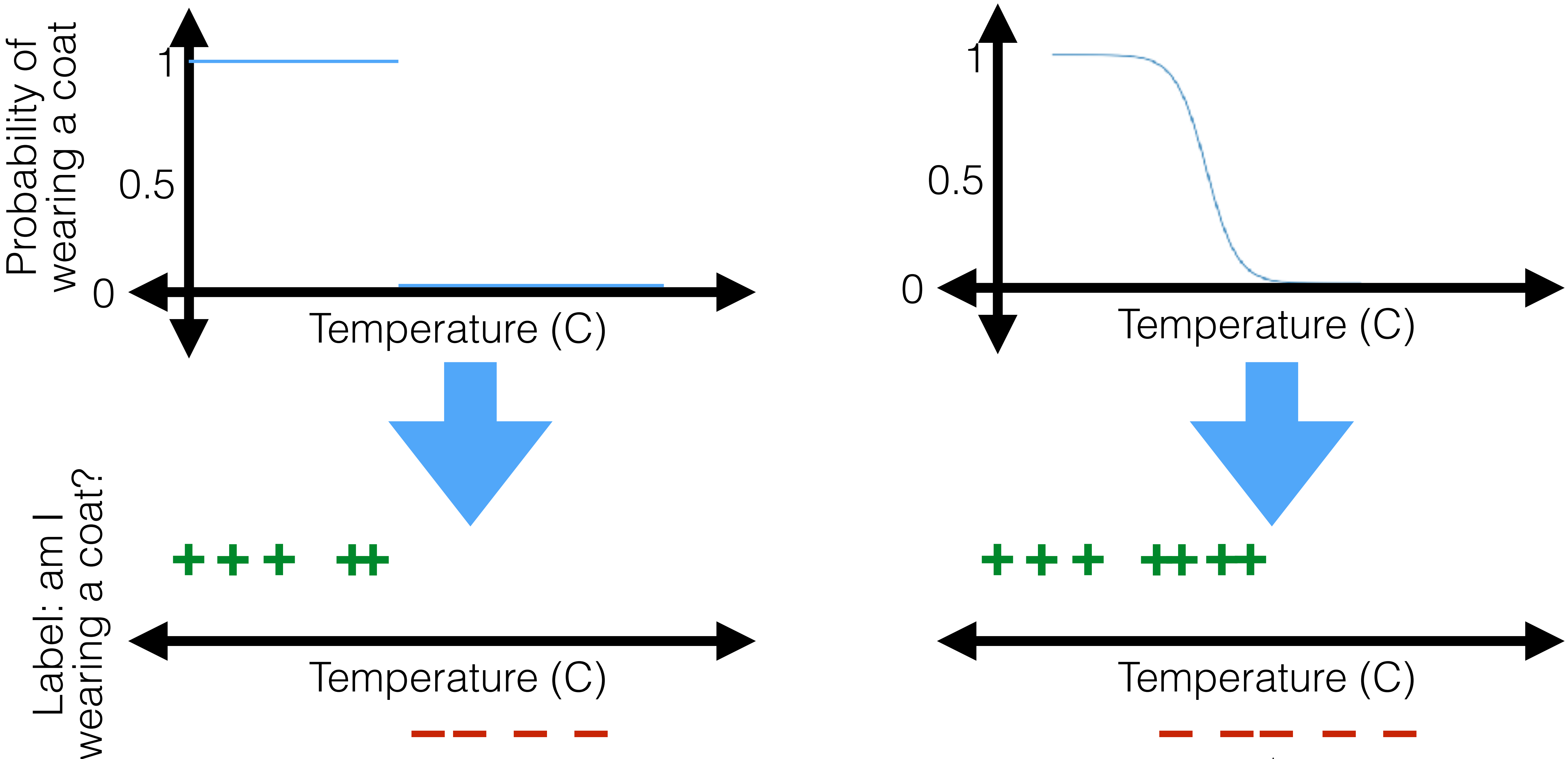

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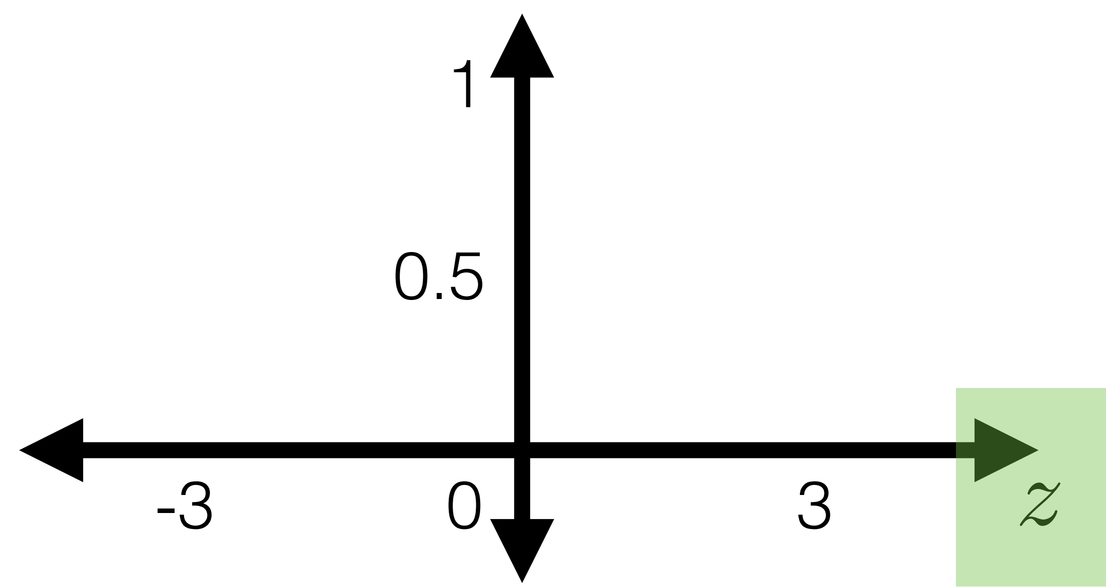

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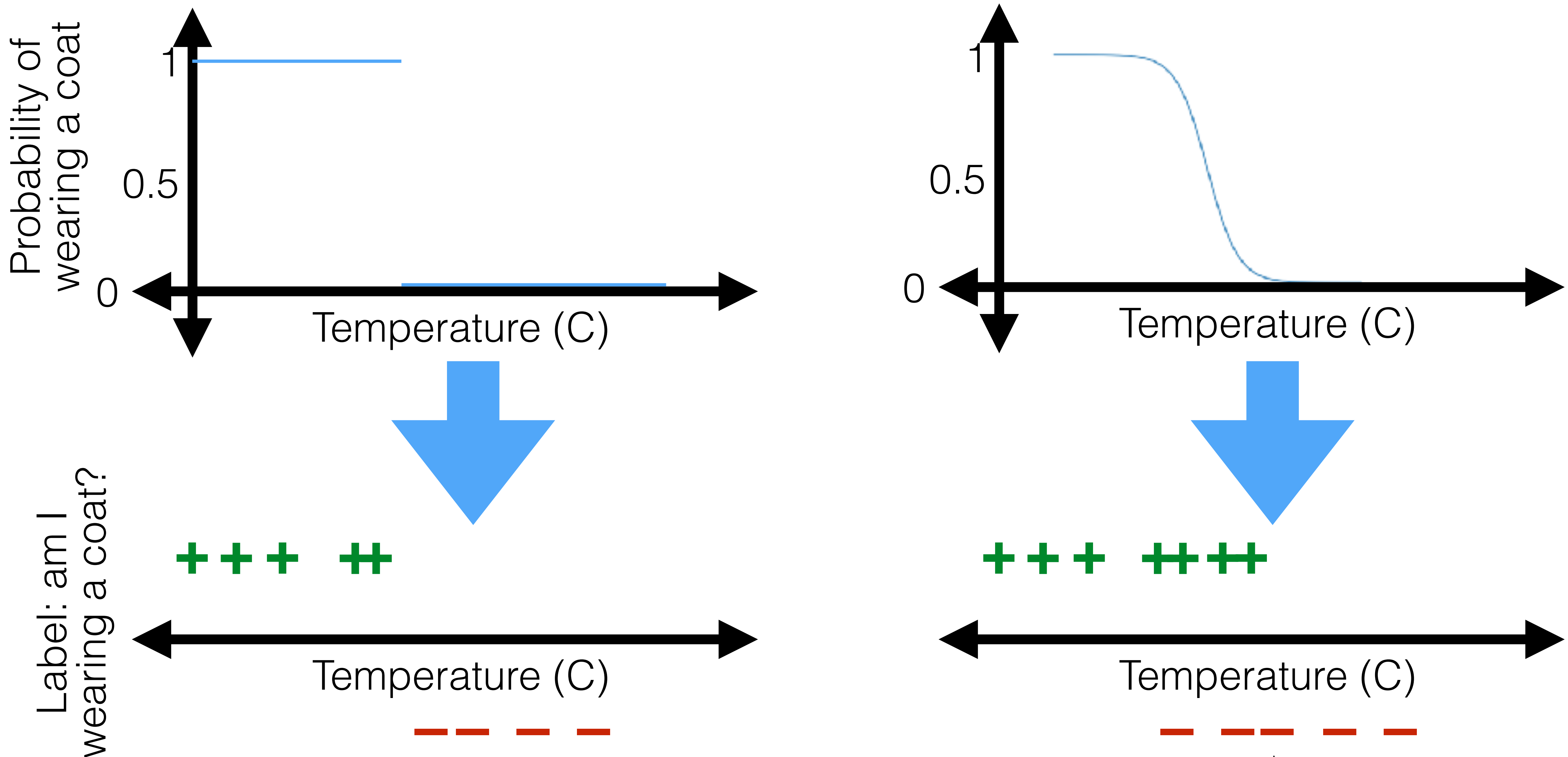

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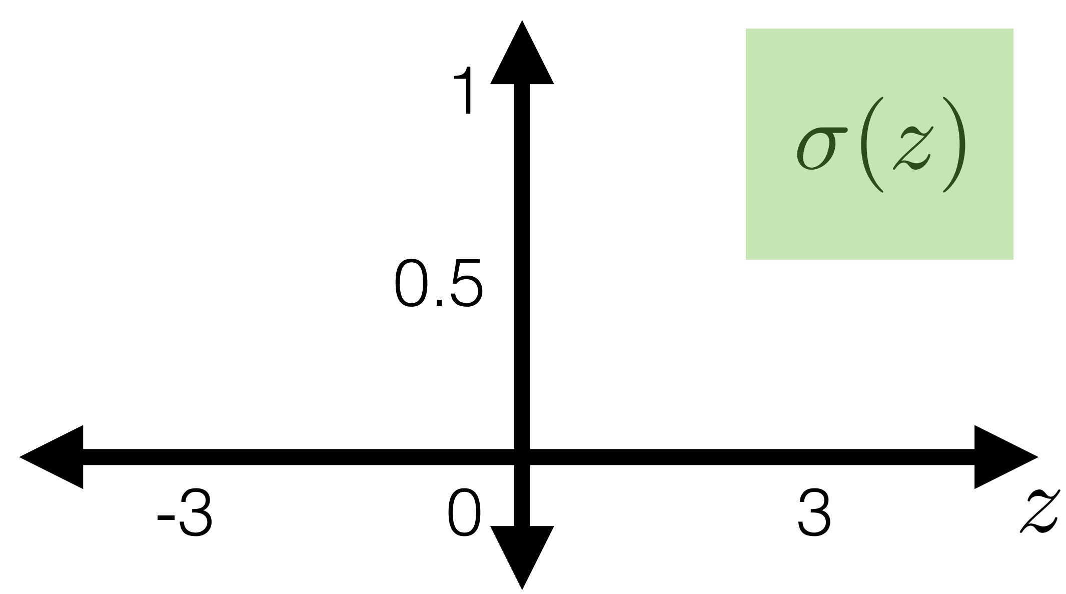

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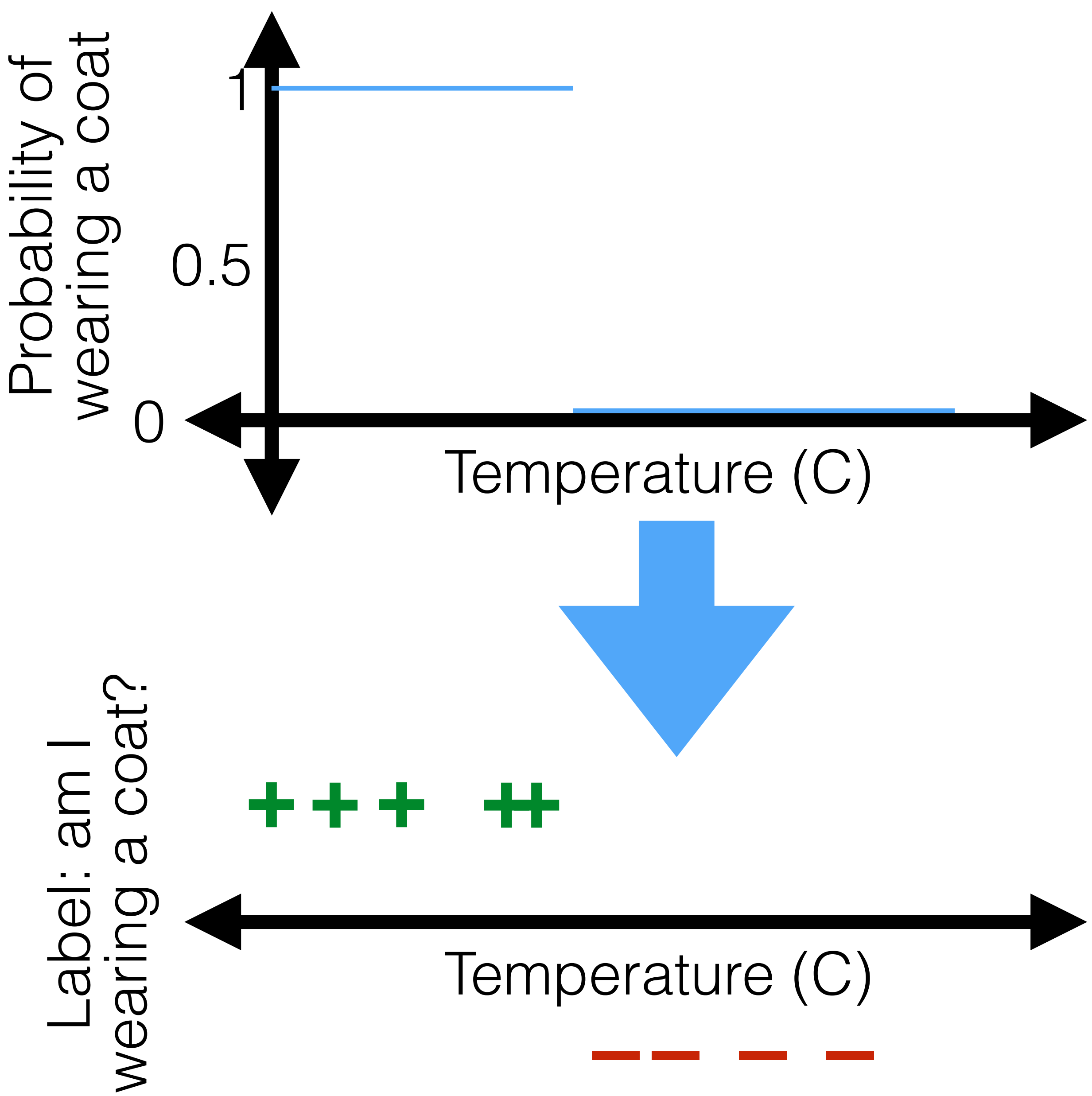

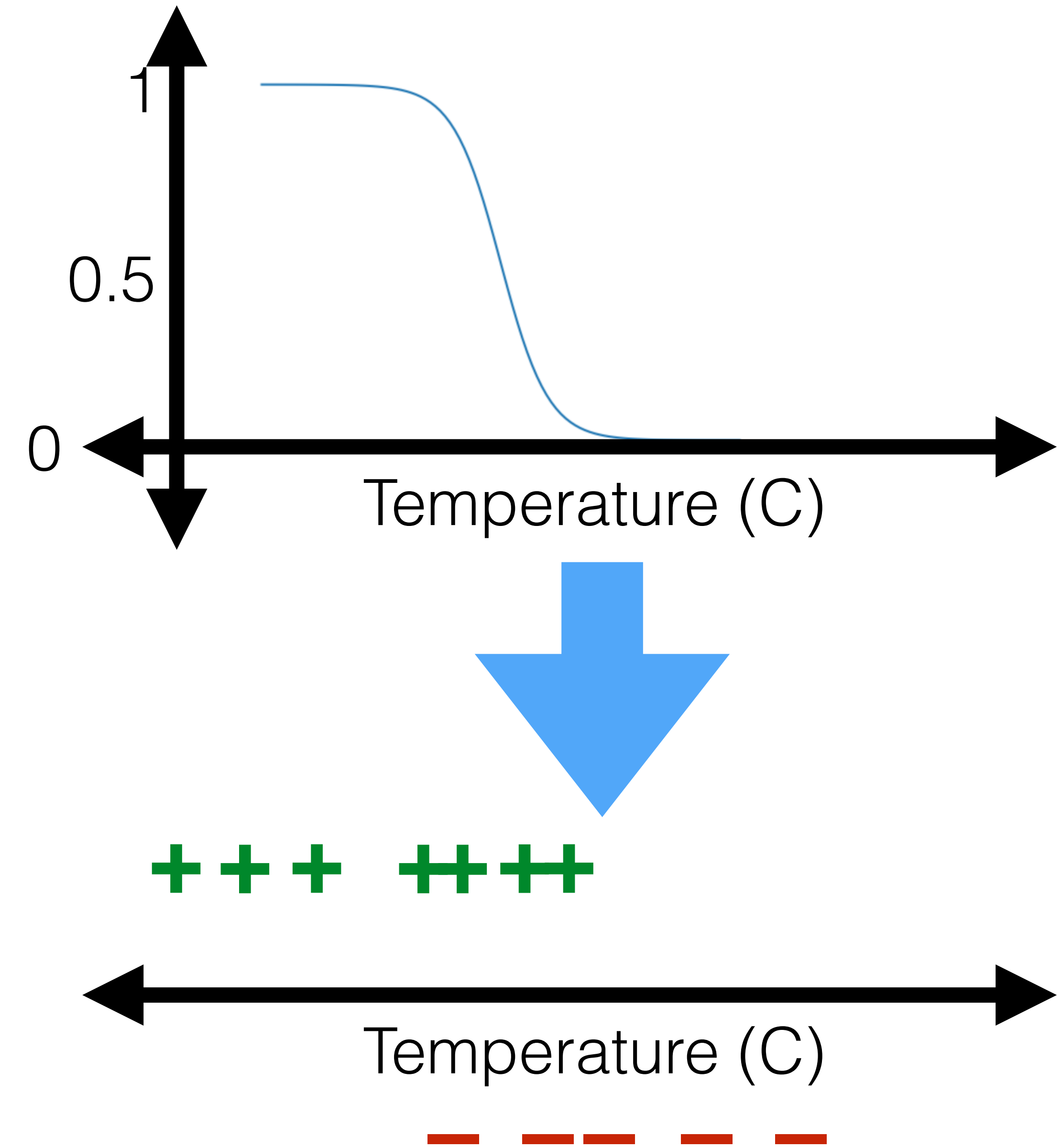

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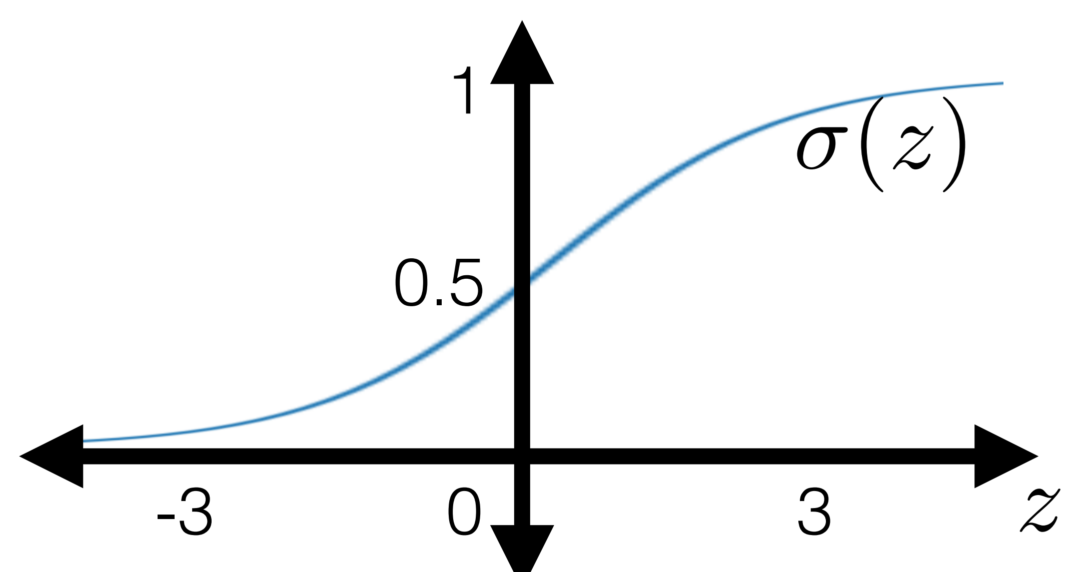

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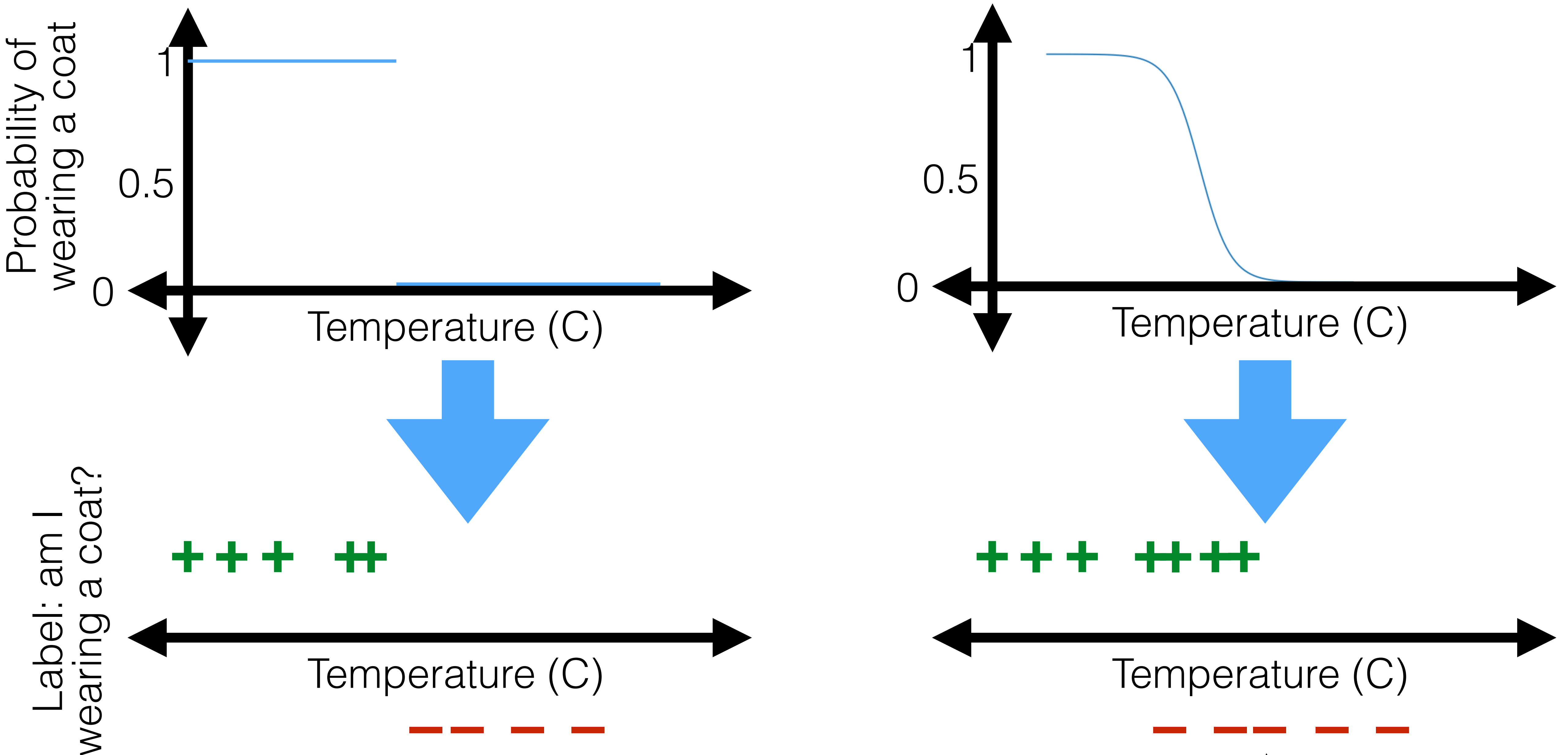

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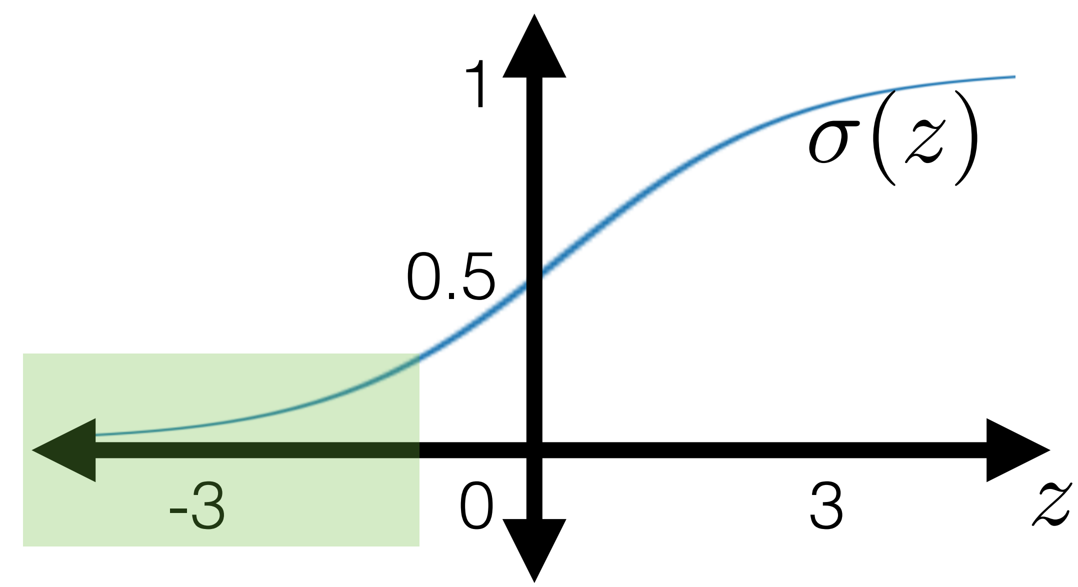

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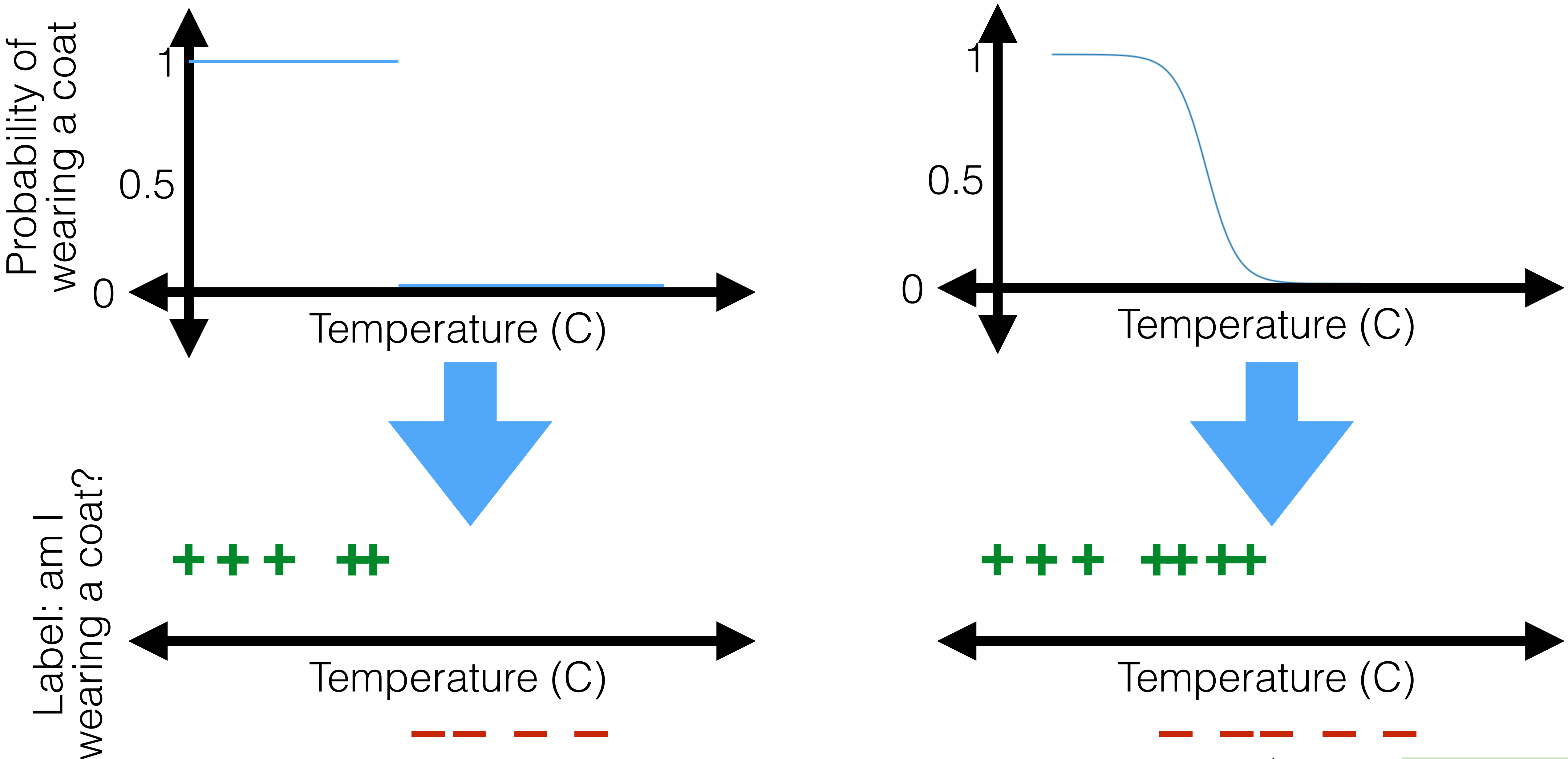

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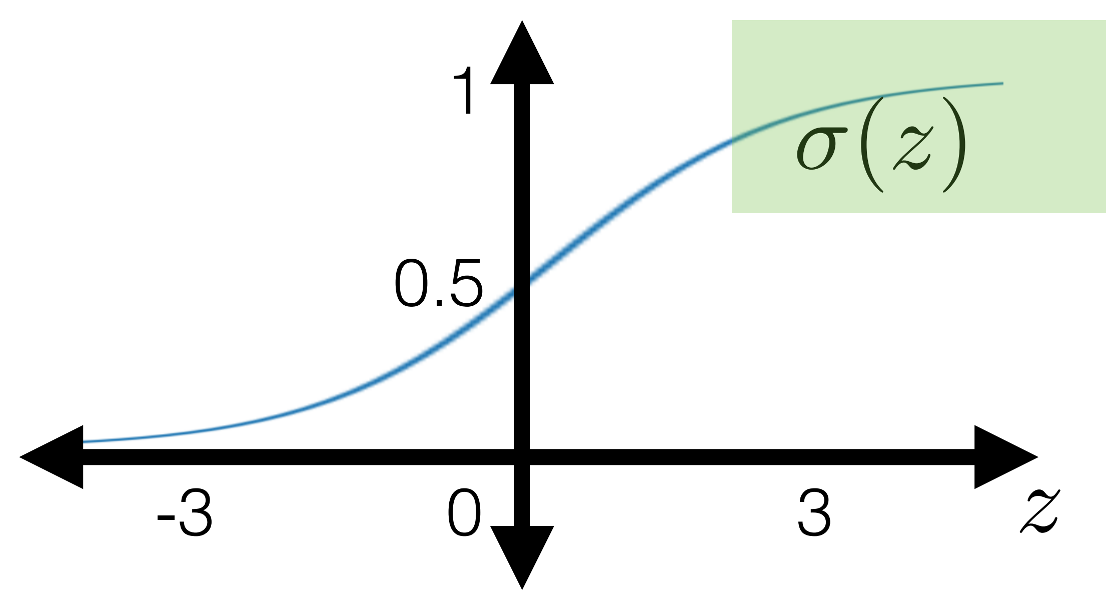

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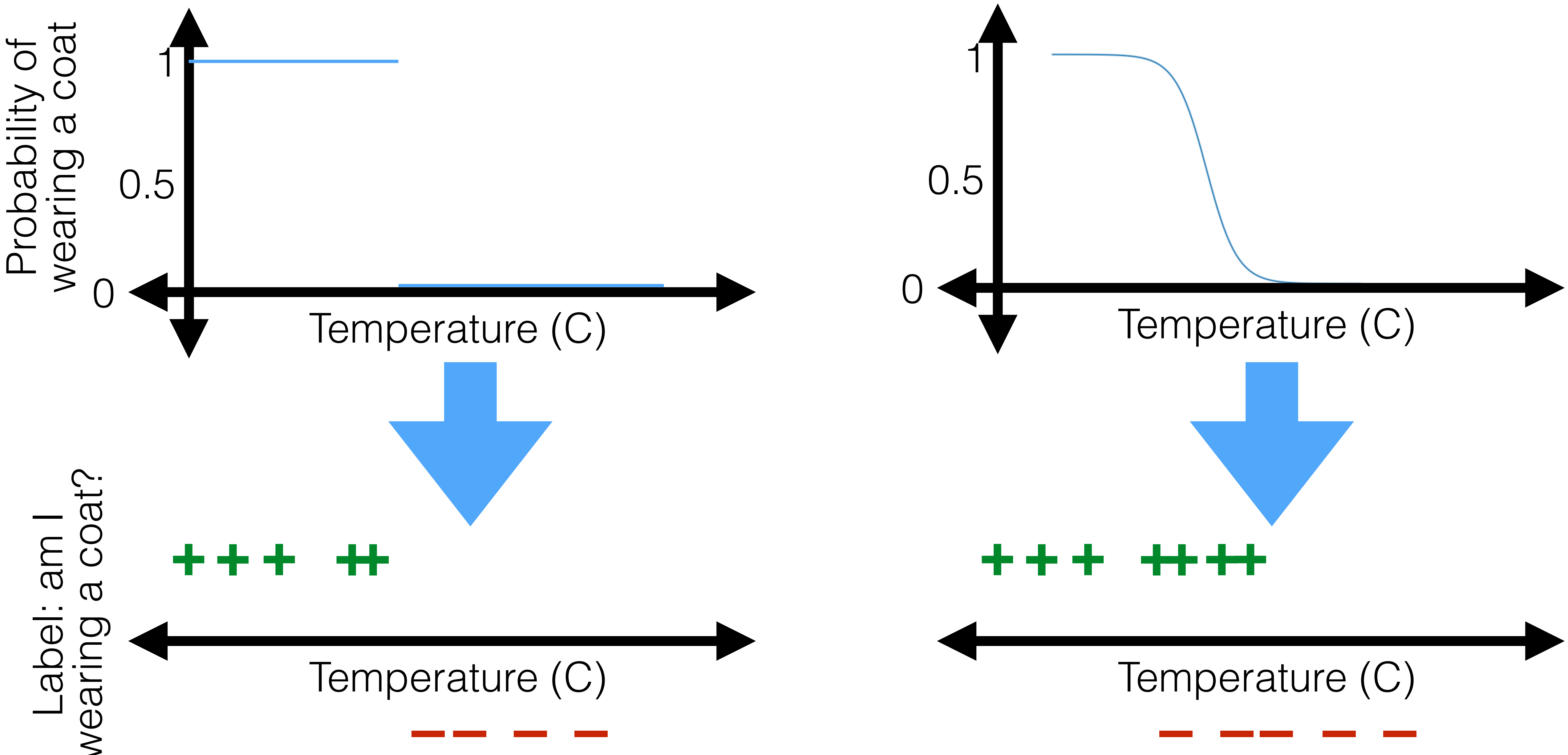

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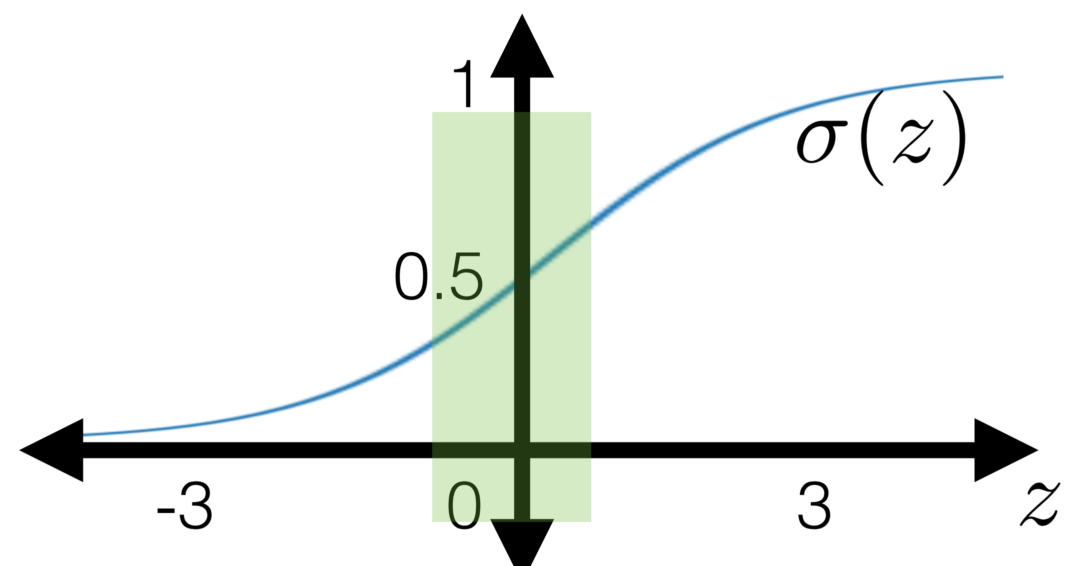
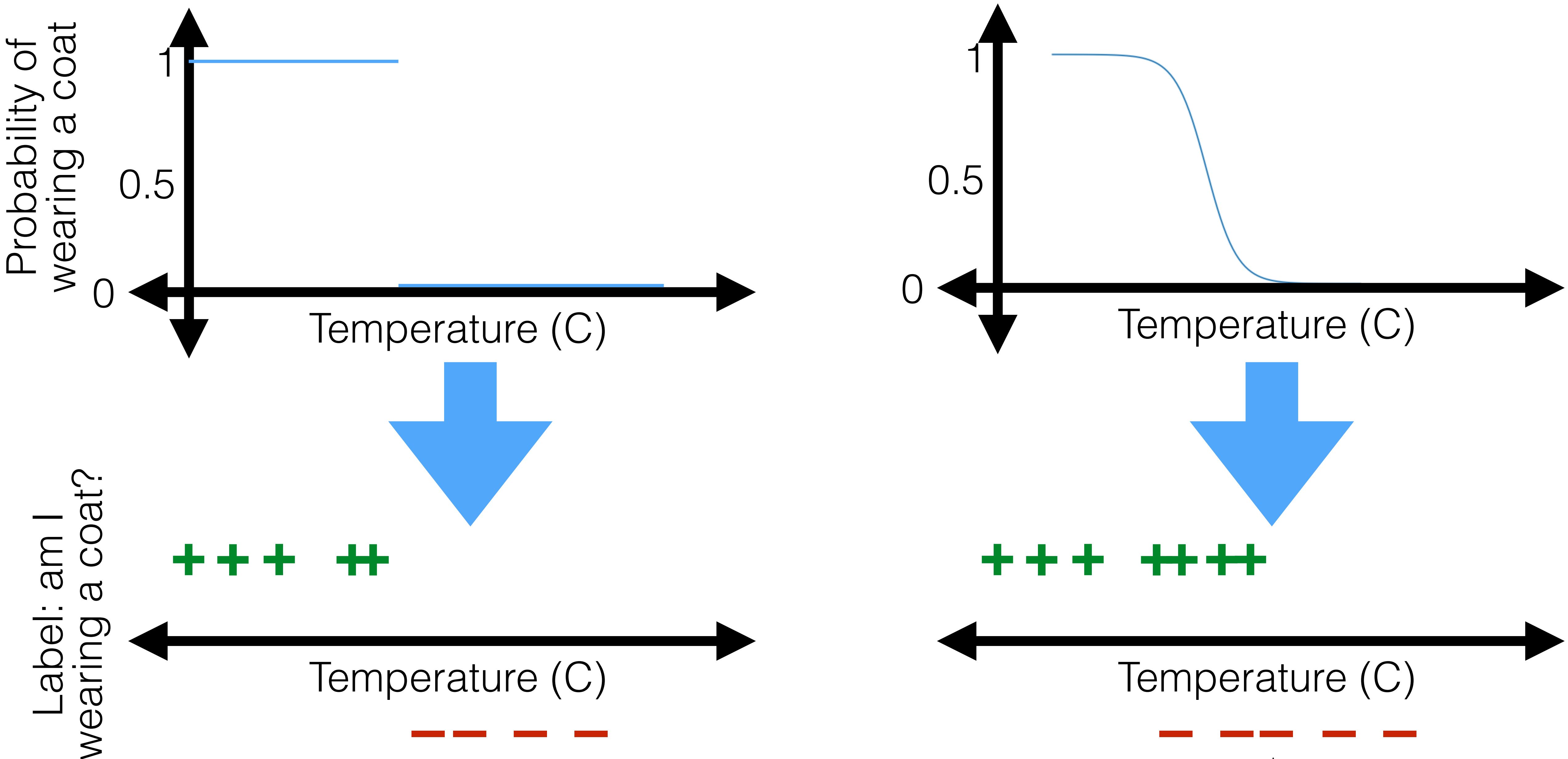


Capturing uncertainty



- How to make this shape?
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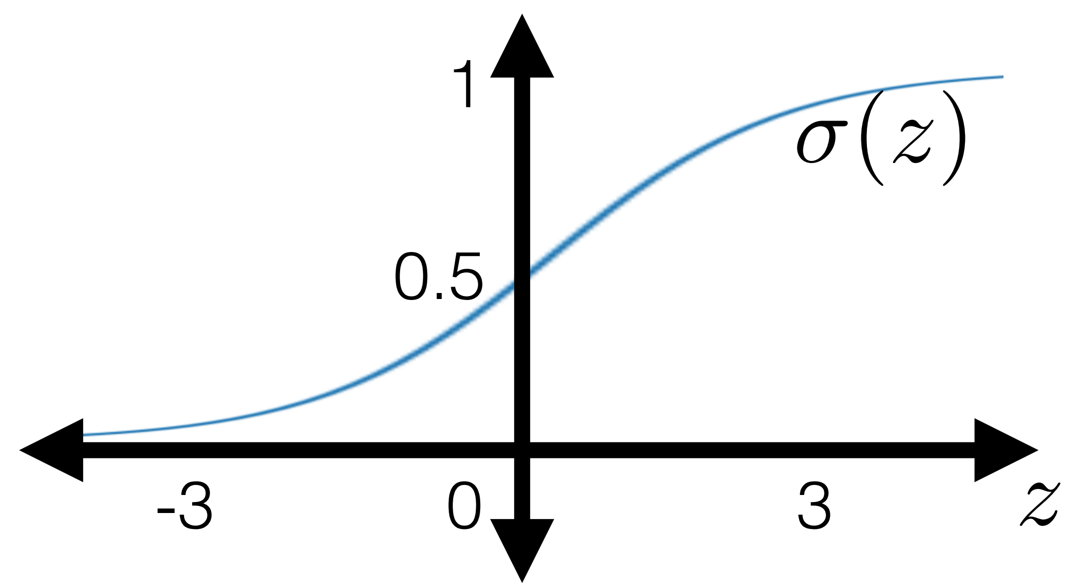

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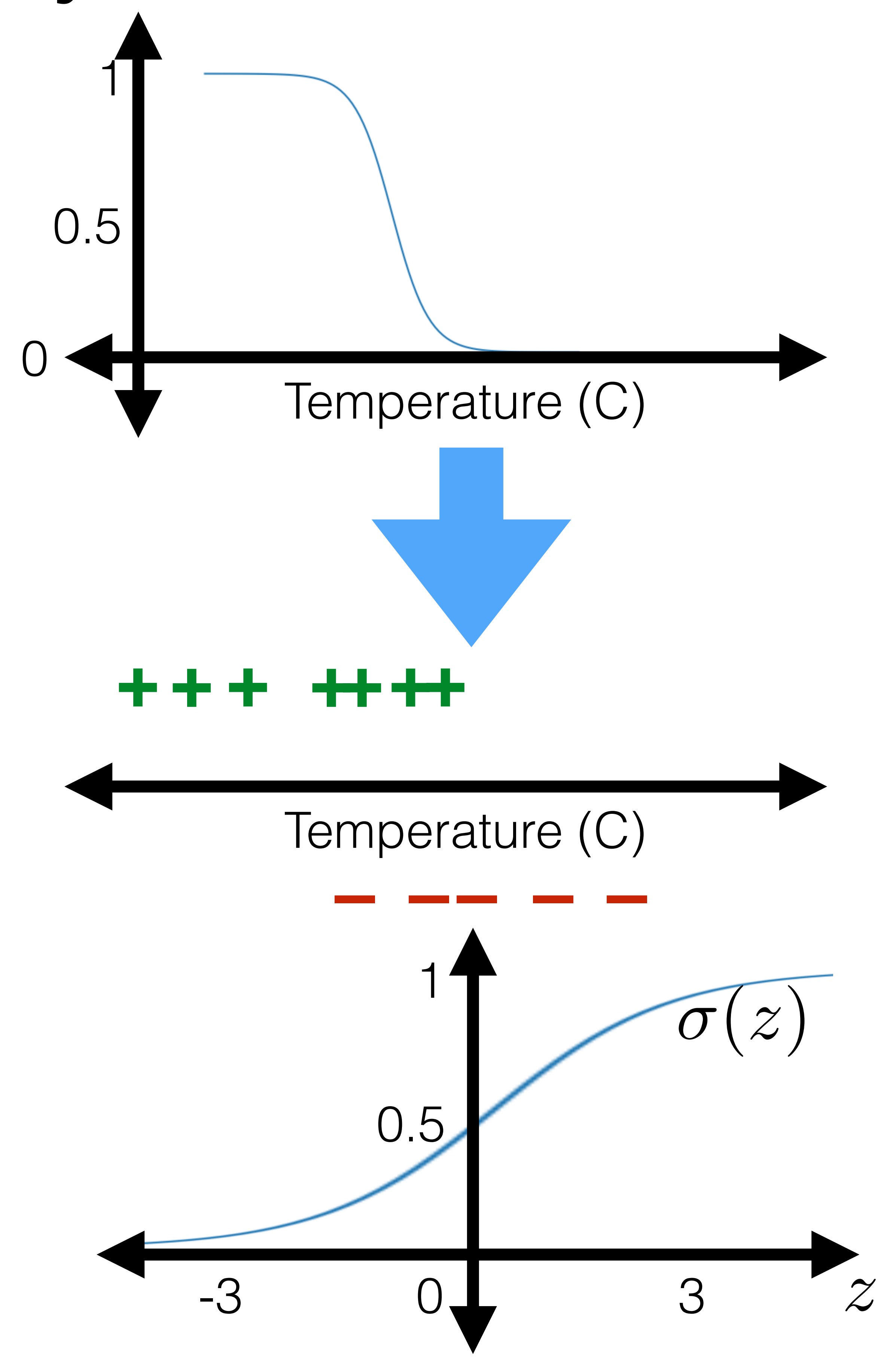

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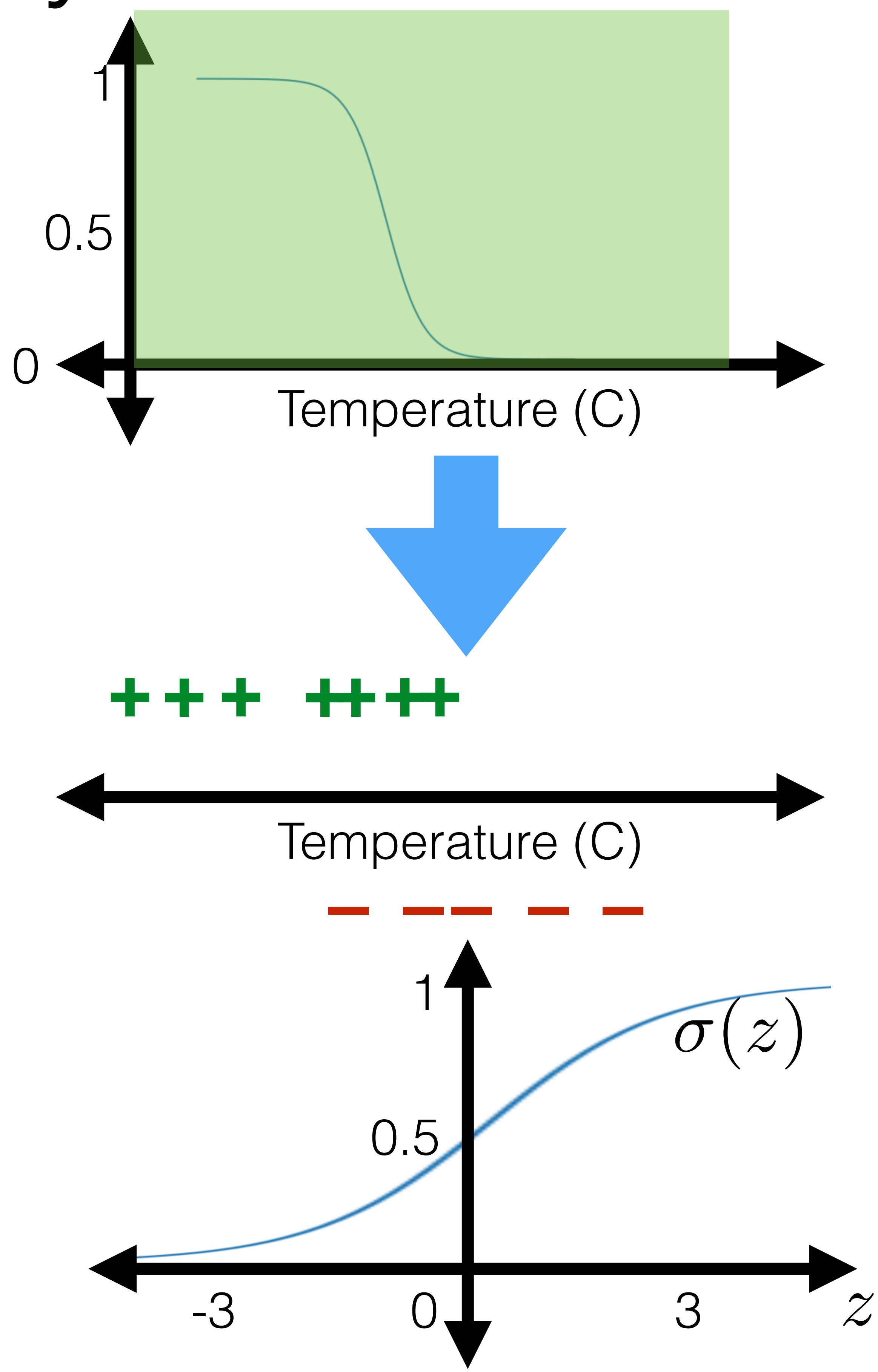

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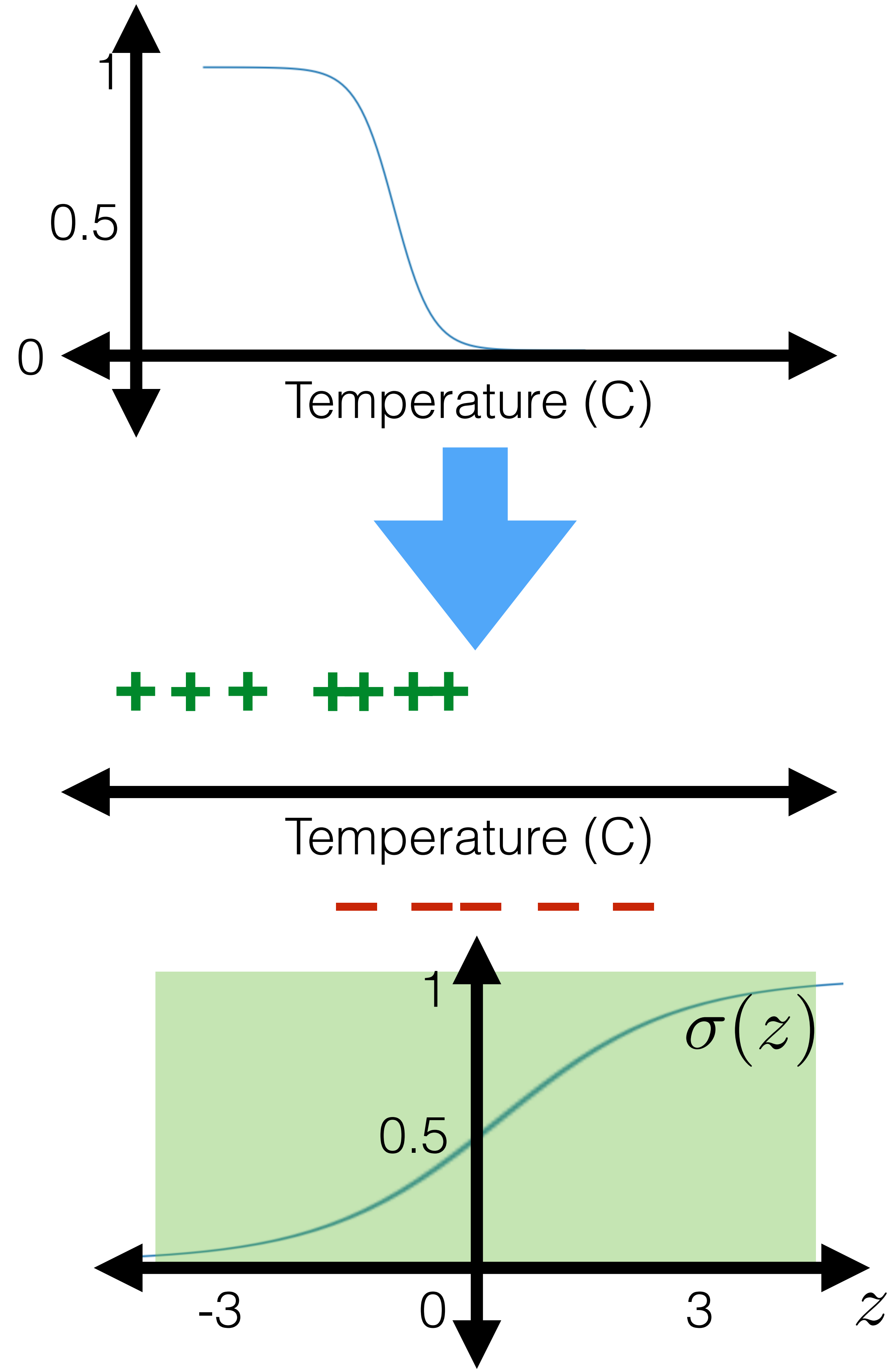

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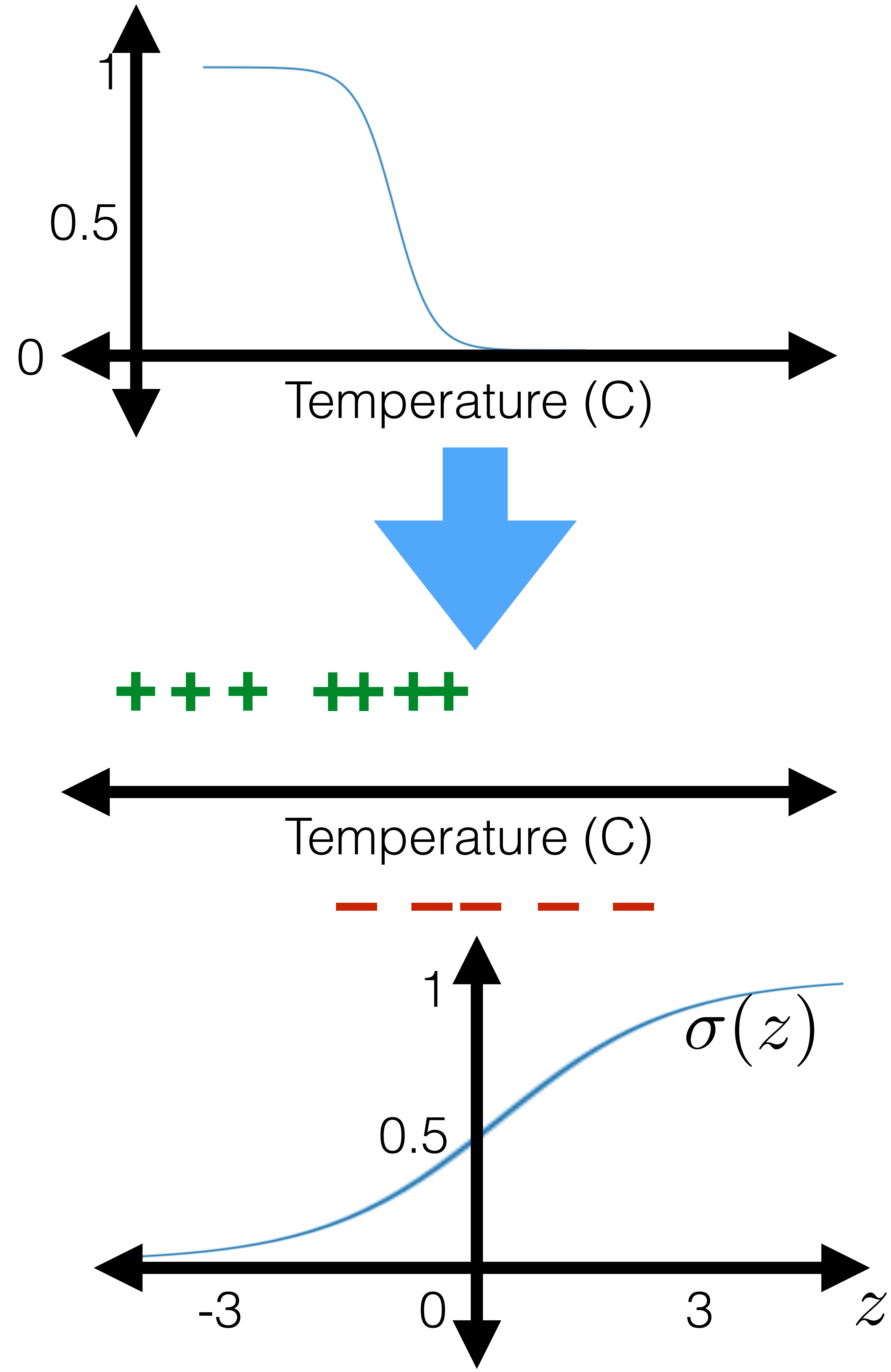

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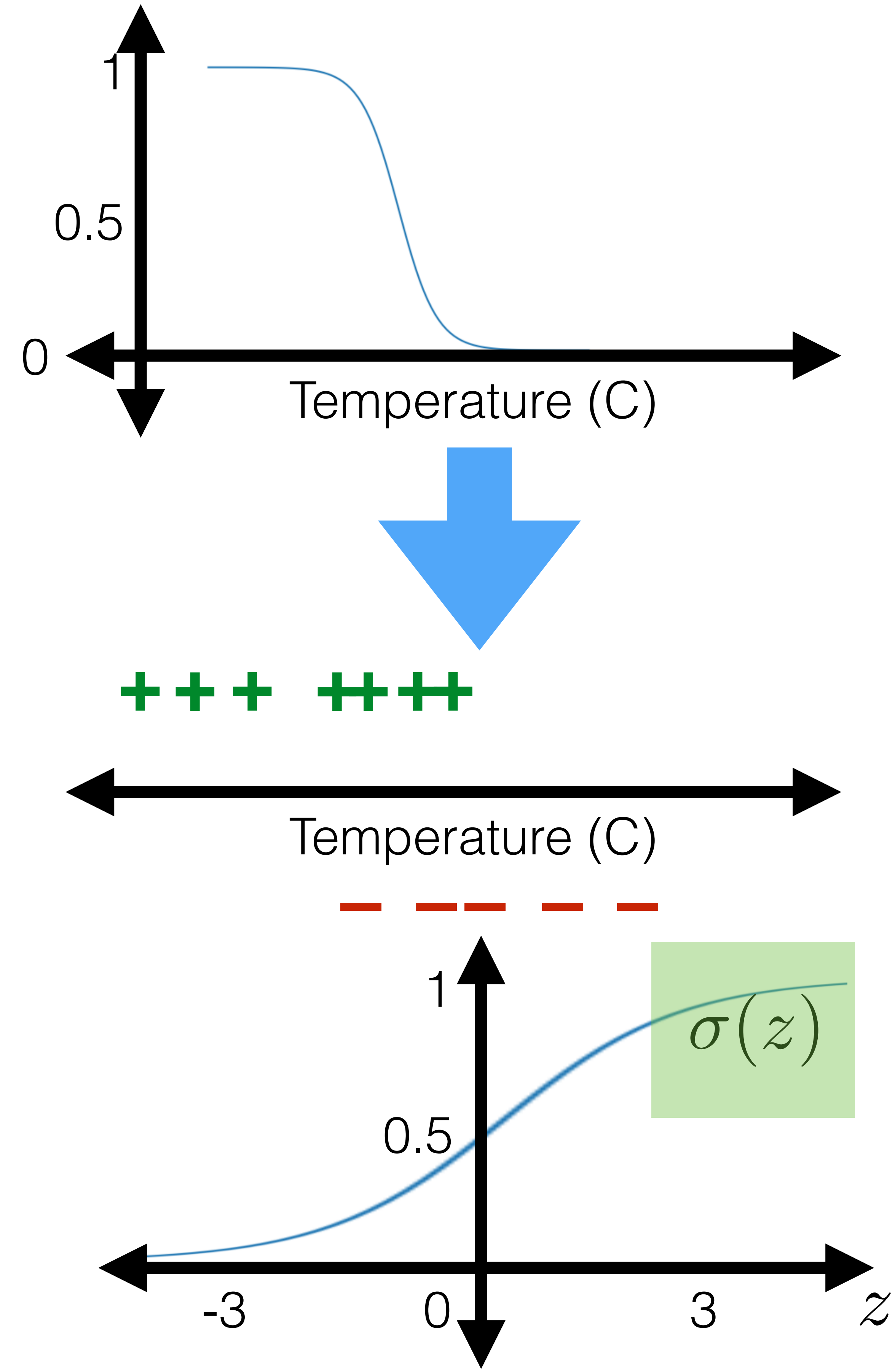

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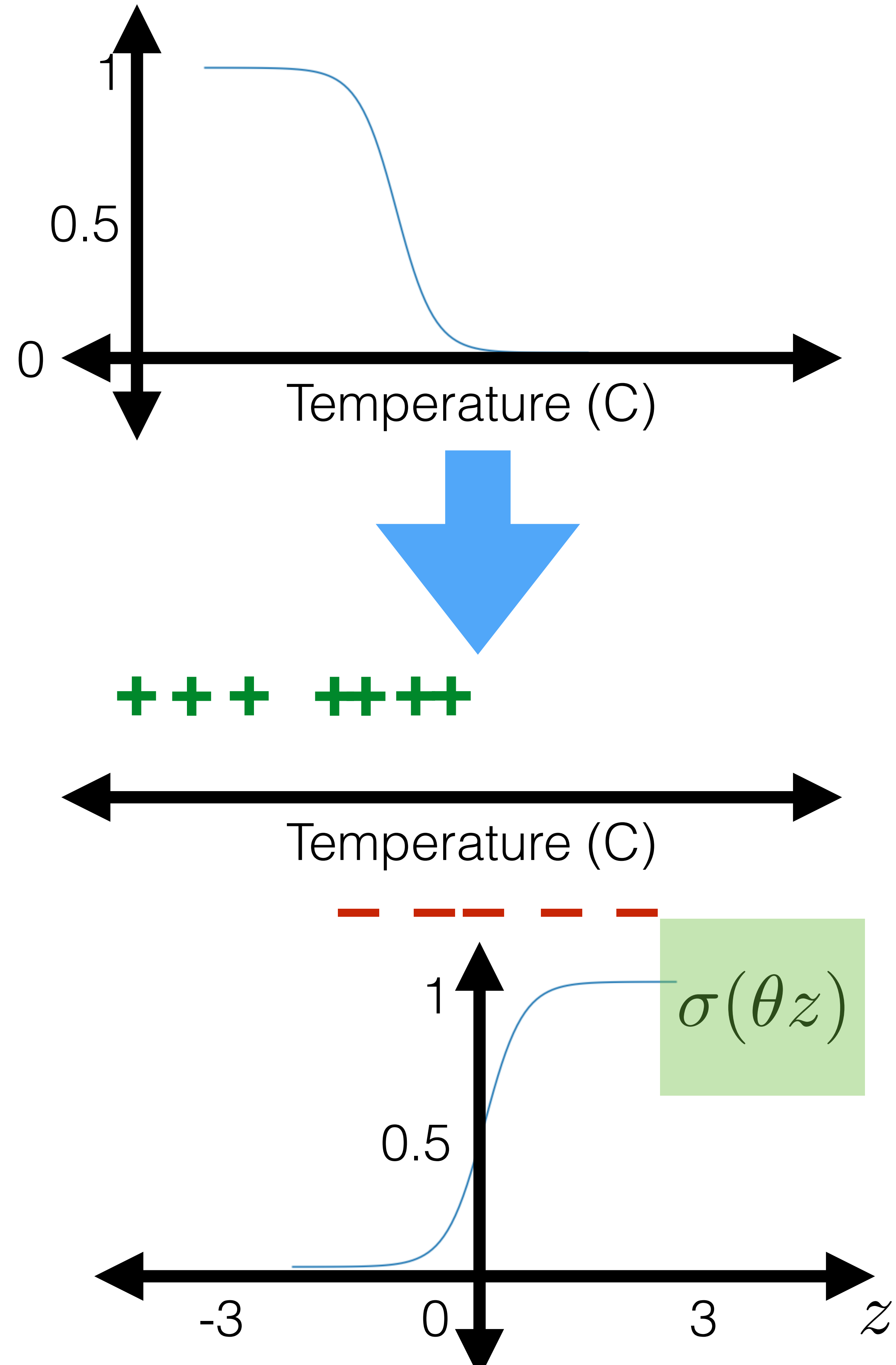

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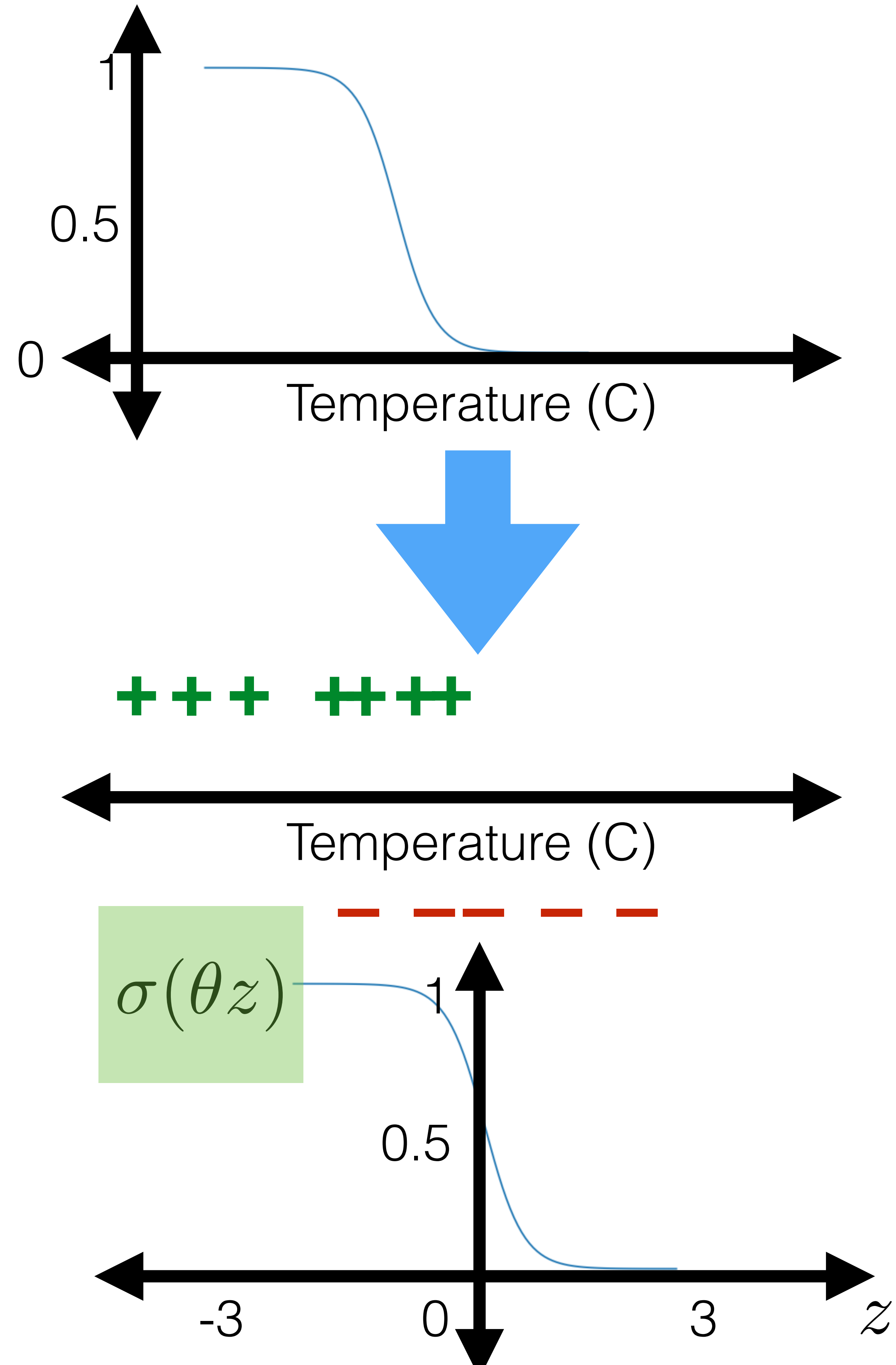

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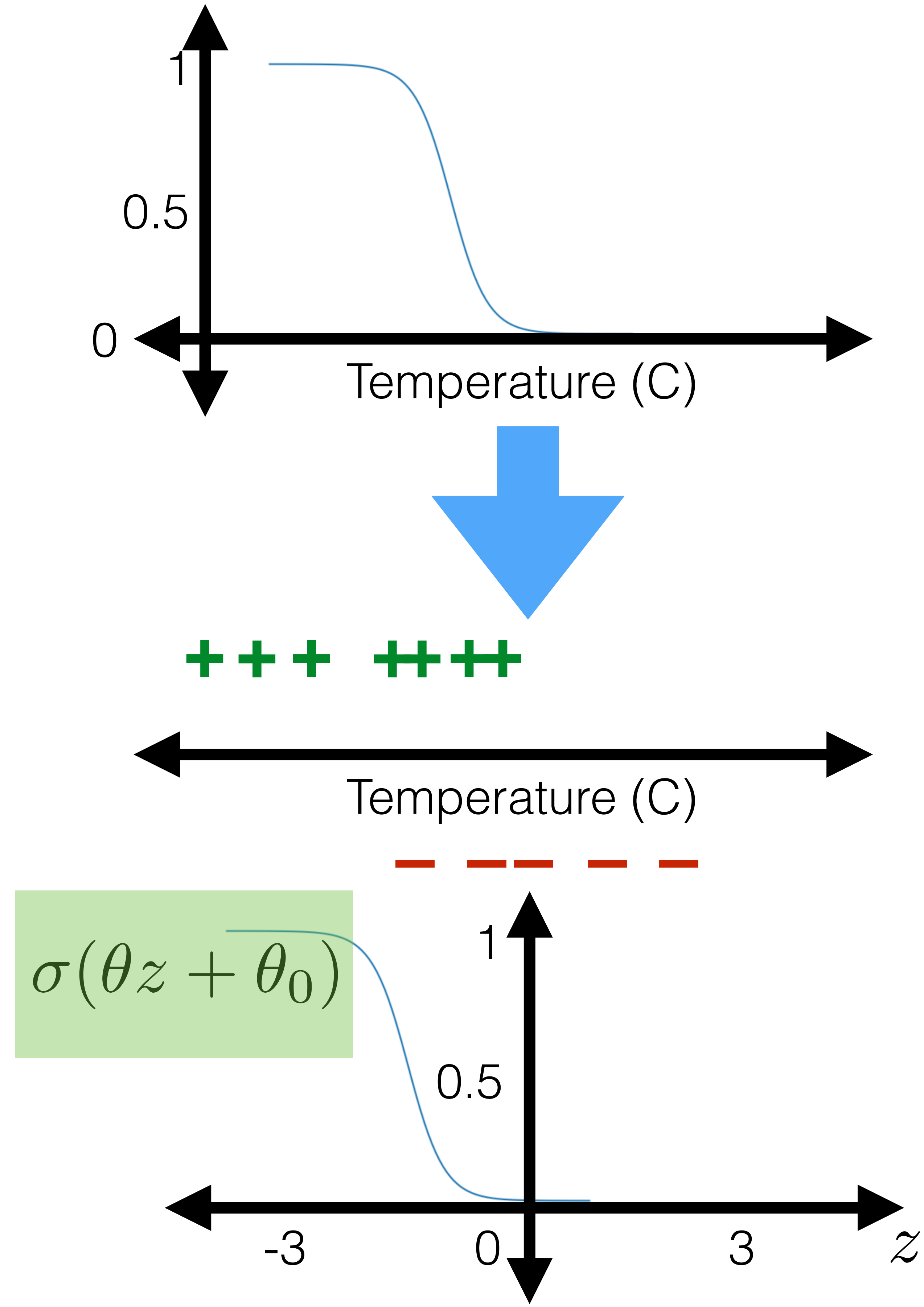

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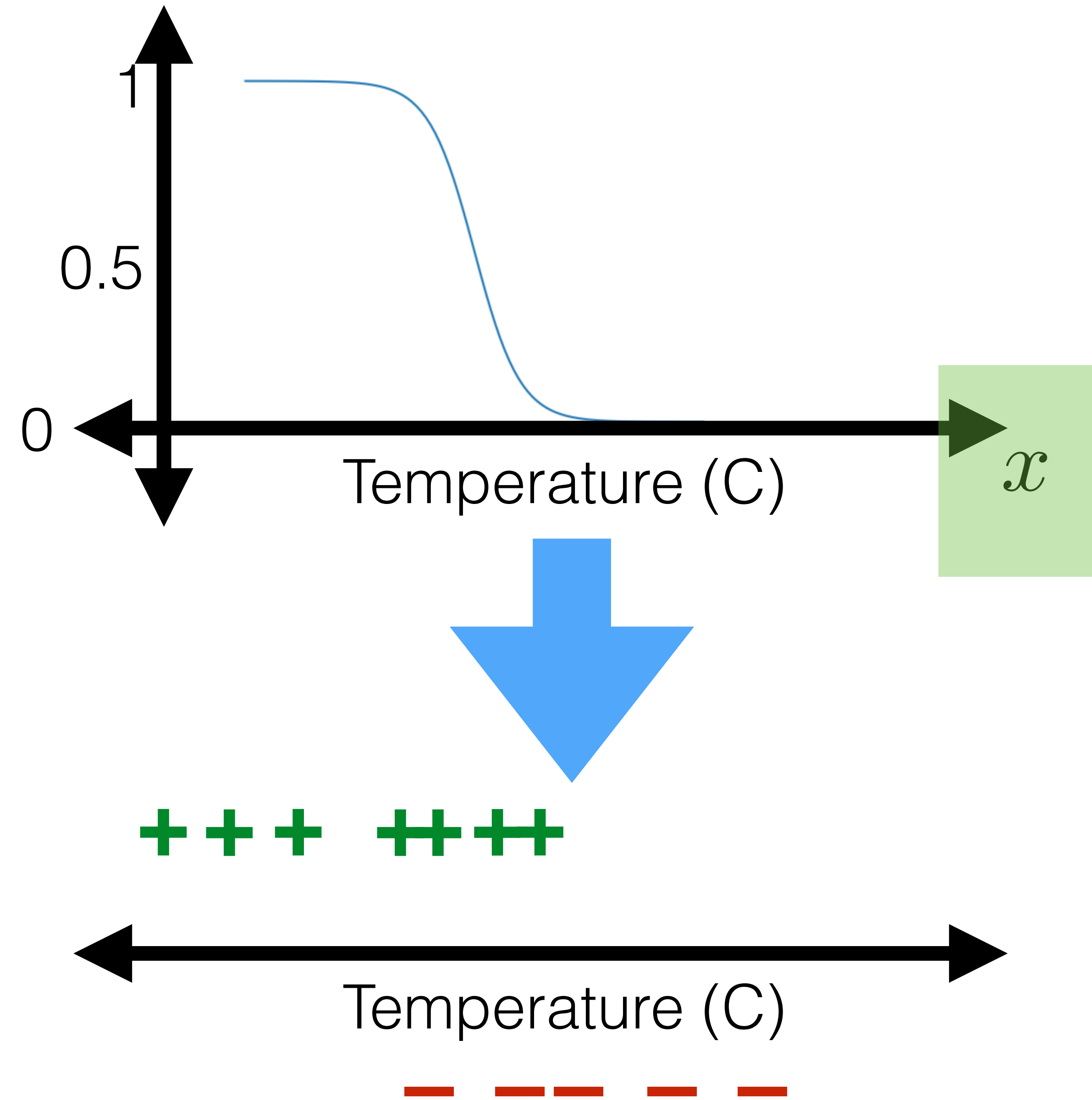

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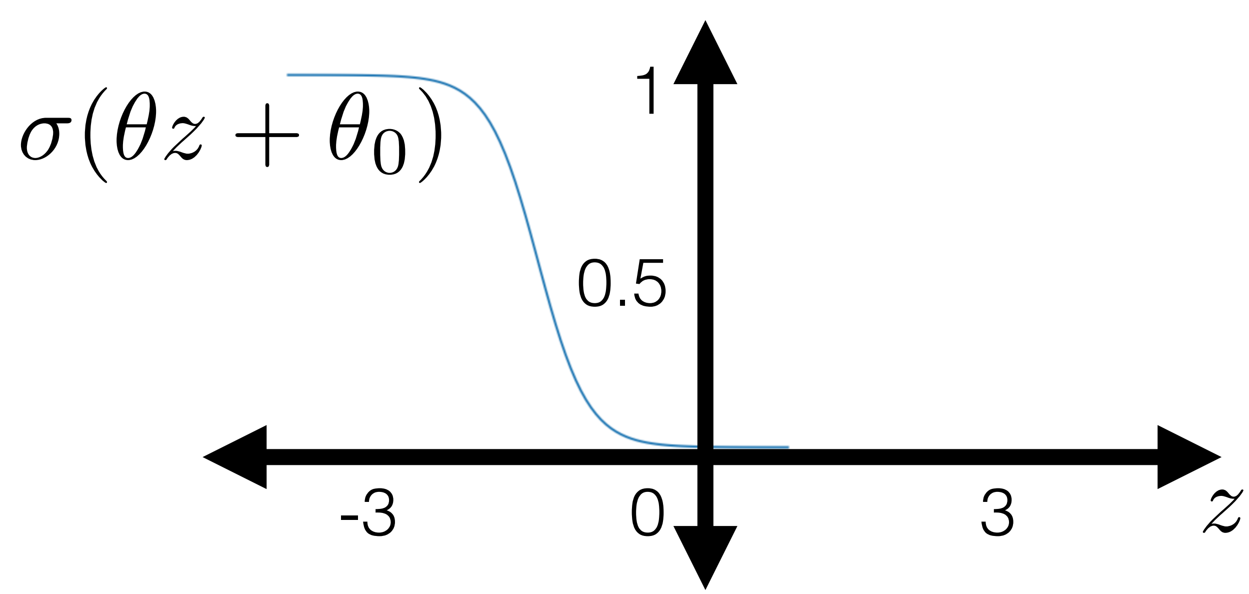

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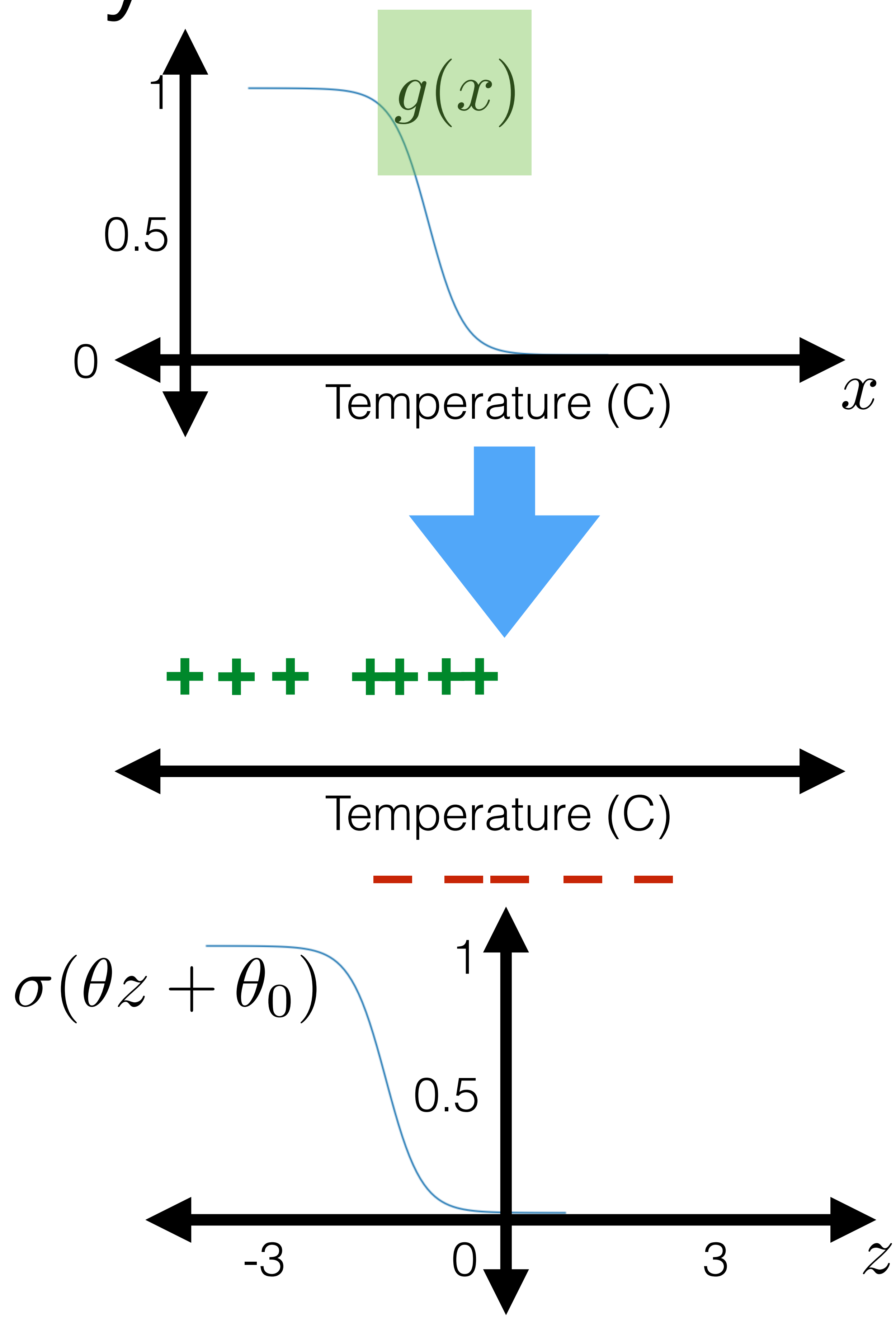

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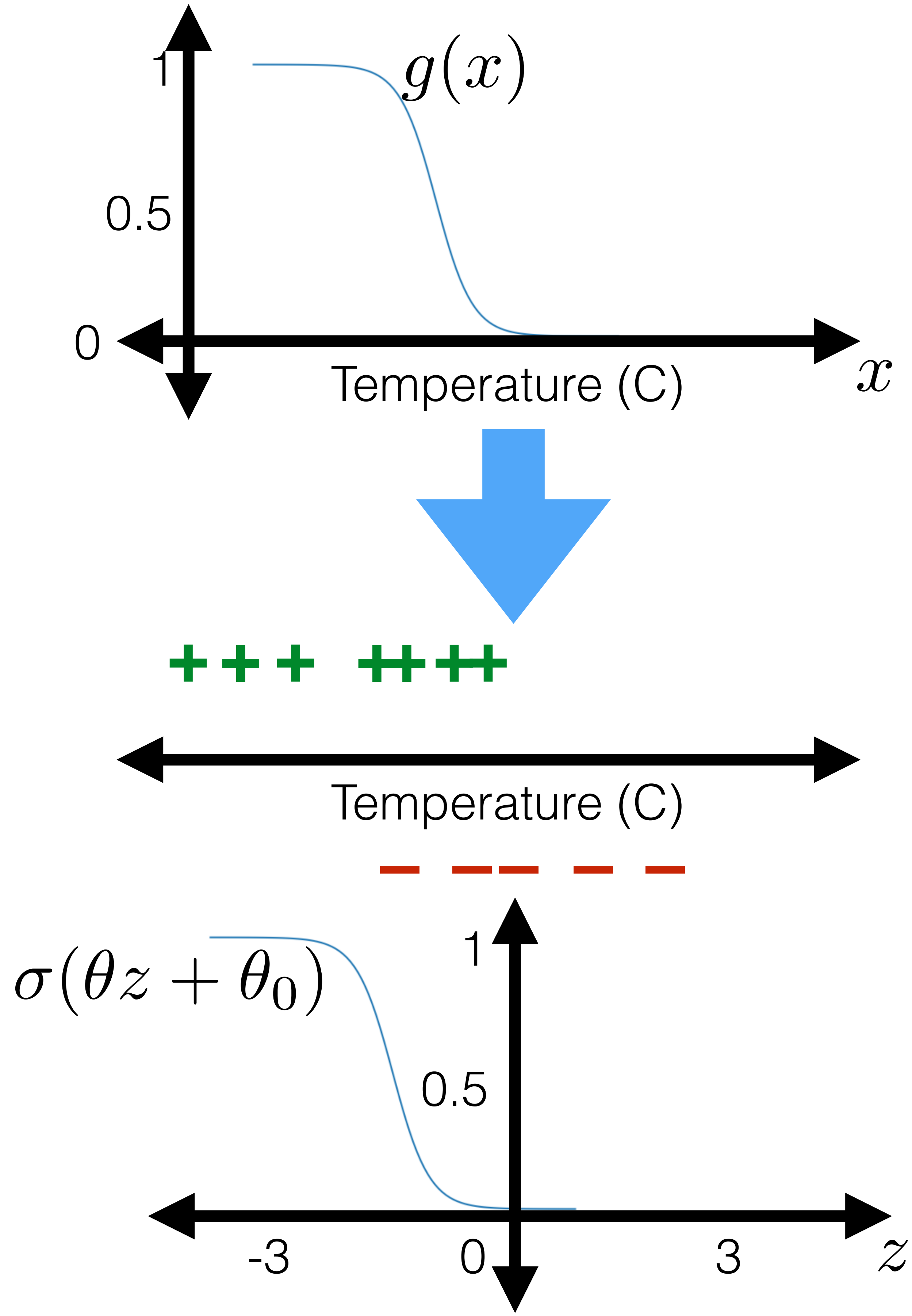
Capturing uncertainty

$$g(x) = \sigma(\theta x + \theta_0)$$



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Capturing uncertainty

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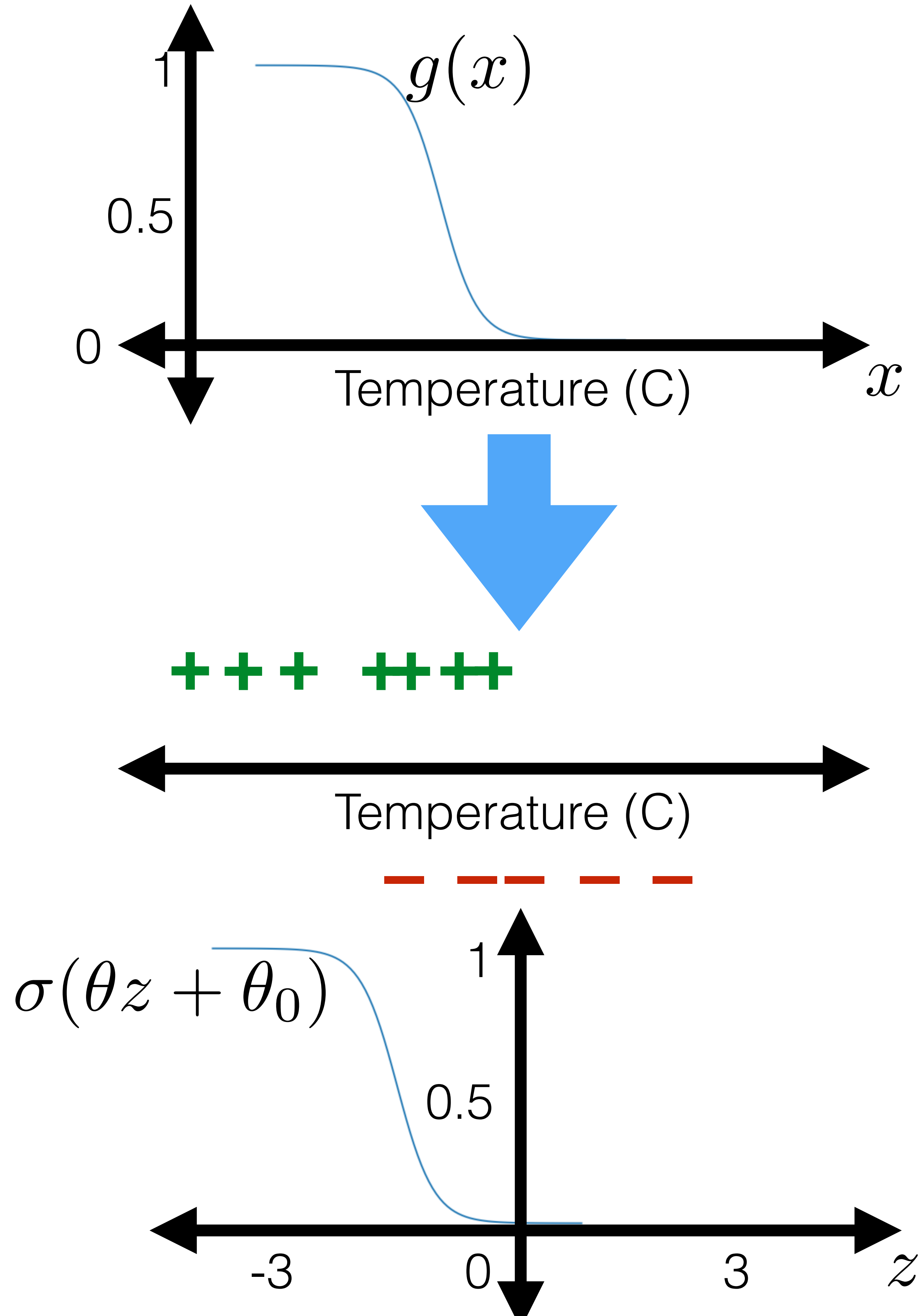

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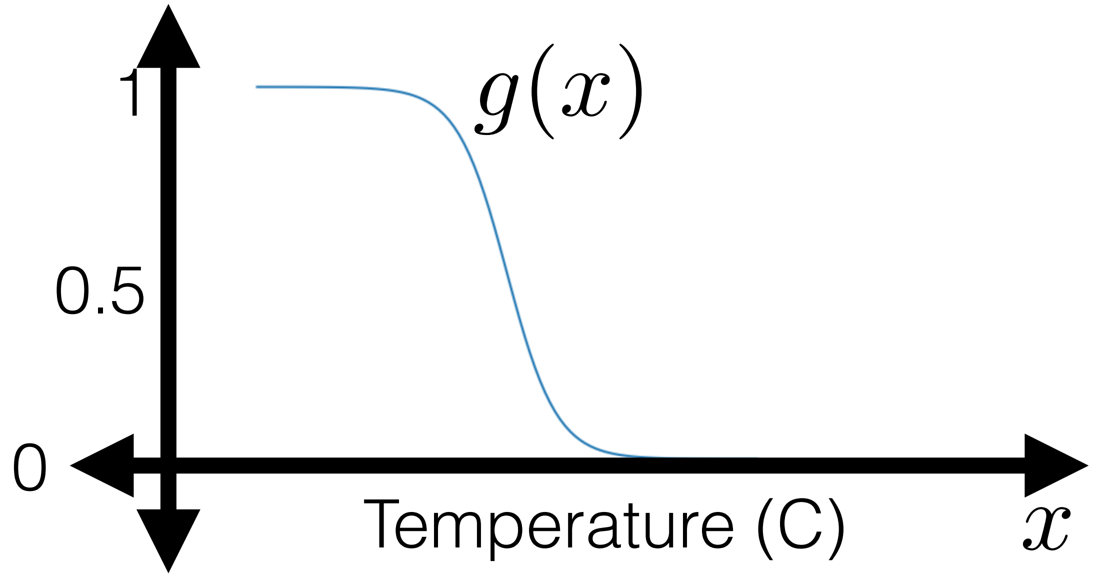

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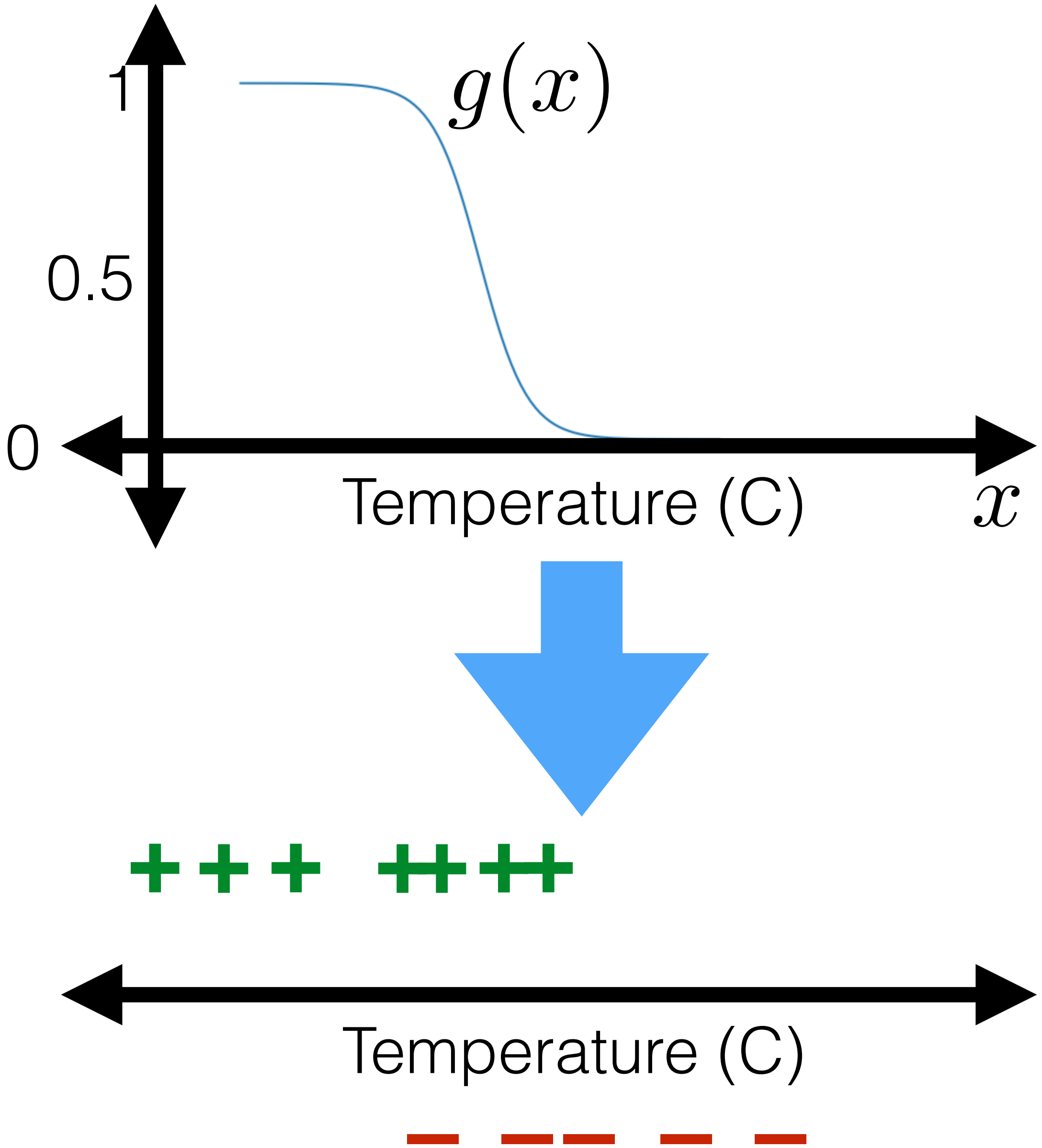

Capturing uncertainty

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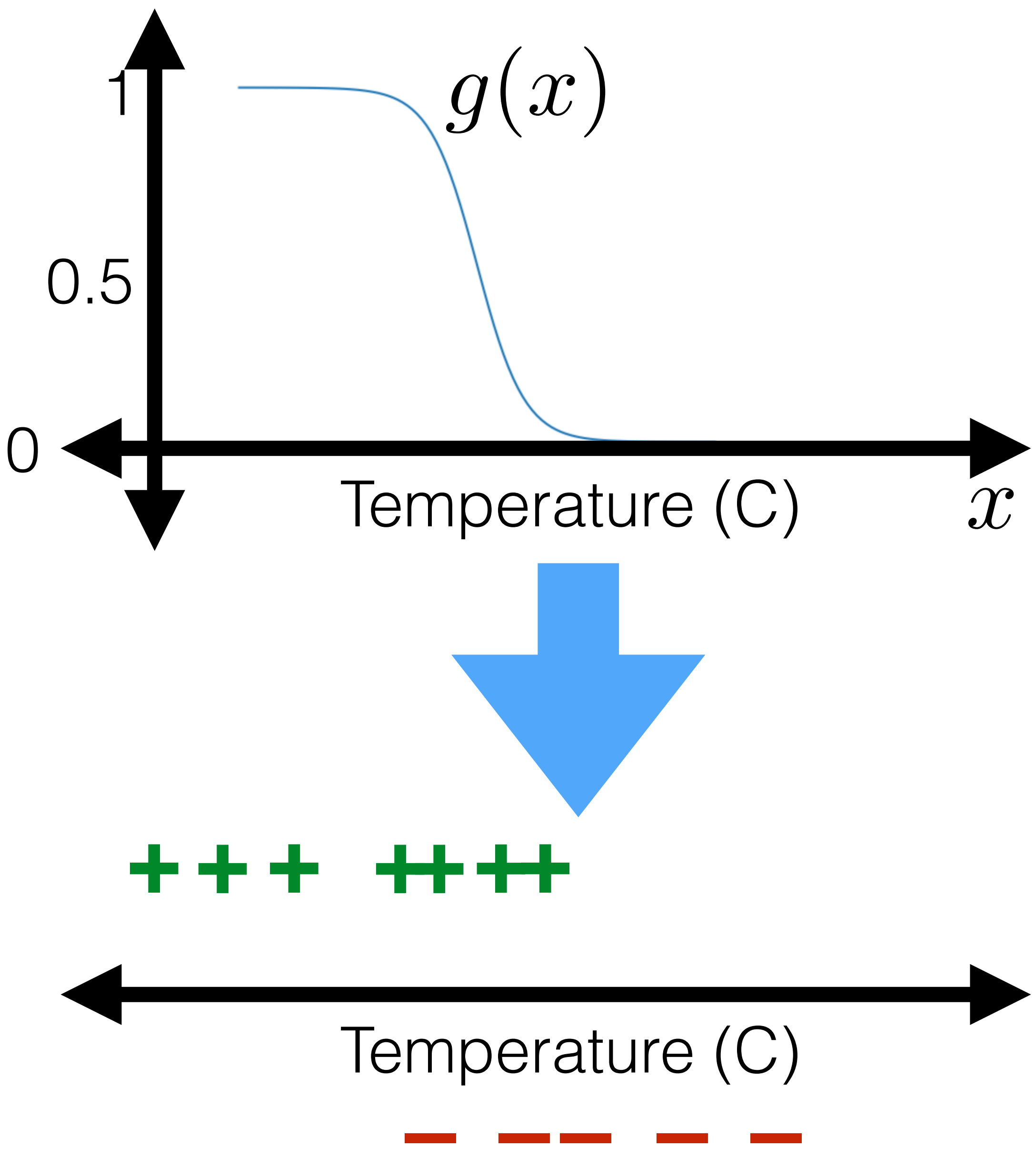

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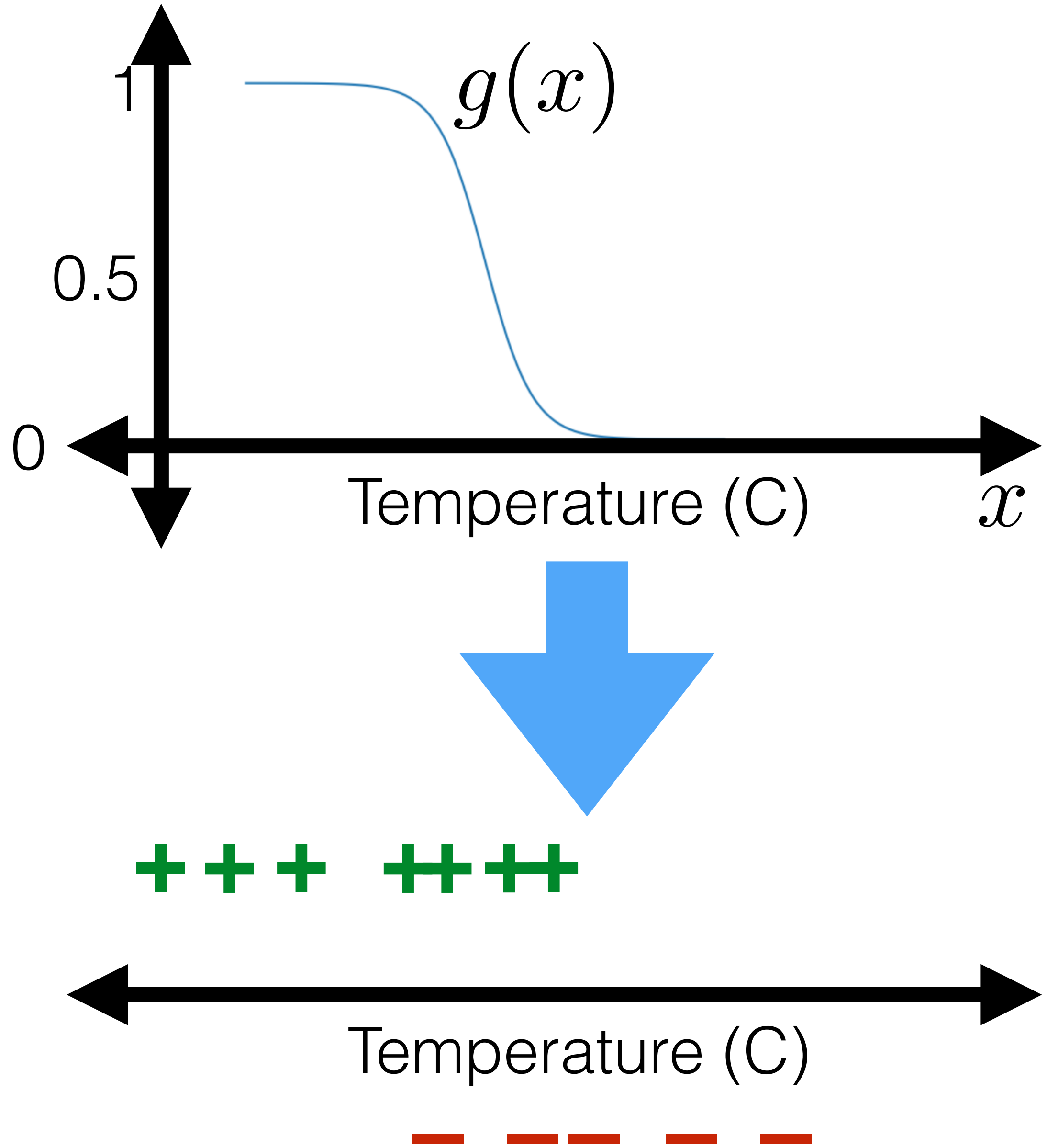

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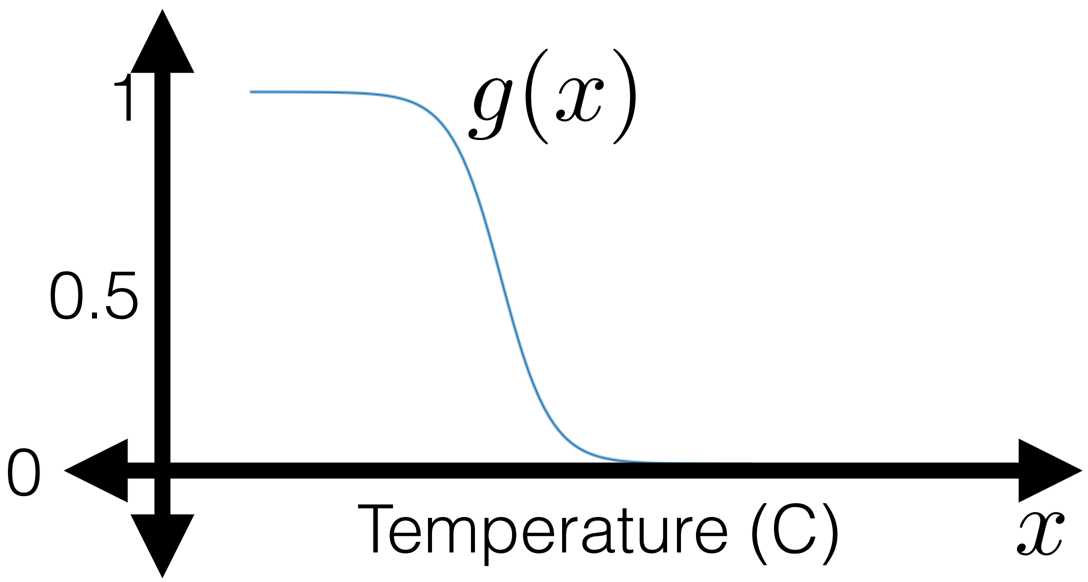

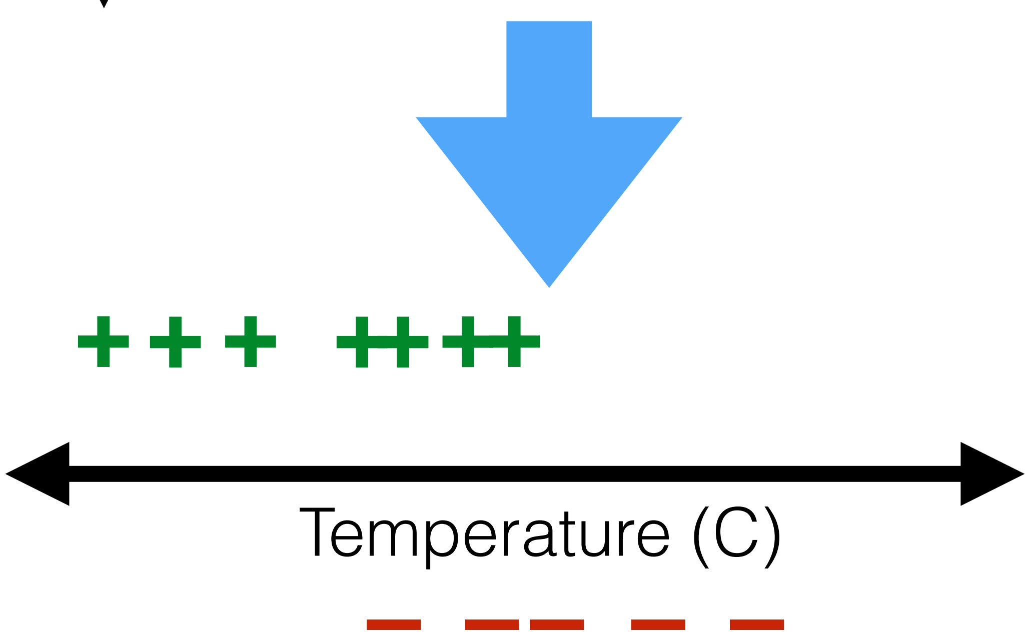

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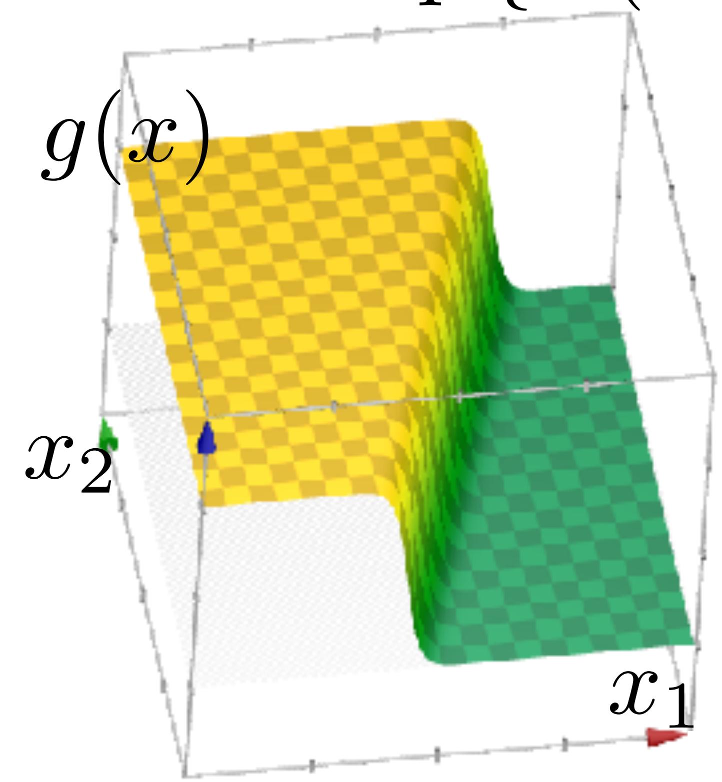

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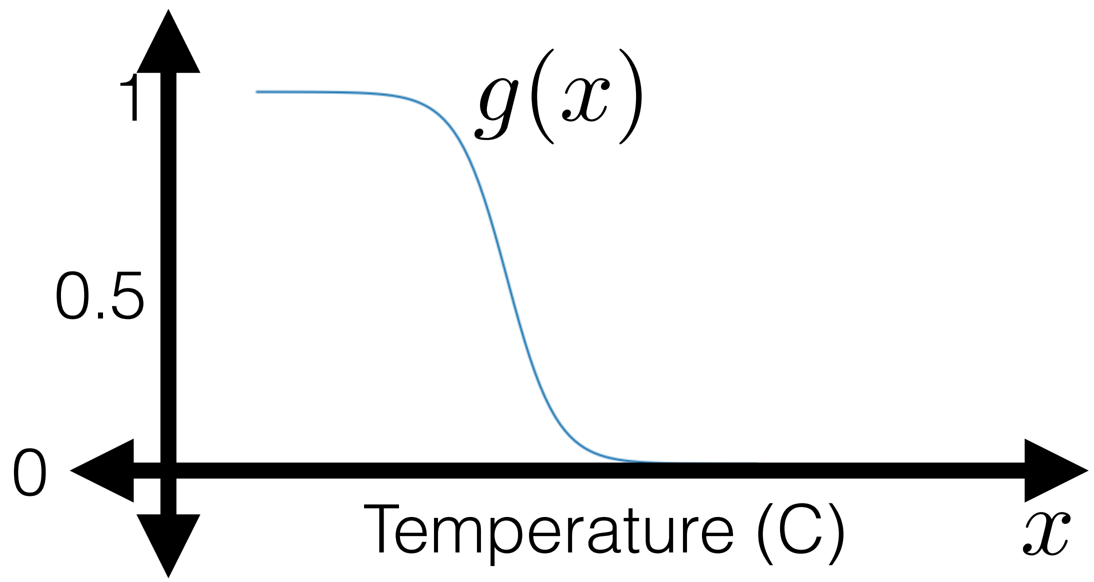

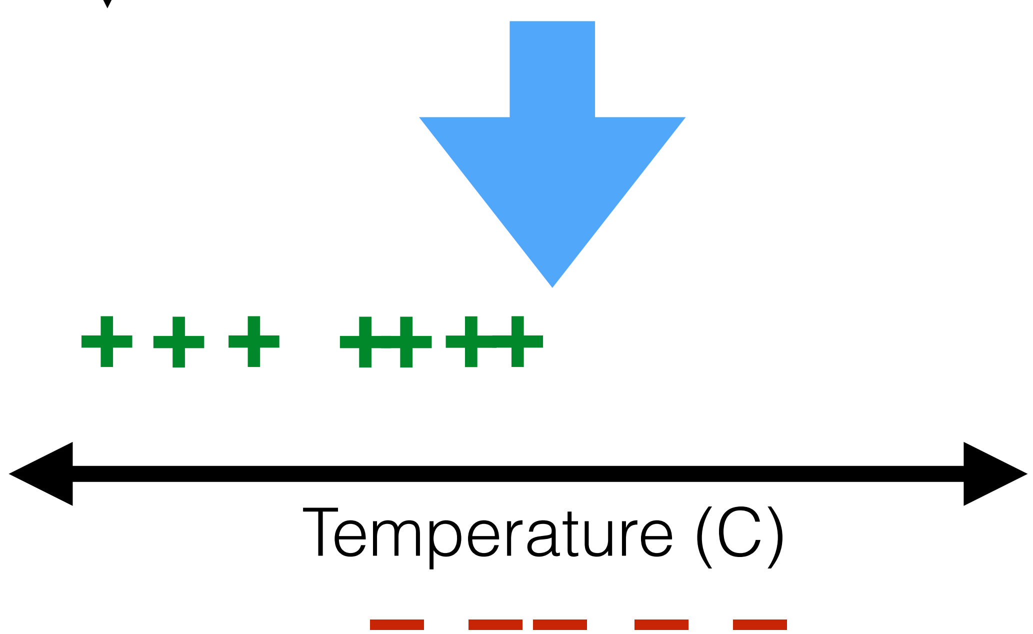

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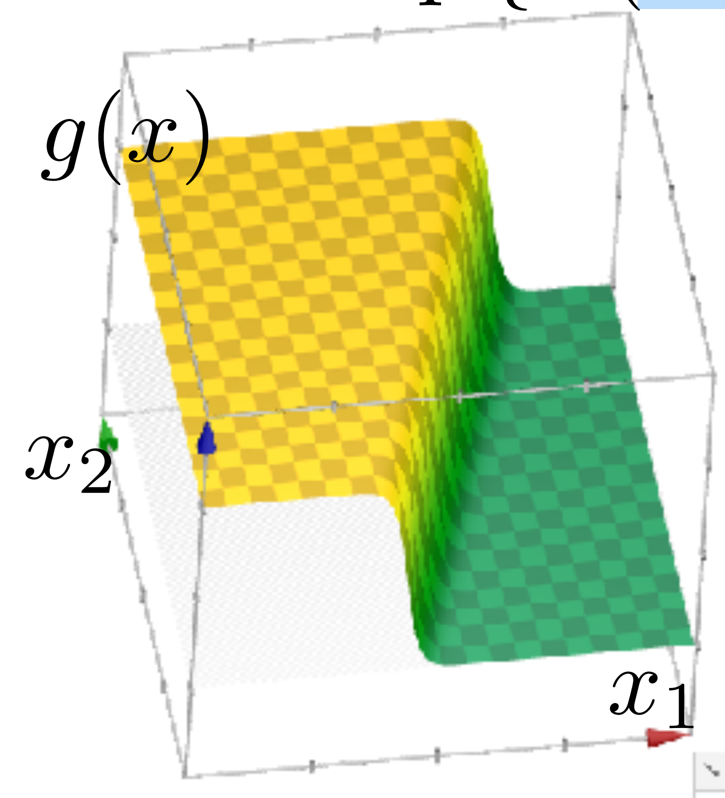

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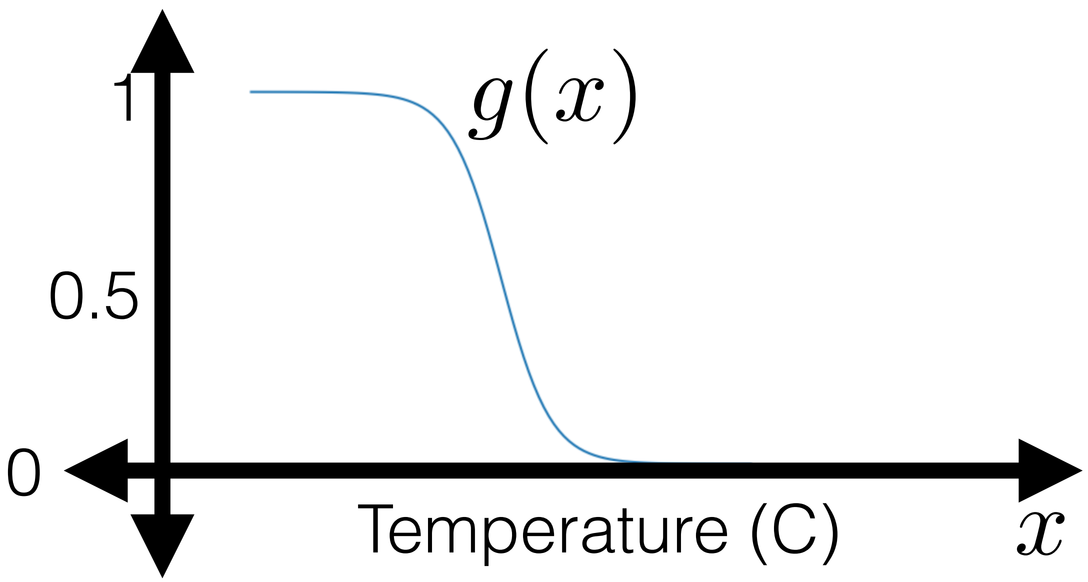

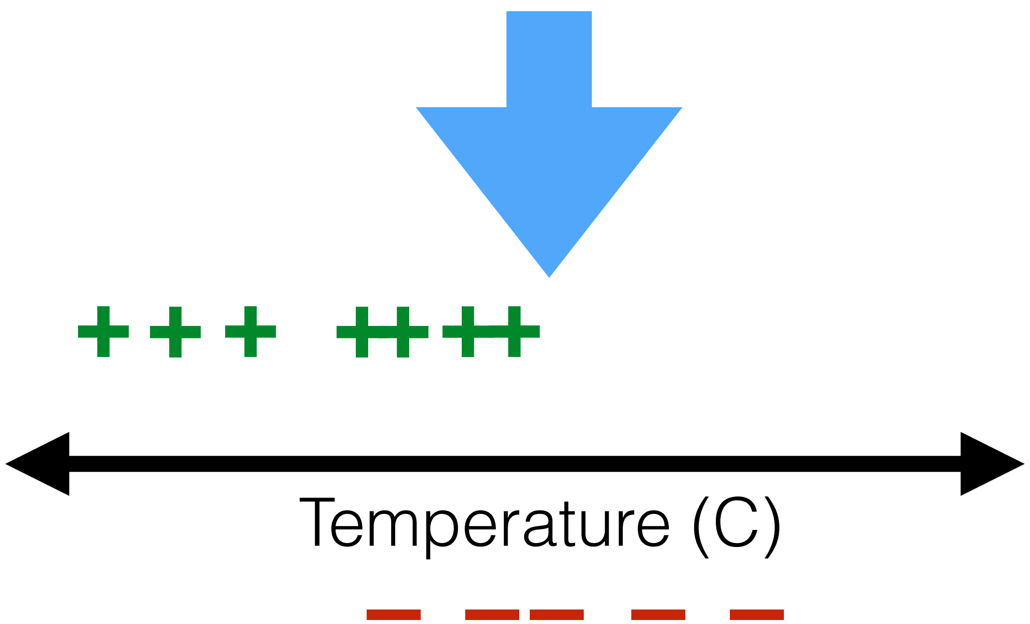

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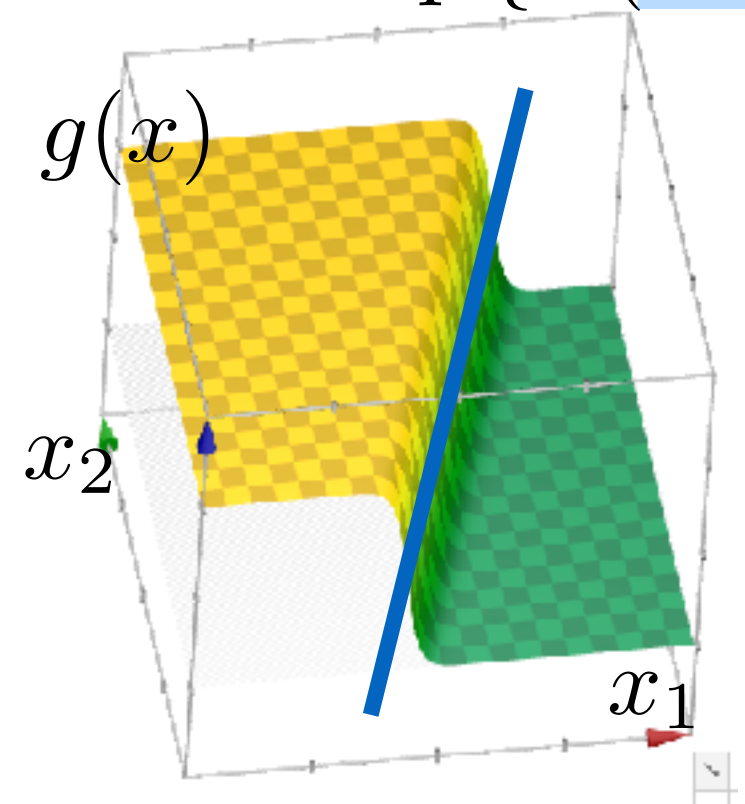

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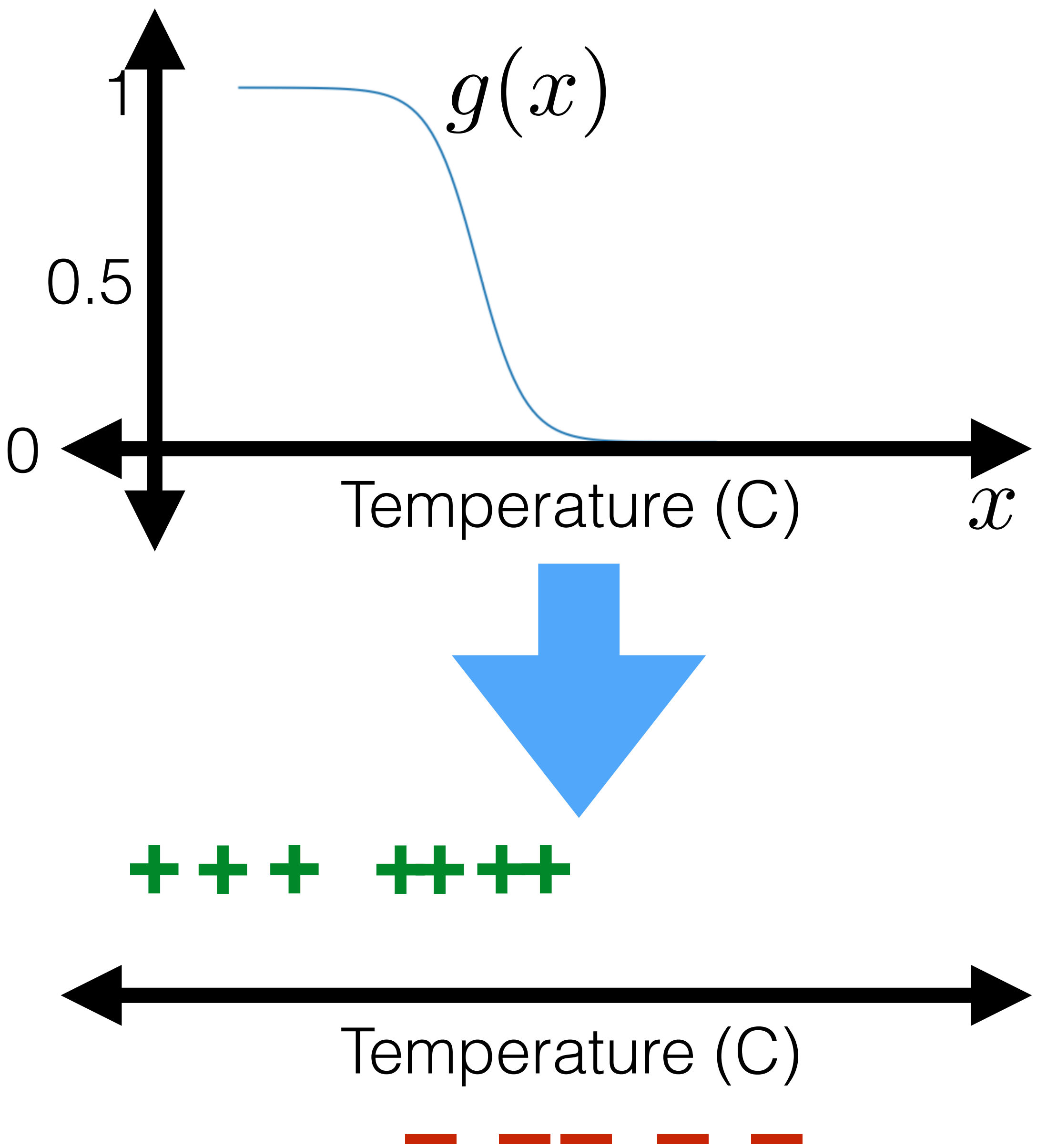

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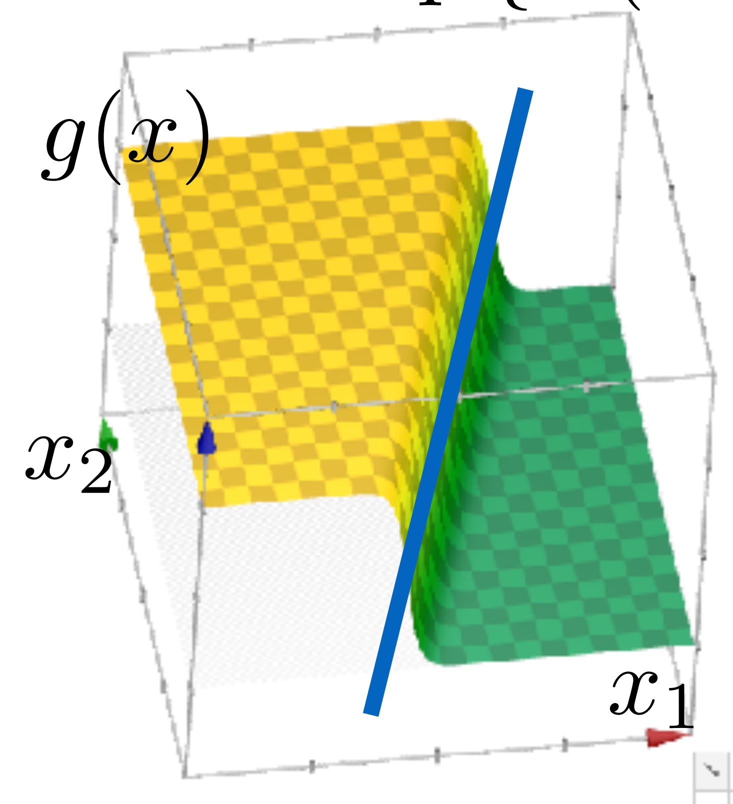

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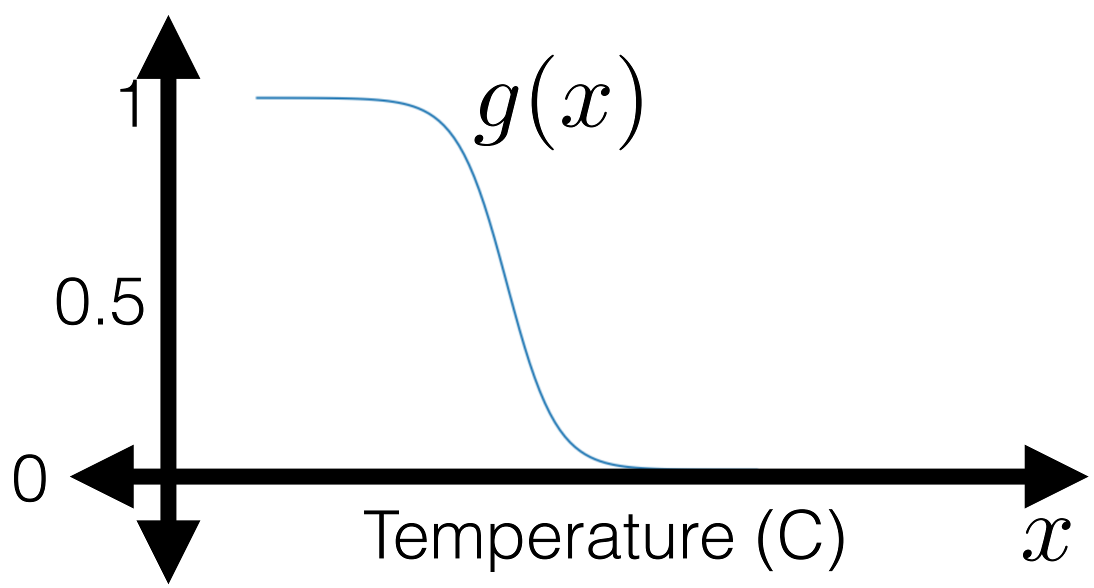

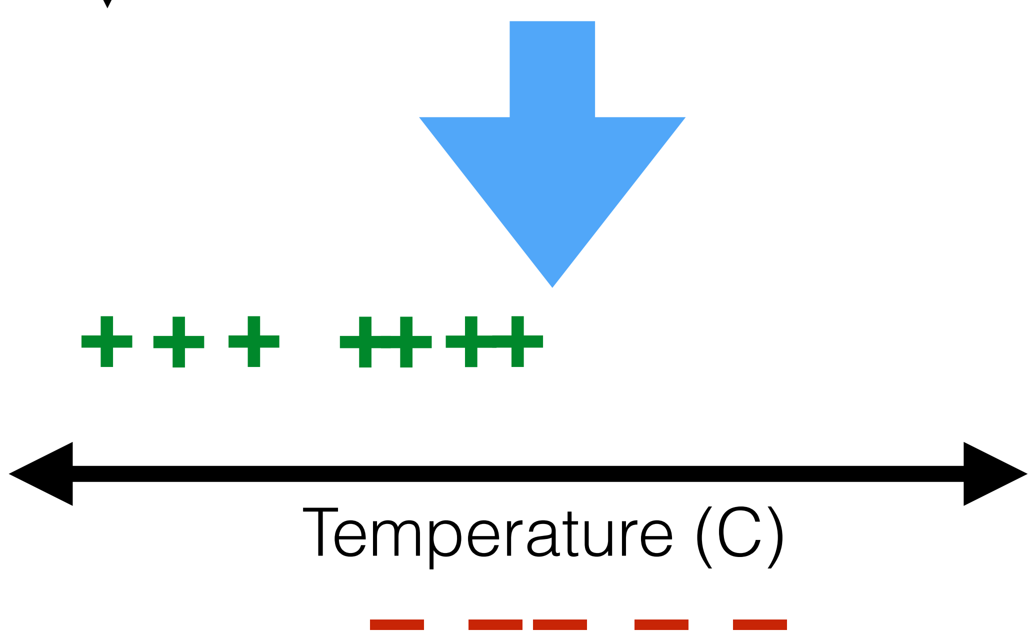

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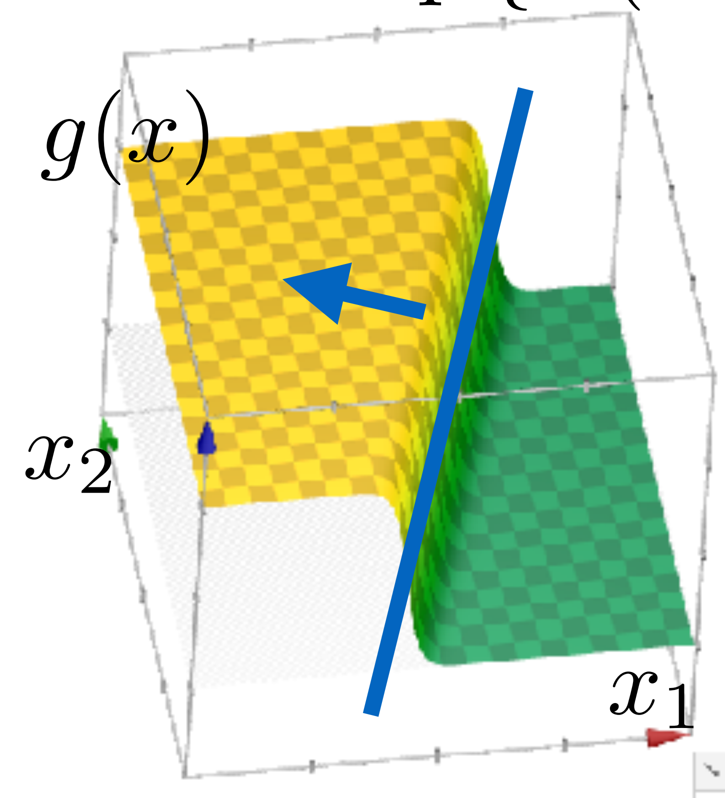

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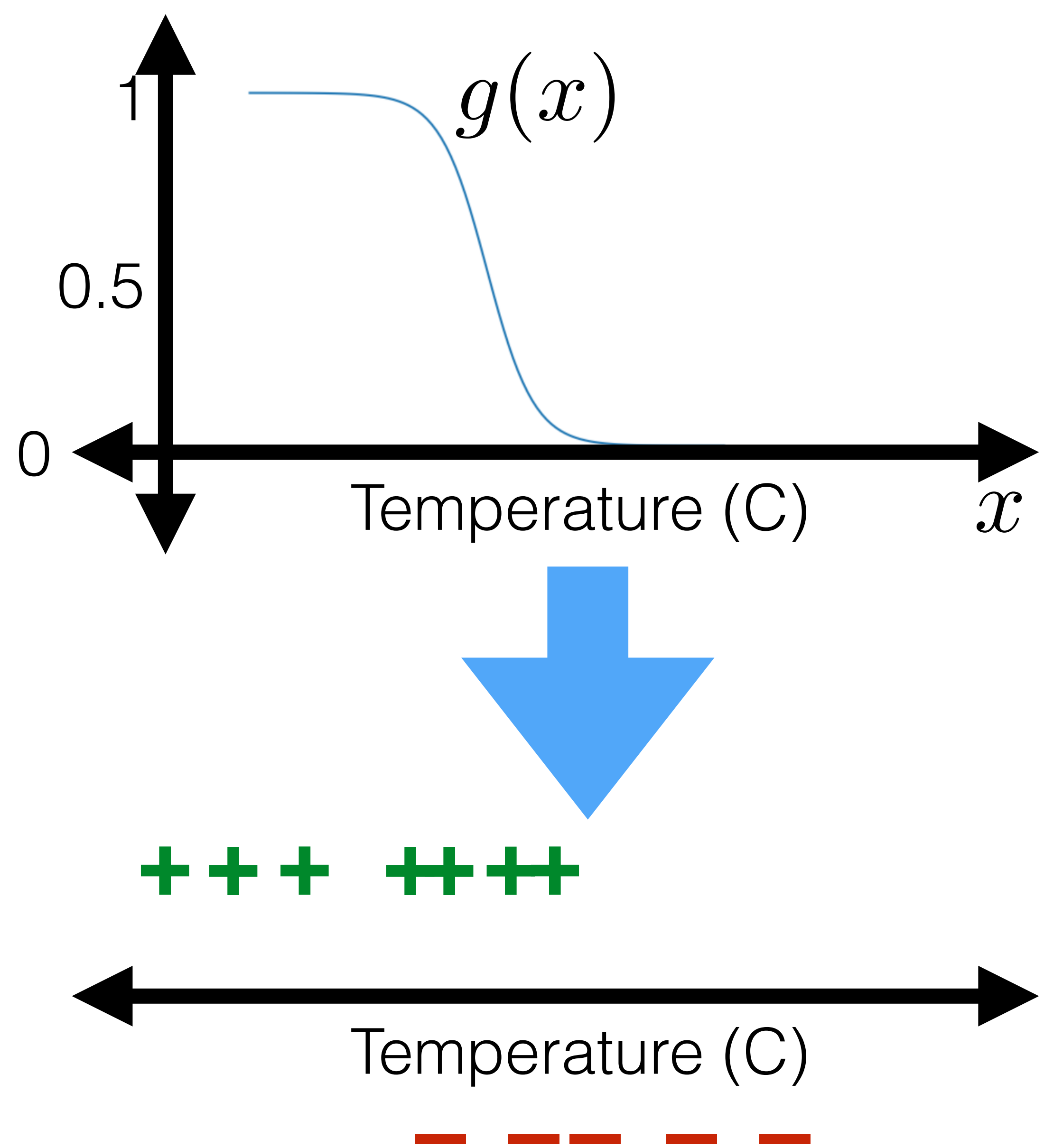

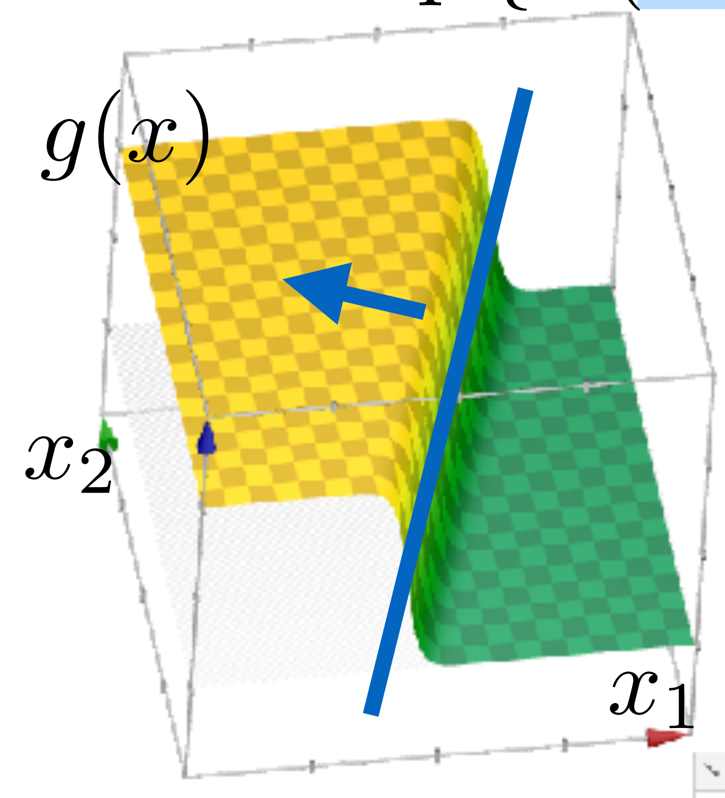

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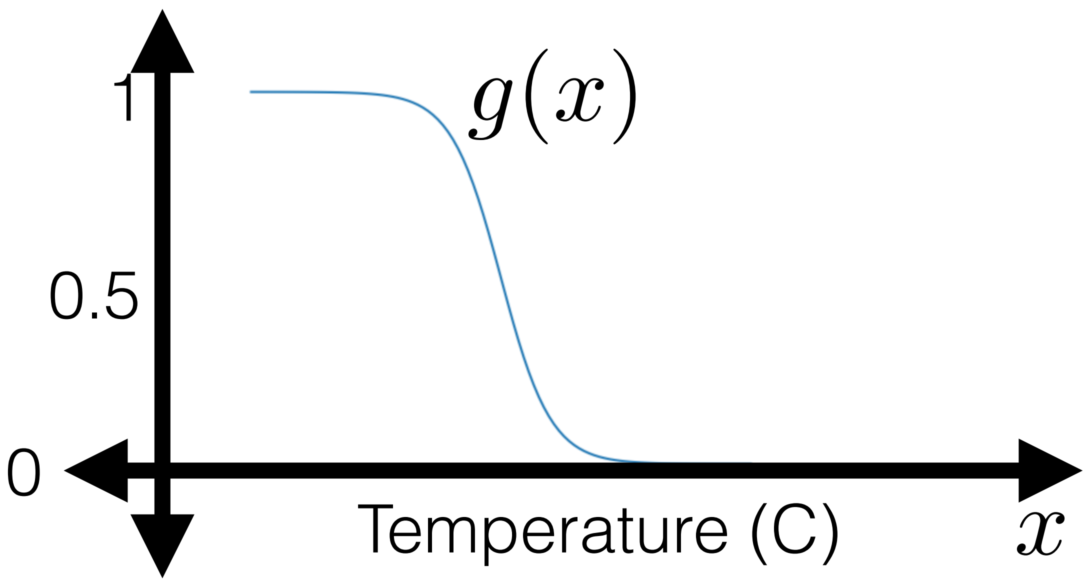

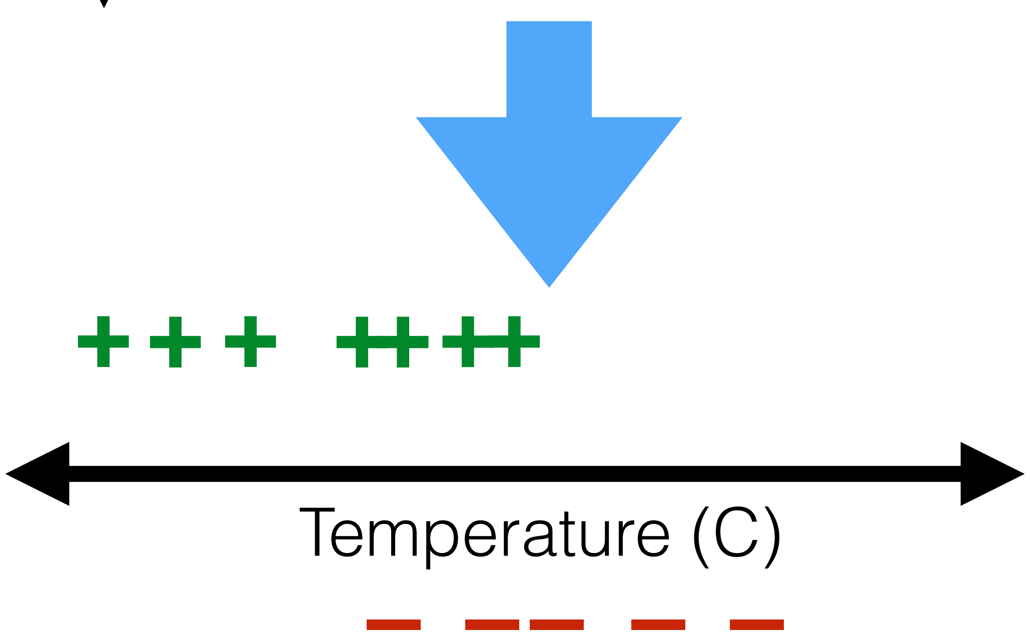

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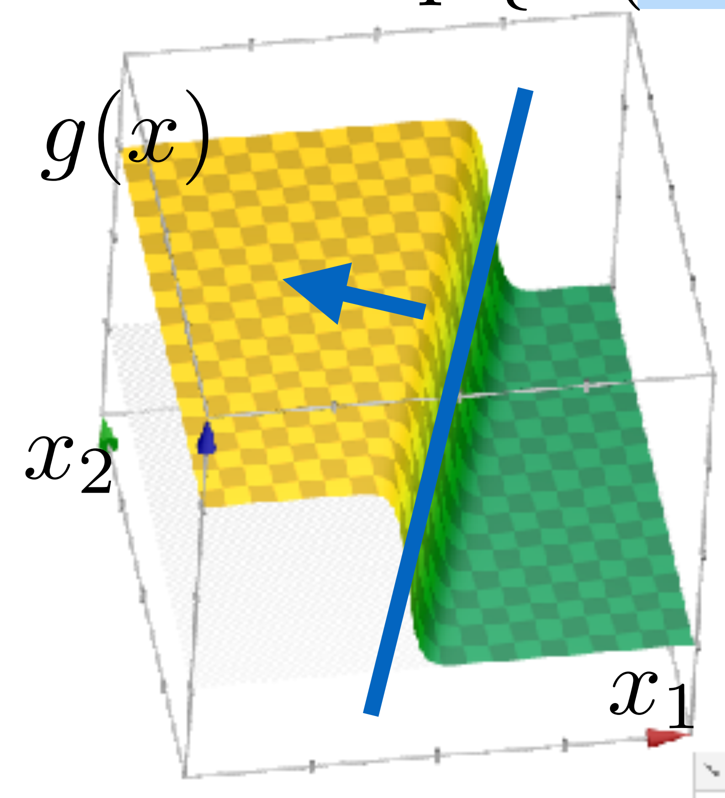

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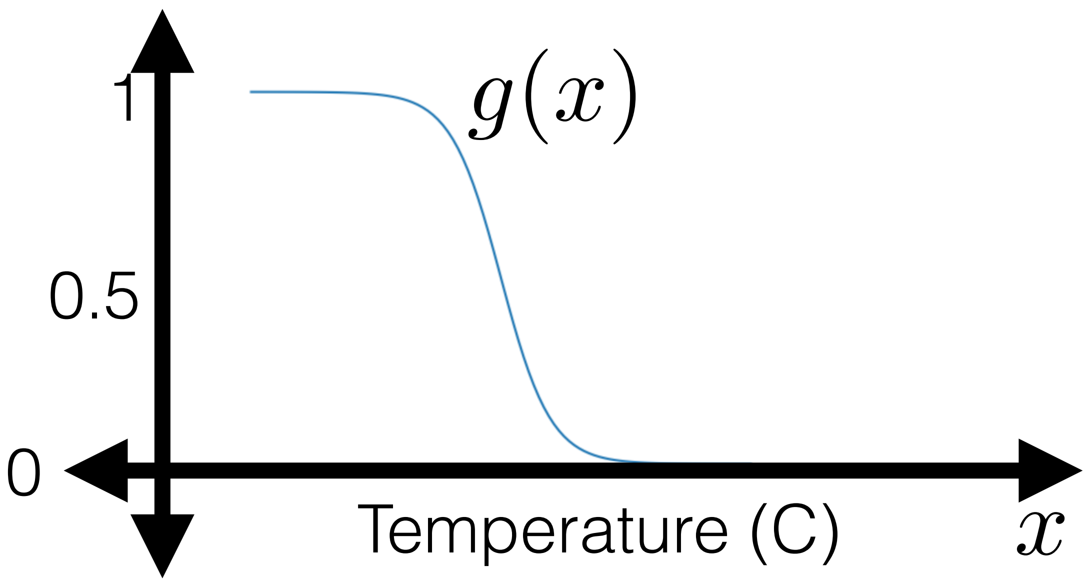

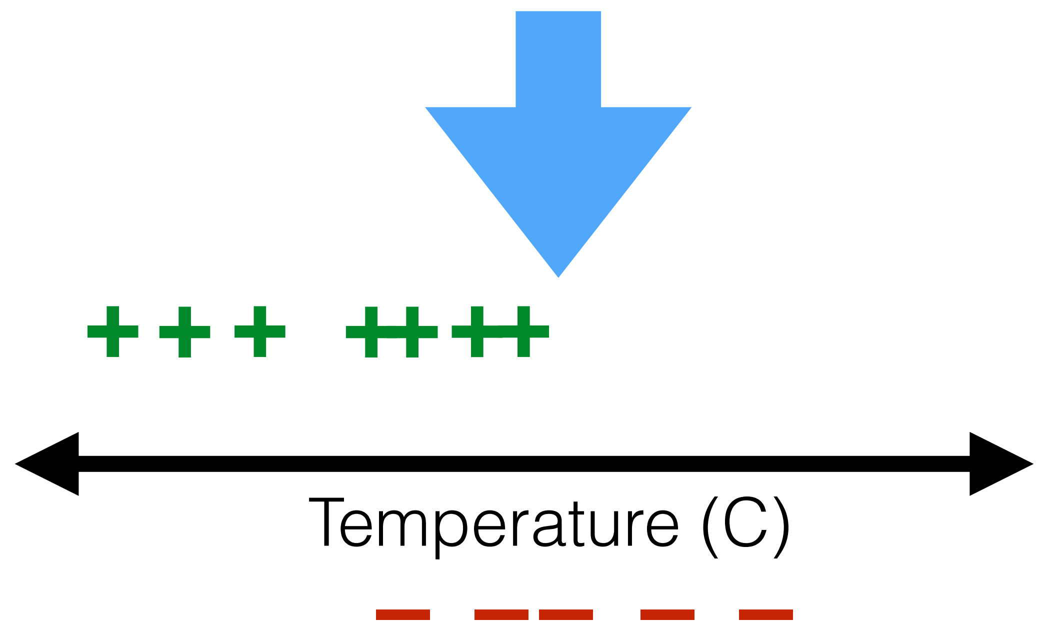

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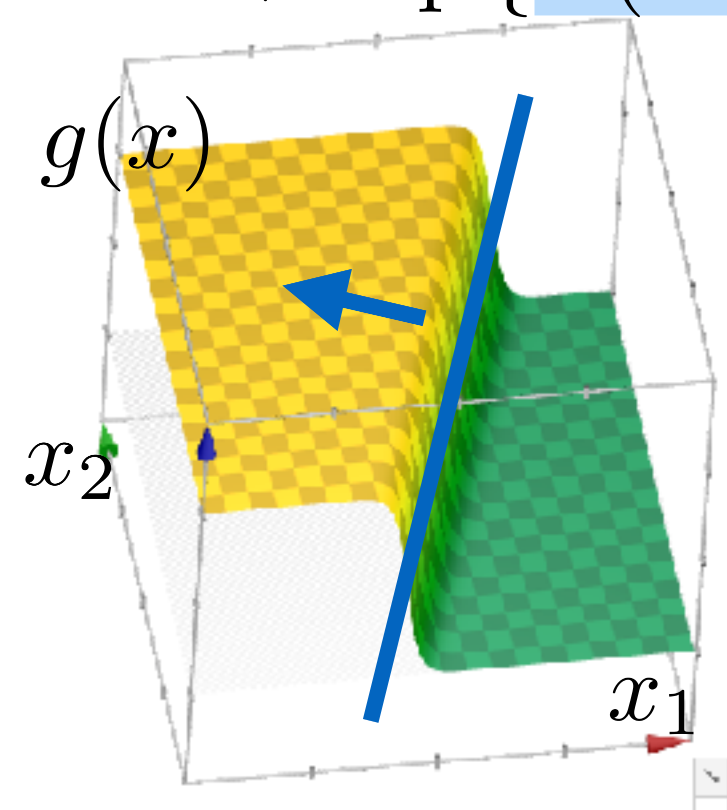

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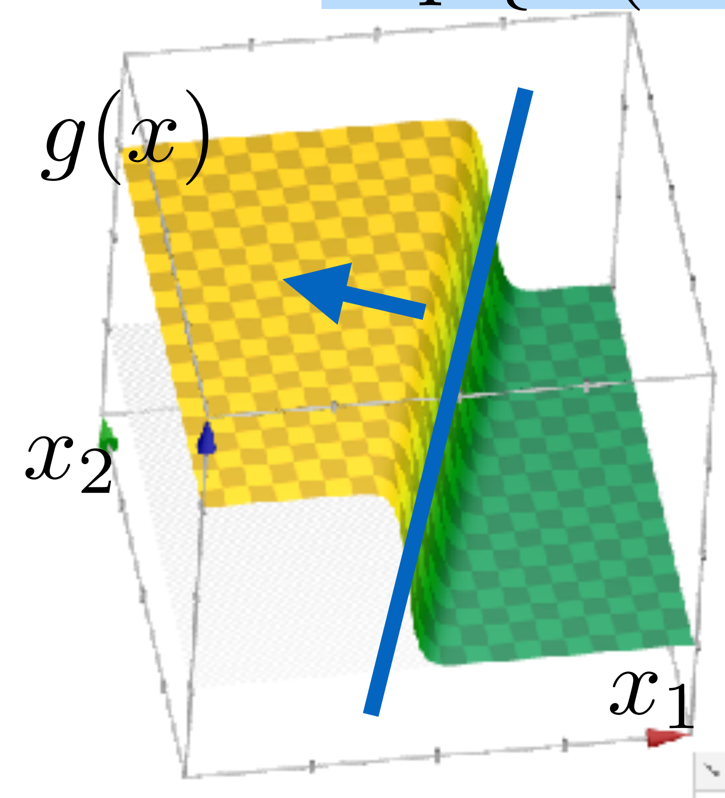

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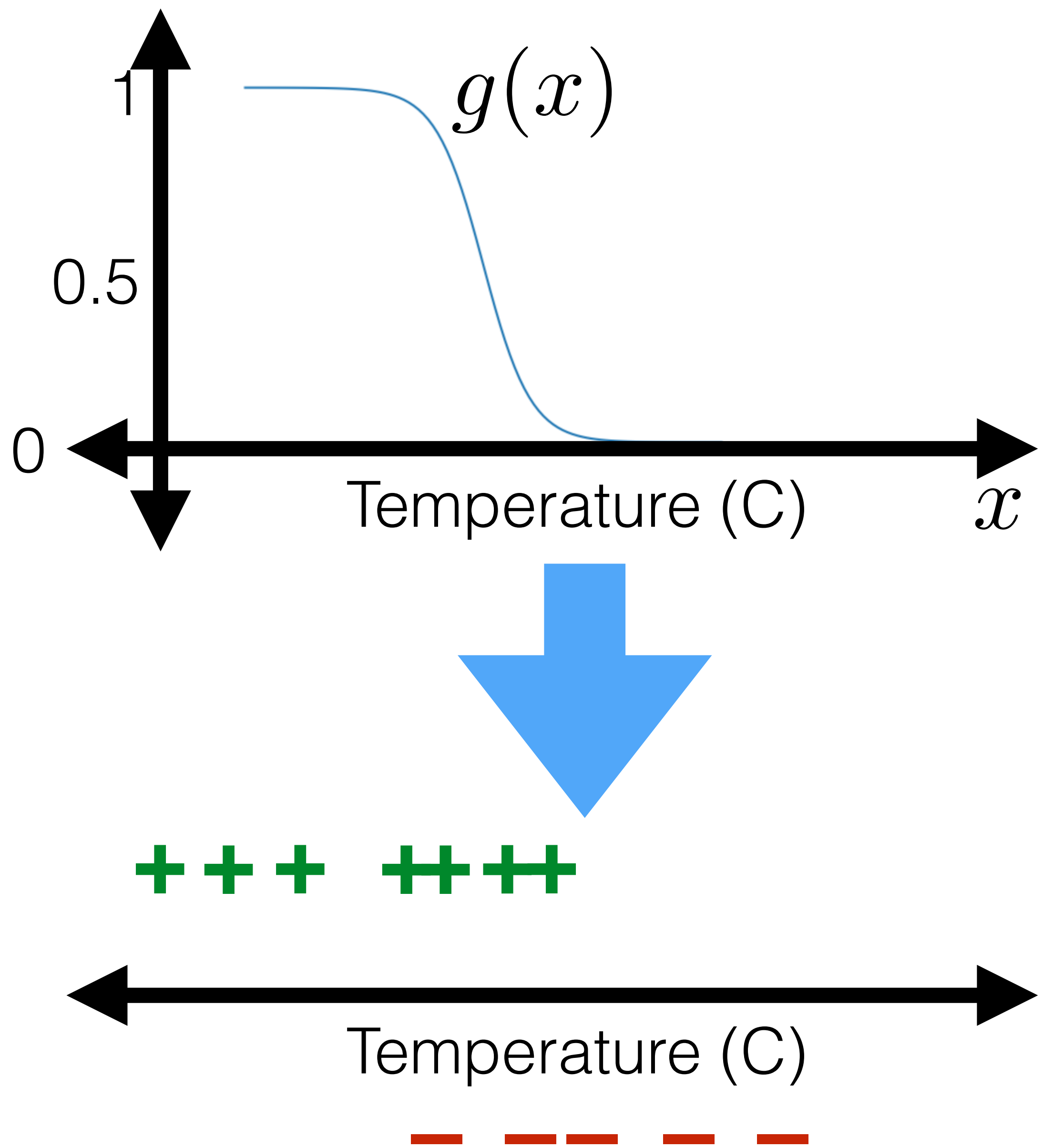

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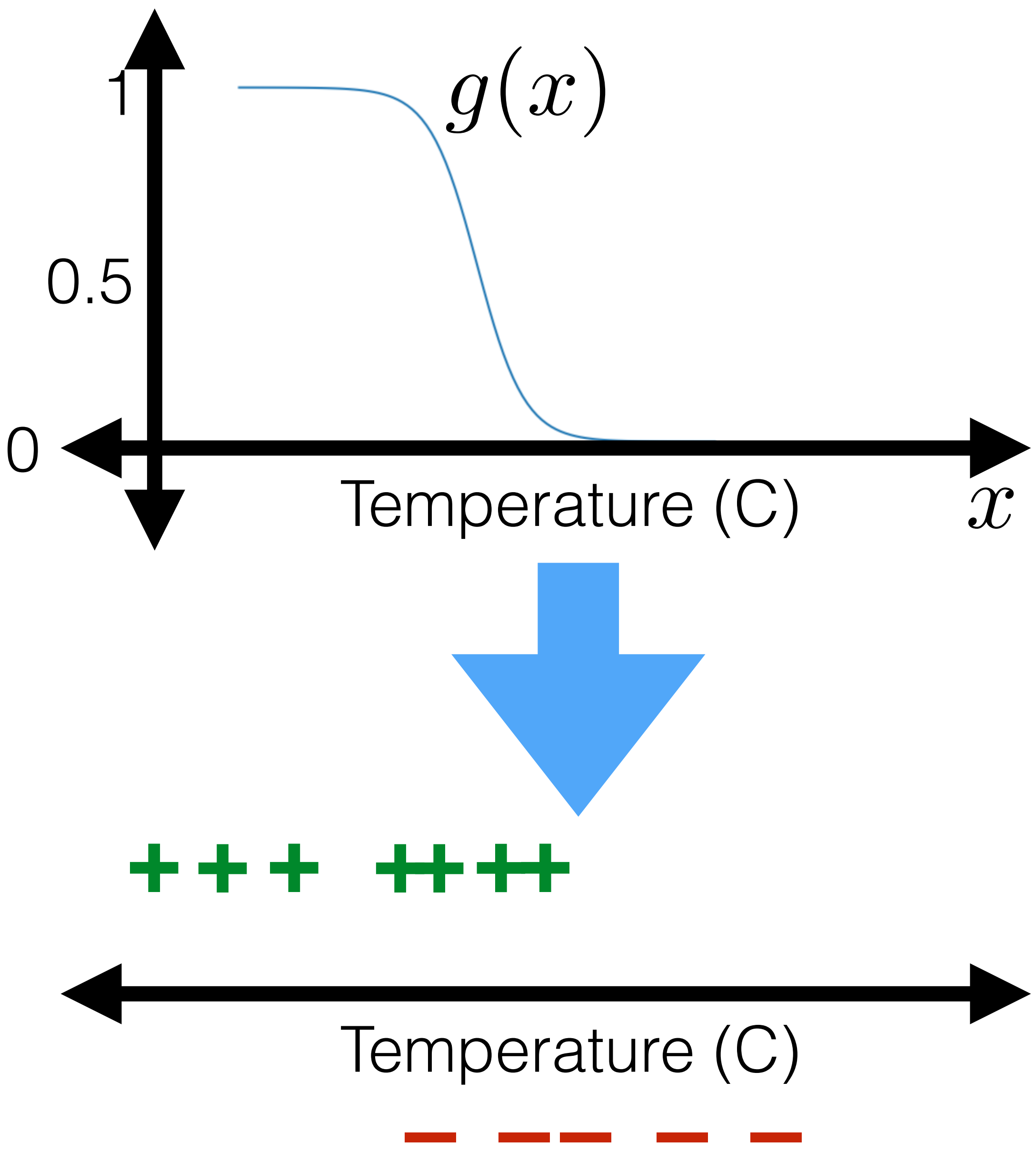

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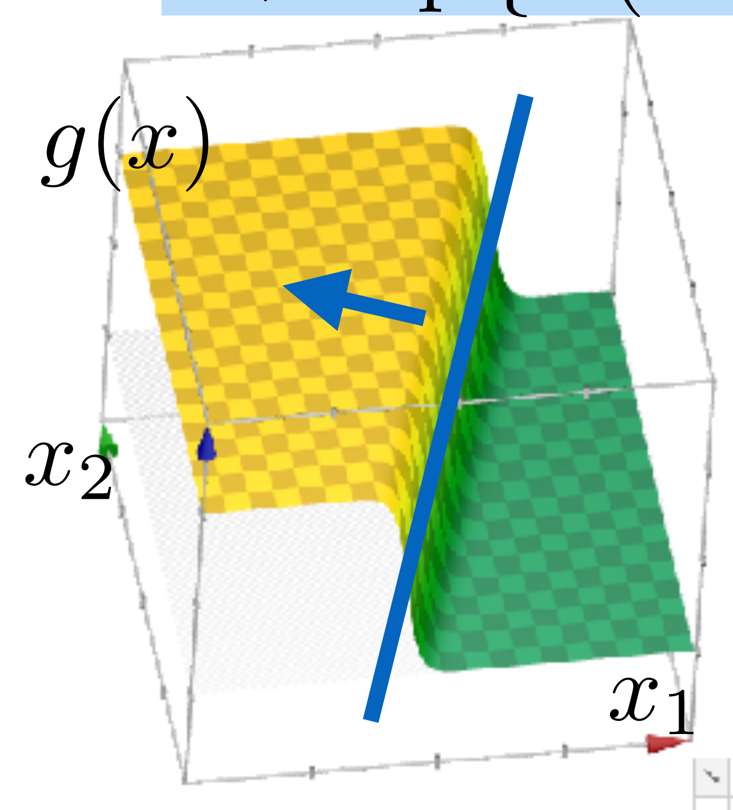

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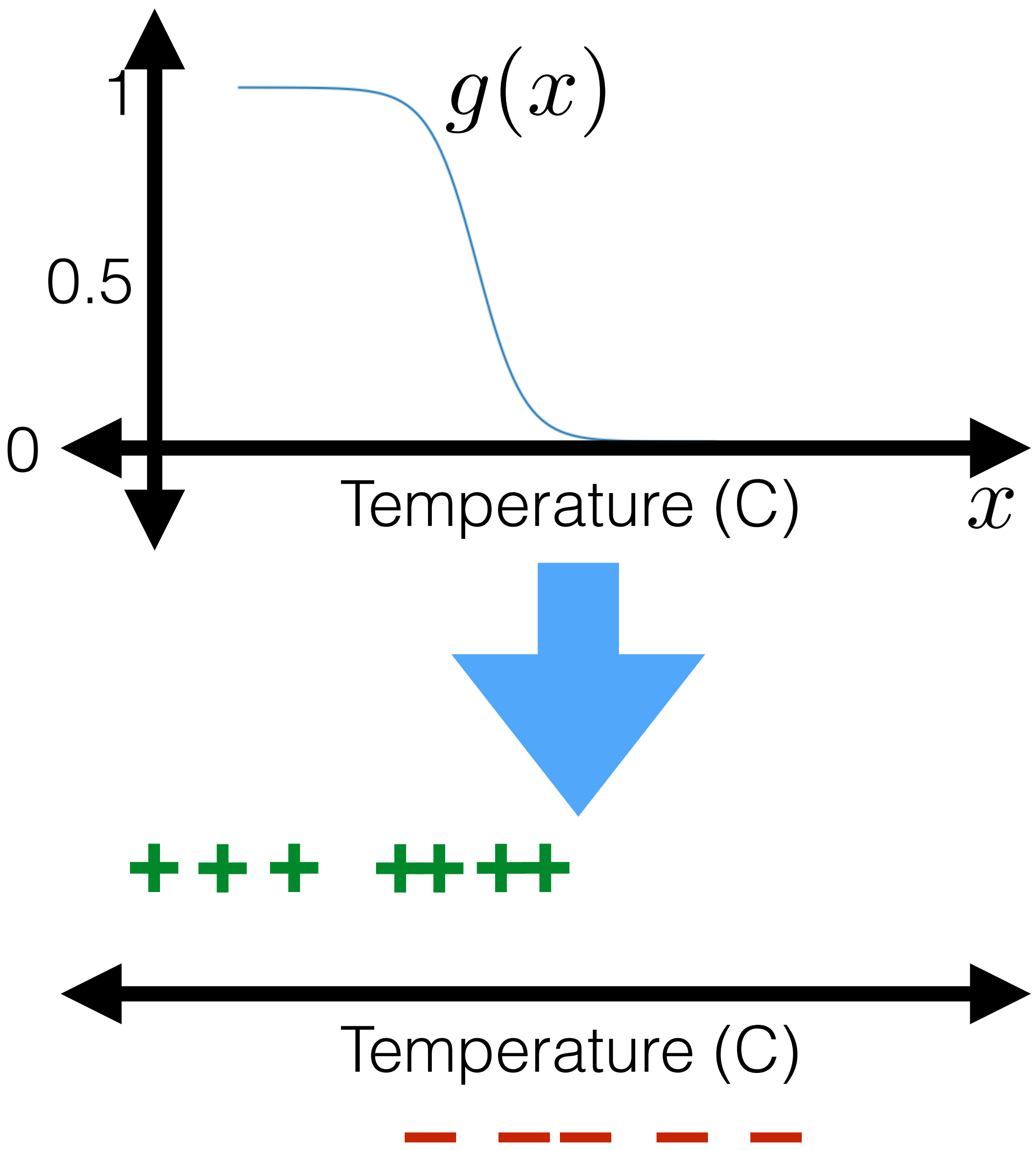

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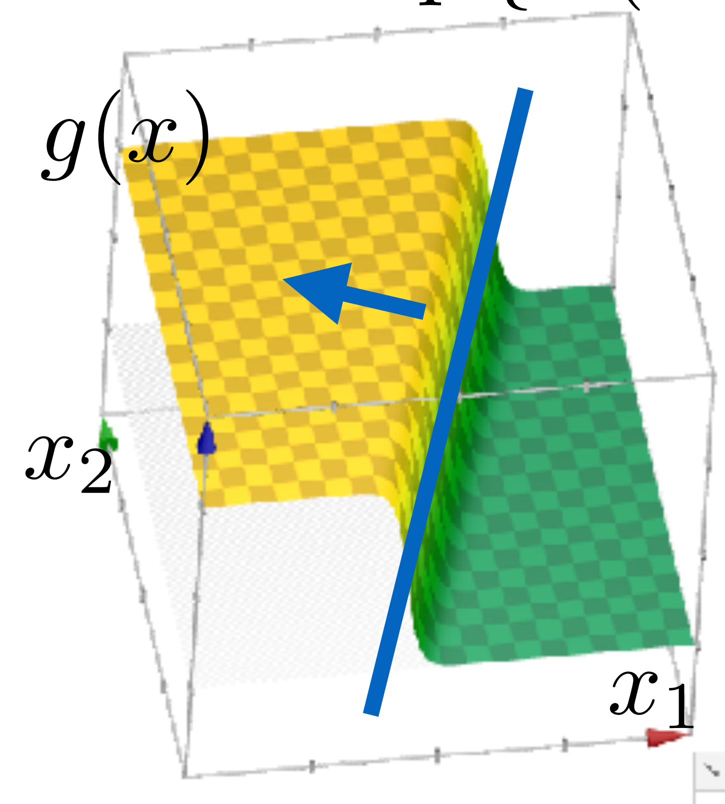

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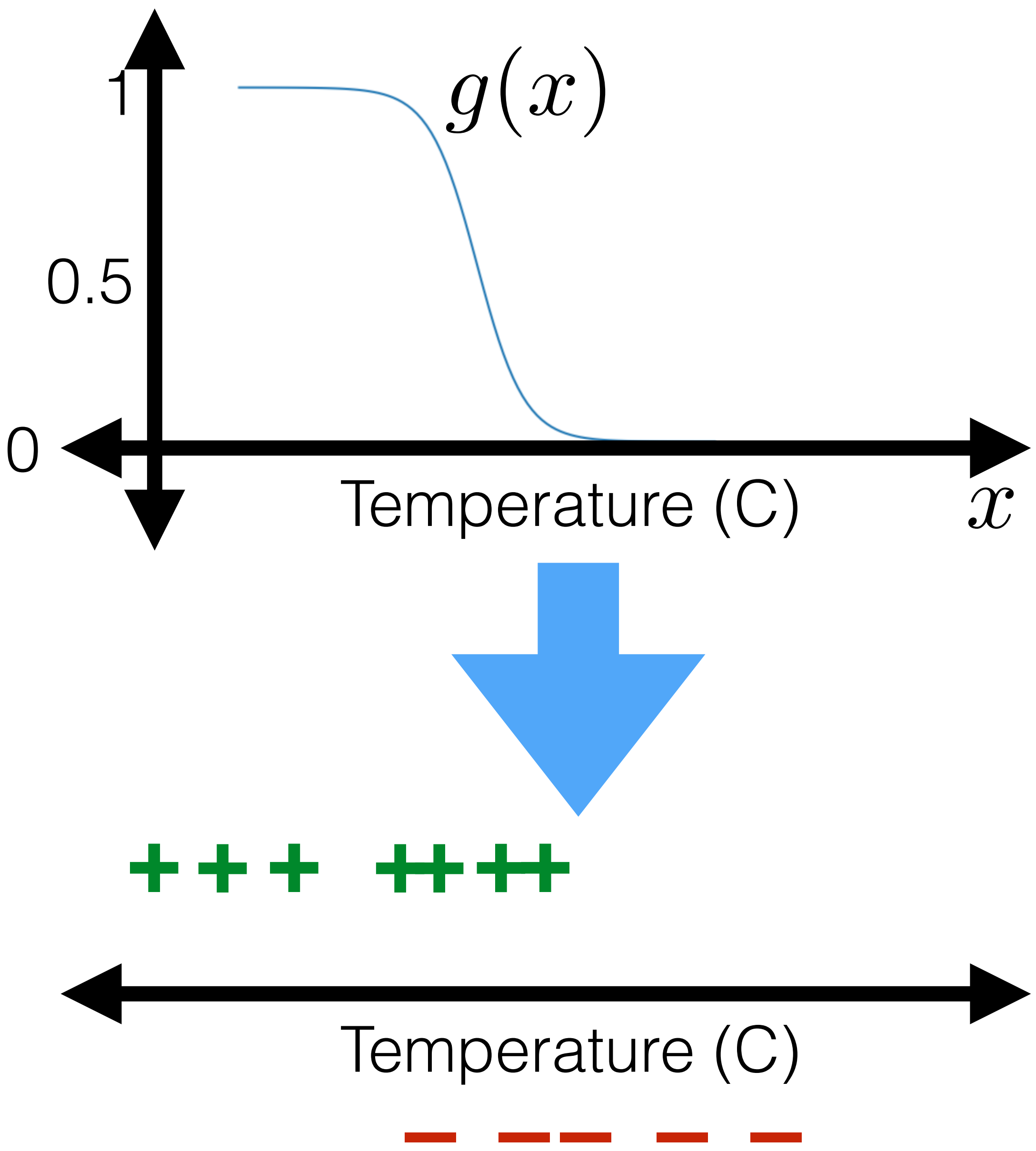

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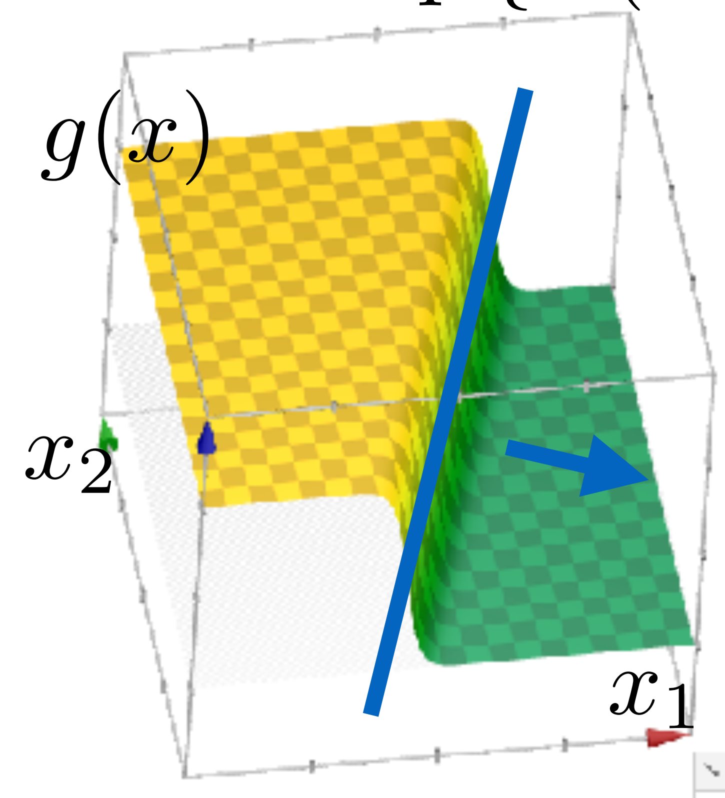

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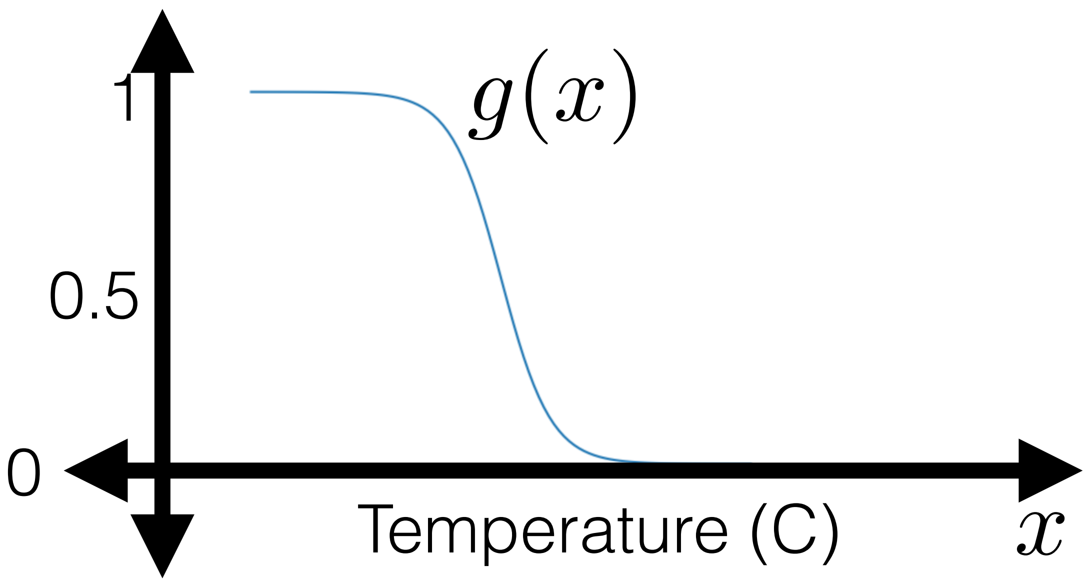

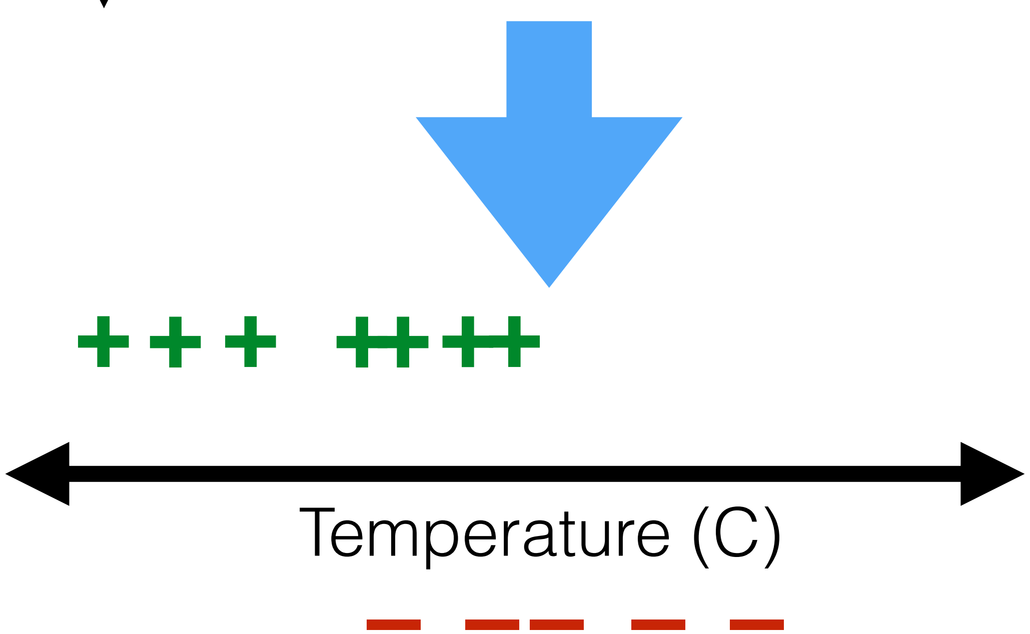

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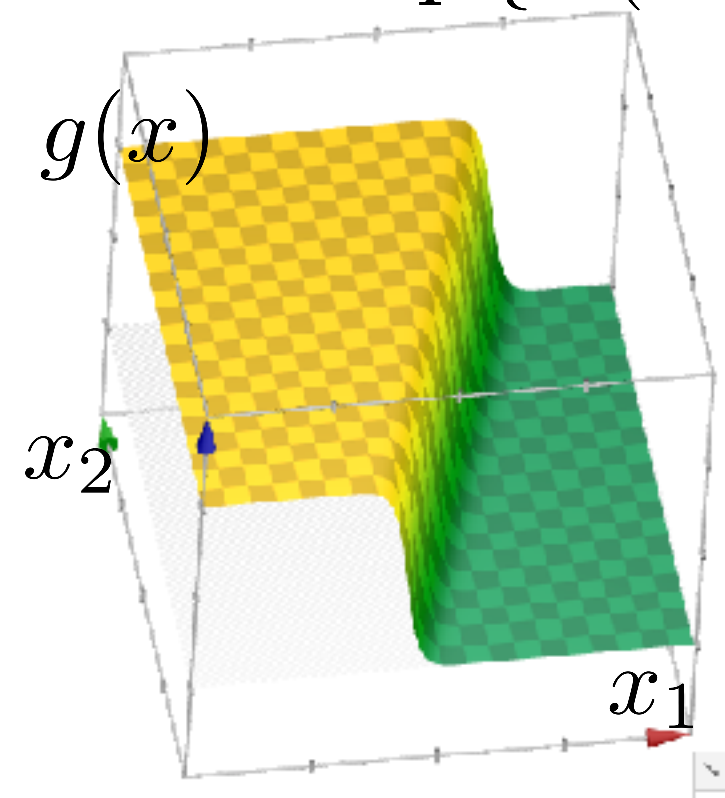

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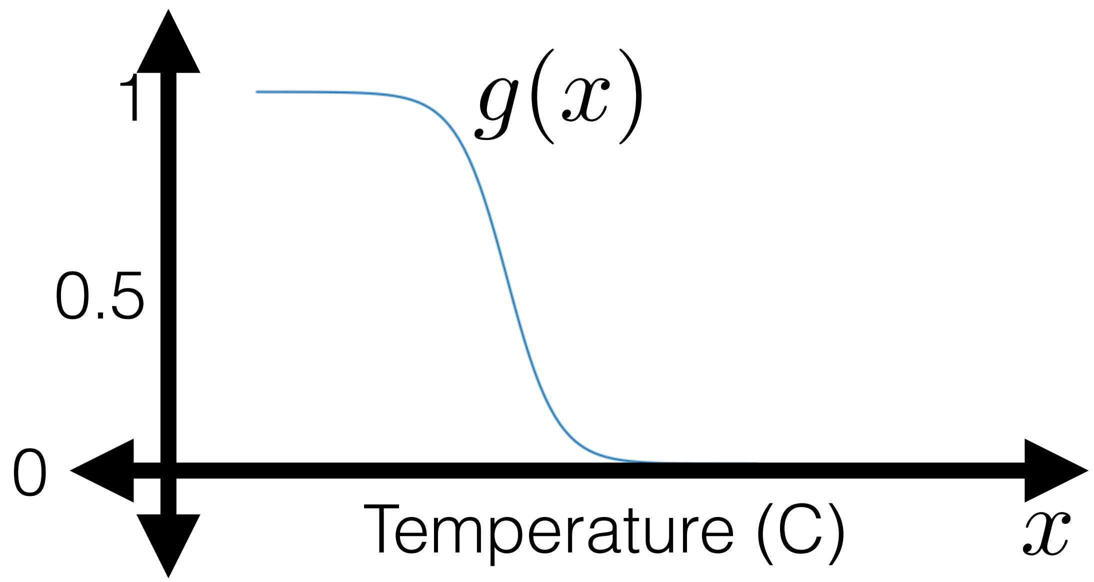

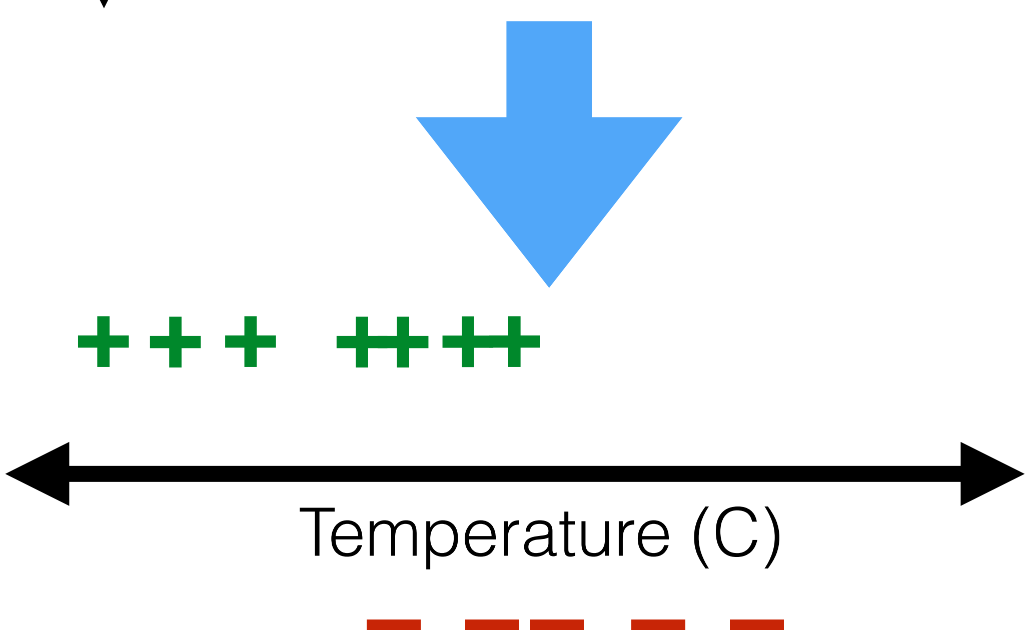

2 features:

$$g(x) = \sigma(\theta^\top x + \theta_0)$$

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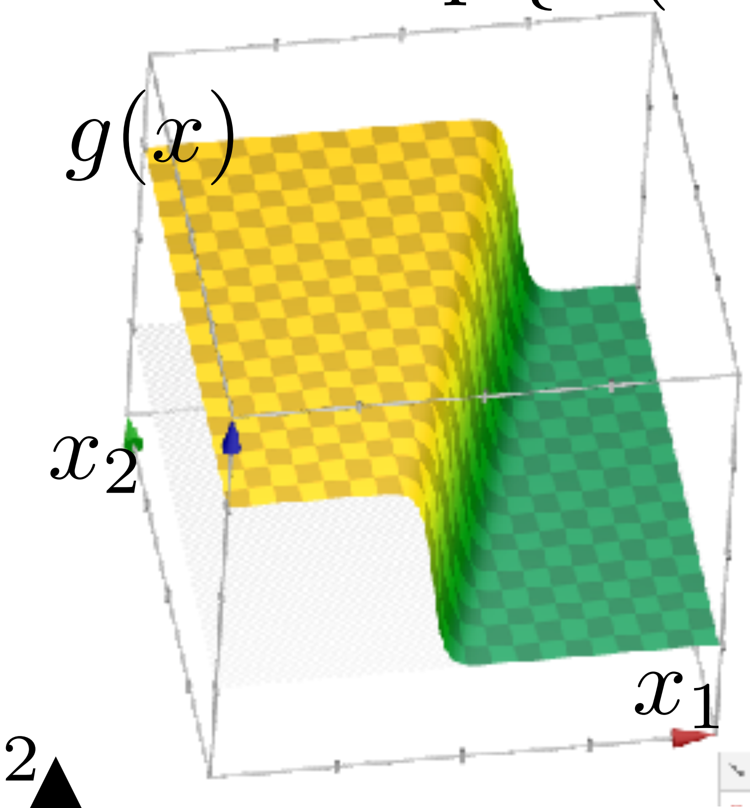

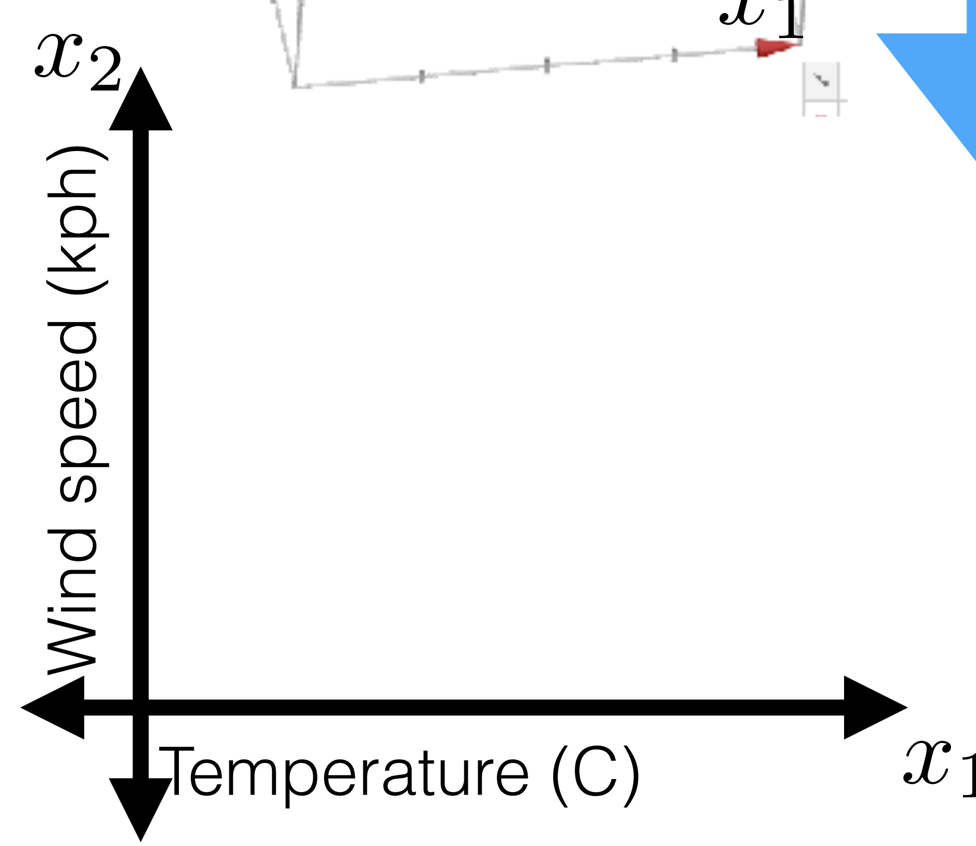

Capturing uncertainty

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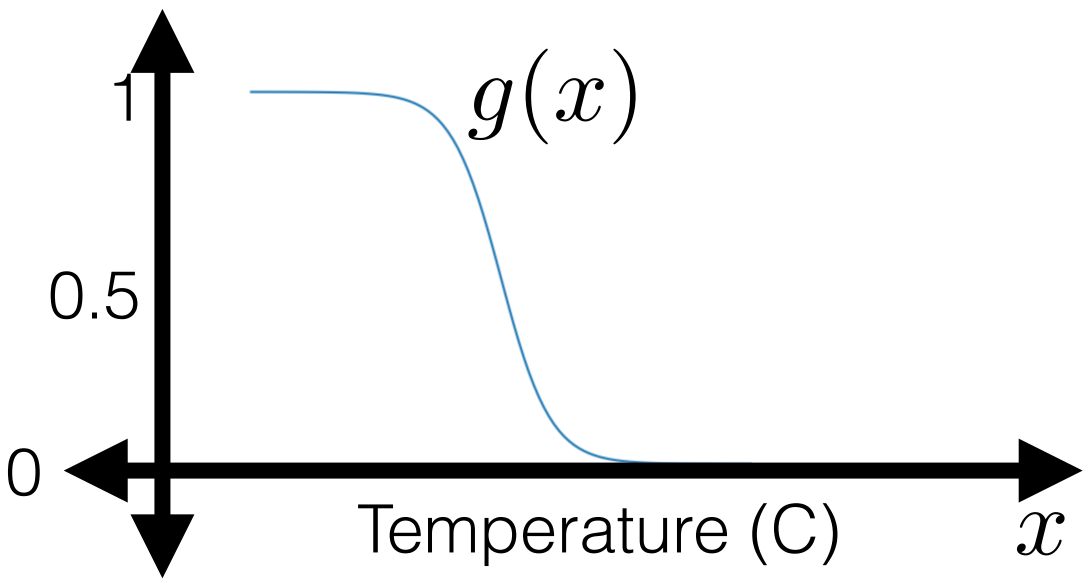

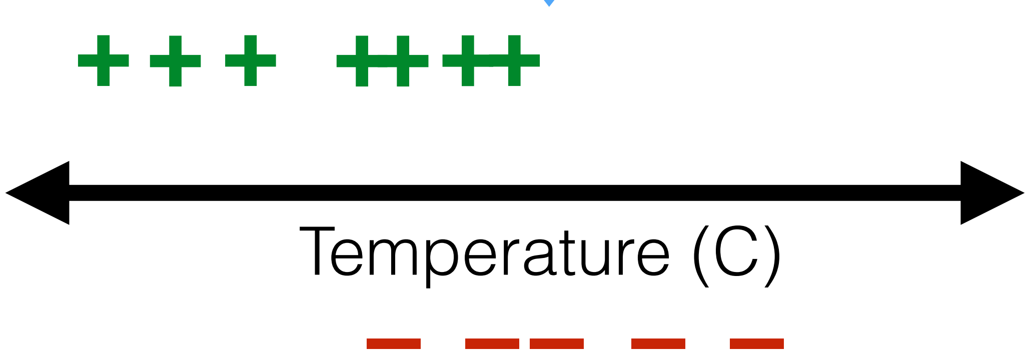

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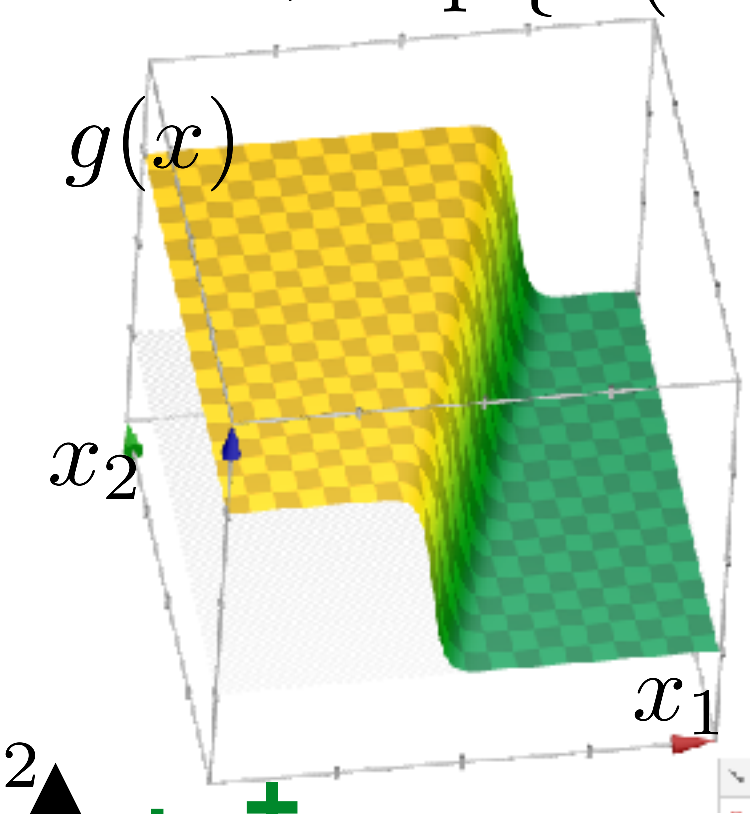

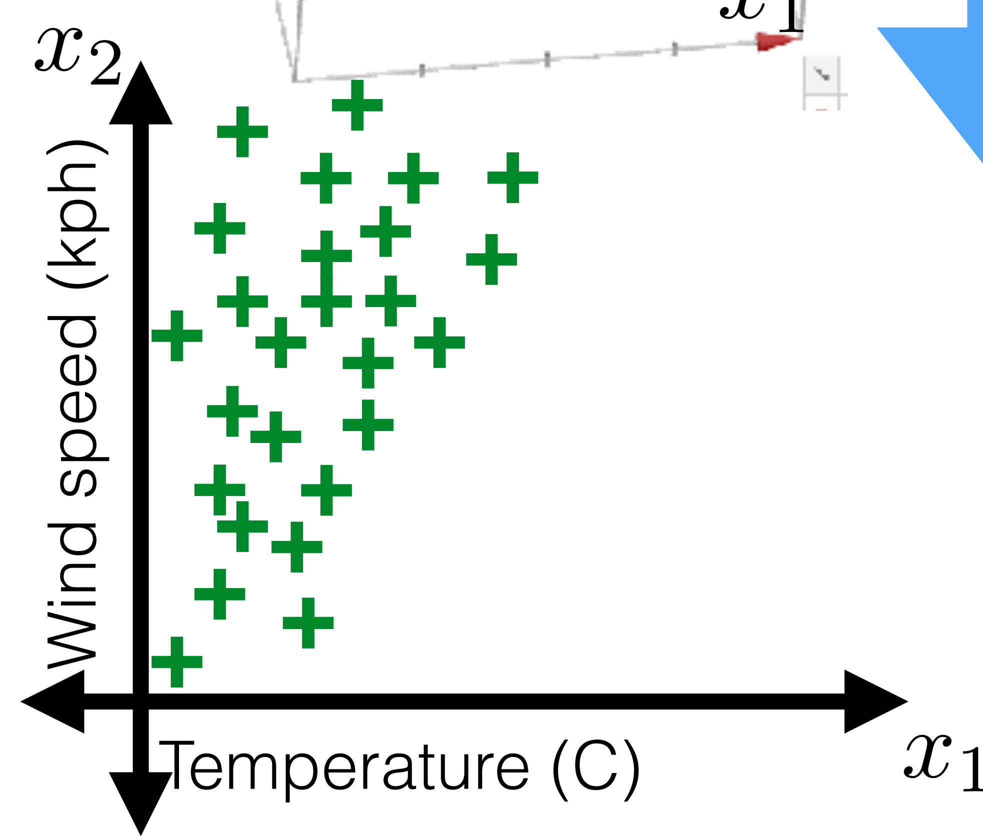

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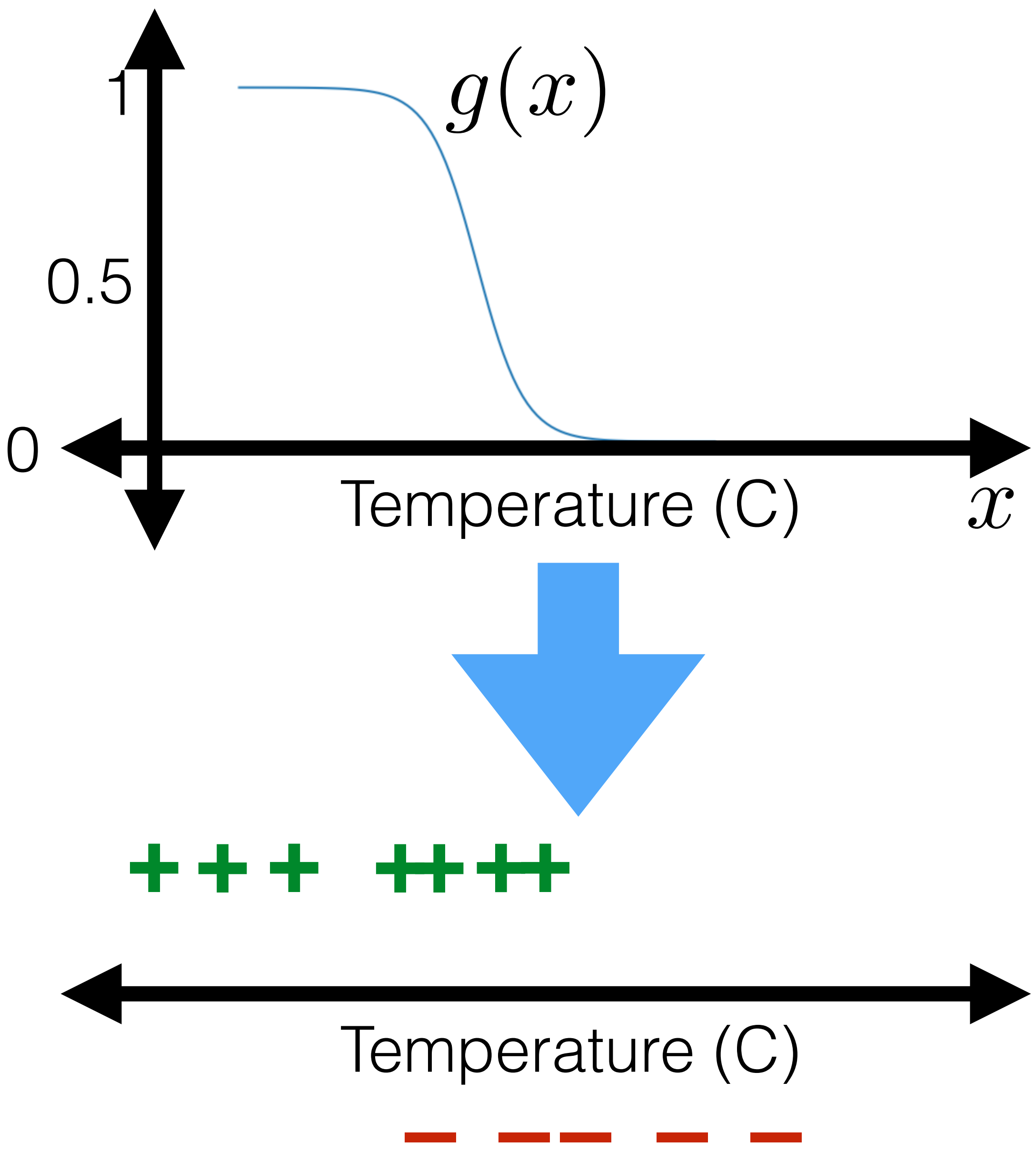

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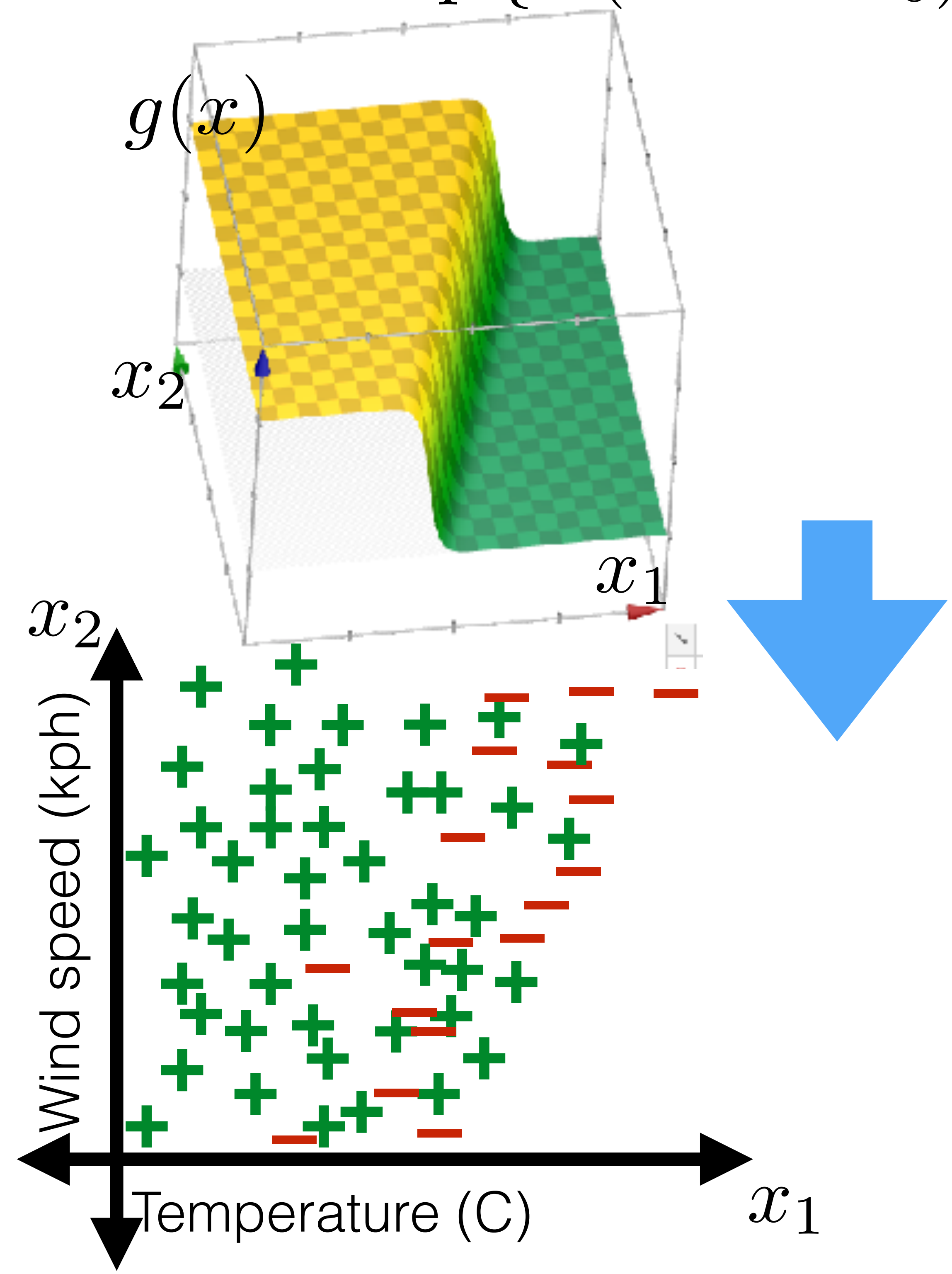

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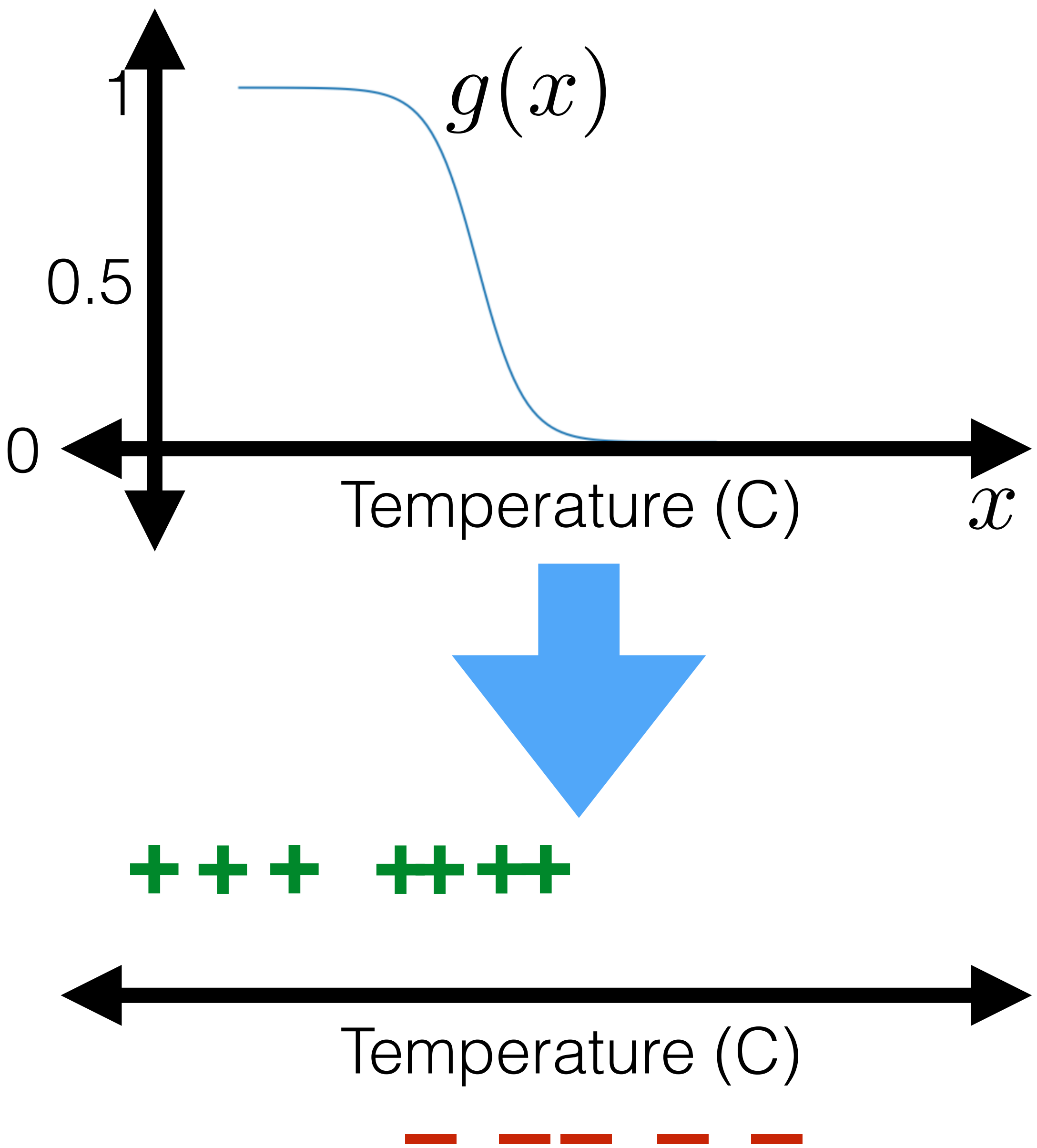

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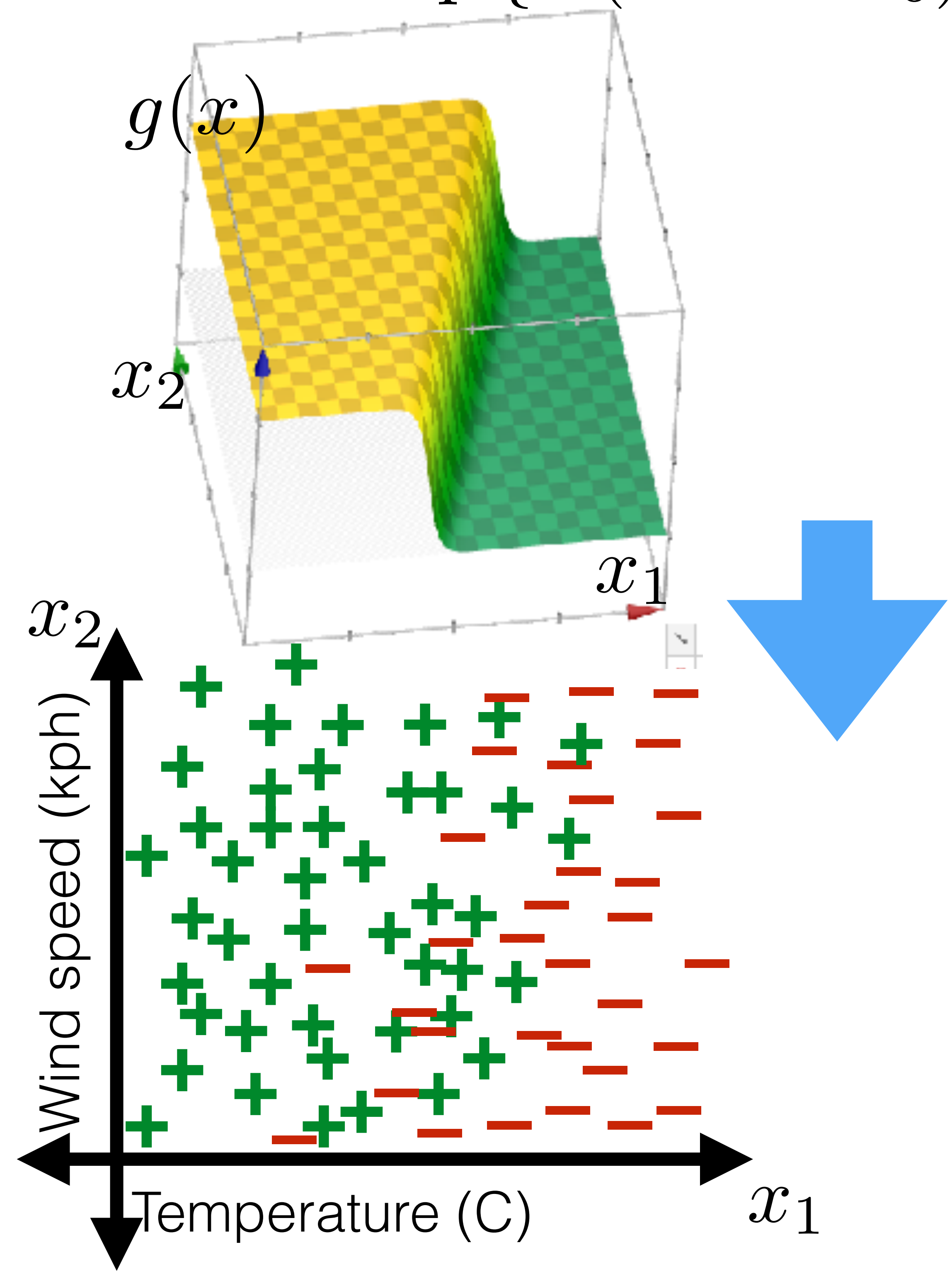

Linear logistic classification

aka logistic regression

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- How do we learn a classifier (i.e. learn θ, θ_0)?

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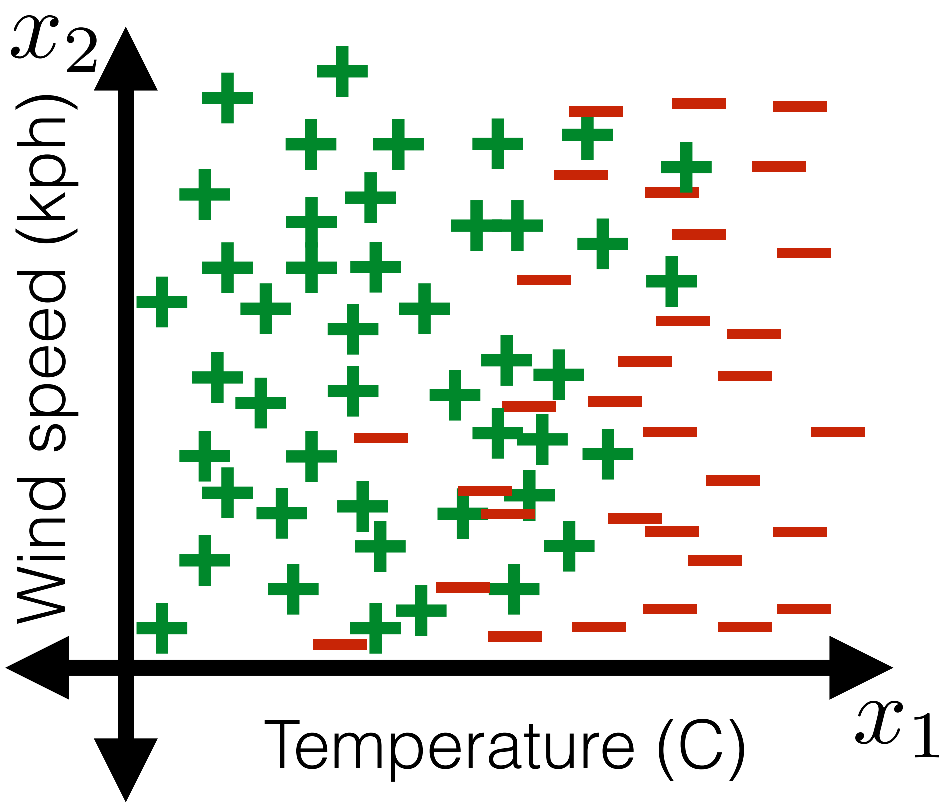

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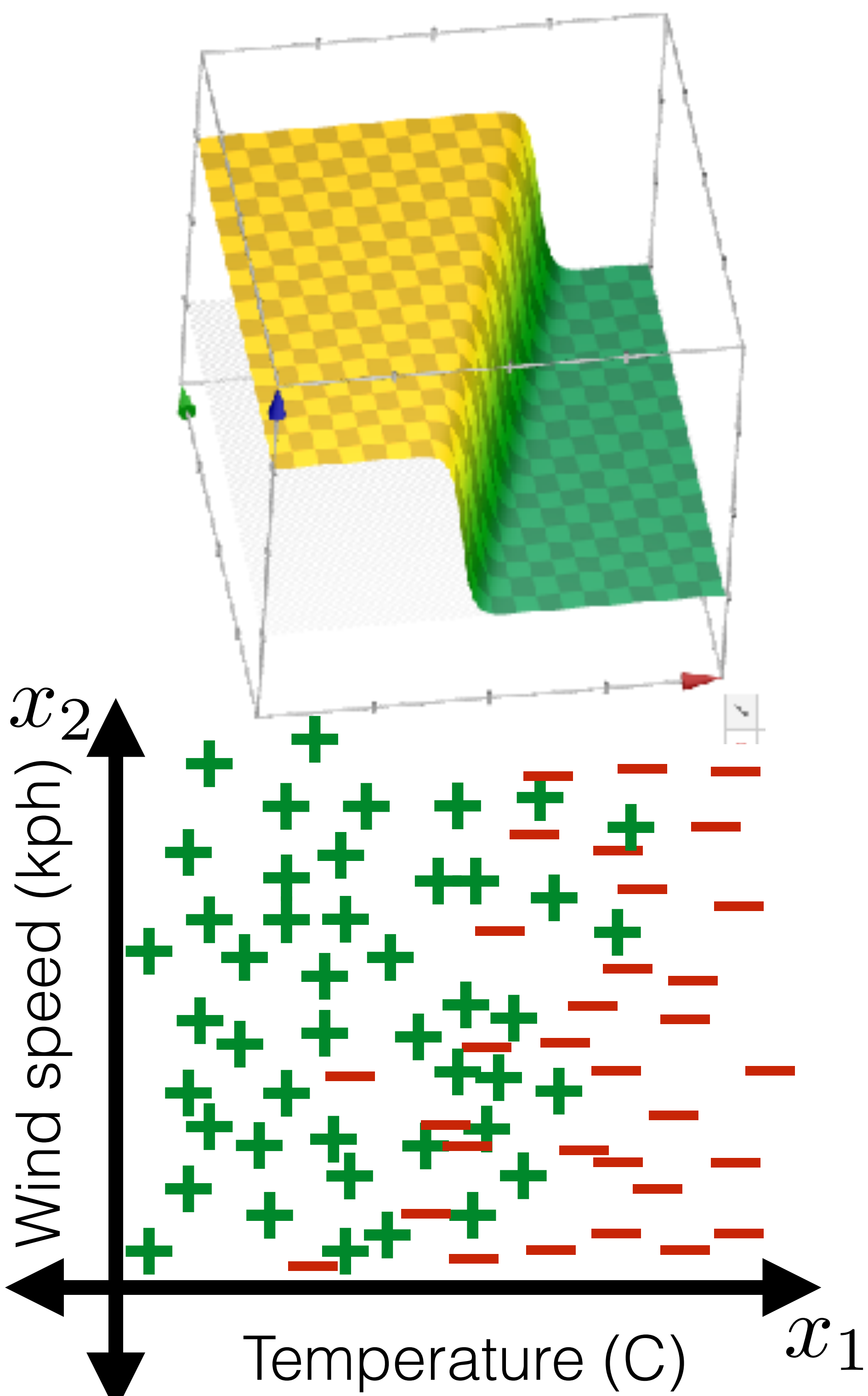

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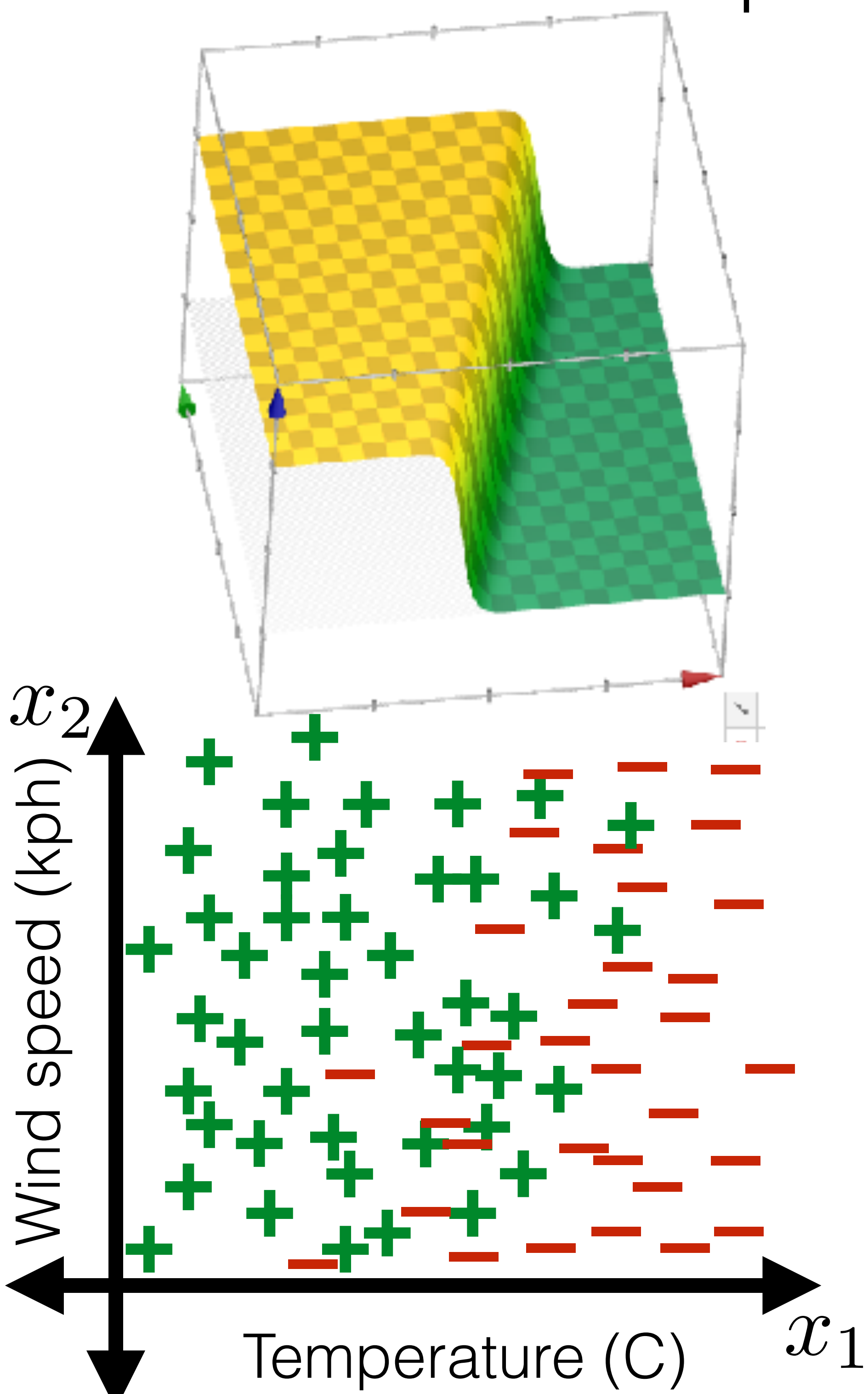

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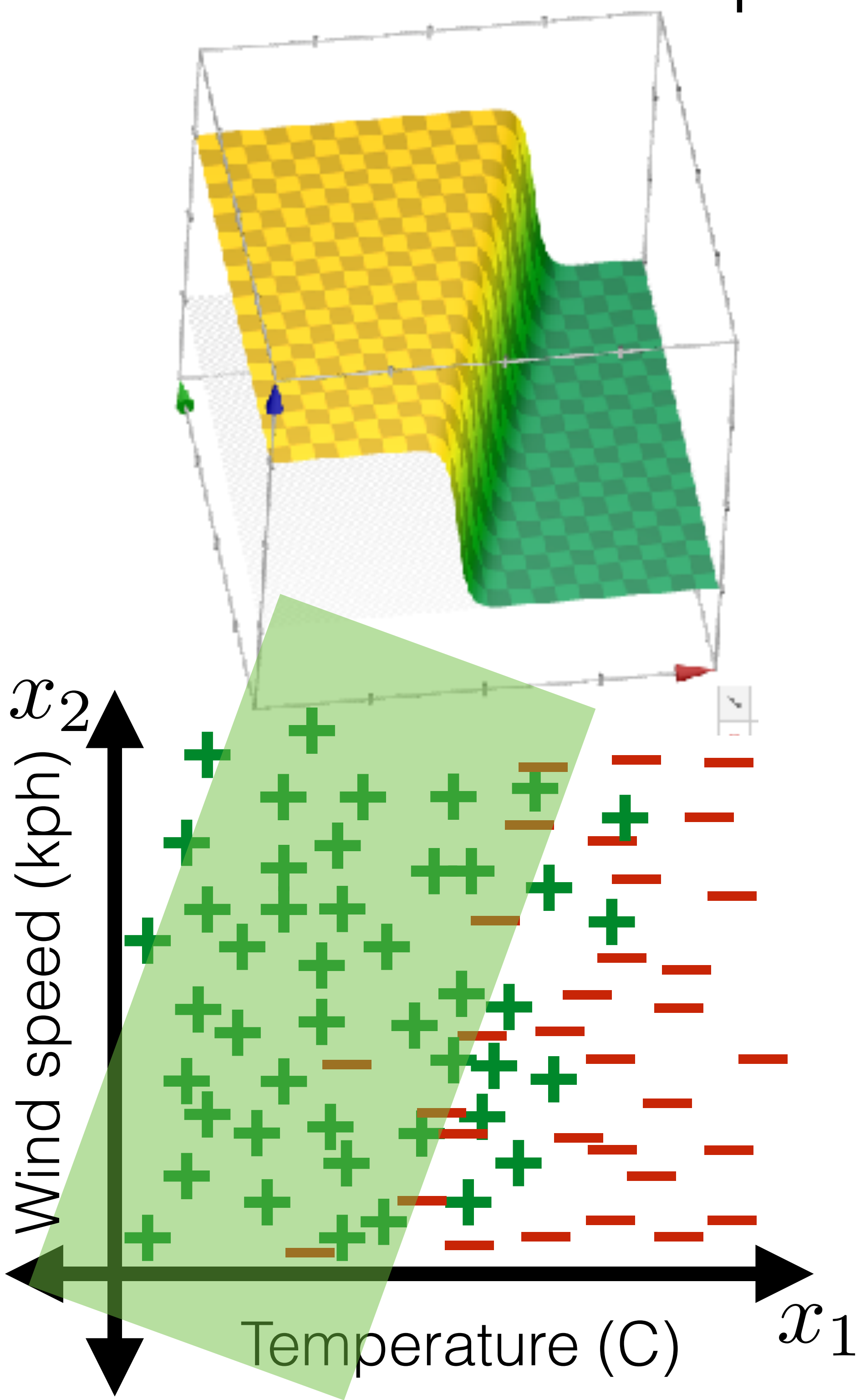

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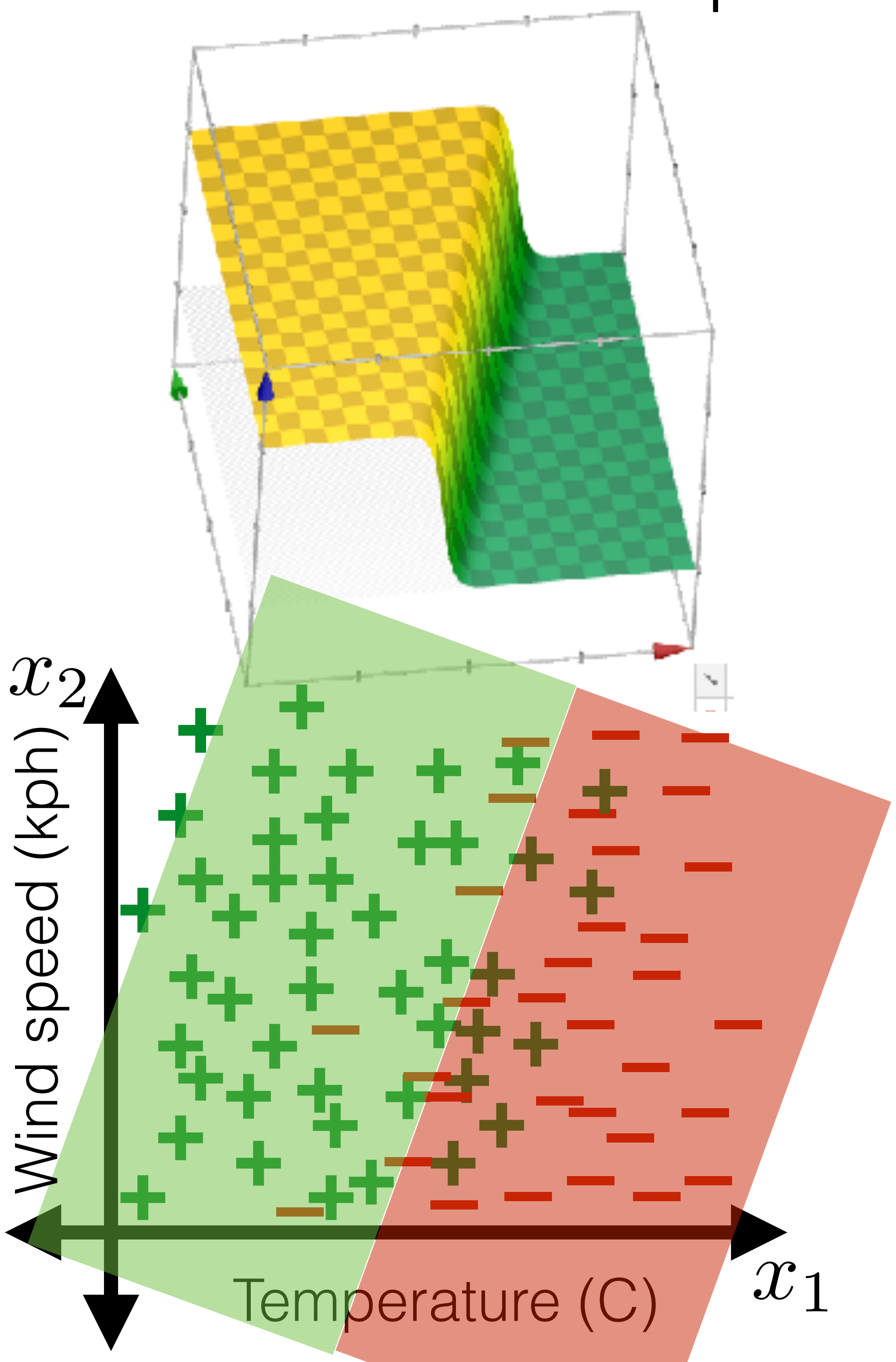

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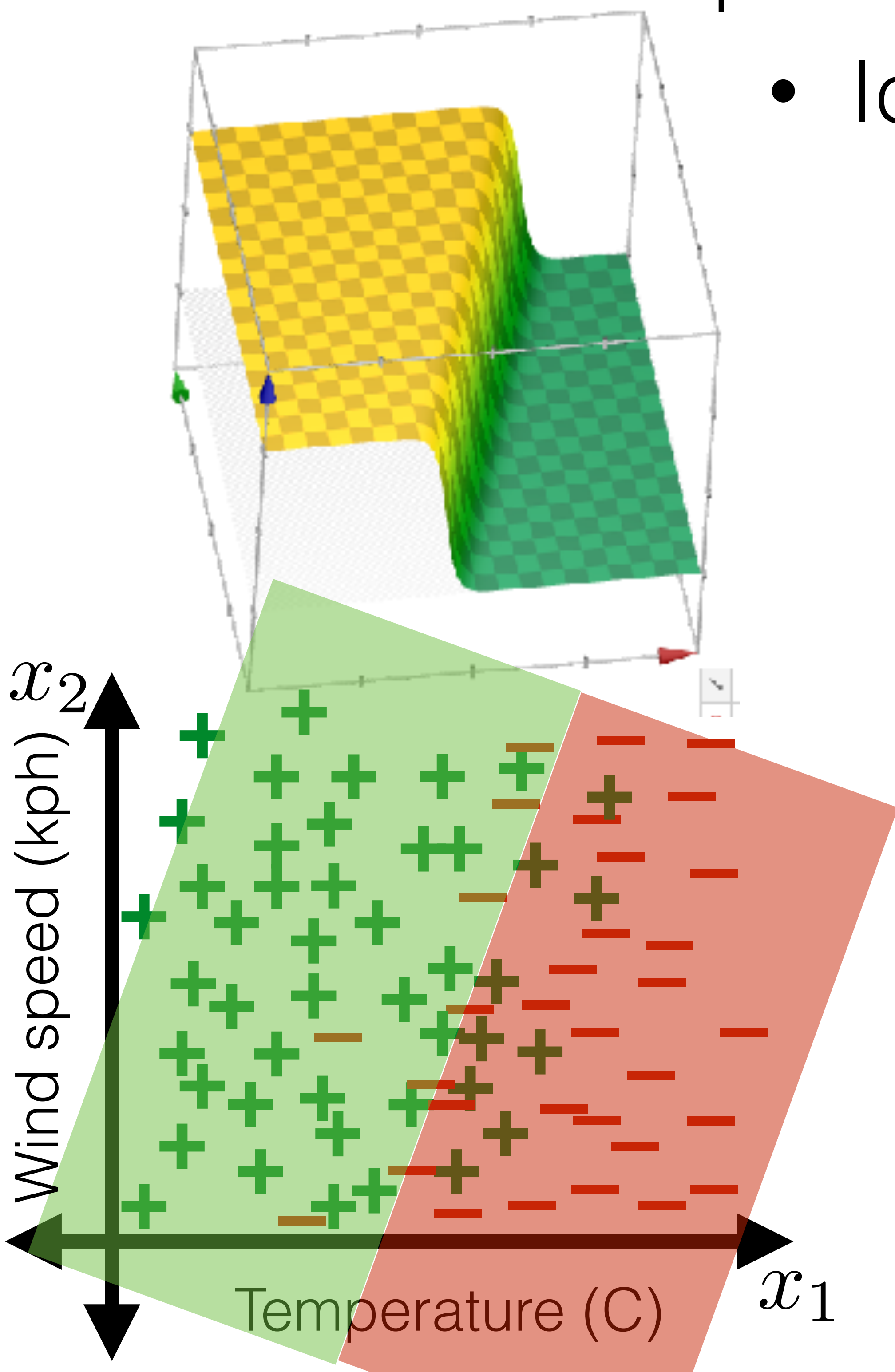

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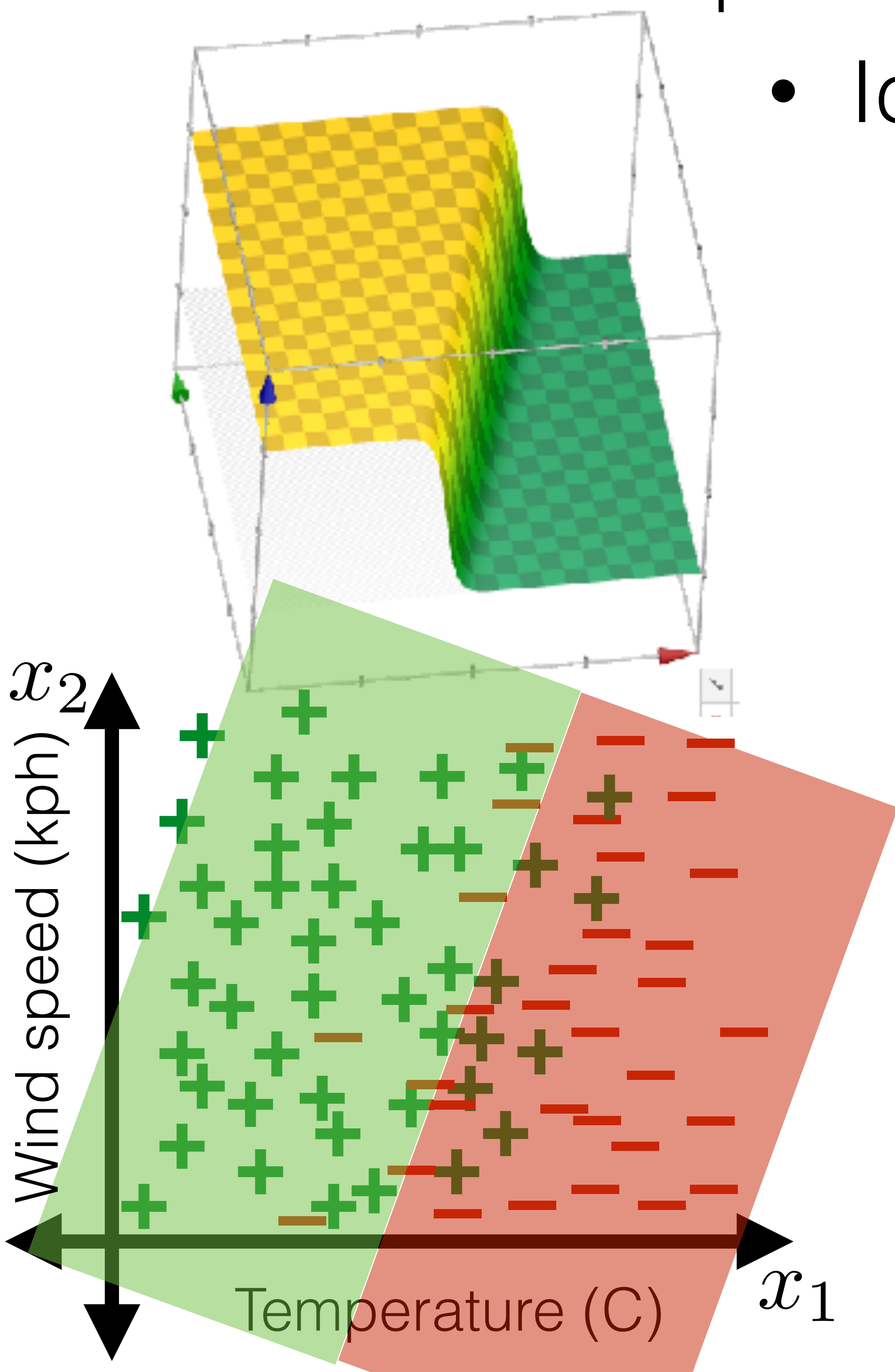

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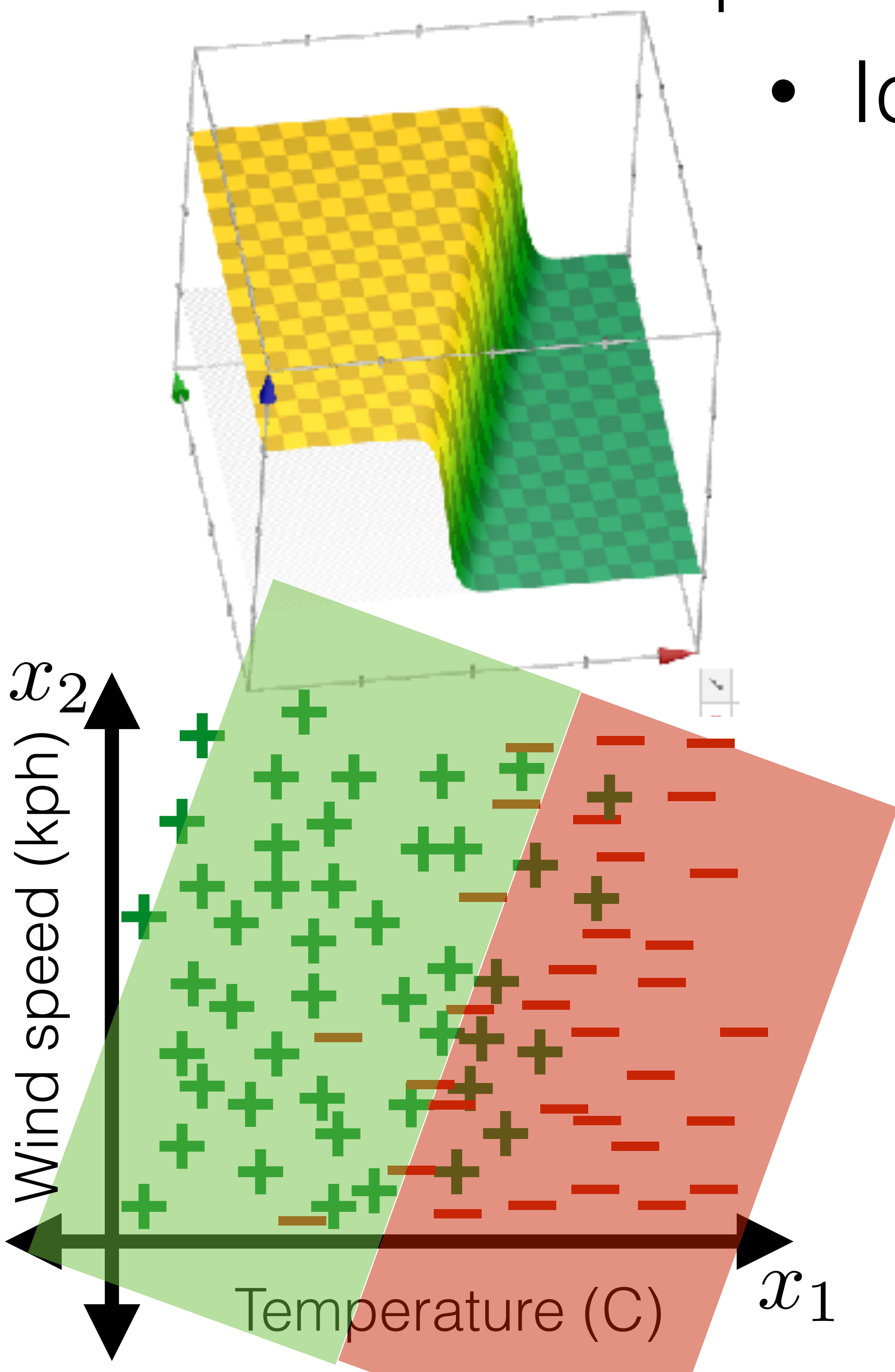

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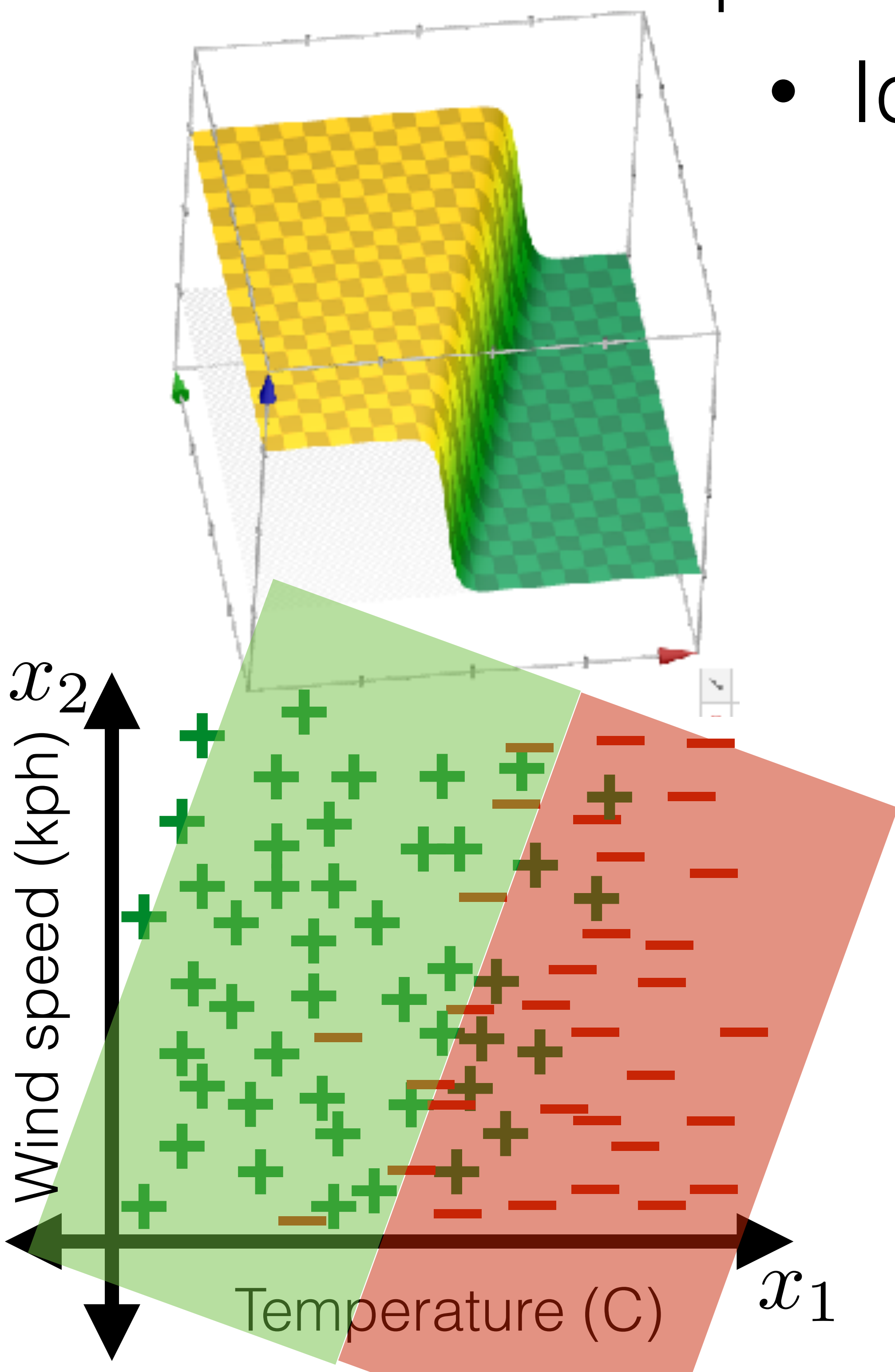

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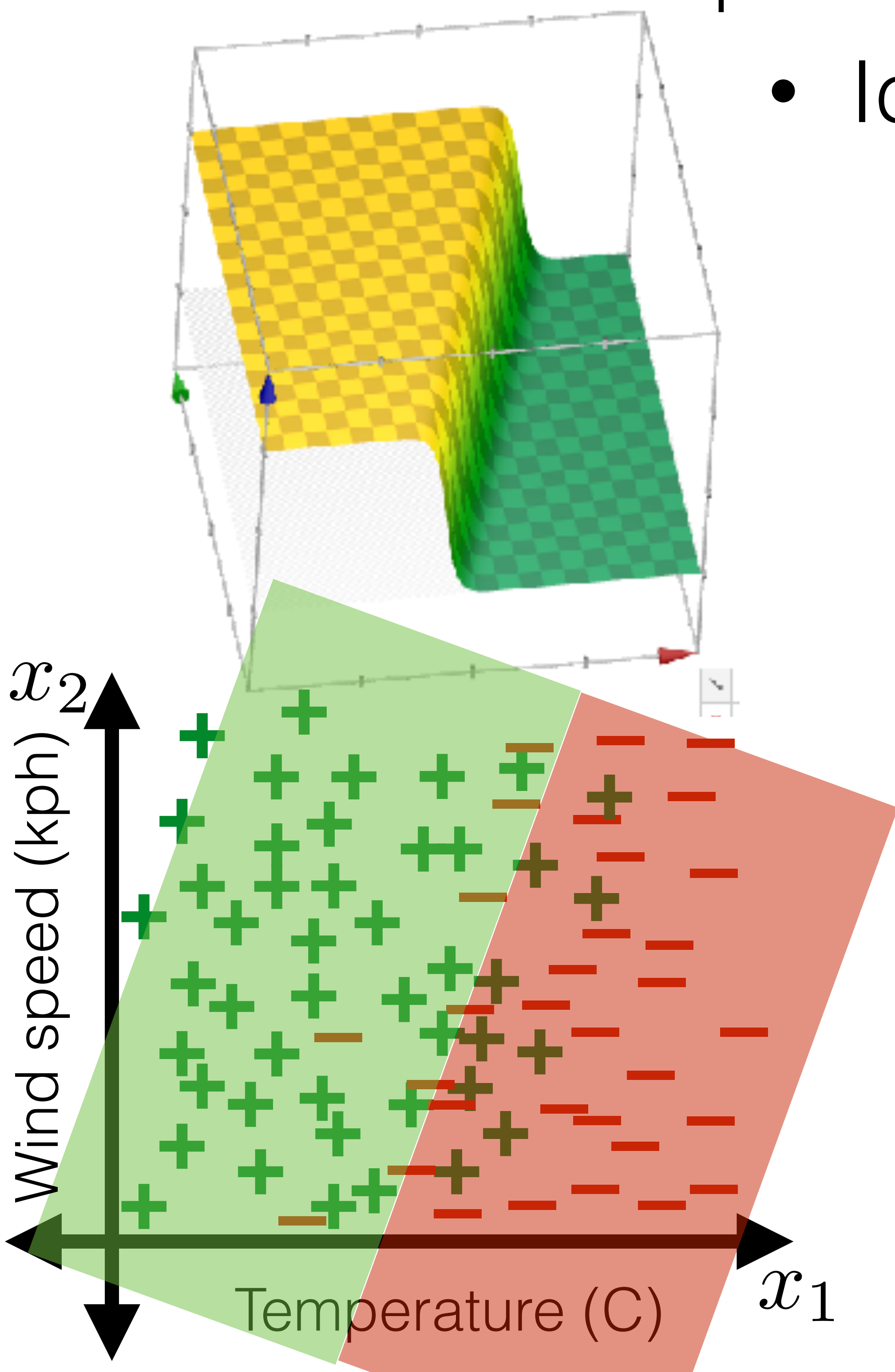

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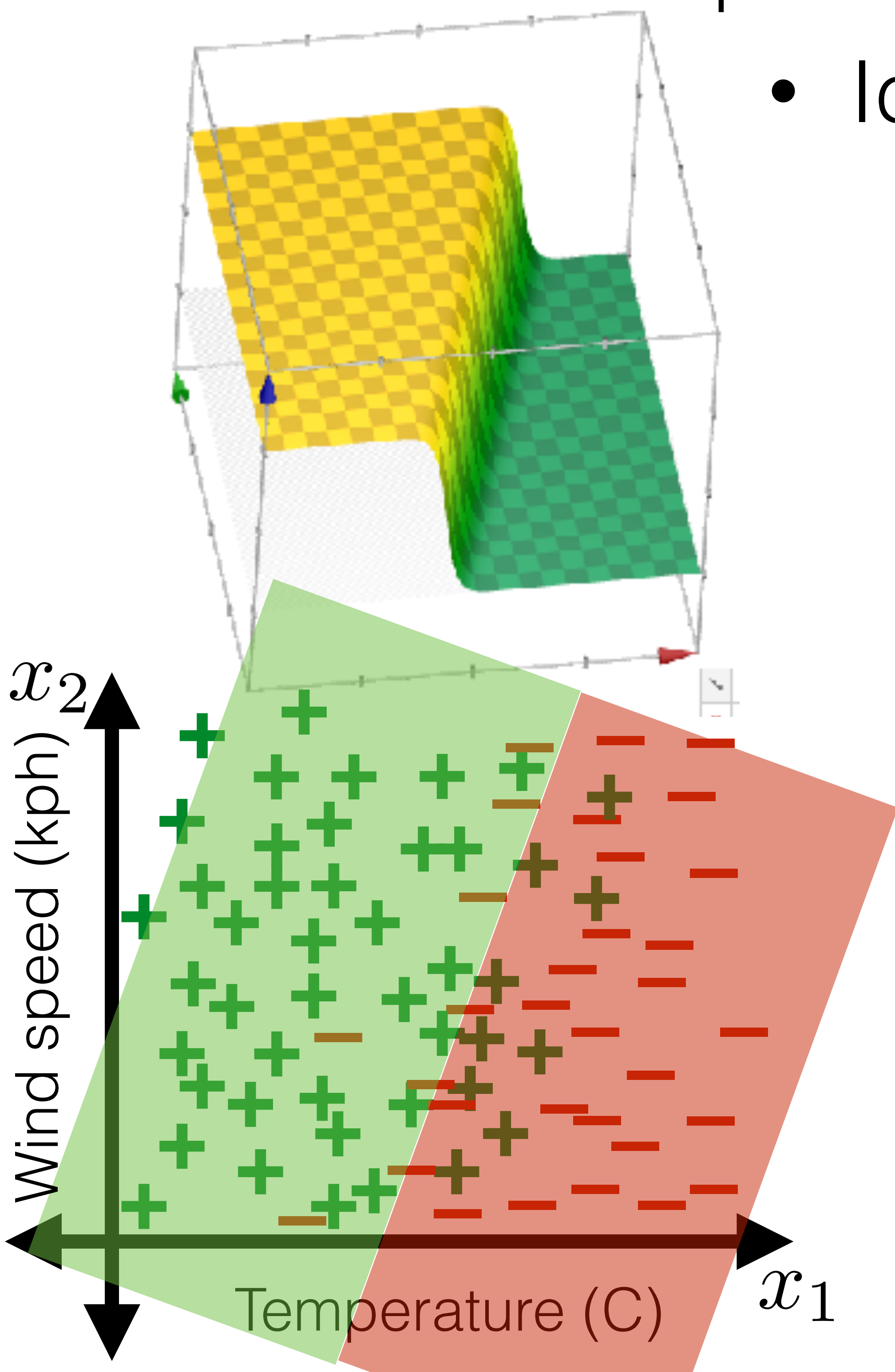

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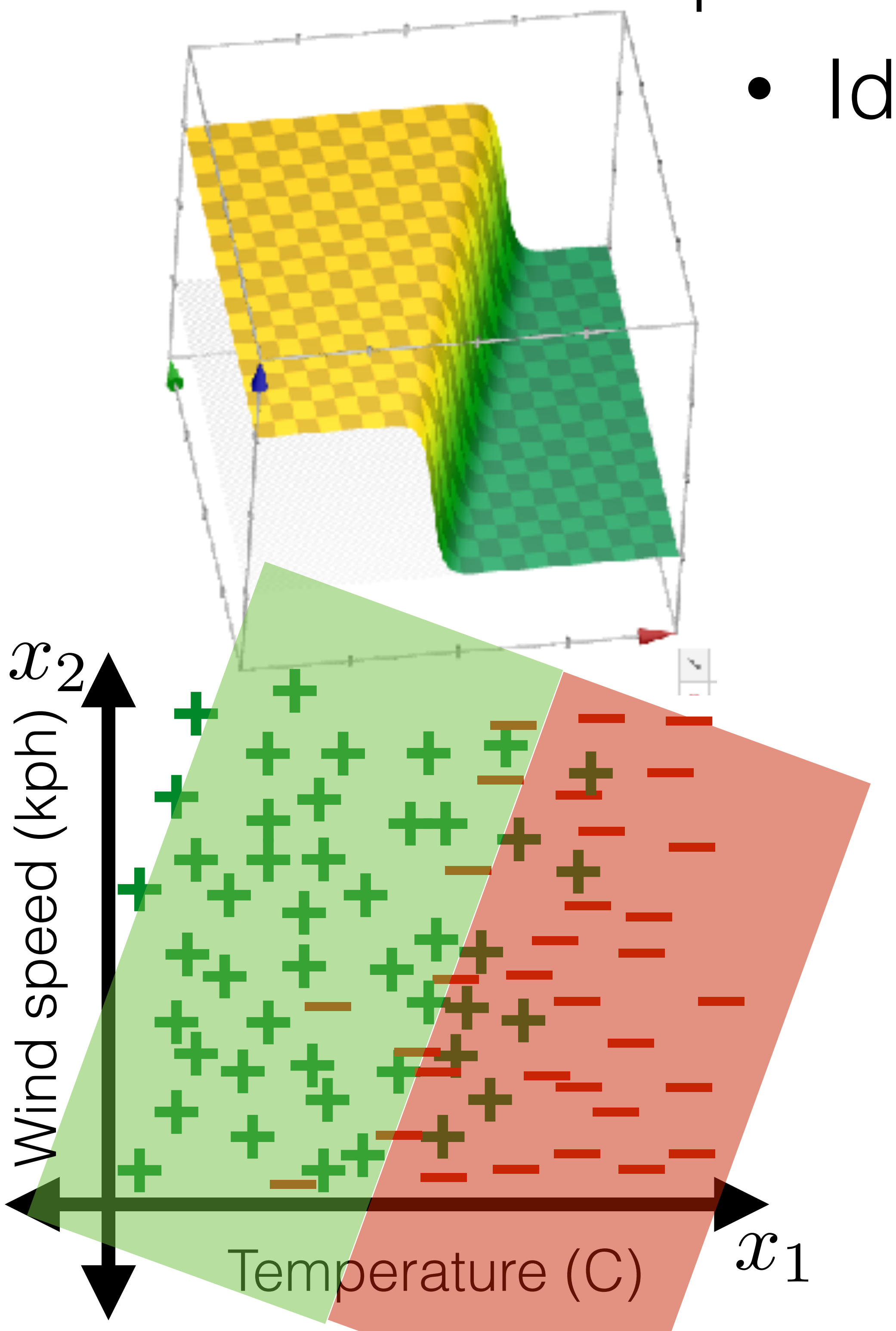

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- Same hypothesis class as before!

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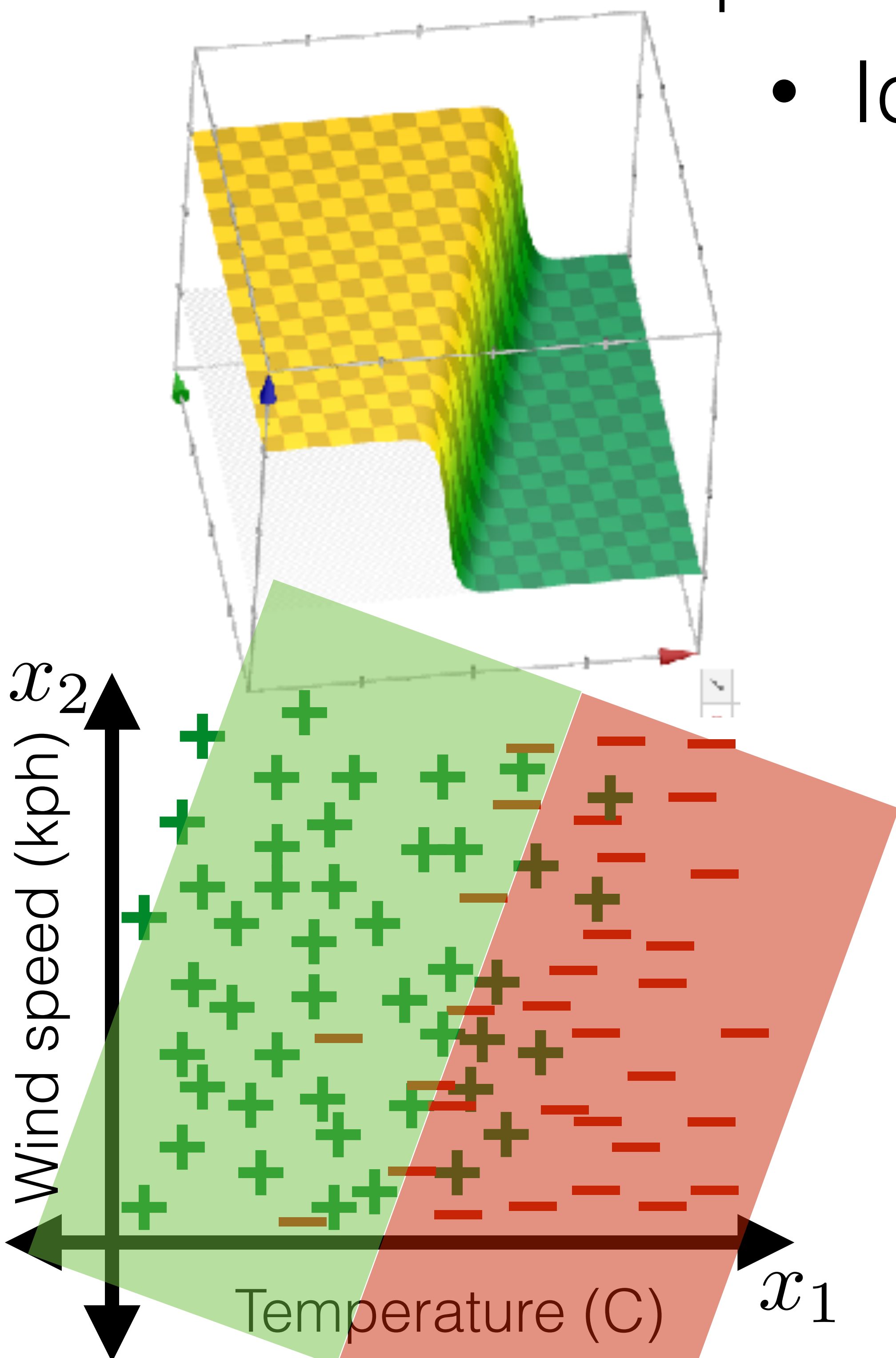

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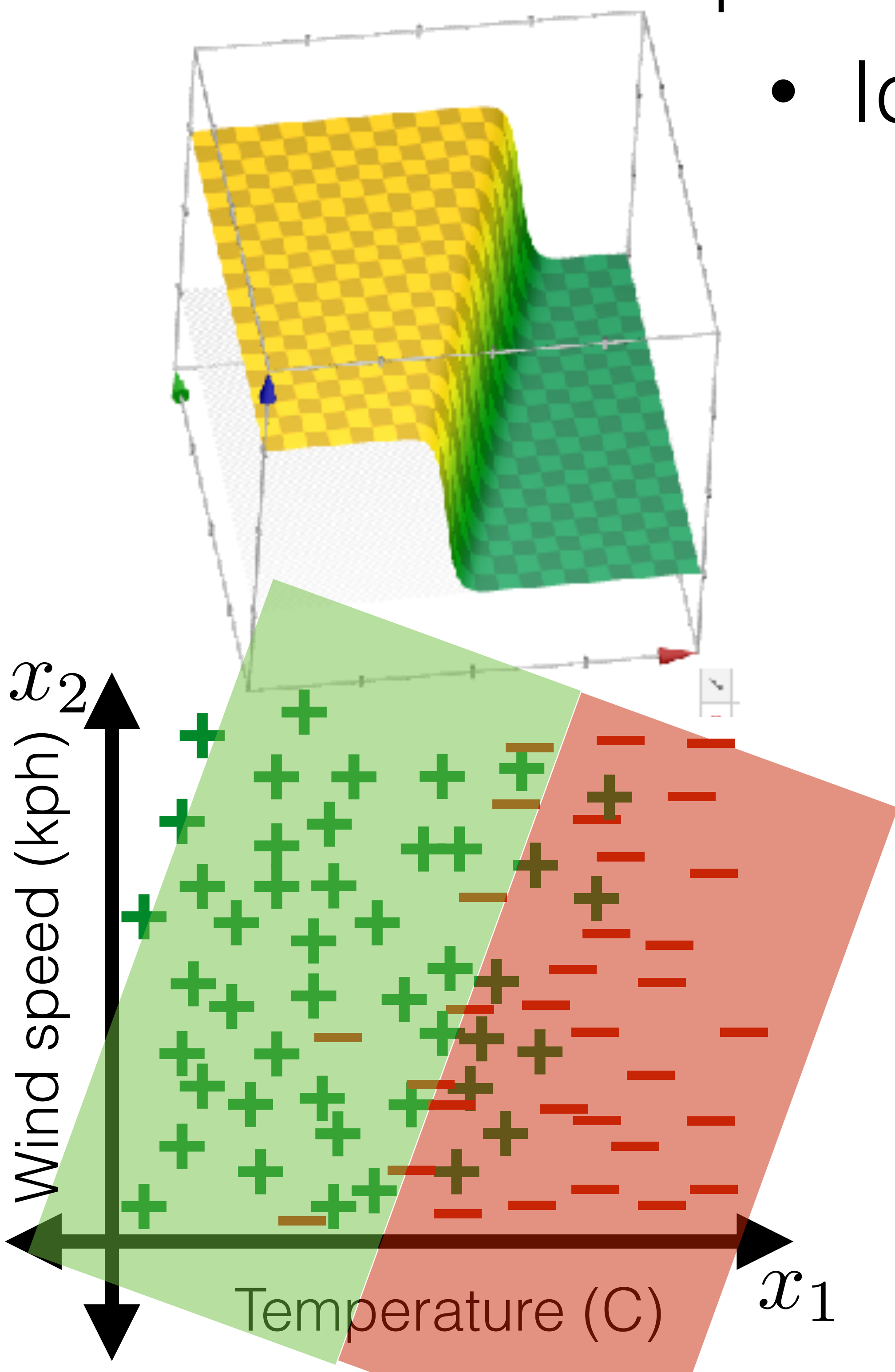

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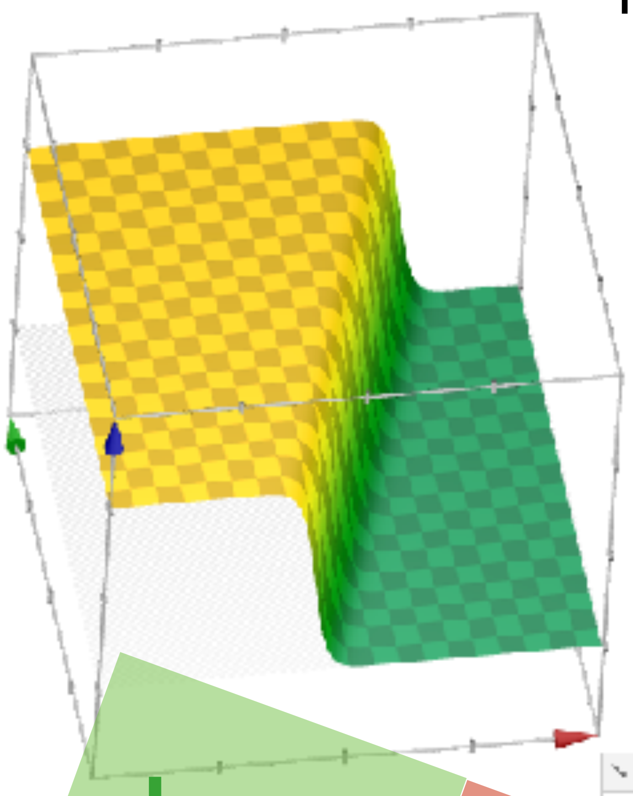

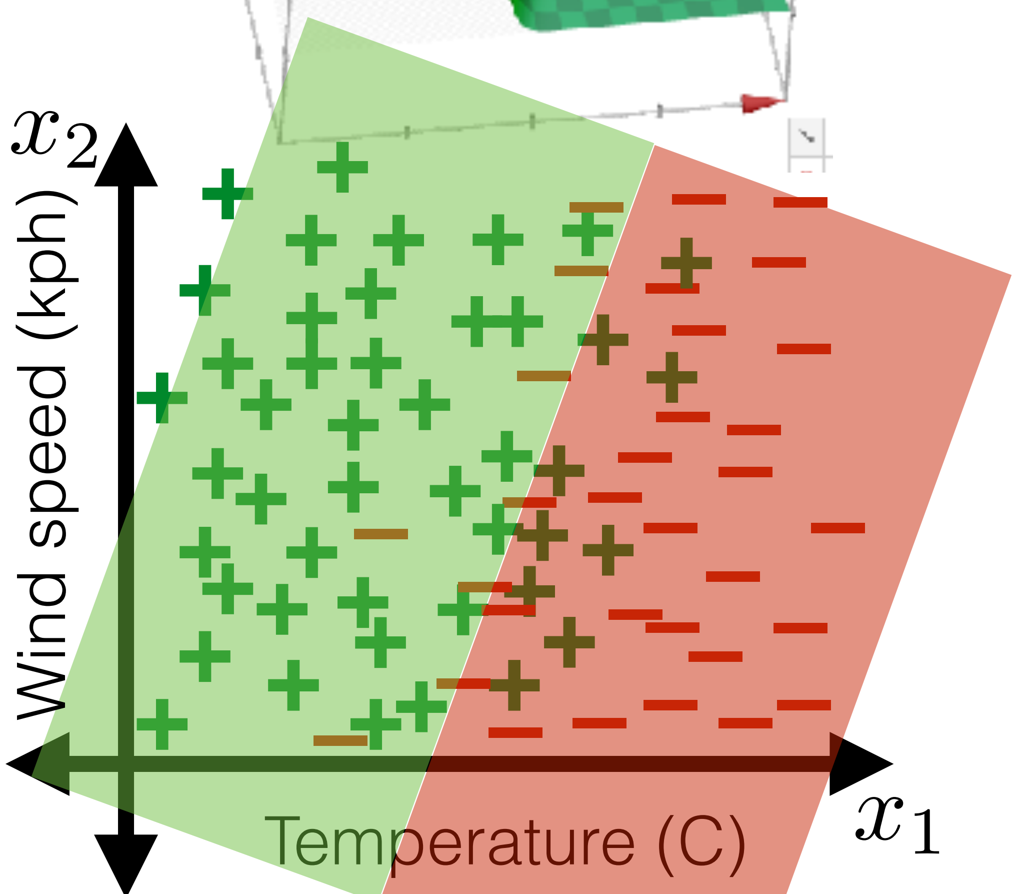

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- Same hypothesis class as before! But we will get:
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  - Quality guarantees when data not linearly separable

Linear logistic classification

aka logistic regression

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Linear logistic classification

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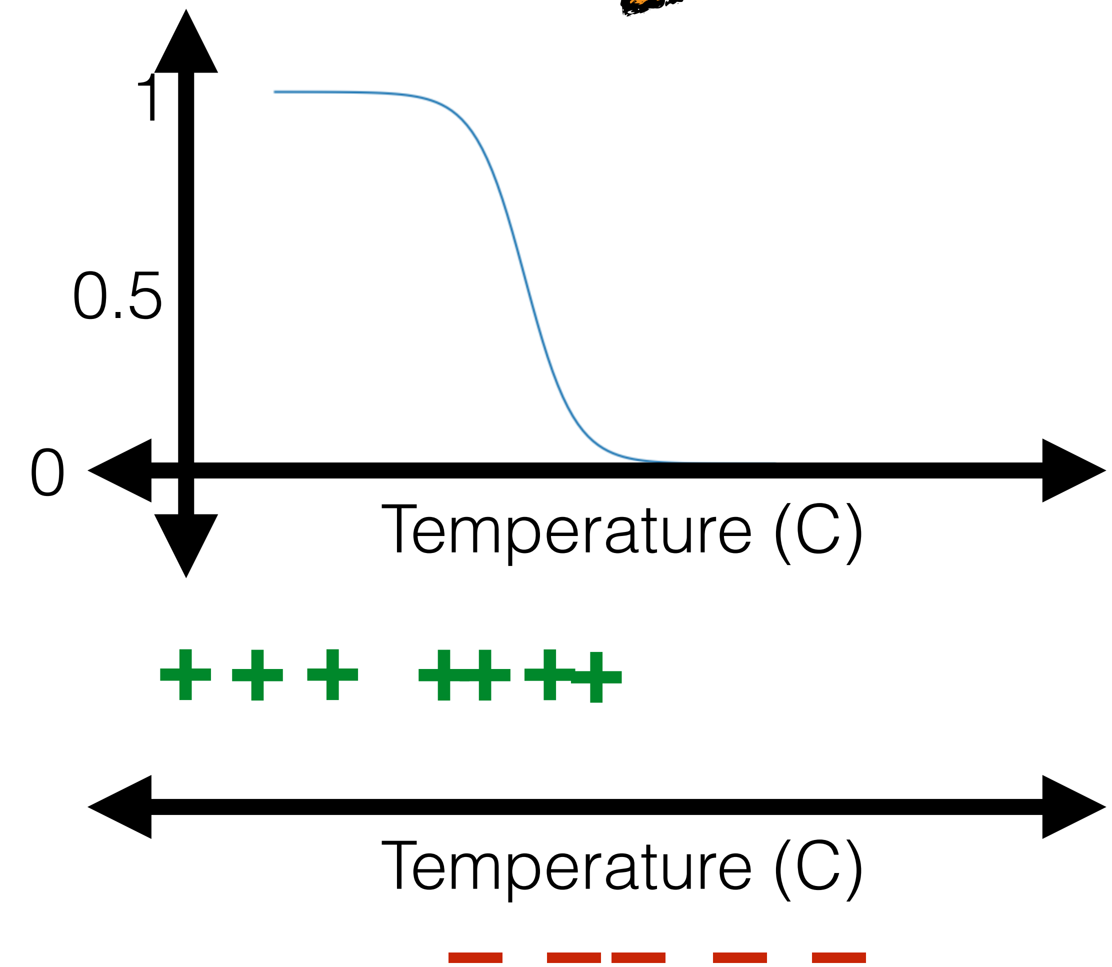

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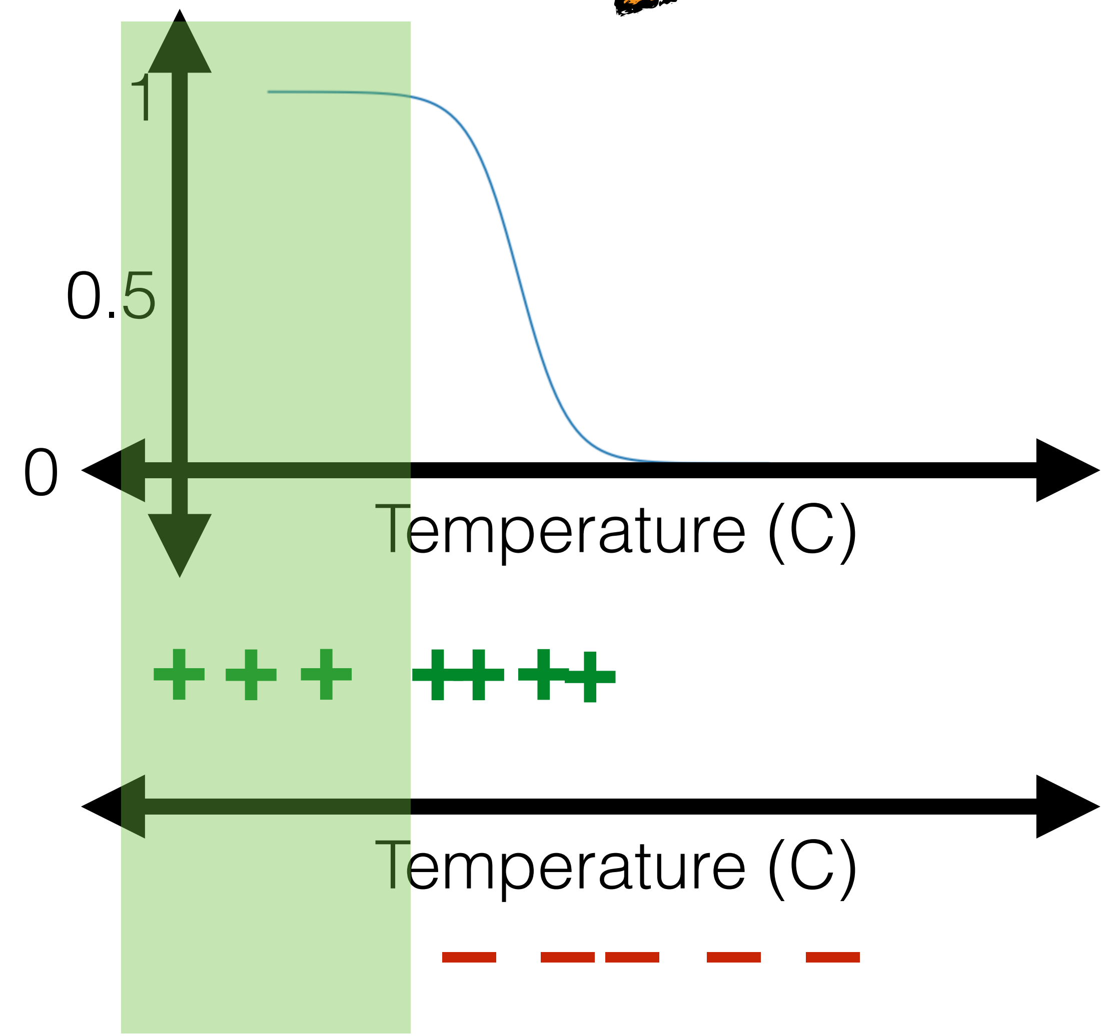

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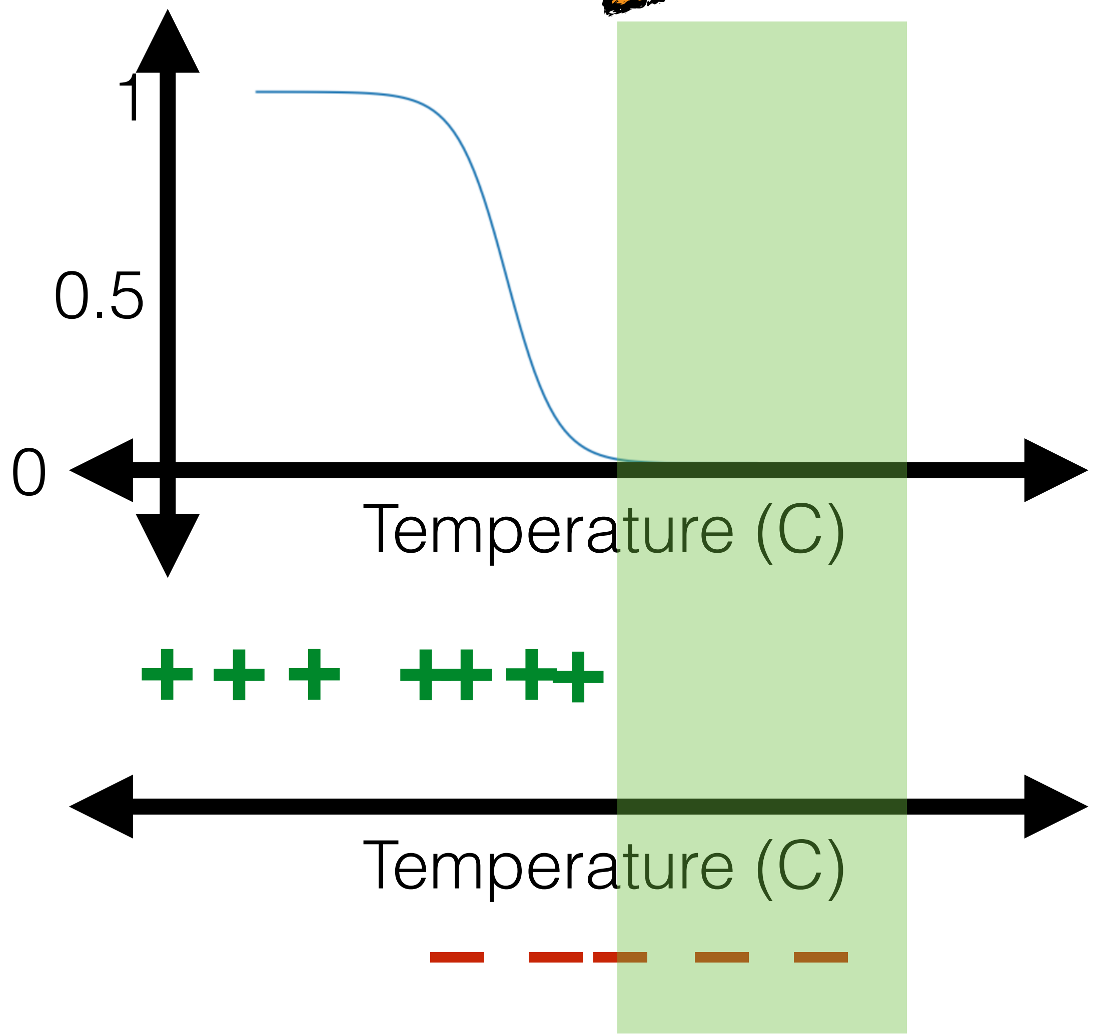

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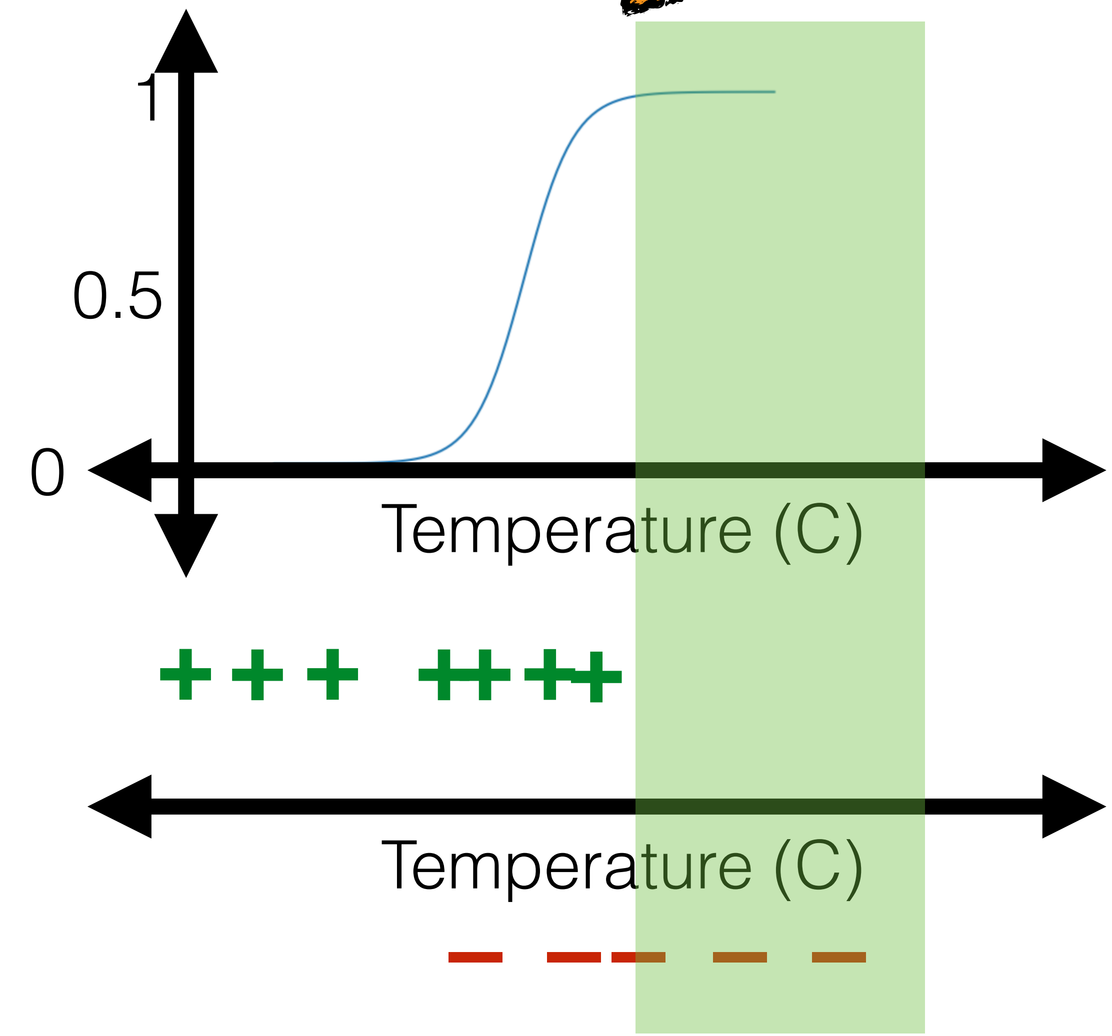

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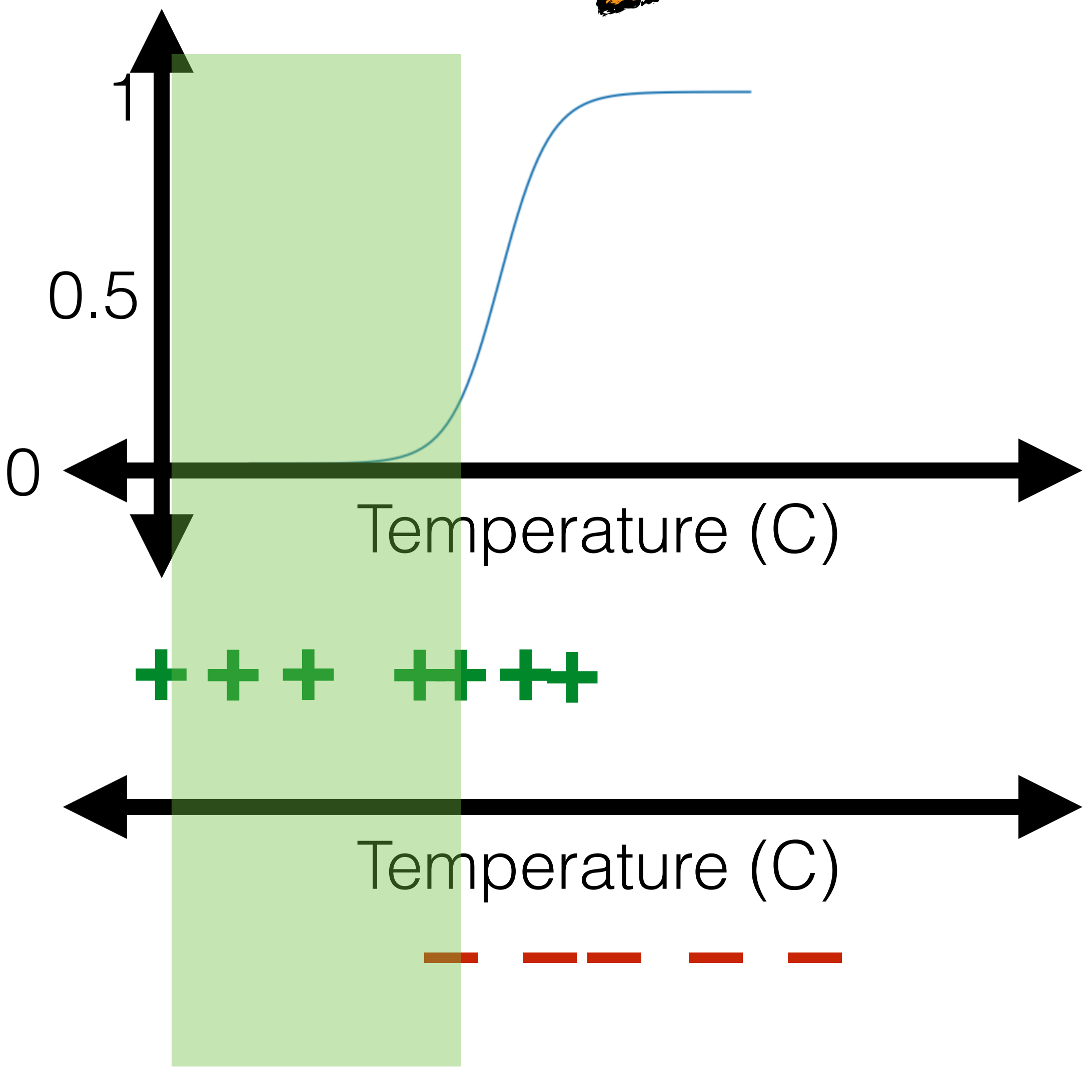

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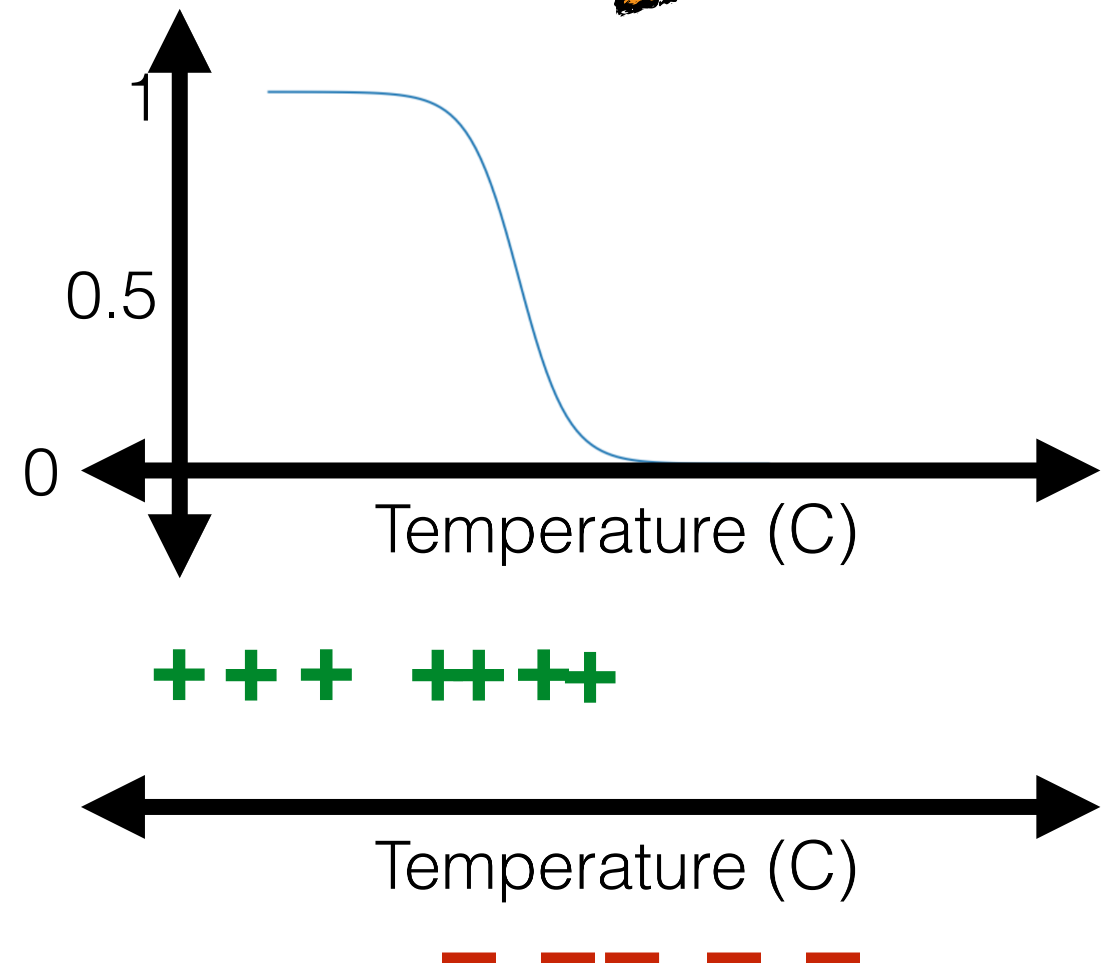

Linear logistic classification

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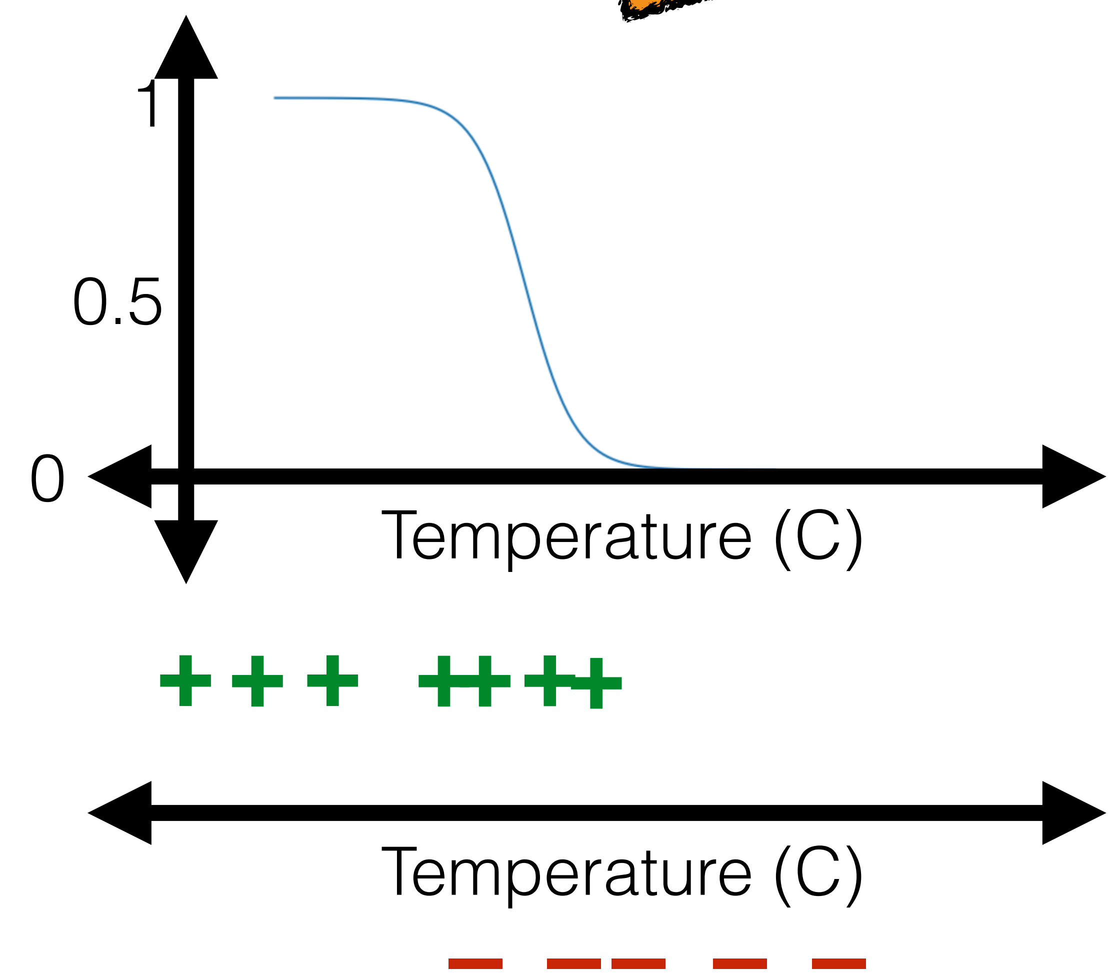
Probability(data)



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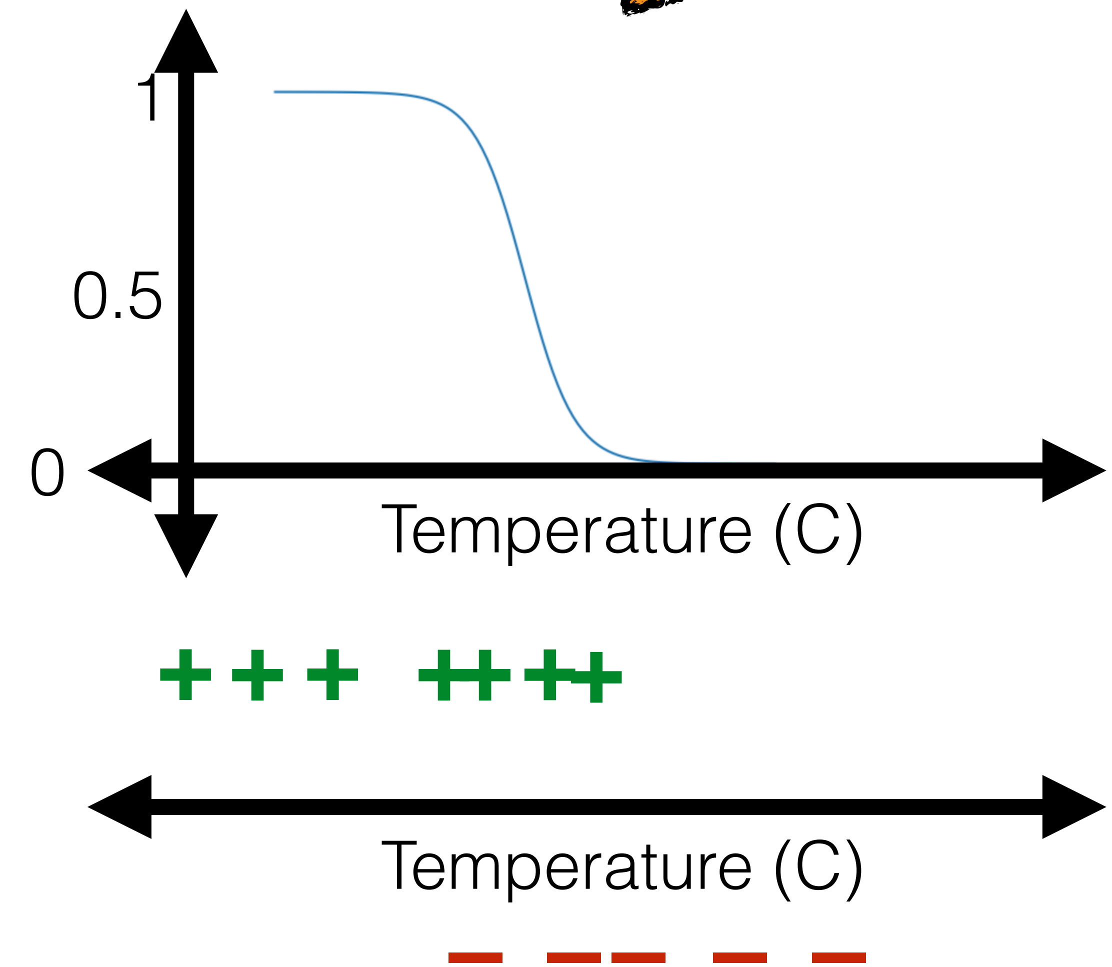

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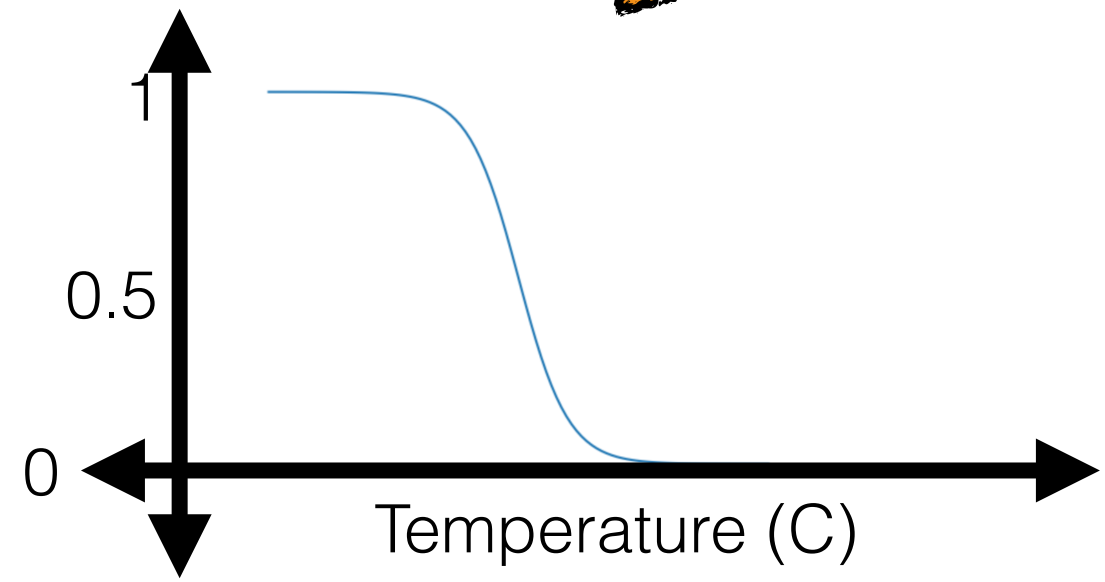

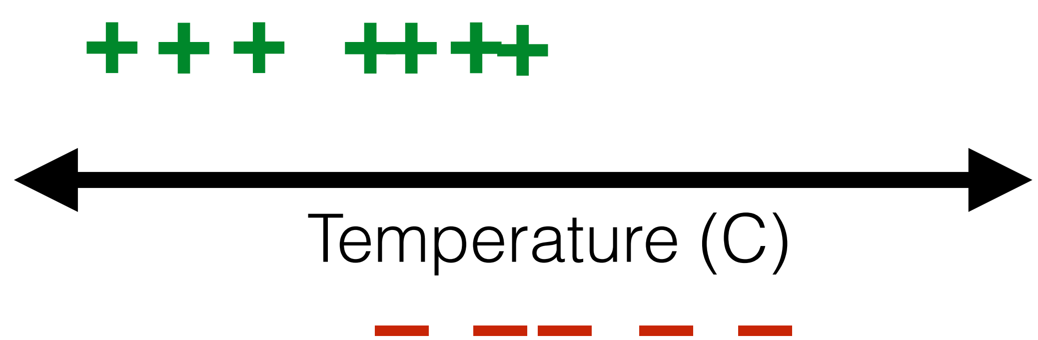

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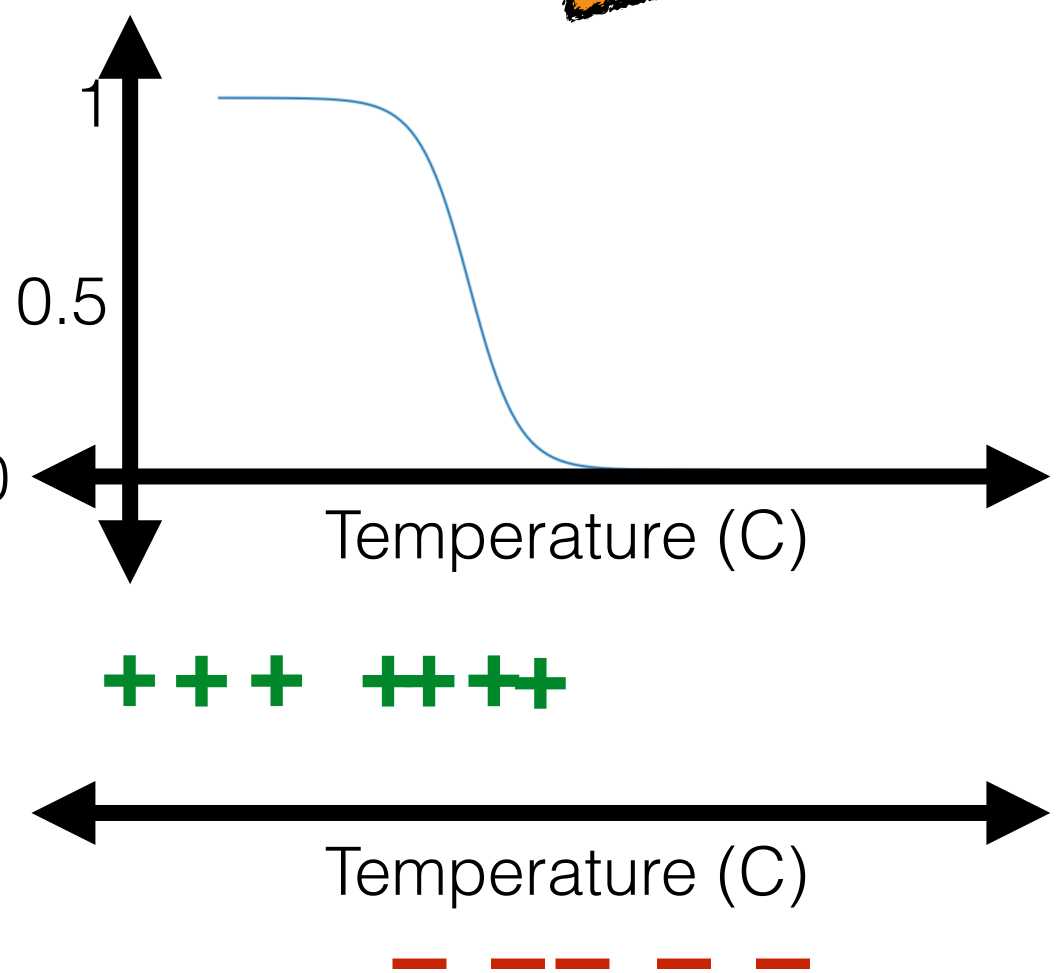

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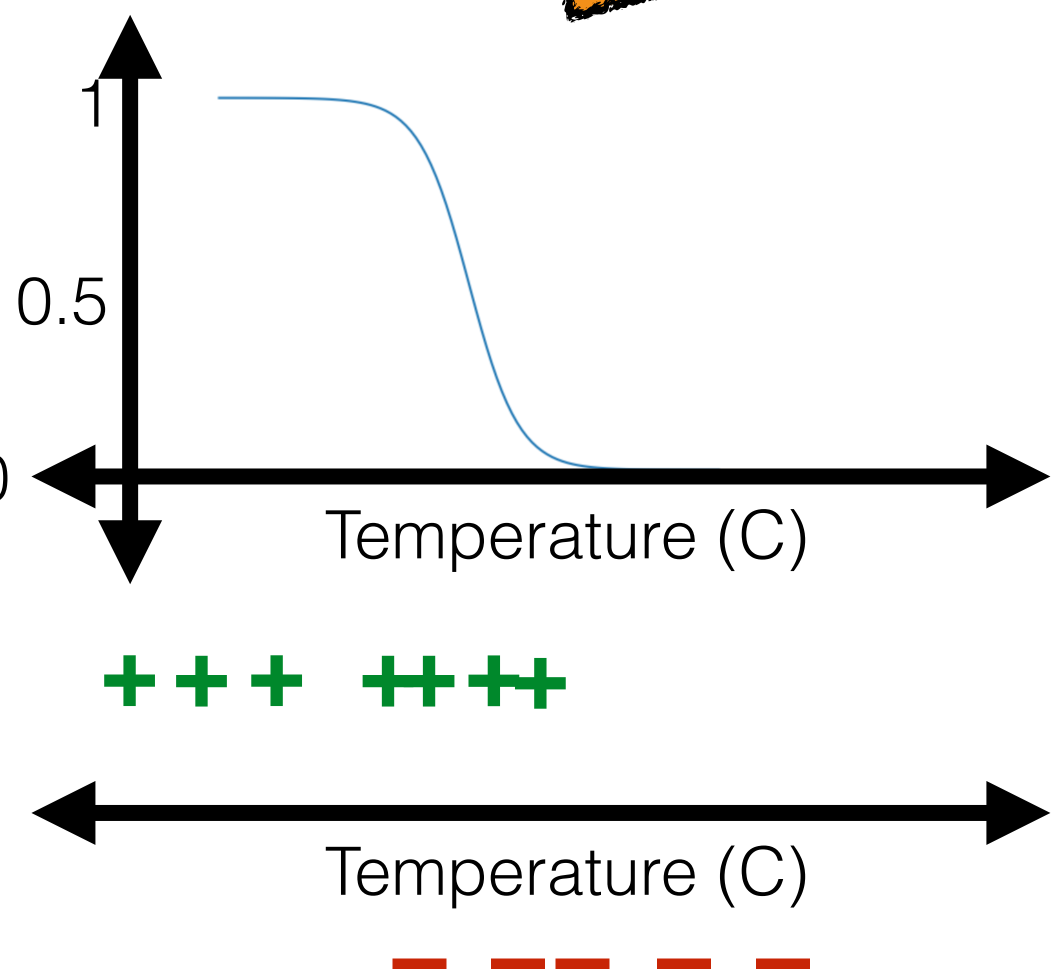

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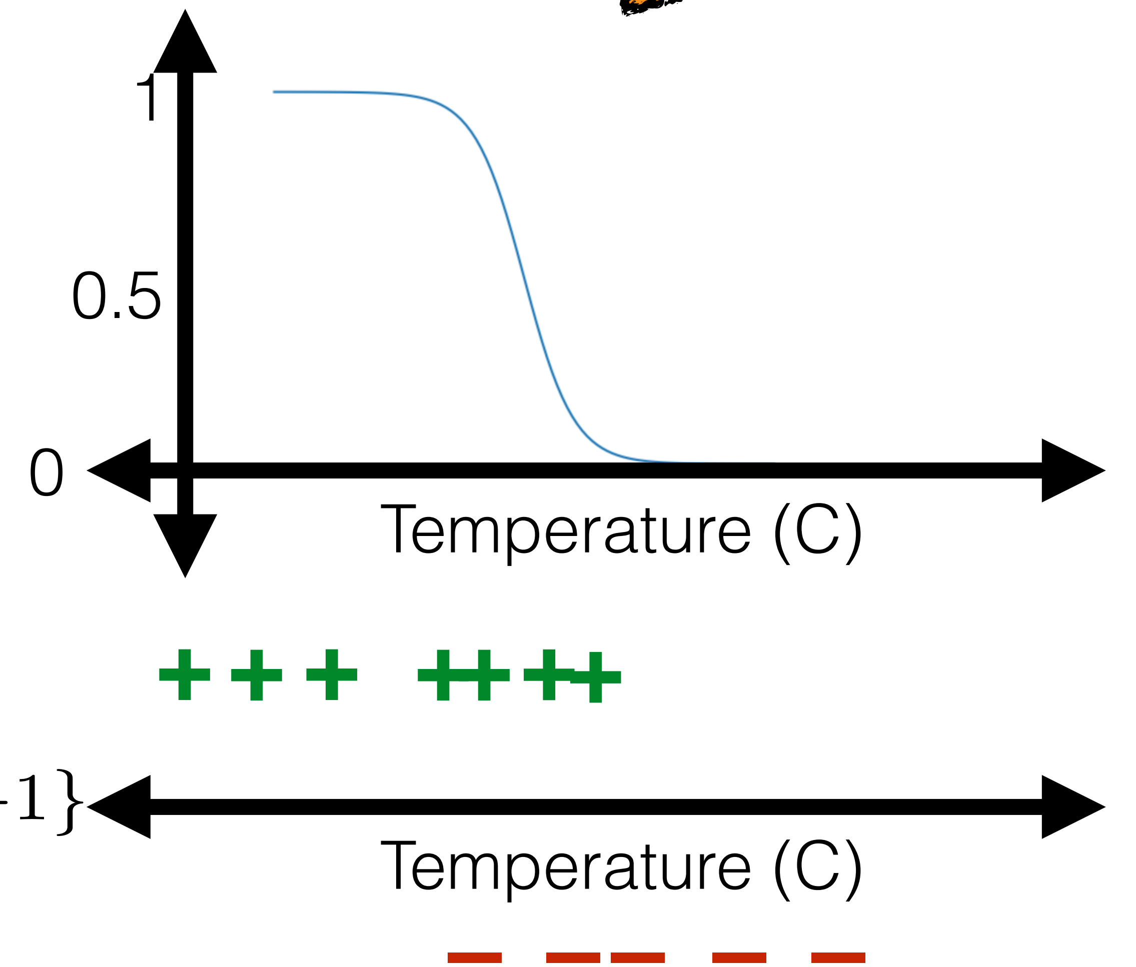

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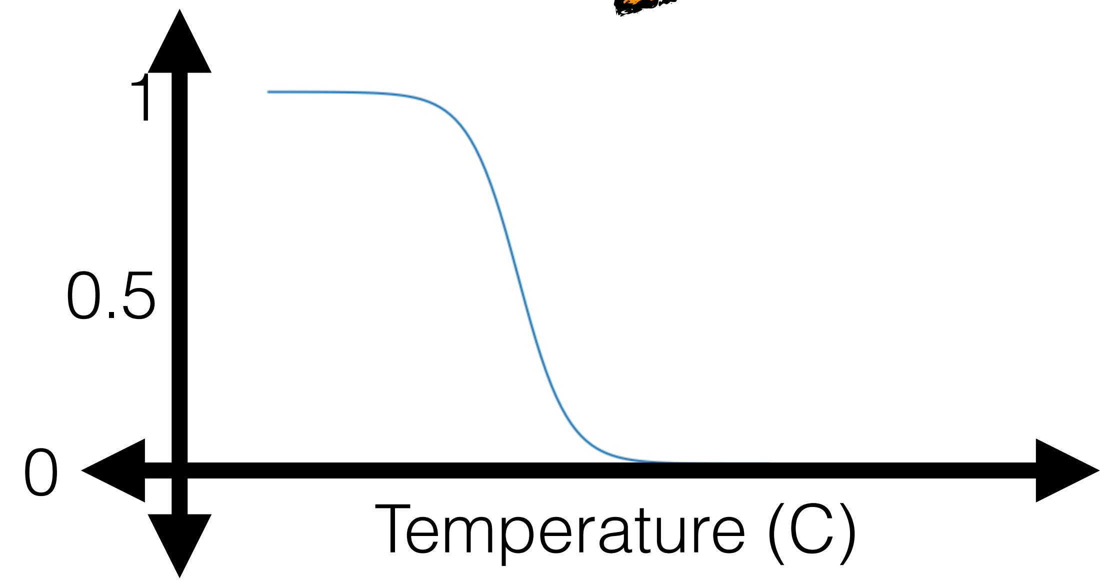

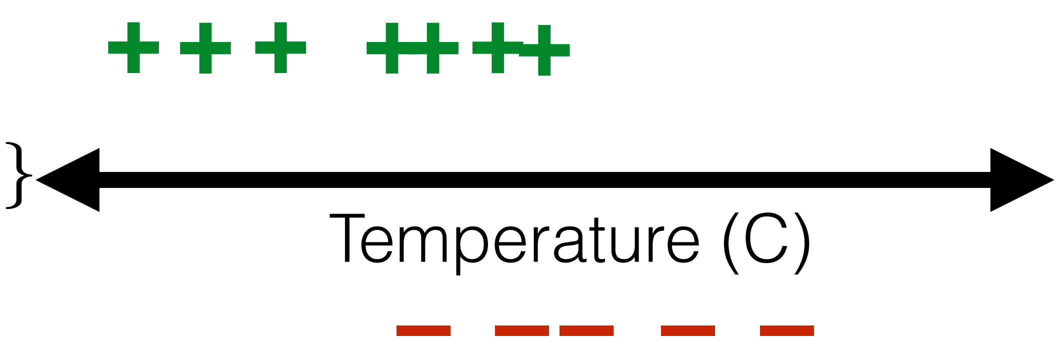

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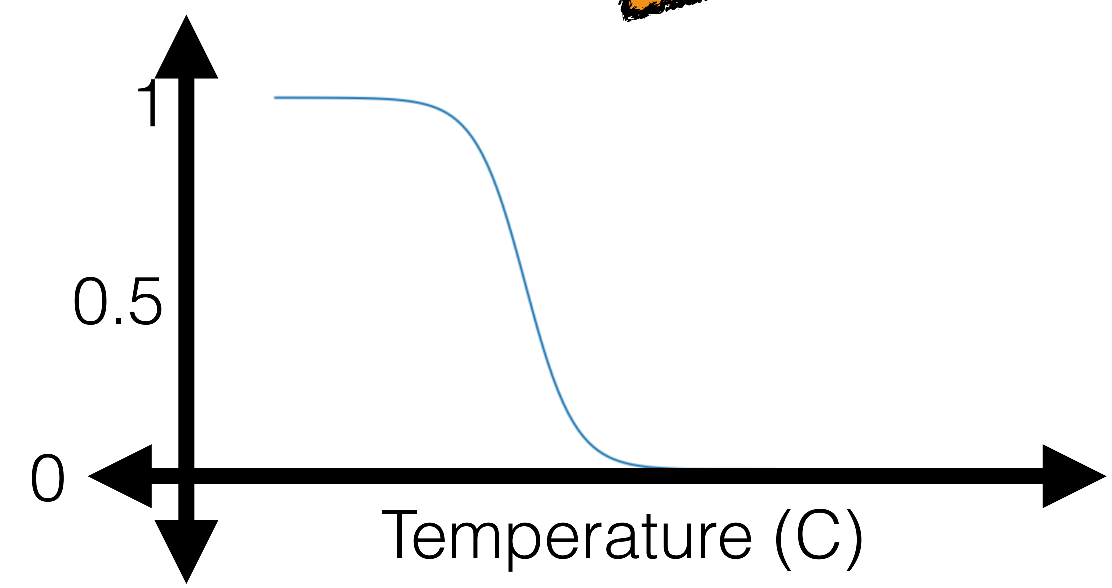

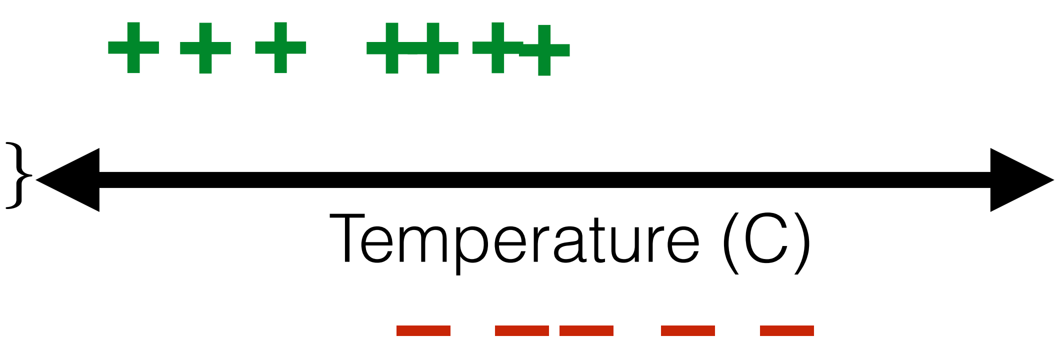

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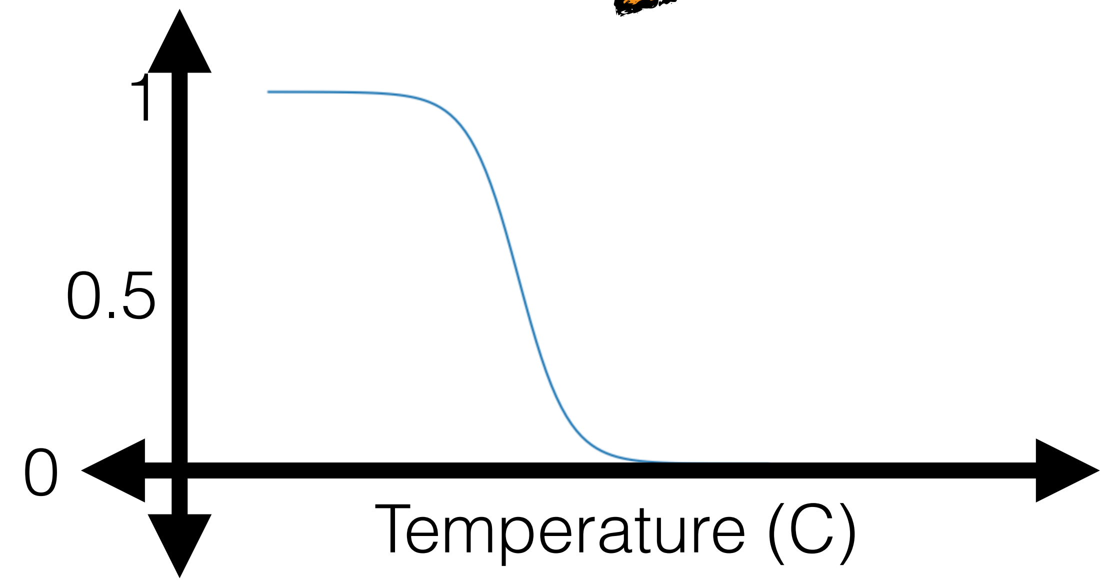

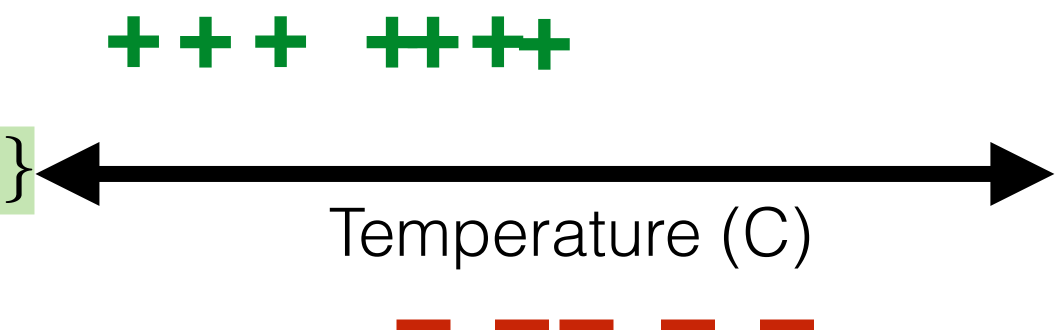

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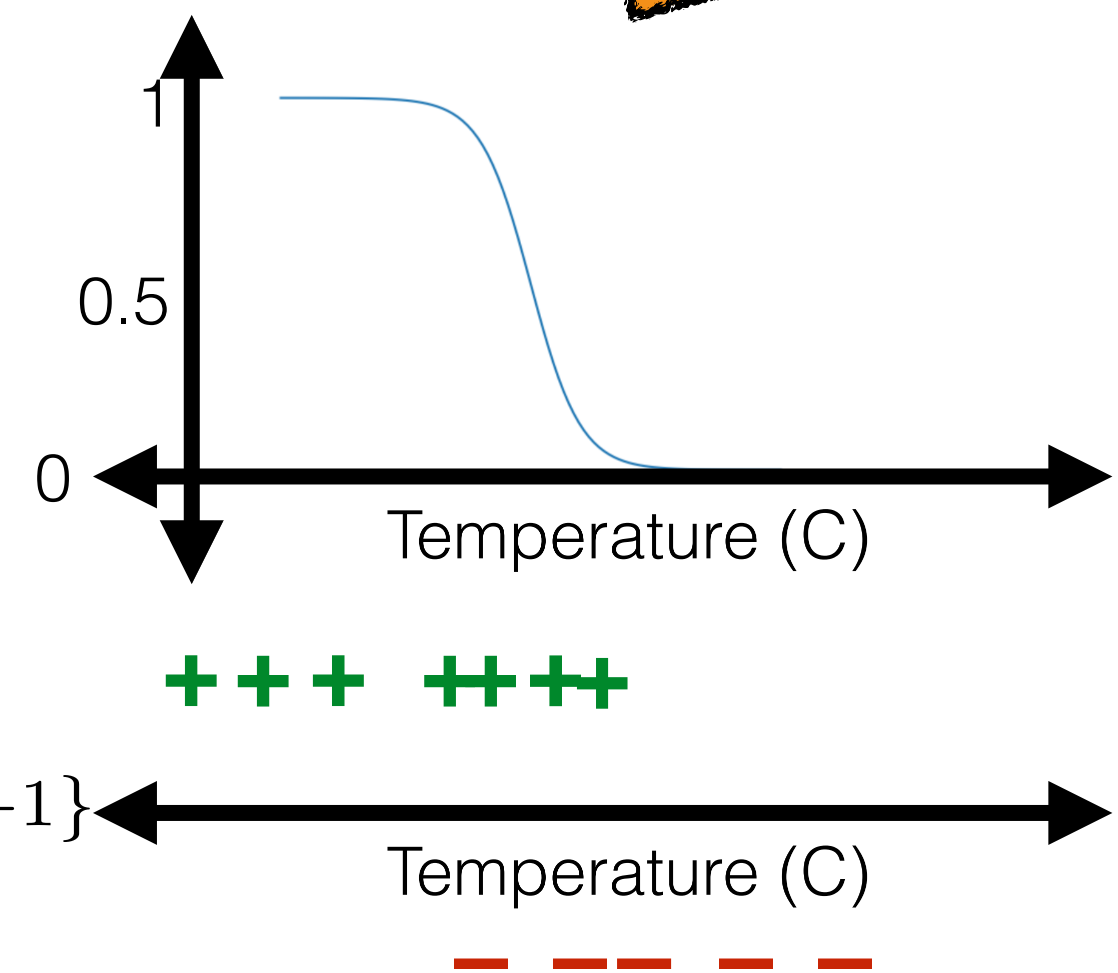

Linear logistic classification

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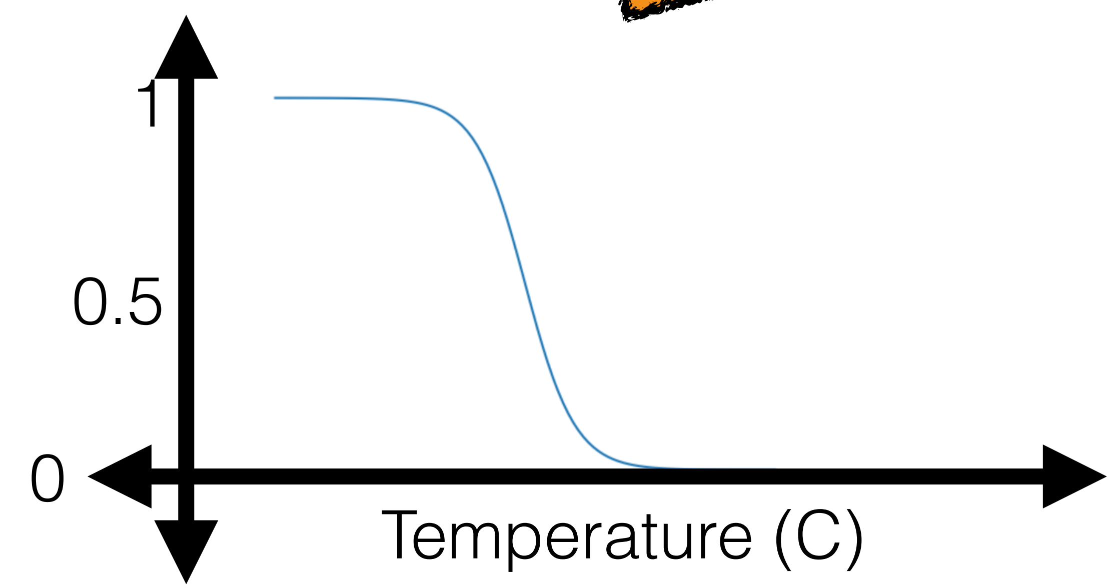

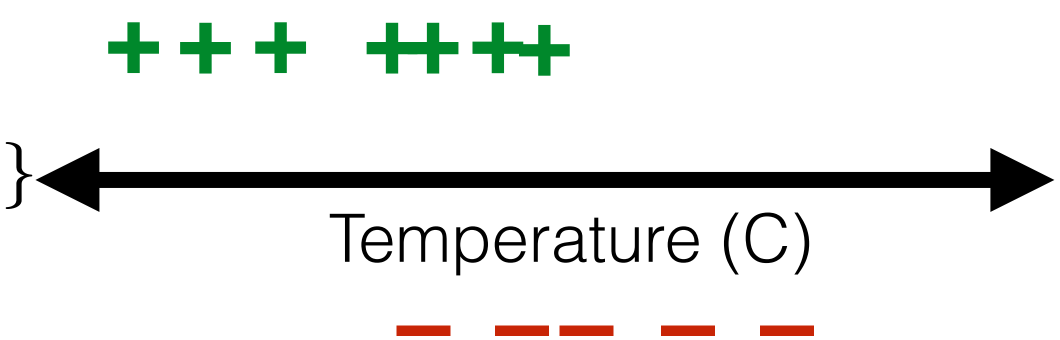

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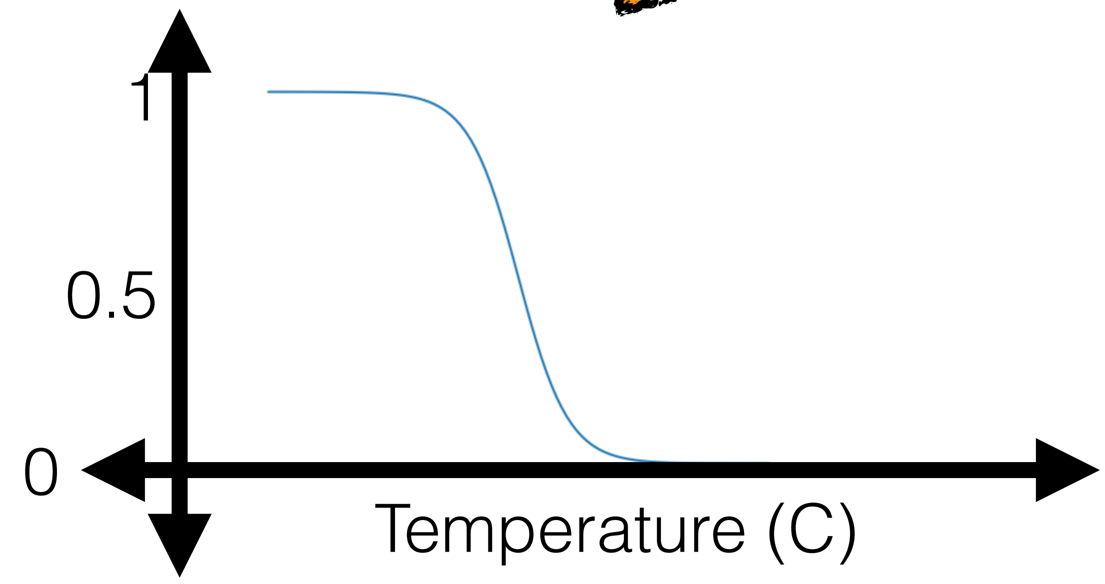

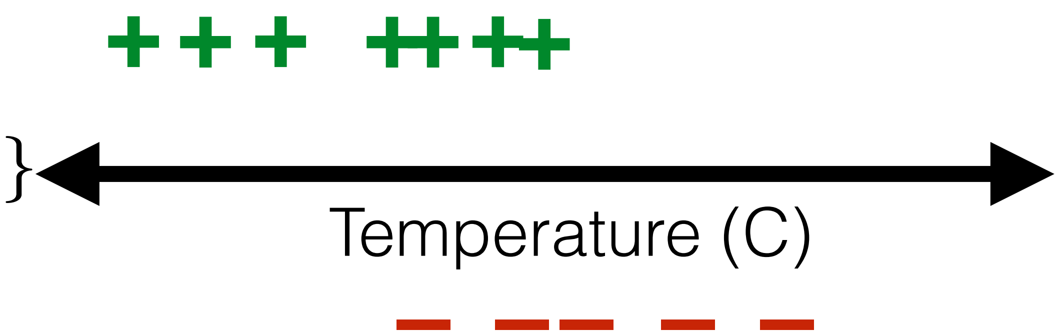

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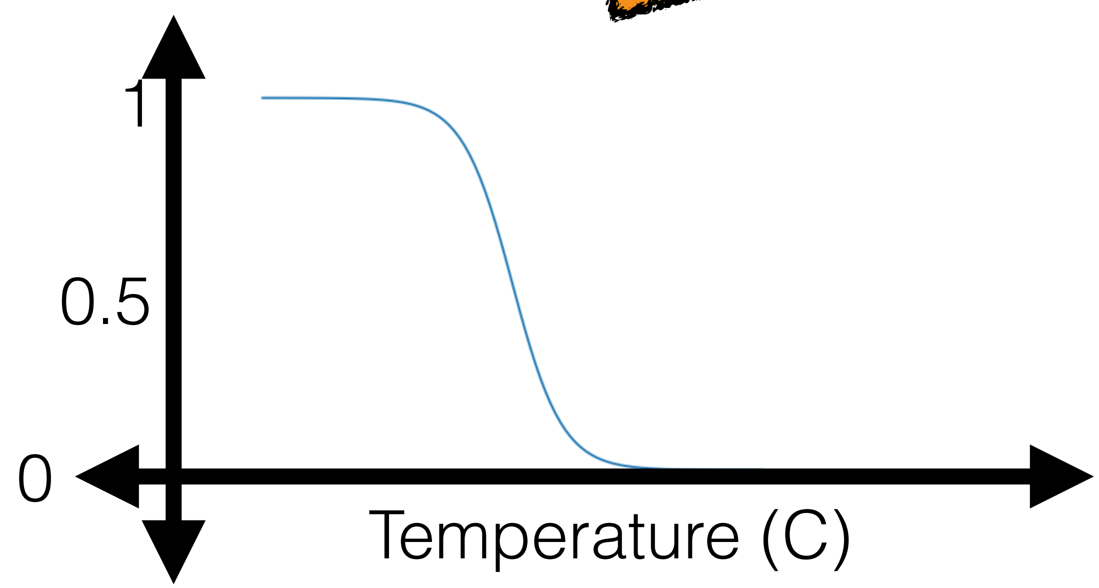

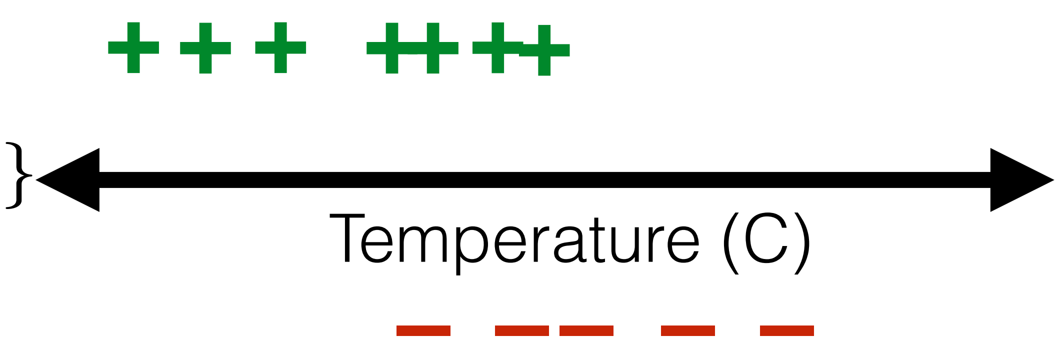

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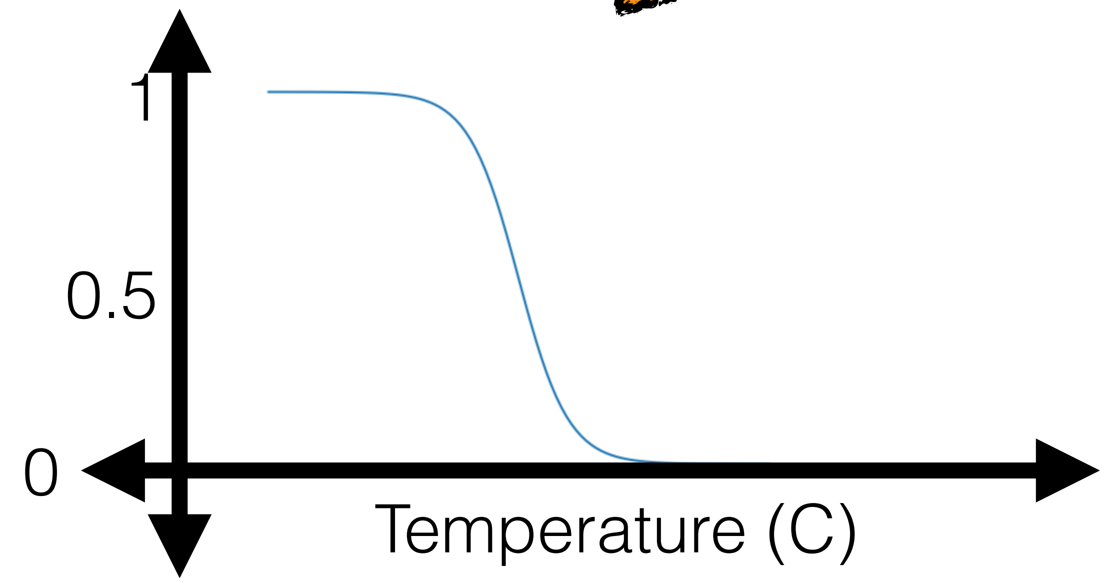

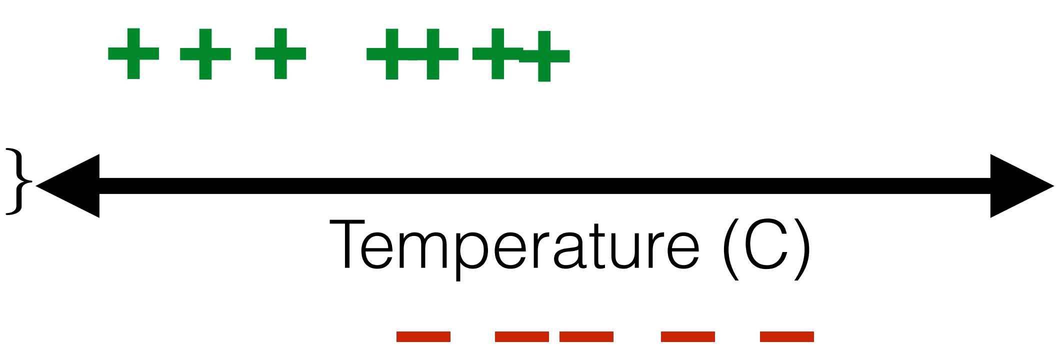

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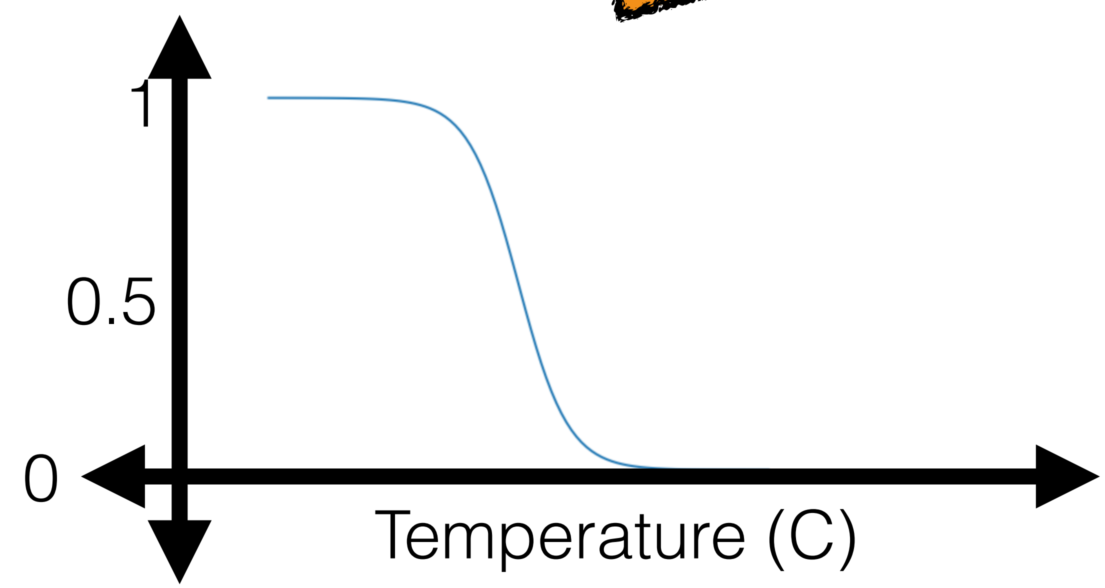

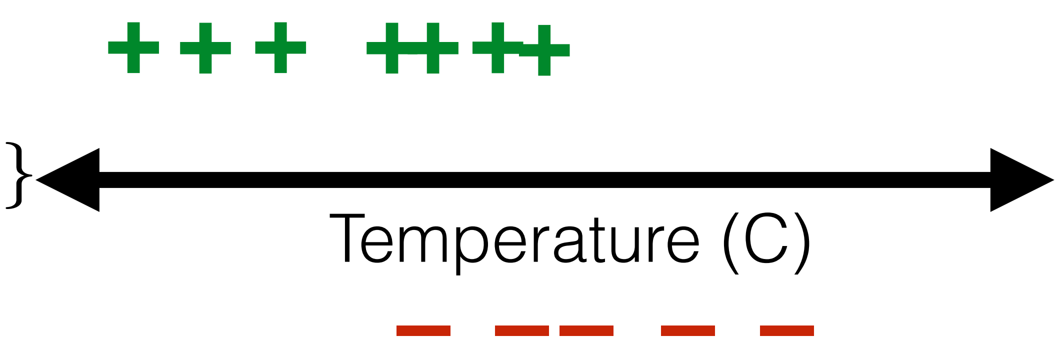

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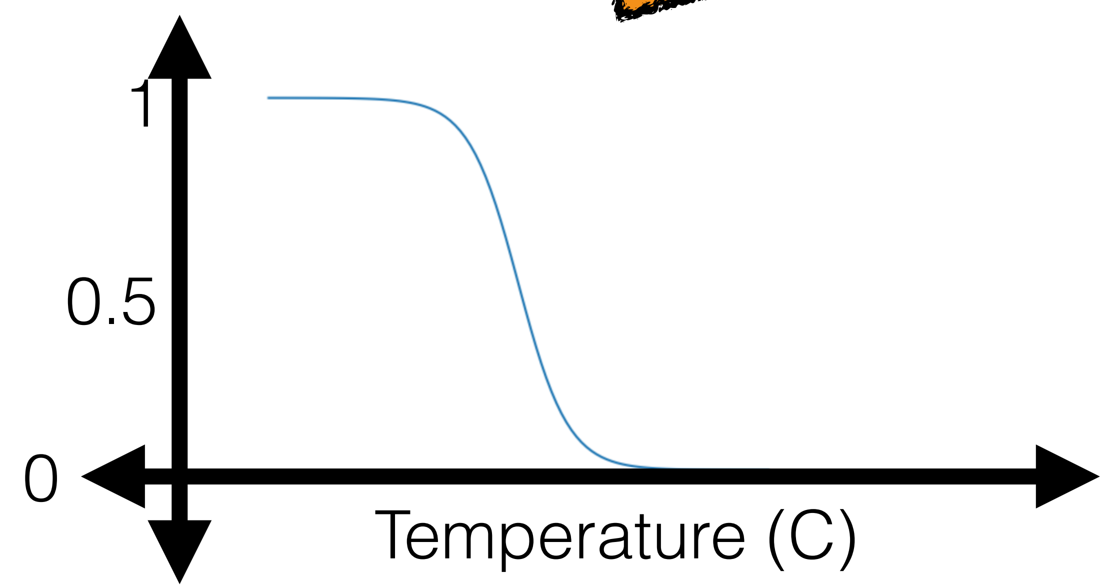

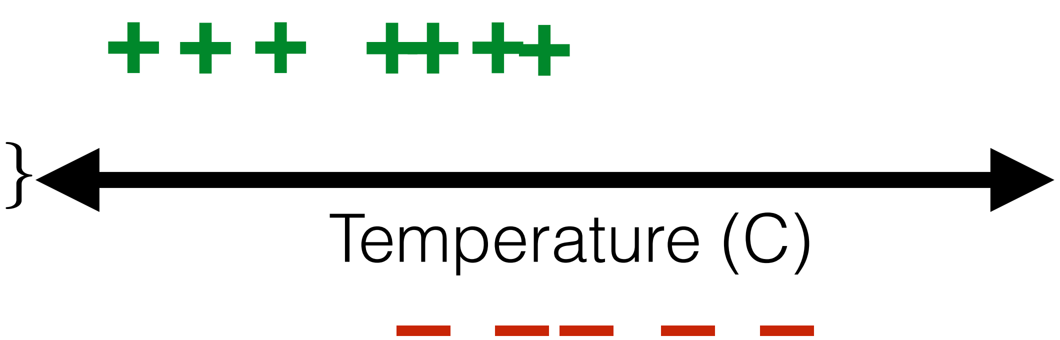

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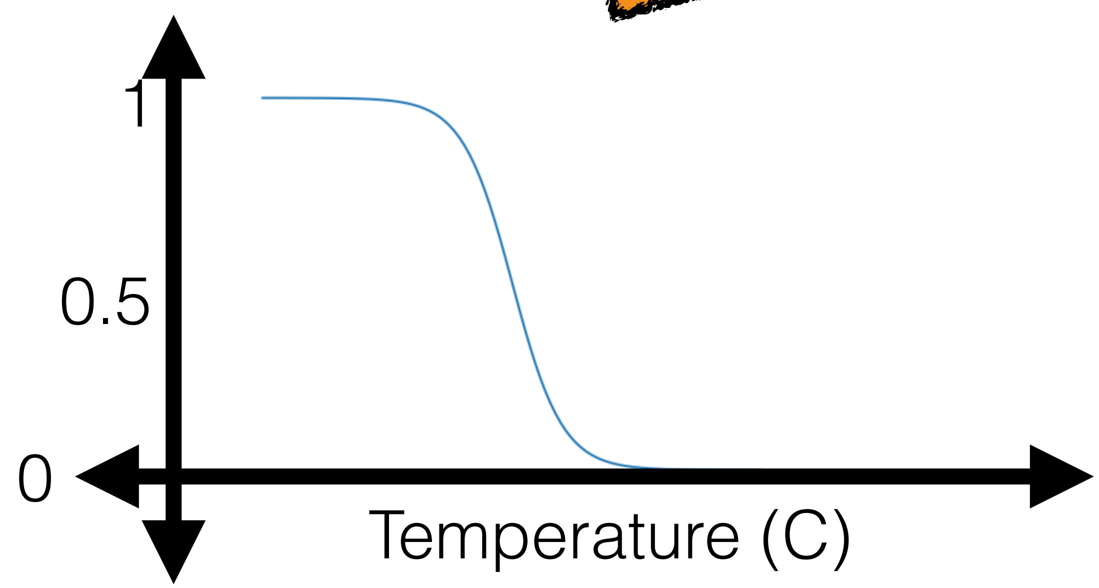

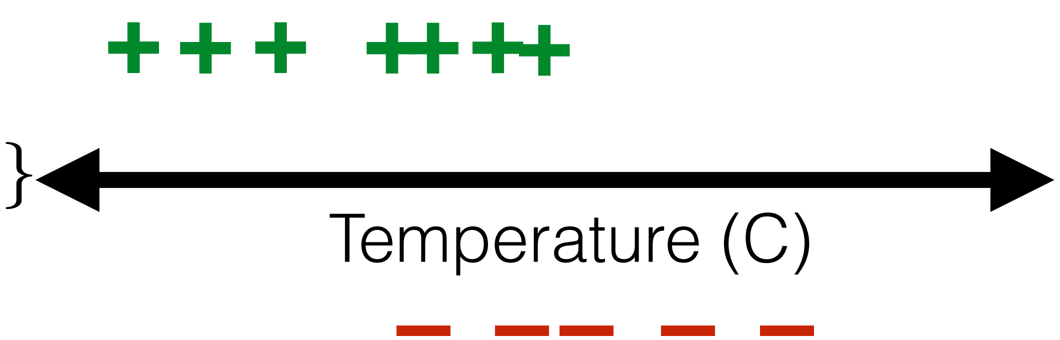

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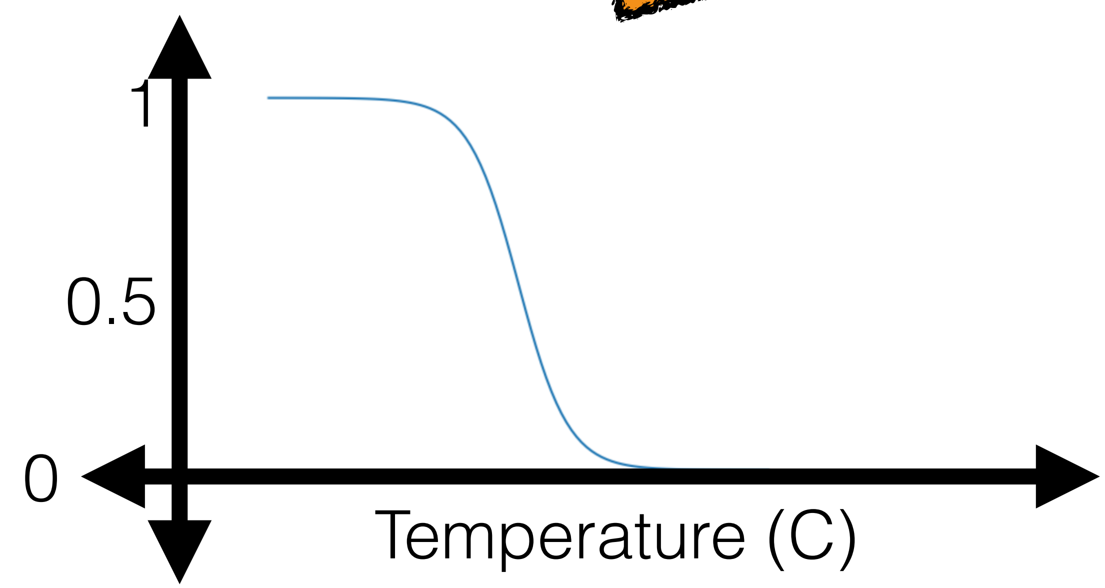

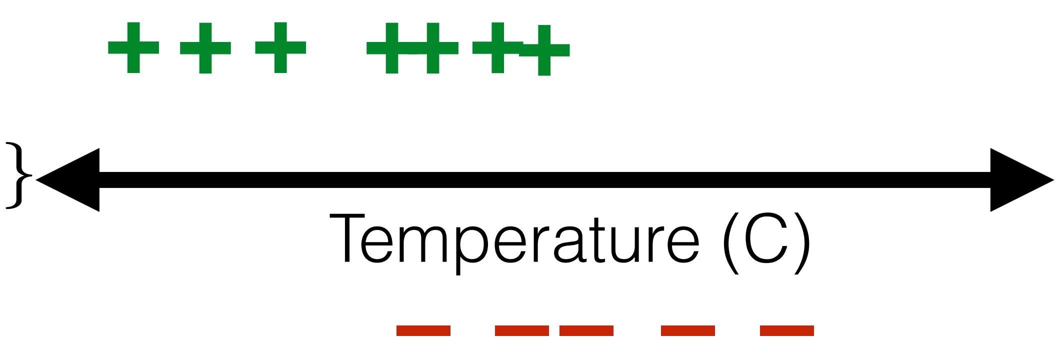

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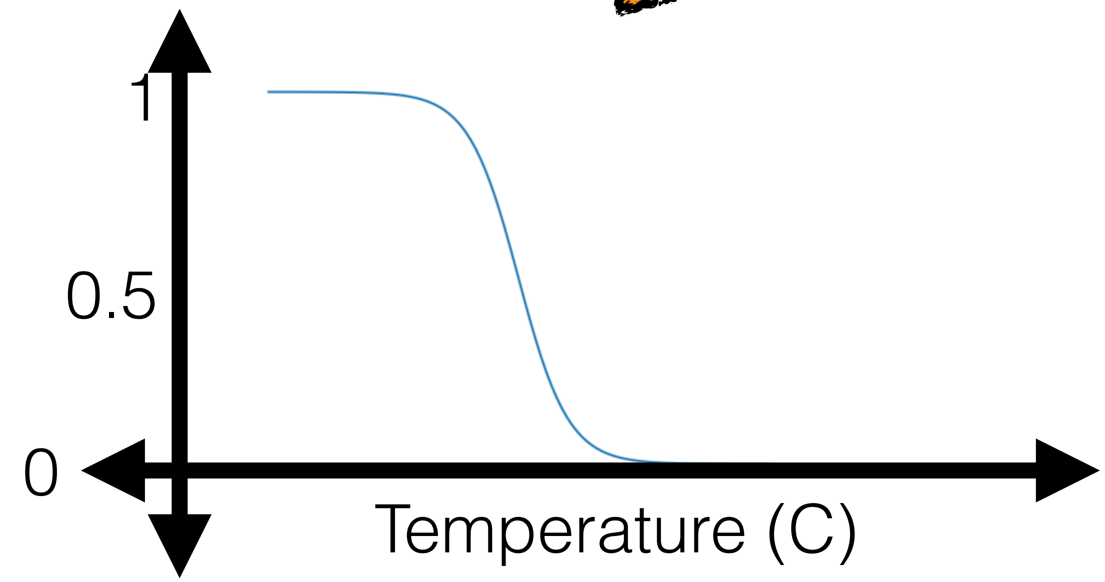

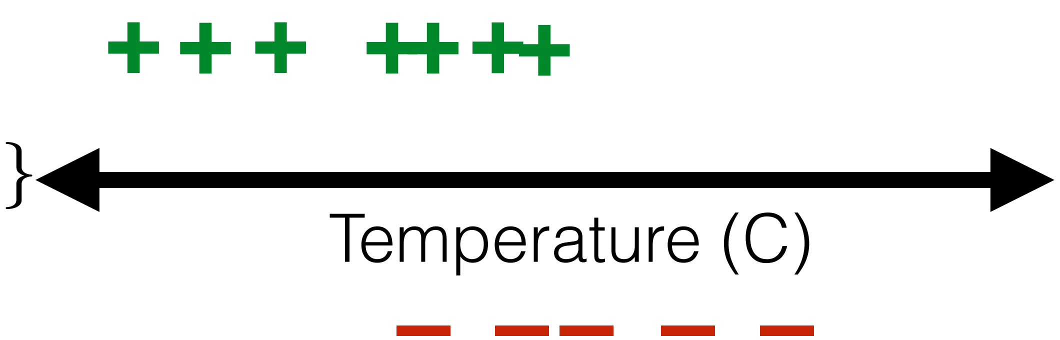

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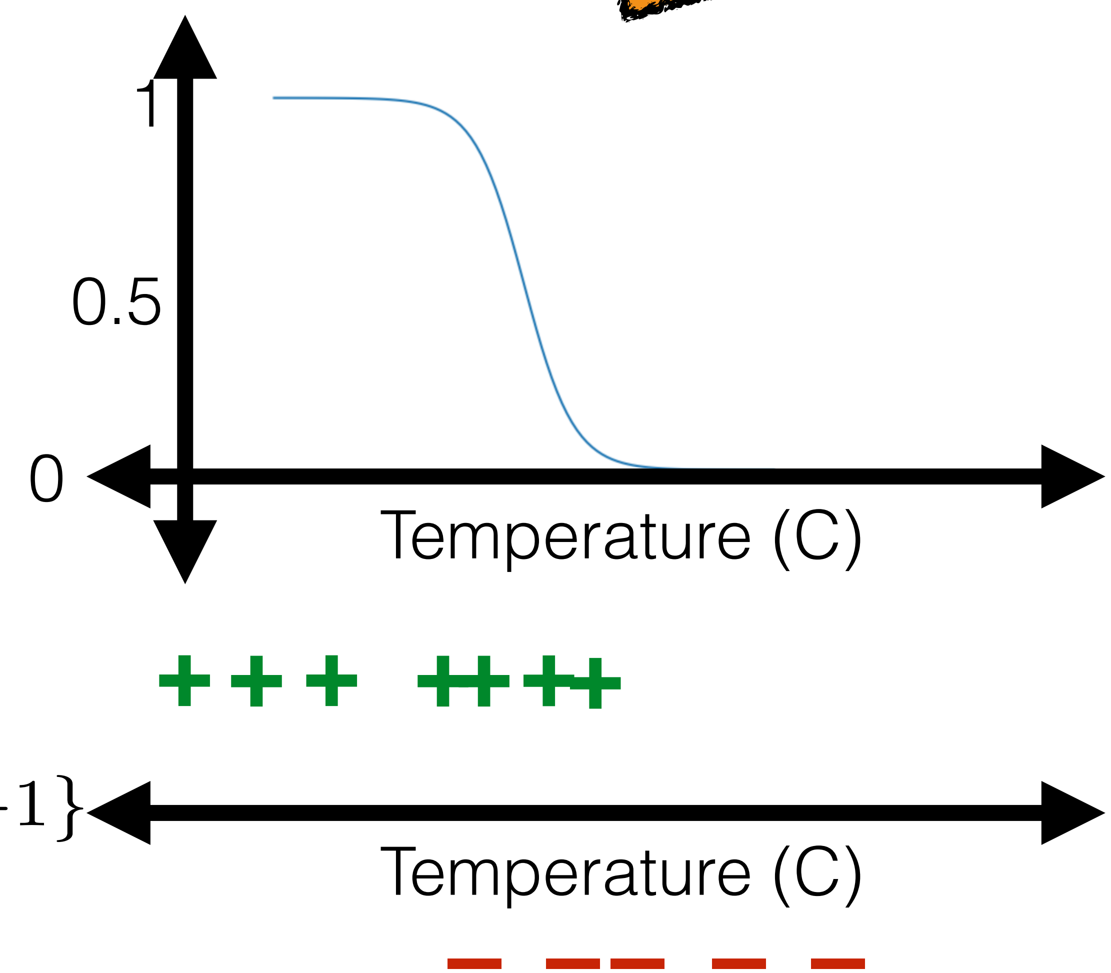

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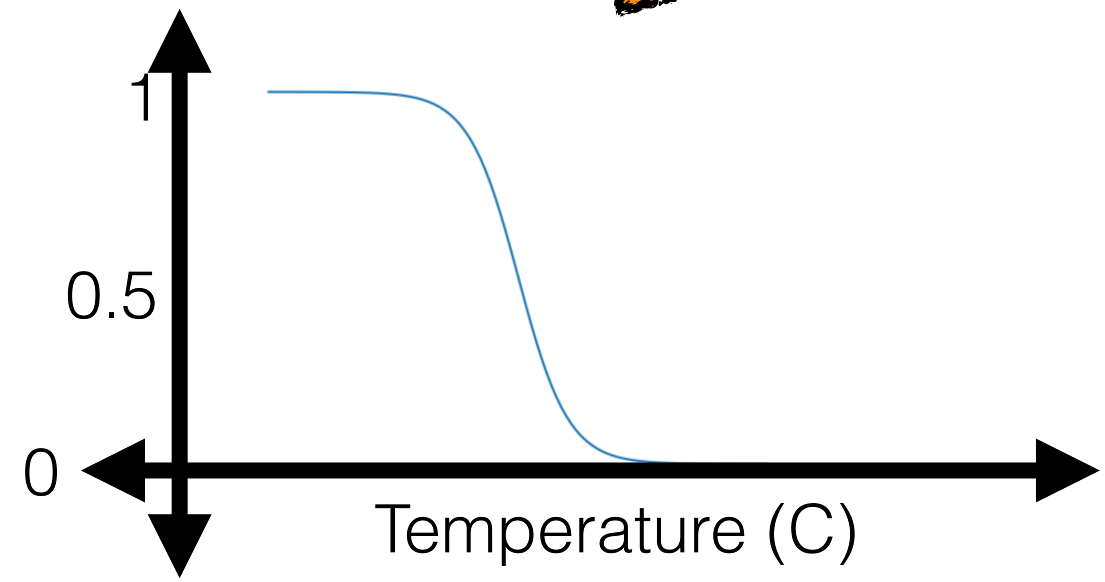

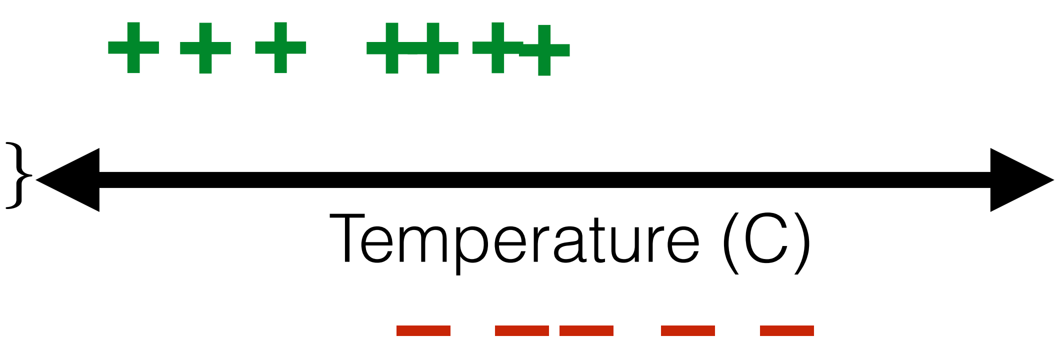

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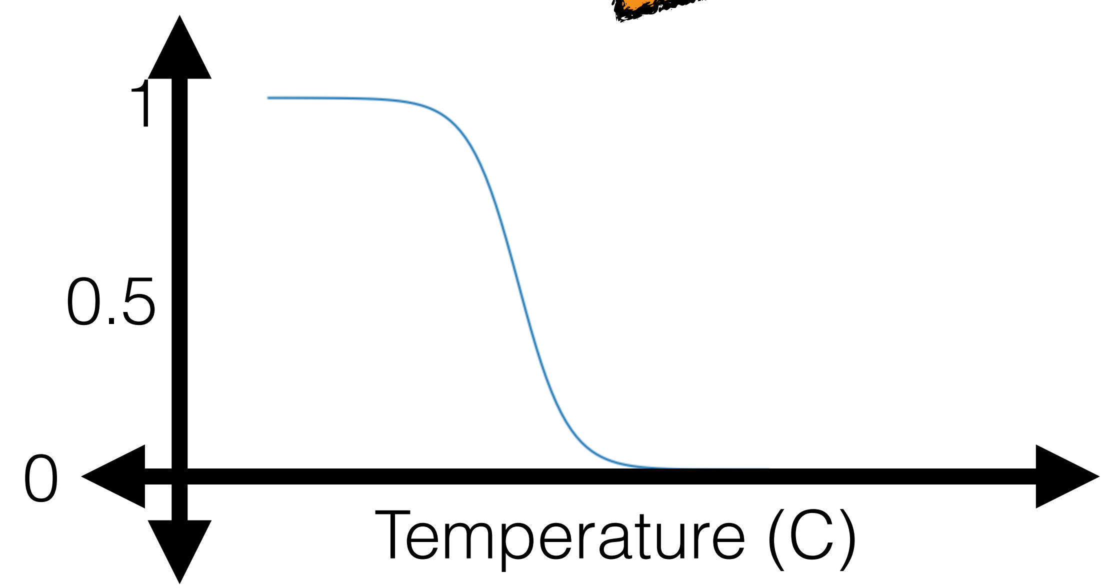

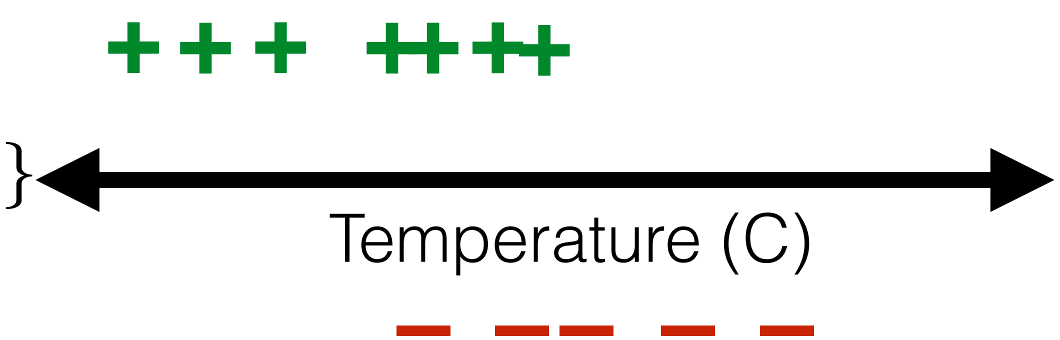

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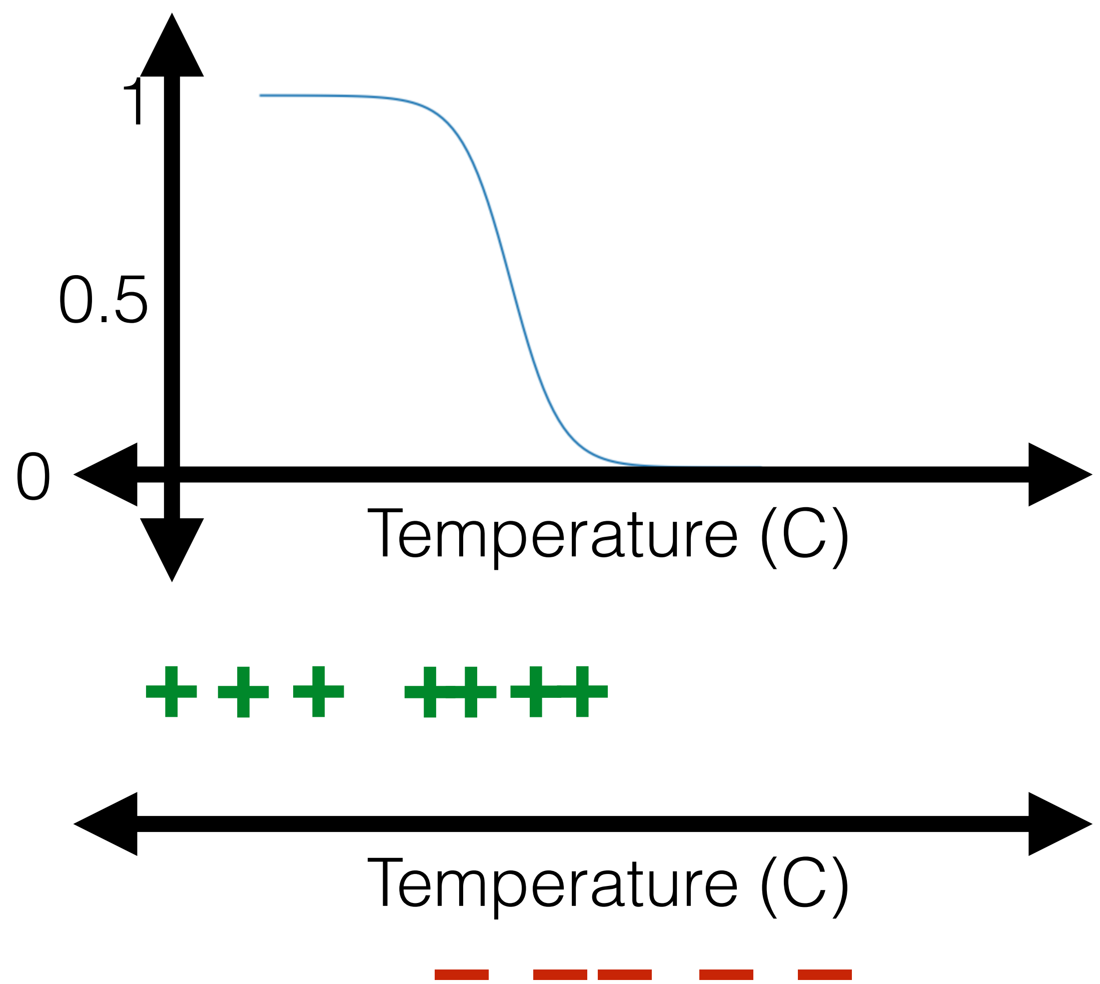

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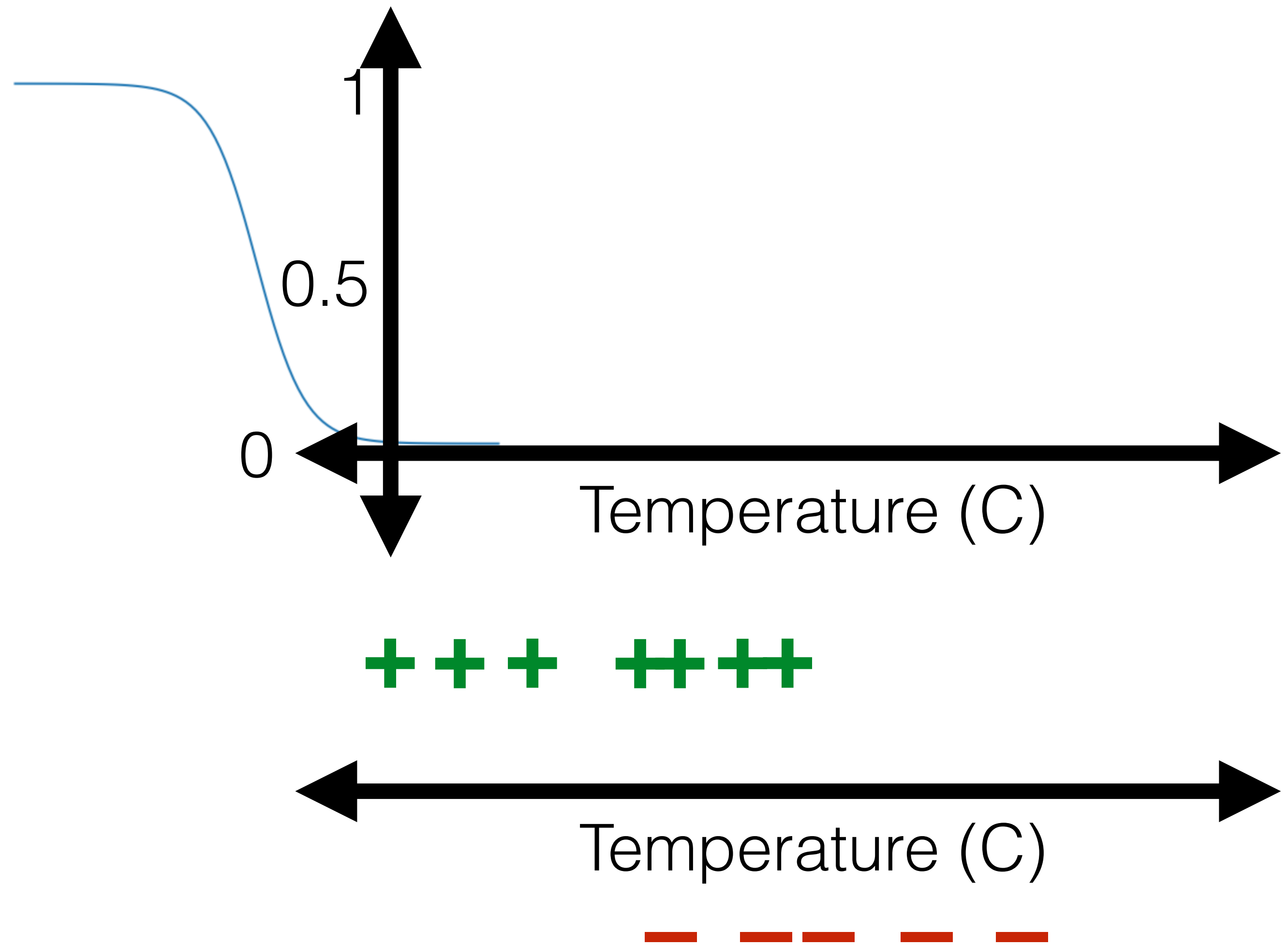

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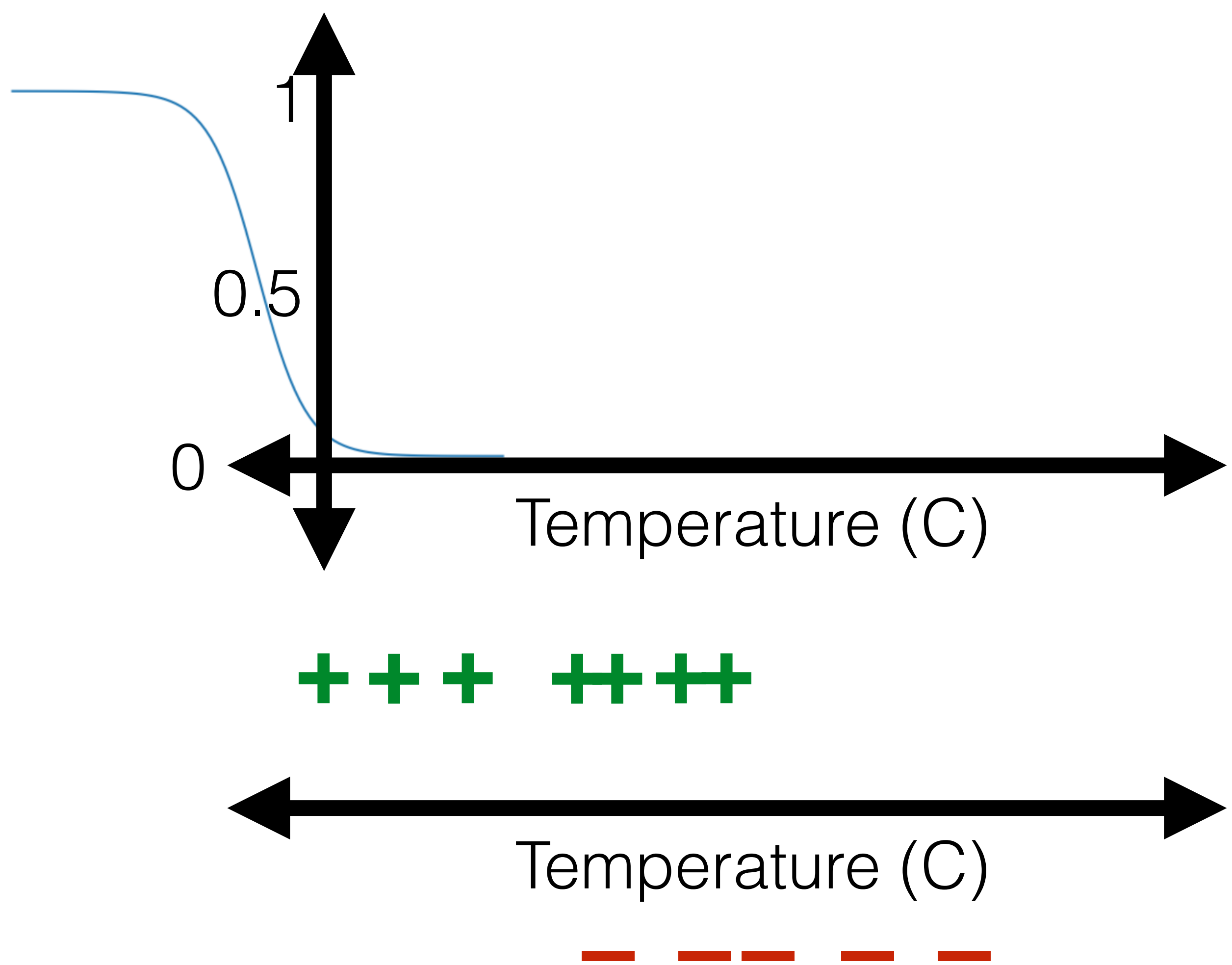

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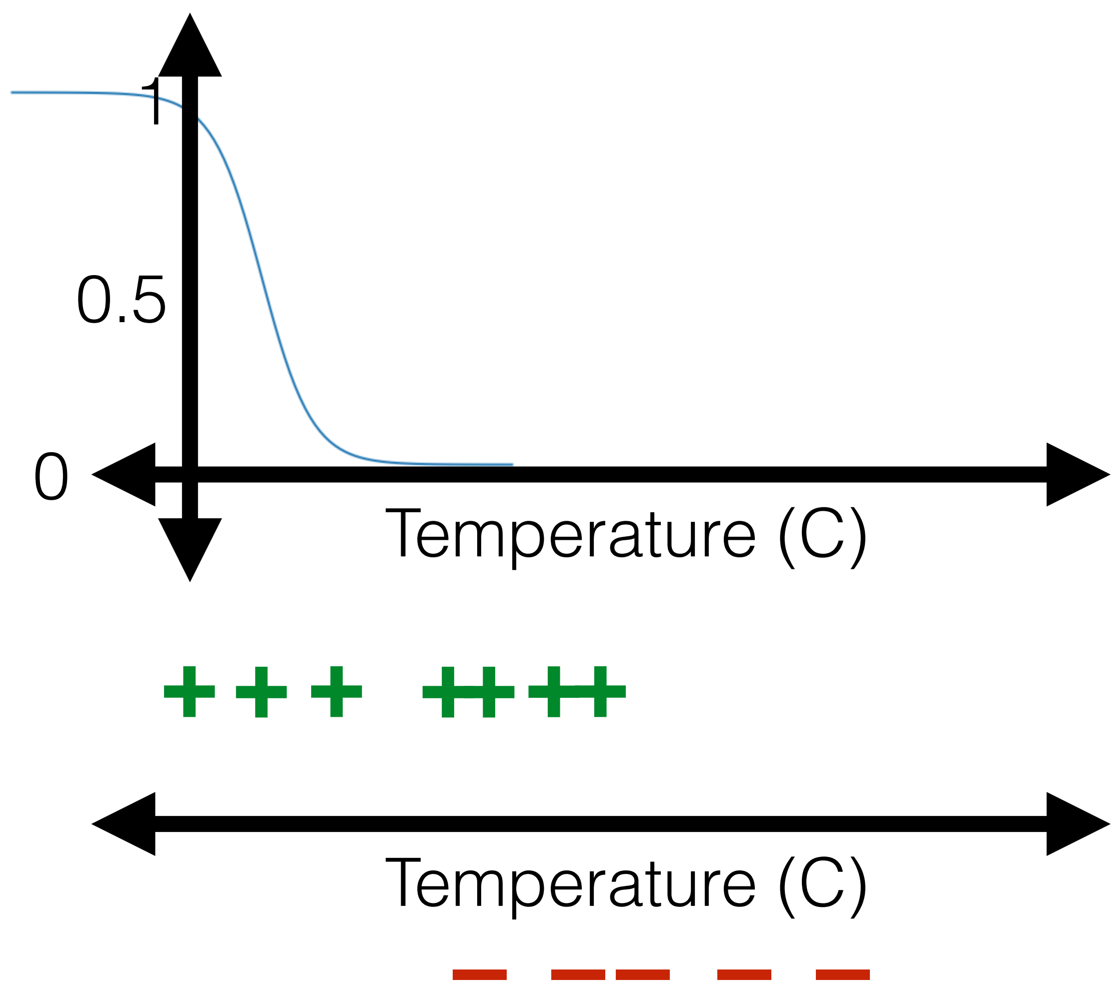

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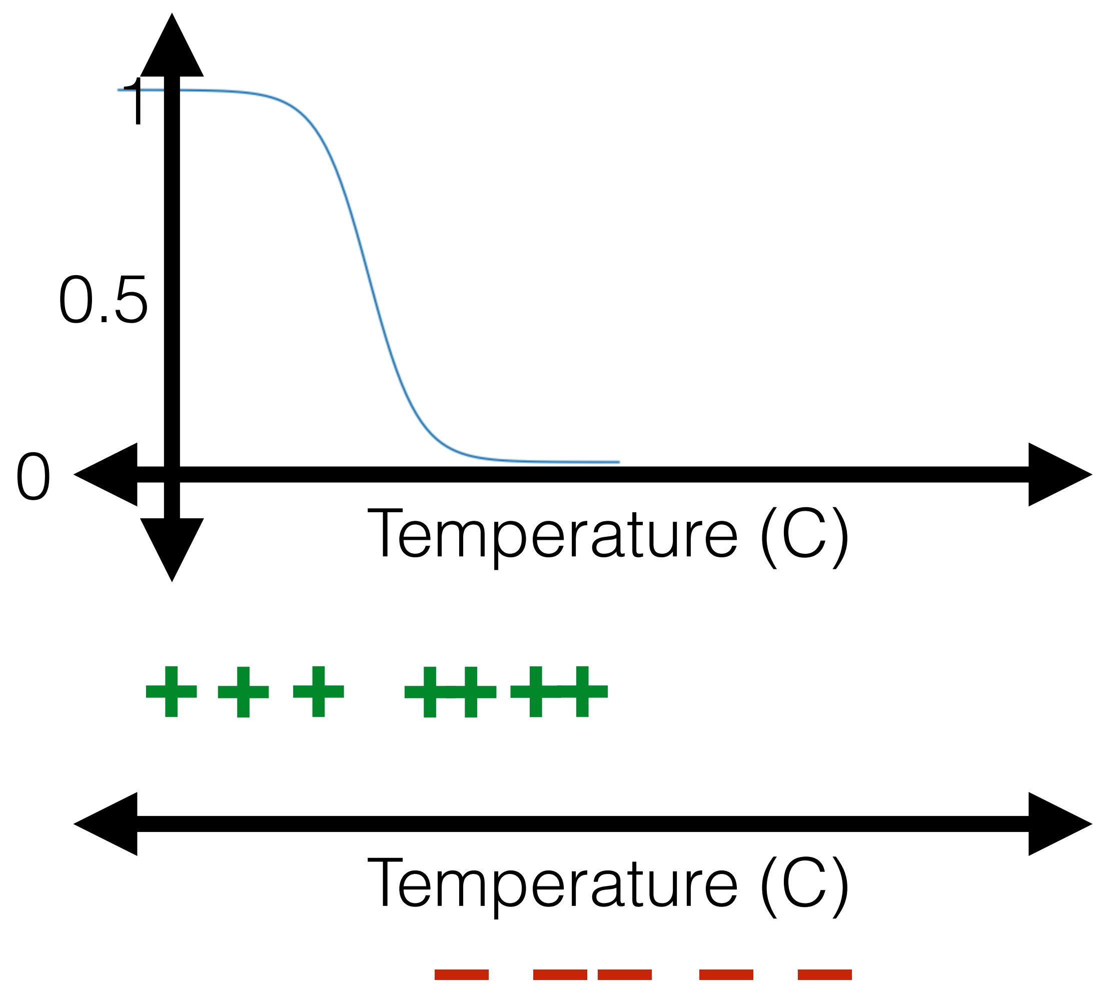

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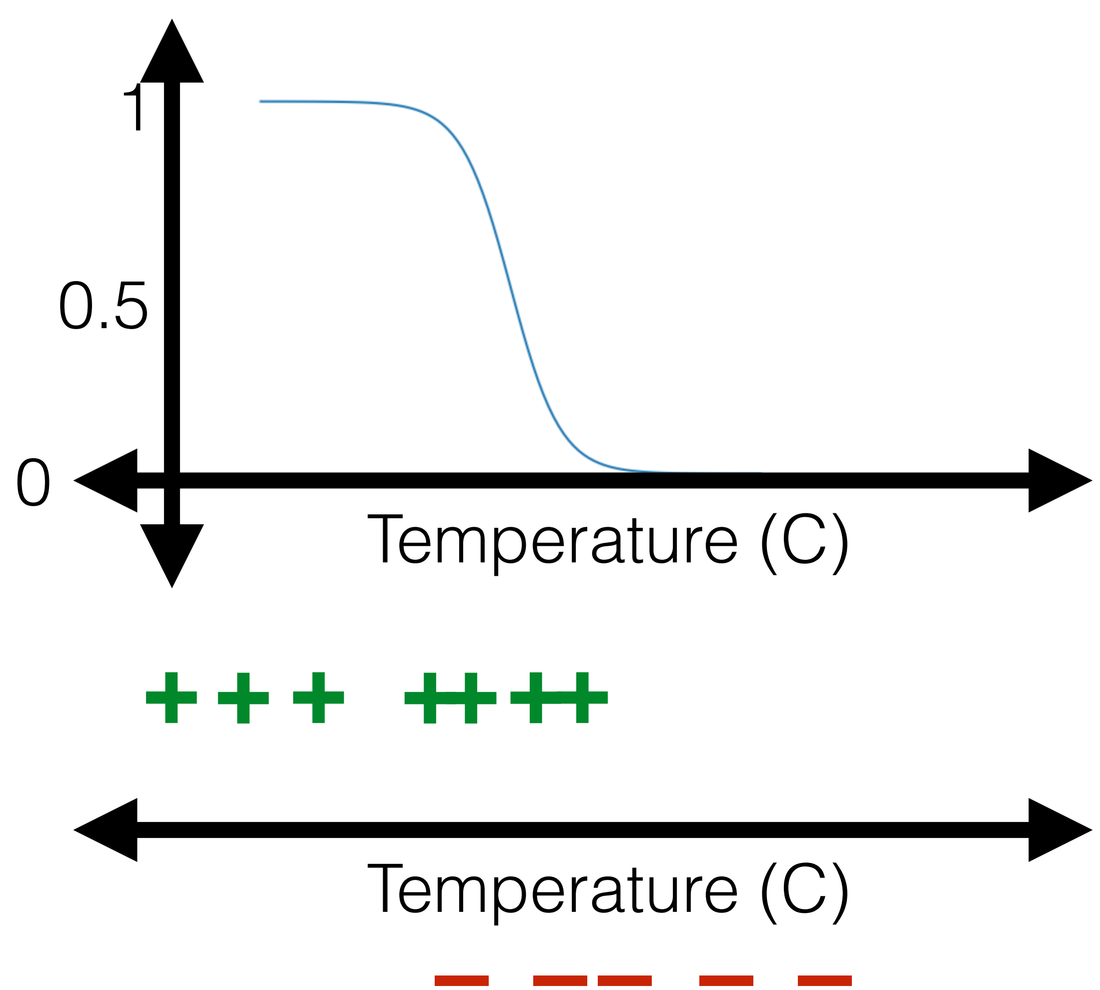

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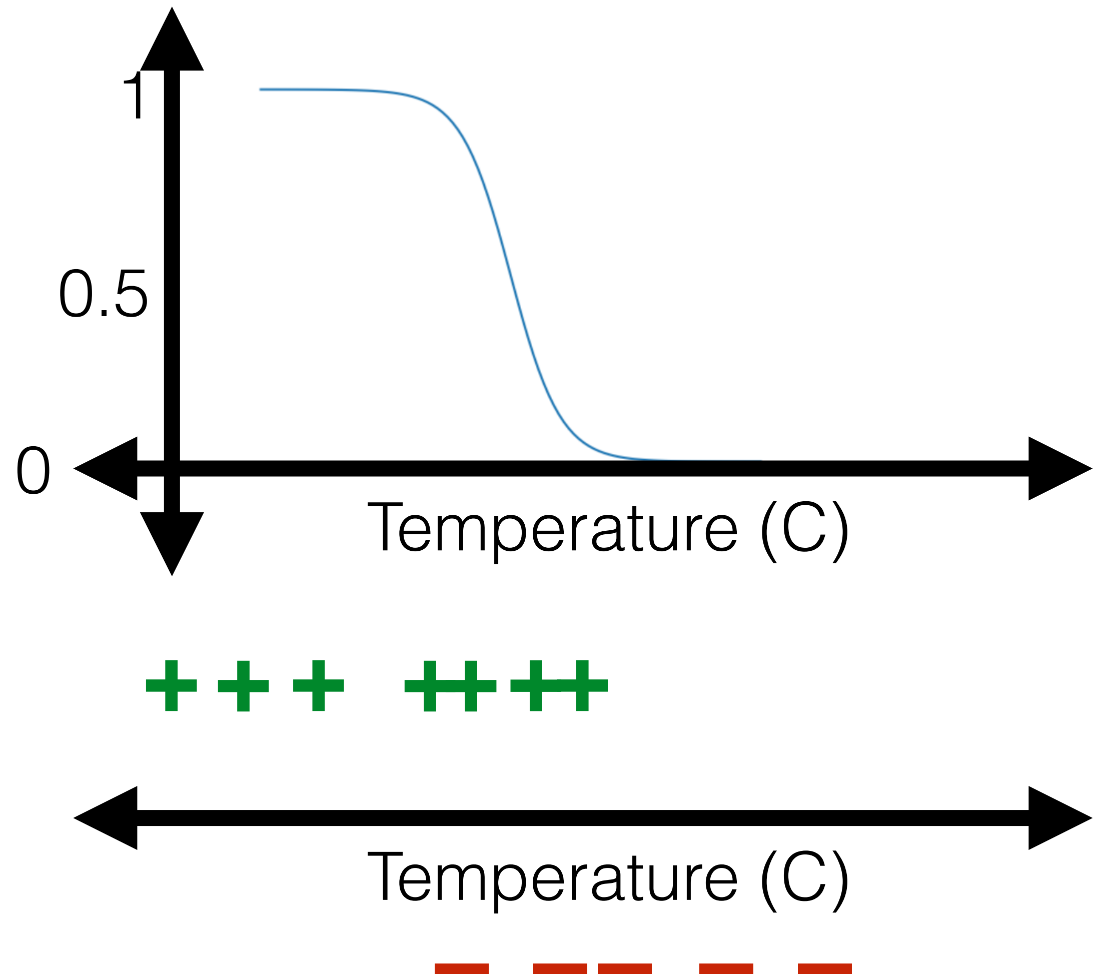

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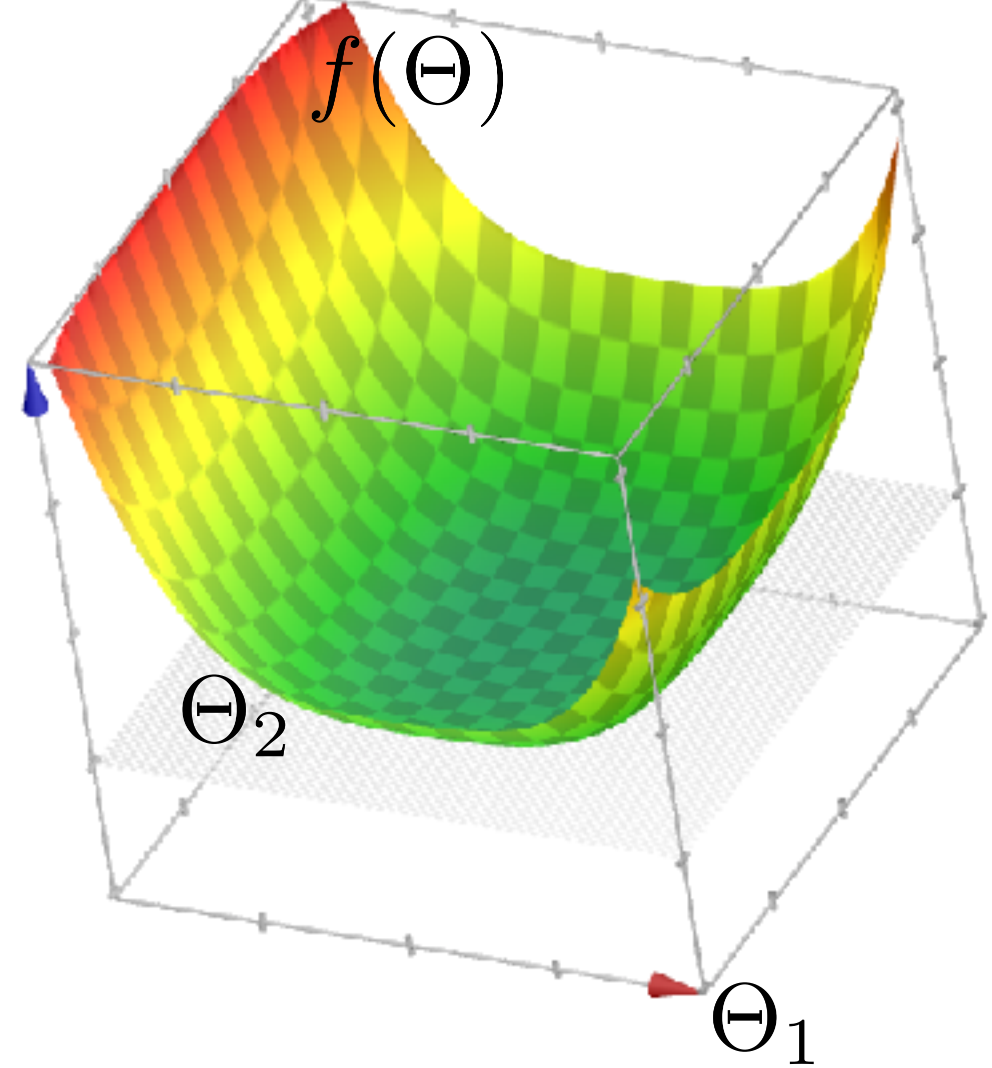

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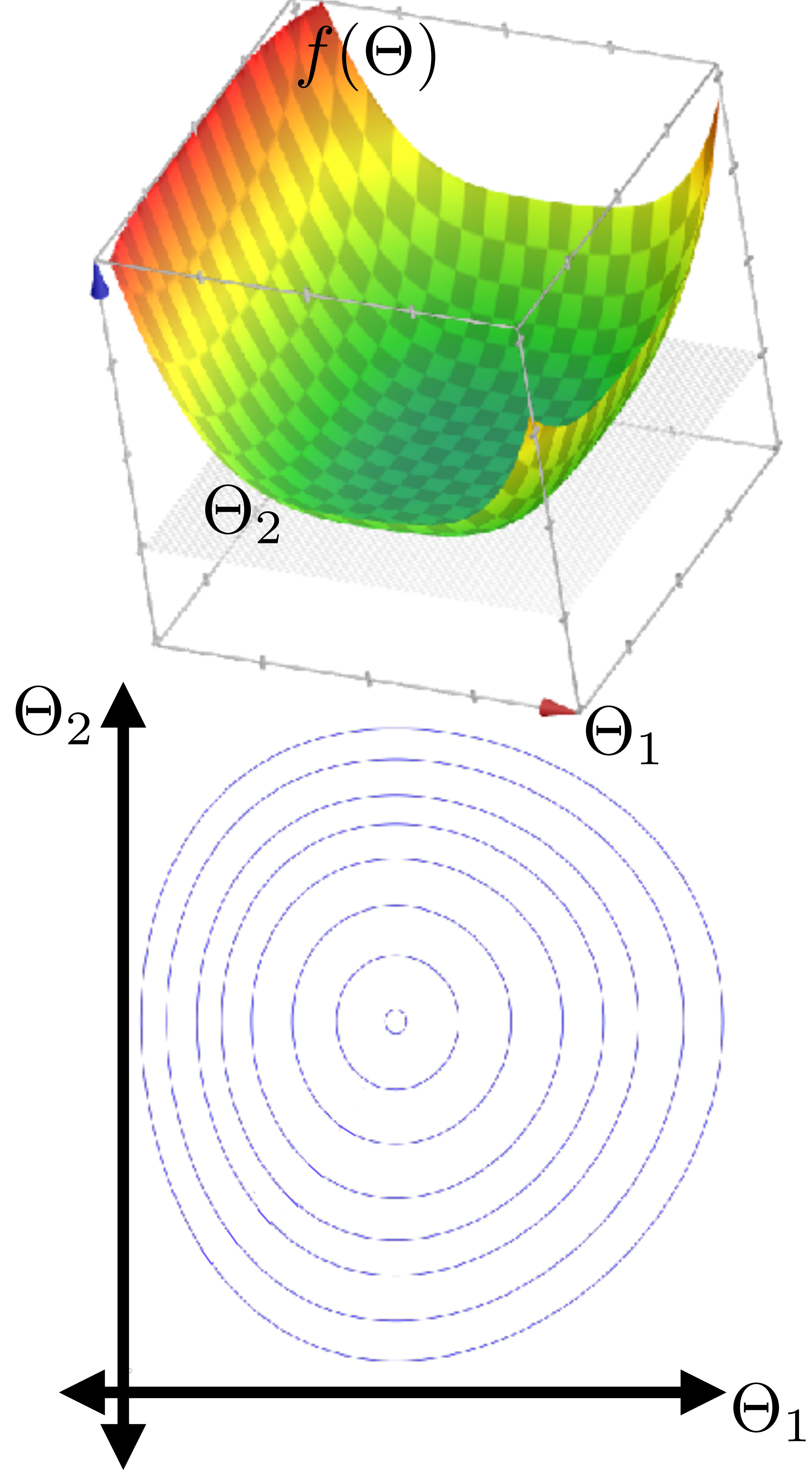

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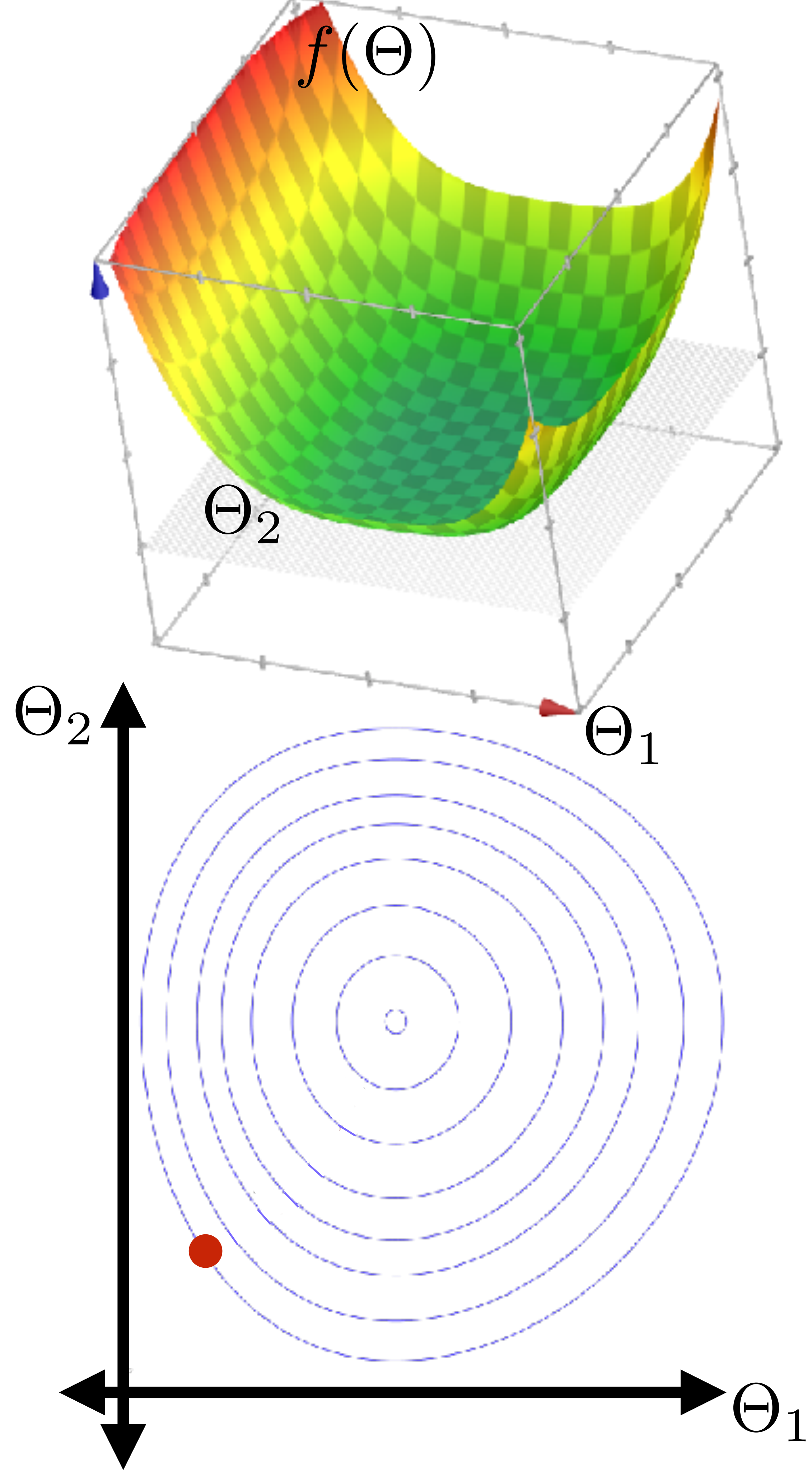

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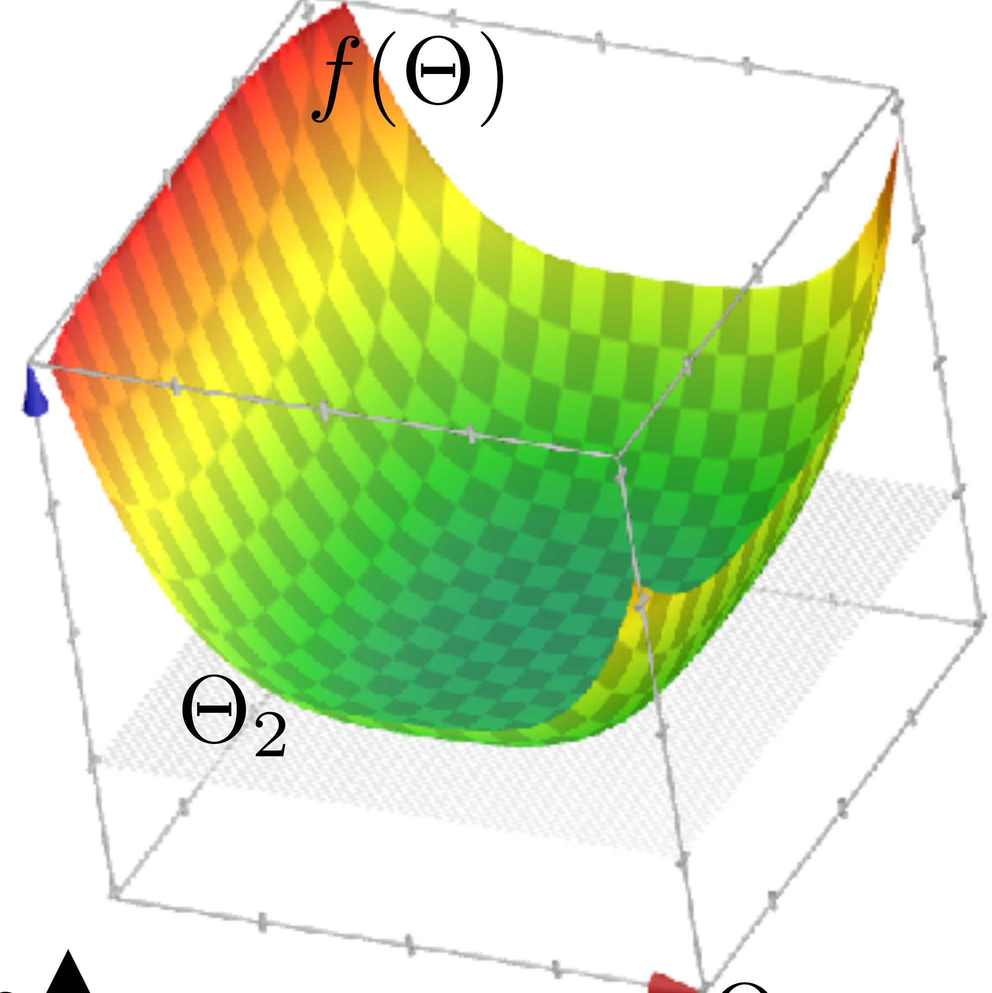

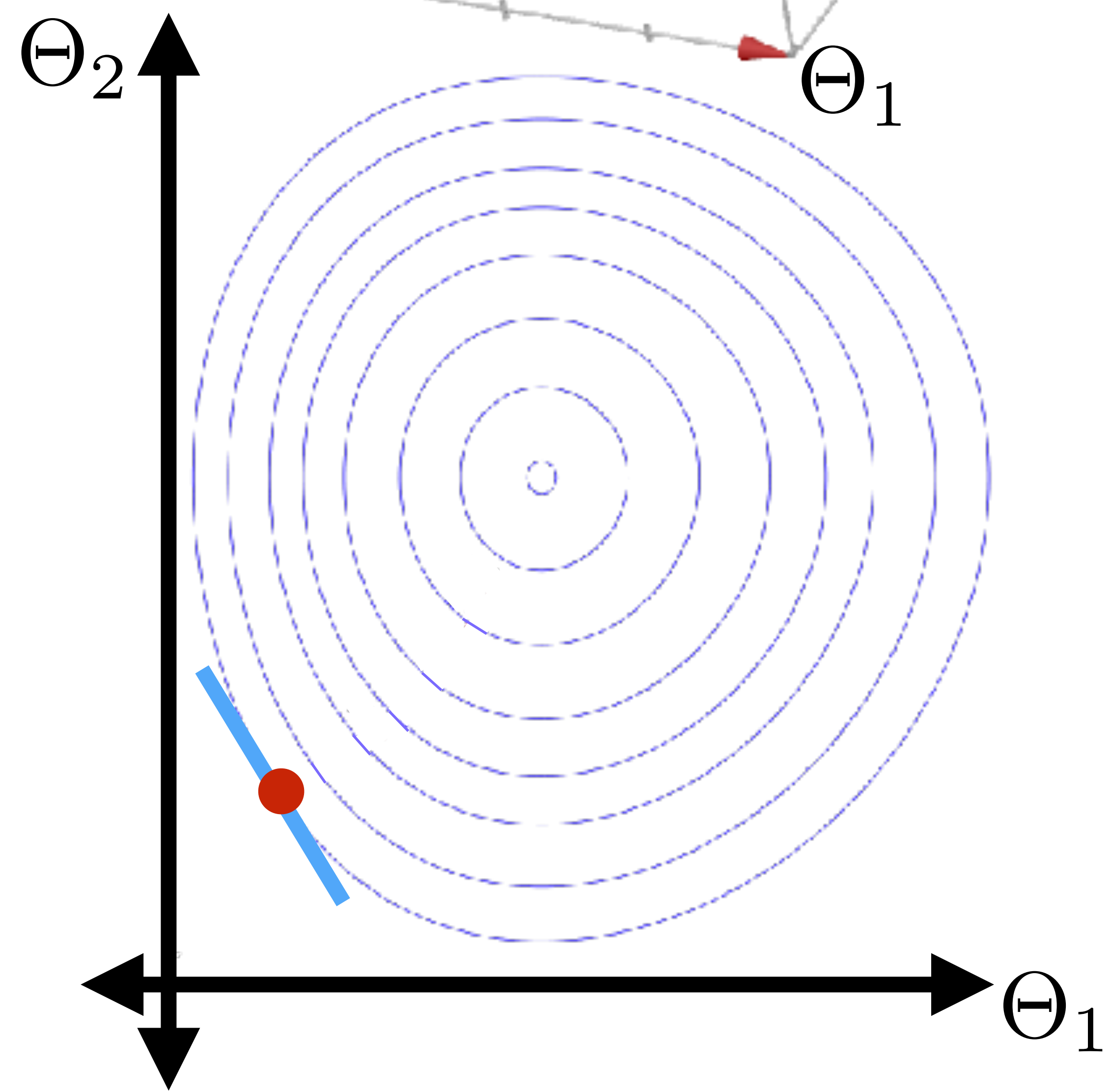

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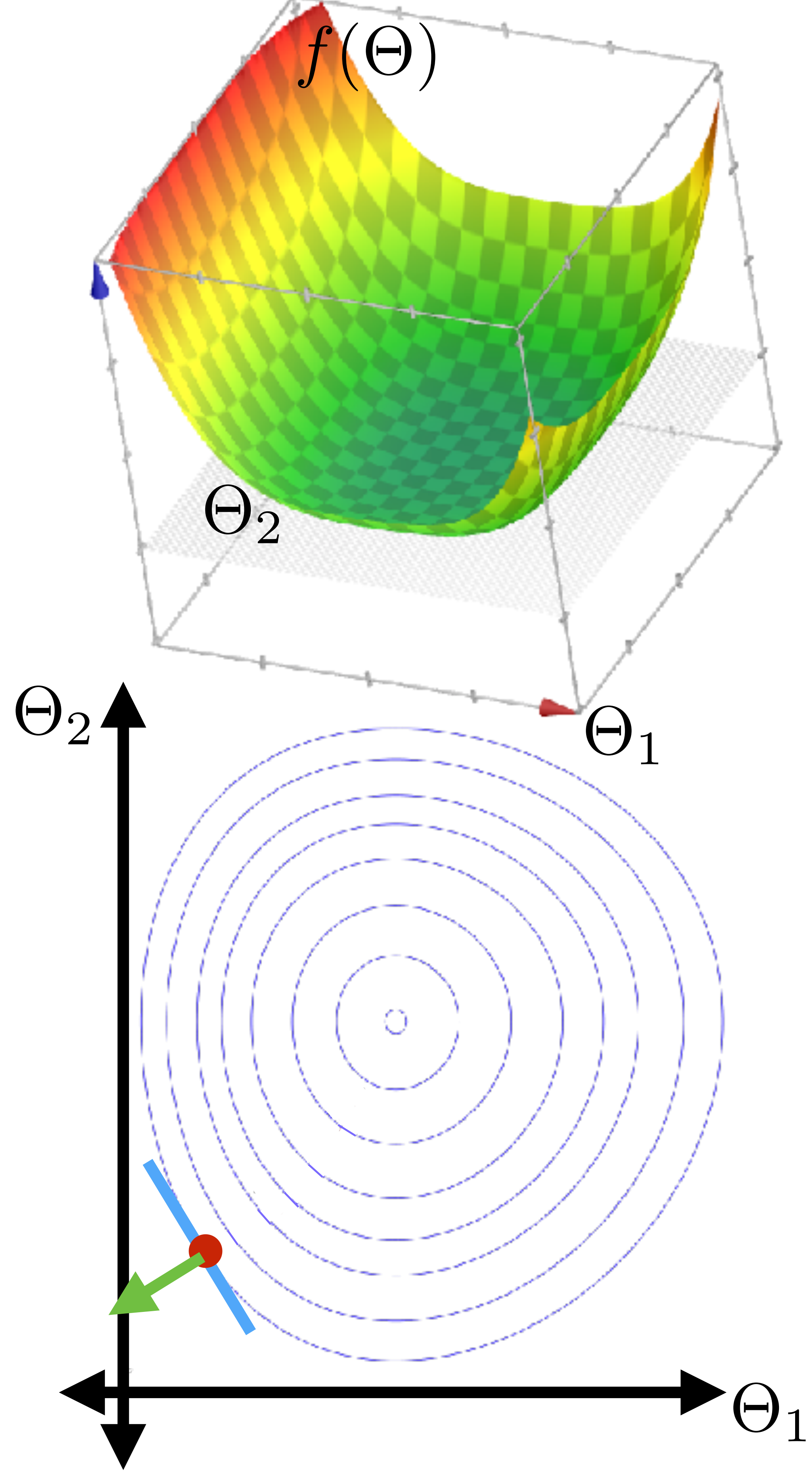

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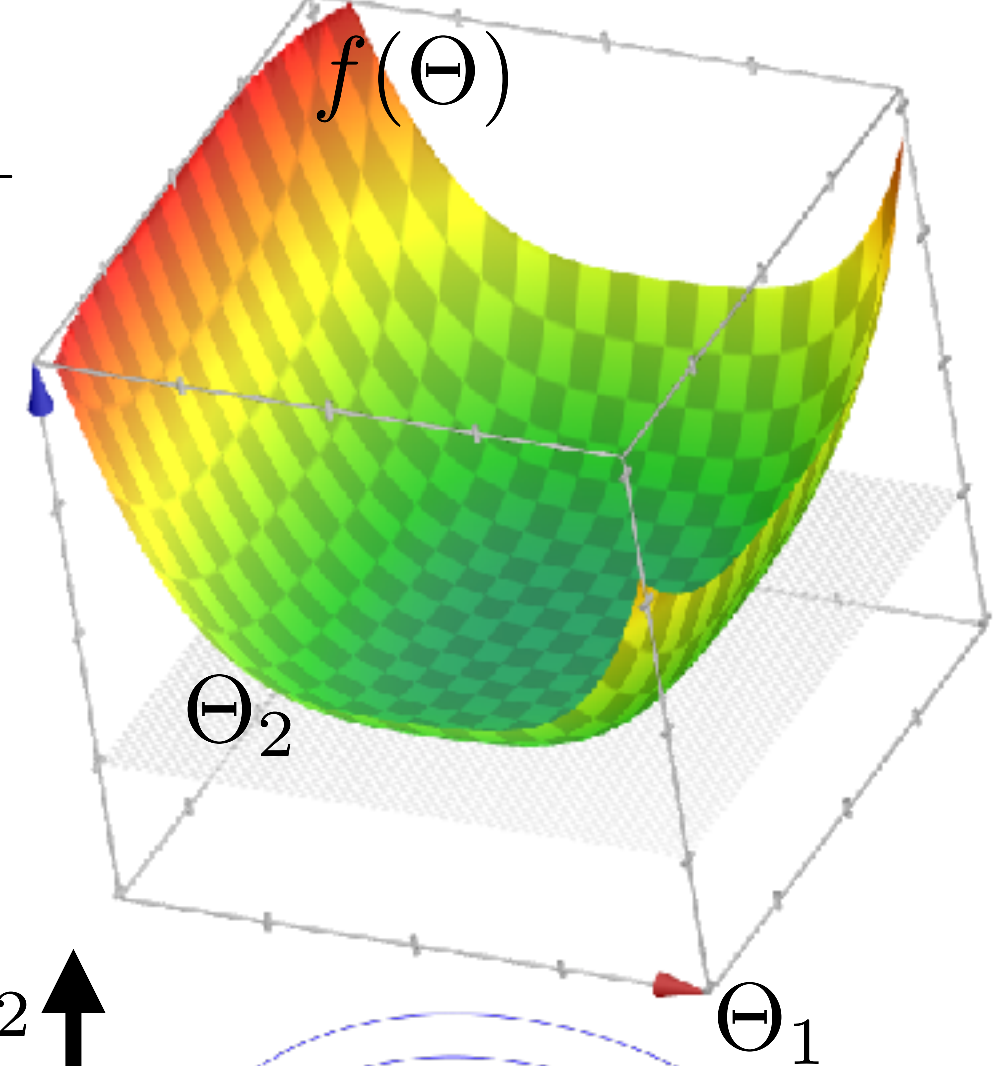

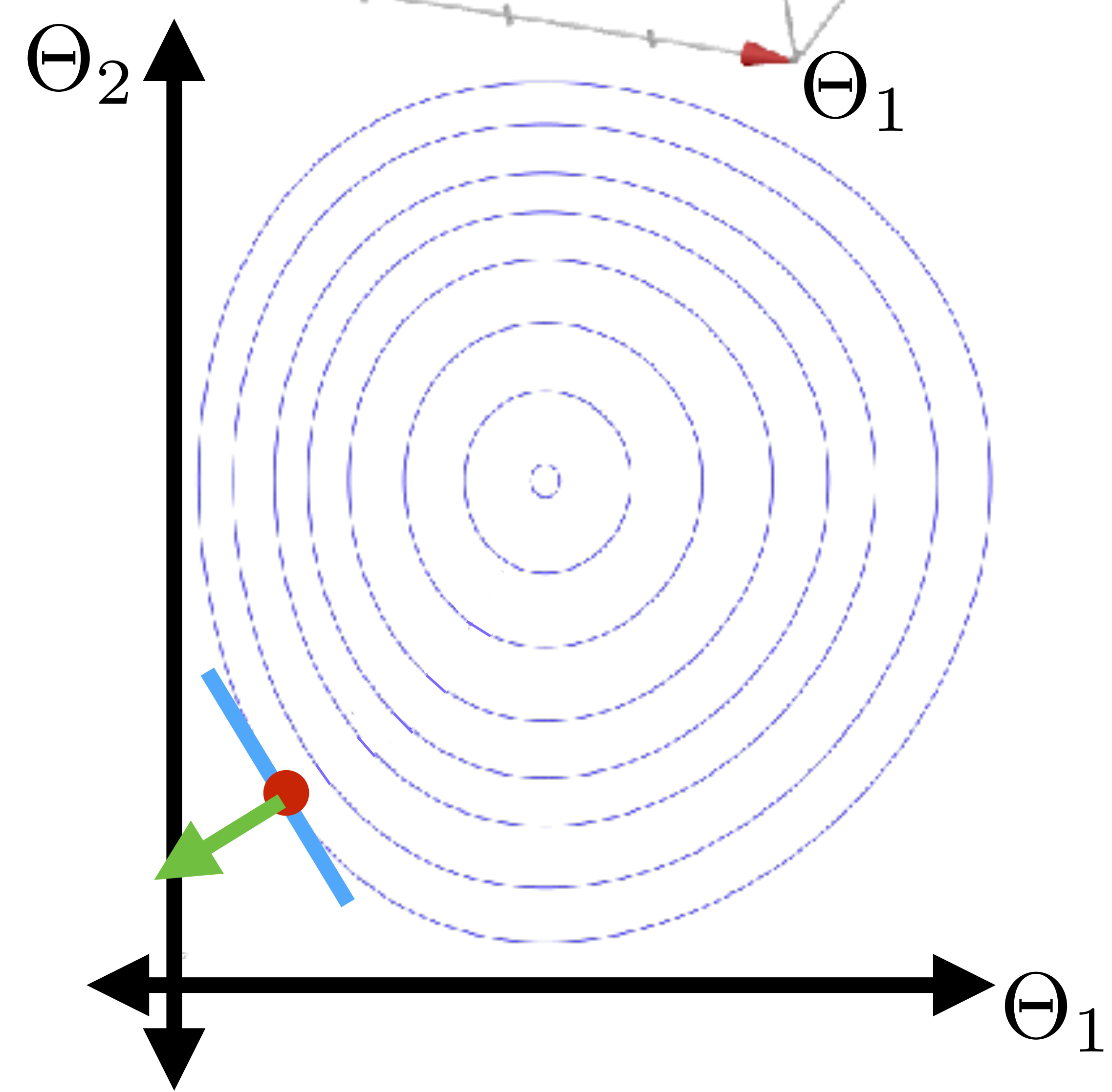

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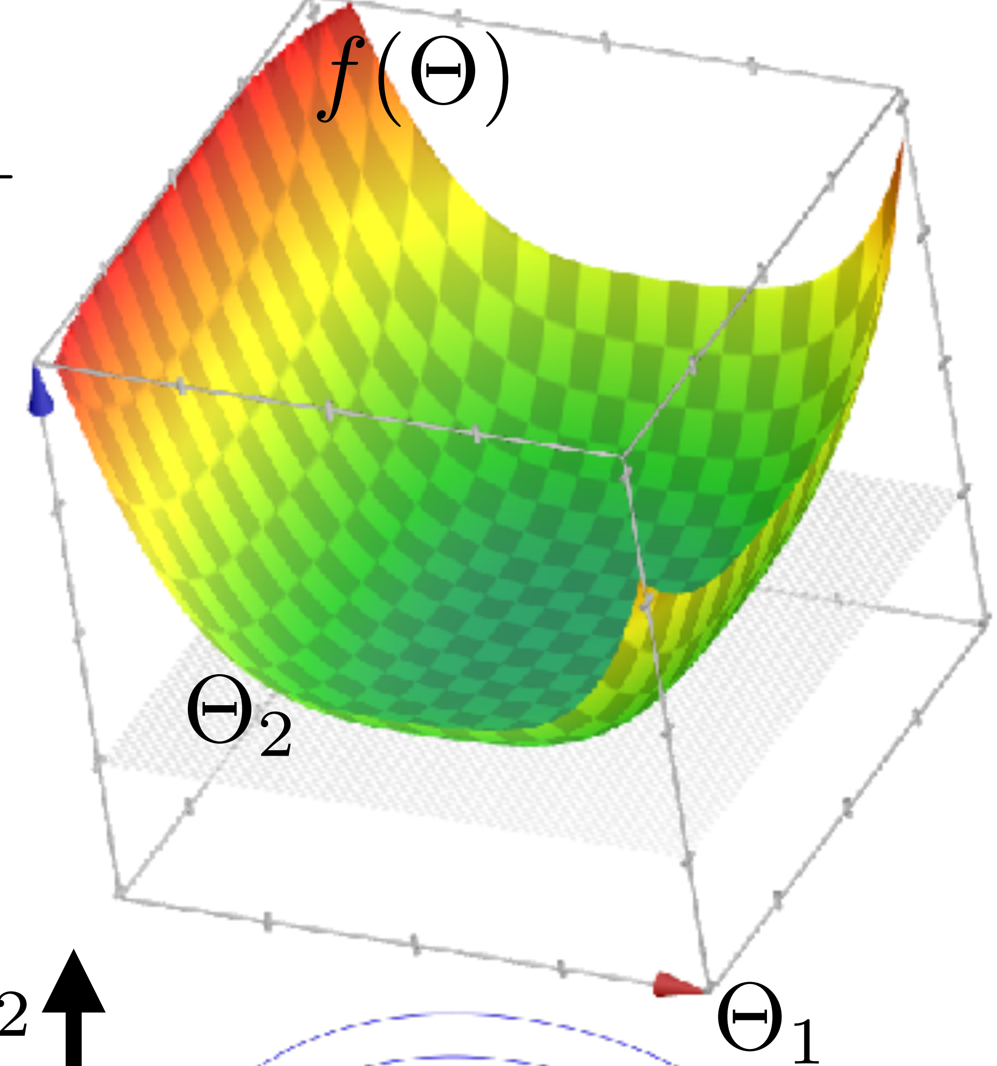

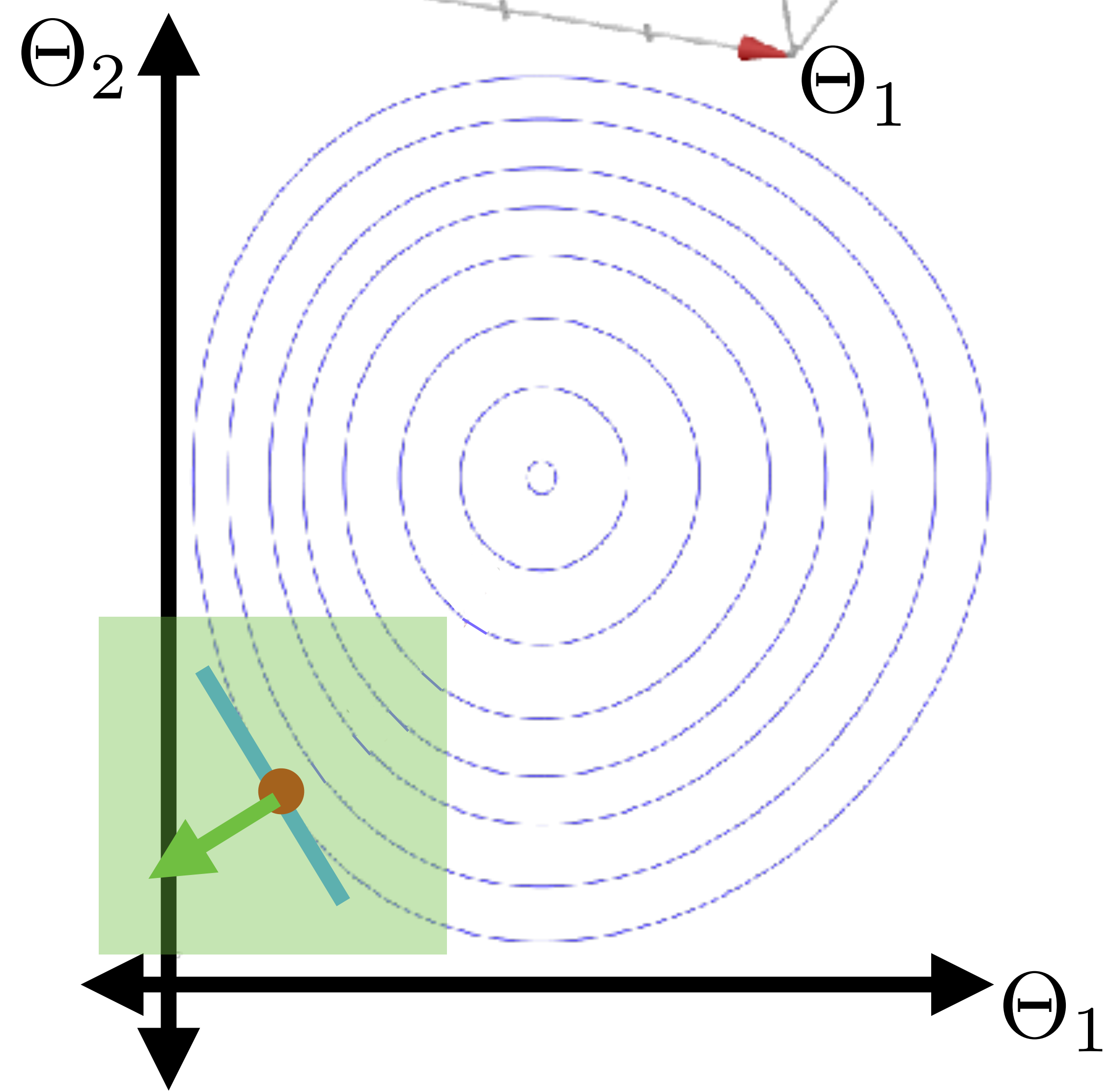

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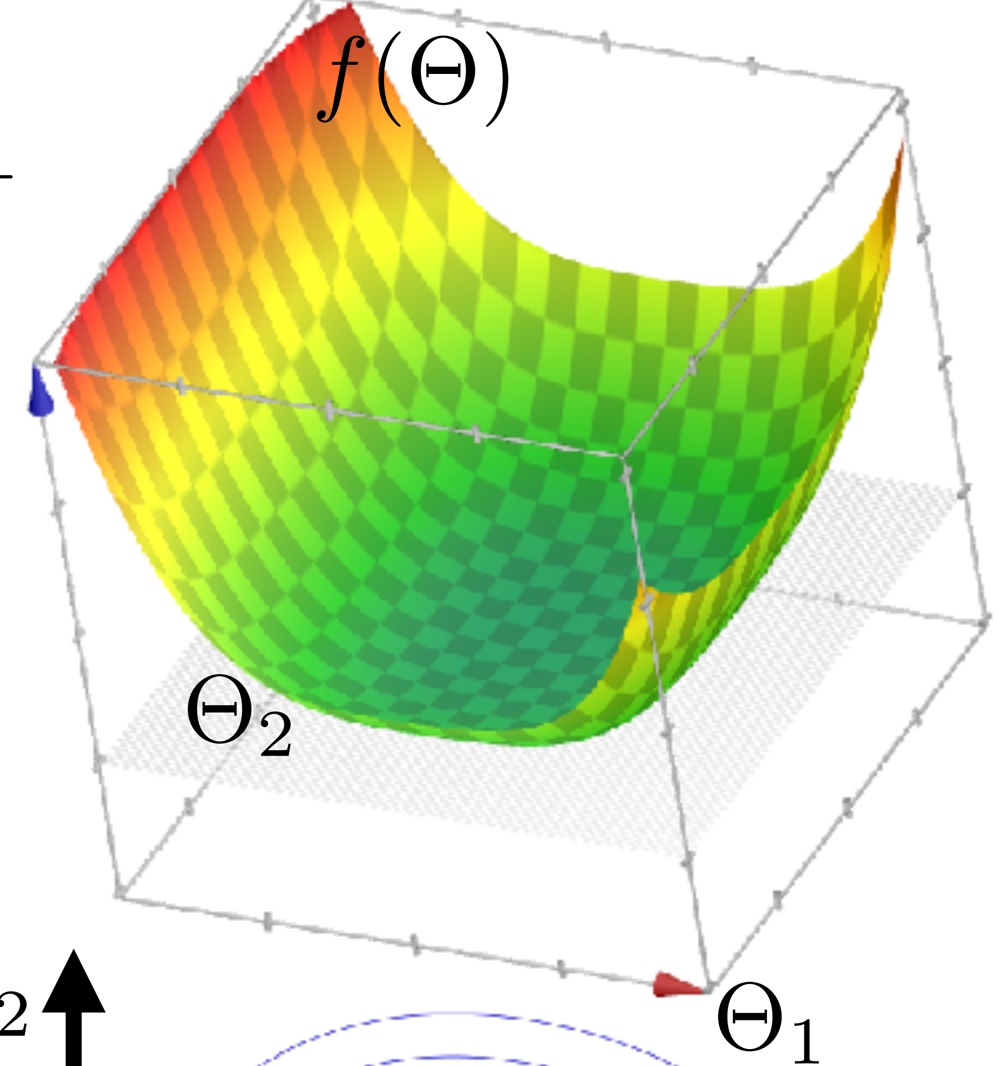

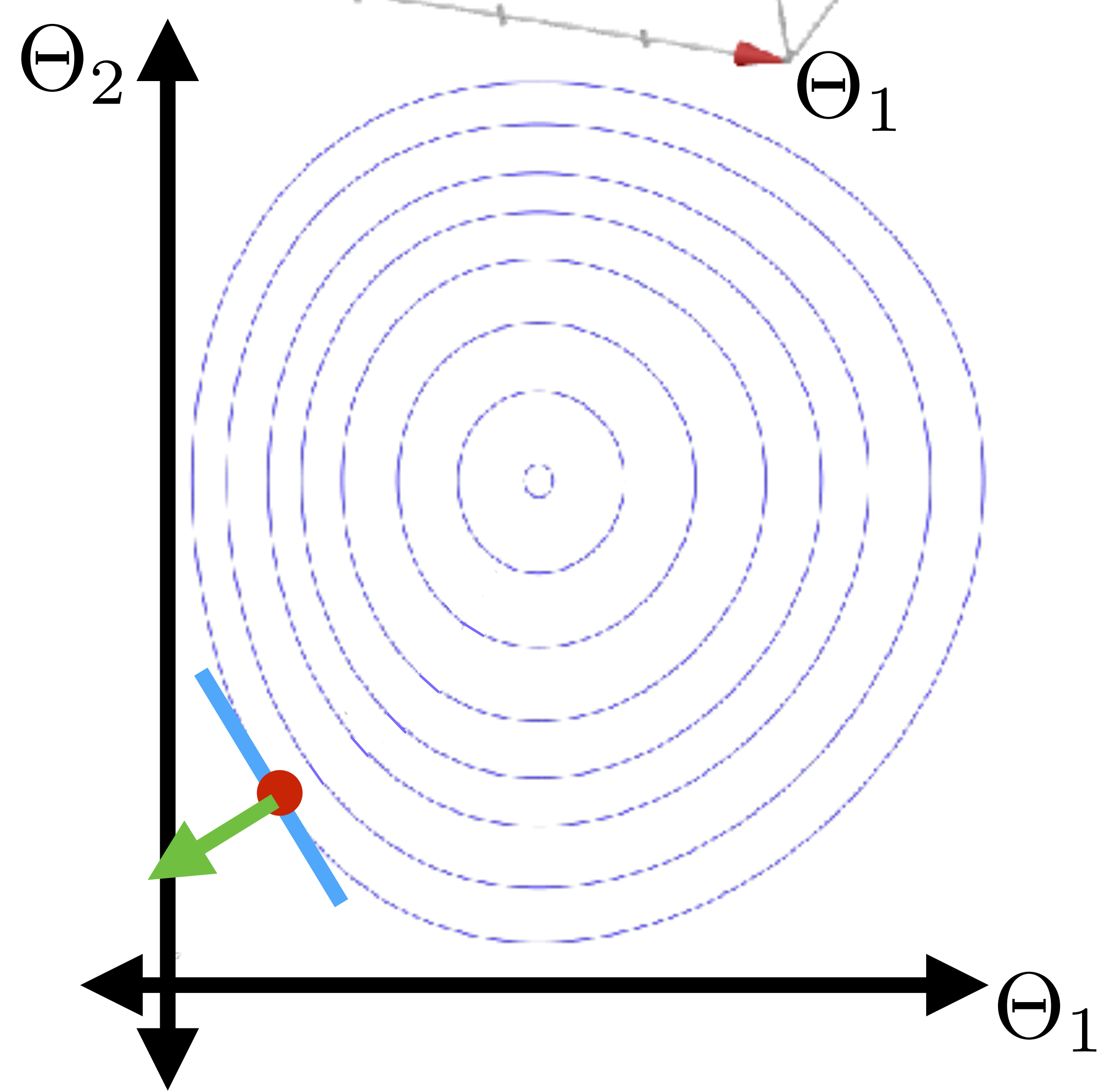

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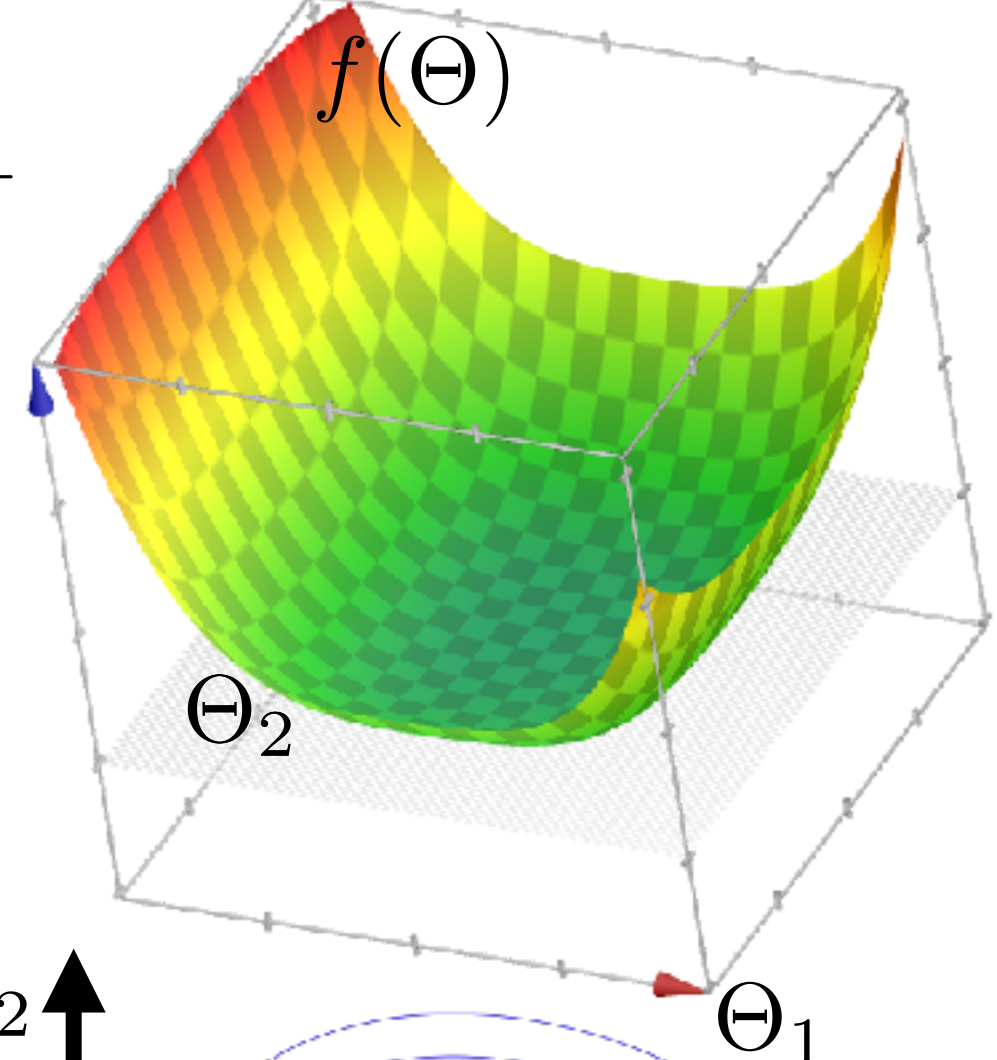

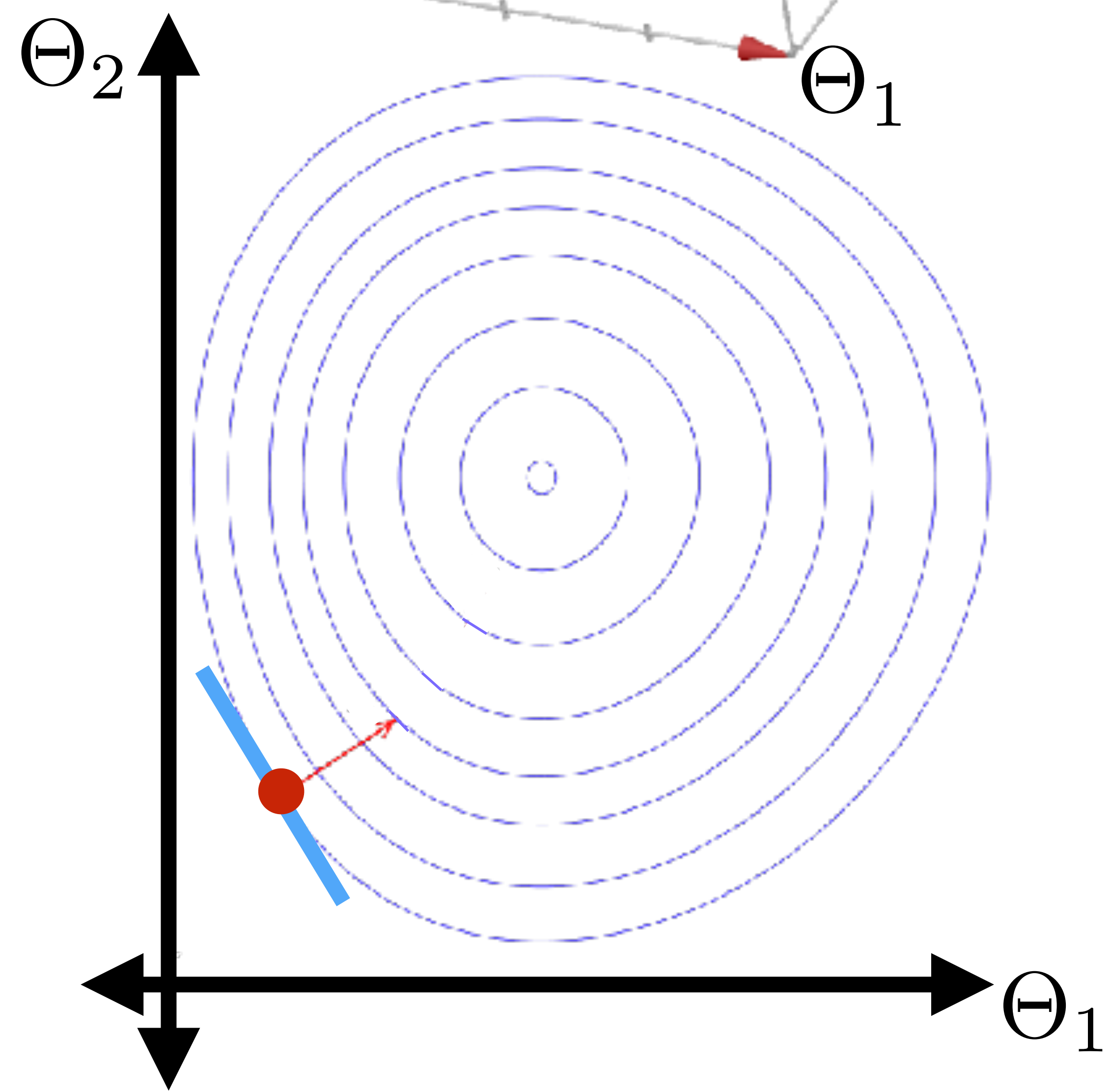

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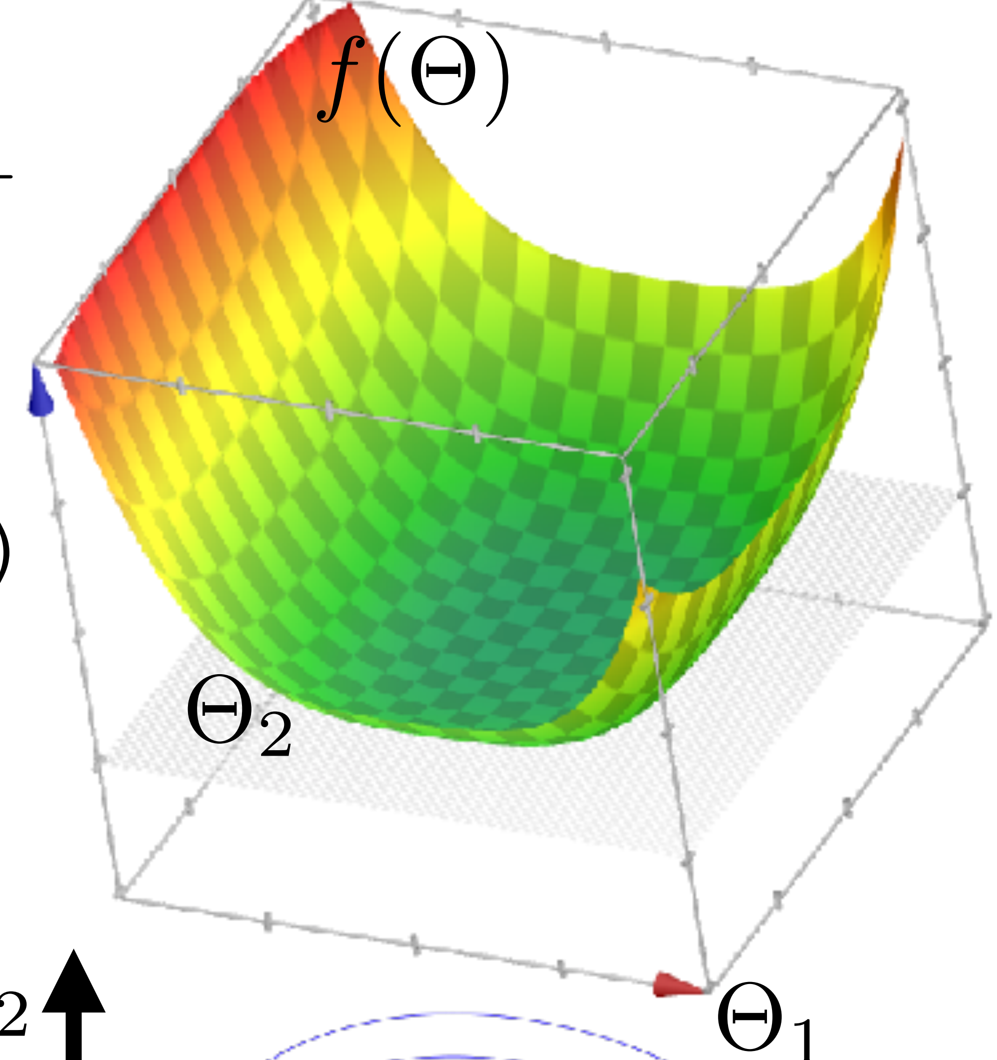

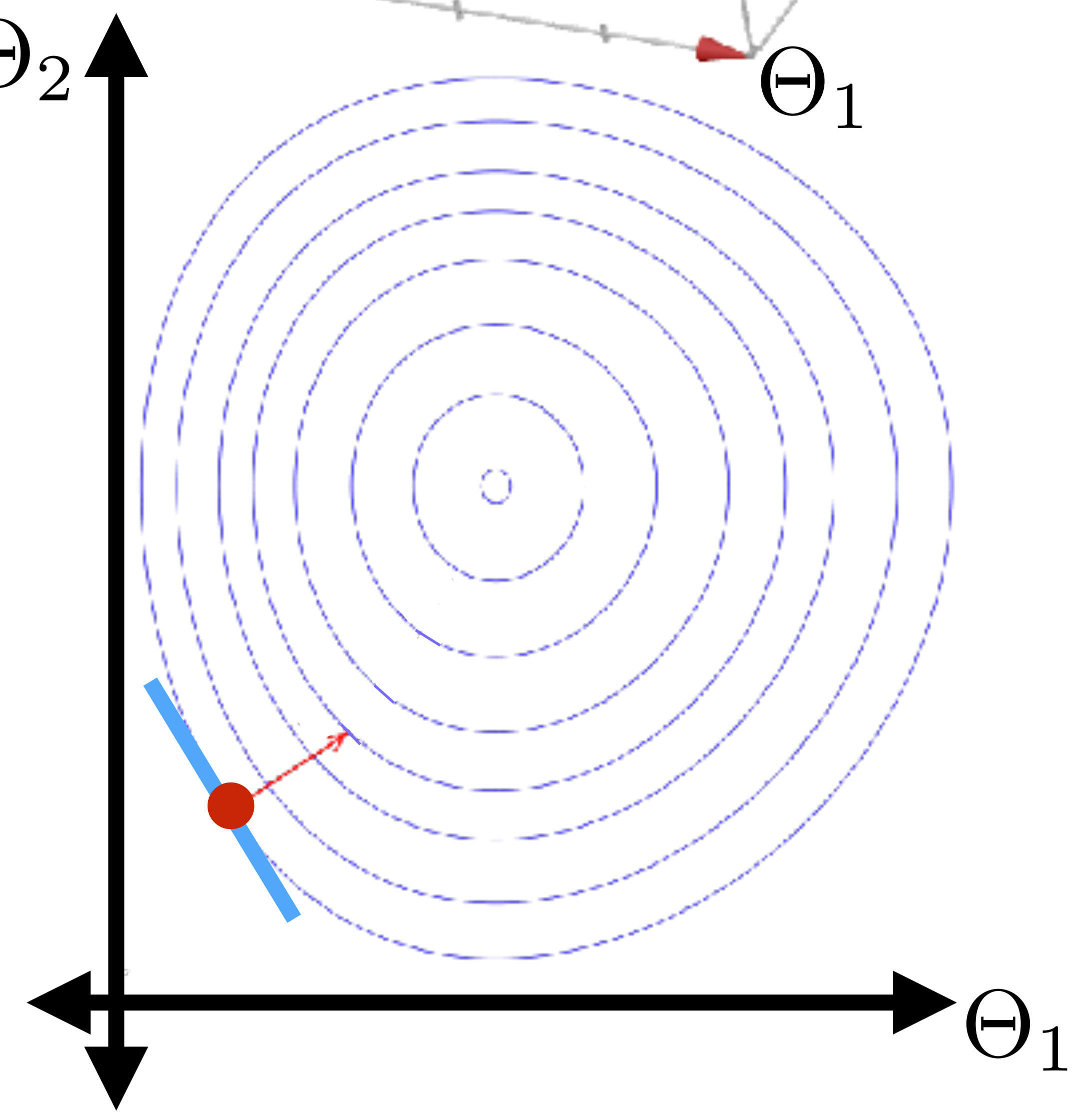

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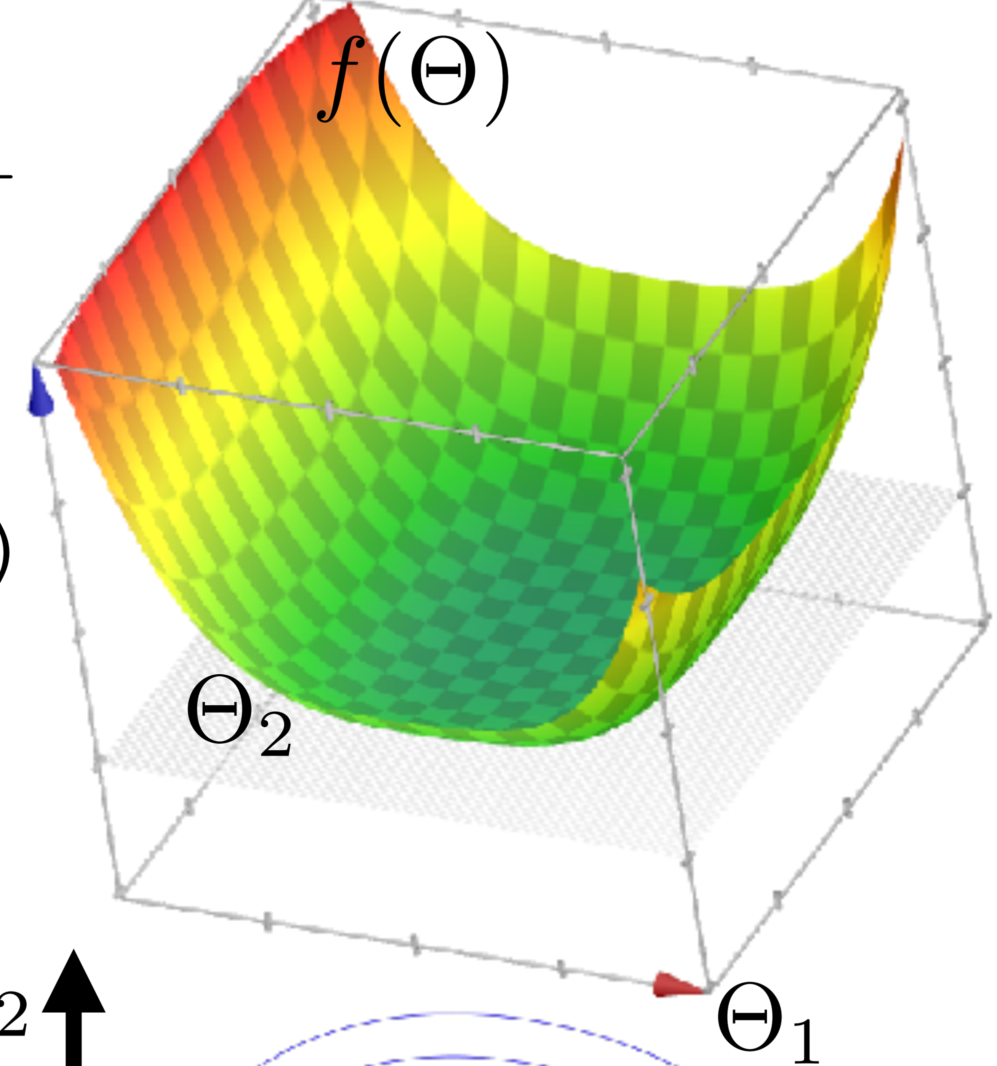

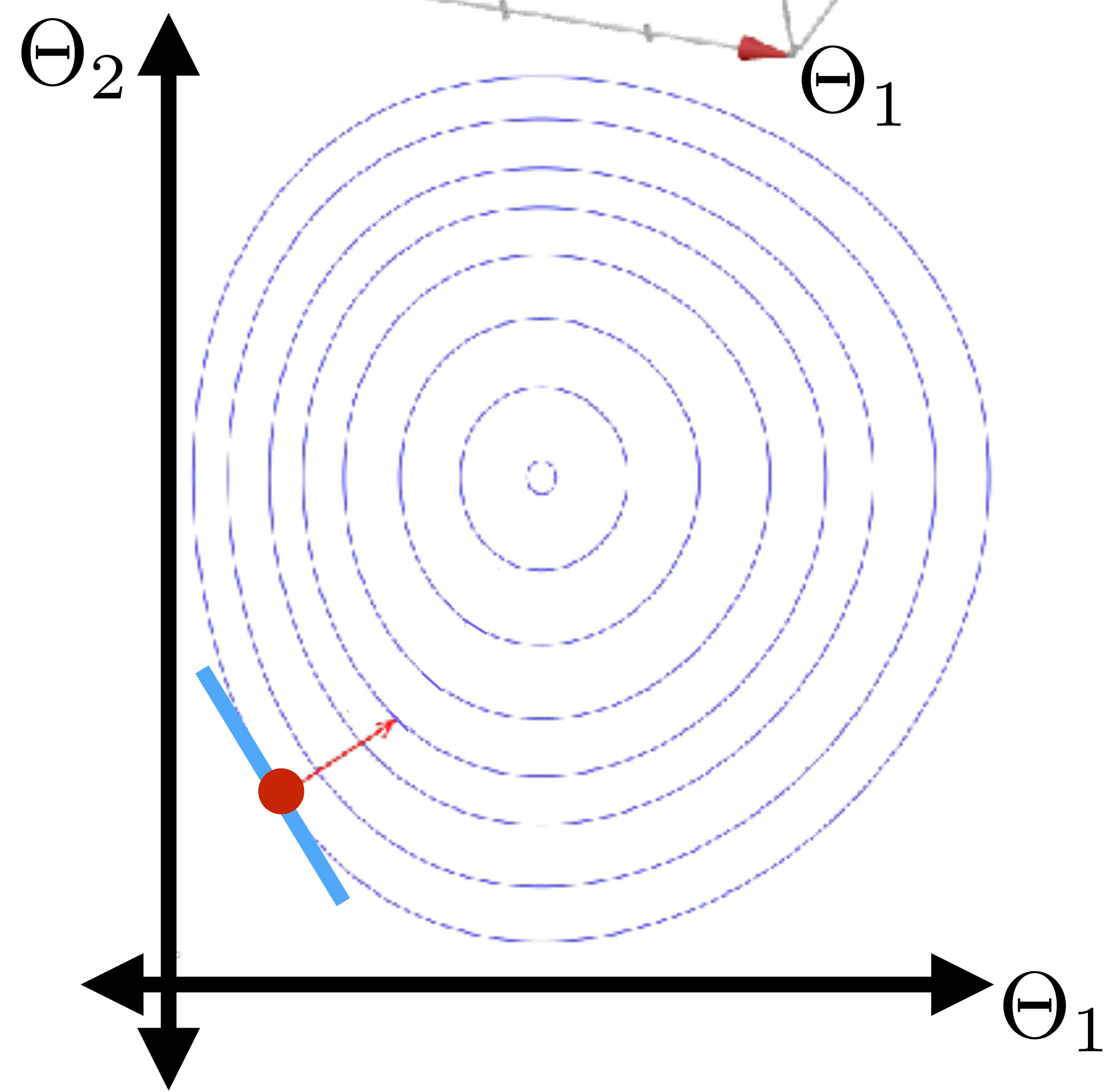

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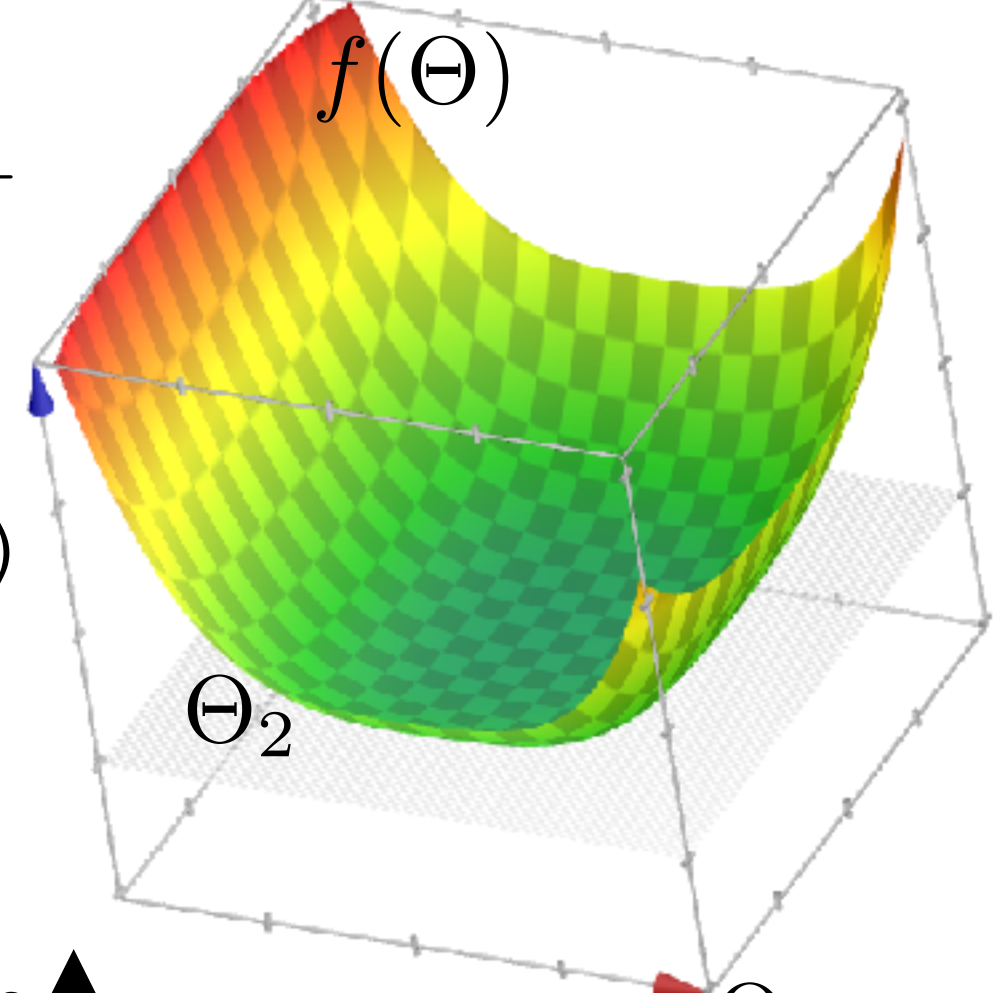

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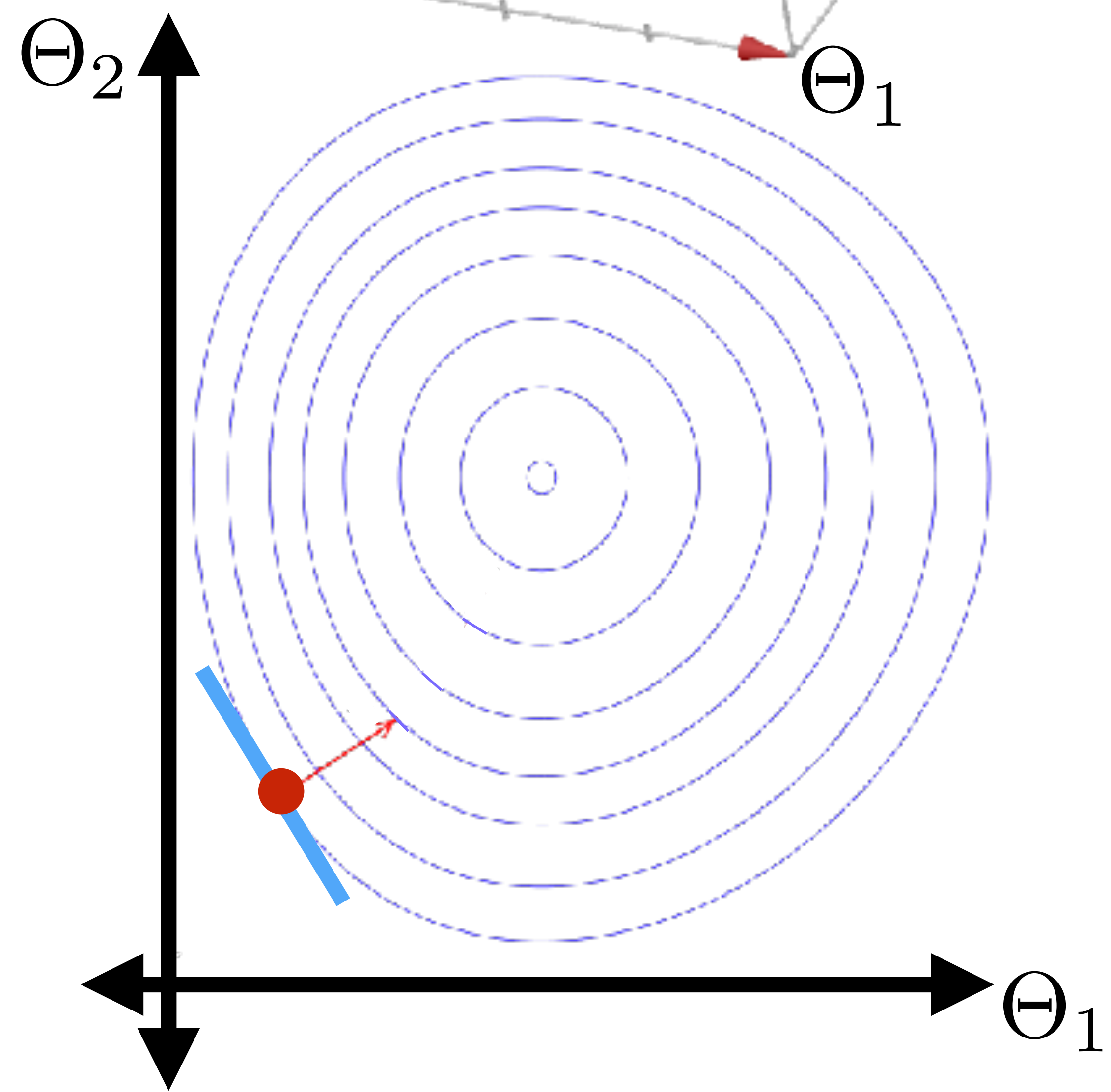

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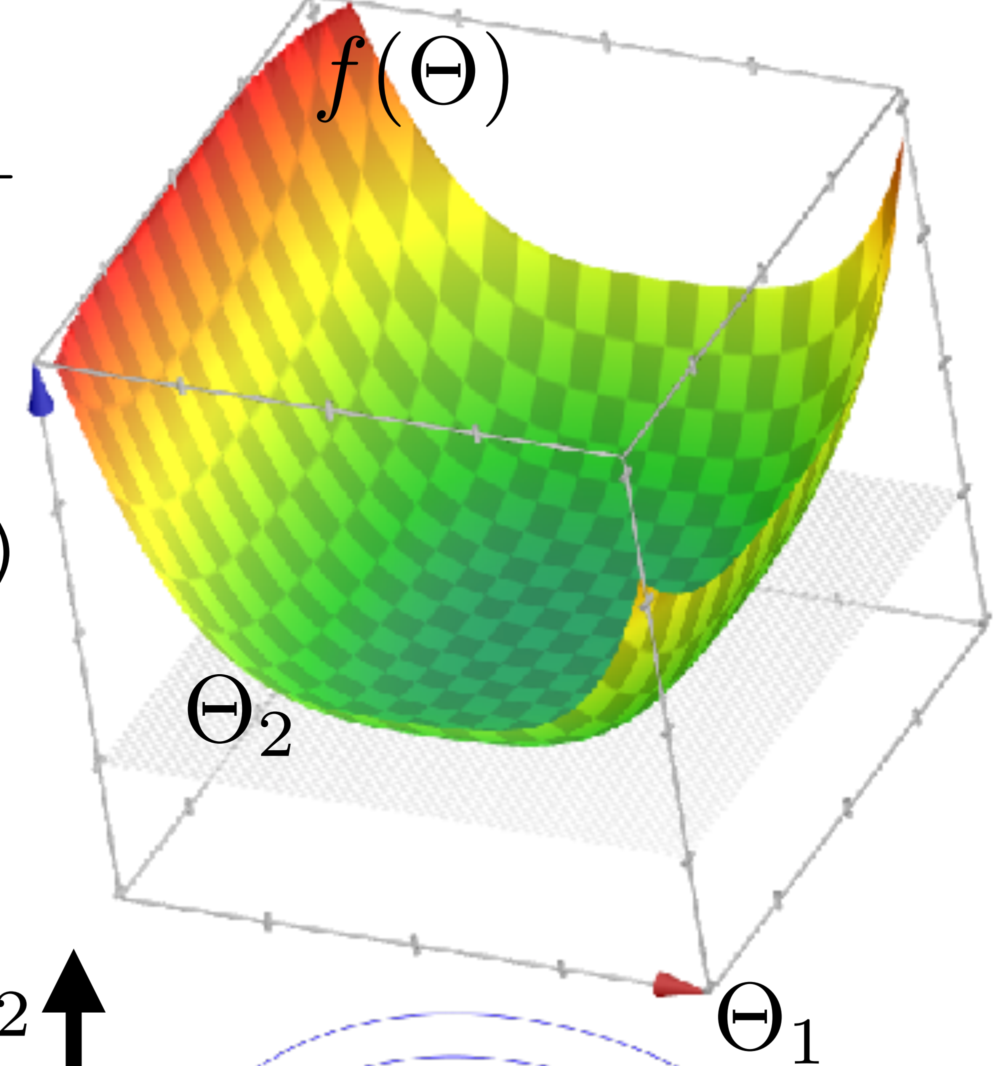

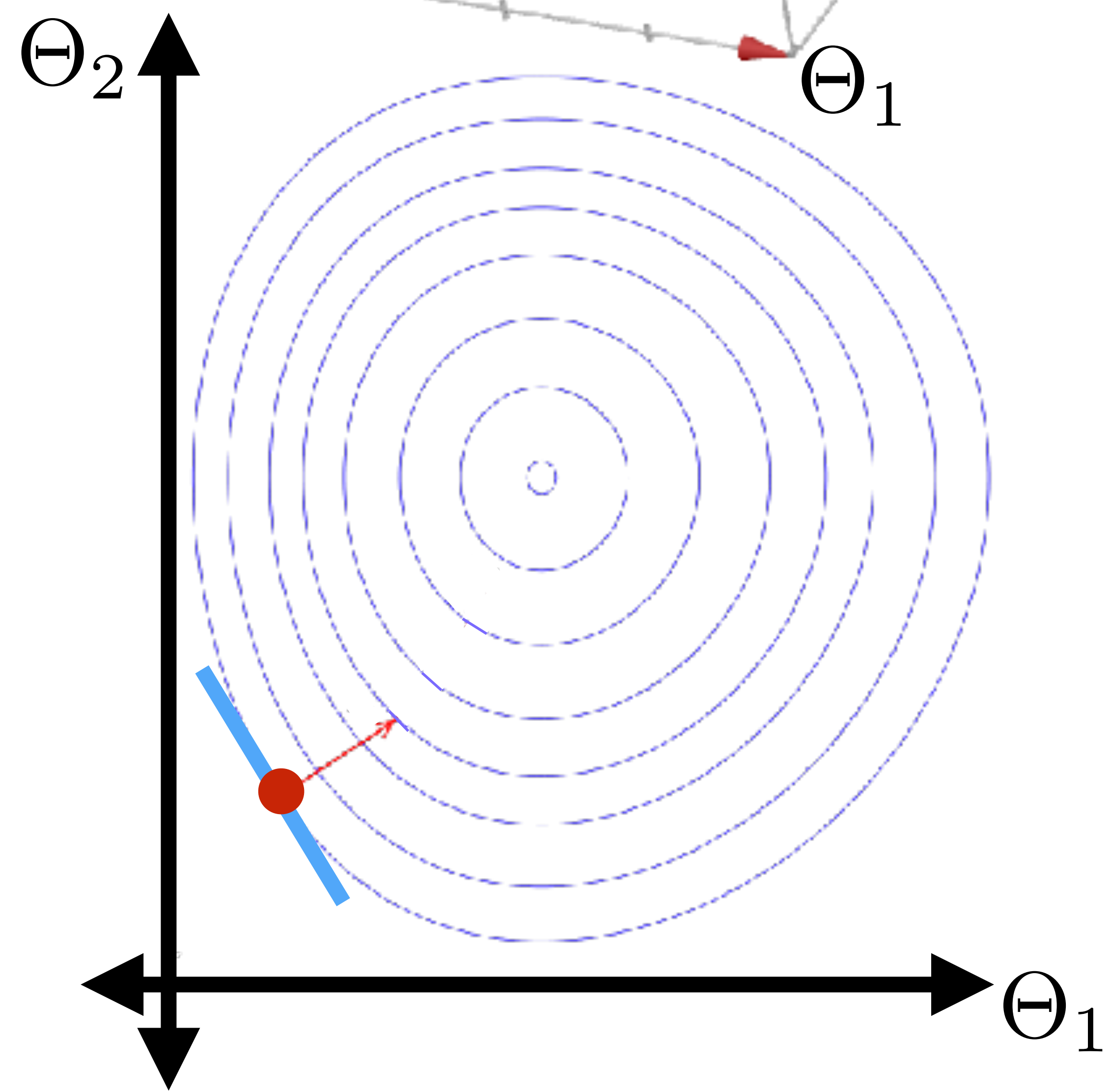

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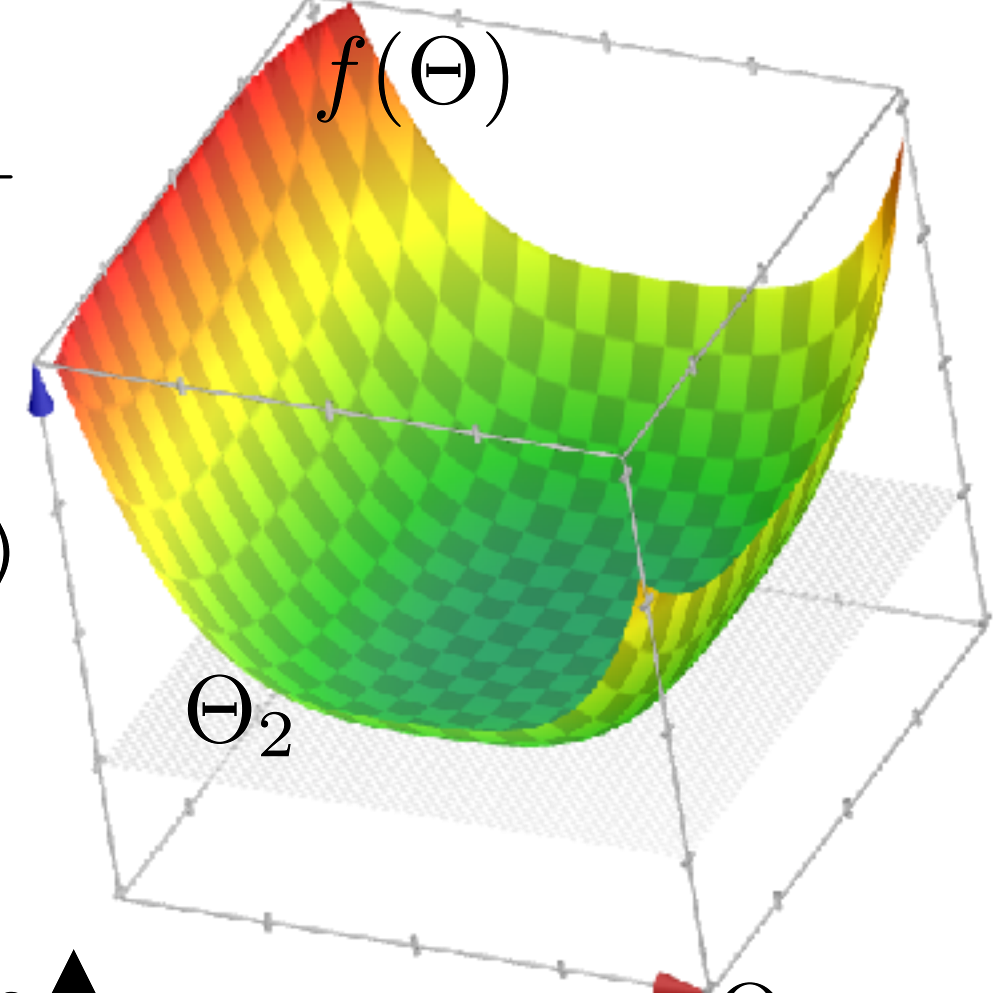

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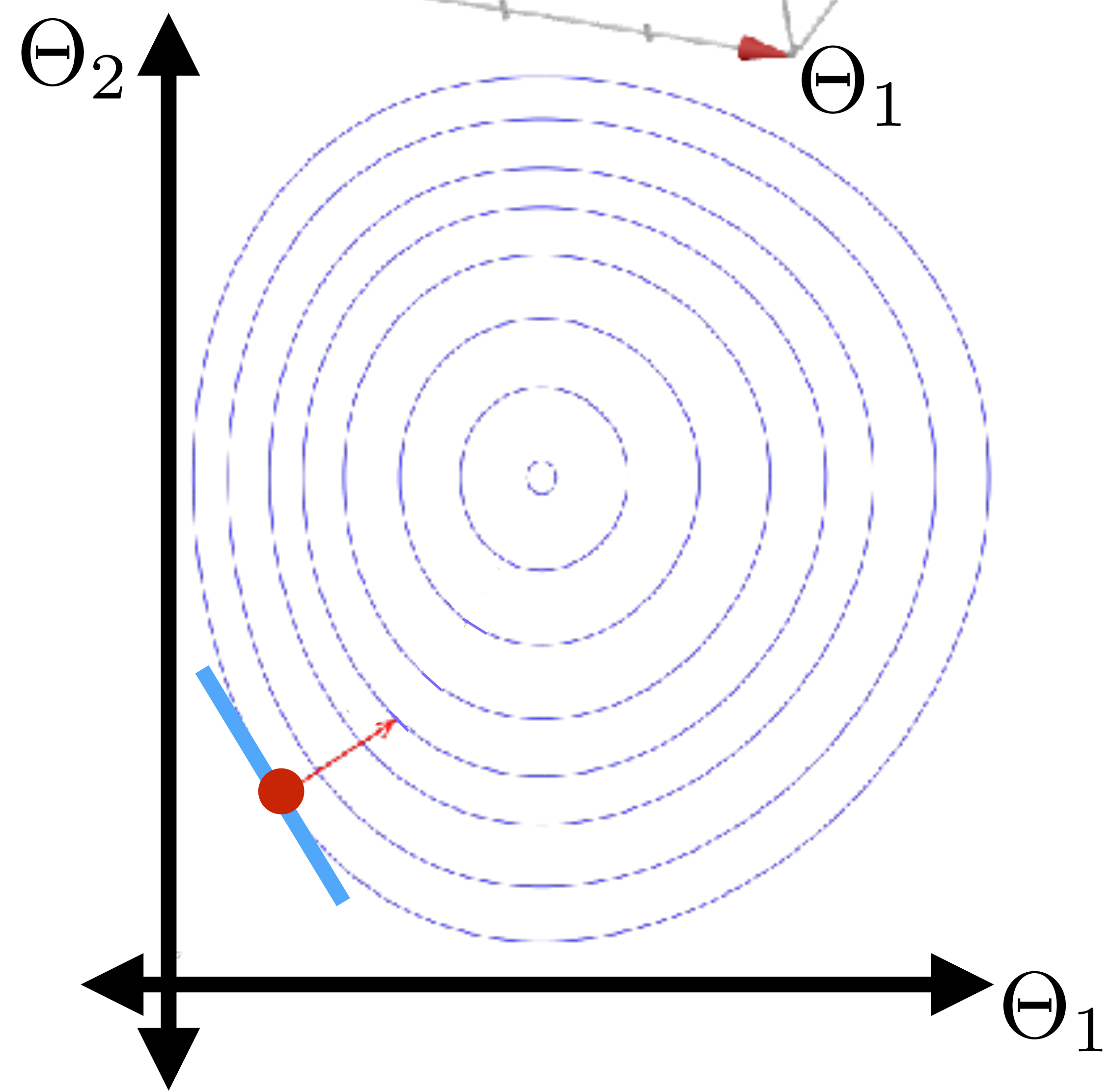

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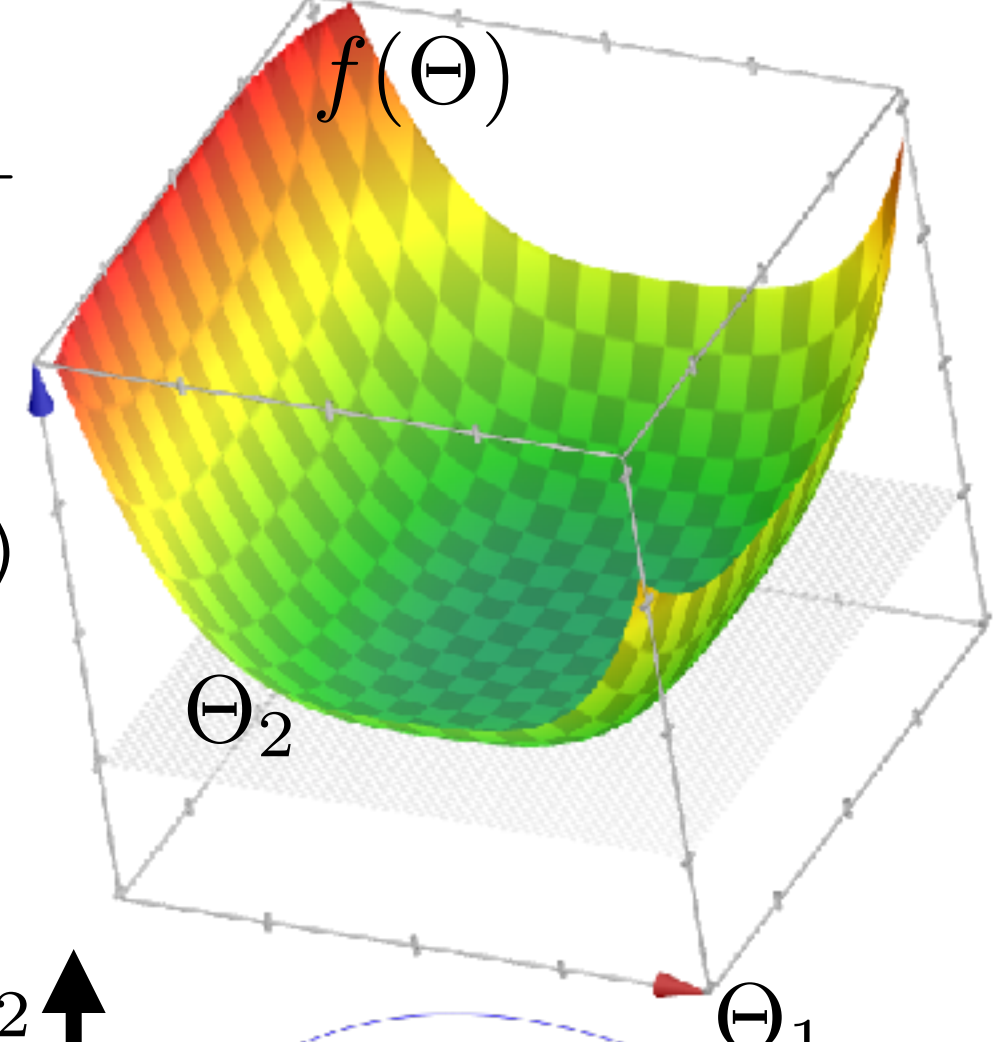

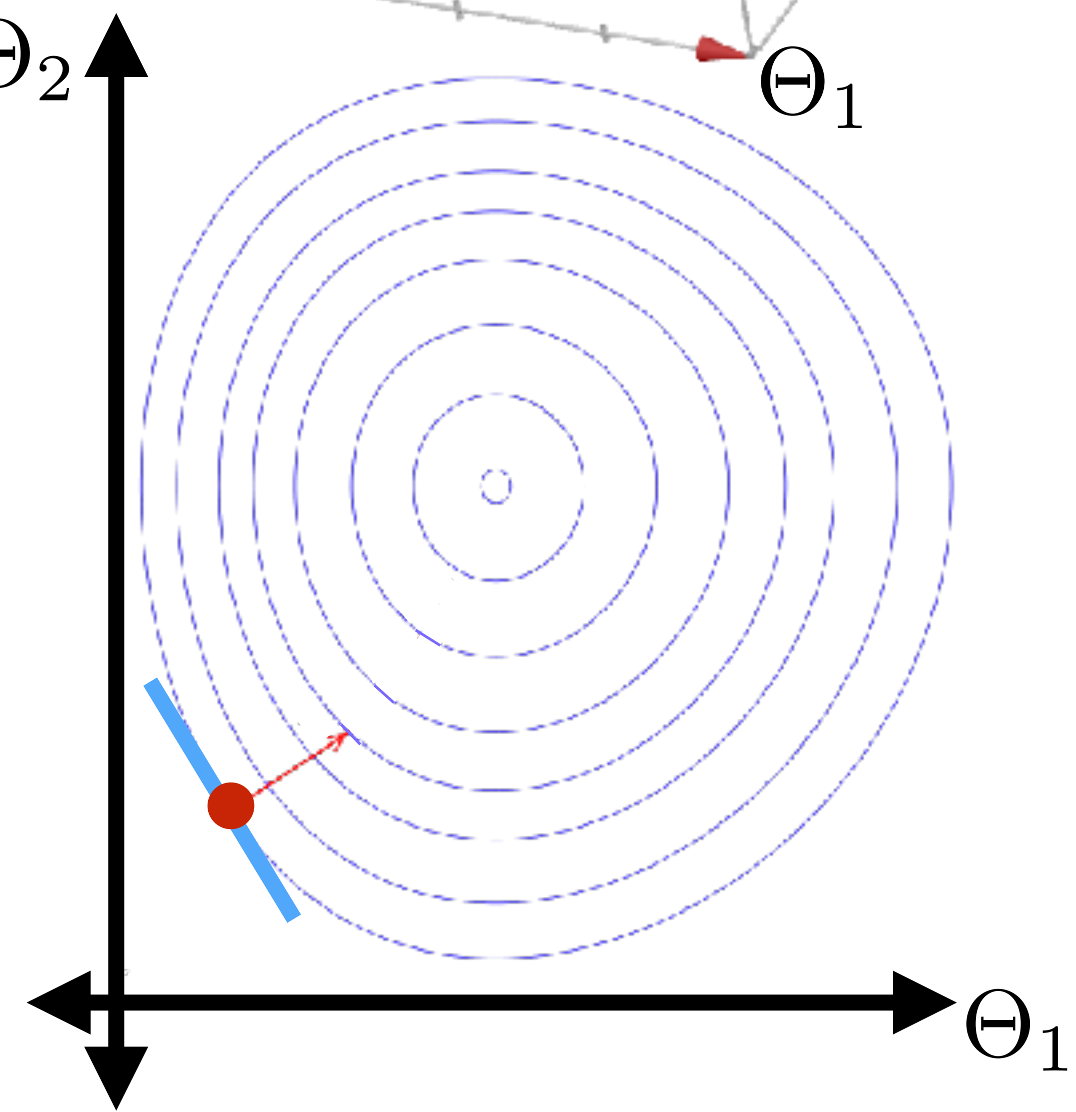

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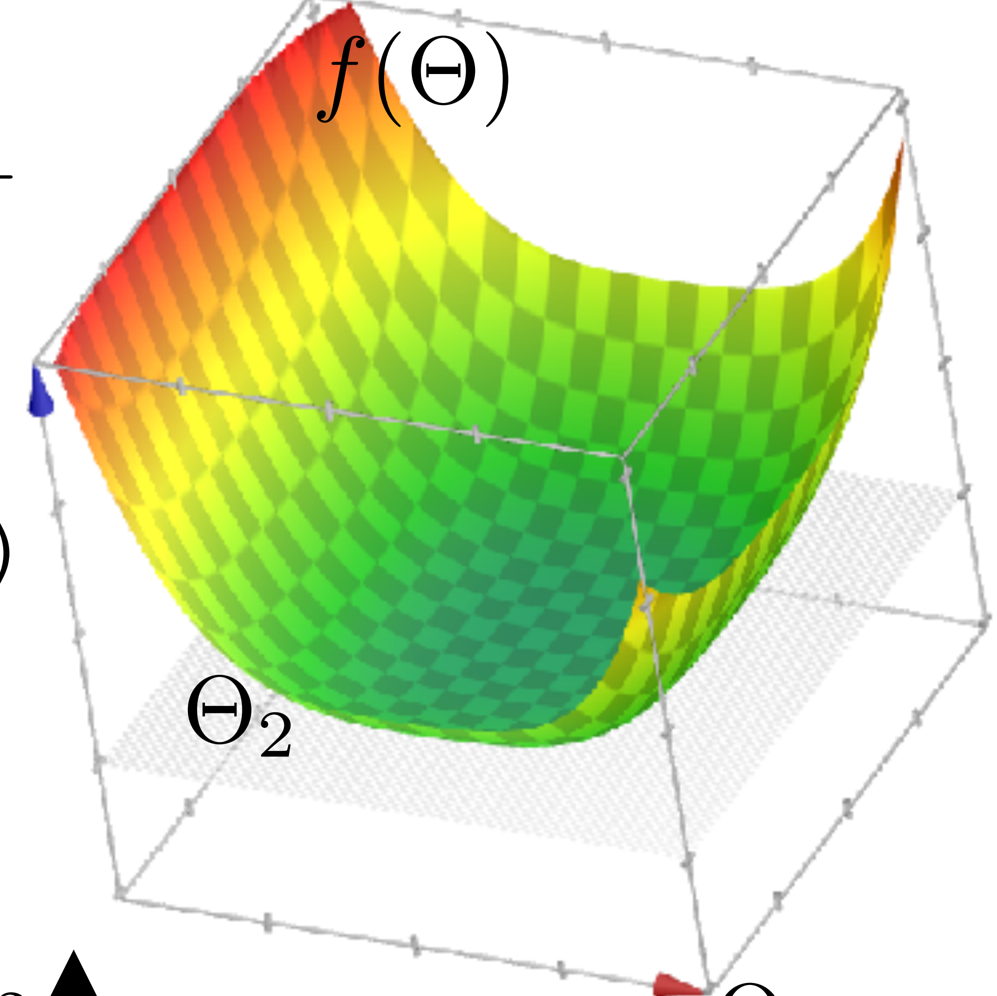

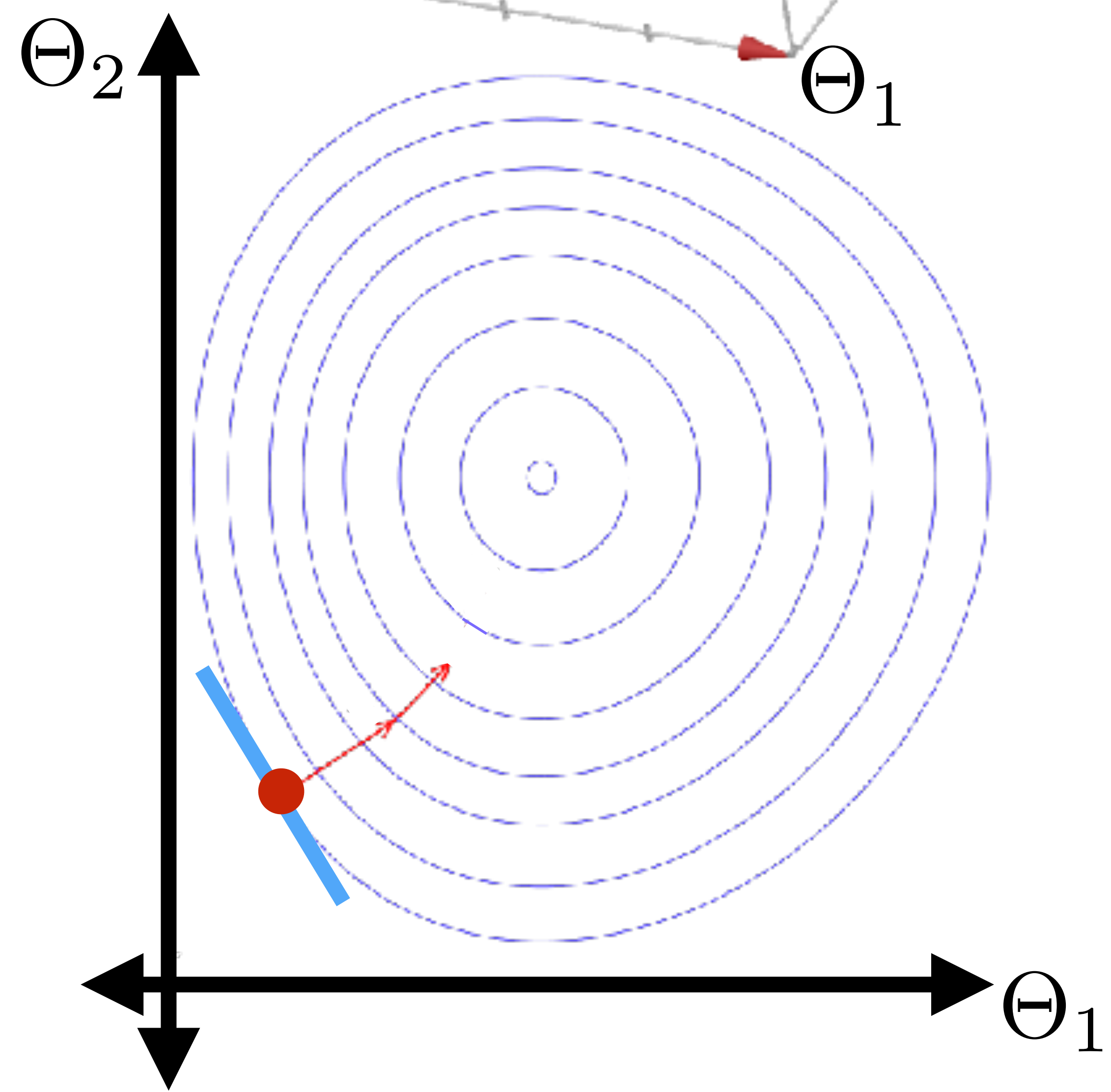

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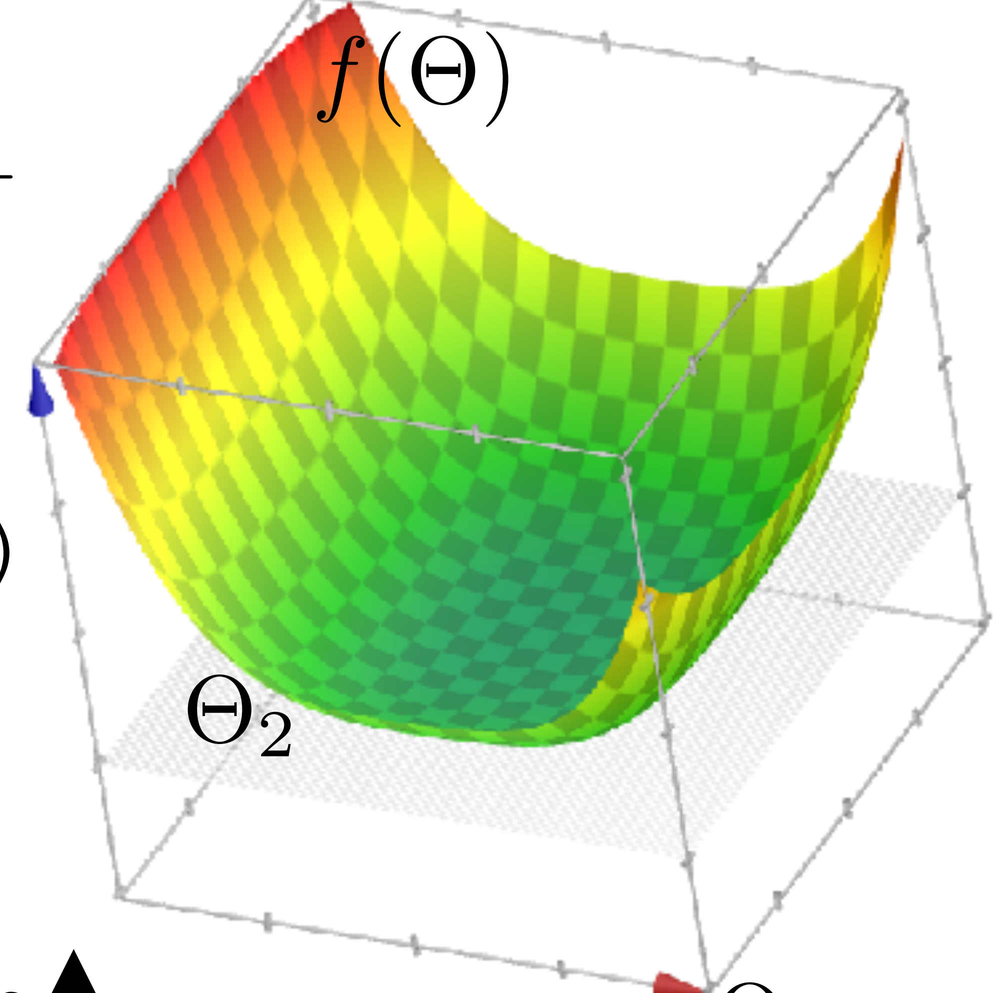


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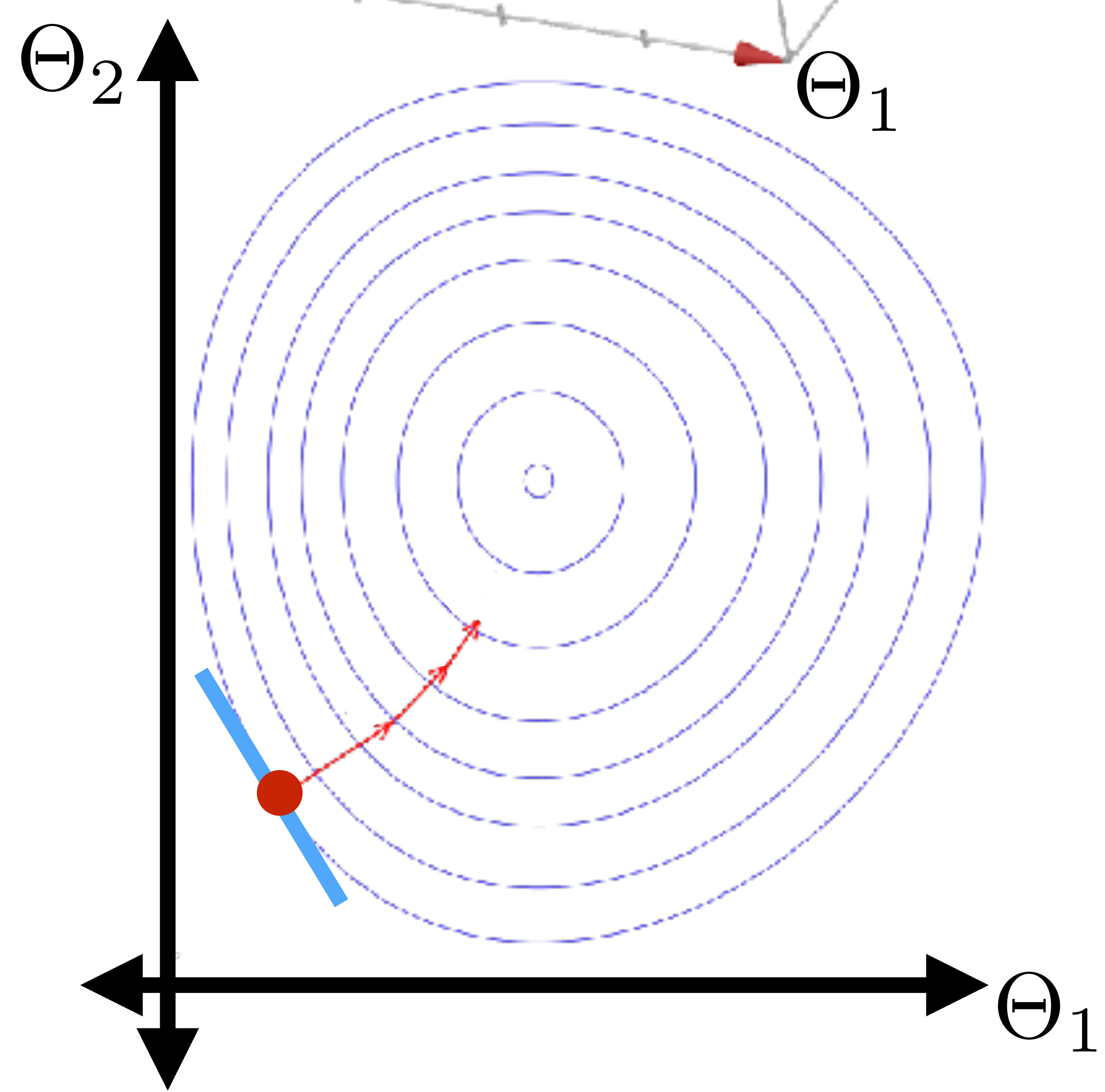

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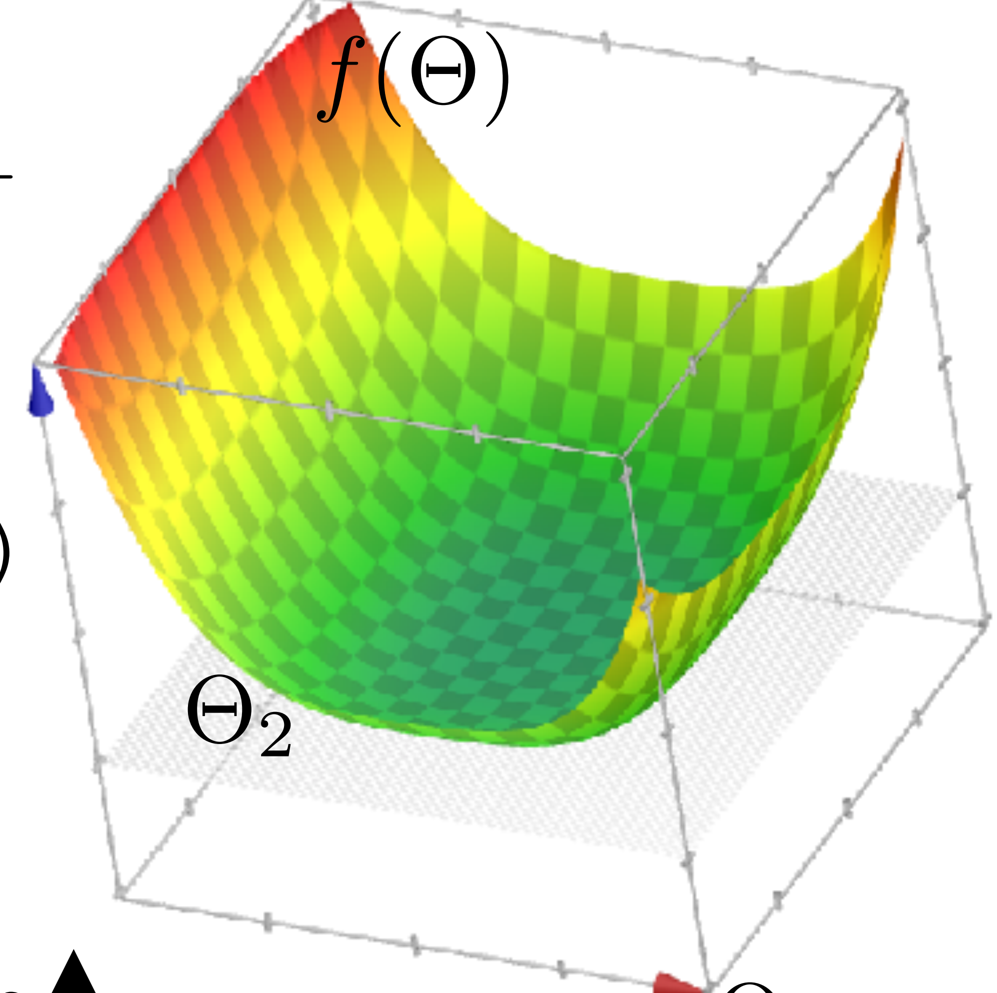

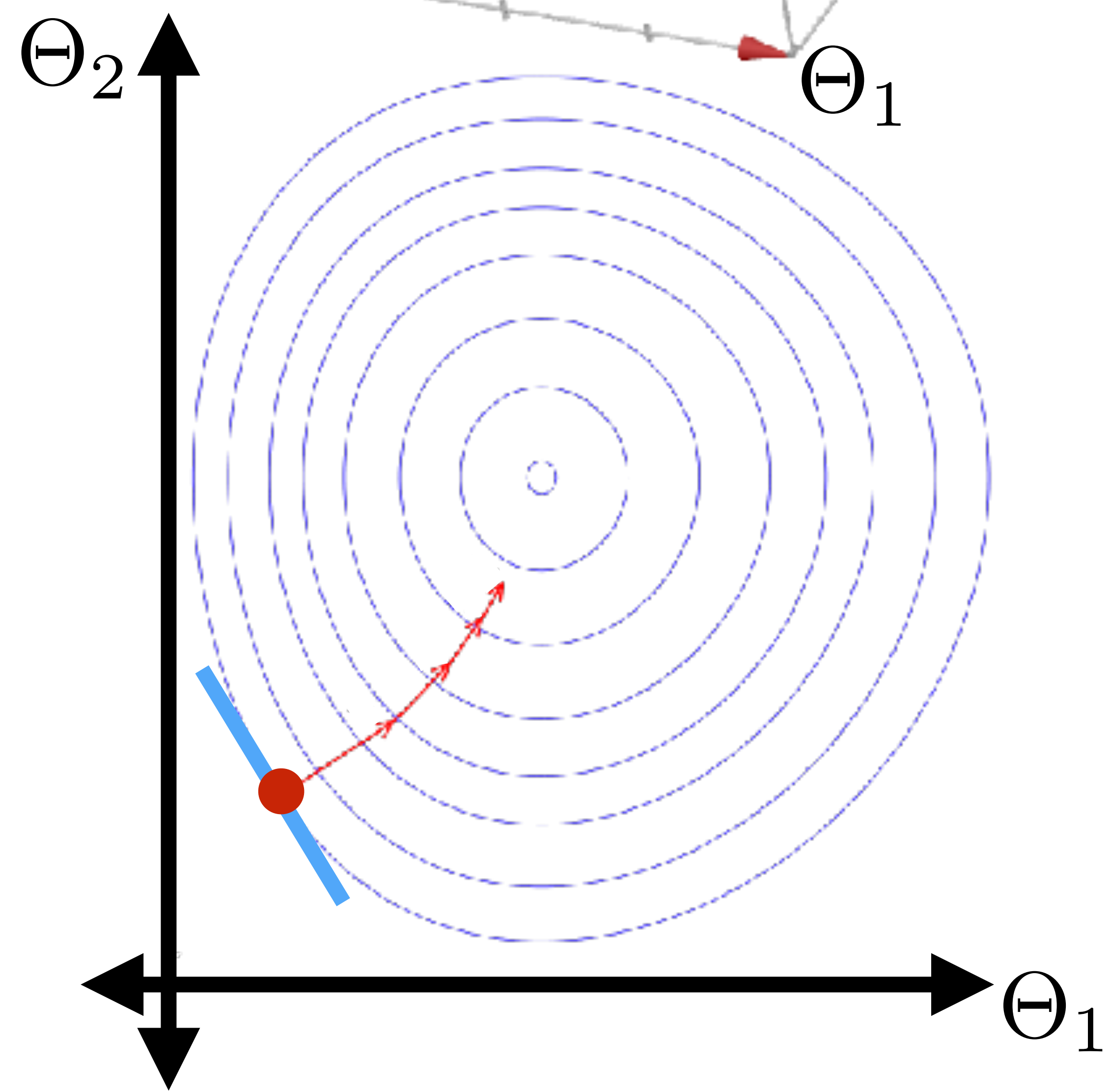

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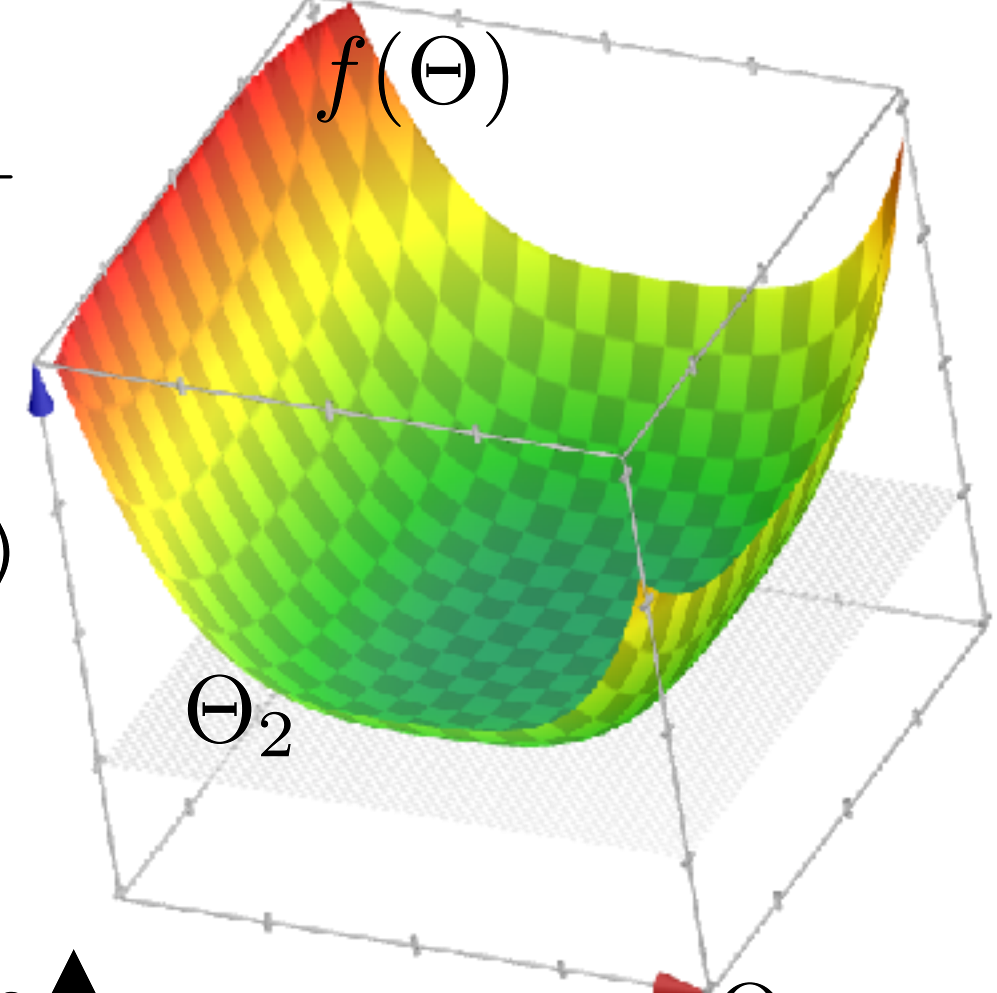

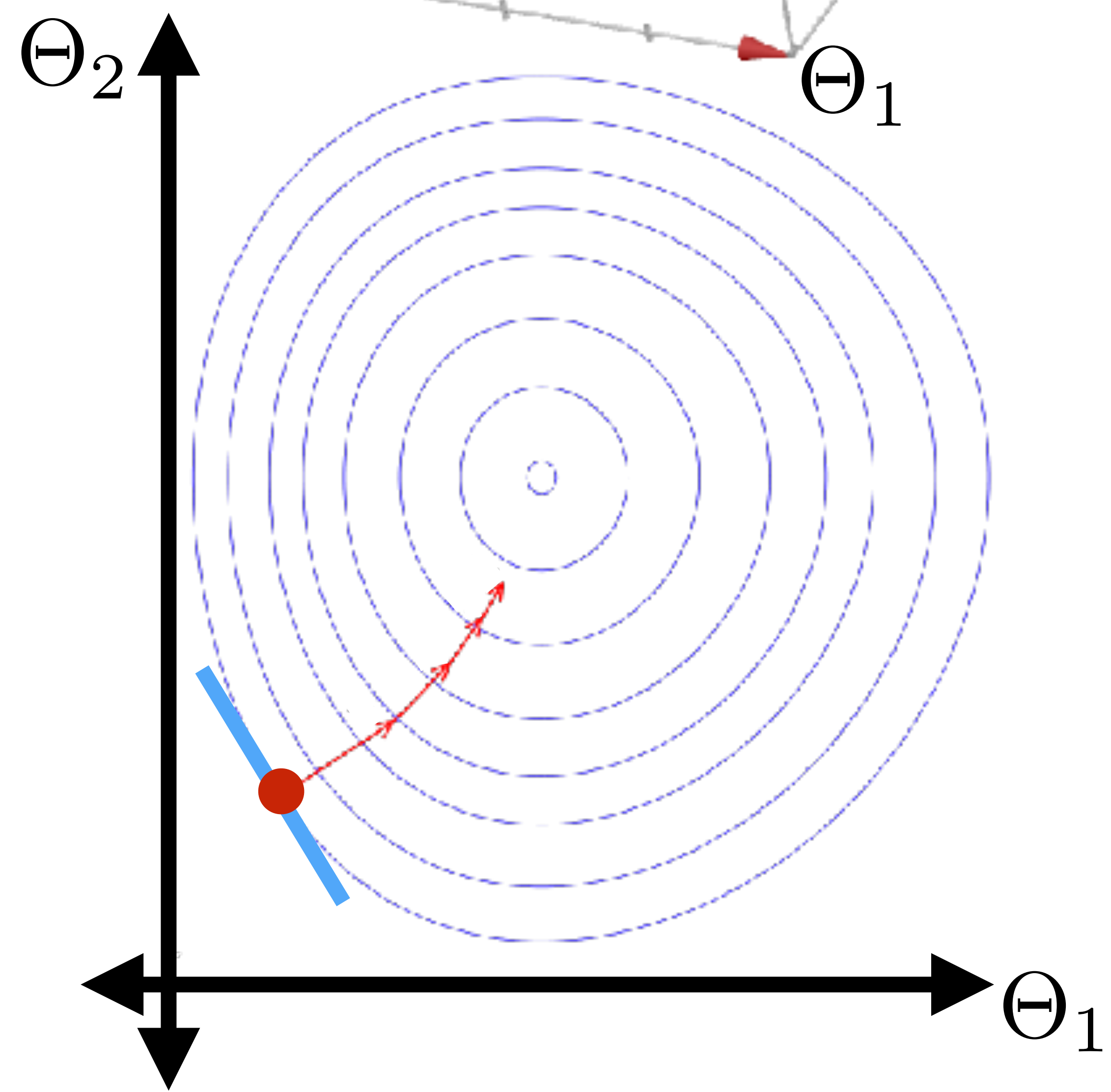

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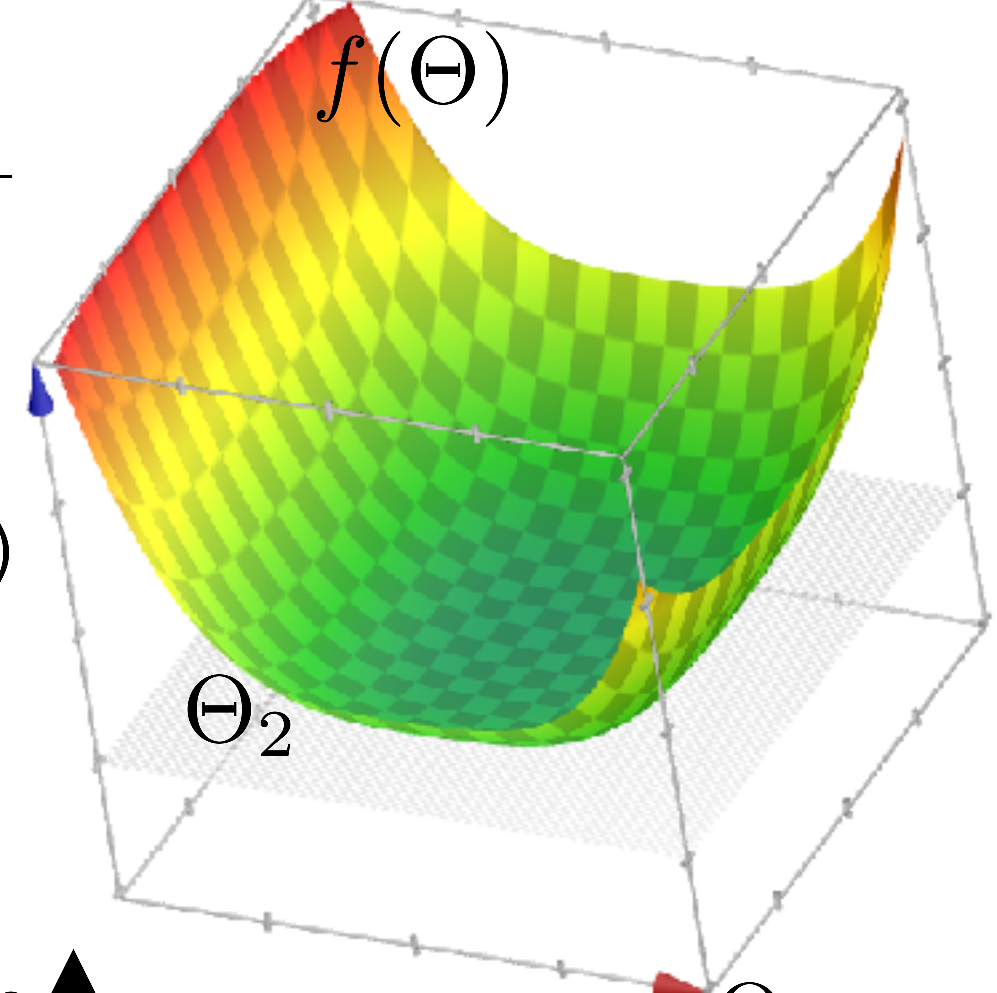

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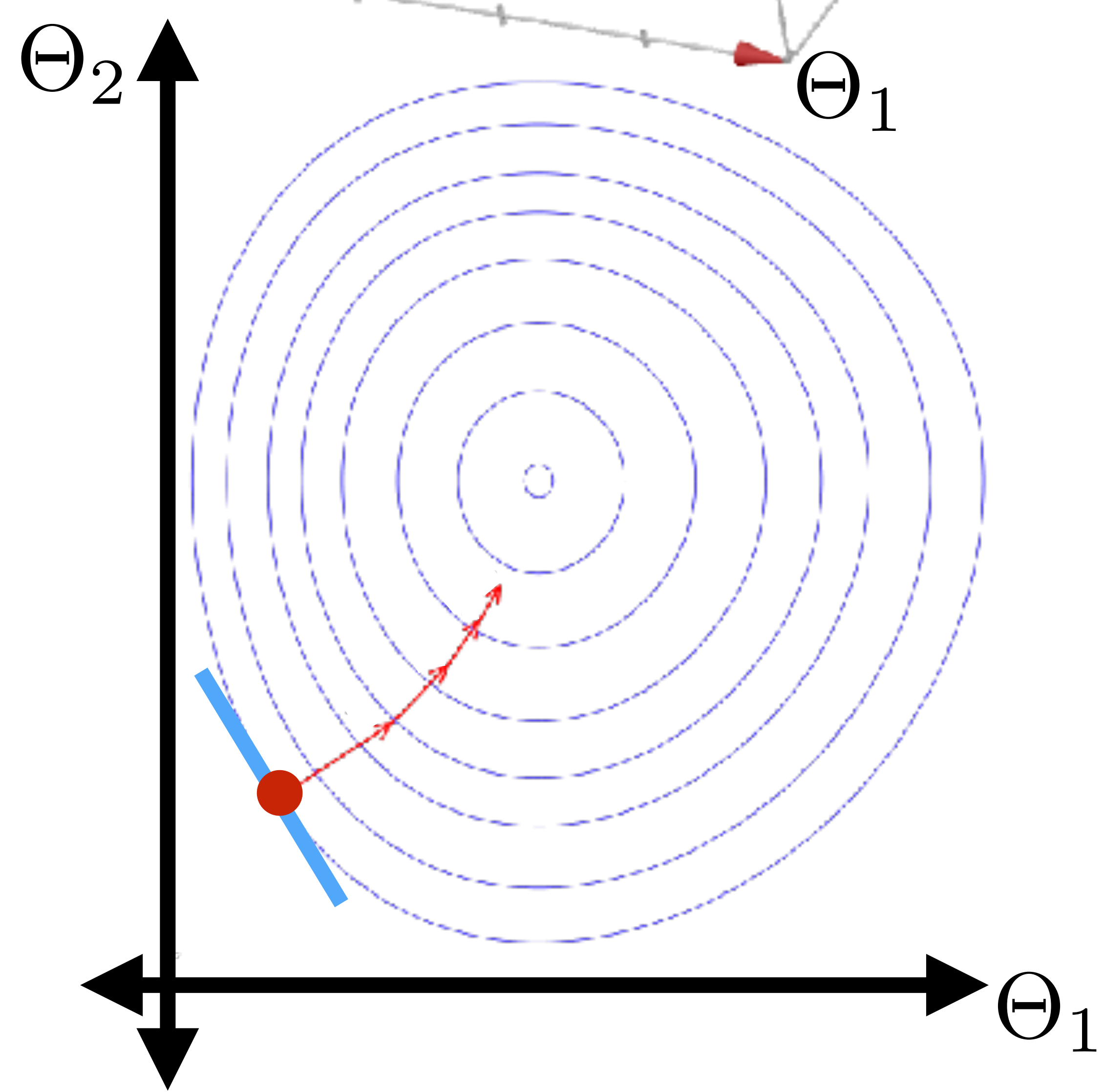

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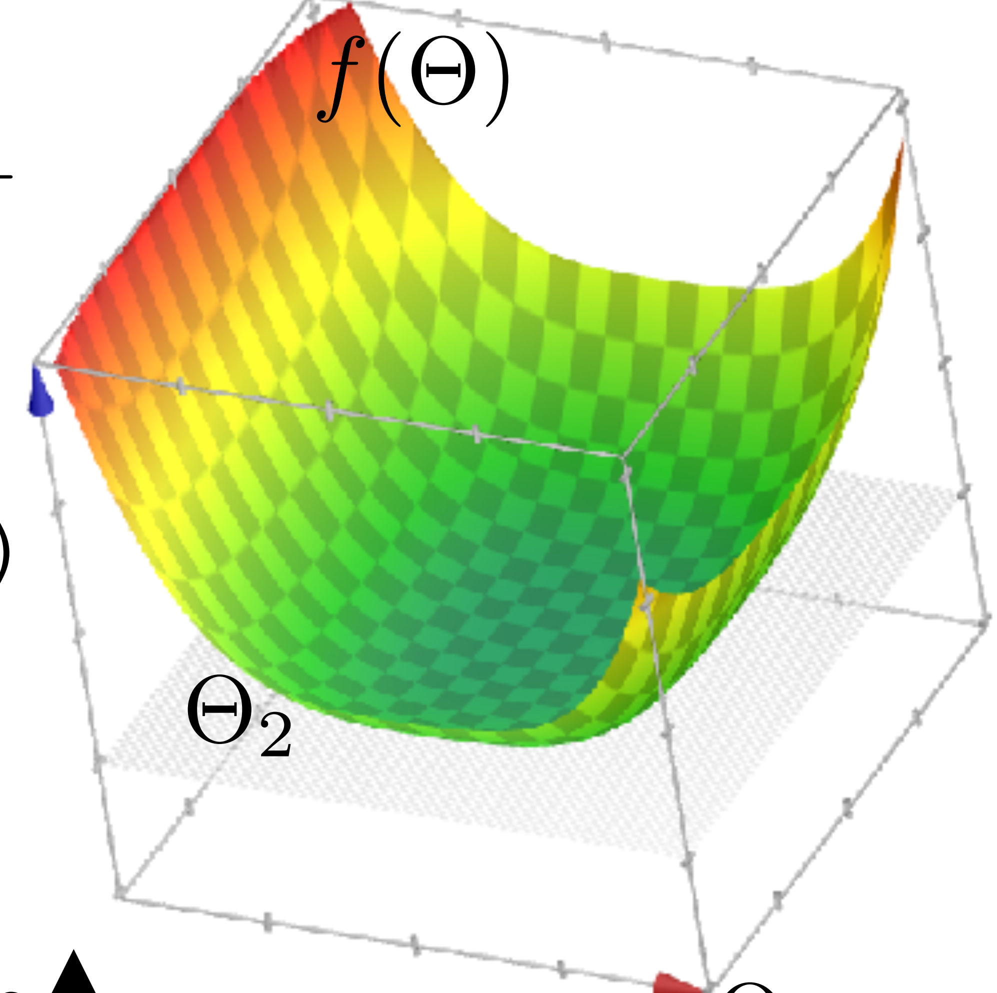

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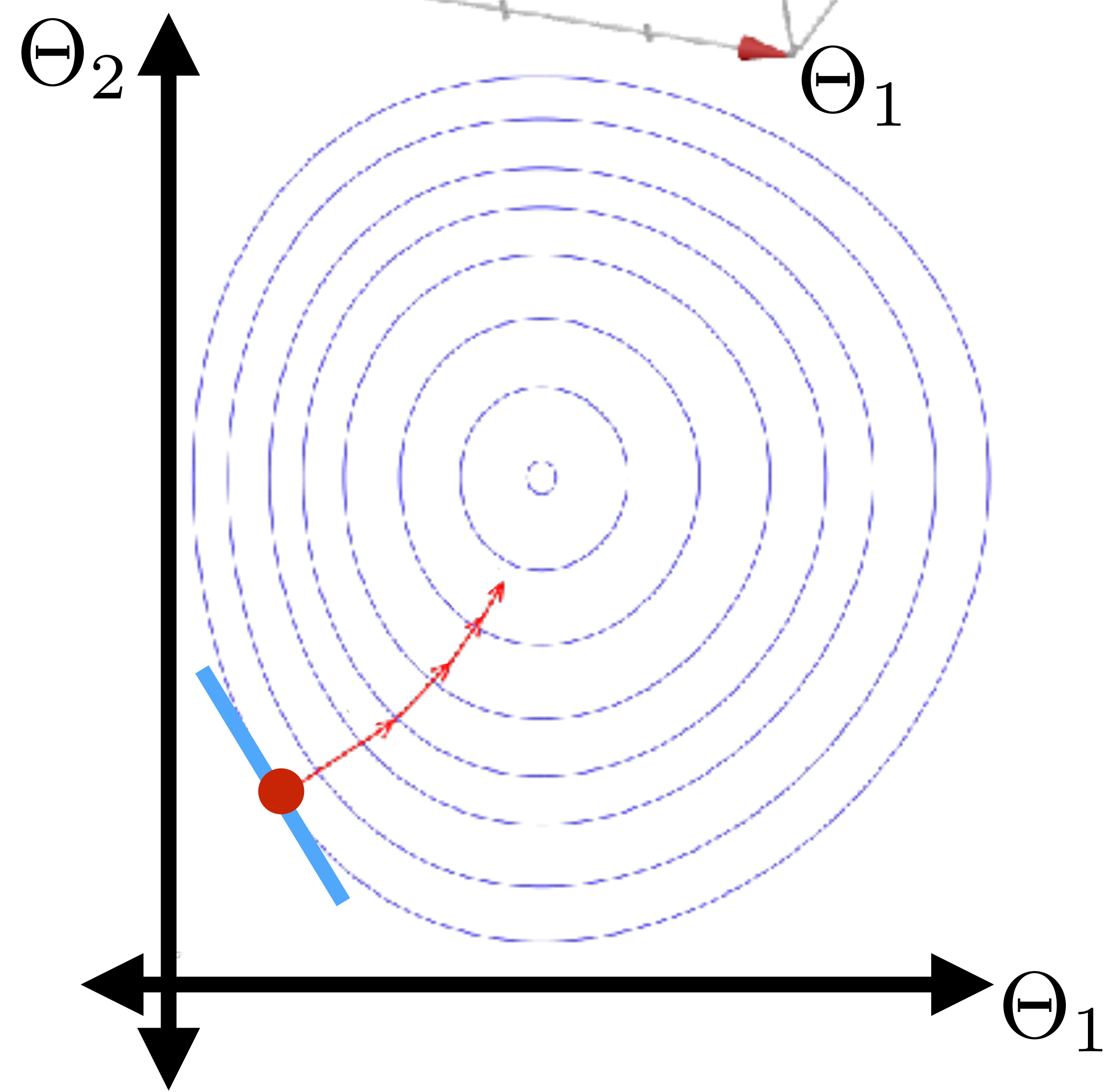

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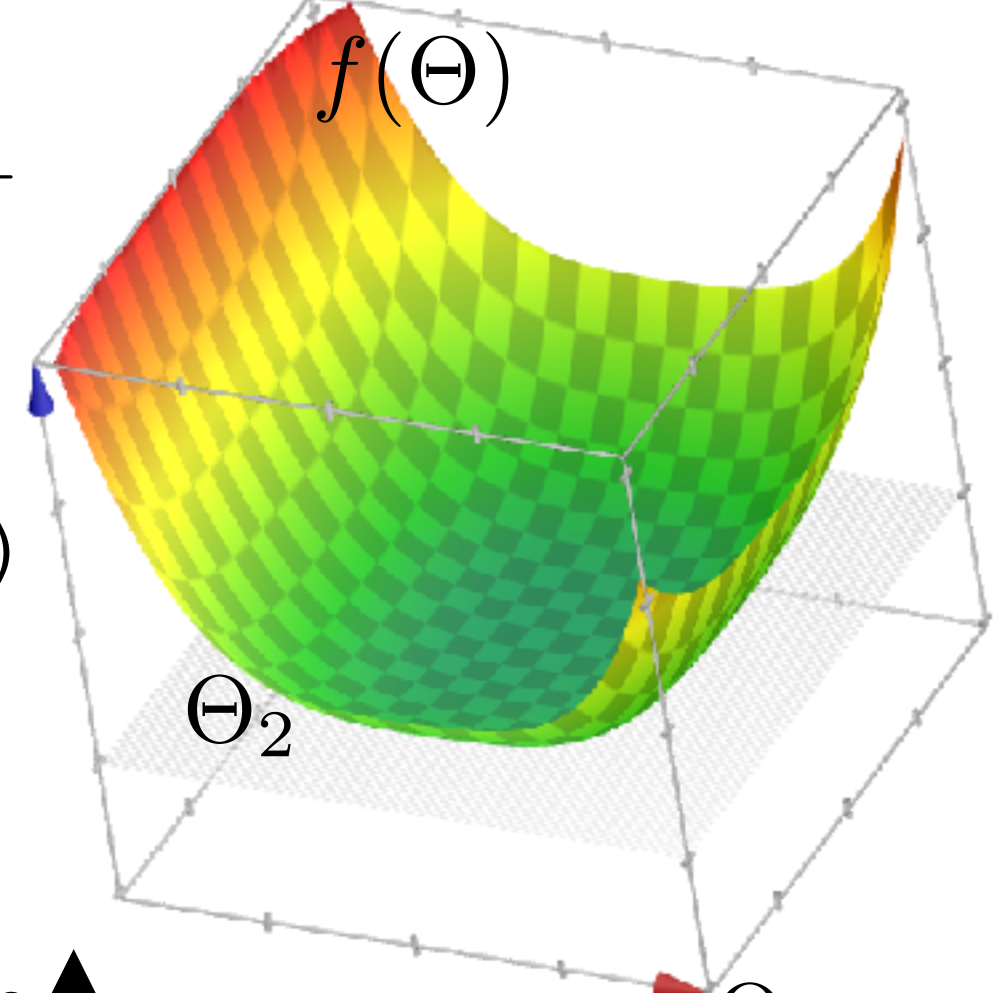


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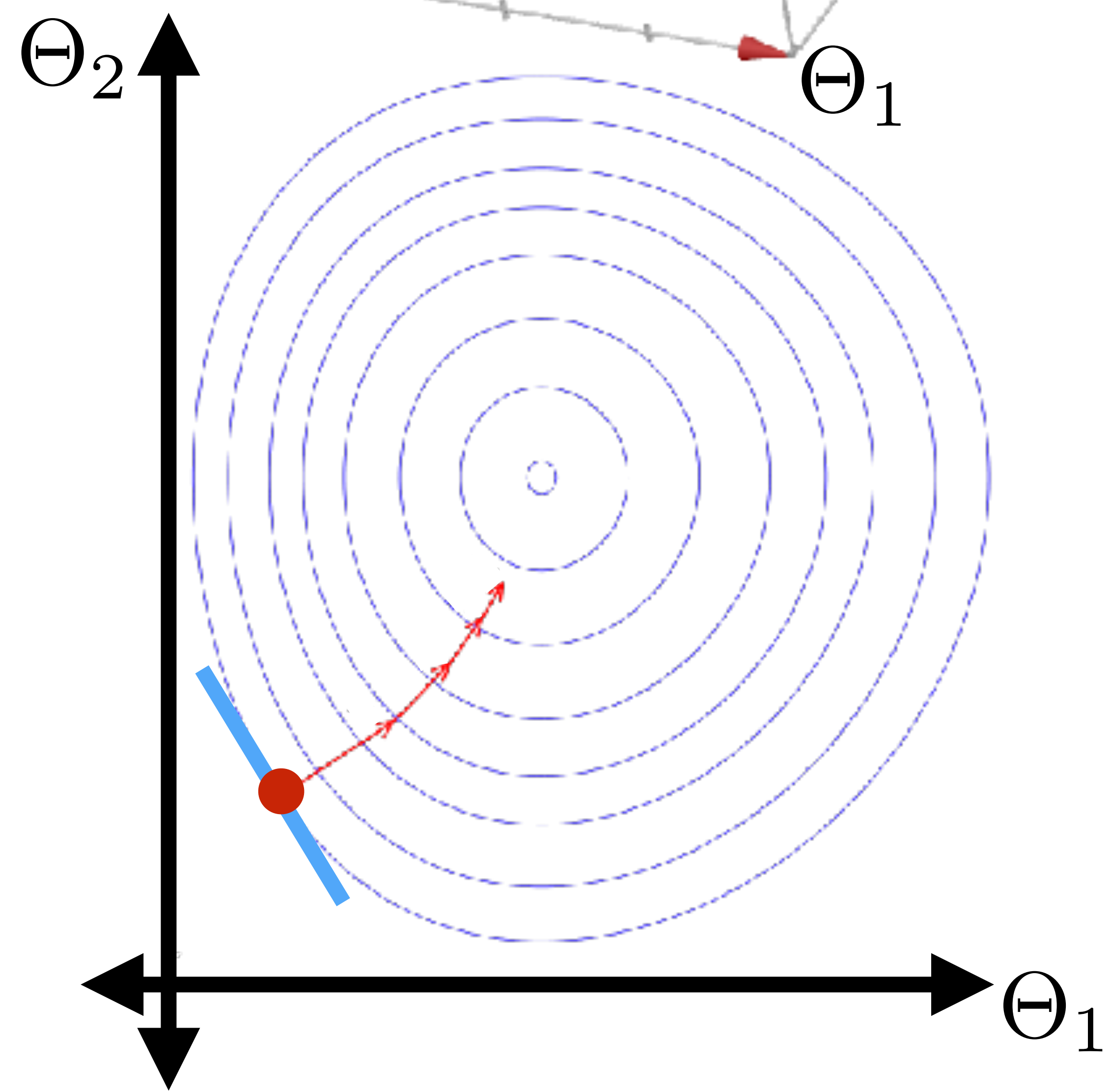
- Other possible stopping criteria:



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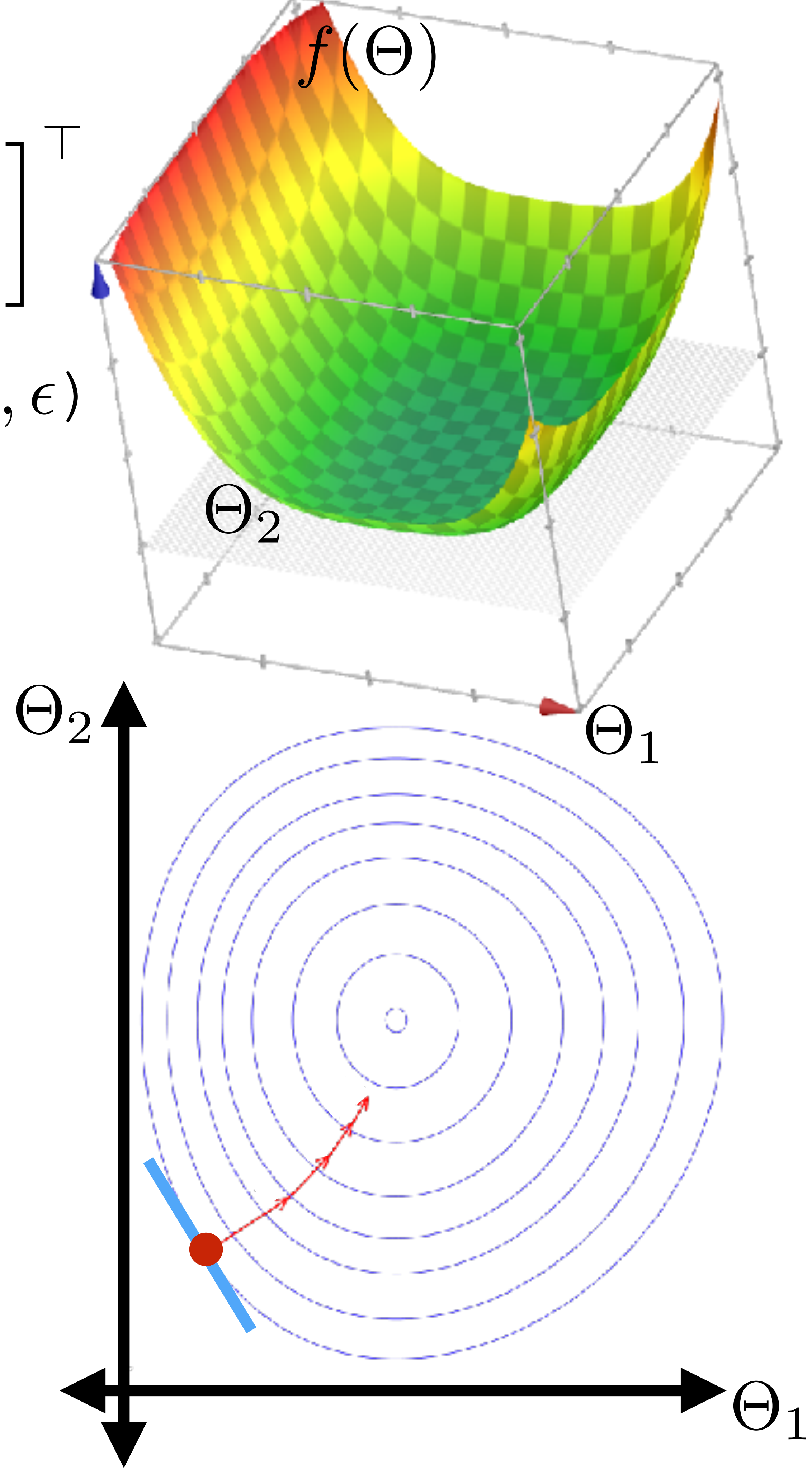

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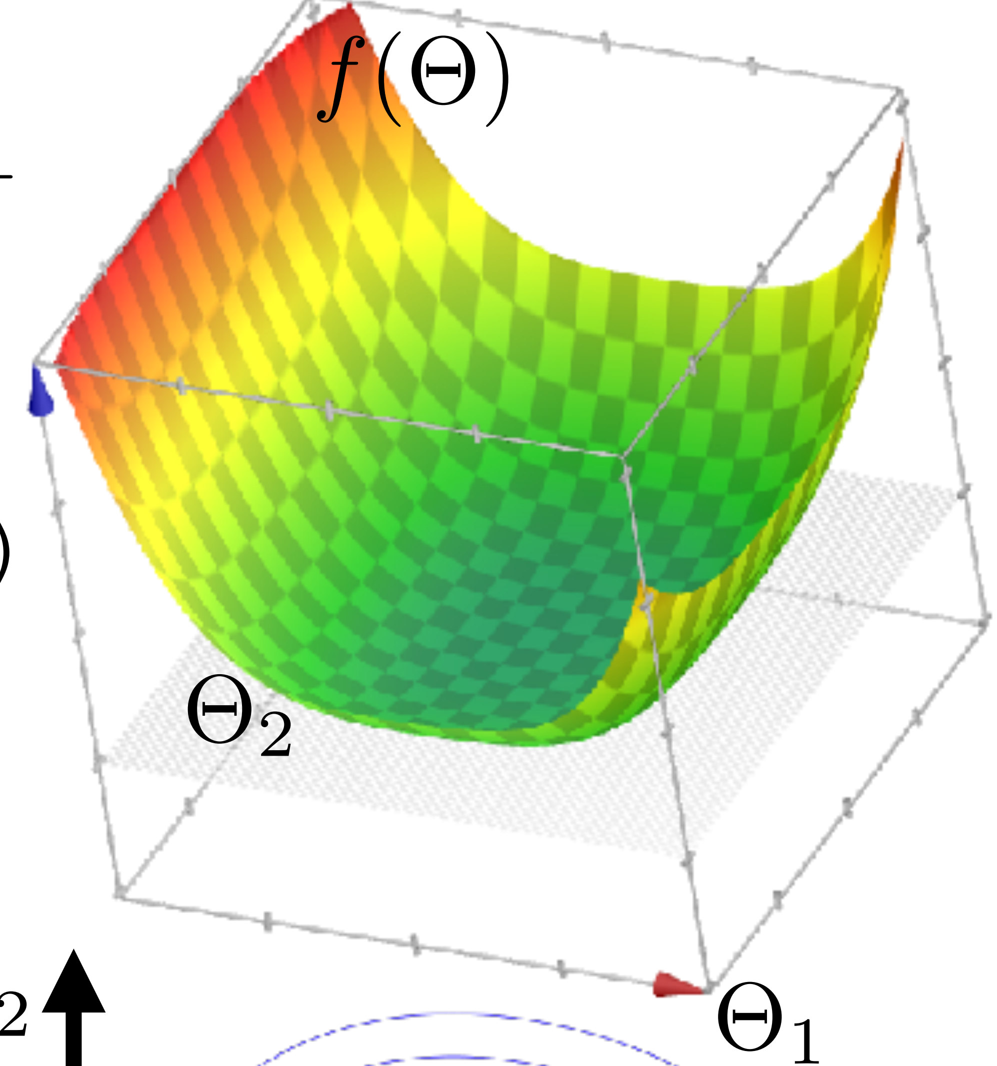

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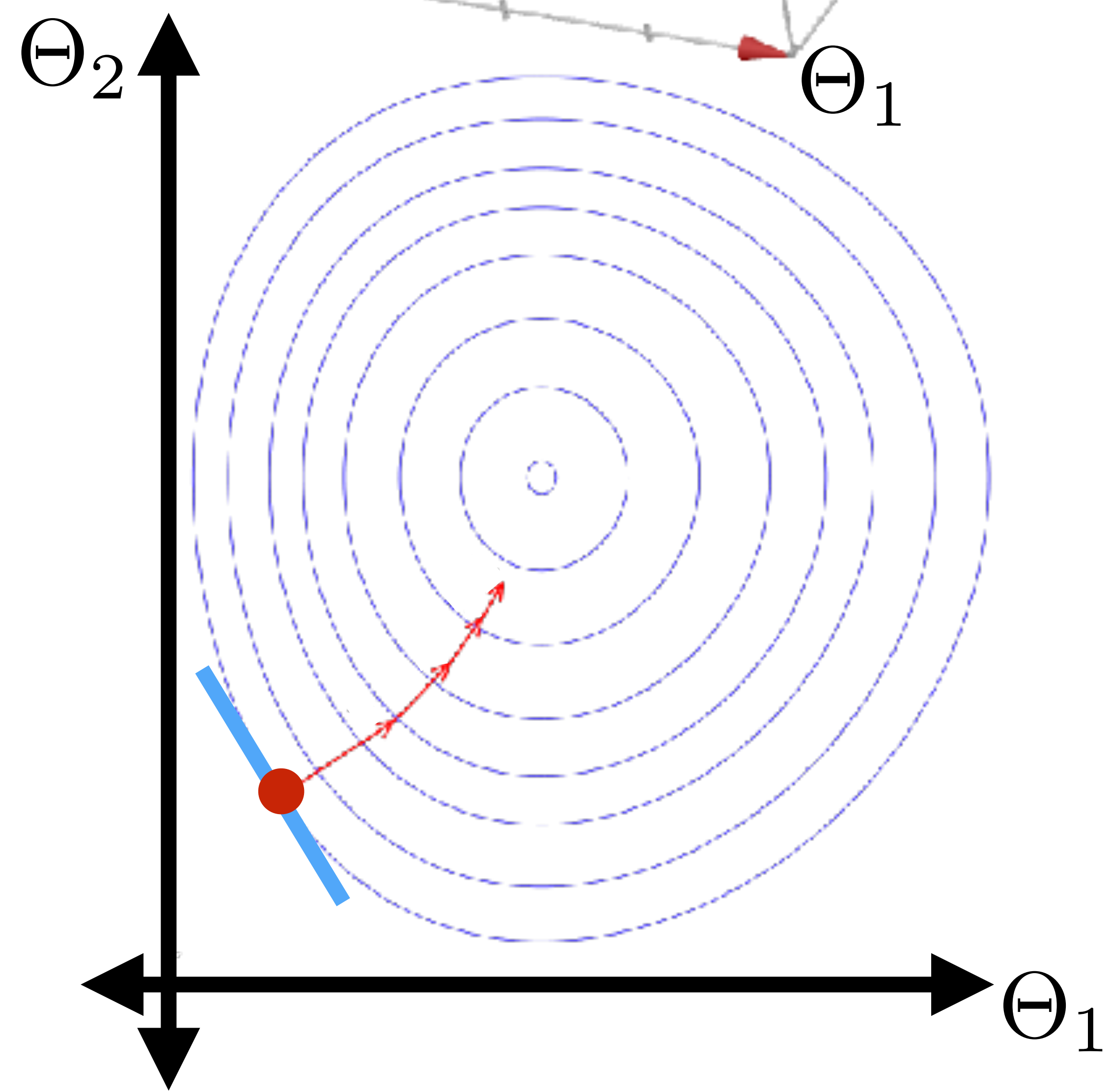

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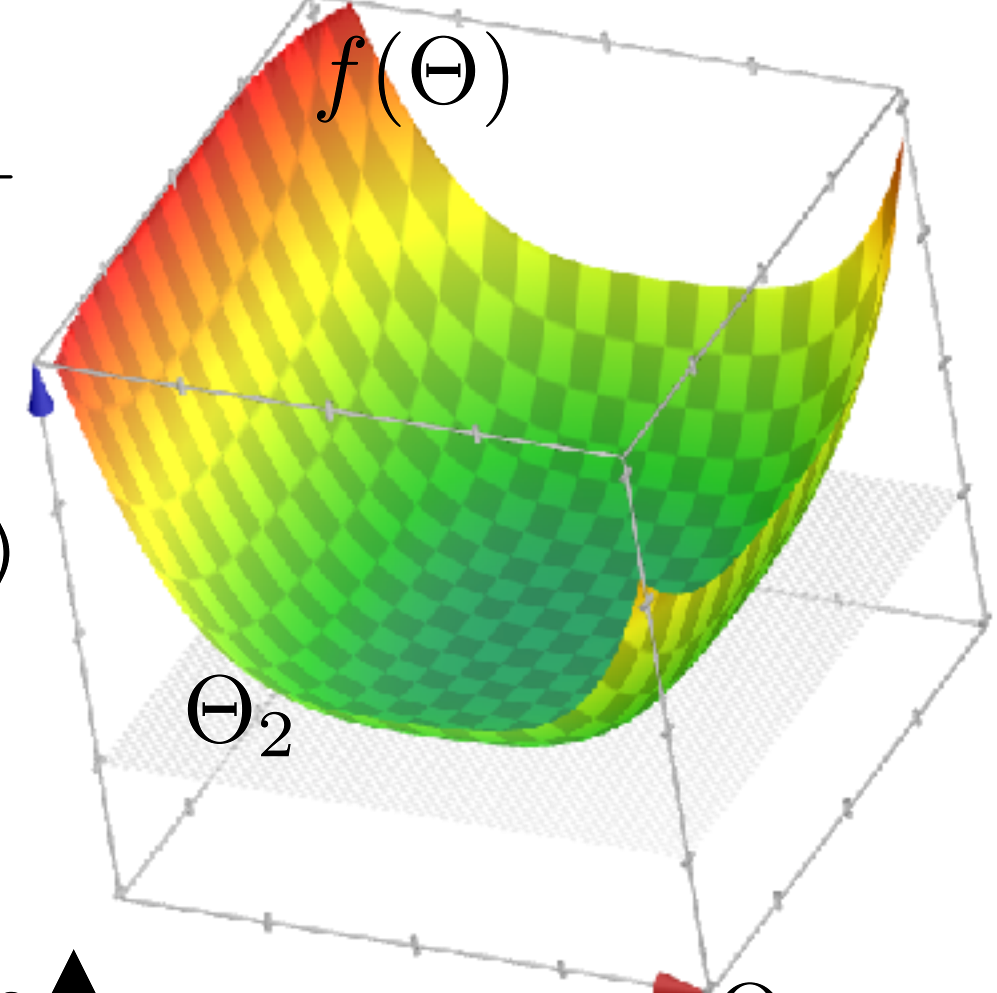

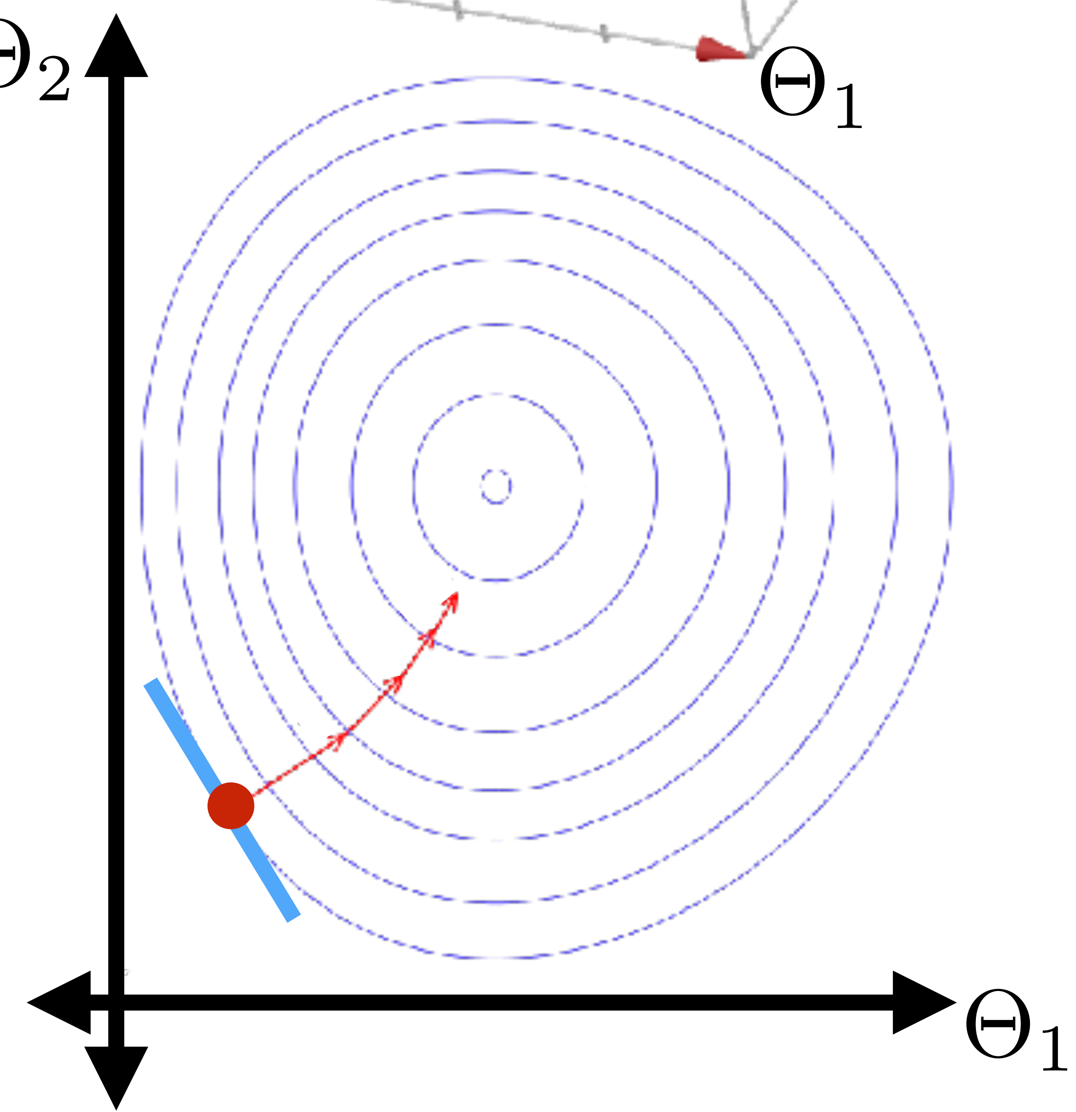

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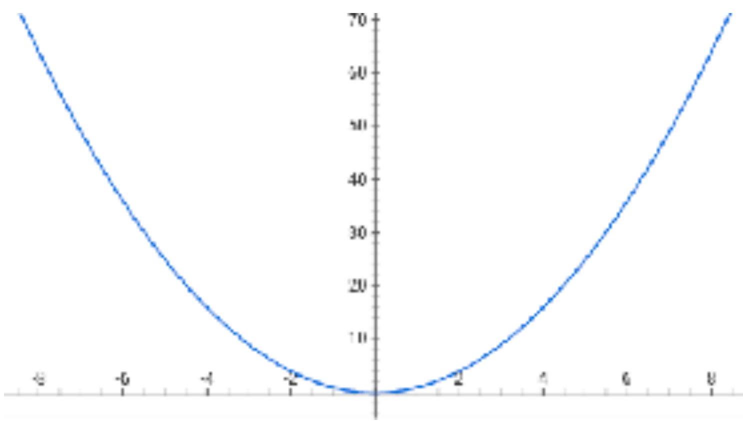

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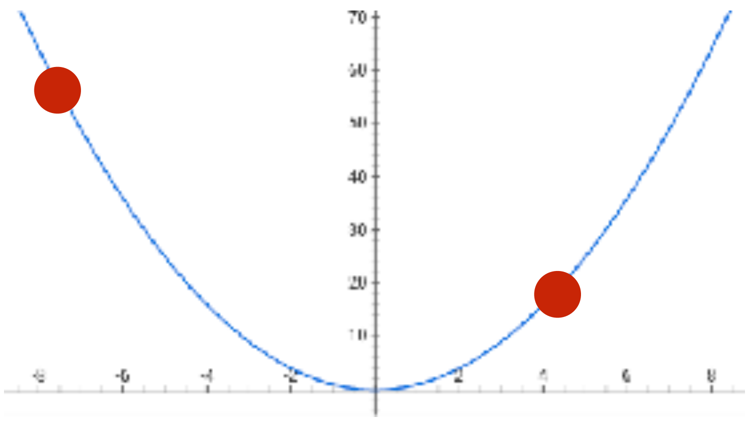

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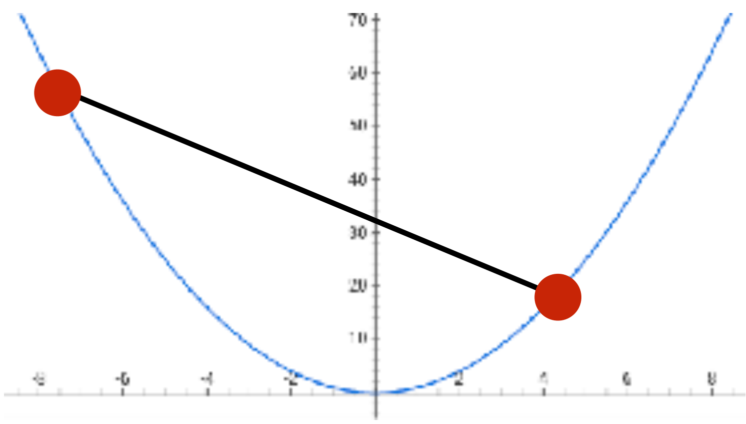

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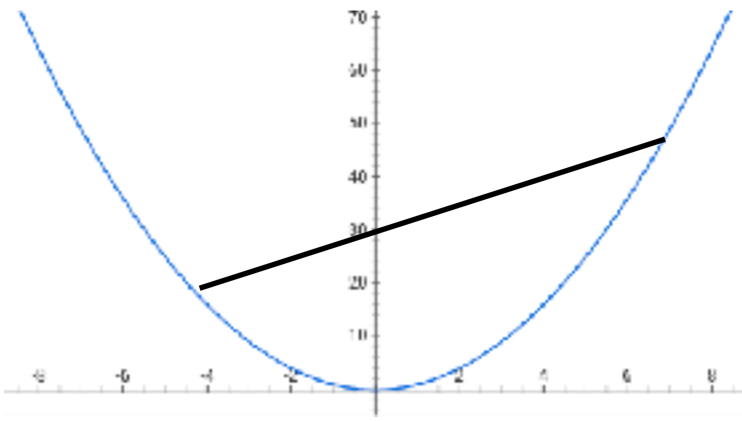

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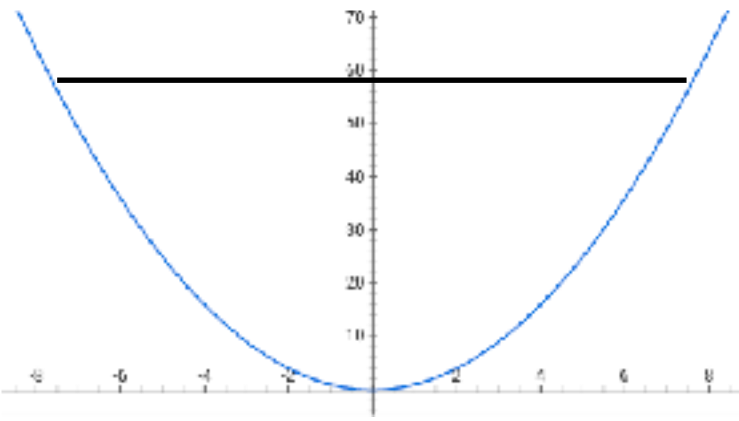

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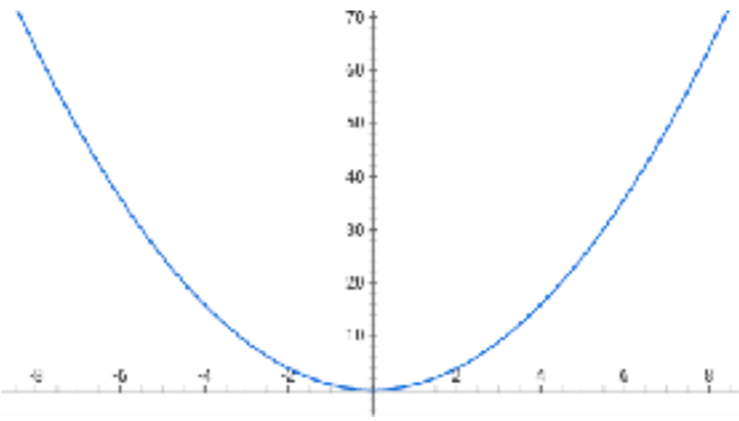

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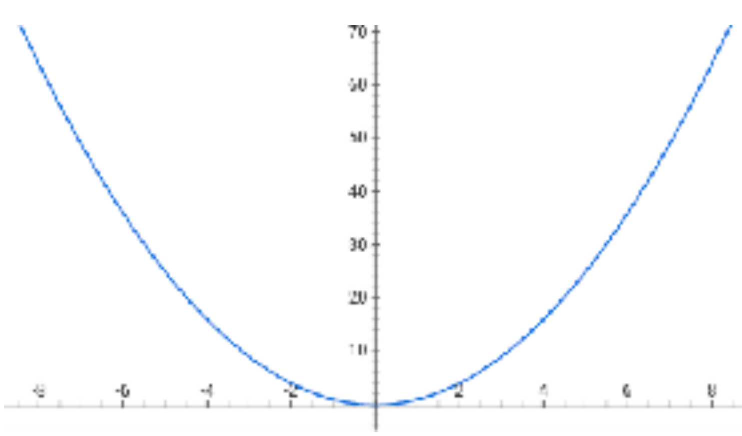

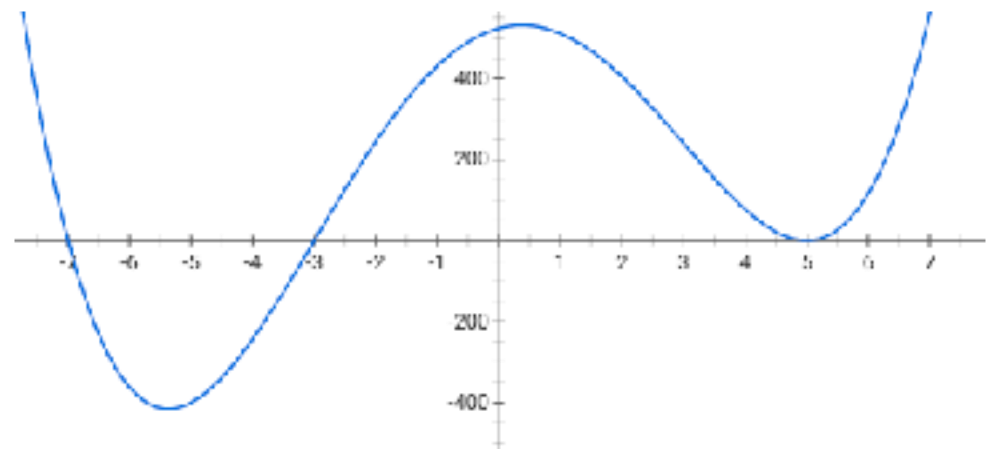

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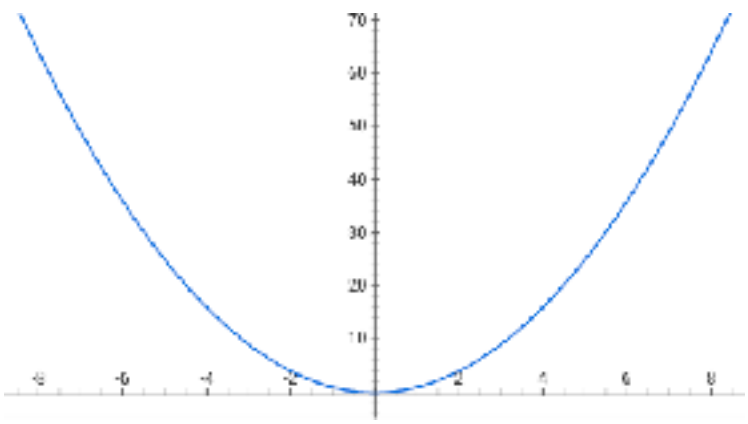

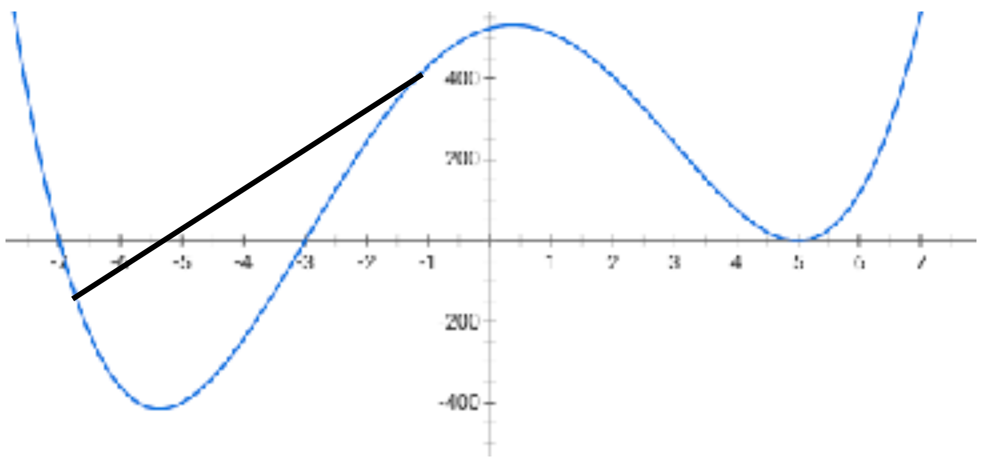

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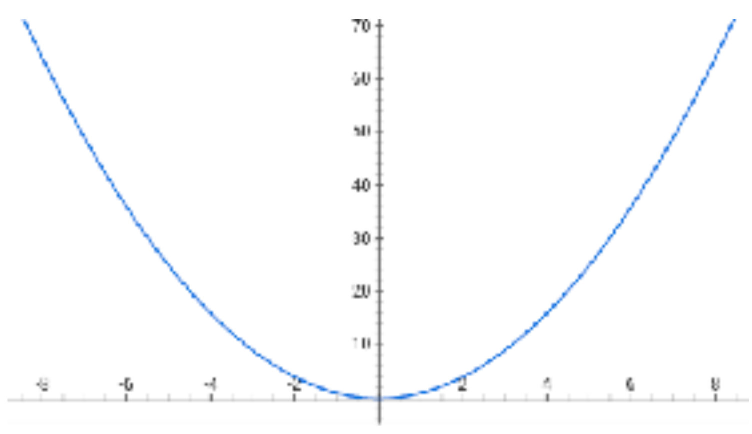

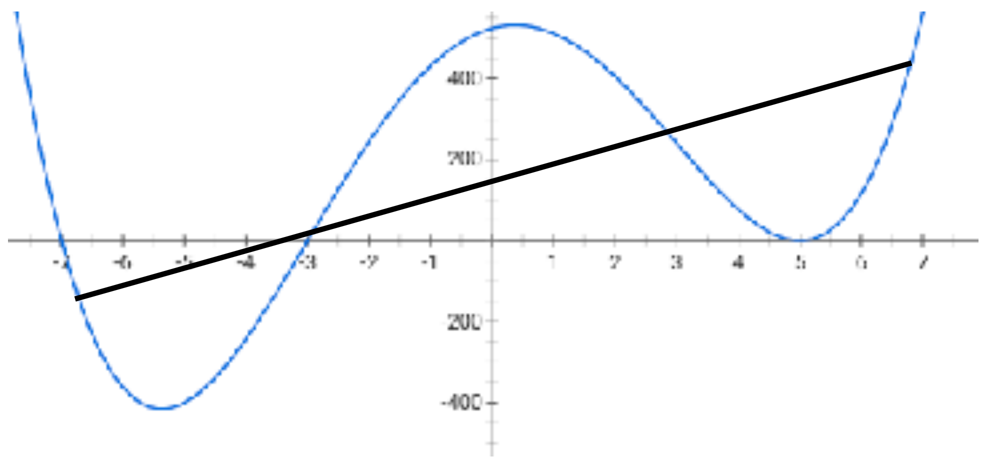

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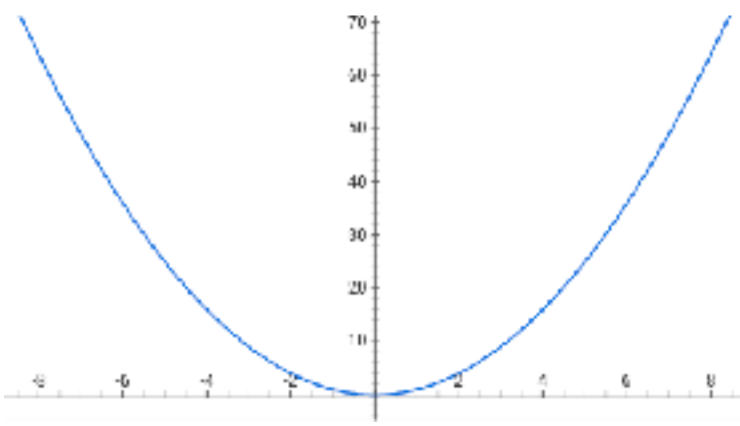

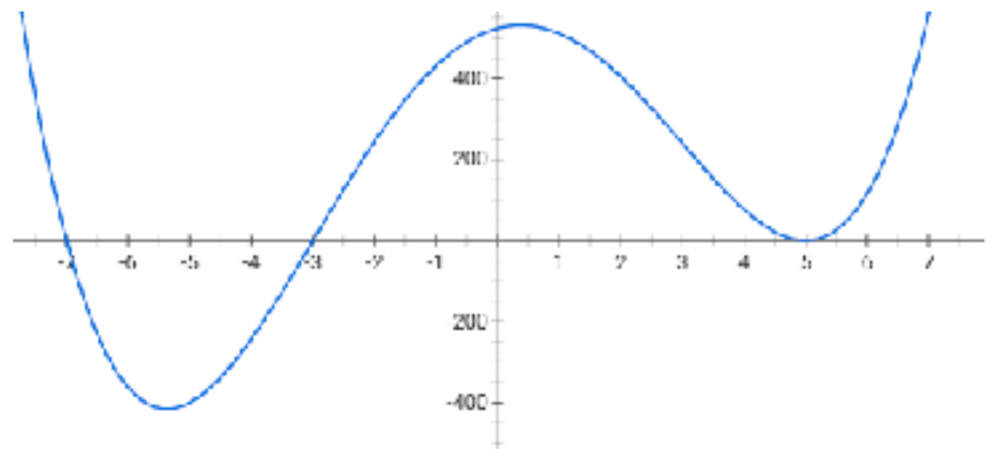

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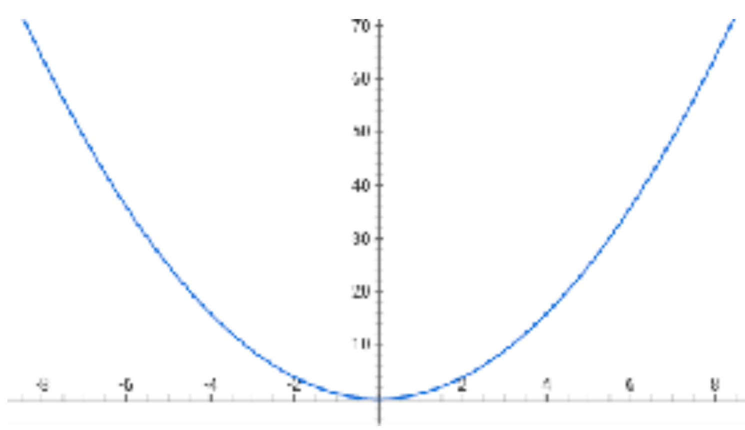

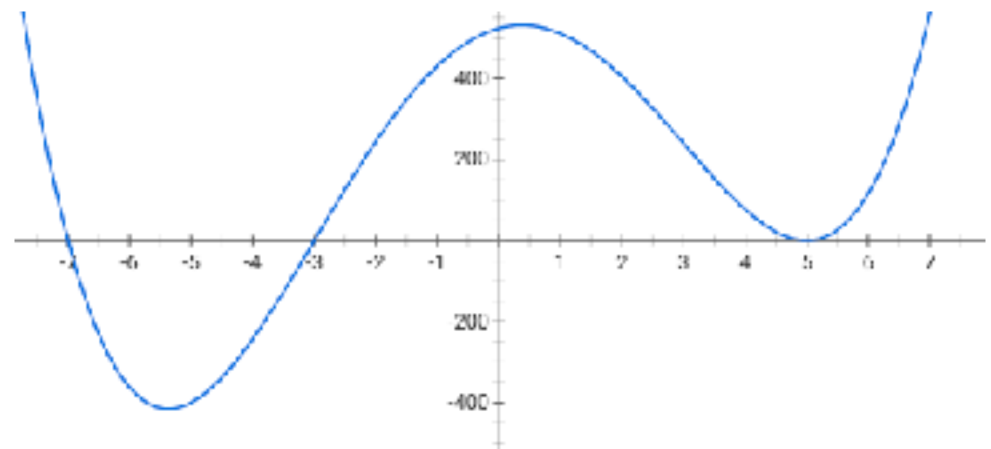

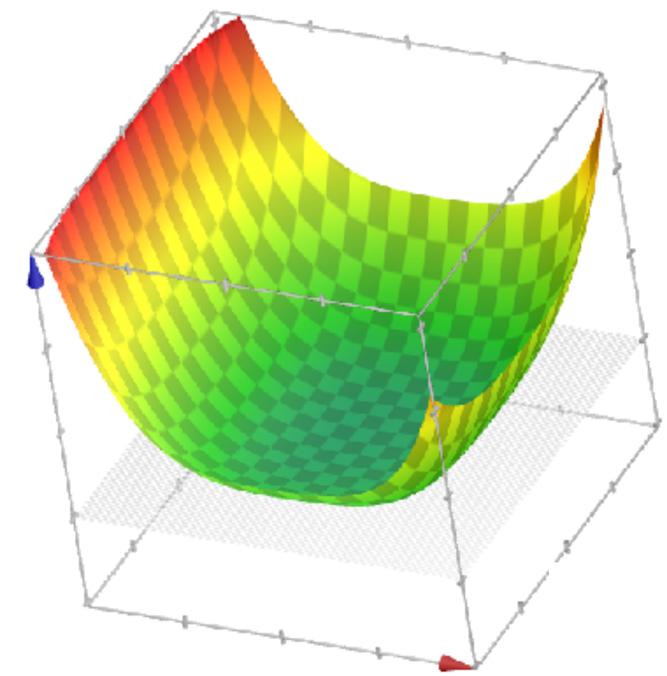

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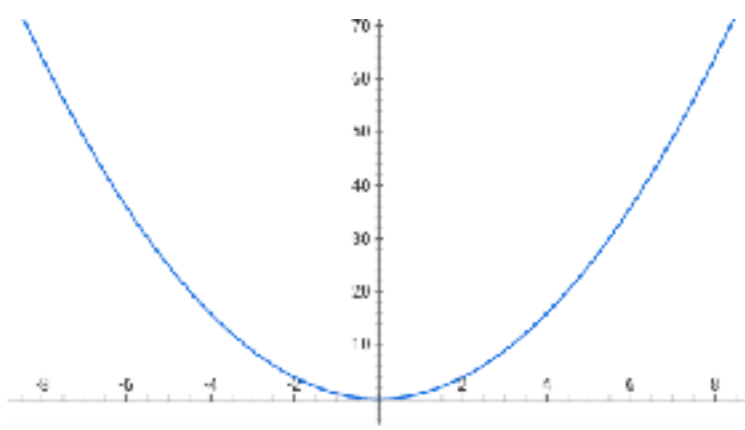

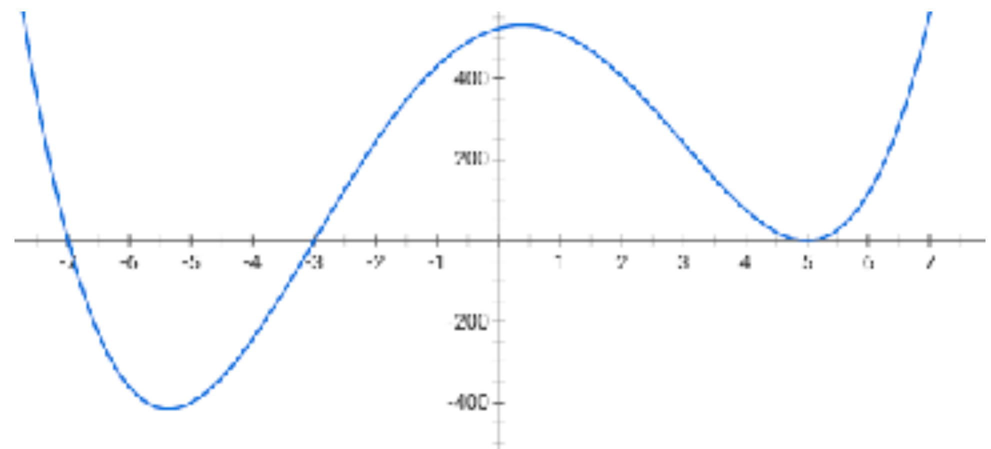

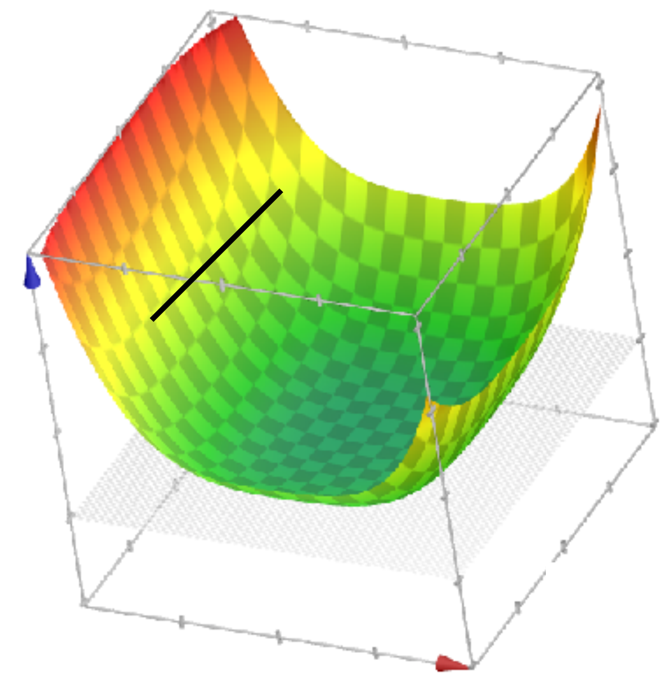

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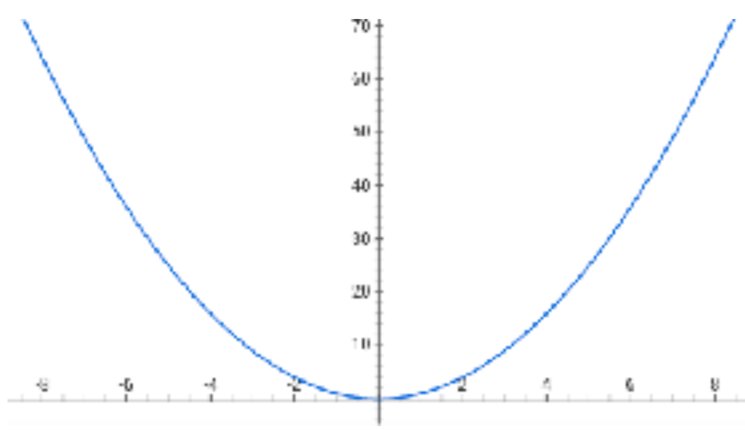

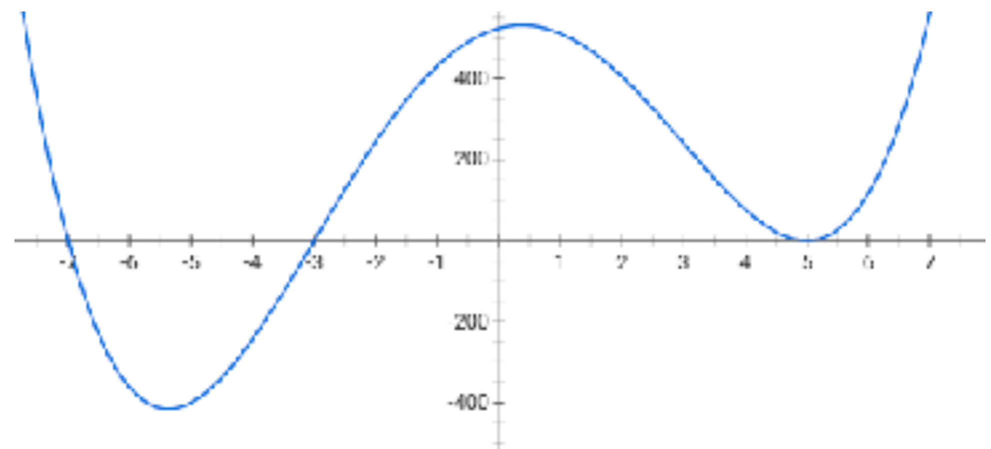

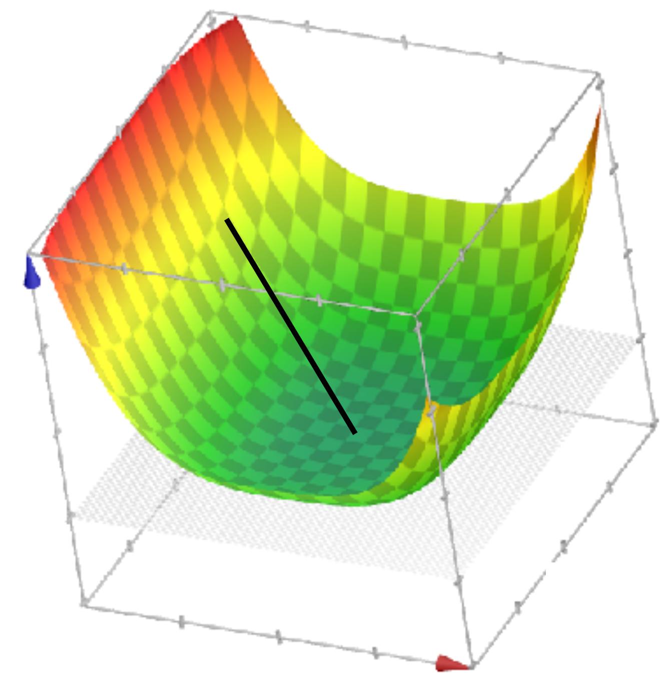

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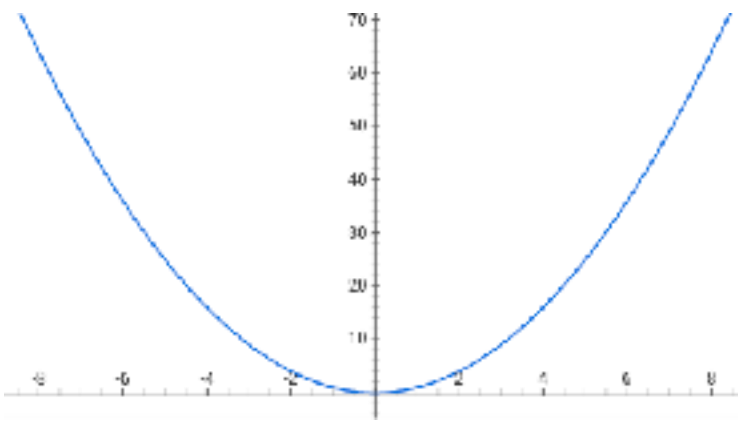

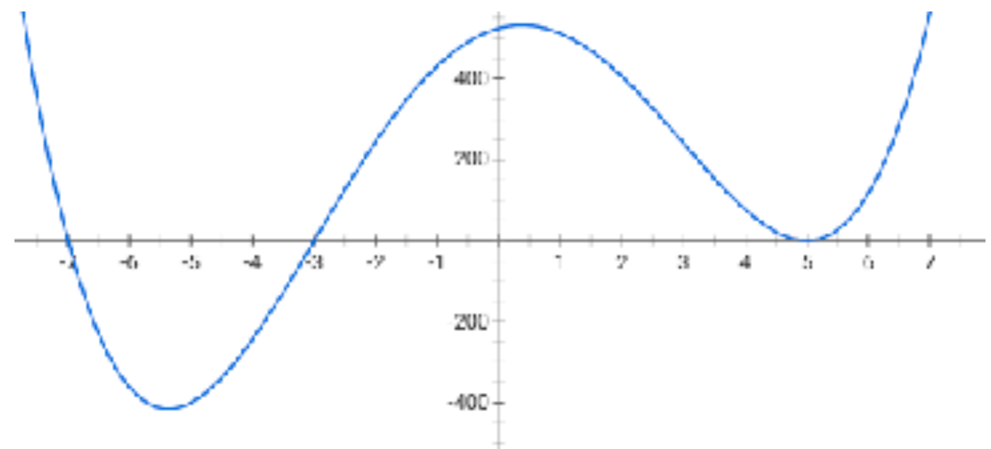

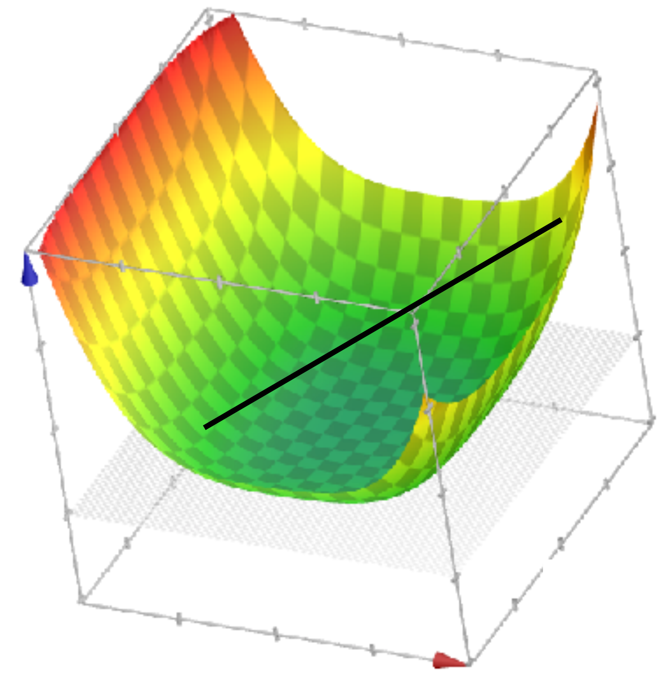

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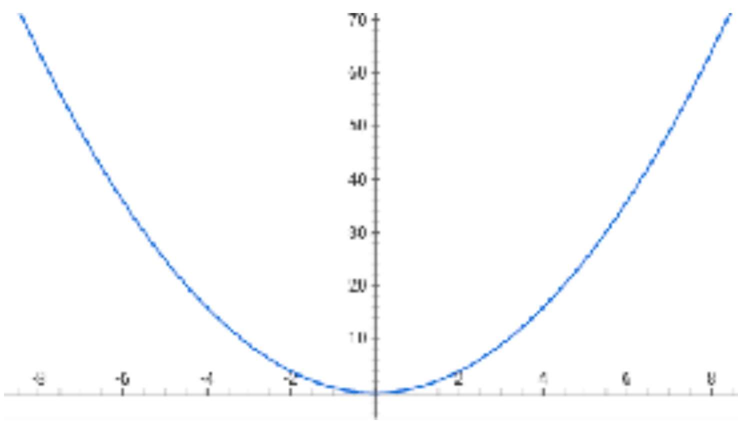

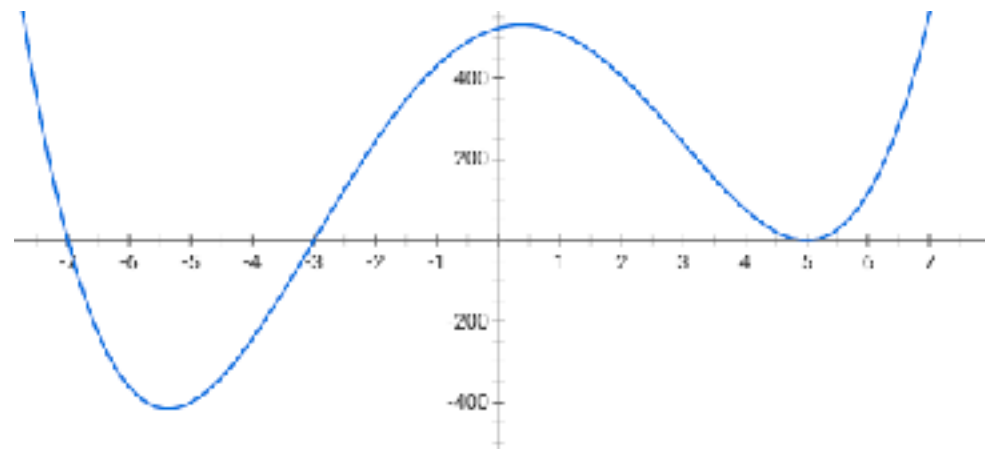

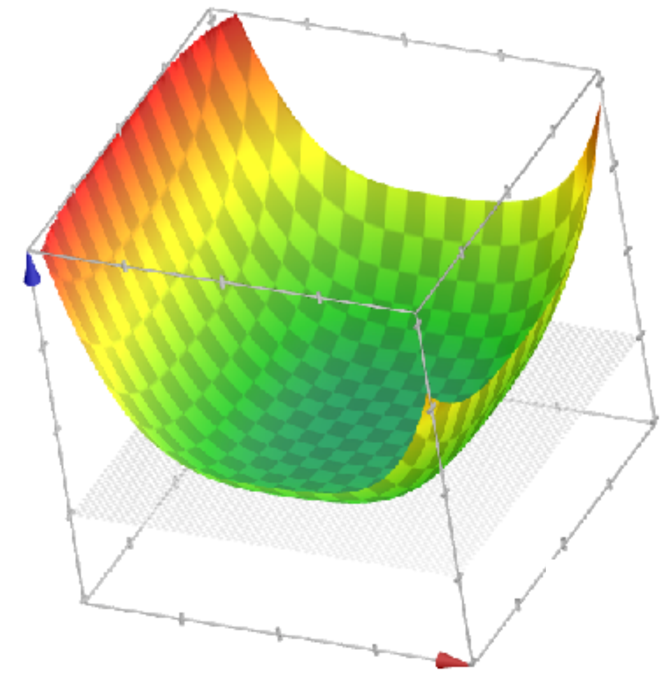

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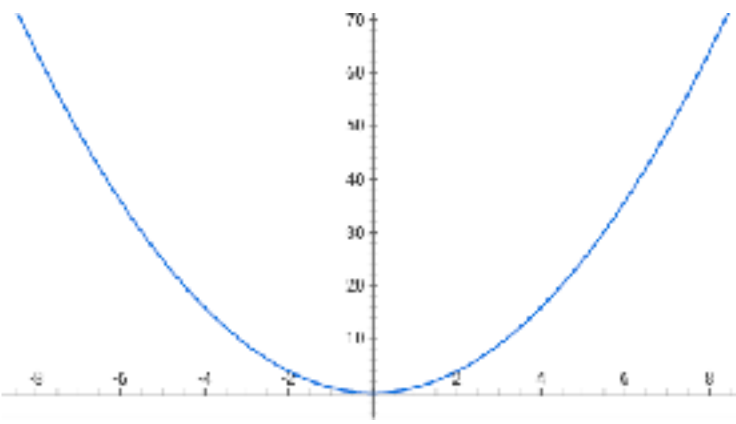

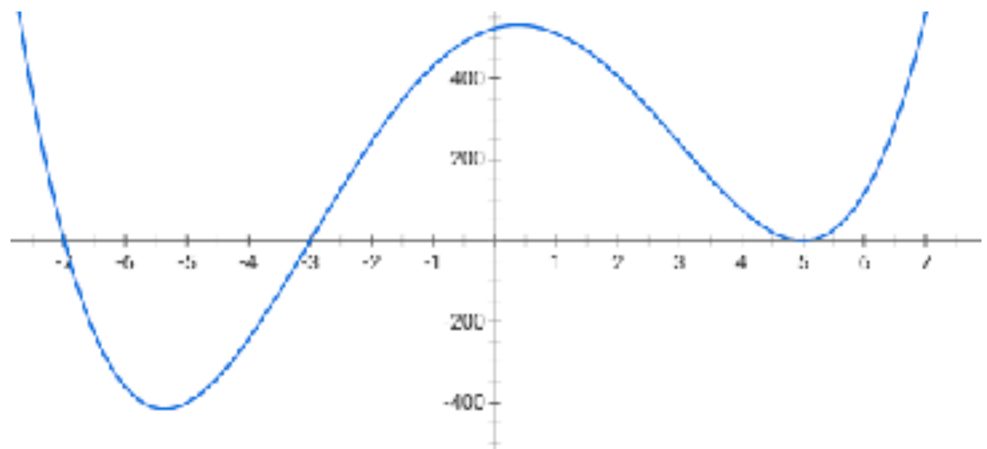

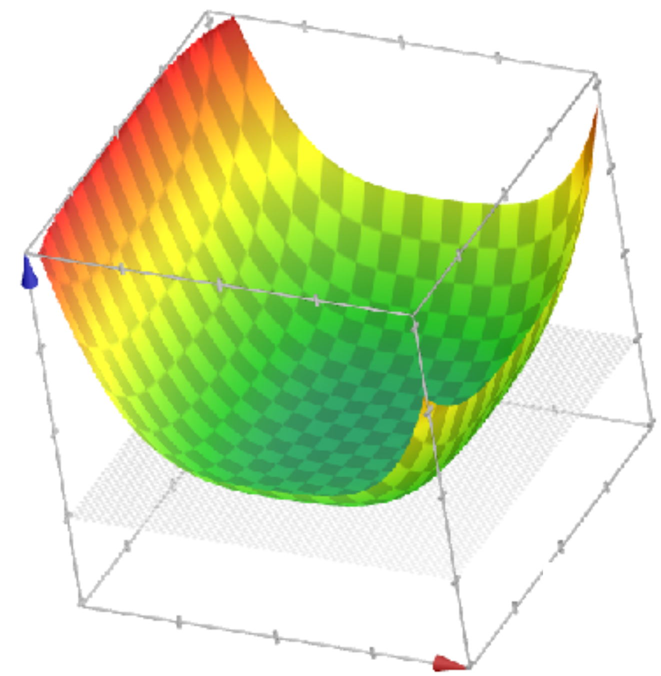

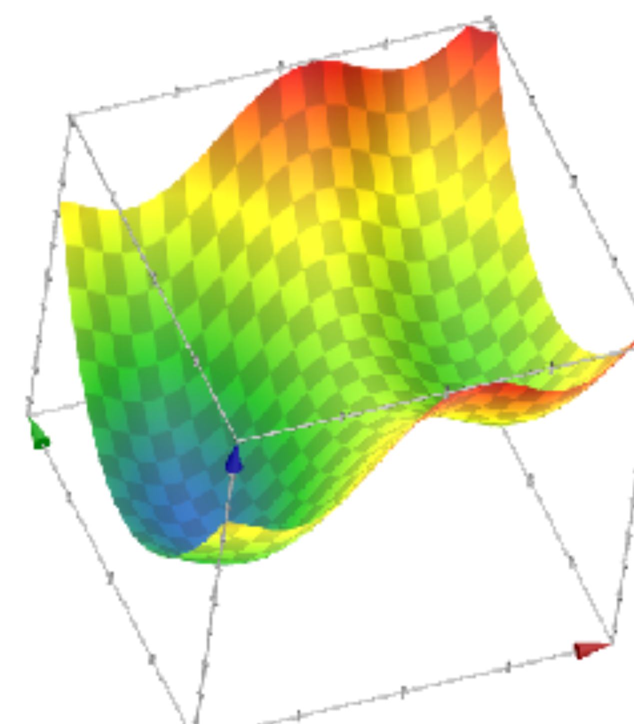

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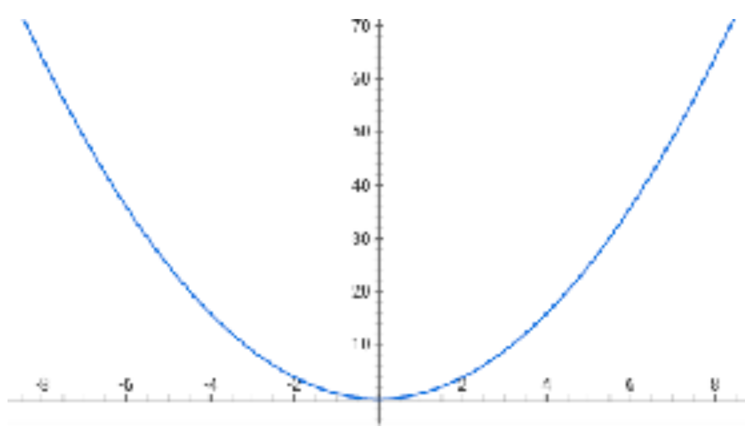
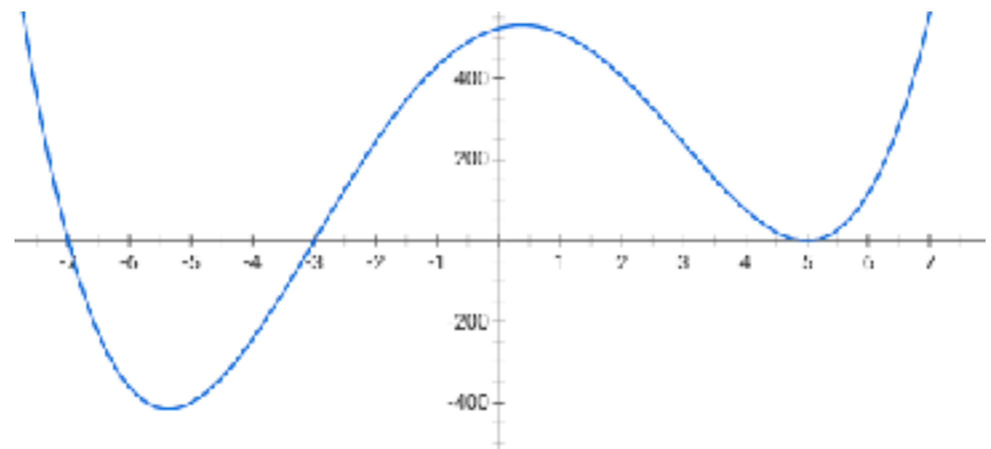
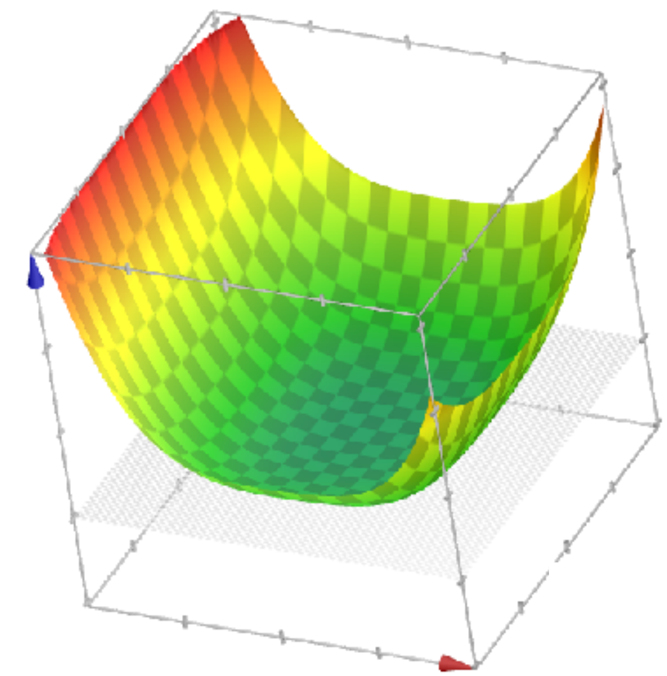
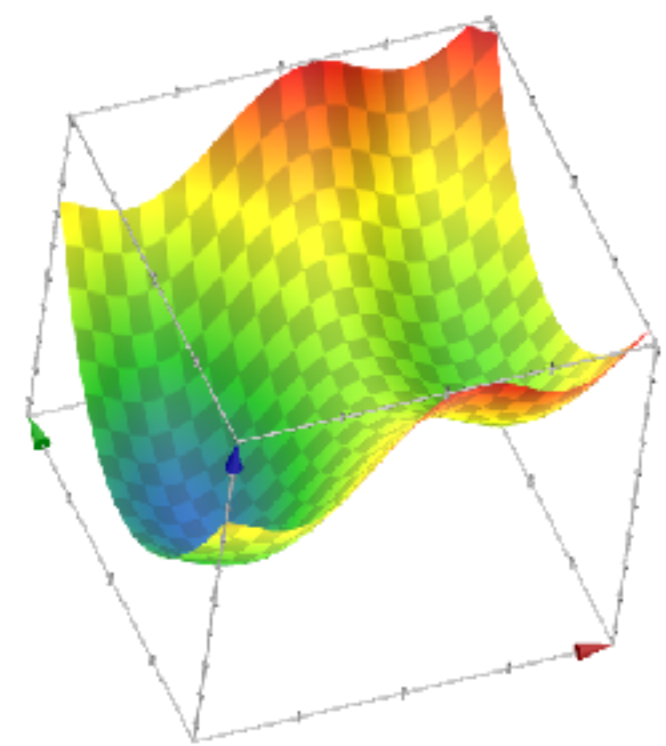






- **Theorem**: Gradient descent performance



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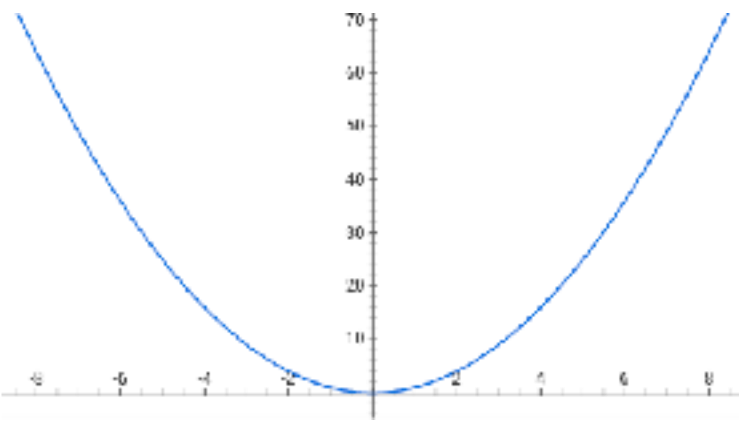

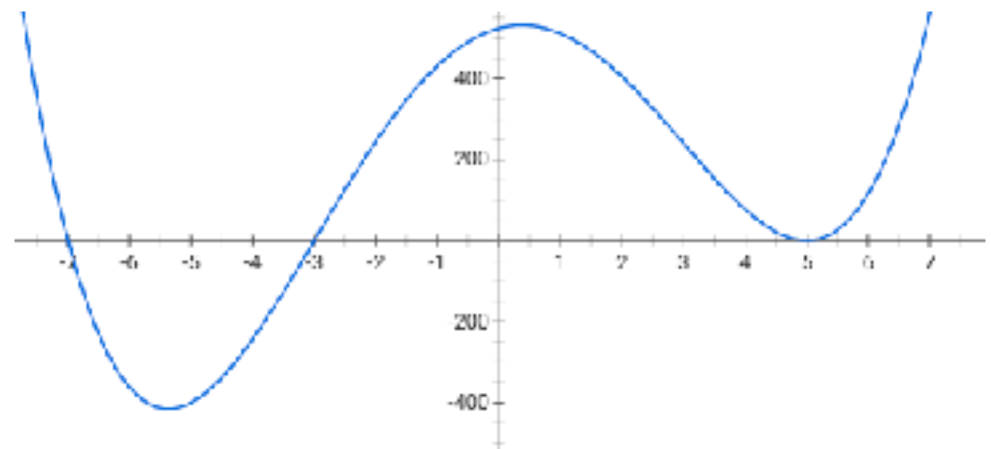

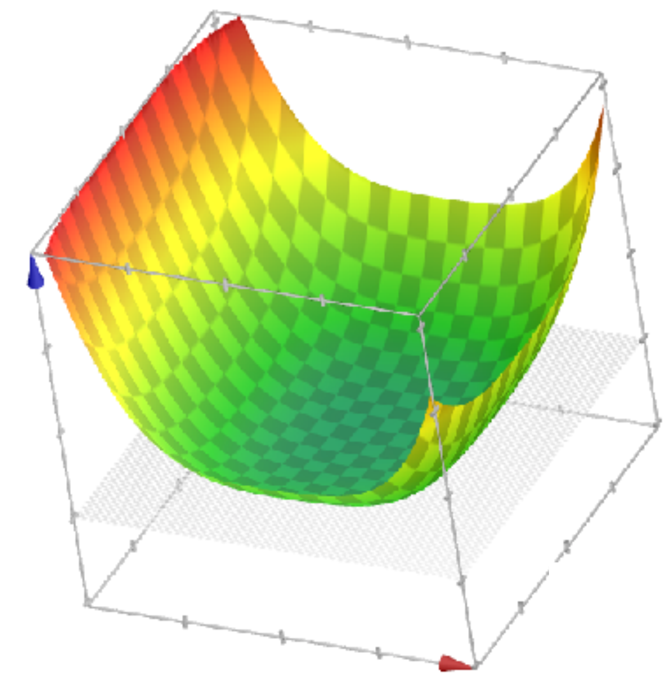

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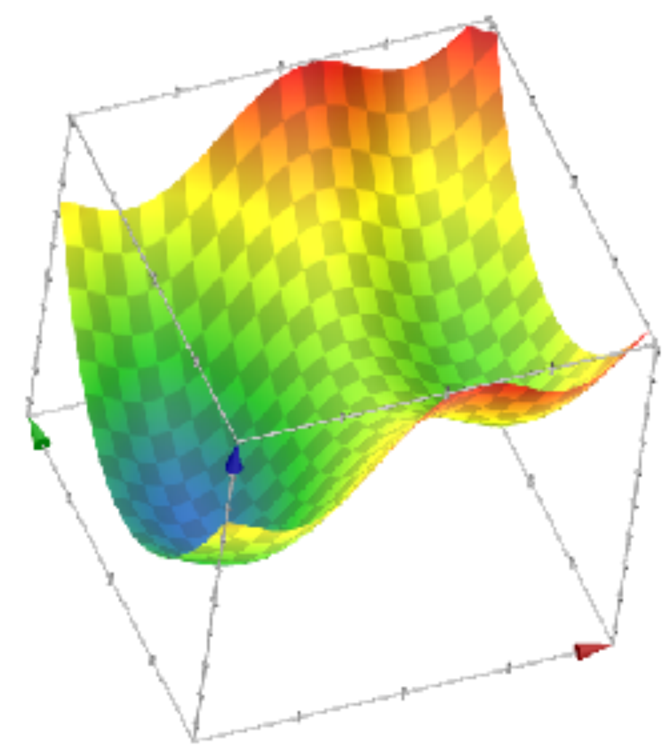

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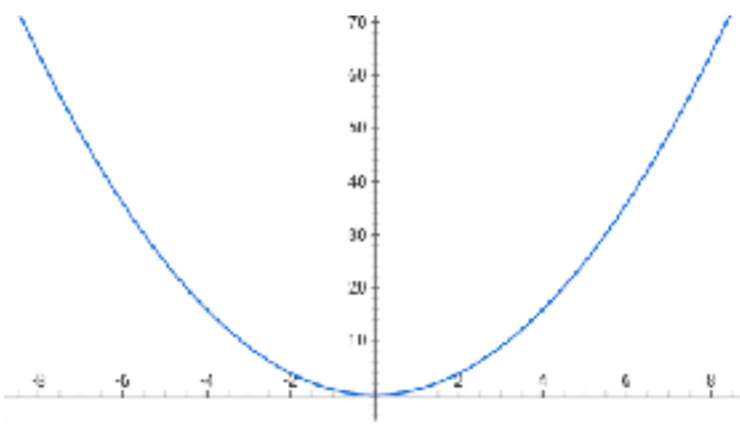
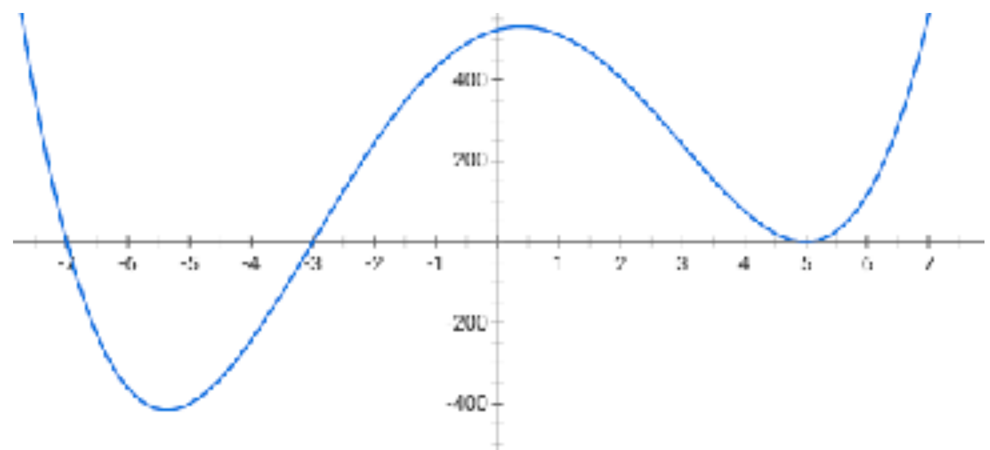
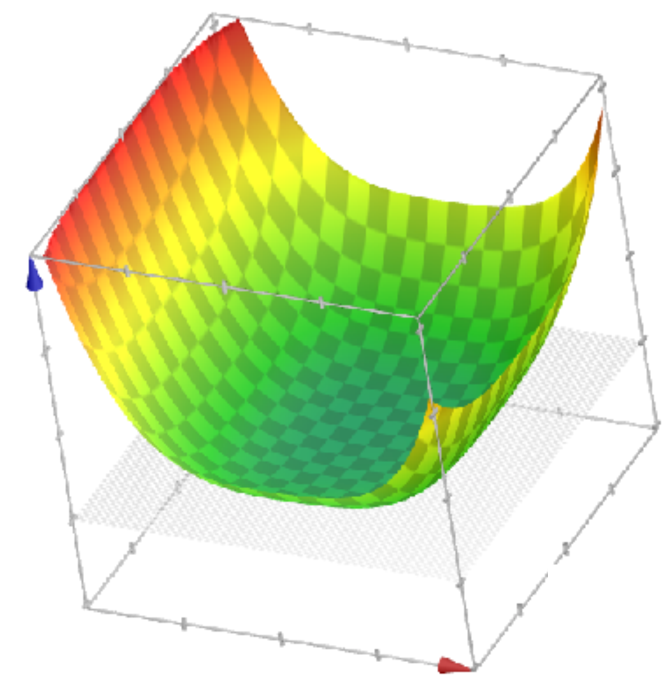

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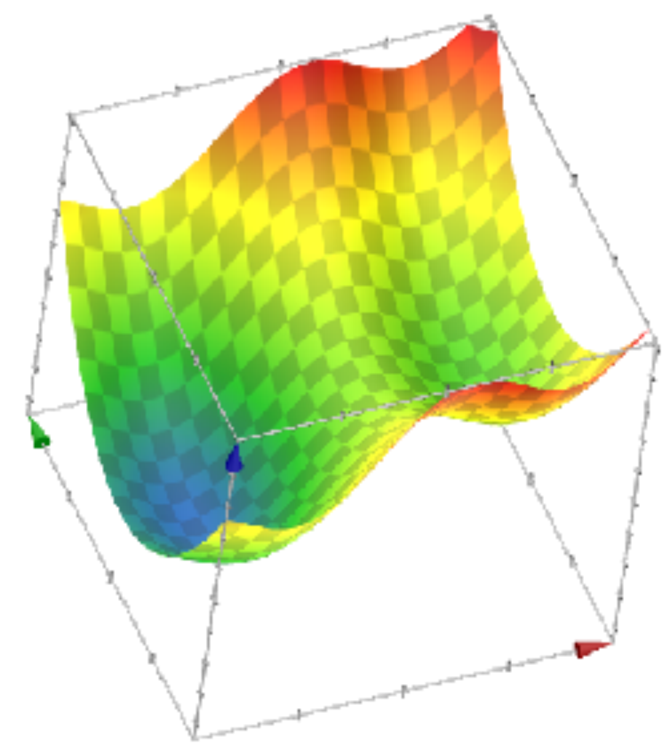

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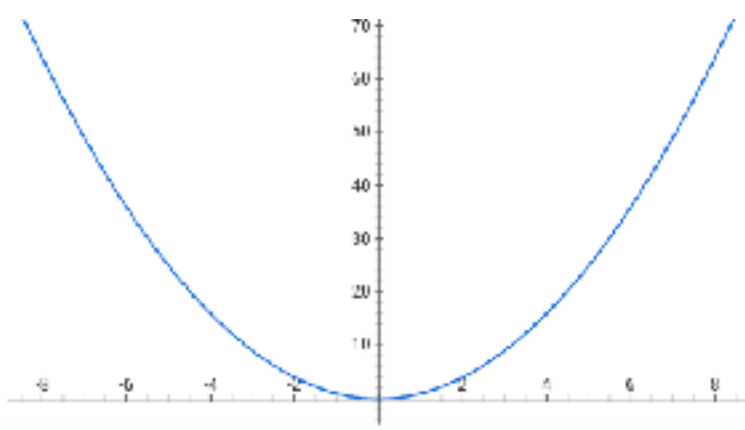
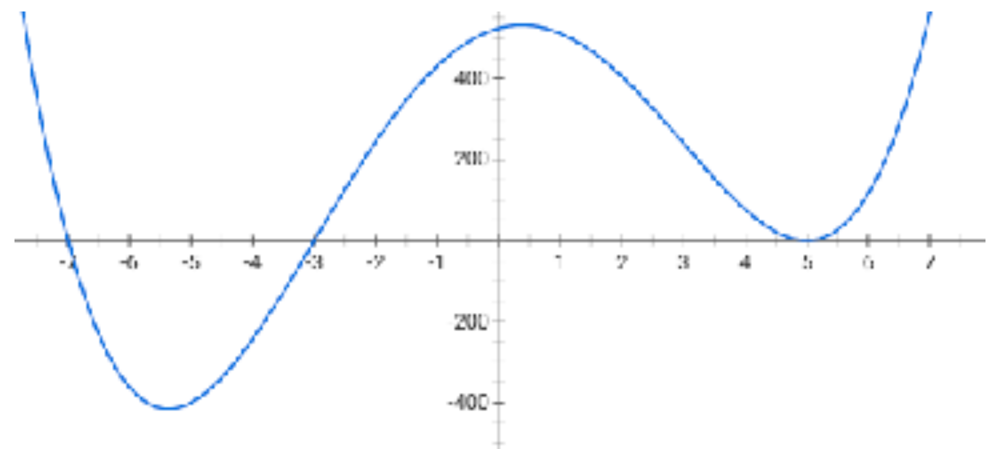
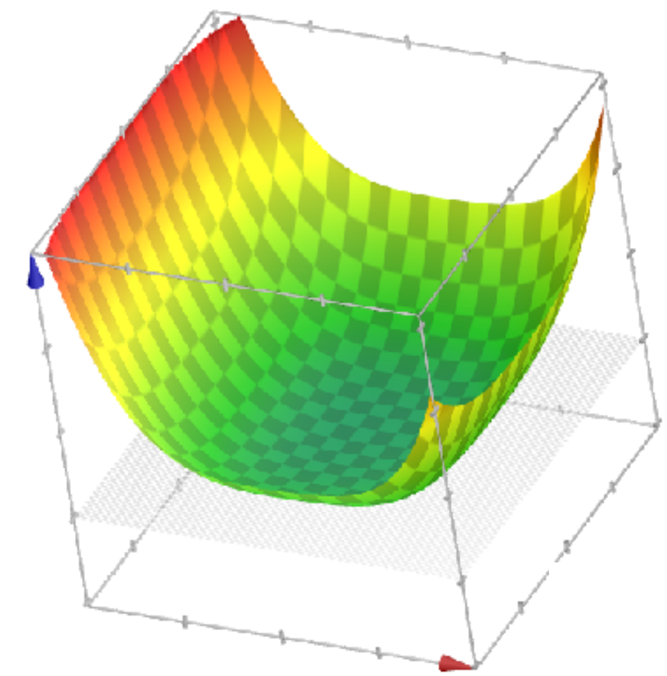

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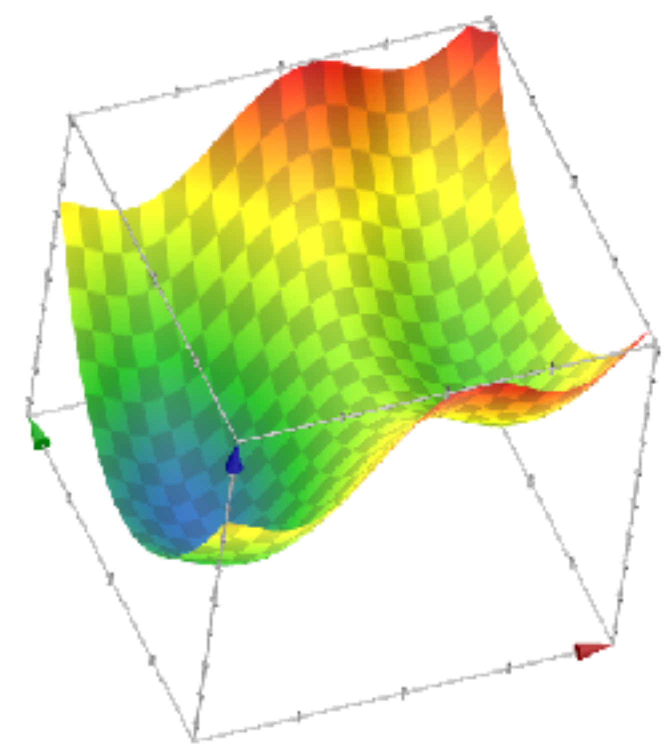

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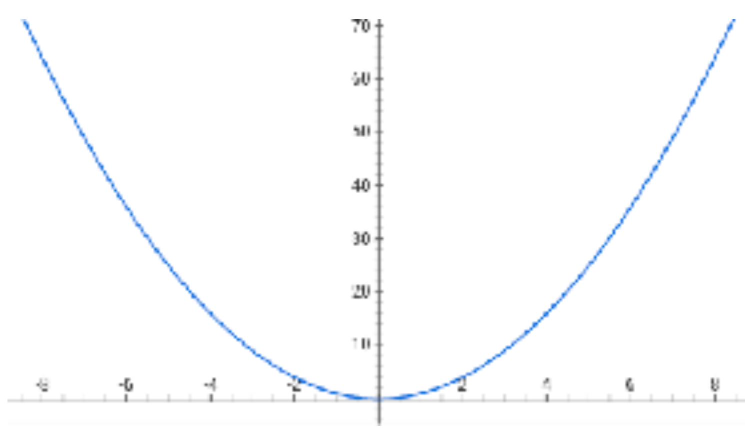

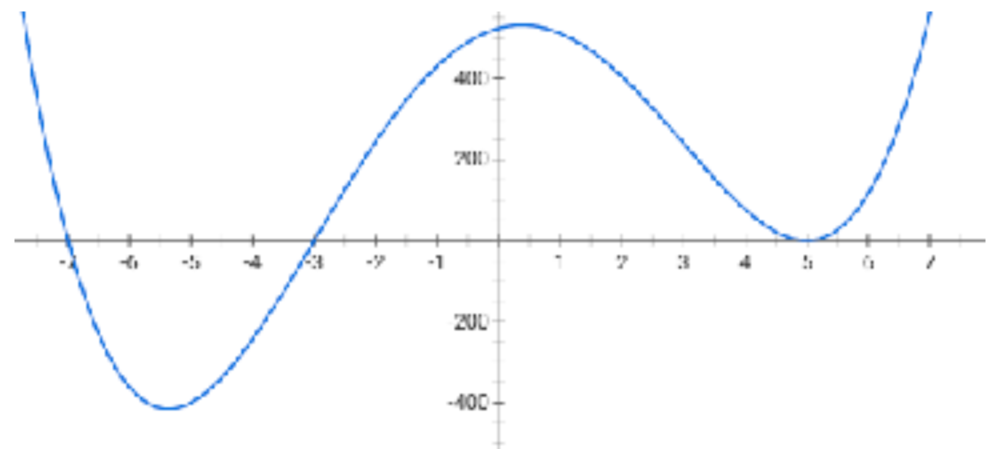

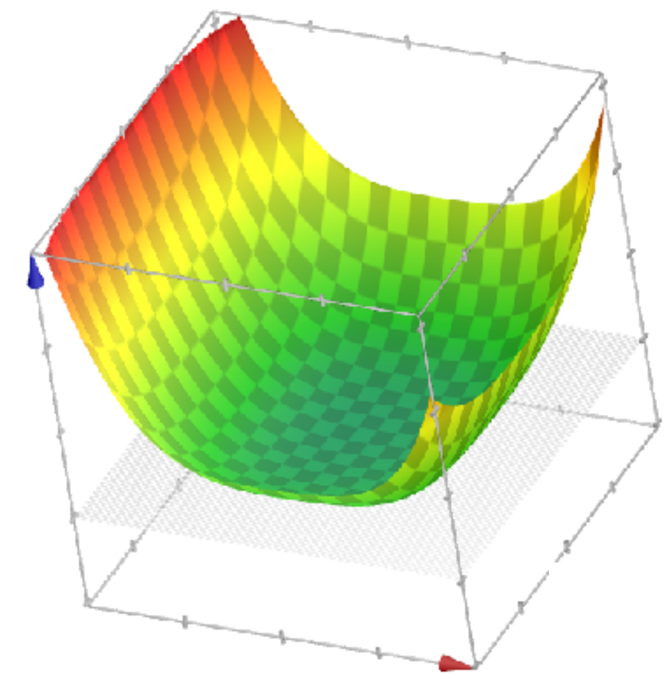

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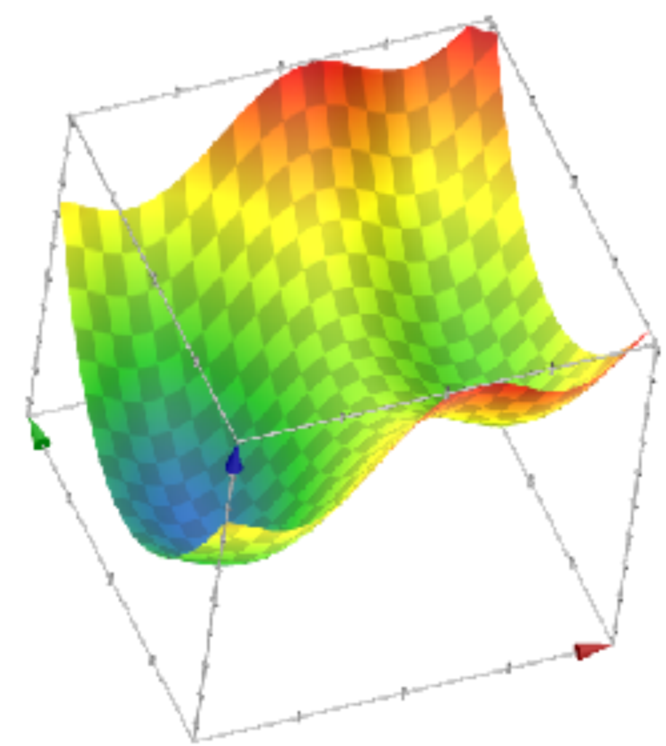

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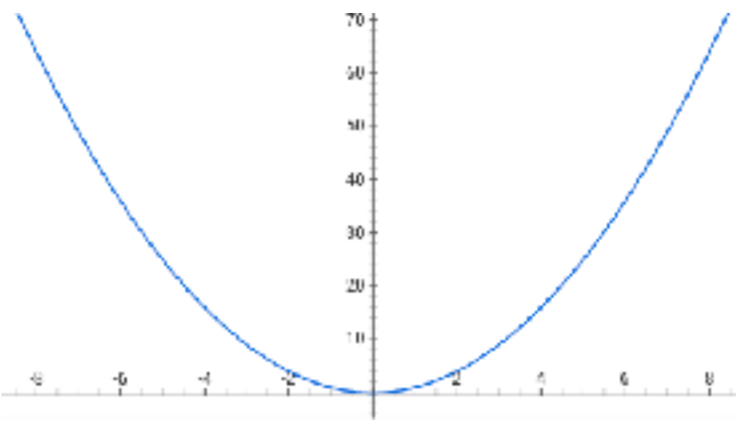

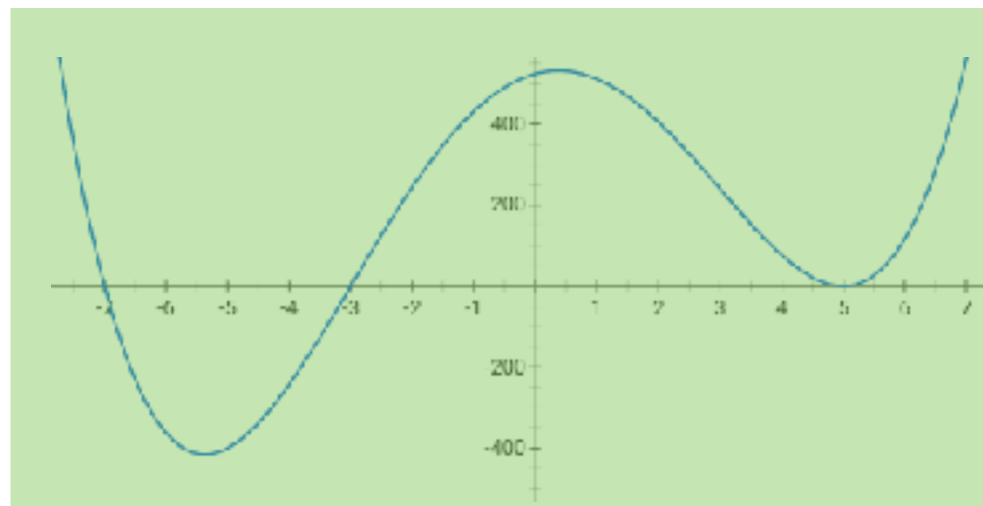

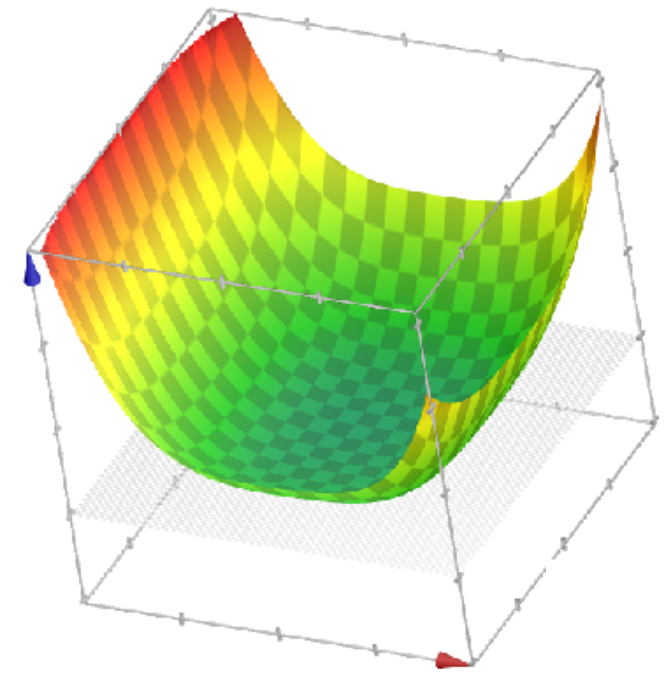

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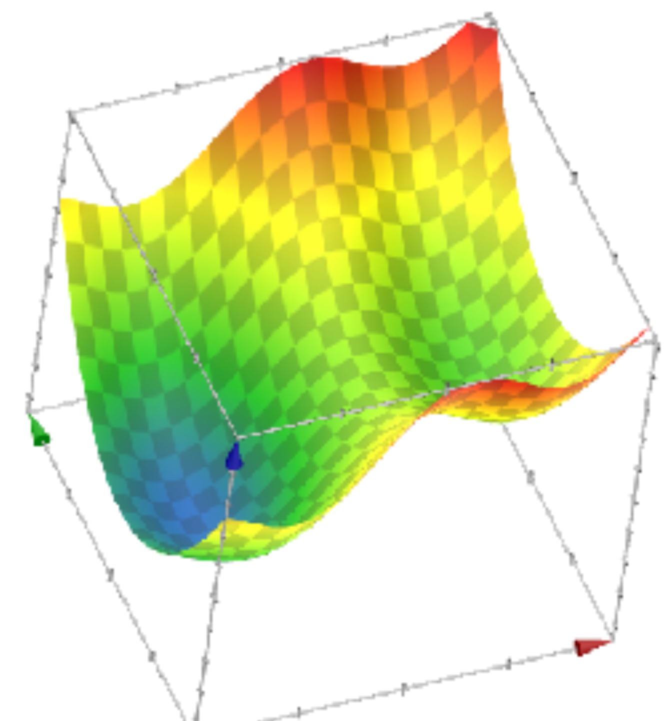

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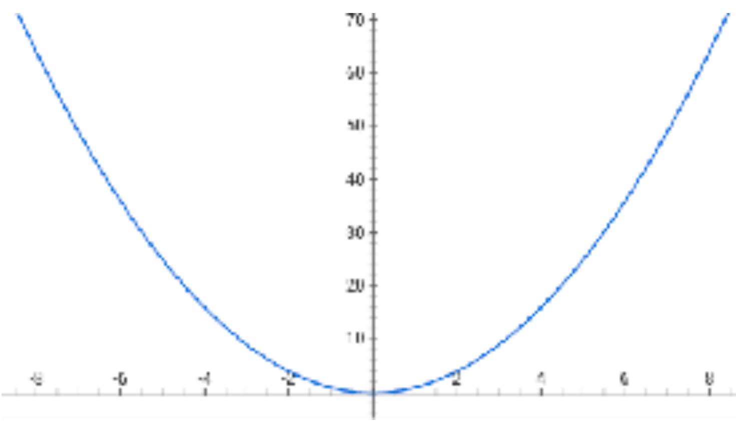

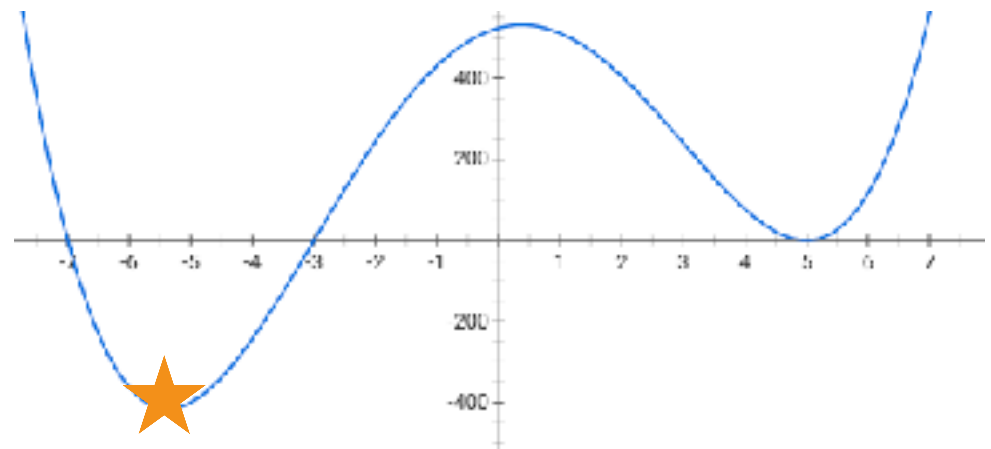

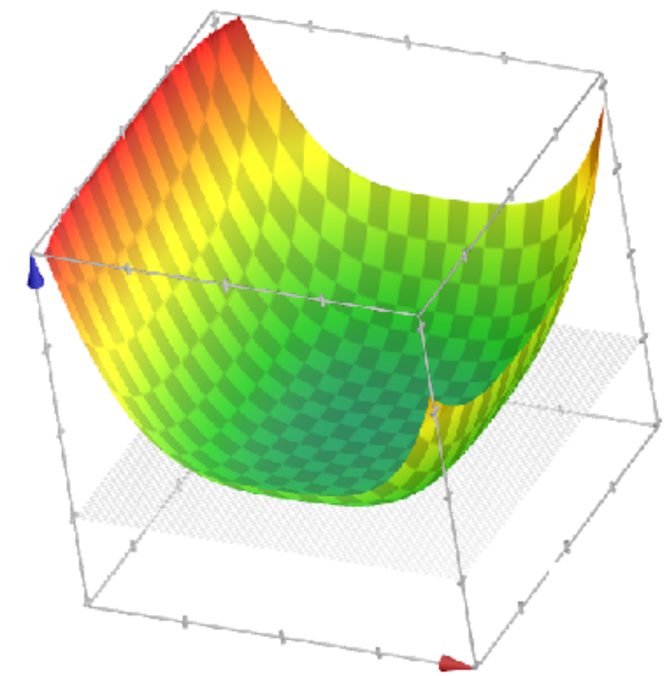

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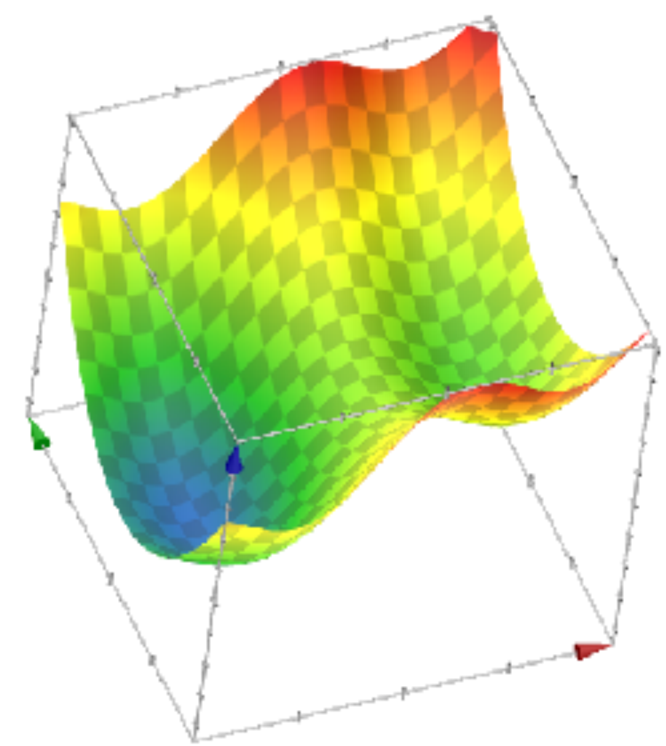

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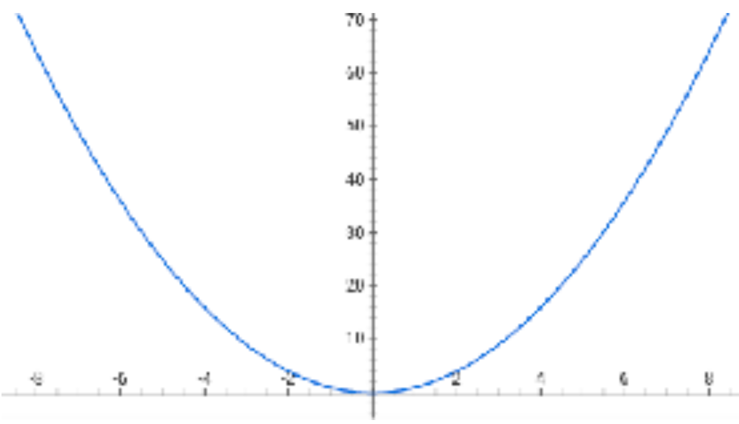

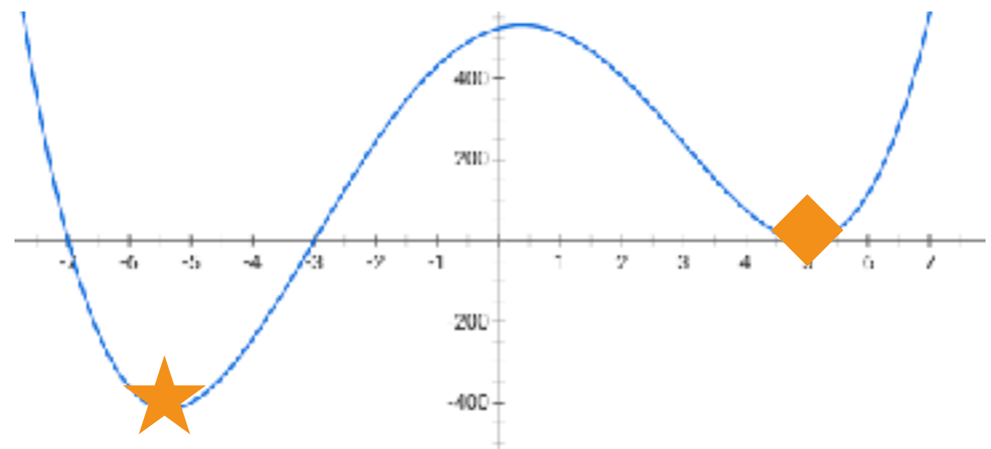

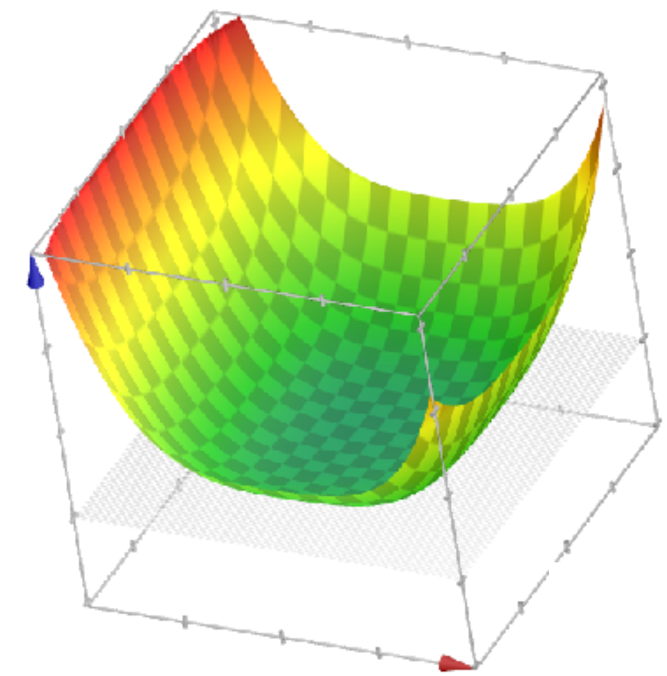

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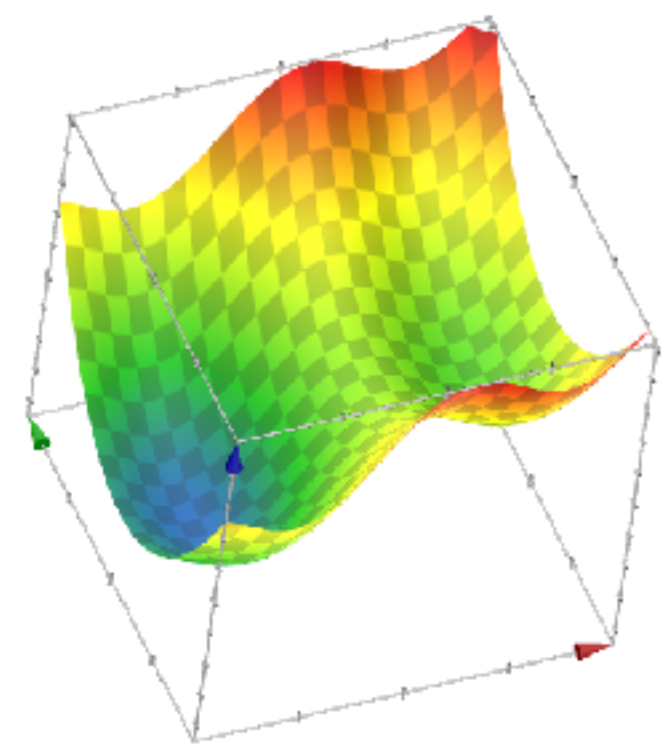

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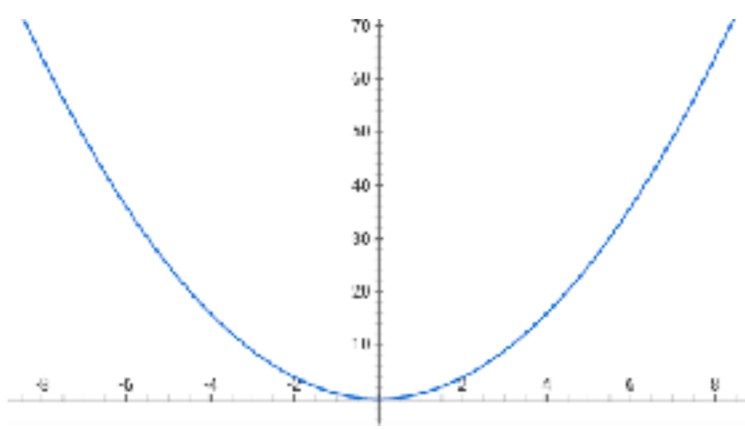
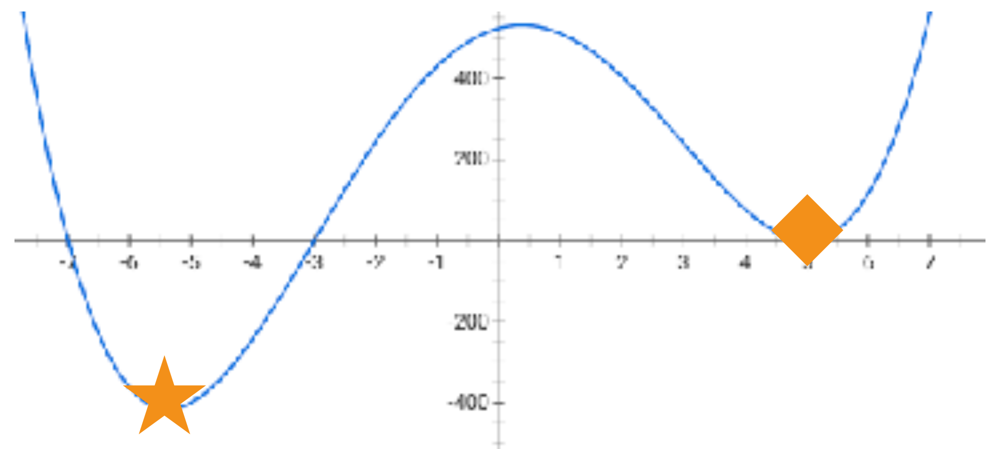
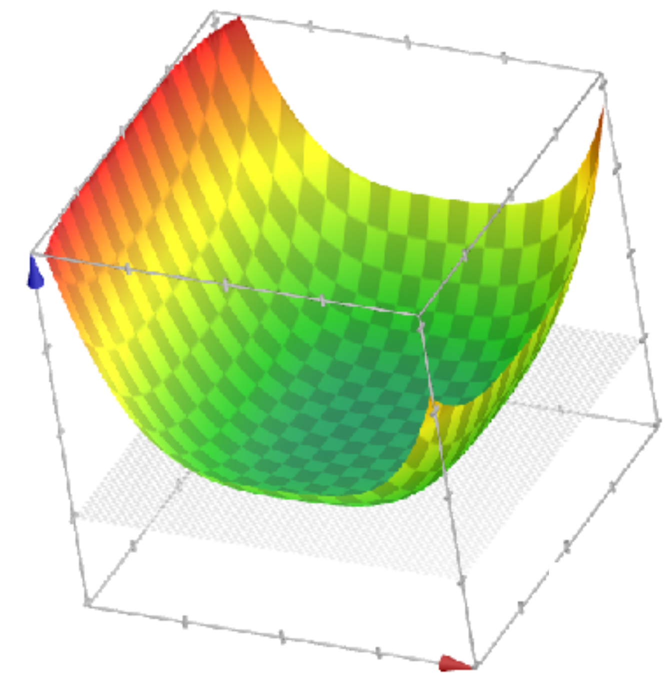
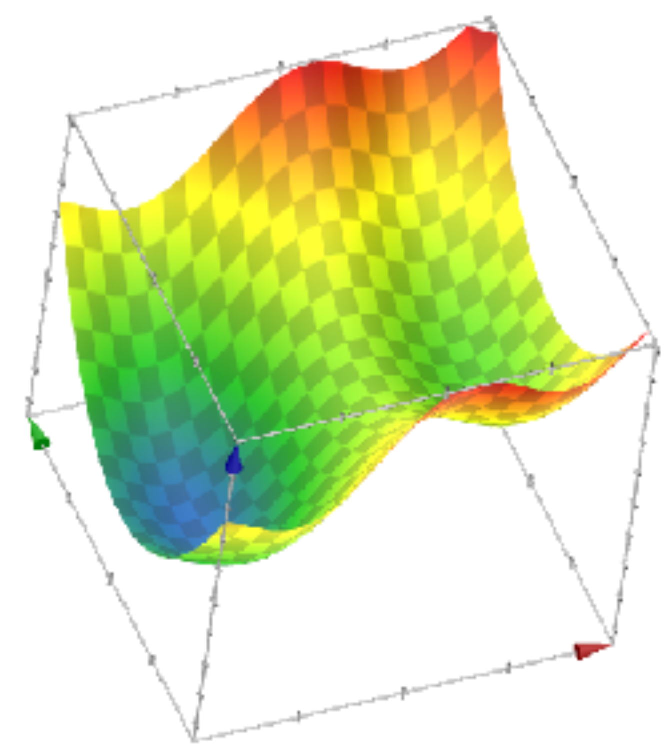






- **Theorem**: Gradient descent performance
  - **Assumptions**: (Choose any $\tilde{\epsilon} > 0$)
    - *f* is sufficiently "smooth" and convex
    - *f* has at least one global optimum
    - η is sufficiently small



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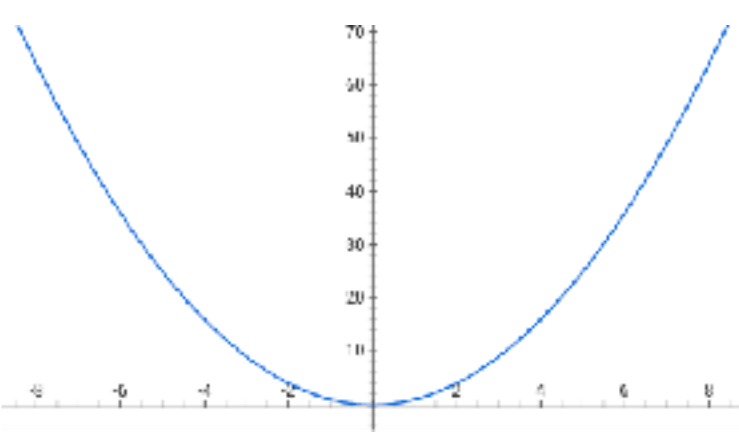

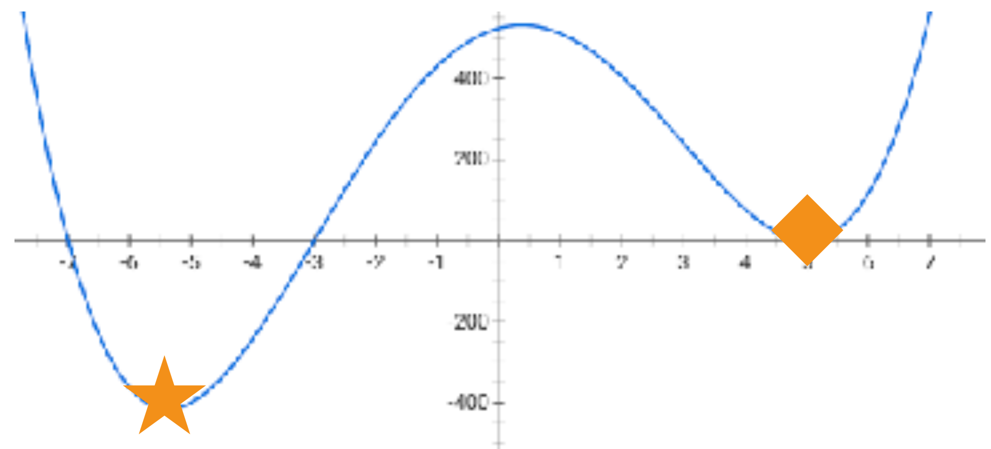

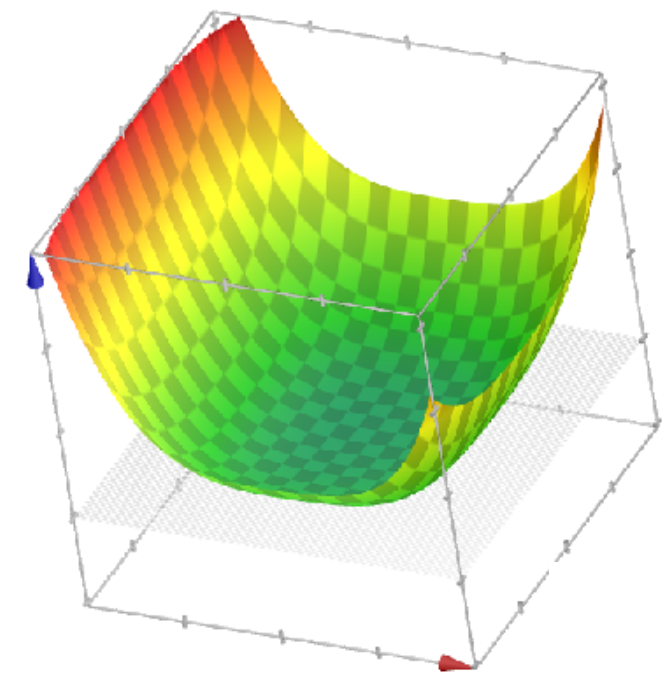

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    - *f* is sufficiently "smooth" and convex
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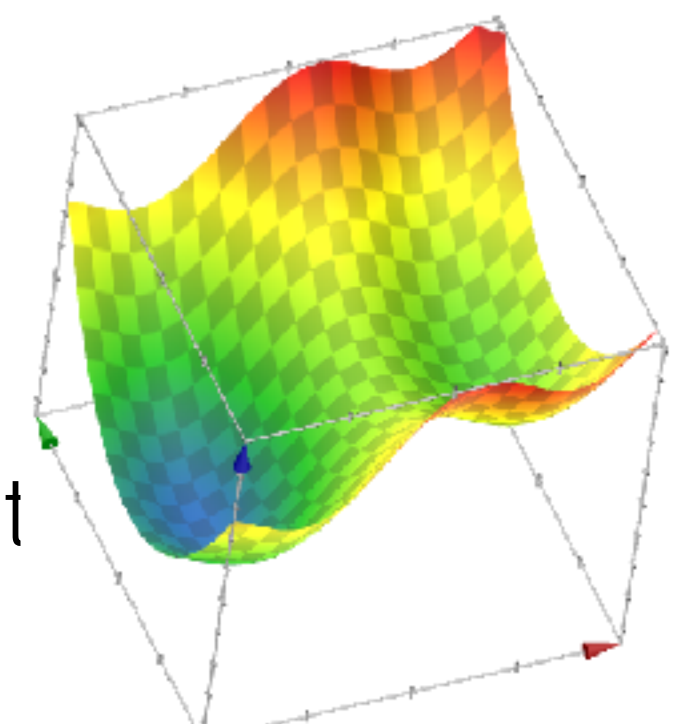
  - **Conclusion**: If run long enough, gradient descent will return a value within $\tilde{\epsilon}$ of a global optimum Θ



Gradient descent for logistic regression

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- Loss $J_{\text{lr}}(\Theta) = J_{\text{lr}}(\theta, \theta_0)$ is differentiable

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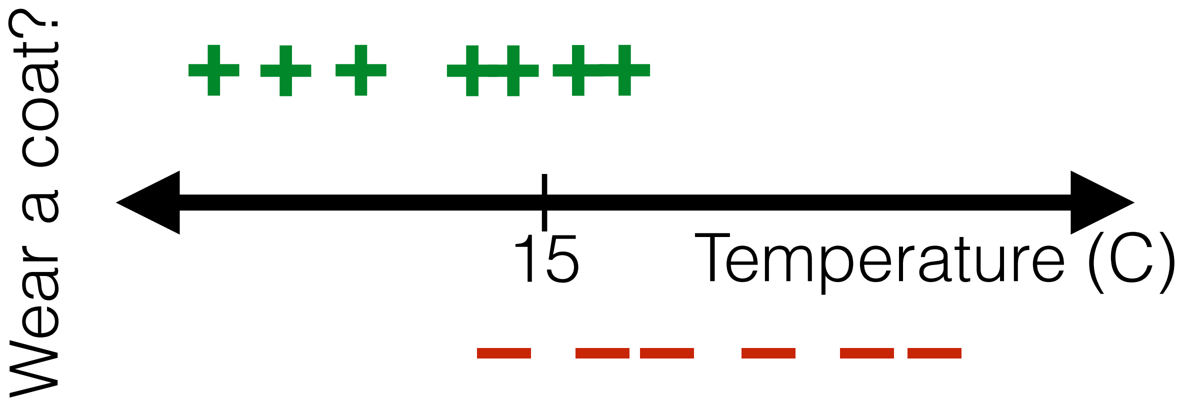

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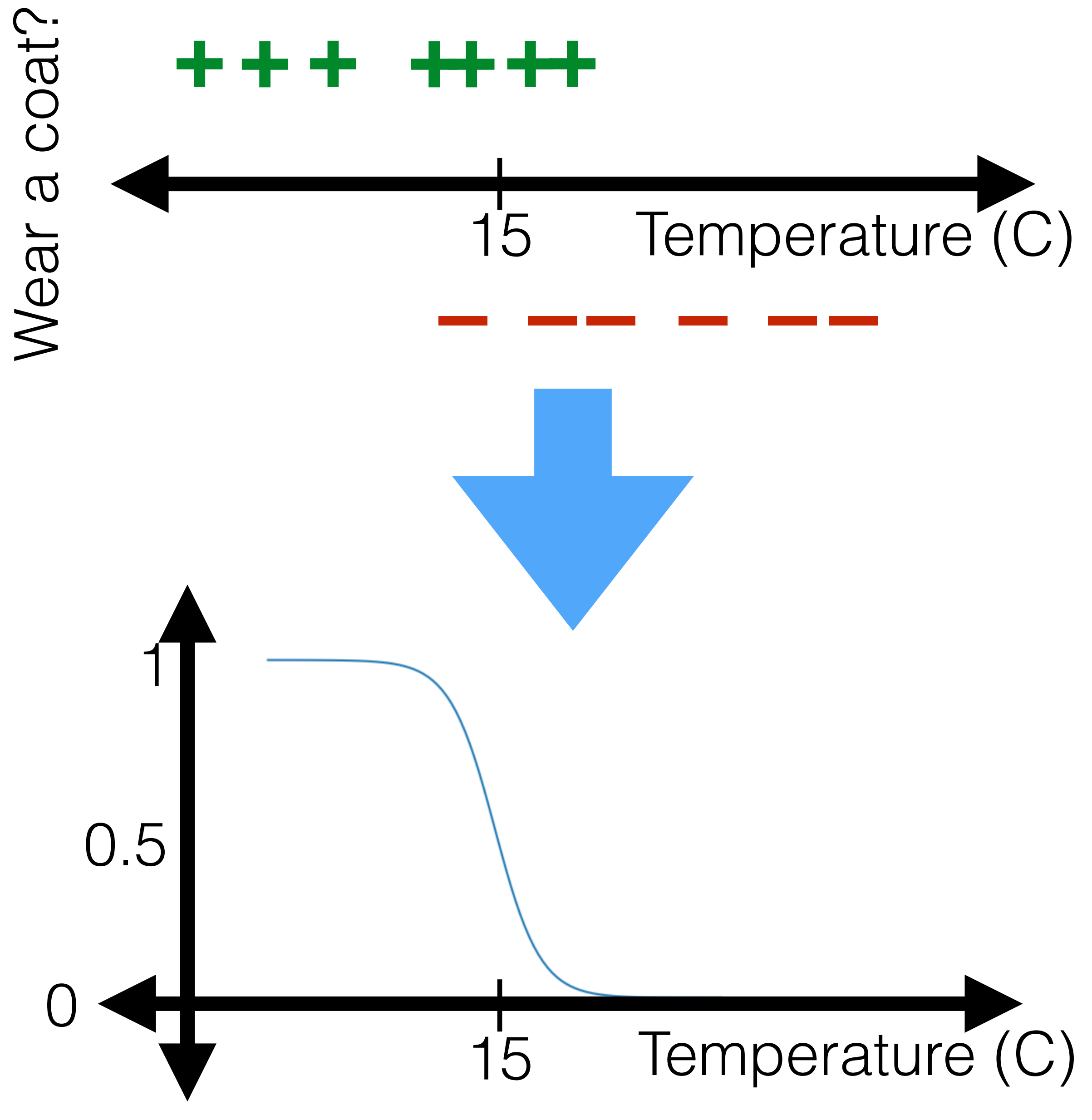

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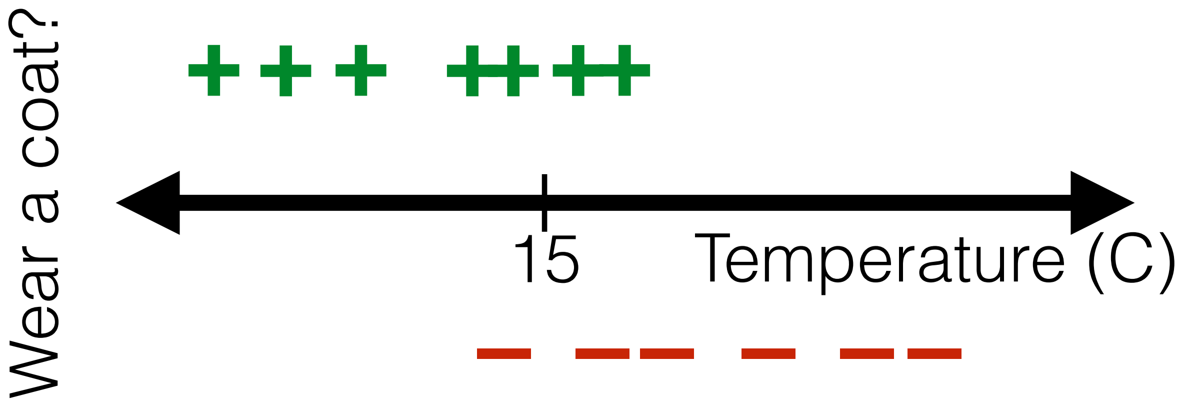

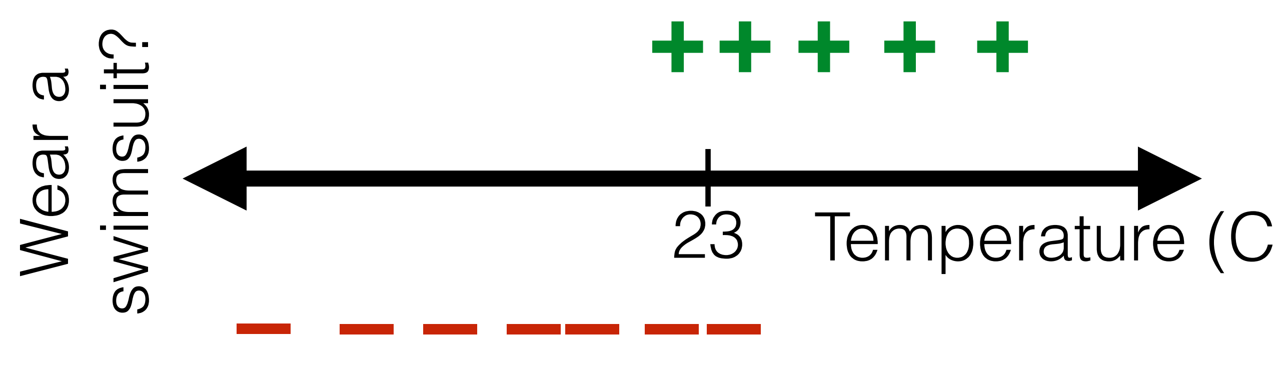

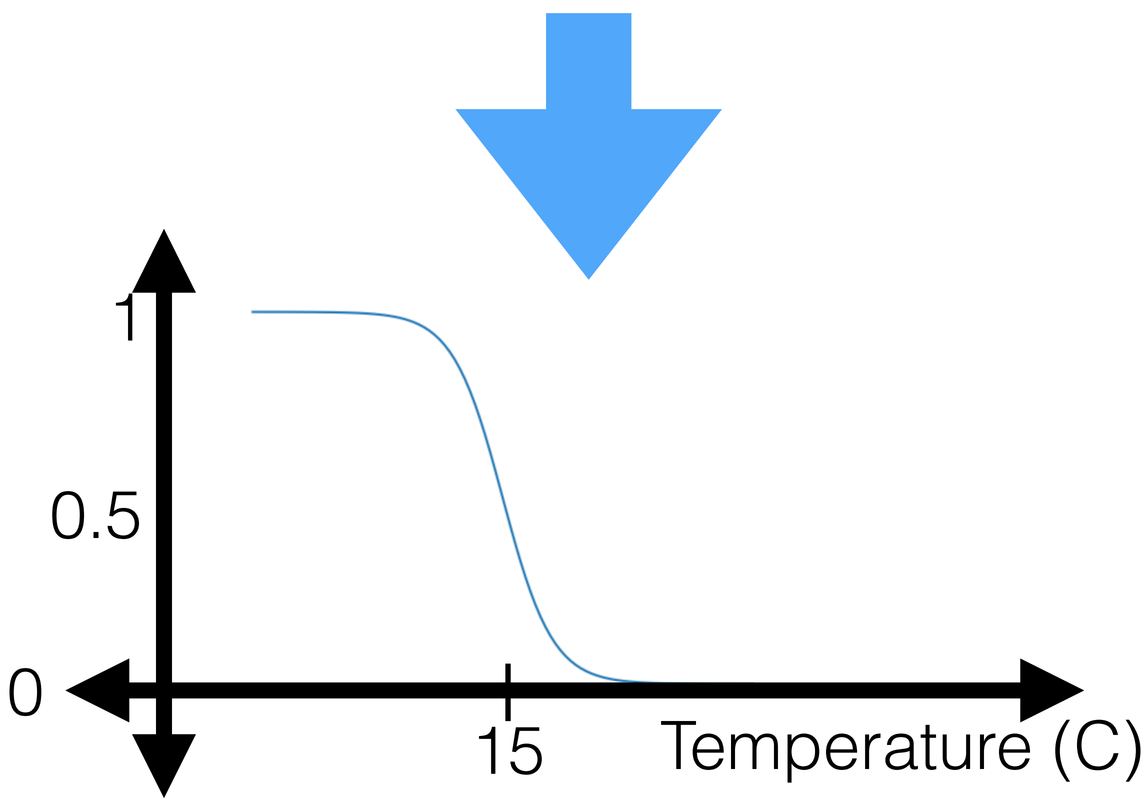

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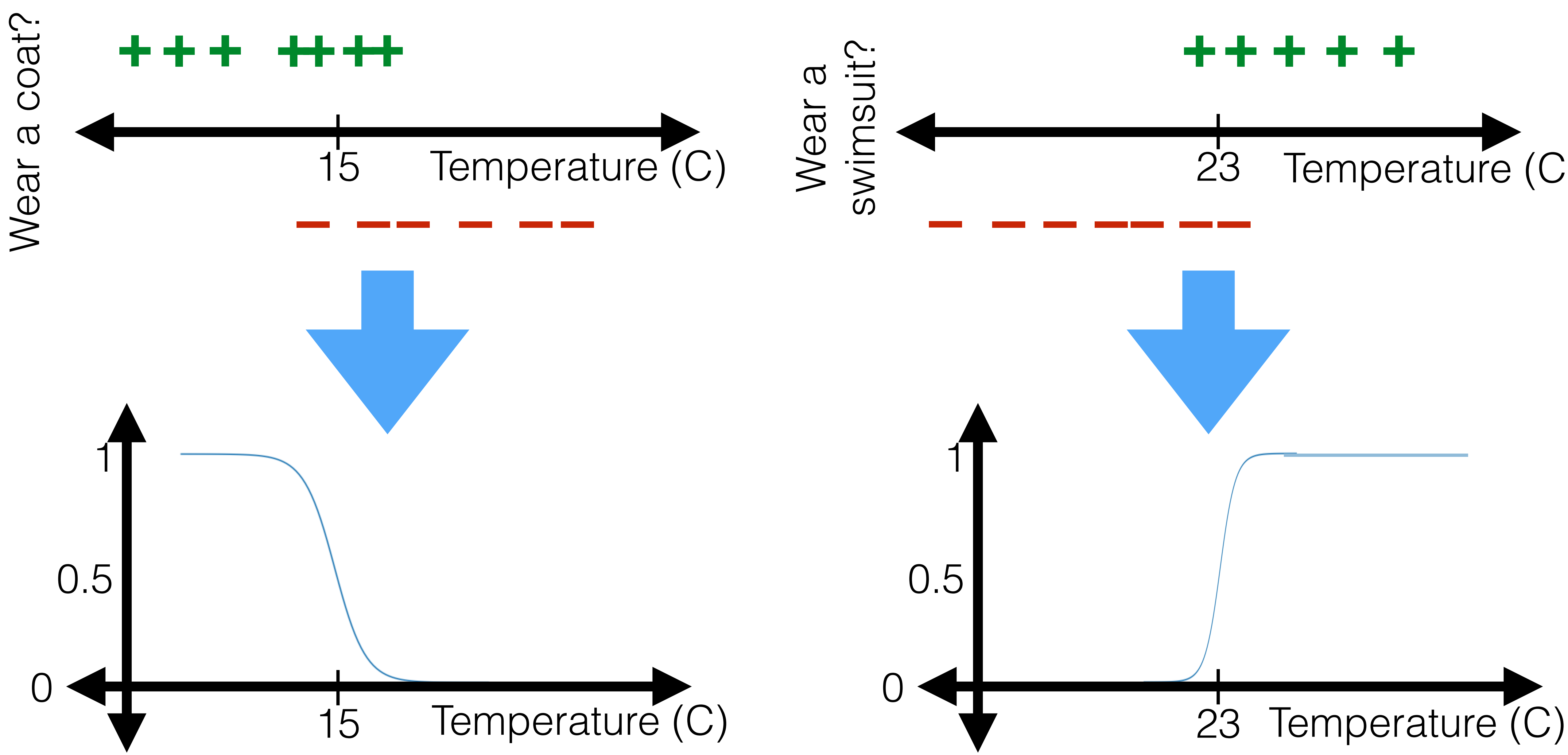

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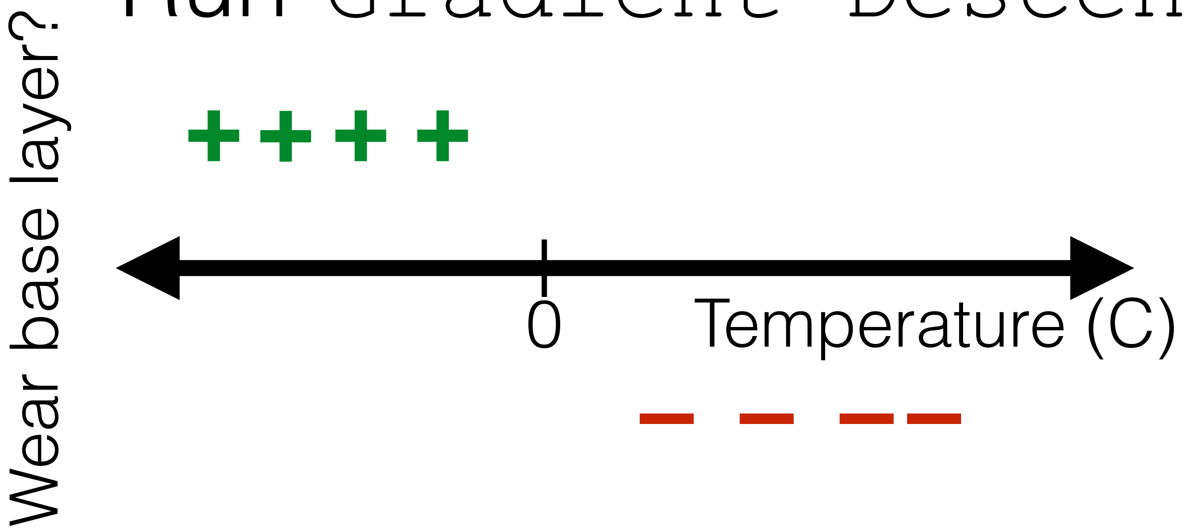

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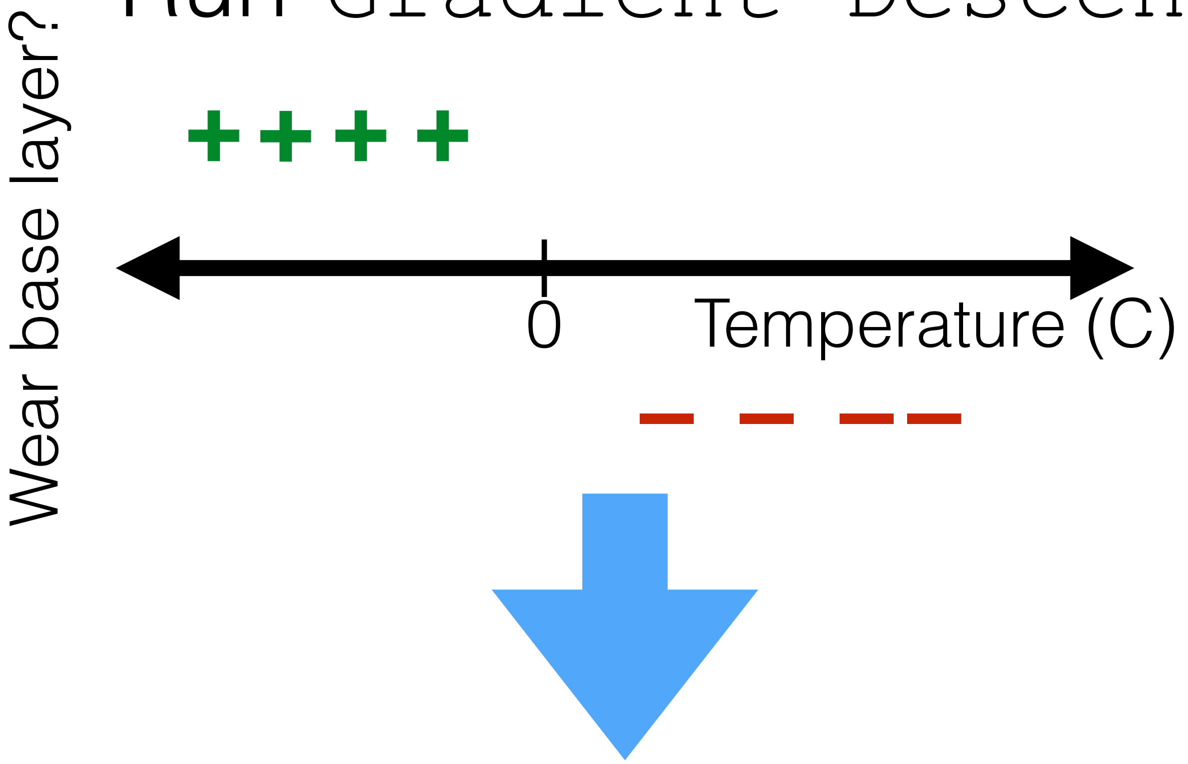

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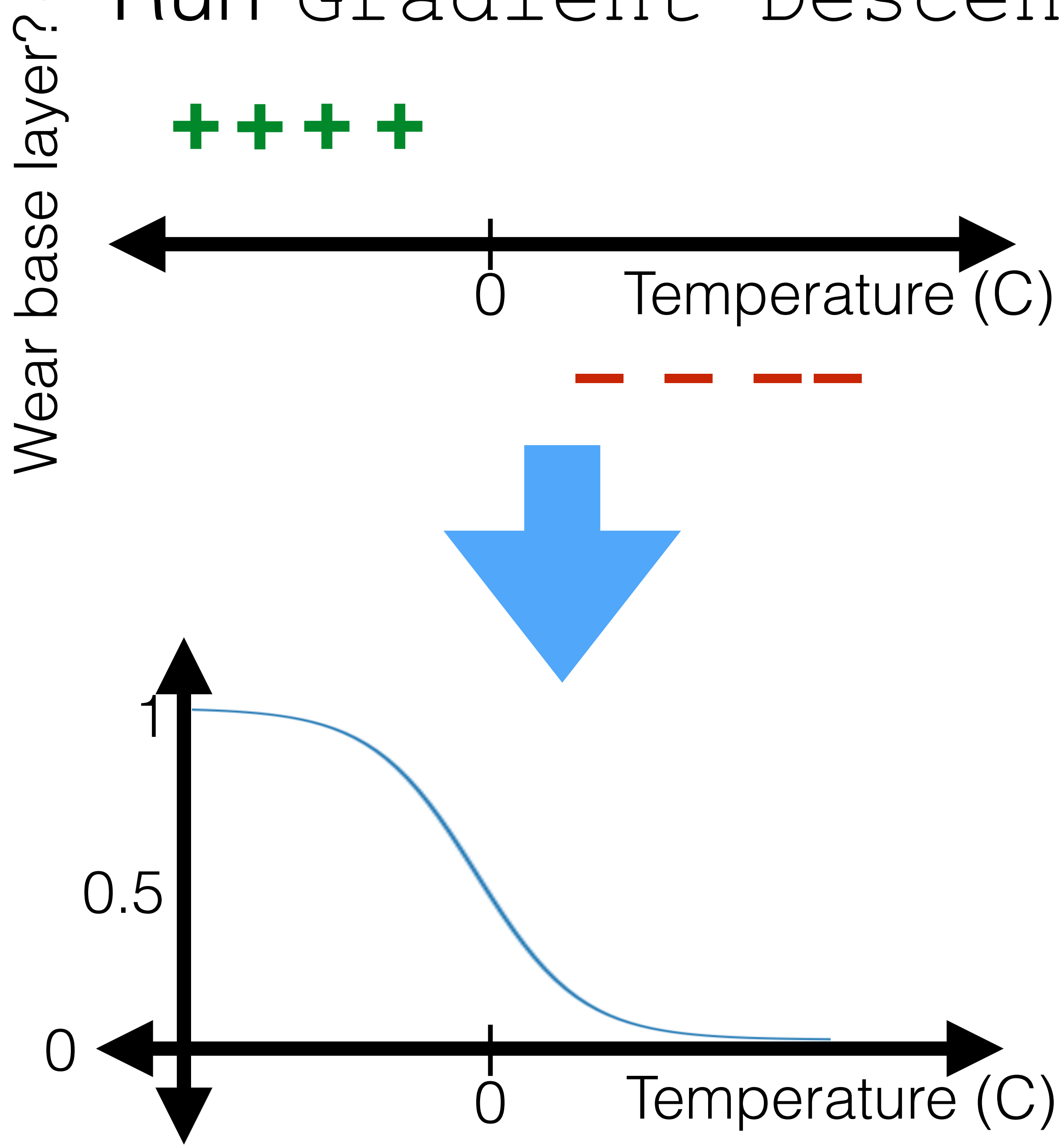

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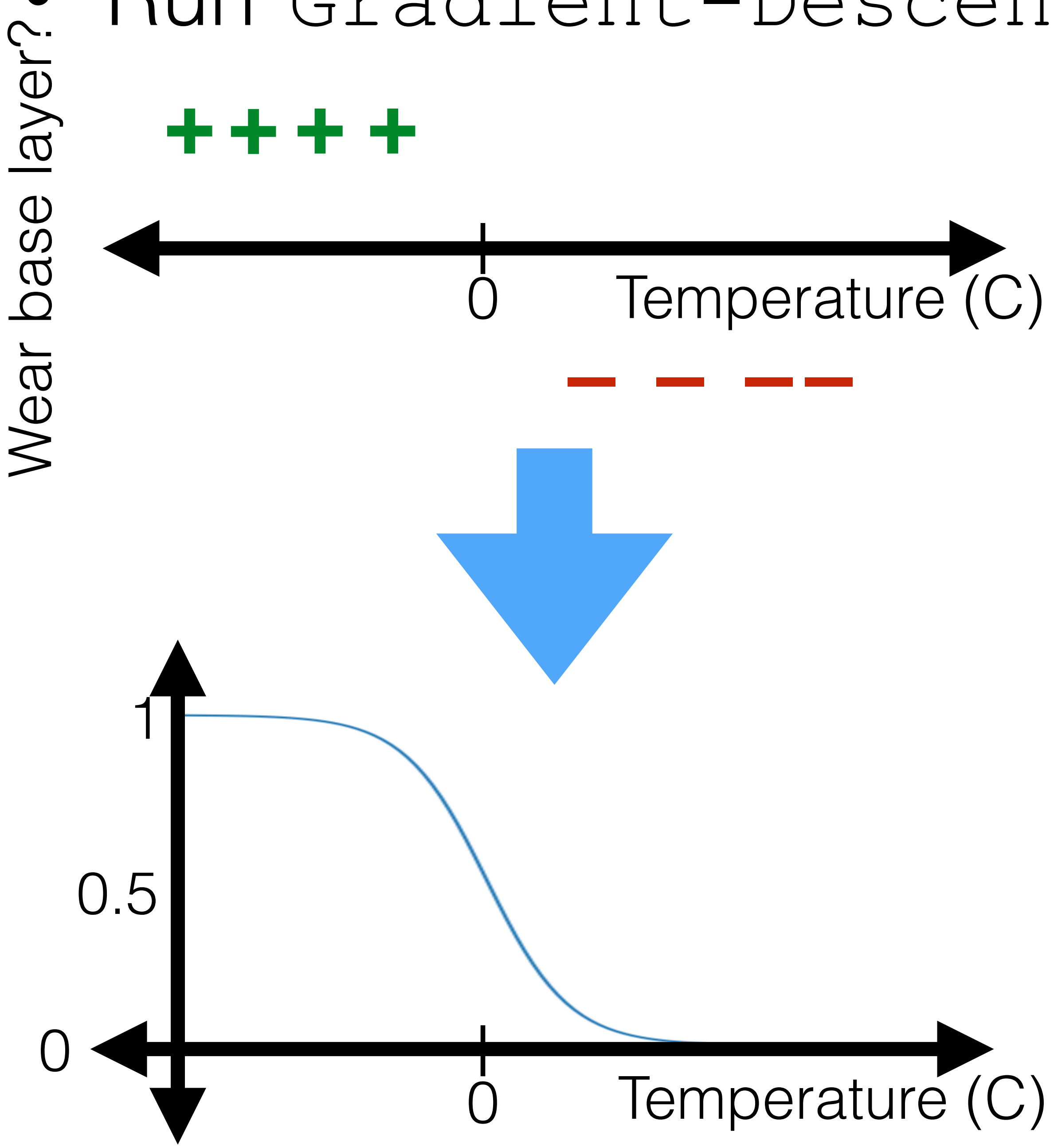

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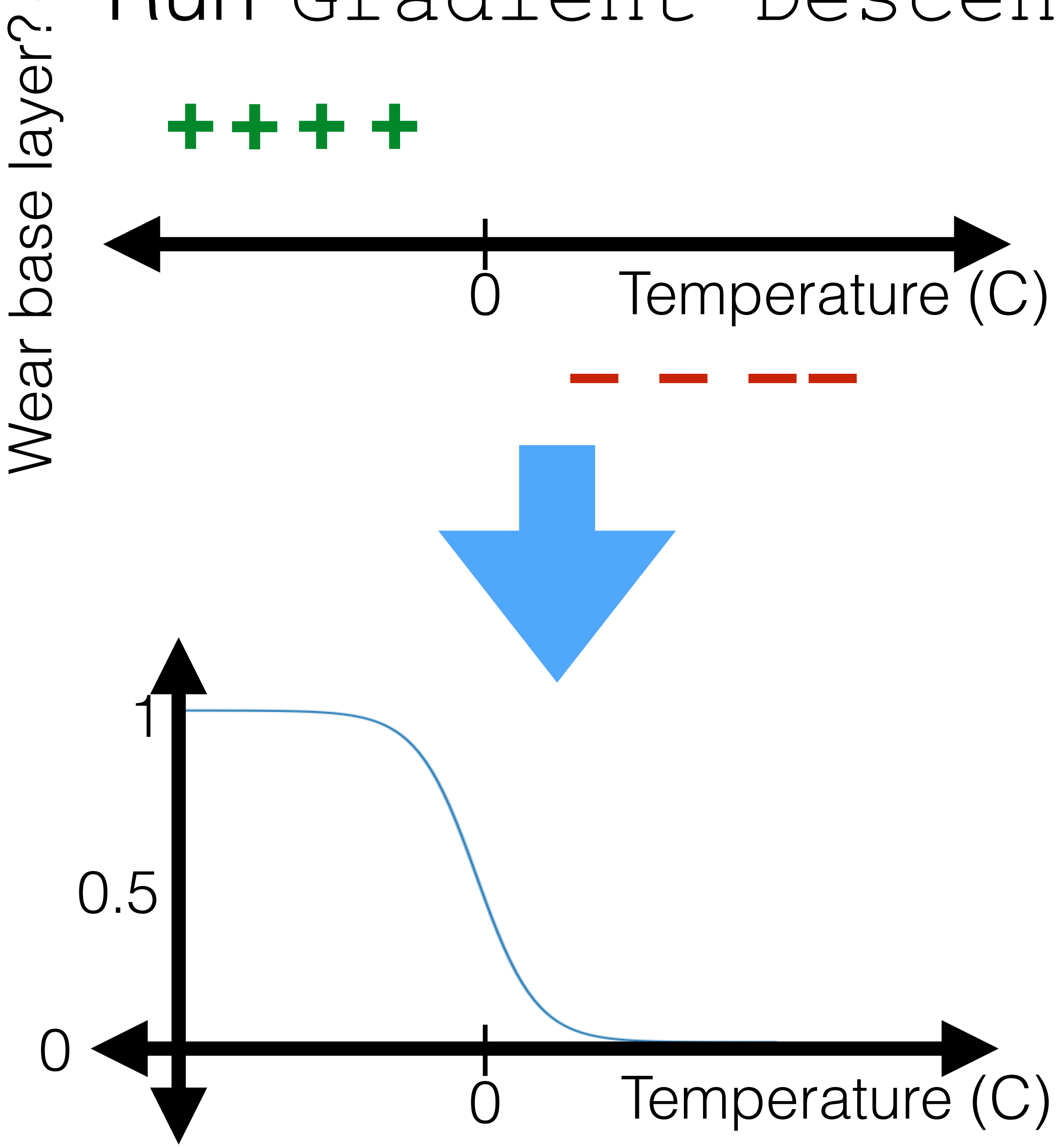

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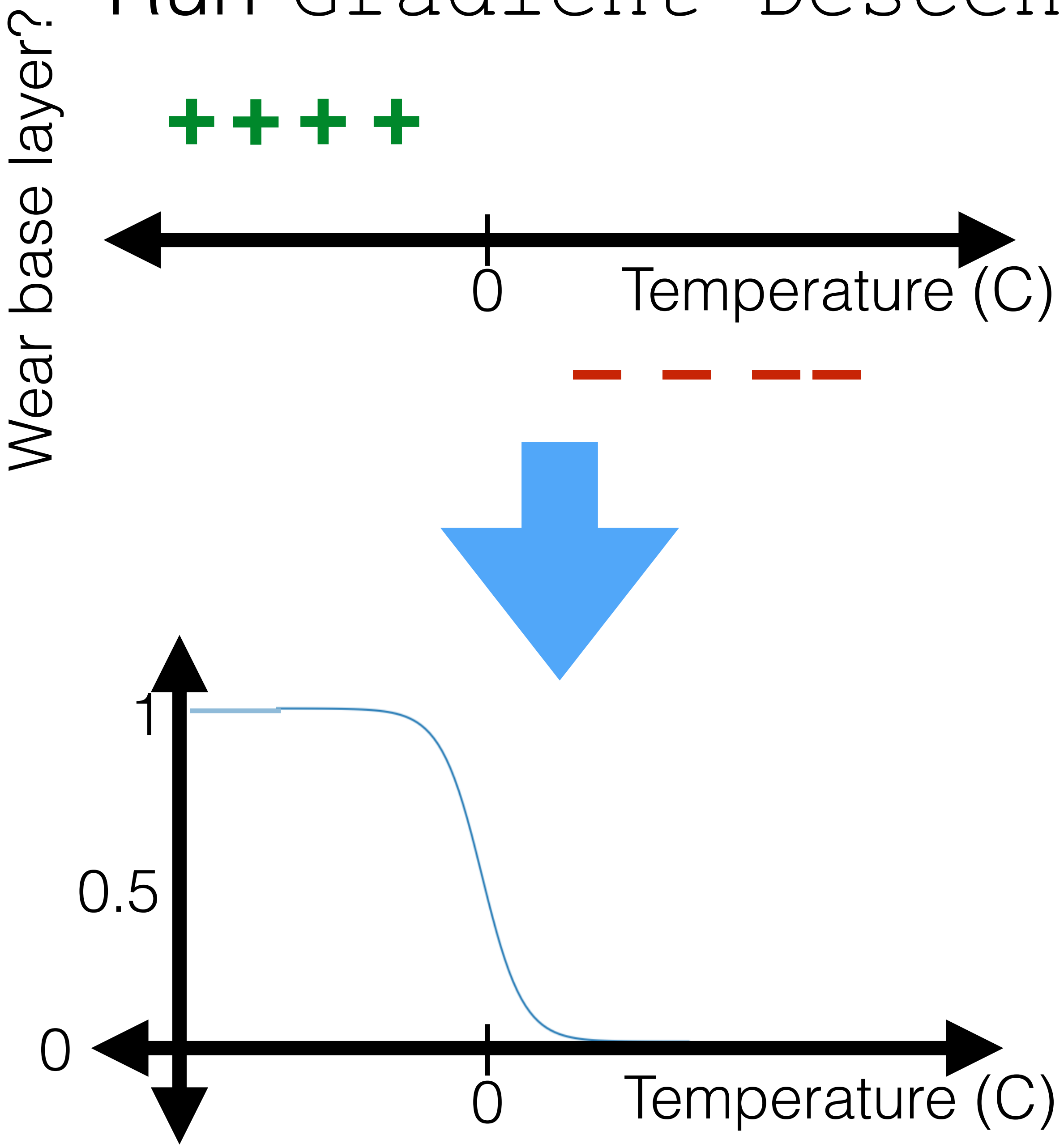

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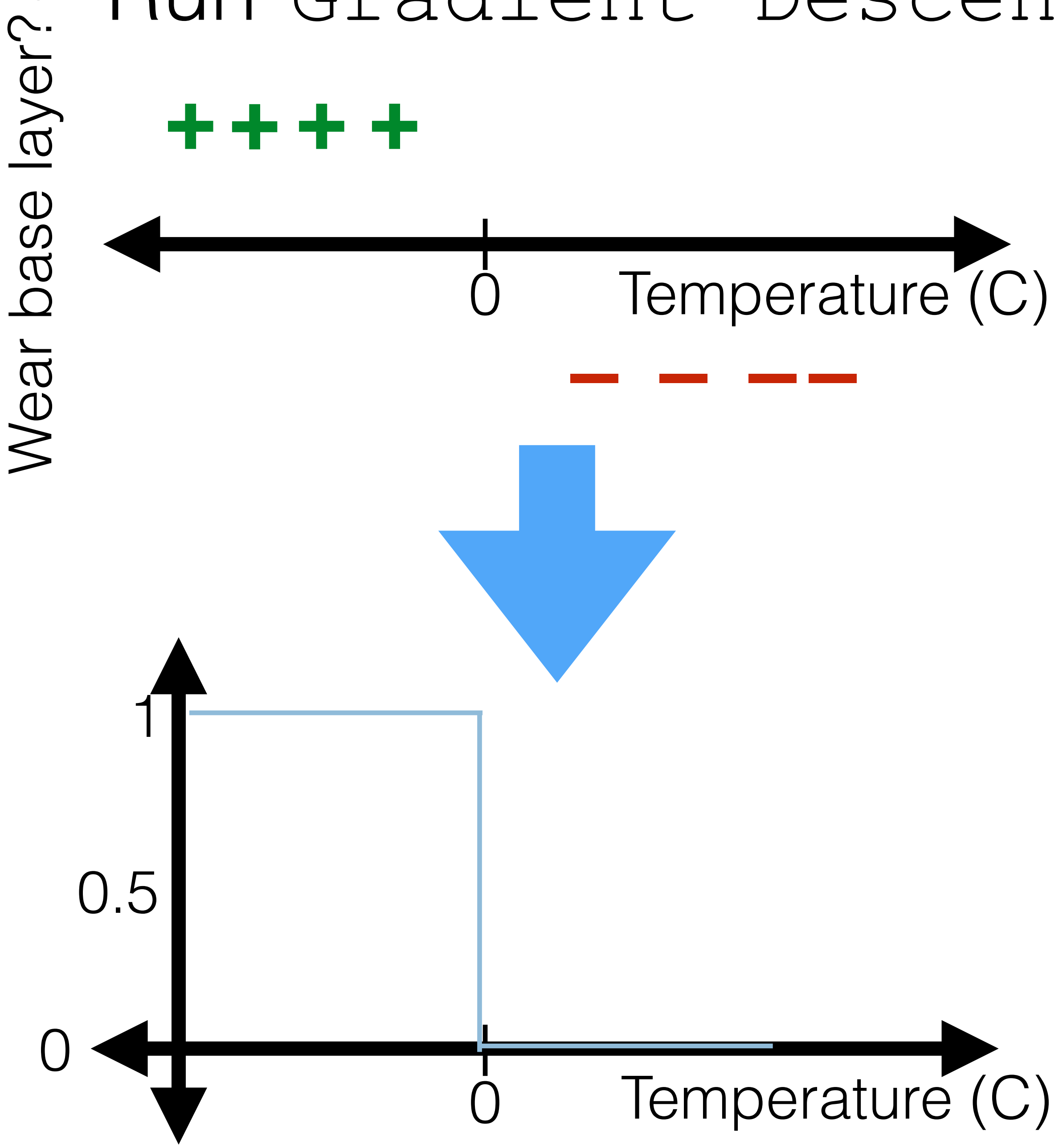

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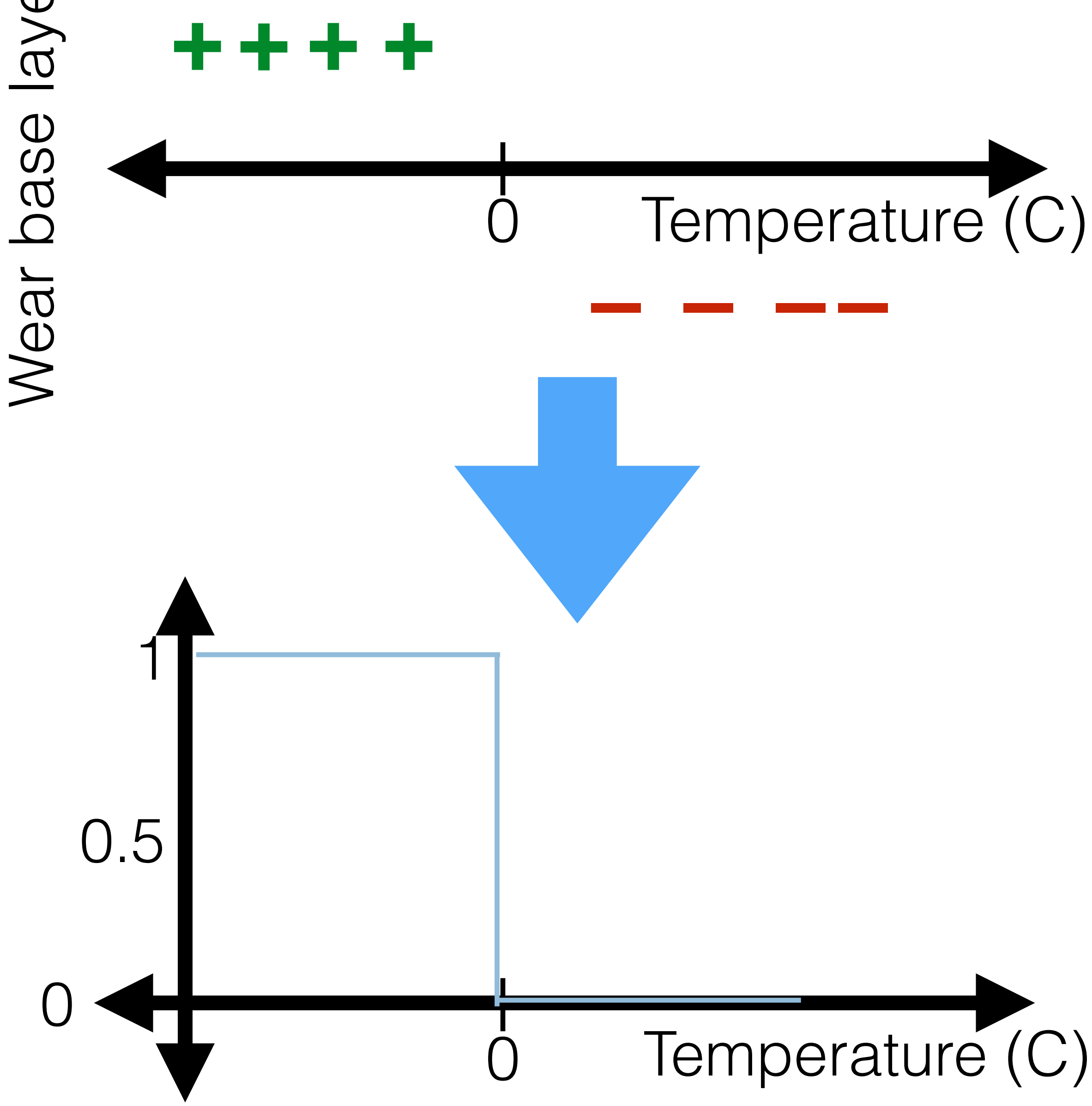

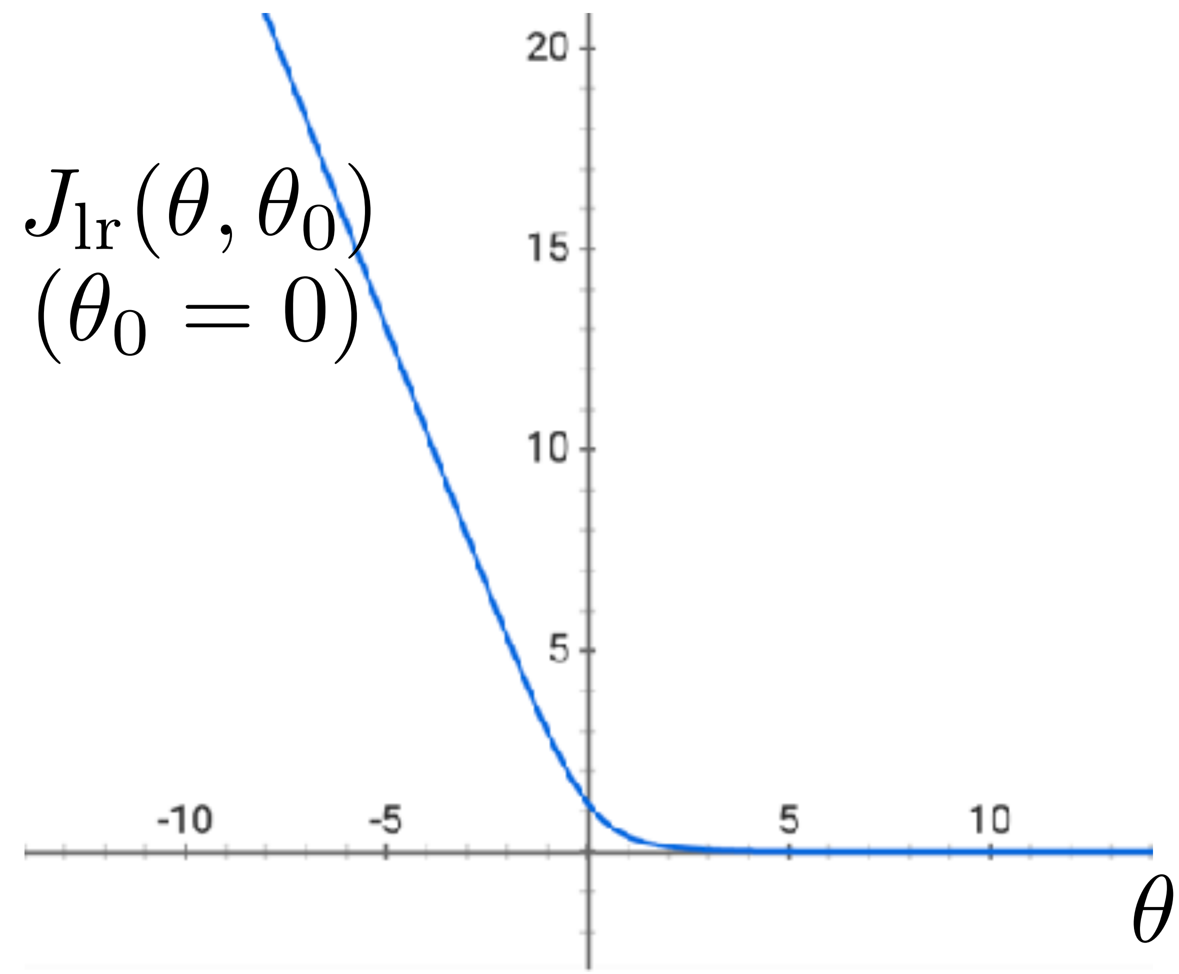

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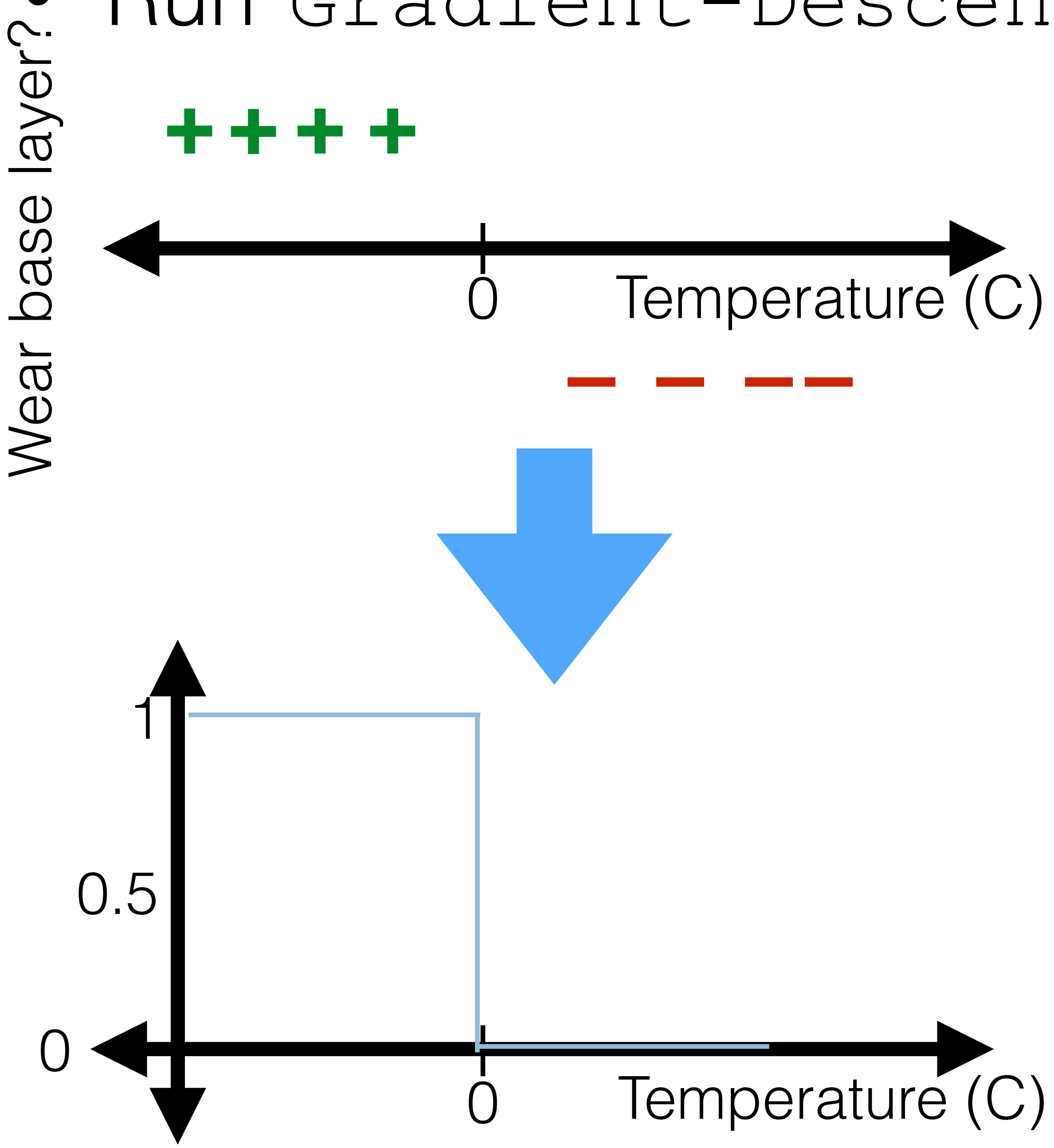

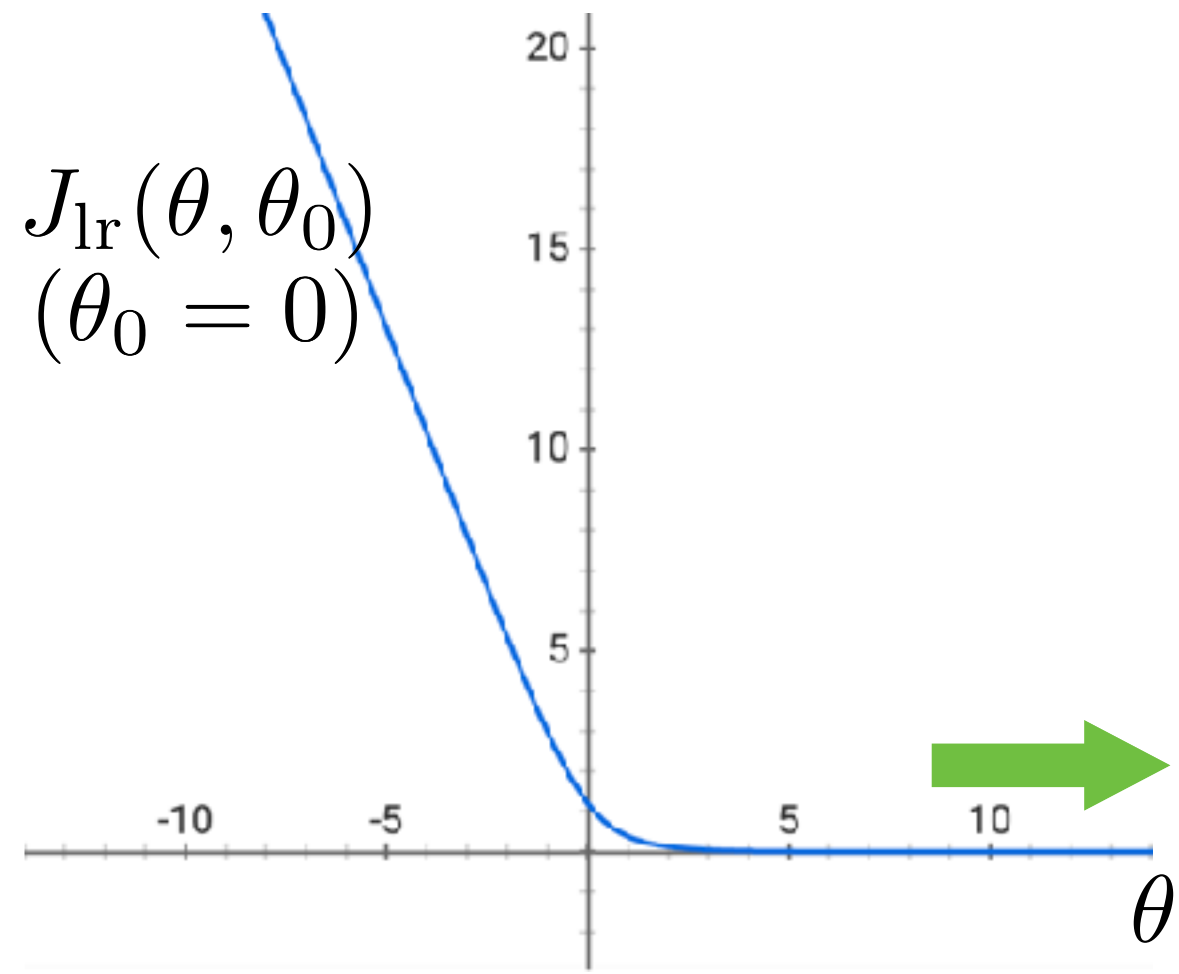

Logistic regression loss revisited

$$
\begin{aligned}
J_{\mathrm{lr}}(\Theta) &= J_{\mathrm{lr}}(\theta, \theta_0) \\
&= \frac{1}{n} \sum_{i=1}^{n} L_{\mathrm{nll}}(\sigma(\theta^\top x^{(i)} + \theta_0), y^{(i)})
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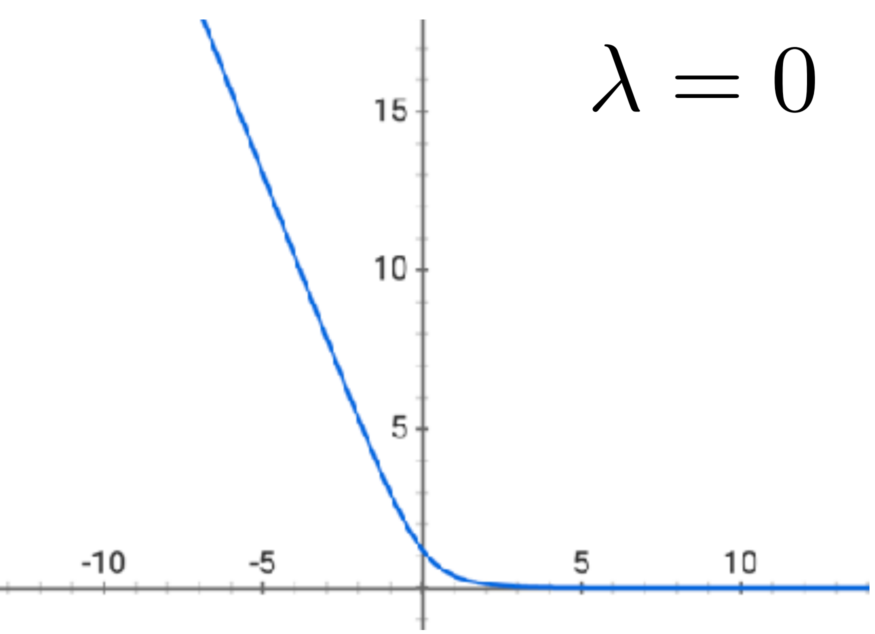

$\lambda = 0$

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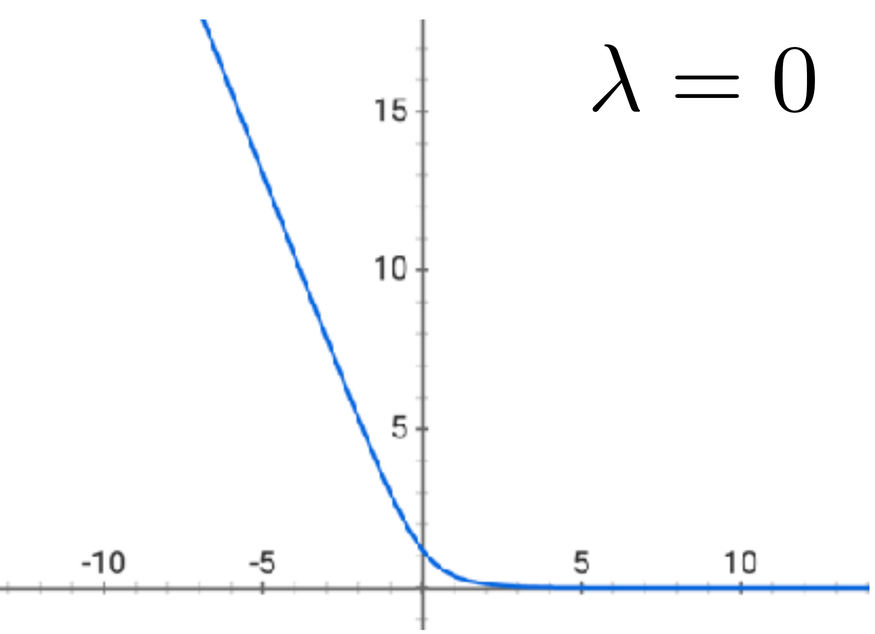

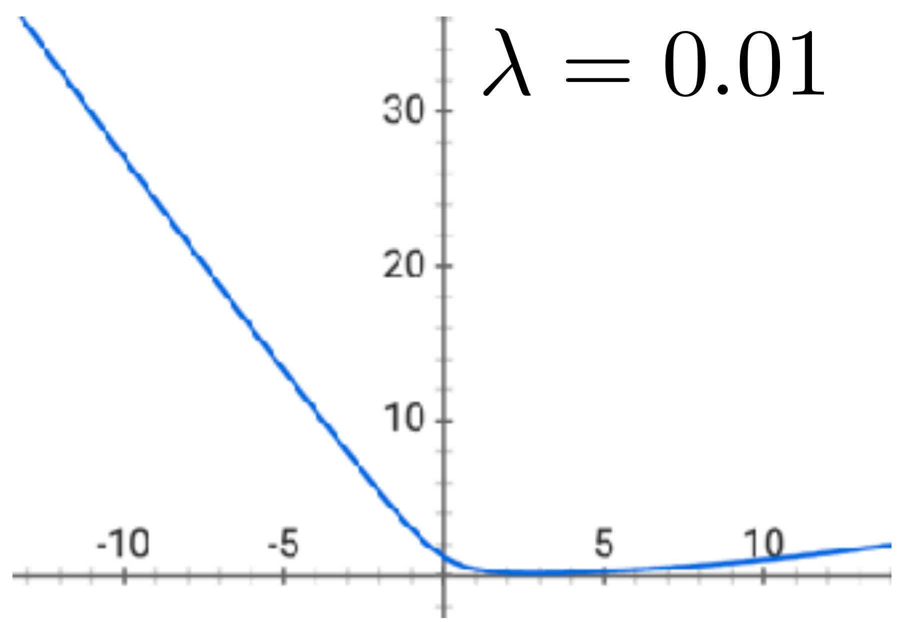

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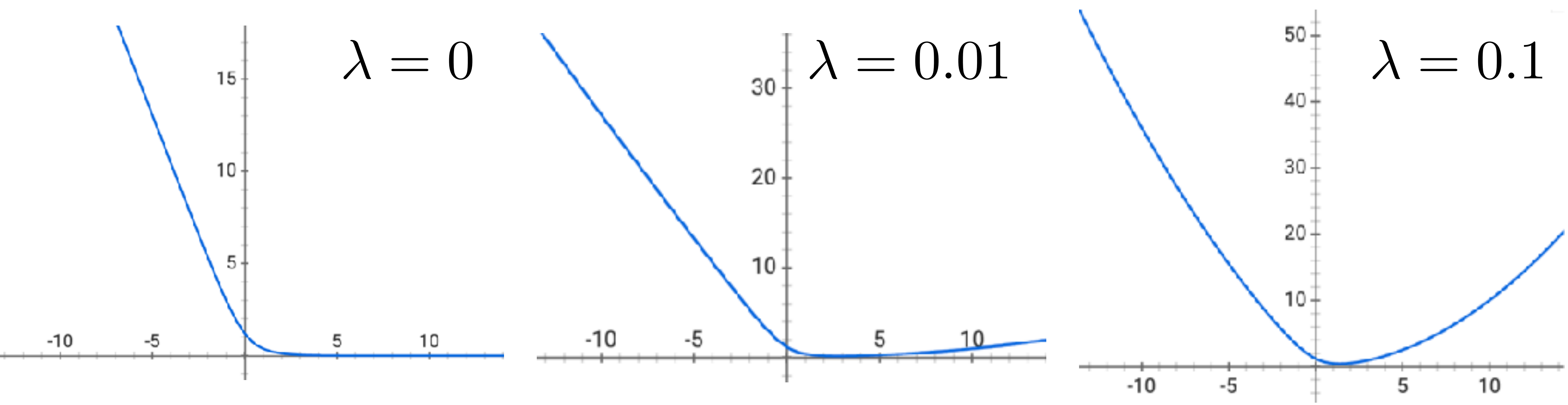

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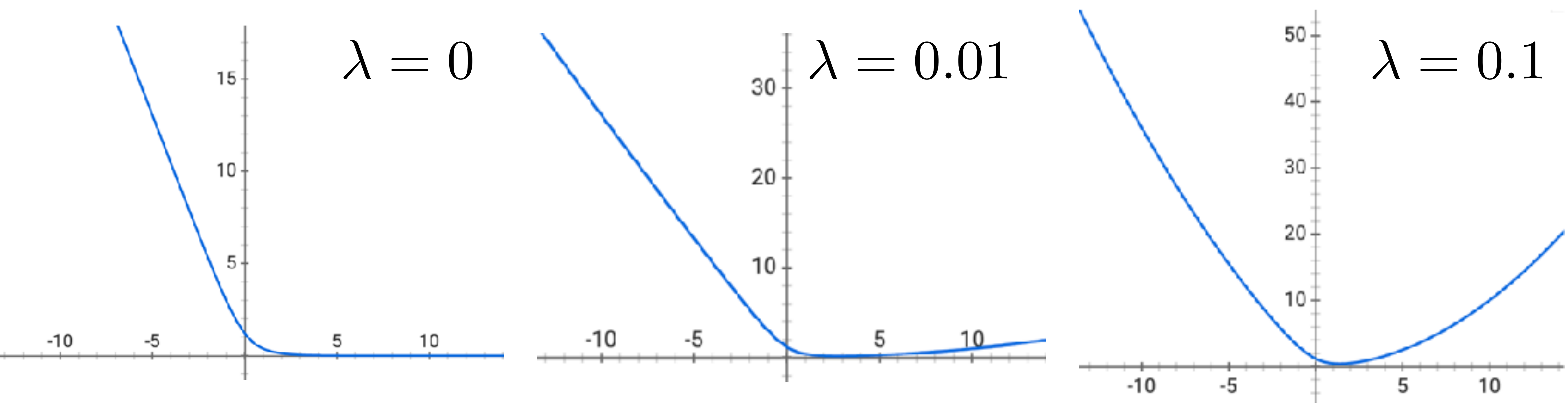

- How to choose hyperparameters? One option: consider a handful of possible values and compare via CV

Logistic regression learning algorithm

Exactly gradient descent with *f* given by logistic regression objective

Logistic regression learning algorithm

LR-Gradient-Descent($\theta_{\text{init}}, \theta_{0,\text{init}}, \eta, \epsilon$)

Exactly gradient descent with *f* given by logistic regression objective

Logistic regression learning algorithm

`LR-Gradient-Descent(`$\theta_{\text{init}}, \theta_{0,\text{init}}, \eta, \epsilon$`)`

`Initialize` $\theta^{(0)} = \theta_{\text{init}}$

`Initialize` $\theta_0^{(0)} = \theta_{0,\text{init}}$

Exactly gradient descent with *f* given by logistic regression objective

Logistic regression learning algorithm

```
LR-Gradient-Descent(θ_init, θ_0,init, η, ε)
  Initialize θ^(0) = θ_init
  Initialize θ_0^(0) = θ_0,init
  Initialize t = 0
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  repeat
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repeat

Exactly gradient descent with *f* given by logistic regression objective

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  repeat
    t = t + 1
```

LR-Gradient-Descent$(\theta_{\text{init}}, \theta_{0,\text{init}}, \eta, \epsilon\,)$

Initialize $\theta^{(0)} = \theta_{\text{init}}$

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t = t + 1

Exactly gradient descent with *f* given by logistic regression objective

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$$\theta^{(t)} = \theta^{(t-1)} - \eta\left\{ \frac{1}{n}\sum_{i=1}^{n}\left[\sigma(\theta^{(t-1)\top}x^{(i)} + \theta_0^{(t-1)}) - y^{(i)}\right]x^{(i)} + 2\lambda\theta^{(t-1)} \right\}$$

$$\theta_0^{(t)} = \theta_0^{(t-1)} - \eta\left\{ \frac{1}{n}\sum_{i=1}^{n}\left[\sigma(\theta^{(t-1)\top}x^{(i)} + \theta_0^{(t-1)}) - y^{(i)}\right] \right\}$$

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until $|J_{\text{lr}}(\theta^{(t)}, \theta_0^{(t)}) - J_{\text{lr}}(\theta^{(t-1)}, \theta_0^{(t-1)})| < \epsilon$

Exactly gradient descent with *f* given by logistic regression objective

Logistic regression learning algorithm

LR-Gradient-Descent($\theta_{\text{init}}, \theta_{0,\text{init}}, \eta, \epsilon$)

Initialize $\theta^{(0)} = \theta_{\text{init}}$

Initialize $\theta_0^{(0)} = \theta_{0,\text{init}}$

Initialize t = 0

repeat

t = t + 1

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until $|J_{\text{lr}}(\theta^{(t)}, \theta_0^{(t)}) - J_{\text{lr}}(\theta^{(t-1)}, \theta_0^{(t-1)})| < \epsilon$

Return $\theta^{(t)}, \theta_0^{(t)}$

Exactly gradient descent with *f* given by logistic regression objective