6.036/6.862: Introduction to Machine Learning

Lecture: starts Tuesdays 9:35am (Boston time zone)

Course website: introml.odl.mit.edu

Who's talking? Prof. Tamara Broderick

Questions? discourse.odl.mit.edu ("Lecture 8" category)

Materials: Will all be available at course website

Last Time

I. Neural networks
   - 2 layers
   - Fully connected
   - Learning

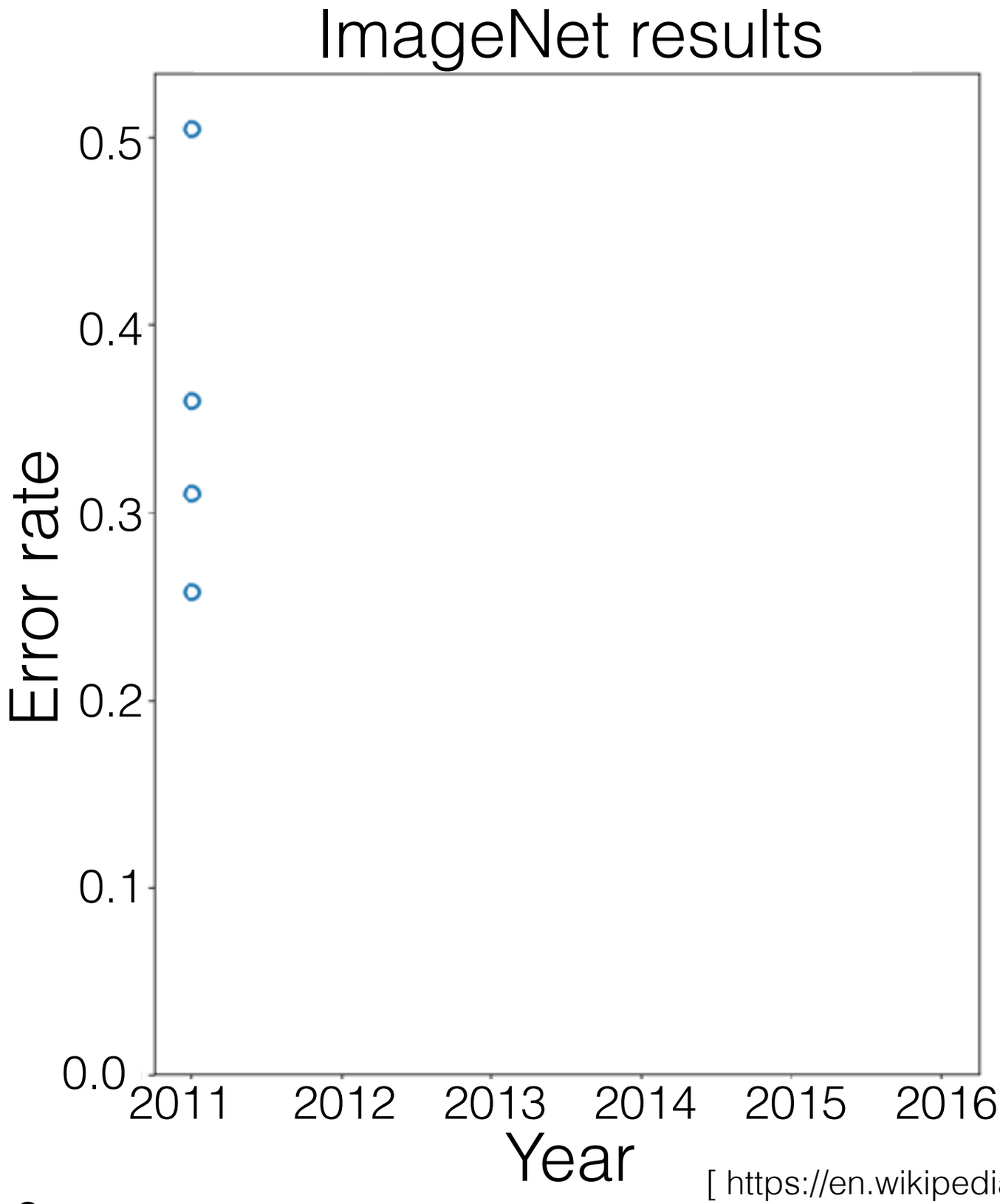
Today's Plan

I. CNNs/ConvNets: hypothesis class

II. Filters & max pooling

III. Learning

Impact of CNNs



- Since 2010: large-scale image classification challenge

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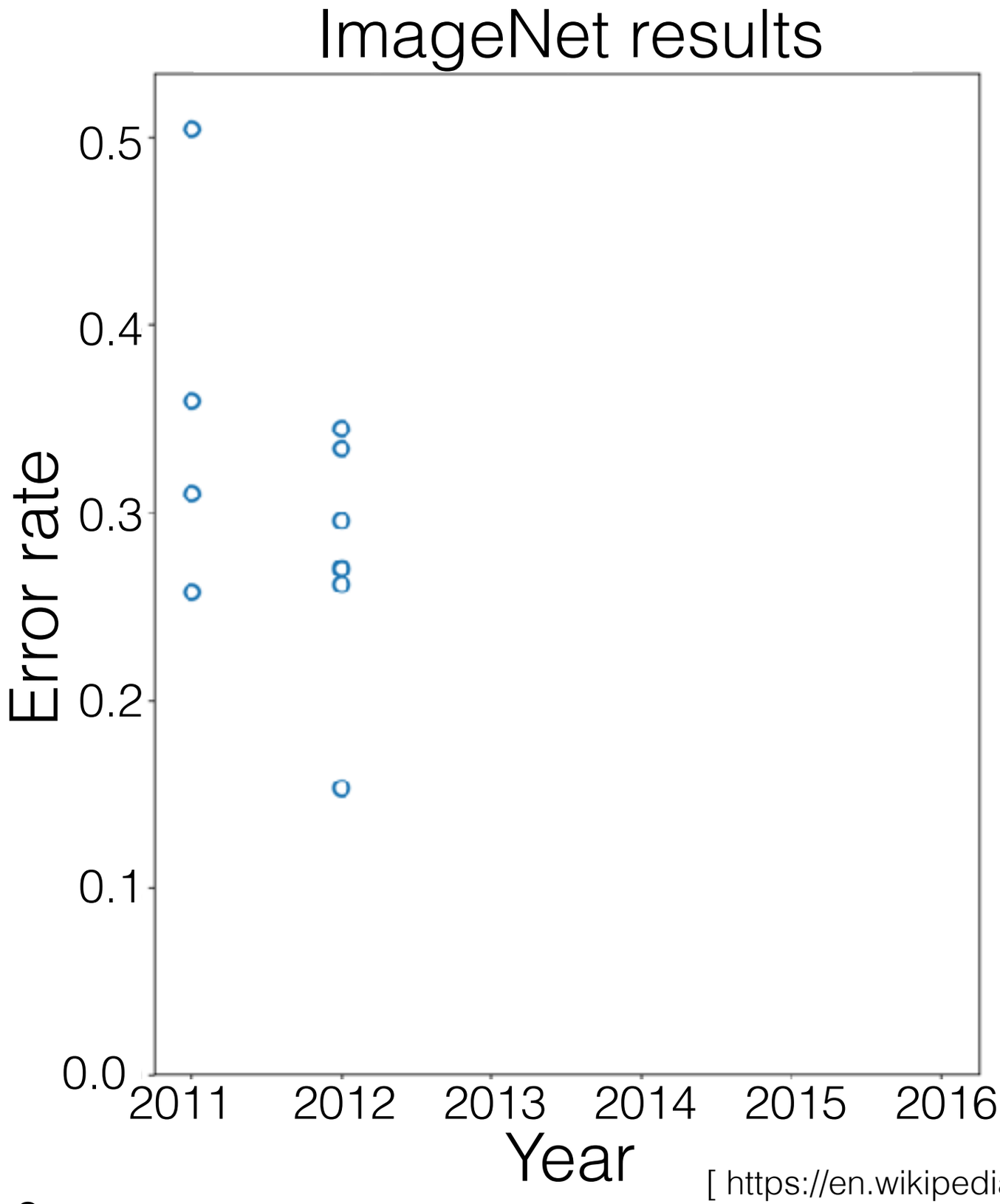

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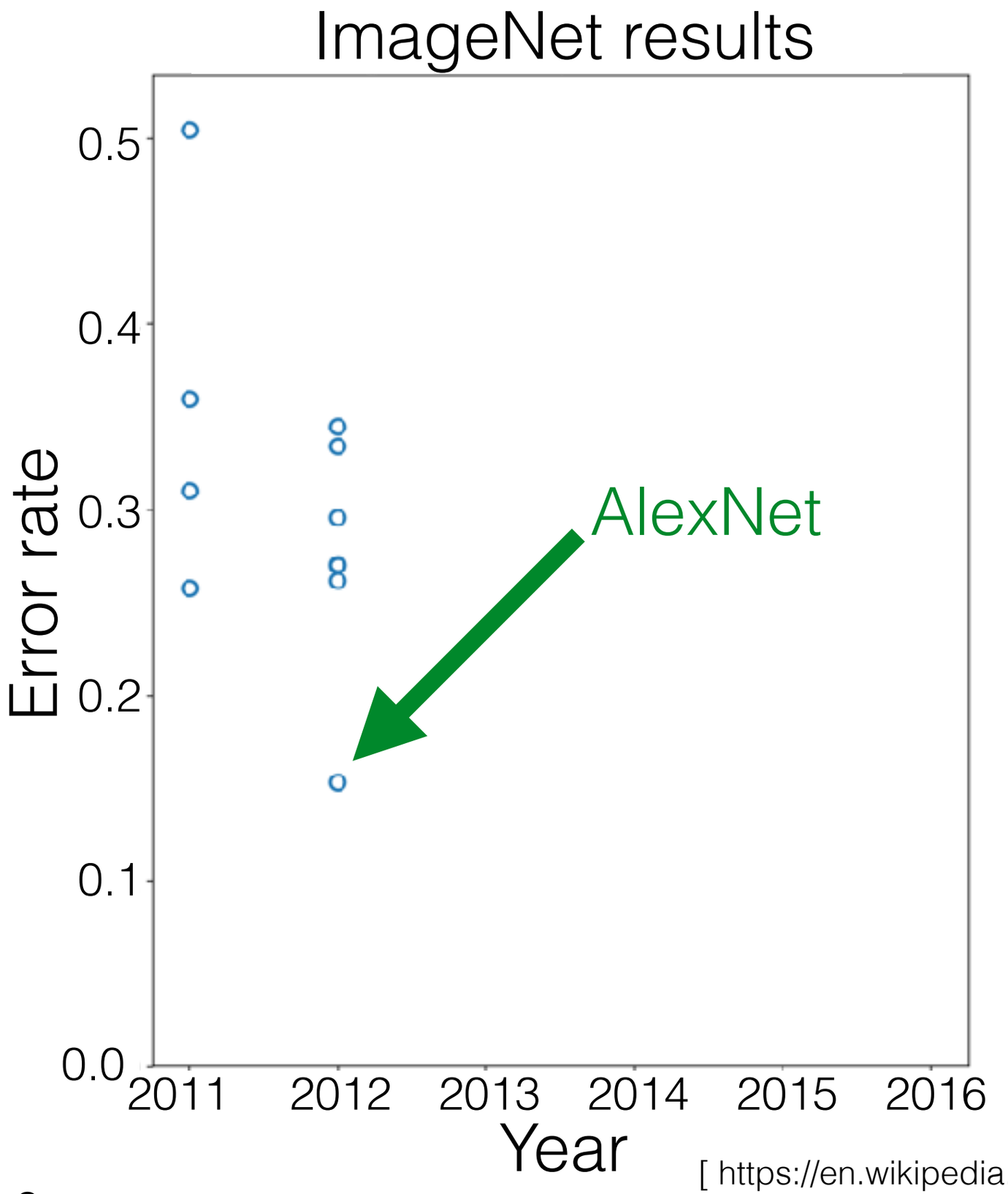

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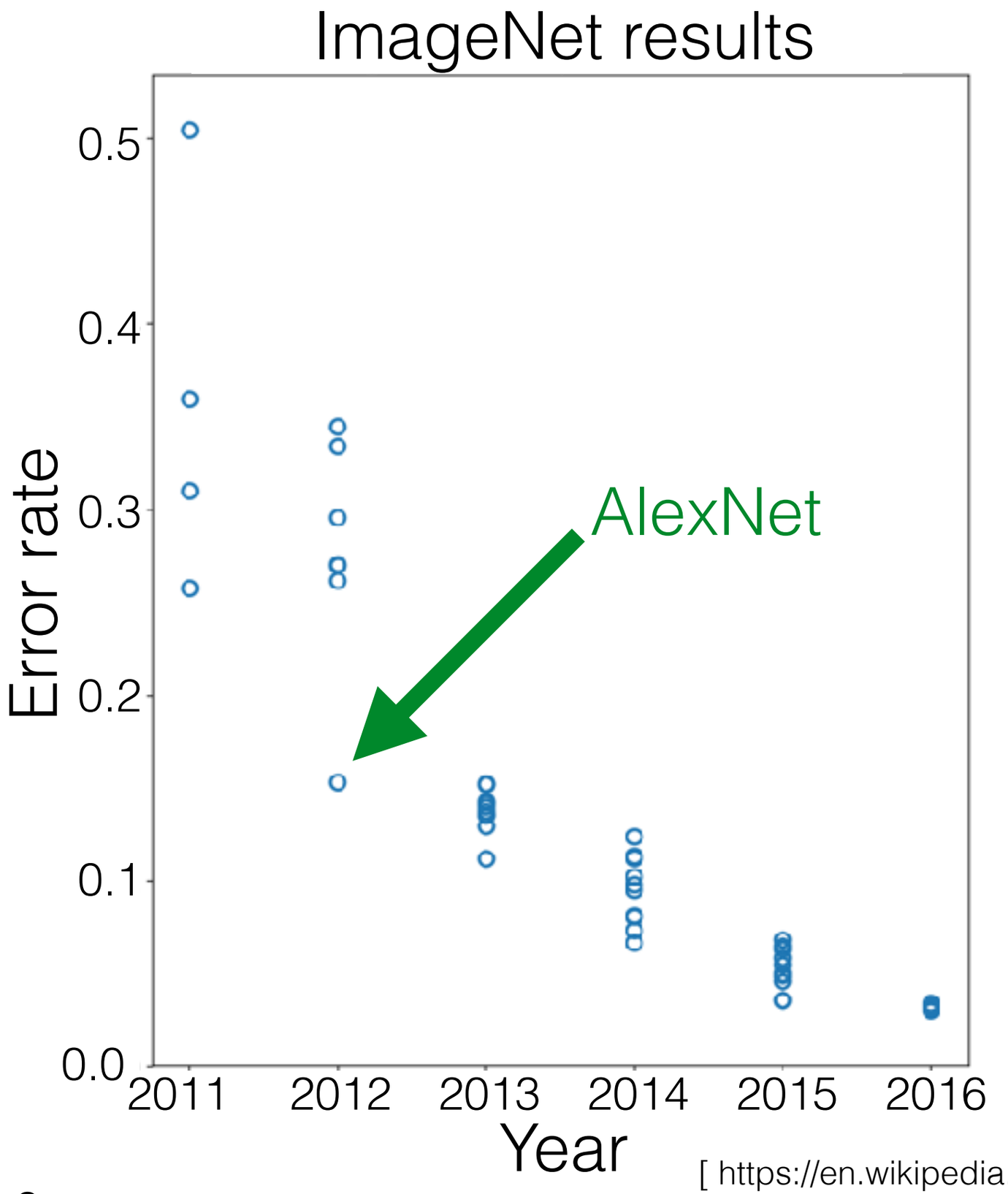

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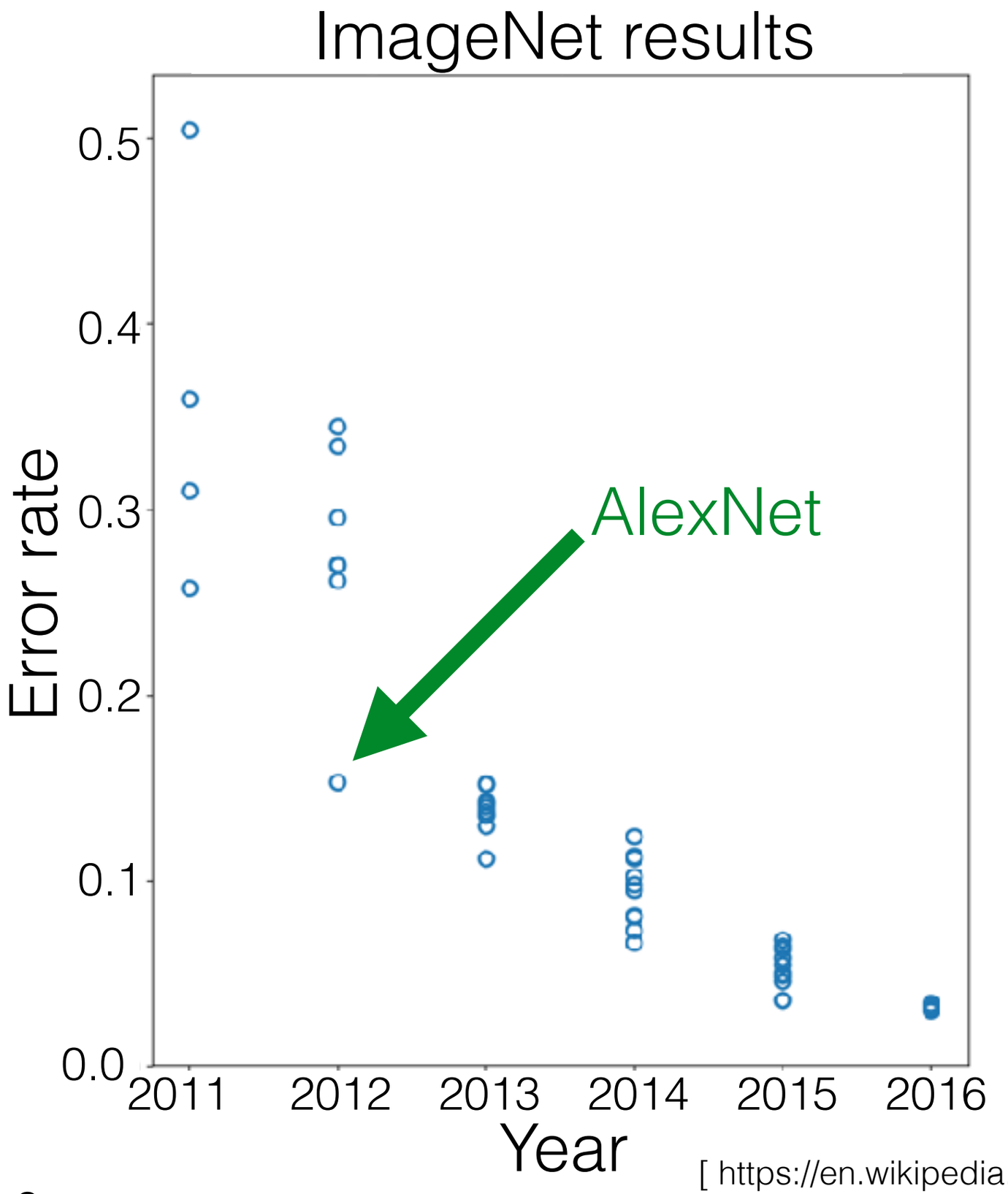

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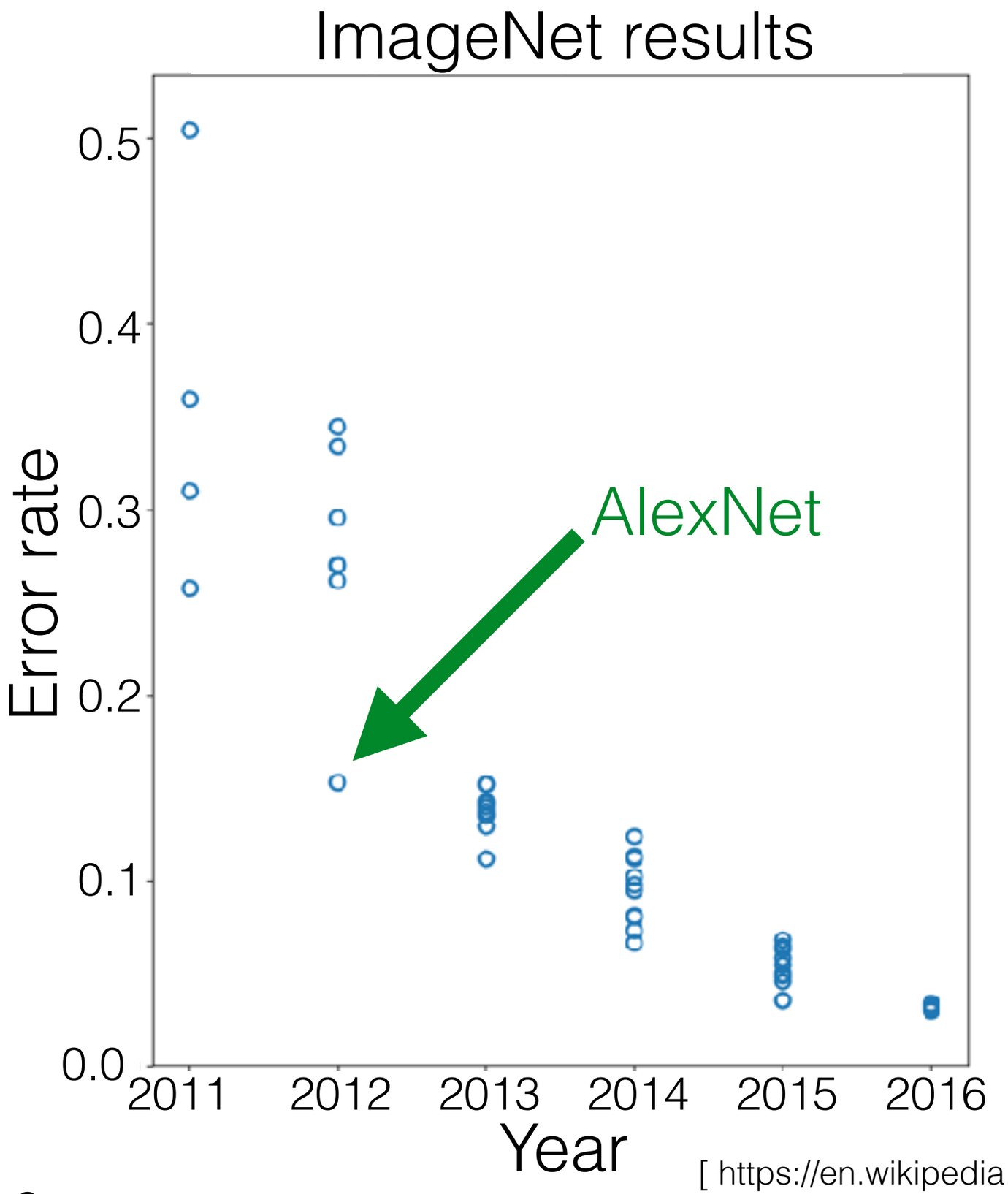

- Since 2010: large-scale image classification challenge
- Recent AI boom
- 1960s, 1980s, today: neural networks
- Since 1980s: CNNs

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Images

- Potential uses of image classification: Detect tumor (type) from medical scans, image search online, autonomous driving

[https://en.wikipedia.org/wiki/Pixel#/media/File:Pixel-example.png]

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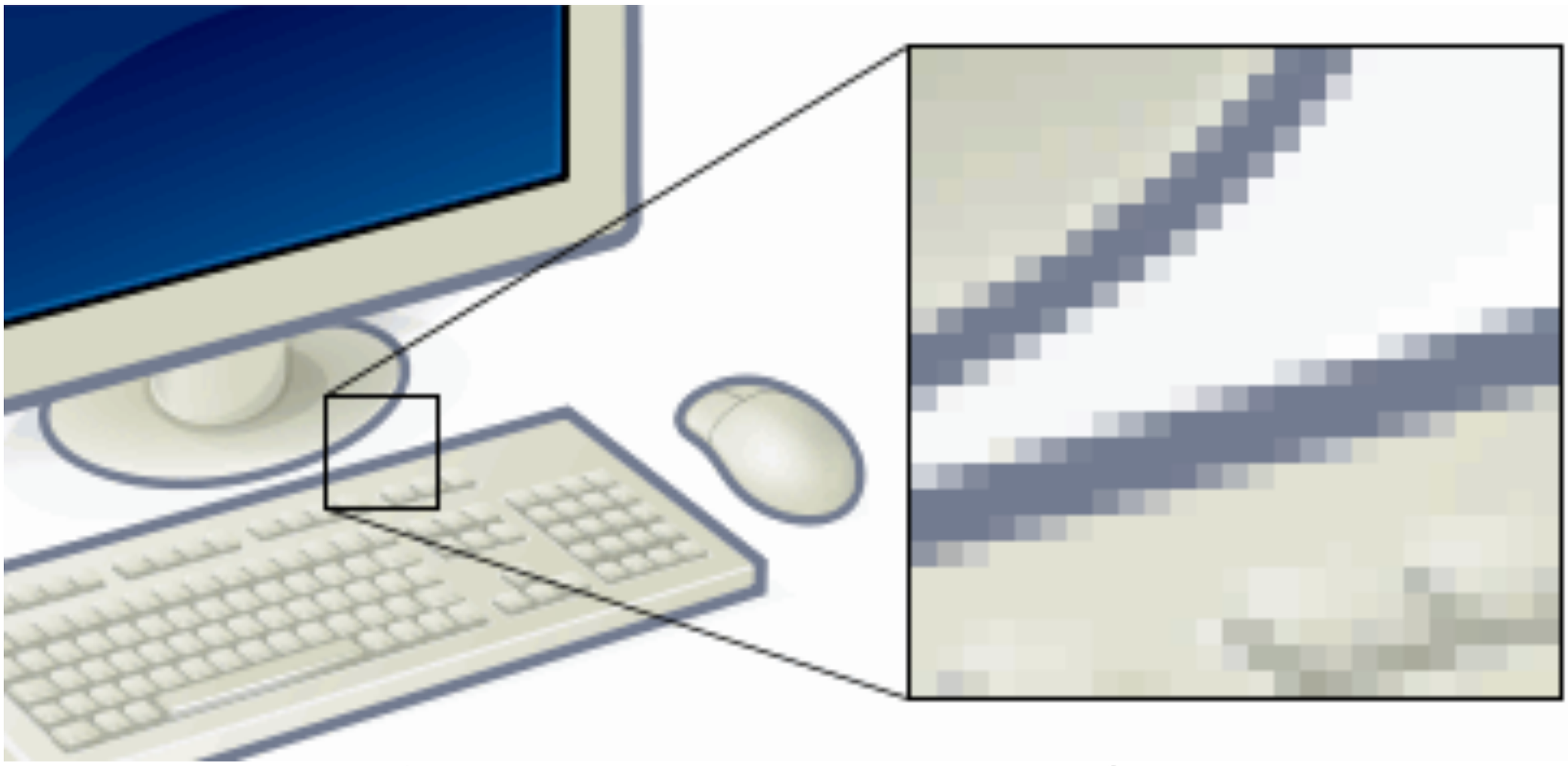


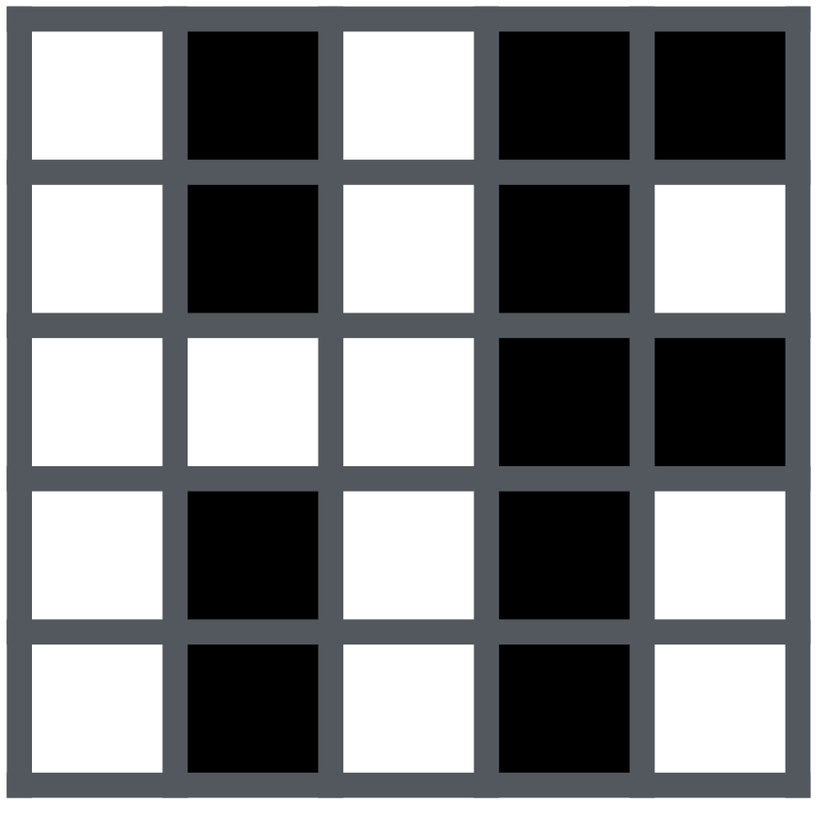
- Recall: images are made of pixels

[https://en.wikipedia.org/wiki/Pixel#/media/File:Pixel-example.png]

Images

- We'll focus on grayscale images

Images



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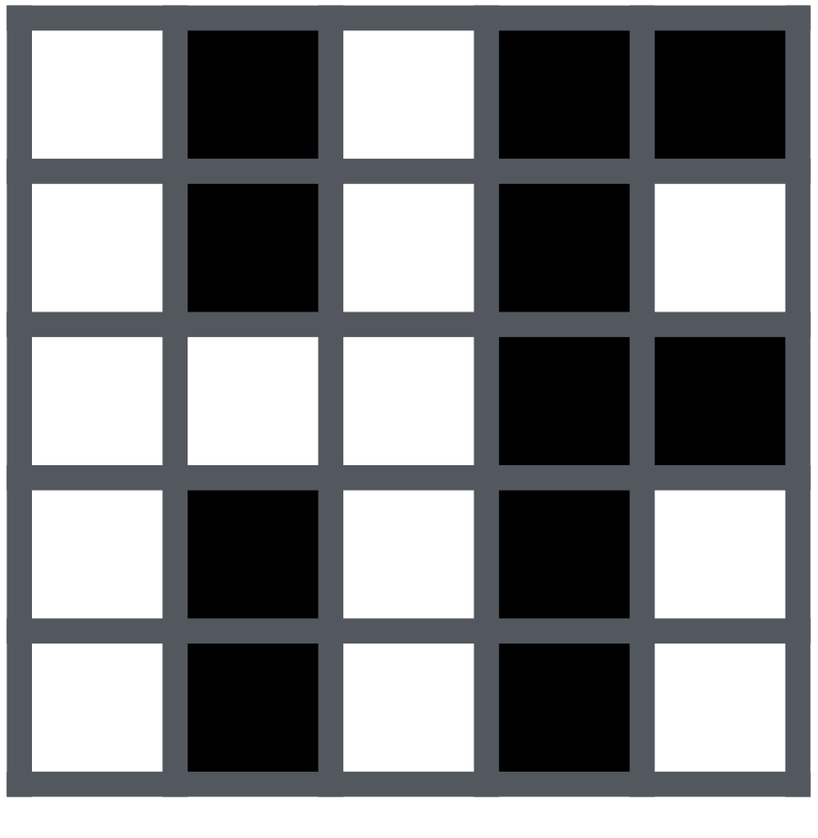

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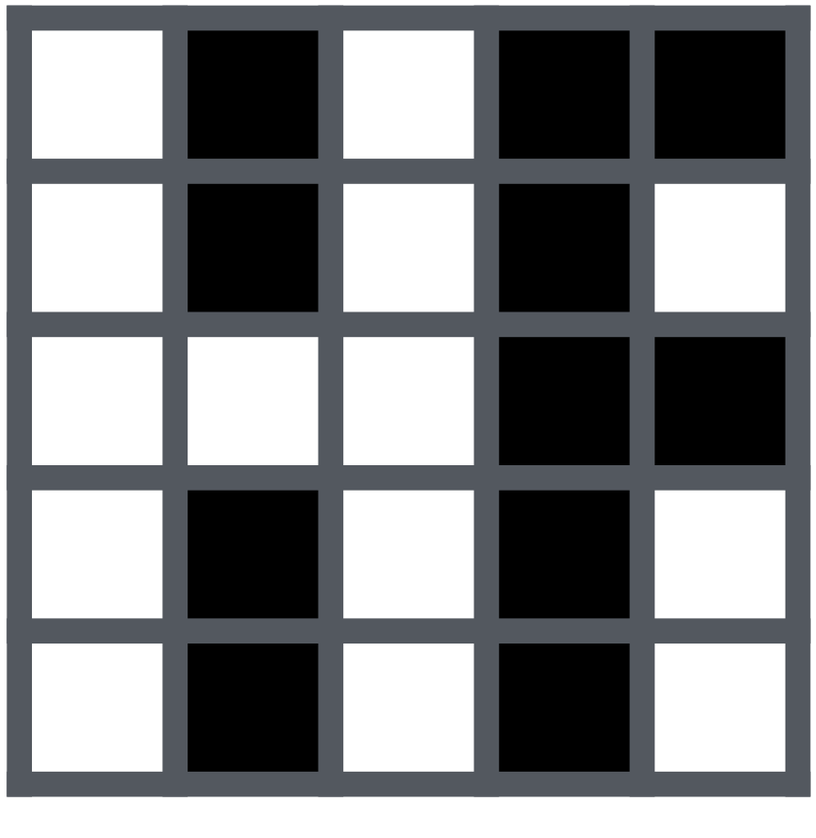

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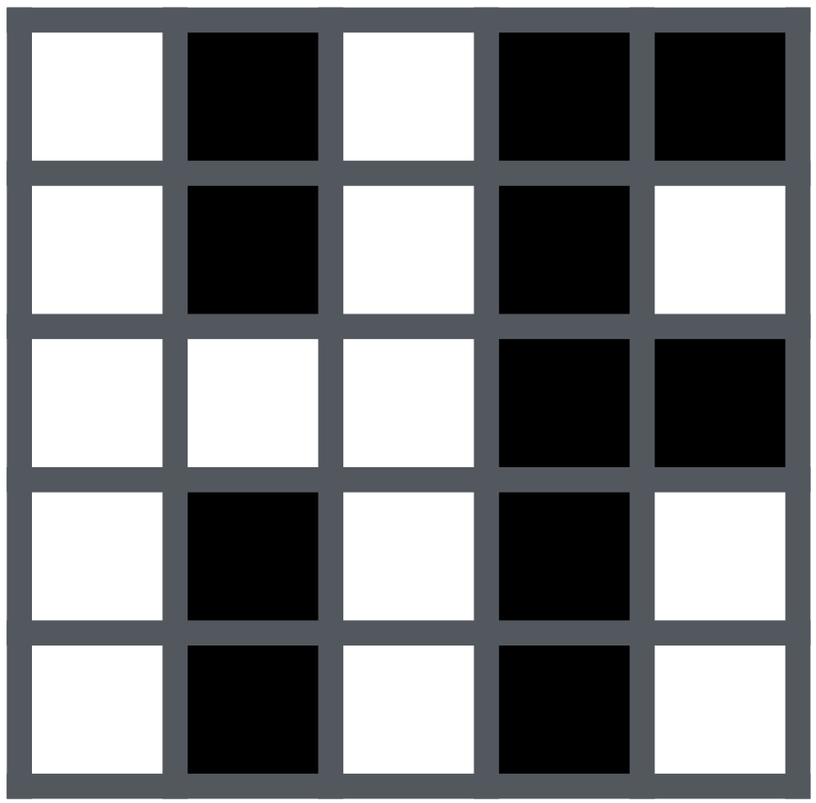

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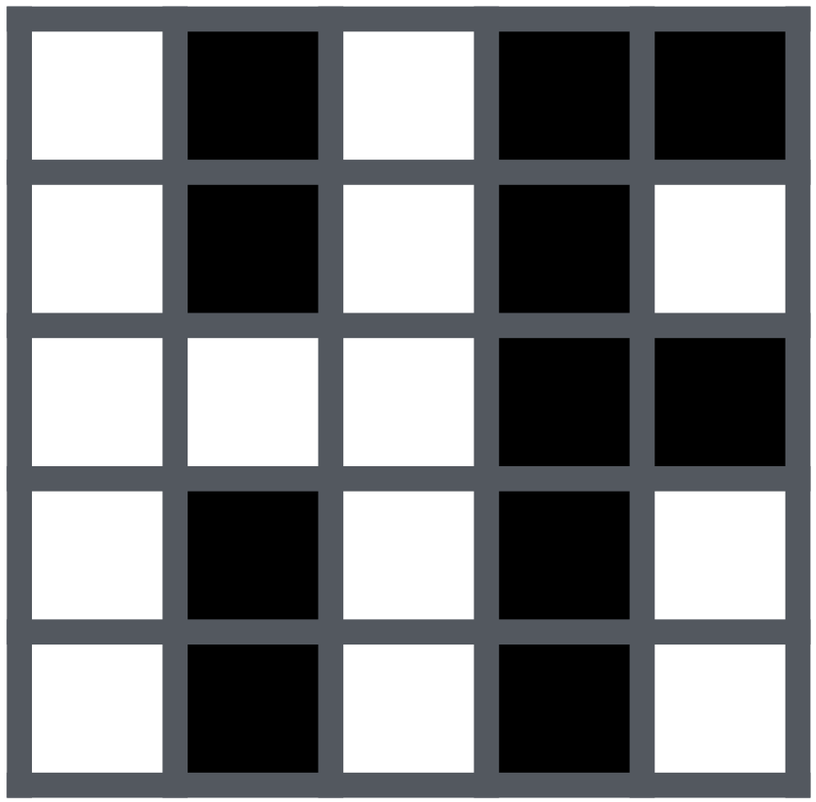

1	0	1	0	0
1	0	1	0	1
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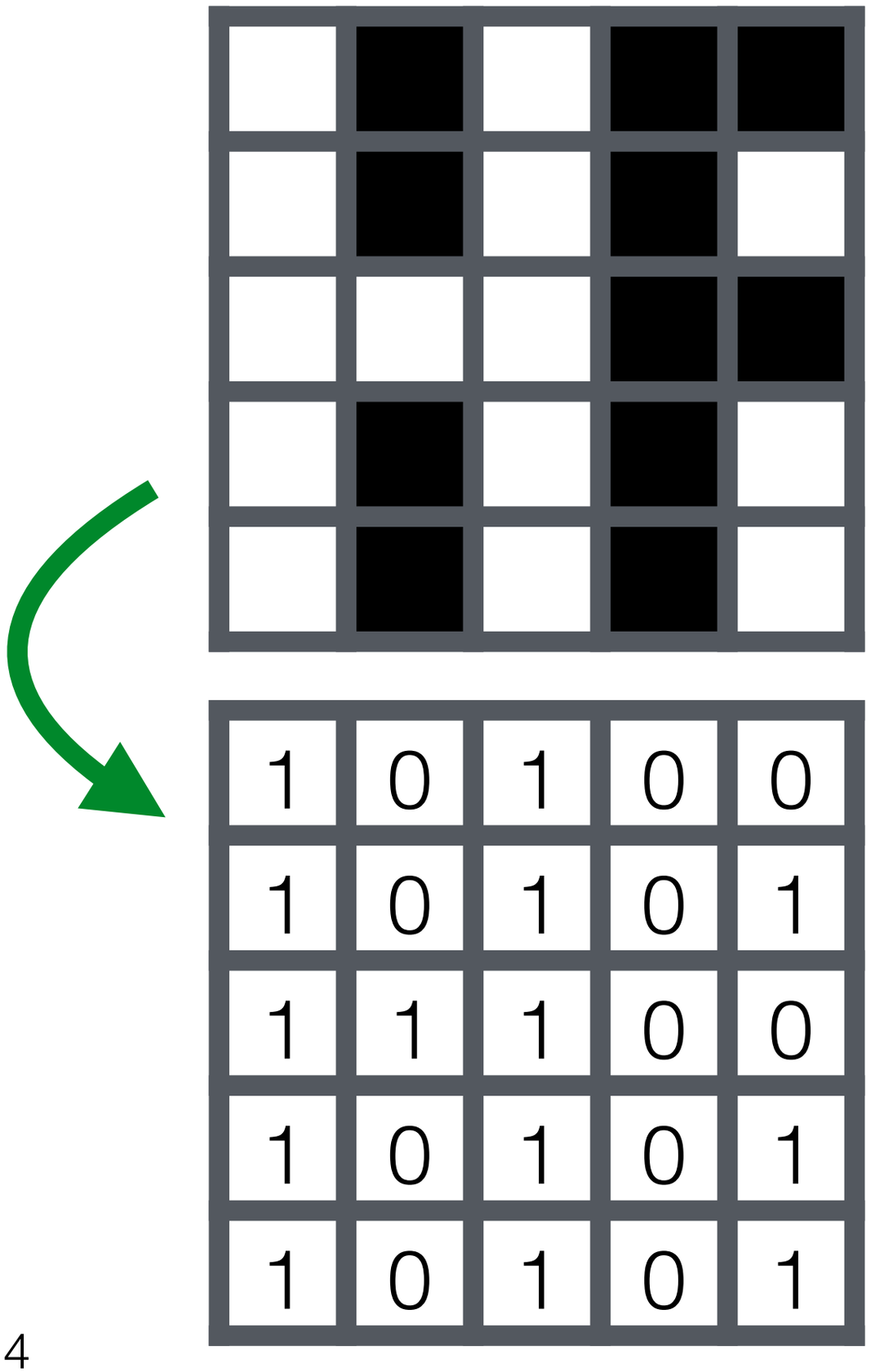

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- How do we use an image as an input for a neural net?

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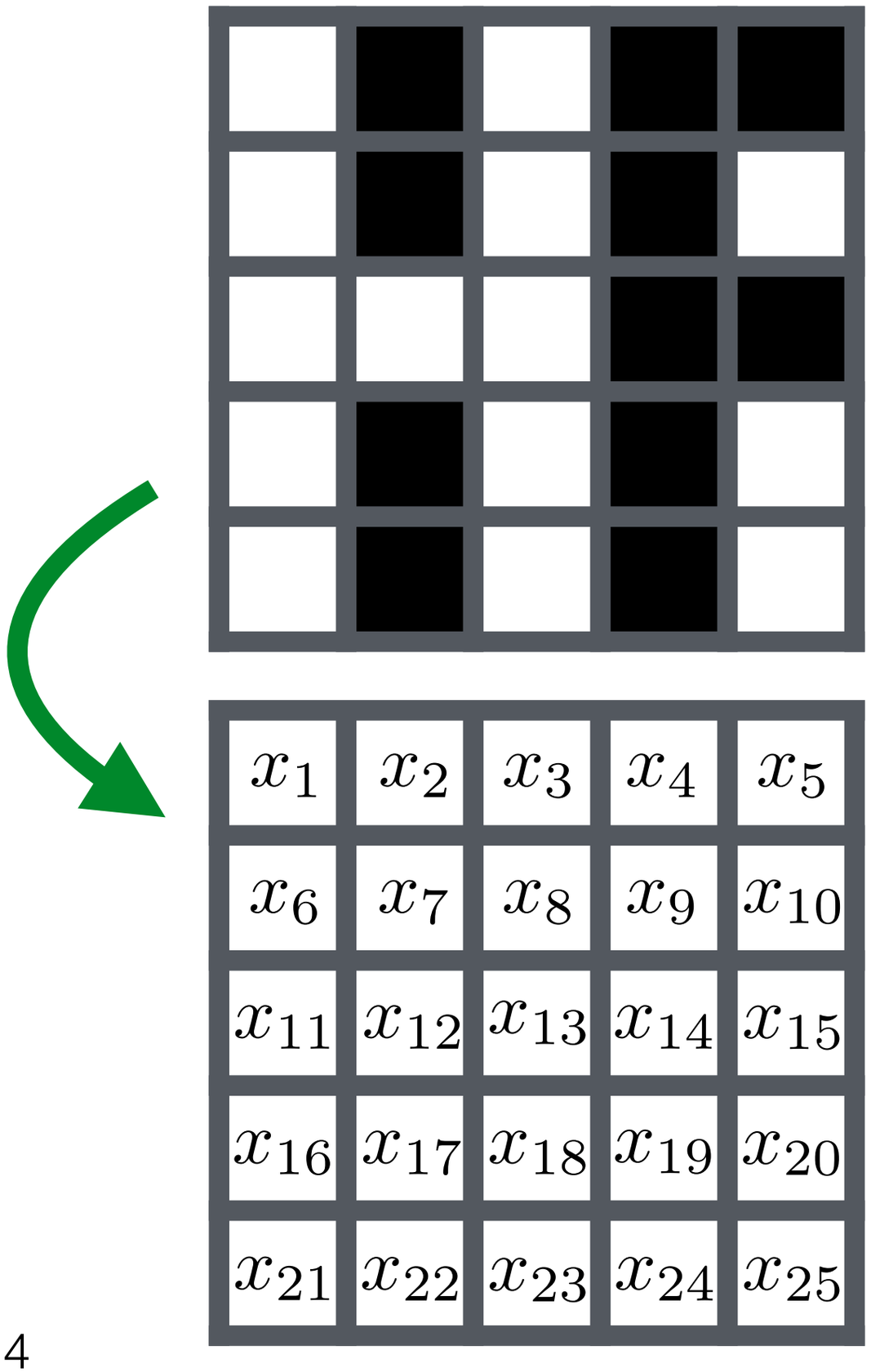

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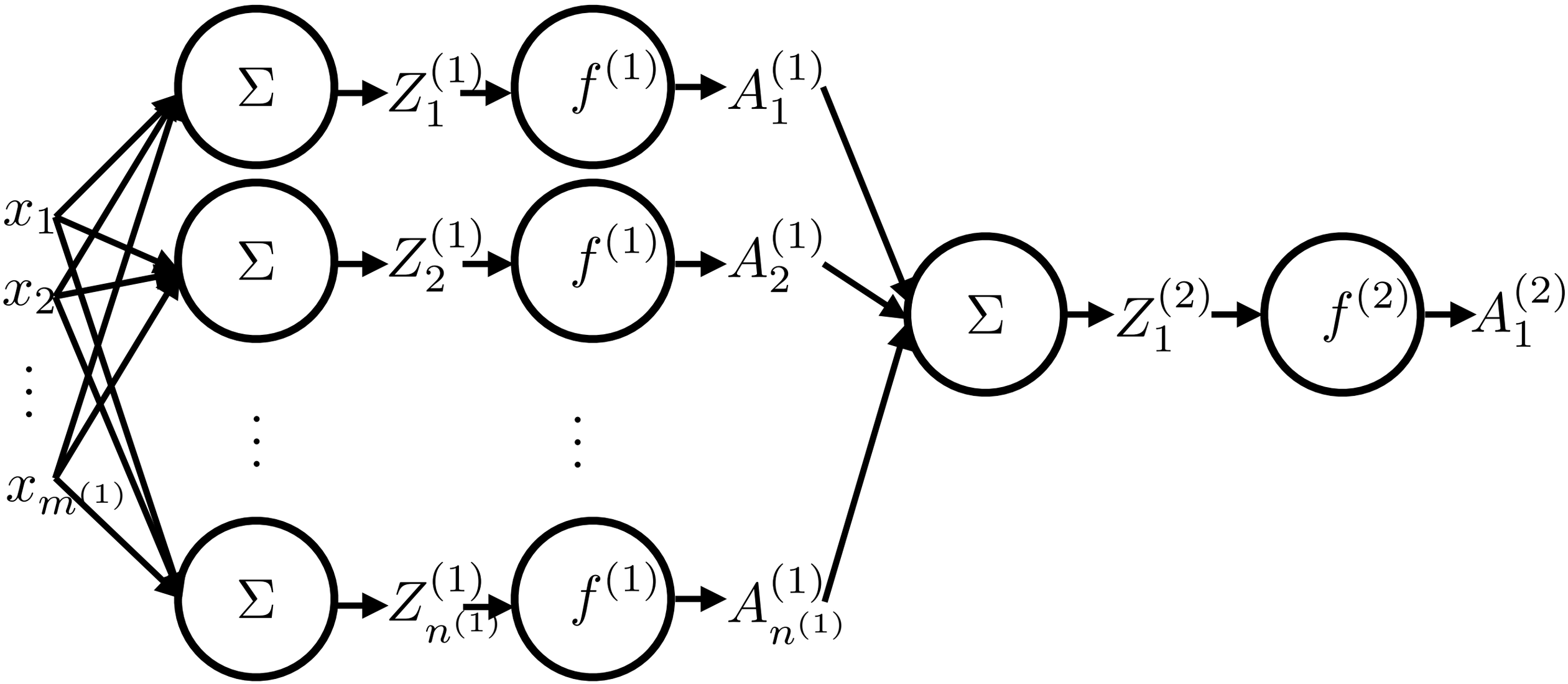

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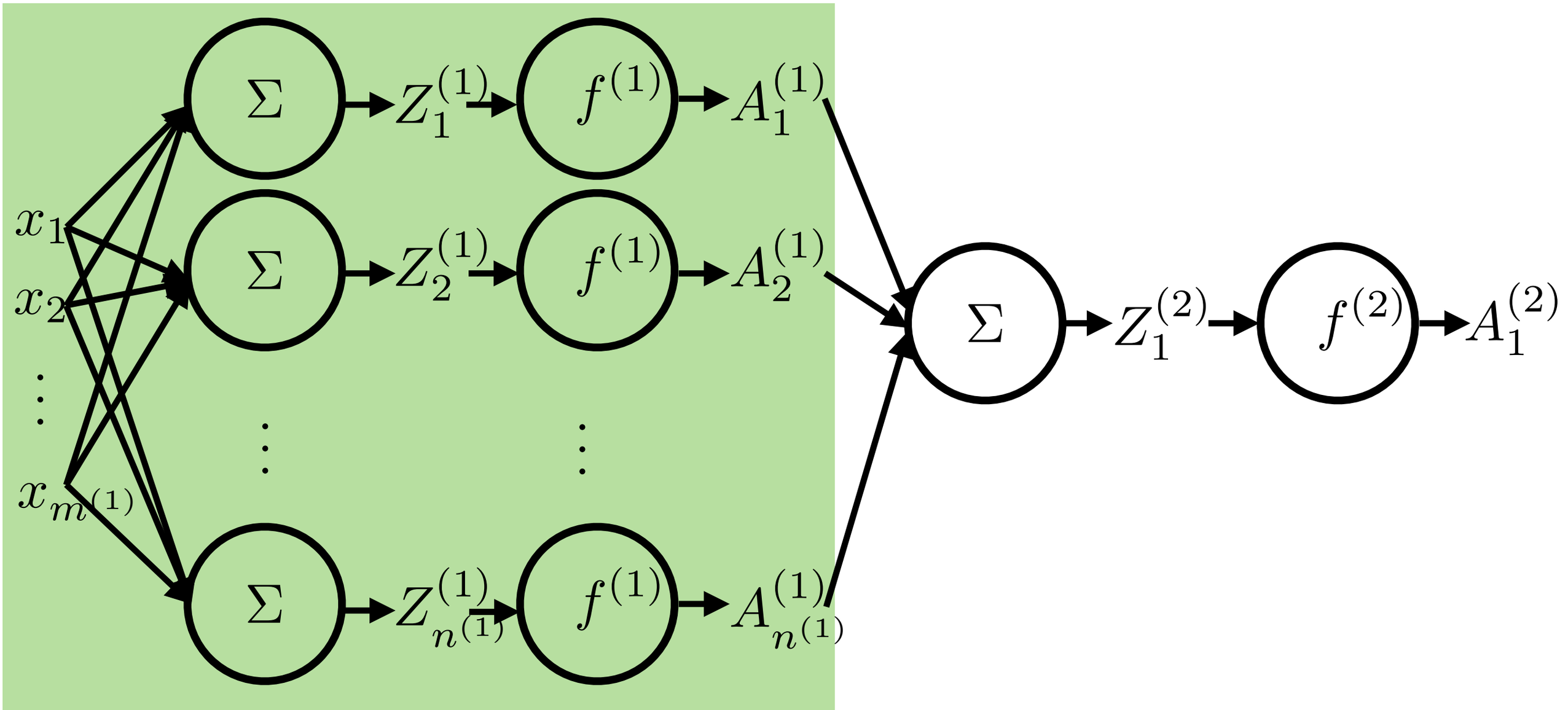

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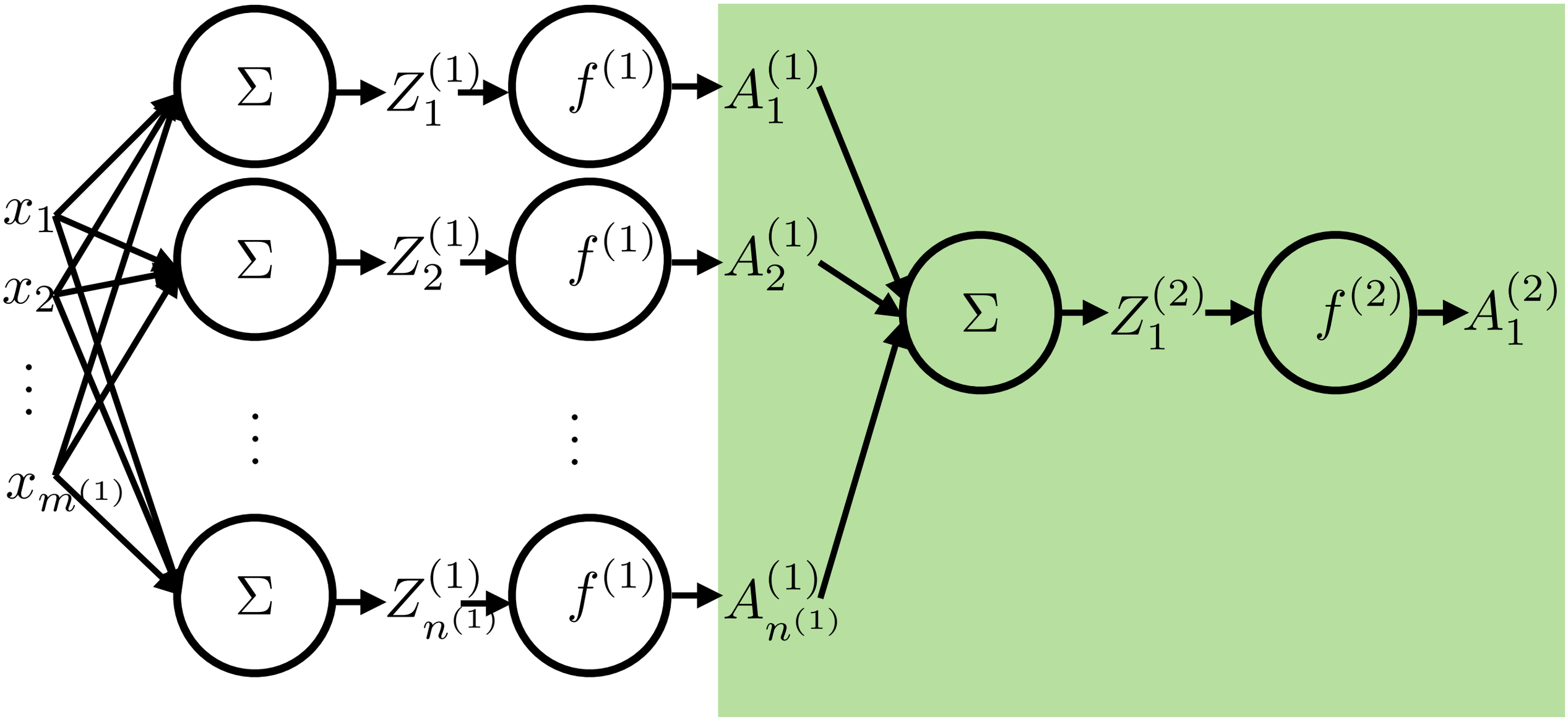

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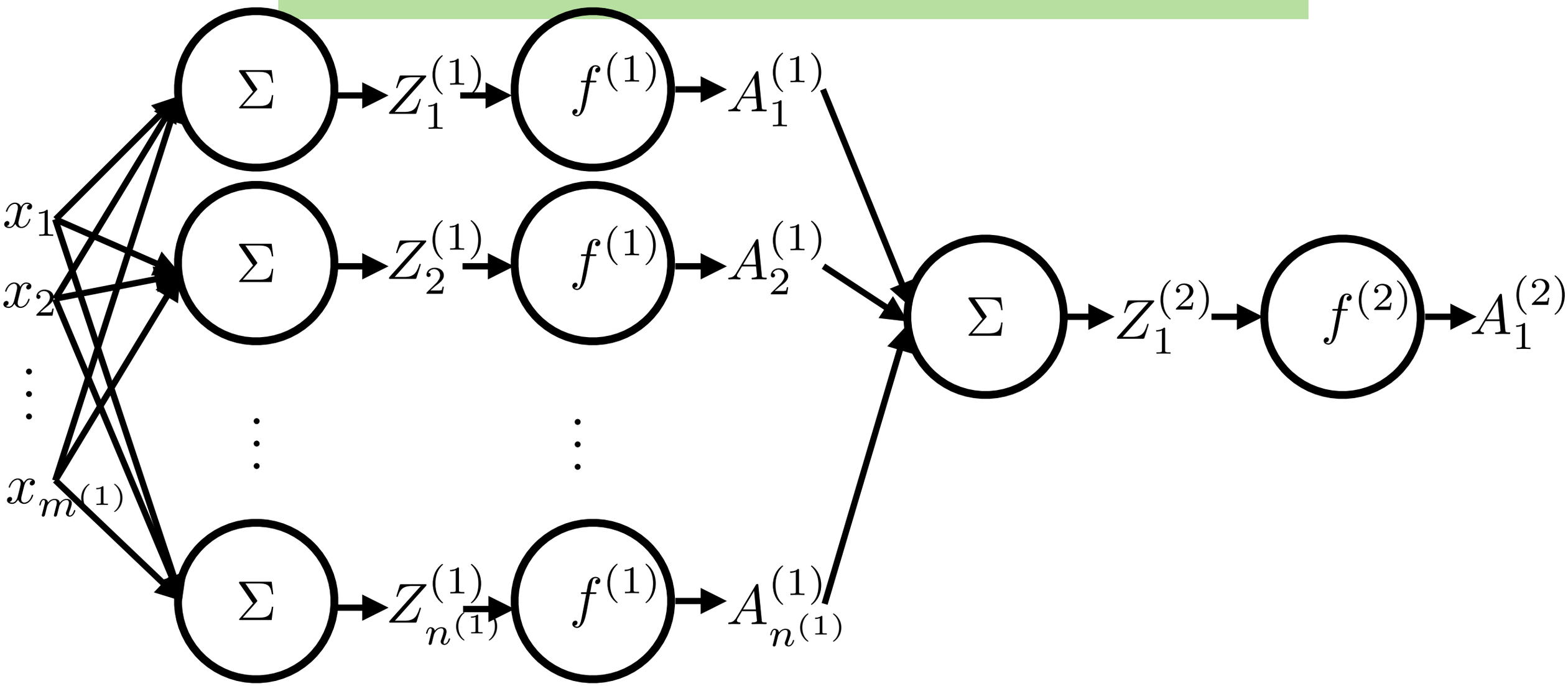
- Recall:

Fully connected layer: every input is connected to every output by a weight



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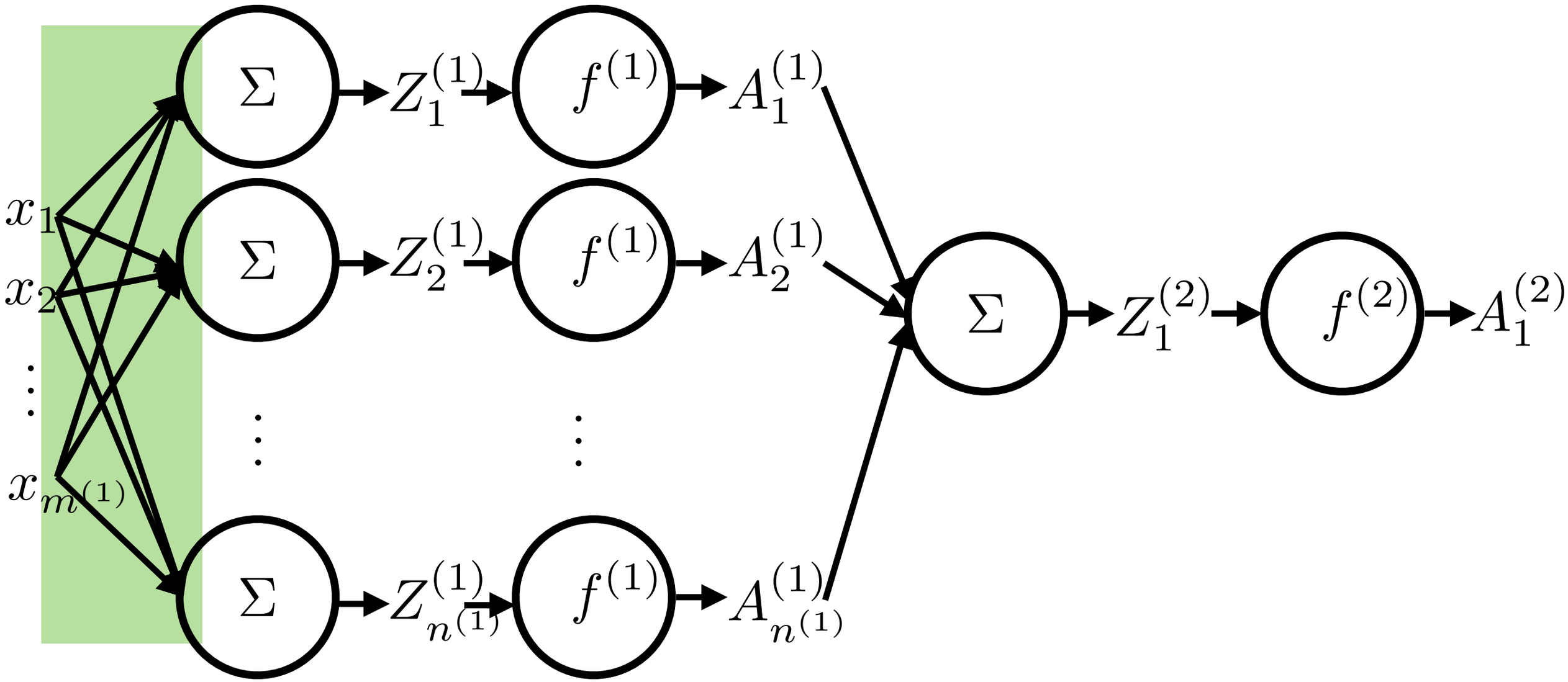

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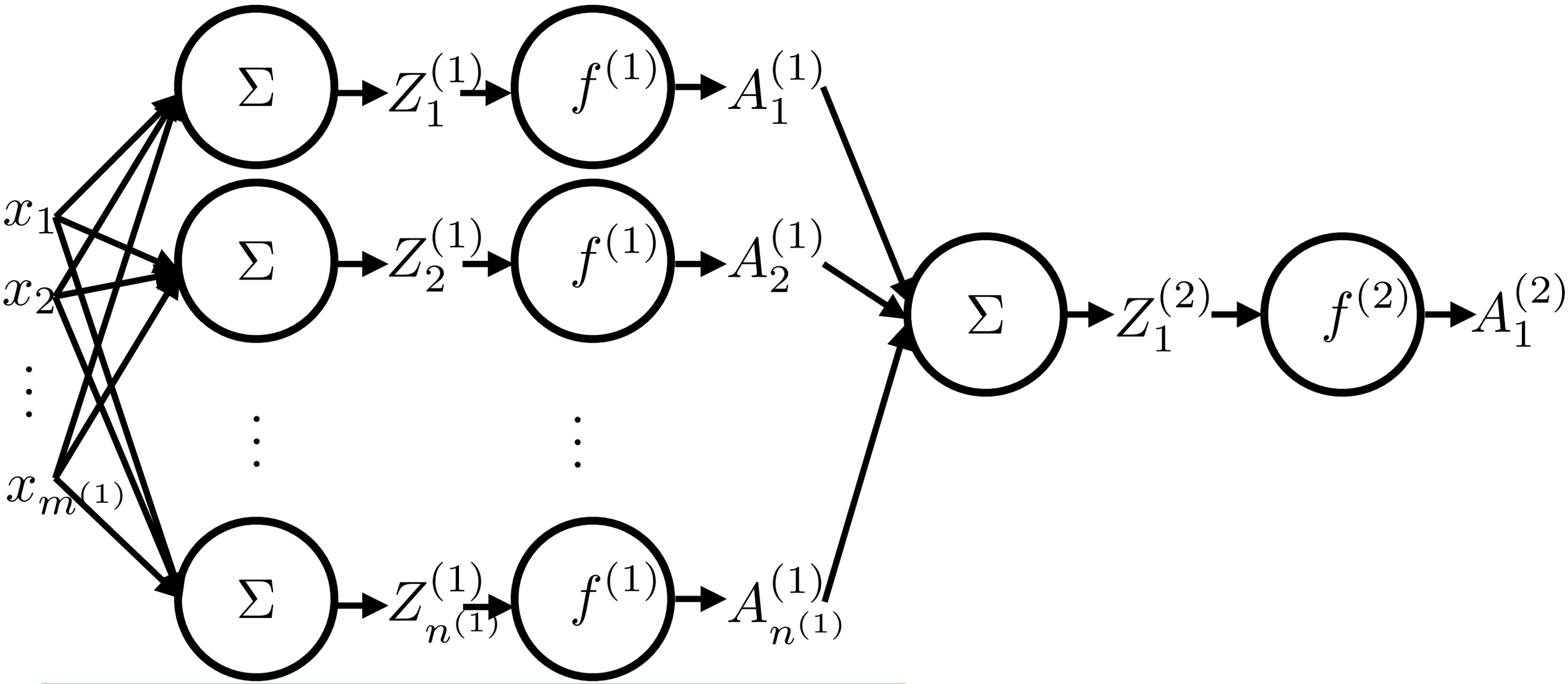

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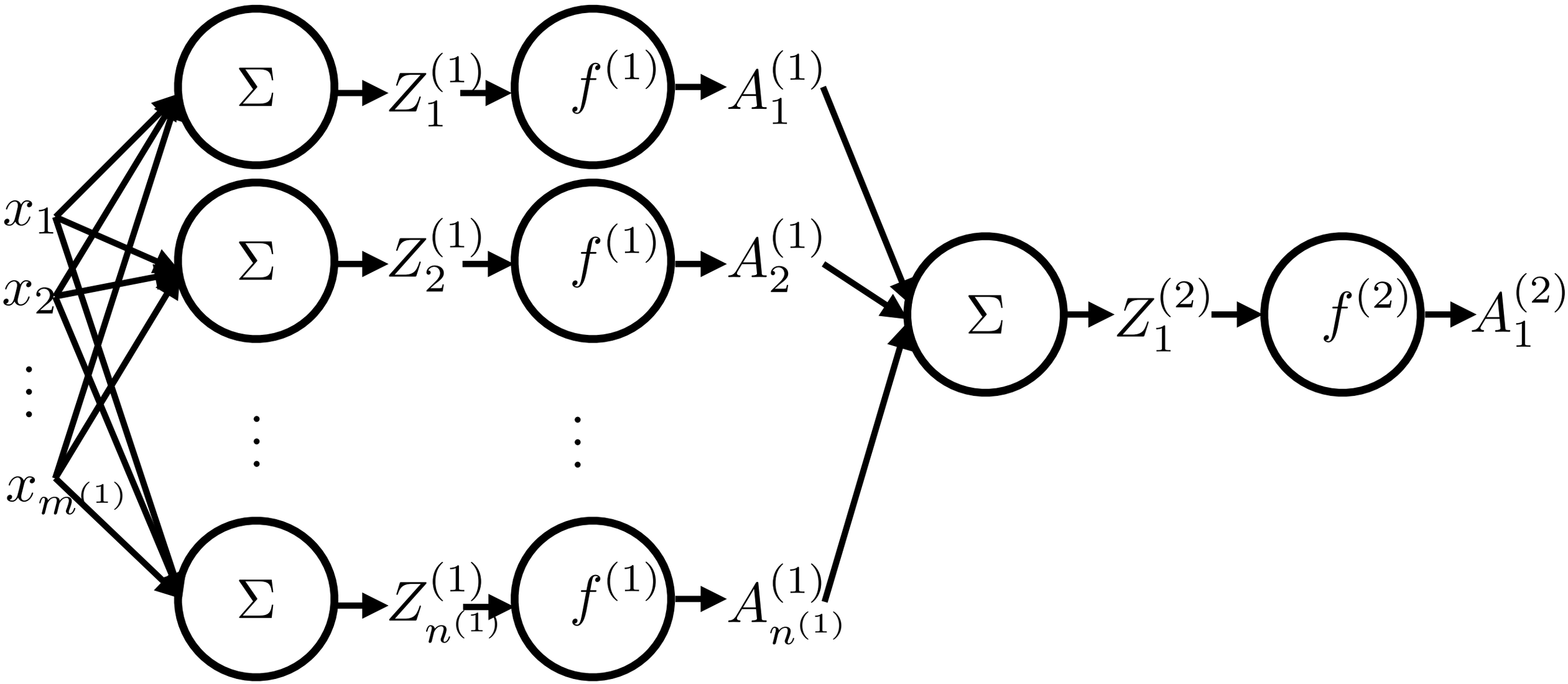

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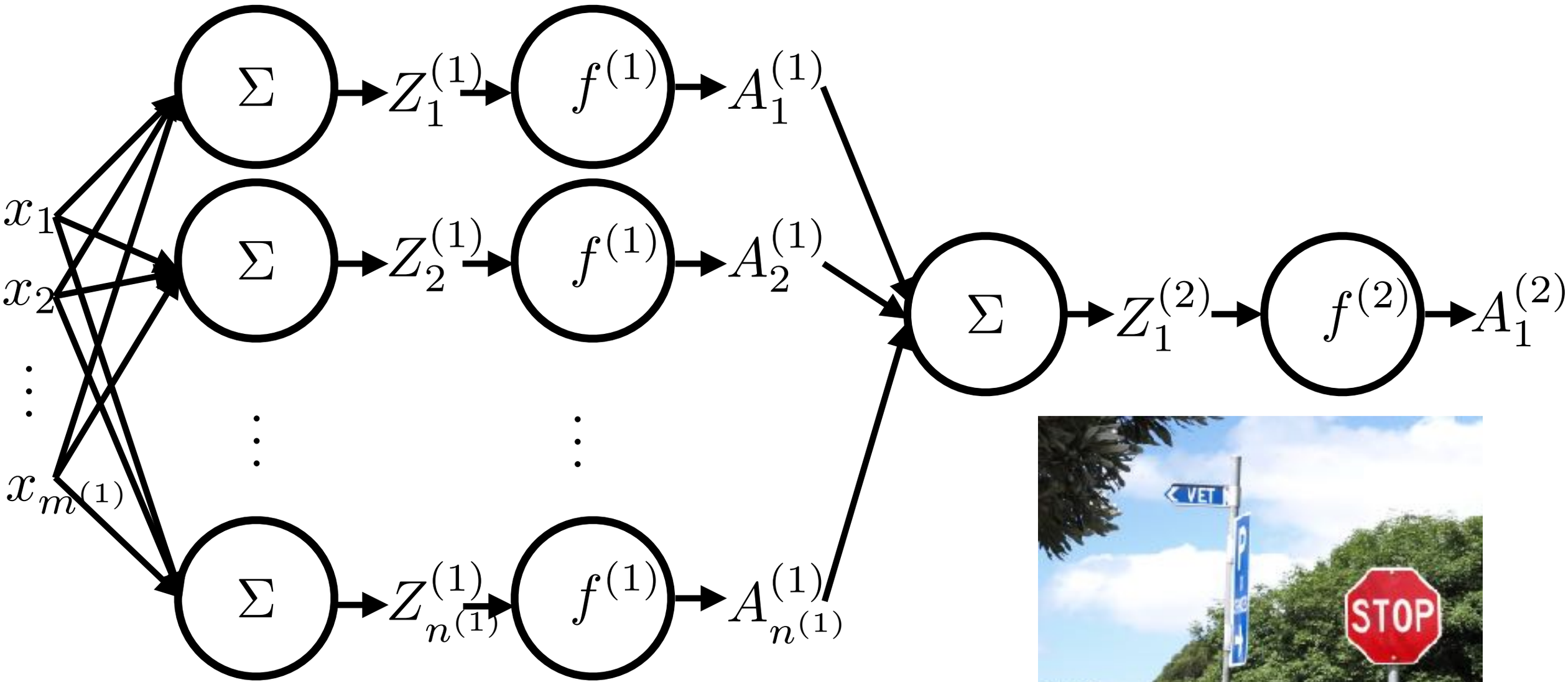

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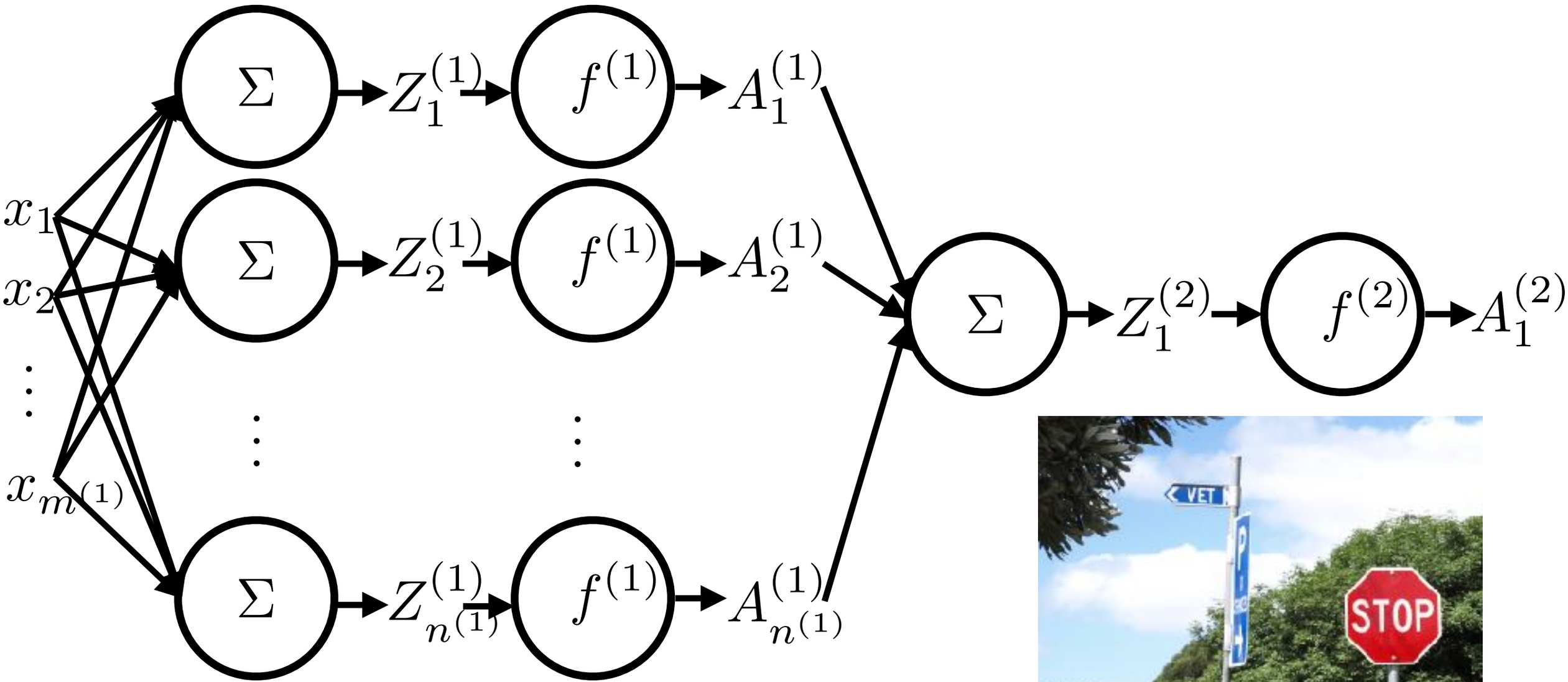

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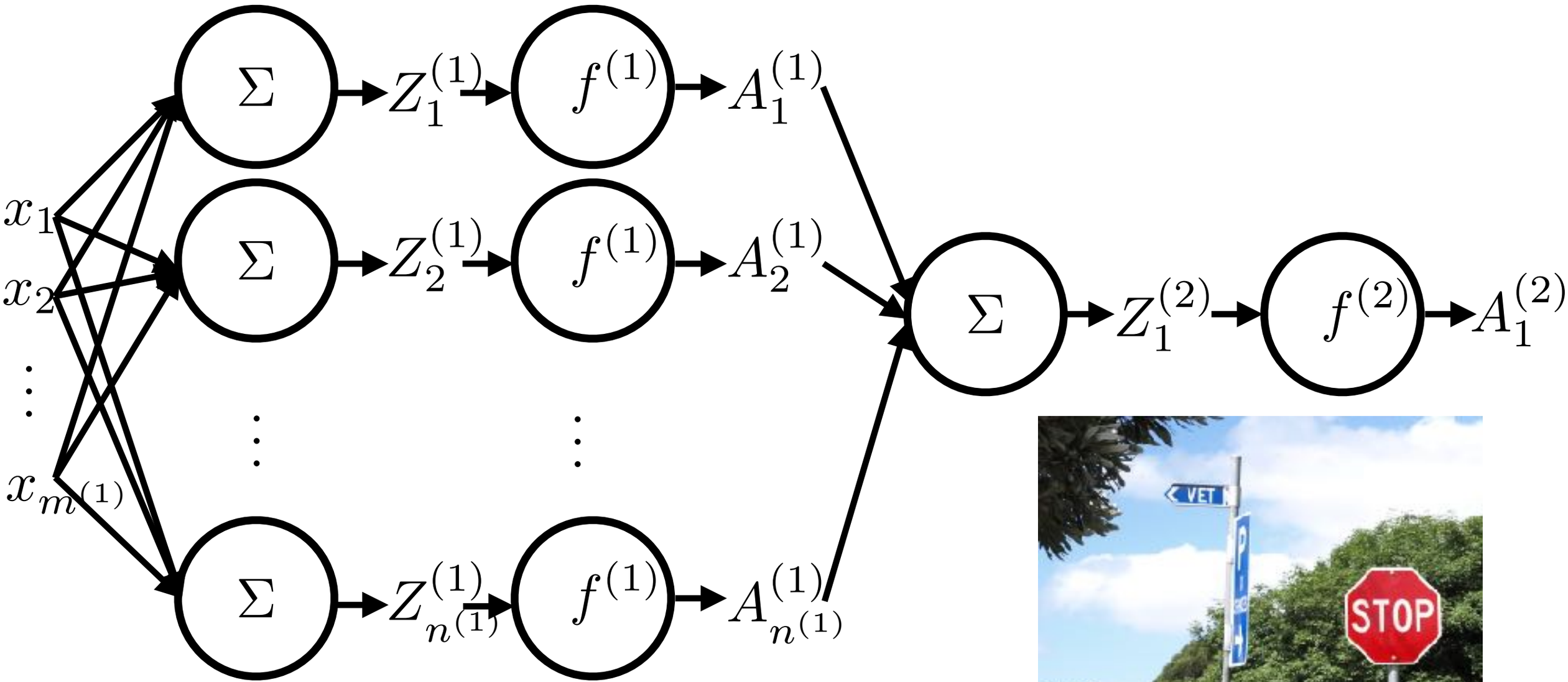

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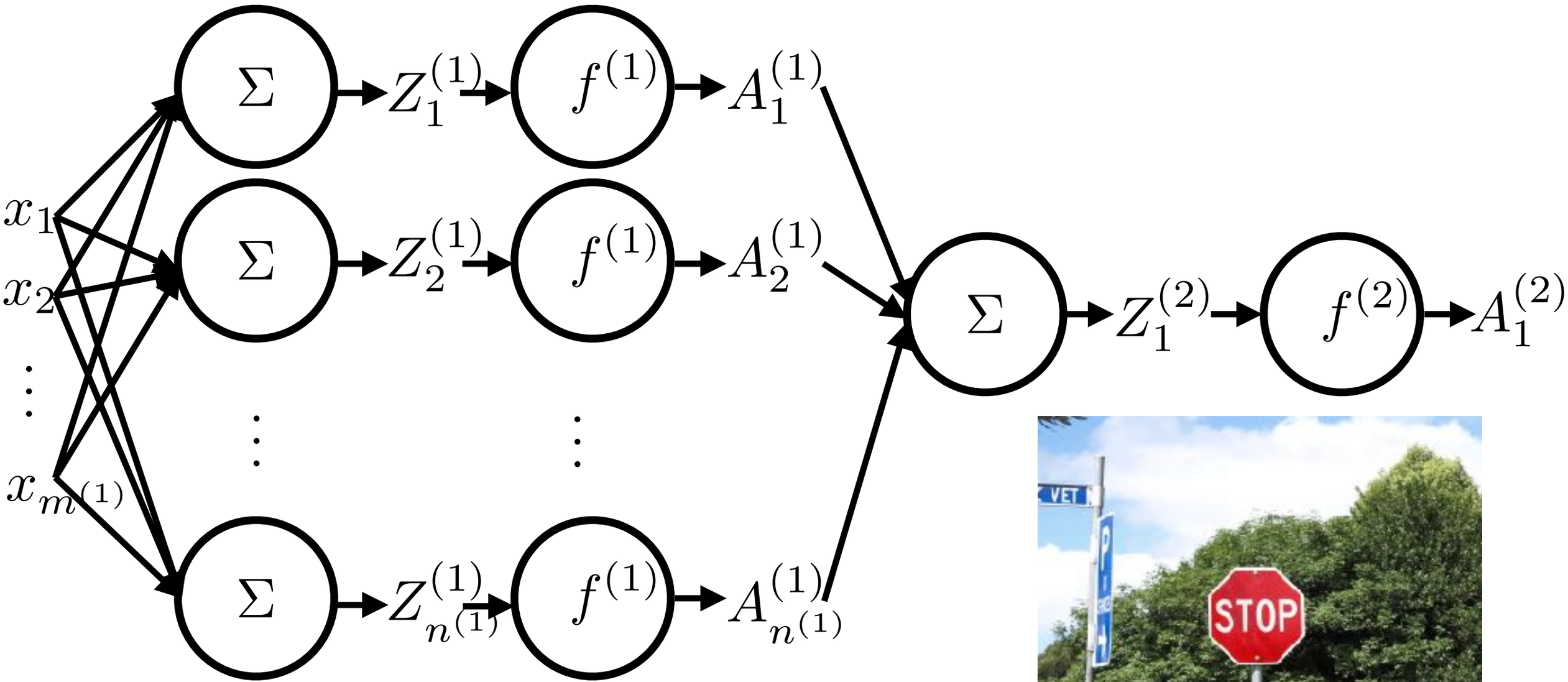

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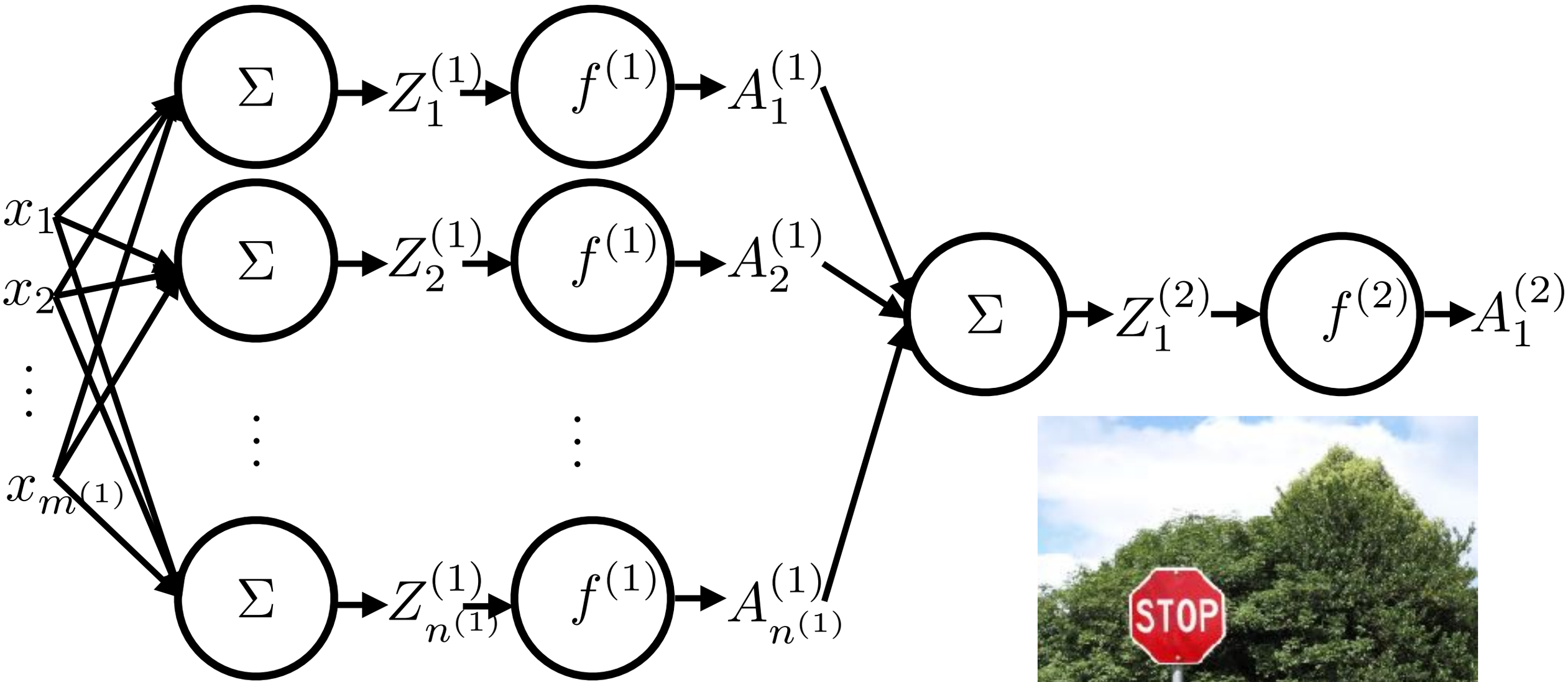

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Convolutional Layer: 1D example

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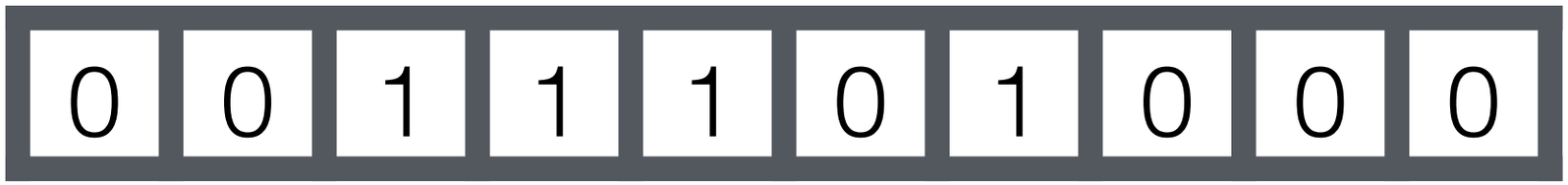

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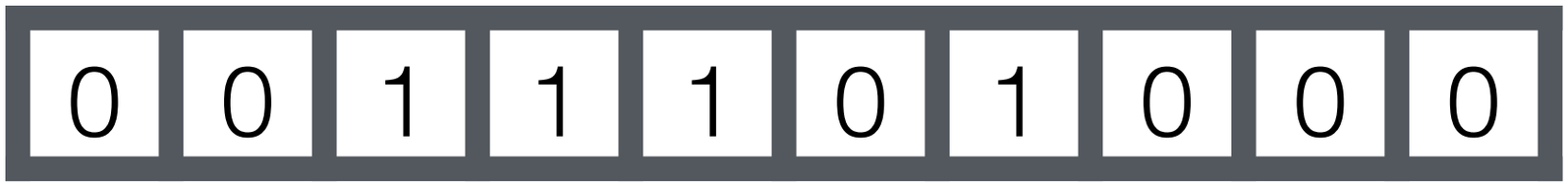

Letter | Published: 07 January 2019

Cardiologist-level arrhythmia detection and classification in ambulatory electrocardiograms using a deep neural network

Awni Y. Hannun ✉, Pranav Rajpurkar, Masoumeh Haghpanahi, Geoffrey H. Tison, Codie Bourn, Mintu P. Turakhia & Andrew Y. Ng

[https://www.nature.com/articles/s41591-018-0268-3]

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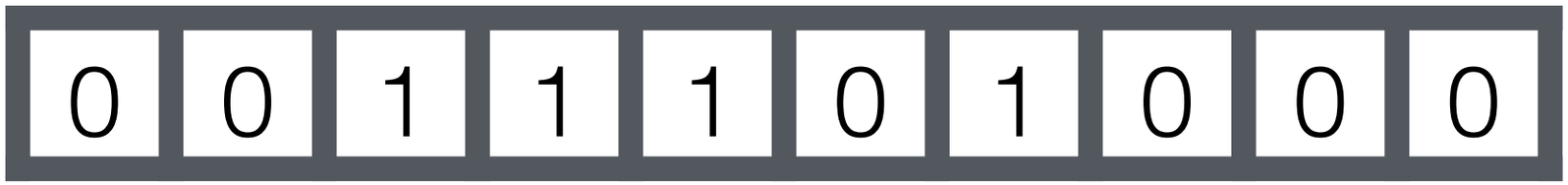

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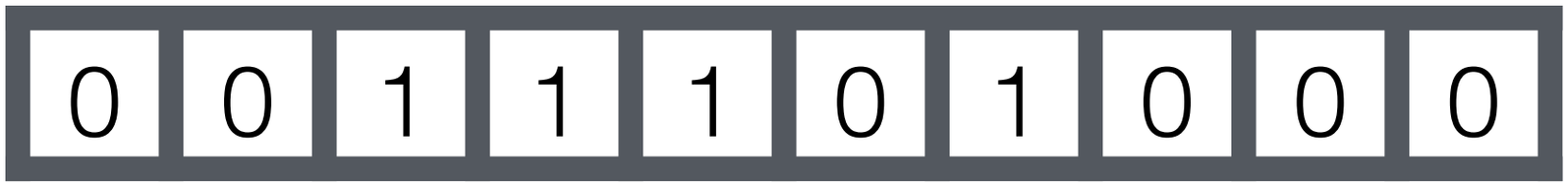

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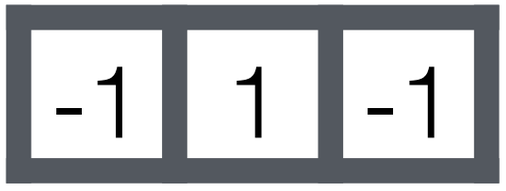

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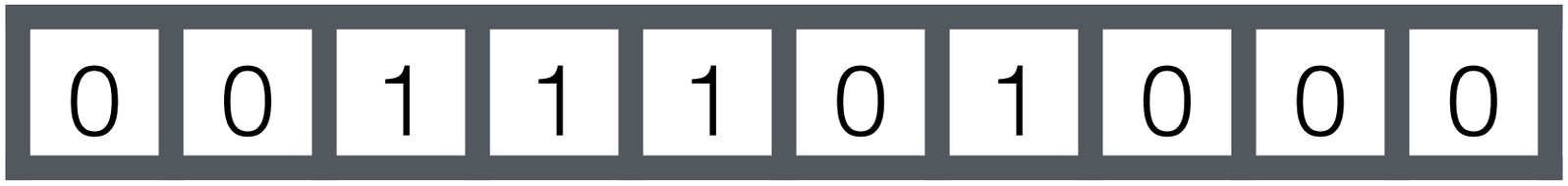

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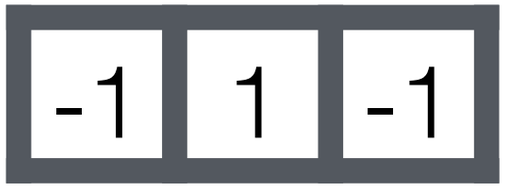

After convolution*:

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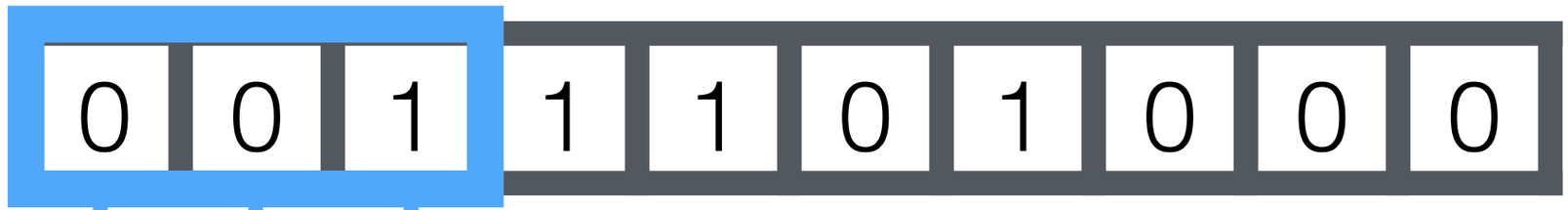

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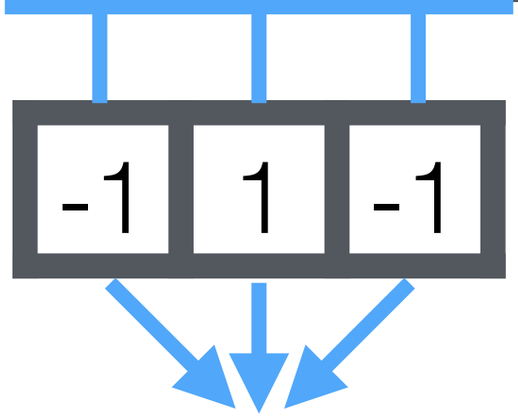

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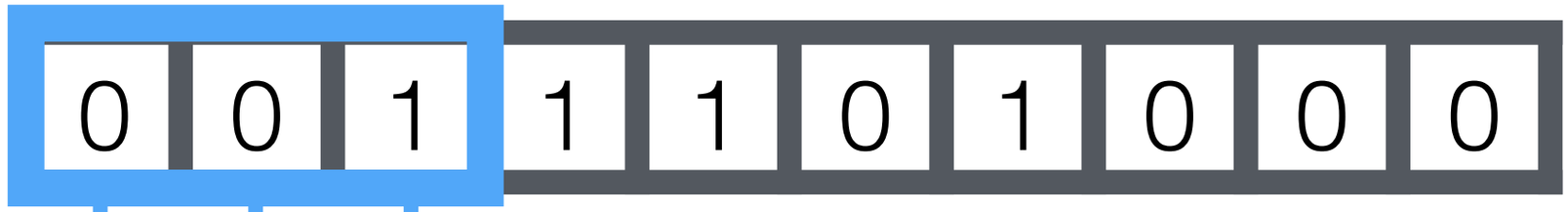

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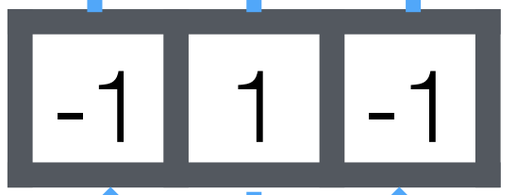

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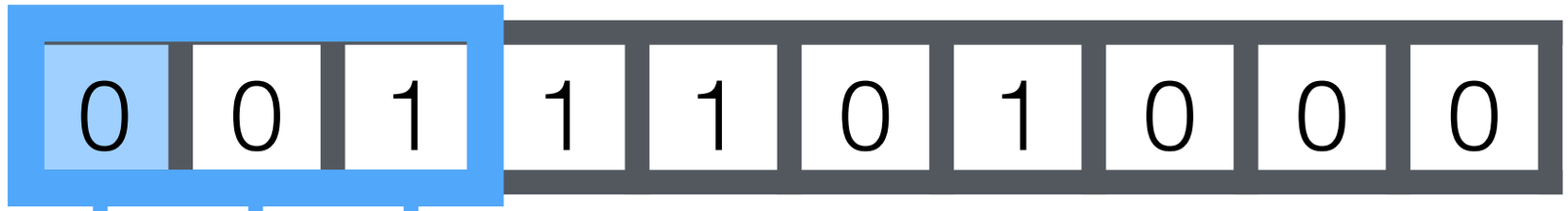

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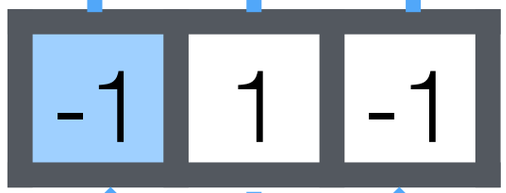

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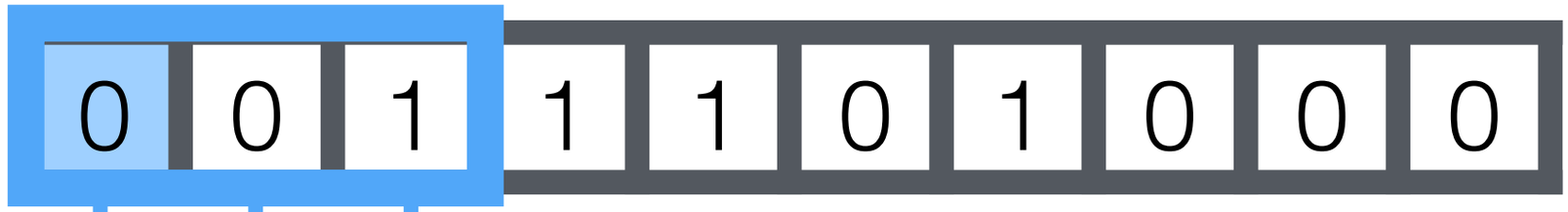

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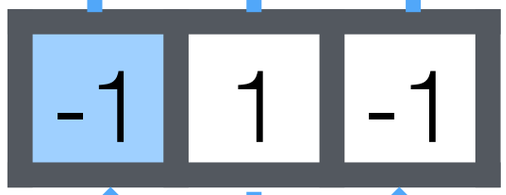

0 * -1

After convolution*:

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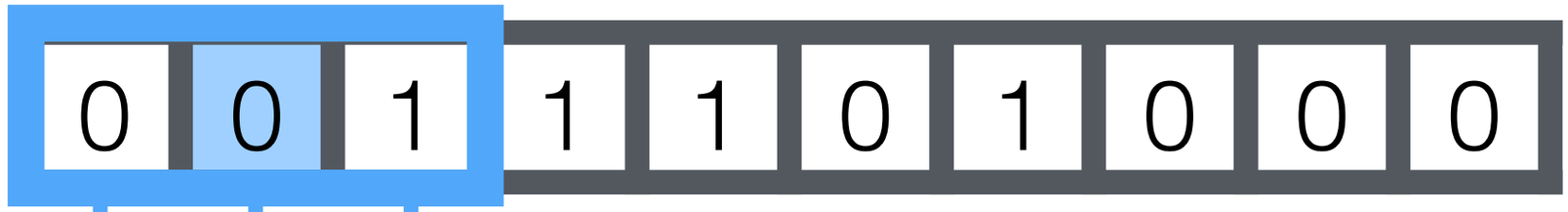

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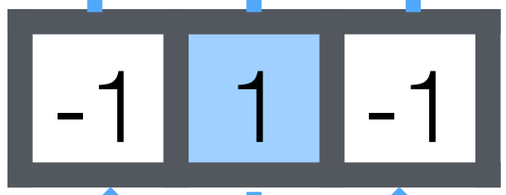

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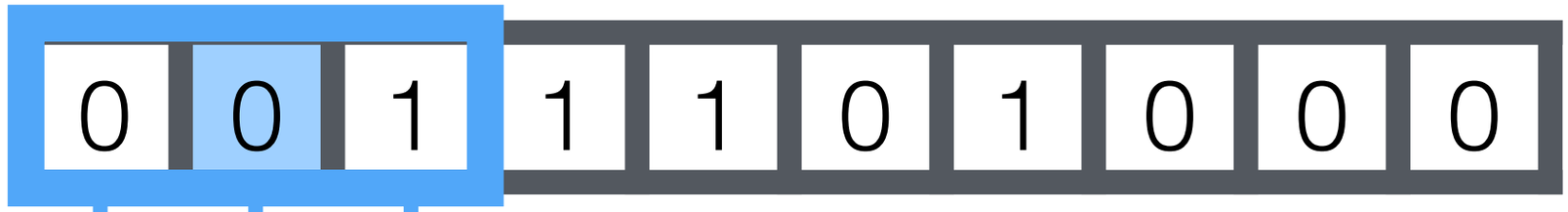

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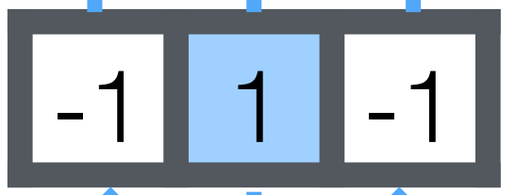

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Convolutional Layer: 1D example

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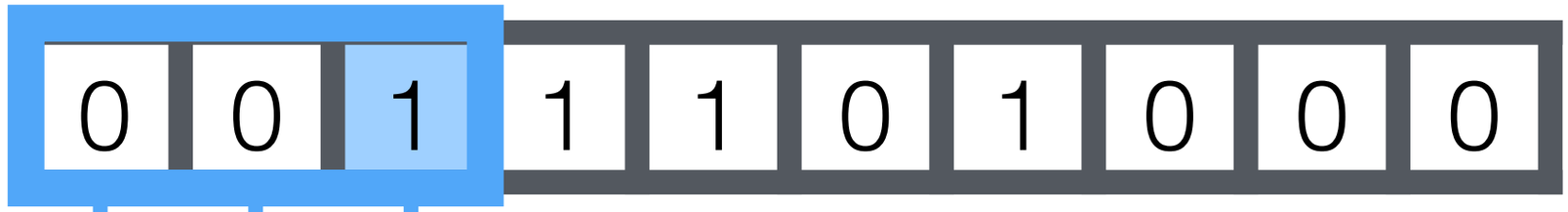

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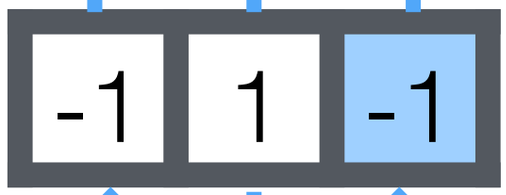

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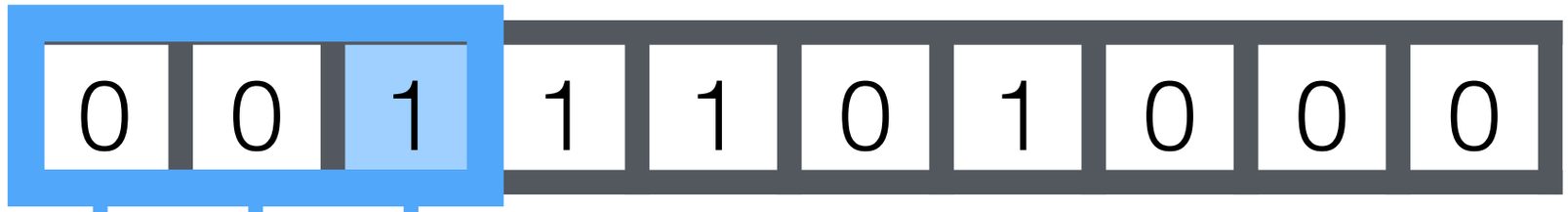
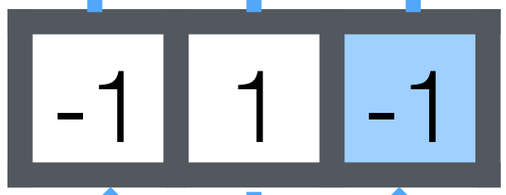
Convolutional Layer: 1D example

A 1D image:



A filter:

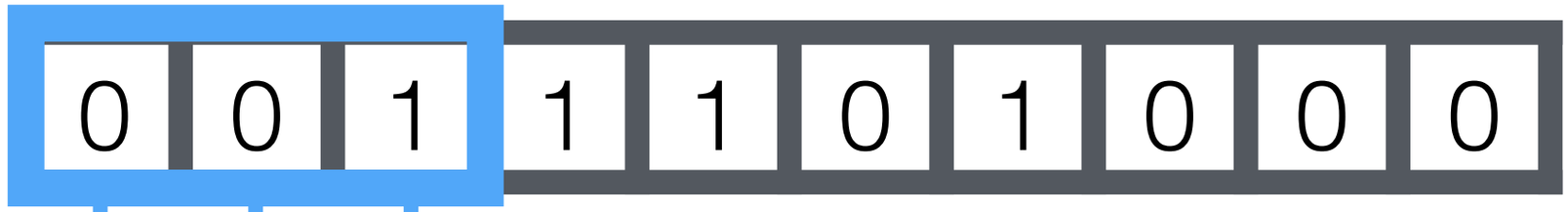


0 * -1 + 0 * 1 + 1 * -1

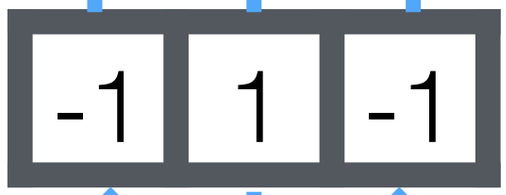
After convolution*:

Convolutional Layer: 1D example

A 1D image:



A filter:

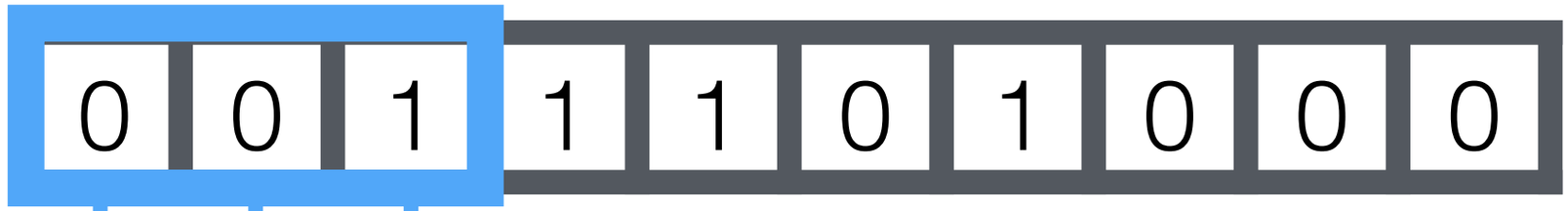


0 * -1 + 0 * 1 + 1 * -1

After convolution*:

Convolutional Layer: 1D example

A 1D image:



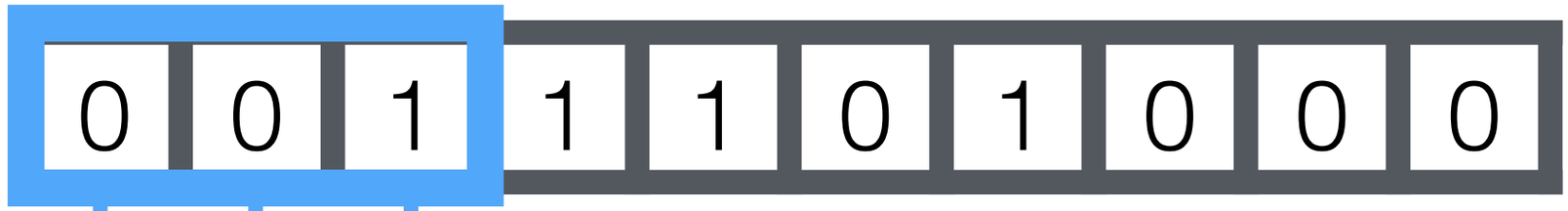
A filter:

0 * -1 + 0 * 1 + 1 * -1 = -1

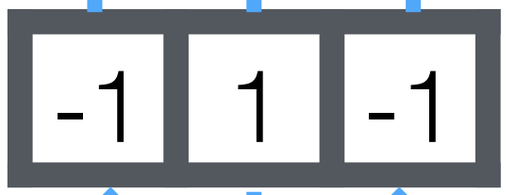
After convolution*:

Convolutional Layer: 1D example

A 1D image:

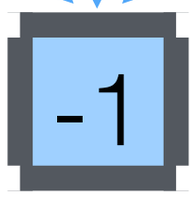


A filter:



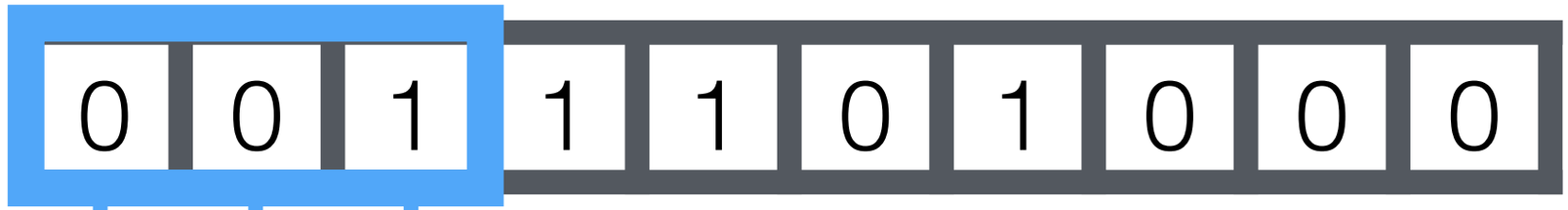
0 * -1 + 0 * 1 + 1 * -1 = -1

After convolution*:

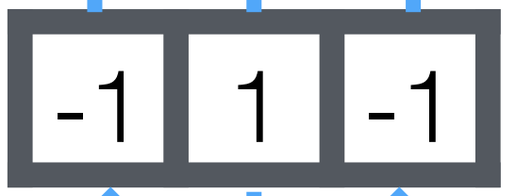


Convolutional Layer: 1D example

A 1D image:



A filter:



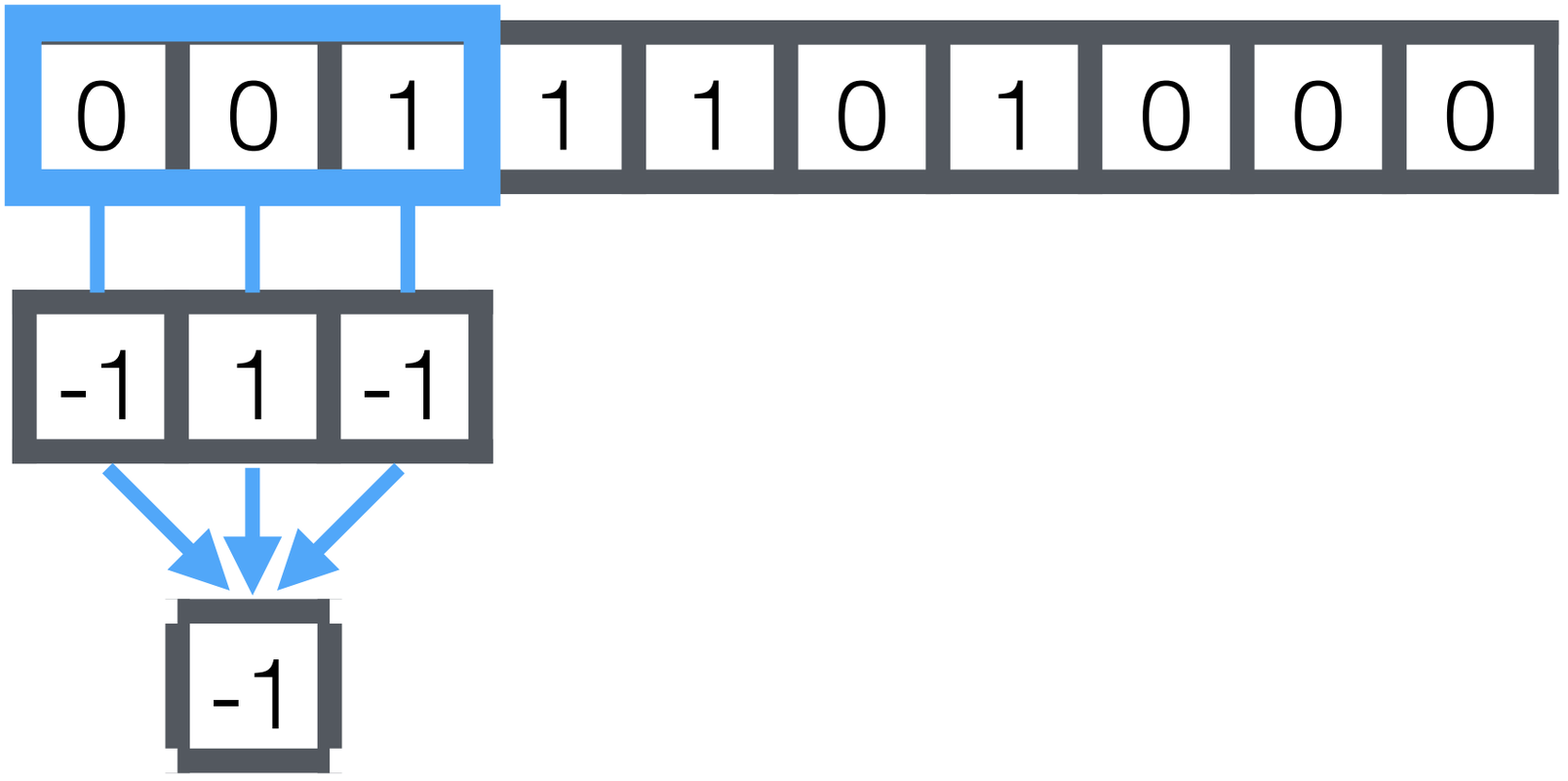
After convolution*:

Convolutional Layer: 1D example

A 1D image:

A filter:

After convolution*:



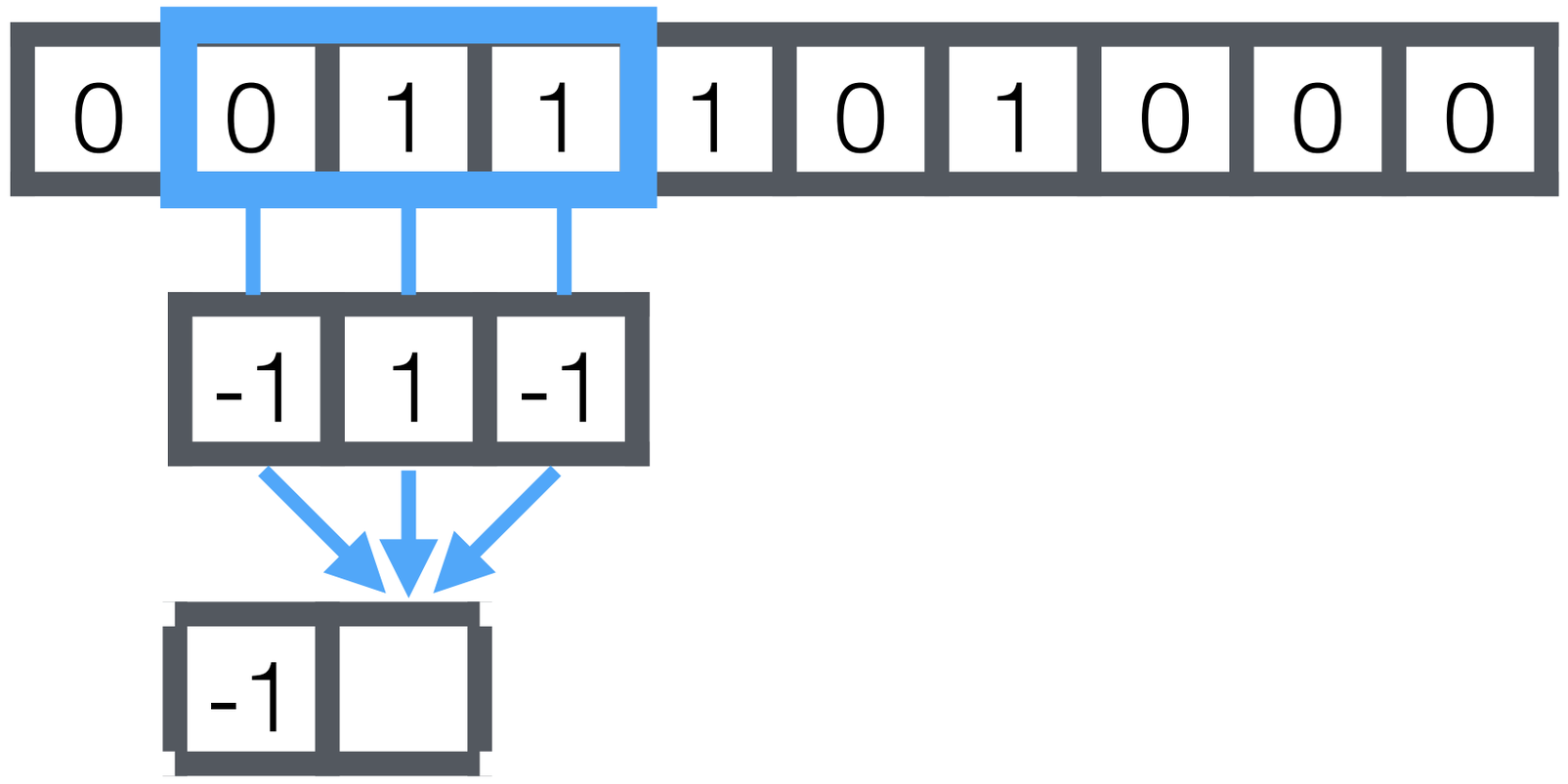
*correlation

Convolutional Layer: 1D example

A 1D image:

A filter:

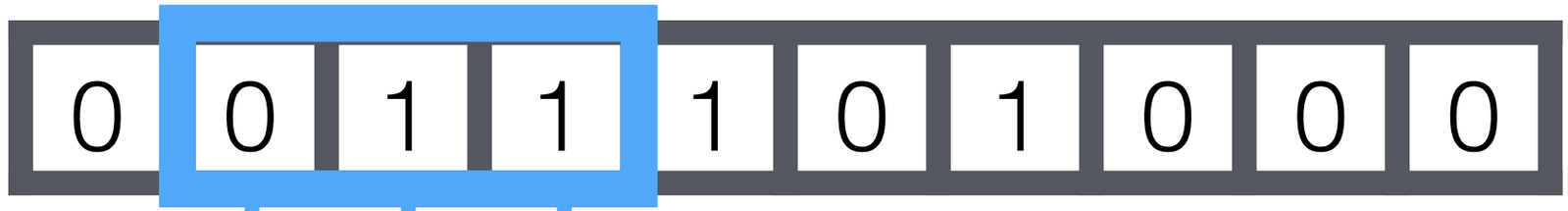
After convolution*:



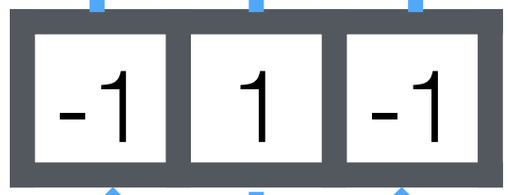
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:

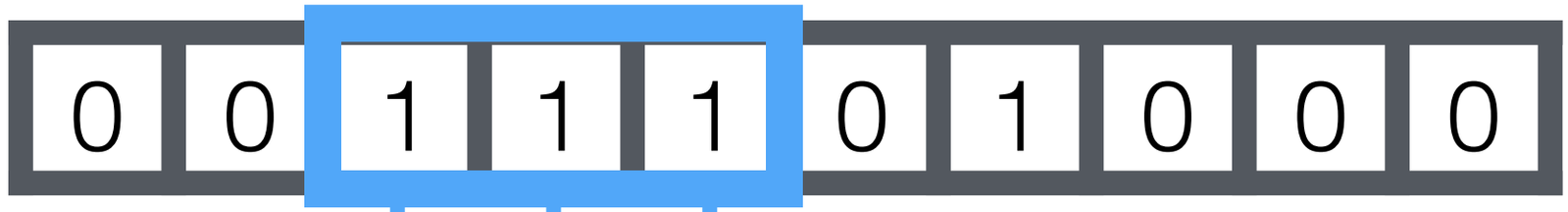


After convolution*:

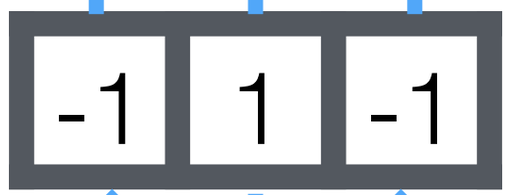
*correlation

Convolutional Layer: 1D example

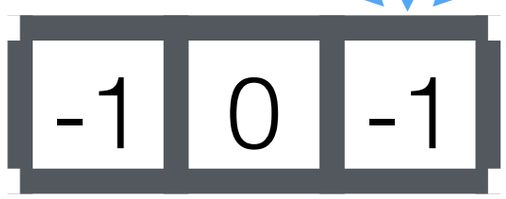
A 1D image:



A filter:



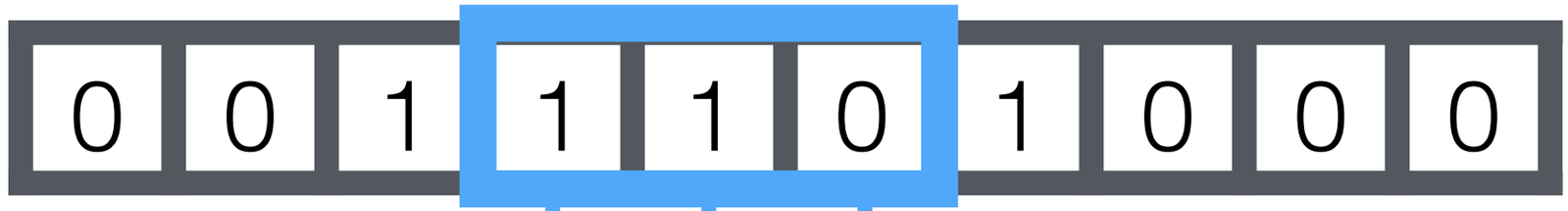
After convolution*:



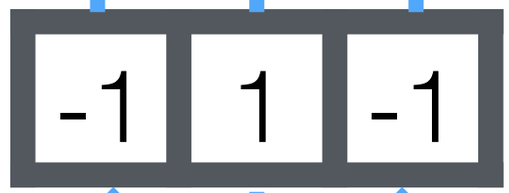
*correlation

Convolutional Layer: 1D example

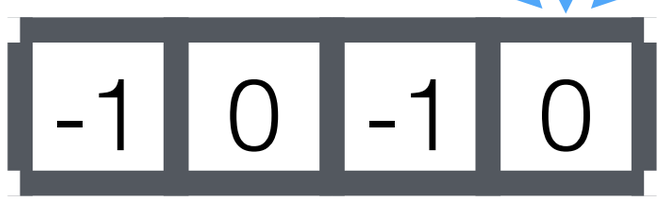
A 1D image:



A filter:



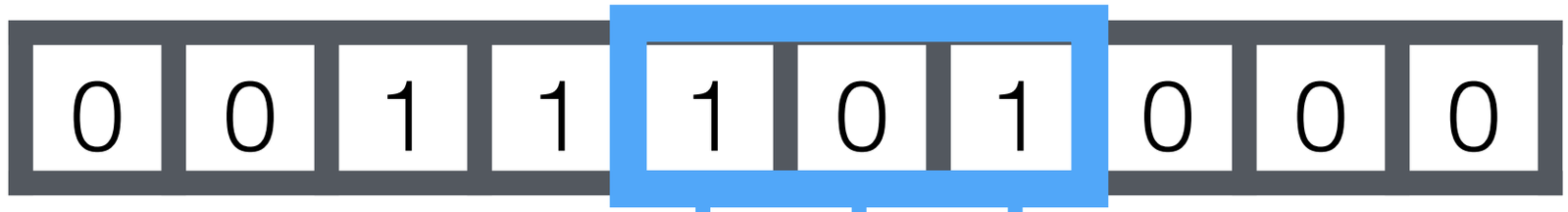
After convolution*:



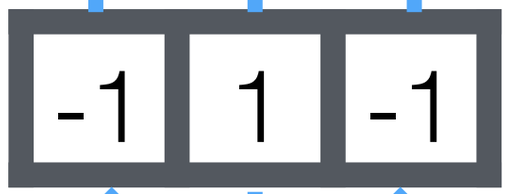
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:

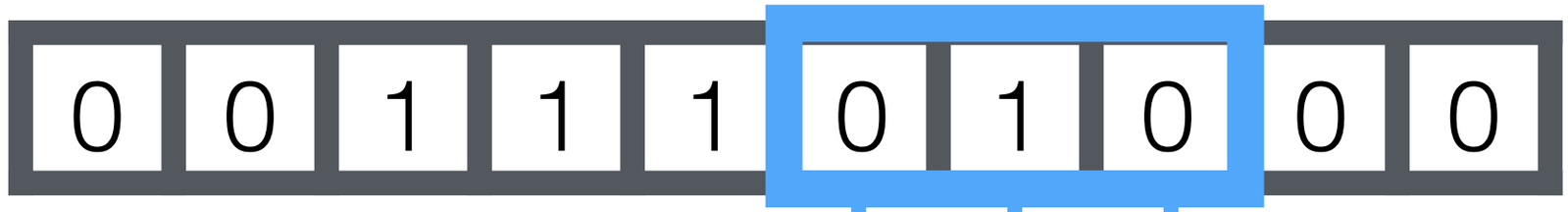


After convolution*:

*correlation

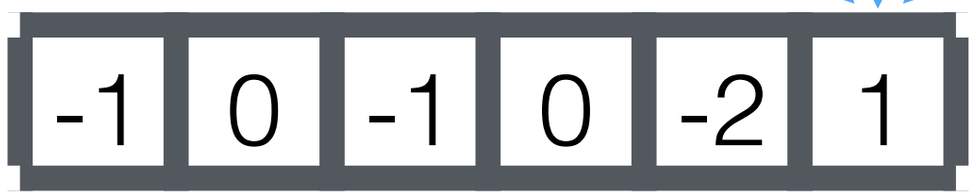
Convolutional Layer: 1D example

A 1D image:



A filter:

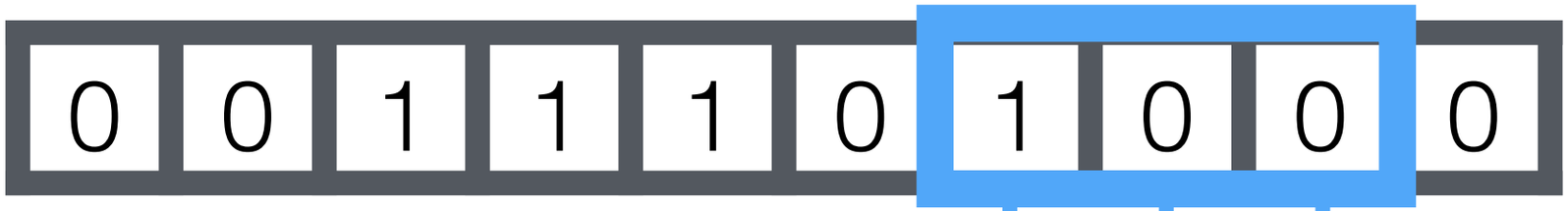
After convolution*:



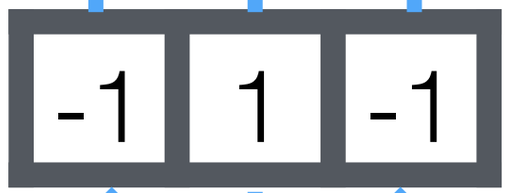
*correlation

Convolutional Layer: 1D example

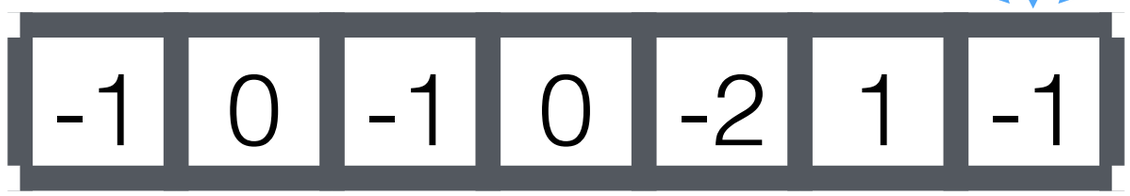
A 1D image:



A filter:



After convolution*:



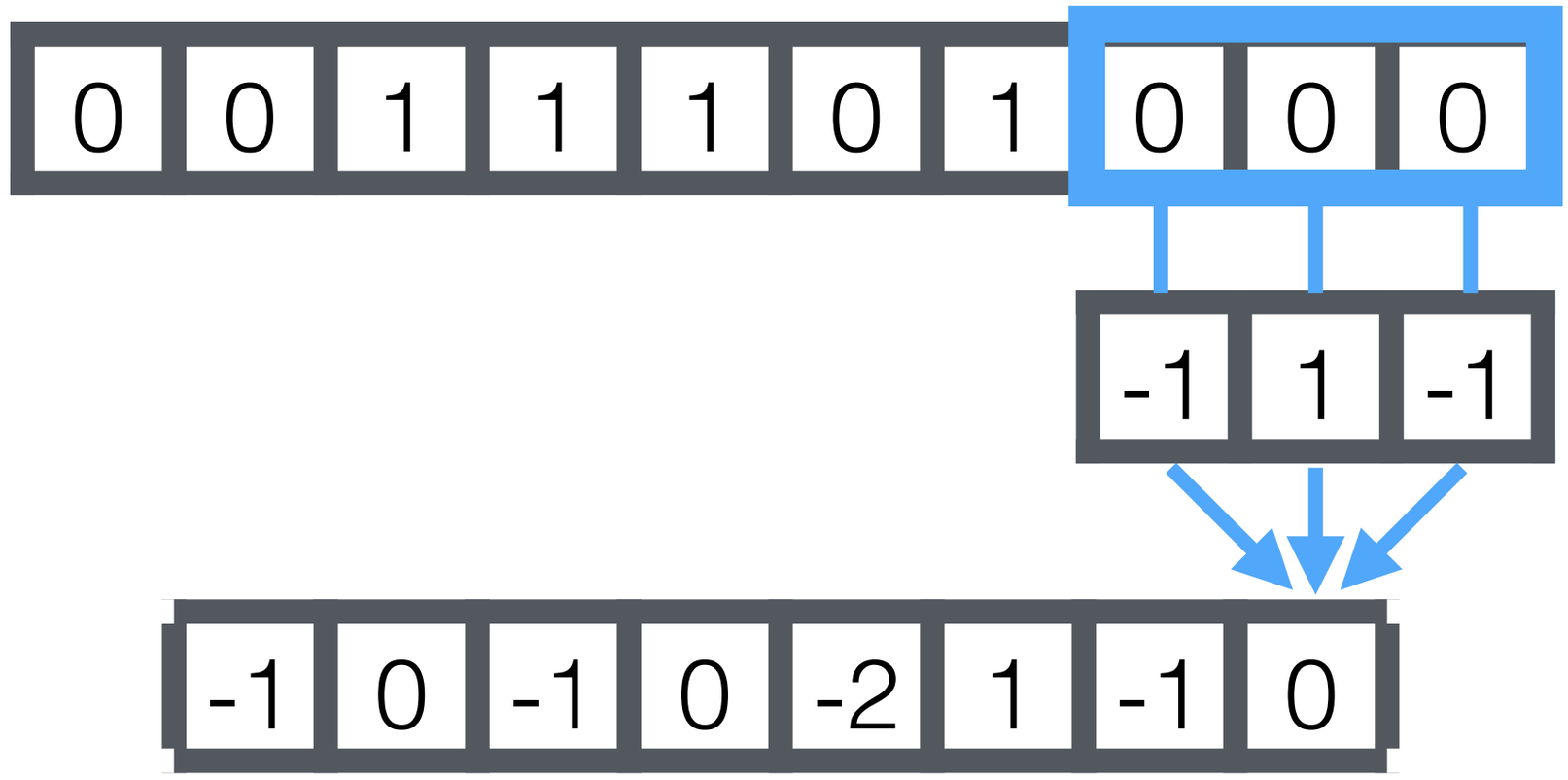
*correlation

Convolutional Layer: 1D example

A 1D image:

A filter:

After convolution*:

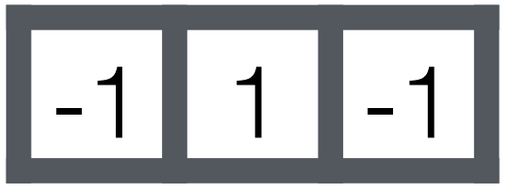


*correlation

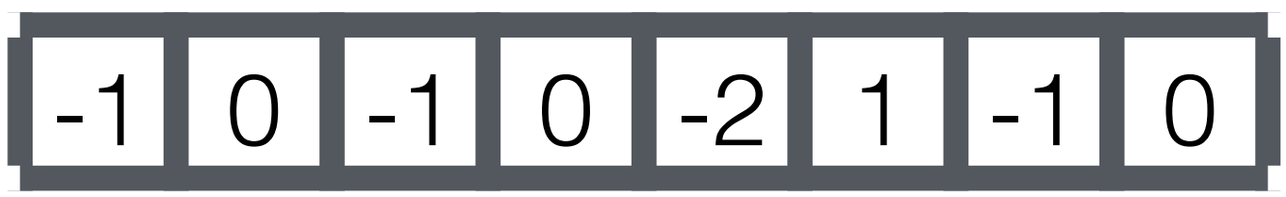
Convolutional Layer: 1D example

A 1D image:

A filter:



After convolution*:

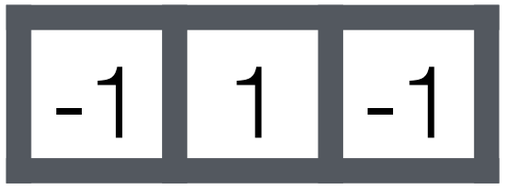


*correlation

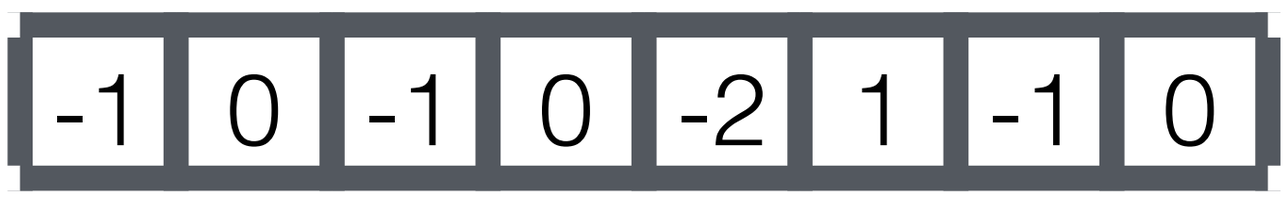
Convolutional Layer: 1D example

A 1D image:

A filter:



After convolution*:

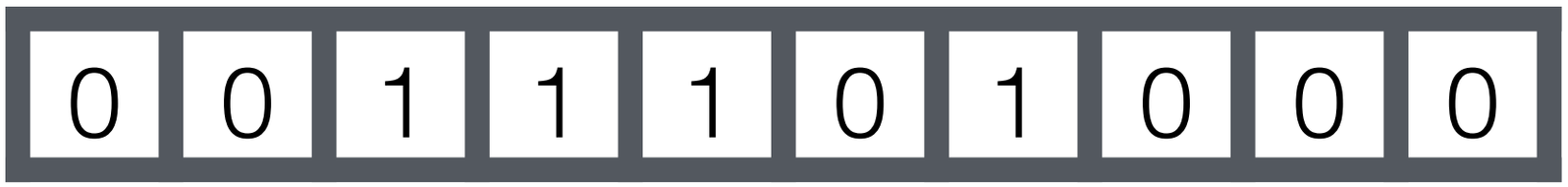


After ReLU:

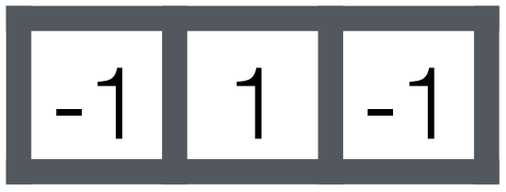
*correlation

Convolutional Layer: 1D example

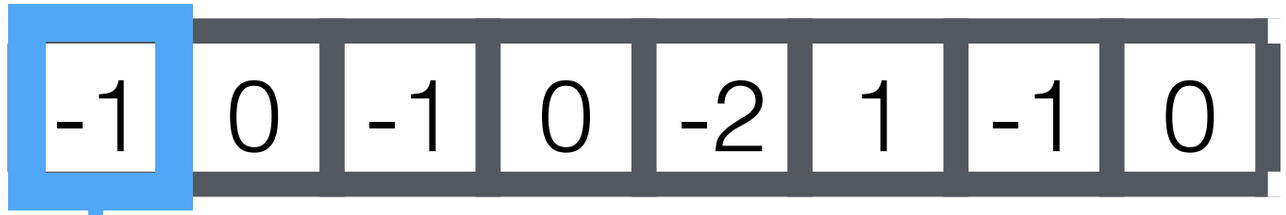
A 1D image:



A filter:



After convolution*:

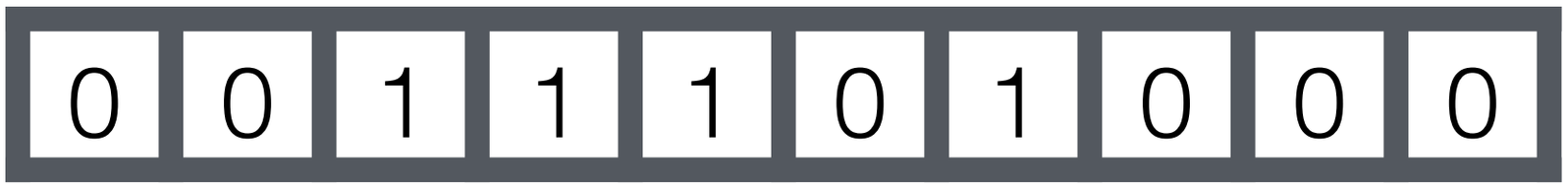


After ReLU:

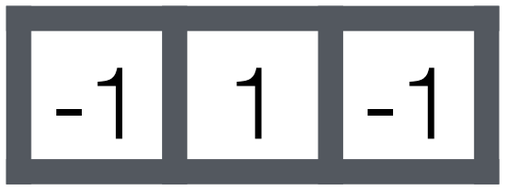
*correlation

Convolutional Layer: 1D example

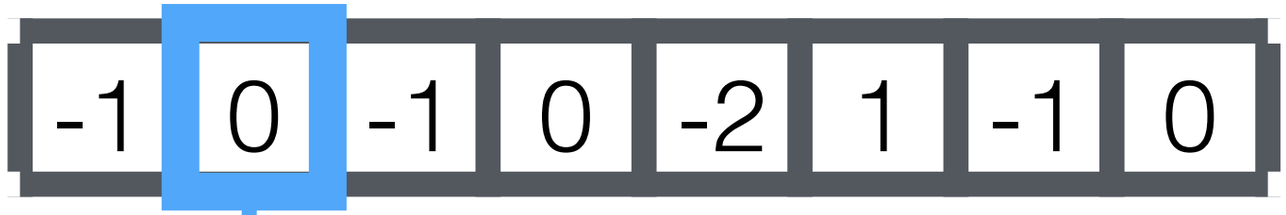
A 1D image:



A filter:



After convolution*:

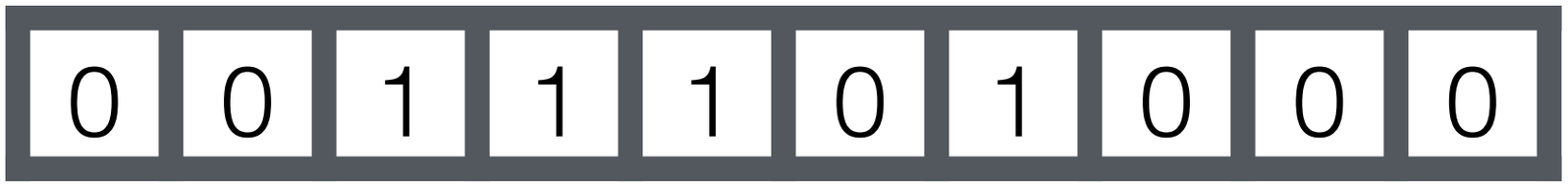


After ReLU:

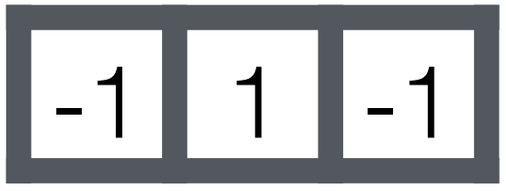
*correlation

Convolutional Layer: 1D example

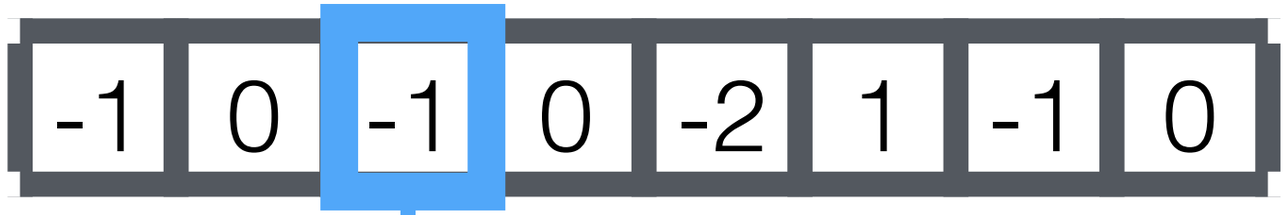
A 1D image:



A filter:



After convolution*:

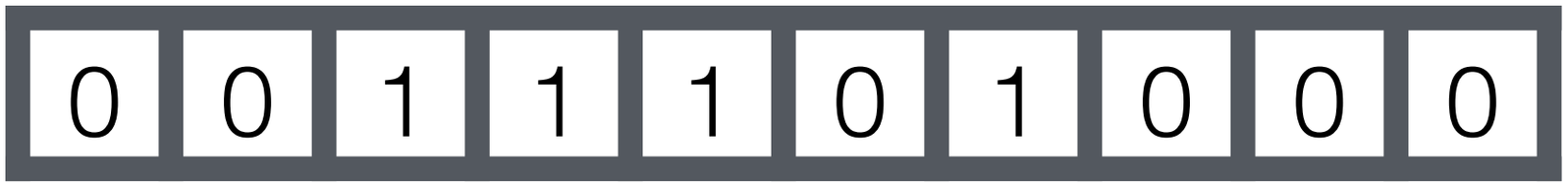


After ReLU:

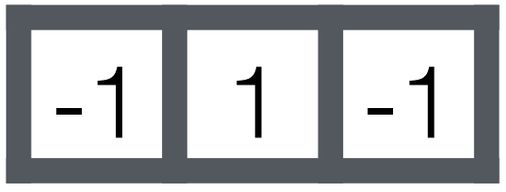
*correlation

Convolutional Layer: 1D example

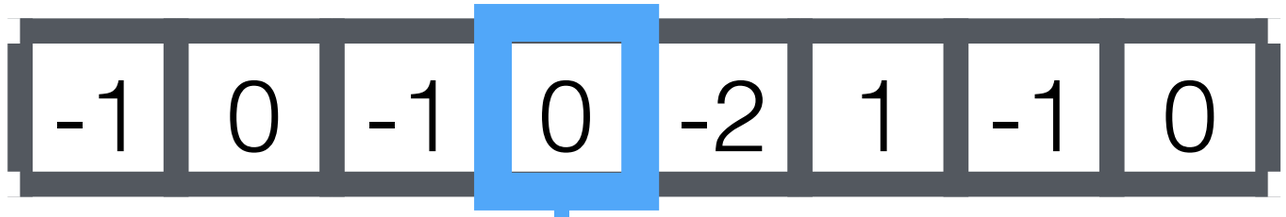
A 1D image:



A filter:



After convolution*:

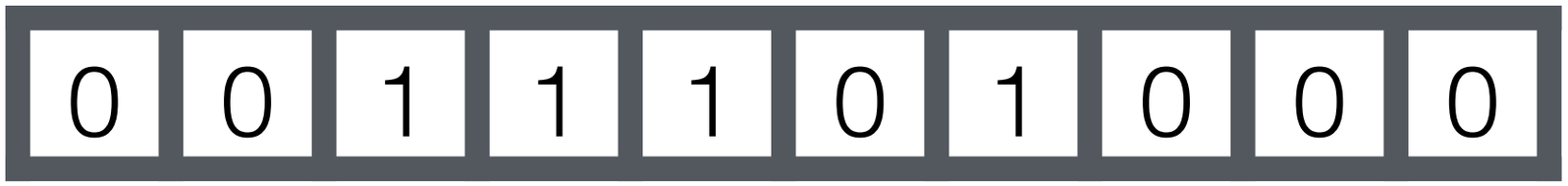


After ReLU:

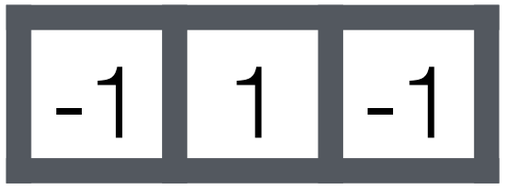
*correlation

Convolutional Layer: 1D example

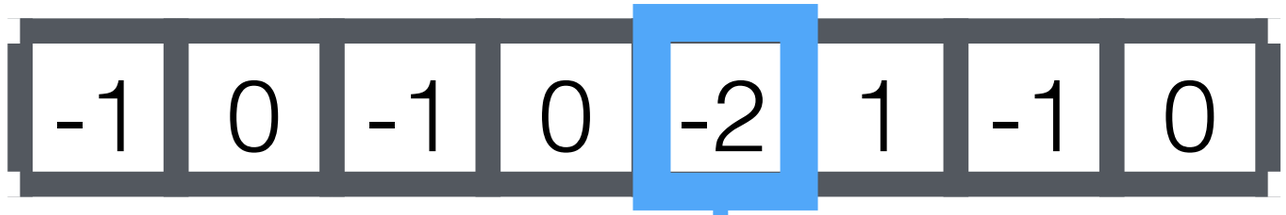
A 1D image:



A filter:



After convolution*:

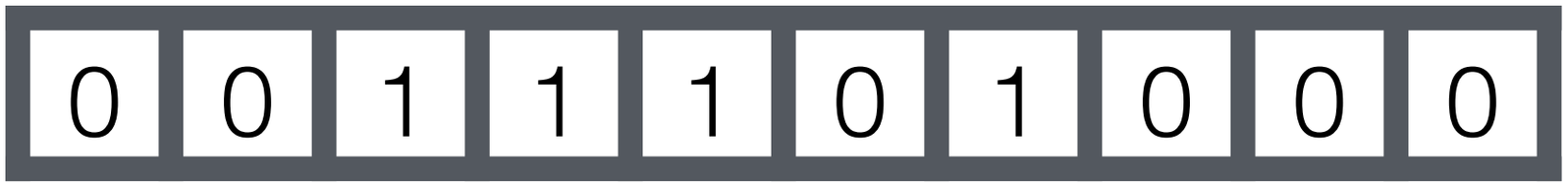


After ReLU:

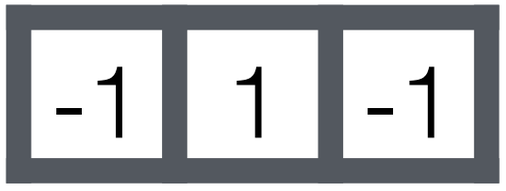
*correlation

Convolutional Layer: 1D example

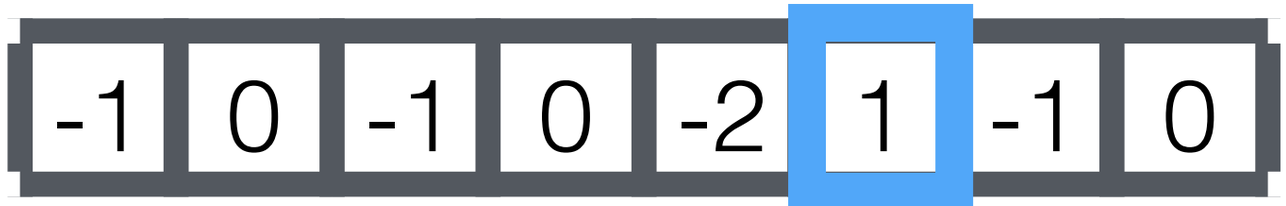
A 1D image:



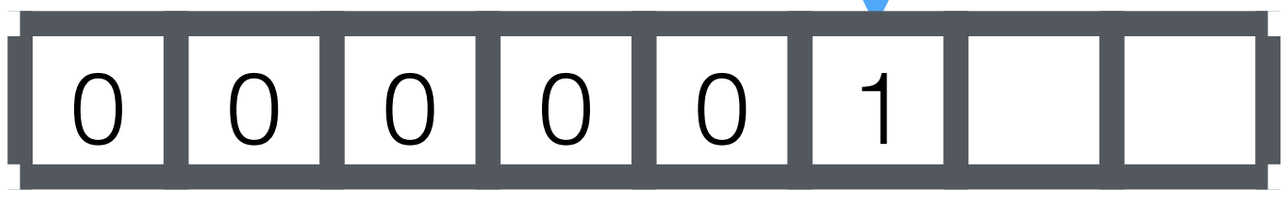
A filter:



After convolution*:



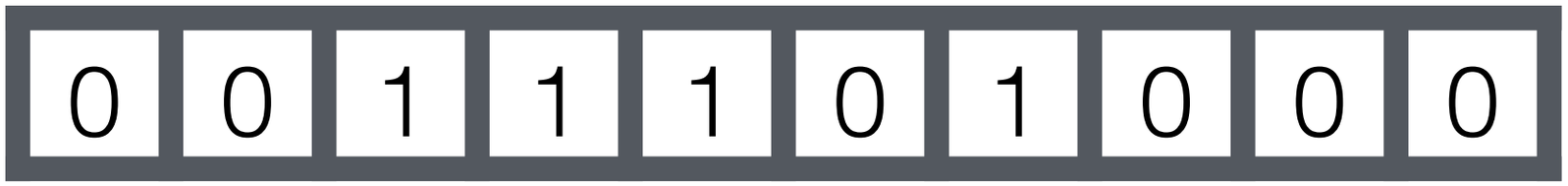
After ReLU:



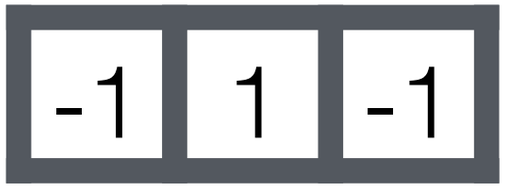
*correlation

Convolutional Layer: 1D example

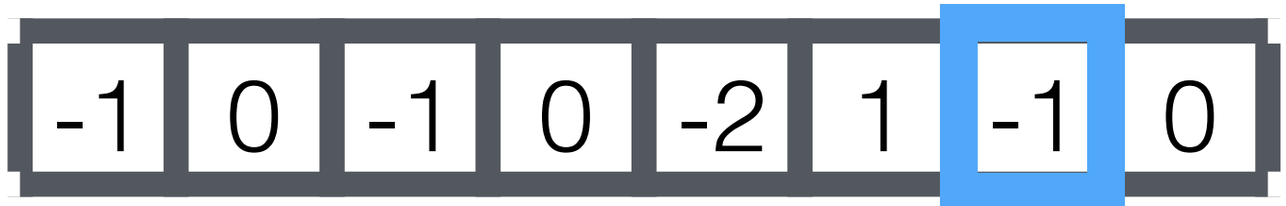
A 1D image:



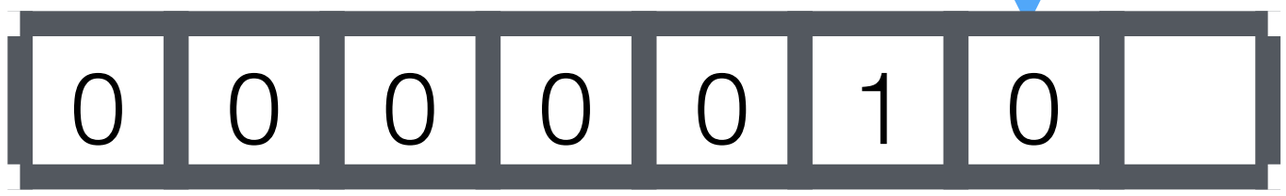
A filter:



After convolution*:



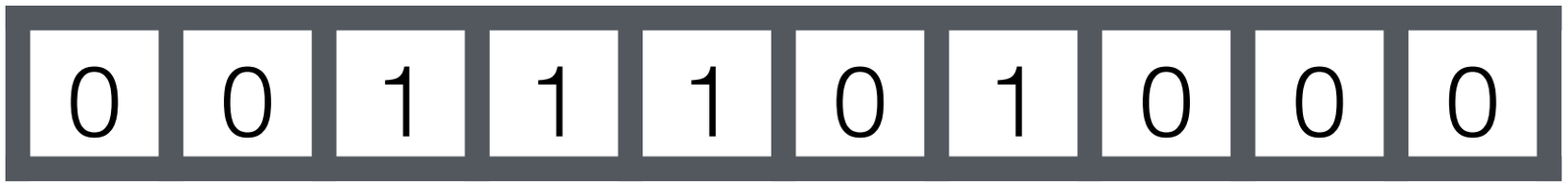
After ReLU:



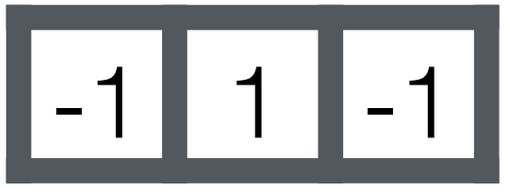
*correlation

Convolutional Layer: 1D example

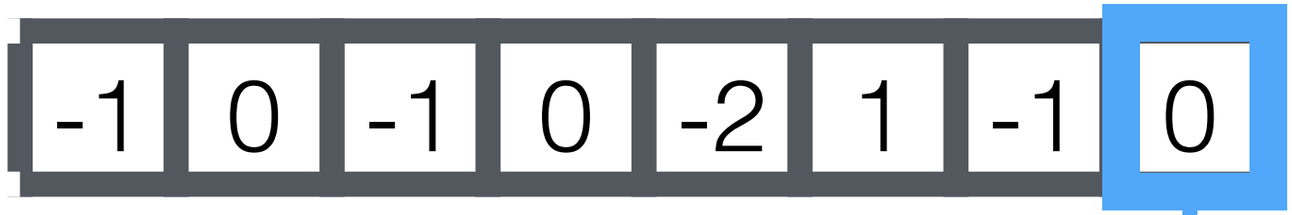
A 1D image:



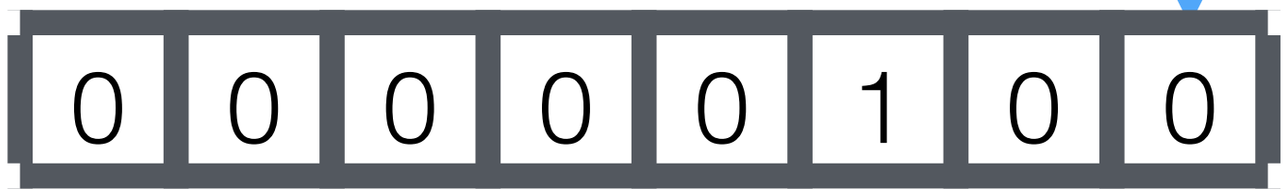
A filter:



After convolution*:



After ReLU:

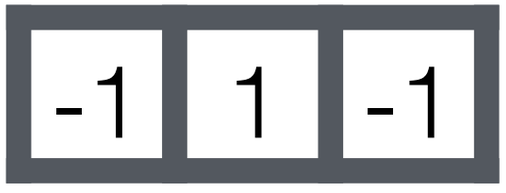


*correlation

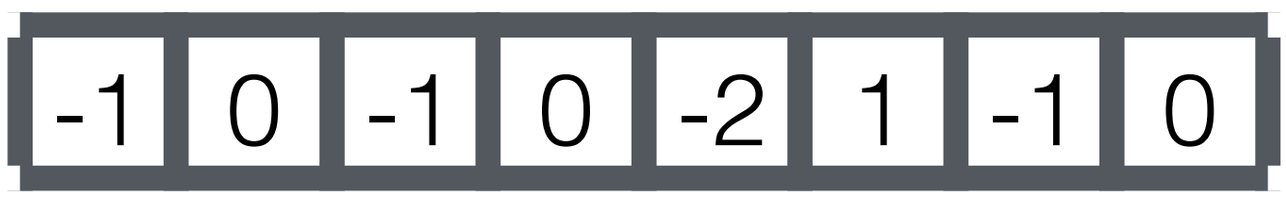
Convolutional Layer: 1D example

A 1D image:

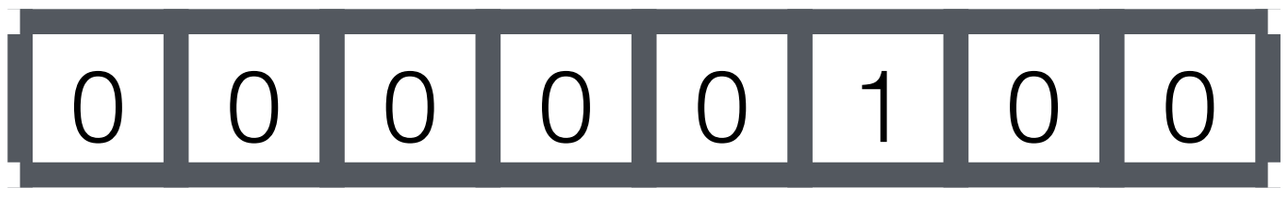
A filter:



After convolution*:



After ReLU:

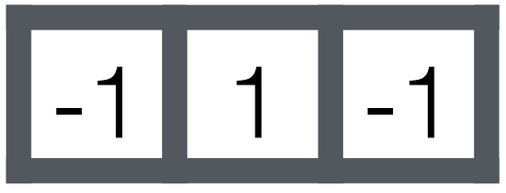


*correlation

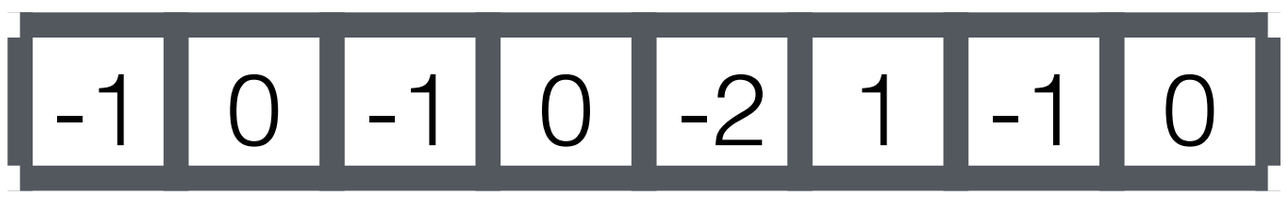
Convolutional Layer: 1D example

A 1D image:

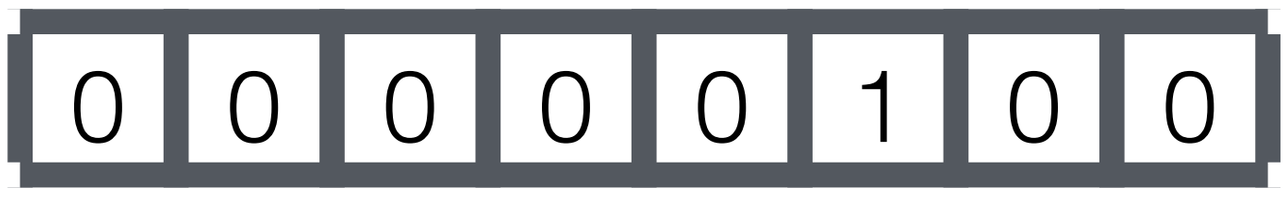
A filter:



After convolution*:



After ReLU:



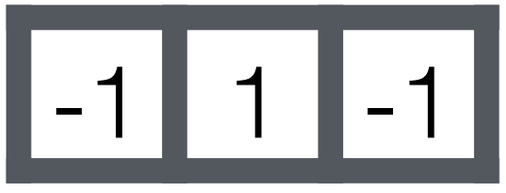
What does the filter do?

*correlation

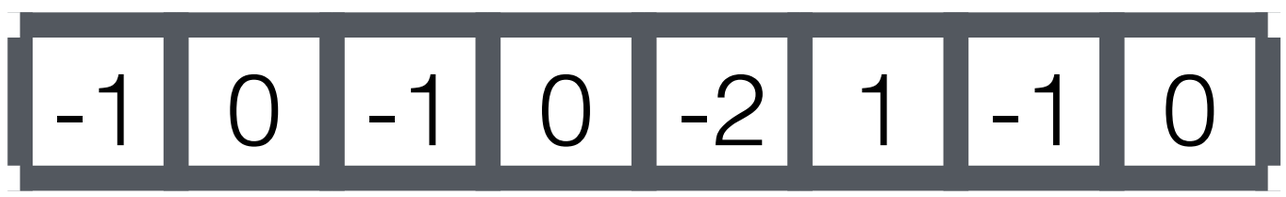
Convolutional Layer: 1D example

A 1D image:

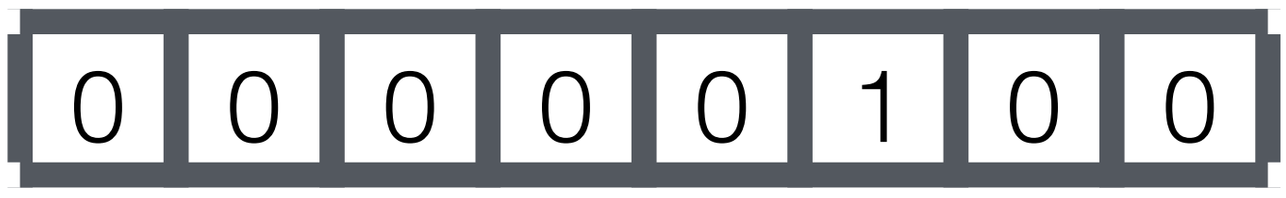
A filter:



After convolution*:



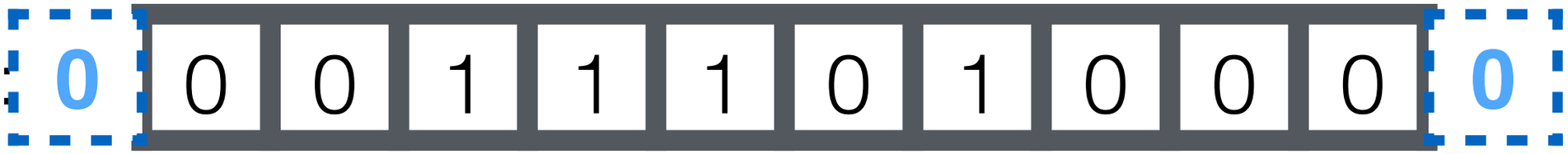
After ReLU:



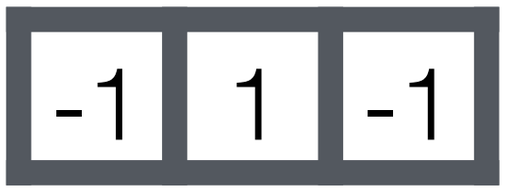
*correlation

Convolutional Layer: 1D example

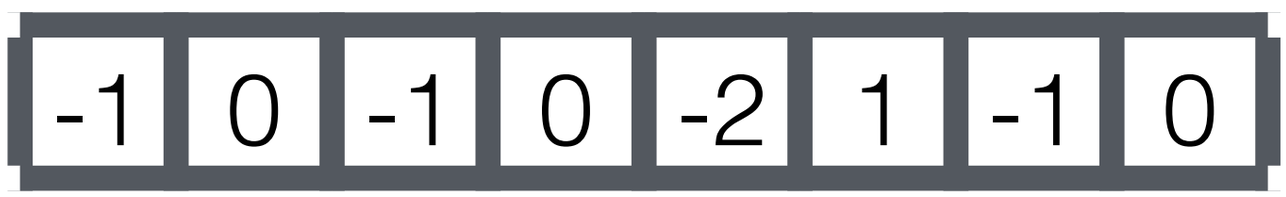
A 1D image:



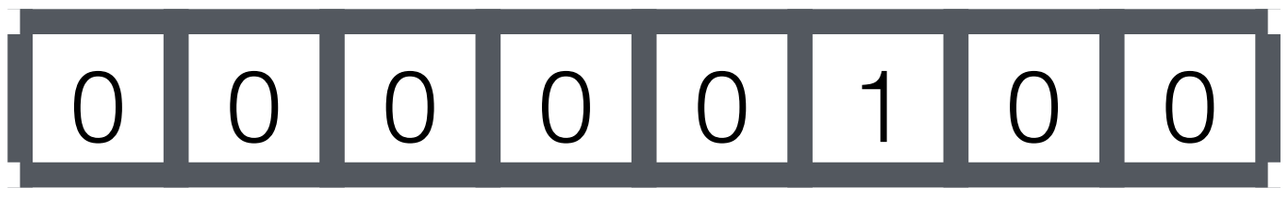
A filter:



After convolution*:

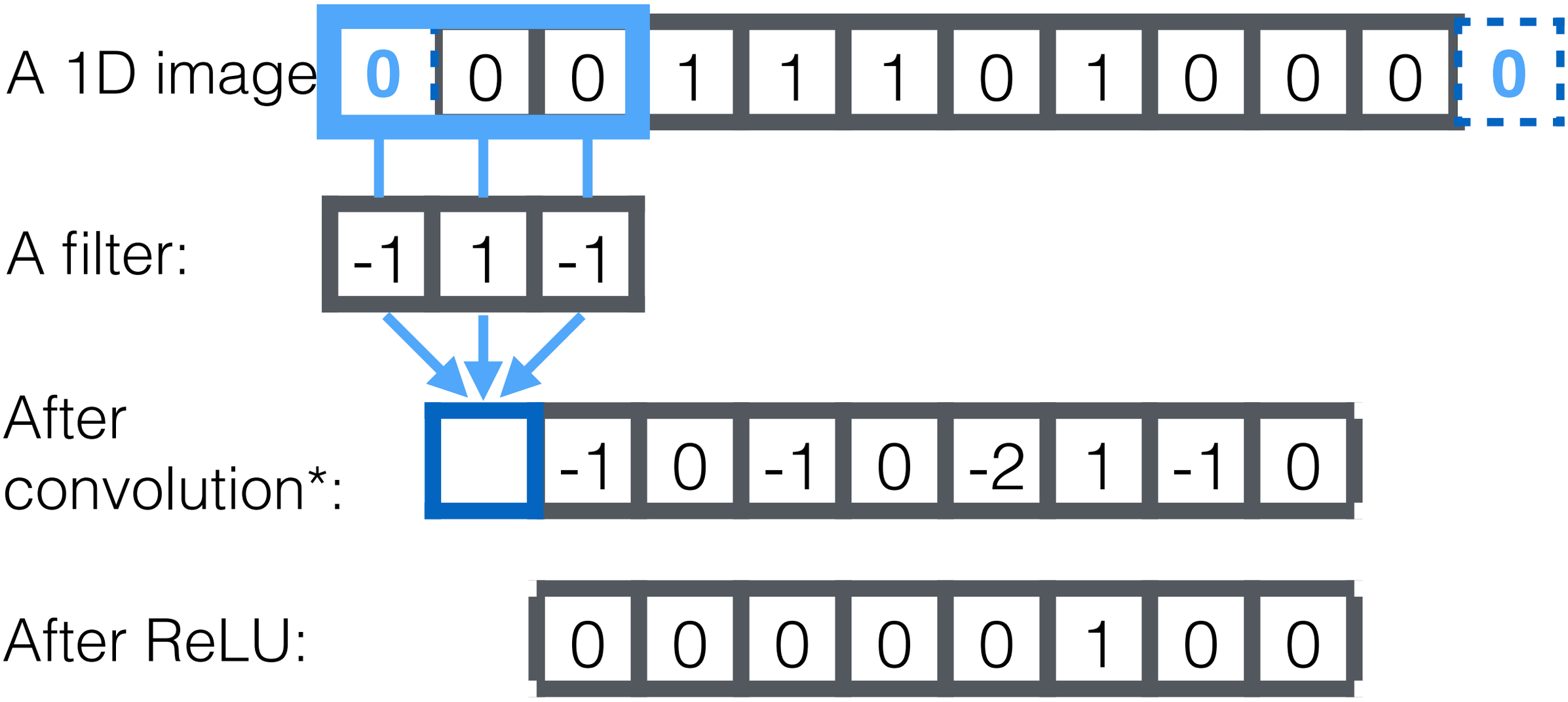


After ReLU:



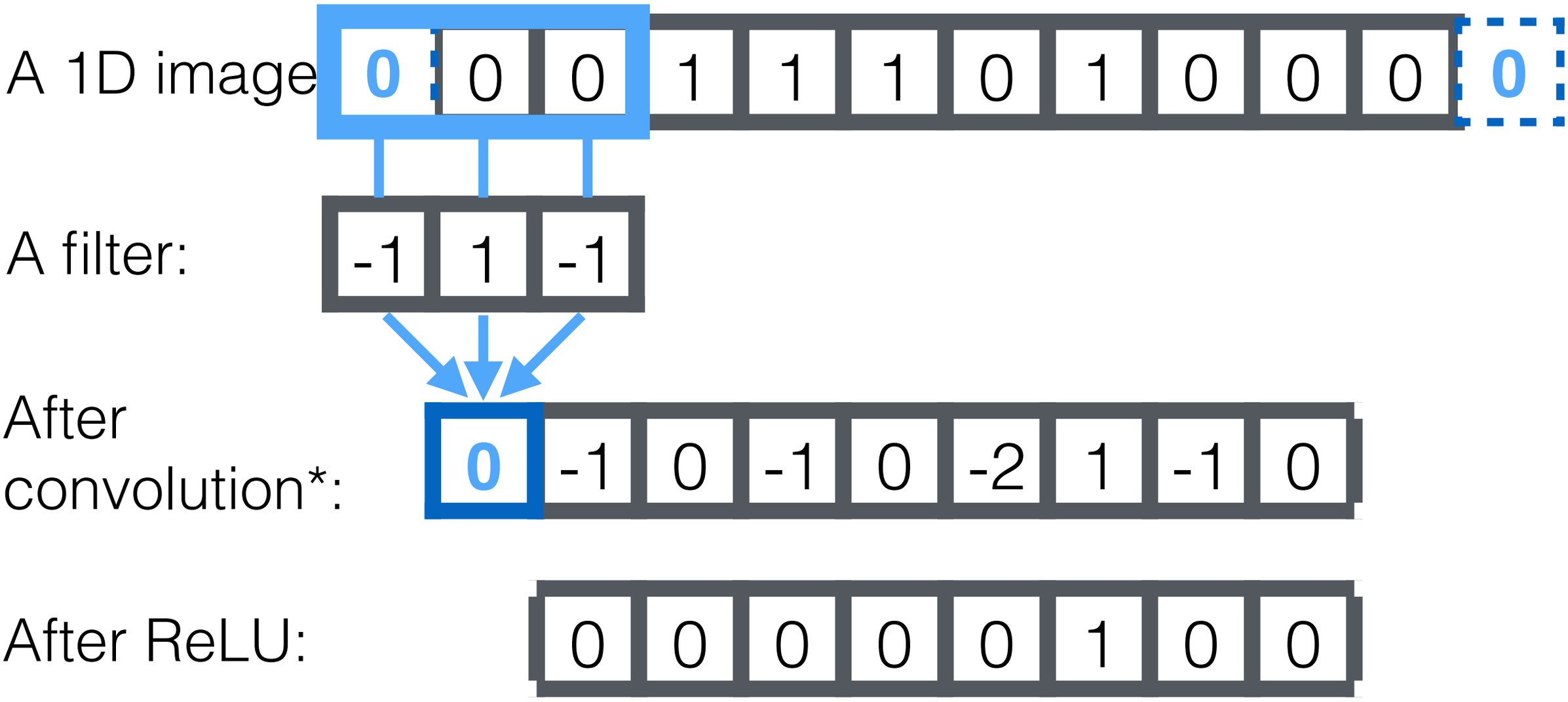
*correlation

Convolutional Layer: 1D example



*correlation

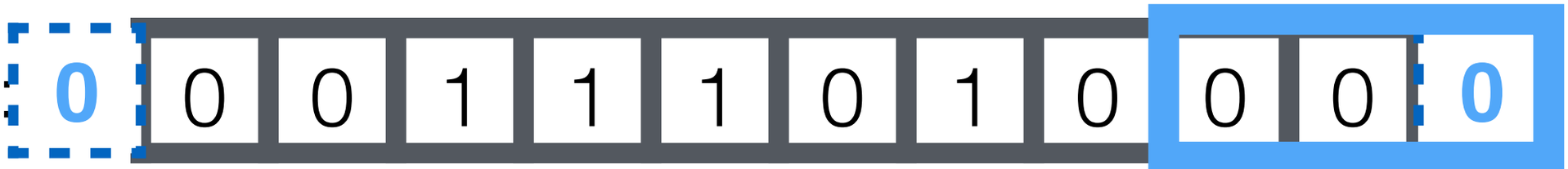
Convolutional Layer: 1D example



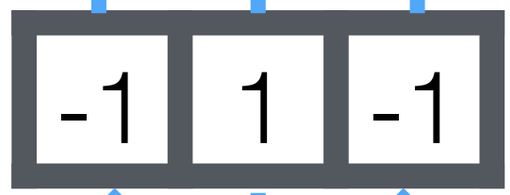
*correlation

Convolutional Layer: 1D example

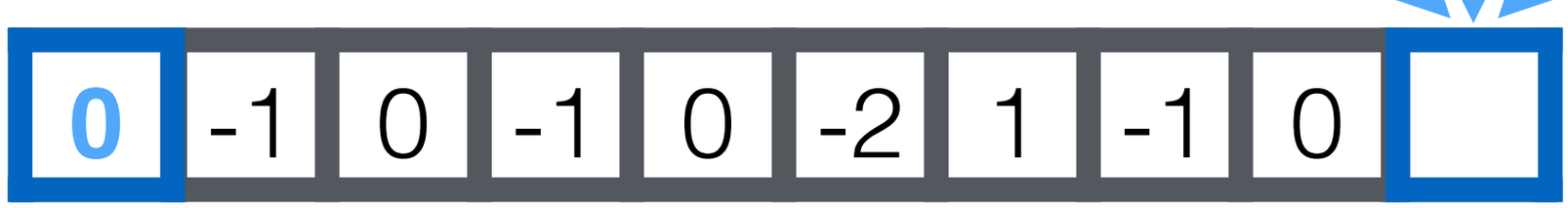
A 1D image:



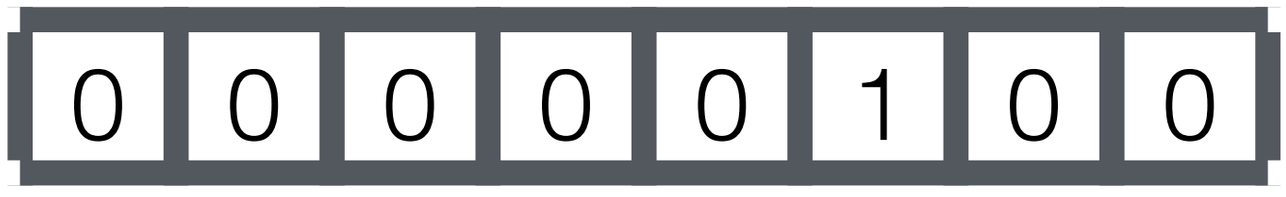
A filter:



After convolution*:



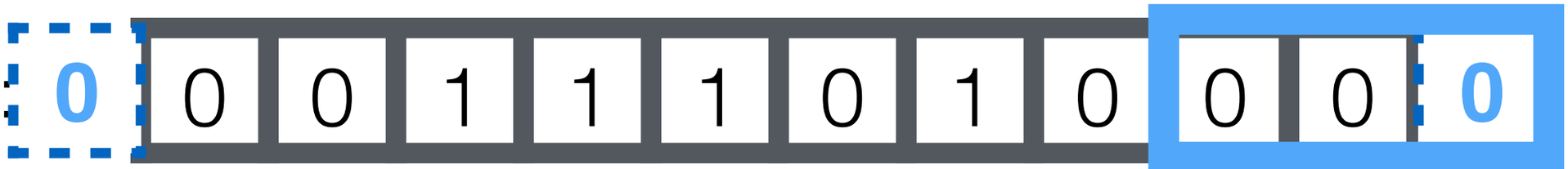
After ReLU:



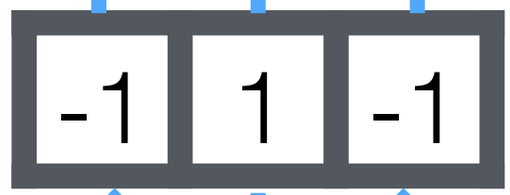
*correlation

Convolutional Layer: 1D example

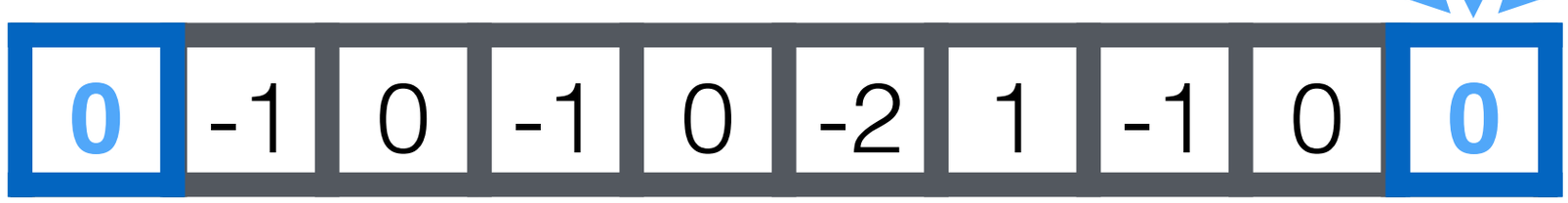
A 1D image:



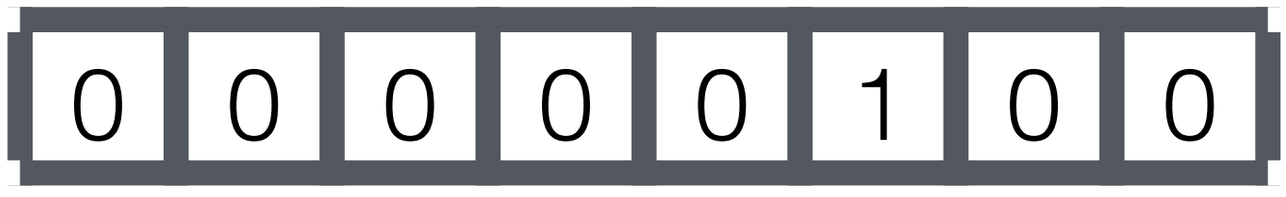
A filter:



After convolution*:



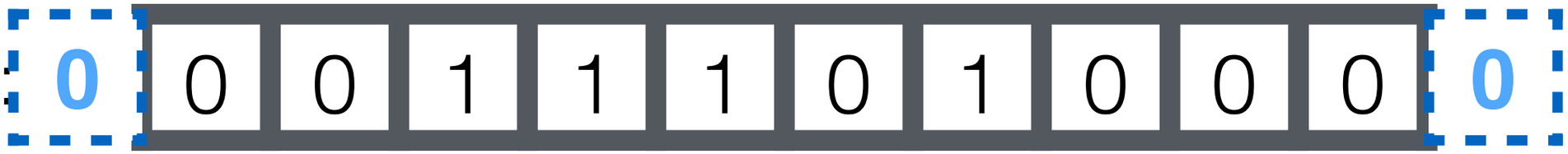
After ReLU:



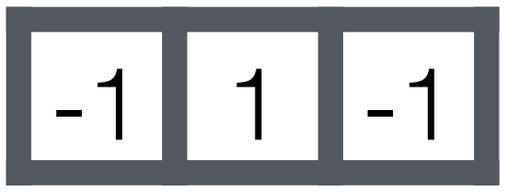
*correlation

Convolutional Layer: 1D example

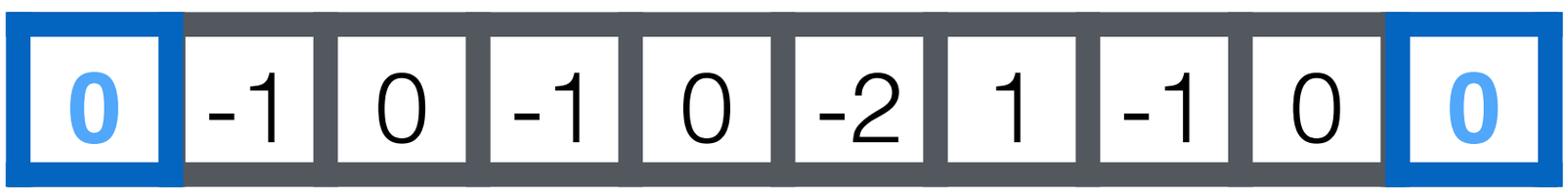
A 1D image:



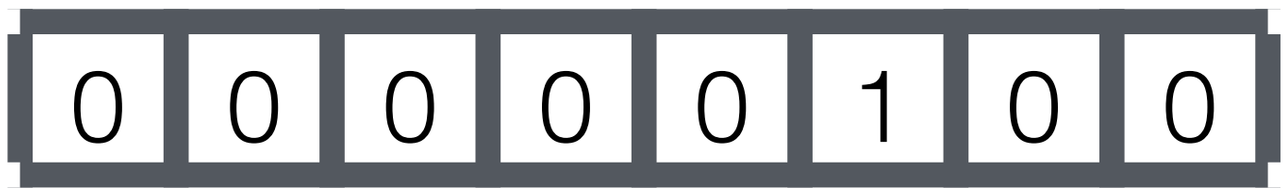
A filter:



After convolution*:



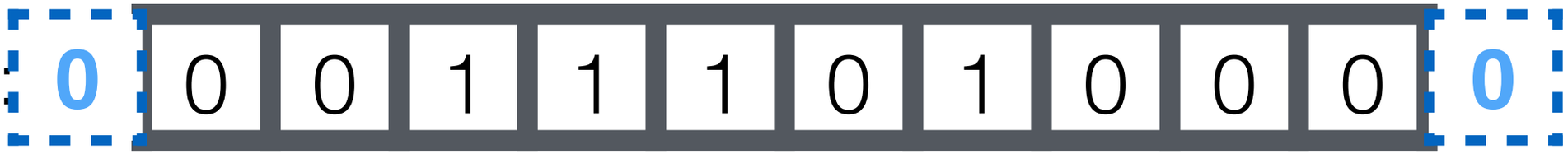
After ReLU:



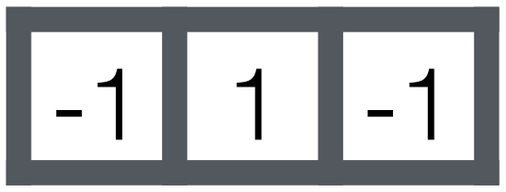
*correlation

Convolutional Layer: 1D example

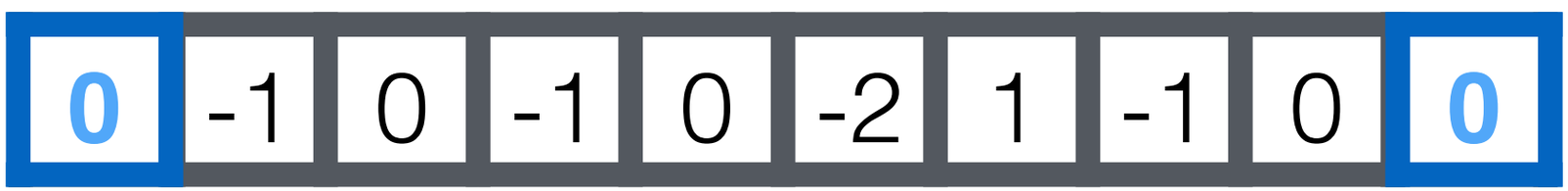
A 1D image:



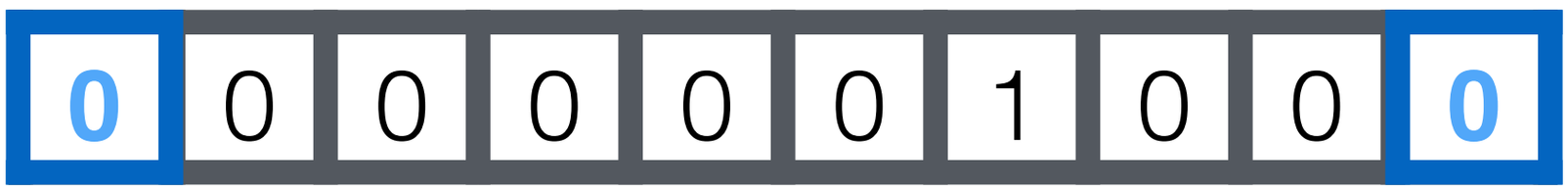
A filter:



After convolution*:



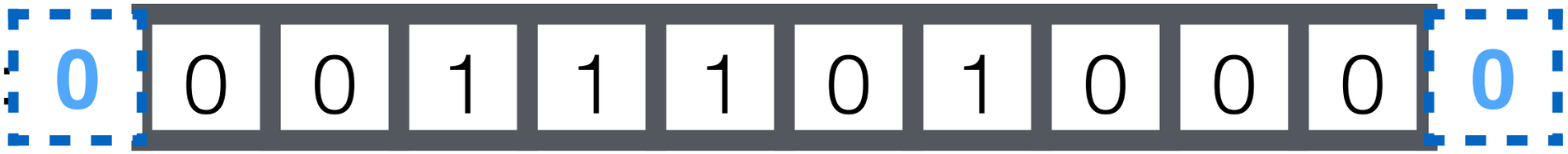
After ReLU:



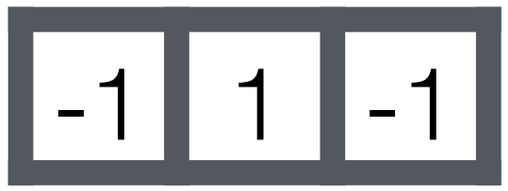
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:



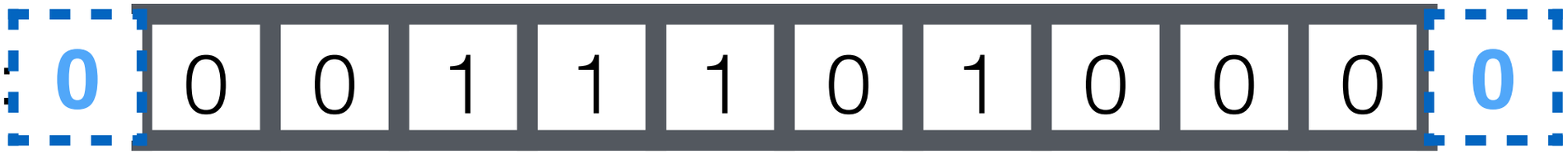
After convolution*:

After ReLU:

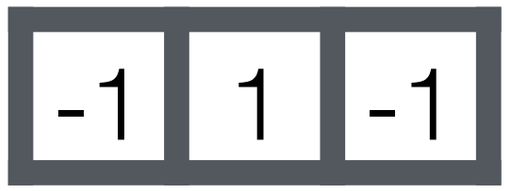
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:



with bias +1

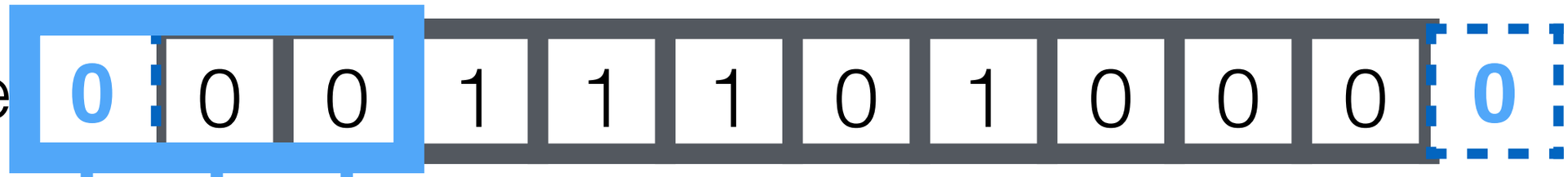
After convolution*:

After ReLU:

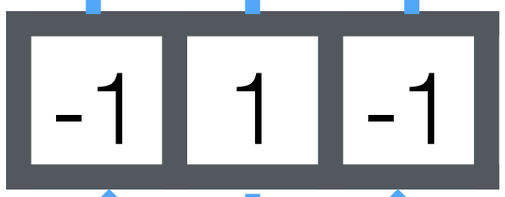
*correlation

Convolutional Layer: 1D example

A 1D image



A filter:



with bias +1

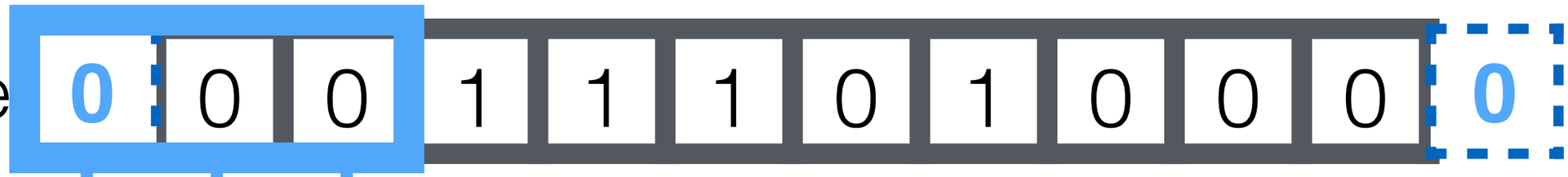
After convolution*:

After ReLU:

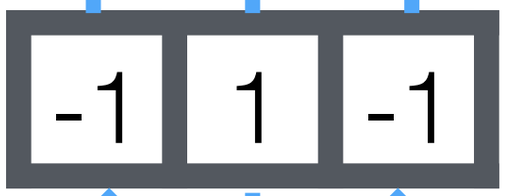
*correlation

Convolutional Layer: 1D example

A 1D image



A filter:



with bias +1

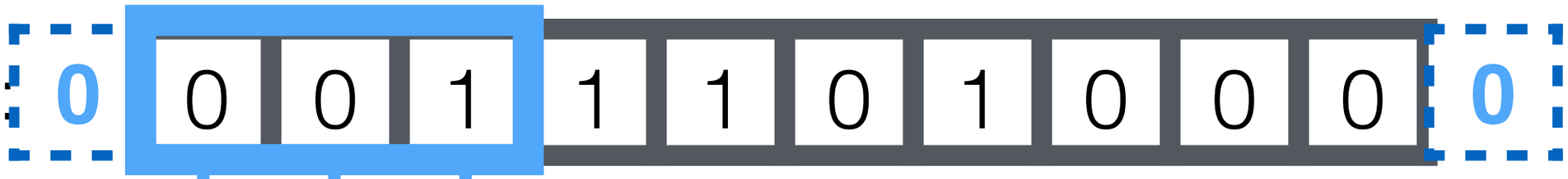
After convolution*:

After ReLU:

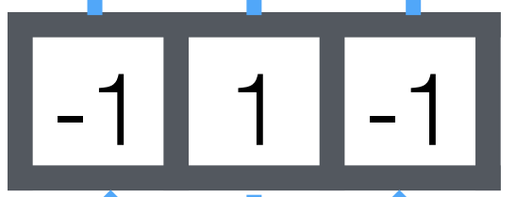
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:



with bias +1

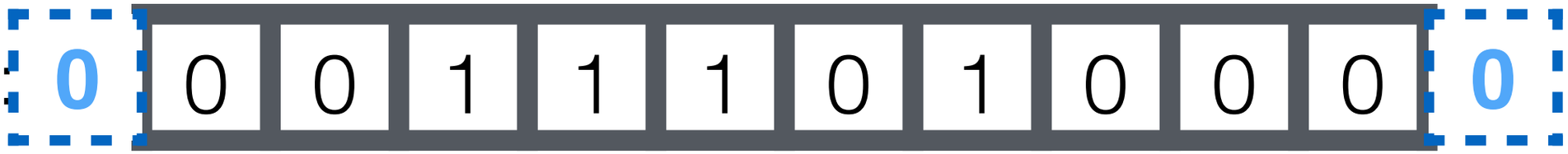
After convolution*:

After ReLU:

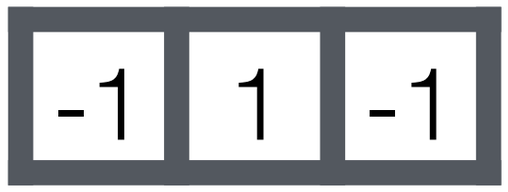
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:



with bias +1

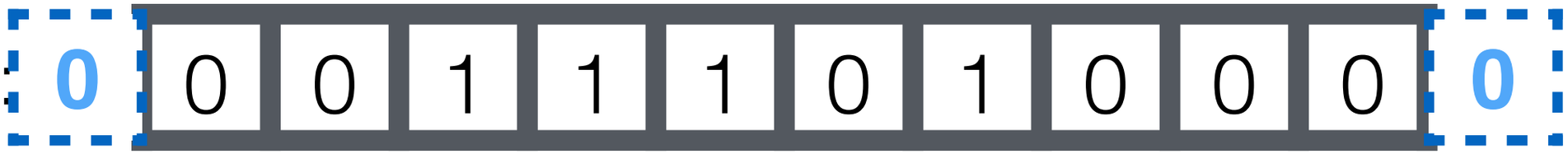
After convolution*:

After ReLU:

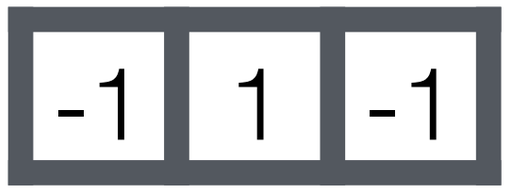
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:



with bias +1

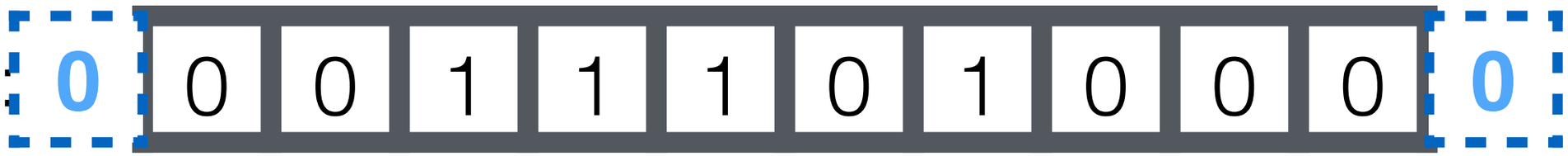
After convolution*:

After ReLU:

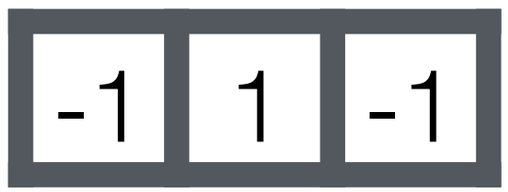
*correlation

Convolutional Layer: 1D example

A 1D image:

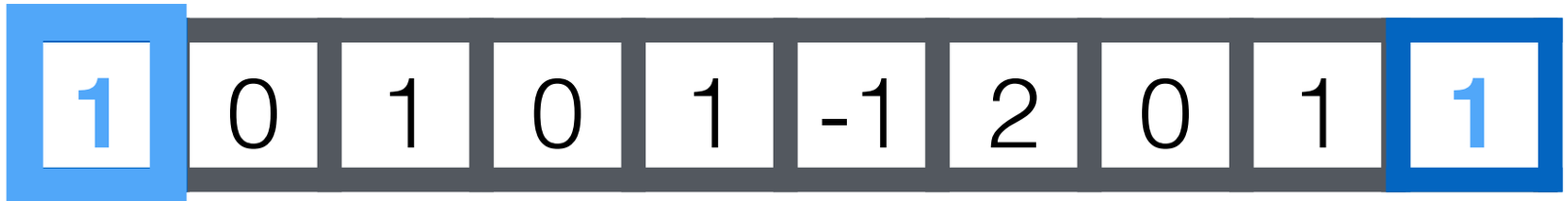


A filter:



with bias +1

After convolution*:

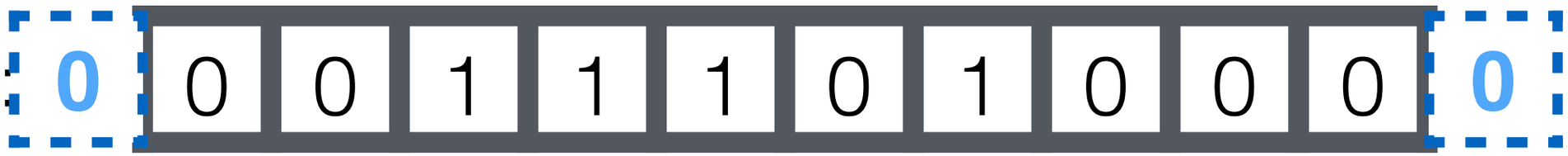


After ReLU:

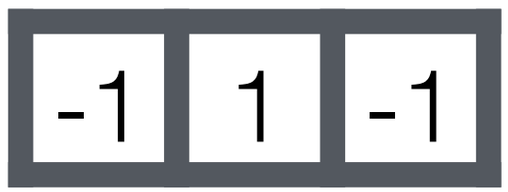
*correlation

Convolutional Layer: 1D example

A 1D image:

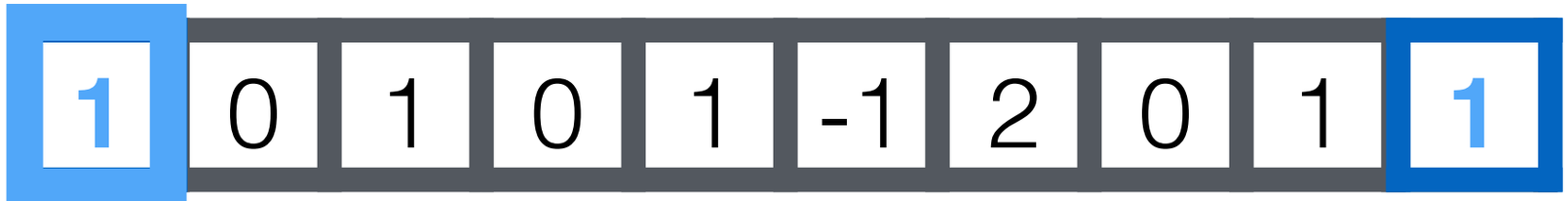


A filter:



with bias +1

After convolution*:

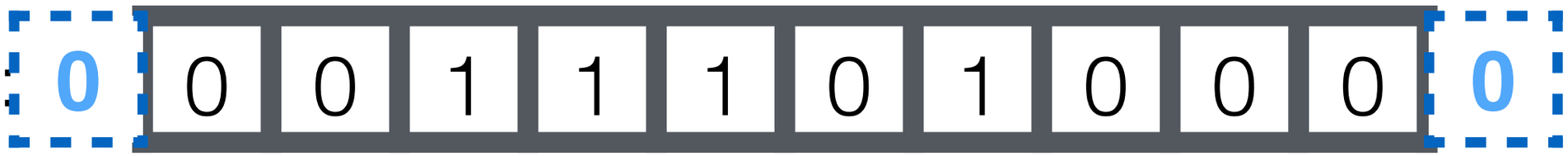


After ReLU:

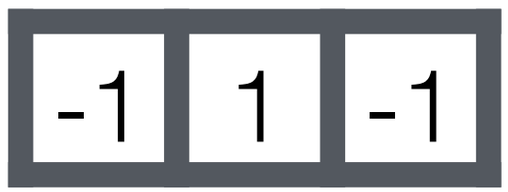
*correlation

Convolutional Layer: 1D example

A 1D image:

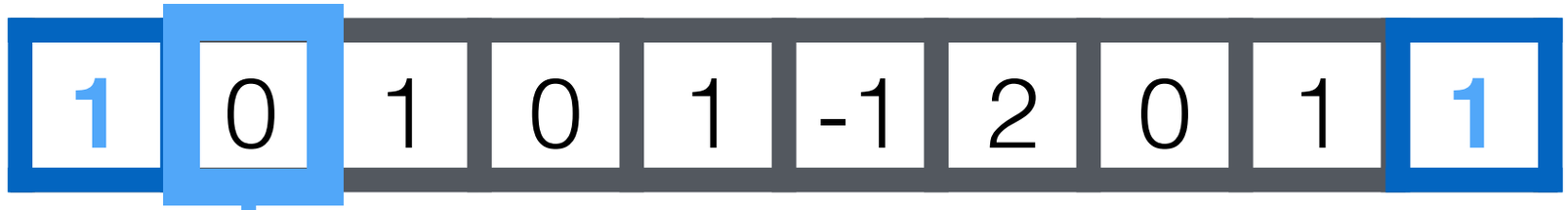


A filter:



with bias +1

After convolution*:

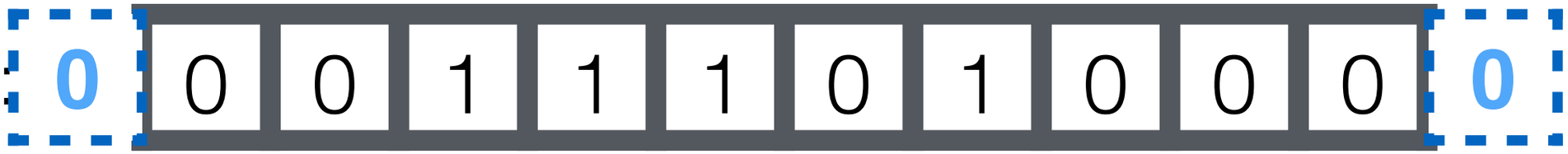


After ReLU:

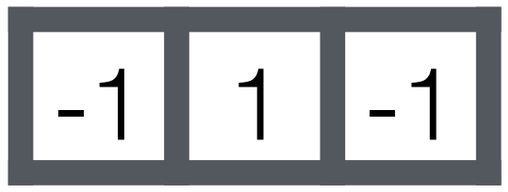
*correlation

Convolutional Layer: 1D example

A 1D image:

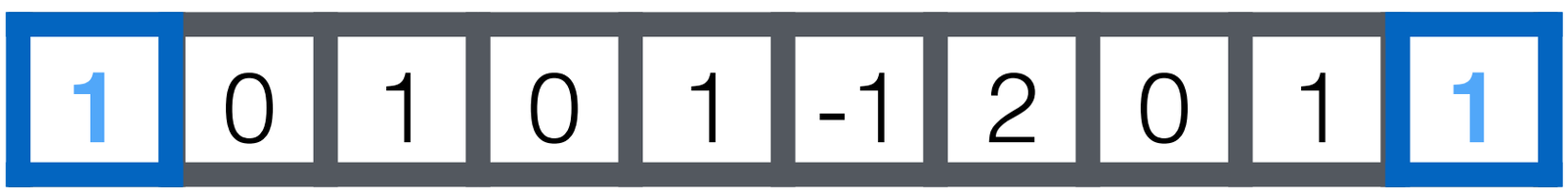


A filter:



with bias +1

After convolution*:

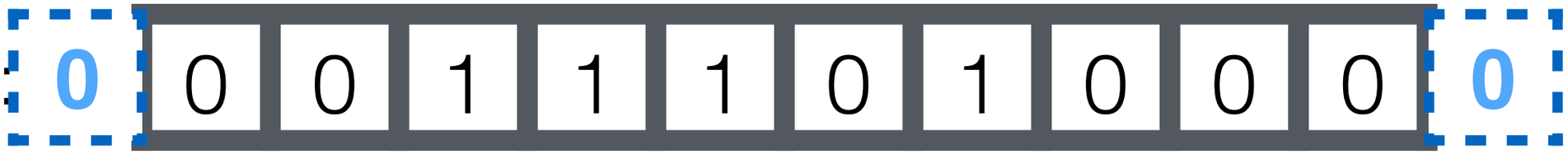


After ReLU:

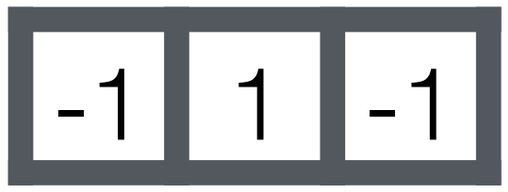
*correlation

Convolutional Layer: 1D example

A 1D image:

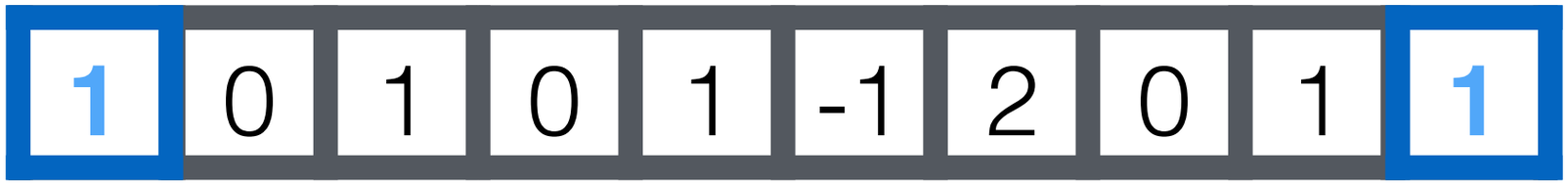


A filter:

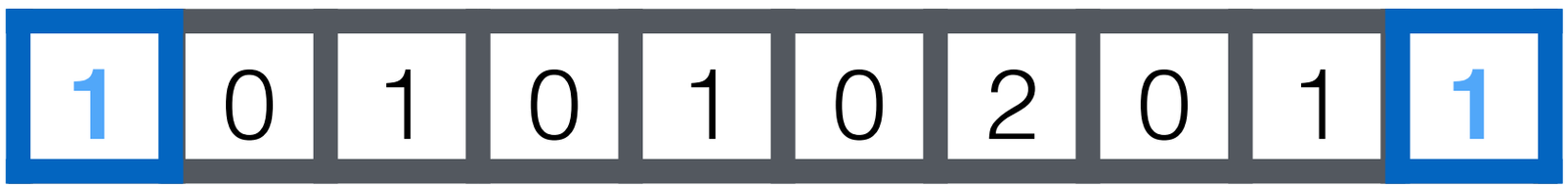


with bias +1

After convolution*:



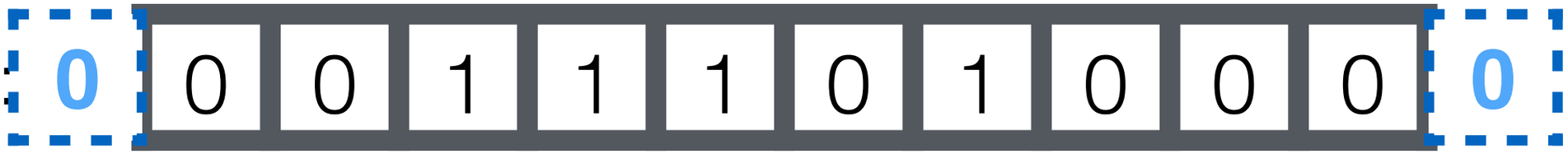
After ReLU:



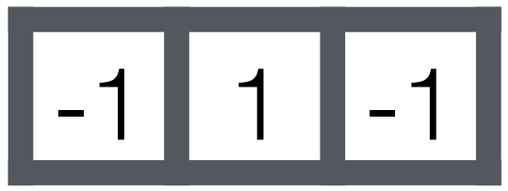
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:



with bias +1

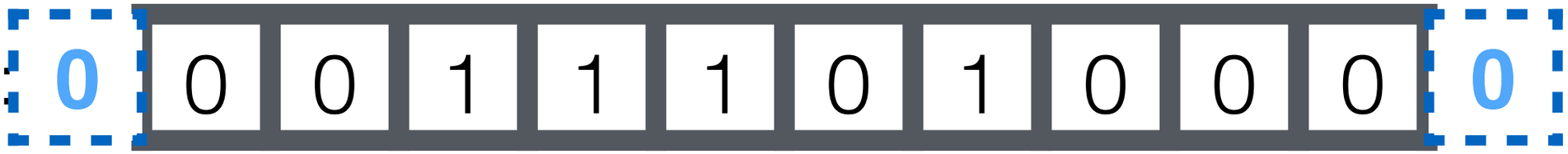
After convolution*:

After ReLU:

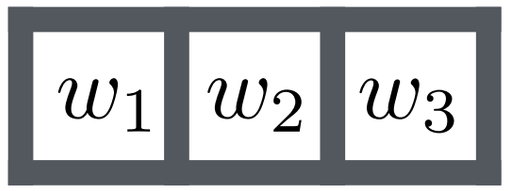
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:



with bias *b*

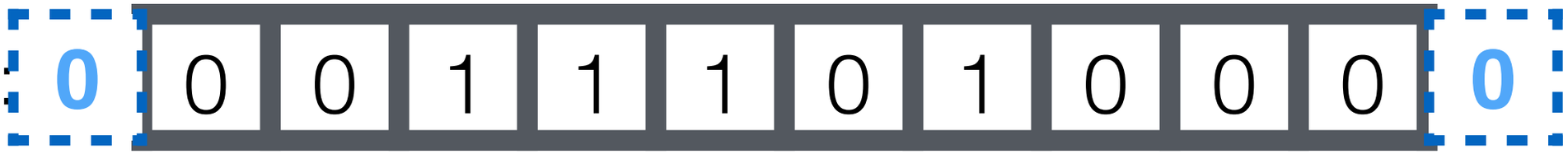
After convolution*:

After ReLU:

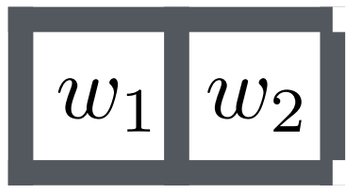
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:



with bias b

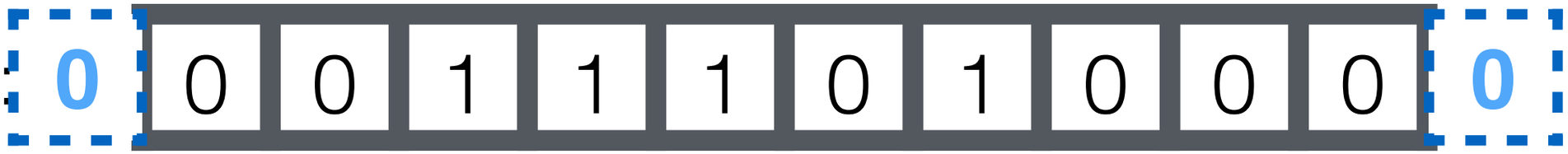
After convolution*:

After ReLU:

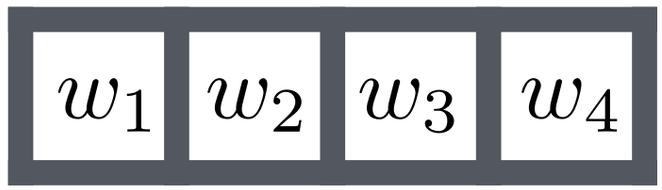
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:



with bias *b*

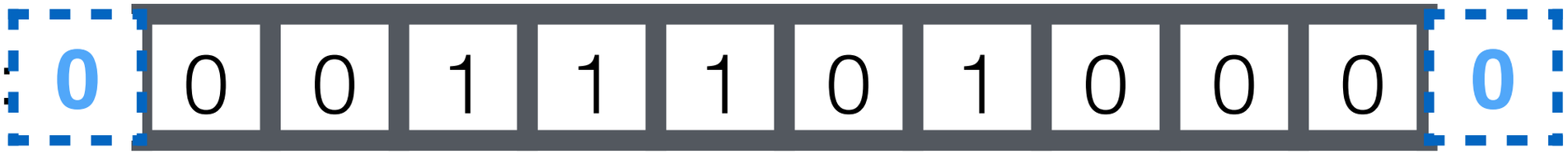
After convolution*:

After ReLU:

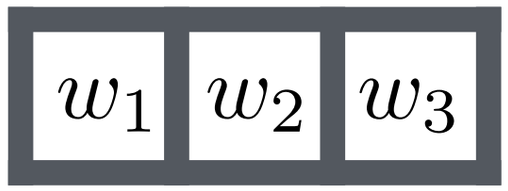
*correlation

Convolutional Layer: 1D example

A 1D image:



A filter:



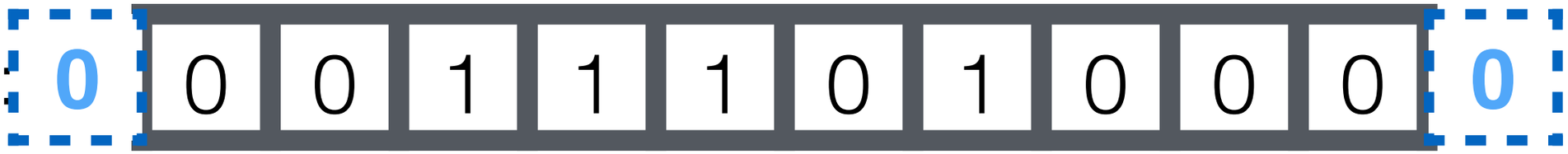
with bias *b*

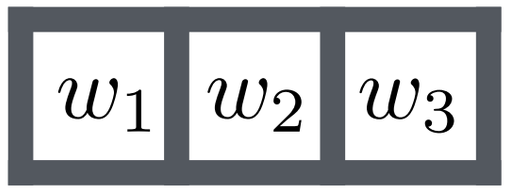
After convolution*:

After ReLU:

*correlation

Convolutional Layer: 1D example

A 1D image: 

A filter: 

with bias *b*

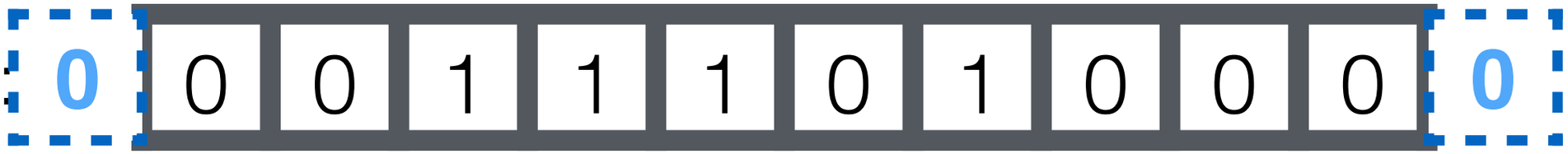
After convolution*: 

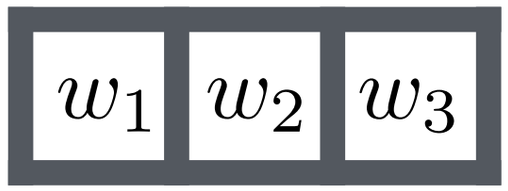
After ReLU: 

- How many weights (including bias)?

*correlation

Convolutional Layer: 1D example

A 1D image: 

A filter: 

with bias *b*

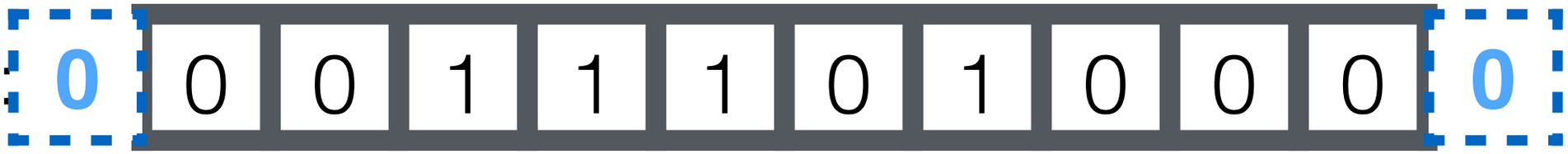
After convolution*: 

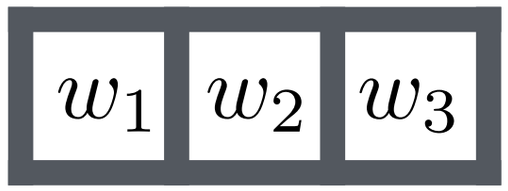
After ReLU: 

- How many weights (including bias)? 4

*correlation

Convolutional Layer: 1D example

A 1D image: 

A filter: 

with bias *b*

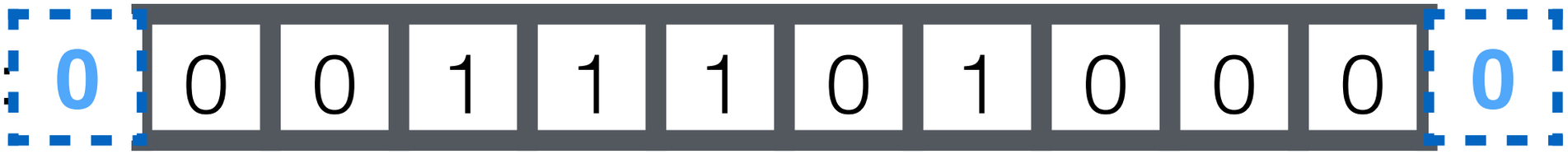
After convolution*: 

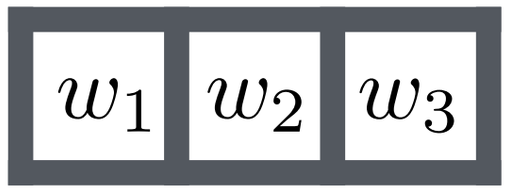
After ReLU: 

- How many weights (including bias)? 4
- How many weights (including biases) for fully connected layer with 10 inputs & 10 outputs?

*correlation

Convolutional Layer: 1D example

A 1D image: 

A filter: 

with bias *b*

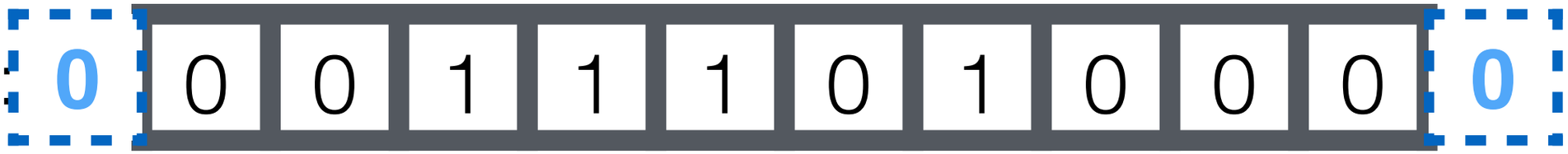
After convolution*: 

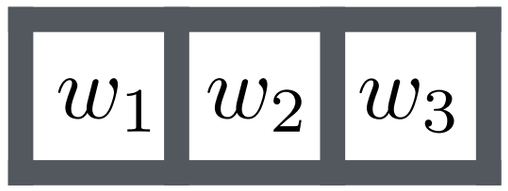
After ReLU: 

- How many weights (including bias)? 4
- How many weights (including biases) for fully connected layer with 10 inputs & 10 outputs? 10 x 11 =

*correlation

Convolutional Layer: 1D example

A 1D image: 

A filter: 

with bias *b*

After convolution*: 

After ReLU: 

- How many weights (including bias)? 4
- How many weights (including biases) for fully connected layer with 10 inputs & 10 outputs? 10 x 11 = 110

*correlation

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

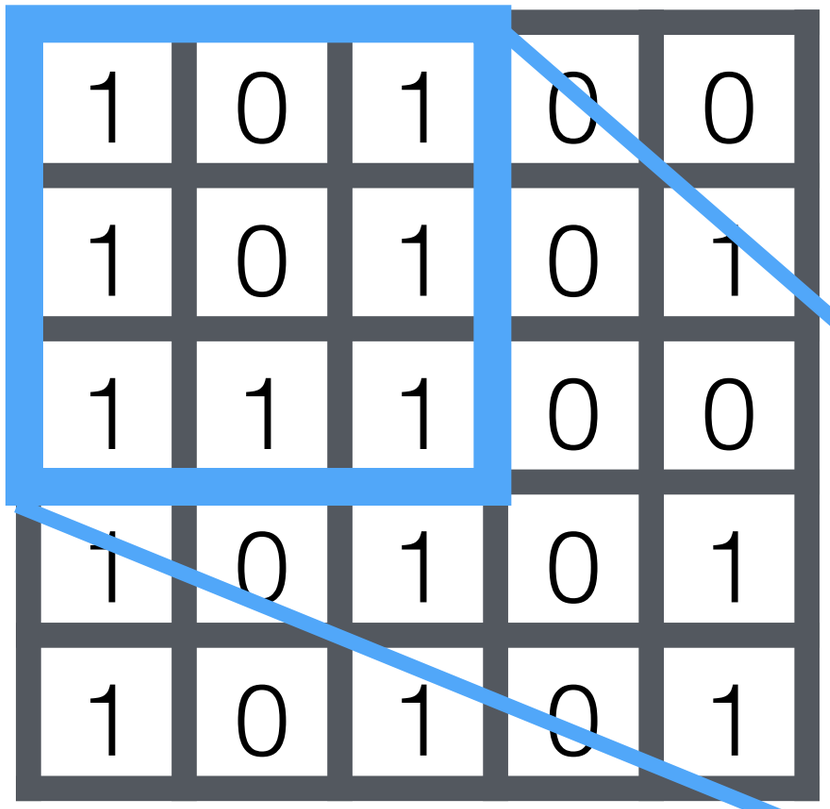
A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

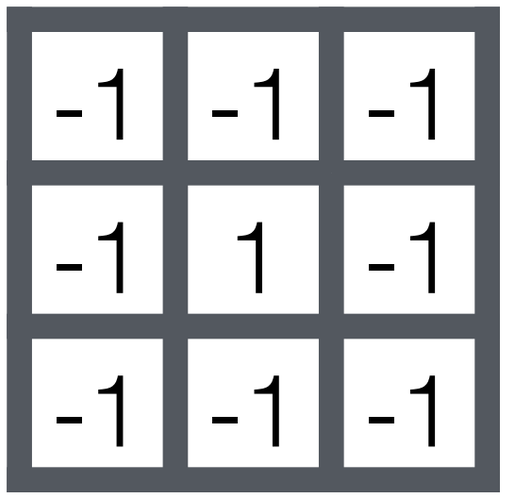
After convolution:

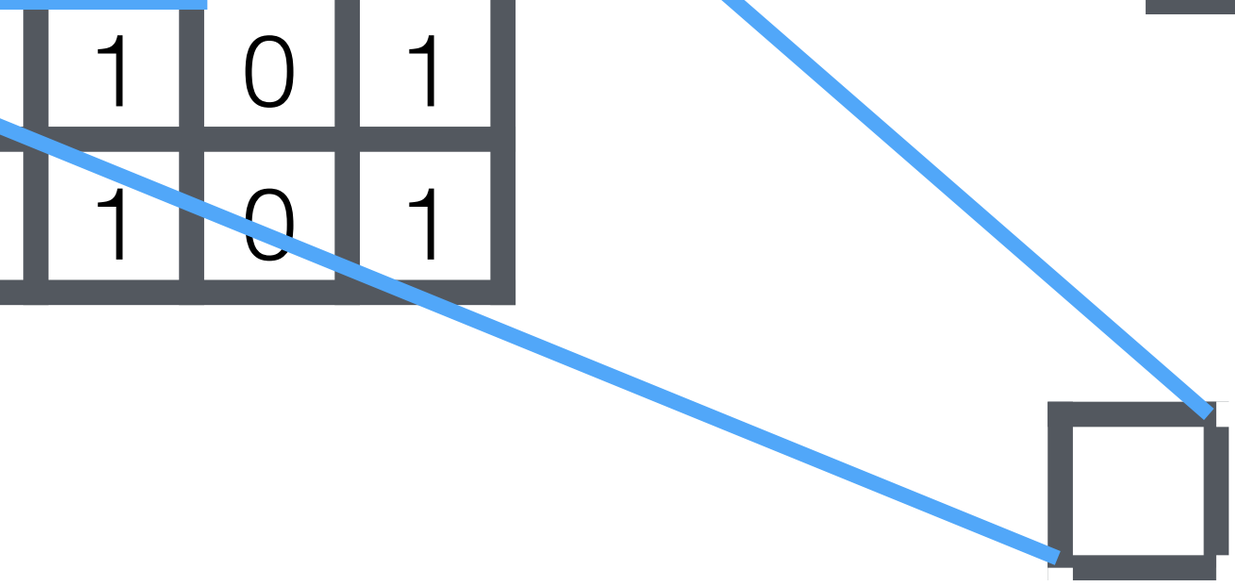
Convolutional Layer: 2D example

A 2D image:



A filter:

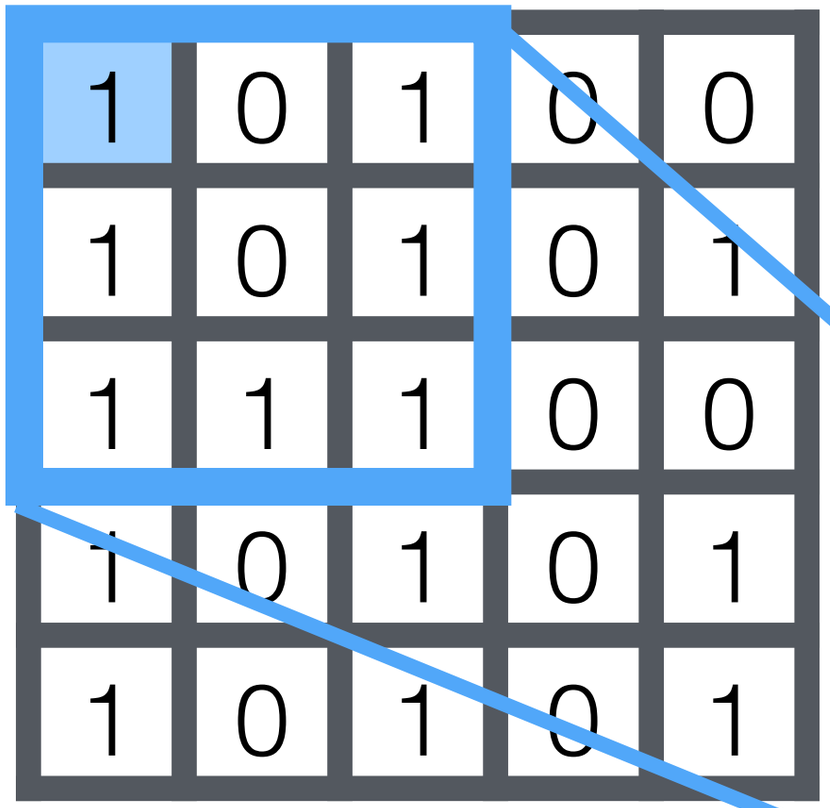




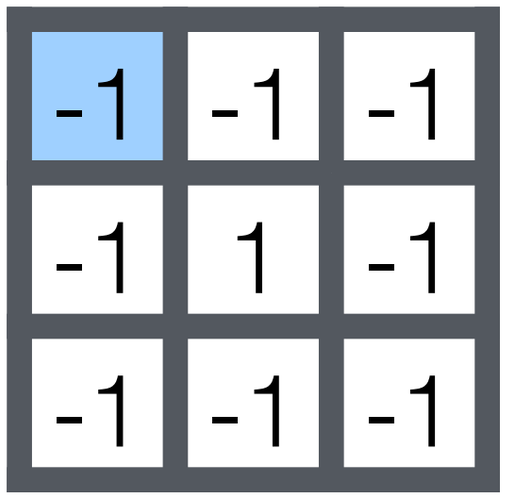
After convolution:

Convolutional Layer: 2D example

A 2D image:



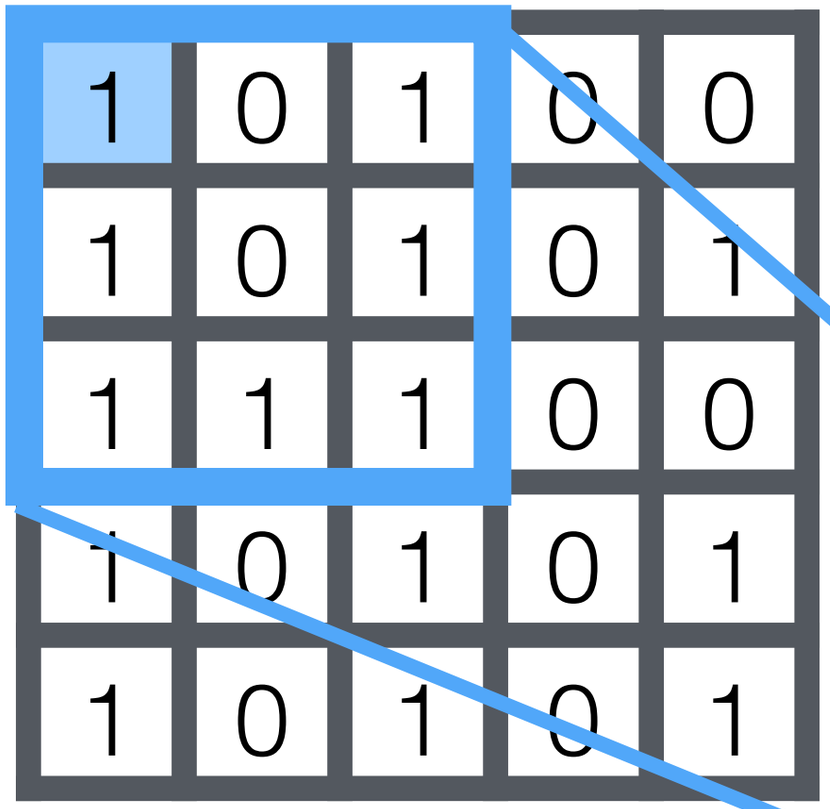
A filter:



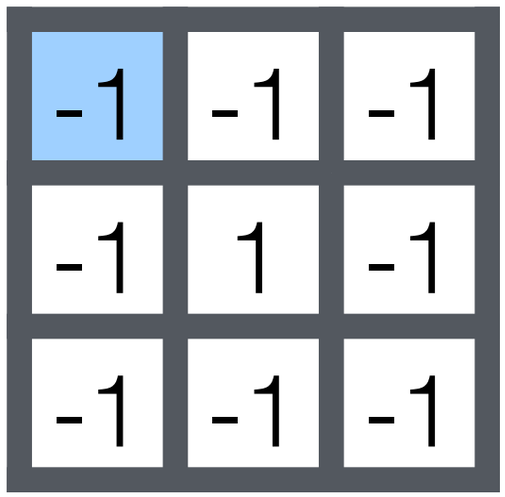
After convolution:

Convolutional Layer: 2D example

A 2D image:



A filter:

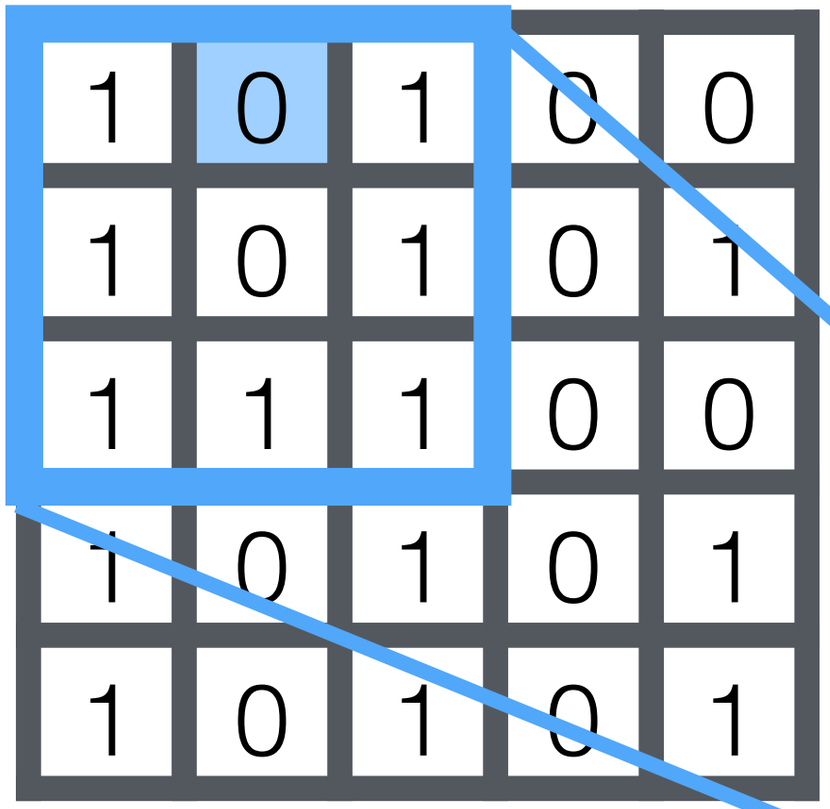


-1

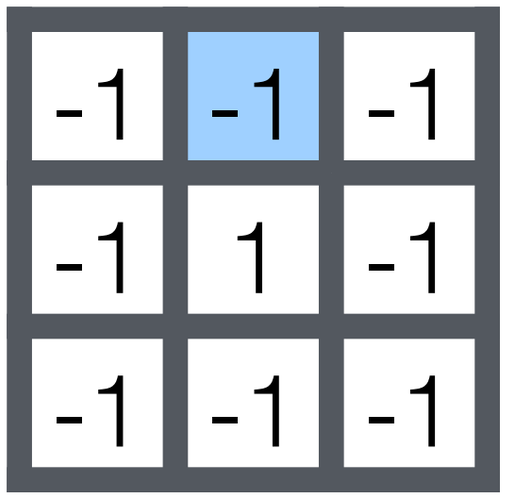
After convolution:

Convolutional Layer: 2D example

A 2D image:



A filter:

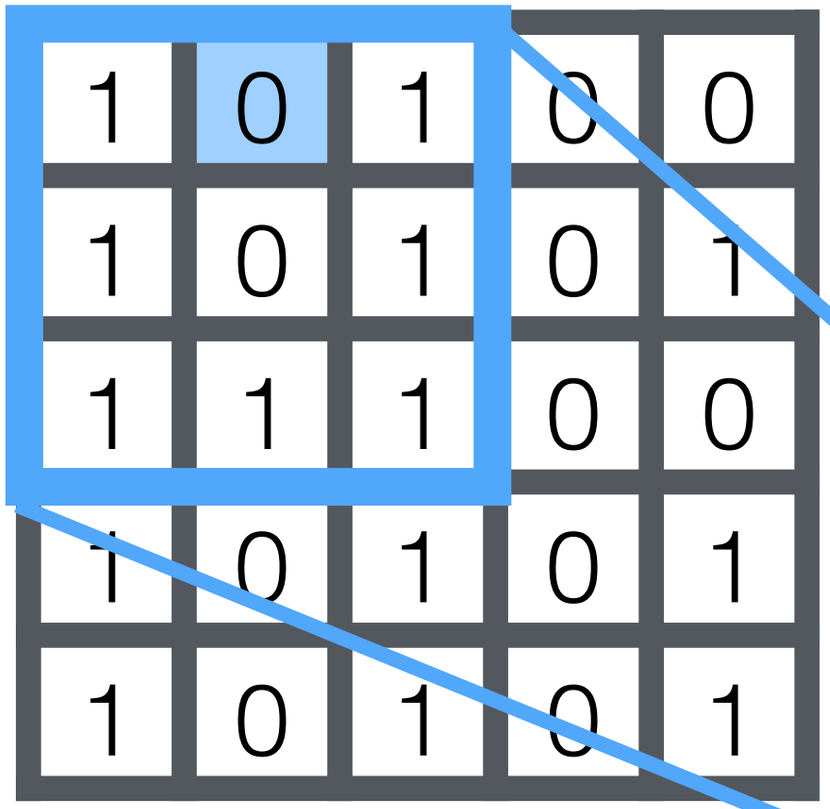


-1

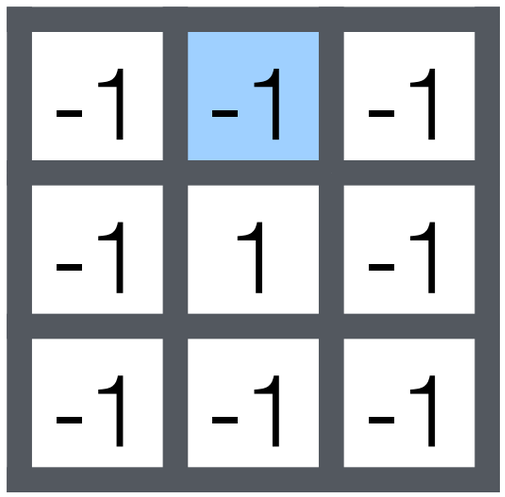
After convolution:

Convolutional Layer: 2D example

A 2D image:



A filter:



-1 + 0

After convolution:

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

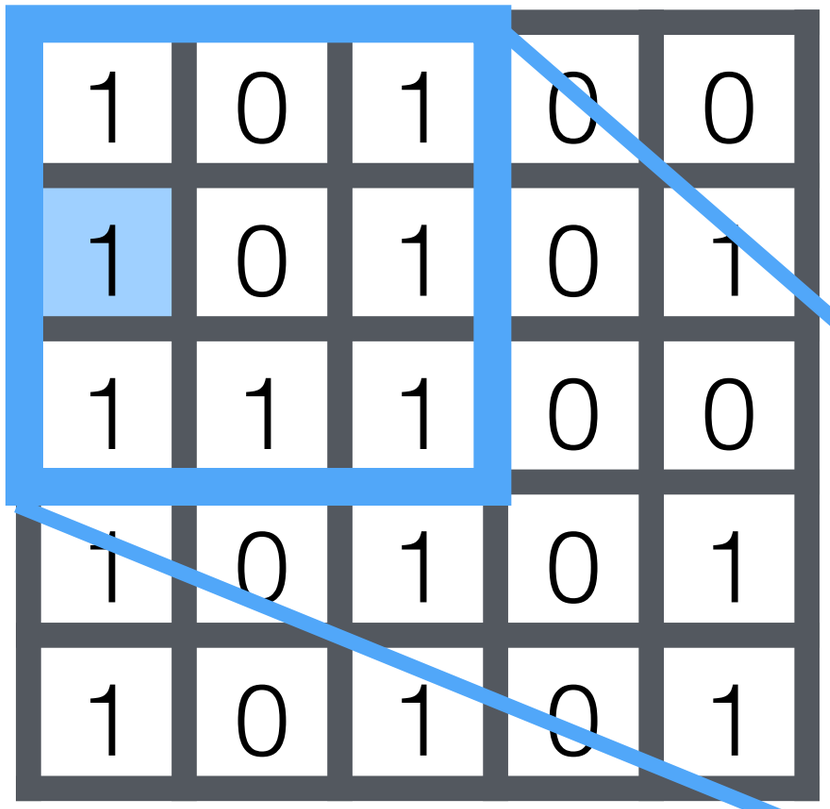
-1	-1	-1
-1	1	-1
-1	-1	-1

-1 + 0 + -1

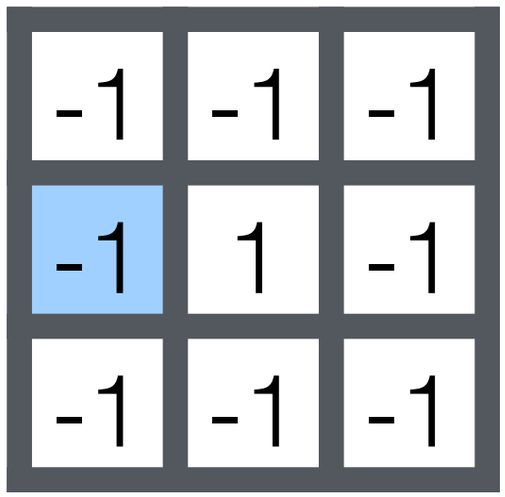
After convolution:

Convolutional Layer: 2D example

A 2D image:



A filter:

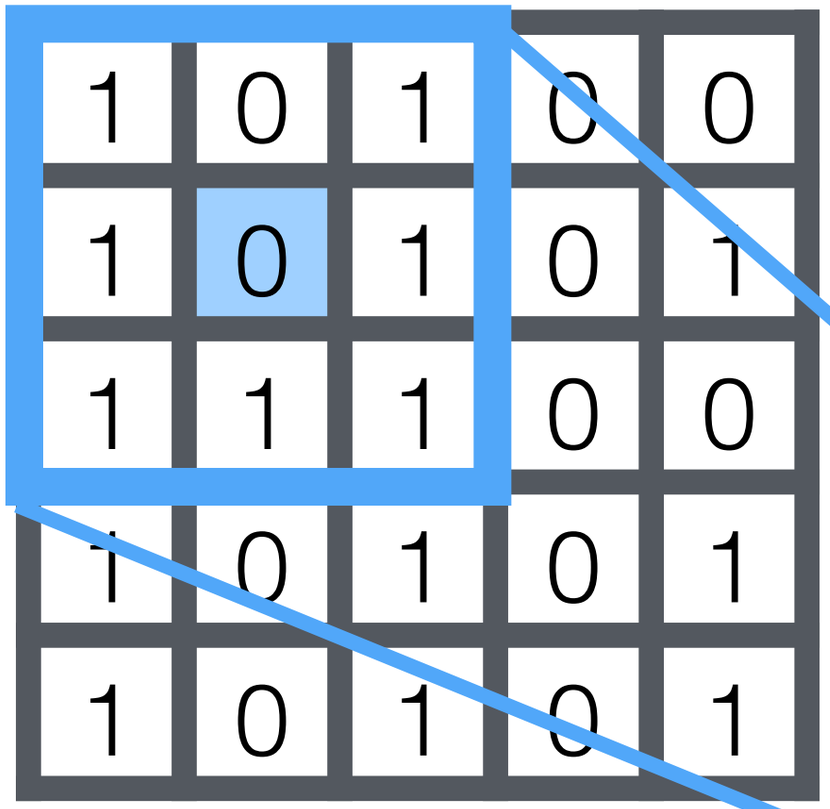


-1 + 0 + -1
+ -1

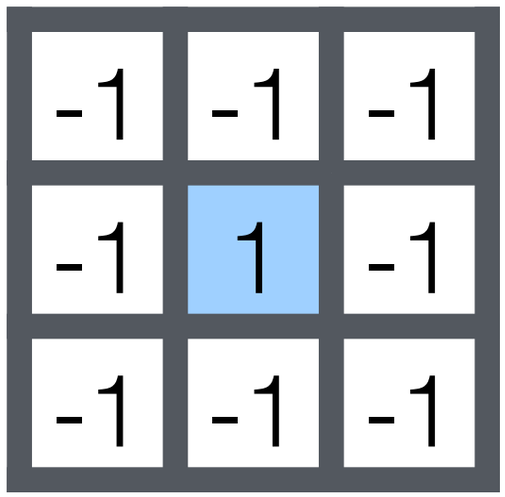
After convolution:

Convolutional Layer: 2D example

A 2D image:



A filter:

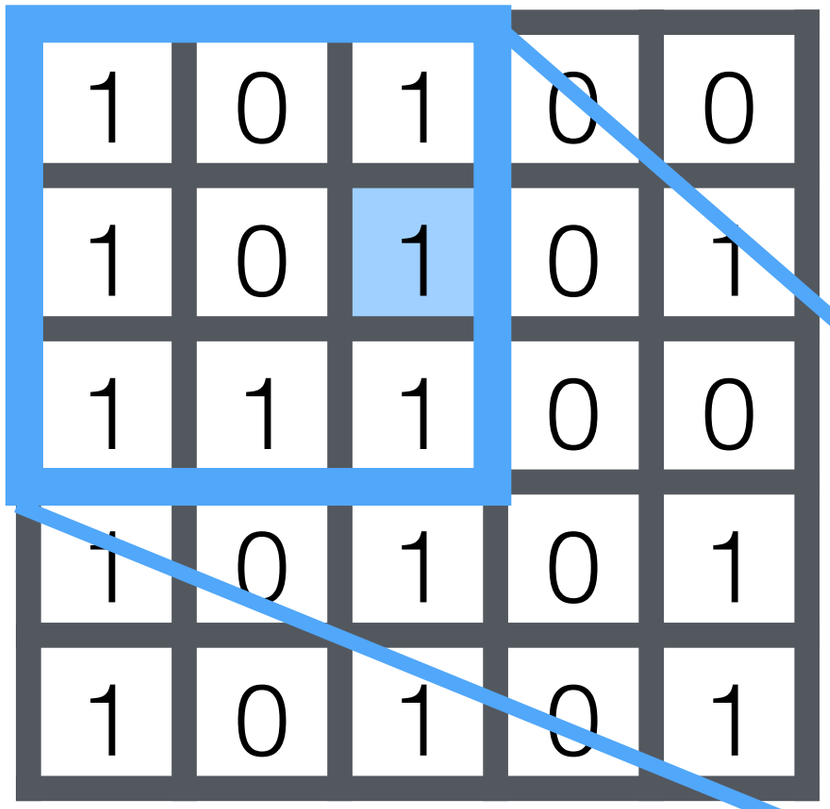


-1 + 0 + -1
+ -1 + 0

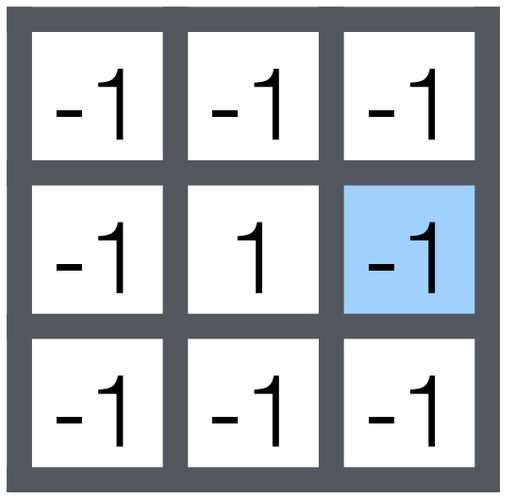
After convolution:

Convolutional Layer: 2D example

A 2D image:



A filter:

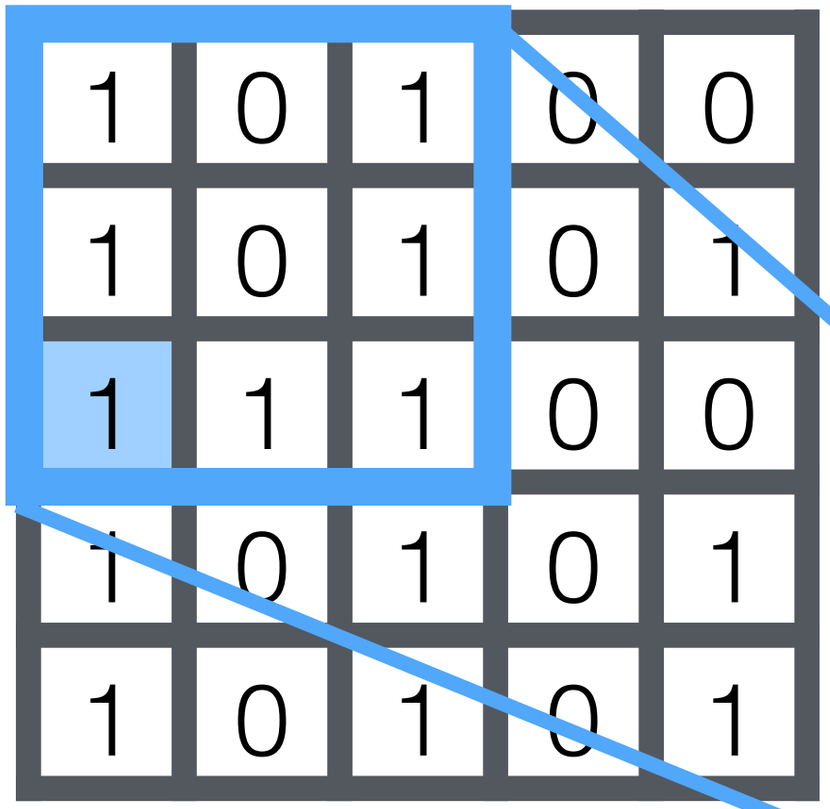


-1 + 0 + -1
+ -1 + 0 + -1

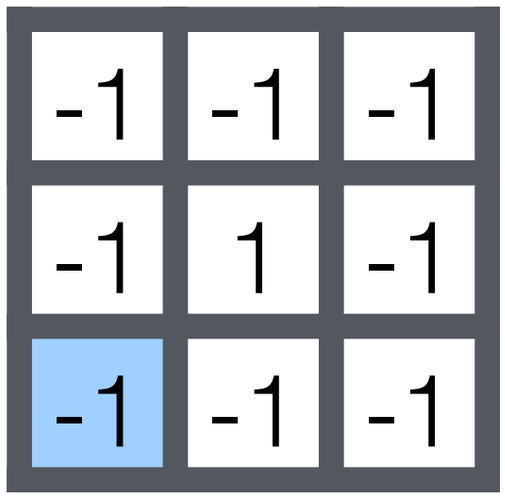
After convolution:

Convolutional Layer: 2D example

A 2D image:



A filter:

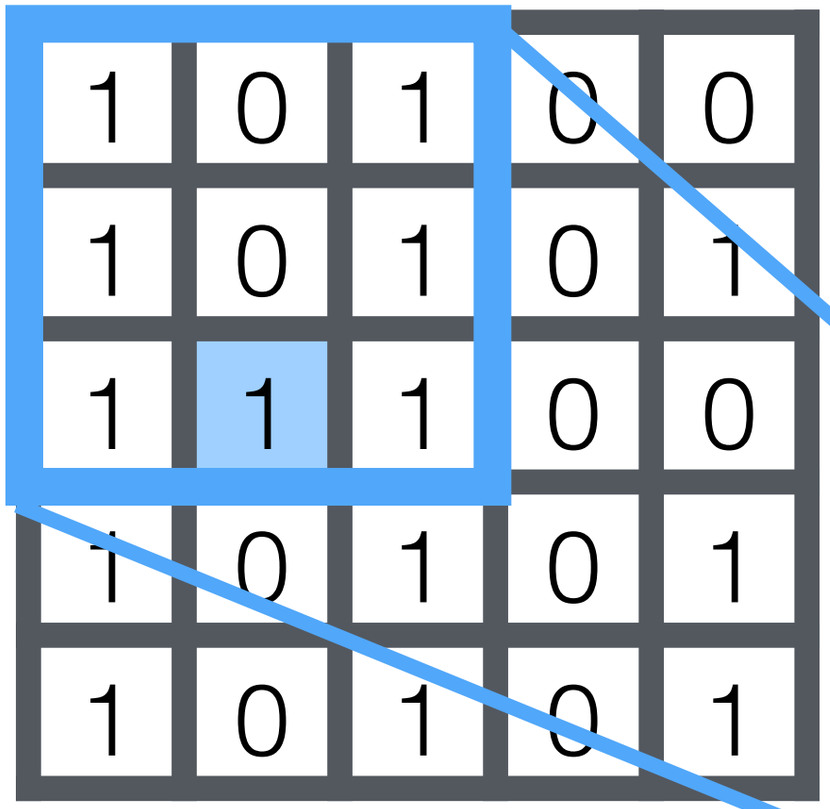


-1 + 0 + -1
+ -1 + 0 + -1
+ -1

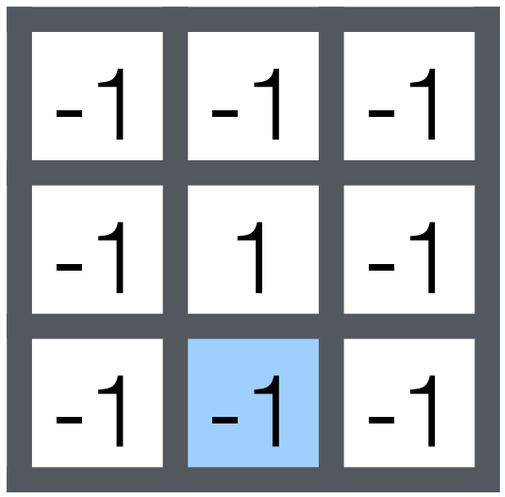
After convolution:

Convolutional Layer: 2D example

A 2D image:



A filter:

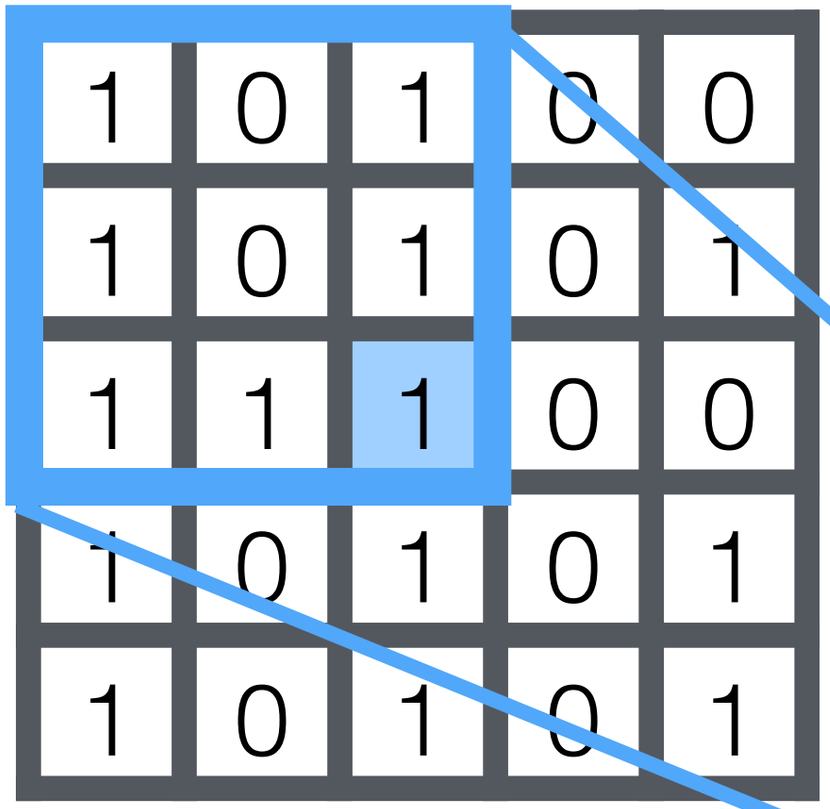


-1 + 0 + -1
+ -1 + 0 + -1
+ -1 + - 1

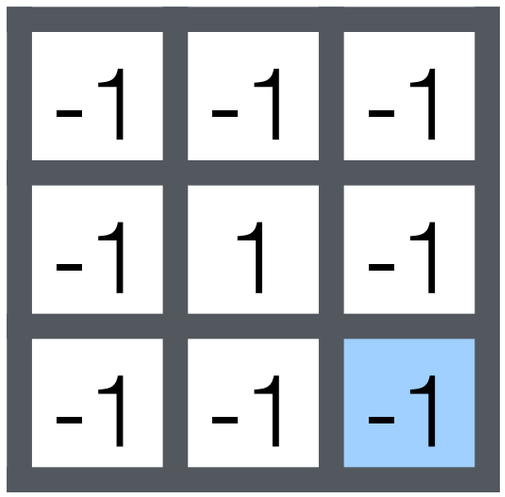
After convolution:

Convolutional Layer: 2D example

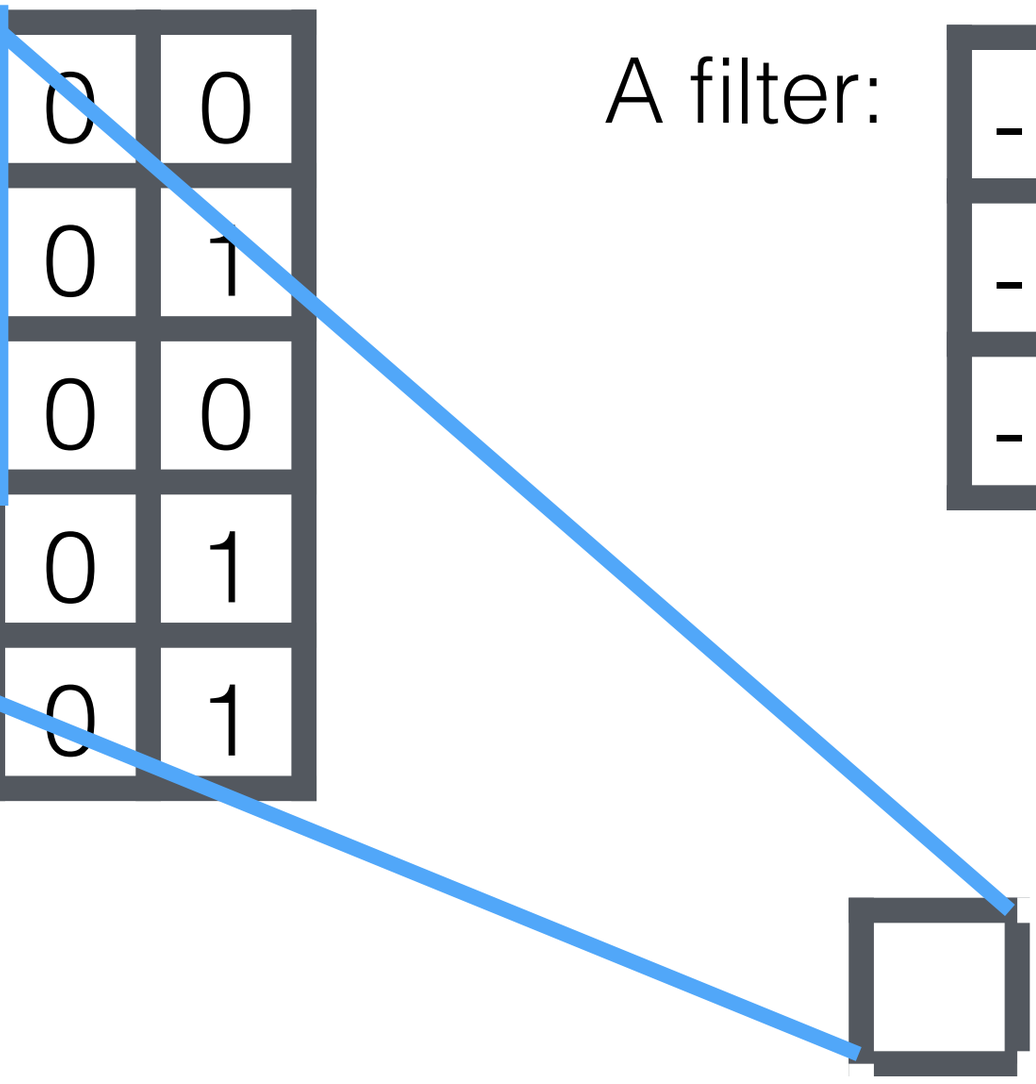
A 2D image:



A filter:



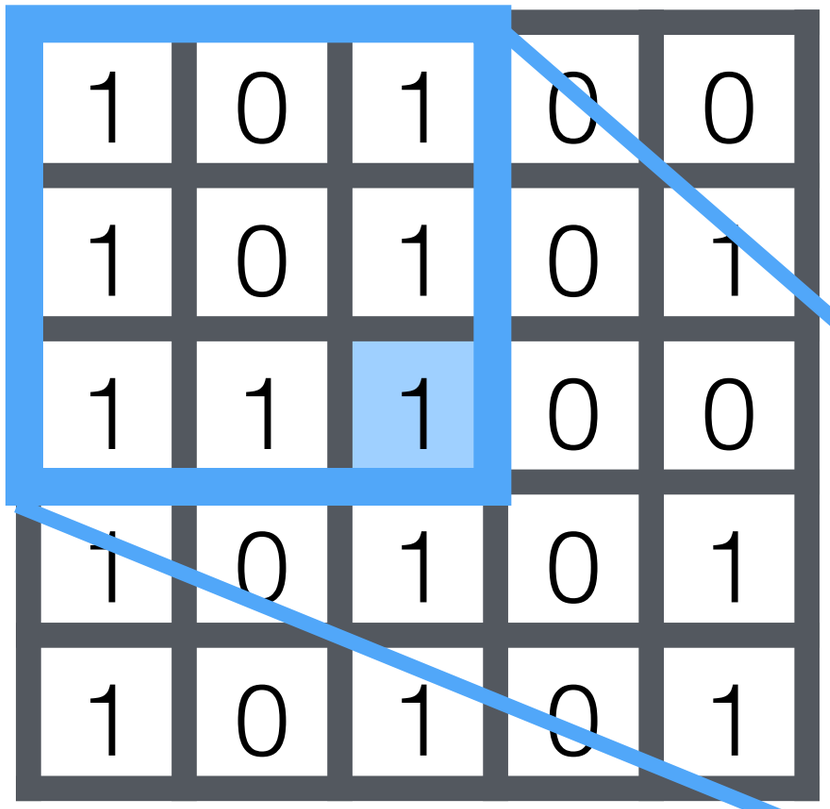
-1 + 0 + -1
+ -1 + 0 + -1
+ -1 + - 1 + -1



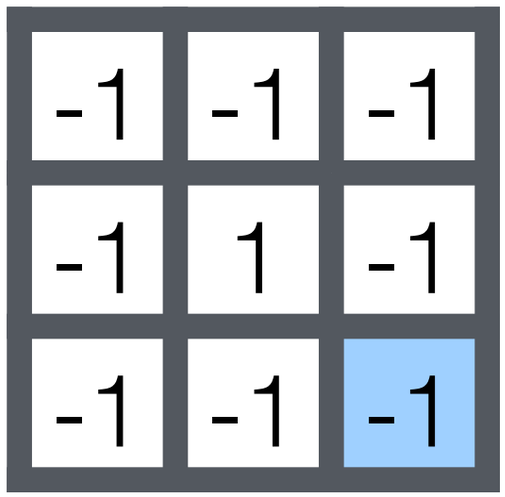
After convolution:

Convolutional Layer: 2D example

A 2D image:



A filter:



-1 + 0 + -1
+ -1 + 0 + -1
+ -1 + - 1 + -1
= -7

After convolution:

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

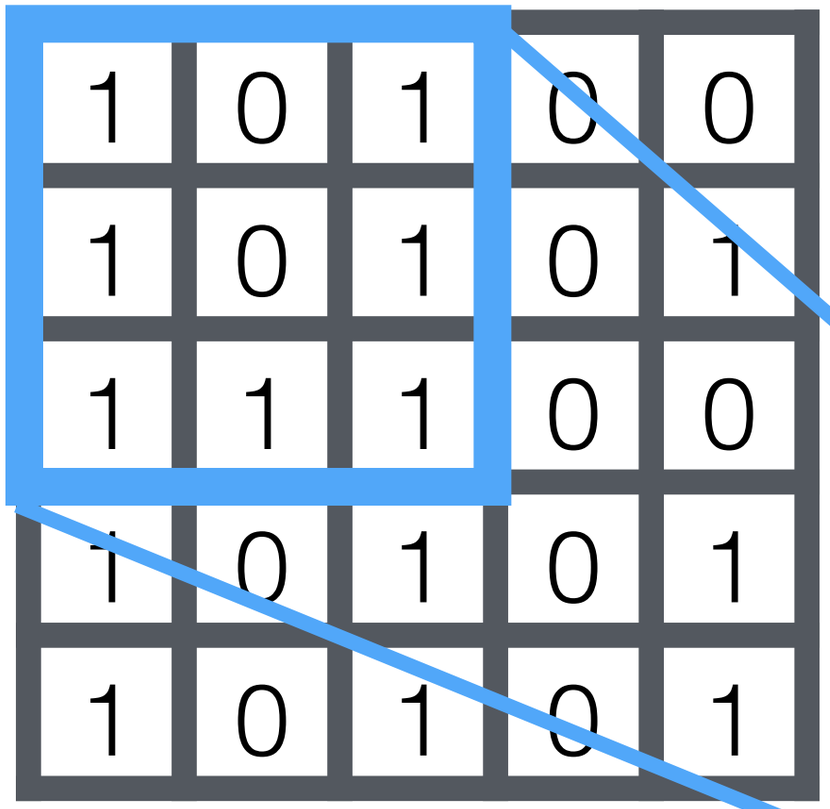
-1 + 0 + -1
+ -1 + 0 + -1
+ -1 + - 1 + -1
= -7

After convolution:

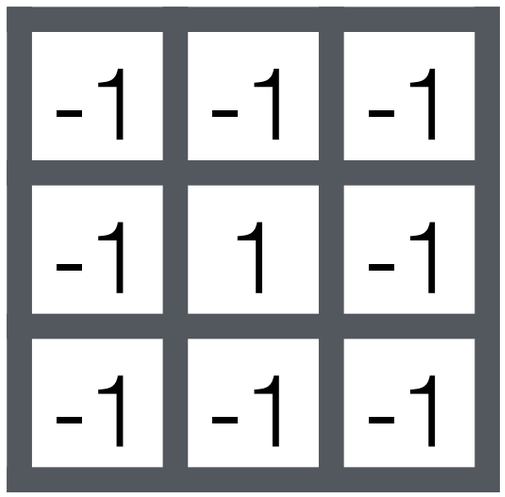
-7

Convolutional Layer: 2D example

A 2D image:



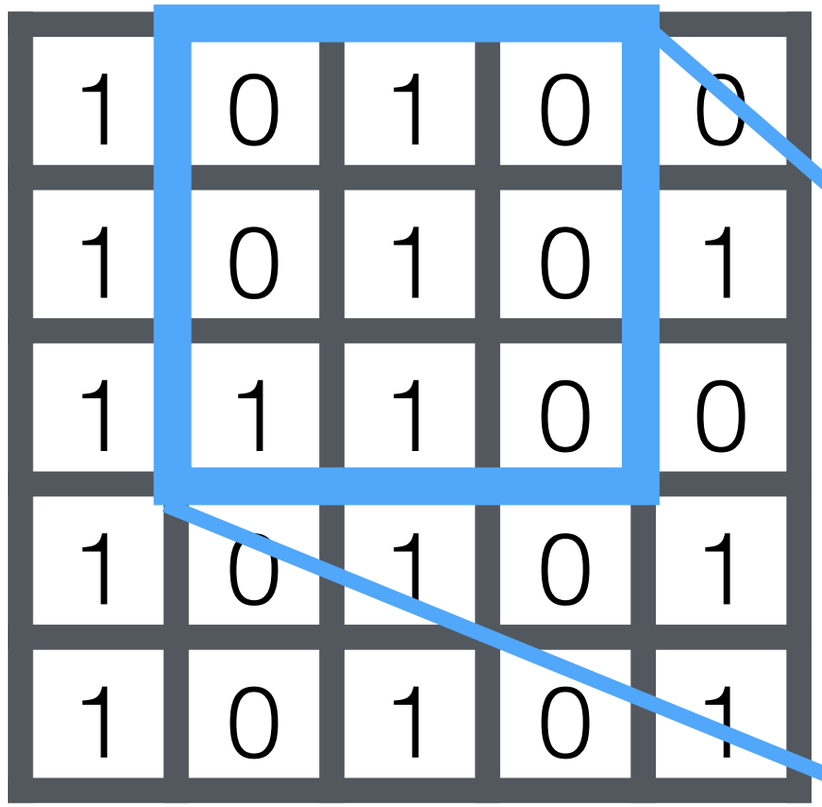
A filter:



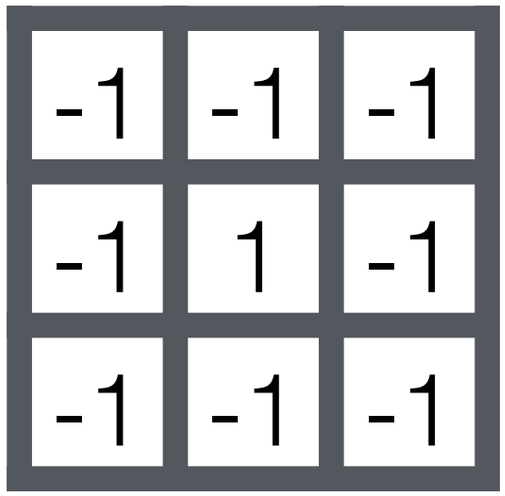
After convolution:

Convolutional Layer: 2D example

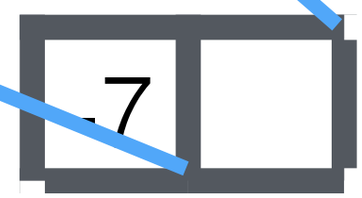
A 2D image:



A filter:



After convolution:



Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

-7	-2

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

-7	-2	

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

-7	-2	-4

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

-7	-2	-4
-5		

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

-7	-2	-4
-5	-2	

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

-7	-2	-4
-5	2	-5

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

-7	-2	-4
-5	-2	-5
-7		

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

-7	-2	-4
-5	-2	-5
-7	-2	

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

-7	-2	-4
-5	-2	-5
-7	-2	-5

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

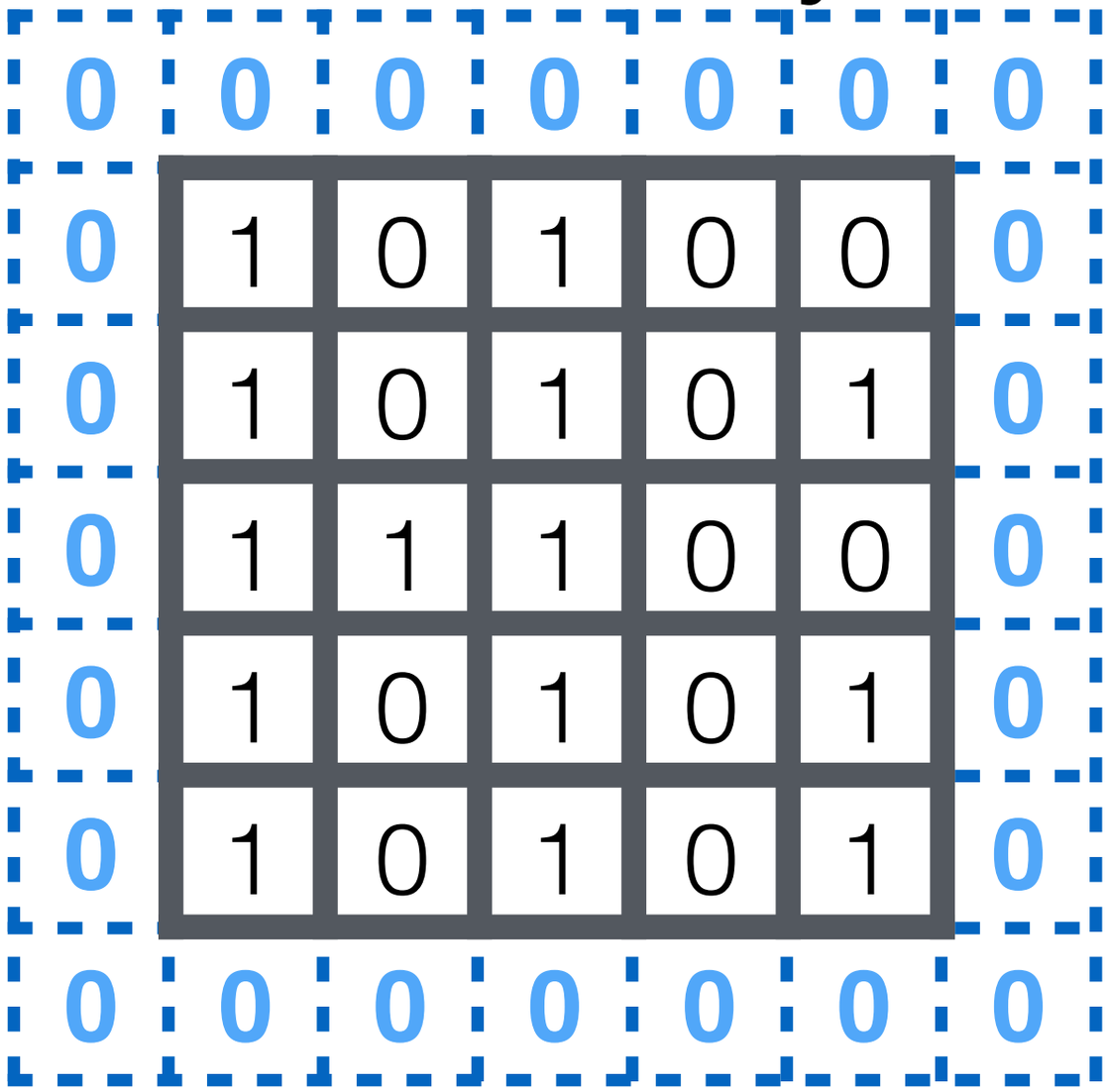
-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

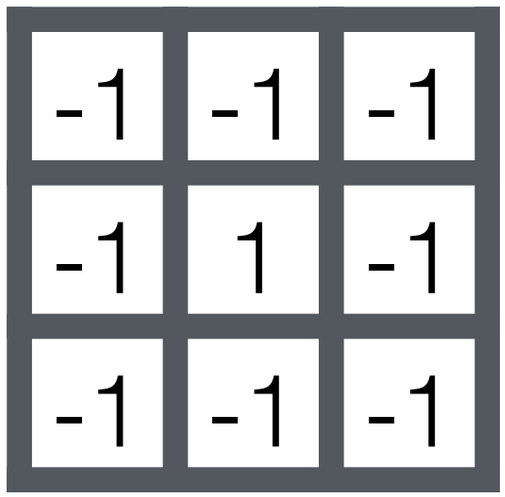
-7	-2	-4
-5	-2	-5
-7	-2	-5

Convolutional Layer: 2D example

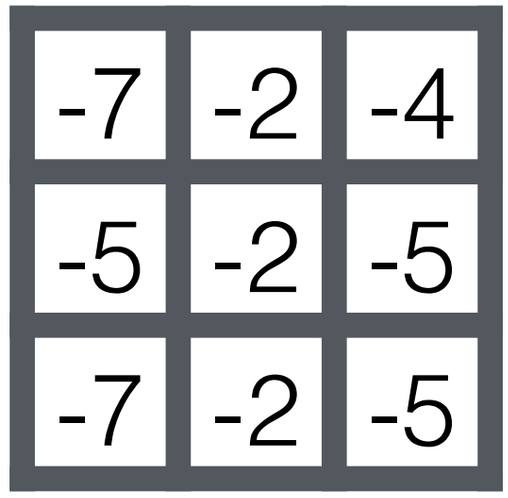
A 2D image:



A filter:



After convolution:



Convolutional Layer: 2D example

A 2D image:

0	0	0	0	0	0	0
0	1	0	1	0	0	0
0	1	0	1	0	1	0
0	1	1	1	0	0	0
0	1	0	1	0	1	0
0	1	0	1	0	1	0
0	0	0	0	0	0	0

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

-7	-2	-4
-5	-2	-5
-7	-2	-5

Convolutional Layer: 2D example

A 2D image:

0	0	0	0	0	0	0
0	1	0	1	0	0	0
0	1	0	1	0	1	0
0	1	1	1	0	0	0
0	1	0	1	0	1	0
0	1	0	1	0	1	0
0	0	0	0	0	0	0

A filter:

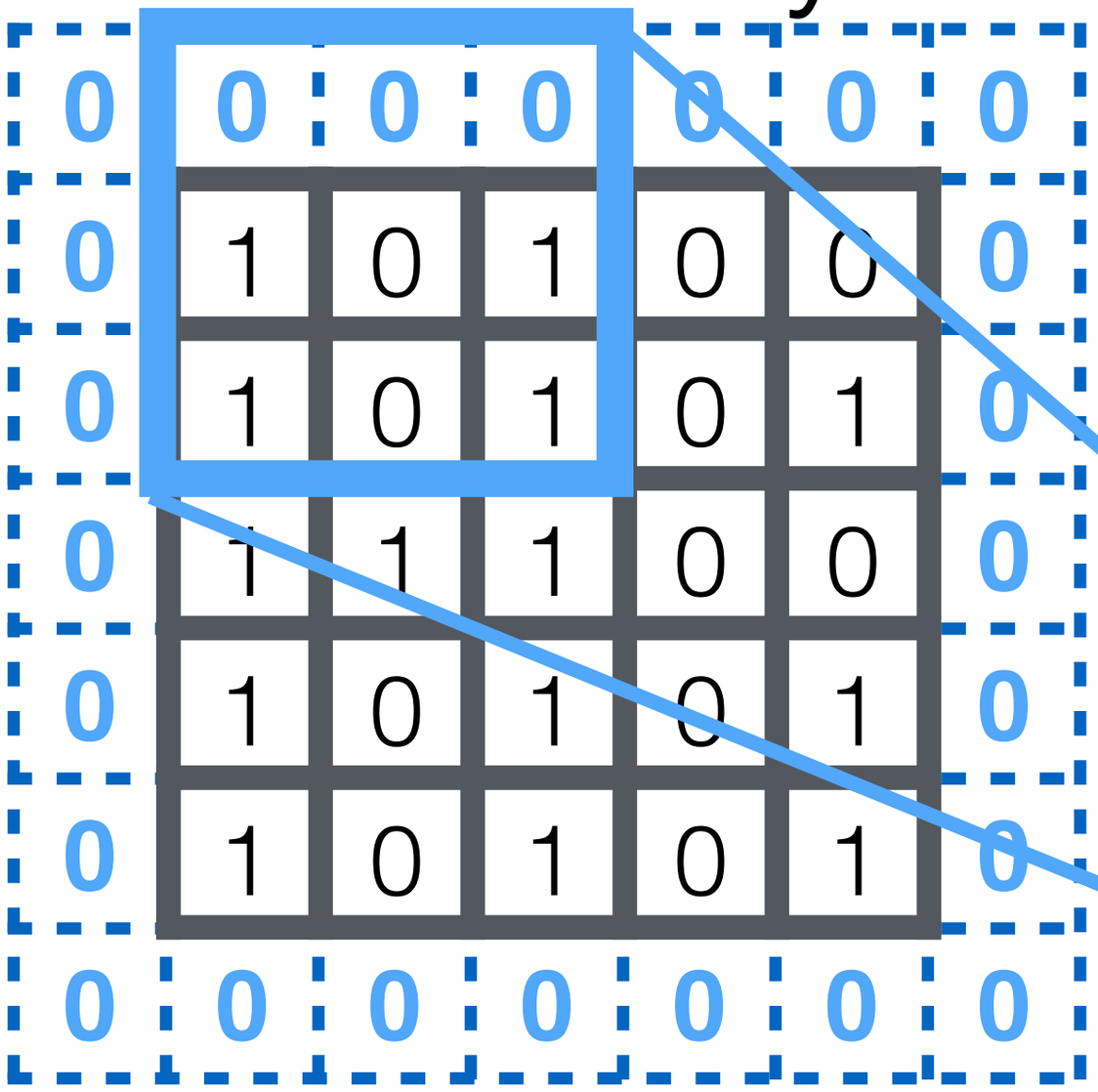
-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

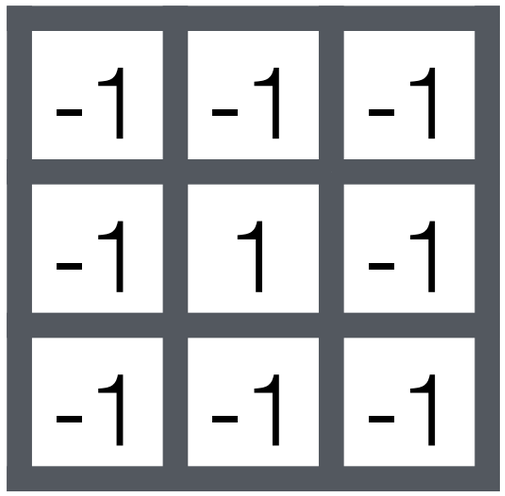
-7	-2	-4
-5	-2	-5
-7	-2	-5

Convolutional Layer: 2D example

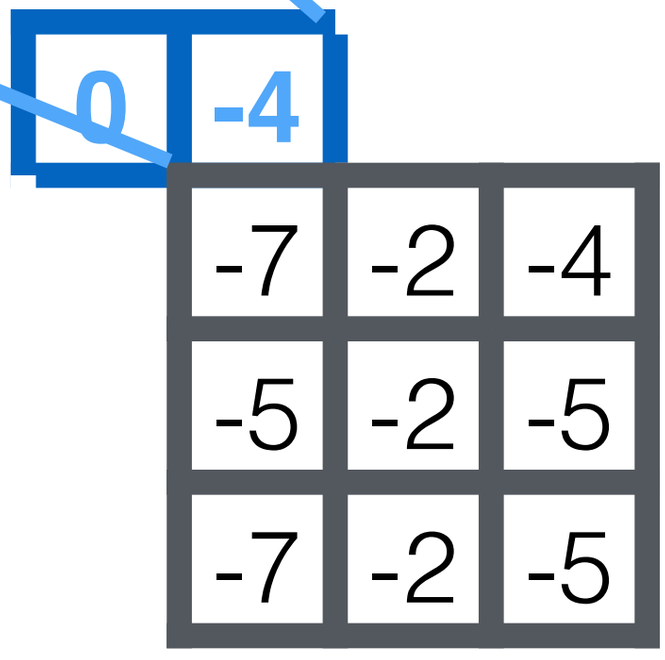
A 2D image:



A filter:

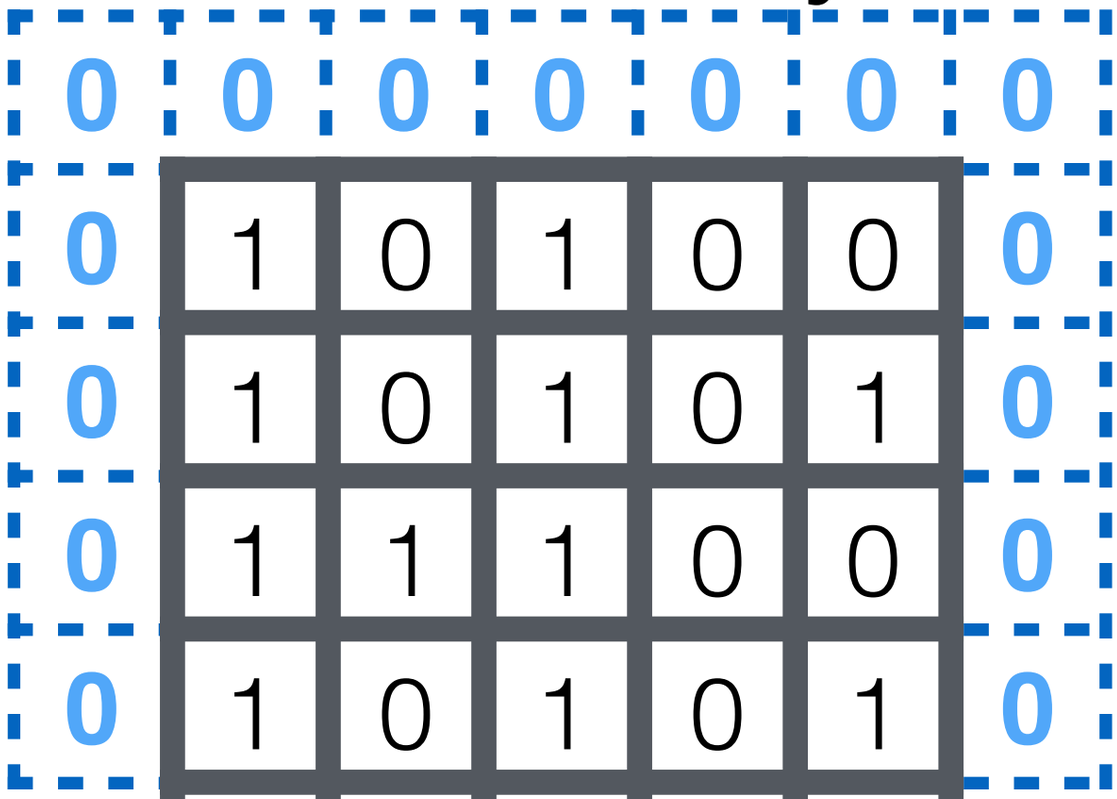


After convolution:

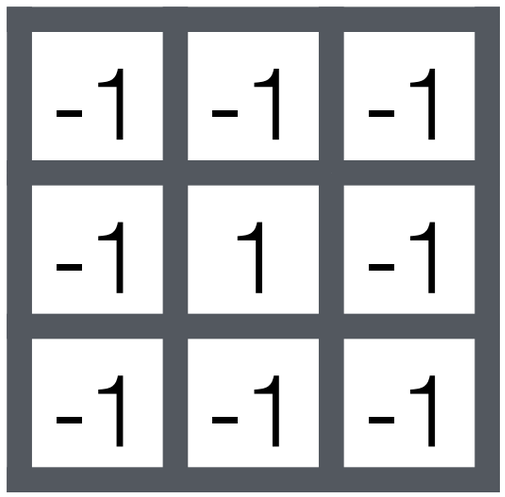


Convolutional Layer: 2D example

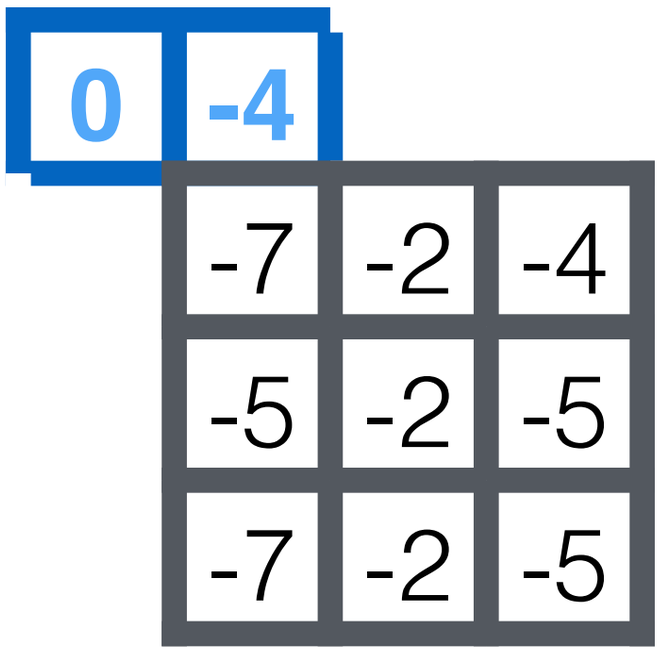
A 2D image:



A filter:



After convolution:



Convolutional Layer: 2D example

A 2D image:

0	0	0	0	0	0	0
0	1	0	1	0	0	0
0	1	0	1	0	1	0
0	1	1	1	0	0	0
0	1	0	1	0	1	0
0	1	0	1	0	1	0
0	0	0	0	0	0	0

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution:

0	-4	0	-3	-1
-2	-7	-2	-4	1
-2	-5	-2	-5	-2
-2	-7	-2	-5	0
0	-4	0	-4	0

Convolutional Layer: 2D example

A 2D image:

0	0	0	0	0	0	0
0	1	0	1	0	0	0
0	1	0	1	0	1	0
0	1	1	1	0	0	0
0	1	0	1	0	1	0
0	1	0	1	0	1	0
0	0	0	0	0	0	0

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution & ReLU:

0	-4	0	-3	-1
-2	-7	-2	-4	1
-2	-5	-2	-5	-2
-2	-7	-2	-5	0
0	-4	0	-4	0

Convolutional Layer: 2D example

A 2D image:

0	0	0	0	0	0	0
0	1	0	1	0	0	0
0	1	0	1	0	1	0
0	1	1	1	0	0	0
0	1	0	1	0	1	0
0	1	0	1	0	1	0
0	0	0	0	0	0	0

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution & ReLU:

0	-4	0	-3	-1
-2	-7	-2	-4	1
-2	-5	-2	-5	-2
-2	-7	-2	-5	0
0	-4	0	-4	0

Convolutional Layer: 2D example

A 2D image:

0	0	0	0	0	0	0
0	1	0	1	0	0	0
0	1	0	1	0	1	0
0	1	1	1	0	0	0
0	1	0	1	0	1	0
0	1	0	1	0	1	0
0	0	0	0	0	0	0

A filter:

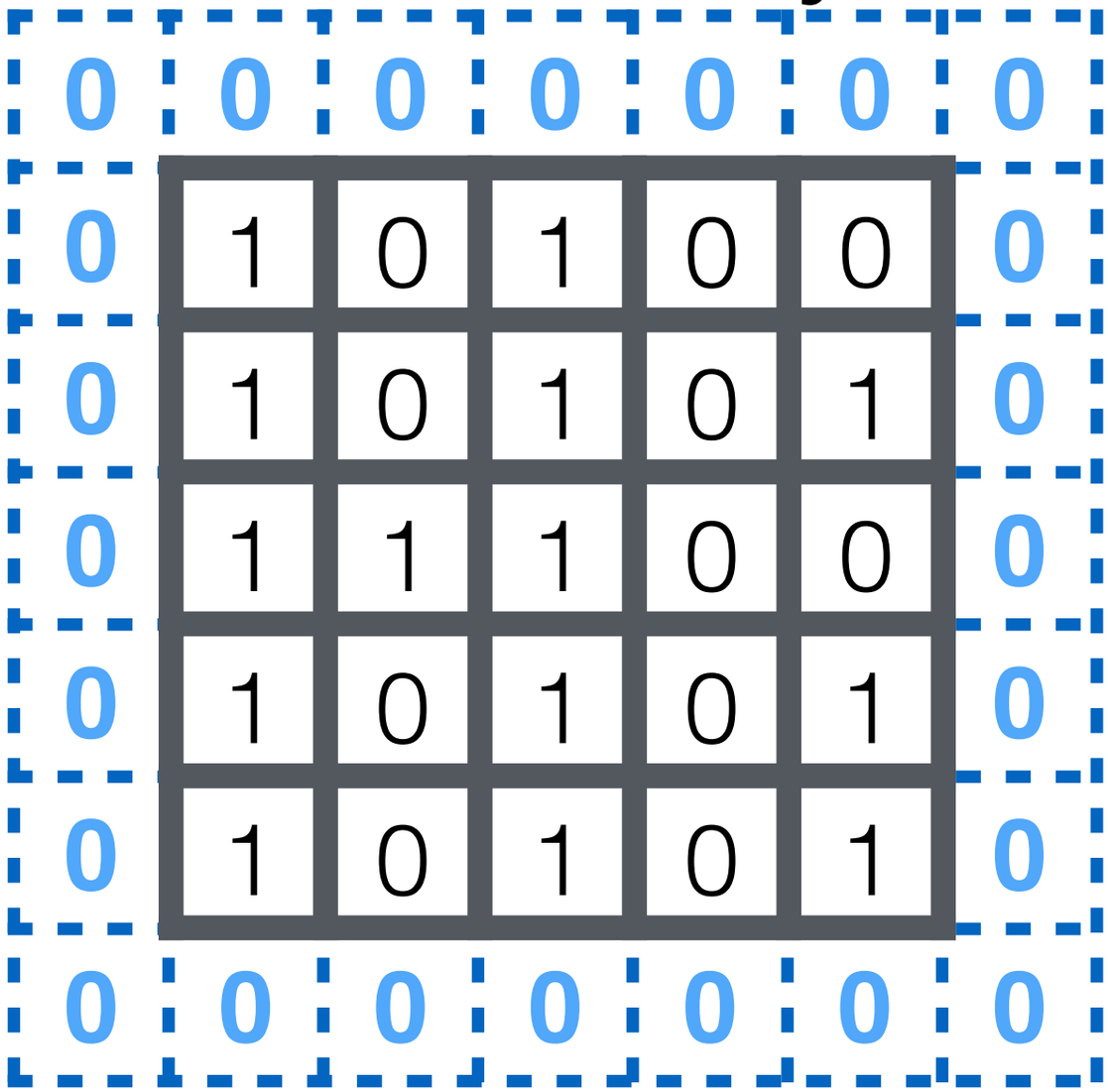
-1	-1	-1
-1	1	-1
-1	-1	-1

After convolution & ReLU:

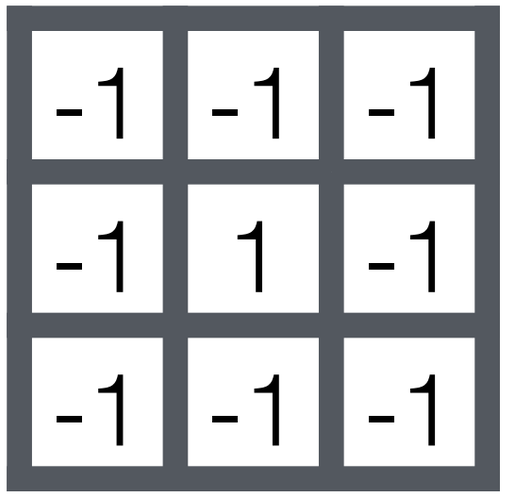
0	-4	0	-3	-1
-2	-7	-2	-4	1
-2	-5	-2	-5	-2
-2	-7	-2	-5	0
0	-4	0	-4	0

Convolutional Layer: 2D example

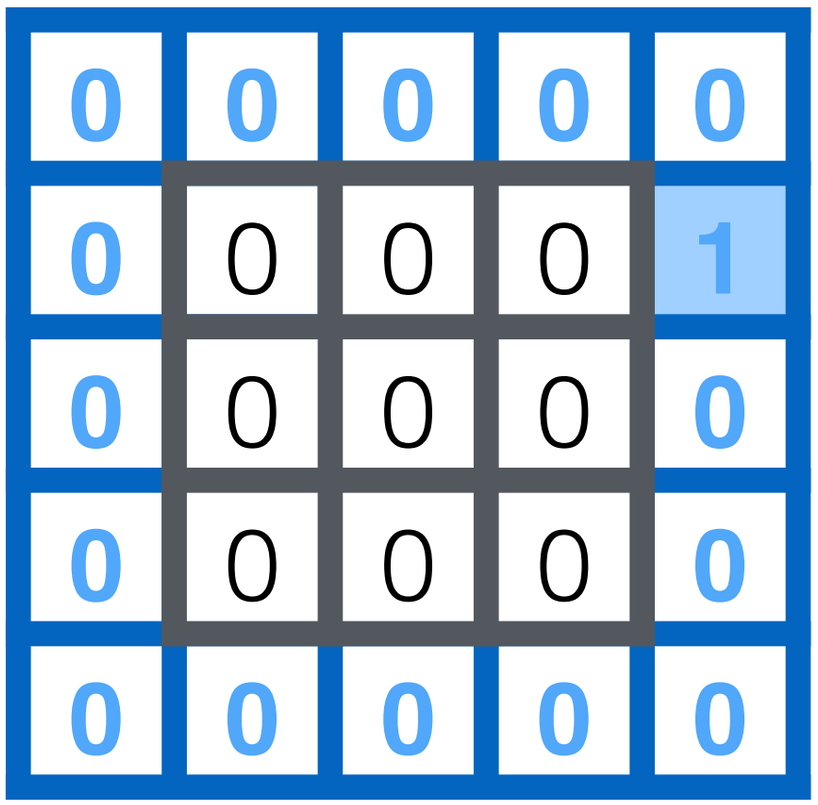
A 2D image:



A filter:

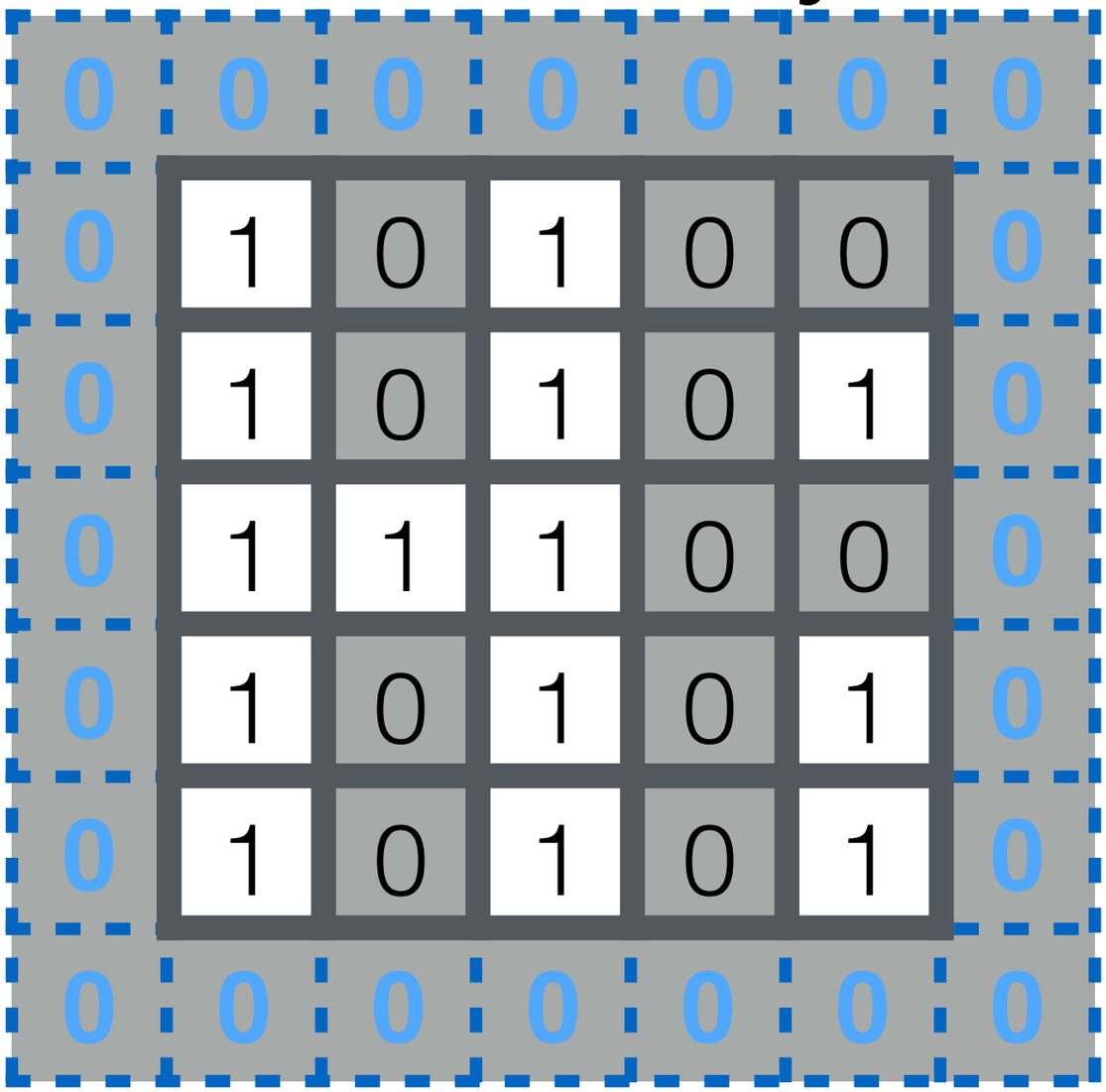


After convolution & ReLU:

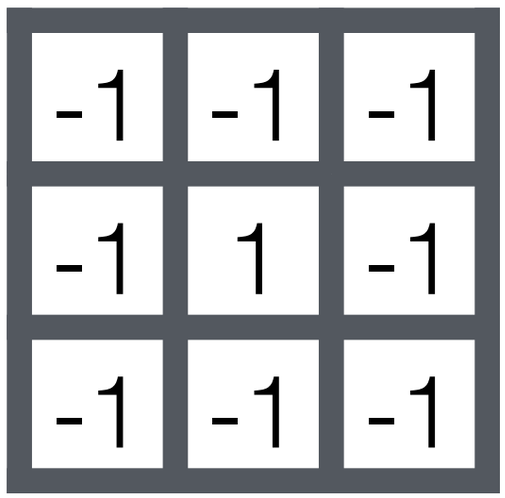


Convolutional Layer: 2D example

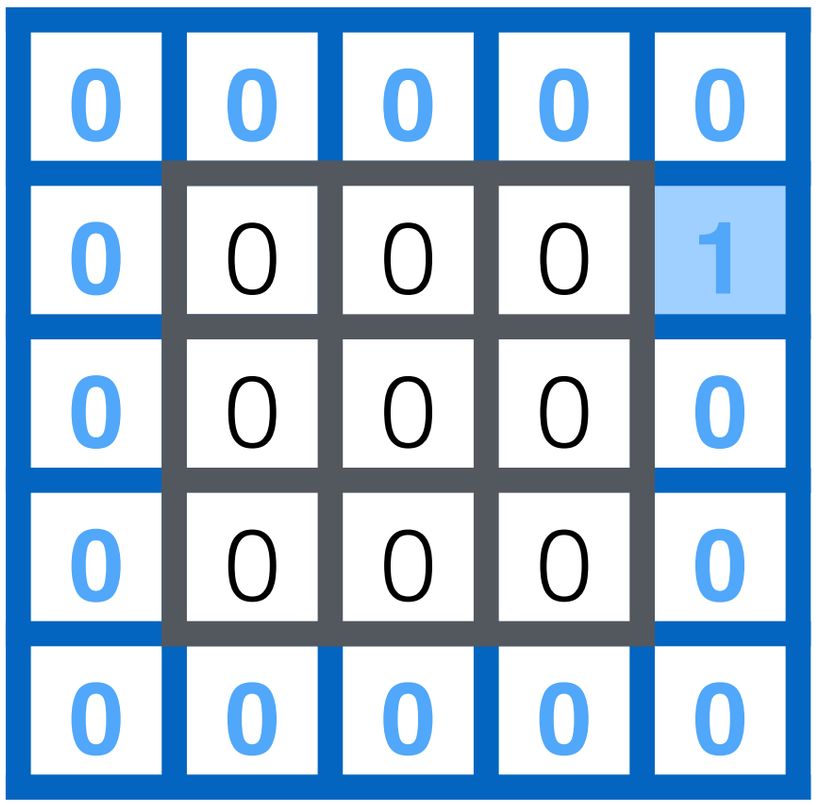
A 2D image:



A filter:

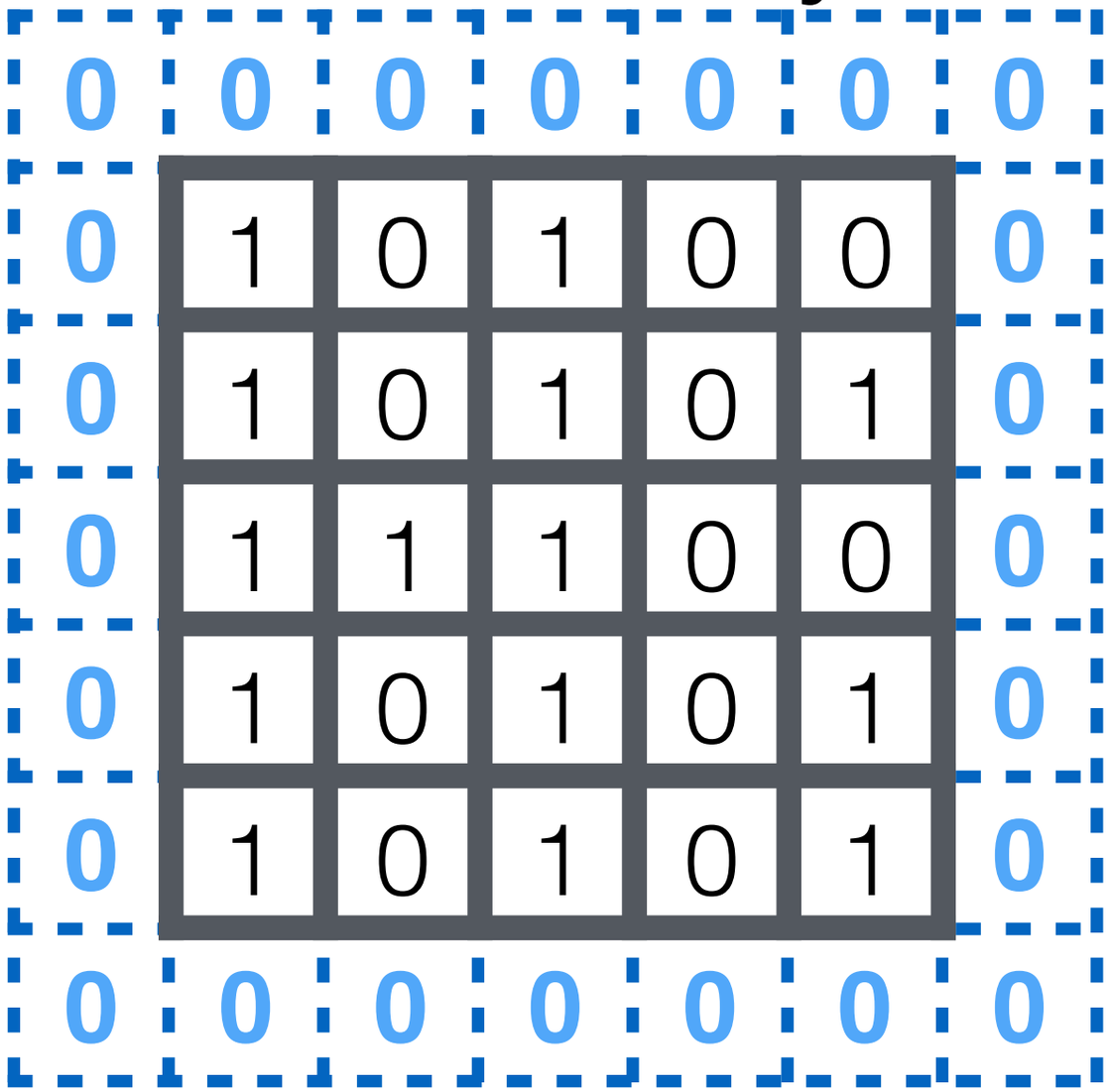


After convolution & ReLU:

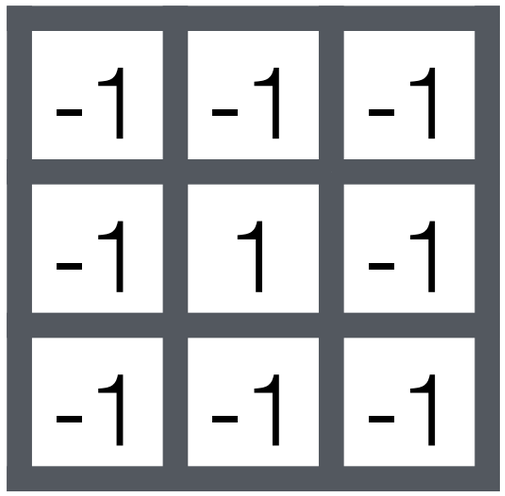


Convolutional Layer: 2D example

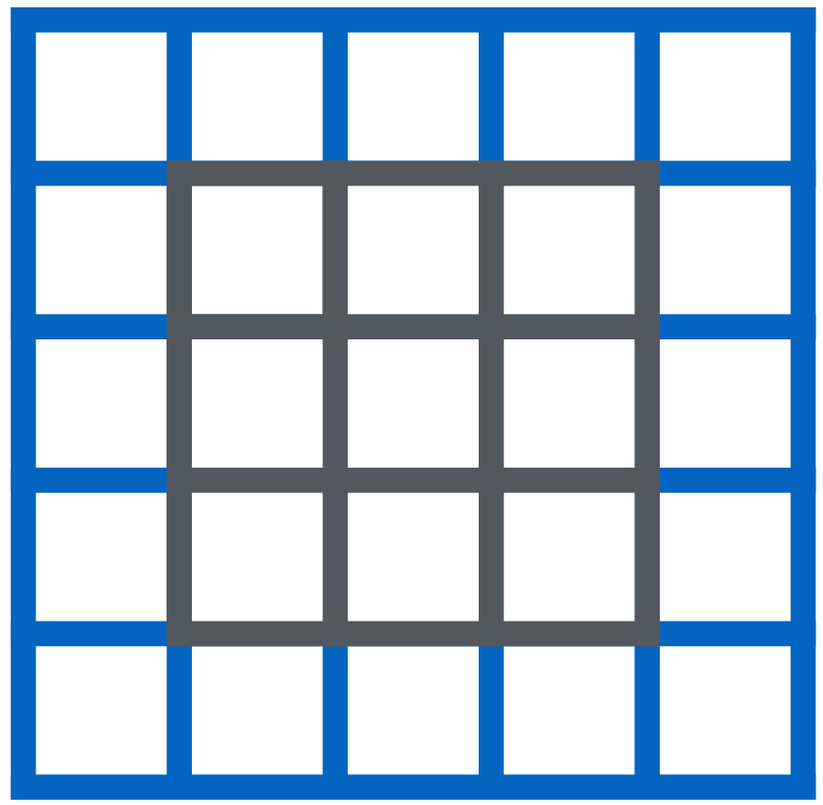
A 2D image:



A filter:

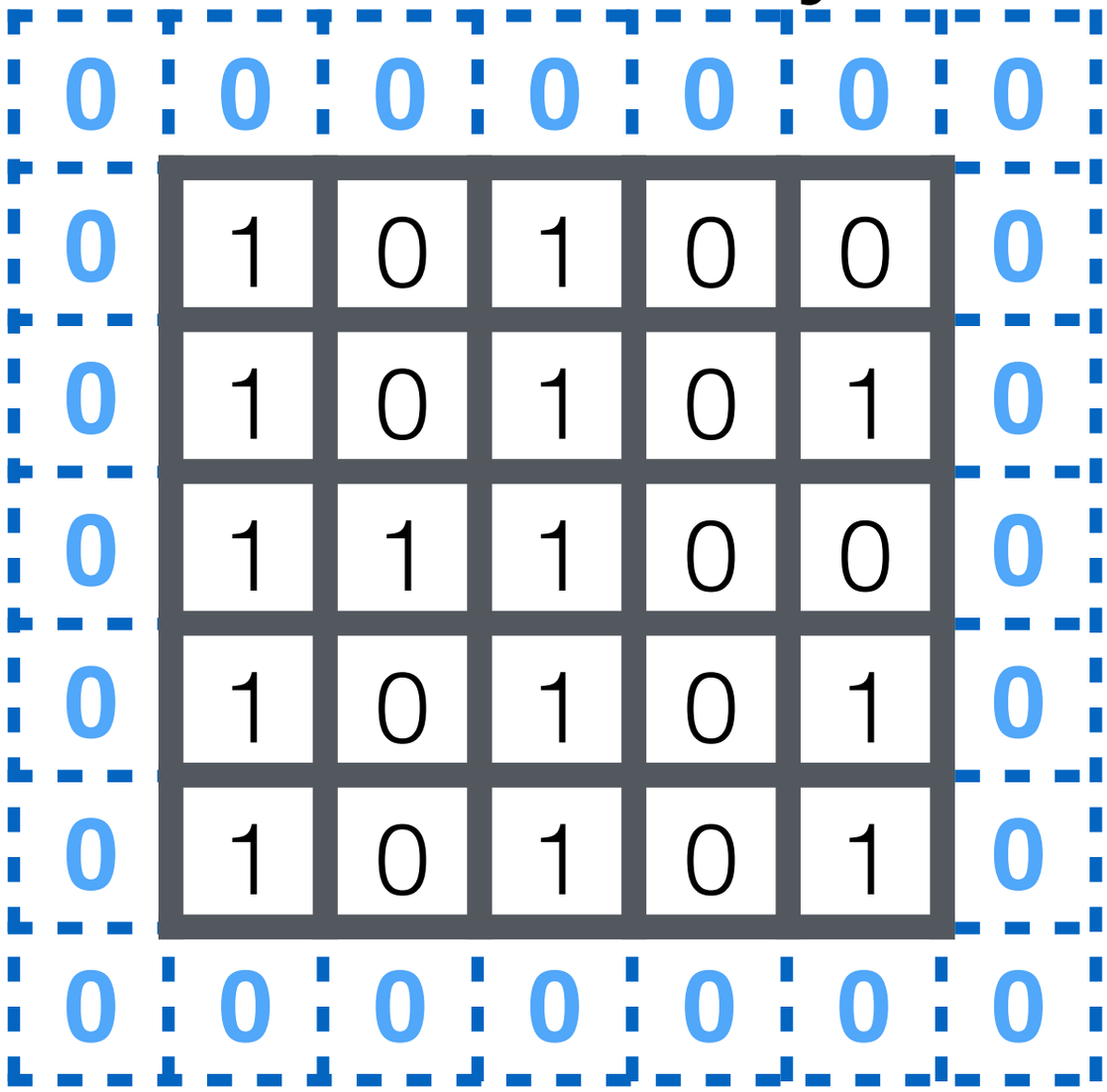


After convolution:

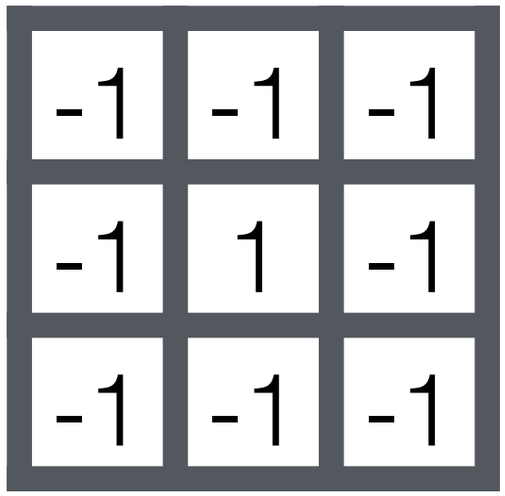


Convolutional Layer: 2D example

A 2D image:

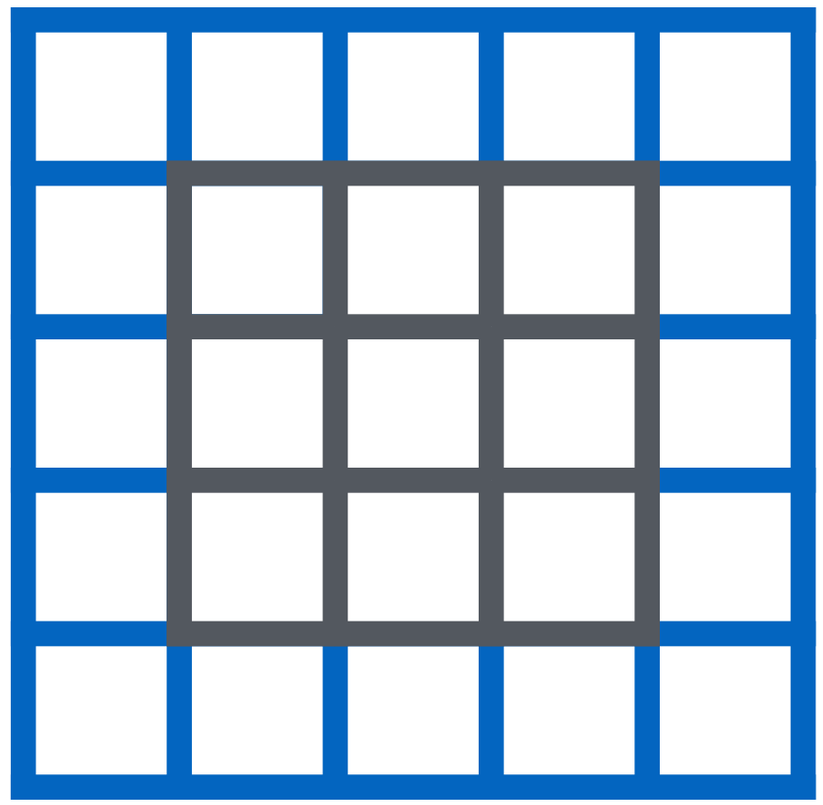


A filter:



with bias 2

After convolution:



Convolutional Layer: 2D example

A 2D image:

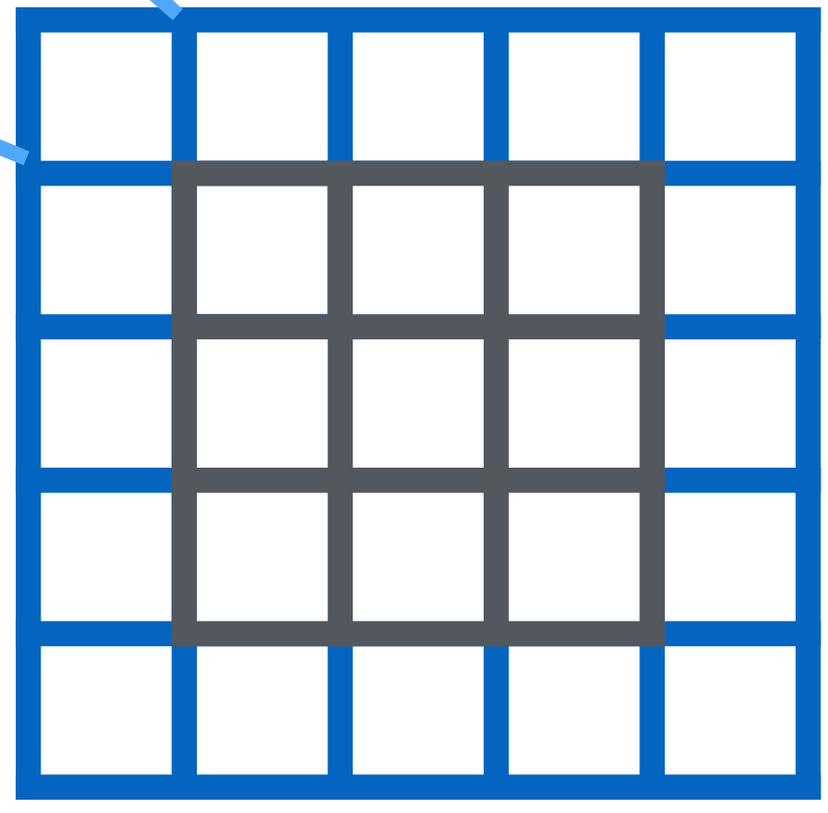
0	0	0	0	0	0	0
0	1	0	1	0	0	0
0	1	0	1	0	1	0
0	1	1	1	0	0	0
0	1	0	1	0	1	0
0	1	0	1	0	1	0
0	0	0	0	0	0	0

A filter:

-1	-1	-1
-1	1	-1
-1	-1	-1

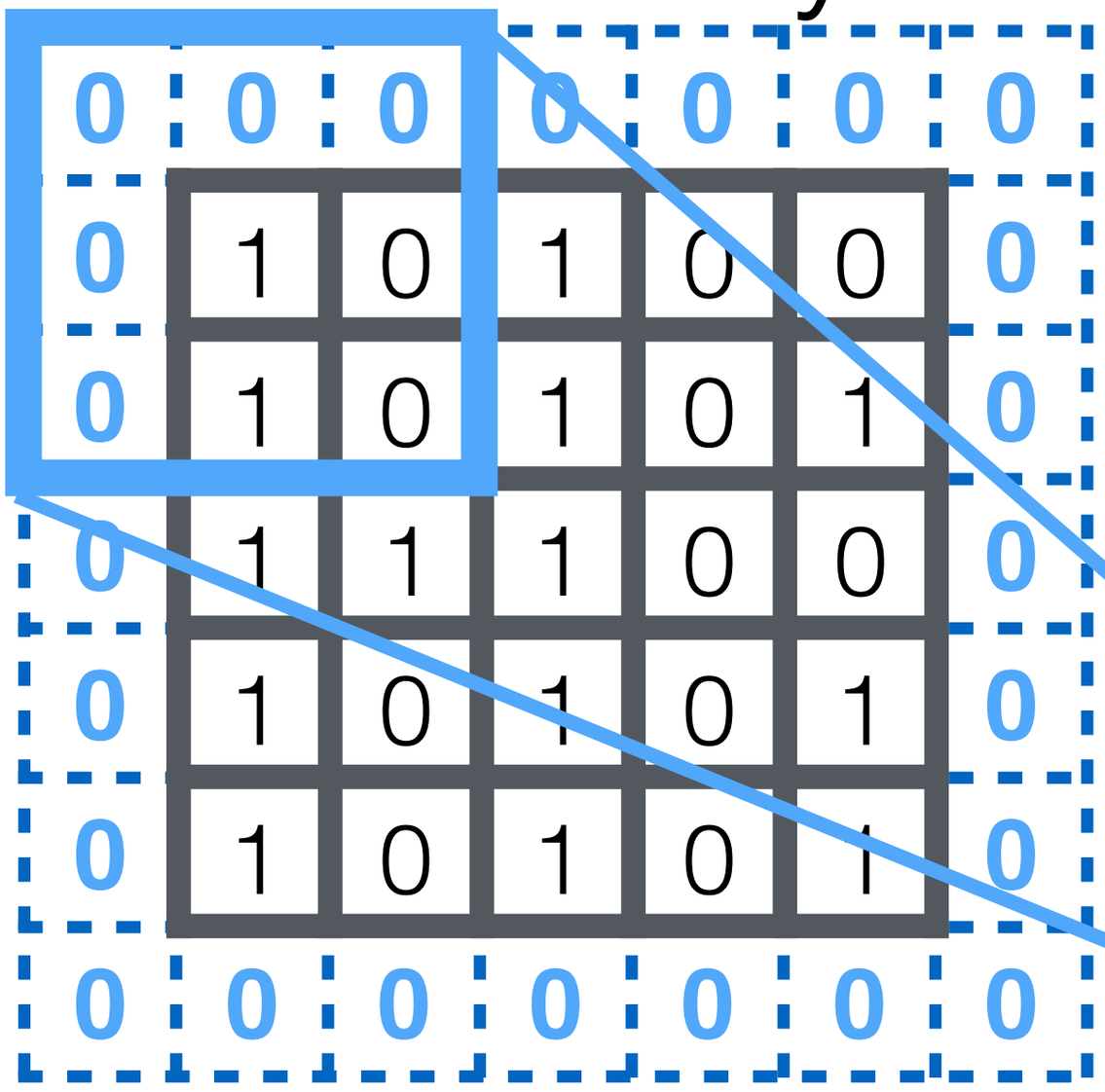
with bias 2

After convolution:

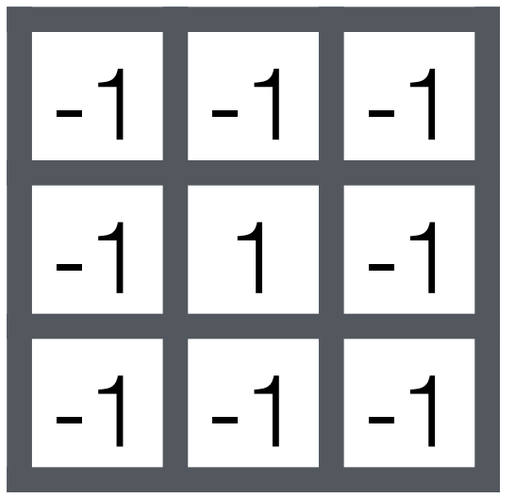


Convolutional Layer: 2D example

A 2D image:

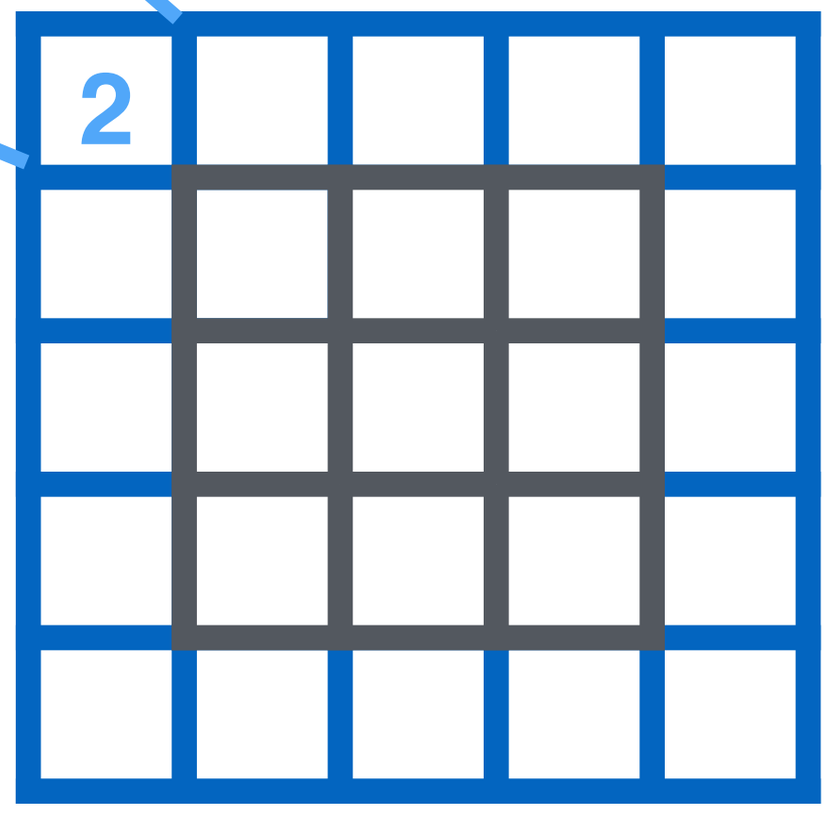


A filter:



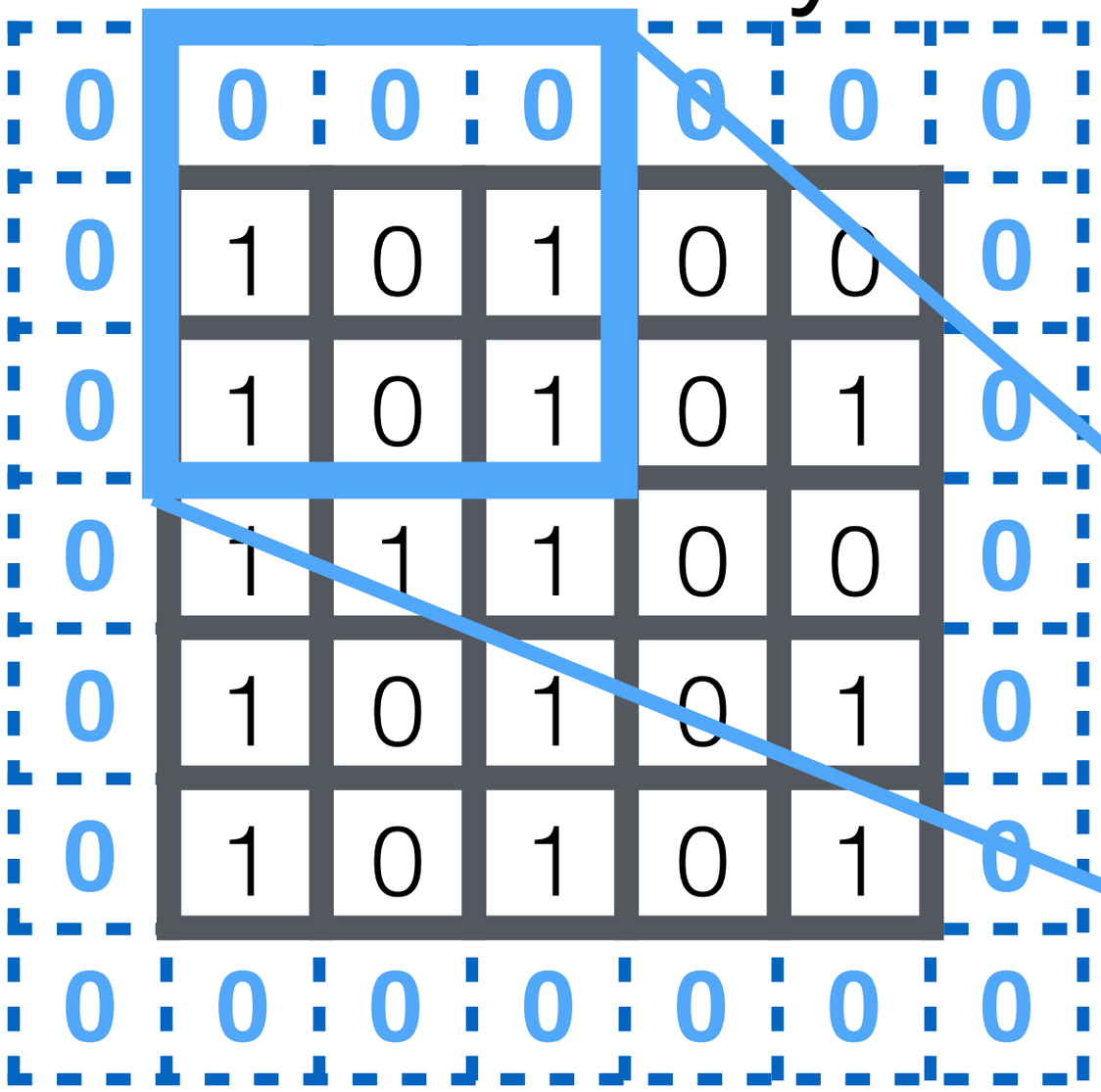
with bias 2

After convolution:

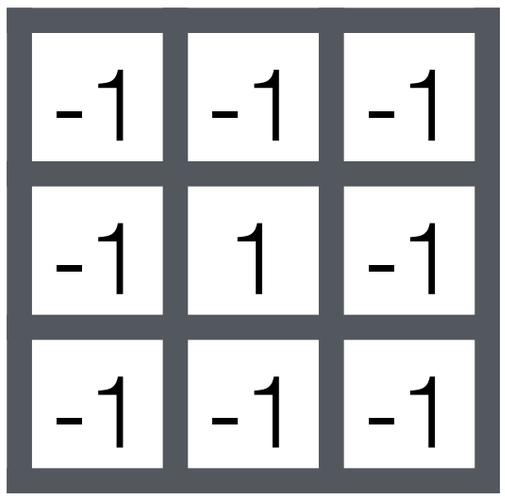


Convolutional Layer: 2D example

A 2D image:

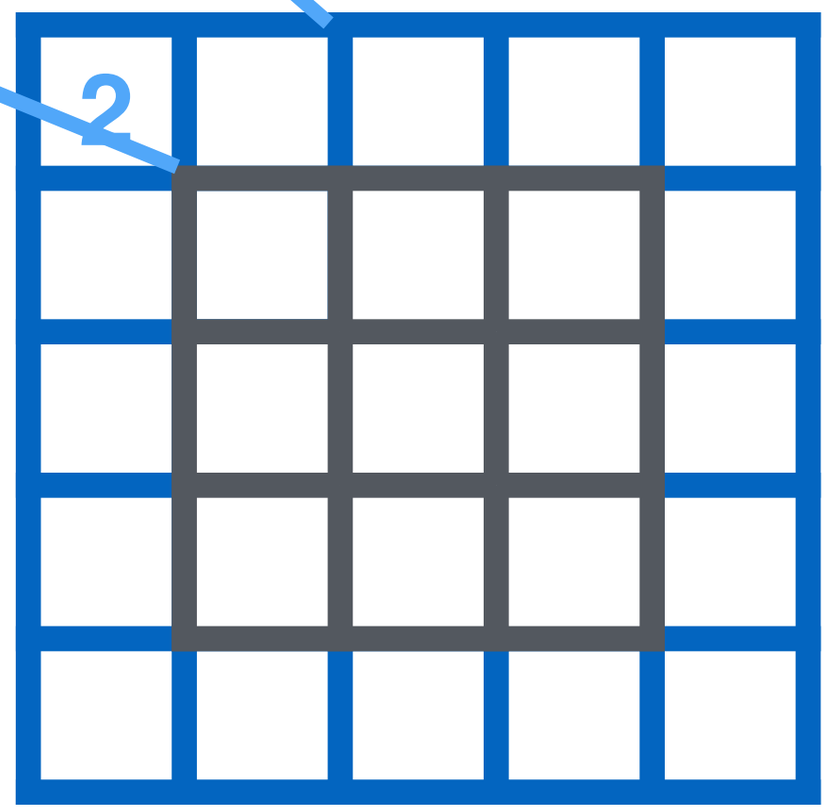


A filter:



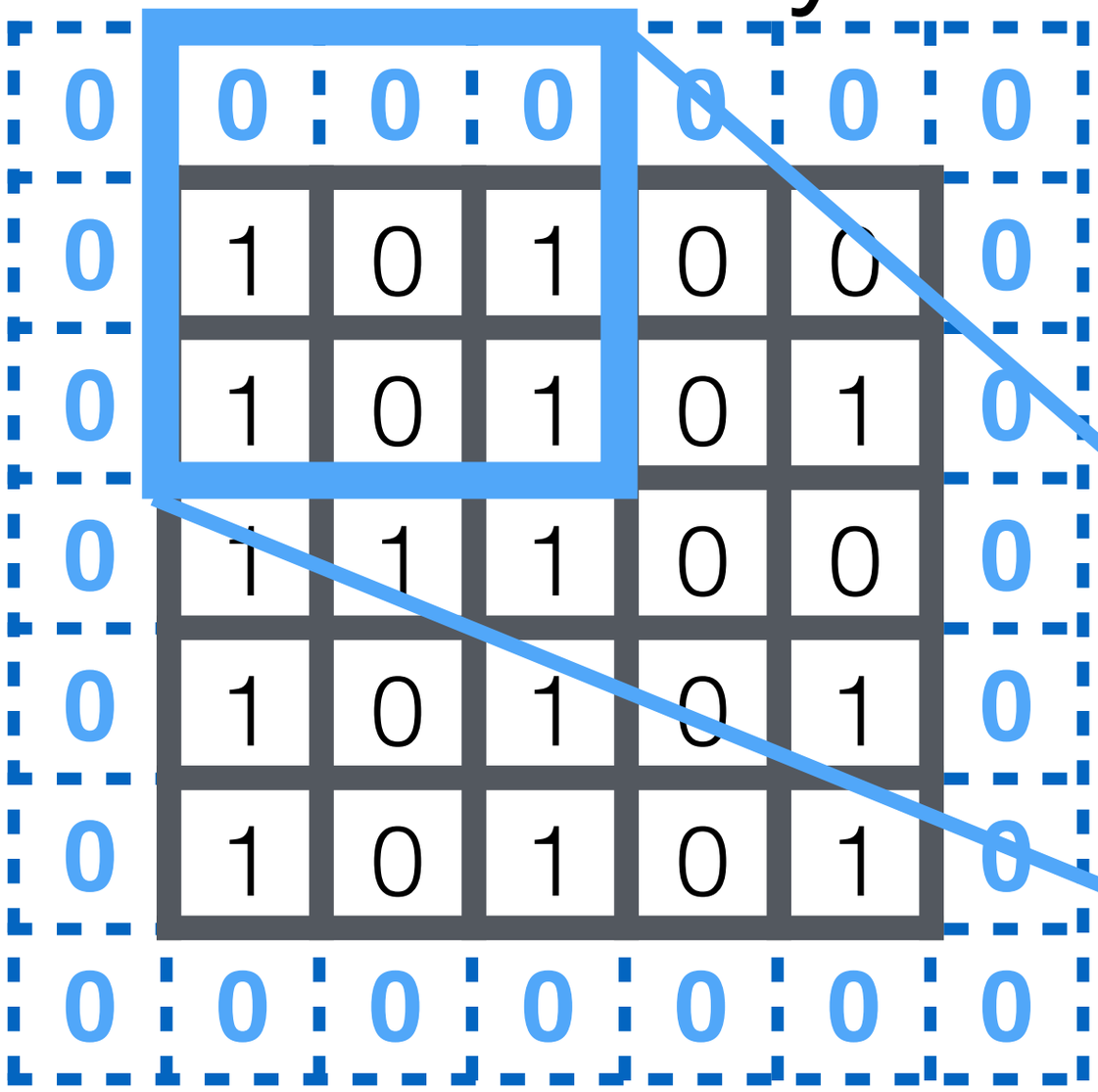
with bias 2

After convolution:

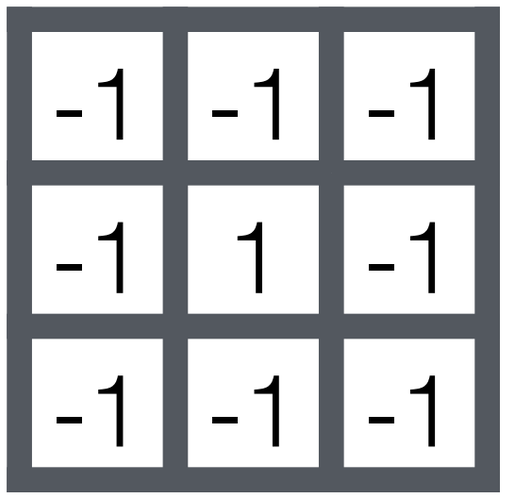


Convolutional Layer: 2D example

A 2D image:

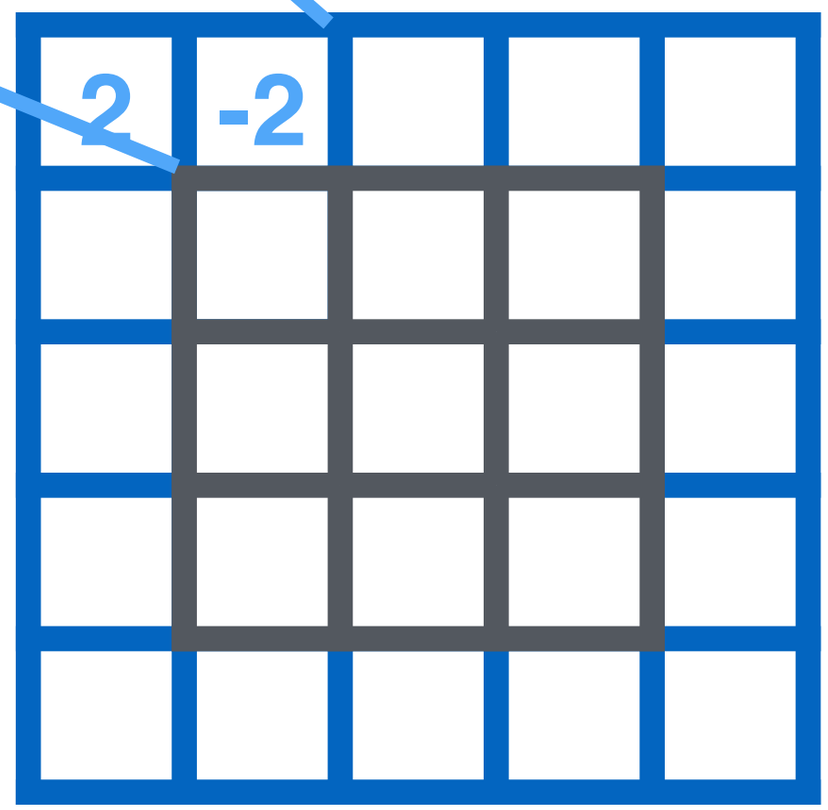


A filter:



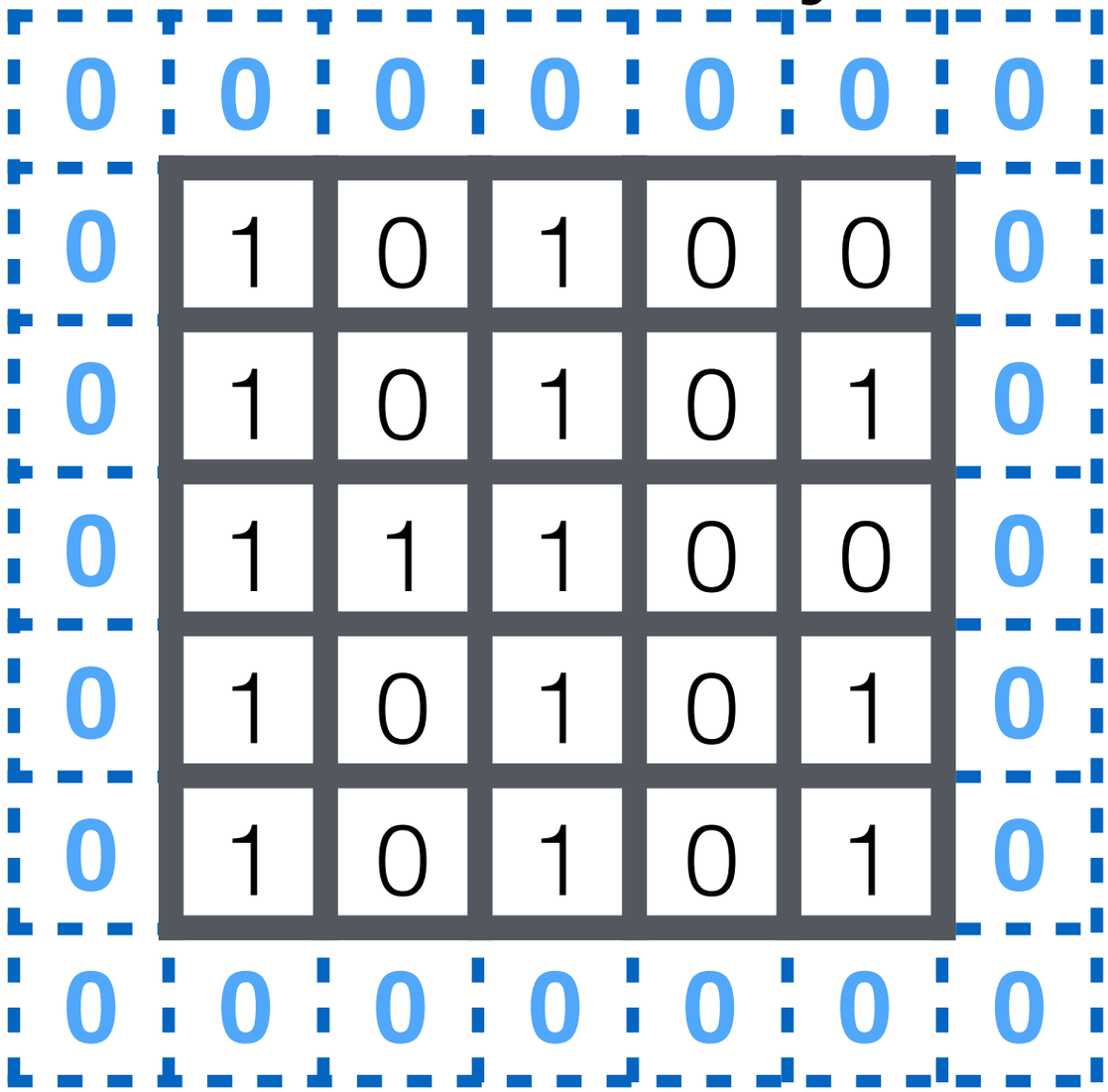
with bias 2

After convolution:

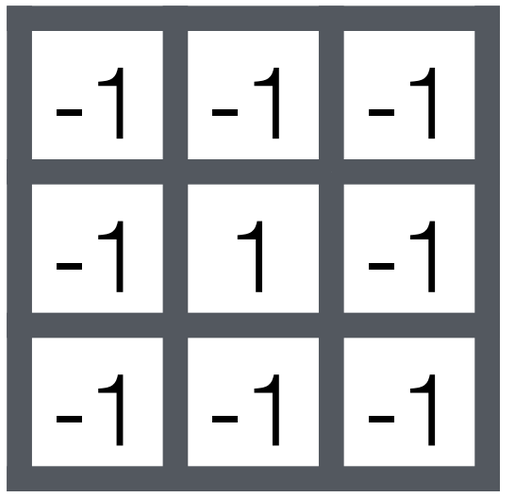


Convolutional Layer: 2D example

A 2D image:

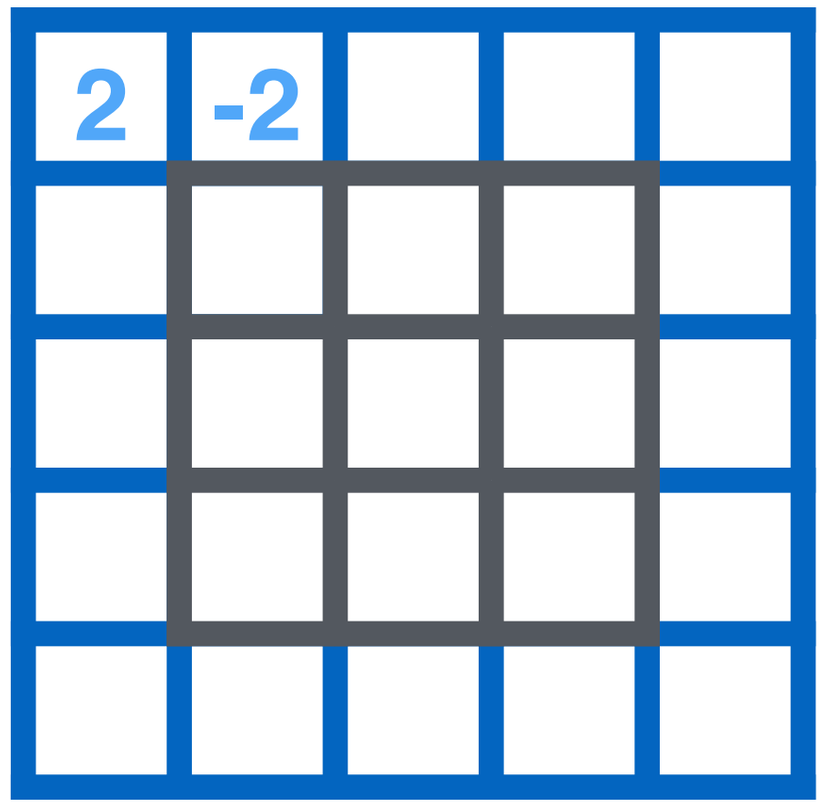


A filter:



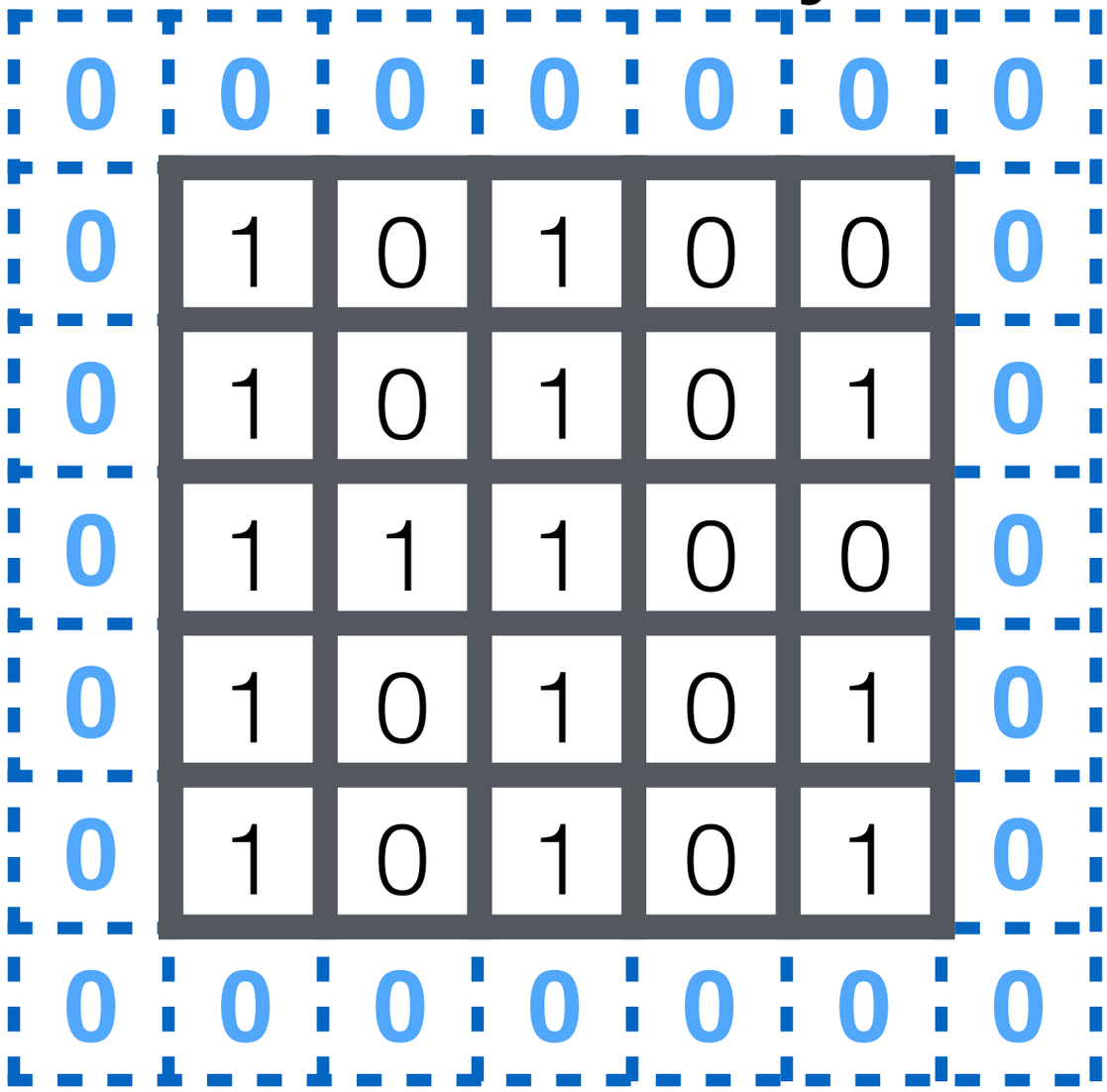
with bias 2

After convolution:

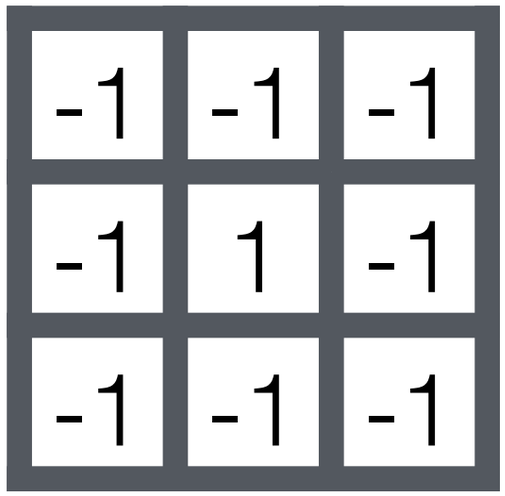


Convolutional Layer: 2D example

A 2D image:

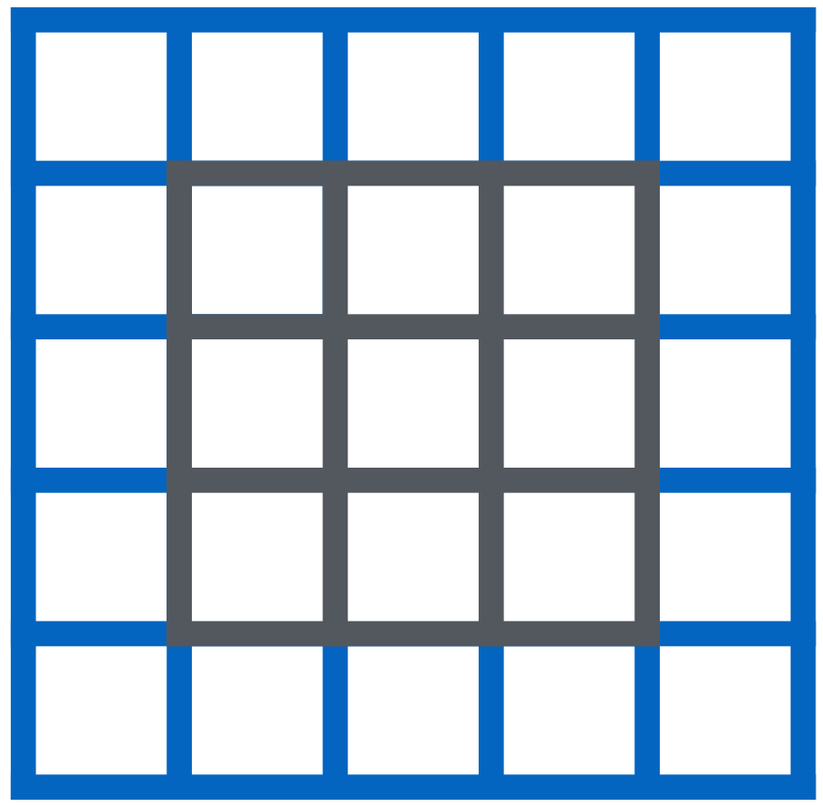


A filter:



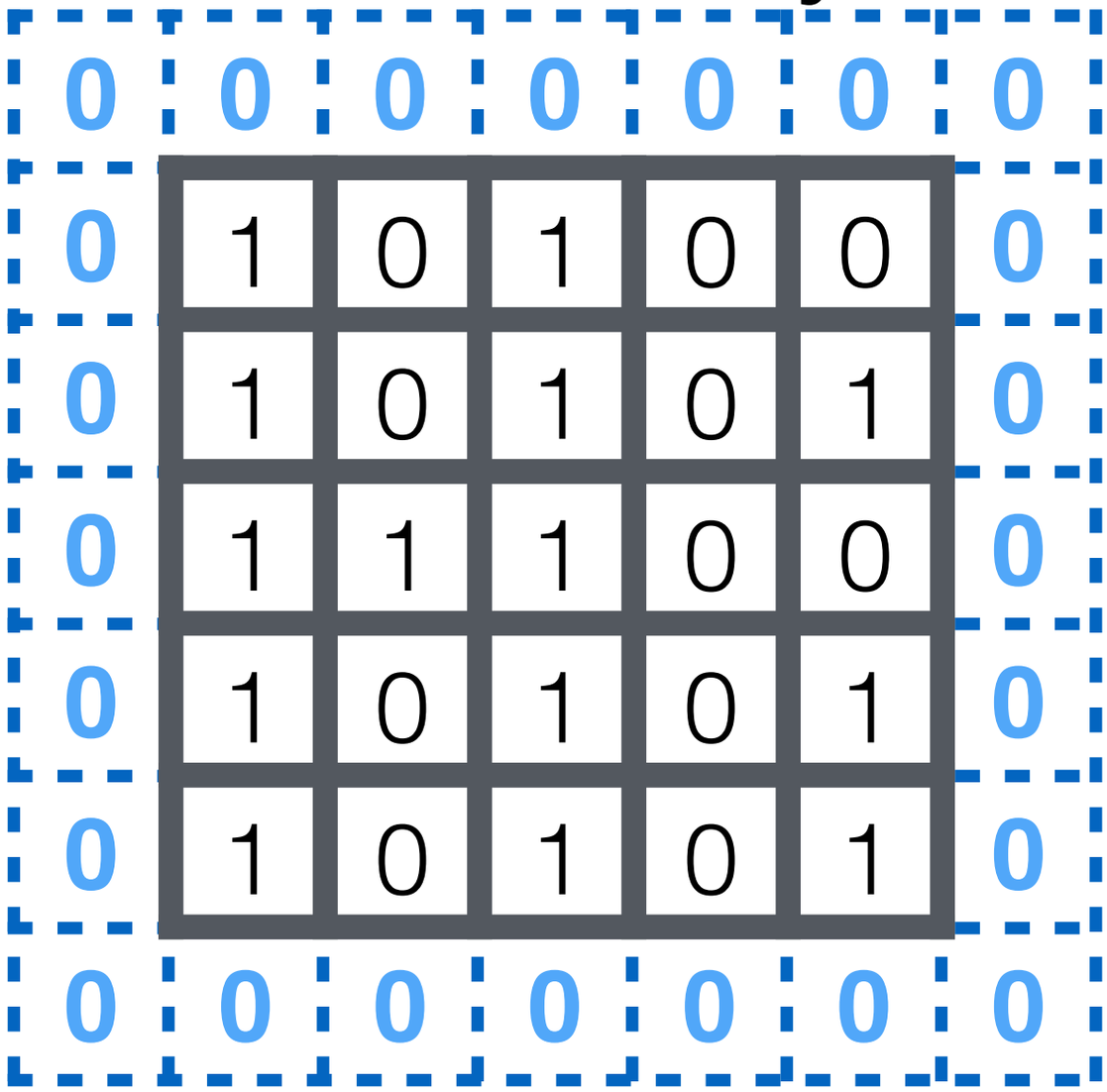
with bias 2

After convolution:

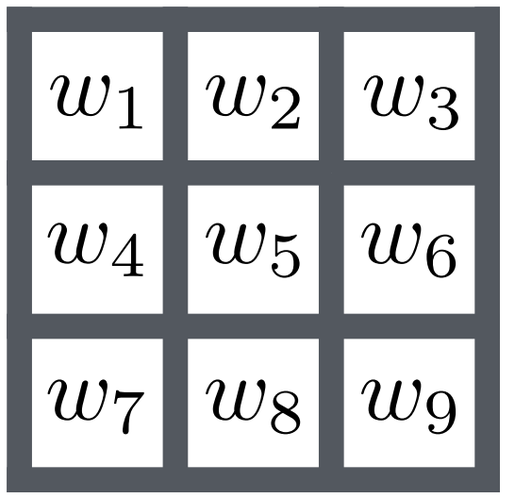


Convolutional Layer: 2D example

A 2D image:

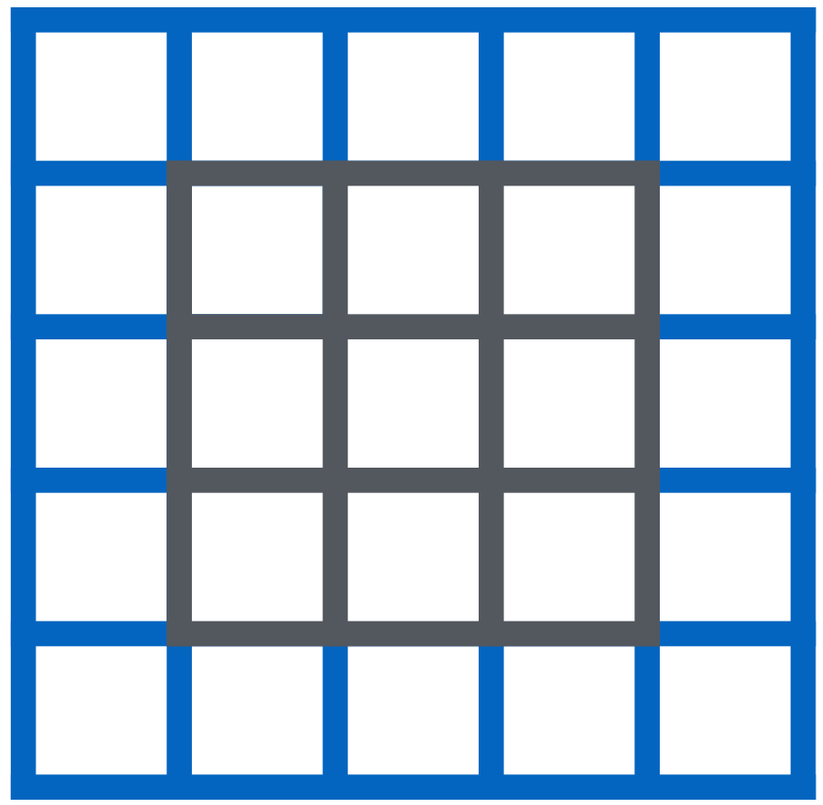


A filter:



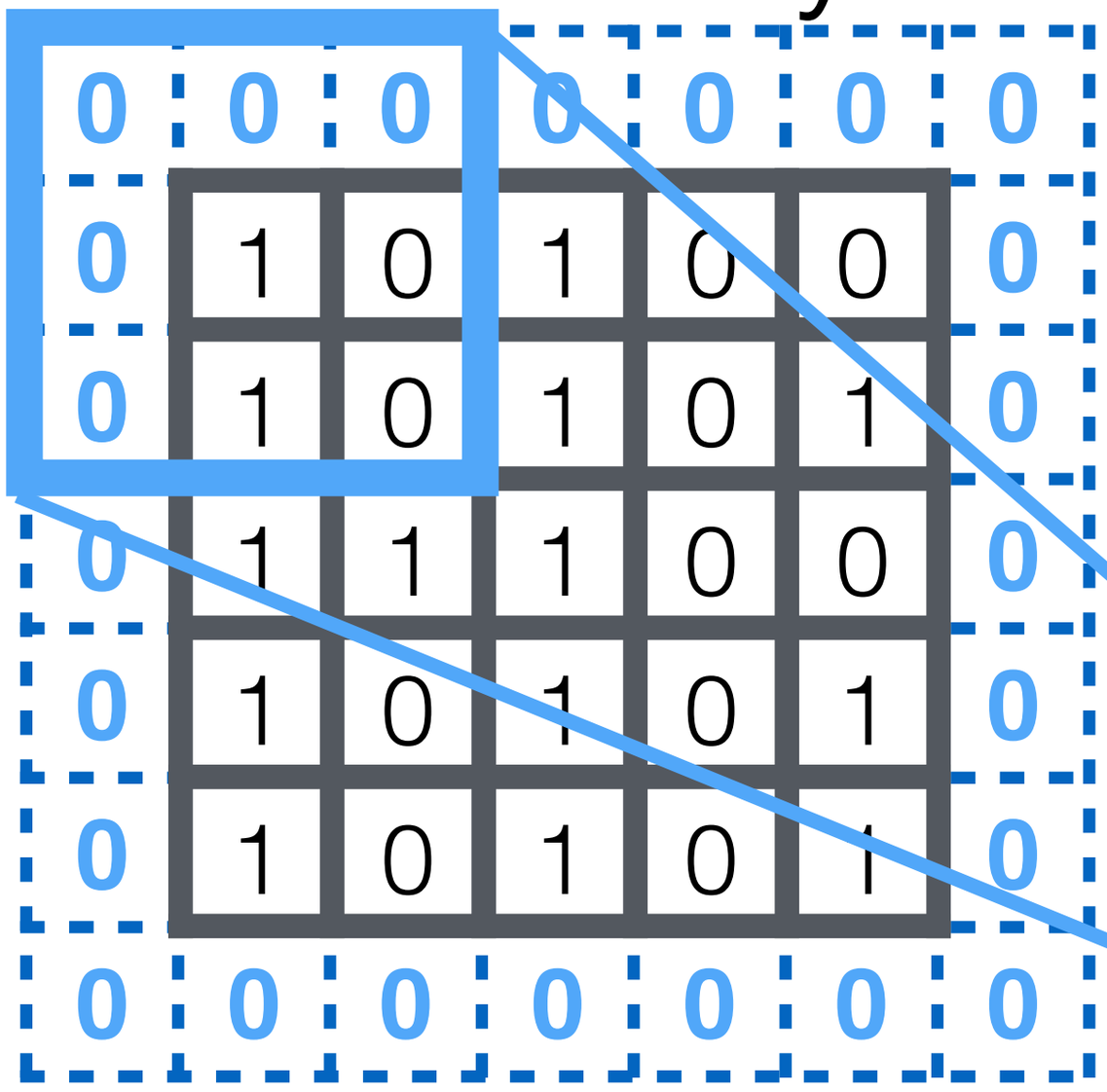
with bias b

After convolution:

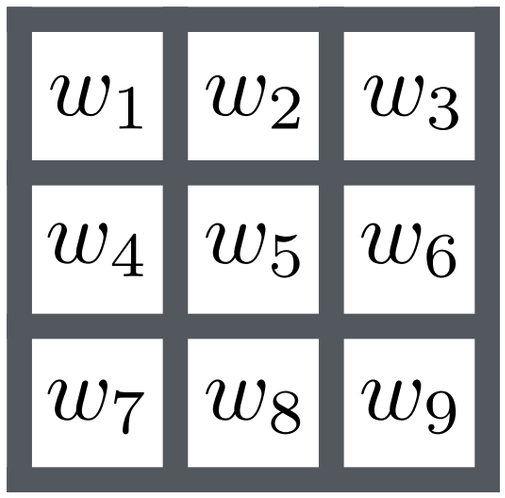


Convolutional Layer: 2D example

A 2D image:

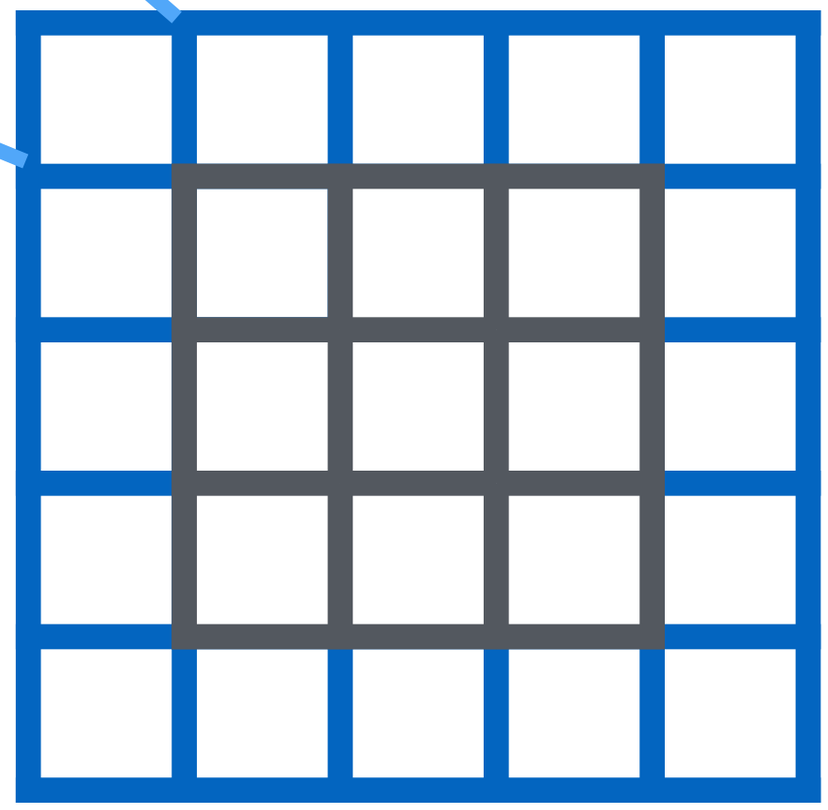


A filter:



with bias b

After convolution:



Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

with bias b

Convolutional Layer: 2D example

A 2D image:

1	0	1	0	0
1	0	1	0	1
1	1	1	0	0
1	0	1	0	1
1	0	1	0	1

A filter:

w_1	w_2
w_3	w_4

with bias b

Convolutional Layer: 3D example

A 3D image:

[https://helpx.adobe.com/photoshop/key-concepts/skew.html]

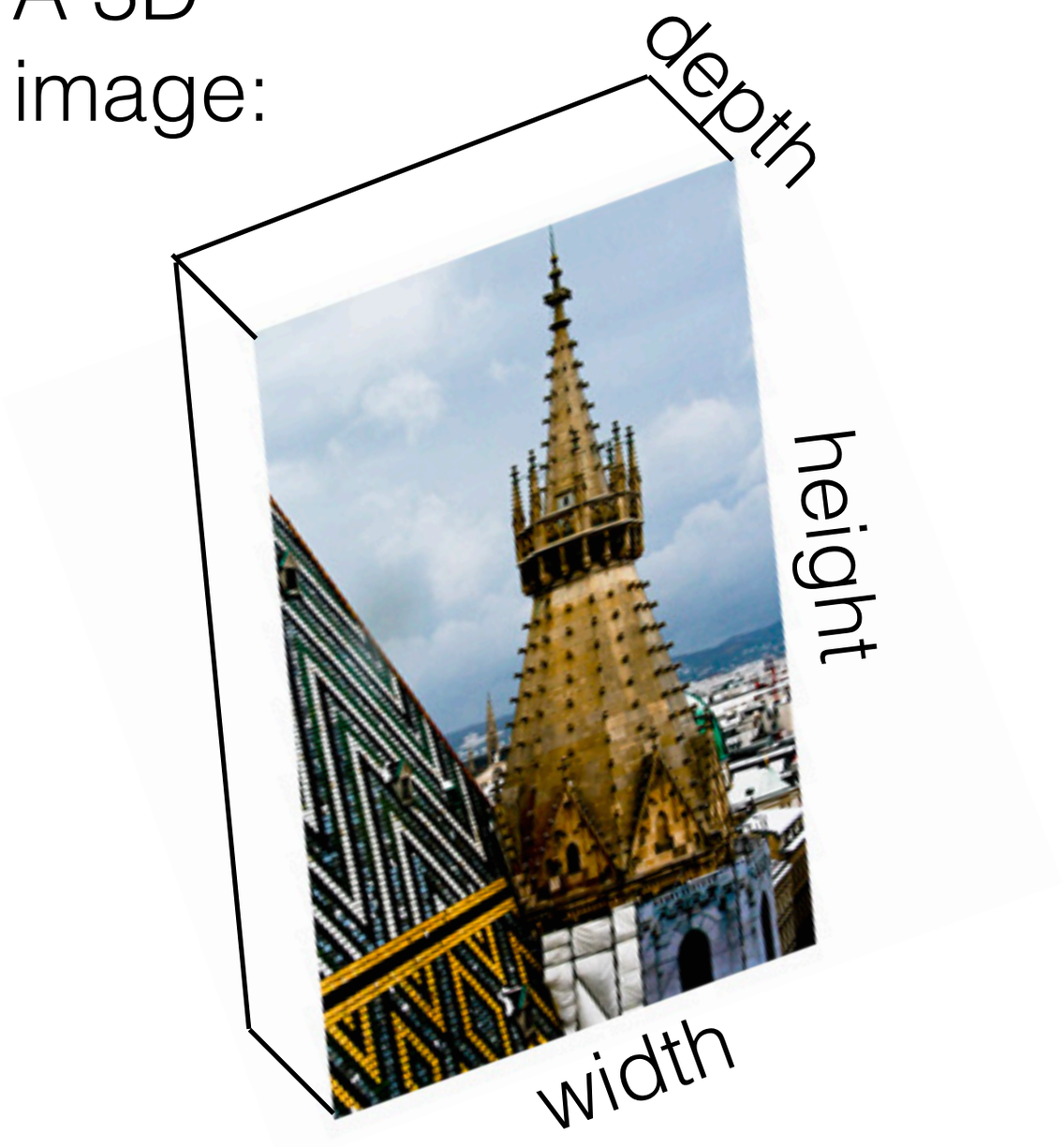
Convolutional Layer: 3D example

A 3D image:

[https://helpx.adobe.com/photoshop/key-concepts/skew.html]

Convolutional Layer: 3D example

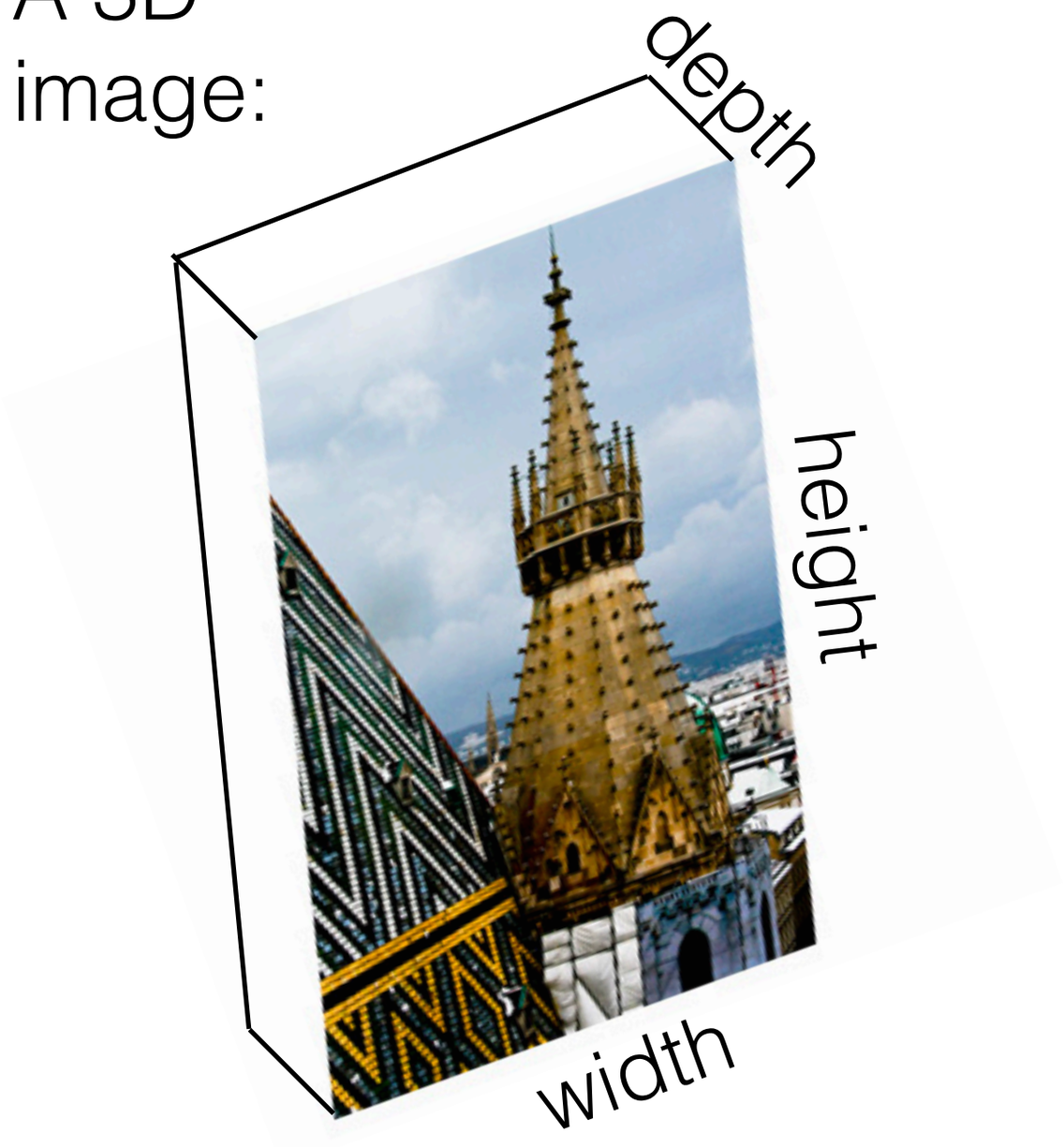
A 3D image:



[https://helpx.adobe.com/photoshop/key-concepts/skew.html]

Convolutional Layer: 3D example

A 3D image:

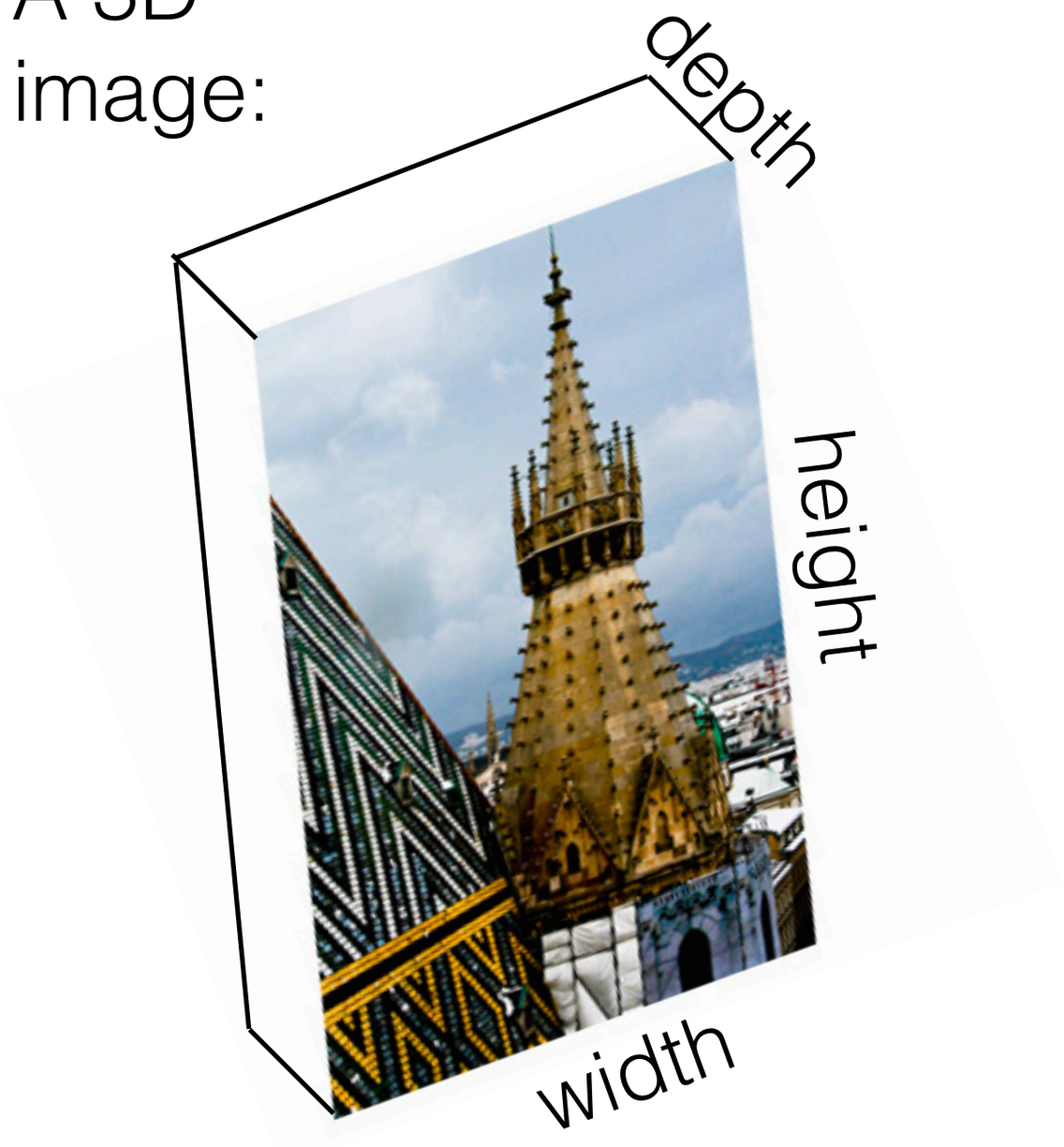


[https://helpx.adobe.com/photoshop/key-concepts/skew.html]

- Tensor: generalization of a matrix
  - E.g. 1D: vector, 2D: matrix

Convolutional Layer: 3D example

A 3D image:



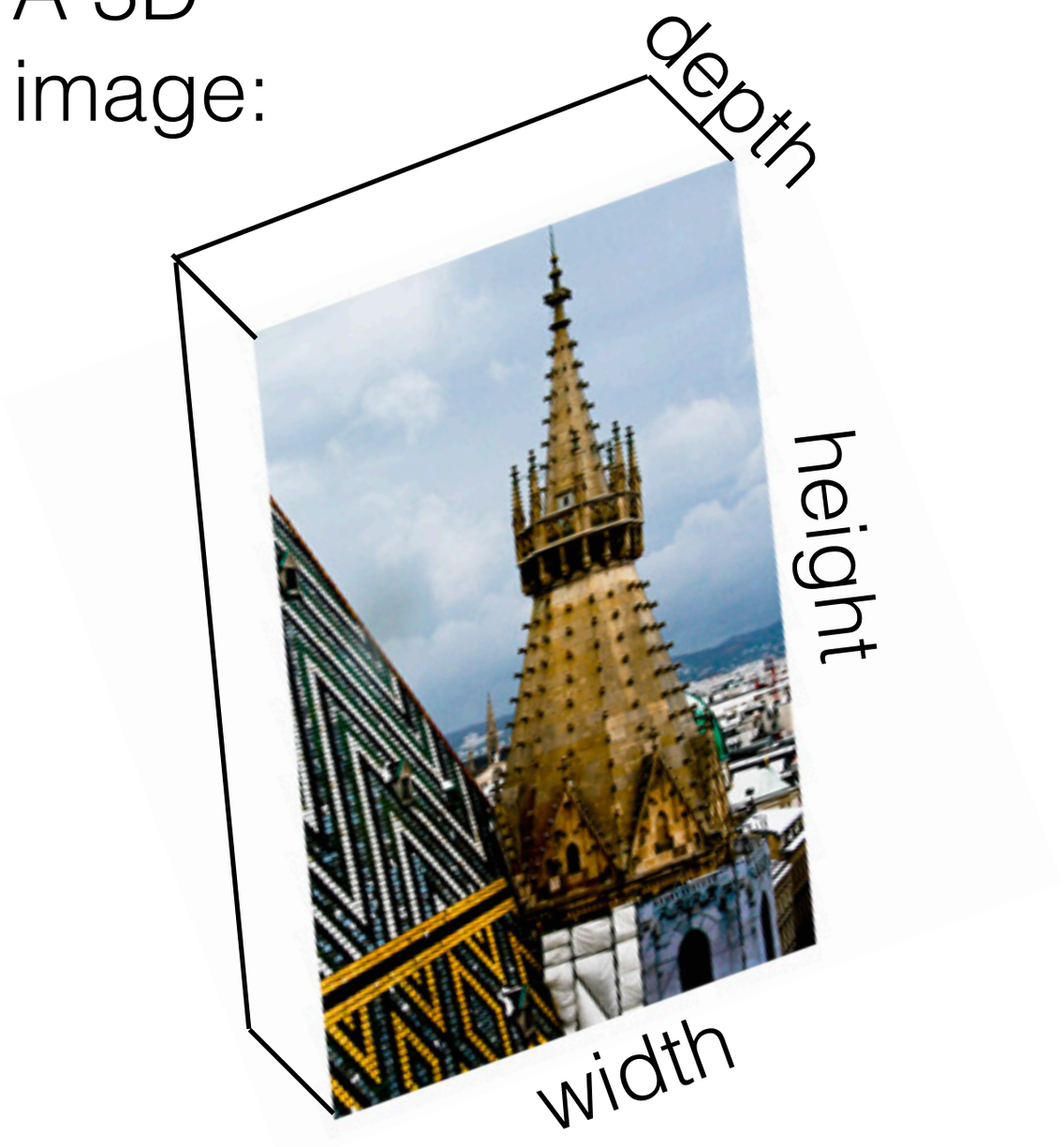
[https://helpx.adobe.com/photoshop/key-concepts/skew.html]

- Tensor: generalization of a matrix
  - E.g. 1D: vector, 2D: matrix

[https://en.wikipedia.org/wiki/TensorFlow]

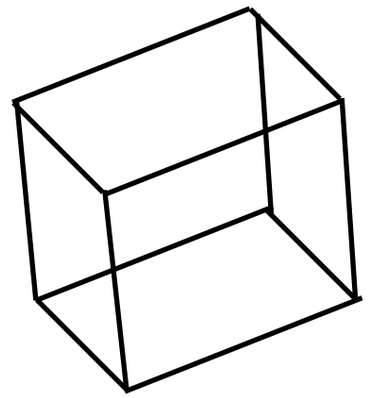
Convolutional Layer: 3D example

A 3D image:



[https://helpx.adobe.com/photoshop/key-concepts/skew.html]

A filter:



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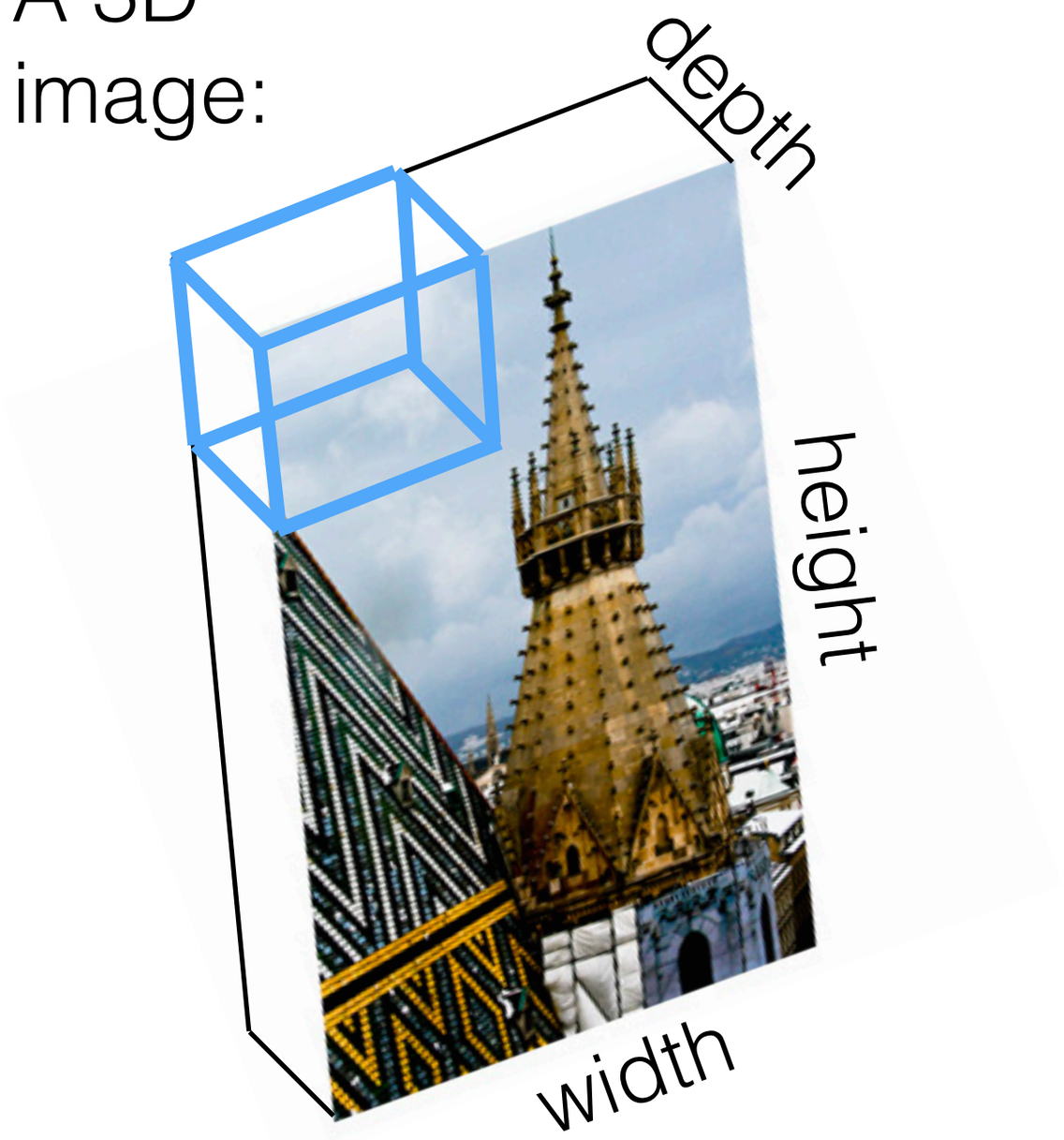
  - E.g. 1D: vector, 2D: matrix

[https://en.wikipedia.org/wiki/TensorFlow]

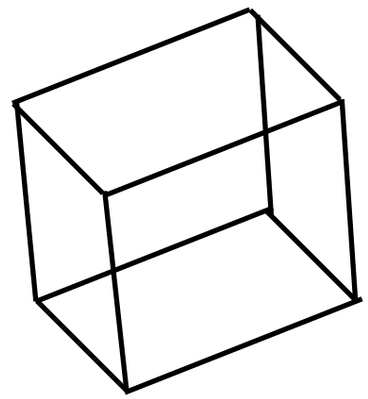
Convolutional Layer: 3D example

A 3D image:



[https://helpx.adobe.com/photoshop/key-concepts/skew.html]

A filter:



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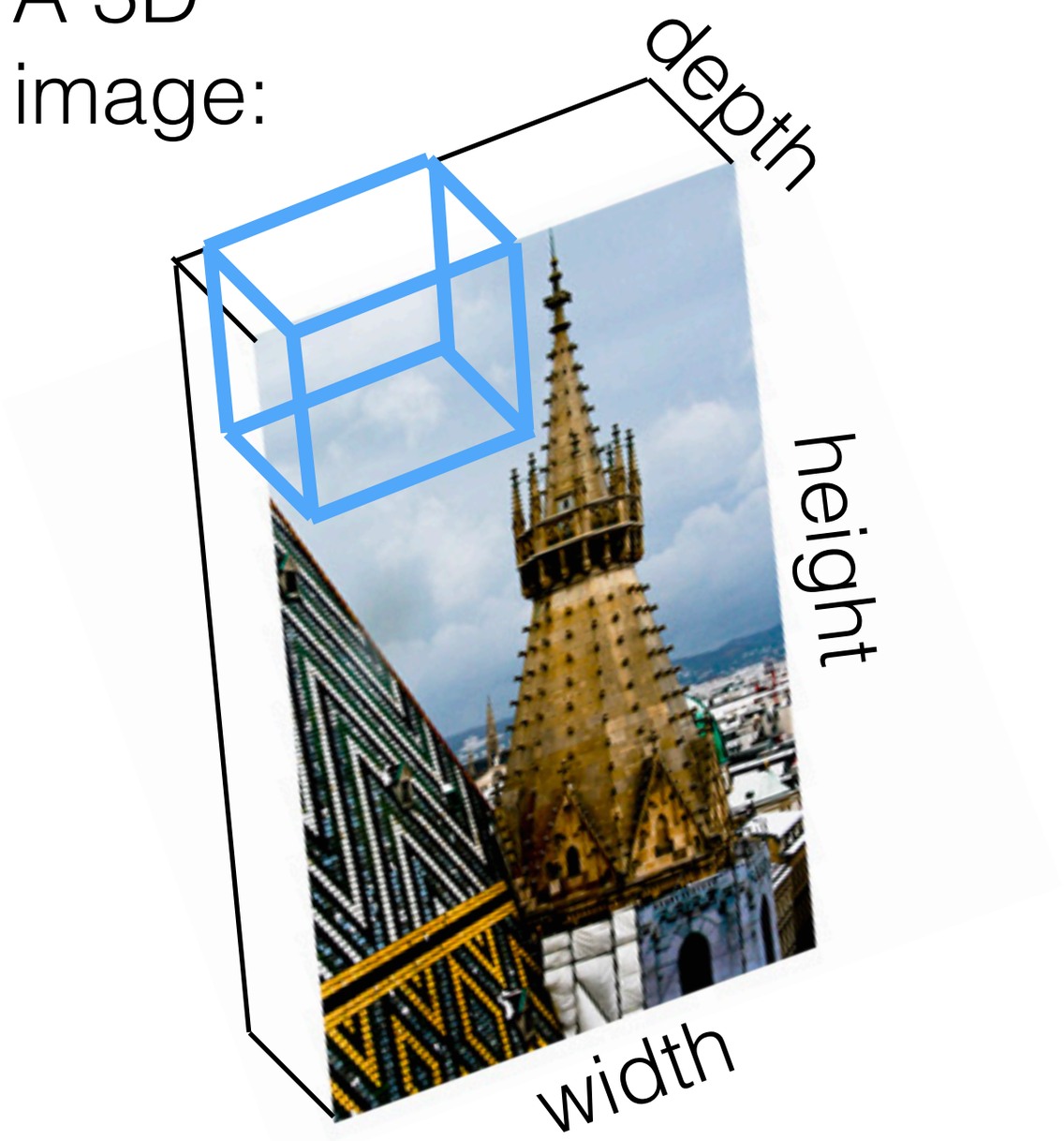
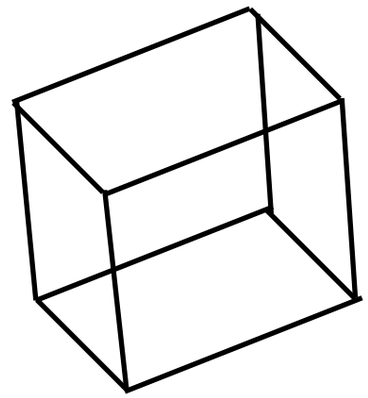
[https://en.wikipedia.org/wiki/TensorFlow]

Convolutional Layer: 3D example

A 3D image:



A filter:



- Tensor: generalization of a matrix
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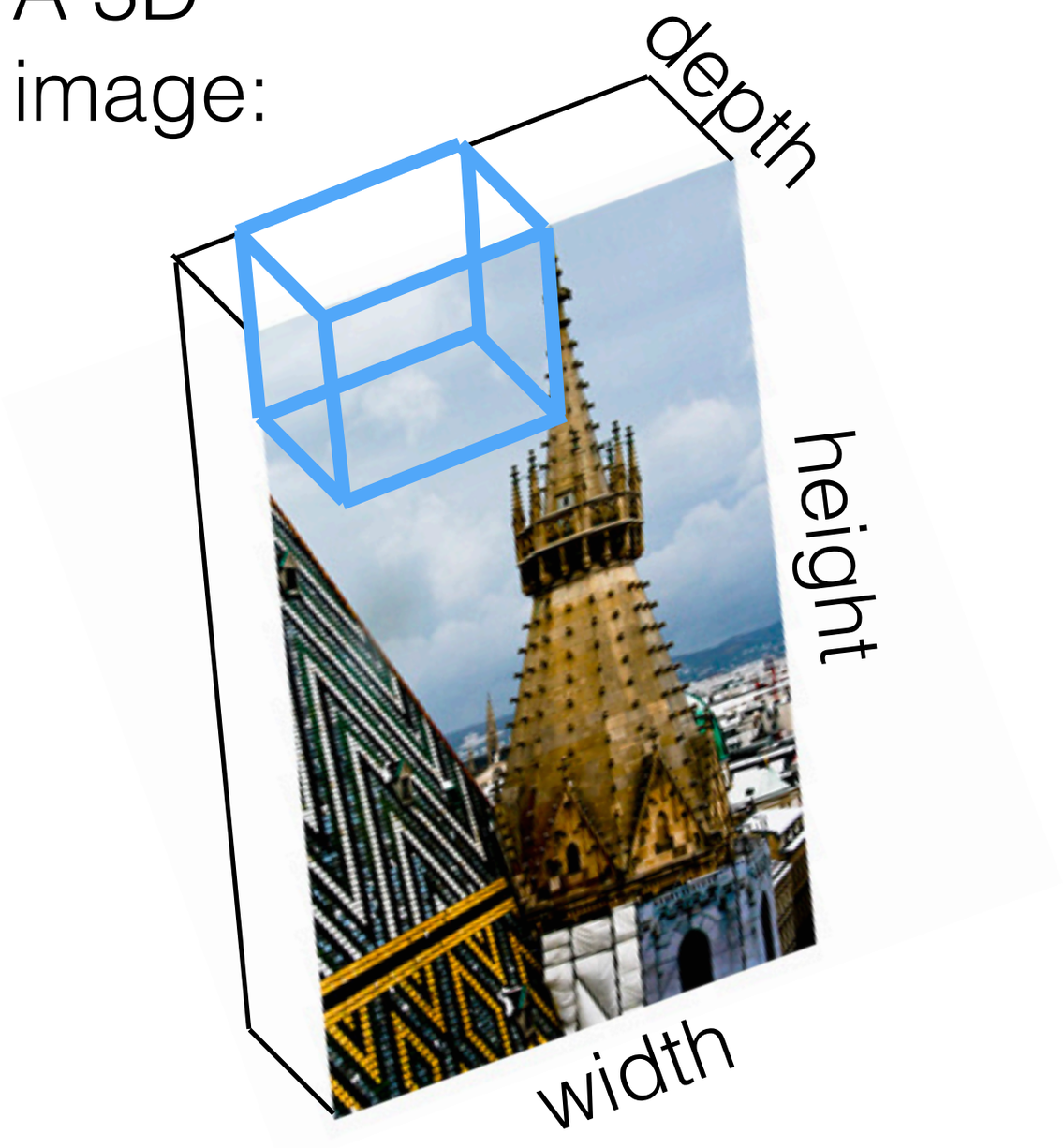
[https://helpx.adobe.com/photoshop/key-concepts/skew.html]

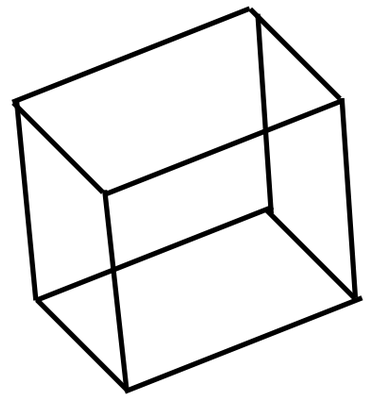
[https://en.wikipedia.org/wiki/TensorFlow]

Convolutional Layer: 3D example

A 3D image:



A filter:



- Tensor: generalization of a matrix
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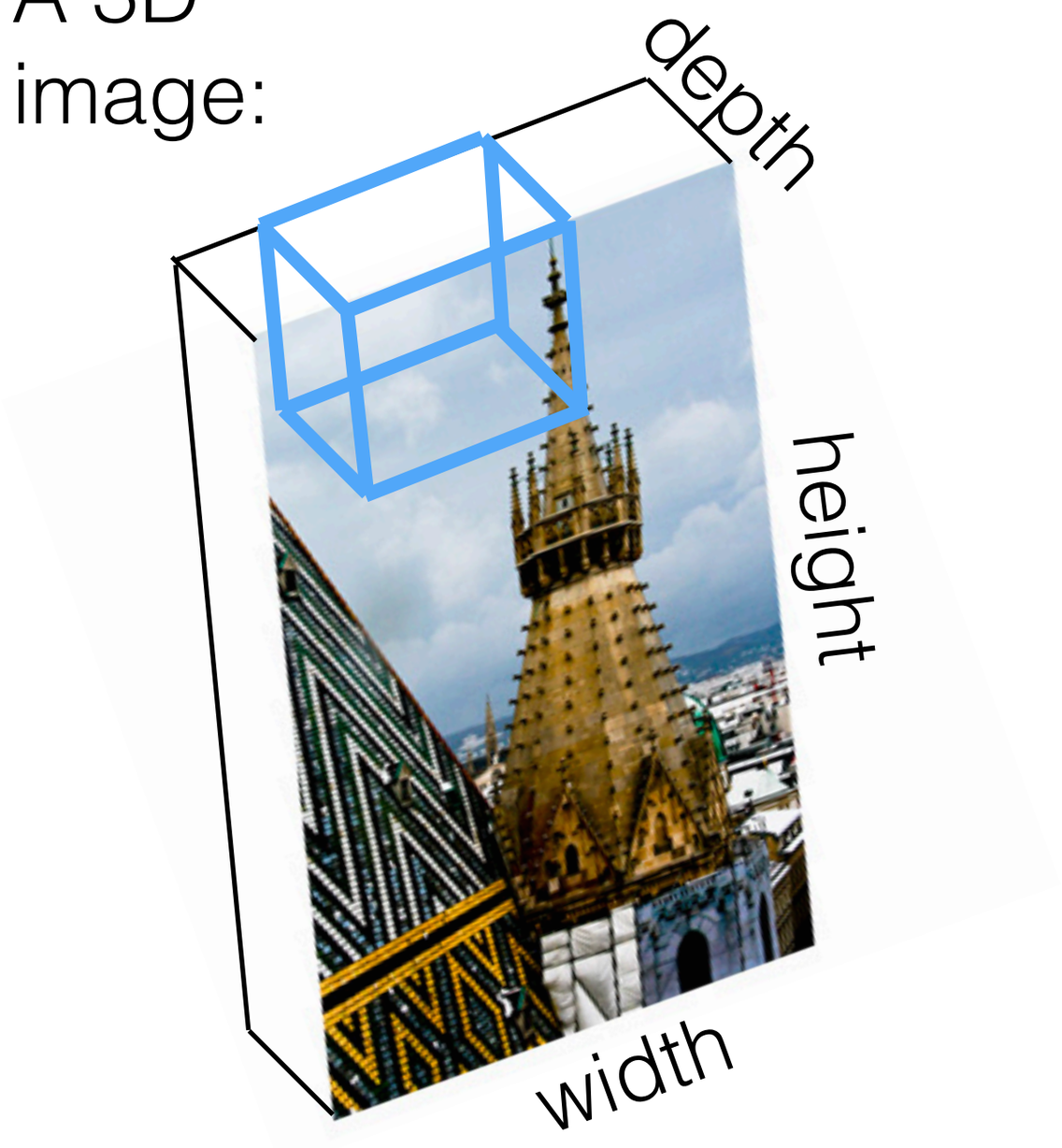
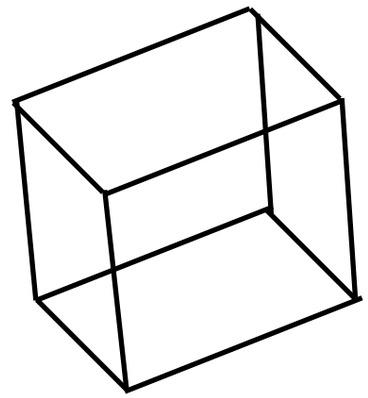
[https://en.wikipedia.org/wiki/TensorFlow]

Convolutional Layer: 3D example

A 3D image:



A filter:



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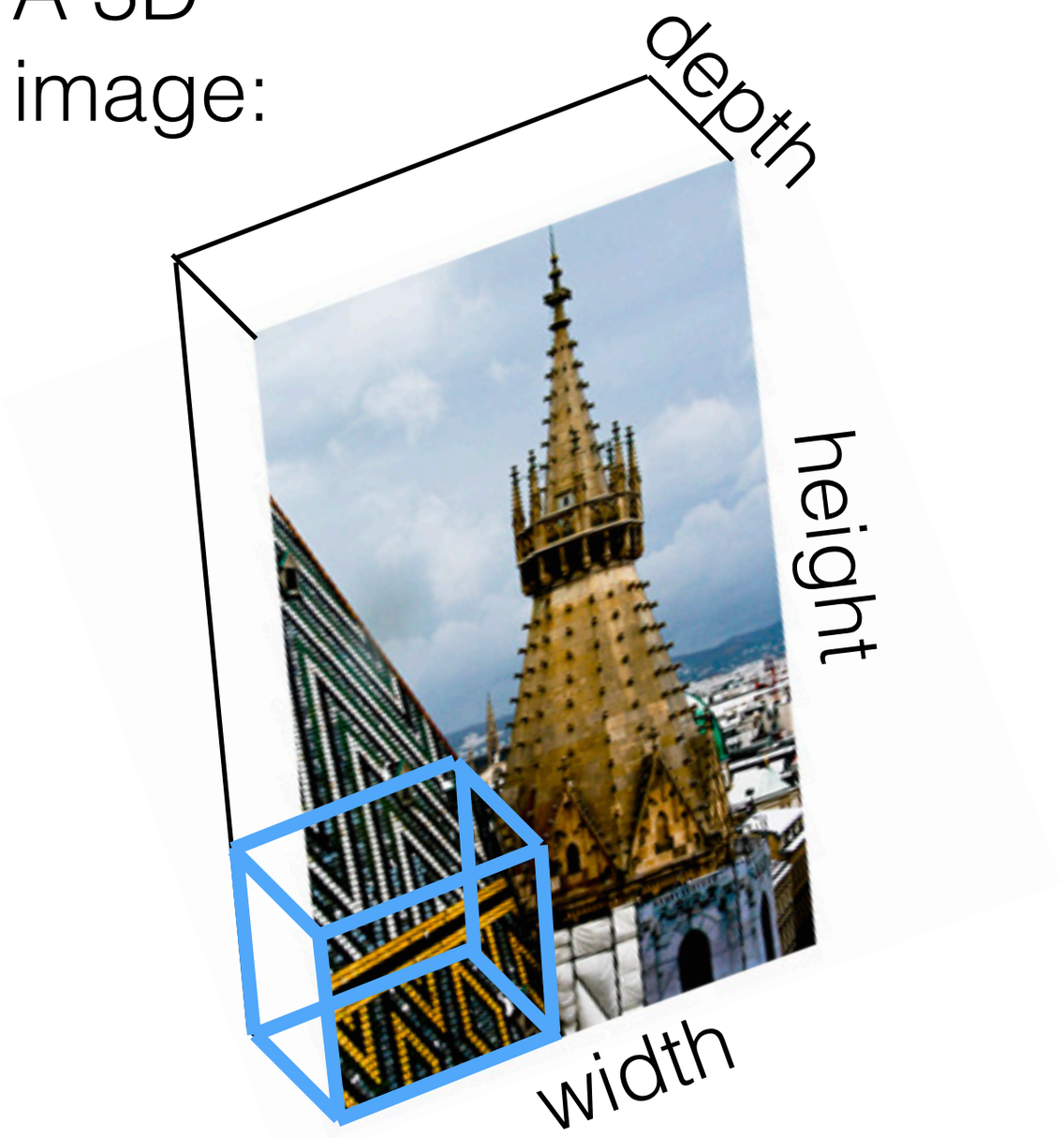
  - E.g. 1D: vector, 2D: matrix

[https://helpx.adobe.com/photoshop/key-concepts/skew.html]

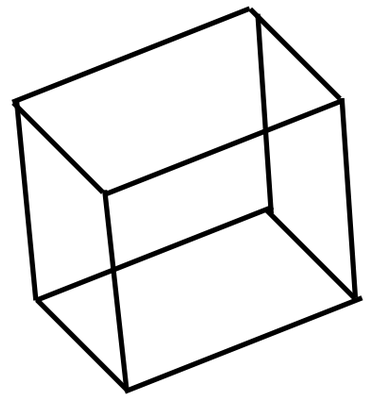
[https://en.wikipedia.org/wiki/TensorFlow]

Convolutional Layer: 3D example

A 3D image:



A filter:



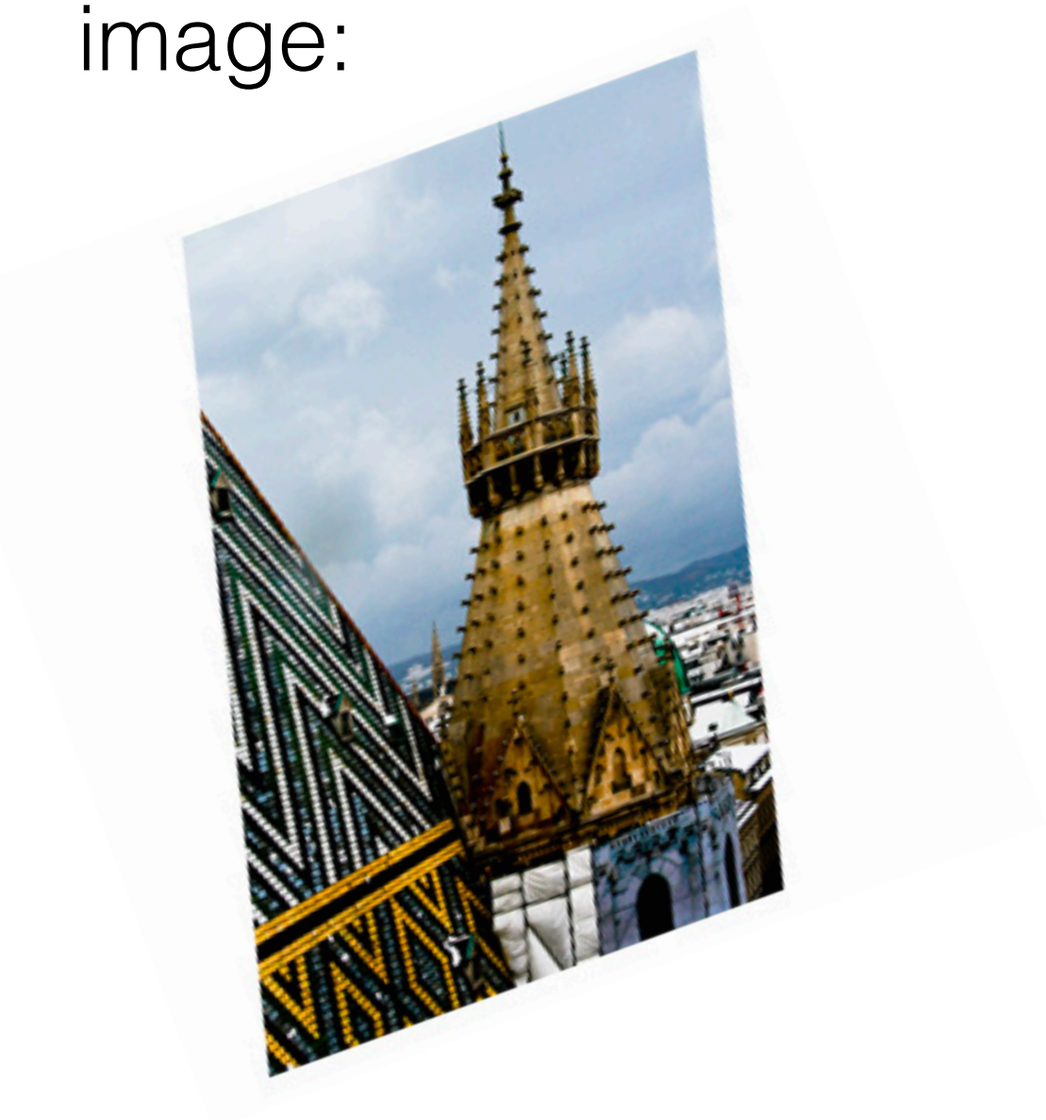
- Tensor: generalization of a matrix
  - E.g. 1D: vector, 2D: matrix

[https://helpx.adobe.com/photoshop/key-concepts/skew.html]

[https://en.wikipedia.org/wiki/TensorFlow]

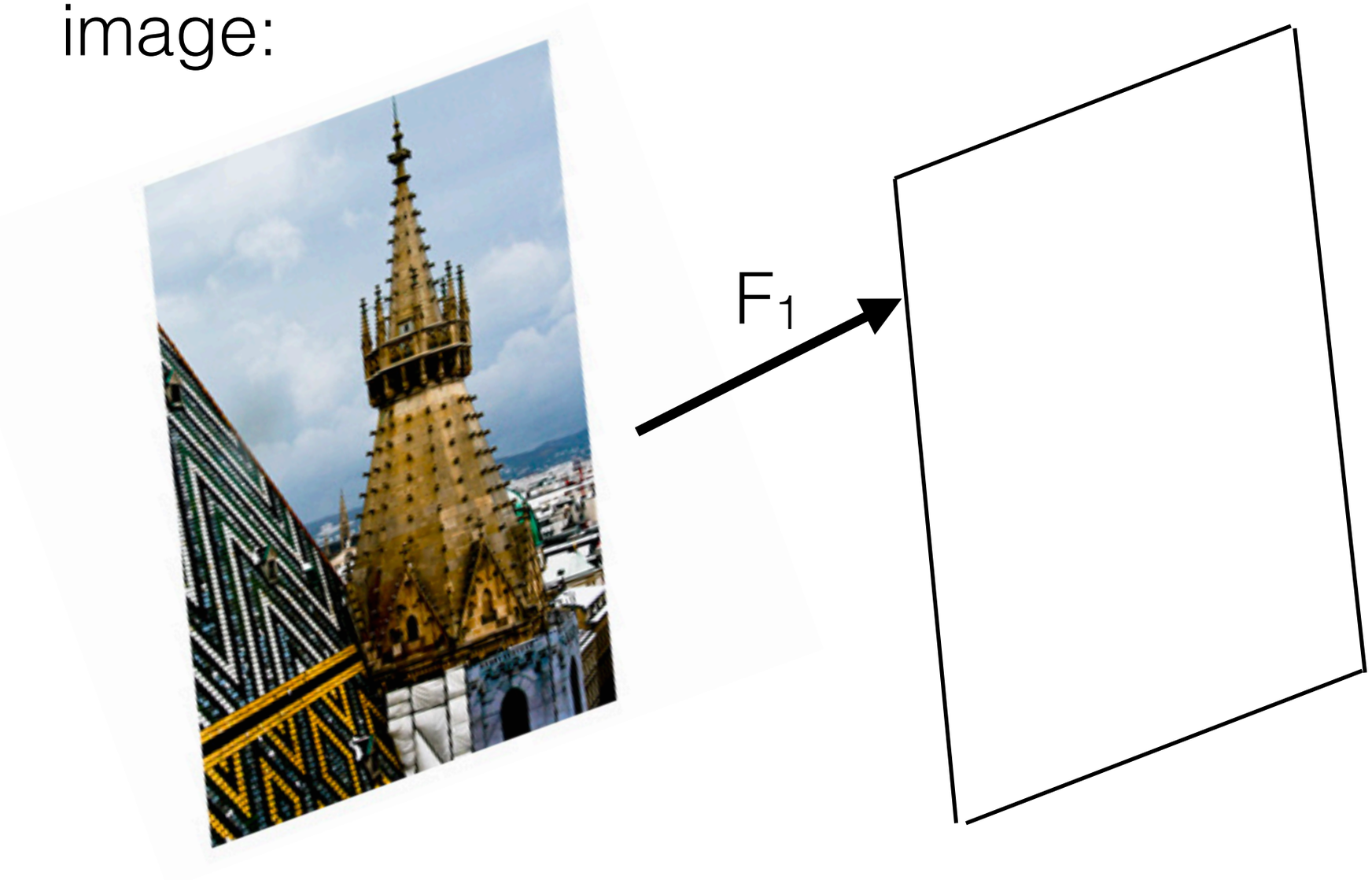
Convolutional Layer: multiple filters

An image:



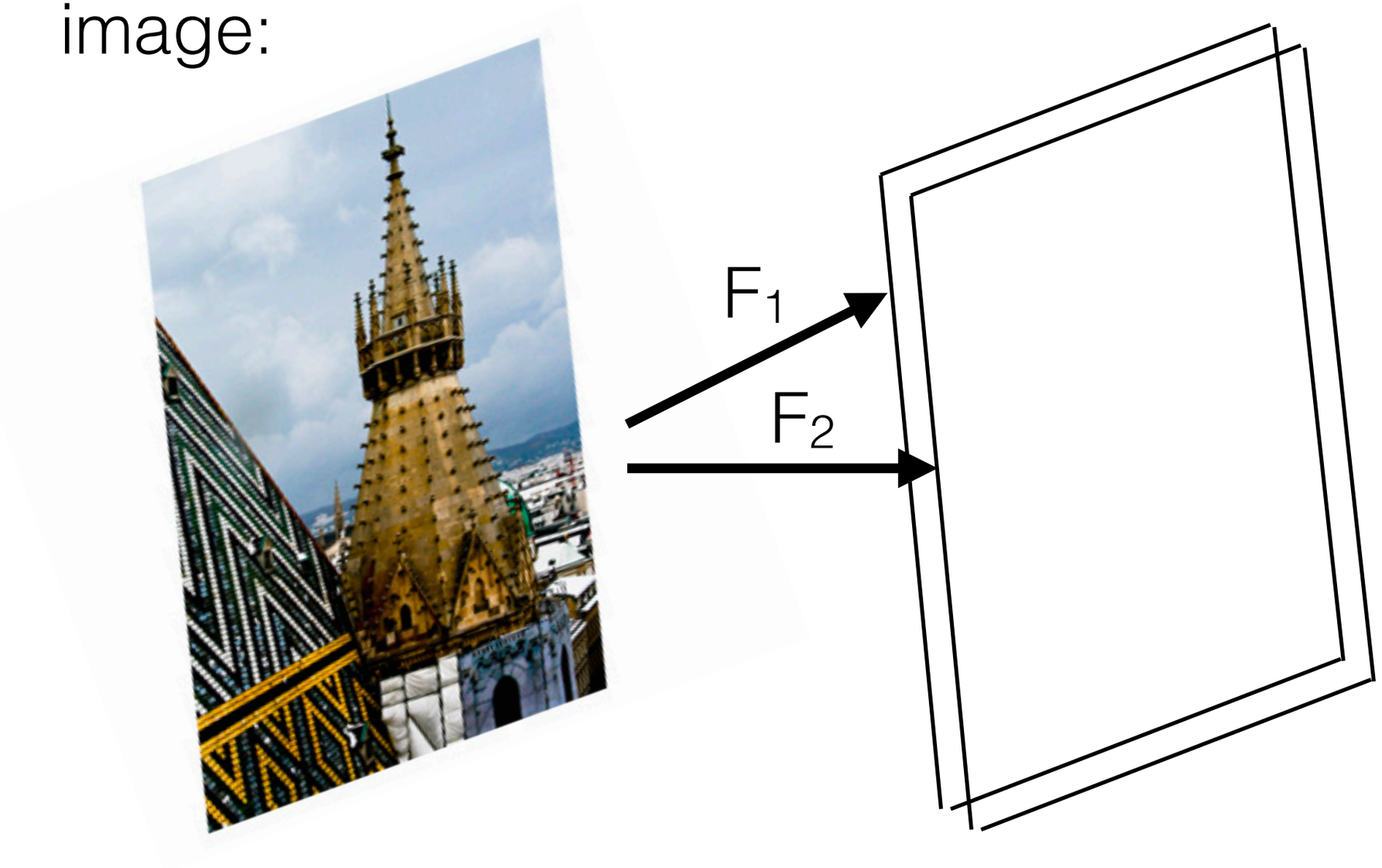
Convolutional Layer: multiple filters

An image:



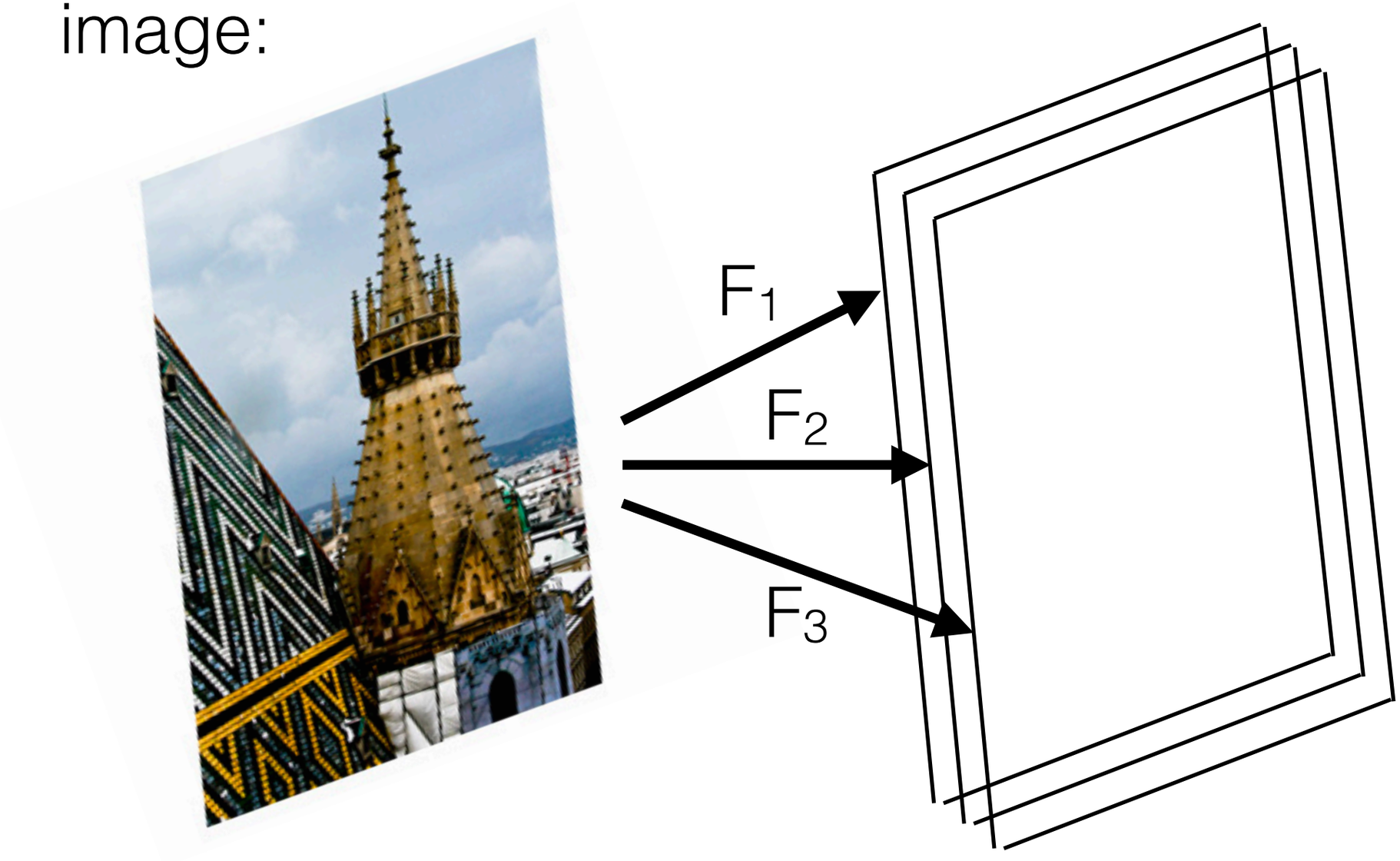
Convolutional Layer: multiple filters

An image:



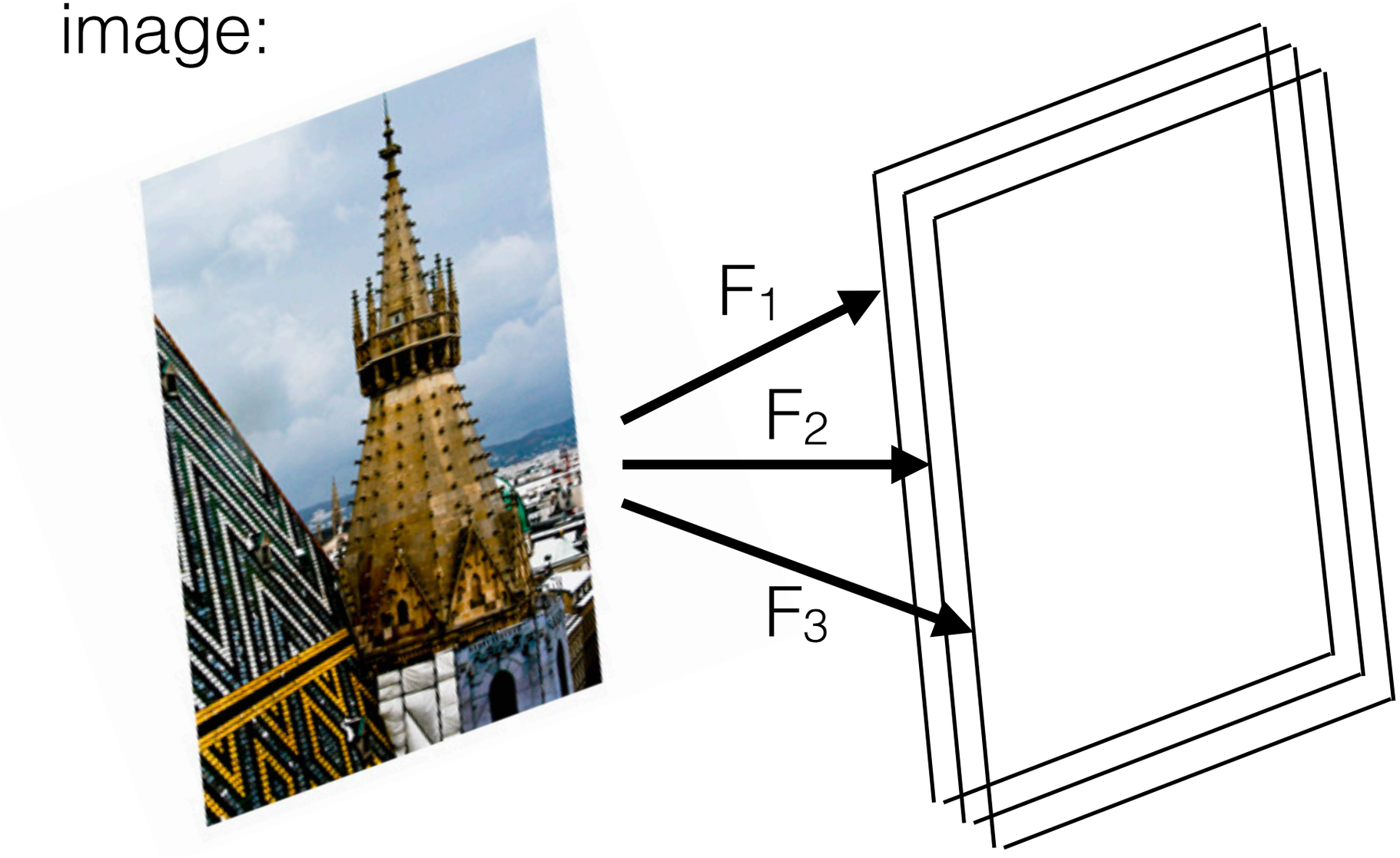
Convolutional Layer: multiple filters

An image:



Convolutional Layer: multiple filters

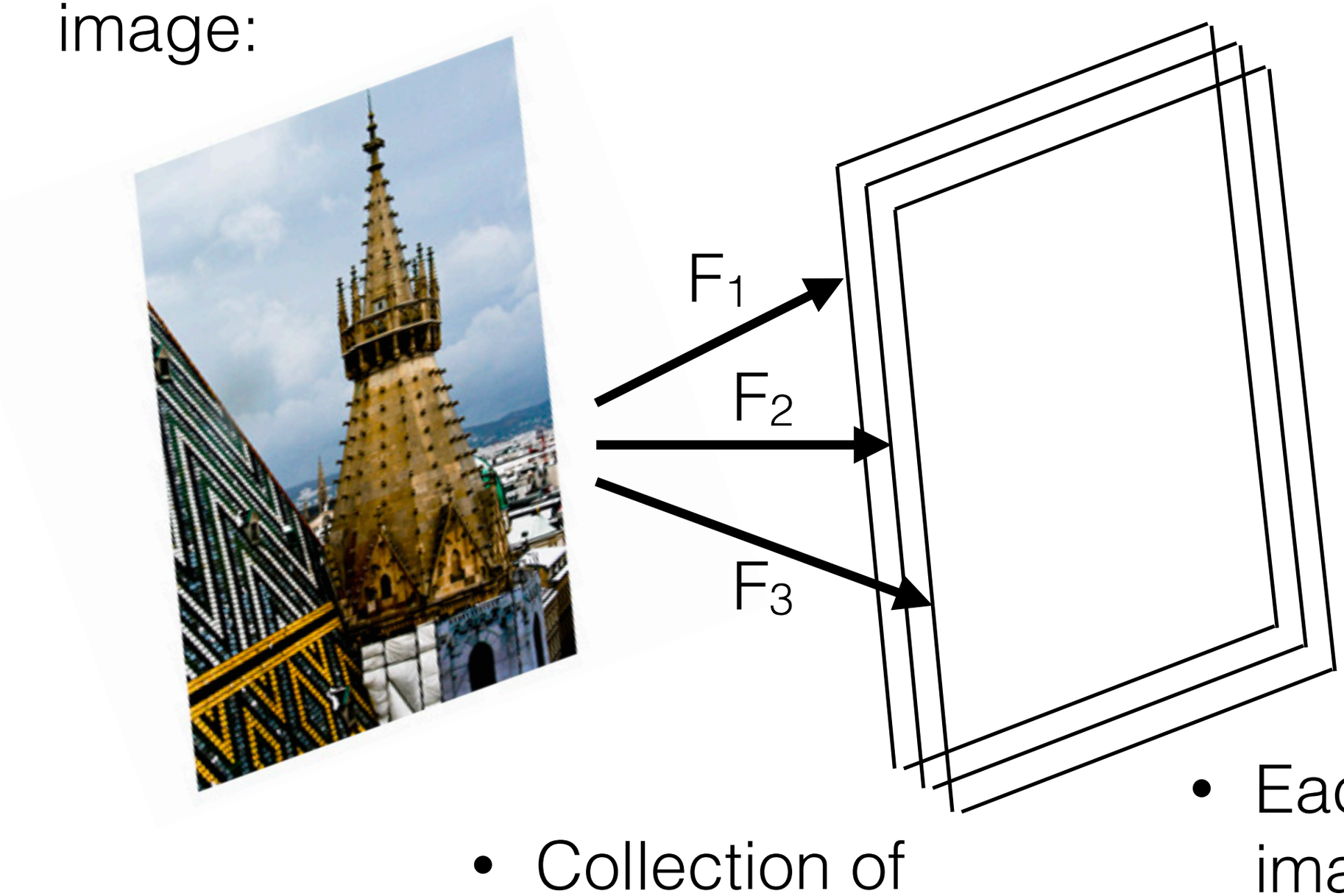
An image:



- Collection of filters in the layer: *filter bank*

Convolutional Layer: multiple filters

An image:



- Collection of filters in the layer: *filter bank*
- Each resulting image is a *channel*

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

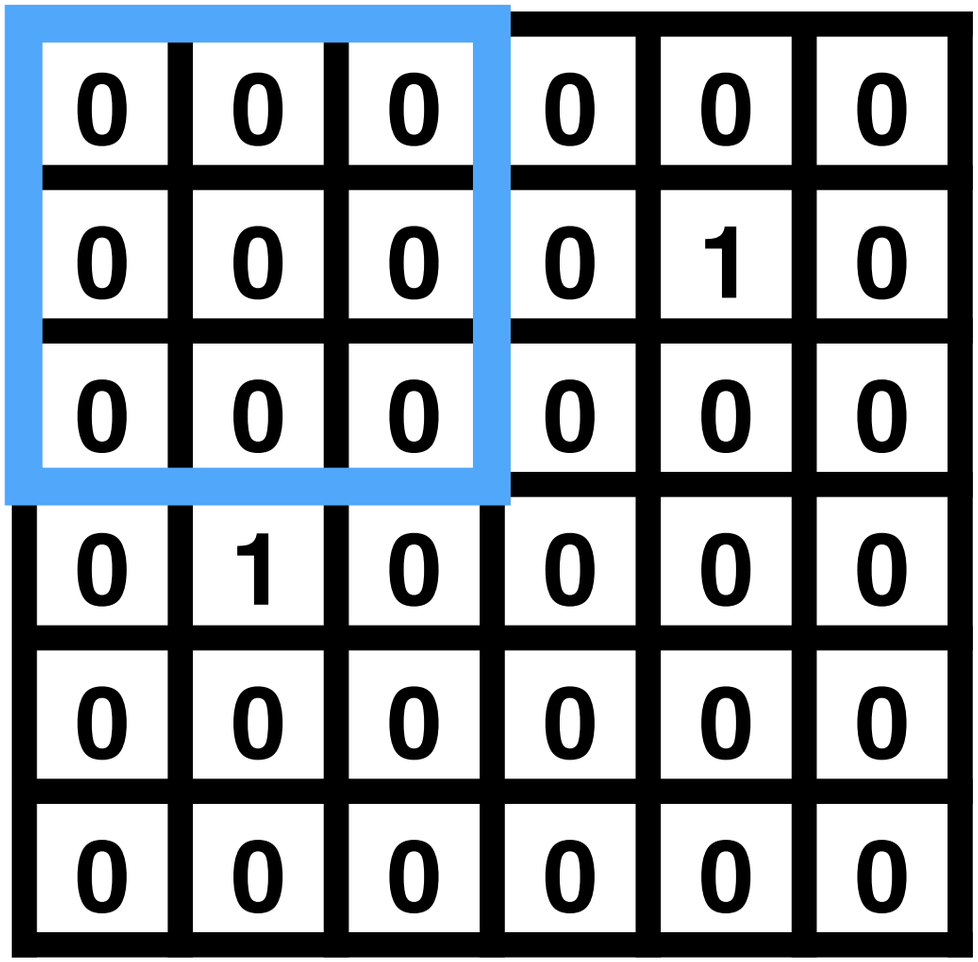
0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:



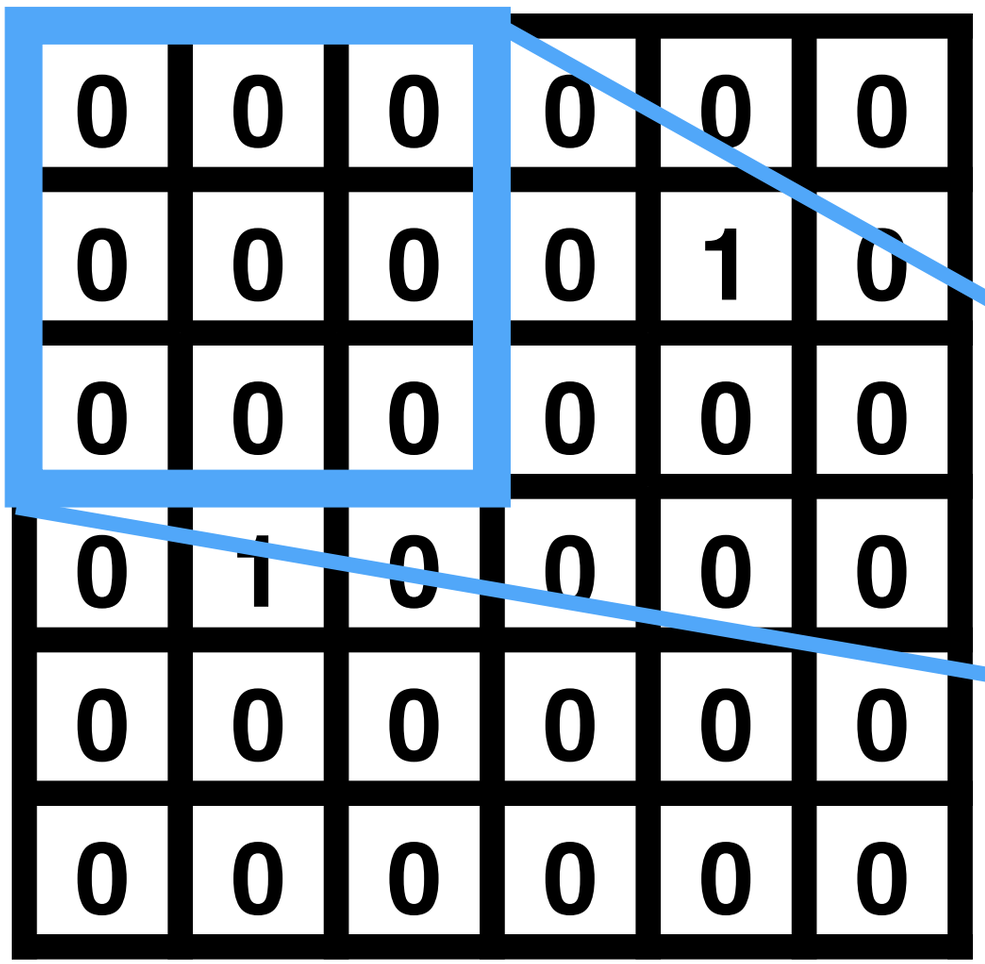
Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)

After max pooling:

Max pooling layer: 2D example

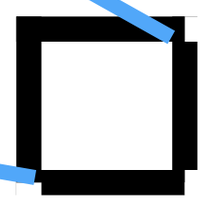
Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

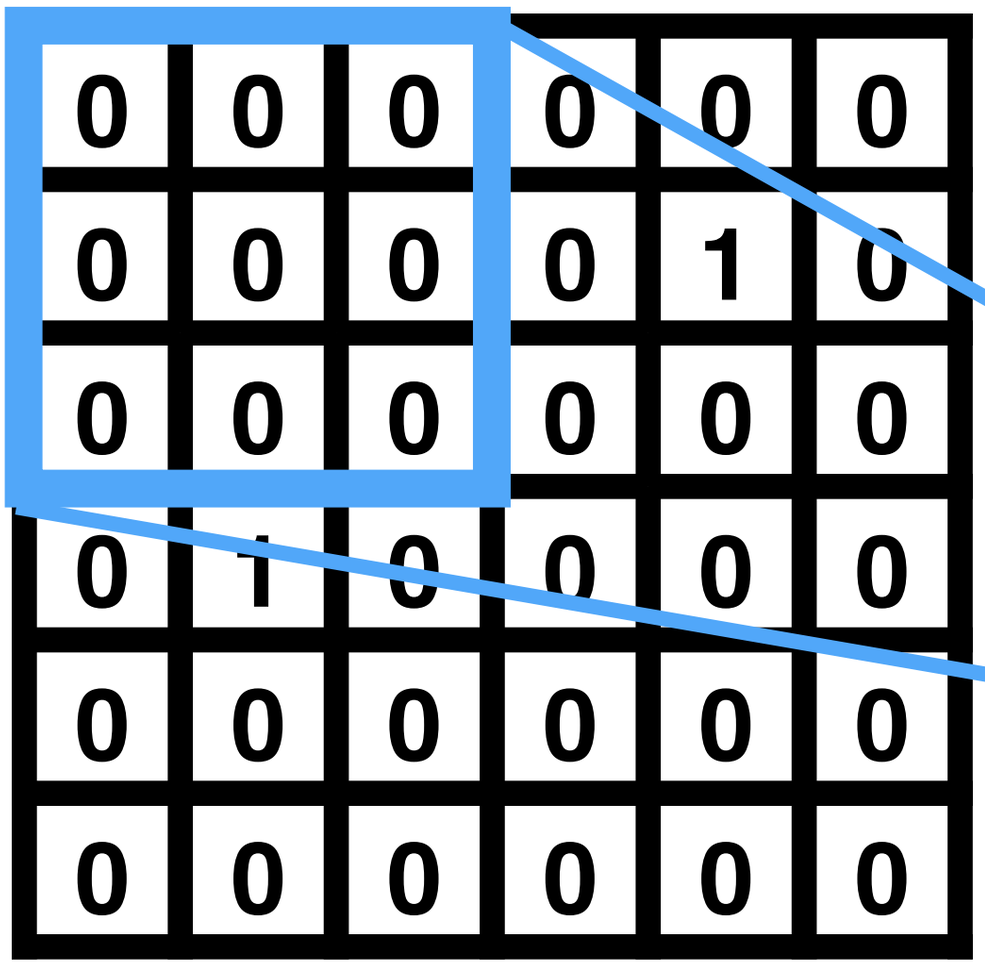
- E.g. size 3x3 (“size 3”)

After max pooling:



Max pooling layer: 2D example

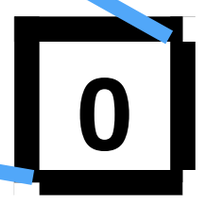
Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)

After max pooling:



Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)

After max pooling:

0	0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)

After max pooling:

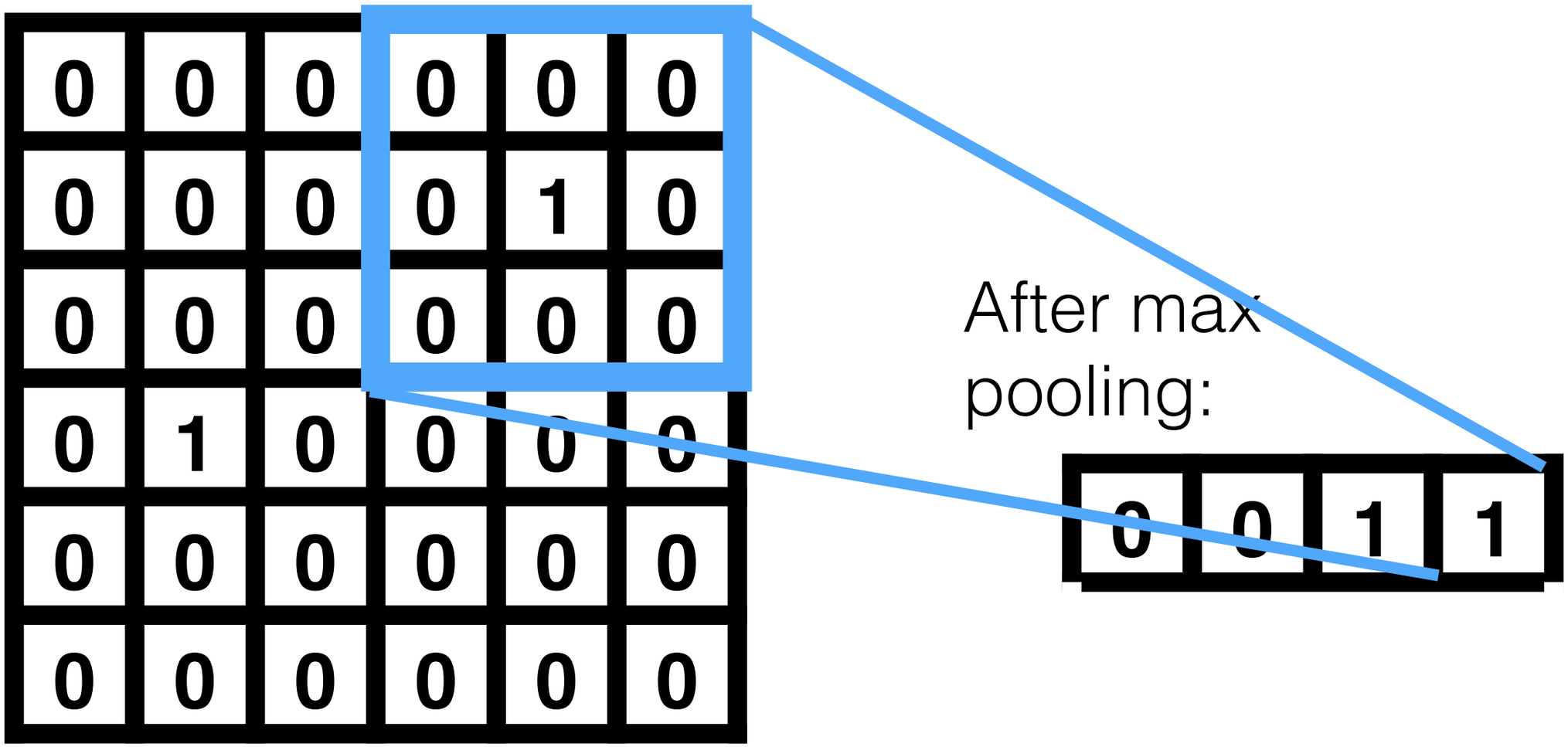
0	0	1

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)



Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)

After max pooling:

0	0	1	1
1	1	1	1
1	1	0	0
1	1	0	0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)

After max pooling:

0	0	1	1
1	1	1	1
1	1	0	0
1	1	0	0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)

After max pooling:

0	0	1	1
1	1	1	1
1	1	0	0
1	1	0	0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)

After max pooling:

0	0	1	1
1	1	1	1
1	1	0	0
1	1	0	0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 1

After max pooling:

0	0	1	1
1	1	1	1
1	1	0	0
1	1	0	0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 1

After max pooling:

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 1

After max pooling:

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

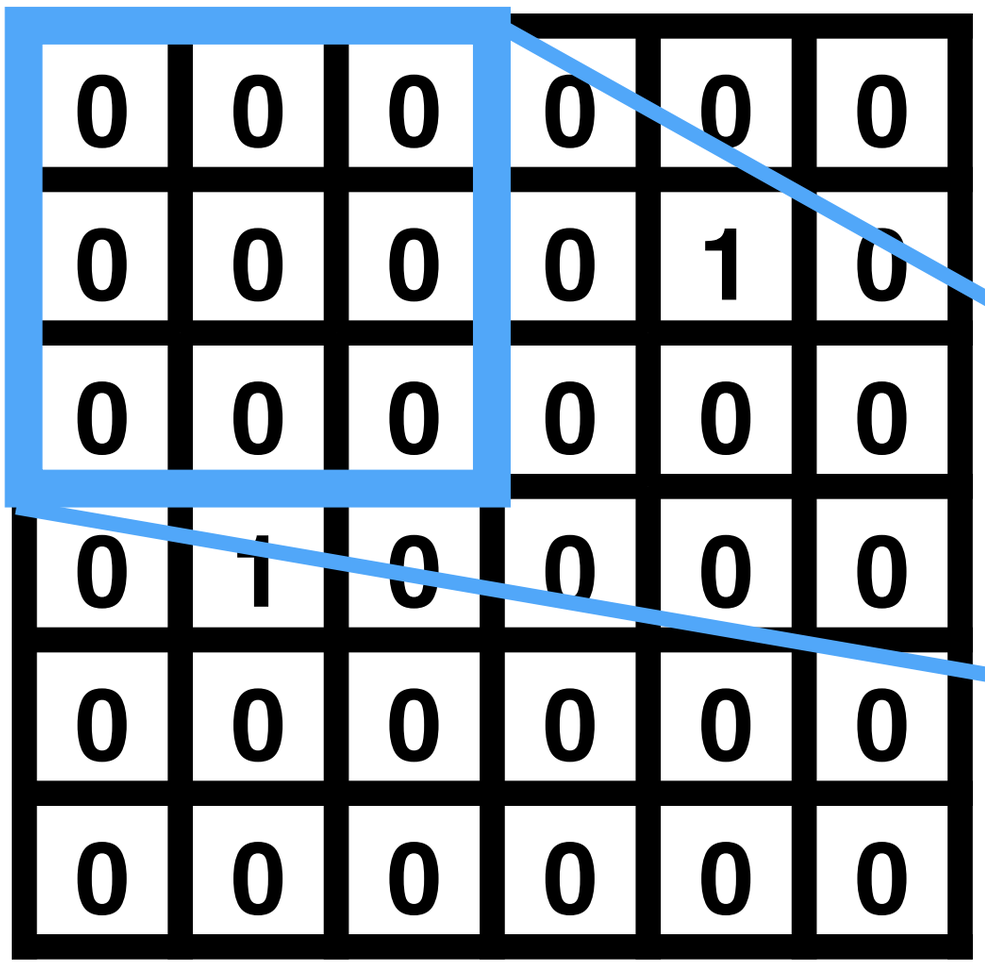
Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

Max pooling layer: 2D example

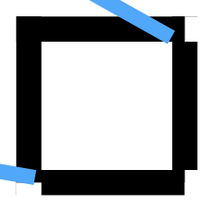
Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

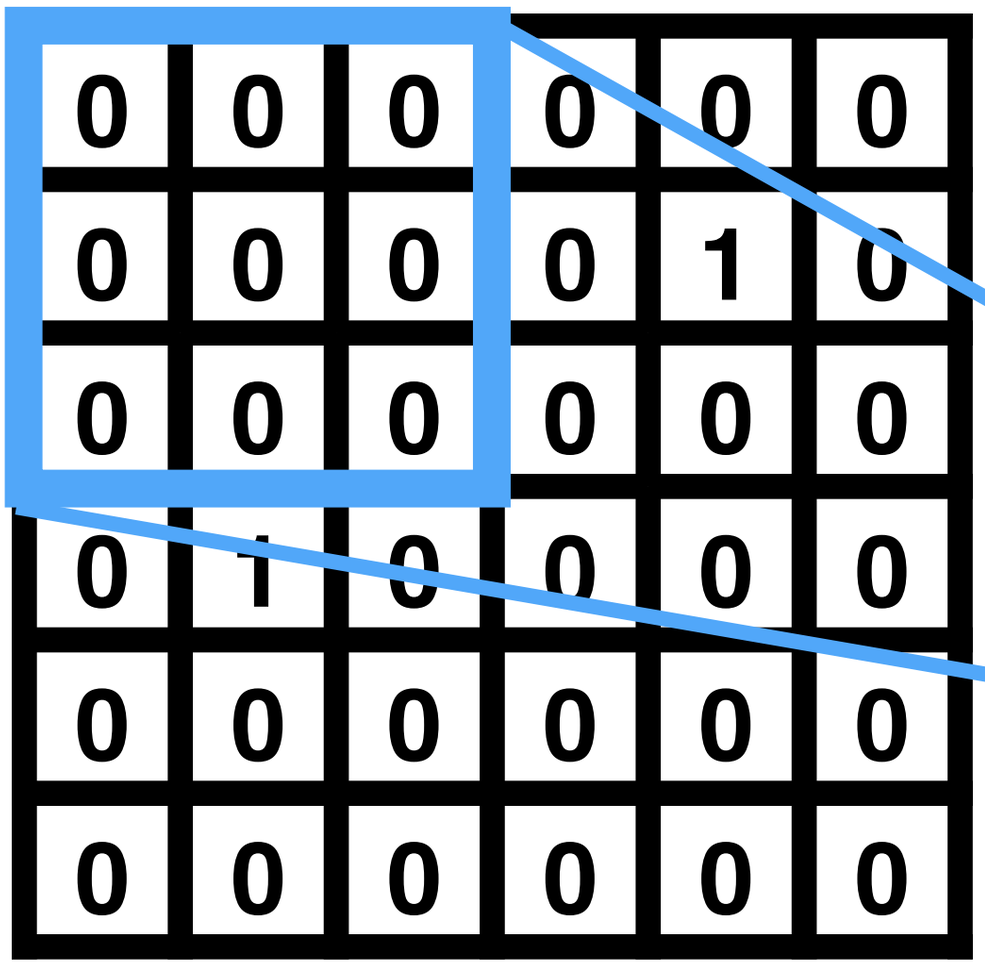
- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:



Max pooling layer: 2D example

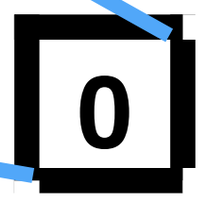
Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

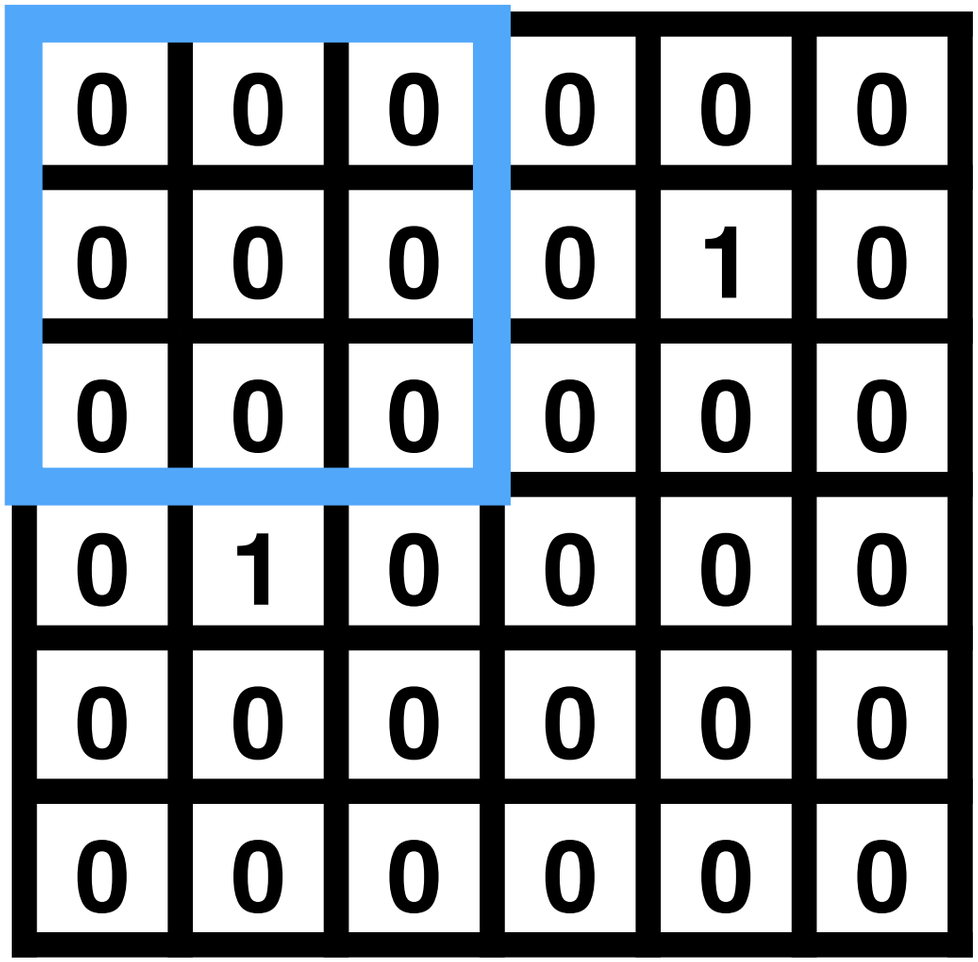
- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:



Max pooling layer: 2D example

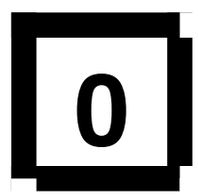
Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

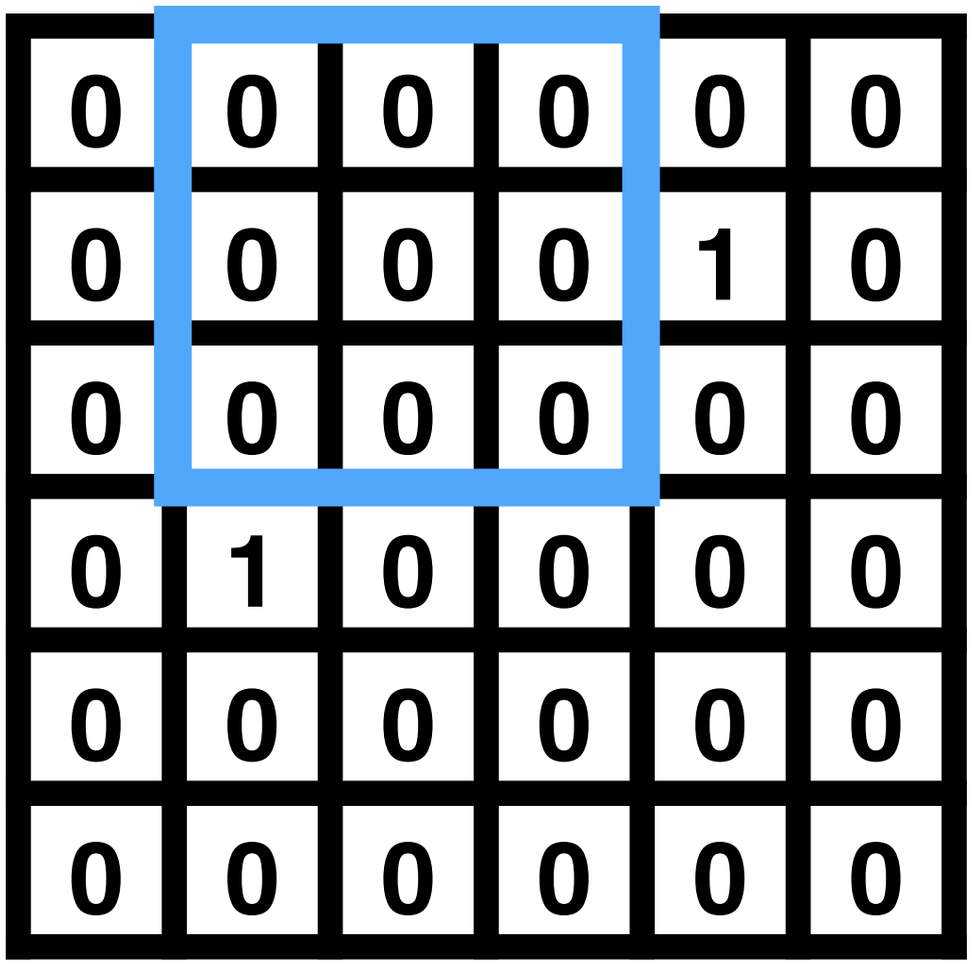
- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:



Max pooling layer: 2D example

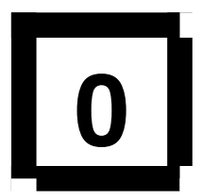
Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

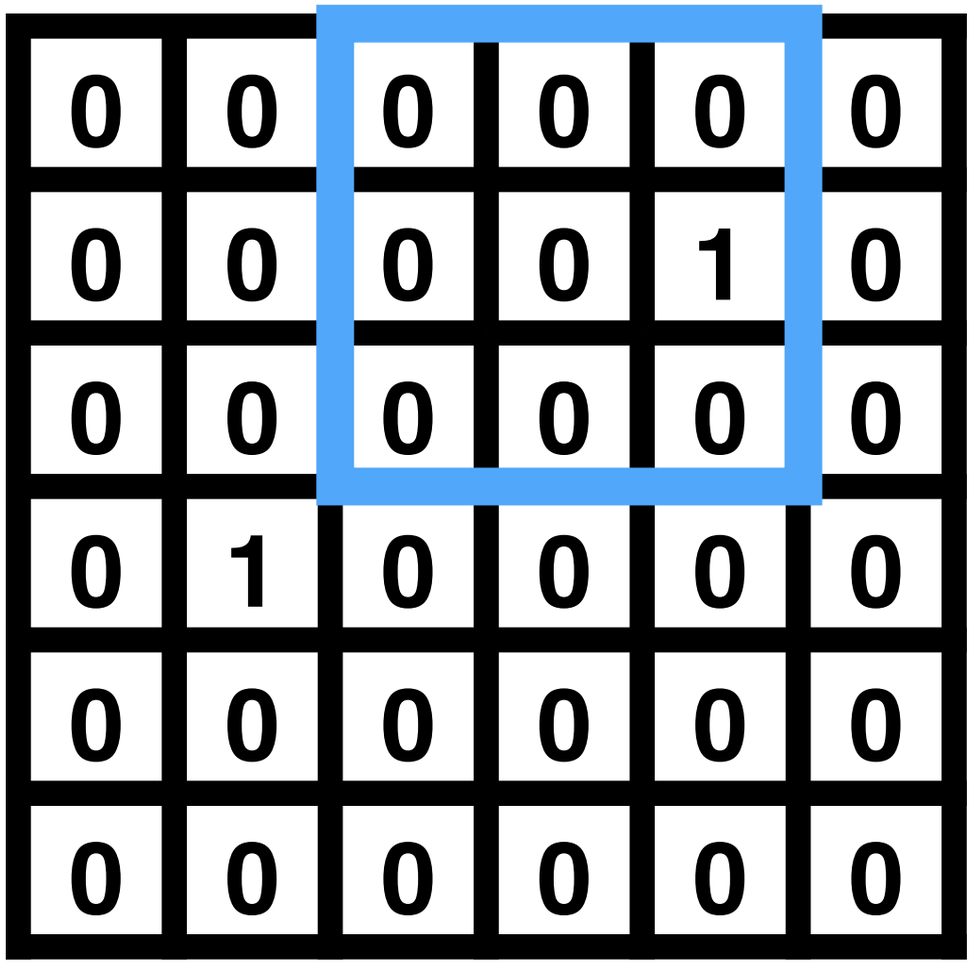
- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:



Max pooling layer: 2D example

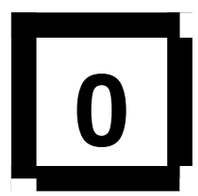
Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:



Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

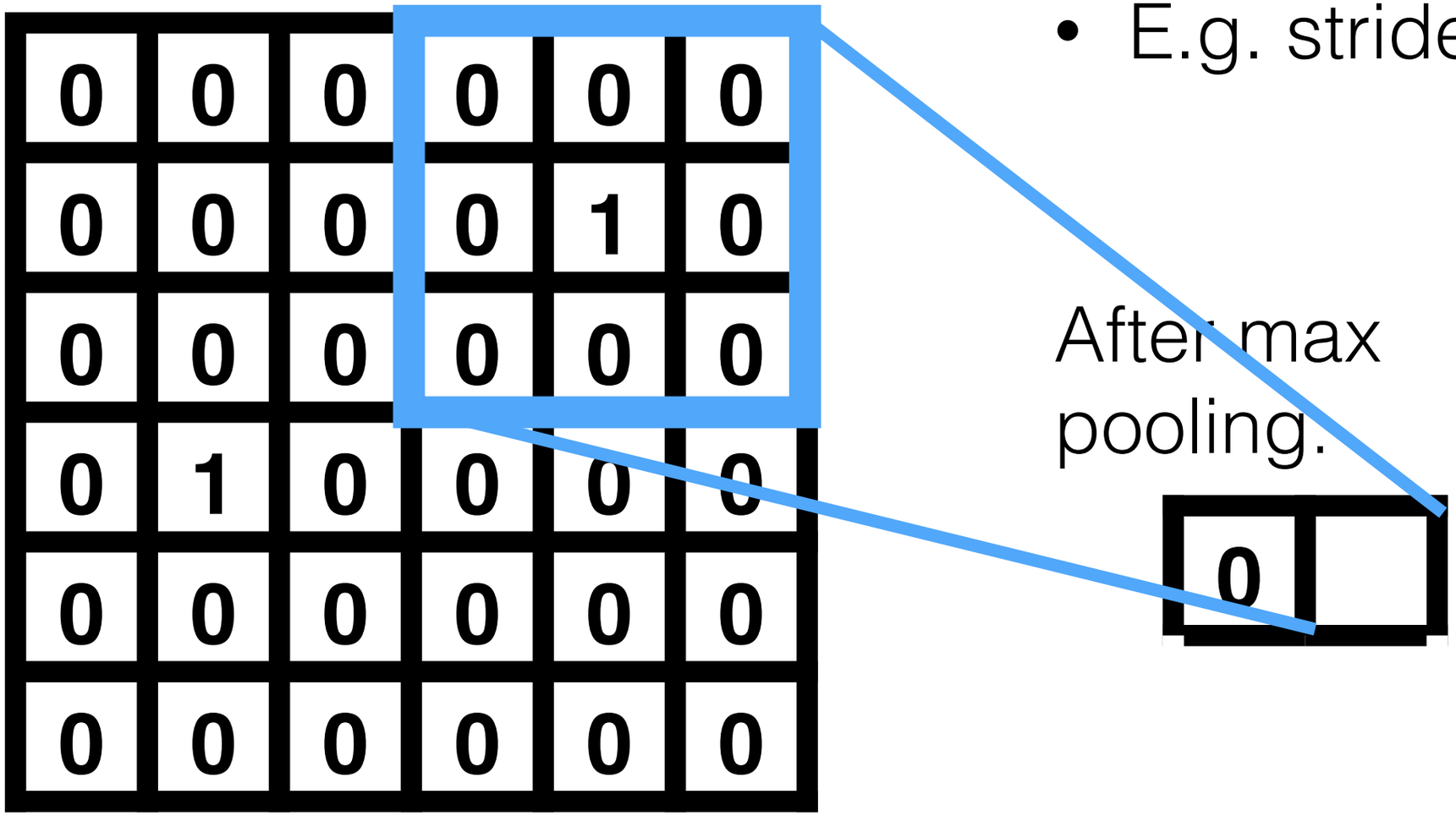
- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

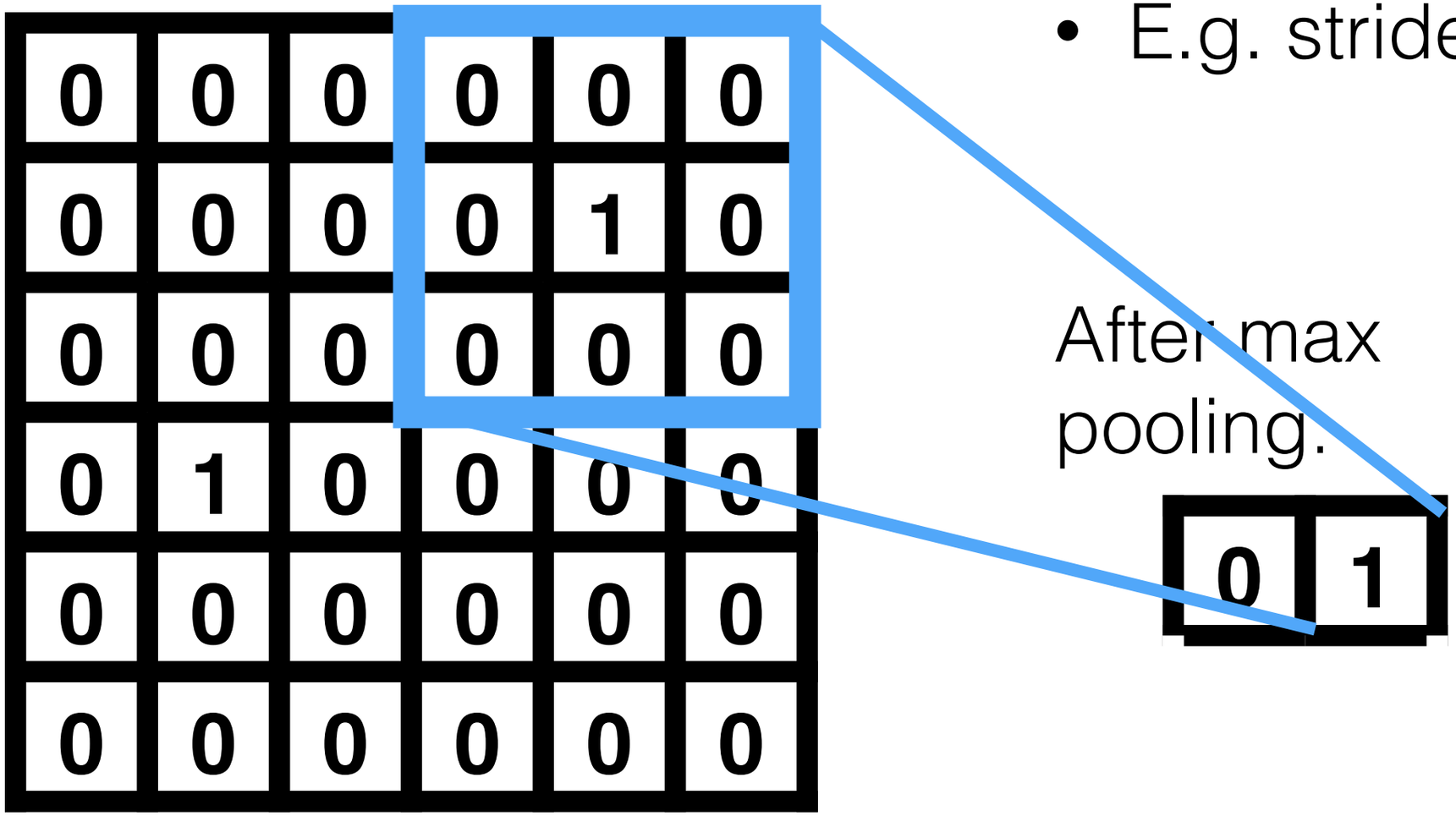


Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0	1

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0	1

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0	1

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0	1

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

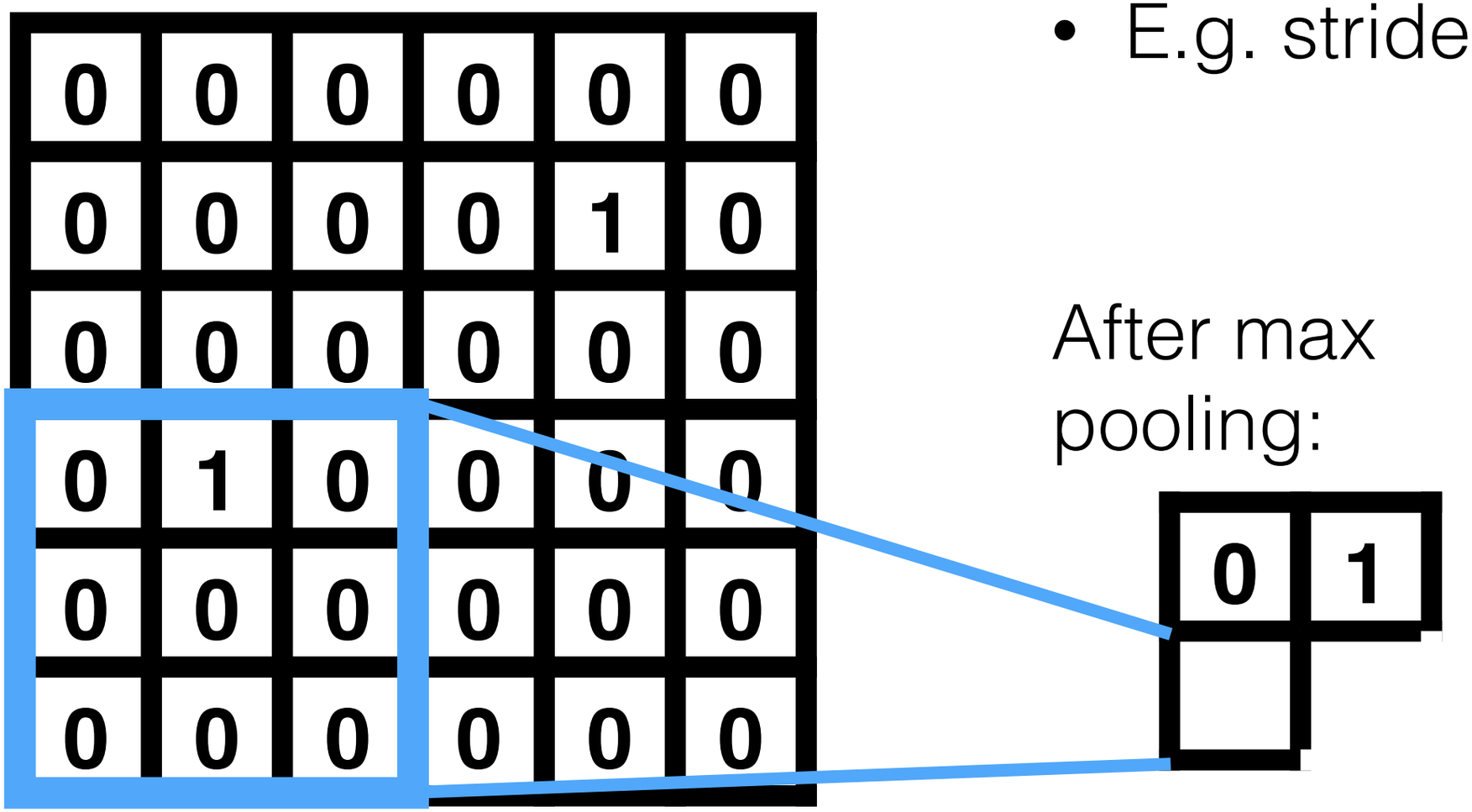
- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0	1

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:



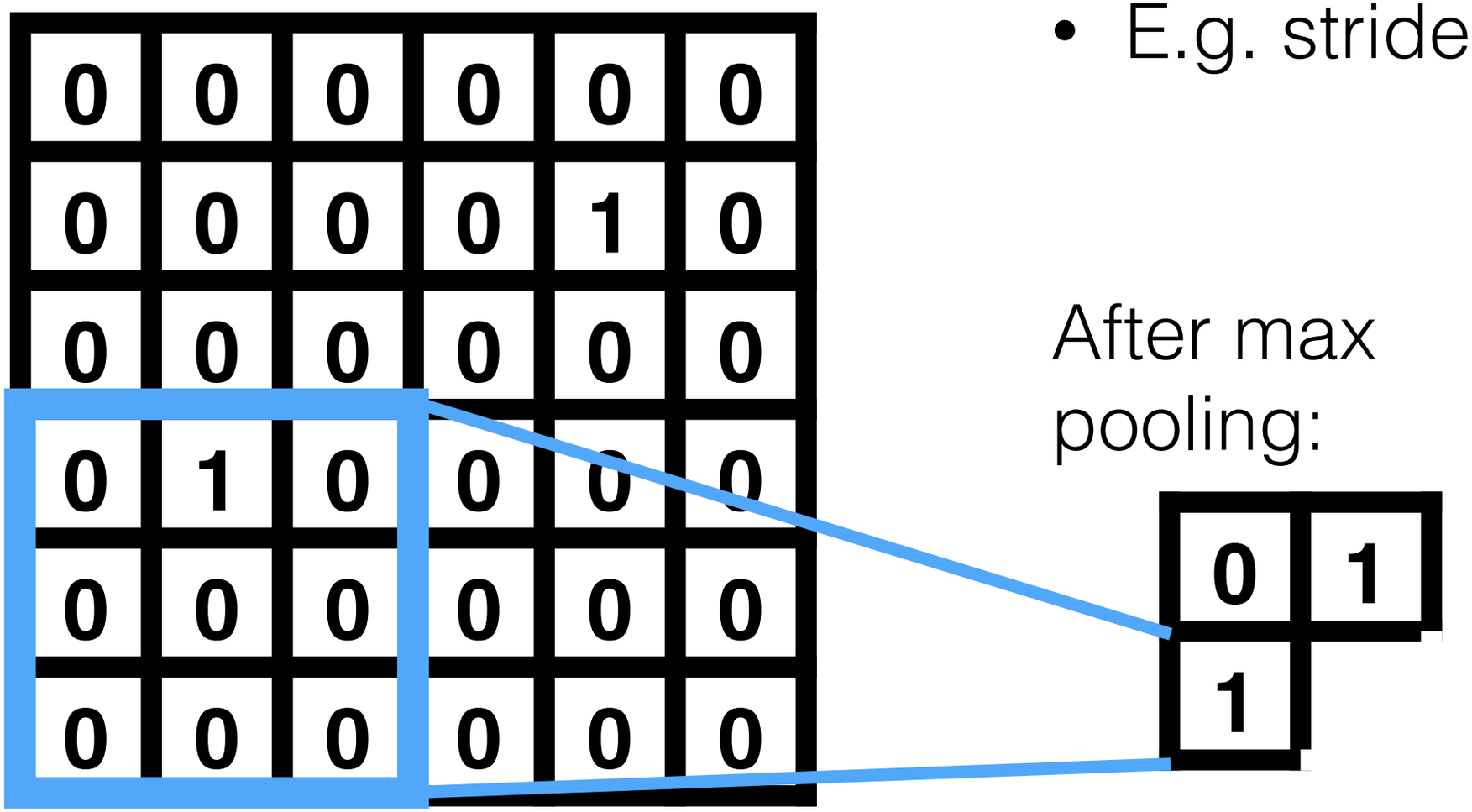
Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

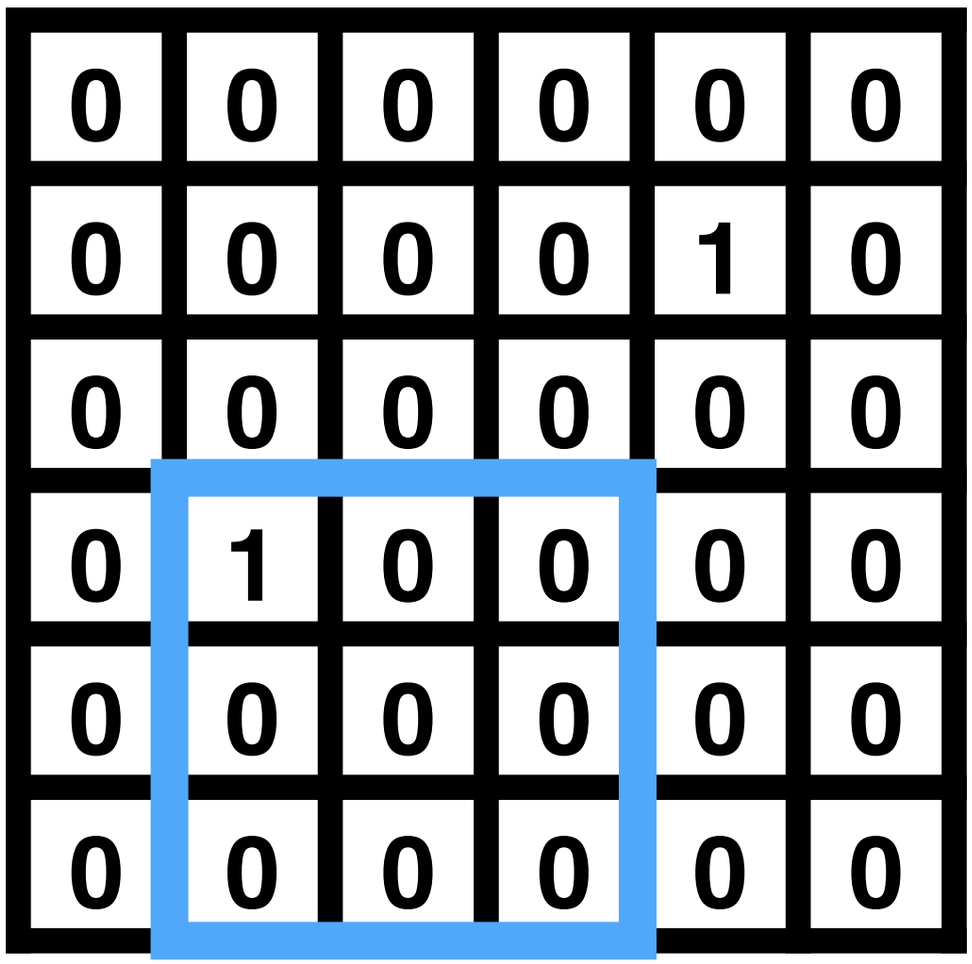
- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0	1
1	

Max pooling layer: 2D example

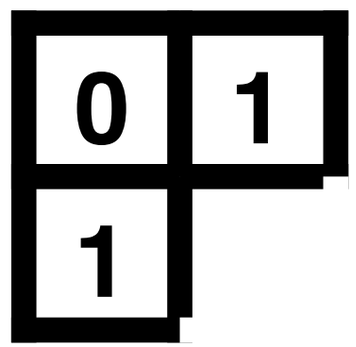
Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

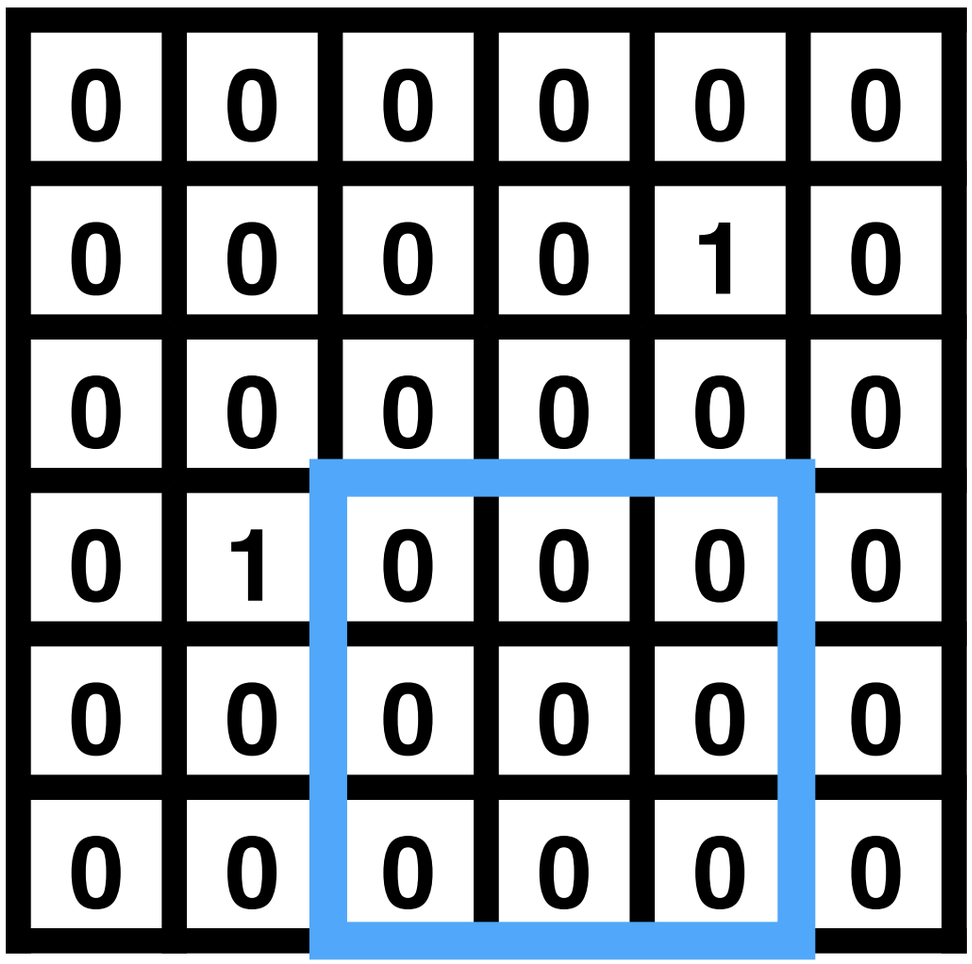
- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:



Max pooling layer: 2D example

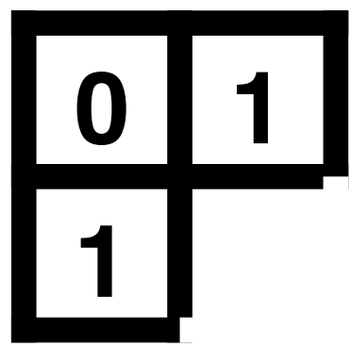
Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:



Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

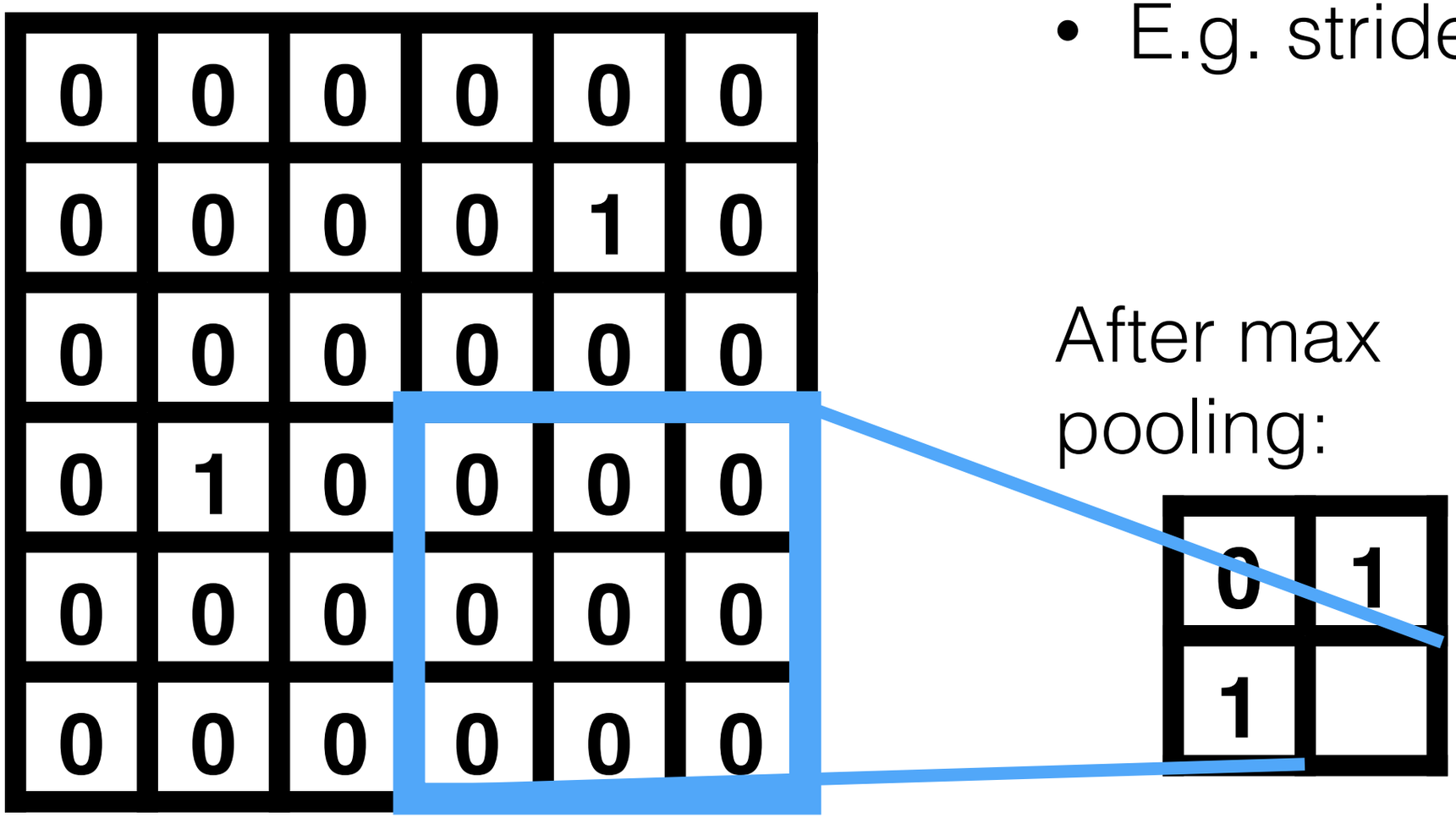
- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0	1
1	

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

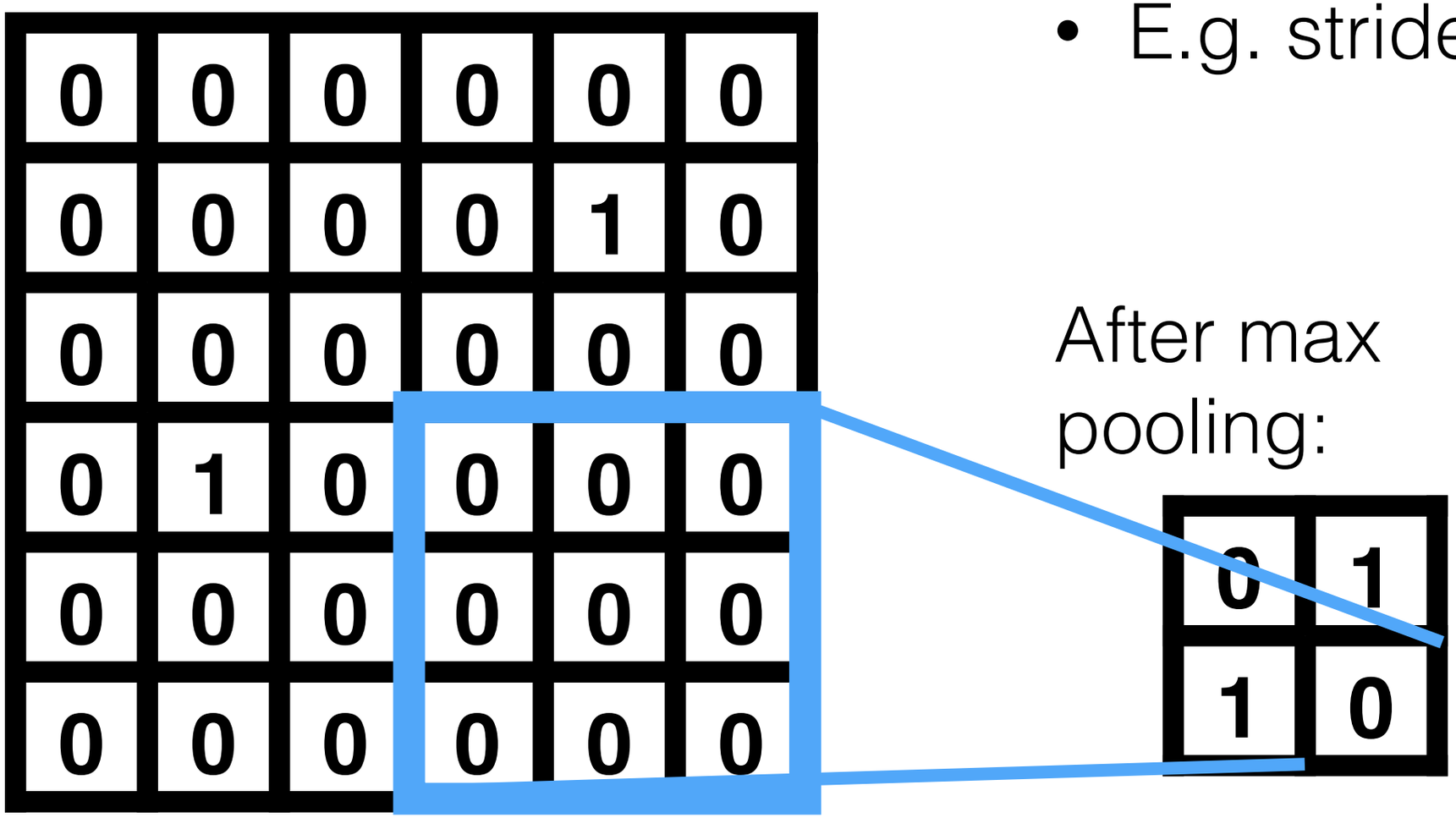


Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:



Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0	1
1	0

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0	1
1	0

- Can use stride with filters too

Max pooling layer: 2D example

Output from the convolutional layer & ReLU:

0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Max pooling: returns max of its arguments

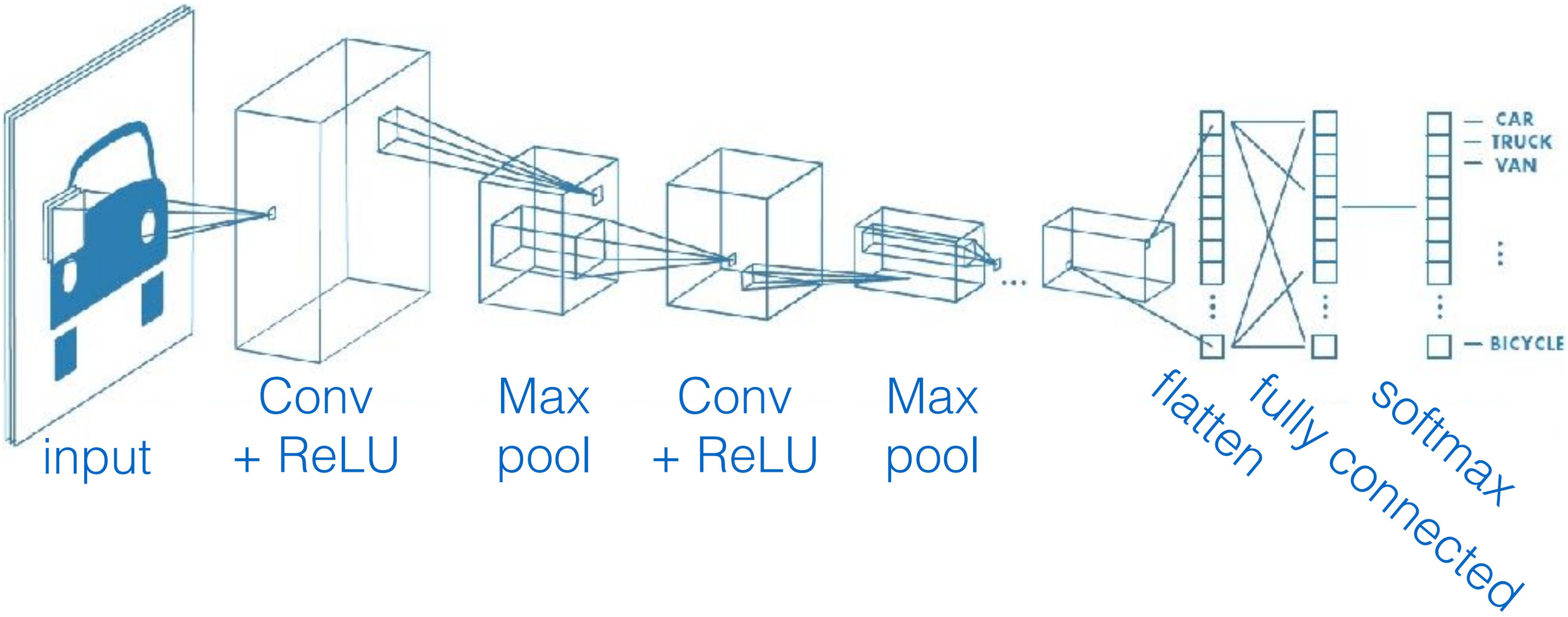
- E.g. size 3x3 (“size 3”)
- E.g. stride 3

After max pooling:

0	1
1	0

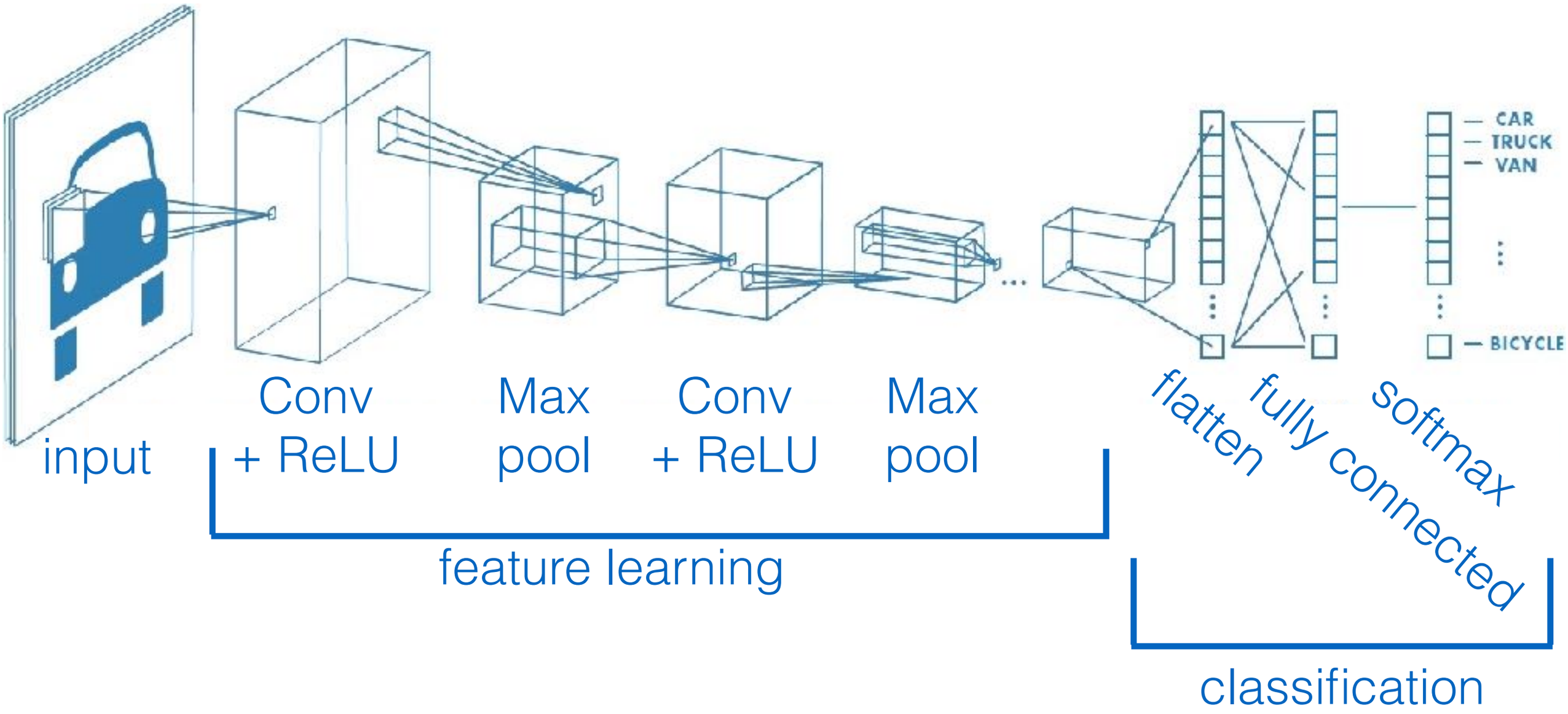
- Can use stride with filters too
- No weights in max pooling

CNNs: typical architecture



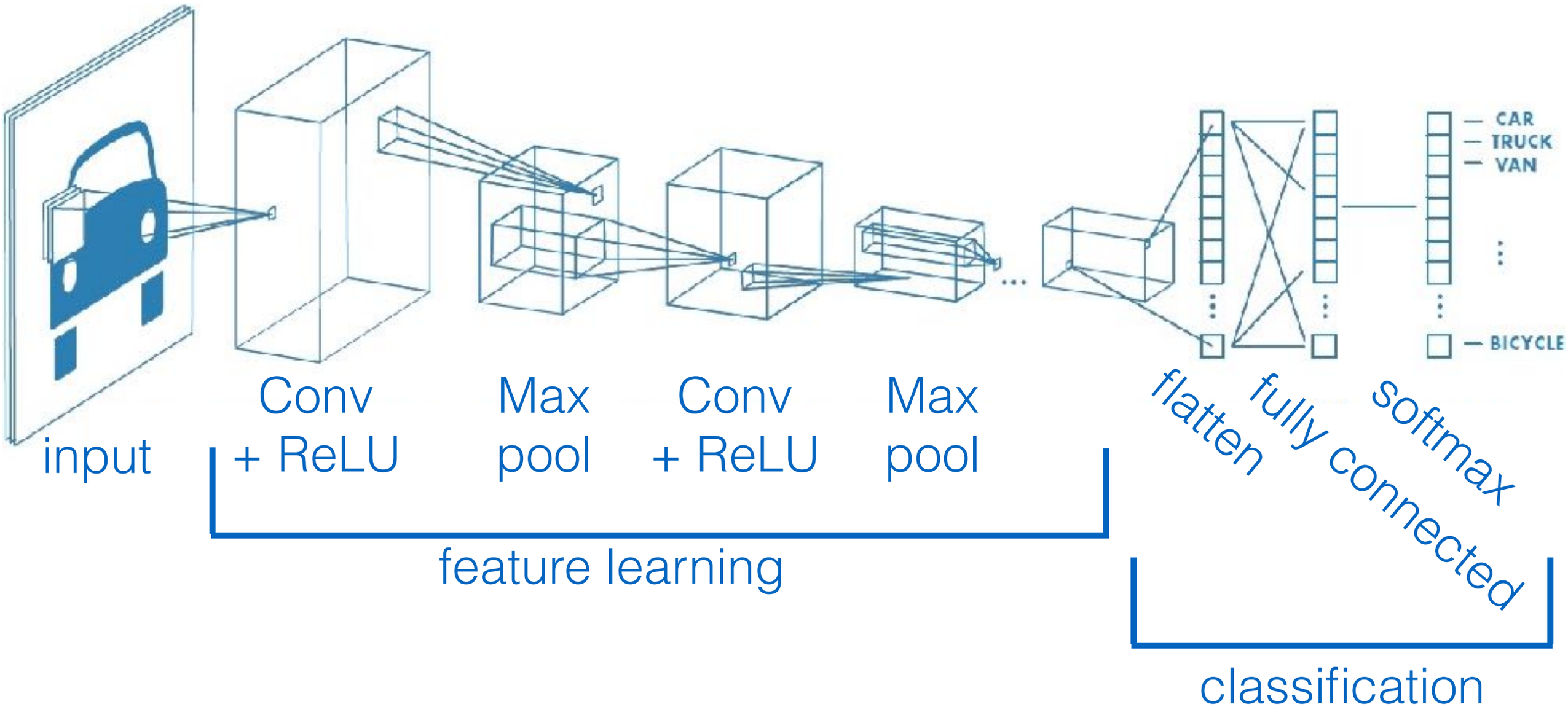
[https://www.mathworks.com/solutions/deep-learning/convolutional-neural-network.html]

CNNs: typical architecture



[https://www.mathworks.com/solutions/deep-learning/convolutional-neural-network.html]

CNNs: typical architecture



x → $\mathrm{NN}(x; W, W_0)$

[https://www.mathworks.com/solutions/deep-learning/convolutional-neural-network.html]

A familiar pattern

A familiar pattern

1. Choose how to predict label (given features & parameters)

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*i*th data point

$x^{(i)}$

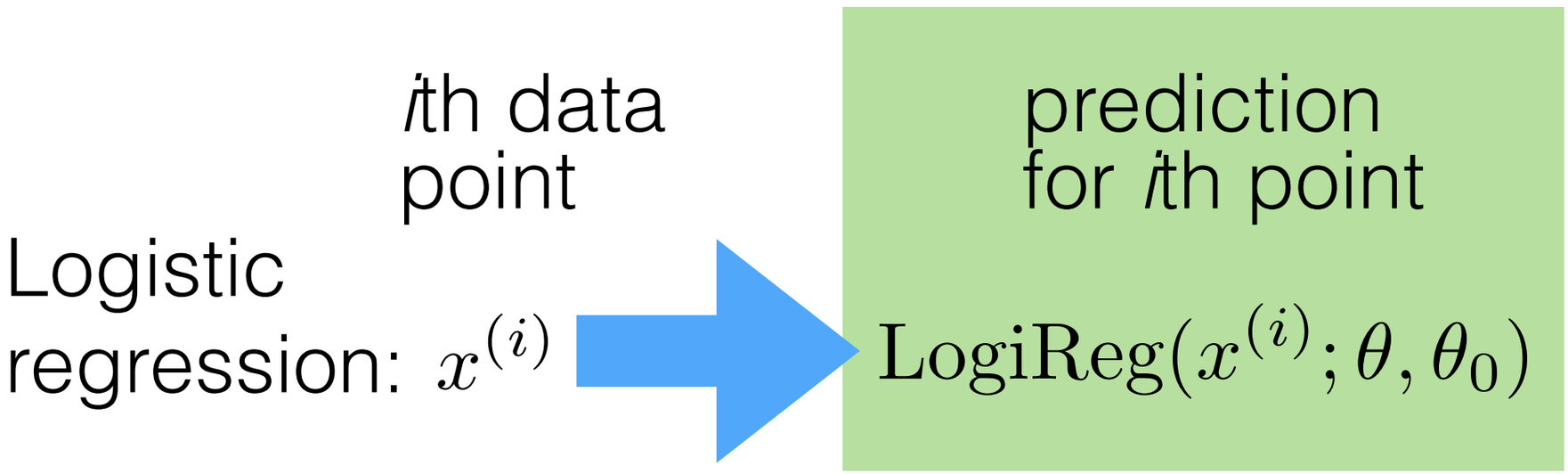
A familiar pattern

1. Choose how to predict label (given features & parameters)

	*i*th data point
Logistic regression:	$x^{(i)}$

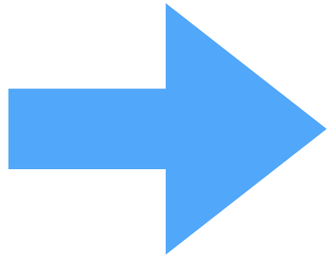
A familiar pattern

1. Choose how to predict label (given features & parameters)



A familiar pattern

1. Choose how to predict label (given features & parameters)
2. Choose a loss (between guessed label & actual label)

	*i*th data point		prediction for *i*th point
Logistic regression:	$x^{(i)}$		$\text{LogiReg}(x^{(i)}; \theta, \theta_0)$

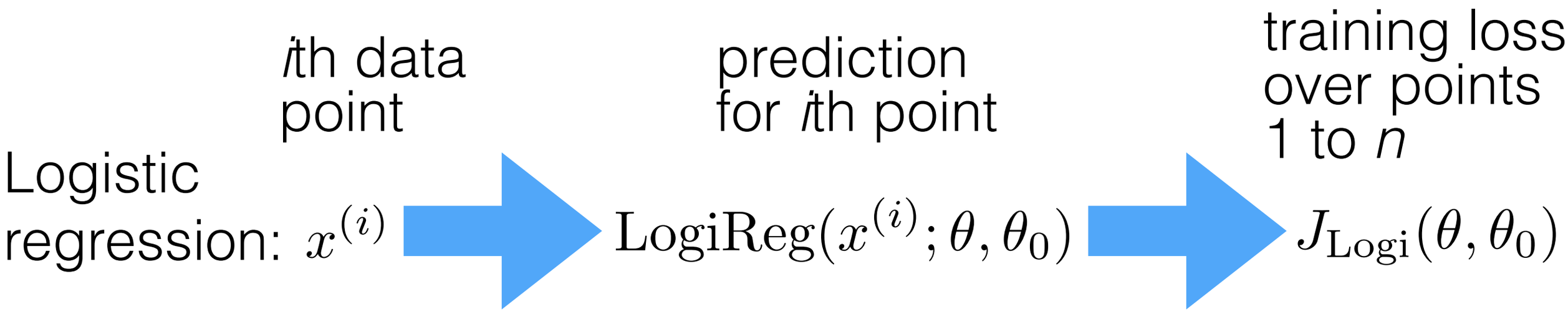
A familiar pattern

1. Choose how to predict label (given features & parameters)
2. Choose a loss (between guessed label & actual label)

	*i*th data point	prediction for *i*th point	training loss over points 1 to *n*
Logistic regression:	$x^{(i)}$	$\text{LogiReg}(x^{(i)}; \theta, \theta_0)$	$J_{\text{Logi}}(\theta, \theta_0)$

A familiar pattern

1. Choose how to predict label (given features & parameters)
2. Choose a loss (between guessed label & actual label)
3. Choose parameters by trying to minimize the training loss



A familiar pattern

1. Choose how to predict label (given features & parameters)
2. Choose a loss (between guessed label & actual label)
3. Choose parameters by trying to minimize the training loss

	*i*th data point	prediction for *i*th point	training loss over points 1 to *n*
Logistic regression:	$x^{(i)}$	$\text{LogiReg}(x^{(i)}; \theta, \theta_0)$	$J_{\text{Logi}}(\theta, \theta_0)$
Linear regression:	$x^{(i)}$	$\text{LinReg}(x^{(i)}; \theta, \theta_0)$	$J_{\text{Lin}}(\theta, \theta_0)$

A familiar pattern

1. Choose how to predict label (given features & parameters)
2. Choose a loss (between guessed label & actual label)
3. Choose parameters by trying to minimize the training loss

	*i*th data point	prediction for *i*th point	training loss over points 1 to *n*
Logistic regression:	$x^{(i)}$	$\text{LogiReg}(x^{(i)}; \theta, \theta_0)$	$J_{\text{Logi}}(\theta, \theta_0)$
Linear regression:	$x^{(i)}$	$\text{LinReg}(x^{(i)}; \theta, \theta_0)$	$J_{\text{Lin}}(\theta, \theta_0)$
Neural networks (e.g. CNNs):	$x^{(i)}$	$\text{NN}(x^{(i)}; W, W_0)$	$J_{\text{NN}}(W, W_0)$

A familiar pattern

1. Choose how to predict label (given features & parameters)
2. Choose a loss (between guessed label & actual label)
3. Choose parameters by trying to minimize the training loss

	*i*th data point	prediction for *i*th point	training loss over points 1 to *n*
Logistic regression:	$x^{(i)}$	$\mathrm{LogiReg}(x^{(i)}; \theta, \theta_0)$	$J_{\mathrm{Logi}}(\theta, \theta_0)$
Linear regression:	$x^{(i)}$	$\mathrm{LinReg}(x^{(i)}; \theta, \theta_0)$	$J_{\mathrm{Lin}}(\theta, \theta_0)$
Neural networks (e.g. CNNs):	$x^{(i)}$	$\mathrm{NN}(x^{(i)}; W, W_0)$	$J_{\mathrm{NN}}(W, W_0)$

* a key difference: NN can mean a lot of different things

CNNs: a taste of backpropagation

Regression. 1 filter: size 3 & padding; $x^{(j)}$ dimension: 5x1

CNNs: a taste of backpropagation

Regression. 1 filter: size 3 & padding; $x^{(j)}$ dimension: 5x1

- Forward pass:

CNNs: a taste of backpropagation

Regression. 1 filter: size 3 & padding; $x^{(j)}$ dimension: 5x1

- Forward pass: $Z_i^1 = (W^1)^\top X_{[i-1,i,i+1]}$

CNNs: a taste of backpropagation

Regression. 1 filter: size 3 & padding; $x^{(j)}$ dimension: 5x1

- Forward pass:

$$Z_i^1 = (W^1)^\top X_{[i-1,i,i+1]}$$

$$A_i^1 = \mathrm{ReLU}(Z_i^1)$$

CNNs: a taste of backpropagation

Regression. 1 filter: size 3 & padding; $x^{(j)}$ dimension: 5x1

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3x1

CNNs: a taste of backpropagation

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- Part of the derivative for SGD:

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$$\frac{\partial Z^1}{\partial W^1} = \begin{bmatrix} Z_1^1 & Z_2^1 & Z_3^1 & Z_4^1 & Z_5^1 \\ & & & & \end{bmatrix} \begin{matrix} W_1^1 \\ W_2^1 \\ W_3^1 \end{matrix}$$

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$Z_1^1 \quad Z_2^1 \quad Z_3^1 \quad Z_4^1 \quad Z_5^1$

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